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by

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Adverse Selection Among Early Adopters and Unraveling Innovation

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Abstract

I provide an equilibrium analysis of "selection markets": where consumers not only vary in how much they are willing to pay, but also in how much they cost to the seller. The model provides a joint explanation for three empirical phenomena: low uptake of existing products, slow demand for new products, and market inactivity despite unmet demand. I characterize when early adopters are more adversely selected in new markets. This lowers demand, increases costs, and leads markets to unravel prematurely. With endogenous market entry for new products (e.g., reverse mortgages, annuities), extended patents serve as de facto time-varying subsidies.

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Selection markets are ubiquitous. These are markets where consumers differ in the costs they impose on a firm when they purchase a good, including annuities, insurance, credit markets, financial securities, used goods, and even labor markets.[1]

Moreover, these markets are often characterized by low utilization, for example in annuities. Empirical work typically rationalizes this under-utilization through low consumer demand.[2] However, this explanation overlooks an essential aspect: the supply side's failure to offer more attractive products in response to unmet consumer demand. I show how adverse selection in immature markets exacerbates initial low demand and causes unraveling. Even if a market would be fully efficient if it could reach maturity, higher adverse selection among early adopters prevents it from maturing.

I show that adverse selection is larger among early adopters in young selection markets. In immature markets, potential consumers have less confidence in the product and benefit less from taking up new products. Consequently, demand in immature markets is lower than in mature markets. Crucially, demand falls, but selection patterns also change. Early adopters, those willing to buy the product despite their lower initial private valuations, are also the consumers that impose the highest costs on firms, making adverse selection generically higher in immature markets.

I thus provide a *joint* explanation for three important empirical regularities in selection markets: first, the low take up of existing products, second, sluggish demand when new products are introduced, and, third, market inactivity despite substantial unmet demand for new or modified alternatives. For example, Zinman (2014) highlights missing rungs in the consumer lending ladder, while Ameriks et al. (2016) establish unmet demand for a better formulation of long-term care insurance. Similarly, Cocco and Lopes (2015) establish demand for alternative reverse mortgage products, while

[1] Einav et al. (2021) provide an overview.

[2] See Lockwood (2012) and Pashchenko (2013) for annuities, Ameriks et al. (2016) and Lockwood (2018) in long-term care insurance, and both Nakajima and Telyukova (2017) and Cocco and Lopes (2020) in reverse mortgage markets. Alternatively see Einav et al. (2010) for results emphasizing adverse selection.

Michaud and Amour (2023) document potential demand for risk management products that bundle existing insurance. Yet, Nakajima and Telyukova (2017) and Webb (2011) document very low take up rates for novel reverse mortgage and annuity products, respectively.[3]

I establish these results in a standard model of insurance demand with multidimensional heterogeneity. While I focus on insurance markets to provide a concrete example, results throughout the paper apply to selection markets in general. In this framework, consumers are heterogeneous along multiple dimensions including costs and risk premia, which determine the surplus from acquiring insurance, as well as other heterogeneity which captures frictions or departures from rationality. I consider adverse selection settings in which consumers' willingness-to-pay for insurance is positively correlated with their costs to insurers.[4]

To study unraveling and entry in novel markets, I focus on mature and immature markets. Mature markets are those that have existed for a long time and are well understood by consumers, while immature markets are new and unfamiliar. The standard framework can be extended to capture the differences between mature and immature markets by allowing consumers' willingness-to-pay for the product to differ across market maturity.

These differences in willingness-to-pay capture objective differences, such as short-term counterparty risk (Briggs et al. 2023) or initial self-insurance patterns that consumers' cannot quickly adjust, which lower the value of purchasing insurance. However, they can also capture subjective differences including consumers' initial perceptions or distrust of new products, for example inertia (see Handel 2013, who studies health insurance) or learning dynamics (see Israel 2005, for the case of learning

[3]More generally, concern about missing innovation in insurance markets dates back over half a century: Rudelius and Wood (1970) identify heterogeneous adoption of life insurance innovations, Dorfman (1971) provides evidence of research activity and barriers to innovation, and Murray (1976) surveys insurance companies to identify innovations.

[4]Analogous results can be obtained for advantageous selection.

in autmobile insurance). These differences between mature and immature markets lower demand and increase consumer sorting based on costs. This increases adverse selection in immature markets and generates under-insurance in equilibrium with lower adoption and higher costs.

A key insight delivered by this approach is that even would-be efficient markets can unravel during their immaturity and fail to mature. This is a new, dynamic source of market failure building on the classic insight of Akerlof (1970). To show this, I derive comparative statics results which allow me to compare mature and immature markets without fully specifying their dynamics. These comparative statics show that, under reasonable assumptions on the primitives of immature markets, demand is lower and the cost to serving the market increases for insurers compared with mature markets. This rationalizes sluggish demand for new products within a framework that can also predict the low utilization of existing products and the complete unraveling of these markets. Subsequently, I build on this insight to study endogenous market entry in the context of novel, but segmented, selection markets.

I show that unraveling in immature markets gives rise to a free-rider problem – leading to permanently dormant markets. Even though firms have dynamic incentives to enter profitable mature markets, firms may be unwilling to bear the cost of familiarizing consumers with new products in adversely selected markets because other firms compete away profits once it reaches maturity. This happens even though firms know lower willingness-to-pay and increased adverse selection in immature markets is transitory. These results explain the empirical phenomena of missing selection markets even when evidence suggests adverse selection may be limited.

Finally, having identified this novel source of market failure, I consider the role of government intervention in promoting entry and innovation within absent selection markets. I discuss how increasing patent lengths can serve as an effective, time-varying subsidy with minimal informational requirements. This solution addresses two fundamental challenges: the inherent absence of data in missing markets and the potential distortions from intervention in mature markets.

A key contribution of this paper is to provide comparative statics results across selection markets using their joint distributions of model primitives. I provide economically interpretable sufficient conditions for non-parametric families of joint distributions which characterize equilibrium adverse selection and demand between markets. While these results are applied to characterize differences between mature and immature markets, the insights apply equally to markets indexed by space and time or for different products.

Related Literature

The comparative static tools I develop provide non-parametric conditions for an ordering over partial unraveling in Akerlof markets and are used to consider the dynamic implications of firm entry decisions in novel selection markets. Akerlof (1970) studies the failure of price mechanisms and market unraveling in selection markets. An extended no-trade theorem for Rothschild and Stiglitz (1976) markets is provided by Hendren (2013), while Attar et al. (2021) provide necessary and sufficient conditions for no-trade theorems in adverse selection markets.[5] Kong et al. (2023) show how adverse selection contributes to non-entry among differentiated firms, even absent fixed costs. This paper extends non-entry results to cases where adverse selection declines over time.

I study changes in the distributions of choice frictions similarly to Handel et al. (2019). I provide comparative statics for non-local changes in both the distribution of choice frictions and the insurance value of contracts across markets. Handel et al., however, study local reforms (including information interventions) that alter consumer's choice

[5]A sizable literature studies endogenous contract terms under adverse selection (e.g. Rothschild and Stiglitz 1976; Handel et al. 2015; Veiga and Weyl 2016; Azevedo and Gottlieb 2017) or screening with both intensive and extensive margins of choice (Geruso et al. 2023). This is not the focus of this paper.

frictions and provide sufficient statistics formulas for their welfare impacts.

I examine the implications of changing adverse selection over time and study policies promoting initial market entry. These results emphasize that inter-temporal cross-subsidization can be optimal under missing markets. These policies are related to, but differ from, those designed to mitigate selection (Geruso and Layton 2017, review these policies in health insurance markets).

1. A Model of Equilibrium in the Insurance Market

I consider an Akerlof (1970) model of insurance in the style of Einav et al. (2010) and Einav and Finkelstein (2011). A single perfectly competitive and risk-neutral insurer sells insurance to a unit continuum of heterogeneous individuals at a common price p. The terms of the insurance contract are exogenous and common across all individuals.

Demand. Individuals are heterogeneous along multiple dimensions. Namely, their risk type π_i which determines the expected cost to the insurer, their risk premium r_i, and potential demand frictions ε_i. These are private information for individual i.[6] An individual buys insurance if and only if their *subjective* willingness-to-pay, defined as

$$\omega_i \equiv u(\pi_i, r_i, \varepsilon_i) = u(\underbrace{\pi_i + r_i}_{\text{insurance value}} + \varepsilon_i), \tag{1}$$

for any strictly increasing utility function $u(\cdot)$, exceeds the price of the insurance. In addition to their risk type, individual i's risk premium, r_i, determines the surplus in the insurance market.

Following Spinnewijn (2017), individual choices depend on additively separable

[6]Conditioning on observable characteristics correlated with π_i is not always legal. Nevertheless, this can be interpreted as a model of insurance contracts in each partition of the observable characteristic space.

demand frictions, ε_i, which can distort insurance demand. This approach is agnostic about the nature of demand frictions, nesting a number of important model features including borrowing constraints, but also inertia, subjective beliefs, and bounded rationality. These frictions are heterogeneous and can lead individuals to overvalue ($\varepsilon > 0$) or undervalue ($\varepsilon < 0$) the insurance contract.

Thus, demand is given by

$$D(p) = 1 - F_\omega(p) = \mathbf{1}[\omega > p], \tag{2}$$

where F_x denotes the CDF of variable x.[7]

Supply. The cost of providing insurance at price p is determined by the risk types of individuals purchasing insurance at that price.[8] Consequently, *average* and *marginal* costs are given by:

$$AC(p) = E(\pi|\omega \geq p) \tag{3}$$

$$MC(p) = E(\pi|\omega = p). \tag{4}$$

When willingness-to-pay, ω, is increasing in the cost to the insurer, π, there is adverse selection; marginal costs increase in price and the MC curve is downward sloping.

With multidimensional heterogeneity, the critical assumption for adverse selection is this dependence between individual willingness-to-pay and costs (See Fang and Wu 2018). In general, this depends on the shape of the copula which represents their joint distribution.[9] Copulas, which specify the shape of rank dependence, encode the

[7]Similarly, μ_x and σ_x^2 will denote the mean and variance of x's distribution.
[8]For simplicity, I abstract from administrative loadings, or fixed costs which don't affect the main result.
[9]Copulas have many uses in economics and finance. Recent examples include competing hazard models (Kim 2021), and multi-dimensional skills (Gola 2021). The main representation result is Sklar's theorem, which I reproduce below.

correlational structure between π_i, r_i, and ε_i without imposing a specific parametric family of distributions and decouple assumptions on dependence properties from those on marginal distributions.[10]

Equilibrium. To simplify notation, I assume demand and cost curves are continuous and monotonic which implies the equilibrium is unique if it exists. The equilibrium price is given by the break even price that pools expected costs

$$p^{\star} = p : D(p) = AC(p) \ . \tag{5}$$

Consequently, the equilibrium allocation may differ from an efficient allocation. An efficient allocation insures everyone whose willingness-to-pay exceeds their expected cost ($\omega_i > \pi_i$), those for whom the demand curve lies above the marginal cost curve as shown in Panel (A) of Figure 1.

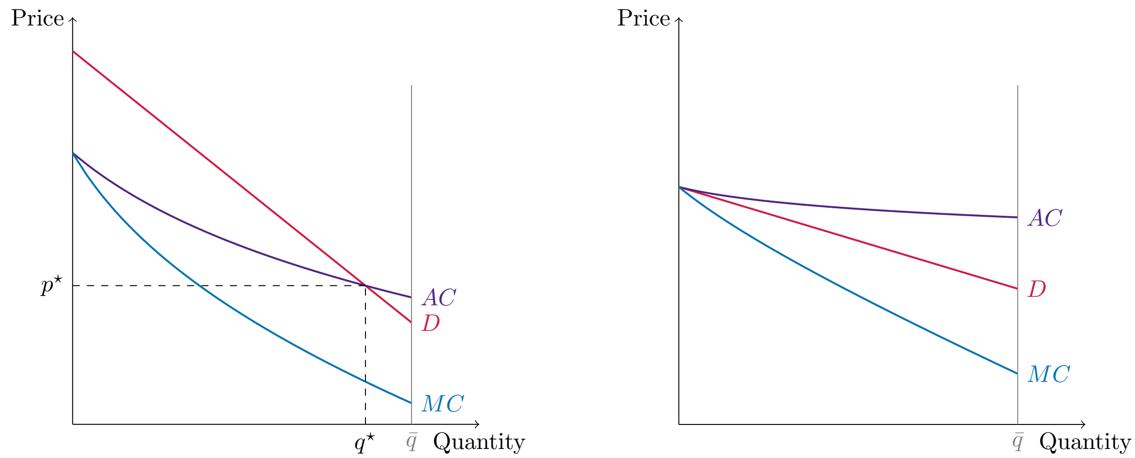
Furthermore, a positive equilibrium price does not necessarily exist. The insurance market unravels (Akerlof 1970) when individuals are not willing to purchase insurance at the pooled cost of those with higher demand. This occurs when the demand curve lies entirely below the average cost curve (Hendren 2013) as shown in Panel (B) of Figure 1. I show below how characteristics of immature and mature markets affect this unraveling.

Sklar's Theorem: Let F be a joint distribution function for a d-dimensional random vector $(X_1, X_2, \ldots, X_d)$, and let F_i be the marginal distribution function for the i-th component X_i. Then, there exists a copula C such that for any $x_1, x_2, \ldots, x_d$ in the real line:

$$F(x_1, x_2, \ldots, x_d) = C(F_1(x_1), F_2(x_2), \ldots, F_d(x_d))$$

Moreover, if X is continuous, then the Copula C is unique.

[10] Copulas are invariant to rank-preserving transformations. Therefore, results immediately apply to multiplicatively separable utility, $u = exp(\cdot)$, and when insurer costs are transformed cost types.



(A) Textbook Adverse Selection

(B) Textbook Unraveling

FIGURE 1. Equilibrium in the Insurance Market Without Demand Frictions

2. Unraveling in Immature Markets

I now provide assumptions on the primitives of insurance markets captured by the joint distribution of π_i, r_i, and ε_i. Below, I discuss possible microfoundations under which these primitives may arise endogenously and how they capture empirically relevant features of the individual's and insurer's problems.

2.1. Differences between Mature and Immature Markets

I use superscripts *MM* to denote the mature insurance market and *IM* to denote the immature insurance market for a new form of innovative insurance coverage.

The key assumption I make is that willingness-to-pay is depressed in immature markets relative to mature markets – lowering demand. This leads to more adversely selected early adopters and generates additional unraveling in immature markets. This relative lack of demand can come from objective differences in risk premia, r, or differences in demand frictions, ε. I formalize this in Assumptions (1) and (2) which define cases where marginal distributions are ordered in a stochastic sense, where

$X \preceq Y$ denotes the relation Y first order stochastic dominates X.

Assumption 1. *The risk premia of insurance contracts are lower for immature markets.* $r^{MM} \succeq r^{IM}$

Three channels generate objective differences between the risk premia for immature and mature markets. First, individual portfolios may not be optimized to take advantage of an insurance product when it is first introduced. For example, an individual may accumulate savings to self-insure in the absence of formal insurance. Thus, the benefits to insurance purchase are larger when other choices can adjust to fully capitalize on the product.

Second, there may be objective uncertainty about the probability that all claims will be covered, the hassle costs associated with claiming, or the likelihood of reclassification risk. For example, in health insurance markets, there is uncertainty over, and heterogeneity in, the coverage of new drugs or procedures. This uncertainty is likely larger for new types of insurance.

Third, objective counterparty risk may be larger in immature markets, relative to mature markets where insurers typically trade with repeated cohorts of potential insurees. In an immature market insurers receive liquid assets now (in the form of premia), but may not make payouts until far in the future (for example, life insurance, deferred annuities, or long term care insurance). This difference in the timing of liquidity and liabilities may distort the insurers portfolio choice (Thompson 2010, models endogenous counterparty risk) and raise the possibility of default or non-payment in immature markets. Assumption 1 allows for insurer moral hazard without specifying their full portfolio problem.

Assumption 2. *Initial demand frictions are shifted to the left.* $\varepsilon^{MM} \succeq \varepsilon^{IM}$

Status quo biases, distrust of new products, or inertia costs all imply smaller biases against products in mature markets compared with immature markets. As does lack of awareness (Boyer et al. 2020) or when new products are less likely to enter consideration sets. Similarly, negative demand frictions that disappear as consumers learn or experience the product[11] can be represented in the same way. Immature markets have *more negative* demand frictions in contrast to mature markets where a larger fraction of consumers have been exposed to a product in the past through peers, advertising, or past purchase.

In contrast to Spinnewijn (2017), Assumption (2) does not require consumers are, on average, unbiased. Systematic optimism or pessimism in mature markets is not ruled out by Assumption 2. Even in established selection markets, such as annuities, there is substantial evidence of choice frictions (Brown et al. 2021) and biased beliefs (O'Dea and Sturrock 2023). Likewise positive systematic biases leading to over-purchase are compatible with Assumption 2, in this case it instead requires advertising or word-of-mouth (see Bronnenberg et al. 2019, for a review) on average increases willingness-to-pay in mature markets.

To make the markets comparable, I hold the distribution of potential and realized risks constant across mature and immature markets. Assumption (3) formalizes that these are mature and immature markets for an identical insurance product with identical potential and realized risk pools.[12]

ASSUMPTION 3. *Risk types, π_i, have identical marginal distributions in mature and immature markets.*

These assumptions restrict how marginal distributions differ between mature and immature markets. Yet, they do not capture how the sorting of consumers into insurance may change as markets mature. These differences in sorting can improve the match of individuals to insurance plans, but also change how purchasing insurance

[11] Evidence on tax minimization (Chetty et al. 2013) and retirement plan participation (Duflo and Saez 2003) suggests peer experience matters.

[12] Neither firms nor individuals can alter the distribution of risks in mature markets.

signals an individual's cost. I now provide restrictions on the structure of the rank dependence properties and how they can change between mature and immature markets. These differences in the dependence properties capture differences in sorting patterns generalizing a restriction on correlation.

Assumption 4. *The dependence between π_i, r_i, and ε_i in mature and immature markets is represented by trivariate copulas C^{IM} and C^{MM}, satisfying* $\frac{\partial^2 C^{MM}(F_\pi(\pi), u_r, u_\varepsilon)}{\partial u_r \partial u_\varepsilon} \leq \frac{\partial^2 C^{IM}(F_\pi(\pi), u_r, u_\varepsilon)}{\partial u_r \partial u_\varepsilon} \quad \forall \pi$ *and* $u_r, u_\varepsilon \in [0, 1]$.

Intuitively, there is positive re-sorting between mature and immature markets and more dependence between the components of willingness-to-pay in immature markets. More formally: as choice frictions, ε, shift right, the multivariate CDF in immature markets increases by more than in mature markets as risk premia, r, also shift right.

Assumption 5. *The dependence between cost type, π_i, and willingness-to-pay, ω_i, in mature and immature markets is represented by copulas $\hat{C}^{IM}$ and $\hat{C}^{MM}$. These copulas satisfy the following conditions:*

i. *Component-wise Concavity: $\hat{C}(u, v)$ is concave in v for all $u \in (0, 1)$*

ii. *Corner Set Monotonicity: $(u - \hat{C}(u, 1-v))/v$ is increasing in v for all $u \in (0, 1)$.*

iii. *Concordance Ordering: $\hat{C}^{IM}(u, v) \geq \hat{C}^{MM}(u, v)\ \forall u, v \in [0, 1]$*

iv. *Monotone Regression Dependence Ordering: $\tilde{g}_u^{IM}(g_u^{MM}(v))$, where $g_x(y) = \partial \hat{C}(x, y)/\partial y$ and $\tilde{g_x} \equiv g_x^{-1}$, is increasing in v for all $u \in (0, 1)$*

Conditions (i) and (ii) restrict the (scale invariant) rank dependence *within* markets and formalize the intuitive notion that larger values of X tend to go with larger values of Y. It implies positive quadrant dependence and positive values of standard correlation coefficients (e.g. Pearson, Kendall's τ, and Spearman's ρ). For parametric copula families these conditions restrict the parameter space, for example requiring the correlation coefficient is positive for the normal copula.

While these conditions are easily verifiable in any empirical setting or application, they are somewhat unintuitive. Lemma 1 provides a sufficient condition for the first two properties using the familiar monotone likelihood ratio property (Milgrom 1981).[13]

LEMMA 1 (MLRP). *All joint distributions satisfying the Monotone Likelihood Ratio Property, that is for any* $\omega_1 \geq \omega_2$ *and* $\pi_1 \geq \pi_2$*, all densities satisfying* $f(\omega_1|\pi_1)/f(\omega_1|\pi_2) \geq f(\omega_2|\pi_1)/f(\omega_2|\pi_2)$ *satisfy conditions (i) and (ii) in Assumption 5.*

Conditions (iii) and (iv) in Assumption 5 impose orderings *between* markets. In economic terms, they rule out self insurance patterns that reverse risk exposure between mature and immature markets. Identical copulas, which imply no re-sorting effects, immediately satisfy these conditions. While I state these conditions in general technical terms, for parametric copula families these are restrictions on their parameters. For example, normal copulas satisfy conditions (iii) and (iv) when $corr^{IM}(\pi, \omega) \geq corr^{MM}(\pi, \omega)$ (Fang and Joe 1992, provide additional examples).

Finally, this general representation does not impose that choice frictions are independent of other characteristics. Consequently, choice frictions are a reduced form compatible with a range of behavioral microfoundations where frictions are not independent, including rational inattention models of endogenous information acquisition.

While the results of this paper are derived under these general assumptions, closed form solutions are available under the assumption that copulas and marginals are normally distributed. This satisfies Assumptions 4 and 5 while increasing transparency, but imposes strong additional restrictions, including allowing consumers to impose negative costs on the insurer and limiting tail risk.

[13] A weaker sufficient condition is a log-concave density – a common assumption when studying insurance (Chade and Schlee 2012).

Assumption 6 (Joint Normality). *π_i, r_i, and ε_i are jointly normally distributed. Furthermore, frictions are independent of the insurance value $(\pi, r) \perp \varepsilon$.*

Additionally, the variance of willingness-to-pay is constant, $\sigma_\omega^{IM} = \sigma_\omega^{MM}$, and correlation between risk type and willingness-to-pay is weaker in mature markets, $corr(\pi, \omega^{IM}) \geq corr(\pi, \omega^{MM}) > 0$.

Assumption 6 rules out changes in the dispersion of willingness-to-pay across mature and immature markets.[14] To simplify exposition in the main body of the text and to build intuition for the general results, I provide proofs of the paper's results under Assumption 6 for the special case of linear utility, $u(x) = x$, and provide proofs for the general non-parametric case in Appendix A.1.

2.2. Equilibrium in Mature and Immature Markets

I now turn to understanding the implication of Assumptions (1) - (5) on quantities, costs, and the market equilibrium.

First, at any given price, the average individual has a larger willingness-to-pay in mature markets and consequently more individuals choose to purchase insurance. Panel A of Figure 2 displays the implications of this *level effect* on demand and Proposition 1 formalizes it.

Proposition 1 (Demand Contraction). *Willingness-to-pay is on average larger and demand expands in mature markets. $D^{IM}(p) \leq D^{MM}(p) \quad \forall p$*

Proof. As the sum of the normal variables is itself normal, the demand curve is

$$D(p) = 1 - F_\omega(p) = 1 - \Phi\left(\frac{p - \mu_\omega}{\sigma_\omega}\right). \tag{6}$$

[14] Under this condition, changes to the primitives take the form of location shifts for the marginal distributions. This is weaker than the stochastic ordering of normals as it does not impose a common variance across mature and immature markets for each of π_i, r_i, and ε_i.

Constant dispersion in willingness-to-pay is required for the normal distribution as the support is unbounded. Without this, equivalent results are available with truncated tails (Levy 1982).

Assumptions (1) - (3) and (6) imply $\mu_\omega^{IM} \leq \mu_\omega^{MM}$ and σ_ω is constant, then distributions of ω for mature markets first order stochastic dominate those of immature markets, $F_\omega^{MM}(p) \leq F_\omega^{IM}(p)$, which delivers the result. □

This level effect on demand holds average and marginal costs constant for any given quantity. Hence, the cost curves faced by insurers in Panel A do not change. Although households need lower prices to induce purchase, the ordering of consumers willingness-to-pay is unchanged. Therefore the identity of marginal (and inframarginal) consumers at any quantity demanded is the same and, as Panel A shows the equilibrium price increases and demand falls (See Proposition 5).

Moreover, when there is *only* a level effect the utility function can be generalized to allow for arbitrary substitution patterns between π_i, r_i, and ε_i. This is formalized in the following lemma with the proof given in Appendix A.1.

LEMMA 2 (Arbitrary Substitutes). *When there is no re-sorting ($C^{IM} = C^{MM}$), Proposition 1 can be extended to any willingness-to-pay satisfying*

$$\omega_i = u(\pi_i, r_i, \varepsilon_i) \tag{7}$$

where $u(\cdot,\cdot,\cdot)$ is an increasing function.

Early adopters differ from the population of consumers who purchase in mature markets in important ways. First, they are more adversely selected as captured by the marginal cost of insurance.

PROPOSITION 2 (Increased Adverse Selection). *Consumers are adversely selected in both mature and immature markets and more adversely selected in immature markets* $\left(\frac{\partial MC^{IM}(p)}{\partial p} \geq \frac{\partial MC^{MM}(p)}{\partial p} > 0\right)$

PROOF. First, consider the joint (normal) distribution of cost type and willingness-to-pay,

$$\begin{pmatrix} \pi_i \\ \omega_i \end{pmatrix} \sim N \left[\begin{pmatrix} \mu_\pi \\ \mu_\omega \end{pmatrix}, \begin{pmatrix} \sigma_\pi^2 & \rho\sigma_\pi\sigma_\omega \\ \rho\sigma_\pi\sigma_\omega & \sigma_\omega^2 \end{pmatrix} \right],$$

where

$$\rho = \frac{\sigma_\pi^2 + \rho_{\pi,r}\sigma_\pi\sigma_r}{\sigma_\omega\sigma_\pi} > 0 \,. \tag{8}$$

Then the marginal cost curve has the following closed form expression:

$$MC(p) = E(\pi|\omega = p) = \mu_\pi + \rho\frac{\sigma_\pi}{\sigma_\omega}(p - \mu_\omega), \tag{9}$$

which is increasing in p when ρ is positive. The second part of the proposition then follows from Assumption 6. □

Intuitively, those who have the highest willingness-to-pay for insurance impose the highest costs on insurers for their coverage – making early adopters more costly.

Further comparing the marginal cost curves in mature and immature markets gives the following lemma relating changes in adverse selection to changes in sorting between markets.

LEMMA 3 (Adverse Selection In Levels). *When there is no re-sorting ($\widehat{C} = \widehat{C}^{IM} = \widehat{C}^{MM}$) marginal costs are higher at any price. However, when re-sorting occurs, effects on margin cost are ambiguous.*

The differences in marginal costs across markets depend on a level effect which holds consumer sorting fixed and captures differences in the level of demand at any price. However, while the slope of the marginal cost curve increases, the effect of re-sorting has an ambiguous effect on the level of marginal costs.

I make this formal using expression (9) for the marginal cost in the normal case:

COROLLARY 1 (Decomposing Marginal Costs). *Under Assumption 6, the differences in marginal costs between mature and immature markets can be decomposed into the level effect of demand and a re-sorting effect*

$$MC^{IM}(p) - MC^{MM}(p) = \frac{\sigma_\pi}{\sigma_\omega}\rho^{IM}\left[\underbrace{(\mu_\omega^{MM} - \mu_\omega^{IM})}_{\text{level effect}\geq 0} + \underbrace{\left(1 - \frac{\rho^{MM}}{\rho^{IM}}\right)(p - \mu_\omega^{MM})}_{\text{re-sorting effect}}\right]. \quad (10)$$

This result makes the intuition for the preceding lemma explicit. Absent re-sorting, the increase in marginal costs depends only on the contraction in demand as the second term is equal to zero. However, when there is re-sorting the sign of the second term is ambiguous. For prices above the average willingness-to-pay in mature markets, re-sorting increases the difference in marginal costs between mature and immature markets. When prices are lower, however, the effect is negative as those purchasing insurance at high prices in immature markets are more adversely selected. Consequently, the pool of remaining consumers at low prices have lower costs.

Nevertheless, Proposition 3 highlights that, despite ambiguity at the margin, when consumers are reordered and selection on cost increases, then insurers face higher *average* costs.

PROPOSITION 3 (Cost Expansion). *Average costs are increasing in price and early adopters are higher cost on average, $AC^{IM}(p) \geq AC^{MM}(p)$.*

PROOF. Under joint normality, average costs are the expected value of a truncated normal distribution. The average cost curve is

$$AC(p) = E(\pi|\omega \geq p) = \mu_\pi + \rho\sigma_\pi \frac{\phi(\frac{p-\mu_\omega}{\sigma_\omega})}{1 - \Phi(\frac{p-\mu_\omega}{\sigma_\omega})}. \quad (11)$$

The average cost depends on the inverse Mills ratio of the subjective willingness-to-pay. This captures the extent to which selection truncates the distribution of cost types relative to its full support.

Comparing across markets gives $\forall\, p$

$$\frac{AC^{IM}(p) - AC^{MM}(p)}{\sigma_\pi \rho^{IM}} = \underbrace{\left(\frac{\phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})}{1-\Phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})} - \frac{\phi(\frac{p-\mu_\omega^{MM}}{\sigma_\omega})}{1-\Phi(\frac{p-\mu_\omega^{MM}}{\sigma_\omega})} \right)}_{\text{level effect}\geq 0} + \underbrace{\left(1 - \frac{\rho^{MM}}{\rho^{IM}}\right) \frac{\phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})}{1-\Phi(\frac{p-\mu_\omega^{IM}}{\sigma_\omega})}}_{\text{re-sorting effect}\geq 0} \geq 0 \tag{12}$$

The inverse Mills ratio and the difference in the inverse Mills ratio are always positive by the log concavity of the normal distribution ensuring non-negativity. □

Again this expression can be decomposed into the contribution from the level and re-sorting effects. In contrast to marginal costs, both effects work in the same direction (raising average costs at a given price), because the effect of additional selection on costs among the inframarginal consumers dominates the ambiguous effect on the marginal consumer.

The first term in (12) captures changes in the level of demand, holding the consumer sorting constant, on the costs of early adopters. The second term in (12), which captures re-sorting, holds fixed the level of selection in immature markets and accounts for the additional sorting on costs which occurs.

The size of both effects are increasing in the share of heterogeneity in willingness-to-pay coming from cost type, π, while resorting effects depend on the amount of selection in immature markets. When this selection is large, insurance purchase is a stronger signal of cost. Consequently, increased sorting magnifies existing selection.

Panel (A) of Figure 2 shows how the decline in willingness-to-pay affects equilibrium price and quantity in the absence of an effect on costs. Crucially, although average costs increase at any price when there is a level effect, a change in the sorting patterns of individuals is necessary to induce a change in the cost curves insurers face conditional on quantity.

Suppose individuals were ordered by their willingness-to-pay. Without re-sorting, this ordering is identical in mature and immature markets. Thus when q individuals purchase insurance, the marginal consumer is identical (q along this order) and imposes identical expected costs on the insurer. If, instead, there is re-sorting, individuals are reshuffled and the identity of the marginal consumer changes. Increased sorting on cost in immature markets, relative to mature markets, results in this new marginal consumer having higher expected costs. Proposition 4 formalizes this intuition.

PROPOSITION 4 (Cost Expansion Fixing Demand). *Average and Marginal Costs as a function of quantity are larger in immature markets,* $AC^{IM}(q) \geq AC^{MM}(q)$ *and* $MC^{IM}(q) \geq MC^{MM}(q)$. *Absent re-sorting, they are identical.*

PROOF. Substituting inverse demand into (9) gives marginal costs as a function of quantity,

$$MC(q) \equiv E\left(\pi | \omega = D^{-1}(q)\right) = \mu_\pi + \rho\sigma_\pi\Phi^{-1}(1-q). \tag{13}$$

It follows that

$$MC^{IM}(q) - MC^{MM}(q) = \underbrace{(\rho^{IM} - \rho^{MM})(\sigma_\pi\Phi^{-1}(1-q))}_{\text{re-sorting effect}} \geq 0 \;\; \forall q,$$

which shows that the marginal costs are higher at any quantity demanded.

Substituting inverse demand into (11) gives average costs as a function of the quantity demanded,

$$AC(q) \equiv E\left(\pi|\omega \geq D^{-1}(q)\right) = \mu_\pi + \rho\sigma_\pi \frac{\phi(\Phi^{-1}(1-q))}{1-\Phi(\Phi^{-1}(1-q))} = \mu_\pi + \rho\sigma_\pi \frac{\phi(\Phi^{-1}(1-q))}{q}. \quad (14)$$

Differences in average costs are then

$$AC^{IM}(q) - AC^{MM}(q) = \rho^{IM}\sigma_\pi \underbrace{\left(1 - \frac{\rho^{MM}}{\rho^{IM}}\right)\frac{\phi(\Phi^{-1}(1-q))}{q}}_{\text{re-sorting effect}} \geq 0 \;\; \forall q. \quad (15)$$

□

Panel B of Figure 2 shows this outward expansion of the average cost curve in immature markets.[15] Even holding the level of demand constant, the increase in adverse selection among the inframarginal consumers of the insurance product drives up expected costs for the insurer. Consequently, insurers set higher prices to cover the cost of providing insurance, even though some lower cost individuals select out of the pool. Thus, the equilibrium price increases and quantity falls. There is (additional) partial unraveling in the market.[16]

PROPOSITION 5 (Partial Unraveling). *If an equilibrium exists, the equilibrium price is higher in immature markets and fewer consumers are insured,* $q^{MM} \geq q^{IM}$ *and* $p^{MM} \leq p^{IM}$.

PROOF. Let p^{MM} be the equilibrium price in the mature market,

$$D^{MM}(p^{MM}) = AC^{MM}(p^{MM}). \quad (16)$$

[15]This is a rotation of the average cost curve around full insurance as in Mahoney and Weyl (2017). The difference in average costs in (15) is decreasing in quantity demanded, reaching zero at full coverage. Intuitively, at full coverage the entire population is insured and $AC^{IM}(1) = AC^{MM}(1) = \mu_\pi$. In contrast, Starc (2014) considers rotation around equilibrium demand.

[16]This can occur even if marginal costs in immature markets fall below demand which, absent choice frictions, implies full coverage is efficient.

If it is also an equilibrium price in the immature market, then

$$D^{IM}(p^{MM}) = AC^{IM}(p^{MM}) \tag{17}$$

also holds. But, from Propositions 1 and 3, the following inequality must also hold

$$D^{IM}(p^{MM}) \leq D^{MM}(p^{MM}) = AC^{MM}(p^{MM}) \leq AC^{IM}(p^{MM}) \,, \tag{18}$$

which is a contradiction unless the immature and mature markets are identical.

Furthermore, if there exists a price $p^{IM} \neq p^{MM}$ that clears the immature market then differences across markets satisfy *strict* dominance. Therefore

$$D^{IM}(p^{IM}) = AC^{IM}(p^{IM}) \longleftrightarrow p^{IM} > p^{MM} \,, \tag{19}$$

and

$$q^{IM} = D^{IM}(p^{IM}) < D^{MM}(p^{IM}) < D^{MM}(p^{MM}) = q^{MM} \,. \tag{20}$$

□

Panel C of Figure 2 illustrates the new equilibrium graphically. Marginal costs are below demand in both mature and immature markets, thus full insurance is the efficient outcome in both regimes. Despite this, there is additional under-insurance relative to the mature market due to both the level effect of demand contraction and the increased adverse selection due to additional sorting. The increase in equilibrium price and reduction in quantity demanded is larger than either effect in isolation (Panels A and B).

The presence of under-insurance ($q^{IM} < q^{MM}$) does not depend on whether changes in willingness-to-pay are driven by shifting risk premia or choice frictions. Calculating welfare costs, however, require a stance on their relative importance and the source of the change.

An alternative consequence of immaturity is that markets may unravel.

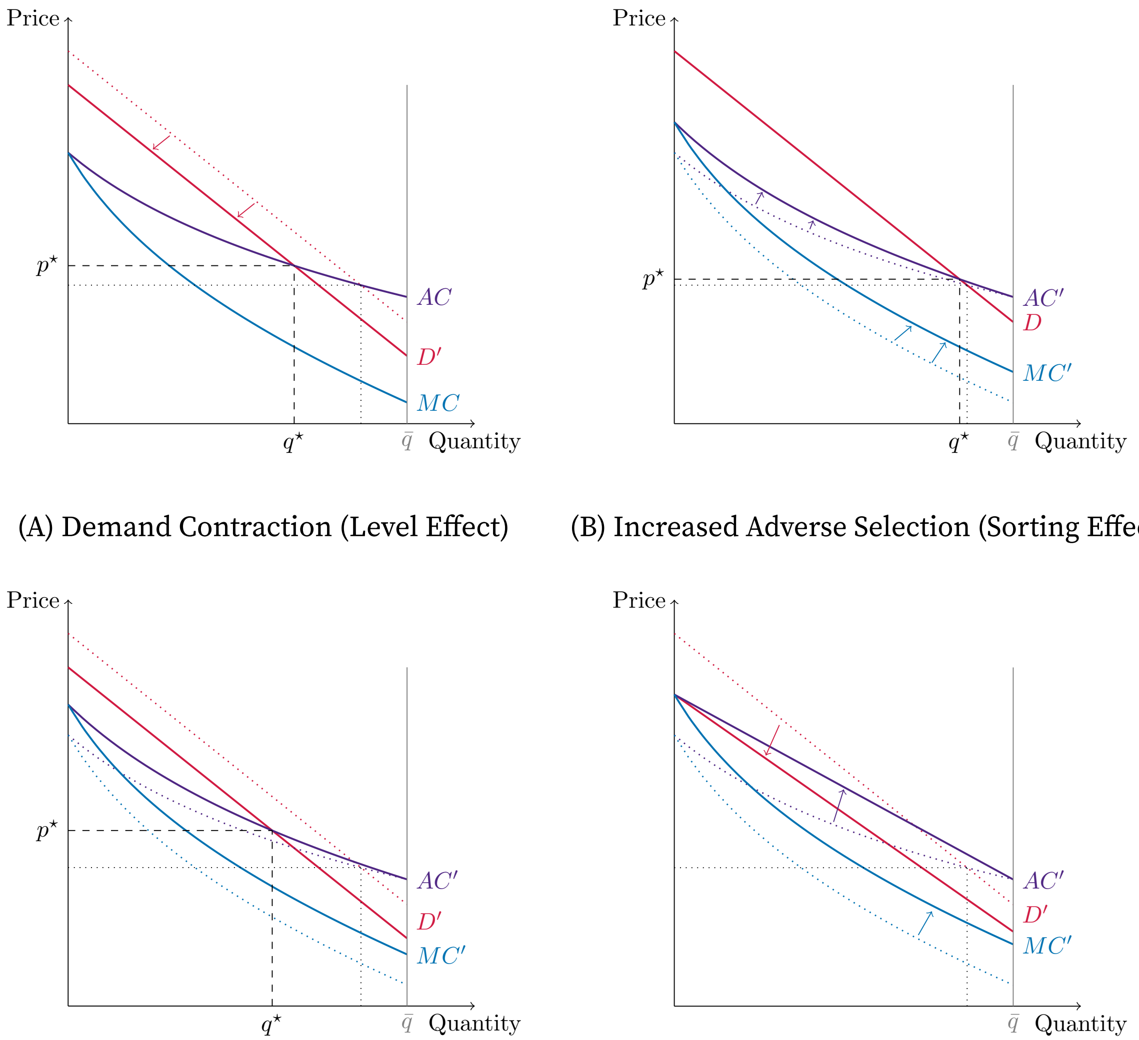
COROLLARY 2 (Complete Unraveling). *Would-be efficient mature markets can unravel before they reach maturity.*

This happens when the average cost curve lies above demand for all prices (Panel D of Figure 2). Thus, even a market that would be efficient if it could mature can fully unravel before maturing when the systematic differences between mature and immature demand (and the adverse selection that follows) are large enough. Again, this can occur irrespective of whether under-insurance in immature markets is driven by rational differences in consumer valuations or irrational systematic biases. This abstracts from administrative loads and fixed costs for insurers, however, by increasing the costs of providing insurance at any given demand, these features reinforce this mechanism.

3. Will Deep Pocketed Insurers Enter When Immature Markets Unravel?

Section 2 establishes that differences between mature and immature markets can lead to the unraveling of competitive equilibria when market immaturity exacerbates adverse selection. This naturally raises the question of whether dynamic incentives counteract these forces. To answer this, I illustrate the key economic forces determining insurer entry into new markets using a two firm, two period model. I show that, when adverse selection is severe enough, even the promise of market power is not enough to entice insurers to enter. Despite the fact that mature markets may be efficient, *temporary and predictable* increases in adverse selection give rise to missing markets.

In each period, firms choose whether to offer an insurance product taking as given contract terms. They observe entry decisions and post prices, competing a la betrand.



(A) Demand Contraction (Level Effect)

(B) Increased Adverse Selection (Sorting Effect)

(C) Under-insurance in the Short Run

(D) Unraveling in the Short Run

FIGURE 2. The Long and Short Run of Insurance Markets

Consequently, prices equal average costs if both firms enter (equation 5). If, however, only one firm enters, they are able to exploit their market power and equate marginal revenue with marginal cost, delivering the usual monopolist pricing rule.

To close the model, I assume the market begins in an immature state. This state of the world updates from immature to mature in the second period if and only if at least one firm enters in the first period. This tractable framework captures the interaction between market power, adverse selection, and pricing, as well as that firms who want the market to reach maturity are engaged in a war of attrition with consumers.[17]

The following proposition characterizes how market immaturity interacts with market forces and can endogenously give rise to market incompleteness in this setting. The proof is in Appendix A.2.

PROPOSITION 6. *If a monopolist cannot earn positive profits in immature markets, there exists a subgame perfect equilibrium where no firm enters. Moreover, this is the unique extensive-form trembling hand perfect equilibria.*

Importantly, the statement depends only on primitives of the immature market and, thus, holds even when the mature market is efficient. In other words, dynamic incentives do not preclude missing efficient mature markets. Therefore, this rationalizes the absence of better insurance products including redesigned long-term care insurance or reverse mortgage products.

The intuition for the result is simple, but reveals a new free-riding problem preventing insurance markets from maturing. When consumer valuations of insurance contracts in immature markets are different from mature markets, those who wish to purchase insurance are adversely selected. When adverse selection is sufficiently large, a firm entering into an immature market is effectively investing (by earning negative profits) in the market reaching maturity.[18] If they could guarantee monopolist profits in the mature market, it may be profitable to absorb the temporary hit to their bottom line. Herein lies the free-rider problem. The entrant pays a cost to familiarize consumers with the new product, but another firm always has an incentive to wait and enter the mature market. Consequently, no firm is willing to enter the immature market.

Crucially, with adverse selection there is no guarantee positive profits exist – even for

[17] Consumers learn about the generic “product” offered by firms instead of developing brand loyalty.

[18] Indeed, a major Canadian auto-insurer offers individuals a subsidy (above rating incentives) to enrol in new contracts using telematic monitoring.

a monopolist. To see this, consider the following standard expression for the insurer's profit margin (Starc 2014; Kong et al. 2023):

$$p - AC(p) = \frac{1}{\eta_{D,p}} - (AC(p) - MC(p)), \tag{21}$$

where the first term is the usual Lerner markup and the second term captures the influence of adverse selection on firm pricing through their desire to "cream skim" low cost consumers at the margin (see Starc 2014; Mahoney and Weyl 2017). Thus, adverse selection disciplines market power. However, when either the price elasticity of demand is large or adverse selection is severe enough, firms may earn negative monopolist profits in the absence of fixed costs.[19]

Implications For Market Intervention: Patents as Time Varying Subsidies. If there is surplus in mature markets, it may be socially optimal for a market to mature even if there is no-trade in its infancy. However, if primitives of markets change over time, permanent policy solutions, like a subsidy, may be inefficient. For instance, in the entry and exit game above there is a welfare cost associated with missing mature markets, but once the market reaches maturity the competitive allocation can be efficient.

In this case, phasing out subsidies over time when there is deadweight loss from government intervention in mature markets is optimal. However, implementing a time varying subsidy may impose unreasonable information requirements on a social planner. Tautologically, there is no choice data to observe if a market is missing. A simple decentralized alternative policy is to remove barriers to patenting or to increase the length of patent protection in insurance markets. Intuitively, this raises the monopolist profit in immature markets by protecting firms against free riders and encourages entry. It offers an effective time-varying subsidy with minimal information requirements.

[19] Kong et al., similarly discuss entry under imperfect competition and derive a threshold condition on the slope of the average cost curve to sustain a given market structure.

4. Discussion

This paper derives general comparative static results for the equilibrium of selection markets under direct assumptions on the joint density of model primitives. It allows for multi-dimensional heterogeneity and develops a partial ordering of markets for non-parametric families of densities.

I use these tools to study the competitive equilibrium in mature and immature markets for identical insurance contracts. Under economically motivated and plausible assumptions on risk premia and the potential biases of consumers, early adopters are more adversely selected and impose higher costs on insurers even though the distribution of potential costs to the insurer are constant.

I use these results to understand missing markets, where firms do not offer potentially valuable insurance contracts. This extends the classic study of market unraveling in insurance markets to consider this dynamic form of market failure. This jointly explains low utilization, slow demand for new products, and market inactivity. Furthermore, I embed these results within a simple model of firm exit and entry to understand the benefits of patenting in insurance markets.

While I apply these tools to the study of insurance markets, the results apply more generally for a class of selection markets – where consumers can impose heterogeneous costs on the seller when they choose to purchase – under multidimensional heterogeneity. Crucially, they do not depend on whether market immaturity affects the true welfare value of insurance or only consumer's perception of the value. It thus encompasses a variety of applications in economics. These same tools, for example, can be used to understand why shifts in risk premia lower adverse selection and derive sharp predictions on the lemons penalty as in Blundell et al. (2019).

Finally, the results in this paper emphasize that in addition to intra-temporal concerns, there is an inter-temporal motive for intervention in selection markets. It is crucial, therefore, to consider the welfare of consumers over the full transition path from immature to mature markets as well as drawing on estimates from a single point in time.

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Appendix A. Online Appendix

A.1. General Proofs for Section 2

PROOF OF PROPOPSITION 1. First consider linear utility with $\omega_i = \pi_i^{IM} + r_i^{IM} + \varepsilon_i^{IM}$. A direct application of Theorem 3.3 in Navarro and Sarabia (2022) using Assumption 4 gives

$$\omega_i^{IM} = \pi_i^{IM} + r_i^{IM} + \varepsilon_i^{IM} \preceq \pi_i^{MM} + r_i^{MM} + \varepsilon_i^{MM} = \omega_i^{MM} \longrightarrow F_\omega^{MM}(p) \leq F_\omega^{IM}(p) \ \forall p\,. \quad \text{(A1)}$$

As the usual stochastic order is preserved under increasing transformations, the stated proposition then follows immediately for any increasing $u(\cdot)$ □

PROOF OF LEMMA 2. Identical in distribution implies $\pi^{MM} \succeq \pi^{IM}$. Thus, each element of the vector $(\pi_i, r_i, \varepsilon_i)$ in mature markets is larger in the usual stochastic order than in immature markets. It follows that the vector $\mathbf{X}^{IM} \preceq \mathbf{X}^{MM}$ in the usual *multivariate* stochastic ordering (Shaked and Shanthikumar 2007, Theorem 6.B.14).

This order is preserved for any increasing function $u : \mathcal{R}^3 \to \mathcal{R}$, then $u(\mathbf{X}^{IM}) \preceq u(\mathbf{X}^{MM})$. Thus, $F_\omega^{IM}(p) \geq F_\omega^{MM}(p) \ \forall p$ and the stated lemma follows. □

PROOF OF PROPOSITION 2. The CDF of the conditional random variable $\pi|\omega = p$ is

$$\begin{aligned} F_{\pi|\omega}(\pi|\omega) &\equiv \int_{-\infty}^{\pi} f_{\pi|\omega}(u|\omega)du = \int_{-\infty}^{\pi} \frac{f_{\pi,\omega}(u,\omega)}{f_\omega(\omega)}du \\ &= \widehat{C}_2(u, F_\omega(\omega)) = g(F_\pi(\pi)), \end{aligned} \quad \text{(A2)}$$

for a given copula $\widehat{C}$ where $\widehat{C}_2(x, y) = \partial\widehat{C}(x,y)/\partial y$.

As the marginal cost curve is a conditional expectation $MC(p) = E(\pi|\omega = p)$, stochastic dominance of the conditional distributions is a sufficient condition to order marginal costs in price

$$F_{\pi|\omega}(\pi|\omega=v_1) \succeq F_{\pi|\omega}(\pi|\omega=v_2) \quad \forall v_1 \geq v_2 \tag{A3}$$

$$\longrightarrow MC(v_1) = E(\pi|\omega=v_1) \geq E(\pi|\omega=v_2) = MC(v_2) \quad \forall v_1 \geq v_2 \,. \tag{A4}$$

Substituting equation (A2) gives the following sufficient condition:

$$\widehat{C}_2(u, F_\omega(v_1)) \leq \widehat{C}_2(u, F_\omega(v_2)) \; \forall v_1 \geq v_2 \text{ and } \forall u \tag{A5}$$

$$\longleftrightarrow \widehat{C}_2(u,a) \leq \widehat{C}_2(u,b) \; \forall a \geq b \text{ and } \; \forall u \tag{A6}$$

as $v_1 \geq v_2 \longrightarrow F_\omega(v_1) \geq F_\omega(v_2)$. This is satisfied for all copulas where $C_2(,)$ is decreasing in its second argument. Condition (i) of Assumption (5) guarantees $\widehat{C}$ is concave and satisfies this property.

This establishes that $MC(p)$ is increasing in p. The remainder of the proposition follows from condition (iv) in Assumption (5) which is a sufficient condition for Proposition 2.1 in Fang and Joe (1992) to hold (see also Yanagimoto and Okamoto 1969). □

PROOF OF LEMMA 3. Analogously to the previous proof

$$F^{IM}_{\pi|\omega}(\pi|\omega) \succeq F^{MM}_{\pi|\omega}(\pi|\omega) \longrightarrow MC^{IM}(p) \geq MC^{MM}(p) \; \forall \, p. \tag{A7}$$

As the unconditional first order stochastic dominance of private valuations in mature and immature markets implies $F^{MM}_\omega(\omega) \leq F^{IM}_\omega(\omega)$, (A7) holds if and only if the copulas satisfy

$$\widehat{C}^{MM}_2(u,a) \leq \widehat{C}^{IM}_2(u,b) \; \forall a \geq b \text{ and } \forall u. \tag{A8}$$

This is trivially satisfied under identical concave copulas ($\widehat{C} = \widehat{C}^{IM} = \widehat{C}^{MM}$). However, it cannot be satisfied for different copulas that are both concave in their second argument (Assumption 5 condition(i)). □

PROOF OF PROPOSITION 3. The CDF of the conditional random variable $\pi|\omega > p$ is

$$F_{\pi|\omega>p}(\pi|\omega > p) \equiv \frac{u - \widehat{C}(u, F_\omega(p))}{1 - F_\omega(p)}, \tag{A9}$$

for a given copula $\widehat{C}$.

As the average cost curve is defined as a conditional expectation $AC(p) = E(\pi|\omega \geq p)$, stochastic dominance of the conditional distributions is a sufficient condition to order average costs in price

$$F_{\pi|\omega\geq p}(\pi|\omega \geq v_1) \succeq F_{\pi|\omega\geq p}(\pi|\omega \geq v_2)\ \forall v_1 \geq v_2 \tag{A10}$$

$$\longrightarrow AC(v_1) = E(\pi|\omega \geq v_1) \geq E(\pi|\omega \geq v_2) = AC(v_2)\ \forall v_1 \geq v_2 \tag{A11}$$

Substituting equation (A9) gives the following sufficient condition:

$$F_{\pi|\omega\geq p}(\pi|\omega \geq v_1) \leq F_{\pi|\omega\geq p}(\pi|\omega \geq v_2)\ \forall v_1 \geq v_2 \textit{ and } \forall \pi \tag{A12}$$

$$\frac{u - \widehat{C}(u, F_\omega(v_1))}{1 - F_\omega(v_1)} \leq \frac{u - \widehat{C}(u, F_\omega(v_2))}{1 - F_\omega(v_2)}\ \forall v_1 \geq v_2 \textit{ and } \forall u \tag{A13}$$

as $v_1 \geq v_2 \longrightarrow F_\omega(v_1) \geq F_\omega(v_2)$, condition (ii) of Assumption (5) guarantees $\widehat{C}$ satisfies this property, delivering an ordering of average costs for a given copula.

As $F_\omega^{MM}(\omega) \leq F_\omega^{IM}(\omega)$, established in Proposition 1, it follows that

$$\frac{u - \widehat{C}^{MM}(u, F_\omega^{IM}(v))}{1 - F_\omega^{IM}(v)} \leq \frac{u - \widehat{C}^{MM}(u, F_\omega^{MM}(v))}{1 - F_\omega^{MM}(v)}\ \forall v \textit{ and } \forall u, \tag{A14}$$

and by property (iii) of Assumption (5)

$$\frac{u - \widehat{C}^{IM}(u, F_{\omega}^{IM}(v))}{1 - F_{\omega}^{IM}(v)} \leq \frac{u - \widehat{C}^{MM}(u, F_{\omega}^{IM}(v))}{1 - F_{\omega}^{IM}(v)} \; \forall v \textit{ and } u \in [0, 1], \tag{A15}$$

which completes the proof. □

PROOF OF PROPOSITION 4. Note that the copula defines *rank* dependence. Consequently, they are invariant under strictly monotone transformations, including the inverse demand transformation considered here. Thus, to order average costs given quantity, q, we have the following condition:

$$\frac{u - \widehat{C}^{IM}(u, q)}{1 - q} \leq \frac{u - \widehat{C}^{MM}(u, q)}{1 - q} \; \forall q \in [0, 1) \textit{ and } u \in [0, 1], \tag{A16}$$

which simplifies to

$$\widehat{C}^{IM}(u, q) \geq \widehat{C}^{MM}(u, q) \; \forall q \in [0, 1) \textit{ and } u \in [0, 1]. \tag{A17}$$

This only depends in the change in sorting ($\widehat{C}^{IM} \neq \widehat{C}^{MM}$) and holds with equality under no re-sorting ($\widehat{C} = \widehat{C}^{IM} = \widehat{C}^{MM}$). Equation (A8) gives an identical condition for marginal costs. □

A.2. Proofs for Section 3

Proof of the first part. Let $-C < 0$ denote the loss when both firms enter and $\Pi_M^{IM} < 0$ denote the monopolist profit in immature markets, respectively. Equivalently Π_M^{MM} denotes the monopolist profit and $\Pi^{MM} = 0$ denotes the (competitive) profit in mature markets.

(a) Mature Market

	enter	exit
enter	(0, 0)	(Π_M^{MM},0)
exit	(0, Π_M^{MM})	(0,0)

(b) Immature Market

	enter	exit
enter	(-C, -C)	(Π_M^{IM},0)
exit	(0, Π_M^{IM})	(0,0)

TABLE A1. Second Period Payoff Matrix

Payoffs in the second subgame when at least one firm entered in the first period are given in Panel A of Table A1.

In a mature market that does not fully unravel $\Pi_M^{MM} > 0$ and entering is a profitable deviation for both firms if neither enter, thus this cannot be a nash equilibrium of the subgame. There is no profitable deviation from the symmetric strategy profile (enter, enter).

Next, consider the second subgame when neither firm entered in the first period (Panel B). When $\Pi_M^{IM} < 0$, exiting is a strictly dominant strategy and there is a unique equilibrium.

Taking these contingent strategy profiles as given, the normal form representation of the first subgame is in Table A2. As the market is immature and firms earn zero equilibrium profits in mature markets, these payoffs are identical to the immature market second period subgame and exit is strictly dominant.

Therefore, the symmetric strategy profile is an SPE:

- First period: Play *exit*

	enter	exit
enter	$(-C+0, -C+0)$	$(\Pi_M^{IM}+0, 0+0)$
exit	$(0+0, \Pi_M^{IM}+0)$	$(0+0, 0+0)$

TABLE A2. First Period

- Second period: Play *exit* if no firm has entered, otherwise *enter*

and in equilibrium no firm enters in either period.

Proof of second part. When the market is mature in the second period, the SPE has firms playing strictly dominant strategies. Therefore, only refinements for the immature second period subgame need to be considered.

As this is a two player, two action game, the set of trembling hand perfect equilibria is equivalent to the set of equilibria in which no player plays a weakly dominated strategy. The equilibria considered above (exit, exit) satisfies this. However, either (entry, exit) or (exit, entry), do not because entering is weakly dominated by exiting.

This leaves a unique equilibria surviving the trembling-hand refinement in this subgame. The same argument can be applied to the first period delivering a unique equilibria of the extensive form game.