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IN AN IMPERFECT MARKET SETTING

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IMPERFECT MARKET SETTING

It has been well known from the time of Edgeworth [10] that growth
can be immiserizing, that is, economic expansion can result in a loss of welfare,
if it leads to a large deterioration in the terms of trade of a growing country
enjoying monopoly power in trade [3], [16], [18]. Recently, however, Johnson
[14] and Bhagwati [4] have shown that the phenomenon of immiserizing growth
can occur even in a small country with no monopoly power in trade (so that its
terms of trade remain unchanged), provided that it suffers from market distor-
tions such that there is a divergence between private and social opportunity
costs or, what is the same thing, the commodity-price ratio is not equal to
the marginal rate of transformation. The result is of great interest to a small
underdeveloped economy where product or factor markets are likely to be char-
acterized by distortions.

The purpose of this paper is to provide a rigorous demonstration of,
what we call, the Bhagwati-Johnson paradox, namely the phenomenon of immiserizing
growth with constant terms of trade. The consequences of two types of economic
growth--one stemming from factor growth and the other from technical improve-
ments--are analyzed for changes in national income in the presence of an inter-
industry wage-differential, and necessary conditions for the Bhagwati-Johnson
paradox are obtained. Furthermore, it is shown that in some cases, the
presence of market distortions may actually raise the rate of economic growth
at constant terms of trade.

I. The Model with the Wage-Differential

Unless otherwise specified, the following assumptions are made throughout
the analysis:
There are two industries, called 1 and 2, with outputs denoted by $X_1$ and $X_2$, and two factors of production, capital ($K$) and labor ($L$). There is perfect competition in the product markets, but factor markets are imperfect in the sense that although rates of return on capital are the same, the wage rates in the two industries differ. Full employment, perfect internal factor mobility, constant returns to scale and diminishing returns to factor proportions, inelastic factor supplies, and constancy of the foreign terms of trade are also assumed.

The two production functions are:

(1) $X_1 = L_1 f_1 (k_1)$
(2) $X_2 = L_2 f_2 (k_2)$

where $K_i$ and $L_i$ are the capital and labor inputs, and $k_i = K_i / L_i$ is the capital/labor ratio in the $i^{th}$ industry. The marginal product of capital in the $i^{th}$ industry is given by $f'_i$ and that of labor by $f_i - k_i f'_i$. Let commodity 1 serve as the numeraire, and let $p$ be the price of the second commodity in terms of the first. Let $r$ be the rate of return for capital, and $w_i$ the wage rate.

Then factor rewards in terms of the first commodity are:

(3) $r = f'_1 = pf'_2$
(4) $w_1 = (f_i - k_i f'_i) = \alpha p (f_2 - k_2 f'_2) = \alpha w_2$;

where $\alpha$ is a positive constant less or greater than unity. Initially, $p$ is assumed to be unity. With full employment,

(5) $L_1 + L_2 = \bar{L}$
(6) $L_1 k_1 + L_2 k_2 = \bar{K}$.

Let $Y$ be national income in terms of the first commodity. Then

(7) $Y = X_1 + pX_2$. 
With this last equation, the production side of our model is complete. The demand side is not needed, because we assume that \( p \), which also represents the free trade terms of trade, is given to the small home country.

A. **The Case of Factor Growth:** Suppose growth occurs in the small country under consideration due to an exogenous increase in the supply of one of the two factors. By differentiating the system of equations (1)-(7), we can obtain the effects of such growth on the two outputs and national income. Since \( p \) is constant, the factor rewards and hence the capital/labor ratios in both industries will also be constant. Differentiating (1) and (2) with respect to, say, \( K \), we have:

\[
\frac{dX_i}{dK} = f_i \frac{dL_i}{dK}, \quad i = 1, 2
\]

\[
\frac{dL_1}{dK} = \frac{1}{k_2 - k_1}
\]

\[
\frac{dL_2}{dK} = \frac{1}{k_2 - k_1}
\]

Substituting (9) and (10) in (8), we have:

\[
\frac{dX_1}{dK} = -f_1 \frac{1}{k_2 - k_1}
\]

\[
\frac{dX_2}{dK} = \frac{1}{k_2 - k_1}
\]

Differentiating (7) with respect to \( K \), and utilizing (11), (12), (3) and (4), then yields:

\[
\frac{dY}{dK} = r \left[ 1 + \frac{\omega_2 (1-\alpha)}{k_2 - k_1} \right],
\]

where \( \omega_2 \) is the wage/rental ratio in the second industry. Equation (13) shows
the effect of an increase in the supply of capital alone on national income at constant terms of trade. The following results can now be derived:

i) If there is no wage differential, i.e., if \( \alpha = 1 \), the rise in the supply of capital raises national income at constant terms of trade. The increase in national income equals the rise in the capital stock multiplied by the rate of return on capital.

ii) If \( k_2 > k_1 \) and \( \alpha < 1 \), that is to say, \( w_1 < w_2 \), then a rise in the supply of capital still raises national income, and this rise in income is even greater than the case would be in the absence of the wage differential. In other words, when the wage differential works against the second commodity, the conventional result that a rise in the supply of a factor will raise national income if terms of trade are constant is reinforced.

On the other hand, if \( k_2 < k_1 \), the traditional result is reinforced if \( \alpha > 1 \), or if the wage differential is unfavorable to the first commodity.

iii) If \( k_2 > k_1 \) and \( \alpha > 1 \), the increase in national income is smaller than would be the case in the absence of the wage differential. Equation (13) also leads to the paradoxical possibility of national income declining as a result of the rise in the stock of capital. Specifically, the Bhagwati-Johnson paradox of immiserizing growth with constant terms of trade requires that

\[
\frac{w_2(1-\alpha)}{k_2 - k_1} < 0,
\]

or

\[
\alpha > 1 + \frac{k_2 - k_1}{w_2}. \tag{14}
\]

The following theorem may now be derived:
Theorem 1: The Bhagwati-Johnson paradox requires that the output of the commodity suffering from the wage-differential decline absolutely as a result of growth. In other words, economic growth should be ultra-biased against the industry which suffers from the wage differential.

This theorem can be proved by examining (11) and (12). If $k_2 > k_1$, the output of $X_1$ declines and that of $X_2$ rises as a result of the rise in the supply of capital alone. In other words, economic growth is ultra-biased against the first commodity, when the second commodity is relatively capital-intensive. From (14) we know that with $k_2 > k_1$, $\alpha$ should exceed one if national income is to decline as a result of the rise in the capital stock, which means that the wage differential should be unfavorable to the first industry. Similarly, if $k_2 < k_1$, then the output of $X_1$ rises and that of $X_2$ declines when capital stock increases, and from (14), $\alpha < 1$ if national income is to decline, which means that the wage-differential is unfavorable to the second commodity.

In other words, in both cases, immiserizing growth at constant terms of trade requires the wage-differential to be against the commodity whose output declines as a result of the rise in the supply of capital. This proves theorem 1.

Further comprehension of theorem 1 may be achieved through simple diagrams. Consider Figures 1 and 2, which are drawn under the assumption that $\alpha > 1$, or that the wage-differential is unfavorable to the first commodity; $TT'$ and $GG'$ are, respectively, the transformation curves before and after a rise in the supply of capital alone has occurred. In Figure 1, $k_2 > k_1$, and the original production point, $P$, may shift to $P'$ or $P''$, both of which display a lower output for $X_1$ at the constant terms of trade given by the slope of lines $FP'$ and $FP''$, both of which are parallel to $FP$. If the output of $X_1$ declines only by $PR$ and the new production point is given
\[ a > 1, \ k_2 > k_1 \]
\[ \alpha > 1, \ k_2 < k_1 \]
by \( P' \), national income, measured in terms of \( x_1 \), rises from OA to OA'. However, if the new production point is given by \( P'' \), so that the output of \( x_1 \) declines by PR', national income in terms of \( x_1 \) actually declines from OA to OA''. However in Figure 2, where \( k_2 < k_1 \), national income in terms of \( x_1 \) must rise above OA. This is because the new production point here will show a rise in the output of \( x_1 \) and a decline in the output of \( x_2 \) at constant terms of trade. One such point is given by \( P' \), which in Figure 2 shows that national income in terms of \( x_1 \) has risen to OA'. In other words, immiserizing growth at constant terms of trade is possible only in terms of conditions described in Figure 2, where the output of the first commodity which suffers from the wage-differential declines as a result of an increase in capital stock.

B. The Case of Technical Progress: So far we have assumed that economic growth in the country under consideration takes place as a result of an increase in the supply of a factor of production. We now consider the case where growth occurs because of technical improvements instead of increases in factor supplies. For simplicity, consider the case where technical progress occurs in only one industry, say, the industry producing the first commodity, and is denoted by \( \beta \), where \( \beta \) stands for the incidence of Hicks' neutral technical progress which increases the marginal productivity of both factors in the same proportion. The first four equations of the model are now rewritten as follows:

\[
(1*) \quad x_1 = \beta l_1 f_1 (k_1) \\
(2) \quad x_2 = l_2 f_2 (k_2) \\
(3*) \quad \beta f_1 = pf_2
\]
(4*) \( \beta(f_1-k_1f_1') = \alpha p (f_2-k_2f_2') \),

where \( \beta = 1 \) initially. The remaining equations of the model are unchanged.

Differentiating (3*) and (4*) totally with respect to \( \beta \), remembering that \( p \) is constant and that \( \beta = p = 1 \) initially, we have:

\[
\frac{dk_1}{d\beta} = \frac{\alpha f_2}{f''_1(k_1-\alpha k_2)}
\]

\[
\frac{dk_2}{d\beta} = \frac{f_1}{f''_2(k_1-\alpha k_2)}
\]

Differentiating (1*), (2), (5), and (6) and utilizing (15) and (16), we obtain:

\[
\frac{dx_1}{d\beta} = L_1f_1 - \left[ \frac{\alpha L_1 f_2(w_1+rk_2)}{f''_1(k_1-k_2)(k_1-\alpha k_2)} + \frac{L_2 f_1^2}{f''_2(k_1-k_2)(k_1-\alpha k_2)} \right]
\]

\[
\frac{dx_2}{d\beta} = \frac{\alpha L_1 f_2^2}{f''_1(k_1-k_2)(k_1-\alpha k_2)} + \frac{L_2 f_1 w_1+rk_1}{f''_2(k_1-k_2)(k_1-\alpha k_2)}
\]

Differentiating (7) and utilizing (17) and (18), we get,

\[
\frac{dy}{d\beta} = L_1 f_1 + \frac{w_2(1-\alpha)}{(k_1-k_2)(k_1-\alpha k_2)} \left( \frac{\alpha L_1 f_2}{f''_1} + \frac{L_2 f_1}{f''_2} \right)
\]

Given our assumption of diminishing returns to factor proportions, \( f''_1 < 0 \), \( i=1,2 \). From (17) and (18) we can obtain the effect of a change in \( \beta \) or technical progress in \( x_1 \) alone on the outputs of both commodities. It is clear that if \( (k_1-k_2) \) and \( (k_1-\alpha k_2) \) have the same signs, \( dx_1/d\beta > 0 \) and \( dx_2/d\beta < 0 \). In other words, if the original factor-intensities do not
get reversed because of the presence of the wage-differential, neutral technical progress in $X_1$ alone at constant terms of trade, will raise the output of $X_1$ and lower the output of $X_2$. This is a well known result which has been previously derived in the absence of the wage-differential by Johnson [15], Bhagwati [5], Corden [9] and Findlay and Grubert [11], and we have shown that the result remains unchanged if the factor-intensities do not get reversed in spite of the wage differential. However, if the presence of the wage-differential results in a factor-intensity reversal, that is, if $(k_1-k_2)$ and $(k_1-\alpha k_2)$ do not have the same sign, then from (18) it is clear that $dX_2/d\beta > 0$, whereas from (17), $dX_1/d\beta$ may be positive or negative, because even though the second term will now be negative, the first term is positive. Thus we may conclude that neutral technical progress in $X_1$ alone may raise the output of both commodities, in which case national income will unambiguously rise, or raise the output of $X_2$, the non-progressive commodity, and lower the output of $X_1$, the progressive commodity.

Let us now proceed to the effect of a change in $\beta$ on national income given by (19). There are four possibilities:

i) There is no wage differential, $(\alpha=1)$; here $dY/d\beta = L_1 f_1 > 0$, so that national income at constant terms of trade rises as a result of technical progress in $X_1$ alone.

ii) The wage-differential is against the first industry, i.e., $\alpha > 1$. In this case $dY/d\beta > 0$, if the original factor intensities are not reversed, so that $(k_1-k_2)(k_1-\alpha k_2) > 0$. Moreover, technical progress in $X_1$ not only increases national income but also raises it more than the case would be in the absence of the wage differential.
iii) The wage-differential is against the second industry, i.e., \( \alpha < 1 \). Here if the original factor-intensities do not get reversed, the rise in national income at constant terms of trade is smaller than would be the case in the absence of the wage-differential.

iv) If \( \alpha > 1 \), but the original factor-intensities are reversed due to the wage differential, equation (19) shows the possibility of immiserizing growth at constant terms of trade. On the other hand, if \( \alpha < 1 \), immiserizing growth may occur in the absence of factor-intensity reversal. It can be further seen that in both of these cases, growth has to be ultra-biased against the commodity which suffers from the wage-differential.

II. Concluding Remarks

In the preceding analysis we have shown that the necessary conditions for the Bhagwati-Johnson paradox of immiserizing growth at constant terms of trade is that growth be ultra-biased against the industry suffering from the wage-differential. If growth is ultra-biased against the commodity which is favored by the wage-differential, national income rises even more than the case would be in the absence of the wage differential. If growth results from an increase in the supply of a factor, the conventional result, derived originally by Rybczynski [19], that the output of the commodity using the growing factor intensively rises at the expense of the output of the other commodity when terms of trade are constant is valid even when factor markets are distorted. However, if growth results from technical progress, the traditional result that the output of the commodity enjoying Hicks neutral technical progress, at constant terms of trade, rises at the expense of the output of the other commodity may not hold if the original
factor-intensities are reversed as a result of the wage-differential. In the latter case, it is possible that the output of both commodities may increase.
Footnotes

1 Other analyses of market distortions in the context of trade theory include Hagen [12], Bhagwati and Ramaswami [6], Bhagwati, Ramaswami and Srinivasan [8], Kemp and Herberg [17], Johnson [13], Batra and Pattanaik [1], and Batra and Scully [2], among others.

2 For the sake of diagrammatical simplicity, the transformation curves are drawn concave to the origin, although it has been recently shown by Kemp and Herberg [17] and Bhagwati and Srinivasan [7] that in the presence of the wage differential, the transformation curve may become locally or globally convex to the origin. It may be noted that our argument does not depend on the shape of the transformation curve.
References


