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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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FOREIGN EXCHANGE CONTROLS IN A BLACK MARKET ECONOMY

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I. INTRODUCTION

The use of foreign exchange controls is widespread in today's world economy. The artificial scarcity of foreign exchange induced by controls often drives the value of foreign exchange significantly above the price at which it can be officially purchased. This divergence between the user value of foreign exchange and its official price creates incentives for arbitragers to emerge who can divert funds away from the low cost official foreign exchange market to higher value users. Thus in economies with foreign exchange controls in effect black markets more often than not emerge. Consequently, it seems likely a meaningful analysis of the effect of foreign exchange controls on an economy's general equilibrium should formally incorporate a black market into the modelling apparatus.

An examination of the impact of foreign exchange controls in a black market setting is undertaken here. The investigation utilizes the cash-in-advance general equilibrium model developed by Helpman (1981) to evaluate alternative exchange rate institutions. This line of analysis has been chosen for three basic reasons. To begin with, in order to study foreign exchange controls a general equilibrium model of the economy which provides a careful articulation of the role money would seem desirable. This would seem to be important if, as is done here, a discussion of the impact of exchange controls on monetary phenomena such as the balance of payments or the exchange rate is to be undertaken.

Next, the choice-theoretic nature of the approach highlights how the imposition of foreign exchange controls limits the opportunity sets facing
private agents and affects their decision-making. The incentive for a black market to develop in order to evade the restrictions placed on private agents' trading opportunities, and the nature of its operation, can be clearly seen. It is then easy to infer the general equilibrium ramifications of foreign exchange controls in the presence of a black market. In the paper the theoretical underpinnings of a black market for foreign currency are studied, and the effect on the incentive structure facing individuals by the presence of such markets is discussed. The consequences of foreign exchange controls for the macro economy's general equilibrium, or for such variables as the domestic terms of trade, the level of imports, the trade balance, and the balance of payments are addressed. Since the model's foundations rest directly on specifications of individuals' tastes and endowments the welfare implications of exchange controls and black market are readily assessed, as is done.

Third, the model employed is an intertemporal one which has its merits. Recent theorizing in international finance [see for instance, Sachs (1983), Svensson and Razin (1983), and Greenwood (1983)] has viewed the trade balance as reflecting optimizing agents' consumption-saving choices. Exchange controls are often imposed temporarily to overcome balance of payments problems. This case is analyzed in the text and introduces a fundamental asymmetry across time, which presumably affects agents' consumption-savings decisions and which is best captured in a dynamic model.

Finally, before proceeding on to the formal model it should be mentioned that foreign exchange controls in the absence of a black market have been discussed recently within the context of cash-in-advance general equilibrium models by Greenwood and Kimbrough (1985) and Kowalczyk (1985). Also, in the real trade literature smuggling has been addressed within the standard static
two-sector real trade model by Bhagwati and Hansen (1973) and Falvey (1978). As has been mentioned, where there are foreign exchange controls present more than often there are black markets also. Thus, the novelty of the current work lies in formally introducing a black market into the recent line of explicitly monetary international finance models dealing with the consequences of foreign exchange controls. This synthesis seems important to undertake given the prevalence of foreign exchange controls in black market settings.

II. THE ECONOMIC ENVIRONMENT

Imagine a small open economy with a life span of two periods that has adopted a fixed exchange rate system. The government of this economy has imposed foreign exchange controls. Private agents have the option, however, of evading the government's foreign exchange restrictions by purchasing foreign currency illegally on a black market. The economy is inhabited by a representative agent whose goal is to maximize his lifetime utility, \( U(\cdot) \), as specified by

\[
U(\cdot) = \sum_{t=1}^{2} \rho^{t-1} \left[ U(X_t) + V(Z_t + \frac{Z}{D}) \right]
\]

where \( \rho \) is his subjective discount factor, and \( X_t \) and \( Z_t + \frac{Z}{D} \) are his period-\( t \) consumption of an exported and imported good. For future reference \( Z_t \) denotes the quantity of the imported good purchased domestically in period \( t \), and \( \frac{Z}{D} \) from foreign sources.

The representative agent has three sources of income. First, in each period \( t \) the individual is endowed with a certain quantity of the exported good \( X_t \), and the imported good, \( Z_t \). The exported good sells in world markets in period \( t \) at the world terms of trade, \( p_{st}^t \), by which is meant the relative
price of imports in terms of exports. Second, the agent earns profits from
the ownership of an illegal firm which sells foreign exchange on the black
market. The profits of this firm in period t amount in real, or exported
denominated, terms to \( v^t \). Third, the agent receives a nominal transfer
payment, \( T^t \), from the domestic government in each period t.

Domestic residents can also freely participate on an international bond
market. In the first period the representative agent can purchase or sell
real bonds which are denominated in terms of the exported good and pay the
fixed internationally determined real rate of return, \( r^* \). For example, if
during the first period the agent purchased one unit of real bonds, he would
receive the equivalent of \( 1 + r^* \) units of the exported good during period two.

In the model, the individual must use currency to purchase goods.
Furthermore, domestic currency must be used to acquire domestic output while
foreign currency is required to buy foreign produced goods. Thus, if in
period t the agent purchases from domestic sources \( X^t \) units of the exported
good and \( Z_D^t \) units of the imported good he must buy them using currency from
his current holdings of domestic money, \( M^t \). Likewise, in this period his
purchases from abroad of the imported good, \( Z^t \), must be bought using currency
from his holdings of foreign money, \( M^*^t \).

A time profile of the individual's life in period t will now be given so
as to highlight the circulation of money in the model. The sequencing of
monetary transactions outlined here is similar to that adopted by Helpman
(1981). The representative agent enters period t with a certain amount of
domestic or foreign money to spend left over from the previous period, \( t-1 \).
Now, at the beginning of period t the individual receives in domestic currency
the income from his sales of the imported and exported good during the
previous period. This amounts to \( P^t X^{t-1} + P^t Z^{t-1} \) where \( P^t \) and \( P^*^{t-1} \)
represent the domestic nominal prices of the exported and imported goods during $t-1$. At this time the agent also earns a dividend payment in the nominal amount $p_{t-1}v_{t-1}$ arising from the operation of his black market firm during the previous period. Finally, the individual also gets a nominal transfer payment from the government in the amount $T^t$.

After receiving this cash the individual then goes to the international bond cum foreign exchange market and redeems the bonds he purchased during the previous period. These bonds are now worth $(1+r^{xt-1})p^tb^{t-1}$ units of domestic currency. The individual then purchases new export denominated bonds worth $p^tb^t$ in domestic currency terms. His resulting new holdings of cash are then allocated between holding domestic and foreign currency in the magnitudes $M^t$ and $M^{xt}$. On the official foreign exchange market a unit of foreign currency can be bought or sold for $\bar{e}$ units of domestic currency where $\bar{e}$ is the (fixed) official foreign exchange rate. Since foreign exchange controls are in effect, the maximum amount of foreign currency the agent can legally acquire is $M^{xt}$. Any purchases of foreign exchange over and above this fixed quota must be effected illegally on the black market. The black market domestic currency price of foreign exchange in period $t$ is denoted by $s^t$.

During the remainder of the period, the individual uses his holding of domestic and foreign currency to purchase his consumption quantities of the exported good, $X^t$, and the imported good, $Z^t + Z_D^t$. He then will enter period $t+1$ with $M^t - P^tX^t - P^tI^tD$ units of domestic currency and $M^{xt} - P^{xt}I^tZ^t$ units of foreign currency where $P^t_I$ is the foreign nominal price of the imported good, and the whole process begins again.

As was previously mentioned, there is a government in this economy. Its sole purpose is to maintain the official exchange rate at the fixed level, $\bar{e}$,
to provide transfer payments to the representative agent, and (attempt) to
regulate the amount of foreign cash balances held by individuals. From the
individual's perspective these transfer payments are unrelated to his holdings
of domestic cash balances.

Finally, note that the representative agent is free to sell whatever
quantities of the export good he chooses either domestically or
internationally as long as the sales are effected using domestic currency.
Now assuming that foreign residents can acquire as much domestic currency as
they desire at the going foreign exchange rate, $\bar{e}$, the law of one price
must hold for the exported

$$P^t = \bar{e}^t P^t$$

$$\forall t = 1, 2$$

(2.2)

good, where $P^t$ is defined as the foreign currency price of the exported good.
Due to the presence of foreign exchange controls the law of one price does not
have to hold—at the official exchange rate—for imports. This occurs because
domestic residents are not free to acquire as much foreign exchange, which is
required to import goods, at the official exchange rate as they desire. This
fact will be returned to later on.

III. THE AGENT'S OPTIMIZATION PROBLEM

The agent's goal in life is to attain the highest possible level of
welfare in the above environment. The mathematical transliteration of this
goal is to solve the following maximization problem where the choice variables
are $x^1, x^2, z^1, z^2, z^{1D}, z^{2D}, m \equiv m^1 / P^1, m^2 \equiv m^2 / P^2, m^{*1} \equiv m^{*1} / P^{*1}, m^{*2} \equiv m^{*2} / P^{*2}$
and $b^1$:

$$\begin{align*}
\text{Max} & \quad U(x^1) + V(z^1 + z^{1D}) + \rho[U(x^2) + V(z^2 + z^{2D})] \\
\text{s.t.} & \\
\end{align*}$$
\[ \begin{align*}
1 & \sim \ast_1 & 1 & \sim \ast_1 \\
m + m + (s \div s)(m - m) + b & = r & \quad (3.1) \\
2 & \sim \ast_2 & 2 & \sim \ast_2 \\
m + m + (s \div s)(m - m) - (P'P)x + (P'P)Z + v & = r & \quad (3.2) \\
& & 1 & \sim \ast_2 & 1 & \sim \ast_2 & 1 & \sim \ast_2 \\
& & (P'P)x - (P'P)Z + (P'P)x & = r & \quad (3.3) \\
& & & \quad & (P'P)x & = r & \quad (3.4) \\
\end{align*} \]

where \( \ast_t \equiv \frac{N^t}{P^t} \) and \( \tau_t \equiv \frac{\tau_t}{P^t} \) (recall that \( P^t \equiv \frac{P^t}{P^t} \)).

The first two constraints, (3.1) and (3.2), are the individual's first- and second-period budget constraints. The next two set of equations, (3.3) and (3.4), are agent's cash-in-advance constraints in each period and reflect the fact that the individual must use domestic currency to purchase the home produced good and foreign currency to purchase the imported one.

In the framework used here money is required by the exchange mechanism in order to effect consumption purchases. Agents can choose whether to hold money as a store of value, however, independently of their consumption decisions. This portfolio decision will be made so as to maximize wealth, and hence the asset (or assets) with the highest real return will serve as a store of value. To see the issues involved here, define \( \pi_t \) as the domestic rate of inflation in period \( t \) and \( \pi^* \) as the foreign rate of inflation so that \( \pi_t = (P_{t+1}^t - P_t^t) / P_t^t \) and \( \pi^* = (P^*_{t+1} - P^*_t) / P^*_t \). As long as \( \pi_t \) and \( \pi^* \) are both greater than \(-\pi^* / (1 + \pi^*)\) the period-\( t \) cash-in-advance constraints will hold as strict equalities. That is, so long as the domestic and foreign economies are inflating faster than the optimum quantity of money prescribes, bonds will dominate money as an abode of purchasing power. This condition is assumed to hold in the remainder of the analysis and hence (3.3) and (3.4) are treated as equalities.
By undertaking the maximization routine involved, the following set of first-order conditions can be obtained:

\[ V(Z + Z_D) = [1 + (s - \bar{e}) / \bar{e}] p^* U(X) \]  
\[ = \left( \frac{P}{P^*} \right) U(X) \]  
\[ V(Z^2 + Z_D^2) = [1 + (s - \bar{e}) / \bar{e}] p^{*2} U(X^2) \]  
\[ = \left( \frac{P}{P^*} \right)^2 U(X^2) \]  
\[ U(X^1) = (1 + r^*) p U(X^2) \]  

(3.5)  
(3.6)  
(3.7)

To begin with, equation (3.5) describes how the agent should optimally divide his first-period expenditure on consumption between imported and exported goods. In an economy with both foreign exchange restrictions in place and a functional black market operating during the first period a wedge in the amount \([1 + (s^1 - \bar{e}) / \bar{e}]\) separates the first-period marginal rate of substitution between import and export consumption, \(V_1(Z^1 + Z_D^1) / U_1(X^1)\), from equality with the world terms of trade \(p^{*1}\). It is easy to see that in the foreign exchange controlled-cum-black market economy \([1 + (s^1 - \bar{e}) / \bar{e}] p^{*1}\) is the effective domestic relative price for the imported good so that \(P / P = (1 + 1 / 1) (s - \bar{e}) / \bar{e}p^{*1}\). Recall that in order to purchase an additional unit of the imported good the agent must first acquire an additional \(P_1^{*1}\) unit of the foreign currency. Since there are (quasi) effective foreign exchange controls in place this must be bought on the black market for a domestic currency price of \(s P^*\). In real terms this is worth \(1 / 1 (s / \bar{e}) p^{*1}\). In real terms this is worth \([1 + 1 / 1 (s - \bar{e}) / \bar{e}] p^{*1}\) units of the exported good. Note that the term \([\left( \frac{P}{P^*} \right) (s / \bar{e}) p^{*1}\)
measures the black market premium for foreign exchange and operates to raise the domestic relative price for imports above the world levels in exactly the same manner as a tariff would.

Equation (3.6) shows that a similar condition governs the optimal division of second-period expenditure between imports and exports. Finally, (3.7) is the familiar intertemporal efficiency condition characterizing the agent's consumption-savings decision.

The agent's optimization problem implies that his compensated demand functions for the imported and exported goods will have the following forms:

\[
Z + Z = Z^D (p (1+\eta), p (1+\eta)/(1+r), 1/(1+r), U) + Z
\]

\[
Z + Z = Z^D (p (1+\eta), p (1+\eta)/(1+r), 1/(1+r), U) + Z
\]

\[
X = X^D (p (1+\eta), p (1+\eta)/(1+r), 1/(1+r), U)
\]

\[
X = X^D (p (1+\eta), p (1+\eta)/(1+r), 1/(1+r), U)
\]

where \(\eta^t \equiv (s^t - e)/e\) is the (proportionate) black premium on foreign exchange. The sign under an argument in one of the demand functions shows the sign of the partial derivative of that demand function with respect to the argument in question. The sign pattern shown obtains from separability in the individual's utility function (2.1). It is needless to say that the agent's level of welfare, \(U\), in general equilibrium is dependent on such things as his endowment of the export and import goods, the terms of trade, the world real interest rate, the extent of foreign exchange controls and the cost of operating a black market. The nature of this dependence is discussed in fuller detail later on.
IV. THE BLACK MARKET FOR FOREIGN EXCHANGE

As was previously mentioned, the agent is free to purchase on a black market any desired holdings of foreign exchange over and above the fixed legal quota allocated to him by the government. The black market is run by a representative firm—dubbed the black marketeer—whose goal is to maximize its profits, \( v \). The black marketeer has the ability to channel foreign currency away from the official foreign exchange market to the black market. This diversion however can only be done at increasing real resource cost.

Specifically, in terms of the exported good, the period-\( t \) real resource costs, \( \phi^t \), is given by \( \phi^t = \phi(m^t/p^t - \tilde{m}^t/p^t) \) where \( \phi(\cdot) \) is a positive convex function with \( \phi(0) = \phi_1(0) = 0 \). In other words, in order to divert \( m^t/p^t - \tilde{m}^t/p^t \) units of foreign currency—denominated in terms of imports—away from the official to the black market the black marketeer must purchase \( \phi(m^t/p^t - \tilde{m}^t/p^t) \) units of the exported good to use as an input in his production process. The black market production process \( \phi^{-1}(\cdot) \) transforms units of exports into imports, so to speak.\(^2\)

The black market firm's period-\( t \) decision problem is shown below where the volume of illegal foreign exchange sales, \( m^t - \tilde{m}^t \), is its decision variable

\[
\max_v \left\{ \frac{(s - \bar{e})/\bar{e}}{(m - \bar{m})} - \phi(m/p - \bar{m}/p) \right\} (4.1)
\]

The first-order condition arising from this maximization problem is

\[
\frac{(s - \bar{e})/\bar{e}}{\phi_1(m/p - \bar{m}/p)/p} = \phi_1(m^t/p^t - \tilde{m}^t/p^t)/p \quad \forall t = 1, 2 \quad (4.2)
\]

The left-hand side of this equation represents the net real marginal revenue obtained from the sale of an additional (real) unit of foreign exchange on the
black market. This term is readily explained. For each real unit of foreign exchange the black marketeer sells he receives $s^t P^t$ nominal units of domestic currency worth $s^t/\bar{e}$ in terms of the export good. This real unit of foreign exchange costs the black market firm $\bar{e}P^t$ units of domestic currency to acquire on the official foreign exchange market, which represents a real cost in terms of the export good of one unit. Thus the net real marginal revenue derived from the transaction is $s^t/\bar{e}-1$. The right-hand side of the above equation represents the real marginal cost of an additional sale of foreign exchange on the black market. The profit-maximizing competitive black market firm of course sets marginal revenue equal to marginal cost when determining the volume of sales it will undertake on the black market.

From the firm's choice problem the following supply curve for black market foreign exchange can be obtained

$$m^t/p - m^t/p = Z(\eta^t P^t) \equiv \phi_t (\eta^t P^t)$$

(4.3)

where $\eta^t \equiv (s - \bar{e})/\bar{e}$ and $Z = l/\phi^t l > 0$.

Note, not surprisingly, that the supply of foreign exchange in the black market is an increasing function of the black market premium, $t P^t$.

V. THE ECONOMY'S GENERAL EQUILIBRIUM

The economy under consideration is effectively composed of three agents: a representative consumer, a black marketeer, and the government. The government, like the "stand in" consumer, must satisfy a budget constraint. Its budget constraints for the first and second periods are:

$$m^1_s = r^1 + h^1_R$$

(5.1)

and
\[
\frac{m_s}{t} = \frac{v}{t} + \frac{1}{m_s} \left( 1 + \pi \right) - b_R^1 (1 + \pi^*)
\] (5.2)

where \( m_s^t \) is the real supply of money in period \( t \), \( \pi^t \) plays the role of period-\( t \) domestic credit and \( b_R^1 \) represents the government's acquisition of interest-bearing reserves in the first period.

Next, equilibrium in the domestic money market implies that the demand and supply for money in each period must be equal. Thus, taking tally of the economy's various actor's holding of money at the time the financial market closes yields
\[
m_t^t + m_F^t + v^t + \phi^t(t) = m_s^t
\] (5.3)

where \( m_F^t \) is the rest-of-the-world's (the foreign country's) period-\( t \) real holdings of domestic cash balances. As can be seen, at the time the financial market closes in period \( t \) the domestic consumer will be holding \( m_t^t \) real units of domestic cash balances, the foreign consumer \( m_F^t \), while the black marketeer will be in possession of \( v^t \) real units of domestic money, and the representative agent's firm will have \( \phi^t(t) \) real units of domestic currency arising from its sales of output to the black marketeer. Now recall that the cash-in-advance constraints will hold as strict equalities as long as the domestic and foreign rates of inflation exceed the values dictated by the optimum quantity of money rule, so from (3.3) and (3.4), \( m_t^t = x_t^t + \left( P_t^t / P_t^t \right) z_D^t \) and \( m^* = p^* z \). If, as will be assumed, foreign residents are solving an analogous optimization problem, except without the foreign exchange restriction, then \( m_F^t = x_F^t \), where \( x_F^t \) is the foreign consumption of the domestic exported good. Hence, (5.3) can be rewritten as
\[
X_t^t + X_F^t + \phi^t(t) + \left( P_t^I / P_t^t \right) Z^t + v^t = m_s^t
\]
\[
X^t + \left( P_t^I / P_t^t \right) Z _t^t + v^t = m_s^t
\] (5.4)

where the left-hand side of (5.4) follows from the fact that the domestic
exported and imported goods markets must clear each period so that
\[ t \quad X + X_F + \phi(t) = X \quad \text{and} \quad Z = Z_D. \]
This equation is a standard condition often obtained in cash-in-advance models stating that the value of output, which here includes the value-added from black market activity, should equal the value of the money stock. For latter use it will be noted that an equation for the current balance of payments, \( b^{1}_R \), can be obtained from (5.4) plus the government's first-period budget constraint (5.1). Specifically,
\[ b^{1}_R = X + \left( \frac{P}{P^*} \right) Z + v - \tau. \] (5.5)

Finally, in the model international trade must balance intertemporally. This fact is easily deduced by first discounting equation (3.2) by \((1+r^*)\) and subsequently adding it to (3.1). Next, the transfer payment terms on the right-hand side of the new equation can be removed by noting that (5.1), (5.2), and (5.4) imply \( \tau^1 + \tau^2/(1+r^*) = [\pi^* \pi^*/(1+r^*)] [(\tau^1 + \tau^2 + \tau^2/1+r^*)] = \left( \frac{P}{P^*} \right) Z + v \right) + \left( 1/(1+r^*) \right) [ \left( \frac{P}{P^*} \right) Z + v \right]. \] Last, eliminate the money terms, \( m^1, m^2, \) on both sides of the resulting equation by using the fact that the cash-in-advance constraints given by (3.3) and (3.4) hold as strict equalities and substitute out for \( v^1 \) and \( v^2 \) through the use of (4.1) to obtain
\[ \frac{\phi^2}{p^*} \frac{2}{Z + X} = \frac{2}{X^\phi + X^\phi} \] (5.6)

How foreign exchange controls in the presence of a black market impinge on the model's general equilibrium has yet to be specified; this is the subject of the next section.
VI. FOREIGN EXCHANGE CONTROLS IN A BLACK MARKET SETTING

Suppose that the government has temporarily instituted a system of foreign exchange controls in the first period. Furthermore, assume that there is a fully operational black market functioning in this period. Now, the imported goods market must clear domestically in the first period in the sense that the demand for imports, $Z^1$, must equal the combined supply of imports from legal and black market sources, $\tilde{m}^1/p^1 + [m^1 - \tilde{m}^1]/p^1$. Thus, the following first-period import market clearing condition must hold

$$Z^1(p^1(1+\eta^1), p^2/(1+r^1), 1/(1+r^1), U^1) = \tilde{m}^1/p^1 + Z^B(\eta^1 p^1) \quad (6.1)$$

In response to various shocks to the economy the domestic market price for first-period imports, $p^1(1+\eta^1)$, must adjust in order to maintain equilibrium in the imported goods market.

As an aid in understanding how an economy with foreign exchange controls and a black market operates consider the impact of an anticipated increase in the agent’s second-period endowment of the exported good, $X^2$. As a consequence of this beneficial change in the second-period endowment of imports, the representative agent immediately realizes an improvement in his welfare $U$ in the amount [cf. eqs. (2.1), (3.5), (3.6), (3.7) and (5.6)]

$$\frac{dU}{dX^2} = \frac{1}{1+r^1}u_1(X^1) > 0 \quad (6.2)$$

This improvement in the agent's real welfare will of course lead, at the original set of relative prices, to increases in his demands for the imported and exported goods in both periods, a fact easily discerned by "eyeballing" the set of demand functions (3.8). The first-period imported goods market must clear domestically, however, and consequently the relative price of first-period imports increases to restore equilibrium in this market by both reducing the demand and stimulating the supply for them. Specifically, from
(6.1) and (6.2) it can be seen that

$$d\left[p^{*1}\frac{1}{1+\eta}\right]/dx = -\frac{1}{(1+\eta)}\mu_U^1 \left[p^{*1}\frac{1}{1+\eta}\frac{1}{Z} - \frac{1}{Z} \right] > 0 \quad (6.3)$$

where $\mu_U^1 \equiv p^{*1}(1+\eta^1)U_1(X^1)Z_4^1$ is the marginal propensity to consume first-period imports. Note that the increase in the black market price for first-period imports is larger the bigger the marginal propensity to consume first-period imports, $\mu_U^1$, is. Also note that it is inversely related to the size of $Z_1^1 - Z_1^B$ which represents the substitution effect of a change in $p^{*1}(1+\eta^1)$ on the excess demand for current import consumption. The rise in the black market premium on foreign exchange, $\eta^1 p^*$, elicits a greater supply of foreign exchange, and consequently of imports too. Thus, an anticipated increase in the agent's second-period endowment of the exported good results in a higher black market price for current imports and a larger consumption/black market supply of them.

The above comparative statics exercise has a simple graphical portrayal which is shown in Figure One. The initial equilibrium is represented by the point E where the demand and supply functions, $Z^1(\cdot 1)$ and $m^{*1}/p^{*1} + Z^B(\cdot 1)$ respectively, intersect. The controls are just binding in this initial position. The improvement in the agent's welfare due to the anticipated rise in his second-period endowment of exported good shifts the demand curve to the right to the position shown by $Z^1(\cdot 1)'$. It can be seen, from the new equilibrium point E', that as a result both the price and quantity of imports increase. It is interesting to compare the results just obtained to those that would occur in the above economy first, without a black market and second, in the absence of foreign exchange controls. When there is not an operational black market the supply curve for current imports would
\[ \frac{m^*}{p^*} \cdot Z^0(0) \]

\[ p^1(1 + \eta^1) \]

\[ p^* \]

\[ Z^1(0) \]

\[ Z^1(0) \]

\[ Z^1 = \frac{m^*}{p^*} \]

\[ Z^1' \]

\[ \text{FIGURE 1} \]
be perfectly inelastic at the quantity \( m / p \). Consequently, an upward movement in the representative agent's second-period endowment of exports would lead solely to an increase in \( p^{*1} (1+\eta^1) \) with there being no accompanying shift in \( Z^1 \), as is shown by the equilibrium point Q. In the absence of foreign exchange controls the supply curve for current imports is perfectly elastic at the world terms of trade, \( p^{*1} \). Here the improvement in the agent's second-period income level leads to a new equilibrium at F which is characterized by an increase in first-period import consumption, with there being no accompanying change in the domestic relative price of imports. Thus, perhaps not surprisingly, the introduction of a black market into an economy with foreign exchange controls creates a hybrid economy which is a cross between the "pure" foreign exchange controlled and free economies.

Finally, the improvement in the representative agent's second-period level of income causes the current trade balance, \( tb^{1} \), to worsen and balance of payments, \( b^{1} \), to improve. Utilizing the definition for the current trade balance \( tb^{1} \equiv X^{1} - X - p^{*1} Z^{1} - \phi^{(1)} \) in conjunction with (6.1), (6.2) and (6.3) one obtains

\[
\frac{dtb}{dX} = \frac{1}{2} \frac{d}{dX} \left\{ (1/(1+r))^{*1} X^{1} + \frac{1}{2} \left( X^{1} + p^{*1} (1+\phi^{(1)}) Z^{1} \right) d[p^{*1} (1+\eta^{1})]/dX \right\} < 0
\]

where \( \mu^{*1}_X \equiv X^{1} u^{*1}_1 (X^{1}) \) is the propensity to consume first-period exports. The trade balance deteriorates on three accounts. First, current export consumption increases because both the agent's welfare level and the first-period price of imports have risen. This effect on the trade balance is portrayed by the term \( (1/(1+r^{*1})) + X^{1} d[p^{*1} (1+\eta^{1})]/dX \). Second, as mentioned above, import consumption moves upward by the amount

\[
\frac{dZ^{1}}{dX} = Z^{1} d[p^{*1} (1+\eta^{1})]/dX .
\]

Last, the real resource costs associated
with operating a black market rise as the volume of imports increase as is shown by the expression \( \phi Z \frac{d[p \ (1+\eta)]}{dX} \). Note that the upwards trend in the current domestic relative price of imports causes the balance of payment to improve since it induces an increase in the demand for money. In particular, from (5.5) and (6.3) it follows that

\[
\frac{1}{R} \frac{db}{dX} = [\frac{1}{Z} + v] \frac{d[p \ (1+\eta)]}{dX}^2 \\
= [\frac{1}{Z} + \frac{\eta \ U \ (X) \ dm}{\eta \ U \ (X) \ dm}] \frac{d[p \ (1+\eta)]}{dX}^2 \geq 0
\]

where the second equality comes from an application of the standard envelope theorem to the profit function \( v^1(*) \) given by (4.1).

Next consider the impact of relaxing the first-period exchange controls by the amount \( \tilde{dm} \). It is easy to see that this will result in the current supply of imports increasing by the amount \( (1/p^\eta) \tilde{dm} \), ceteris paribus, as shown in Figure Two. As a result of the liberalization in the level of foreign exchange controls the agent realizes a welfare gain in the amount \( \frac{1}{\eta} \ U \ (X) \ dm \). This causes demand for imports, other things equal, to rise by \( (1/p^\eta)[\eta /(1+\eta)]\mu \ dm \) which is also portrayed in Figure Two.

Since at the initial relative price for imports supply increases more than demand, the price of imports falls so as to clear the market. Note that the consumption of imports increases. These facts are represented in Figure Two by the change in the equilibrium point from A to A'. Following the line of argument employed in the previous comparative statics exercise, the balance of payments deteriorates due to the fall in the demand for money brought about by the reduction in the current domestic relative price for imports. Formally
FIGURE 2
\[
\frac{1}{R} \frac{db}{dm} = \left[ \frac{\mu + (m - m^*)}{m} \right] p \left( 1 + \eta \right) \ln \left( \frac{1}{1 - \phi} \right) \frac{d[p(1+\eta)]}{dm}.
\]

Finally, the change in trade balance in response to a liberalization in exchange controls is shown below.

\[
\frac{dtb}{dm} = \frac{1}{x} \eta \mu + \frac{1}{1 - \phi(\phi)} p + \frac{p(1+\phi(\phi))}{1 - \phi(\phi)} \ln \frac{1}{1 - \phi} \frac{d[p(1+\eta)]}{dm} + \eta \frac{d[p(1+\eta)]}{dm} < 0
\]

The sign of the above expression is in general ambiguous. There are five terms to consider on the right-hand side of the above expression. The first term illustrates the rise in the agent's current consumption of the exported good stimulated by the increase in his welfare level. This has a negative impact on the trade balance. The second term shows the direct one-to-one deleterious effect on the trade balance associated with the increase in imports induced by the liberalization in exchange controls. When exchange controls are loosened the real resource cost associated with operating a black market is reduced and, as the third term shows, this has a beneficial effect on the trade balance. The drop in the current price of imports results in fewer goods being channeled through the black market and this operates to improve the trade balance, as is portrayed by the fourth term. Finally, the fall in the price of current imports retards export consumption and, as the last term illustrates, this works to improve the trade balance. As has been mentioned, in general a change in the level of controls has ambiguous implications for the trade balance; however, it is shown in the Appendix that starting from an initial position of free trade a slight restriction in the level of foreign exchange domestic residents can hold, will unambiguously improve the trade balance. Thus, an important observation can be drawn
from the above comparative statics exercise. By temporarily imposing a system of foreign exchange controls a country may improve both its trade balance and balance of payments, which is presumably one reason why foreign exchange controls are so popular. The imposition of foreign exchange controls, however, is associated with a welfare loss.

VII. BLACK MARKETS AND ECONOMIC WELFARE

An obvious question which comes to mind is whether the introduction of a black market into an economy with foreign exchange controls improves economic welfare. The answer in the current setting seems to be ambiguous and crucially dependent upon the government's underlying rational for imposing foreign exchange controls. This point will be demonstrated through the use of two examples. To begin with, suppose that the government's simple objective is to limit agents' legal holdings of foreign exchange (or equivalently imports) below some ceiling level. In this circumstance the presence of a black market will improve welfare even though its operation incurs a real resource cost. To see this notice that the real general equilibrium of the decentralized competitive foreign exchange controlled economy with a functioning black market can be compactly summarized as the solution to the following programming problem with \( X^1, X^2, Z^1 \) and \( Z^2 \) being the decision variables. \(^5\)

Problem One

\[
\begin{align*}
\text{Max } & \ U(X^1) + V(Z^1 + Z^2) + \rho[U(X^1) + V(Z^1 + Z^2)] \\
\text{s.t. } & \quad \frac{X^1}{p} \cdot \frac{X^2}{Z^1 + X} + \frac{X^2}{Z^2} = X^1 \cdot \frac{1 - \phi(Z^1 - m/p)}{1 + r} + \frac{X^2}{Z^2 - \phi(Z^1 - m/p)} \\
& \quad \frac{X^1}{1 + r} \cdot \frac{X^2}{1 + r} \\
\end{align*}
\]
The first-order necessary and sufficient conditions—in addition to the constraint (7.2)—are given by (7.3), (7.4) and (7.5) below.

\[ \begin{align*}
&\psi_1^1(Z + Z^1) = [1 + \phi_1^1(1/p) \cdot \phi_1^1] \cdot U_1^1(X^1) \\
&\psi_1^2(Z + Z^2) = [1 + \phi_1^2(2/p) \cdot \phi_2^2] \cdot U_1^2(X^2) \\
&U_1^1(X^1) = (1+r^*)^\phi U_1^1(X^2)
\end{align*} \tag{7.3} \tag{7.4} \tag{7.5}

The above set of first-order is identical to that obtained earlier from the representative agent's maximization problem in the decentralized monetary economy with two exceptions. First, \((s'-\delta)/\delta\) has been replaced by \(\phi_1^t\), a condition implied by the maximizing behavior of the black marketeer in the private economy, and second \(Z^1\) appears instead of \(Z^D\) in the above set of equations; a fact which held in the decentralized economy if equilibrium in the domestic imported goods market was to prevail. Note that the decentralized monetary economy also had to abide by the intertemporal general equilibrium budget constraint (7.2) so that the set of equations (7.2), (7.3), (7.4), and (7.5) provides a unique solution to its real general equilibrium.

Now, without an operational black market in place \(p \cdot Z = m\) for each period \(t\). Clearly, this restriction is in the feasible set of solutions to the above programming problem. In general, this is not the optimal solution, as given by (7.2), (7.3), (7.4), and (7.5), to the economy's programming problem. Thus, the welfare level of the foreign exchange controlled economy with a black market will be at least as high as the welfare level of the same economy without a black market. The intuition behind this result is straightforward. Giving private agents the option to trade on a black market increases their level of welfare since it allows them to partially evade the quantitative restrictions levied on their trading opportunities by the government. In the standard trade literature Falvey (1978) has demonstrated
that smuggling improves welfare in the presence of import quotas for the same reason.

Now consider the alternative story lying behind the government's choice for $m$. Specifically, imagine that the government is actively managing $m$ and $m$ so that some policy objective, say a target level for total first- and second-period imports, $Z$ and $Z$, will be obtained. This type of motivation for foreign exchange controls seems more realistic unless one believes that the policymaking process is fundamentally inept. To obtain the benchmark first-best policy for limiting imports imagine that the economy is run by a central planner who can dictate individual behavior. The central planner's goal is to maximize societal welfare (7.6) subject to the economy's intertemporal budget constraint (7.7), and the import restrictions (7.8). This problem is shown below where the central planner is choosing values for $X, X, Z, Z, m$, and $m$. Note that the central planner has been allowed the option of running a black market if desirable.

**Problem Two**

$$\begin{align*}
\text{Max} & \quad U(X) + V(Z + Z) + \rho[U(X) + V(Z + Z)] \\
\text{s.t.} & \quad \frac{1}{p} \left( Z + X + \frac{2}{Z + X + \frac{1}{\rho}} \right) = \frac{1}{1+m} - \phi(\max(Z - m / \rho, 0)) \\
& \quad \frac{2}{X - \phi(\max(Z - m / \rho, 0))} \\
& \quad Z = Z \\
& \quad Z = Z
\end{align*}$$

(7.6)
The solution to this programming problem yields the unique first-best policy for attaining maximal societal welfare given the government's policy objective. Such a first-best policy may not be implementable in a decentralized competitive equilibrium. Trivially, it can be seen that the central planner will choose not to operate a black market since the right-hand side of (7.7), or the present value of the economy's endowment of the export good, is maximized when \( m^{*} \) is chosen so that \( p^t Z^t = m^{*t} \).

In the decentralized competitive economy the policymaker is regulating \( m^{*1} \) and \( m^{*2} \) so as to attain the target levels for imports, \( Z^{*1} \) and \( Z^{*2} \). Denote the values chosen for these policy instruments by \( m^{*1} \) and \( m^{*2} \). Now, the outcomes of the decentralized competitive equilibrium can be obtained from a modified version of the first programming problem in this section. Specifically, in the new programming problem \( m^{*1} \) and \( m^{*2} \) replace \( m^{*1} \) and \( m^{*2} \) in (7.2). Also, notice that the constraints (7.8) can actually be added to this new problem at no cost since \( m^{*1} \) and \( m^{*2} \) have been picked by the government to ensure that \( Z^{*1} \) and \( Z^{*2} \) are in fact equal to \( Z^{*1} \) and \( Z^{*2} \). Thus, the imposition of these constraints is in fact redundant. Clearly, the level of welfare attained in the decentralized economy with a black market is lower than that achieved in the centrally planned economy since in the second problem \( m^{*1}_Z \) and \( m^{*2}_Z \) are choice variables determined optimally so that \( \phi(t^*) = 0 \) for all \( t^* \). For a given level of import restrictions, the decentralized economy incurs the deadweight cost of running a black market, \( \phi(\cdot_1^*) + \phi(\cdot_2^*)/(1+r^*) \). Note that each individual in the competitive equilibrium avails himself of the opportunity to participate on the black market because from his perspective it is optimal to do so, as the first problem in this section clearly
illustrated. From the social perspective this isn't the case because any leakage of imports into the economy that the black market allows is met by a tightening of exchange controls so that the government's policy target is still met. The individual fails to internalize this last aspect of the problem into his decision-making. Bhagwati and Hansen (1973) have noted for similar reasons, in the real trade literature, that smuggling in the presence of tariffs can reduce welfare if the government is using the tariffs to achieve certain policy objectives such as a target level of domestic production for the import good.

VIII. CONCLUSIONS

A choice-theoretic model of a small open economy has been developed to study the impact of foreign exchange controls in settings where black markets are present. It was found in such environments that the imposition of foreign exchange controls tends to raise the domestic relative price of imports above the world level in exactly the same manner as a tariff would. For a given level of exchange controls a black market mitigated the effect of such quantitative restrictions on the domestic relative price and consumption of imports. While foreign exchange controls may improve the trade balance and balance of payments of an economy even with black markets, they unambiguously lower economic welfare. Finally, the welfare implications of a black market were found to be ambiguous and contingent on the particular policy objectives the government has in mind. For instance, if the government's, perhaps uninspired, goal was merely to limit agents' legal holdings of foreign exchange then the presence of a black market allows the private sector to partially evade the policy and consequently can improve welfare. Alternatively if the government's desire was to regulate agents' holdings of
foreign exchange as a tool to achieve some objective such as regulating the total volume of imports then a black market may reduce welfare. Basically this follows since the same level of imports obtains with or without a black market, but now the economy must suffer the deadweight loss of operating the black market.
APPENDIX

As was mentioned in Section VIII of the text the model's general equilibrium, or a determination of $X^1, X^2, Z^1$ and $Z^2$ is completely specified by the system of equations (7.2), (7.3), (7.4) and (7.5). In the situation where foreign exchange controls are only temporarily in place $\phi(\cdot 2)$ and $\phi_1(\cdot 2)$ should be eliminated from equations (7.2) and (7.4), respectively. To determine how a liberalization in the level of foreign exchange controls affects the current trade balance, $tb^1$, one should first solve the above system of equations for $dX/dm$ and $dZ/dm$ since

$$dtb / dm = (1/(1+r'))[dX / dm + p\ dZ / dm]$$

(A1)

The resulting expression obtained after doing this

$$dtb / dm = \frac{1}{U_{11}}[V_{11}(\cdot 2) + (p_{21})U_{11}(\cdot 2)]\frac{1}{U_{11}}[V_{11}(\cdot 1)\phi_{11}(\cdot 1)/p_{11}]
+ \phi_{11}(\cdot 1)U_{11}(X^1)J
$$

$$\geq 0 \text{ as } -V_{11}(\cdot 1)\phi_{11}(\cdot 1)/p_{11}U_{11}(X^1)\phi_{11}(\cdot 1) < 1 \text{ } 11$$

(A2)

where $J > 0$ is the Jacobian of the system of equations in question. From an initial position of free trade it can easily be seen that slight restriction in the amount of foreign currency domestic residents can hold will improve the trade balance since here $\phi_{11}(\cdot 1) = 0$ so that $-dtb / dm > 0$.

That a change in the level of temporary foreign exchange restrictions has an ambiguous effect on the current trade balance is not without intuitive appeal. On the one hand, a relaxation in the level of foreign exchange controls improves welfare which increases future export and import consumption.
via an income effect and this tends to drive the current trade balance into surplus [c.f. (A1)]. This income effect is proportional in size to $-V_{11}(\cdot 1) \phi_1(\cdot 1)/p^{*1}$. On the other hand, the liberalization of exchange controls reduces the marginal cost of current import consumption, i.e.,

$$p^{*1} \sim \phi_1(Z - \frac{m}{p'}) \text{ falls in value for given } z.$$  

This generates a substitution effect retarding second-period export and import consumption which is proportional in magnitude to $U_1(X^1)\phi_{11}(\cdot 1)$. The net effect on the current trade balance depends on whether income or substitution effect on exchange control liberalization dominates, as (A2) illustrates.
FOOTNOTES

1. This notation can be translated into the above two-period endowment by utilizing the following definitions: \( r^{*1} \equiv r^* \), \( r^{*2} \equiv 0 \), \( v^1 \equiv (P^2 - P^1)/P^1 \), \( v^2 \equiv \infty \), \( v^{*1} \equiv (P^2 - P^*)/P^* \), and \( v^{*2} \equiv \infty \).

2. The normalization of \( m^t \) and \( m^{*t} \) by \( p \) in the function \( \phi(\cdot^t) \) was done for technical convenience only and is devoid of any implications for the paper's conclusions.

3. The cases of anticipated future controls and permanent exchange controls are also easily handled by the method employed in this section. The temporary exchange controls case has a simple diagrammatic representation which is presented in this section. Rather than be taxonomic the other two cases are left to the interested reader to analyze.

4. The Appendix discusses the nature of this ambiguity in greater detail and relates it to the underlying taste and technology parameters in the model.

5. That the outcomes of decentralized competitive economy can often be mimicked by a simple programming problem is a well known fact. See Negishi (1960) for the formal details.

6. The notation \( (\max(Z - m^t) / p, 0) \) inside the function \( \phi(\cdot^t) \) has been employed to emphasize the point that when the exchange controls are not binding, so that \( Z \leq m^t / p \), there are no resources devoted to running a black market.

7. The government presumably should pick the policy instruments \( m^{*1} \) and \( m^{*2} \) so that the target levels for imports, \( Z \) and \( Z \) are attained optimally. Define \( V(V(X, X, X, X, X, Z, Z, p, p, p, r, m, m) \) is the value function associated with the first problem in this section. Also, note that
the solution to this problem implies that $z^t$ can be written as the following

$$
t^{t} \cdot X_1^{1/2} \cdot X_2^{1/2} \cdot \alpha_1 \cdot \alpha_2 \cdot \beta_1 \cdot \beta_2 \cdot \gamma_1 \cdot \gamma_2 \cdot \zeta_1 \cdot \zeta_2$$

function $Z = Z(X, X, Z, Z, p, p, r, m, m)$. The government's goal should be to choose $m^t$ so as to maximize $V(\cdot)$ subject to the constraint that $Z(\cdot) = Z^t$ for all $t$. 

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