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Perceived and Actual Option Values of College Enrollment

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Abstract

An important feature of post-secondary schooling is the experimentation that accompanies sequential decision-making. Specifically, by entering college, a student gains the option to decide at a future time whether it is optimal to remain in college or to drop out, after resolving uncertainty that existed at entrance about factors that affect the return to college. This paper uses data from the Berea Panel Study to quantify the value of this option. The unique nature of the data allows us to make a distinction between "actual" option values and "perceived" option values and to examine the accuracy of students' perceptions. We find that the average perceived option value is 65% smaller than the average actual option value ($8,670 versus $25,040). A further investigation suggests that this understatement is not due to misperceptions about how much uncertainty is resolved during college, but, rather, because of overoptimism at entrance about the returns to college. In terms of policy implications related to college entrance, we do not find evidence that students understate the overall value of college, which depends on the sum of the option value and expectations at entrance about the returns to college.

Keywords: College Education, Dropout, Option Value, Learning Model, Expectations Data

JEL: I21, I26, J24, D83, D84

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1 Introduction

An important feature of post-secondary schooling is the experimentation that accompanies sequential decision-making.[1] Specifically, by entering college, a student gains the option to decide at a future time ($t = t^*$) whether it is optimal to remain in college or to drop out, after resolving uncertainty that existed at entrance ($t = t_0$) about academic ability or other factors that affect her return to college. This paper uses data from the Berea Panel Study to contribute to a literature that has recognized the importance of quantifying the value of this option (Heckman, Lochner, and Todd, 2006, Heckman and Navarro, 2007, and Stange, 2012). The unique nature of the data allows an examination of whether students' perceptions about option values tend to be accurate by allowing, for the first time, a distinction to be made between "actual" option values and "perceived" option values.

For the purpose of illustration, consider a scenario where all that occurs between t_0 and t^* is that students resolve uncertainty that existed at entrance. In this scenario, in the absence of the option to make decisions after receiving new information, the decision of whether to enter college after high school is equivalent to a decision of whether to commit to staying in school until college graduation.[2] The value of the option quantifies how beneficial it is to be able to delay the graduation decision until after some uncertainty is resolved during the early portion of college. For a student who would not enter college in the absence of the option, the expected lifetime utility at t_0 of graduating is lower than the expected utility at t_0 of not graduating. Roughly speaking, the option value for this student tends to be substantial when, given the magnitude of the (negative) difference between these two expected utilities at t_0, the information she will obtain after entering college will often push her across the margin of indifference to a situation where the expected utility at t^* of graduating is non-trivially higher than the expected utility at t^* of not graduating. Similarly, for a student who would enter college in the absence of the option, the expected utility at t_0 of graduating is higher than the expected utility at t_0 of not graduating. Roughly speaking, the option value for this student tends to be substantial when, given the size of the (positive) difference between these two expected utilities at t_0, the information she will obtain after entering college will often push her across the margin of indifference to a situation where the expected utility at t^* of graduating is non-trivially lower than the expected utility at t^* of not graduating.

[1] This notion that education can be considered as a sequential choice that is made under uncertainty has been widely accepted in the literature since the seminal work in Manski (1989) and Altonji (1993).

[2] If there are also direct net benefits/costs associated with staying in school between t_0 and t^* (e.g., tuition, utility or disutility of schooling, foregone earnings), students' entrance decisions would also depend on these benefits/costs. This could slightly complicate the illustrative discussion in the introduction. If students derive substantial utility from staying in school between t_0 and t^*, in the form of, for example, amenities and consumption benefits (e.g., Jacob, et al., 2018, Gong, et al., 2019), they might decide to start school and drop out after a couple of years even if they do not resolve uncertainty during school. However, we note that our formal approach for quantifying the option value does not rely on the assumption that there are no direct benefits/costs between t_0 and t^*.

The importance of quantifying the option value comes from its fundamental importance for understanding/interpreting college attendance and college dropout decisions; while policy discussion often suggests that college attendance rates are too low or college dropout rates are too high, it is difficult to reach an informed view of these rates without understanding the option value's importance.[3] In terms of college entrance, as implied by the discussion in the previous paragraph, the number of high school graduates who should find it optimal to enter will depend directly on the option value; when option values are close to zero, students will tend to enter college only if the expected utility at t_0 of graduating is greater than the expected utility at t_0 of not graduating, while substantially higher option values can induce entrance even for students for which the difference between these expected utilities, hereafter referred to as the "initial expectations gap" at t_0, is substantially negative. Further, this effect on who attends college also leads to a very direct link between option values and dropout rates. Indeed, inconsistent with policy discussion that tends to view dropout as inherently bad, if high option values imply that students with substantial negative initial expectations gap find it useful to enter college, then a non-trivial amount of dropout would be a natural part of a healthy environment in which schools are providing useful information to students.

Our primary contribution comes from being able to compute both the actual option value for each student and each student's perceptions about the option value. We formalize the discussion above through the lens of a stylized college dropout model. We show that the option value uniquely depends on (1) the initial expectations gap, which measures how far away a student is from the margin of indifference at entrance and (2) how much uncertainty the student resolves before making dropout decisions. Then, as we discuss in Section 2, our ability to compare actual and perceived option values arises from the fact that the unique combination of administrative data and expectations data available in the Berea Panel Study allows perceived and actual values of (1) and (2) to be constructed.

In a baseline scenario where students only resolve uncertainty about the pecuniary benefits of college completion, we find that, on average, students' perceptions about the value of the option understate the actual value of the option substantially: The average perceived option value is \$8,670, roughly 65% smaller than the average actual option value, \$25,040. We examine whether there exist gender differences in option values by conducting our analysis separately for male and female students. We find that while, on average, males and females have similar perceptions about the option value (\$8,440 for males, \$7,660 for females), there exists a substantial gender gap in the average actual option value (\$39,690 for males, \$15,200 for females). Thus, while there do exist some differences by gender, our general conclusion that students underestimate the option value holds for both groups of students. As a robustness check, we examine the implications

[3] As one of many examples, Hess (2018) suggests, in a recent article in the Forbes (June 6, 2018), that "The sad reality is that far too many students invest scarce time and money pursuing a degree they never finish, frequently winding up worse off than if they'd never set foot on campus in the first place."

of allowing students to learn about non-pecuniary factors and also about the pecuniary benefits associated with their non-college option.

One important aspect of our approach is that it allows us to examine why an understatement of the option value occurs. We find that it is not driven by an understatement of the amount of earnings uncertainty that is resolved in college - both the actual and the perceived fraction of initial earnings uncertainty that is resolved in college are 0.51. Instead, we find that students' perceptions tend to substantially overstate the initial expectations gap. Our findings about the reason for misperceptions about the option value are important because, while it may seem at a first glance that an understatement of the option value would necessarily lead to too few students entering college, in reality whether this is true depends critically on why misperceptions exist.[4] This is the case because the overall value of college, which is the relevant object for the college entrance decision, is strongly related but not identical to the option value. Under the illustrative scenario in the second paragraph - where all that occurs between t_0 and t^* is that students resolve uncertainty that existed at entrance - the overall value of college is equal to the sum of the option value and the initial expectations gap, under the most likely scenario where the initial expectations gap is positive. We find that the understatement of the option value is more than offset by the optimism about the initial expectations gap. Thus, once one takes into account both components of the overall value of college, concerns that too few students enter college tend to dissipate.

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2 Overview of Our Approach and Related Literature

The well-recognized difficulty of characterizing option values can be viewed as arising, to a large extent, because of data issues. As noted in the introduction, in Section 4 we use a stylized model to show that the option value is determined by (1) the initial expectations gap and (2) the amount of uncertainty about the gap in expected utilities that will be resolved before making the dropout decision at t^*. Then, because (1) and (2) completely determine the dropout probability in the stylized model, what is needed to characterize the option value is any two of (1), (2), and the dropout probability.

Unfortunately, while administrative data sources can provide direct evidence about the dropout probability, they are not well-suited for providing direct evidence about the other two objects. For example, it is hard to provide information about the initial expectations gap because this gap includes not only the financial return to schooling but

[4] The relevance of this concern is apparent in related research which, for example, examines whether higher-education decisions are influenced by misperceptions about college costs (Bleemer and Zafar, 2018) or by misperceptions about available opportunities (Hoxby and Turner, 2013).

also non-pecuniary benefits of schooling.[5] As such, research characterizing option values typically has turned to fully specified models (often dynamic discrete choice models) to estimate the option value.[6] In contrast, the Berea Panel Study data allow the option value to be computed in a more direct way; in addition to containing information about dropout, evidence about uncertainty resolution, which arises in our baseline model because of learning about pecuniary factors under the scenario in which a student graduates from college, comes from the fact that the distribution describing beliefs about future earnings is collected at multiple times during school.

A feature of the models traditionally used to estimate option values is that Rational Expectations (RE) assumptions are employed to link actual outcomes to choices that depend on students' subjective expectations. Consequently, these approaches do not make a distinction between students' perceptions about option values (hereafter referred to as "perceived" option values) and their values implied by rational expectations (hereafter referred to as "actual" option values); roughly speaking, the option values computed using these models are a mix of perceived and actual option values. Generally, the potential importance of this distinction is highlighted by a recent expectations literature, which has found that perceptions about objects of relevance for educational decisions are often inaccurate.[7] In the particular context of interest here, it seems quite possible that students may not entirely appreciate the benefits of experimentation. Indeed, the importance of learning models was not even widely recognized in the economics of education literature until quite recently, and policy discussion does not tend to extol the virtues of experimentation.[8] Our ability to differentiate between perceived and actual option values comes from the fact that (1) in addition to observing actual dropout rates, the Berea Panel Study collected information about perceived dropout rates and (2) in addition to being able to characterize students' actual uncertainty resolution from longitudinal earnings expectations data, students' perceptions about how much uncertainty will be resolved can be estimated using a simple model describing the relationship between the perceived dropout probability, the perceived initial expectations gap, and the perceived

[5]These non-pecuniary benefits are inherently difficult to observe directly. Instead, many researchers have treated them as the "residual" in the contemporaneous utility function and have identified/estimated their values from the component of schooling attendance decisions that is not explained by pecuniary factors (e.g., Keane and Wolpin, 1997, Cunha, Heckman, and Navarro, 2005, Heckman, Lochner, and Todd, 2006, and Abbott, et al., forthcoming).

[6]Estimation of σ_i typically requires researchers to either impose or estimate the structure of agent information sets at college entrance and the end of college. As one example of the former, Stange (2012) assumes that students update their beliefs about the benefit of college mainly through observing grades as signals. As one example of the latter, Heckman and Navarro (2007) estimate students' information sets using a method developed by Cunha, Heckman, and Navarro (2005).

[7]The importance of whether perceptions tend to be accurate can be seen in recent research emphasizing the value of supplementing expectations data with data on actual outcomes (e.g., Arcidiacono, Hotz, Maurel and Romano, 2019, Stinebrickner and Stinebrickner, 2014a, Wiswall and Zafar, 2016, D'Haultfoeuille, Gaillac, and Maurel, 2018, and Giustinelli and Shapiro, 2019).

[8]The Berea Panel Study was designed (in 1998) with the specific objective of understanding the importance of learning in educational decisions. At the time, Altonji (1993) and Manski (1989) represented some of the only research specifically focusing on the importance of learning models for understanding dropout. See, e.g., Stinebrickner (2012, 2014a/b) for BPS analyses involving dropout.

amount of uncertainty resolution.[9]

3 The Berea Panel Study

Our empirical analysis takes advantage of the Berea Panel Study (BPS). Initiated by Todd Stinebrickner and Ralph Stinebrickner, the BPS is a longitudinal survey that closely followed two cohorts of students at Berea College from the time they entered college, in 2000 and 2001, until 2014. We focus on the 2001 cohort because the 2000 cohort did not answer the survey question about perceived dropout probability in the baseline survey.

Students were surveyed multiple times each year while in college. The baseline survey, which took place immediately after students arrived for their freshman year, was completed in our presence after students received classroom training. Subsequent in-school surveys were distributed through the campus mail system. Students returned completed surveys to Ralph Stinebrickner, who, after ensuring that surveys were completed in a conscientious manner, immediately provided compensation. We found that this survey approach led to, not only high response rates, but also to, for example, virtually no item nonresponse.[10]

The BPS had a specific focus on the collection of students' expectations about various academic and labor market outcomes. Much of our previous work using the BPS contributed to an early expectations literature that was interested in the quality of answers to expectations questions. As one example, Stinebrickner and Stinebrickner (2012) finds that a simple theoretical implication related to college dropout - that the dropout decision should depend on both a student's cumulative GPA and beliefs about future GPA - is satisfied when beliefs are directly elicited through survey questions, but is not satisfied when beliefs are constructed under a version of Rational Expectations. As a second example, Gong, Stinebrickner and Stinebrickner (2019) propose and implement a method for characterizing the amount of measurement error in responses to expectations questions, which takes advantage of the fact that the BPS data often allow the unconditional perceived probability of a particular outcome to be characterized using two different sets of expectations questions.[11] In the context here, of particular importance are survey questions eliciting students' perceptions about the probability of dropping out

[9] Our approach is related to the literature noting the usefulness of expectations data that allow individuals to express uncertainty about outcomes that would occur in the future. The BPS data of this type has been used in papers such as Stinebrickner and Stinebrickner (2014a) to study college major and Stinebrickner and Stinebrickner (2012, 2014b) to study dropout. For other research recognizing this use see, e.g., Blass, Lach, and Manski (2010), van der Klaauw and Wolpin (2008), and van der Klaauw (2012), Wiswall and Zafar, (2014), Delavande and Zafar, (forthcoming).

[10] BPS response rates were very high. Approximately 90% of all students who entered Berea College in 2001 responded to the baseline survey, and response rates were around 85% for subsequent in-school surveys.

[11] Intuitively, differences in the unconditional probabilities computed using the two different sets of expectations questions are informative about the amount of measurement error present in the underlying survey questions.

and perceptions about future earnings under a scenario in which the student graduates and under a scenario in which the student drops out). Unless otherwise noted, the analyses in the paper involve the 337 students (from the 2001 cohort) who provided complete answers to these questions on the baseline survey. Providing evidence in support of the notion that the elicited dropout probabilities contain useful content, we find that the null hypothesis that perceived dropout probabilities are unrelated to actual dropout outcomes is rejected at a .10 level of significance.[12]

Berea College is a four-year college located in central Kentucky. The college focuses on providing educational opportunities to students from relatively low-income backgrounds, and, as part of this focus, offers full-tuition scholarships to all students. This feature supports our parsimonious conceptual setting in which dropout is a result of information acquisition rather than, for example, a result of financial hardship (Stinebrickner and Stinebrickner, 2003, 2008). Despite certain unique features, important for the notion that the basic lessons from our work are likely to be useful for thinking about what takes place elsewhere, Berea operates under a standard liberal arts curriculum and students at Berea are similar in academic quality, for example, to students at the University of Kentucky (Stinebrickner and Stinebrickner, 2008). Perhaps even more importantly, academic decisions and outcomes that are closely related to the option value at Berea are similar to those found elsewhere (Stinebrickner and Stinebrickner, 2014a). For example, dropout rates are similar to the dropout rates at other schools (for students from similar backgrounds) and patterns of major choice and major-switching are similar to those found in the NLSY by Arcidiacono (2004).

4 Defining the Option Value in a Stylized Learning Model

In this section, we define the option value in the context of a stylized model that captures the key features of learning in the college environment of interest. When entering college at t_0, a student knows that she will have the option to choose between college completion ($s = 1$) and dropping out ($s = 0$) at a future time t^*, after resolving a certain fraction of her initial uncertainty (i.e., uncertainty at $t = t_0$) about the discounted lifetime utility, or value, of each alternative, which we denote as V_1 and V_0, respectively. We note that, throughout this paper, a subscript on any object denotes the choice of the schooling outcome s, where $s = 0, 1$.

Even after resolving a certain amount of uncertainty between t_0 and t^*, some uncer-

[12] Of course, from a conceptual standpoint, a strong relationship between perceptions about an object of interest and the actual outcomes of that object are not necessary for expectations data to be useful. Indeed, much of the motivation for the direct elicitation of expectations comes from the possibility that beliefs may be incorrect. Nonetheless, given the difficulty of providing evidence in support of the quality of expectations data, much previous research has examined whether a relationship exists between perceptions and actual outcomes.

tainty may remain at t^* about V_1 and V_0. Thus, standard theory implies that a student's decision at t^* will be made by comparing the expected utilities associated with the two alternatives at t^*. Given that these expected utilities at t^* are simply expectations of V_1 and V_0 at t^*, we denote them $\bar{V}_1$ and $\bar{V}_0$, respectively. We stress that it is important to keep in mind that these expectations are taken at t^*, but that adding an additional t^* subscript to these terms is superfluous because decisions in our model are made only at t^*.

At t_0, a person knows that her decision at t^* will be determined by $\bar{V}_1$ and $\bar{V}_0$, but, because she will resolve uncertainty between t_0 and t^*, there exists uncertainty at t_0 about what these objects will turn out to be. Thus, the option value depends on the distribution describing a student's beliefs at t_0 about $\bar{V}_1$ and the distribution describing the student's beliefs at t_0 about $\bar{V}_0$. We refer to these two belief distributions as $\bar{V}_1^B$ and $\bar{V}_0^B$, respectively. We note that, throughout this paper, a superscript of B on any object denotes students' beliefs about the object at t_0, unless otherwise specified.

Intuitively, the value of the option, OV^B, comes from the fact that students can make dropout decisions based on information available at t^*, rather than based on information available at t_0. Formally, OV^B can be defined as:

$$OV^B \equiv E_{t=t_0} \max(\bar{V}_1^B, \bar{V}_0^B) - \max(E_{t=t_0}(\bar{V}_1^B), E_{t=t_0}(\bar{V}_0^B)), \text{for } B = P, A. \tag{1}$$

The first term shows, on average, how well a student would do if she were able to choose s after seeing which option turned out to be the best at t^*. The second term shows, on average, how well a student would do if she were forced to choose the s with the highest expected value at $t = t_0$, i.e., before any additional uncertainty is resolved. Note that, while strictly unnecessary, we include a "$t = t_0$" subscript to the expectation operators to emphasize the point that $\bar{V}_s^B$ characterizes students' beliefs at t_0.

The fact that B is seen to take on two values (P and A) in Equation (1) relates to our contribution of differentiating between the perceived option value ($B = P$) and the actual option value ($B = A$). This contribution requires that we examine two different sets of belief distributions for B. When $B = P$, the distributions $\bar{V}_1^P$ and $\bar{V}_0^P$ represent a student's perceived distributions at t_0 about $\bar{V}_1$ and $\bar{V}_0$. When $B = A$, the distributions $\bar{V}_1^A$ and $\bar{V}_0^A$ represent the actual distributions at t_0 of $\bar{V}_1$ and $\bar{V}_0$. In general, the assumption of Rational Expectations implies that an individual's perceived distribution of a future outcome coincides with the actual distribution of that outcome. Thus, in our context, these two sets of belief distributions ($\bar{V}_s^P$ and $\bar{V}_s^A$, $s = 0, 1$) are identical to each other if and only if the students have Rational Expectations about $\bar{V}_s$.

Let $\Delta = (\bar{V}_1 - \bar{V}_0) - E_{t=t_0}(\bar{V}_1 - \bar{V}_0)$ represent the new information received between t_0 and t^*. We let Δ^B denote a student's beliefs about Δ at t_0. Naturally, Δ^B is given by:

$$\Delta^B = (\bar{V}_1^B - \bar{V}_0^B) - E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B), \text{ for } B = P, A. \tag{2}$$

We assume that Δ^B is normally distributed for $B = P, A$.[13] It has a mean of zero by construction, and we denote its variance as $(\sigma^B)^2$.[14]

At time t^*, the student chooses to drop out if and only if $\bar{V}_0 > \bar{V}_1$. Given the normality assumed for Δ^B, her belief about the dropout probability P_0^B is given by:

$$P_0^B = \Phi(\frac{-E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)}{\sigma^B}), \text{ for } B = P, A, \tag{3}$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution.

In Equation (1), $E_{t=t_0}\max(\bar{V}_1^B, \bar{V}_0^B)$, which can be referred to as the continuation value of college enrollment, is given by:

$$E_{t=t_0}\max(\bar{V}_1^B, \bar{V}_0^B) = P_1^B E_{t=t_0}(\bar{V}_1^B) + P_0^B E_{t=t_0}(\bar{V}_0^B) + \sigma^B \phi(\frac{E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)}{\sigma^B}), \tag{4}$$

where $P_1^B \equiv 1 - P_0^B = \Phi(\frac{E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)}{\sigma^B})$ is the probability of completing college, and $\phi(\cdot)$ is the pdf of the standard normal distribution.[15]

Combining Equation (1) and (4), we obtain the following expression for the option value OV^B as a function of $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$, which we refer to as the initial expectations gap hereafter, the student's beliefs about the dropout probability P_0^B, and the amount of uncertainty resolved σ^B:

$$\begin{aligned} OV^B &\equiv E_{t=t_0}\max(\bar{V}_1^B, \bar{V}_0^B) - \max(E_{t=t_0}(\bar{V}_1^B), E_{t=t_0}(\bar{V}_0^B)) \\ &= \begin{cases} -P_0^B E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B) + \sigma^B \phi(\frac{-E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)}{\sigma^B}) & \text{if } E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B) > 0 \\ (1 - P_0^B) E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B) + \sigma^B \phi(\frac{E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)}{\sigma^B}) & \text{if } E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B) \le 0 \end{cases} . \end{aligned} \tag{5}$$

Equation (3) shows that any two of P_0^B, σ^B, and $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$ uniquely determine the third. Hence, Equation (5) shows that, as discussed in the introduction, the option value is uniquely determined by σ^B and $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$. However, because it is difficult to obtain direct information about $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$, we instead write OV^B as a function

[13] Later in Section 5.2.1, to obtain baseline results, we impose an assumption that uncertainty resolution in school is through learning about future earnings. In this case, the normality assumption for Δ^B can be motivated by the finding in Gong, Stinebrickner, and Stinebrickner (2019) that a normal distribution fits students' responses to earnings expectations question better than a log-normal distribution.

[14] For both $B = P$ and $B = A$, we have $E_{t=t_0}\Delta^B = E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B) - E_{t=t_0}[E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)] = 0$. We note that, if the student does not have rational expectations about $\bar{V}_s$, then $E_{t=t_0}(\bar{V}_1^P - \bar{V}_0^P)$ is not necessarily equal to $E_{t=t_0}(\bar{V}_1^A - \bar{V}_0^A)$. In this case, the difference between these two terms measures the systematic overoptimism of this student. By construction, such systematic overoptimism was not anticipated by the student at t_0. This is the case because, if it was anticipated, it should be incorporated into $E_{t=t_0}(\bar{V}_1^P - \bar{V}_0^P)$, which would then be correct, on average.

[15] Equation (4) is equivalent to a well-known alternative formulation: $E_{t=t_0}\max(\bar{V}_1^B, \bar{V}_0^B) = E_{t=t_0}(\bar{V}_1^B | \bar{V}_1^B \ge \bar{V}_0^B) P_1^B + E_{t=t_0}(\bar{V}_0^B | \bar{V}_0^B > \bar{V}_1^B) P_0^B$. A comparison between the two formulations reveals that the last term in Equation (4) captures the difference between the conditional and unconditional means of $\bar{V}_1^B$ and $\bar{V}_0^B$.

of P_0^B and σ^B:

$$OV^B = \begin{cases} P_0^B \sigma^B \Phi^{-1}(P_0^B) + \sigma^B \phi(\Phi^{-1}(P_0^B)) & \text{if } P_0^B < 0.5 \\ (1-P_0^B)\sigma^B \Phi^{-1}(1-P_0^B) + \sigma^B \phi(\Phi^{-1}(1-P_0^B)) & \text{if } P_0^B \geq 0.5 \end{cases}$$
$$\equiv \begin{cases} \sigma^B G(P_0^B) & \text{if } P_0^B < 0.5 \\ \sigma^B G(1-P_0^B) & \text{if } P_0^B \geq 0.5 \end{cases}, \tag{6}$$

where $G(P) \equiv P\Phi^{-1}(P) + \phi(\Phi^{-1}(P))$ is a known function of P, which has the easily verifiable property:

Lemma 1. *$G(P)$ is monotonically increasing in P for $P \in (0, 1)$.*

Lemma 1 implies the following propositions.

Proposition 2. *The option value, OV^B, has the following properties with respect to the amount of uncertainty resolved before t^*, σ^B, and the probability of dropping out, P_0^B.*

1. *The OV^B is uniquely determined by σ^B and P_0^B;*
2. *The OV^B is multiplicatively separable in σ^B and P_0^B;*
3. *The OV^B is linearly increasing in σ^B;*
4. *The OV^B is monotonically increasing in P_0^B for $P_0^B \in (0, 0.5)$ and monotonically decreasing in P_0^B for $P_0^B \in [0.5, 1)$.*

Proposition 2.1 shows that data on the dropout probability, P_0^B, and the amount of uncertainty resolved during college, σ^B, are sufficient for determining the option value, with Equation (3) detailing how the initial expectations gap is uniquely characterized by these two terms. Important for our analysis in Section 5, Proposition 2.2 shows that σ^B and P_0^B enter the expression of OV^B in a multiplicatively separable fashion. Proposition 2.3 and Proposition 2.4 qualitatively describe how σ^B and P_0^B affect the value of OV^B.

5 Characterizing the Option Value

Proposition 2 shows that the option value OV^B is uniquely determined by a student's beliefs about the dropout probability, P_0^B, and the amount of uncertainty that is resolved during college, σ^B. In Section 5.1, we describe the direct information available in the BPS about both the actual and perceived values of P_0^B. In Section 5.2, we impose more structure on the general model described in Section 4 in order to estimate the actual and perceived values of σ^B. In Section 5.3, combining information about P_0^B and σ^B, we compute both actual and perceived option values for each student. Comparing actual option values (obtained using P_0^A and σ^A) to perceived option values (obtained using P_0^B and σ^B) provides evidence about the accuracy of beliefs about the option value at the time of entrance. Finally, in Section 5.4, we discuss the policy implications of potential misperceptions.

5.1 Actual and Perceived Dropout Probabilities

Both actual dropout outcomes and perceived dropout probabilities can be obtained directly from the BPS data. 218 out of the 337 students in the sample eventually graduated from Berea College, which implies a dropout rate, or equivalently an average actual dropout probability P_0^A, of 0.353. Question 1 in Appendix A elicits a student's perceived probability of graduating from Berea College. Subtracting this number from 1 yields the perceived dropout probability, P_0^P. We find that the average perceived dropout probability of students in our sample is 0.147, 58% smaller than the average actual dropout probability.

Proposition 2 is useful for examining how the underestimation of the dropout probability influences the size of the perceived option value relative to the size of the actual option value. Suppose students have correct perceptions about σ^B. Since the option value is multiplicatively separable in P_0^B and σ^B, without loss of generality, we set $\sigma^B = 1$. As implied by proposition 2.4, Figure 1 shows that the option value is increasing in the dropout probability over the range $(0, 0.5)$. Evaluating the option value at the average actual dropout probability leads to an actual option value of 0.238. Evaluating the option value at the average perceived dropout probability leads to a perceived option value of 0.076. Then, for a "representative" student, the perceived value of the option is 68% lower than the actual value of the option.

Of course, in reality there is no reason that individuals would necessarily have Rational Expectations about σ^B. Proposition 2.1 indicates that obtaining point estimates for the actual and perceived values of the option requires knowledge of actual and perceived values of σ^B. In the next section, we discuss our approach for taking advantage of additional unique data to obtain these objects. Nonetheless, the evidence presented in the previous paragraph strongly suggests that we are likely to find that students at Berea College tend to underestimate the option value at the time of entrance. Indeed, using Proposition 2.3, we see that the representative student would need to overestimate σ^B by at least 214% in order to not underestimate the option value.

Before we turn to the characterization of σ^B for $B = P, A$, we note that, in order to compute the option value for each student, individual-specific measures of actual and perceived dropout probabilities are required. As mentioned earlier, individual-specific perceived dropout probabilities can be directly obtained from students' responses to Question 1 in Appendix A. The sample standard deviation of perceived dropout probabilities is 0.180. In contrast, individual-specific measures of actual dropout probabilities are not directly available. We allow for individual heterogeneity by assuming that a student's actual dropout probability is equal to the predicted probability from a probit regression of a dropout dummy on observables.[16]

[16] The observables in the probit regression include gender, race, high school GPA, ACT score, and a student's perceived dropout probability.

5.2 Actual and Perceived Earnings Uncertainty Resolution

In this section, we describe the construction of the actual and perceived values of σ^B. In Section 5.2.1, we show that, under the assumption that the learning of relevance during college is about future earnings associated with college completion, σ^B can be computed by combining: 1) data characterizing a student's uncertainty at the time of entrance (i.e., initial uncertainty) about future earnings under the scenario in which she graduates from college and 2) a parameter ρ^B capturing the fraction of this initial uncertainty that is resolved between t_0 and t^*. Section 5.2.2 describes how we can construct measures of initial earnings uncertainty from survey questions eliciting subjective beliefs about future earnings. Section 5.2.3 describes how we can consistently estimate the actual fraction of uncertainty resolution, which we denote ρ^A, and therefore the actual σ^A, by taking advantage of the longitudinal feature of our expectations data. Finally, Section 5.2.4 shows that, by taking advantage of data on students' perceived dropout probabilities and students' initial subjective beliefs about future earnings, our model permits us to estimate the perceived fraction of uncertainty resolution, which we denote ρ^P, and, therefore the perceived σ^P.

5.2.1 Defining σ^B in a Fully Specified Model

We consider a model in which the value of alternative s, $\bar{V}_s$, is equal to the expectation, at time t^*, of the sum of the discounted lifetime earnings associated with this alternative, Y_s, and an additional term γ_s summarizing a student's overall non-pecuniary benefit from s:

$$\bar{V}_s = E_{t=t^*}(Y_s + \gamma_s). \tag{7}$$

We start by specifying the discounted lifetime earnings, Y_s, for each alternative. If a student chooses $s = 1$, the student stays in college until graduation ($t = \bar{t}$), then starts to work. For ease of notation, we index time t by a student's age a. Y_1 is then given by $Y_1 = \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} w_1^a$, where w_s^a represents the earnings that the student receives at age a given her choice of s, β is the discount factor and $\bar{T}$ is the age of retirement. Similarly, if the student chooses $s = 0$, she leaves college and starts working immediately. The discounted lifetime earnings associated with this alternative, Y_0, is given by $Y_0 = \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*} w_0^a$.

Turning to the non-pecuniary benefit/utility associated with the choice of s, the immediate exit from school that accompanies a choice of $s = 0$ implies that γ_0 will tend to capture a person's preferences about working in jobs that do not require a college degree. On the other hand, γ_1 will capture not only preferences for working in the types of jobs that are obtained with a college degree, but also a person's non-pecuniary costs/benefits from staying in college until graduation.

For our primary results, we make the simplifying assumption that the only updating that occurs during college is about the future earnings that would be received under the graduation scenario. That is, students learn only about Y_1 while in college. Abstracting away from learning about earnings under the dropout scenario, Y_0, allows for a more transparent discussion of identification, but is also consistent with the intuitively appealing notion that college is best suited for providing information about one's ability to perform high skilled jobs. Further, when relaxing this assumption as a robustness check in Appendix C, we find strong evidence in support of this notion; 1) Students resolve substantially less uncertainty about earnings under the dropout scenario than under the graduation scenario, and 2) our main results remain quantitatively similar when we relax this assumption.

Abstracting away from learning about the non-pecuniary benefits, γ_s, while obviously not literally correct, would tend to not be particularly problematic if students tend to have a good sense of how much they like school by the end of high school or if the overall non-pecuniary benefit of the graduation alternative ($s = 1$) arises largely because a college degree affects the non-wage aspects of one's work over her lifetime - since individuals presumably learn the most about these non-wage aspects when they actually hold these jobs after graduation.[17] Nonetheless, in Appendix D, we discuss how relaxing this assumption would affect our results. In particular, we show that, if, as in Stinebrickner and Stinebrickner (2012), a common set of signals (e.g., grades) influences what a student learns about both pecuniary and non-pecuniary benefits, our estimates of actual option values tend to be downward biased while our estimates of perceived option values remain consistent.

Assuming that students do not learn about non-pecuniary benefits implies that $E_{t=t_0}(\gamma_s) = E_{t=t^*}(\gamma_s)$ for $s = 0, 1$. Assuming that students do not learn about earnings under the non-graduation scenario implies that $E_{t=t_0}(Y_0) = E_{t=t^*}(Y_0)$. Then, the relevant new information Δ is given by:

$$\begin{aligned}
\Delta &= (\bar{V}_1 - \bar{V}_0) - E_{t=t_0}(\bar{V}_1 - \bar{V}_0) \\
&= E_{t=t^*}[(Y_1 + \gamma_1) - (Y_0 + \gamma_0)] - E_{t=t_0}[(Y_1 + \gamma_1) - (Y_0 + \gamma_0)] \\
&= E_{t=t^*}(\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} w_1^a) - E_{t=t_0}(\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} w_1^a) \\
&= \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} [E_{t=t^*}(w_1^a) - E_{t=t_0}(w_1^a)]. \qquad (8)
\end{aligned}$$

Taking the variance of the last line of Equation (8) shows that variation in Δ depends on variation in how much a student updates her expectations about earnings under the graduation scenario ($E_{t=t^*}(w_1^a) - E_{t=t_0}(w_1^a)$), or equivalently, on the amount of initial earnings uncertainty that is resolved between t_0 and t^*. Recall that σ^B represents the standard deviation of Δ^B, which describes a student's beliefs about Δ. Then, σ^B is de-

[17] While students do likely learn something about how much they like school after entrance, this learning only affects utility for the short period of time between t^* and $\bar{t}$. In contrast, the non-wage aspects of one's future work would have a lifelong impact on her utility.

termined by a student's beliefs about variation in how much she updates her expectations about earnings under the graduation scenario.

We begin the process of characterizing this updating by writing w_1^a, without loss of generality, as the sum of three independently distributed factors, ϵ_1^{a,τ_1}, ϵ_1^{a,τ_2}, and ϵ_1^{a,τ_3}, that are observed by the student in the period before t_0 (denoted τ_1), in the period between t_0 and t^* (denoted τ_2), and in the period after t^* (denoted τ_3), respectively:

$$w_1^a = \epsilon_1^{a,\tau_1} + \epsilon_1^{a,\tau_2} + \epsilon_1^{a,\tau_3}. \tag{9}$$

At the time of entrance, there exists no uncertainty about ϵ_1^{a,τ_1} because, by definition, students have observed its realization. On the other hand, uncertainty does exist about ϵ_1^{a,τ_2} and ϵ_1^{a,τ_3}. Let ϵ_1^{B,a,τ_2} and ϵ_1^{B,a,τ_3} denote a student's beliefs about these two factors at t_0. We assume that ϵ_1^{B,a,τ_2} and ϵ_1^{B,a,τ_3} are each normally distributed for $B = P, A$. We denote their standard deviations as σ_1^{B,a,τ_2} and σ_1^{B,a,τ_3}, respectively. We also denote the mean of σ_1^{B,a,τ_2} as μ_1^{B,a,τ_2} and normalize the mean of ϵ_1^{B,a,τ_3} to be zero. Then, Equation (8) implies that $\Delta^B \sim N(0, \sigma^B)$ is given by:

$$\begin{aligned}\Delta^B &= \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}[E_{t=t^*}(\epsilon_1^{a,\tau_1} + \epsilon_1^{B,a,\tau_2} + \epsilon_1^{B,a,\tau_3}) - E_{t=t_0}(\epsilon_1^{a,\tau_1} + \epsilon_1^{B,a,\tau_2} + \epsilon_1^{B,a,\tau_3})] \\ &= \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{B,a,\tau_2}) - \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\mu_1^{B,a,\tau_2}), \text{for } B = P, A,\end{aligned} \tag{10}$$

where ϵ_1^{a,τ_1} does not appear on the last line because there does not exist any uncertainty about this object at t_0, and ϵ_1^{B,a,τ_3} does not appear because no uncertainty about this object is resolved before t^*.

Motivated by the notion that, during college, learning about future earnings is mostly through permanent factors such as innate ability, we assume that the ϵ_1^{B,a,τ_2} are perfectly correlated across all future ages a. Under this assumption, computing the standard deviation of Δ^B from Equation (10) implies that σ^B is given by:

$$\sigma^B = \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\sigma_1^{B,a,\tau_2}), \text{for } B = P, A. \tag{11}$$

As is standard, BPS survey questions, such as Question 2 in Appendix A, elicit uncertainty about earnings in the future, but do not directly elicit information about how quickly uncertainty is resolved. Thus, what is observed at t_0 is students' perceptions about the distribution of $w_1^a = \epsilon_1^{a,\tau_1} + \epsilon_1^{a,\tau_2} + \epsilon_1^{a,\tau_3}$. In our setting, the mean and the standard deviation of this perceived distribution correspond to $E_{t=t_0}(\epsilon_1^{a,\tau_1} + \epsilon_1^{P,a,\tau_2} + \epsilon_1^{P,a,\tau_3}) = \epsilon_1^{a,\tau_1} + \mu_1^{P,a,\tau_2}$ and $std_{t=t_0}(\epsilon_1^{a,\tau_1} + \epsilon_1^{P,a,\tau_2} + \epsilon_1^{P,a,\tau_3}) = \sqrt{(\sigma_1^{P,a,\tau_2})^2 + (\sigma_1^{P,a,\tau_3})^2}$, respectively.

Motivated by data availability, we proceed under the assumption that all students expect to resolve a fraction ρ^P of their perceived initial uncertainty about earnings at age a before t^*, but actually resolve a fraction ρ^A of their perceived initial uncertainty about earnings at age a before t^*. Formally, with B continuing to take on the values of either P or A, we have:

$$\sigma_1^{B,a,\tau_2} = \rho^B \sqrt{(\sigma_1^{P,a,\tau_2})^2 + (\sigma_1^{P,a,\tau_3})^2}. \tag{12}$$

Then, Equation (11) becomes:

$$\sigma^B = \rho^B \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} (\sqrt{(\sigma_1^{P,a,\tau_2})^2 + (\sigma_1^{P,a,\tau_3})^2}), \tag{13}$$

where parameters ρ^B and β are common across all students and variables σ^B, σ_1^{P,a,τ_2}, and σ_1^{P,a,τ_3} are individual specific.

The computation of the components of Equation (13) is discussed in the remainder of Section 5.2. Section 5.2.2 describes the computation of $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} (\sqrt{(\sigma_1^{P,a,\tau_2})^2 + (\sigma_1^{P,a,\tau_3})^2})$, taking advantage of a sequence of survey questions eliciting subjective beliefs about earnings at different future ages. Section 5.2.3 describes the estimation of ρ^A. Section 5.2.4 describes the estimation of ρ^P.

5.2.2 Computing $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} (\sqrt{(\sigma_1^{P,a,\tau_2})^2 + (\sigma_1^{P,a,\tau_3})^2})$ from Survey Data

In this section, we describe the computation of $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} (\sqrt{(\sigma_1^{P,a,\tau_2})^2 + (\sigma_1^{P,a,\tau_3})^2})$. Under the assumptions in Section 5.2.1, this term corresponds to the standard deviation of the random variable describing a student's perceived distribution at t_0 of the discounted lifetime earnings associated with the graduation alternative, Y_1. As a result, we denote this term $\tilde{\sigma}_1^{P,Y}$. Similarly, we denote $\sqrt{(\sigma_1^{P,a,\tau_2})^2 + (\sigma_1^{P,a,\tau_3})^2}$ as $\tilde{\sigma}_1^{P,a}$. With this notation, Equation (13) can be written as:

$$\begin{aligned} \sigma^B &= \rho^B \tilde{\sigma}_1^{P,Y} \\ &= \rho^B \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} \tilde{\sigma}_1^{P,a}. \end{aligned} \tag{14}$$

Our approach for computing $\tilde{\sigma}_1^{P,a}$, and therefore $\tilde{\sigma}_1^{P,Y}$, takes advantage of a sequence of survey questions that elicits information about a student's perception at t_0 about the distribution of w_1^a. Specifically, following the format of Question 2 in Appendix A, a respondent reports, at t_0, the three quartiles, $Q_s^{k,a}$, $k = 1, 2, 3$, of the distribution describing her subjective beliefs about what her earnings will be at a particular future age a under choice s. Maintaining the assumption that this distribution is normal, the standard deviation ($\tilde{\sigma}_s^{P,a}$) of the distribution is given by:

$$\tilde{\sigma}_s^{P,a} = (Q_s^{3,a} - Q_s^{1,a}) / \left[\Phi(0.75) - \Phi(0.25)\right], \tag{15}$$

where $\Phi(\cdot)$ is the standard normal cdf.

Equation (14) shows that the computation of $\tilde{\sigma}_1^{P,Y}$ requires taking into account a student's uncertainty about earnings, $\tilde{\sigma}_1^{P,a}$, for all future ages a. As can be seen in Question 2, the earnings expectations questions in the BPS were asked for three specific ages a: the first year after graduation (age 23), age 28, and age 38. Following Stinebrickner and Stinebrickner (2014b), we assume that $\tilde{\sigma}_1^{P,a}$ grows linearly between the first post-college year and age 28, grows linearly between ages 28 and 38, and does not change after age 38 (until the age of retirement, $\bar{T} = 65$). We operationalize our stylized model by assuming that a student enters college at age 19 ($t_0 = 19$), decides whether to drop out at the end of the third year ($t^* = t_0 + 3$), and graduates at age 23 ($\bar{t} = 23$) if she chooses to remain in school.[18] Focusing on the case where $s = 1$, Equation (15), together with the interpolation and timing assumptions above, allows the computation of $\tilde{\sigma}_1^{P,Y}$.[19] We report all values in 2001 dollars. The first column of Table 1 shows that the average value of $\tilde{\sigma}_1^{P,Y}$ is \$226,000 for our primary sample.[20]

Table 1: Descriptive Statistics

# of Observations: 337	$\tilde{\sigma}_1^{P,Y}$	$\tilde{\sigma}_0^{P,Y}$	$\tilde{\mu}_1^{P,Y}$	$\tilde{\mu}_0^{P,Y}$
Sample Mean	226	163	954	680
Sample Std	201	145	436	333

Note: The unit of measurement is one thousand dollars.

5.2.3 Computing ρ^A from Longitudinal Beliefs Data

In this section, we describe the estimation of the actual fraction of perceived initial earnings uncertainty that is resolved before t^*, ρ^A. Our approach takes advantage of the fact that the longitudinal nature of the BPS expectations data provides direct evidence about the extent to which uncertainty decreases over time.

Section 5.2.2 shows that $\tilde{\sigma}_1^{P,Y}$, the standard deviation of a student's perceived distribution of Y_1 at t_0, can be constructed from the expectations data reported at the time of entrance ($t = t_0$). Using the same method, the expectations data collected at t^* allows us to also construct $\tilde{\sigma}_1^{P^*,Y}$, the standard deviation of a student's perceived distribution of Y_1 at t^*. Of interest here is the relationship between these values. Our timing assumption

[18]Our choice of $t^* = t_0 + 3$ was informed by Gong, Stinebrickner and Stinebrickner (2019) who found that the vast majority of uncertainty resolution during college takes place before the end of the third year. However, perhaps more importantly, we find that, because uncertainty resolution tends to take place rather quickly, our results change little if we assume that dropout takes place at the end of the second year, i.e., $t^* = t_0 + 2$.

[19]We assume that the discount factor β is equal to 0.95.

[20]Using the same method, we can also compute $\tilde{\sigma}_0^{P,Y} \equiv \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\tilde{\sigma}_0^{P,a})$, the standard deviation of the random variable describing a student's subjective beliefs about the discounted lifetime earnings associated with the dropout alternative. As reported in the second column of Table 1, the sample average of $\tilde{\sigma}_0^{P,Y}$ is \$163,000, implying that students on average are more uncertain about earnings associated with the graduation alternative.

in Section 5.2.1 suggests that a student's perceived distribution of Y_1 changed between t_0 and t^* because of the realization of $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{a,\tau_2})$. Hence, the reduction in the perceived uncertainty about Y_1 between t_0 and t^* (measured by $(\tilde{\sigma}_1^{P,Y})^2 - (\tilde{\sigma}_1^{P^*,Y})^2$) is equal to the variance of the actual distribution of $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{a,\tau_2})$, which directly corresponds to $(\sigma^A)^2 = [\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\sigma_1^{A,a,\tau_2})]^2$. Formally, it implies that $\tilde{\sigma}_1^{P^*,Y}$ is given by:

$$\begin{aligned} \tilde{\sigma}_1^{P^*,Y} &= \sqrt{(\tilde{\sigma}_1^{P,Y})^2 - (\sigma^A)^2} \\ &= \sqrt{1-(\rho^A)^2}\tilde{\sigma}_1^{P,Y}, \end{aligned} \tag{16}$$

where the second line in Equation (16) follows from the assumption that all students actually resolve the same fraction of perceived initial earnings uncertainty, i.e. $\sigma^A = \rho^A \tilde{\sigma}_1^{P,Y}$ for all students.

Equation (16) shows that $\sqrt{1-(\rho^A)^2}$ can be computed using the ratio of the average of $\tilde{\sigma}_1^{P^*,Y}$ to the average of $\tilde{\sigma}_1^{P,Y}$ for the same sample of students.[21] Using the sample of students who were still in school at $t^* = 3$, the estimated value of $\sqrt{1-(\rho^A)^2}$ is 0.86.[22] Hence, the estimated value of ρ^A is 0.51.[23] Then, Equation (14) can be used to compute σ^A for each student in our sample.

5.2.4 Computing ρ^P Using a Dropout Model

In this section, we describe how the perceived fraction of initial earnings uncertainty that is resolved before t^*, ρ^P, can be estimated using a simple model of dropout.

At the time of entrance (t_0), a student reports her perceived dropout probability, P_0^P. Equation (3) shows that this perceived probability depends on a student's perceived initial expectations gap, $E_{t=t_0}(\bar{V}_1^P - \bar{V}_0^P)$, and her perceived amount of uncertainty resolved between t_0 and t^*, σ^P.

[21] We choose to use the ratio of the average of $\tilde{\sigma}_1^{P^*,Y}$ to the average of $\tilde{\sigma}_1^{P,Y}$ rather than, for example, the average of the ratio of $\tilde{\sigma}_1^{P^*,Y}$ to $\tilde{\sigma}_1^{P,Y}$, because the former tends to be a consistent estimator of $\sqrt{1-(\rho^A)^2}$ even when individual uncertainty measures might potentially contain measurement error.

[22] In practice, some students dropped out of college before $t^* = 3$, and, therefore, were not included in the estimation of ρ^A. One might be concerned that those who dropped out before t^* might have resolved systematically different fractions of their initial uncertainty under the counterfactual in which they stayed until t^* than those who actually remained in our sample until t^*. As a robustness check, it would be desirable to add students who dropped out before t^* to our estimation sample. We do this by using a student's last observed earnings uncertainty as a proxy for what her earnings uncertainty would have been at t^*. Given that students who dropped out before t^* would have resolved additional uncertainty between the time of dropout and t^* if they had remained in school, the resulting estimator should produce a lower bound for ρ^A. We find that this lower bound is 0.41 and that the corresponding lower bound for the average actual option value is $19,990. As we show later in Section 5.3, this lower bound is still substantially higher than the estimated average perceived option value, suggesting that our main conclusion that students vastly underestimate the option value is robust to the selection issue.

[23] Our results about actual earnings uncertainty resolution are comparable in magnitude to what was found in Gong, Stinebrickner, and Stinebrickner (2019), which also take advantage of the BPS dataset. Using data for both the 2000 and the 2001 cohorts, they find that the sample average of the standard deviation of the distribution describing students' beliefs about w_1^{28} at the end of the third year ($t = t^*$) is roughly 82% of the sample average of the standard deviation of the distribution describing students' beliefs about w_1^{28} at the beginning of college ($t = t_0$).

Equation (7) implies that $E_{t=t_0}\bar{V}_s^P = E_{t=t_0}(Y_s^P + \gamma_s^P)$, where Y_s^P and γ_s^P describe a student's perceived distributions at t_0 of the pecuniary and non-pecuniary benefits of schooling alternative s, respectively. We denote $E_{t=t_0}Y_s^P$ as $\tilde{\mu}_s^{P,Y}$ for $s = 0, 1$ and let $\tilde{\gamma}^P \equiv E_{t=t_0}(\gamma_0^P - \gamma_1^P)$ represent the student's subjective expectation at t_0 about the difference in the non-pecuniary benefits associated with the two alternatives. Thus, we have $E_{t=t_0}(\bar{V}_1^P - \bar{V}_0^P) = -(\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y} + \tilde{\gamma}^P)$.

Then, with the expression for σ^P coming from Equation (14), we obtain:

$$P_0^P = \Phi(\frac{\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y} + \tilde{\gamma}^P}{\rho^P \tilde{\sigma}_1^{P,Y}}). \tag{17}$$

The intuition underlying the role of ρ^P in Equation (17) is clear. The numerator in the probability expression is the difference between the expected utility of $s = 0$ and the expected utility of $s = 1$, at t_0. Thus, for example, a negative numerator represents the distance that a student is "above" the margin of dropping out at the time of entrance. A larger denominator implies that a student resolves more uncertainty about earnings between t_0 and t^*, thereby increasing the probability that the new information she receives will push her across the margin into a dropout decision; all else equal, in the seemingly most likely scenario in which the numerator is negative, the dropout probability will tend to be increasing in the denominator.[24] Roughly speaking, identification of ρ^P comes from the fact that the relationship between the amount of perceived uncertainty at the time of entrance, $\tilde{\sigma}_1^{P,Y}$, and the perceived dropout probability, P_0^P, will tend to be stronger when ρ^P is high (than when ρ^P is low) because ρ^P maps the amount of initial uncertainty into the amount of uncertainty that the students believes will be resolved.

As described in previous sections, P_0^P and $\tilde{\sigma}_1^{P,Y}$ can be obtained using students' responses to survey Questions 1 and 2, respectively. Appendix B shows that $\tilde{\mu}_s^{P,Y}$ can also be computed using survey Question 2, in a manner similar to that used for the computation of $\tilde{\sigma}_1^{P,Y}$. As reported in the last two columns of Table 1, at t_0, the sample average of expected lifetime earnings associated with the graduation scenario ($\tilde{\mu}_1^{P,Y}$) and the dropout scenario ($\tilde{\mu}_0^{P,Y}$) are approximately \$954,000 and \$680,000, respectively.

The only components in Equation (17) that are yet known to us are a common parameter ρ^P and individual-specific net non-pecuniary benefits $\tilde{\gamma}^P$. To estimate the value of ρ^P (and the distribution of $\tilde{\gamma}^P$), we rewrite Equation (17) as follows:

$$\Phi^{-1}(P_0^P)\tilde{\sigma}_1^{P,Y} = \frac{\bar{\gamma}^P}{\rho^P} + [\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y}]\frac{1}{\rho^P} + \frac{\tilde{\gamma}^P - \bar{\gamma}^P}{\rho^P}, \tag{18}$$

where $\bar{\gamma}^P$ represents the population mean of $\tilde{\gamma}^P$.

The only common unknown parameters in Equation (18) are ρ^P and $\bar{\gamma}^P$. Hence, if all the expectations variables (P_0^P, $\tilde{\mu}_1^{P,Y}$, $\tilde{\mu}_0^{P,Y}$, and $\tilde{\sigma}_1^{P,Y}$) are measured perfectly, then $\frac{\bar{\gamma}^P}{\rho^P}$ and $\frac{1}{\rho^P}$ can be estimated via an easy-to-implement OLS regression of $\Phi^{-1}(P_0^P)\tilde{\sigma}_1^{P,Y}$ on

[24] Of course, from a theoretical standpoint, when experimentation plays a role in the decision to enter school, a student might enter even if she has a positive numerator.

$[\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y}]$. However, it is worthwhile to address the concern that responses to survey questions eliciting expectations may contain a non-trivial amount of measurement error (e.g., Manski and Molinari, 2010, Ameriks et al., 2019, Giustinelli, Manski, and Molinari, 2019, and Gong, Stinebrickner, and Stinebrickner, 2019), which can lead to well-known attenuation bias in the estimation of linear models such as Equation (18). We first modify Equation (18) to accommodate measurement error:

$$\Phi^{-1}(P_0^P)\tilde{\sigma}_1^{P,Y} + \delta^y = \frac{\bar{\gamma}^P}{\rho^P} + [\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y} + \delta^x]\frac{1}{\rho^P} + \frac{\tilde{\gamma}^P - \bar{\gamma}^P}{\rho^P}. \tag{19}$$

In this specification, the observed measure of the pecuniary component of the initial expectations gap, $\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y}$, contains individual-specific classical measurement error δ^x. In addition, we also allow the computed value of $\Phi^{-1}(P_0^P)\tilde{\sigma}_1^{P,Y}$ to contain individual-specific classical measurement error δ^y.[25] In Appendix E.2, we show that, under these assumptions, the attenuation bias in the estimation of $\frac{\bar{\gamma}^P}{\rho^P}$ and $\frac{1}{\rho^P}$ can be corrected if the variance of δ^x is known.[26] In Appendix E.1, we describe how to utilize the method developed in Gong, Stinebrickner and Stinebrickner (2019) to estimate $var(\delta^x)$.[27] We find that, after correcting for the attenuation bias, the estimate of ρ^P is 0.51, which is almost identical to the second decimal to its actual counterpart, ρ^A.

5.3 Actual and Perceived Option Values

Given individual-specific actual and perceived values for students' beliefs about σ^B and P_0^B, we are able to compute the actual and perceived option value for each student using Equation (6). The solid line in Figure 3 shows the cdf for the estimated actual option values. The sample average and standard deviation of the actual option values are $25,040 and $28,440, respectively. Our finding about the average actual option value is generally similar to what has been found in the literature using very different methods. For example, estimating a schooling decision model under Rational Expectations assumptions, Stange (2012) finds that the option value is roughly $19,000 (in 2001 dollars) for an average high school graduate in the United States.

The "+" line in Figure 3 shows the cdf for estimated perceived options values. The sample average and standard deviation of the perceived option values are $8,670 and

[25] δ^y may be relevant because either answers to survey questions eliciting perceived dropout probabilities or answers to survey questions eliciting earnings expectations (specifically, initial uncertainty) may be measured with error.

[26] The presence of classical measurement error in dependent variable (δ^y) does not affect the consistency of the OLS estimator.

[27] The BPS contains two sets of survey questions that can be used to compute a student's unconditional subjective expectation about earnings at age 28, $\tilde{\mu}_1^{P,28}$. Intuitively, differences in the unconditional expectations computed using these two sets of expectations questions are informative about the amount of measurement error, $\delta_1^{P,28}$, present in the observed measure of $\tilde{\mu}_1^{P,28}$. Gong, Stinebrickner and Stinebrickner (2019) formalize this intuition and develop a method to estimate $var(\delta_1^{P,28})$ under the assumption that the measurement error is classical. In Appendix E.1,, we adopt the same method to estimate $var(\delta_1^{P,28})$ using our sample (the 2001 cohort) and detail the assumptions that are required to compute $var(\delta^x)$ using $var(\delta_1^{P,28})$

$16,400, respectively. Consistent with what was suggested by a comparison between actual and perceived dropout probabilities in Section 5.1, students at Berea College do indeed vastly underestimate the option value of college enrollment.

Perhaps the most tenuous parameter to estimate is the perceived fraction of initial uncertainty that is resolved between t_0 and t^*, ρ^P, which in turn determines σ^P. However, the finding that students underestimate the option value is robust to the estimate of σ^P. An upper bound on σ^P can be obtained from Equation (14) by assuming that students believe they will fully resolve their initial uncertainty about lifetime earnings ($\rho^P = 1$). Combining this upper bound of σ^P and data on the perceived dropout probability, we can compute an upper bound for the perceived option value. As shown in Figure 2, the upper bound for the average perceived option value would still be roughly $8,000 lower than the average actual option value, due to the considerable underestimation of the dropout probability.

Our results about actual and perceived option values are obtained under a simplifying assumption that ρ^A and ρ^P are homogeneous across students. Motivated by Stinebrickner and Stinebrickner (2012), who show that the amount of learning during college tends to be different between males and females, we also redo our analysis separately for males and females. We find that, while male students understate the amount of earnings uncertainty resolution ($\rho^A = 0.64$, $\rho^P = 0.40$), female students overstate the amount of earnings uncertainty resolution somewhat ($\rho^A = 0.40$, $\rho^P = 0.56$). We combine these gender-specific estimates of ρ^A and ρ^P with information on dropout probabilities and initial earnings uncertainty to compute actual and perceived option values. Our main result that students underestimate the option value holds for both males and females. However, we do find some gender differences; while, on average, perceptions about the option value are similar for males and females ($8,440 for males, $7,660 for females), there exists a substantial gender gap in the average actual option value ($39,690 for males, $15,200 for females). [28]

5.4 Policy Implications

Our finding that students' perceptions understate the actual option value of college enrollment raises a question fundamental to the general policy concern that informational problems may cause too few students to enter college: what would happen if misperceptions about the option value were corrected? Importantly, our approach allows us to examine not only whether misperceptions about the option value exist, but also why

[28] We also conducted a similar analysis to examine whether ρ^A and ρ^P depend on other observed characteristics. For example, dividing students in our sample into two equal-sized subgroups based on their high school GPA (HSGPA), we find that, while students with high HSGPA expect to resolve a slightly larger fraction of initial earnings uncertainty ($\rho^P = 0.60$) than students with low HSGPA ($\rho^P = 0.46$), the actual fraction ρ^A is very similar for these two groups of students ($\rho^A = 0.51$ for high HSGPA, $\rho^A = 0.52$ for low HSGPA). For each group, we again find that, on average, perceptions about the option value ($9,680 for high HSGPA, $8,310 for low HSGPA) understate the actual option value ($14,500 for high HSGPA, $37,420 for low HSGPA).

they exist. As highlighted by the simple conceptual model in the second paragraph of the introduction, the understatement of the option value could be caused by either an understatement of how much uncertainty is resolved during college or an overly optimistic view about the size of the initial expectations gap. Then, our finding that perceptions about uncertainty resolution are accurate implies that individuals overstate the size of the initial expectations gap. Correcting misperceptions about the option value would involve providing information about, for example, the returns to college.

These findings about the reason for misperceptions about the option value are important because, while it may seem at a first glance that an understatement of the option value would necessarily lead to too few students entering college, in reality whether this is true depends critically on why misperceptions exist. This is the case because the overall value of college, which is the relevant object for the college entrance decision, is strongly related but not identical to the option value. Under the illustrative scenario in the second paragraph of the introduction - where all that occurs between t_0 and t^* is that students resolve uncertainty that existed at entrance - the overall value of college is given by the net continuation value (Heckman, Lochner, and Todd, 2006, Heckman and Navarro, 2007, and Heckman and Urzua, 2008).[29] Defined as $E_{t=t_0}[\max(\bar{V}_1^B, \bar{V}_0^B)] - E_{t=t_0}(\bar{V}_0^B)$, the net continuation value (NCV) captures the expected continuation value of college enrollment net of the value of the outside option (dropout). In the scenario where the initial expectations gap, $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$, is negative, the net continuation value and the option value are identical. However, in the more likely case where the initial expectations gap is positive, the net continuation value is equal to the sum of the option value and the initial expectations gap. Therefore, with the option value computed using methods described in previous sections and $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$ uniquely determined by data on σ^B and P_0^B, we can compute the actual and perceived NCV for each student.[30]

We start by computing the perceived and actual value of $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$ for each student. Equation (3) implies that $E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B)$ is given by:

$$E_{t=t_0}(\bar{V}_1^B - \bar{V}_0^B) = -\Phi^{-1}(P_0^B)\sigma^B. \tag{20}$$

Using the actual dropout probability, P_0^A, and the actual amount of uncertainty resolution, σ^A, we find that, on average, the actual value $E_{t=t_0}(\bar{V}_1^A - \bar{V}_0^A)$ is \$45,120. Similarly, using perceived dropout probability P_0^P and the perceived amount of uncertainty resolution, σ^P, we find that the average perceived value $E_{t=t_0}(\bar{V}_1^P - \bar{V}_0^P)$ is \$160,930. Thus, at the time of entrance, students overestimated the expected net benefit of college completion (i.e., the initial expectations gap) by more than \$115,000. This implies that misperceptions about the option value and misperceptions about the initial expectations

[29]In the more general case, the overall value of college continues to take into account the NCV, but also takes into account the direct utility differences between the two options over the period t_0 to t^*.

[30]Alternatively, similar to what we did for the option value, we can directly express the net continuation value as a function of σ^B and P_0^B. We can show that the NCV is increasing in σ^B and decreasing in P_0^B.

gap work in an offsetting manner when computing the NCV. Taking both into account, we find that perceptions about the NCV somewhat overstate its actual value; the actual and perceived NCV are $76,130 and $173,110, respectively. Thus, while students underestimate the option value, once one considers the NCV, concerns that there might be too few students attending college tend to dissipate.

6 Conclusion

From a student's perspective, the return to college education is likely to be uncertain when she makes the college attendance decision. Having the option to decide whether to remain in college or to drop out after receiving relevant new information can potentially help students insure against this uncertainty. Complementing administrative data on college completion with data describing students' beliefs, at the time of entrance, about the probability of dropping out and data describing students' beliefs, at multiple points in college, about future earnings allows us to pay careful attention to the distinction between perceived and actual option values.

We find strong evidence that students substantially underestimate the experimentation benefits of enrolling in college. However, importantly, we find that this underestimate is caused by an overly optimistic view about the size of the initial expectations gap, rather than an understatement of the amount of uncertainty that is resolved during college. This has important implications for whether inaccurate perceptions create a situation where too few students are entering college. In the calculation of the overall value of college, the underappreciation of the experimental benefit is more than offset by overoptimism about the initial expectations gap. Once one considers the overall value of college, concerns that there might be too few students attending college tend to dissipate.

As in our other work using the BPS, we feel it is important to be appropriately cautious when thinking about exactly how the results from our study would generalize to other institutions. Our results are perhaps most relevant for thinking about students from low income backgrounds, who are a primary focus of the educational mission at Berea College. This group is of particular policy interest, in part because they may be more likely to be affected by informational problems. In addition, from a methodological standpoint, we feel that our paper provides a concrete example of how unique expectations data can be useful for characterizing difficult-to-identify objects of direct policy relevance.

Figure 1: Option Value and Dropout Probability

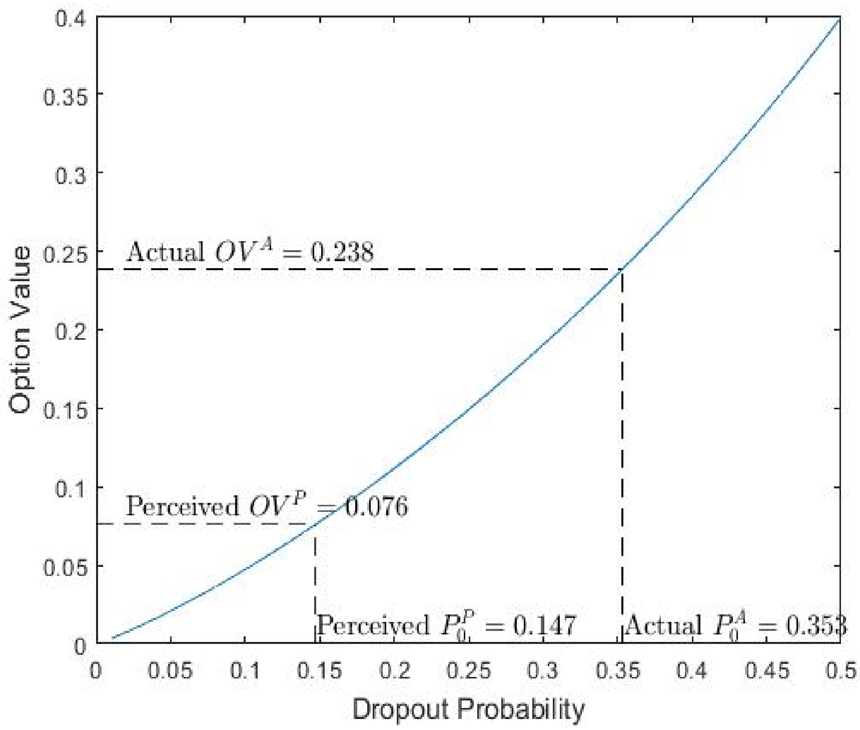


Figure 2: Perceived Option Value and ρ^P

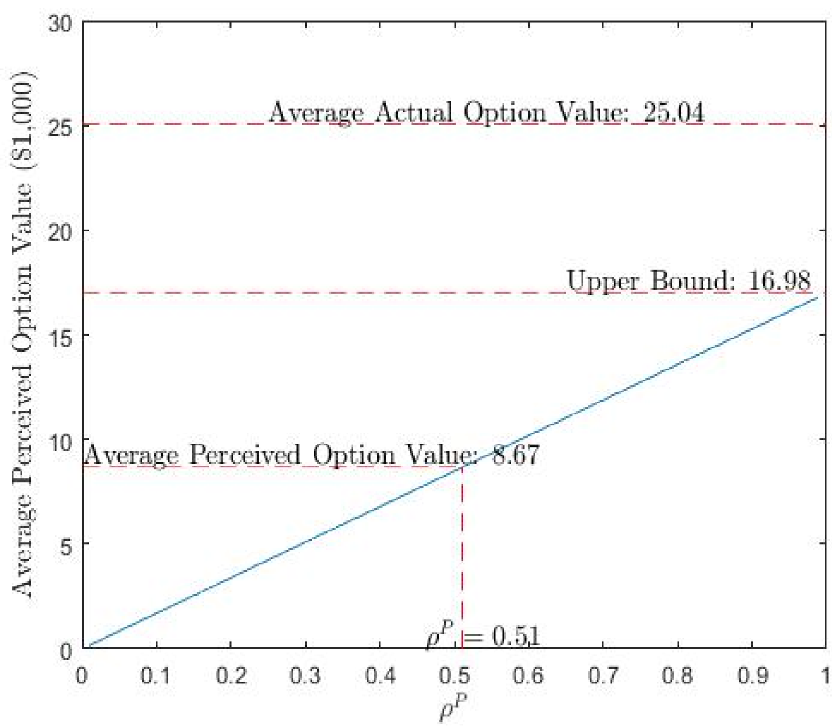
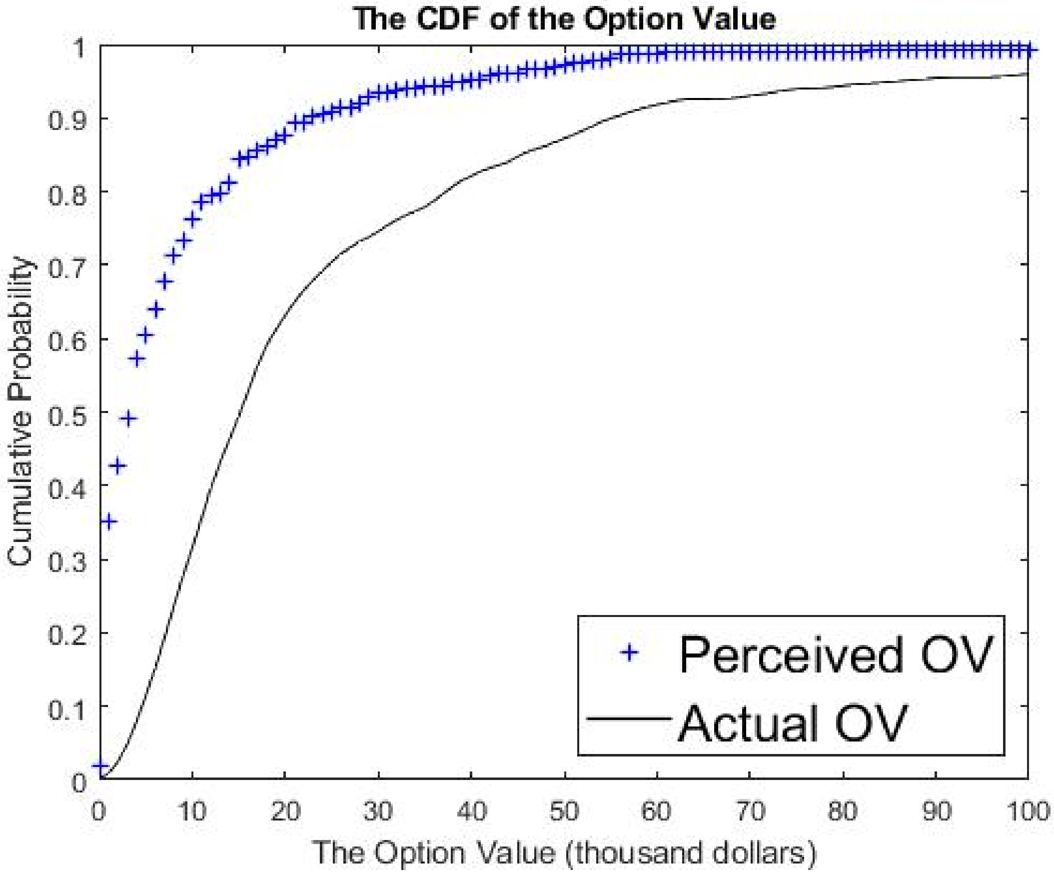


Figure 3: The CDF of the Option Value



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Appendices

A Survey Questions

Question 1. What is the percent chance that you will eventually graduate from Berea College? ____________Note: Number should be between 0 and 100 (could be 0 or 100).

Question 2. For ALL of question 2, **assume that you graduate from Berea**. Think about the kinds of jobs that will be available for you and those that you would accept. Please write the **FIVE NUMBERS** that describe the income which you would expect to earn at the following ages or times under this hypothetical scenario.

I. Your income during the first full year after you leave school

lowest highest

II. Your income at age 28 (note: if you are 20 years of age or older, give your income 10 years from now)

lowest highest

III. Your income at age 38 (note: if you are 20 years of age or older, give your income 20 years from now)

lowest highest

NOTE TO READER: In the paper, we also use close variants of Question 2, in which students were asked to consider scenarios in which they leave Berea after three years of study. Before answering Question 2, students received classroom training related to these specific questions and received the following written instructions, which relate strongly to the classroom training.

INSTRUCTIONS The following questions will ask you about the income you might earn in the future at different ages under several hypothetical scenarios. We realize that you will not know exactly how much money you would make at a particular point in time. However, you may believe that some amounts of money are quite likely while others are quite unlikely. We would like to know what you think. We first ask you to indicate the lowest possible amount of money you might make and the highest amount of money you might make. We then ask you to divide the values between the lowest and the highest into four intervals. Please mark the intervals so that there is a 25% chance that your income will be in each of the intervals.

Example To illustrate what we are asking you to do, consider the following example. A student is asked to describe what she thinks about how well she will do on an exam before taking it. Before the exam the person will not know exactly what grade she will receive. However, she will have some idea of what grade she will receive. Suppose that the person believes that the lowest possible grade she will receive is a 14 and the highest possible grade is 100 (so she believes that there is no chance that she will receive less than a 14 and some chance she will earn as high as 100).

1) The above person would begin by indicating the lowest and highest value on the line. (We will provide the lines for you whenever they are needed.)

14	100
lowest	highest

2) The person would then divide the values between 14 and 100 into four intervals so that she thinks that there is a 25% chance that her grade will be in each interval. For example, suppose that the person marked three points between 14 and 100 and labeled them 52, 80 and 92.

14	54	80	92	100
lowest				highest

This would mean that the person thinks there is a 25% chance she will get a grade between 14 and 52. Similarly, the person thinks there is a 25% chance she will get a grade between 52 and 80, a 25% chance she will get a grade between 80 and 92, and there is a 25% chance she will get a grade between 92 and 100. (This also means that the person thinks that there is a 50% chance she will get a grade less than 80 and a 50% chance that she will get a grade higher than 80.)

NOTE that the intervals do not have to have the same widths. For example, the interval between 14 and 52 is wider than the other intervals. This suggests that the student believes that she has a smaller chance of receiving a particular grade in this interval than a particular grade in the higher intervals. For example, the person may think that she is less likely to receive a 30 than 82.

A different person taking the exam might have very different views about how he might do on the exam. For example, a student might fill in the line to look like

0	32	51	63	90

lowest highest

This student thinks that the smallest possible grade is 0 and the highest possible grade he will receive is 90. When compared to the other student, this student thinks he is more likely to get a lower grade. For example, he thinks that there is a 25% chance he will get a grade less than 32. There is a 25% chance he will get a grade between 32 and 51. The chance that he gets a grade higher than 63 is only 25%. This person thinks there is a 50% chance he will get less than 51 and a 50% chance he will get more than 51.

We will be asking you questions about income instead of grades. However, the process will be the same as above. **For each question, please do the following:**

1) **Write the lowest and highest possible incomes above the words lowest and highest on the line.** Give the salary in thousands of dollars. If you write 15, you will mean $15,000. If you write 120, you will mean $120,000.

2) **Mark three points on the line between the lowest and highest values and write an income above each point.** These income values should divide the line into four intervals. As in the previous example, the numbers should be chosen so that there is a 25% chance that your income will be in each interval. The middle value you write should be the number such that there is a 50% chance that you will make more money and a 50% chance you will make less money.

Note: For each line you should enter five numbers.

The following questions will ask you about the income you would expect to earn under several hypothetical scenarios. Each of the questions will have the same format. In particular, each question will be divided into three parts. Each part will ask you the income that you will earn at a particular time in your life. The questions will differ in their assumptions about how far you go in school an how well you do in classes. In the first three questions, we will ask you about your income under several scenarios in which you do not graduate. In the last four questions, we ask you about your income under several scenarios in which you graduate with different grade point averages. When reporting incomes, take into account the possibility that you will work full-time, the possibility that you will work part-time, the possibility that you will not be working, and (for the hypothetical scenarios which involve graduation) the possibility that you will attend graduate or professional school. When reporting income you should ignore the effects of price inflation.

B Computation of $\tilde{\mu}_s^{P,Y}$

In this appendix we show how $\tilde{\mu}_s^{P,Y}$ can be computed using students' responses to Question 2. Recall that $Y_1 = \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} w_1^a$ and $Y_0 = \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*} w_0^a$. Denoting the mean of a student's perceived distribution at t_0 of w_s^a as $\tilde{\mu}_s^{P,a}$, we have:

$$\tilde{\mu}_1^{P,Y} = \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} \tilde{\mu}_1^{P,a}, \qquad \tilde{\mu}_0^{P,Y} = \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*} \tilde{\mu}_0^{P,a}. \tag{21}$$

Similar to $\tilde{\sigma}_s^{P,a}$, we can obtain $\tilde{\mu}_s^{P,a}$ using the reported quartiles of the distribution describing a student's subjective beliefs about what her earnings will be at a particular future age a under choice s. Specifically, the normality assumption that we imposed on this distribution implies that $\tilde{\mu}_s^{P,a}$ is equal to $Q_s^{2,a}$, the second quartile (median) of the distribution. Hence, adopting the same interpolation and timing assumptions as in Section 5.2.2, Equation (21) allows us to compute $\tilde{\mu}_s^{P,Y}$ for $s = 0, 1$.

C Robustness: Allowing for Learning about Y_0

Our analysis in Section 5.2-5.4 assumed that students learn only about the future earnings associated with the graduation alternative, Y_1. The simplifying assumption that students do not learn about the future earnings associated with the dropout alternative, Y_0, has the virtue of allowing for a more transparent discussion of identification and the virtue of allowing results to be discussed in a straightforward manner. It is also consistent with the intuitively appealing notion that college is best suited for providing information about one's ability to perform high skilled jobs. Nonetheless, this section recognizes the benefit of providing some evidence that this is a reasonable assumption. We find that this is the case. Both the actual and perceived amounts of uncertainty resolved about Y_0 are much smaller than the corresponding amounts resolved about Y_1. Further, in part because of this result and in part because what a student learns about earnings under the graduation scenario is informative about earnings under the dropout alternative, allowing students to also resolve uncertainty about Y_0 does not change our substantive conclusion in Section 5.3 and Section 5.4 - that students underestimate the option value and overestimate the net continuation value.

C.1 Defining σ^B in a Correlated Learning Environment

Allowing students to learn about the future earnings associated with the dropout alternative leads to a modification of Equation (10). A student's beliefs about the relevant new information Δ^B, is now given by:

$$\Delta^B = [\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{B,a,\tau_2}) - \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\mu_1^{B,a,\tau_2})] - [\sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\epsilon_0^{B,a,\tau_2}) - \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\mu_0^{B,a,\tau_2})], \tag{22}$$

where, analogous to ϵ_1^{B,a,τ_2}, ϵ_0^{B,a,τ_2} is the student's beliefs at t_0 about the component of w_0^a that is observed between t_0 and t^*. Similarly, we assume that the ϵ_0^{B,a,τ_2} is normally distributed with mean μ_0^{B,a,τ_2} and standard deviation σ_0^{B,a,τ_2} and are perfectly correlated across all a.

Motivated by recent work suggesting the importance of correlated learning (Arcidiacono et al., 2016), we allow $\epsilon_1^{B,a,\tau_2} - \mu_1^{B,a,\tau_2}$ and $\epsilon_0^{B,a',\tau_2} - \mu_0^{B,a',\tau_2}$ to have correlation κ^B for all a, a' pairs. Under these assumptions, Equation (22) implies that the standard deviation of Δ^B, σ^B, is given by:

$$\sigma^B = \sqrt{\left[\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\sigma_1^{B,a,\tau_2})\right]^2 + \left[\sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\sigma_0^{B,a,\tau_2})\right]^2 - 2\kappa^B \left[\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\sigma_1^{B,a,\tau_2})\right]\left[\sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\sigma_0^{B,a,\tau_2})\right]}. \tag{23}$$

As shown in Section 5.2.2, $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\sigma_1^{B,a,\tau_2})$ can be written as a fraction ρ^B of $\tilde{\sigma}_1^{P,Y}$, the student's perceived initial uncertainty about lifetime earnings associated with alternative $s = 1$. Similarly, we can write $\sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\sigma_0^{B,a,\tau_2})$ as a fraction ρ_0^B of $\tilde{\sigma}_0^{P,Y} \equiv \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\tilde{\sigma}_0^{P,a})$, the student's perceived initial uncertainty about lifetime earnings associated with alternative $s = 0$. Equation (23) becomes:

$$\sigma^B = \sqrt{(\rho^B \tilde{\sigma}_1^{P,Y})^2 + (\rho_0^B \tilde{\sigma}_0^{P,Y})^2 - 2\kappa^B \rho^B \rho_0^B \tilde{\sigma}_1^{P,Y} \tilde{\sigma}_0^{P,Y}}. \tag{24}$$

In Section 5.2.2, we showed how to obtain $\tilde{\sigma}_1^{P,Y}$ from students' responses to earnings expectations questions in the BPS. Since students report their beliefs about future earnings under both alternatives ($s = 0$ and $s = 1$), $\tilde{\sigma}_0^{P,Y}$ can be obtained using the same method. The second column of Table 1 shows that the sample average of $\tilde{\sigma}_0^{P,Y}$ is \$163,000, roughly 30% smaller than the sample average of $\tilde{\sigma}_1^{P,Y}$, implying that, at t_0, on average there is more uncertainty about earnings under the graduation scenario than there is about earnings under the dropout scenario.

With data on $\tilde{\sigma}_s^{P,Y}$ for $s = 0, 1$, computation of σ^B, and therefore option values, requires information on ρ^B, ρ_0^B, and κ^B. In the next two subsections we discuss how to estimate the actual and perceived values of these objects.

C.2 Actual Option Values

Allowing for learning about the value of the dropout alternative has no bearing on our estimation of ρ^A; the estimate of ρ^A remains 0.51. The value of ρ_0^A can be estimated in the same manner. We find an estimate of 0.28 for ρ_0^A, suggesting that students resolve a smaller fraction of their initial uncertainty about Y_0 than about Y_1. Since students were less uncertain about Y_0 than about Y_1 to begin with, we conclude that the actual uncertainty resolution about Y_0 is much smaller than that about Y_1.

The actual correlation κ^A can be estimated from the evolution of individual earnings beliefs. Appendix B shows that $\tilde{\mu}_s^{P,Y}$, the mean of a student's perceived distribution of Y_s at t_0, can be constructed from the expectations data reported at the time of entrance ($t = t_0$). Using the same method, the expectations data collected at t^* allows us to also construct $\tilde{\mu}_s^{P^*,Y}$, the mean of a student's perceived distribution of Y_s at t^*. Equation (9) along with our timing assumptions imply that $\tilde{\mu}_s^{P,a} = \epsilon_s^{a,\tau_1} + \mu_s^{P,a,\tau_2}$ and $\tilde{\mu}_s^{P^*,a} = \epsilon_s^{a,\tau_1} + \epsilon_s^{a,\tau_2}$. Hence, Equation (21) shows that:

$$\begin{aligned}
\tilde{\mu}_1^{P^*,Y} - \tilde{\mu}_1^{P,Y} &= \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{a,\tau_2} - \mu_1^{P,a,\tau_2}) = \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{a,\tau_2} - \mu_1^{A,a,\tau_2}) + \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\mu_1^{A,a,\tau_2} - \mu_1^{P,a,\tau_2}), \\
\tilde{\mu}_0^{P^*,Y} - \tilde{\mu}_0^{P,Y} &= \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\epsilon_0^{a,\tau_2} - \mu_0^{P,a,\tau_2}) = \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\epsilon_0^{a,\tau_2} - \mu_0^{A,a,\tau_2}) + \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\mu_0^{A,a,\tau_2} - \mu_0^{P,a,\tau_2}),
\end{aligned} \tag{25}$$

where $\mu_s^{A,a,\tau_2} - \mu_s^{P,a,\tau_2}$ measures the systematic bias in the student's expectation at t_0 about earnings at age a given schooling outcome s.

Note that the population distribution of ϵ_s^{a,τ_2} coincides with the actual belief distribution ϵ_s^{A,a,τ_2}. Hence, our assumptions on ϵ_s^{A,a,τ_2} in Section C.1 imply that 1) $\epsilon_1^{a,\tau_2} - \mu_1^{A,a,\tau_2}$ and $\epsilon_0^{a',\tau_2} - \mu_0^{A,a',\tau_2}$ have a correlation of κ^A for any pair $(a,\ a')$, and 2) $\epsilon_s^{a,\tau_2} - \mu_s^{A,a,\tau_2}$ are perfectly correlated across a (for a given s). Thus, the population correlation of $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{a,\tau_2} - \mu_1^{A,a,\tau_2})$ and $\sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\epsilon_0^{a,\tau_2} - \mu_0^{A,a,\tau_2})$ is also κ^A. Under an additional assumption that the systematic bias $\mu_s^{A,a,\tau_2} - \mu_s^{P,a,\tau_2}$ is homogeneous across students for $s = 0, 1$, we can show that the population correlation of $\tilde{\mu}_1^{P^*,Y} - \tilde{\mu}_1^{P,Y}$ and $\tilde{\mu}_0^{P^*,Y} - \tilde{\mu}_0^{P,Y}$ is κ^A as well. Hence, for a random sample of students, κ^A can be consistently estimated by the sample correlation of $\tilde{\mu}_1^{P^*,Y} - \tilde{\mu}_1^{P,Y}$ and $\tilde{\mu}_0^{P^*,Y} - \tilde{\mu}_0^{P,Y}$.

However, in practice, a complication exists because the sample of students who remained at the end of third year is, by construction, not random. Indeed, in the context of our model, students choose to remain in school precisely because the realization of $\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{a,\tau_2}) - \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\epsilon_0^{a,\tau_2})$ is sufficiently high. To deal with this selection issue, we take advantage of the fact that selection should not be problematic when estimating the correlation between $\tilde{\mu}_1^{P(t_0+1),Y} - \tilde{\mu}_1^{P,Y}$ and $\tilde{\mu}_0^{P(t_0+1),Y} - \tilde{\mu}_0^{P,Y}$, where $\tilde{\mu}_s^{P(t_0+1),Y}$ represents the mean of a student's perceived distribution of Y_s at the end of the first year ($t = t_0 + 1$). This is the case because very few students drop out before the end of the first year (i.e., we have a random sample for the first year). Data on $\tilde{\mu}_s^{P,Y}$ and $\tilde{\mu}_s^{P(t_0+1),Y}$ are collected at the beginning and end of the first year, respectively. We compute this correlation to be 0.63. In the end of this subsection (Section C.2), we show that, with additional assumptions on how uncertainty about future earnings is resolved over time between t_0 and t^*, this correlation represents a consistent estimator of κ^A.

With ρ^A, ρ_0^A, and κ^A estimated using the methods described above, we compute the actual amount of uncertainty resolution, σ^A, for each student. The average value of σ^A is \$96,780. This is smaller than the value of \$115,430 obtained using the values of ρ^A and $\tilde{\sigma}_1^{P,Y}$ from Section 5.2 under the previous assumption that students only resolve uncertainty about Y_1. Hence, allowing students to also resolve uncertainty about Y_0 leads students to learn less about the gap between the value of the two alternatives. This is primarily because students learn about the two alternatives in a positively correlated fashion: a positive information shock to the graduation alternative is likely to be accompanied with a positive information shock to the dropout alternative. Consequently, the actual option values computed under this correlated learning environment are also somewhat smaller than their counterparts in the baseline scenario. The average actual option value and NCV are now \$21,020 and \$63,720, respectively (versus \$25,040 and \$76,130, respectively, in Section 5.3 and Section 5.4).

$\kappa^{A(1)} = \kappa^A$: Assumptions and Proof

We show that, with additional assumptions on how uncertainty about future earnings are resolved between t_0 and t^*, we can consistently estimate κ^A using the correlation between $\tilde{\mu}_1^{P(t_0+1),Y} - \tilde{\mu}_1^{P,Y}$ and $\tilde{\mu}_0^{P(t_0+1),Y} - \tilde{\mu}_0^{P,Y}$. We start by further decomposing ϵ_s^{a,τ_2} into independently distributed factors that are realized in Year 1, Year 2 and Year 3, respectively;

$$\epsilon_s^{a,\tau_2} = \sum_{j=1}^{3} \epsilon_s^{a,\tau_2^j}. \tag{26}$$

As usual, we let $\epsilon_s^{B,a,\tau_2^j}$ denote a student's beliefs about the distribution of ϵ_s^{a,τ_2^j} at t_0 and assume that $\epsilon_s^{B,a,\tau_2^j}$ is normally distributed with mean μ_s^{B,a,τ_2^j} and standard deviation σ_s^{B,a,τ_2^j}. It follows that:

$$\begin{aligned}
\tilde{\mu}_1^{P(t_0+1),Y} - \tilde{\mu}_1^{P,Y} &= \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\epsilon_1^{a,\tau_2^1} - \mu_1^{A,a,\tau_2^1}) + \sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}(\mu_1^{A,a,\tau_2^1} - \mu_1^{P,a,\tau_2^1}), \\
\tilde{\mu}_0^{P(t_0+1),Y} - \tilde{\mu}_0^{P,Y} &= \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\epsilon_0^{a,\tau_2^1} - \mu_0^{A,a,\tau_2^1}) + \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}(\mu_0^{A,a,\tau_2^1} - \mu_0^{P,a,\tau_2^1}),
\end{aligned} \tag{27}$$

Similarly, we assume that 1) the correlation between $\epsilon_1^{a,\tau_2^j} - \mu_1^{A,a,\tau_2^j}$ and $\epsilon_0^{a',\tau_2^j} - \mu_0^{A,a',\tau_2^j}$, given any a, a' pair, is $\kappa^{A(j)}$, 2) $\epsilon_s^{a,\tau_2^j} - \mu_s^{A,a,\tau_j}$ are perfectly correlated across a (for a given s), and 3) $\mu_s^{A,a,\tau_2^j} - \mu_s^{P,a,\tau_2^j}$ is homogeneous across students for $s = 0, 1$. Under additional assumptions that both the correlation $\kappa^{A(j)}$ and the ratio of signal strength $\frac{\sigma_1^{A,a,\tau_2^j}}{\sigma_0^{A,a,\tau_2^j}}$ are constant over j, it can be shown that $\kappa^A = \kappa^{A(1)}$.

Proof. We first show that $\frac{\sigma_1^{A,a,\tau_2}}{\sigma_0^{A,a,\tau_2}} = \frac{\sigma_1^{A,a,\tau_2^1}}{\sigma_0^{A,a,\tau_2^1}}$;

$$\begin{aligned}
\frac{\sigma_1^{A,a,\tau_2}}{\sigma_0^{A,a,\tau_2}} &= \frac{\sqrt{\sum_{j=1}^{3}(\sigma_1^{A,a,\tau_2^j})^2}}{\sqrt{\sum_{j=1}^{3}(\sigma_0^{A,a,\tau_2^j})^2}} \\
&= \sqrt{\frac{\sum_{j=1}^{3}(\sigma_0^{A,a,\tau_2^j})^2(\frac{\sigma_1^{A,a,\tau_2^j}}{\sigma_0^{A,a,\tau_2^j}})^2}{\sum_{j=1}^{3}(\sigma_0^{A,a,\tau_2^j})^2}} \\
&= \sqrt{\frac{\sum_{j=1}^{3}(\sigma_0^{A,a,\tau_2^j})^2(\frac{\sigma_1^{A,a,\tau_2^1}}{\sigma_0^{A,a,\tau_2^1}})^2}{\sum_{j=1}^{3}(\sigma_0^{A,a,\tau_2^j})^2}} \\
&= \frac{\sigma_1^{A,a,\tau_2^1}}{\sigma_0^{A,a,\tau_2^1}}.
\end{aligned} \tag{28}$$

Then, we can show that:

$$\begin{aligned}
\kappa^A &= corr(\epsilon_1^{a,\tau_2} - \mu_1^{A,a,\tau_2}, \epsilon_0^{a,\tau_2} - \mu_0^{A,a,\tau_2}) = \frac{cov(\epsilon_1^{a,\tau_2}, \epsilon_0^{a,\tau_2})}{\sqrt{var(\epsilon_1^{a,\tau_2})var(\epsilon_0^{a,\tau_2})}} \\
&= \frac{cov(\sum_{j=1}^{3}\epsilon_1^{a,\tau_2^j}, \sum_{j=1}^{3}\epsilon_0^{a,\tau_2^j})}{\sqrt{(\sigma_1^{A,a,\tau_2})^2(\sigma_0^{A,a,\tau_2})^2}} \\
&= \frac{\sum_{j=1}^{3} cov(\epsilon_1^{a,\tau_2^j}, \epsilon_0^{a,\tau_2^j})}{\sigma_1^{A,a,\tau_2}\sigma_0^{A,a,\tau_2}} \\
&= \frac{\sum_{j=1}^{3} \kappa_j^A \sigma_1^{A,a,\tau_2^j}\sigma_0^{A,a,\tau_2^j}}{\sigma_1^{A,a,\tau_2}\sigma_0^{A,a,\tau_2}} \\
&= \kappa^{A(1)} \frac{\sum_{j=1}^{3}(\sigma_1^{A,a,\tau_2^j}/\sigma_0^{A,a,\tau_2^j})(\sigma_0^{A,a,\tau_2^j})^2}{(\sigma_1^{A,a,\tau_2}/\sigma_0^{A,a,\tau_2})(\sigma_0^{A,a,\tau_2})^2} \\
&= \kappa^{A(1)} \frac{(\sigma_1^{A,a,\tau_2^1}/\sigma_0^{A,a,\tau_2^1})\sum_{j=1}^{3}(\sigma_0^{A,a,\tau_2^j})^2}{(\sigma_1^{A,a,\tau_2}/\sigma_0^{A,a,\tau_2})(\sigma_0^{A,a,\tau_2})^2} \\
&= \kappa^{A(1)}
\end{aligned} \tag{29}$$

□

C.3 Perceived Option Values

Analogous to Equation (17), substituting the new expression σ^P (shown in Equation 24 with B taking the value P) into Equation (3), we obtain:

$$P_0^P = \Phi\left(\frac{\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y} + \tilde{\gamma}^P}{\sqrt{(\rho^P \tilde{\sigma}_1^{P,Y})^2 + (\rho_0^P \tilde{\sigma}_0^{P,Y})^2 - 2\kappa^P \rho^P \rho_0^P \tilde{\sigma}_1^{P,Y} \tilde{\sigma}_0^{P,Y}}}\right). \tag{30}$$

Parallel to Section 5.2.4, here we rewrite Equation (30) as a linear equation and explicitly allow for measurement error in expectations variables.

$$\Phi^{-1}(P_0^P)\tilde{\sigma}^{P,Y} + \delta^y = \frac{\bar{\gamma}^P}{\rho^P} + [\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y} + \delta^x]\frac{1}{\rho^P} + \frac{\tilde{\gamma}^P - \bar{\gamma}^P}{\rho^P}, \tag{31}$$

where $\tilde{\sigma}^{P,Y} \equiv \sqrt{(\tilde{\sigma}_1^{P,Y})^2 + (\theta^P \tilde{\sigma}_0^{P,Y})^2 - 2\kappa^P \theta^P \tilde{\sigma}_1^{P,Y} \tilde{\sigma}_0^{P,Y}}$, and $\theta^P \equiv \frac{\rho_0^P}{\rho^P}$. Similarly, we assume that the observed measure of $\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y}$ contains individual-specific classical measurement error δ^x and that the computed value of $\Phi^{-1}(P_0^P)\tilde{\sigma}^{P,Y}$ contains individual-specific classical measurement error δ^y.

To apply the measurement-error-robust approach detailed in Section 5.2.4 and Appendix E, we need to compute $\tilde{\sigma}^{P,Y}$ for each student. Note that $\tilde{\sigma}_1^{P,Y}$ and $\tilde{\sigma}_0^{P,Y}$ can be directly computed from the data. We impose the assumption that the perceived values of the ratio of signal strength, θ^P, and the correlation, κ^P, are equal to their actual counterparts, which have been estimated in Section C.2 ($\theta^A \equiv \frac{\rho_0^A}{\rho^A} = 0.55$ and $\kappa^A = 0.63$).

With $\tilde{\mu}_0^{P,Y}$ and $\tilde{\mu}_1^{P,Y}$ directly constructed from the data and $\Phi^{-1}(P_0^P)\tilde{\sigma}^{P,Y}$ computed as above, we consistently estimate $\frac{\bar{\gamma}^P}{\rho^P}$ and $\frac{1}{\rho^P}$ using the approach described in Section 5.2.4. The estimates of ρ^P and ρ_0^P are 0.55 and 0.29, respectively. Comparing $\rho^P = 0.55$ and $\rho_0^P = 0.29$ to $\rho^A = 0.51$ and $\rho_0^A = 0.28$ (Section C.2), we continue to find, as in Section 5, that students have quite accurate perceptions about the magnitude of uncertainty resolution. Then, as expected, the perceived amount of uncertainty resolution, σ^P, is equal to \$103,140, which is very close to its actual counterpart (\$96,780). The resulting average perceived option value and average perceived NCV are \$7,680 and \$155,590, respectively, which are almost identical to the average values computed in Section 5. Comparing these numbers to the actual analogs found in Section C.2, (\$21,020 and \$63,720), our main conclusion that students underestimate the option value and overestimate the net continuation value remains appropriate in this slightly modified learning environment.

D Robustness: Allowing for Learning about γ_1

In this appendix, we examine the implications of allowing students to also obtain relevant information about the non-pecuniary benefits associated with the graduation scenario, γ_1. In particular, we show that, under assumptions that are broadly consistent with the setting in Stinebrickner and Stinebrickner (2012, 2014b), our estimates of actual option values in Section 5.3 tend to be downward biased while our estimates of perceived option values in Section 5.3 remain consistent.

Recall that Section 4 shows that the option value is multiplicatively separable in a student's beliefs about the dropout probability P_0^B and the amount of uncertainty resolved in college σ^B. Since both actual and perceived values of P_0^B are obtained from the data in somewhat direct ways, we only need to examine whether our estimates of the actual and perceived σ^B tend to be consistent when students are also learning about γ_1. For the purpose of clarity, here we denote the estimates of actual and perceived ρ^B computed in Section 5.2 as $\underline{\rho}^A$ and $\underline{\rho}^P$, respectively, and denote the estimates of actual and perceived σ^B computed in Section 5.2 as $\underline{\sigma}^A$ and $\underline{\sigma}^P$, respectively.

The relevant new information Δ is given by:

$$
\begin{aligned}
\Delta &= (\bar{V}_1 - \bar{V}_0) - E_{t=t_0}(\bar{V}_1 - \bar{V}_0) \\
&= [E_{t=t^*}(Y_1) - E_{t=t_0}(Y_1)] + [E_{t=t^*}(\gamma_1) - E_{t=t_0}(\gamma_1)] \\
&\equiv \Delta Y_1 + \Delta\gamma_1.
\end{aligned} \tag{32}
$$

Motivated by Stinebrickner and Stinebrickner (2012, 2014b), we consider a case where, between t_0 and t^*, students resolve uncertainty about Y_1 and γ_1 through a common signal. For example, in their setting, grade performance is a signal that is found to influence both beliefs about earnings and the non-pecuniary benefits of school. In this case, both ΔY_1 and $\Delta\gamma_1$ are functions of this signal. Under a linearity assumption for the two functions, we have that $\Delta\gamma_1$ is proportional to ΔY_1. Let ΔY_1^B and $\Delta\gamma_1^B$ represent the student's beliefs at t_0 about ΔY_1 and $\Delta\gamma_1$, respectively. Then, we have $\Delta\gamma_1^B = \alpha(\Delta Y_1^B)$.[31] It implies that:

$$
\Delta^B = (1+\alpha)\Delta Y_1^B \text{ and } \sigma^B = (1+\alpha)std(\Delta Y_1^B) = (1+\alpha)\rho^B\tilde{\sigma}_1^{P,Y}, \text{ for } B = P, A. \tag{33}
$$

We first examine the consistency of our estimates of actual option values in Section 5.3. Recall from Section 5.2.3 that the actual fraction ρ^A is estimated using observed data on $\tilde{\sigma}_1^{P,Y}$ and $\tilde{\sigma}_1^{P^*,Y}$. Therefore, our estimates of ρ^A and $\rho^A\tilde{\sigma}_1^{P,Y}$ are consistent regardless of whether students are also resolving uncertainty about non-pecuniary benefits γ_1, i.e. $\underline{\sigma}^A$ consistently estimates $\rho^A\tilde{\sigma}_1^{P,Y}$. In the likely case where $\alpha > 0$, the actual value of σ would be greater than $\rho^A\tilde{\sigma}_1^{P,Y}$.[32] Thus, $\underline{\sigma}^A$ underestimates the actual value σ^A, which implies that, for each student, our estimate of actual option value reported in Section 5.3 underestimates its true value.

We then examine the consistency of our estimates of perceived option values in Sec-

[31]Both ΔY_1^B and $\Delta\gamma_1^B$ have a mean of zero, by construction.

[32]This is consistent with a scenario where the common factor is grade performance; Having a high realized grade would tend to positively influence a student's perceptions about both the pecuniary and non-pecuniary benefits associated with the graduation scenario.

tion 5.3. Allowing students to learn about non-pecuniary benefits associated with the graduation scenario leads to a modification of Equation (17).

$$P_0^P = \Phi\left(\frac{\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y} + \tilde{\gamma}^P}{(1+\alpha)\rho^P \tilde{\sigma}_1^{P,Y}}\right). \tag{34}$$

Consequently, the main estimation equation (Equation 19) can be modified as follows:

$$\Phi^{-1}(P_0^P)\tilde{\sigma}_1^{P,Y} + \delta^y = \frac{\bar{\gamma}^P}{(1+\alpha)\rho^P} + [\tilde{\mu}_0^{P,Y} - \tilde{\mu}_1^{P,Y} + \delta^x]\frac{1}{(1+\alpha)\rho^P} + \frac{\tilde{\gamma}^P - \bar{\gamma}^P}{(1+\alpha)\rho^P}. \tag{35}$$

The only difference between Equation (19) and Equation (35) is that $(1+\alpha)\rho^P$ shows up in Equation (35) at places where ρ^P shows up in Equation (19). Therefore, $\underline{\rho}^P$ consistently estimates $(1+\alpha)\rho^P$, which implies that $\underline{\sigma}^P$ consistently estimates σ^P as well. Hence, for each student, the estimate of perceived option value reported in Section 5.3 consistently estimates its true value.

E Measurement Error Correction

E.1 Estimating the Variance of δ^x

Appendix B describes how to obtain measures of $\tilde{\mu}_1^{P,Y}$ and $\tilde{\mu}_0^{P,Y}$ using our measures of $\tilde{\mu}_1^{P,a}$ and $\tilde{\mu}_0^{P,a}$. Let $\delta_s^{P,a}$ denote the individual-specific measurement error that is present in our measure of $\tilde{\mu}_s^{P,a}$. Equation (21) implies that $var(\delta^x)$ is given by:

$$var(\delta^x) = var(\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*}\delta_1^{P,a} - \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*}\delta_0^{P,a}). \tag{36}$$

Recall that the unconditional earnings expectations questions in the BPS were asked for three specific ages a: the first year after graduation (age 23), age 28, and age 38, and for both schooling scenarios: graduation ($s=1$) and dropout ($s=0$). The linear interpolation assumption we employed to impute $\tilde{\mu}_s^{P,a}$ for other ages implies that $\delta_s^{P,a}$ is a linear combination of a subset of $\{\delta_s^{P,23}, \delta_s^{P,28}, \delta_s^{P,38}\}$ for all a.

We further assume that (1) the distribution of measurement error is the same for each of the six unconditional earnings expectations questions; (2) measurement errors are uncorrelated across schooling scenarios s, but are perfectly correlated within schooling scenarios.[33] Under these assumptions, we have:

$$
\begin{aligned}
var(\delta^x) &= var(\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} \delta_1^{P,a} - \sum_{a=t^*}^{\bar{T}} \beta^{a-t^*} \delta_0^{P,a}) \\
&= var(\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*} \delta_1^{P,28}) + var(\sum_{a=t^*}^{\bar{T}} \beta^{a-t^*} \delta_0^{P,28}) \\
&= var(\delta_1^{P,28})[(\sum_{a=\bar{t}}^{\bar{T}} \beta^{a-t^*})^2 + (\sum_{a=t^*}^{\bar{T}} \beta^{a-t^*})^2]. \qquad (37)
\end{aligned}
$$

Following the method developed in Gong, Stinebrickner and Stinebrickner (2019), we estimate the variance of the measurement error contained in students' reported value of $\tilde{\mu}_1^{P,28} \equiv E_{t=t_0}(w_1^{P,28})$ for the 2001 cohort, $\delta_1^{P,28}$. The approach takes advantage of the fact that the BPS includes two separate sets of expectations questions that can be used to compute $\tilde{\mu}_1^{P,28}$. The difference between the two computed values of $\tilde{\mu}_1^{P,28}$ provides evidence about the magnitude of measurement error. The estimate of $var(\delta_1^{P,28})$ is 109.54 (earnings measured in \$1,000 units). Using Equation (37), we estimate that $var(\delta^x)$ is 67236 (earnings measured in \$1,000 units).

E.2 ME Correction Formula

Let vector $\boldsymbol{z}_i$ denote the independent variables that are accurately measured and x_i denote the independent variable that is measured with classical measurement error η_i. We allow the variance of η_i to depend on observable $\boldsymbol{g}_i$ and denote this variance $\sigma^2_{ME}(\boldsymbol{g}_i)$. Let $\tilde{x}_i = x_i + \eta_i$ denote the measured value of x_i. Then, the dependent variable y_i is given by:

$$
\begin{aligned}
y_i &= \boldsymbol{z}_i'\boldsymbol{a} + bx_i + \epsilon \\
&= \boldsymbol{z}_i'\boldsymbol{a} + b\tilde{x}_i + (\epsilon - b\eta_i). \qquad (38)
\end{aligned}
$$

By construction, $\tilde{x}$ and $\epsilon - b\eta_i$ are correlated. Hence, the OLS estimator is biased. To correct for this bias, we notice that:

$$
E\left[(y_i - (\boldsymbol{z}_i'\boldsymbol{a} + b\tilde{x}_i))\begin{pmatrix} \boldsymbol{z}_i \\ \tilde{x}_i \end{pmatrix} + \begin{pmatrix} \boldsymbol{0} \\ b\sigma^2_{ME}(\boldsymbol{g}_i) \end{pmatrix}\right] = E\left[(\epsilon - b\eta_i)\begin{pmatrix} \boldsymbol{z}_i \\ \tilde{x}_i \end{pmatrix} + \begin{pmatrix} \boldsymbol{0} \\ b\sigma^2_{ME}(\boldsymbol{g}_i) \end{pmatrix}\right] = \boldsymbol{0}. \qquad (39)
$$

Equation system (39) has the same number of equations and parameters which are

[33] Assumption (2) captures the notion that factors that affect students' beliefs about earnings under the college alternative ($s = 1$) are likely different from those affecting students' beliefs about earnings under the non-college alternative ($s = 0$).

equal to the number of observables. Hence, it can be estimated using the Method of Moments, i.e., the estimator of $\begin{pmatrix} \boldsymbol{a} \\ b \end{pmatrix}$ is the solution to the sample analog of the moment conditions defined by Equation (39). It is easy to show that this estimator has an easy-to-implement matrix-form expression. Letting $\boldsymbol{c}$ denote $\begin{pmatrix} \boldsymbol{a} \\ b \end{pmatrix}$ and $\boldsymbol{q}_i$ denote $\begin{pmatrix} \boldsymbol{z}_i \\ \tilde{x}_i \end{pmatrix}$, we have:

$$\hat{\boldsymbol{c}} = \left[Q'Q - \begin{pmatrix} \mathbf{0} & 0 \\ 0 & \sum_i \sigma^2_{ME}(\boldsymbol{g}_i) \end{pmatrix} \right]^{-1} Q'Y, \tag{40}$$

where and Y and Q are the matrices of y_i and $\boldsymbol{q}_i$, respectively.