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Essays on Conflict Mediation

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Abstract

An important barrier to conflict resolution is asymmetric information. That is adversaries have private information about their objectives, resources, and strengths during the conflict and have incentives to misrepresent this information during the negotiations. This can lead to the breakdown of negotiations. Third-party institutions, like mediators, can help adversaries to reach an agreement by making a peace proposal. In this thesis, I explore the implication of asymmetric information and the lack of commitment of players to the design of an optimal peace proposal by a mediator.

In Chapter 2, co-authored with Charles Zheng, I study a problem of conflict mediation where a mediator proposes a split of a good between two ex-ante identical contestants thereby preempting a conflict if and only if both accept the proposal. A contestant worries that accepting the proposal may signal weakness that may be exploited in the event of conflict. Thus, when conflict cannot be fully preempted, the mediator leans toward a proposal that shuts down the signaling channel for one of the contestants. Consequently, the socially optimal proposal offers to one of the contestants a minimally larger share of the good than what it offers to the other party so that the former contestant always accepts it without fearing any part of its private information being revealed.

Chapter 3 examines the participation decisions of the players in a mediation process. The action of participating in the mediator's negotiation mechanism conveys information to the opponent. The mediator wants to minimize the probability of conflict subject to incentive compatibility and full participation conditions of players. I find that despite ex-ante identical players, a certain class of biased proposals augmented by a randomization device, that incentivizes the favored player to always accept, satisfy full participation. The mechanism that proposes the equal proposal cannot satisfy full participation even with randomization. These results are true when the type distribution is binary or a continuum of types. When the type distribution is binary, the lopsided proposals also minimize the probability of conflict subject to the full participation constraint.

Chapter 4 studies a conflict model where adversaries lack commitment and can renege on an accepted agreement. A mediator whose objective is to maximize welfare subject to renege-proof constraint proposes a peaceful split of a contested prize between two players. Despite ex-ante identical players, the renege-proof optimal proposal is a biased proposal that the favored player always accepts. This proposal is even more biased than the optimal lopsided proposal that maximizes welfare when players have full commitment.

Keywords: Conflict Mediation, Endogenous Conflict, Information Design, Full Participation, Renege

Summary for Lay Audience

An important barrier to a peaceful settlement between adversaries is asymmetric information, i.e., each involved party is uncertain about the strength, objective, or resources of the opponent. The difficulty of assessing the cost of conflict and the strategic interaction of involved parties can lead to the collapse of negotiations and escalation of disputes. Mediators can help adversaries to reach an agreement by making peace proposals. In this thesis, I examine the design of optimal peace proposals when two adversaries are competing over a prize and if they cannot settle their dispute they will go into a conflict.

Chapter 2, co-authored with Charles Zheng, studies conflict mediation problems where a mediator cannot enforce a peace settlement and cannot provide any economic incentives. She proposes a split of a prize between two players that if and only if they both accept, they avoid a conflict. The conflict is winner-take-all. Whoever puts more effort will win it. Players have private information about their cost of exerting effort. They can be either strong or weak. These types are drawn from a common prior distribution. We find that, when conflict cannot be fully preempted, despite ex-ante identical players, the optimal proposal that maximizes social welfare, the sum of the ex-ante payoffs of the players, is a specific biased proposal. It is the smallest proposal that the favored player always accepts, without worrying to signal any weakness by accepting the proposal.

Chapter 3 considers a conflict mediation problem where adversaries can decide to participate in a mediation procedure or trigger conflict immediately. The act of participation reveals information about players' private information. A mediator, who can provide economic incentives, offers a peace surplus of an agreement to two players to prevent a conflict if both accept the proposal. The mediator is interested in designing proposals that guarantee the full participation of the players in the mediation. I find that despite ex-ante identical players the peace proposals that guarantee the full participation of players are a class of biased proposals, augmented with a fair coin, that incentivize the favored player to always accept them. These results are true when the type distribution is binary or a continuum of types. In the former case, these stochastic biased proposals also minimize the probability of conflict among the fully participating proposals.

Chapter 4 studies conflict mediation problems where players lack commitment and can renege on an accepted proposal. Players can learn about each other by observing each other's decisions in the mediation. In particular, they can learn about each other after a successful round of mediation and use this information to reassess the cost of conflict and renege on the previously accepted proposal. I find that a mediator who is interested in the design of renege-proof welfare-maximizing proposals should propose even more biased proposals compared to the environments where players have full commitment.

Co-Authorship Statement

This thesis contains a co-authored material. Chapter 2 is co-authored with Charles Zheng. All authors are equally responsible for the work. A version of Chapter 2 has been submitted for publication.

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Chapter 1

Introduction

Social scientists have long been concerned with the design of contracts that lift societies from pervasive conflicts and minimize the social cost associated with them. A crucial impediment to peaceful settlement is asymmetric information, i.e., each involved party is uncertain about the strength, objective, or resources of the opponent. The difficulty of assessing the cost of conflict and strategic interaction of involved parties can lead to the collapse of negotiations and escalation of disputes. Examples abound. For instance, international negotiation over natural resources heightened to war, pre-litigation negotiations escalated to court, union bargaining ascended to strike, or trade negotiations escalated to tariff wars.

Asymmetric information of the types mentioned above provides scope and incentive for adversaries to misrepresent their strength. Thus, it could hinder peaceful settlements and exacerbate the cost of conflict. Third-party institutions such as mediators can mitigate this problem and help these adversaries to reach an agreement by making peace proposals that can set a status quo or a focal point for an agreement in a situation that may otherwise lack one. My Ph.D. research examines the optimal peace proposal in a dynamic environment with asymmetric information where two adversaries may agree to settle disputes that would otherwise escalate to a costly conflict. Even when a mediator cannot prevent a conflict, by designing peace proposals she can manipulate adversaries' beliefs about each other's strengths. Hence, she could affect their strategies in the conflict and minimize the social cost of conflict.

My thesis contains three chapters that explore the design of peace proposals by a mediator in three different contexts. In Chapter 2, co-authored with Charles Zheng, I study situations where a mediator, with very limited power, wants to design peace proposals that maximize the social welfare of adversaries who have full commitment if they accept a proposal. In Chapter 3, I study the implication of information revelation from the mediation procedures on the participation decisions of players in a mediation mechanism. In the final chapter, I study conflict mediation problems where adversaries lack commitment and can renege on an accepted proposal.

In Chapter 2, co-authored with Charles Zheng, I study a problem of conflict mediation where a mediator proposes a split of a good between two ex-ante identical contestants thereby preempting a conflict if and only if both accept the proposal. The mediator's decisions are not binding, she cannot provide economic incentives, and if the conflict happens, it is beyond her control. Conflict is modeled as an all-pay auction. It is costly and winner-take-all. Each contestant simultaneously chooses a level of effort or resources to devote to the conflict and whoever puts the greatest resources wins. Contestants have private information about their marginal cost of exerting effort, i.e., their strength, in the conflict, independently drawn from the same prior distribution. The outcome of conflict depends on the contestants' warring efforts, which is determined by the posterior beliefs that players form about each other after observing the mediation's outcome. A contestant worries that accepting the proposal may signal weakness that may be exploited in the event of conflict. Thus, when conflict cannot be fully preempted, the mediator leans toward a proposal that shuts down the signaling channel for one of the contestants. Consequently, the socially optimal proposal offers to one of the contestants a minimally larger share of the good than what it offers to the other party so that the former contestant always accepts it without fearing any part of its private information being revealed. That is, even though the contestants are ex ante identical, and are assigned equal welfare weights, the socially optimal proposal lopsidedly favors one side against the other.

Chapter 3 studies a conflict model where a mediator, who can offer economic incentives and for reputation and practical motivations would like to guarantee full participation in her negotiation mechanism, proposes a split of peace surplus to two players. Players can participate in the nonbinding mediation that can result in conflict or they can trigger conflict immediately. Conflict is modeled as an endogenous continuation game. The action of participating in the mediator's mechanism can convey information to the opponent. Thus, full participation in the negotiation mechanism cannot be assumed without loss of generality. The mediator wants to minimize the probability of conflict subject to incentive compatibility and full participation conditions of players. The players are ex-ante identical. I find that when the type distribution is binary, a certain class of biased proposals augmented by a randomization device, that incentivizes the favored player to always accept, are the constrained optimal proposals and satisfy full participation. The mechanism that proposes the equal proposal cannot satisfy full participation even with randomization. When the type distribution is continuous, the randomized biased proposals satisfy full participation while the equal proposal cannot satisfy it.

Chapter 4 studies a conflict model where adversaries lack commitment and can renege on an accepted agreement. The primitives, similar to the previous chapters, are such that no negotiation mechanism exists that fully preempts conflict. A mediator whose objective is to maximize welfare subject to renege-proof constraint proposes a peaceful split of the contested prize to two ex-ante identical rivals. By observing each other's decisions, after a successful or failed mediation, adversaries learn about each other and update their forecast of conflict. Despite ex-ante identical players, the renege-proof optimal proposal is a biased proposal where the favored player always accepts and reveals no further information. This biased proposal is even more extreme than the biased proposal that maximizes welfare in a renege-banning model where players have full commitment. The same results hold when the mediator's objective is to minimize the probability of conflict.

The possibility of fully preventing conflict has been vastly studied in the economic literature. This literature models conflicts as the outside option of a negotiation procedure that imposes a cost on it. Conflicts are either modeled as exogenous lotteries for the players (e.g., Hörner et al., 2015) that impose an exogenous cost on the negotiations or they are modeled as an endogenous continuation game after the breakdown of negotiations (e.g., Zheng, 2019) so that the posterior belief conditional on the outcome of negotiations becomes crucial. This thesis also models conflict as an endogenous continuation game, but different from both of the previously mentioned strands of literature, the assumptions on primitives of the model are such that full preemption of conflict is impossible. Therefore, conflict happens on the path of equilibrium. The mediator that wants to maximize welfare or minimize the probability of conflict should take into account the payoff from both the event of peace and conflict. The only channel through which a mediator can affect the players' payoffs in the conflict is by indirectly manipulating the posterior beliefs that they have about each other after the mediation. However, these beliefs are not policy instruments and they are interdependent with the equilibrium strategy profiles via Bayes's rule. This poses a challenge in finding an optimal solution for the mediator's mechanism. Balzer and Schneider (2021) have considered a conflict management model where full preemption of conflict is also impossible. They consider communication mechanisms that minimize the probability of conflict and focus on the case where the designer is an arbitrator with full commitment power. While they also consider a mediation case, the mediator is assumed able to communicate separately and confidentially to the contestants so that the two can learn from each other only if the mediation fails. In this thesis, by contrast, a mediator can only indirectly influence the posterior systems through a message-independent peace proposal, to which the contestants' responses, commonly observed, cannot be misrepresented.

The general result I find in this thesis is that the optimal proposal sometimes treats the two equal adversaries unequally. Therefore, it should not be taken for granted that a neutral mediator should offer an equal share to equal contestants. The insight conveyed by these results is that a lopsidedly biased proposal is conducive to peacemaking because the favored side is willing to accept the peace deal without fearing being viewed to be weak and subsequently exploited in the event of a conflict so that the mediator is less constrained and can devote more resources to compensate the unfavored side.

Bibliography

Balzer, B. and J. Schneider (2021). Managing a conflict: optimal alternative dispute resolution. *The RAND Journal of Economics 52*(2), 415–445.

Hörner, J., M. Morelli, and F. Squintani (2015). Mediation and peace. *The Review of Economic Studies 82*(4), 1483–1501.

Zheng, C. Z. (2019). Necessary and sufficient conditions for peace: Implementability versus security. *Journal of Economic Theory 180*, 135–166.

Chapter 2

Unequal Peace Proposals for Equal Contestants: Designing Information Indirectly

2.1 Introduction

In international conflicts, a mediator's power is limited. A mediator may find itself having no extra value to offer to the two sides and no special skills or understanding of the complex situation, as exemplified by how Israel is described by the media during its mediation effort in the current Russian-Ukraine conflict (Kershner, 2022). Even when a mediator is powerful enough to use various instruments ranging from proposing peace deals to providing economic incentives, such as the United States Presidency during its intensive involvement in the Oslo Process, it is still unable to enforce a peace settlement (cf. Lasensky, 2004). As much as a mediator might try to conduct the talks with confidentiality, leaks are commonplace (cf. Feerick, 2003). Furthermore, as noted in the conflict management literature such as Kydd (2003), Rauchhaus (2006), and Smith and Stam (2003), when a mediator uses a communication mechanism to conduct the negotiation, the incentive compatibility of the mediator is at issue because the peacekeeping intention might drive mediators to exaggerate some information—regarding the cost of conflict—when they convey it from one contestant to the other. The question is: What can and what should a mediator do given severely limited instruments?

This chapter therefore considers a stylized model of conflict mediation where the mediator can only propose a peaceful split of the prize between the two contestants so that the only response a contestant can make is to accept or to reject it. If neither of them reject it, the proposed split becomes the status quo. Otherwise, conflict ensues, in which the marginal cost of warring efforts that a contestant incurs is the contestant's private information. A main assumption in this setup is that the mediator makes the proposal without any more information than the common prior, and that the peaceful split in the proposal is independent of the messages that the mediator might have been able to collect from the contestants. This assumption is to capture the situations where a mediator's instruments are limited, especially in international conflicts.[1] With the proposal independent of any message from any party involved, the aforementioned incentive issue of the mediator is avoided and hence we do not need to make the restrictive assumption in the mechanism- or information-design literature that the mediator is trustworthy throughout the negotiation.

Despite the simplicity of the negotiation mechanism that we assume, the mediator can still affect the outcome. This, roughly speaking, is in line with Schelling's (Schelling, 1980) original idea that through making a peace proposal a mediator creates a focal point for agreement in a situation that would have otherwise lacked one. More precisely, a contestant's response to the proposal can signal to the other contestant the former's willingness or hesitancy to fight in the conflict. Since we model the conflict as an all-pay auction where each contestant chooses an amount of effort to exert and, win or lose, bears the cost of its effort, these signals affect how the conflict unfolds. Thus, despite the restriction to merely proposing a settlement that has room only for a binary response, a mediator can still indirectly influence the contestants' beliefs about each other, thereby affecting the outcome and social welfare.

The possibility of fully preempting the conflict between two contestants is considered by Bester and Wärneryd (2006), Compte and Jehiel (2009), Fey and Ramsay (2011), Hörner et al. (2015), Meirowitz et al. (2019), and Spier (1994), who model conflicts as exogenous lotteries for the contestants. A recent literature models conflict as an endogenous continuation game after the breakdown of the negotiation, so that the posterior beliefs conditional on the outcome of the negotiation become crucial. In this endogenous conflict literature, the possibility of full preemption of conflict is characterized by Zheng (2019a,b) and Celik and Peters (2011) when the peace or collusion proposal is from a mediator, and by Lu et al. (2021) when the peace proposal is from an exogenously designated contestant.

This chapter also models conflict as an endogenous continuation game. Different from

[1]Mediation through proposing a peaceful split corresponds roughly to the formulative style in real-world mediation, as opposed to the manipulative style where the mediator offers economic incentives such as side payments. According to Wilkenfeld et al. (2007), the formulative style accounts for almost half of all mediated cases in their dataset of international conflict.

both of the above literatures, however, the chapter is based on an environment where full preemption is impossible. That is, no proposal exists—be it message-independent as in our model or message-dependent as in the general framework—that admits a perfect Bayesian equilibrium where conflict occurs with zero probability. Thus conflict is an on-path event in any proposal-equilibrium pair. Consequently, a benevolent mediator cares about the contestants' welfare not only in the event of peace, but also in the event of conflict. Therefore, a received insight in the above literatures needs to be reexamined. The insight goes that a mediator should minimize each contestant's expected payoff in the conflict thereby incentivizing the contestant to accept the peace proposal. Now that conflict is not off-path and hence contestants' conflict payoffs are part of their ex ante expected payoffs, such a policy may hurt the overall welfare of the contestants. Our analysis takes into account the contestants' expected payoffs in all possible events, be they conflict or peace. That is, the objective is to maximize the *social surplus*, or the sum of ex ante equilibrium payoffs for both contestants.

We focus on the question whether the mediator should treat equal parties equally. More precisely, we assume that the two contestants are ex ante symmetric so that their types—private information about their marginal costs of warring efforts in the conflict—are drawn from the same distribution, assumed binary for tractability, and that they have the same valuation, commonly known, of the contested prize. In the spirit of the symmetry axiom in the Nash bargaining solution, there is a sense that the mediator should propose to split the prize equally between the two symmetric players. In a model where the proposed split depends on the contestants' messages (which the mediator is assumed to trustworthily collect and convey), Hörner et al. (2015) restrict attention to symmetric mechanisms. In our model, which considers only message-independent proposals, their symmetric mechanisms would all be the equal-split proposal. In addition, the notion of equal split between equal claimants dates back to the Talmud (cf. Aumann and Maschler, 1985). Yaari and Bar-Hillel (1984) suggest several ways to justify the equal split between contesting claimants (including the equal treatment property in general equilibrium). Recently, Keniston et al. (2021) provide a rationale for, and conduct an experimental study of, the equal split of the perceived surplus between two bargainers in a dynamic game.

By contrast, despite the ex ante symmetry between the two contestants, we find that when the prior probability for a contestant to be the weak type is in an intermediary range, the social-surplus maximizing proposal is not the equal split but rather a particularly lopsided one. It proposes to split the prize in such an unequal way that the contestant offered the larger share would always accept it while the other contestant would reject it for sure when the latter's type is strong (namely, incurring a low marginal cost of warring efforts), and mix between rejection and acceptance when its type is weak.

We obtain this result through analyzing the mediator's indirect choice of the posterior belief system. A posterior system here associates to each player a posterior conditional on the player's action (accepting or rejecting the peace proposal). Obviously these posteriors are jointly constrained by the equilibrium condition including Bayes's rule and mutual best responses. The shortcoming of the equal-split proposal is that its equilibrium constraint leaves little room for the designer to effect a desirable posterior system. That is because, given the equal-split proposal, both rejecting and accepting the proposal are on-path actions for each player and hence the posteriors of both players are constrained by Bayes's rule. By contrast, the mediator can propose a lopsidedly biased split that offers a much larger share of the prize to one of the players so that the favored player always accepts it. That means the posterior about this player conditional on its rejecting the proposal becomes off-path and hence free of the Bayesian restriction. With one less constraint to satisfy, the mediator in proposing the biased split gains some leeway to manipulate the posteriors.

This shortcoming of the equal-split proposal is shared by all other proposals whose equilibria prescribe both Accept and Reject as on-path actions for each player. That leaves us only those proposals whose equilibria have one of the players choose Accept always, similar to the lopsided one described above. Among such lopsided proposals, the one that offers the unfavored contestant the largest possible share is our optimum, as it maxes out the welfare for the unfavored contestant while still securing acceptance from the favored one. This lopsided solution also satisfies both the Intuitive and D1 Criteria of refinement.

Albeit based on a stylized model, our result conveys a new insight that it should not be taken for granted, even from a benevolent social planner's standpoint, that a peace proposal should offer a fair share to each contestant. Counterintuitively, a lopsidedly biased proposal is conducive to peacemaking because it makes the favored party willing to accept the deal without fearing that the acceptance might reveal some information that might be used against it later. The favored party fully incentivized, the mediator is less constrained and hence able to devote resources for the other party. From this perspective, it is not surprising to see the number of Arab League countries that agree to establish diplomatic relations with Israel jump suddenly[2] soon after the United States announced its embassy relocation to Jerusalem in 2018. The US embassy relocation can be viewed as a proposal—and a message-independent one, as the US President did not appear to

[2]The number jumped from two—Egypt and Jordan—to six, including Bahrain, the United Arab Emirates, Sudan and Morocco.

have consulted either side before making the announcement—for a new status quo that recognizes Israel's full ownership of Jerusalem, the most contested "prize" between the Arab League and Israel. Another example of a lopsided proposal is the Vatican mediation of the Beagle Channel Dispute between Argentina and Chile. In the shadow of a war between the two countries, the Pope issued a proposal that awarded Chile all of the disputed islands, granting Argentina only the navigation rights in the area waters and a shared resource right in a part of the sea. Chile immediately accepted the proposal while Argentina was initially reluctant but eventually accepted it. We shall describe it in more detail in Section 2.5[3]

Endogenizing the initial status quo through a mediator's decision, our study complements the conflicts literature where contestants themselves take the initiative to mitigate or escalate the conflict with an implicitly exogenous status quo that defines the sequence of actions. In Baliga and Sjöström (2004), the two contestants decide simultaneously whether to escalate the conflict. In Baliga and Sjöström (2020), given an exogenous initial status quo, each contestant decides whether to challenge it. In Lu et al. (2021), one of the two contestants has the bargaining power to make a take-it-or-leave offer to the other player for a peace settlement. The focus in this literature is the dynamic interaction between the contestants given the implicit status quo. We simplify this interaction into a static all-pay auction game and focus instead on the determination of the initial status quo.

The method in this chapter is related to information design in the sense that the mediator's choice of posterior systems amounts to "splitting" each player's prior distribution into two posterior distributions, one conditional on the action Accept, the other conditional on the action Reject. Correspondingly, a player's interim expected payoff—which guides the player's action in response to the proposal—becomes a convex combination of the player's post-mediation payoffs that are determined by such posteriors. Differently, the designer in information design frameworks can split a prior distribution into any convex combination of any posterior distributions as long as the combination satisfies Bayes's rule (cf. Kamenica and Gentzkow, 2011 and, more recently, Doval and Smolin, 2021). While Le Treust and Tomala (2019) have extended the framework to allow for capacity constraints, the choice set for the designer is still completely determined by the primitives of the agent under consideration. In our model, by contrast, the convex combination of the posteriors for one player is not only constrained by Bayes's rule but is also

[3]If we view trade unions as settlements among countries to avoid potential trade conflicts, the Maestricht Treaty for the UK to join the European Union is yet another episode of lopsided proposals. The treaty offered UK the opt-outs from the single currency mandate and the Social Chapter of employment regulations, while none of the other member nations were offered such opt-outs (cf. Baun, 1995 and Burton, 2021, Chapter 5).

interdependent with the convex combination of the posteriors for the other player. That is because the former convex combination needs to rationalize the equilibrium strategy as a best response to the equilibrium strategy rationalized by the latter convex combination, and vice versa. Constrained by Bayes's rule only in a separate, agent-by-agent manner, the designer in information design frameworks attains the optimum on either the convex or the concave closure of an agent's ex post payoff as a function of the posterior distribution. The mediator in our model, by contrast, is subject to an endogenous constraint consisting of Bayes's rule for each player and mutual best response across players. Consequently, neither the convex nor the concave closure attains the optimum, as neither minimizing a player's interim expected payoff in the conflict, nor maximizing thereof, is necessarily on the right direction of maximizing the sum of ex ante payoffs across the players.[4]

Such equilibrium constraints faced by the mediator are different in nature from those faced by the sender in the information design models with multiple, interacting receivers such as Mathevet et al. (2020). In Mathevet et al. (2020), as in its single-receiver counterpart, receivers each get their signals directly from the sender and then interact with one another conditional on the signals. By contrast, in our model, the mediator cannot directly send any signal to a player. Instead, it is the players who send signals to one another through their responses to the mediator's proposal, and their responses are chosen to best-reply one another.

Balzer and Schneider (2021a) have considered a conflict management model where full preemption of conflict is also impossible. They consider communication mechanisms that minimize the probability of conflict and focus on the case where the designer is an arbitrator with full commitment power. While they also consider a mediation case, the mediator is assumed able to communicate separately and confidentially to the contestants so that the two can learn from each other only if the mediation fails. These assumptions allow them to apply the information design method commented above. In our model, by contrast, a mediator wants to maximize the sum of the contestants' ex ante expected payoffs, and can only indirectly influence the posterior systems through a message-independent peace proposal, to which the contestants' responses, commonly observed, cannot be misrepresented.

[4]The only exception is the case where full preemption of conflict is possible and one considers those equilibria where both players for sure accept the peace/collusion proposal on path, as do Celik and Peters (2011), Zheng (2019b), and Balzer and Schneider (2021b). Then Reject becomes off-path for each player, and the designer needs only to choose for each player a convex combination of off-path posteriors that minimizes the player's interim expected payoff from rejecting the peace proposal. As long as the size of the prize is larger than the sum of such interim expected payoffs across the players, full preemption of conflict obtains as an equilibrium. This chapter, by contrast, consider a model where full preemption of conflict is mathematically impossible.

After defining the model next, we derive in Section 2.3 the contestants' interim expected payoffs through the semi-information-design method just mentioned, illustrated by Figures 2.3–2.5, and then derive the ex ante expected payoffs. Section 2.4 presents the result. It starts with the statement of the proposition and continues with the main structure of the proof. Section 2.5 describes the Vatican mediation of the Beagle Channel Dispute as an example for lopsided proposals in the real world. Section 2.6 concludes and mentions a couple of possible extensions. The appendix contains all omitted details.

2.2 The Model

Two players, named 1 and 2, compete for a prize. Each player's type is independently drawn from the same binary distribution, whose realization is either w ("weak") with probability θ, or s ("strong") with probability $1-\theta$, such that $0 < \theta < 1$ and $s > w > 0$.[5] Denote

$$\alpha := 1 - w/s.$$

Thus $0 < \alpha < 1$. After each player's type t_i is drawn and privately known to the player, a neutral mediator proposes a peaceful split of the prize of size one:[6]

$$(x_1, x_2) \in [0,1]^2 \quad \text{such that} \quad x_1 + x_2 = 1.$$

Then each player independently and publicly announces whether to accept (A) or reject (R) the proposal. If both choose A, the game ends with player i getting a payoff equal to x_i ($\forall i$). If at least one chooses R, then conflict takes place in the form of an all-pay auction: Each player i, after observing the actions (choices between A and R) of both, submits a sealed bid $b_i \in \mathbb{R}_+$; the higher bidder wins the prize, with ties broken randomly with equal probabilities; the payoff for player i of type t_i is equal to $\frac{1}{\alpha}\left(1 - b_i/t_i\right)$ if i wins, and equal to $\frac{1}{\alpha}(-b_i/t_i)$ otherwise. Then the game ends. Thus, a player's bid represents the player's total amount of warring efforts in the conflict, and the reciprocal $1/t_i$ of a player's type t_i represents the player's marginal cost of warring efforts in the conflict.[7]

[5] Our assumption of binary types is in line with much of the conflict resolution literature such as Balzer and Schneider (2021a,b), Hörner et al. (2015), and Meirowitz et al. (2019),

[6] Our formulation of a peaceful split implicitly assumes that the mediator cannot diminish the size of the prize thereby making $x_1 + x_2 < 1$. This assumption is in line with our motivation of capturing the situations where the mediator's instruments are severely restricted, which is the case in most international conflicts.

[7] We scale up the payoff from the conflict by the parameter $1/\alpha$ purely for notational cleanliness. That is because α emerges as a multiple of each player's expected payoff from any equilibrium of the conflict continuation game (Section 2.3.1), and our scalar $1/\alpha$ cancels out the multiple. Without the scalar $1/\alpha$ to cancel out α, α would appear in most expressions in the chapter thereby complicating

Each peace proposal (x_1, x_2) determines a two-stage game, for which perfect Bayesian equilibrium (PBE) is the solution concept. We measure the social welfare achieved by a peace proposal by the social surplus generated on path of the PBE admitted by the proposal. By *social surplus* we mean the sum of the two players' ex ante expected payoffs (before realization of types). A peace proposal of particular interest is the *equal split* $(1/2, 1/2)$, treating the two ex ante identical players equally. Another proposal of interest is $(\theta, 1-\theta)$, splitting the prize according to the prior probabilities assigned to the weak and strong types.

Throughout the chapter we assume

$$\theta > 1/2. \tag{2.1}$$

The assumption is to avoid triviality of the problem. Otherwise, $\theta \leq 1/2$ would guarantee existence of a peace proposal accepted by both players for sure at equilibrium, attaining the largest possible social surplus.[8]

2.3 Interim Payoffs and Posterior Beliefs

2.3.1 The Post-Mediation Payoff in the Conflict

Let us start by considering the continuation game where conflict ensues. Recall that this stage is reached if at least one player has chosen Reject to the peace proposal. At the start of the stage, the belief about a player is updated conditional on the player's response to the proposal. For each player $i \in \{1, 2\}$, denote p_i for the posterior probability of player i being type s (strong). This, together with the players' private information of their own types t_i, defines a Bayesian game. Consider any Bayesian Nash equilibrium (BNE) of the all-pay auction given the posteriors (p_1, p_2). If G_{-i} is the c.d.f. of the bid from player $-i$ at equilibrium, and if i's type is t_i, then i's expected payoff from bidding b is equal to

$$\frac{1}{\alpha}\left(G_{-i}(b) - \frac{b}{t_i}\right)$$

them, though all our results remain true.

[8] To see this, apply (Zheng, 2019b, Example 4). Since we have scaled up the payoff in the conflict to $1/\alpha$ times the quantity assumed in Zheng (2019b), the peace-implementability threshold $c_* = \alpha\theta$ there becomes $(1/\alpha)c_* = \theta$. Thus the necessary and sufficient condition for peace implementability becomes $2\theta \leq 1$. If $2\theta \leq 1$, one can split the prize such that each player gets a share at least as large as θ, and it is an equilibrium for both to accept any such splits, the equal split $(1/2, 1/2)$ being one of them.

unless b is an atom of G_{-i}. According to the all-pay auction literature, there exists a unique equilibrium and the (G_1, G_2) is characterized by the first-order condition

$$G_i'(b) = \begin{cases} 1/s & \text{if } G_{-i}(b) > 1 - p_{-i} \\ 1/w & \text{if } G_{-i}(b) < 1 - p_{-i} \end{cases}$$

for each $i \in \{1, 2\}$. Without loss of generality, let $p_1 \geq p_2$. Coupled with the boundary condition that $G_i(0) = 0$ for at least one player, this differential system admits a unique solution.[9] One way to solve it is to start with the maximum bid $\bar{b}$, common to both players, and trace the graphs of G_1 and G_2 according to the differential system when the bid decreases from $\bar{b}$ to zero. One can see that their graphs are as depicted in Figure 2.1. Both graphs start by decreasing at the rate equal to $1/s$. Then the graph of G_1 changes to

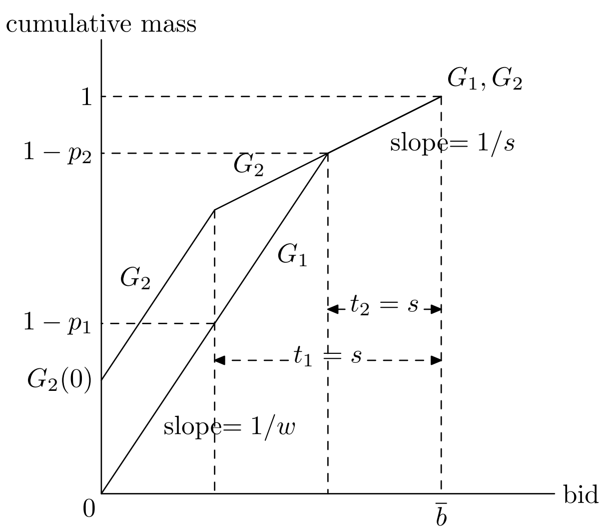


Figure 2.1: The equilibrium in the all-pay auction

the steeper slope $1/w$ at the bid b for which $G_2(b) = 1 - p_2$, while G_2 remains decreasing at the $1/s$, because $p_1 \geq p_2$, until $G_1(b) = 1 - p_1$. Thus, when the bid decreases down to zero, $G_2(0) \geq G_1(0)$. Since the zero bid cannot be an atom for both bidders (or an equilibrium condition is violated), $G_1(0) = 0$. That pins down $\bar{b}$ and $G_2(0)$:

$$\begin{aligned} \bar{b}/s &= 1 - (1 - w/s)(1 - p_2) = 1 - \alpha(1 - p_2), \\ G_2(0) &= (1 - w/s)(p_1 - p_2) = \alpha(p_1 - p_2), \end{aligned}$$

[9]Since G_i and G_{-i} need not be differentiable, the differential system holds only for almost all b in their common support. However, one can prove that G_i and G_{-i} are each absolutely continuous and hence the system coupled with a boundary condition admits a unique solution. See Zheng (2019b) for details.

where we have used the notation $\alpha := 1 - w/s$. Thus the equilibrium is determined.[10]

From the above derivation we obtain the expected payoff $U_i^t(p_i, p_{-i})$ in any BNE of the continuation game, for all players $i \in \{1, 2\}$ and all types $t \in \{s, w\}$:

$$\begin{aligned} U_1^s(p_1, p_2) &= \frac{1}{\alpha}\left(1 - \bar{b}/s\right) = 1 - p_2 = 1 - \min\{p_1, p_2\}, \\ U_1^w(p_1, p_2) &= \frac{1}{\alpha}\left(G_2(0) - 0/w\right) = p_1 - p_2 = p_1 - \min\{p_1, p_2\}, \\ U_2^w(p_2, p_1) &= 0 = p_2 - \min\{p_2, p_1\}. \end{aligned}$$

Without the assumption $p_1 \geq p_2$, one can easily generalize the above to:

$$U_i^s(p_i, p_{-i}) = 1 - \min\{p_i, p_{-i}\}, \quad (2.2)$$

$$U_i^w(p_i, p_{-i}) = p_i - \min\{p_i, p_{-i}\}, \quad (2.3)$$

which is the expected payoff, for each player $i \in \{1, 2\}$ and each type $t \in \{s, w\}$, in any equilibrium of the continuation game given any posterior beliefs (p_1, p_2).

The functions $U_i^s(p_i, \cdot)$ and $U_i^w(p_i, \cdot)$ are graphed in Figure 2.2. These conflict payoffs

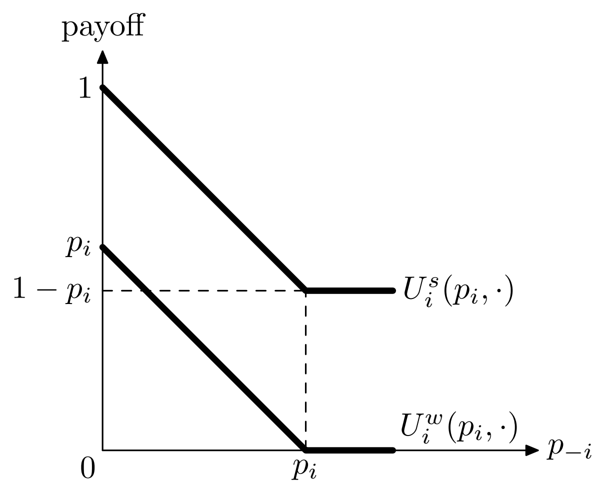


Figure 2.2: Payoff in the conflict as a function of the opponent's posterior

play a similar role as the ex post payoff that a designer would like to concavify in the information design framework, except that in our game concavification need not bring about larger social surplus, as they are the payoffs only in the event of conflict.

[10] In a nutshell, there is a unique BNE. Both players randomly select their bids (effort levels) from an interval $\left[0, \bar{b}\right]$. The strong type of a player selects its bid from an upper subinterval of $\left[0, \bar{b}\right]$, and the weak type of the player, from the complement of the upper subinterval. The player whose posterior probability p_i of being the strong type is lower than the other's bids zero (exerting zero effort) with a positive probability when its type is weak, while the other player bids zero with zero probability and hence enjoys a positive probability of winning even with zero effort.

Remark 2.1 It is clear from Figure 2.2 that an increase in p_i hurts the strong type of player i—shifting the graph of $U_i^s(p_i,\cdot)$ downward—and benefits the weak type of i—shifting the graph of $U_i^w(p_i,\cdot)$ upward. That is, due to the all-pay nature of the conflict, a strong type would hide, and a weak type would exaggerate, its strength in the conflict.

2.3.2 Interim Payoffs in Mediation

Given any proposal-PBE pair, denote q_i for player i's ($\forall i \in \{1,2\}$) ex ante probability (before realization of i's type) of choosing Reject, and p_i^A (resp. p_i^R) for the posterior probability of player i being type s conditional on i's having chosen Accept (resp. Reject) in response to the peace proposal. Given type $t \in \{w,s\}$ and anticipating the continuation payoff U_i^t in the event of conflict, player i's expected payoff from choosing Accept is equal to

$$V_i^A(t) := q_{-i}U_i^t\left(p_i^A, p_{-i}^R\right) + (1-q_{-i})x_i, \tag{2.4}$$

and that from choosing Reject is equal to

$$V_i^R(t) := q_{-i}U_i^t\left(p_i^R, p_{-i}^R\right) + (1-q_{-i})U_i^t\left(p_i^R, p_{-i}^A\right). \tag{2.5}$$

By Bayes's rule, we have $q_i p_i^R = (1-\theta)\sigma_i(s)$ and $(1-q_i)p_i^A = (1-\theta)(1-\sigma_i(s))$, with $\sigma_i(s)$ the equilibrium probability with which player i of type s chooses Reject. Sum the two equalities to obtain the next condition, called *Bayesian plausibility* in the information design literature.

$$q_i p_i^R + (1-q_i)\,p_i^A = 1-\theta. \tag{2.6}$$

Thus, the point $(1-\theta, V_i^R(t))$ is the convex combination between the two points on the graph of $U_i^t(p_i^R,\cdot)$ whose horizontal coordinates are p_{-i}^R and p_{-i}^A. This is illustrated by Figure 2.3, where the positioning of $p_{-i}^A \leq 1-\theta \leq p_{-i}^R$ comes from an intuitive fact that Reject (thereby triggering conflict) signals one's strength more than Accept does (Lemma A.2, Appendix A.1).

Remark 2.2 From Figure 2.3 the followings are obvious: (a) The interim payoff for the weak (w) type in the conflict is bounded from above by θ, and attains this upper bound when $p_i = 1$. (b) The interim payoff for the strong (s) type in the conflict is bounded from below by θ, and attains this lower bound when $p_i \geq p_{-i}$. (c) It follows from (b) that, in any proposal-PBE pair, the strong type of each player can always secure an interim payoff no less than θ through choosing Reject thereby triggering conflict.

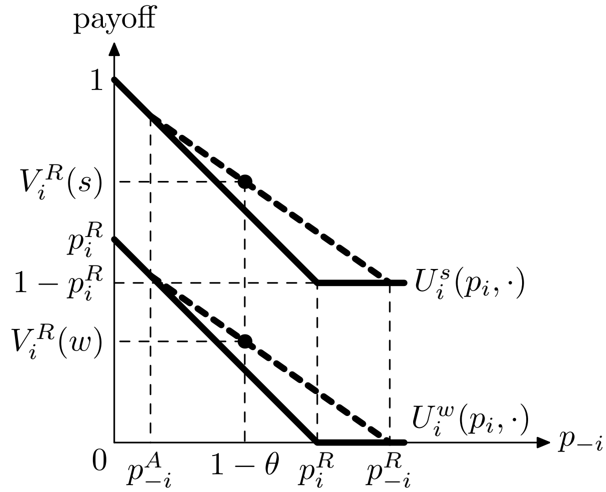


Figure 2.3: Interim expected payoffs as convex combinations

2.3.3 Lopsided versus Non-Lopsided Solutions

By *solution* we mean a pair of peace proposal (x_1, x_2) and a PBE $(\sigma_i, p_i^A, p_i^R)_{i=1}^2$ admitted by the game given the proposal, with $\sigma_i := (\sigma_i(w), \sigma_i(s))$ mapping player i's type $t \in \{w, s\}$ to a probability $\sigma_i(t)$ of choosing Reject, p_i^R the posterior probability that his type is strong (s) conditional on his having chosen Reject, and p_i^A the counterpart conditional on having chosen Accept. Given any solution, the ex ante probability q_i with which player i chooses Reject is determined. Thus we shall denote a solution by $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$.

One can show (Lemma A.1, Appendix A.1) that there are exactly two kinds of solutions in our model. One kind, called *lopsided*, consists of the solutions where one player accepts the proposal for sure regardless of his type, and the other player rejects it for sure if her type is strong, and mixes between Accept and Reject if her type is weak. The other kind, called *non-lopsided*, consists of all the solutions where Reject is a best response for both types of each player. Except for the trivial solutions where conflict occurs for sure (e.g., a player always rejects a proposal because he expects the same from the opponent), both Accept and Reject are on-path actions for each player in any non-lopsided solution. Roughly speaking, the proposed split of the prize in a non-lopsided solution is even-handed enough for each type of each player to find it a best response to reject it.

2.3.4 The Social Surplus in Any Non-Lopsided Solution

By *social surplus* we mean the sum of equilibrium ex ante expected payoffs (before realization of types) across the two players. Given any non-lopsided solution $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$, since Reject is a best response for both types of each player, the social surplus is equal to $\sum_{i=1}^2 \left(\theta V_i^R(w) + (1-\theta)V_i^R(s)\right)$. The next lemma provides a tractable formula for this sum.

Lemma 2.1 *In any non-lopsided solution $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ such that $p_i^R \geq p_{-i}^R$, the social surplus is equal to $2\theta p_i^R + (q_i - \theta)\left(p_i^R - p_{-i}^R\right)$.*

The lemma stems from the idea behind Figure 2.3. First, the condition $p_i^R \geq p_{-i}^R$ means that the positions of p_i^R and p_{-i}^R in that figure are switched, so that the figure becomes Figure 2.4 for the weak type, and Figure 2.5 for the strong type. Then one readily obtains

$$V_i^R(w) = p_i^R - 1 + \theta, \tag{2.7}$$

$$V_i^R(s) = \theta. \tag{2.8}$$

Second, note that $V_i^R(w) - V_{-i}^R(w)$ is equal to the length $|B'C'|$ of segment $B'C'$ in Figure 2.4. By similar triangles,

$$\frac{|B'C'|}{|BC|} = \frac{p_i^R - (1-\theta)}{p_i^R - p_i^A} = 1 - q_i,$$

where the second equality follows from the Bayesian plausibility condition (2.6). Since $|BC| = p_i^R - p_{-i}^R$, we have

$$V_{-i}^R(w) - V_i^R(w) = -(1-q_i)\left(p_i^R - p_{-i}^R\right). \tag{2.9}$$

Analogously, from inspection of Figure 2.5 and the similar triangles therein we obtain

$$V_{-i}^R(s) - V_i^R(s) = q_i\left(p_i^R - p_{-i}^R\right). \tag{2.10}$$

By Eqs. (2.9) and (2.10), due to the posterior probability difference $\Delta p^R := p_i^R - p_{-i}^R$, the interim payoff for player $-i$ is smaller than that for player i by $(1-q_i)\Delta p^R$ if both are of the weak type, and larger than that for player i by $q_i \Delta p^R$ if both are strong. Sum

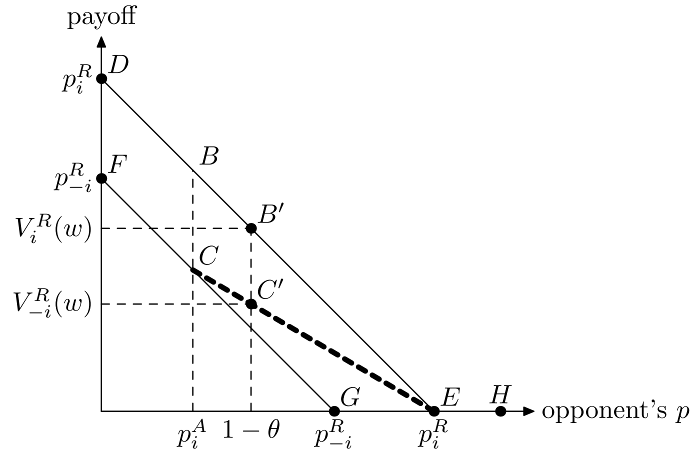


Figure 2.4: Rejection payoffs for the weak type

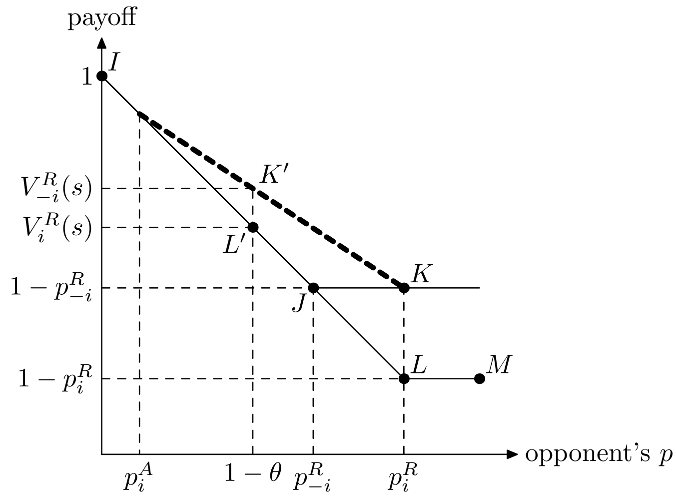


Figure 2.5: Rejection payoffs for the strong type

the two differences to obtain the ex ante payoff difference between the two players as

$$\begin{aligned}\left(\theta V_{-i}^R(w)+(1-\theta)V_{-i}^R(s)\right)-\left(\theta V_i^R(w)+(1-\theta)V_i^R(s)\right) &= \left(-(1-q_i)\theta+q_i(1-\theta)\right)\Delta p^R \\ &= (q_i-\theta)\Delta p^R.\end{aligned}$$

This, combined with (2.7) and (2.8), implies that $\sum_{i=1}^{2}\left(\theta V_i^R(w)+(1-\theta)V_i^R(s)\right)$ is equal to $2\theta p_i^R+(q_i-\theta)\left(p_i^R-p_{-i}^R\right)$.

2.4 The Optimality of a Lopsided Solution

A lopsided solution (defined in Section 2.3.3) has the advantage that one of the players chooses Accept independently of his own type. Thus, the player accepts the peace proposal without fearing that his acceptance may betray some information that the opponent may use against him later. Put differently, the mediator can manipulate the posterior about the other player's type without having to watch out for the posterior about the former player, as the on-path posterior about the former is fixed at the prior, and the off-path posterior can be chosen in whatever level that the mediator desires.

Choosing among the lopsided solutions, in which the favored player accepts the proposal for sure, the mediator would transfer a tiny bit of the share from the favored player to the other player thereby increasing the chance for the latter to accept the proposal as well. Since the strong type of a player can always secure an expected payoff no less than θ through Reject (Remark 2.2), the mediator cannot offer a share less than θ to the favored player and still guarantee acceptance from him. Thus the mediator would reduce the share offered to the favored player down to θ (note that θ is still the larger share due to (2.1)):

Proposition 2.1 *If $2/3 \leq \theta \leq 3/4$, the proposal that maximizes the sum of the ex ante expected payoffs for the two players (among all proposals that admit PBEs) is to offer θ to one player and $1 - \theta$ to the other player.*

Before proving the proposition, we make two remarks:

First, the assumption $2/3 \leq \theta \leq 3/4$ in the proposition, albeit partially relaxable with more calculations, reflects an intuition that the equal-split $(1/2, 1/2)$ proposal is likely to be optimal when θ is close to $1/2$ or 1. Since conflict can be fully preempted by the equal split when $\theta \leq 1/2$ (cf. Section 2.4.1 or Footnote 8), the equal-split proposal might remain optimal when θ is just slightly above $1/2$. When $\theta \approx 1$, the social surplus puts a heavy weight on the weak type, and one can show that the total expected payoff for the weak type of both players under the equal-split proposal is almost equal to the full size of the prize.[11]

Second, while the PBE in a lopsided solution involves an off-path posterior, with the favored player expected to accept the proposal always, the optimal solution stated in the proposition satisfies both the Intuitive and D1 criteria of refinement (Appendix A.2).

The proof of the proposition is essentially to show how the social surplus—the sum of ex ante expected payoffs for the two players—varies with the larger share $\max_i x_i$ offered in a peace proposal. The relationship between the two is depicted in Figure 2.6. When $\max_i x_i$ is larger than or equal to θ, the PBE is lopsided. Section 2.4.2 constructs the PBE

[11]In a PBE under the equal-split proposal, $p_1^R = p_2^R = 1/2$ (Lemma A.5, Appendix A.5) and hence each player's weak type gets $p_1^R - (1 - \theta) = \theta - 1/2$ (Figure 2.4). Thus the total expected payoff for them, $2\theta - 1$, converges to one as $\theta \to 1$.

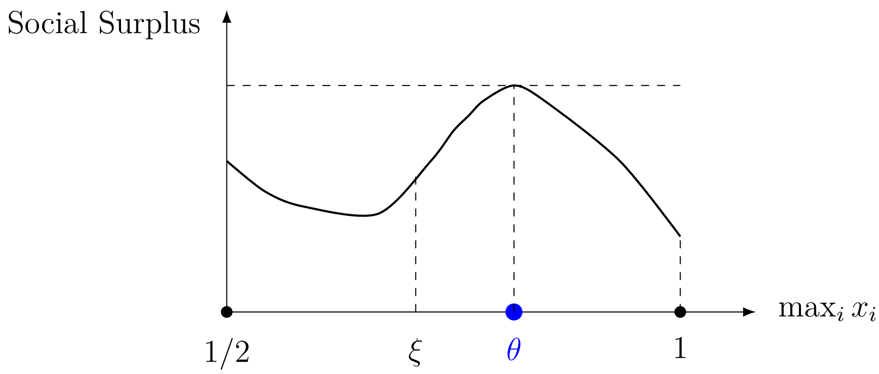


Figure 2.6: The lopsided proposal $(\theta, 1-\theta)$ as the global optimum

for any such lopsided proposal. Section 2.4.3 shows that any proposal with $\max_i x_i > \theta$ is outperformed by $\max_i x_i = \theta$. When $\max_i x_i$ is smaller than θ, the PBE is non-lopsided and converges to the lopsided one at $\max_i x_i = \theta$ when $\max_i x_i$ converges to θ from below. Section 2.4.4 shows that the graph is upward-sloping and concave on a sufficiently small interval (ξ, θ) to the left of θ. When $\max_i x_i$ falls below ξ, the corresponding PBE changes to other kinds of non-lopsided solutions. Section 2.4.5 outlines our treatments of those cases, the details of which are relegated to the appendix.

In the rest of the chapter, without loss of generality, let player 1 be the one offered θ in the proposal stated in the proposition. That is, write the proposal as $(\theta, 1-\theta)$.

2.4.1 A More Detailed Intuition

While the proof of the proposition involves lengthy calculations, the gist of the proof is an elaboration of the intuitive idea before the statement of the proposition: The advantage of lopsided solutions is that they provide more leeway for the mediator to manipulate the posterior beliefs. In a non-lopsided solution, both Accept and Reject being on-path for each player, each component of the posterior system $(p_i^A, p_i^R)_{i=1}^2$ is constrained by Bayes's rule. In a lopsided PBE, by contrast, Reject is off-path for one of the players, say player 1; hence the posterior probability p_1^R for this player to be the strong type conditional on his having played Reject is unconstrained by Bayes's rule.

How does such unconstrained posterior turn into a gain in the social surplus? Let us recall the main insight in the previous literature (e.g., Zheng, 2019b) that the optimal posterior system is to have $p_i^R = 1$, namely, taking Reject to mean that the player is the strong type for sure. As noted in Remark 2.2, with $p_i^R = 1$, the interim payoff from Reject is minimized to θ for the strong type of player i, and maximized to θ for the weak type of i. This would have constituted an optimal solution should each player be offered a share at least as large as θ so that each is willing to accept the proposal and the interim payoffs from Reject, being off-path, do not matter to the social surplus. Given our assumption $\theta > 1/2$, however, such proposals do not exist, as any split of the prize (of size one) renders the share for some player below θ. Thus, any PBE of any proposal sees some player choose Reject on path. Consequently, a player's interim payoff from Reject becomes part of the social surplus. This, coupled with the fact that an increase in p_i^R benefits the weak and hurts the strong (Remark 2.1), means that the calculus of p_i^R is more complicated than that in the previous literature.

Nonetheless, there are two intuitive reasons why the previous insight of achieving optimality through maximizing p_i^R might still work. First, since a strong type incurs less marginal cost in the conflict than a weak type does, one would expect that a strong type is more inclined than a weak type to reject a peace proposal. Thus, if we are to pick a type to deter it from Reject, it would be the strong type. To deter it from Reject, we minimize its interim payoff from Reject. Second, the assumption $\theta > 1/2$ implies that, from the ex ante viewpoint, any quantity of payoff to a weak type contributes more to the social surplus than the same quantity of payoff to a strong type does. Thus, one would expect that the social surplus enlarges with an increase in p_i^R, which benefits the weak at the expense of the strong. It is therefore conceivable that, the less constrained is p_i^R, the more can p_i^R be maxed out and hence the larger is the social surplus.

2.4.2 Construction of Lopsided PBEs

Consider any proposal (x_1, x_2) for which $x_1 \geq \theta$ (and hence $x_1 > 1/2 > x_2$). We shall construct a PBE where player 1 chooses Accept always, and player 2 chooses Reject for sure when the type is strong and mixes between Accept and Reject when the type is weak.

First, the strong type of player 2 chooses Reject for sure and gets an expected payoff equal to θ on path: Since player 1 always accepts the proposal on path, the posterior probability for player 1 to be the strong type remains to be the prior $1 - \theta$. Thus the strong type of player 2 gets θ from rejecting the proposal (Eq. (2.2) together with $p_2^R \geq 1 - \theta = p_1^A$), and she gets only $x_2 \leq 1 - \theta < \theta$ (since $x_2 = 1 - x_1$ and $\theta > 1/2$) from accepting it.

Second, the strong type of player 1 chooses Accept for sure. By playing Accept, his expected payoff is at least θ: If player 2 also accepts the proposal, player 1 gets the share $x_1 \geq \theta$ as offered; if player 2 rejects the proposal, the post-mediation payoff for the strong type of player 1 (in the conflict) is equal to θ (Eq. (2.2) coupled with $p_2^R \geq 1-\theta = p_1^A$). If player 1 deviates to Reject, the off-path posterior p_1^R can be chosen such that $p_1^R \geq p_2^R$, which according to Figure 2.5—or Eq. (2.8)—yields θ as the interim payoff for the strong type of player 1. Thus Accept is a best response for the strong type of player 1.

Third, the weak type of player 2 is indifferent between Accept and Reject, and she gets an expected payoff equal to $p_2^R - 1 + \theta$ on path. Since player 1 is expected to always choose Accept, player 2 gets the offered share x_2 from choosing Accept. If she plays Reject instead, player 2, of the weak type, gets the payoff $p_2^R - (1-\theta)$ that results from the conflict (Eq. (2.3) together with $p_2^R \geq 1-\theta = p_1^A$). Thus, the weak type of player 2 is indifferent between Accept and Reject if and only if

$$p_2^R = 1 - \theta + x_2. \tag{2.11}$$

Since the strong type of player 2 chooses Reject for sure, the Bayesian formula of p_2^R is $p_2^R = (1-\theta)/q_2$, namely, the ex ante probability q_2 for player 2 to choose Reject is equal to

$$q_2 = \frac{1-\theta}{1-\theta+x_2}.$$

Fourth, the weak type of player 1 chooses Accept for sure. His interim payoff from Accept is equal to $(1-q_2)x_1 = (1-q_2)(1-x_2)$: If player 2 chooses Accept, player 1 gets the offered share x_1; else player 1's post-mediation payoff in the conflict is equal to zero according to Eq. (2.3) and the fact $p_2^R \geq 1-\theta = p_1^A$. The interim payoff from Reject for the weak type of player 1 is equal to $p_1^R - (1-\theta)$: This follows from the condition $p_1^R \geq p_2^R$, which we use to incentivize the strong type of player 1 (the second step in the above), combined with Figure 2.4 (or Eq. (2.7)). Thus, for the weak type of player 1 to choose Accept, it suffices to have $(1-q_2)(1-x_2) \geq p_1^R - (1-\theta)$, namely,

$$p_1^R \leq 1 - \theta + \frac{x_2}{1-\theta+x_2}(1-x_2).$$

This, combined with the conditions $p_1^R \geq p_2^R$ and $p_2^R = 1-\theta+x_2$ explained above, implies that the PBE is valid if $x_2 \leq \theta/2$. Since $x_2 \leq 1-\theta$ by the choice of the proposal and $\theta \geq 2/3$ by the assumption of the proposition, $x_2 \leq \theta/2$ holds and the PBE is valid.

2.4.3 Why Raising the Larger Share above θ Is Suboptimal

First, observe that, for any proposal (x_1, x_2) such that $x_1 \geq \theta$, the social surplus based on the PBE constructed above is strictly increasing in p_2^R. The ex ante expected payoff for player 2 is strictly increasing in p_2^R because her on-path interim expected payoff is equal to $p_2^R - 1 + \theta$ when her type is weak (Step 3, Section 2.4.2), and θ when her type is strong (Step 1, Section 2.4.2). To see the same monotonicity property for player 1, observe that player 1—as long as he always plays Accept according to the equilibrium—prefers smaller q_2 (probability of player 2 choosing Reject) to larger q_2: If player 2 chooses Accept, player 1 gets $x_1 \geq \theta$; else player 1 gets θ if his type is strong (Step 2, Section 2.4.2), and zero if his type is weak (Step 4, Section 2.4.2); hence smaller q_2 makes player 1's ex ante expected payoff strictly larger. Thus, both players considered, the social surplus is maximized among all proposals (x_1, x_2) with $x_1 \geq \theta$ when p_2^R is maximized.

By (2.11), maximizing p_2^R is equivalent to maximizing x_2 subject to $x_1 \geq \theta$, namely, $x_2 \leq 1 - \theta$. Thus, social surplus attains its maximum at $x_2 = 1 - \theta$. That is, any proposal with $x_1 > \theta$ is outperformed by setting $x_1 = \theta$.

2.4.4 Why Perturbing the Larger Share below θ Is Suboptimal

When the larger share x_1 offered in the proposal falls below θ, the lopsided PBE ceases to exist. That is because the strong type of the favored player 1 can always secure an interim payoff at least θ from Reject, while Accept gives him an interim payoff less than θ: From Accept, he gets $x_1 < \theta$ if player 2 chooses Accept, and θ if player 2 chooses Reject (Step 2, Section 2.4.2). Consequently, the strong type of player 1 chooses Reject sometimes. Then the weak type of player 1 would chooses Reject sometimes as well. Otherwise, the action Reject from the player would reveal that he is for sure the strong type, $p_1^R = 1$. Given this posterior, the weak type of the player would deviate to Reject thereby getting an interim payoff equal to θ (Remark 2.2.a) rather than a smaller payoff from Accept.

Thus, when the larger offered share x_1 is less than θ, the lopsided PBE is not valid, and both types of the favored player 1 would mix between Accept and Reject. Meanwhile, player 2's strategy remains similar to those in the lopsided ones provided that the share x_2 offered to her is sufficiently near to $1 - \theta$: As in the lopsided PBE, she plays Reject for sure if her type is strong, and mixes between Accept and Reject if her type is weak. Specifically, when $x_2 < x_1 < \theta$ (hence $x_2 > 1 - \theta$) and x_2 is sufficiently close to $1 - \theta$, the PBE satisfies:

$$\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0, 1), \quad \sigma_2(s) = 1, \quad \text{and} \quad p_1^R \geq p_2^R. \tag{2.12}$$

Furthermore, as x_2 converges to $1-\theta$ from above, one can show that the PBE converges to the lopsided PBE under the proposal $(\theta, 1-\theta)$ (Eqs. (A.34)–(A.38), Appendix A.7.1). It follows that the social surplus under the proposal (x_1, x_2) for which $x_1 > x_2 > 1-\theta$ converges to the social surplus produced by $(\theta, 1-\theta)$ when x_2 converges to $1-\theta$ from above.

Therefore, to show that the proposal $(\theta, 1-\theta)$ outperforms any proposal (x_1, x_2) for which $x_1 > x_2 > 1-\theta$ and x_2 is sufficiently close to $1-\theta$, it suffices to show that the social surplus is strictly decreasing when x_2 enlarges from $1-\theta$ as long as (2.12) remains valid in the PBE. To that end, one can calculate the PBE $(\sigma_i, p_i^R, p_i^A, q_i)_{i=1}^2$ according to (2.12) (detailed in Lemma A.11, Appendix A.7.1) and then obtain

$$\begin{aligned} p_1^R &= \frac{3-2\theta-x_2}{2}, \\ p_2^R &= 2-2\theta, \\ q_1 &= \frac{2(\theta-1+x_2)}{2\theta+x_2-1}. \end{aligned}$$

(In the above displayed, the first equation signifies a main difference between non-lopsided PBEs and lopsided ones: In a lopsided PBE, there is no equation to constrain the posterior p_1^R of the favored player 1 when he chooses Reject.) By Lemma 2.1, the social surplus given any PBE $(\sigma_i, p_i^R, p_i^A, q_i)_{i=1}^2$ that satisfies (2.12) is

$$S(x_2) := 2\theta p_1^R + (q_1 - \theta)\left(p_1^R - p_2^R\right),$$

where we denote the social surplus as a function of x_2 because the variables on the right-hand side are each a function of x_2 according to the above-displayed equations. It suffices to show $\frac{d}{dx_2}S(x_2) < 0$ for any $x_2 > 1-\theta$ sufficiently close to $1-\theta$. To that end, use the above-displayed equations to obtain

$$\frac{d}{dx_2}S(x_2) = (q_1+\theta)\frac{dp_1^R}{dx_2} + \left(p_1^R - p_2^R\right)\frac{dq_1}{dx_2} = -\frac{q_1+\theta}{2} + \left(p_1^R - p_2^R\right)\frac{2\theta}{(2\theta+x_2-1)^2}.$$

In other words, an increase of x_2 (decrease of x_1) makes player 1 more willing to choose Reject. This has two opposite effects on the social surplus. On one hand, with player 1 more willing to choose Reject, Reject signals the strength of player 1 less and so p_1^R decreases (whereas in a lopsided solution the p_1^R, off path, is subject to no such influence), which reduces the social surplus by Lemma 2.1. On the other hand, player 1 choosing Reject more often means that q_1 increases, which enlarges the social surplus (by Lemma 2.1). Despite the countervailing effects, from the above equations one readily sees that $\frac{d}{dx_2}S(x_2)$ is strictly decreasing when x_2 increases, as $-(q_1+\theta)/2$, $p_1^R - p_2^R$ (nonnegative), and $\frac{dq_1}{dx_2}$ (nonnegative) are each strictly decreasing in x_2. Furthermore, from the above equations one can show

$$\lim_{x_2 \downarrow 1-\theta} \frac{d}{dx_2} S(x_2) = -\frac{1}{2\theta}\left((\theta-3)^2 - 5\right),$$

which is negative because the assumption $\theta \leq 3/4$ in the proposition implies $\theta < 3 - \sqrt{5}$. It follows that $\frac{d}{dx_2}S(x_2) < 0$ for all $x_2 > 1-\theta$ such that (2.12) holds, as desired.

2.4.5 Why Any Drop of the Larger Share below θ Is Suboptimal

When x_1 is further below θ, the PBE changes to other kinds of non-lopsided ones. First, since the further decrease of the share x_1 offered to the favored player 1 implies that he is willing to reject the offer more even if his type is weak, the posterior p_1^R of his type being strong drops further so that p_1^R is less than p_2^R. Thus the PBE changes from (2.12) to

$$\sigma_1(s), \sigma_1(w), \sigma_2(w) \in (0,1), \quad \sigma_2(s) = 1, \quad \text{and} \quad p_1^R < p_2^R. \tag{2.13}$$

Second, when x_1 is further lower so that it is near to (or equal to) 1/2 (the equal-split share), one of two changes happens: either the favored player 1 finds the share x_1 offered to him so near to the equal split that he rejects it for sure when his type is strong:

$$\sigma_1(w), \sigma_2(w) \in (0,1) \quad \text{and} \quad \sigma_1(s) = \sigma_2(s) = 1; \tag{2.14}$$

or the unfavored player 2 finds the share x_2 $(= 1 - x_1)$ offered to her large enough so that she mixes between Accept and Reject even when her type is strong:

$$\forall i \in \{1,2\}: \ \sigma_i(w), \sigma_i(s) \in (0,1). \tag{2.15}$$

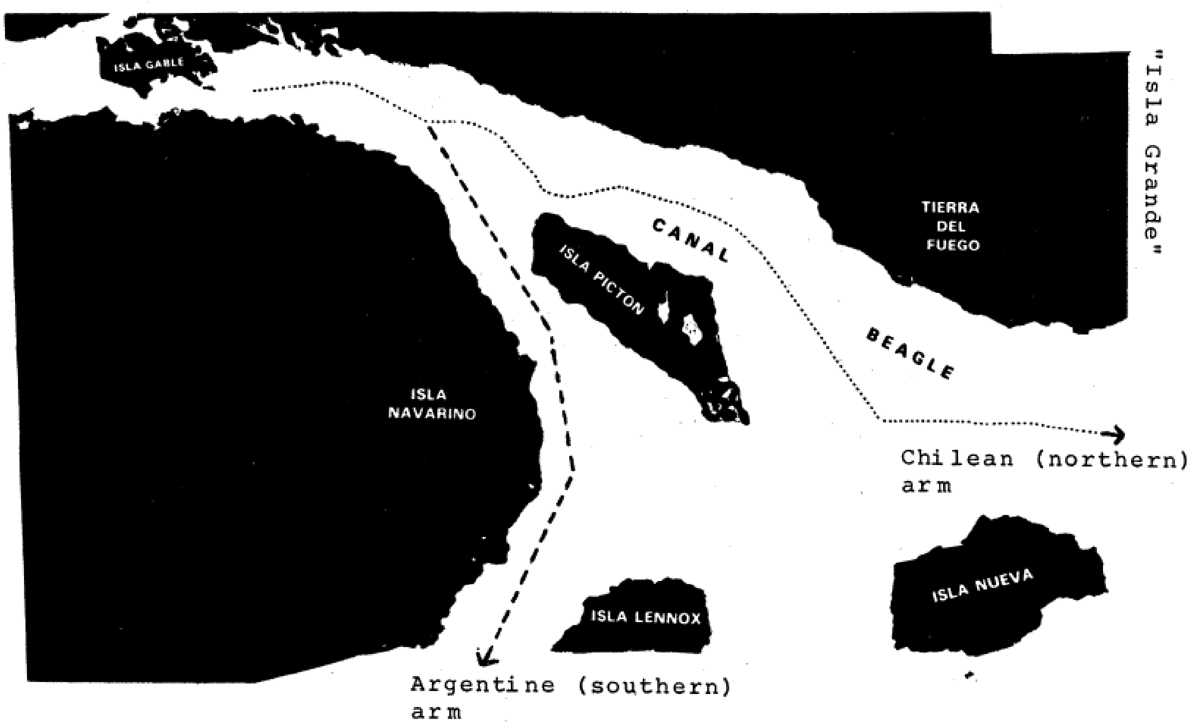
The PBEs of the form (2.13) are handled by Lemma A.14, Appendix A.7.2: For any solution in this case, the smaller share x_2 in the proposal is greater than $2\theta - 1$ and less than $1/2$. With $x_2 < 1/2$ and the assumption $2/3 \leq \theta \leq 3/4$ in the proposition, one can show $q_2 < \theta$. Consequently, the social surplus, by Lemma 2.1 applied to the case $i = 2$, is less than $2\theta p_2^R$. This quantity can be shown less than the social surplus generated by the optimal proposal $(\theta, 1-\theta)$, due to $x_2 > 2\theta - 1$.

The PBEs of the form (2.14) are handled by Appendix A.5. There, we show that the PBE of the form (2.14) under the equal-split proposal $(1/2, 1/2)$ maximizes the social surplus among all the solutions of the form (2.14). Then we show that the social surplus generated by this local maximum is still less than the one generated by the lopsided solution under $(\theta, 1-\theta)$ (where we use the assumption $\theta \leq 3/4$.)

The PBEs of the form (2.15) are handled by Appendix A.6. The method is similar to that for (2.13): One can show that PBEs of the form (2.15) satisfy $p_2^R \geq p_1^R$ and $q_2 < \theta$. Consequently, the social surplus, according to Lemma 2.1, is less than $2\theta p_2^R$. This quantity can be shown less than the social surplus under the optimal proposal $(\theta, 1-\theta)$, due to our assumption $\theta \geq 2/3$.

2.5 Example: The Beagle Channel Dispute

A lopsided peace proposal was crucial to the eventual peace settlement in the Beagle Channel Dispute between Argentina and Chile. The dispute centered on the claims by Argentina and Chile over the three islands south of Tierra Del Fuego and the surrounding waters. Figure 2.7 is a map from Garrett (1985) showing the area under dispute. The



Source: Republic of Argentina. Diferendo Austral. Texto del Tratado, 1984. Map IV.

Figure 2.7: Argentina claimed from the northeast down to the dashed line; Chile claimed from the southwest up to the dotted line.

dispute dated back to the Boundary Treaty of 1881, aimed at resolving all the boundary disputes between the two countries upon independence. However, it fell short of its aim for the Beagle Channel and both countries claimed sovereignty over it. Under the treaty all the area south of the Beagle channel would belong to Chile. However, the treaty was ambiguous on path of the Beagle Channel and where it should end. In Figure 2.7, the dashed line is what Argentine perceived as path of the channel, and the dotted line represents Chile's claim. The disputed area is rich in natural resources and is strategically important in defining the maritime zones and territorial claims in Antarctica.

After several decades of failed negotiations and arbitration, the two countries were on the verge of a war in December 1978 when Pope John Paul II intervened. The Pope launched a mediation effort and in December 1980 issued a proposal that awarded Chile all the three disputed islands and offered Argentina only the right, which it would need to share with Chile, of navigation and resource in the surrounding seas (cf. Garrett Garrett (1985), and Greig and Diehl Greig and Diehl (2012)). This is clearly a proposal lopsidedly favoring Chile. Meanwhile, the proposal did not grant Chile as much navigation and resource right as the legal maritime zone would have if it were implied by Chile's ownership of the three disputed islands. That is consistent with our theoretical finding that the optimal peace proposal, while offering a larger share of the prize to one side, still keeps the larger share in check.

Chile accepted the Pope's proposal immediately, which is understandable in light of our theoretical insight that the party favored by a peace proposal is willing to accept it without fearing that its acceptance might betray its weakness. Argentina, also understandably in light of our theoretical finding that the unfavored party would mix between Reject and Accept, was initially reluctant to accept. After several years of ups and downs, the two country agreed to the Vatican proposal and signed a peace treaty in November 1984 thereby officially ending the Beagle Channel Dispute (ibid).

It is reasonable to regard the Pope as the neutral, benevolent mediator in our model, the majorities of both countries in the Beagle Dispute identifying as Roman Catholic. Also similar to the mediator with limited power in our model, the Pope had no resource to enforce his decision on the two countries, nor any control over the military conflict had it erupted. The main instrument the Pope resorted to was to propose a split of the disputed region and request the two countries to respond to the proposal by a certain date (mid-January 1981, ibid). That resembles the peace proposal in our model.

2.6 Conclusion

Humanity is often trapped in conflict situations where full preemption of conflict is impossible. In such situations, it is inadequate for a benevolent social planner to aim merely at minimizing the likelihood of conflict, as the social welfare in both the event of peace and the event of conflict should be taken into account. This chapter contributes to the conflict mediation literature by incorporating both conflict and peace into maximization of social surplus and presenting an explicit solution for the maximization problem. Our solution respects a realistically relevant constraint that a mediator, restricted in instruments, cannot effect any information structure deemed desirable with tailor-made communication mechanisms, but rather can only indirectly influence the outcome through simple mechanisms whose integrity is easy to trust. Thus, techniques in the information-design literature are not readily available, and this chapter contributes an explicit analysis on how a mediator can nonetheless achieve a constrained optimal posterior information structure given simple, message-independent mechanisms.

Our solution produces a surprising implication: Even though the adversaries are ex ante identical, and are assigned equal welfare weights, the socially optimal peace proposal is to lopsidedly favor one adversary against the other. Thus it should not be taken for granted that a peace proposal should offer a fair share to each contestant even from the viewpoint of a benevolent mediator. The insight conveyed by our result is that a peace proposal lopsidedly biased towards one side may, counterintuitively, achieve better social welfare than an unbiased one because the favored side is willing to accept the peace deal without fearing being viewed to be weak and taken advantage of later, so that the mediator can devote more resources to compensate the unfavored side.

While the design objective considered in this chapter is to maximize the social surplus, which incorporates the players' ex ante payoffs in both peace and conflict, the social welfare merit of a lopsided peace proposal demonstrated by our result is extendable to models where the design objective is to minimize the probability of conflict. In fact, given the same intermediary range of the weak-type probability θ for which the lopsided proposal maximizes social surplus, one can show that the lopsided proposal also minimizes the probability of conflict. In addition, the equal-split proposal minimizes the probability of conflict when the probability of being weak is very high or when it is low enough to be near to the region where peace can be guaranteed. This is similar to the pattern when the objective is to maximize the social surplus (cf. the remark below the statement of the Proposition).

An open question is what happens if a contestant can renege on its acceptance of a peace deal. After Iran accepted the nuclear deal in 2015, the United States withdrew from the agreement in 2018 thereby resuming the hostile relationship. It is conceivable that Iran, in retrospect, would attribute the US withdrawal to Iran's acceptance of the deal in 2015, which might have revealed Iran's weak position in the conflict. That taken into account, Iran will be more reluctant to accept any nuclear deal in the future than before, for fear of its weakness being further revealed and exploited. Thus we conjecture that the inscrutability of a contestant's response to a peace proposal can only become more important when contestants may renege. In the sense that a lopsided solution guarantees acceptance from the favored side thereby making its private information inscrutable from its acceptance, the optimality of lopsided solutions may be robust to such limited commitment situations.

For tractability, and for a clear contrast with the lopsided solution, we assume that the two contestants are ex ante identical with a common value of the contested prize. A natural question is to what extent a lopsided solution may remain optimal when ex ante asymmetry or private values are considered. While we conjecture that the inscrutability advantage that a lopsided solution provides for the favored party remains crucial, the ex ante asymmetry between the two sides is likely to bring about new questions such as which side should be favored and which side could benefit more from being inscrutable during negotiation.

Bibliography

Aumann, R. J. and M. Maschler (1985). Game theoretic analysis of a bankruptcy problem from the talmud. *Journal of economic theory 36*(2), 195–213.

Baliga, S. and T. Sjöström (2004). Arms races and negotiations. *The Review of Economic Studies 71*(2), 351–369.

Baliga, S. and T. Sjöström (2020). The strategy and technology of conflict. *Journal of Political Economy 128*(8), 3186–3219.

Balzer, B. and J. Schneider (2021a). Managing a conflict: optimal alternative dispute resolution. *The RAND Journal of Economics 52*(2), 415–445.

Balzer, B. and J. Schneider (2021b). Persuading to participate: Coordination on a standard. *International Journal of Industrial Organization 78*, 102764.

Baun, M. J. (1995). The maastricht treaty as high politics: Germany, france, and european integration. *Political Science Quarterly 110*(4), 605–624.

Bester, H. and K. Wärneryd (2006). Conflict and the social contract. *Scandinavian Journal of Economics 108*(2), 231–249.

Burton, M. (2021). *From Broke to Brexit: Britain's Lost Decade*. Springer International Publishing.

Celik, G. and M. Peters (2011). Equilibrium rejection of a mechanism. *Games and Economic Behavior 73*(2), 375–387.

Compte, O. and P. Jehiel (2009). Veto constraint in mechanism design: inefficiency with correlated types. *American Economic Journal: Microeconomics 1*(1), 182–206.

Doval, L. and A. Smolin (2021, September). Information payoffs: An interim perspective. CEPR Discussion Paper No. DP16543.

Feerick, J. D. (2003). The peace-making role of a mediator. *Ohio St. J. on Disp. Resol. 19*, 229.

Fey, M. and K. W. Ramsay (2011). Uncertainty and incentives in crisis bargaining: Game-free analysis of international conflict. *American Journal of Political Science 55*(1), 149–169.

Garrett, J. L. (1985). The beagle channel: Confrontation and negotiation in the southern cone. *Journal of Interamerican Studies and World Affairs 27*(3), 81–109.

Greig, J. M. and P. F. Diehl (2012). *International mediation*. Polity.

Hörner, J., M. Morelli, and F. Squintani (2015). Mediation and peace. *The Review of Economic Studies 82*(4), 1483–1501.

Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *American Economic Review 101*(6), 2590–2615.

Keniston, D., B. J. Larsen, S. Li, J. J. Prescott, B. S. Silveira, and C. Yu (2021). Fairness in incomplete information bargaining: Theory and widespread evidence from the field. Working Paper 29111, National Bureau of Economic Research.

Kershner, I. (2022, March 6). Israel's prime minister says he wants to keep both sides talking, but holds out little hope of peace. *The New York Times*.

Kydd, A. (2003). Which side are you on? bias, credibility, and mediation. *American Journal of Political Science 47*(4), 597–611.

Lasensky, S. (2004). Paying for peace: The oslo process and the limits of american foreign aid. *The Middle East Journal 58*(2), 210–234.

Le Treust, M. and T. Tomala (2019). Persuasion with limited communication capacity. *Journal of Economic Theory 184*, 104940.

Lu, J., Z. Lu, and C. Riis (2021, July 27,). Peace through bribing. Working Paper, National University of Singapore.

Mathevet, L., J. Perego, and I. Taneva (2020). On information design in games. *Journal of Political Economy 128*(4), 1370–1404.

Meirowitz, A., M. Morelli, K. W. Ramsay, and F. Squintani (2019). Dispute resolution institutions and strategic militarization. *Journal of Political Economy 127*(1), 378–418.

Rauchhaus, R. W. (2006). Asymmetric information, mediation, and conflict management. *World Politics 58*(2), 207–241.

Schelling, T. C. (1980). *The Strategy of Conflict*. Harvard university press. The 1980 edition.

Smith, A. and A. Stam (2003). Mediation and peacekeeping in a random walk model of civil and interstate war. *International Studies Review 5*(4), 115–135.

Spier, K. E. (1994). Pretrial bargaining and the design of fee-shifting rules. *The RAND Journal of Economics*, 197–214.

Wilkenfeld, J., K. Young, D. Quinn, and V. Asal (2007). *Mediating international crises*. Routledge.

Yaari, M. E. and M. Bar-Hillel (1984). On dividing justly. *Social choice and welfare 1*(1), 1–24.

Zheng, C. Z. (2019a). Bidding collusion without passive updating. *Journal of Mathematical Economics 85*, 70–77.

Zheng, C. Z. (2019b). Necessary and sufficient conditions for peace: Implementability versus security. *Journal of Economic Theory 180*, 135–166.

Chapter 3

Favoritism in Manipulative Conflict Mediation

3.1 Introduction

International conflicts among countries are frequently subject to third-party mediation, the goal of which is usually conflict resolution or mitigating the probability of conflict. The mediator of such conflicts faces two important challenges. First, conflicting parties are sovereign countries, and the mediator cannot enforce her decision on them. Second, conflict mediation is informative. Adversaries fear that by participating in the mediation and announcing their decisions they might reveal information that might be used against them in the conflict. This chapter is interested in a situation where two adversaries are having a dispute over a prize and face the possibility of conflict if they cannot settle their dispute. A mediator, who can put forward economic incentives and threats, proposes a split of peace surplus of an agreement that if the players do not agree on conflict occurs. Participation in the mediation is voluntary and nonparticipation triggers conflict. The mediator's objective is to design proposals that minimize the probability of conflict among those that guarantee the full participation of players in her mechanism. Although full participation, in general, cannot be assumed without loss of generality, it is a realistic assumption to make in the context of international conflicts as part of the practical and political motivations of a mediator.[1] In such an environment, this chapter asks should a mediator shows favoritism among ex-ante identical rivals to minimize the probability of conflict subject to the full participation constraint?

[1]In Henry Kissinger words "A reputation for success tends to be self-fulfilling. Equally, failure feeds on itself: A Secretary of State who undertakes too many journeys that lead nowhere depreciates his coin."(Kissinger, 2011).

To attain conflict minimizing goal, the mediator should somehow make conflict, which happens if the mediation breaks down, more costly. But conflict rules are beyond the mediator's control. The mediator can only indirectly, through the manipulation of posterior beliefs that players form about each other after observing each others' decisions, affect players' strategies in the conflict. A common intuition is that the less disparity between the players and the more symmetric players' beliefs about each other types the larger the cost of conflict.[2] Asymmetry between players can make conflict more appealing: the player that looks stochastically dominant would be complacent and the stochastically dominated one would be intimidated leading to devoting fewer costly efforts and resources by both to the conflict. Therefore, if players are ex-ante identical, it seems intuitive that the mediator offers unbiased proposals: (i) directly, biased proposals could increase the chance of rejection by the less favored player, (ii) indirectly, biased proposals could induce asymmetry between players; making the outside option appealing.

If the peace surplus from the agreement is not large enough to fully preempt conflict, then any proposal will sometimes be rejected by at least one player. In such situations, players anticipate that by participating and making decisions at the mediation they might reveal information that can be used against them once the mediation fails. This can render participation more costly compared to nonparticipation that shuts down such communications. The mediator should take into account this information revelation effect in the design of her fully participating conflict minimizing proposals. Therefore, full participation constraint poses important challenges to the mediator's design of conflict minimizing proposals. With nonparticipation off-path in any fully participating equilibrium, the deviating player will not learn anything about her opponent while her opponent, who observes this off-path behavior, will form an arbitrary belief about the deviating player's type distribution. The off-path posterior belief information structure can take any arbitrary asymmetric form while, given the ex-ante identical players, the equal proposal admits symmetric posterior beliefs information structure for both players. As explained above, asymmetric off-path posterior beliefs can make nonparticipation, which triggers conflict, appealing. Can the equal proposal by inducing symmetric information structure makes nonparticipation more appealing? If yes, are there biased proposals that can induce asymmetric information structure and make players better off compared to nonparticipation? Can the indirect information effect, i.e., the asymmetric beliefs, override the direct equal treatment effect and leads to a higher probability of peace?

To answer these questions, this chapter considers a conflict mediation model where two players are disputing over a prize for which each has a private valuation drawn from

[2] This intuition is rooted in the theory of the Balance of Power in Political Science literature and the Economics literature in which conflict is modeled as an all-pay auction game.

an i.i.d binary distribution, i.e., players are ex-ante identical. They have either a high or low valuation for the prize. A neutral mediator announces a peace proposal that is a type-independent split of the peace surplus of an agreement. Proposals can be deterministic or stochastic. The primitives of the model are so chosen that the mediator cannot fully preempt conflict: there does not exist any peace proposal that admits a perfect Bayesian equilibrium (PBE) in which conflict occurs with zero probability. If parties do not participate in the mediation or do not agree on the peace proposal, conflict ensues. Conflict is costly for both and winner-take-all; whoever devotes the highest level of effort and resources wins. Since the two players are ex-ante identical, and the mediator is neutral, the unbiased proposal is the equal split of the peace surplus. Any other proposals mean favoritism toward the player who is receiving the lion's share.

The mediator considers proposals admissible if the ensuing multistage game admits a fully participating PBE. Albeit stylized, our model highlights the important informational ramifications of the mediation by allowing the mediation procedure to affect the posterior belief information structure at the conflict stage and the strategies that are pursued at it.

I examine the set of all equilibria, each consisting of a peace proposal and a PBE of the continuation game given the proposal. Participation in the mediation for a player depends on the payoff from deviating to nonparticipation and triggering conflict. The payoff at this zero probability event is endogenous and depends on beliefs at this event, which for the deviating player does not follow Bayes's rule. To provide a full characterization of admissible peace proposals, I examine players' nonparticipation incentives given all possible off-path beliefs. The presence of a type-dependent outside option poses a challenge. The worst and the best off-path belief about the deviating player depends on her type. The high type benefits from being perceived as a "weak" player, i.e., being perceived to be a low type with a higher probability than the prior probability, and the low type benefits from being perceived as a"strong" player, i.e., being perceived to be a low type with a lower probability than the prior. In brief, the participation constraint is not necessarily monotone. Therefore, the type for which the participation constraint binds cannot be identified a priori.[3]

This chapter's results show that if the probability of being a low type is high and the peace surplus from the agreement is lower than a threshold, then admissible conflict minimizing proposals are a specific category of biased proposals, which I call lopsided

[3]Albeit in a different environment, the non-monotone participation constraint resembles the countervailing incentives literature on the adverse selection with exogenous type-dependent outside options. For further references see Jullien (2000), Lewis and Sappington (1989), and Maggi and Rodriguez-Clare (1995).

proposals, and if the peace surplus is above the threshold, then the equal proposal minimizes the probability of conflict (Theorem 3.1). The lopsided proposals are such that they all admit the same probability of peace. Given these proposals, in equilibrium, the favored player always accepts and thereby does not reveal any information about her type, while the less favored player follows a fully revealing strategy to reject if she is a high type and accept if she is a low type.

The deterministic proposals are admissible if the off-path posterior about the deviating player is that she is a low type with a lower probability than the prior. However, if the reverse holds, then the high type of a player will benefit from being perceived as a low type with a higher probability than the prior upon nonparticipation. In that case, the equal proposal is not admissible because it reveals too much information and lowers the interim payoff of the high type. The lopsided proposals, by not revealing any information about the favored player, make her better off compared to nonparticipation. However, these proposals by admitting a fully revealing strategy for the less favored player, harm her and provide her high type an incentive to not participate. This asymmetric revelation of information provides room for the mediator to make these proposals admissible by using an equal probability randomization device (Theorem 3.2). Intuitively, randomization between the roles of the two players helps to subsidize the type that is hurt by participation through taxing the type that gains from the participation. The equal proposal by treating the two players equally does not provide such a possibility for the mediator, while the lopsided proposals do.

The mediation style studied in this chapter is sometimes referred to as manipulative mediation (Zartman, 2007). This is the most intensive form of mediation where the mediator not only facilitates discussions and develops peace proposals but also actively uses its resources to leverage an agreement with incentives like financial aid or threats like economic sanctions. According to the International Crisis Behavior Project, almost forty percent of conflicts mediated in 1918-2001 were conducted by such powerful mediators (See Wilkenfeld et al., 2007). The examples are many, among which is the Camp David Accords mediated by the United States between Egypt and Israel in 1978.[4] The Camp David mediation agenda included several issues including the dispute over the Sinai Peninsula, which Israel had captured during the Six-Day War in 1967, security arrangements between the two countries, and the Israel-Palestine conflict. The media-

[4]Other examples for further reference are the numerous events that the United States mediated between Israel and Palestine such as the Oslo Process (Lasensky, 2004) as well as the most recent peace plan proposed in 2020, the numerous treaties mediated by major powers in the nineteenth century Europe at the time known as the Concert of Europe like the Treaty of Constantinople (1832) between the Ottoman Empire and Greece (Brewer, 2011), and the Dayton Accords (1995) that end the Bosnian War.

tion ended in a peace agreement which led to the normalization of relations between the two countries and the withdrawal of Israel from the Sinai Peninsula. The economic aid promised by the USA was calculated roughly to be $2 billion in the case of Egypt and $ 3 billion for Israel annually (Quandt, 2015). The agreement can be seen as arguably a lopsided proposal where the contested prize, the Sinai Peninsula, is recognized in the full ownership of Egypt.

As a robustness check on these surprising results, I extend the analysis to a continuum of types and study monotone cutoff mediation strategies. I find that equal proposal is not admissible (Theorem 3.3). This holds for a non-degenerate set of off-path beliefs including the passive updating assumption, i.e., whenever a player observes nonparticipation behavior she maintains her prior belief about the opponent. I find that depending on the assumption on the off-path beliefs, either the high types or mid-range types of a player have incentives to not participate. Given mild assumptions on the prior's CDF that ensure its concavity, stochastic lopsided proposals are implementable (Theorem 3.4). Given such proposals, the favored player always accepts, and the less favored player follows a monotone cutoff strategy where she accepts if her type is below a threshold and rejects if above. Thus, the mechanism is not too revealing.

To study the information effect of mediation, it is germane that conflict is viewed as an endogenous outcome. The economics literature in studying conflict management (Baliga and Sjöström, 2020, Bester and Wärneryd, 2006, Compte and Jehiel, 2009, Fey and Ramsay, 2011, Hörner et al., 2015, Meirowitz et al., 2019, and Spier (1994)) assumes that the outcome of conflict is determined by an exogenous lottery, abstracting from the information externality of mediation on the conflict's payoff. Among them, Hörner et al. (2015), in a binary type environment and using a more general class of mechanisms, show that a mediator, whose objective is conflict minimization, offers an equal split of the prize when the players' types are the same, whereas in this chapter I show that equal proposal is not always implementable.

The closest papers in the literature to this one are Balzer and Schneider (2019, 2021a), Kamranzadeh and Zheng (2022), Lu et al. (2021a), and Zheng (2019b). Given a binary type distribution, Balzer and Schneider consider the problem of an arbitrator with enforcement power and a mediator who wants to minimize the probability of litigation. The mediator makes separate confidential proposals to the players. Allowing for more general classes of mechanisms and conflict games, they characterize optimal proposals in terms of optimal information structure once the mediation fails. In my model, the mediator's proposals are public, which means players can learn from both accept and reject decisions of their opponent, which reduces the mediator's control of the information structure once mediation fails.

Kamranzadeh and Zheng (2022) consider the problem of a mediator who proposes a peaceful split of a common value prize to players. With social surplus maximization as the objective, they show suboptimality of the equal proposal when the type distribution is binary. In my model, participation in the mediation is voluntary which in turn affects the admissibility of the equal proposal. Zheng (2019b) identifies necessary and sufficient conditions, in terms of the prior distributions, under which there exists a mechanism for the mediator to fully preempt conflict. In the current chapter, the prior distribution does not satisfy those conditions and the mediator cannot propose any mechanism that would fully avoid conflict. Lu et al. (2021a) study similar conditions for peace as Zheng (2019b) but in a setup where one player, instead of a mediator, proposes a peace proposal to the other.

In the received literature on mechanism design when the outside option is type-dependent or endogenous, it is not without loss of generality to restrict attention to full participation in the mechanism even for general negotiation mechanisms (Jullien, 2000, Celik and Peters, 2011). Similarly, in the conflict mediation literature with an endogenous outside option, full participation is not implied by the revelation principle. The exceptions are Zheng (2019b) which focuses attention on peace guaranteeing equilibria, which are not mathematically feasible in the current chapter, and Balzer and Schneider (2021a) where the arbitrator has enforcement power.

The full participation objective of the mediator is related to the mechanism design literature on bidding collusion (Balzer and Schneider, 2021b, Lu et al., 2021b, Pavlov, 2008, and Zheng, 2019a), where if privately informed firms cannot agree on a collusion mechanism, a default game is triggered. To study players' incentive to collude, one must calculate the default game payoffs. In my model, the mediator lacks enforcement power and the primitives are so chosen that the default game cannot be fully preempted. Moreover, I consider a non-degenerate set of off-path beliefs that include both passive updating and extreme off-path beliefs, usually assumed in this literature.[5]

The conflict game in my model is analogous to independent private value all-pay auction. In solving the continuum of type all-pay auction game, I extend the approach introduced in Amann and Leininger (1996) and Kirkegaard (2008). Amann and Leininger solve a class of two bidder all-pay auctions in which types are drawn from different distributions with common type space. Kirkegaard solves similar games with partially overlapping type spaces that have different upper bounds. I extend their analysis to

[5]The exception is Zheng (2019a) that characterizes the possibility of collusion given the off-path belief most conducive to it.

handle all asymmetric cases that arise endogenously in my model, where the posteriors are a truncated distribution of the prior, type spaces overlap, and have different lower bounds and upper bounds (Appendix B.7.2).

Political science literature also studies the effectiveness of a biased mediator (see Kydd, 2003 and Kydd, 2006, Smith and Stam, 2003, and Rauchhaus, 2006). They consider the mediator as a strategic player with biased preferences among disputants or the issue at stake. Whereas, in my model, the mediator is neutral and favoritism arises endogenously.[6]

I shall present the model and preliminary analysis in Section 3.2, report the findings and intuitions for the binary type distributions in Section 3.3 and the continuum of types distribution in Section 3.4. Appendix B presents the formal arguments and calculation details.

3.2 The Model

Two players, named 1 and 2, compete for a prize. Each player has a private valuation, or type, for the prize. Each player's type is independently drawn from the same binary distribution, whose realization is either a ("low"), with probability θ, or 1 ("high") with probability $1-\theta$, such that $\theta \in (0,1)$, $0 < a < 1$.[7] After privately learning their types, players simultaneously and publicly announce whether they participate in the *mediation*. Conditional on participation, if a peace proposal is accepted by both players, they avoid conflict. If at least one player chooses nonparticipation or if no peace proposal is accepted by both, then the game enters a *conflict* stage.

The conflict is a winner-take-all. Each player i, after observing the announced actions of both, simultaneously chooses a level of effort $b_i \in \mathbb{R}_+$ to devote to conflict. The player that exerts the greatest efforts wins the prize, with ties broken randomly with equal probabilities; the payoff for player i of type t_i is equal to $t_i - b_i$ if i wins, and equal to $-b_i$ otherwise.

At the mediation, a neutral mediator makes a type independent *peace split*

$$\nu := (\nu_1, \nu_2) \in [0, S]^2 \quad \text{such that} \quad \nu_1 + \nu_2 = S, \tag{3.1}$$

[6]The political science literature on conflict mediation is vast and cannot be done justice here. For a recent review of bargaining models of war, see Ramsay (2017), Kydd (2010), and Baliga and Sjostrom (2013). For further reference, including empirical literature, see Wilkenfeld et al. (2007).

[7]Normalizing one of the types to 1 is without loss of generality because only the ratio of the types matter. It helps to clean up the calculations.

where S is the expected surplus of a multi-dimensional peace agreement and (ν_1, ν_2) are interpreted as the splits of the peace surplus between two players. The mediator can augment her mechanism with an equal probability randomization device.

At the mediation, each player simultaneously announces whether she will Accept or Reject the proposal. If both accept, the game ends with player i getting a payoff equal to ν_i. If at least one of them rejects the proposal, then the game enters the conflict stage.

Once the mediator has chosen a mediation mechanism in the form of a (stochastic) peace proposal, the ensuing multistage game is defined for which I use perfect Bayesian equilibrium (PBE) as the solution concept. The mediator's objective is to minimize the probability of conflict subject to the full participation of players in her mechanism. Hence, nonparticipation is an off-equilibrium path event.

A PBE is said to be *fully participating* if and only if on its path deviation to nonparticipation occurs with zero probability relative to the prior distribution. A (stochastic) peace proposal is said to be *admissible* if and only if the multistage game given the proposal admits a fully participating PBE. A peace proposal is *optimal* if it minimizes the probability of conflict subject to admissibility.

The following assumption about the parameters is maintained throughout this chapter:

$$0 < S < 2\theta(1-a). \tag{3.2}$$

As demonstrated in Zheng (2019), the minimum peace proposal that is required to guarantee peace in our game is $\theta(1-a)$, i.e., once each player is offered this peace split, they will Reject it with zero probability on the path of equilibrium. Hence, if $S = 2\theta(1-a)$, then an equal split that offers $\theta(1-a)$ to each player guarantees peace. The above assumption on the parameters is to avoide the triviality of the problem.

3.2.1 The Continuation Equilibrium during Conflict

Given any proposed split (ν_i, ν_{-i}), let $\sigma_i(\nu_i; t)$ denote the probability with which player i of type t rejects the proposal ($\forall i \in \{1,2\}, \forall t \in \{a,1\}$) in the mediation stage. Given any strategy profile $(\sigma_i)_{i=1}^2$, one can obtain player i's ex-ante probability q_i^A of accepting the proposal (before realization of her type), and his probability q_i^R of rejecting it at a fully participating PBE:

$$\begin{aligned} q_i^A &= \theta\left(1-\sigma_i(\nu_i;a)\right) + (1-\theta)\left(1-\sigma_i(\nu_i;1)\right), && (3.3) \\ q_i^R &= \theta\sigma_i(\nu_i;a) + (1-\theta)\sigma_i(\nu_i;1). && (3.4) \end{aligned}$$

Denote π_i^A (resp. π_i^R) for the posterior probability of "$t = a$" conditional on i's accepting (resp. rejecting) the proposal. By Bayes's rule,

$$\pi_i^A q_i^A = \theta\left(1 - \sigma_i(\nu_i; a)\right), \tag{3.5}$$

$$\pi_i^R q_i^R = \theta\sigma_i(\nu_i; a). \tag{3.6}$$

The posteriors satisfy the Bayes's consistency condition, which means that the expected posteriors equal to the prior

$$q_i^A \pi_i^A + q_i^R \pi_i^R = \theta. \tag{3.7}$$

To understand the working of the model, I start with the last stage which is the conflict stage. Whether this stage is entered because someone rejected the proposal, or one deviated and did not participate in the mediation, the game is the same. Each player knows their type, the history of the game, and has a posterior belief about the rival's type based on that history. Denote π_i for the posterior probability of player i being type $t_i = a$. Denote $\mathscr{G}\left(\pi_i, \pi_{-i}\right)$ for the continuation game at the conflict stage such that π_i is the posterior distribution of player i's type for each $i \in \{1, 2\}$. At any $\mathscr{G}\left(\pi_i, \pi_{-i}\right)$ each player simultaneously chooses a level of effort/resources b_i to devote to the conflict and the outcome is determined. This conflict game is analogous to an independent private value all-pay auction where each player submits a sealed bid $b_i \in \mathbb{R}_+$.

The solution to such games is well known and is solved in Kamranzadeh and Zheng (2022).[8] There it has been shown that one can characterize the cumulative distribution function (CDF) of player i's effort at the BNE of the conflict game. In any BNE, the union of all types' efforts' (bidding) support is a bounded interval that is the same for each player, and both players mix down to zero. There is a unique monotone BNE, where the high type of each player exerts higher effort than the low type, i.e., the interior of the support of the effort distribution is disjoint and the high type's support ranges over higher effort levels than the low type. There will be common maximal effort; at most one atom at zero; and no gap. Thus, given any pair $(\pi_i)_{i=1}^2$ of posterior probabilities, the expected payoff for each player-type in the continuation game of conflict is determined according to the next lemma:

Lemma 3.1 *(Kamranzadeh and Zheng (2022)) Given any pair $(\pi_i)_{i=1}^2$ of posterior probabilities at the start of the conflict stage, the expected payoff for each player i at any*

[8] For a general treatment of all-pay auction games given any arbitrary type distribution see Zheng (2019b).

Bayesian Nash equilibrium in the conflict stage is equal to

$$U_i^1(\pi_i, \pi_{-i}) = (1-a)\max\{\pi_i, \pi_{-i}\}, \tag{3.8}$$
$$U_i^a(\pi_i, \pi_{-i}) = a(1-a)\left(\pi_{-i} - \pi_i\right)^+. \tag{3.9}$$

For player i, given a belief about her opponent type distribution π_{-i}, the expected payoff of $t = a$ is weakly decreasing in π_i and that of $t = 1$ is weakly increasing in π_i. Player i is referred to as "weak", and player $-i$ as "strong", if $\pi_i > \pi_{-i}$.

This chapter studies admissible peace proposals as defined above. Nonparticipation is an off-path event that triggers conflict. This outside option is endogenous and depends on the beliefs over the distribution of types. Suppose player $-i$ unilaterally deviates and does not participate in the mediation. Denote the off-path belief upon nonparticipation by $\tilde{\pi}_{-i}^{np}$. In that event, Bayes's rule implies $\pi_i = \theta$, as player i participates in the mechanism almost surely. $\tilde{\pi}_{-i}^{np}$ is off-path and hence arbitrary. I maintain the assumption that the off-path belief $\tilde{\pi}_{-i}^{np}$ is independent of π_i along with the "no signaling what you don't know" assumption of Fudenberg and Tirole (1991). Payoff of the deviating player depends on the posterior beliefs at the continuation game $\mathscr{G}\left(\pi_i, \tilde{\pi}_{-i}^{np}\right)$, where $\pi_i = \theta$ and $\tilde{\pi}_{-i}^{np}$ is arbitrary and in $[0,1]$.

Given any (stochastic) peace proposal, a fully participating PBE of the game amounts to a pair of mediation strategy $(\sigma_i)_{i=1}^{i=2}$, and belief system $\left(\pi_i^A, \pi_i^R, \tilde{\pi}_i^{np}\right)_{i=1}^{i=2}$, such that, for each player i, σ_i best replies to σ_{-i} given the continuation values determined by $\left(\pi_i^A, \pi_i^R, \tilde{\pi}_i^{np}\right)_{i=1}^{i=2}$ according to Eqs. (3.8)-(3.9), and $\left(\pi_i^A, \pi_i^R, \tilde{\pi}_i^{np}\right)_{i=1}^{i=2}$ obey Bayes's rule whenever applies.

Call a PBE *always-conflict* if and only if $\sigma_i(\nu_i; a) = \sigma_i(\nu_i; 1) = 1$ for some player $i \in \{1, 2\}$. It is easy to show that any peace proposal admits an always-conflict PBE, supported by posteriors $\pi_i^A = \pi_i^R = \theta$, on or off path, for each $i \in \{1, 2\}$.

Remark 3.1 Due to the dynamic nature of the multistage game, the action to participate and announce Reject for a player is not equivalent to nonparticipation. By participating and announcing Reject, the player observes her opponent's decisions and thereby learns about her type, which affects strategies in conflict. Whereas, nonparticipation, a unilateral deviation, shuts down all communication and the player does not learn about her opponent. Thus, full participation translates itself into an interim participation constraint (Section 3.2.2). Moreover, any peace proposal admits a PBE where each type of each player fully participates and announces Reject. At this PBE the on-path posterior is the same as the prior (c.f. proof of Lemma B.1). This is a suboptimal PBE because it admits conflict with probability one.

3.2.2 The Equilibrium Condition during Mediation

For notational convenience, denote

$$r := 1/(1-a). \tag{3.10}$$

Using the definition of r, a neutral mediator makes a *peace proposal*

$$x := (x_1, x_2) \in [0, M]^2 \quad \text{such that} \quad x_1 + x_2 = M, \tag{3.11}$$

interpreted as a peaceful split $(x_1/r, x_2/r)$ of the peace surplus, where $M = Sr$.

Conditional on participation in the mediation, given any strategy profile $(\sigma_i)_{i=1}^2$ and the associated probability and belief system $(q_i^A, q_i^R, \pi_i^A, \pi_i^R)_{i=1}^2$ defined by Eqs. (3.3)–(3.6), each player i's interim expected payoff from rejecting or accepting a peace proposal (x_i, x_{-i}) for each $i \in \{1, 2\}$ is determined. Using the definition of r,

$$V_i^A(x_i; t) := q_{-i}^A x_i + q_{-i}^R r U_i^t \left(\pi_i^A, \pi_{-i}^R\right), \tag{3.12}$$

$$V_i^R(x_i; t) := q_{-i}^A r U_i^t \left(\pi_i^R, \pi_{-i}^A\right) + q_{-i}^R r U_i^t \left(\pi_i^R, \pi_{-i}^R\right). \tag{3.13}$$

That is, $V_i^d(x_i; t)$ is r times the expected payoff of player i of type t from choosing $d \in \{A, R\}$. The difference between the two expected payoffs is summarized by a vector:

$$\begin{bmatrix} \Delta_i(1) \\ \Delta_i(a) \end{bmatrix} := q_{-i}^A \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^A\right\} - x_i \\ a\left(\pi_{-i}^A - \pi_i^R\right)^+ - x_i \end{bmatrix} + q_{-i}^R \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^R\right\} - \max\left\{\pi_i^A, \pi_{-i}^R\right\} \\ a\left(\pi_{-i}^R - \pi_i^R\right)^+ - a\left(\pi_{-i}^R - \pi_i^A\right)^+ \end{bmatrix}. \tag{3.14}$$

Hence,

$$\sigma_i(\nu_i; t) > 0 \quad \Rightarrow \quad \Delta_i(t) \geq 0, \tag{3.15}$$

$$\sigma_i(\nu_i; t) < 1 \quad \Rightarrow \quad \Delta_i(t) \leq 0. \tag{3.16}$$

Suppose player i unilaterally deviates to nonparticipation. Then, Bayes's rule implies $\pi_{-i} = \theta$. $\tilde{\pi}_i^{np}$ is off-path and arbitrary. Let $\tilde{U}_i^t\left(\tilde{\pi}_i^{np}, \theta\right)$ denote the maximum expected off-path payoff for type t of player i. Let $\tilde{V}_i^{np}(t) := r\tilde{U}_i^t\left(\tilde{\pi}_i^{np}, \theta\right)$. Hence, by Eqs. (3.8) and (3.9)

$$\tilde{V}_i^{np}(1) = \max\{\tilde{\pi}_i^{np}, \theta\}, \tag{3.17}$$

$$\tilde{V}_i^{np}(a) = a\left(\theta - \tilde{\pi}_i^{np}\right)^+. \tag{3.18}$$

If $\tilde{\pi}_i^{np} \leq \theta$, then $\tilde{V}_i^{np}(1) = \theta$, $\tilde{V}_i^{np}(a) = a\left(\theta - \tilde{\pi}_i^{np}\right)$. If $\tilde{\pi}_i^{np} \geq \theta$, then $\tilde{V}_i^{np}(1) = \tilde{\pi}_i^{np}$, $\tilde{V}_i^{np}(a) = 0$.

Let $V_i(t)$ denote r times the interim payoff of type t of player i at the participation stage. With probability 1/2 player i is the favored one and is offered $x_i > x_{-i}$ and with the complementary probability she is offered x_{-i}. Conditional on participation and once the proposals are announced, player i announces $d \in \{A, R\}$ and $m \in \{A, R\}$. Thus,

$$V_i(t) := V_i^d(x_i; t)/2 + V_i^m(x_{-i}; t)/2. \tag{3.19}$$

where $V_i^d(x_i; t)$ for each $d \in \{A, R\}$ and $V_i^m(x_i; t)$ for each $m \in \{A, R\}$ is defined by Eqs. (3.12) and (3.13). A proposal satisfy full participation condition if and only if for each $t \in \{a, 1\}$

$$V_i(t) \geq \tilde{V}_i^{np}(t) \tag{3.20}$$

3.3 Main Findings

For tractability I assume that the parameters (M, θ, a) satisfy:

$$\theta + a \leq M < 2\theta \quad \text{and} \quad a < \theta. \tag{3.21}$$

where by Eq. (3.11), $M = Sr$. By the definition of r in Eq. (3.10), Ineq. (3.2) is equivalent to:

$$0 < M < 2\theta. \tag{3.22}$$

If $M = 2\theta$ then the equal split can guarantee peace. Therefore, the first clause of Ineq. (3.21) states that the level of peace surplus is low enough that peace cannot be guaranteed. If $a \geq \theta$ and $\theta + a \leq M$, then $M \geq 2\theta$, implying there exists a peace proposal that fully prevents conflict.

Note that $\theta + a \leq M$ guarantees that the level of peace surplus is larger than a threshold which is the sum of the minimum expected payoff that the high type can get by triggering conflict, i.e., θ, and the highest payoff that is achievable for the low type, a. Accordingly, $a < \theta$ implies that the minimum expected payoff of the high type from triggering conflict is larger than the maximum payoff that the low type can get from such an action. Together with $M < 2\theta$, they imply that the level of peace surplus is high but not too high that peace can be fully guaranteed.

I find that if some player $i \in \{1, 2\}$ rejects peace proposal with strictly positive probability, then the mediation strategies are increasing in type, i.e., $\sigma_i(\nu_i, a) < \sigma_i(\nu_i, 1)$ (Lemma B.4, Appendix B.1). Given the dynamic nature of the game, the monotone mediation strategies are not obvious a priori. On the one hand, the high type may have the incentive to feign weakness and play the same action as the low type at the mediation. Being perceived as a low type would make the conflict less intense and costly, and the high type can gain a larger payoff (Eq. 3.8). This strategic behavior is sometimes called sandbagging. On the other hand, the weak type may have the incentive to bluff and play the same action as the high type. Being perceived as high type increases her payoff in the conflict that ensues the mediation (Eq. 3.9). The reasons for monotone mediation strategies are monotonicity of the conflict's payoff in type (Eqs. 3.8 and 3.9) and that proposals are type independent, i.e., types only matter at conflict.

Monotone mediation strategies coupled with Eqs. (3.5) and (3.6) implies that in any fully participating PBE such that $0 < q_i^R < 1$,

$$\pi_i^R < (\leq)\ \pi_i^A \iff \sigma_i(\nu_i; 1) > (\geq)\ \sigma_i(\nu_i; a) \iff \pi_i^R < (\leq)\ \theta < (\leq)\ \pi_i^A. \tag{3.23}$$

If player i announces Accept (resp. Reject), then her opponent by observing this decision infers that she is a low type with a higher (resp. lower) probability compared to the prior. The following proposition summarizes all possible fully participating PBEs that are not sub-optimal and do not admit always-conflict PBEs. The proof for this proposition, and all other proofs, are provided in Appendix B.

Proposition 3.1 *Suppose 3.21. Given any peace proposal* (ν_1, ν_2)*, there are two possible classes of fully participating PBEs that are not always-conflict and not suboptimal:*
(i) Lopsided: For some $i \in \{1, 2\}$, $\sigma_{-i}(\nu_i; a) = \sigma_{-i}(\nu_i; 1) = 0 = \sigma_i(\nu_i; a) < 1 = \sigma_i(\nu_i; 1)$*,*
(ii) Mutually partially mixed (MPM): For each $i \in \{1, 2\}$, $\sigma_i(\nu_i; a) = 0$, $\sigma_i(\nu_i; 1) \in (0, 1)$*.*

MPM PBE is admitted by equal proposal (Lemma B.8, Appendix B.2) and Lopsided PBEs by lopsided proposals (hence the denomination) such that $\min\{x_1, x_2\} \in \left[a\theta, \bar{M}\right]$, where

$$\bar{M} := M - \left(\theta + \frac{1-\theta}{\theta}\tilde{\pi}_{-i}^R\right) \tag{3.24}$$

for some off-path posterior $\tilde{\pi}_{-i}^R \in [0, 1]$ (Lemma B.7, Appendix B.2). By Ineq (3.22), one can verify that $\bar{M} < \frac{M}{2}$.

To see intuitions on possible classes of PBEs that are not suboptimal (Appendix B.1) note that by the Bayes's consistency requirement (3.7), the ex-ante probability of accept is equal to $q_i^A = \frac{\theta - \pi_i^R}{\pi_i^A - \pi_i^R}$. Thus, maximizing probability of peace, is equivalent to maximizing $q_i^A q_{-i}^A = \prod_{i=1}^2 \left(\frac{\theta - \pi_i^R}{\pi_i^A - \pi_i^R} \right)$. If the mediator can somehow decrease π_i^A and π_i^R of each player, it increases probability of peace. By the monotone mediation strategies, if the players participate and Reject is a best reply for at least one type of a player, then $\pi_i^R \leq \theta \leq \pi_i^A$ (Eq. 3.23). But the mediator only indirectly and through the proposals can affect the posterior beliefs.

The ex-ante probability of Accept, by Eq. (3.3), is a convex combination of $(1 - \sigma_i(\nu_i; a))$ and $(1 - \sigma_i(\nu_i; 1))$. If the mediator can somehow decrease π_i^R, then by Eqs. (3.8) and (3.9), it increases payoff at conflict for the high type of player i, incentivizing her to announce Reject, while hurt the low type, discouraging her from announcing Reject. Therefore, in choosing the proposal that admits these beliefs, the mediator must make a trade off which type she wants to discourage from Reject. The mediator cannot induce strategies that high type of both players Accept (Lemma B.1, Appendix B.1). Then, to maximize probability of peace, the mediator may want to minimize $\sigma_i(\nu_i; a) = 0$. This, by Eq. (3.3), means $q_i^A \geq \theta$, a large probability of Accept especially for large values of θ. Moreover, by Eq. (3.6), $\sigma_i(\nu_i; a) = 0$ means $\pi_i^R = 0$, which is the best belief for the low type of player i and the worst belief for her high type. Thus, it minimizes the interim payoff of Reject for high type of player i and maximizes that of her low type.

If player i plays a fully revealing strategy, i.e., $\sigma_i(\nu_i; a) = 0$ and $\sigma_i(\nu_i, 1) = 1$ and her rival always Accept, then $q_i^A = \theta$ and $q_{-i}^A = 1$ (Eq. (3.3)) and the probability of peace is θ. It turns out such a class of PBEs works as a benchmark for comparison of probability of peace. Then, any PBE where both players Reject with strictly positive probability and $\sigma_i(\nu_i; 1) = 1$ for at least one, is outperformed by this benchmark (Lemma B.3, Appendix B.1). Thus, one can conclude that to find proposals that are admissible and maximize probability of peace it suffices to investigate four possible classes of PBEs (Appendix B.1, Table B.1). Moreover, by assumption (3.21), this class of PBEs shrinks further (Lemmas B.5 and B.6, Appendix B.2) as stated in Proposition 3.1.

3.3.1 Conflict Minimizing Proposals

By Proposition 3.1, given any peace proposal (x_1, x_2), there are two possible PBEs that are not always conflict and not sub-optimal: MPM PBEs admitted by the equal split (Lemma B.8, Appendix B.2) and Lopsided PBEs admitted by lopsided splits (Lemma B.7, Appendix B.2). Hence, for each of these PBEs, one can characterize the strategy profile $(\sigma_i)_{i=1}^2$, which in turn, by Eq. (3.3), determines an ex-ante probability of Accept. Let $P(x_{\min})$ denote the ex-ante probability of conflict generated by any not-always-conflict PBE given a peace proposal (x_1, x_2) where $x_{\min} := \min\{x_1, x_2\}$. Let $P(0) := 1$, as the equilibrium given $x_{\min} = 0$ is always conflict. Ex-ante probability of conflict is uniquely determined for the equal proposals by $P\left(\frac{M}{2}\right) = 1 - (q^A)^2$, where q^A is characterized by Lemma B.8, and for lopsided proposals by $P(x_{\min}) = 1 - \theta$ (Corollary B.1).

Suppose player i unilaterally deviates to nonparticipation. Given an off-path posterior belief $\tilde{\pi}_i^{np}$, a proposal, to be admissible, must satisfy the participation constraint (3.20). The payoff upon deviation to nonparticipation depends on how the deviating player is perceived compared to her rival. The deviating player does not learn more than prior about her rival while her rival forms an arbitrary belief about her. Hence the off-path continuation game is $\mathscr{G}\left(\tilde{\pi}_i^{np}, \theta\right)$. If upon nonparticipation she is perceived to be low type with a higher probability than her rival, i.e., $\tilde{\pi}_i^{np} > \theta$, then these beliefs are optimistic forecast of deviation for the high type (Eq. (3.17)) and pessimistic for the low type (Eq. (3.18)) delivering the low type zero payoff and the high type's payoff is equal to $\tilde{\pi}_i^{np}$. If $\tilde{\pi}_i^{np} \leq \theta$, then the high type's deviation payoff is equal to θ and that of low type is equal to $a(\theta - \tilde{\pi}_i^{np})$.

By the definition of MPM PBEs, given in Proposition 3.1, the weak type of both players always accept. Thus, by Bayes's rule, in MPM PBEs $\pi_1^R = \pi_2^R = 0$. These are the best beliefs for the weak type of each player and gains her the highest feasible interim payoff if she announces Reject, i.e., $V_i^R(x_i; a) = a\theta$ and the worst belief for the high type of each player, delivering $V_i^R(x_i; 1) = \theta$. By Eqs. (3.17) and (3.18), if $\tilde{\pi}_i^{np} \leq \theta$ then the nonparticipation payoffs for each type are $\tilde{V}_i^{np}(1) = \theta$ and $\tilde{V}_i^{np}(a) = a(\theta - \tilde{\pi}_i^{np})$ and if $\tilde{\pi}_i^{np} > \theta$ then the nonparticipation payoffs are $\tilde{V}_i^{np}(1) = \tilde{\pi}_i^{np} > \theta$ and $\tilde{V}_i^{np}(a) = 0$. Thus, in MPM PBEs, if $\tilde{\pi}_i^{np} > \theta$, the high type deviates to nonparticipation. If $\tilde{\pi}_i^{np} \leq \theta$, both types fully participate.

Proposition 3.2 *For any $\tilde{\pi}^{np} \leq \theta$ the equal proposal is admissible. For any $\tilde{\pi}^{np} > \theta$, given the equal proposal, the high type prefers nonparticipation and the proposal is not admissible.*

Intuitively, randomization between the roles of the two players can help to subsidize the type that is hurt by participation through taxing the type that gains from the participation. The equal proposal by treating the two players equally does not provide such a possibility for the mediator. Hence, the equal proposal is not admissible even with randomization, if the off-path beliefs penalize the high types of both players and incentivize them to not participate.

In Lopsided PBEs, the favored player, labeled as $-i$, receives a larger share, always Accept, and reveals no further information than the prior, i.e., $\pi_{-i}^A = \theta$. The larger share that the favored player receives and the fact that she does not reveal any information about her type while learns about her opponent by participation results in her full participation. The less favored player, labeled as i, follows a fully revealing pure strategy: Accept if she is low type and Reject if she is high type. Thus, $\pi_i^A = 1$ and $\pi_i^R = 0$ by Bayes's rule. By participation, the high type's interim payoff is $V_i^R(x;1) = \theta$ (Eqs. (3.12) and (3.13)) and that of her low type, by revealed preference argument $V_i^A(x_i;a) \geq V_i^R(x_i;a)$, is at least $V_i^R(x_i;a) = a\theta$. By Eqs. (3.17) and (3.18), if $\tilde{\pi}_i^{np} \leq \theta$ then the nonparticipation payoffs for this player are $\tilde{V}_i^{np}(1) = \theta$ and $\tilde{V}_i^{np}(a) = a(\theta - \tilde{\pi}_i^{np})$ and if $\tilde{\pi}_i^{np} > \theta$ then the nonparticipation payoffs are $\tilde{V}_i^{np}(1) = \tilde{\pi}_i^{np} > \theta$ and $\tilde{V}_i^{np}(a) = 0$. Thus, if $\tilde{\pi}_i^{np} \leq \theta$, both players fully participate and if $\tilde{\pi}_i^{np} > \theta$, the less favored player's high type prefers nonparticipation.

Proposition 3.3 *For any $\tilde{\pi}^{np} \leq \theta$ lopsided proposals are admissible. For any $\tilde{\pi}^{np} > \theta$, given lopsided proposals, the favored player and the low type of the less favored player prefer participation while the less favored player's high type prefers nonparticipation.*

In contrast to the equal proposal that treats two players equally, lopsided proposals provide different payoffs for the players. Hence, the mediator can randomize between the roles of the two players to subsidize the type that is hurt by participation by taxing the type that gains from the participation. Through this channel, she can guarantee the full participation of both players in the mechanism.

Given (M, θ, a) and $\tilde{\pi}^{np}$, one can characterize admissible conflict minimizing proposals. By Propositions 3.2 and 3.3, if $\tilde{\pi}^{np} \leq \theta$, then both the equal and lopsided proposals are admissible. The following theorem provides the optimal proposal for such off-path beliefs. Denote,

$$\theta^* := 6 - a - 4\sqrt{2-a}, \tag{3.25}$$

Theorem 3.1 *Suppose (M, a, θ) satisfy Ineq. (3.21). For any $\tilde{\pi}^{np} \leq \theta$:*

a. if $a < \theta < \theta^*$ then the equal proposal is the unique optimal proposal;

b. if $\theta^* \leq \theta$ then there exists a unique $M^* \in [\theta + a, 2\theta)$ such that: (i) if $M > M^*$ then the equal proposal is the unique optimal proposal, (ii) if $M < M^*$ then there will be multiple optimal proposals each being lopsided and admitting the same probability of conflict, (iii) if $M = M^*$ both the equal and lopsided proposals are optimal.

By Proposition 3.2, if $\tilde{\pi}^{np} > \theta$, then the equal proposal is not admissible. Since the equal proposal provides both players with similar payoffs, randomization does not help.

However, by Proposition 3.3, if $\tilde{\pi}^{np} > \theta$, then given the lopsided proposals, the favored player's high type prefers participation while that type of the less favored player prefers nonparticipation. This unequal treatment provides room for the mediator to randomize the role of players and satisfy full participation constraint by averaging the participation payoffs among the player that is hurt by participation with the one that benefits from it. The following Theorem states this result.

Theorem 3.2 *Suppose (M, a, θ) satisfy Ineq. (3.21), $\theta \geq \frac{1}{2-a}$, and any $\tilde{\pi}^{np} \in \left(\theta, \frac{2\theta}{1+\theta}\right]$. Then, lopsided proposals augmented with an equal probability randomization device are the only admissible conflict-minimizing proposals.*

Remark 3.2 Note that stochastic lopsided proposals from an ex-ante point of view, by using a fair coin to assign the role of players, treat players symmetrically. However, the splits, after the realization of the role of each player, are lopsided. The unequal interim payoff admitted by the lopsided proposals provides room for randomization by a fair coin. Without this interim favoritism, the lopsided proposals would not be admissible. This is precisely the reason that the equal proposal is not admissible even if augmented by a fair coin.

3.4 Robustness: Continuum of Types Distributions

Previous sections show that for identical binary type prior distributions the equal proposal is sometimes suboptimal or not admissible and the mediator may show favoritism to admit a conflict minimizing proposal. To verify that these results are not driven by the assumption of two types, as a robustness check, I extend the analysis to a continuum of types distribution.

Two players, indexed by $i \in \{1, 2\}$, compete for a prize. Each player i's type t_i, privately known to i, is drawn independently from a commonly known cumulative distribution function F with the support $[0, 1]$. F possesses positive density f. At the outset, and after knowing their types, each player decides whether to participate in a mediation mechanism or pursue conflict directly. The rest of the model is identical to the binary type private model (Section 3.2).

The following assumption about the parameters is maintained throughout this chapter:

$$0 < S < 2c^*, \tag{3.26}$$

where c^* is a function of the primitives and defined as

$$c^* := \inf\left\{c \in [0,1] : \int_c^1 \frac{1}{F^{-1}(s)} ds \leq 1\right\}. \tag{3.27}$$

c^* is the payoff that type $t = 1$ of player i gains in a continuation game $\mathscr{G}(\delta_1, F)$, where she is perceived to be type $\{1\}$, i.e., the Dirac measure at 1 denoted by δ_1, and her rival's type distribution is the prior. As demonstrated in Zheng (2019b), δ_1 is the most penalizing belief for type $t = 1$ at $\mathscr{G}(\delta_1, F)$, and c^* is the minimum proposal required to guarantee peace, i.e., once it is offered to each player they Reject it with zero probability on the path of equilibrium. Thus, Ineq. (3.26) (analogous to (3.2) for the binary type distributions) states that if $S < 2c^*$, any peace proposal would be rejected with positive probability on the path of equilibrium.

Given any proposed split, a multi stage game is defined. This chapter studies equilibria in which each player $i \in \{1, 2\}$, conditional on participation in the mediations, employs monotone cutoff strategy $\sigma_i(\nu_i; t_i)$ with cutoff value $\lambda_i \in [0, 1]$:

$$\sigma_i(\nu_i; t_i) = \begin{cases} \text{Accept} & \text{if } t_i \in [0, \lambda_i] \\ \text{Reject} & \text{if } t_i \in [\lambda_i, 1]. \end{cases} \tag{3.28}$$

Given cutoff strategies, beliefs are updated via Bayes's rule, whenever it applies. Therefore, conditional on participation, for each player $i \in \{1, 2\}$ at the continuation game of conflict the posterior probabilities are truncated distribution denoted by CDF F_i^m, where $m \in \{A, R\}$ is the announced decisions at the mediation stage. A stands for Accept and R for Reject. Hence, whenever Bayes's rule applies, F_i^A (resp. F_i^R) is a truncation of the prior F and it has the support $[0, \lambda_i]$ (resp. $[\lambda_i, 1]$). The densities are defined respectively. Deviation to non-participation by player i is an off-path event, where the off-path belief about i is denoted by $\tilde{F}_i^{np}$. Analogous to the binary case, I maintain the assumption that the off-path belief $\tilde{F}_i^{np}$ is independent of F_{-i}. Player 1 is referred to as "weak", and player 2 as "strong", if F_2^h first order stochastically dominates F_1^l, where $h \in \{A, R, np\}$ and $l \in \{A, R, np\}$.

Whether the conflict is triggered because someone rejected the mediator's offer, or because one deviated and did not participate, the game is the same. Each player knows their type, the history of the game, and has a posterior belief about the rival's types based on that history. I extend the methodology of Amann and Leininger (1996) and Kirkegaard (2008) to handle all asymmetric continuation games of conflicts that arise endogenously where the posterior beliefs can have overlapping supports. This game has a unique BNE; both players exert effort (bid) over common support; one player exerts effort that is strictly increasing in type while that of the other is weakly increasing in type: types below a threshold put zero effort and those above the threshold exert strictly increasing effort in type (Appendix B.7.2).

Thus, given any (stochastic) peace proposal, a full participation PBE of the game amounts to, a pair of mediation cutoff strategy $(\sigma_i)_{i=1}^{i=2}$ defined by (3.28), and belief system $\left(F_i^A, F_i^R, \tilde{F}_i^{np}\right)_{i=1}^{i=2}$, such that, for each player i, σ_i best replies to σ_{-i} given the continuation values determined by the belief system that obeys Bayes's rule whenever applies.

I study two classes of proposals that admit equilibria resembling those of the binary type distribution. One is the equal proposal that admits symmetric PBEs. The other one is the lopsided proposals, which admit Lopsided PBEs where player i who is offered the larger share always announces Accept, and player $-i$ follows cutoff strategies at the mediation.

3.4.1 Symmetric Equilibrium

A symmetric equilibrium means a PBE where the two players fully participate and use the same cutoff strategy at the mediation. To be explicit:

$$\sigma(\nu_i; t) = \begin{cases} \text{Accept} & \text{if } t \in [0, \lambda] \\ \text{Reject} & \text{if } t \in [\lambda, 1], \end{cases} \tag{3.29}$$

and the associated posteriors are F_i^A and F_i^R. The cutoff type is indifferent between Accept and Reject. Conditional on participation, equal proposal admits symmetric mediation strategies (Appendix B.7.3). Analysis of participation decisions of players is based on the characterization of payoffs at the continuation game of conflicts and comparative statics analysis of how these payoffs change if one player is stochastically perceived as stronger or weaker by the rival. The general insight is that the low (resp. high) types of a player are better off if she is perceived as stronger (resp. weaker) at the conflict (implications of Corollaries B.3 and B.4, Appendix B.7.2). Thus, in analyzing the incentives to deviate to nonparticipation, one needs to calculate on-path and off-path payoffs of all types for different off-path beliefs that can take any arbitrary form, a demanding task with a continuum of types. In particular, the difficulty arises because the participation constraint is not necessarily monotone in type. If players are perceived to be strong upon the off-path event of nonparticipation, then these beliefs penalize the strong types the most and provide incentives for them to participate in the mediation while the same beliefs are beneficial for the weak types in the event of nonparticipation and can encourage them to not participate.

One of the most commonly used off-path beliefs in the literature is the passive updating assumption: whenever a player observes any off-path behavior she does not learn anything about her opponent, i.e., $\tilde{F}_i^{np} = F$. The equal proposal admits symmetric strategies defined by Eq. (3.29), where associated posterior belief upon announcing Reject F_i^R has the support $[\lambda, 1]$ and stochastically dominates the prior distribution F while F_i^A has the support $[0, \lambda]$ is stochastically dominated by F. Hence, conditional on participation, the high types including $t_i = 1$, who always announces Reject (Eq. (3.29)) and signal strength, at the event $\mathscr{G}\left(F_i^R, F_{-i}^A\right)$, face a stochastically weaker opponent and at the event $\mathscr{G}\left(F_i^R, F_{-i}^R\right)$ a stochastically of similar strength opponent. One can show a non-degenerate set of high types are worse off by this information revelation compared to nonparticipation, where the players do not learn anything about each other. With the same token, since high types are better off if they are perceived weaker at the conflict, off-path beliefs with the support $\left[0, \bar{t}\right]$ such that $\bar{t} \leq 1$ also rationalize nonparticipation by benefiting these types at $\mathscr{G}(\tilde{F}_i^{np}, F)$. These intuitive ideas are used to prove the following Proposition (in Appendix B.7.3) which states that given passive updating upon nonparticipation or off-path beliefs that are a truncation of the prior distribution from above, the equal proposal is not admissible. This is because the high types have will be better off by nonparticipation where they do not learn anything about their opponent but are also perceived weakly weaker compared to the prior distribution and benefit from this perception. Moreover, since the equilibrium is symmetric randomization between players cannot help to make the equal split admissible.

Proposition 3.4 *Suppose the assumption on off-path posterior belief is passive updating or any truncation of the prior with the support* $\left[0, \bar{t}\right]$ *for any* $\bar{t} < 1$*. Given any* $0 < S < 2c^*$*, the equal proposal is not admissible.*

Suppose upon nonparticipation the off-path beliefs are truncation of the prior with support $[\underline{t}, 1]$ where $\underline{t} \in (\lambda, 1]$ meaning that it is perceived that the deviating player's type is above the cutoff defined by Eq. (3.29). These off-path beliefs incentivize mid-range types including the cutoff λ to not participate in the mediation. At the off-path event of nonparticipation, these types are perceived stochastically stronger compared to the prior distribution while at the same time they do not learn anything about their opponent. If these types participate, they reveal that they are weak (if they are below threshold λ) and learn that the opponent is strong upon observing her Reject announcement and thereby be exploited by her in the event of a conflict. Therefore, this information revelation makes these types worse off compared to nonparticipation. For instance, the extreme case is

$\tilde{F}_i^{np} = \delta_1$: whenever a player observes any off-path behavior, she believes her opponent is the highest type $\{1\}$, i.e., $\tilde{F}^{np} = \delta_1$. This is the most penalizing off-path belief for $t = 1$ at $\mathscr{G}(\delta_1, F)$ (Zheng, 2019b) and makes participation appealing for her. However, it renders the mid-range types high nonparticipation payoffs that even the possibility of gaining an equal proposal cannot compensate. These arguments are used to prove the following proposition (In Appendix B.7.3) which states that given the equal proposal and the aforementioned off-path beliefs mid-range types do not participate in the mediation and since the equilibrium is symmetric randomization cannot help to make the equal proposal admissible.

Proposition 3.5 *Suppose off-path posterior belief is any truncation of the prior with the support $[\underline{t}, 1]$ for any $\underline{t} \in (\lambda, 1]$, where λ is the cutoff type at the symmetric continuation game of mediation. Given any $0 < S < 2c^*$, the equal proposal is not admissible.*

To summarize these results, given the equal proposal, I assume the off-path posterior belief $\tilde{F}_i^{np}$ is any truncation of the prior distribution with supports that are specified in:

$$\xi_i := \left\{ \tilde{F}_i^{np} : \text{supp } \tilde{F}_i^{np} = \left[0, \bar{t}\right] \ \forall \bar{t} \in (0, 1] \text{ or supp } \tilde{F}_i^{np} = [\underline{t}, 1] \ \forall \underline{t} \in (\lambda, 1] \right\}, \tag{3.30}$$

where λ is the cutoff of symmetric strategies defined by Eq. (3.29). This set of off-path beliefs includes passive updating and the Dirac measure at $\{1\}$. The following Theorem is a direct implication of Propositions 3.4 and 3.5.

Theorem 3.3 *Given any $0 < S < 2c^*$ and any $\tilde{F}_i^{np} \in \xi_i$, the equal proposal is not admissible.*

3.4.2 Lopsided Equilibrium

This section, motivated by the binary type results, studies biased proposals that admit PBEs, called Lopsided PBEs, where a player that receives the larger share always announces Accept. To be explicit, given any peace proposal (ν_1, ν_2), relabeling the players, if necessary, suppose $\nu_2 < \nu_1$, i.e., $\nu_2 \in \left[0, \frac{S}{2}\right)$. Hereafter, without loss of generality, I denote the favored player as player 1 and the less favored player as player 2. Given (ν_1, ν_2), in any fully participating Lopsided PBEs both players fully participate, player 1 always announces Accept while player 2 follows a monotone cutoff strategy where she would announce Accept if her type is below a threshold and announces Reject otherwise:

$$\sigma_1(\nu_1;t) = \text{Accept for all } t \in [0,1] \quad \text{and} \quad \sigma_2(\nu_2;t) = \begin{cases} \text{Accept} & \text{if } t \in [0,\lambda] \\ \text{Reject} & \text{if } t \in [\lambda,1]. \end{cases} \tag{3.31}$$

The cutoff type of player 2 is indifferent between Accept and Reject. Conflict happens on the path of equilibrium only when player 2 announces Reject. Since player 1 always announces Accept, one needs to make an assumption on $\tilde{F}_1^R$, the off-path belief about her if she deviates to Reject. Also, an assumption on $\tilde{F}_i^{np}$ at the off-path event that player i deviates to nonparticipation is needed. I construct the Lopsided PBEs by picking the arbitrary off-path beliefs $F_1^R = \tilde{F}_i^{np} = \delta_1$: whenever a player observes any off-path behavior, she believes her opponent is the highest type $\{1\}$.

Given the off-path belief δ_1, let $C_{RA}^*(1;\lambda)$ and $C_{RR}^*(1;\lambda)$ denote payoff of type $t=1$ of player 1 at off-path continuation games $\mathscr{G}\left(\delta_1, F_2^A\right)$ and $\mathscr{G}\left(\delta_1, F_2^R\right)$. For tractability, I assume:

Assumption 3.4.1 *For any $\lambda \in (0,1)$, the prior distribution's CDF, F, satisfies*

$$F(\lambda)C_{RA}^*(1;\lambda) + (1-F(\lambda))C_{RR}^*(1;\lambda) > c^*.$$

c^*, $C_{RA}^*(.)$, and $C_{RR}^*(.)$, all functions of primitives, are defined by Eqs. (3.27), (B.58), and (B.59).

This assumption, for example, is satisfied by power distributions where $F(t) = t^\alpha$ for $\alpha \in (0,1)$ and support $t \in [0,1]$ (Lemma B.20, Appendix B.7.6). An intuitive explanation of this assumption can help. Suppose player 1, that is supposed to always announces Accept at Lopsided PBEs, deviates to Reject and triggers conflict, and that the belief about her upon this deviation is δ_1, i.e., she is perceived to be the highest type $t=1$. Player 2 follows cutoff strategy and announces Accept if her type is below the cutoff type λ and announces Reject if her type is above λ. Player 1 by observing these decisions updates her belief about her opponent and her highest type $t=1$ at the continuation game of conflict $\mathscr{G}\left(\delta_1, F_2^A\right)$ gains the expected payoff $C_{RA}^*(1;\lambda)$ (defined by Eq. (B.58)) and at the continuation game of conflict $\mathscr{G}\left(\delta_1, F_2^R\right)$ gains the expected payoff $C_{RR}^*(1;\lambda)$ (defined by Eq. (B.59)). Thus, the interim payoff of $t=1$ of player 1 would be $F(\lambda)C_{RA}^*(1;\lambda) + (1 - F(\lambda))C_{RR}^*(1;\lambda)$. Moreover, suppose the continuation game of conflict $\mathscr{G}(\delta_1, F)$, where, as before, the belief about player 1 is δ_1, while the belief about player 2 is the prior distribution. In this case player 1 has not learned anything new about her opponent and the payoff of the highest type of player 1 at this continuation game is c^* (defined by Eq. ((3.27)). Assumption 3.4.1 implies that the highest type of player 1 is better off under the scenario that she learns about her opponent compared to when she does not learn anything new. Since c^*, $C^*_{RA}(1;\lambda)$, and $C^*_{RR}(1;\lambda)$ are all functions of primitives (defined by Eqs. (3.27), (B.58), and (B.59)) and the cutoff type λ can takes any value in $(0,1)$, this assumption is solely on the primitives. In construction of Lopsided PBEs in Appendix B.7.4 and B.7.5, Assumption 3.4.1 is utilized to verify incentive compatibility and participation constraint of $t=1$ of the favored player, i.e., player 1.

The off-path belief δ_1 is the most penalizing for $t=1$ if she deviates to nonparticipation. Given lopsided proposals and off-path beliefs δ_1, the following lemma states the favored player always participates while mid-range types of the less favored prefer nonparticipation. One can show that there exist lopsided proposals that admit Lopsided PBEs where all types of the favored player prefer participation and always announce Accept because of receiving a larger share. The off-path belief δ_1 is beneficial for the mid-range type of the less favored player because they will be perceived to be the highest type at the off-path event of non-participation. Since this player is less favored and receives a small share if she participates and announces Accept, then these mid-range types of her prefer nonparticipation and triggering conflict. Thus, the participation constraint for the less favored player is not monotone in type. These arguments are used to prove this Lemma in Appendix B.7.5.

Lemma 3.2 *Suppose Assumption 3.4.1 and $F^{np}=\delta_1$. For any $c^* \in \left(\frac{1}{2},1\right)$, and any $S \in [S', 2c^*)$, where S' is a function of primitives defined by Eq. (B.60), given lopsided proposals, favored player prefers participation and mid-range types of less favored player such that $t \in \left(\underline{t}, \bar{t}\right)$ where $0<\underline{t}<\bar{t}<1$ prefer nonparticipation.*

Similar to the binary type distribution, the unequal treatment of players by the lopsided proposals allows the mediator to randomly assign the role of the favored player, with equal probability, and satisfy the full participation constraint. The following Theorem states this result. In essence, the randomization allows the mediator to transfer the payoffs from the favored players to the less favored and distribute payoffs evenly between them such that they are both better off compared to nonparticipation. These arguments are used to prove the following Theorem in Appendix B.7.5.

Theorem 3.4 *Suppose Assumption 3.4.1 and $F^{np}=\delta_1$. For any $c^* \in \left[\frac{3}{4},1\right)$, and any $S \in [S', 2c^*)$, where S' is a function of primitives defined by Eq. (B.60), stochastic lopsided proposals are admissible.*

c^* is the payoff of $t=1$ of the deviating player at $\mathscr{G}(\delta_1, F)$ and it is a function of the primitives. The stochastically weaker F the larger c^* (Zheng, 2019b). $c^* \geq \frac{3}{4}$, means

F has a high weight on low types. Note that $S' \leq 1$ (Eqs. (B.58) and (B.60)). Hence, the lopsided proposals are admissible even if the expected peace surplus is less than the maximum valuation of the prize by players. The idea of the proof (provided in detail in Appendix B.7.5) is based on the observation that Lemma 3.2 implies the participation constraint for the less favored player is not monotone in type. By randomization between the two players, one can establish a monotone and weakly decreasing participation constraint that is satisfied if the ex-ante probability of announcing Accept by the less favored player is higher than a threshold $F(\lambda) \geq 2-2c^*$ (Lemma B.17, Appendix B.7.5). Intuitively, since the off-path belief δ_1 is the most penalizing for the highest type and rewarding for the low types, the weakly monotone participation constraint implies that high types receive the lowest information rent by participation. Also, one can show that the lopsided proposals that admit Lopsided PBEs admit a cutoff type λ such that the ex-ante probability of announcing Accept by the less favored player is $F(\lambda) > 1/2$ (Lemma B.15, Appendix B.7.4). Hence, to satisfy the condition $F(\lambda) \geq 2 - 2c^*$ that guarantees full participation, it suffices that $1/2 \geq 2 - 2c^*$, or in other words $c^* \geq 3/4$ as stated in Theorem 3.4. Technically, the higher c^* the easier to satisfy condition $F(\lambda) \geq 2 - 2c^*$. Intuitively speaking, the higher c^* the lower the information rent for the strong types by participation in the mediation. Given the stochastic lopsided proposals the participation constraint is weakly decreasing in type, this also means full participation is guaranteed at the expense of the high types.

Remark 3.3 One can show that given passive updating assumption, if the prior is power distribution $F(t) = t^\alpha$ where $\alpha \in (0, .5)$ and $1 \leq S < 2c^*$, then stochastic lopsided proposals are admissible. This prior satisfies Assumption 3.4.1 and $c^* > 1/2$. Given passive updating, the less favored player's high types prefer nonparticipation while the favored player prefers participation. By participating, the less favored player does not learn about her rival while revealing information by announcing Reject, making her high types worse off compared to nonparticipation. Because of the unequal treatment, randomization can help with admissibility.

3.5 Conclusion

Pre-conflict negotiations are often hindered by asymmetric information of adversaries and their strategic incentive to not reveal their hidden information. This, throughout history, has provided room for intervention by third parties to resolve or reduce the probability of conflict. These institutes, especially those with "muscle", design mediation mechanisms coupled with incentives and threats. Yet, according to the International Crisis Behavior

Project data set, only 67 percent of this mediation style terminated in agreement (See Wilkenfeld et al., 2007).

This chapter emphasizes on information effect of mediation on the design of conflict minimizing proposals for a mediator who wants to guarantee full participation of adversaries in her negotiation mechanism. To study these effects, conflict is modeled as an endogenous outcome. Despite ex-ante identical players, if the peace surplus is not large, optimal proposals among those that guarantee full participation are so lopsided that one player always accepts and the other pursues a fully revealing strategy (Theorem 3.1). Even if the peace surplus is large yet the high type of players has an optimistic forecast of vetoing a mediation and triggering conflict, then only the stochastic lopsided proposals are optimal (Theorem 3.2). These results are robust even when the distribution has continuum of types: the equal proposal is not admissible (Theorem 3.3) yet the stochastic lopsided proposals are (Theorem 3.4). In all these results, the peace surplus from the multidimensional agreement can be less than the highest valuation of the adversaries for the prize. Thus, the agreement can contain both punishment and incentives.

This chapter investigates fully participating mediation mechanisms. A question for future research is how the mediation mechanism is affected if non-participation is admissible. Then beliefs at such events are not arbitrary and follow Bayes's rule, posing a challenge for the design. Can the mediator achieve an even lower probability of conflict? Could on-path nonparticipation decisions lead to strategic behaviors such as bluffing and sandbagging?

This chapter studies the information effects of failed mediation. But even successful mediation reveals crucial information. Does such revelation lead to reneging of an agreement by a player who infers her rival is weak by acceptance of a biased proposal? Could this explain the short-term success but the long-term failure of mediation documented by empirical literature (Beardsley, 2008)?

Bibliography

Amann, E. and W. Leininger (1996). Asymmetric all-pay auctions with incomplete information: the two-player case. *Games and economic behavior 14*(1), 1–18.

Baliga, S. and T. Sjostrom (2013). Bargaining and war: A review of some formal models. *Korean Economic Review 29*(2), 235–266.

Baliga, S. and T. Sjöström (2020). The strategy and technology of conflict. *Journal of Political Economy 128*(8), 3186–3219.

Balzer, B. and J. Schneider (2019, September 20). Belief management and optimal arbitration. Mimeo.

Balzer, B. and J. Schneider (2021a). Managing a conflict: optimal alternative dispute resolution. *The RAND Journal of Economics 52*(2), 415–445.

Balzer, B. and J. Schneider (2021b). Persuading to participate: Coordination on a standard. *International Journal of Industrial Organization 78*, 102764.

Beardsley, K. (2008). Agreement without peace? international mediation and time inconsistency problems. *American Journal of Political Science 52*(4), 723–740.

Bester, H. and K. Wärneryd (2006). Conflict and the social contract. *Scandinavian Journal of Economics 108*(2), 231–249.

Brewer, D. (2011). *The Greek War of Independence: The Struggle for Freedom and the Birth of Modern Greece*. Abrams.

Celik, G. and M. Peters (2011). Equilibrium rejection of a mechanism. *Games and Economic Behavior 73*(2), 375–387.

Compte, O. and P. Jehiel (2009). Veto constraint in mechanism design: inefficiency with correlated types. *American Economic Journal: Microeconomics 1*(1), 182–206.

Fey, M. and K. W. Ramsay (2011). Uncertainty and incentives in crisis bargaining: Game-free analysis of international conflict. *American Journal of Political Science 55*(1), 149–169.

Fudenberg, D. and J. Tirole (1991). Perfect bayesian equilibrium and sequential equilibrium. *journal of Economic Theory 53*(2), 236–260.

Hörner, J., M. Morelli, and F. Squintani (2015). Mediation and peace. *The Review of Economic Studies 82*(4), 1483–1501.

Jullien, B. (2000). Participation constraints in adverse selection models. *Journal of Economic Theory 93*(1), 1–47.

Kamranzadeh, A. and C. Z. Zheng (2022, March). Unequal peace proposals for equal contestants: designing information indirectly. Mimeo, Department of Economics, University of Western Ontario.

Kirkegaard, R. (2008). Comparative statics and welfare in heterogeneous all-pay auctions: Bribes, caps, and performance thresholds. *The BE Journal of Theoretical Economics 8*(1), 1–32.

Kissinger, H. (2011). *Years of upheaval.* Simon and Schuster.

Kydd, A. (2003). Which side are you on? bias, credibility, and mediation. *American Journal of Political Science 47*(4), 597–611.

Kydd, A. H. (2006). When can mediators build trust? *American Political Science Review 100*(3), 449–462.

Kydd, A. H. (2010). Rationalist approaches to conflict prevention and resolution. *Annual Review of Political Science 13*, 101–121.

Lasensky, S. (2004). Paying for peace: The oslo process and the limits of american foreign aid. *The Middle East Journal 58*(2), 210–234.

Lewis, T. R. and D. E. Sappington (1989). Countervailing incentives in agency problems. *Journal of economic theory 49*(2), 294–313.

Lu, J., Z. Lu, and C. Riis (2021a, July 27,). Peace through bribing. Working Paper, National University of Singapore.

Lu, J., Z. Lu, and C. Riis (2021b). Perfect bidder collusion through bribe and request. *Games and Economic Behavior 129*, 1–14.

Maggi, G. and A. Rodriguez-Clare (1995). On countervailing incentives. *Journal of Economic Theory 66*(1), 238–263.

Meirowitz, A., M. Morelli, K. W. Ramsay, and F. Squintani (2019). Dispute resolution institutions and strategic militarization. *Journal of Political Economy 127*(1), 378–418.

Pavlov, G. (2008). Auction design in the presence of collusion. *Theoretical Economics 3*(3), 383–429.

Quandt, W. B. (2015). *Camp David: peacemaking and politics.* Brookings Institution Press.

Ramsay, K. W. (2017). Information, uncertainty, and war. *Annual Review of Political Science 20*, 505–527.

Rauchhaus, R. W. (2006). Asymmetric information, mediation, and conflict management. *World Politics 58*(2), 207–241.

Smith, A. and A. Stam (2003). Mediation and peacekeeping in a random walk model of civil and interstate war. *International Studies Review 5*(4), 115–135.

Spier, K. E. (1994). Pretrial bargaining and the design of fee-shifting rules. *The RAND Journal of Economics*, 197–214.

Wilkenfeld, J., K. Young, D. Quinn, and V. Asal (2007). *Mediating international crises*. Routledge.

Zartman, I. W. (2007). *Negotiation and conflict management: Essays on theory and practice*. Routledge.

Zheng, C. Z. (2019a). Bidding collusion without passive updating. *Journal of Mathematical Economics 85*, 70–77.

Zheng, C. Z. (2019b). Necessary and sufficient conditions for peace: Implementability versus security. *Journal of Economic Theory 180*, 135–166.

Chapter 4

Peace Settlements with Possibilities to Renege

4.1 Introduction

Conflicts are prevalent and costly. One ubiquitous practice in conflict management is intervention by third parties like mediators to help adversaries settle their dispute by setting up negotiation mechanisms and proposing peace agreements. In the context of international conflicts, the effectiveness and short-term and long-term impact of these interventions have been vastly studied (Beardsley, 2008, Gartner, 2008). Although mediators have short-run success in securing an agreement and avoiding conflict, in the long run, these agreements are prone to reneging by adversaries and hence recurring conflicts. For example, the Russia-Ukraine conflict in 2022 resumed a few years after Minsk Agreement II (2015), which was mediated by European countries to secure peace between the two countries. Russia reneged on this agreement by officially recognizing the independence of Luhansk and Donetsk in Ukraine's eastern region of Donbas and initiated a war (Kramer, 2022). Beardsley (2008) found that half of all mediated crises in the International Crises Behavior data project, which includes crises from 1918 to 2003, recur.

One common explanation for the recurrence of conflict is that mediators, especially those with leverage, by providing economic incentives or threats directly manipulate adversaries' incentives for pursuing conflict and thereby secure peace agreements. If these incentives cannot be sustained over time then there is a chance for recurring conflicts. However, even a mediation mechanism in which a mediator has very limited power and only formulates a peace proposal provides valuable information to adversaries, reduces their uncertainty about each others' private information, and thereby indirectly provides incentives for them to renege on a previously reached agreement in the light of the new information. A real-world episode is the United States withdrawal from the Iran Nuclear Deal in 2018. Iran accepted the agreement in 2015. After the change of administration in the US in 2017, the new administration perceived Iran's acceptance of the deal as a sign of weakness, withdrew from the agreement, and resumed hostility (Landler, 2018).

This chapter studies the conflict mediation problem where the mediator has very limited power and cannot provide any economic incentives or impose any threats and the adversaries lack commitment. The mediator merely proposes a nonbinding peace proposal that if adversaries do not agree upon or renege on them after they made an initial agreement, they can trigger conflict. This style of mediation is often called formulative mediation and it is prevalent in international conflicts (Wilkenfeld et al., 2007). The only way that the mediator can affect the decision of the adversaries is through her peace proposal and indirectly through the information that players learn about each other in the mediation process. They can use this information to assess the expected payoff from conflict. In this sense, when players lack commitment, even in a successful mediation they can reveal important information about themselves that they can use to evaluate the benefits of reneging on an accepted agreement.

In this chapter, I ask how should a mediator with very limited power design peace proposals that leads to renege-proof agreements in an environment where adversaries lack commitment? The renege-proof motivation of a mediator can be justified as part of a long-term motivation of a mediator who does not want to develop a reputation that she proposes agreements that the adversaries later renege on.

To answer this question, conflict mediation is modeled as a multi-stage game. In the *mediation* stage, a neutral mediator proposes a peace settlement. If the rivals accept the proposal, then either with some exogenous probability the game ends and each rival gets a payoff equal to their share of the prize or with the complementary probability, the game enters a rectification stage. In the *rectification* stage, any player can renege on the peace settlement and trigger conflict. Decisions are simultaneously announced. If the deal is ratified the game ends with the rivals getting a payoff equal to the peaceful split. If the proposal is rejected by at least one of the players at the mediation stage or the agreement is reneged on at the rectification stage, conflict ensues. The *conflict* stage is modeled as an all-pay auction. It is costly for both and winner-take-all; whoever devotes the highest level of resources and efforts wins. The rivals have private information about their marginal cost of exerting effort. They are either strong or weak. Conflict is less costly for the strong type. These types are drawn from a common prior distribution.

Therefore, the players are ex-ante identical.

The exogenous probability that the rectification stage occurs is motivated by exogenous changes in the environment, e.g., changes of decision-makers on one side. It can also be motivated by the time that it takes until an agreement becomes official and reneging on it become prohibitively costly. To keep the problem tractable and focus on the dynamic features of conflict mediation, I assume that mediation mechanisms are message-independent splits, and adversaries' types are binary. The primitives of the model are such that conflict cannot be fully avoided. In other words, there does not exist any negotiation mechanism that the mediator can use and fully prevent conflict. Any proposal would be rejected with a positive probability by at least one of the players. This is because the prior probability of being weak is relatively high. Therefore, the strong type of one player always has an incentive to reject a peace proposal and triggers conflict. Since conflict happens on the path of equilibrium, the mediation mechanism is informative in the sense that players can signal information about their type, i.e., their willingness or hesitancy to go to conflict, by announcing their decisions.

This chapter contributes to the conflict mediation literature by presenting an explicit answer to the following policy question: What are the renege-proof mediation settlements that a neutral benevolent mediator should propose when the mediator lacks enforcement power and rivals lack commitment? The objective of the mediator is to maximize social surplus, or the sum of the two adversaries' expected payoffs before the realization of types, subject to renege-proof constraint.

To characterize the optimal renege-proof peace proposals, I characterize all possible cases of renege-proof equilibria. For a peace settlement to be renege-proof, i.e., renege does not occur on the path of equilibrium, either each type of each adversary should find the accepted proposal better than triggering conflict or the type that has an incentive to renege would reject the proposal, to begin with, at the mediation stage, meaning that the tempted type is not present at the rectification stage. I show that only two sets of perfect Bayesian equilibrium (hereafter, PBE) survive renege-proof conditions (Lemma 4.2). One case of renege-proof PBEs is *Mutually Partially Mixed* , hereafter MPM. These PBEs are such that strong types of both players reject the settlement at the mediation, while their weak types mix between accept and reject and do not renege at the rectification. The other case of PBEs is *Lopsided*. The peace settlements that admit these PBEs are so biased that the favored player always accepts and ratifies it. The less favored player rejects this proposal if she is strong, mixes if she is weak, and ratifies it at the rectification stage.

I show the main result of this chapter by Theorem 4.1. It states that if the probability of the rectification stage is higher than a threshold or the prior probability of being weak is lower than a threshold, then the only peace settlement that is renege-proof and maximizes social surplus is the lopsided split. If the probability of the rectification stage is lower than a threshold then the social-surplus maximizer is either the lopsided split or the equal split. In this case, if the prior probability of being weak is higher than a threshold, the equal split is the social surplus maximizer, and the lopsided split is optimal when the probability is below this threshold. When the probability of the rectification stage is high and the equal split is proposed, the strong type of both players, similar to their weak type, has an incentive to accept the proposal and then renege at the rectification stage. Hence, the equal split is not renege-proof in such an environment.

Instead of maximizing social surplus, the mediator's objective could be minimizing the ex-ante probability of conflict. This objective is well studied in the conflict management literature. Theorem 4.2 states that the optimal solution for these two different objectives is qualitatively the same. This is surprising because the social surplus maximization objective in this environment in which peace cannot be guaranteed takes into account both the payoffs from peace and conflict while to attain the conflict minimization objective, the mediator wants to minimize the expected payoff from conflict to dissuade players from triggering it. Therefore, these two objectives are not necessarily aligned.

The economics literature in studying conflict management (Bester and Wärneryd, 2006, Compte and Jehiel, 2009, Fey and Ramsay, 2011, Hörner et al., 2015 and Spier, 1994) assume that outcome of the conflict is determined by an exogenous lottery, thereby turning the mediator's decision into a standard mechanism design problem. Among these papers, Hörner et al. (2015), albeit in a static framework, compares the optimal design of arbitration rule to that of mediation. Mediators lack enforcement power compared to arbitrators. They show that mediators can be as effective as arbitrators in preventing conflict. However, in my framework, the conflict is endogenous and it depends on what rivals learn about each other in the mediation, and the players lack commitment.

The outcome of conflict depends on the resources that the adversaries choose to spend on it. The adversaries make these decisions based on the beliefs that they have about each other and what they have learned about each other in the mediation. When adversaries lack commitment, even after the event that they both have agreed on a peace settlement, they learn about each other, update their beliefs, assess the cost of conflict, and decide whether to renege on an accepted proposal and trigger conflict. A mediator with very limited power that seeks renege-proof mediation mechanisms can only indirectly, through manipulating the posterior beliefs at the end of the mediation stage, affect the cost of conflict and adversaries' decisions to renege. As long as the conflict occurs with a positive probability on the path of the equilibrium, these posterior beliefs are interdependent with the equilibrium strategy profile via Bayes's rule. The interdependence between the strategy profiles and the posterior beliefs makes the problem of finding an optimal peace settlement more challenging. With conflict endogenous and adversaries' lack of commitment, no optimal solution to the mediator's social-surplus maximization problem has been found, however stylized is the model, as long as it precludes the possibility of full preemption of conflict.

The closest papers in the literature that considers the design problem of conflict mediation in a dynamic setting similar to this chapter are Balzer and Schneider (2021),Kamranzadeh and Zheng (2022), and Zheng (2019). These papers also model conflict as an endogenous outcome. However, adversaries are committed to the agreement they reach. Balzer and Schneider (2021), taking minimization of the probability of conflict as their design objective, provides a characterization of the conflict-minimizing solution in terms of the on-path posterior belief system in the equilibrium associated with the mediator's mechanism. Zheng (2019) proposes two notions of full preemption of conflict that differ in the mediator's coordination ability regarding off-path continuation plays. For each notion, Zheng provides a necessary and sufficient condition, in terms of the primitives, under which there exists a mechanism for the mediator to fully preempt conflict. This chapter is based on a case that does not satisfy those conditions. Kamranzadeh and Zheng (2022), hereafter KZ, under the assumption that the mediation mechanism takes the form of a fixed split, provides an explicit characterization of optimal peace proposal in a conflict mediation where the outcome of the conflict is endogenous and full preemption of conflict is impossible.[1]

Relaxing rivals' commitment assumption substantially affects the design of the optimal proposal. In section 4.2.3, I show that the optimal lopsided proposal of KZ is not renege-proof. This optimal proposal is so biased that the in the PBE that it admits the favored player always accepts the proposal while her rival, the player who receives a smaller share, rejects it if her type is strong, and rejects it with a probability in $(0, 1)$ if her type is weak. If both players have accepted the proposal, the favored player learns her adversary is weak with probability one while the less favored player does not learn anything new about her opponent (because her rival always accepts the proposal). Thus, the players partially learn about each other's type after a successful round of negotiations.

[1]The term dynamic mechanism design often has been used to refer to the cases where private information evolves exogenously over time and the principal has full control throughout the game, whereas in my setup an agent's private information is given at the outset and the principal lacks control of the game after the mechanism and hence can only indirectly influence the outcome through manipulating the posterior beliefs via her mechanism. For a useful survey of dynamic mechanism design with exogenous evolution of information over time see Pavan (2017).

If reneging is allowed, the strong type of the favored player exploits the information that her opponent is weak and triggers conflict which provides him a higher expected payoff than the split she has previously agreed on. Therefore, the optimal lopsided split of the renege-banning model is not renege-proof. The renege-proof lopsided split is even more biased than the optimal lopsided split in the renege-banning model of KZ. This more biased split satisfies the favored player that always accepts to never renege while the less favored player follows the same mediation strategies as in the renege-banning model. In other words, the mediator internalizes the lack of commitments in its proposal and this constraint leads to more extreme biased proposals.

Another literature that this chapter relates to is the mechanism design problems where agents lack commitment. Bester and Strausz (2001) extend the revelation principle to the environments where the mechanism designer has limited commitment and cannot commit to the outcome admitted by the mechanism. However, compared to them, in this chapter, agents lack commitment and the designer lacks enforcement power which makes the problem more challenging (for a more recent treatment see Doval and Skreta, 2021, Skreta, 2015, and Bester and Strausz, 2007). Hence, for tractability, I consider only message-independent splits. Moreover, this chapter, at a higher level, relates to the notion of posterior implementability introduced by Green and Laffont (1987). They study mechanisms where players have no commitment but they do not change their message in the mechanism and sign the agreement obtained by the mechanism. Similarly, in our environment, albeit given message-independent splits, the renege-proof PBEs are such that the players ratify an agreement even if the chance of reneging on a previously accepted agreement presents itself. [2]

This chapter also relates to the bargaining models of war. For a recent review of these models see Ramsay (2017) and Baliga and Sjostrom (2013). In these models, war is modeled as an exogenous outcome and one of the two players can make a take-it-or-leave-it offer in the form of the split of the prize in dispute to avoid the war. This literature studies the dynamic interaction between adversaries in this environment. Fey et al. (2013) relaxes commitment to make an agreement or fight in this environment and study how this would impact the peace proposals that are made by one of the players. Compared to them, I study the design of optimal splits that are offered by a neutral benevolent mediator. Moreover, to study the effect of information revelation from mediation on the design of peace proposals, I model conflict as an endogenous outcome.

I shall present the model and preliminary analysis in Section 4.2. Section 4.2.3 pro-

[2] Another literature that deals with environments where players lack commitment is an optimal auction with resale. For instance, Zheng (2002) provides the design of seller-optimal auctions while allowing for resale by winning bidders.

vides theoretically compelling evidence that the optimal split of the renege-banning model is not renege-proof. Section 4.3 characterizes all possible cases of renege-proof PBEs. Section 4.4 reports the main findings and intuitions Appendix C presents the formal arguments and calculation details.

4.2 Model

Two players, named 1 and 2, compete for a prize. Each player's type is independently drawn from the same binary distribution, whose realization is either w ("weak"), with probability θ, or s ("strong") with probability $1-\theta$, such that $\theta \in (0,1)$, and $s > w > 0$.

At the outset, each player i's type t_i is drawn from the above distribution and is privately known to i. In the *mediation* stage, a neutral mediator makes a *peaceful split*

$$\nu := (\nu_1, \nu_2) \in [0,1]^2 \quad \text{such that} \quad \nu_1 + \nu_2 = 1, \tag{4.1}$$

Then each player independently and simultaneously announces whether they Accept or Reject the proposal. If both announce Accept, then with an exogenous probability $\alpha \in [0,1)$ the game enters the *rectification stage* where they can renege on an accepted proposal, and with probability $1-\alpha$ the game ends with each player i getting a payoff equal to the agreed split ν_i. If at least one of the players announces Reject at the mediation stage or reneges on an accepted proposal at the rectification stage, then the game enters *conflict* stage.

In the *rectification* stage any player can renege on the agreed peace settlement and trigger conflict. At this stage players simultaneously announce whether they ratify the accepted proposals or renege on them. If any player reneges the game enters the conflict stage. Otherwise, the game ends with player i getting a payoff equal to ν_i according to the peaceful split that they agreed on in the mediation stage.

In the *conflict* stage, each player i submits a sealed bid $b_i \in \mathbb{R}_+$. The player who submits the higher bid (spends higher resources) wins the prize, with ties broken randomly with equal probabilities. The game ends, with player i getting a payoff equal to $1 - b_i/t_i$ if i wins the prize, and equal to $-b_i/t_i$ if otherwise. The bids are interpreted as the level of resources that a player chooses to devote to conflict.

Once the mediator has chosen a mediation mechanism in the form of a peace split, the ensuing multistage game is defined for which I use perfect Bayesian equilibrium (PBE) as the solution concept.

A PBE is said to be *renege-proof* if and only if on its path renege occurs with zero probability relative to the prior distribution. A peace split is said to be *renege-proof* if and only if the multistage game given the split admits a renege-proof PBE. A peace split is *optimal* if it maximizes the social surplus, the sum of the ex-ante payoffs of players, subject to renege-proof constraint. The mediator's objective is to maximize the social surplus subject to the renege-proof constraint.

The following assumption is maintained throughout this chapter:

$$(1 - w/s)\,\theta > 1/2. \tag{4.2}$$

As demonstrated in Zheng (2019), the minimum peace proposal that is required to guarantee peace in our game is $\theta(1 - w/s)$, i.e., once each player is offered this peace split, they will Reject it with zero probability on the path of equilibrium. Hence, if the parameters are such that $2\theta(1 - w/s)$ is less than or equal to 1, i.e., the size of the prize, then an equal split that takes the form of $\theta(1 - w/s)$ and is offered to each payer would guarantee peace. The above assumption is to avoid the triviality of the problem.

As a benchmark, the *renege-banning* model is a variant of the above-defined model that is obtained from removing the rectification stage so that a player cannot renege on his acceptance of a peace proposal, i.e., $\alpha = 0$.

Remark 4.1 From the point of view of the actual conflict mediation procedure, the possibility of reneging can capture the exogenous changes in the environment, e.g., changes in decision-makers, or the fact that once an agreement is reached between parties, the formal procedure until the agreement becomes an official document usually takes time. Once the formal agreement is signed then it is either enforceable by a third party (for example in the context of a dispute between firms) or sufficiently costly to renege (for example in the context of international conflicts). Until the agreement is signed, either side can still refuse to sign and trigger conflict. Even when an agreement is signed, the change in decision-makers on one or both sides can provide a chance for reneging on the agreement and triggering conflict. This is especially common and important in the international conflicts where players are sovereign states and agreements are not usually third-party enforceable. In this context, α can be attributed to the possibility of change in decision-makers. Thus, one expects α to be higher in environments where adversaries are democratic states. From a theoretical point of view, this possibility is important due to information revelation from the mediation process. Not only a failed mediation process reveals important information about players' types that can be exploited against them in the event of a conflict, but also a successful mediation can provide valuable information about players types who have agreed on a proposal. The fact that one player has agreed on a less favorable proposal may convey important information about her hesitancy to go into conflict, i.e., her type being weak, which may provide reason for her rival to renege on the agreement if the possibility presents itself. In Section 4.2.3, I show that the optimal proposal of the renege banning model of KZ is not renege-proof.

4.2.1 The Continuation Equilibrium During Conflict

Given any proposed split (ν_i, ν_{-i}) a multi-stage game is defined. Let $\sigma_i(t_i)$ denote the probability with which each player i of type t_i rejects the proposal in the mediation stage. Given any strategy profile $(\sigma_i)_{i=1}^2$, we obtain player i's ex-ante probability q_i^A of accepting the proposal (before realization of his type), and his probability q_i^R of rejecting it:

$$q_i^A = \theta(1-\sigma_i(w)) + (1-\theta)(1-\sigma_i(s)), \tag{4.3}$$
$$q_i^R = \theta\sigma_i(w) + (1-\theta)\sigma_i(s). \tag{4.4}$$

Denote π_i^A for the posterior probability of "$t_i = w$" conditional on i's accepting the proposal, and π_i^R for the posterior probability of "$t_i = w$" conditional on i's rejecting it. By Bayes's rule,

$$\pi_i^A q_i^A = \theta(1-\sigma_i(w)), \tag{4.5}$$
$$\pi_i^R q_i^R = \theta\sigma_i(w). \tag{4.6}$$

Sum the two equalities to obtain

$$\pi_i^A q_i^A + \pi_i^R q_i^R = \theta. \tag{4.7}$$

Note from Eqs. (4.5) and (4.6) that, for any $i \in \{1, 2\}$, if $0 < \sigma_i(w) < 1$ then

$$\pi_i^R < (\text{resp. } \leq)\ \pi_i^A \Longleftrightarrow \sigma_i(s) > (\text{resp. } \geq)\ \sigma_i(w) \Longleftrightarrow \pi_i^R < (\text{resp. } \leq)\ \theta < (\text{resp. } \leq)\ \pi_i^A. \tag{4.8}$$

To understand the working of the model, I start with the last stage which is the conflict stage. Whether this stage is entered because someone rejected the proposal, or one accepted a proposal at the mediation stage and then reneged at the rectification stage, the game is the same. Each player knows their type, the history of the game, and has a posterior belief about the rival's type based on that history. Denote π_i for the posterior probability of player i being type $t_i = w$. Denote $\mathscr{G}(\pi_i, \pi_{-i})$ for the continuation game at the conflict stage such that π_i is the posterior distribution of player i's type for each $i \in \{1, 2\}$. At any $\mathscr{G}(\pi_i, \pi_{-i})$ each player simultaneously choose b_i and the outcome is determined. This conflict game is analogous to an all-pay auction where each player submits a sealed bid $b_i \in \mathbb{R}_+$ and each player has private information about their marginal cost of exerting effort, which is their types.

Any pair $(\pi_i)_{i=1}^2$ of posterior probabilities determines the expected payoff for each player-type in the continuation game of conflict according to the next lemma:

Lemma 4.1 *(Kamranzadeh and Zheng (2022)) Given any pair $(\pi_i)_{i=1}^2$ of posterior probabilities at the start of the conflict stage, the expected payoff for each player i at any Bayesian Nash equilibrium in the conflict stage is equal to* [3]

$$U_i^s(\pi_i, \pi_{-i}) = (1 - w/s) \max\{\pi_i, \pi_{-i}\}, \tag{4.9}$$

$$U_i^w(\pi_i, \pi_{-i}) = (1 - w/s) (\pi_{-i} - \pi_i)^+ . \tag{4.10}$$

This chapter studies renege-proof peace proposals as defined above. Therefore, renege is an off-path event at the rectification stage and it triggers conflict. This outside option of conflict is endogenous and its payoff depends on the beliefs players have about each others' type distribution. Suppose player $-i$ unilaterally deviates and reneges on a previously accepted proposal in the mediation. Denote the off-path belief upon renege by π_{-i}^D. In that event, Bayes's rule implies $\pi_i = \pi_i^A$, as player i has accepted the proposal before entering the rectification stage. π_{-i}^D is off-path and hence arbitrary. I maintain the assumption that the off-path belief π_{-i}^D is independent of π_i^A along with the "no signaling what you don't know" assumption of Fudenberg and Tirole (1991). Payoff of the deviating player depends on the assumption on the posterior beliefs at the continuation game $\mathscr{G}\left(\pi_i^A, \pi_{-i}^D\right)$, where the type distribution π_{-i}^D is arbitrary and in $[0, 1]$.

Thus, given any peace proposal (x_1, x_2), a PBE of the multi-stage game amounts to a pair of mediation strategy profile $(\sigma_i)_{i=1}^2$ and belief system $(\pi_i^A, \pi_i^R, \pi_i^D)_{i=1}^2$ such that, for each player i, σ_i best replies to σ_{-i} given the continuation values determined by $(\pi_i^A, \pi_i^R, \pi_i^D)_{i=1}^2$ according to Lemma 4.1, and $(\pi_i^A, \pi_i^R, \pi_i^D)$ obey Eqs. (4.5) and (4.6).

4.2.2 The Equilibrium Condition During Mediation

Given any mediation strategy profile, each player i's interim expected payoff from rejecting or accepting a peace proposal (x_1, x_2) is determined. Given type $t \in \{w, s\}$, player i'

[3] $y^+ := \max\{y, 0\}$.

expected payoff from choosing Reject is equal to

$$V_i^R(\nu_i;t) := q_{-i}^A U_i^t\left(\pi_i^R,\pi_{-i}^A\right) + q_{-i}^R U_i^t\left(\pi_i^R,\pi_{-i}^R\right). \tag{4.11}$$

and that from choosing Accept is equal to

$$V_i^A(\nu_i;t) := q_{-i}^A\left[(1-\alpha)\nu_i + \alpha\max\{\nu_i, U_i^t\left(\pi_i^D,\pi_{-i}^A\right)\}\right] + q_{-i}^R U_i^t\left(\pi_i^A,\pi_{-i}^R\right), \tag{4.12}$$

Here, the term $\max\{\nu_i, U_i^t\left(\pi_i^D,\pi_{-i}^A\right)\}$ reflects the type-t of player i's choice at the rectification stage between reneging, thereby getting a payoff equal to $U_i^t\left(\pi_i^D,\pi_{-i}^A\right)$, or remaining committed to the accepted proposal and thereby getting ν_i (as player $-i$ is expected never to renege in a renege-proof PBE). The rectification stage happens with probability α. Thus, the possible payoff from this stage is multiplied by α.

A proposal admits a renege-proof PBE if and only if for each player $i \in \{1,2\}$ and for an off-path belief $\pi_i^D \in [0,1]$ either (i) type t of player i ratifies a proposal at the rectification stage, i.e., $\nu_i \geq U_i^t(\pi_i^D,\pi_{-i}^A)$, or (ii) type t of player i announces Reject at the mediation stage, i.e., $V_i^R(\nu_i,t) \geq V_i^A(\nu_i,t)$. Condition (ii) means that even if type t will renege at the rectification stage, its reneging is not observed on the equilibrium path because it would reject the proposal at the mediation stage, to begin with. Hence, this type is not present at the rectification stage.

Call a PBE *always-conflict* if and only if $\sigma_i(s) = \sigma_i(w) = 1$ for some player $i \in \{1,2\}$. It is easy to show that any peace proposal admits an always conflict PBE, supported by posteriors $\pi_j^A = \pi_j^R = \theta$, on or off path, for each $j \in \{1,2\}$ (Lemma C.2).

4.2.3 The Optimal Solution of the Renege-Banning Model Is Not Renege-Proof

A perfect Bayesian equilibrium (PBE) in the renege-banning model, given some mediation mechanism, is said *renege-proof* iff its strategy profile, coupled with "no type of any player ever reneges in the rectification stage," constitutes a PBE in the renege-allowing model, given the same mediation mechanism. In the renege-banning model of KZ, the lopsided split $\nu_i = \theta$ and $\nu_{-i} = 1-\theta$ maximizes the social-surplus. Lopsided in the sense of giving a larger share, i.e., $\nu_i = \theta$, to some player $i \in \{1,2\}$ such that she always accepts the proposal, while giving a smaller share, i.e., $\nu_{-i} = 1-\theta$, to the less favored player. Player $-i$, rejects this proposal if she is of strong type and mixes between accepting and rejecting if she is weak type (Proposition 1 of Kamranzadeh and Zheng (2022)).

We observe that this lopsided PBE is not renege-proof. Suppose, to the contrary, that it is renege-proof. Consequently, given the same proposed split $(\theta, 1-\theta)$, in the renege-allowing model, at the rectification stage where both players have accepted the proposal, the posterior belief about player i, who always announces Accept, by Bayes's rule remains equal to the prior $\pi_i^A = \theta$, whereas the posterior about player $-i$, who always announces Reject if her type is $t_{-i} = s$, is that her type is equal to $t_{-i} = w$, i.e., $\pi_{-i}^A = 1$. But then in the rectification stage, player i of type s strictly prefers to renege: If she reneges, conflict ensues and by Lemma 4.1, scaled by $1/(1-w/s)$ to be comparable with KZ's conflict payoff, her expected payoff is equal to

$$U_i^s(\pi_i^A, \pi_{-i}^A) = \max\{\pi_i^A, \pi_{-i}^A\} = \max\{\theta, 1\} = 1,$$

wherease, if she does not renege, she gets the peaceful share θ, which is less than 1. Therefore, the strong type of the favored payer always has the incentive to renege, implying the optimal proposal of the renege-banning model is not renege-proof.

Intuitively, the strong type of the favored player at the rectification stage by observing that her opponent has accepted the smaller share of a lopsided split infers that her opponent's type is weak with probability 1. Thus, if the possibility to renege presents itself, this player figures that she can be better off by triggering conflict rather than committing to the share she initially agreed on at the mediation stage. This is despite the fact that this player has already received a favorable split. But, this favorable split is not large enough to convince her to not renege in the presence of the new information she inferred about her opponent after a successful round of mediation. She will be better off by reneging and triggering conflict because she will encounter a weak opponent and she can win such a conflict at a very low cost.

4.3 All Possible Cases of a Renege-Proof Equilibrium

The following lemma categorizes all possible cases of renege-proof PBEs. The proof for this lemma, and all other proofs, are provided in Appendix C.

Lemma 4.2 *Suppose any (θ, w, s) satisfying (4.2) and any $\alpha \in (0,1)$. There are only two possible classes of renege-proof PBEs that are not always-conflict:*

a. *For some $i \in \{1,2\}$, at the mediation $\sigma_i(s) = \sigma_i(w) = 0 < \sigma_{-i}(w) < 1 = \sigma_{-i}(s)$. Player i and type w of player $-i$ ratify at the rectification stage. Call this case*

Lopsided *PBEs.*

b. *For each* $i \in \{1, 2\}$*, at the mediation* $0 < \sigma_i(w) < 1 = \sigma_i(s)$*. Type* w *of both players ratify at the rectification stage. Call this case* mutually partially mixed (MPM) *PBEs.*

In Section 4.2.2, I characterize player i's interim-expected-payoff from announcing Reject and Accept the proposal, expecting the other player to never renege. I also specify the renege-proof conditions. In Appendix C.1, I categorize all possible cases of renege-proof PBEs, and then use the best responses conditions and renege-proof conditions to show that only two classes of renege-proof PBEs are possible. This is shown by Lemmas C.1 -C.5, which are then used to prove Lemma 4.2 in Appendix C.1.1.

In Appendix C.1, I show that if the strong type of both players accepts a proposal with a positive probability then they would renege on a proposal at the rectification stage unless the proposal is very biased such that one player always accepts it. A player by announcing Accept signals weakness. Thus, to convince the strong type of each player to ratify a proposal at the rectification stage one should offer them a large share, which is not possible given the size of the prize. Therefore to have a renege-proof PBE either a strong type of each player should reject the proposal at the mediation stage or one player should receive a very large share that she always accepts and ratifies and her opponent rejects if she is strong and accepts and ratifies if she is weak. Lemma 4.2 summarizes these results in two cases of Lopsided and MPM PBEs respectively. In other words, Lemma 4.2 implies that it is not possible to have a proposal that both players ratify at the rectification stage. But we can have a renege-proof proposal that one player always accepts and ratifies while her opponent rejects if she is a strong type and accepts with positive probability if she is weak, and ratifies at the rectification stage. Moreover, there is another possible class of renege-proof PBEs where the strong type of both players reject a proposal while their weak types follow a mix strategy at the mediation stage and ratify at the rectification stage. The next section characterizes these classes of PBEs.

4.3.1 Characterization of Renege-Proof Equilibria

Lemma 4.2 states that the only two possible classes of renege-proof PBEs are Lopsided and MPM PBE. In this section, I characterize these two classes of PBEs. To that end and for notational convenience, denote

$$r := 1/(1 - w/s). \tag{4.13}$$

Using the definition of r, a neutral mediator makes a *peace proposal*

$$x := (x_1, x_2) \in [0, r]^2 \quad \text{such that} \quad x_1 + x_2 = r, \tag{4.14}$$

interpreted as a peaceful split $(x_1/r, x_2/r)$ of the prize. Thus, using the definition of r the maintained assumption (4.2) on the parameters is equivalent to

$$1 < r < 2\theta < 2. \tag{4.15}$$

Given any mediation strategy profile, each player i's interim expected payoff from rejecting or accepting a peace proposal (x_1, x_2) is determined by Eqs. (4.11) and (4.12). Plugging in the conflict payoffs for the strong and weak type, i.e., Eqs. (4.9) and (4.10) , into Eqs. (4.11) and (4.12) and using the above mentioned definition of r, the difference between the two payoffs is summarized by a vector:

$$\begin{aligned}\begin{bmatrix} \Delta_i(s) \\ \Delta_i(w) \end{bmatrix} = \; & q_{-i}^A \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^A\right\} - \left((1-\alpha)x_i + \alpha \max\left\{x_i, \pi_i^D, \pi_{-i}^A\right\}\right) \\ \left(\pi_{-i}^A - \pi_i^R\right)^+ - \left((1-\alpha)x_i + \alpha \max\{x_i, \left(\pi_{-i}^A - \pi_i^D\right)^+\}\right) \end{bmatrix} \\ & + q_{-i}^R \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^R\right\} - \max\left\{\pi_i^A, \pi_{-i}^R\right\} \\ \left(\pi_{-i}^R - \pi_i^R\right)^+ - \left(\pi_{-i}^R - \pi_i^A\right)^+ \end{bmatrix}. \end{aligned} \tag{4.16}$$

Here, the term $\max\left\{x_i, \pi_i^D, \pi_{-i}^A\right\}$ on the top line reflects the type s of player i's choice between reneging, provided that rectification is possible with probability α , thereby getting a payoff equal to $\alpha \max\{\pi_i^D, \pi_{-i}^A\}$, and not reneging thereby getting αx_i. Analogously, the term $\max\left\{x_i, \left(\pi_{-i}^A - \pi_i^D\right)^+\right\}$ on the bottom line is for type w of player i.

Thus, a proposal admits a renege-proof PBE if and only if for each player $i \in \{1, 2\}$ for an off-path belief $\pi_i^D \in [0, 1]$ all the followings hold:

a. either (i) type w ratifies at the rectification stage, i.e., $x_i \geq \left(\pi_{-i}^A - \pi_i^D\right)^+$, or (ii) type w announces Reject in the mediation, i.e., $\Delta_i(w) \geq 0$;

b. either (i) type s ratifies at the rectification stage, i.e., $x_i \geq \max\left\{\pi_i^D, \pi_{-i}^A\right\}$, or (ii) type s announces Reject in the mediation, i.e., $\Delta_i(s) \geq 0$.

Alternative (ii) in each of these conditions means that even if the type would renege its reneging is not observed on the path because it would reject the proposal, to begin with. This means that either each type finds the accepted proposal better than triggering conflict or she would have rejected it at the mediation stage.

Lopsided PBEs

As defined in Lemma 4.2, a PBE belongs to renege-proof Lopsided PBEs if and only if

$$\sigma_i(w) = \sigma_i(s) = 0 < \sigma_{-i}(w) < 1 = \sigma_{-i}(s), \text{ and player } i \text{ and type } w \text{ of } -i \text{ ratify.} \quad (4.17)$$

The Bayes's rule implies $\pi_i^A = \theta$ and $\pi_{-i}^A = 1$. The best response conditions for (4.17) to constitute a renege-proof PBE are that $V_i^R(s) - V_i^A(s) \leq 0$, $V_i^R(w) - V_i^A(w) \leq 0$, $V_{-i}^R(s) - V_{-i}^A(s) \geq 0$, $V_{-i}^R(w) - V_{-i}^A(w) = 0$, $x_i \geq \max\{\pi_i^D, \pi_{-i}^A\}$, $x_i \geq \left(\pi_{-i}^A - \pi_i^D\right)^+$, and $x_{-i} \geq \left(\pi_i^A - \pi_{-i}^D\right)^+$.

First, we verify that $x_i \geq 1$ in any renege-proof Lopsided PBEs. Player i always announces Accept and ratifies a proposal at the rectification stage. By monotonicity of conflict's payoff in type (Lemma 4.1), the most tempted type of this player to renege at the rectification stage is $t_i = s$. Thus, the best response condition for player i to ratify the peace proposal at the rectification stage is

$$x_i \geq U_i^s(\pi_i^D, \pi_{-i}^A) = \max\{\pi_i^D, \pi_{-i}^A\} = \max\{\pi_i^D, 1\} = 1.$$

Thus, given the split $x_i \geq 1$, both types of player i ratify a proposal in Lopsided PBEs.

By Eq. (4.8), $\pi_{-i}^R < \theta < \pi_{-i}^A$. Moreover, by (4.17), the Bayes's rule implies $\pi_i^A = \theta$ and $\pi_{-i}^A = 1$. By plugging these posterior beliefs in Eq. (4.16) one obtains:

$$\begin{aligned}
\Delta_i(s) &= q_{-i}^A(1 - x_i) + q_{-i}^R \left[\max\left\{\pi_i^R, \pi_{-i}^R\right\} - 1\right], \\
\Delta_i(w) &= q_{-i}^A \left[1 - \pi_i^R - x_i\right] + q_{-i}^R \left[\left(\pi_{-i}^R - \pi_i^R\right)^+\right], \\
\Delta_{-i}(s) &= \left[\max\{\pi_{-i}^R, \theta\} - (1-\alpha)x_{-i} - \alpha \max\{x_{-i}, \pi_{-i}^D, \theta\}\right], \\
\Delta_{-i}(w) &= \left[\left(\theta - \pi_{-i}^R\right)^+ - (1-\alpha)x_{-i} - \alpha \max\{x_{-i}, \left(\theta - \pi_{-i}^D\right)^+\}\right].
\end{aligned}$$

Pick the off-path posteriors $\pi_i^D \in [0,1]$ and $\pi_{-i}^D \in [\theta - x_{-i}, \theta]$. Thus,

$$\Delta_{-i}(w) = \left[\left(\theta - \pi_{-i}^R\right)^+ - (1-\alpha)x_{-i} - \alpha \max\{x_{-i}, \left(\theta - \pi_{-i}^D\right)^+\}\right] = \theta - \pi_{-i}^R - x_{-i}.$$

Agent $-i$ of type w follows mixed strategies at the mediation. Thus, the best response condition is $V_{-i}^R(w) - V_{-i}^A(w) = 0$, or equivalently $\Delta_{-i}(w) = 0$. Equilibrium strategies and beliefs would be uniquely determined by this equation:

$$\sigma_{-i}(w) = \frac{(1 - x_{-i}/\theta)}{(1 + x_{-i}/(1-\theta))}, \; \pi_i^A = \theta, \; \pi_{-i}^A = 1, \; \pi_{-i}^R = \theta - x_{-i}.$$

Third, given $x_i \geq 1$, $x_{-i} \leq r-1$ and the off-path posteriors $\pi_i^D \in [0,1]$, $\pi_{-i}^D \in [\theta - x_{-i}, \theta]$, and $\pi_i^R \in [0,1]$, one can use the above displayed equations for $\Delta_i(s)$, $\Delta_i(w)$, and $\Delta_{-i}(s)$ to verify the best response conditions for mediation strategies of player i and type s of player $-i$ are satisfied:

$$\Delta_i(s) \leq q_{-i}^A(1-1) + q_{-i}^R\left[\max\left\{\pi_i^R, \pi_{-i}^R\right\} - 1\right] < 0,$$
$$\Delta_i(w) \leq q_{-i}^A\left[1 - \pi_i^R - 1\right] + q_{-i}^R\left[\left(\pi_{-i}^R - \pi_i^R\right)^+\right] < 0,$$
$$\Delta_{-i}(s) \geq \left[\theta - (1-\alpha)(r-1) - \alpha\max\{x_{-i}, \pi_{-i}^D, \theta\}\right] = (1-\alpha)(\theta - (r-1)) > 0,$$

where the equality in the last line is due to $\max\{x_{-i}, \pi_{-i}^D, \theta\} = \theta$ by $\pi_{-i}^D \in [\theta - x_{-i}, \theta]$, $x_{-i} \leq r-1$, and $\theta > r/2$ and $r \in (1,2)$ by assumption (4.15), which also verifies the last inequality in the above displayed set of inequalities. Hence, best response conditions such that splits $x_i \geq 1$, $x_{-i} \leq r-1$ admit mediation strategies defined by (4.17) is satisfied.

Lastly, I check that given the lopsided proposals and off-path beliefs π_{-i}^D, renege-proof conditions for type w of player $-i$ is satisfied. Weak type of player $-i$ would get $x_{-i} \leq r-1$ if she stays committed to the accepted proposal. If she reneges she would get $(\theta - \pi_{-i}^D)^+$ which is less than x_{-i} by $\pi_{-i}^D \in [\theta - x_{-i}, \theta]$. Thus, she doe not renege at the rectification stage. At the mediation, weak type of player $-i$ is indifferent between accepting and rejecting the proposal. As shown above, strong type of player $-i$ Reject the proposal at the mediation. These results are summarized in the following Proposition.

Proposition 4.1 *Suppose any (θ, r) satisfying Ineq. (4.15) and any $\alpha \in (0,1)$. For some $i \in \{1,2\}$ any lopsided proposal such that $x_i \geq 1$ and $x_{-i} \leq r-1$, where $x_{-i} < x_i$, admits a renege proof Lopsided PBE that is characterized by:*

$$\sigma_i(w) = \sigma_i(s) = 0,\ \sigma_{-i}(w) = \frac{(1-\theta)(\theta - x_{-i})}{\theta(1-\theta+x_{-i})},\ \sigma_{-i}(s) = 1.$$
$$\pi_i^A = \theta,\ \pi_{-i}^A = 1,\ \pi_{-i}^R = \theta - x_{-i},\ \pi_i^R \in [0,1],\ \pi_i^D \in [0,1],\ \text{and } \pi_{-i}^D \in \left[\pi_{-i}^R, \theta\right].$$

MPM PBEs

A PBE belong to MPM PBEs if and only if its strategy profile satisfies

$$\forall i \in \{1,2\} : 0 < \sigma_i(w) < 1 = \sigma_i(s) \text{ and type } w \text{ of player } i \text{ ratifies.} \tag{4.18}$$

In other words, in this case of PBEs, weak type of both agents mixes between announcing Accept and Reject in the mediation stage and ratifies in the rectification stage while the strong type of both players announces Reject in the mediation. The strong type of both players has rejected the proposal and are not present at the rectification stage.

The best response conditions for (4.18) to constitute a PBE are $V_i^R(w) - V_i^A(w) = 0$, $V_i^R(s) - V_i^A(s) \geq 0$, and $x_i \geq U_i^w\left(\pi_i^D, \pi_{-i}^A\right)$ for each player i. This in turn is equivalent to $\Delta_i(w) = 0$, $\Delta_i(s) \geq 0$, and $x_i \geq \left(\pi_{-i}^A - \pi_i^D\right)^+$.

By (4.18), Bayes's rule implies $\pi_i^A = 1$ for each player i and hence (by Eq. (4.7)), $q_i^R(1-\pi_i^R) = 1-\theta$ for each player i. By Eq. (4.8), $\pi_i^R < \theta < \pi_i^A$ for each $i \in \{1,2\}$. By plugging in these posterior beliefs in Eq. (4.16), one obtains:

$$\begin{bmatrix} \Delta_i(s) \\ \Delta_i(w) \end{bmatrix} = q_{-i}^A \begin{bmatrix} (1-\alpha)(1-x_i) \\ \left(1-\pi_i^R\right) - \left((1-\alpha)x_i + \alpha \max\{x_i, \left(1-\pi_i^D\right)\}\right) \end{bmatrix} + \quad q_{-i}^R \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^R\right\} - \pi_i^A \\ \left(\pi_{-i}^R - \pi_i^R\right)^+ \end{bmatrix}.$$

Recall that π_1^D and π_2^D are off-path beliefs. Pick $1 - x_i \leq \pi_i^D \leq 1$ for each $i \in 1, 2$. Thus, the above set of equations simplify to:

$$\begin{bmatrix} \Delta_i(s) \\ \Delta_i(w) \end{bmatrix} = q_{-i}^A \begin{bmatrix} (1-\alpha)(1-x_i) \\ \left(1-\pi_i^R\right) - x_i \end{bmatrix} + \quad q_{-i}^R \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^R\right\} - \pi_i^A \\ \left(\pi_{-i}^R - \pi_i^R\right)^+ \end{bmatrix}. \tag{4.19}$$

At the MPM PBEs, the equilibrium strategies $\sigma_i(w) \in (0,1)$ implies a system of equations $\Delta_i(w) = 0$ while $\sigma_i(s) = 1$ implies $\Delta_i(s) \geq 0$ for each player i. Since the defining condition of this PBE is symmetric between the two players, let us assume, without loss of generality, that $\pi_2^R \geq \pi_1^R$. Hence, the equation system $\Delta_i(w) = 0$ uniquely determines the strategy profile $(\sigma_i(w))_{i=1}^2$ and the on-path posteriors $(\pi_i^A, \pi_i^R)_{i=1}^2$. Then plug this solution into the system of inequalities to simplify equilibrium conditions $\Delta_1(s) \geq 0$ and $\Delta_2(s) \geq 0$. Hence, a split (x_2, x_1) admit renege-proof MPM equilibria if and only if it satisfies

$$q_i^R = \frac{1-\theta}{1-\pi_i^R} \text{ and } \pi_i^A = 1 \ \forall i \in \{1,2\}, \ \pi_2^R = 1 - x_2, \pi_1^R = \frac{\theta\,(x_2 - x_1) + x_1\,(1-x_2)}{x_2}, \tag{4.20}$$

and,

$$\Delta_1(s) \geq 0 \iff q_2^A(1-x_1)(1-\alpha) \geq (1-q_2^A)x_2, \tag{4.21}$$

$$\Delta_2(s) \geq 0 \iff q_1^A(1-x_2)(1-\alpha) \geq (1-q_1^A)x_2. \tag{4.22}$$

By (4.20), the inequality $\pi_2^R \geq \pi_1^R$ is equivalent to

$$1 - x_2 \geq \frac{\theta\,(x_2 - x_1) + x_1\,(1 - x_2)}{x_2} \iff (1 - x_2 - \theta)\,(x_2 - x_1) > 0,$$

The last inequality displayed above is equivalent to either (i) $r - 2x_2 \geq 0$ and $x_2 \geq 1 - \theta$ or, (ii) $r - 2x_2 \leq 0$ and $x_2 \leq 1 - \theta$, which is impossible due to Ineq. (4.15) . Thus,

$$\pi_2^R \geq \pi_1^R \iff 1 - \theta \leq x_2 \leq r/2. \tag{4.23}$$

Hence, the equal proposal is the upper bound of splits that can admit renege-proof MPM PBEs. One can show that the social surplus function of MPM PBEs and the probability of peace within this class of PBEs is strictly increasing in $x_{\min} = \min\{x_1, x_2\}$ (By Lemmas C.9 and C.11). Therefore, in the rest of this section I characterize conditions under which the equal proposal, the upper bound for x_2, admits renege-proof MPM PBEs.

These results are summarized in the following proposition. It implies that when probability of rectification stage is higher than a threshold or the probability of being weak is lower than a threshold, the equal proposal is not renege-proof. When probability of the rectification stage is high enough, the strong type of each player would rather accept the proposal at the mediation stage and then renege on it at the rectification stage than rejecting the split for sure. Denote this threshold as

$$\Gamma(\theta, r) := \frac{4\theta - 3 - (r-1)^2}{(2-r)(r + 2\theta - 2)}. \tag{4.24}$$

Proposition 4.2 *Suppose any (θ, r) satisfying Ineq. (4.15) and any $\alpha \in (0,1)$. There exists a $\Gamma(\theta, r) \in (0,1)$ such that if and only if $\alpha < \Gamma(\theta, r)$ and $\theta \geq \frac{3+(r-1)^2}{4}$, then the equal proposal admits a symmetric renege-proof MPM PBE that for each player $i \in \{1, 2\}$:*

$$\sigma_i(w) = \frac{(1-\theta)(2-r)}{\theta r}, \;\; \sigma_{-i}(s) = 1, \;\; \pi_i^A = 1, \;\; \pi_i^R = 1 - r/2, \;\; \textit{and } \pi_i^D \in \left[\pi_i^R, 1\right].$$

4.4 Main Findings

In the previous section, I show that there are only two classes of PBEs that are renege-proof: MPM and Lopsided PBEs. Moreover, in Proposition 4.2, I verify the uniqueness and existence of MPM PBEs admitted by the equal split. This split is of interest because by Lemmas C.9 and C.11 in the Appendices C.3 and C.4, it maximizes the social surplus and the probability of peace among the proposals that admit MPM PBEs. This section compares the equal split with the lopsided proposals for two different objectives for the mediator. One is social-surplus maximization and the other is conflict minimization. I show that if the probability of the rectification stage is above a threshold or the prior probability of being weak is below a threshold, then the only renege-proof social-surplus maximizer proposal is the lopsided split $(w/s, 1-w/s)$. These results hold even when the objective is minimizing the ex-ante probability of conflict.

4.4.1 The Social Surplus Maximizing Proposal

Pick any peace proposal (x_1, x_2) such that without loss of generality $x_2 \le x_1$, i.e., $x_2 = \min\{x_1, x_2\} =: x_{\min}$. Let (σ, π) denote the associated equilibrium and q the associated probability system defined in (4.3)–(4.4). Thus, for any $x_{\min} \in (0, r/2]$, let $S(x_{\min})$ denote the social surplus generated by any not-always-conflict PBE given a peace proposal (x_1, x_2) such that $\min\{x_1, x_2\} = x_{\min}$; and let $S(0)$ denote the social surplus generated by the always-conflict PBE (which one can show is the only kind of PBEs given $x_{\min} = 0$). For notation convenience we scale up the social surplus $S(x_{\min})$ by the positive parameter r and denote $\tilde{S}(x_{\min}) := rS(x_{\min})$.

First, by Lemma C.2 (Appendix C.1), we know that always-conflict PBEs admits social surplus equals to $\tilde{S} = 2\theta(1-\theta)$. These PBEs are always outperformed by the equal proposal. This is because by Lemma C.7 (Appendix C.3), the equal proposal admits social surplus $\tilde{S} = rS = \theta$, which by Ineq. (4.15) is higher than that of always-conflict PBEs.

By Lemmas C.6 and C.7, in the Appendix C.3, the social surplus admitted by the Lopsided and the MPM PBEs can be summarized as:

i. Given peace proposal (x_1, x_2) such that $x_2 < x_1$, then Lopsided PBEs admits social surplus that is equal to:

$$rS(x, q, \pi) = q_2^A x_1 + \theta(1-\theta) q_2^R + \theta\left(1 - \pi_2^R\right); \tag{4.25}$$

ii. Given peace proposal (x_1, x_2) such that $x_2 \le x_1$, then MPM PBEs admits social surplus that is equal to:

$$rS(x, q, \pi) = \theta - \theta\left(\pi_1^R + \pi_2^R\right) + q_1^A \pi_1^A + q_1^R \pi_2^R. \tag{4.26}$$

By Lemmas C.8 and C.9, in Appendix C.3 , the social surplus function admitted by the splits admitting the Lopsided and MPM PBE are strictly increasing in $x_{\min} = \min\{x_1, x_2\}$. The equal proposal, i.e., $x_1 = x_2 = r/2$, is the upper bound of $x_{\min} =$

$\min\{x_1, x_2\}$ that admit MPM PBEs (Ineq. 4.23). Moreover, by Propositions 4.1, $x_2 = \min\{x_1, x_2\} = r-1$ is the upper bound of the proposals that admit renege-proof Lopsided PBEs. Thus, the only candidate for social surplus maximum are the equal proposal $(r/2, r/2)$ and the lopsided proposal $(r-1, 1)$, or respectively the equal split $(1/2, 1/2)$ and the lopsided split $(w/s, 1-w/s)$. The following theorem states the main result of this chapter. To that end, define threshold $\phi(r)$:

$$\phi(r) := \max\left\{\frac{r+\sqrt{r^2+8r-8}}{4}, \frac{3+(r-1)^2}{4}\right\}. \tag{4.27}$$

Theorem 4.1 *For any (θ, r) satisfying Ineq. (4.15) and any any $\alpha \in (0,1)$ there exists a $\Gamma(\theta, r) \in (0,1)$ and a $\phi(r) \in (r/2, 1)$ such that a unique renege-proof $\nu^* \in (0, 1/2]$ maximizies the social surplus according to:*

$$\arg\max_{\nu_{\min}\in[0,1/2]} S(\nu_{\min}) = \begin{cases} \{w/s\} & \text{if } \alpha \geq \Gamma(\theta,r) \text{ or } \theta < \frac{3+(r-1)^2}{4} , \\ \{w/s\} & \text{if } \alpha < \Gamma(\theta,r) \text{ and } \theta < \phi(r) , \\ \{w/s, 1/2\} & \text{if } \alpha < \Gamma(\theta,r) \text{ and } \theta = \phi(r) , \\ \{1/2\} & \text{if } \alpha < \Gamma(\theta,r) \text{ and } \theta > \phi(r) . \end{cases}$$

Thus, if the probability of the rectification stage is high, then only the lopsided split $(w/s, 1-w/s)$ is renege-proof and maximizes the social surplus. The same results hold when the prior probability of being weak is lower than a threshold. When there is a very high chance that the rectification stage is available and the equal split is proposed, the strong type of both players prefer to accept a proposal with a positive probability similar to the weak types, partially revealing information by pooling with the weak types, and then renege at the rectification stage. Analogously, when the prior probability of players being weak is relatively high, the strong type of both players prefer to accept a proposal with a positive probability and then renege at the rectification stage.

The lopsided proposal does not provide any such incentives because one player is receiving a very large share of the prize that she always accepts and never reneges while the strong type of her opponent rejects at the mediation stage and is not present at the rectification stage. However, when the rectification stage is available with a lower probability, then depending on how high the prior probability of being weak and the relative strength w/s are, either the lopsided or the equal split is the renege-proof social surplus maximizing proposal. In these cases, when players are weak with a very high probability, the strong types always have an incentive to reject a proposal. By Lemma C.8 (Appendix C.3), the social surplus function in Lopsided PBEs is strictly increasing in $x_{\min} = w/s$, thus the lower w/s the lower the social surplus admitted by it. Hence, the equal split outperforms the lopsided split for high θ and low w/s values. Figure 4.1, shows the region where the lopsided or the equal split is optimal when the probability of the rectification stage is low. Recall that r, on the horizontal axes in this Figure, by Eq. (4.13) is strictly increasing in w/s.

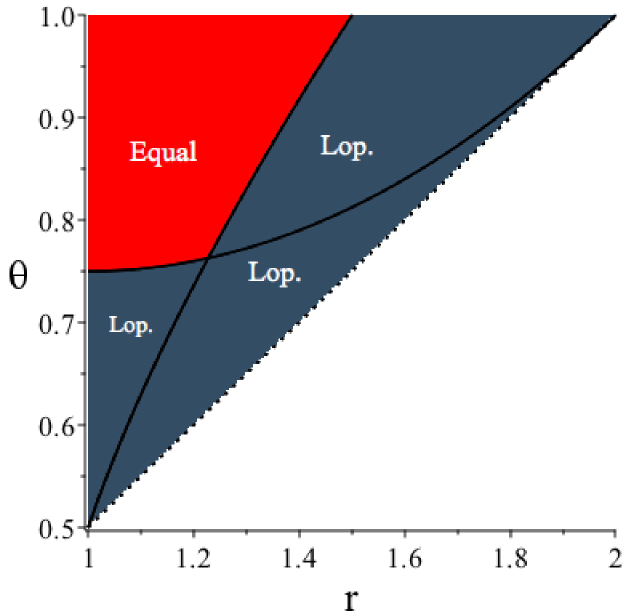


Figure 4.1: Parameter spaces where the equal and the lopsided split are the welfare-maximizing proposal when $\alpha < \Gamma(\theta, r)$.

4.4.2 The Conflict Minimizing Proposal

Instead of maximizing social-surplus the mediator objective could be minimizing the ex-ante probability of conflict. This is an objective well studied in the conflict management literature. For any $x_{\min} \in (0, r/2]$, let $P(x_{\min})$ denote the ex-ante probability for conflict to occur in the not-always-conflict equilibrium given the peace proposal (x_1, x_2) with $\min\{x_1, x_2\}$; let $P(0) := 1$, as the equilibrium given $x_{\min} = 0$ is always conflict. By Lemmas C.10 and C.11 in the Appendix C.4, when $x_{\min}$ increases in $[0, r/2]$, $P(x_{\min})$ is strictly decreasing on $[0, r-1]$ and on $[1-\theta, r/2]$. Hence the lopsided proposal $(r-1, 1)$ and the equal proposal $(r/2, r/2)$, or respectively the lopsided split $(w/s, 1-w/s)$ and the equal split $(1/2, 1/2)$, are again the only two candidates for the optimal solution.

If the probability of the rectification stage is higher than a threshold or the prior probability of being weak is lower than a threshold, i.e., $\alpha \geq \Gamma(\theta, r)$ or $\theta < \frac{3+(r-1)^2}{4}$ then the only renege-proof conflict minimizer proposal is the lopsided split $(1-w/s, w.s)$, or equivalently the lopsided proposal $(r-1, 1)$. If $\alpha < \Gamma(\theta, r)$ then I show that there is a threshold, higher than that in the case of surplus maximization, such that the conflict minimizing proposal is the equal split when θ is above the threshold, and $(w/s, 1-w/s)$ when θ is below the threshold. Denote,

$$\psi(r) := \max\left\{\frac{1+\sqrt{3r^2-4r+1}}{2}, \frac{3+(r-1)^2}{4}\right\}. \tag{4.28}$$

Theorem 4.2 *For any (θ, r) satisfying Ineq. (4.15) and any $\alpha \in (0,1)$ there exists a $\Gamma(\theta, r) \in (0,1)$ and a $\psi(r) \in (r/2, 1)$ such that a unique renege-proof $\nu^* \in (0, 1/2]$ minimizes the probability of conflict according to:*

$$\arg\min_{\nu_{\min}\in[0,1/2]} P(\nu_{\min}) = \begin{cases} \{w/s\} & \text{if } \alpha \geq \Gamma(\theta,r) \text{ or } \theta < \frac{3+(r-1)^2}{4}, \\ \{w/s\} & \text{if } \alpha < \Gamma(\theta,r) \text{ and } \theta < \psi(r), \\ \{w/s, 1/2\} & \text{if } \alpha < \Gamma(\theta,r) \text{ and } \theta = \psi(r), \\ \{1/2\} & \text{if } \alpha < \Gamma(\theta,r) \text{ and } \theta > \psi(r). \end{cases}$$

Figure 4.2 shows the region where the lopsided or the equal split is the conflict minimizing split when the probability of the rectification stage is lower than the threshold specified in Theorem 4.2. Comparing Figure 4.2 to Figure 4.1, it is evident that the equal split is the conflict minimizing proposal when the prior probability of weak is weakly higher compared to when it is the social surplus maximizing proposal.

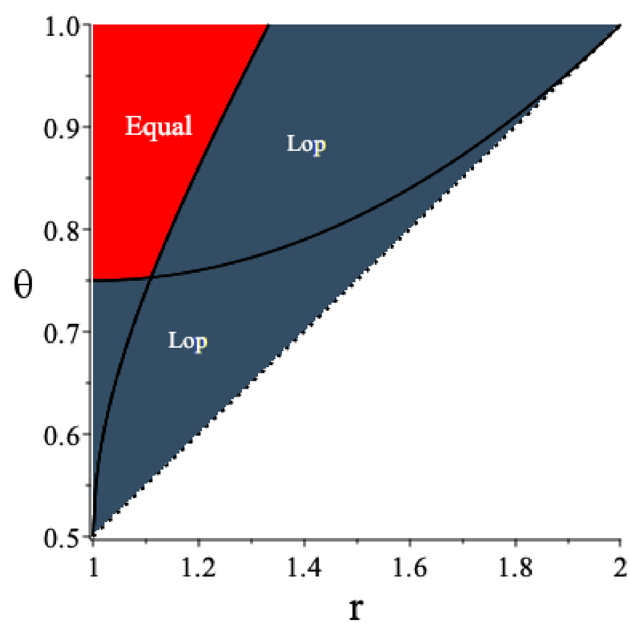


Figure 4.2: Parameter spaces where the equal and the lopsided split are the conflict minimizing proposal when $\alpha < \Gamma(\theta, r)$.

To see intuitions behind these results, note that on the one hand, the optimal lopsided proposal guarantees acceptance by one player. It is the smallest renege-proof proposal such that this favored player never reneges. Therefore, it can increase the probability of peace by securing acceptance from one player, even though this is at the cost of a higher chance of rejection by the less favored player. On the other hand, the equal split treats players equally and admits symmetric posterior belief information structure. Thus, it makes conflict very costly and less appealing for the strong types. Therefore, these two classes of splits are the candidates for conflict minimizing proposals. However, if the probability of the rectification stage is high or the prior probability of being weak is relatively small, then if the players are proposed with an equal split, the strong types of them have an incentive to accept with a positive probability, similar to their weak types, and then renege at the rectification stage. Thus, in these environments, the equal split is not renege-proof. If the probability of the rectification stage is low and the prior probability that agents are weak is very high, then the lopsided proposal does not admit a high probability of peace because the less favored player announces Reject with a very high probability while the equal split does not suffer from the same issue by treating the player symmetrically. Thus, the equal split admits a lower probability of conflict compared to the lopsided split in such an environment.

4.5 Conclusion

The intervention of mediators in settling a dispute between adversaries could alter their incentives and harm the long-term prospect of peace. This is especially true if these interventions take the form of hard interventions like providing economic incentives or imposing threats. In this chapter, I show that even when the mediator has very limited instruments and just makes peace proposals, players can learn about each other in the mediation process and this information can have important impact on recurring conflicts. I study situations where mediators can only through indirect manipulation of beliefs affect the decisions of players in the conflict. I show that the possibility to indirectly affecting the posterior belief information structure of adversaries has an important impact on the design of renege-proof proposals. In particular, I show that the optimal proposal in an environment where reneging is banned is not renege-proof when the possibility of reneging is present (Section 4.2.3).

I show if the possibility of reneging is present with a high probability or if the prior probability of players being weak is relatively low, then a mediator that is interested in maximizing welfare subject to renege-proof constraint should make the lopsided peace proposal to ex-ante symmetric players (Theorem 4.1). This proposal is so biased that the favored player always accepts it and does not renege on it, while her rival accepts with positive probability and does not renege on it if her type is weak and rejects it if she is a strong type. The intuitive equal split offer in this symmetric environment is only optimal when the possibility to renege has a low probability and the prior probability that agents are weak is high. I also show that when the objective of the mediator is changed to conflict minimization the same results hold (Theorem 4.2). Thus, one can observe the optimal proposal given the two different objectives qualitatively align. If the probability of the rectification stage is high or the prior probability of being weak is relatively small, then if the players are proposed with an equal split, the strong types of them have an incentive to accept with a positive probability, similar to their weak types, and then renege at the rectification stage. Thus, in these environments, the equal split is not renege-proof.

The lopsided split by securing acceptance from one player does not reveal any information about her. Therefore, this player is not worried that by announcing Accept she will sign weakness and will be exploited in the event of a conflict. Given the lopsided splits, the only event of a conflict on the path of equilibrium is when the less favored player rejects the proposal. Thus, the mediator by proposing these splits, not only avoids the event that both players reject a proposal and trigger a conflict but also in the only event of a conflict on the path of equilibrium, both players are perceived as relatively strong. Thus such a proposal can increase overall welfare by discouraging players from exerting mutually detrimental efforts. At the same time, this proposal guarantees acceptance by one player. Thus, it can be a candidate for conflict minimization as long as the prior probability that agents are weak is not very high. If this probability is very high, to secure acceptance by the favored player, the mediator should offer her a very large share, which incentivizes the less favored player to reject with a higher probability. Thus, the equal split in these situations by treating players symmetrically can admit the lower probability of rejection by both players and outperform the lopsided split.

In this chapter, I focus on renege-proof proposals due to their importance in international conflicts applications. However, focusing on the class of renege-proof proposals is with loss of generality. An open area to study in the future is establishing a renege-proof principle such that when a mediator announces a general communication mechanism, for example à la Myerson (1986), it would be without loss of generality to focus on direct renege-proof mechanisms. Finding conditions that guarantee such a principle is a non-trivial step given the lack of enforcement power of the mediator, lack of commitment by players, and presence of an endogenous outside option.

Bibliography

Baliga, S. and T. Sjostrom (2013). Bargaining and war: A review of some formal models. *Korean Economic Review 29*(2), 235–266.

Balzer, B. and J. Schneider (2021). Managing a conflict: optimal alternative dispute resolution. *The RAND Journal of Economics 52*(2), 415–445.

Beardsley, K. (2008). Agreement without peace? international mediation and time inconsistency problems. *American Journal of Political Science 52*(4), 723–740.

Bester, H. and R. Strausz (2001). Contracting with imperfect commitment and the revelation principle: the single agent case. *Econometrica 69*(4), 1077–1098.

Bester, H. and R. Strausz (2007). Contracting with imperfect commitment and noisy communication. *Journal of Economic Theory 136*(1), 236–259.

Bester, H. and K. Wärneryd (2006). Conflict and the social contract. *Scandinavian Journal of Economics 108*(2), 231–249.

Compte, O. and P. Jehiel (2009). Veto constraint in mechanism design: inefficiency with correlated types. *American Economic Journal: Microeconomics 1*(1), 182–206.

Doval, L. and V. Skreta (2021). Mechanism design with limited commitment. *Econometrica, Forthcoming*.

Fey, M., A. Meirowitz, and K. W. Ramsay (2013). Credibility and commitment in crisis bargaining. *Political Science Research and Methods 1*(1), 27–52.

Fey, M. and K. W. Ramsay (2011). Uncertainty and incentives in crisis bargaining: Game-free analysis of international conflict. *American Journal of Political Science 55*(1), 149–169.

Fudenberg, D. and J. Tirole (1991). Perfect bayesian equilibrium and sequential equilibrium. *journal of Economic Theory 53*(2), 236–260.

Gartner, S. S. (2008). *International Conflict Mediation: New Approaches and Findings*. Routledge.

Green, J. R. and J.-J. Laffont (1987). Posterior implementability in a two-person decision problem. *Econometrica: Journal of the Econometric Society*, 69–94.

Hörner, J., M. Morelli, and F. Squintani (2015). Mediation and peace. *The Review of Economic Studies 82*(4), 1483–1501.

Kamranzadeh, A. and C. Z. Zheng (2022, March). Unequal peace proposals for equal contestants: designing information indirectly. Mimeo, Department of Economics, University of Western Ontario.

Kramer, A. E. (February 8, 2022). What are the minsk accords, and could they defuse the ukraine crisis? *The New York Times.*

Landler, M. (May 8, 2018). Trump abandons iran nuclear deal he long scorned. *The New York Times.*

Myerson, R. B. (1986). Multistage games with communication. *Econometrica: Journal of the Econometric Society*, 323–358.

Pavan, A. (2017). Dynamic mechanism design: Robustness and endogenous types. In *Advances in Economics and Econometrics: Eleventh World Congress*, Volume 1, pp. 1–62.

Ramsay, K. W. (2017). Information, uncertainty, and war. *Annual Review of Political Science 20*, 505–527.

Skreta, V. (2015). Optimal auction design under non-commitment. *Journal of Economic Theory 159*, 854–890.

Spier, K. E. (1994). Pretrial bargaining and the design of fee-shifting rules. *The RAND Journal of Economics*, 197–214.

Wilkenfeld, J., K. Young, D. Quinn, and V. Asal (2007). *Mediating international crises.* Routledge.

Zheng, C. Z. (2002). Optimal auction with resale. *Econometrica 70*(6), 2197–2224.

Zheng, C. Z. (2019). Necessary and sufficient conditions for peace: Implementability versus security. *Journal of Economic Theory 180*, 135–166.

Appendix A

Appendices to Chapter 2

A.1 Categorization of All Equilibria

The next lemma classifies all the possible cases of solutions (proposal-PBE pairs). Case (b) corresponds to the set of lopsided solutions. The other cases constitute the set of non-lopsided solutions. Among them, Case (a) consists of those where conflict always occur. It contains the trivial equilibrium, which exists under any proposal, where each player chooses Reject for sure, expecting the other to do the same. Case (c) corresponds to those PBEs that satisfy (2.14), Case (d) those satisfying (2.15), and Case (e) those satisfying (2.12) or (2.13).

Lemma A.1 *For any solution $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$, exactly one of the following is true:*

a. $q_i = 1$ *for some player* i*, and the on-path posterior is equal to the prior for both players;*

b. for some $i \in \{1, 2\}$, $\sigma_i(w) = \sigma_i(s) = 0$ *and* $0 < \sigma_{-i}(w) < 1 = \sigma_{-i}(s)$*;*

c. for each $i \in \{1, 2\}$, $0 < \sigma_i(w) < 1 = \sigma_i(s)$*;*

d. for each $i \in \{1, 2\}$, $\sigma_i(w), \sigma_i(s) \in (0, 1)$*;*

e. for some $i \in \{1, 2\}$, $\sigma_i(w), \sigma_i(s), \sigma_{-i}(w) \in (0, 1)$*, and* $\sigma_{-i}(s) = 1$.

Proof The proof is based mainly on an observation summarized by the following table, where each cell that is filled indicates the property of the corresponding combination of a player i's equilibrium probabilities of Reject by the two types.

	$\sigma_i(s) = 0$	$0 < \sigma_i(s) < 1$	$\sigma_i(s) = 1$
$\sigma_i(w) = 0$		impossible	impossible.
$0 < \sigma_i(w) < 1$	impossible		
$\sigma_i(w) = 1$	impossible	impossible	$q_i = 1$

The table shows that, unless $q_i = 1$ for some player i, there are only three possibilities for a player i's equilibrium strategy: $\sigma_i(w) = \sigma_i(w) = 0$, or both $\sigma_i(w)$ and $\sigma_i(s)$ belong to $(0, 1)$, or only $\sigma_i(w) \in (0, 1)$. These are listed as the rows and columns in the next table:

	$\sigma_{-i}(w) = \sigma_{-i}(s) = 0$	σ_{-i} is totally mixed	σ_{-i} is mixed by w
$\sigma_i(w) = \sigma_i(s) = 0$	impossible	impossible	case (b), $\sigma_{-i}(s) = 1$
σ_i is totally mixed	impossible	case (d)	case (e), $\sigma_{-i}(s) = 1$
σ_i is mixed by w	case (b), $\sigma_i(s) = 1$	case (e), $\sigma_i(s) = 1$	case (c)

In the above table, the cells $(3, 1)$ and $(1, 3)$—the first coordinate indicating the row, and the second coordinate the column—corresponds to Case (b). Likewise, the cell $(3, 3)$ corresponds to Case (c), the cell $(2, 2)$ corresponds to Case (d), and the cells $(3, 2)$ and $(2, 3)$ Case (e). The cell $(1, 1)$ is impossible because, as explained in Footnote 8, our assumption $\theta > 1/2$ implies that it is impossible to have $\sigma_i(s) = \sigma_i(w) = 0$ for both players i.

Thus, to complete the proof, it suffices to prove the following claims:

1. If $q_i = 1$ for some player i, the on-path posterior is equal to the prior for each player.

2. If $q_i < 1$ for each player i, then there is no $i \in \{1, 2\}$ for whom:

   i. $\sigma_i(w) = 0 < \sigma_i(s) \leq 1$; or

   ii. $\sigma_i(s) = 0 < \sigma_i(w) \leq 1$; or

   iii. $0 < \sigma_i(s) < 1 = \sigma_i(w)$; or

   iv. $\sigma_i(w) = \sigma_i(s) = 0$ and $\sigma_{-i}(w), \sigma_{-i}(s) \in (0, 1)$.

Claim 1 completes the statement of Case (a). Claim 2.i implies the impossibility of cells $(1, 2)$ and $(1, 3)$ in the first table; Claim 2.ii implies the impossibility of cells $(2, 1)$ and $(3, 1)$ in the first table; Claim 2.iii, that of cell $(3, 2)$ in the first table, and Claim 2.iv, that of cells $(1, 2)$ and $(2, 1)$ in the second table.

Claim 1 Let $q_i = 1$ for some player i. Then the on-path posterior about i is $p_i^R = 1 - \theta$. For player $-i$, suppose that the action A is on path and p_{-i}^A is not equal to the prior $1 - \theta$. Then Bayes's rule requires that the other action R be on path as well such that $p_{-i}^R \neq 1 - \theta$ and (2.6) be satisfied. Thus, one of p_{-i}^A and p_{-i}^R is above $1 - \theta$, and the other below $1 - \theta$. If $p_{-i}^A > 1 - \theta > p_{-i}^R$, then by (2.2) and (2.3) (or simply Figure 2.2),

$$\begin{aligned} U_{-i}^s(p_{-i}^A, 1 - \theta) = \theta \quad &< \quad 1 - p_{-i}^R = U_{-i}^s(p_{-i}^R, 1 - \theta), \\ U_{-i}^w(p_{-i}^R, 1 - \theta) = 0 \quad &< \quad p_{-i}^A - \theta + 1 = U_{-i}^w(p_{-i}^A, 1 - \theta); \end{aligned}$$

thus player $-i$ of type s would choose R for sure, and $-i$ of type w, A for sure. That implies $p_{-i}^R = 1$ and $p_{-i}^A = 0$, contradicting $p_{-i}^A > 1 - \theta > p_{-i}^R$. The other case, where $p_{-i}^A < 1 - \theta < p_{-i}^R$, is self-contradicting analogously. This proves Claim 1.

Claim 2.i Suppose, to the contrary, that $\sigma_i(w) = 0 < \sigma_i(s) \leq 1$ for some player i. By Bayes's rule, $\sigma_i(w) = 0$ implies $p_i^R = 1$. Then the two graphs in Figure 2.2 coincide, with p_i there equal to $p_i^R = 1$, and hence $V_i^R(s) = V_i^R(w) = 1 - (1 - \theta) = \theta$ by (2.5)—simply put, the dashed segment in Figure 2.3 coincides with the solid thick line because any p_{-i}^A and p_{-i}^R are less than or equal to $1 = p_i^R$. Recall from (2.4) that $V_i^A(t)$ denotes i's expected payoff from choosing A given type $t \in \{s, w\}$. By the best response condition,

$$\begin{aligned} \sigma_i(w) = 0 \quad &\Rightarrow \quad V_i^A(w) \geq V_i^R(w) = \theta, \\ \sigma_i(s) > 0 \quad &\Rightarrow \quad V_i^A(s) \leq V_i^R(s) = \theta. \end{aligned}$$

Thus $V_i^A(w) \geq V_i^A(s)$. Meanwhile, (2.4) implies that $V_i^A(w) \leq V_i^A(s)$, as $U_i^w(p_i^A, \cdot) \leq U_i^s(p_i^A, \cdot)$ for any $p_i^A \in [0,1]$. Consequently, $V_i^A(w) = V_i^A(s)$. Then (2.4) coupled with $q_{-i} > 0$ implies that $U_i^w(p_i^A, p_{-i}^R) = U_i^s(p_i^A, p_{-i}^R)$. Compare (2.2) with (2.3)—or simply inspect Figure 2.2—to see that the equation is possible only if $p_i^A = 1$. But that violates Bayes's rule given that $\sigma_i(w) < 1$. Thus Claim 2.i follows.

Claim 2.ii Suppose, to the contrary, that $q_i < 1$ for both players i, and $\sigma_i(s) = 0 < \sigma_i(w) \leq 1$ for some player i. By Bayes's rule, $\sigma_i(s) = 0$ implies $p_i^R = 0$. By (2.2) and (2.3), $U_i^s(p_i^R, \cdot) = 1$ and $U_i^w(p_i^R, \cdot) = 0$. It follows from (2.5) that $V_i^R(s) = 1$ and $V_i^R(w) = 0$. By the best response condition for $\sigma_i(w) > 0$,

$$0 = V_i^R(w) \geq V_i^A(w) \overset{(2.4)}{=} q_{-i} U_i^w(p_i^A, p_{-i}^R) + (1 - q_{-i}) x_i \geq (1 - q_{-i}) x_i$$

and hence $x_i = 0$ (since $1 - q_{-i} > 0$). This coupled with the best response condition for $\sigma_i(s) = 0$ implies

$$1 = V_i^R(s) \leq V_i^A(s) = 0 + q_{-i} U_i^s(p_i^A, p_{-i}^R) \overset{(2.2)}{=} q_{-i} \left(1 - \min\{p_i^A, p_{-i}^R\}\right).$$

Thus, $q_{-i} = 1$, contradiction.

Claim 2.iii Suppose, to the contrary, that $q_i < 1$ for both players i, and $0 < \sigma_i(s) < 1 = \sigma_i(w)$ for some player i. By Bayes's rule, $\sigma_i(w) = 1$ implies $p_i^A = 1$. It then follows from (2.2) and (2.3) that $U_i^s(p_i^A, \cdot) = U_i^w(p_i^A, \cdot)$ and hence, by (2.4), $V_i^A(s) = V_i^A(w)$. By the best response condition, $0 < \sigma_i(s) < 1$ implies $V_i^R(s) = V_i^A(s)$, and $\sigma_i(w) > 0$ implies $V_i^R(w) \geq V_i^A(w)$. Thus, $V_i^R(w) \geq V_i^R(s)$. This, by inspection of Figure 2.3—or (2.5)—is possible only if $p_i^R = 1$. But $p_i^R = 1$ violates Bayes's rule since $\sigma_i(w) > 0$, contradiction.

Claim 2.iv Suppose, to the contrary, that for each player i we have $q_i < 1$ and $\sigma_i(w) = \sigma_i(s) = 0$, $0 < \sigma_{-i}(w) < 1$ and $0 < \sigma_{-i}(s) < 1$. With $\sigma_i(w) = \sigma_i(s) = 0$, we have $q_i = 0$ and $p_i^A = 1 - \theta$. Plug them into (2.5)—or simply noting that the convex combination in Figure 2.3 degenerates to the point $1 - \theta$—to see that $V_{-i}^R(w) = p_{-i}^R - (1 - \theta)$ and $V_{-i}^R(s) = 1 - (1 - \theta) = \theta$. Since $\sigma_{-i}(w) > 0$, $p_{-i}^R < 1$ and hence $p_{-i}^R - (1 - \theta) < \theta$. Consequently, $V_{-i}^R(w) < V_{-i}^R(s)$. Meanwhile, by the best response condition and $q_i = 0$,

$$0 < \sigma_{-i}(w) < 1 \quad \Rightarrow \quad x_{-i} = V_{-i}^A(w) = V_{-i}^R(w),$$
$$0 < \sigma_{-i}(s) < 1 \quad \Rightarrow \quad x_{-i} = V_{-i}^A(s) = V_{-i}^R(s).$$

Thus $V_{-i}^R(w) = V_{-i}^R(s)$, contradiction. ■

An implication of Lemma A.1 is that the condition $p_i^R > 1 - \theta > p_i^A$ in Figures 2.3–2.5 is indeed satisfied by any solution unless the posterior degenerates to the prior $1 - \theta$.

Lemma A.2 *For any solution* $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$*, either* $q_i = 1$ *for some player* i *and the on-path posterior is equal to the prior for both players, or* $q_i < 1$ *for both players* i *and, for each player* i*,* $q_i > 0 \Rightarrow p_i^R > 1 - \theta > p_i^A$*.*

Proof By Lemma A.1, either Case (a) is true, which means the on-path posterior is equal to the prior for both players, or (a) is not true and hence $q_i < 1$ for both players i. In the latter alternative, if $q_i > 0$ then we have either (I) $\sigma_i(w), \sigma_i(s) \in (0,1)$—which is true for both players in case (d), and player i in case (e), in Lemma A.1—or (II) $\sigma_i(s) > \sigma_i(w)$ (which is true for player $-i$

in case (b), both players in case (c), and player $-i$ in case (e)). In (I), the best response condition implies

$$V_i^R(s) - V_i^A(s) = 0 = V_i^R(w) - V_i^A(w),$$

which, by (A.1), simplifies to $1 - p_i^R = q_{-i}(1 - p_i^A)$. This coupled with $q_{-i} < 1$ implies $1 - p_i^R < 1 - p_i^A$, i.e., $p_i^R > p_i^A$. In (II), by Bayes's rule $\sigma_i(s) = q_i p_i^R/(1-\theta)$ and $\sigma_i(w) = q_i(1 - p_i^R)/\theta$, and by $q_i > 0$, we have $p_i^R/(1-\theta) > (1 - p_i^R)/\theta$, namely, $p_i^R > 1 - \theta$. Both cases considered, we have shown that $q_i > 0$ implies $p_i^R > p_i^A$ or $p_i^R > 1 - \theta$. In either case, the Bayesian plausibility condition (2.6) implies $p_i^R > 1 - \theta > p_i^A$. ■

A.2 Verification of the Intuitive and D1 Criteria

To prove that the lopsided equilibrium of the proposal $(\theta, 1-\theta)$ satisfies both the Intuitive and D1 refinement criteria, notice first that the only observable deviation from the equilibrium is player 1 choosing Reject (R). Also note that player 1's expected payoff from this equilibrium is equal to $V_1^A(s) = \theta$ when his type is s, and equal to $V_1^A(w) = \theta/2$ when the type is w (Steps 2 and 4, Section 2.4.2). For each $t \in \{s, w\}$ and any $p_1^R \in [0, 1]$, denote $\tilde{V}_1^R(t, p_1^R)$ for type-t player 1's expected payoff from the deviation provided that the posterior belief is that p_1^R is the probability for player 1 to be type s (and the posterior probability for player 2, who abides by the equilibrium, is $p_2^R = 2(1-\theta)$ according to (2.11) and $x_2 = 1 - \theta$).

Intuitive Criterion Denote J for the set of player 1's types whose equilibrium payoff is higher than any payoff it could get by playing R, as long as player 2's action is rationalizable. That is,

$$J := \left\{ t \in \{s, w\} \;\middle|\; V_1^A(t) > \max_{p_1^R \in [0,1]} \tilde{V}_1^R(t, p_1^R) \right\}.$$

Observe that $J = \varnothing$: $s \notin J$ because the equilibrium payoff θ is the minimum payoff that a strong type s can achieve from playing R (Remark 2.2); $w \notin J$ because the equilibrium payoff $\theta/2$ is less than θ, which is equal to $\tilde{V}_1^R(w, 1)$ because $p_1^R = 1 > 2(1-\theta) = p_2^R$ implies via (2.7) that $\tilde{V}_1^R(w, 1) = 1 - (1 - \theta) = \theta$. Now that $J = \varnothing$, the set of distributions of player 1's type whose supports exclude J (the empty set) contains the posterior distribution that supports the lopsided equilibrium. Thus, the equilibrium satisfies the Intuitive Criterion.

D1 Criterion It suffices to falsify the following inequality for each $t \in \{s, w\}$ (and $\{t'\} := \{s, w\} \setminus \{t\}$):

$$\left\{ p_1^R \in [0,1] \;\middle|\; V_1^A(t) \leq \tilde{V}_1^R(t, p_1^R) \right\} \subsetneq \left\{ p_1^R \in [0,1] \;\middle|\; V_1^A(t') < \tilde{V}_1^R(t', p_1^R) \right\}.$$

To that end, consider first $t = s$ (so $t' = w$). Since $V_1^A(s) = \theta$ is the minimum payoff that a strong type s can achieve from playing R (Remark 2.2), the left-hand side is equal to $[0, 1]$ and hence the (strict) inequality cannot hold. Next consider $t = w$ (and so $t' = s$). Note that $p_1^R = 1$ belongs to the left-hand side, as $V_1^A(w) = \theta/2 < \theta = \tilde{V}_1^R(w, 1)$, shown in the previous paragraph. However, $p_1^R = 1$ does not belong to the right-hand side, because $V_1^A(s) = \theta$ and $\tilde{V}_1^R(s, 1) = \theta$ by (2.8). Thus again the inequality displayed above does not hold. Both cases considered, the D1 Criterion is satisfied.

A.3 Three Useful Equations

Lemma A.3 *In any solution* $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$,

$$V_i^R(s) - V_i^A(s) - \left(V_i^R(w) - V_i^A(w)\right) = 1 - p_i^R - q_{-i}(1 - p_i^A) \tag{A.1}$$

for each player i, *and if* $p_i^R \geq p_{-i}^R \geq 1 - \theta$, *then*

$$\begin{aligned} V_i^R(w) - V_i^A(w) &= p_i^R - (1 - q_{-i})x_i - 1 + \theta, &\text{(A.2)}\\ V_{-i}^R(w) - V_{-i}^A(w) &= p_{-i}^R - p_i^A - x_{-i}. &\text{(A.3)} \end{aligned}$$

Proof To prove (A.1), note from (2.4) and (2.5) that the left-hand side is equal to

$$\begin{aligned} & q_{-i}\left(U_i^s(p_i^R, p_{-i}^R) - U_i^s(p_i^A, p_{-i}^R) - U_i^w(p_i^R, p_{-i}^R) + U_i^w(p_i^A, p_{-i}^R)\right) \\ & +(1 - q_{-i})\left(U_i^s(p_i^R, p_{-i}^A) - U_i^w(p_i^R, p_{-i}^A)\right) \\ \overset{(2.2),(2.3)}{=} \; & q_{-i}\left(1 - \min\{p_i^R, p_{-i}^R\} - 1 + \min\{p_i^A, p_{-i}^R\} - p_i^R + \min\{p_i^R, p_{-i}^R\} + p_i^A - \min\{p_i^A, p_{-i}^R\}\right) \\ & +(1 - q_{-i})\left(1 - \min\{p_i^R, p_{-i}^A\} - p_i^R + \min\{p_i^R, p_{-i}^A\}\right) \\ = \; & q_{-i}\left(-p_i^R + p_i^A\right) + (1 - q_{-i})\left(1 - p_i^R\right), \end{aligned}$$

which is equal to the right-hand side. To prove (A.2), assume without loss that $p_1^R \geq p_2^R$. Thus for each player i, $p_i^R \geq 1 - \theta$ and hence, by the Bayesian plausibility condition (2.6), $p_i^A \leq 1 - \theta$. Use (2.4) and (2.5) to obtain

$$\begin{aligned} V_1^R(w) - V_1^A(w) &= q_2\left(U_1^w(p_1^R, p_2^R) - U_1^w(p_1^A, p_2^R)\right) + (1 - q_2)\left(U_1^w(p_1^R, p_2^A) - x_1\right) \\ &\overset{(2.3)}{=} q_2\left(p_1^R - \min\{p_1^R, p_2^R\} - p_1^A + \min\{p_1^A, p_2^R\}\right) + (1 - q_2)\left(p_1^R - \min\{p_1^R, p_2^A\} - x_1\right) \\ &= q_2\left(p_1^R - p_2^R - p_1^A + p_1^A\right) + (1 - q_2)\left(p_1^R - p_2^A - x_1\right) \\ &= p_1^R - q_2 p_2^R - (1 - q_2)p_2^A - (1 - q_2)x_1 \\ &= p_1^R - (1 - \theta) - (1 - q_2)x_1, \end{aligned}$$

with the third line due to $p_1^R \geq p_2^R \geq 1 - \theta \geq p_j^A$ for each player j, and the last line due to the Bayesian plausibility condition (2.6). Thus (A.2) is true. Analogously, (A.3) follows from

$$\begin{aligned} V_2^R(w) - V_2^A(w) &= q_1\left(p_2^R - p_2^R - p_2^A + p_2^A\right) + (1 - q_1)\left(p_2^R - p_1^A - x_2\right) \\ &= (1 - q_1)\left(p_2^R - p_1^A - x_2\right). \quad \blacksquare \end{aligned}$$

A.4 Supoptimality of Case-(a) Solutions

Since Case-(a) solutions always entail conflict, it is intuitive that they are suboptimal. In particular, they are outperformed by the lopsided proposal $(\theta, 1 - \theta)$. First—

Lemma A.4 *The social surplus generated by the Case-(b) PBE admitted by the proposal* $(\theta, 1-\theta)$ *is equal to* $\theta(3 - 5\theta/2)$.

Proof By definition of any Case-(b) solution, $q_1 = 0$ and $0 < \sigma_2(w) < 1 = \sigma_1(s)$. Thus the social surplus from $(\theta, 1-\theta)$ is equal to

$$\underbrace{(1-q_2)\theta + q_2\left[\theta U_1^w(p_1^A, p_2^R) + (1-\theta)U_1^s(p_1^A, p_2^R)\right]}_{\text{player 1}} + \underbrace{\theta U_2^w(p_2^R, p_1^A) + (1-\theta)U_2^s(p_2^R, p_1^A)}_{\text{player 2}}.$$

By Bayes's rule, $p_1^A = 1-\theta$, $p_2^A = 0$ and $q_2 = (1-\theta)/p_2^R$. By (2.11), $p_2^R = 1-\theta+x_2 = 2(1-\theta)$. Combine them with (2.2) and (2.3) to calculate the above-displayed sum:

$$\begin{aligned}
& (1-q_2)\theta + q_2\left(\theta\cdot 0 + (1-\theta)(1-1+\theta)\right) + \theta(p_2^R - 1 + \theta) + (1-\theta)(1-1+\theta) \\
= & \left(1 - \frac{1-\theta}{p_2^R}\right)\theta + \frac{1-\theta}{p_2^R}(1-\theta)\theta + \theta(p_2^R - 1 + \theta) + (1-\theta)\theta \\
= & \left(1 - \frac{1-\theta}{2(1-\theta)}\right)\theta + \frac{1-\theta}{2(1-\theta)}(1-\theta)\theta + \theta(2(1-\theta) - 1 + \theta) + (1-\theta)\theta \\
= & \theta(3 - 5\theta/2). \quad \blacksquare
\end{aligned}$$

Next, we calculate the social surplus generated by any Case-(a) solution. By the Claim 1 in the proof of Lemma A.1, any PBE that belongs to Case (a) has the on-path posterior equal to the prior $1-\theta$ for each player. Since $q_i = 1$ for some player i, conflict takes place for sure and hence each player's ex ante payoff from the PBE is equal to

$$\theta U_i^w(1-\theta, 1-\theta) + (1-\theta)U_i^s(1-\theta, 1-\theta) = 0 + (1-\theta)(1-(1-\theta)) = \theta(1-\theta).$$

Thus, the social surplus generated by the PBE is equal to $2\theta(1-\theta)$, which is less than $\theta(3-5\theta/2)$, the social surplus generated by the lopsided $(\theta, 1-\theta)$. Thus, any PBE that belongs to Case (a) is suboptimal.

A.5 Suboptimality of Case-(c) Solutions (Eq. (2.14))

First we show that, within the Case-(c) PBEs, the one admitted by the equal-split proposal maximizes the social surplus.

Lemma A.5 *(i) The Case-(c) PBE admitted by the equal-split proposal maximizes the social surplus among all Case-(c) solutions. (ii) At this Case-(c) optimal solution, $p_1^R = p_2^R = 1/2$, $q_2 = 2(1-\theta)$, and the social surplus is equal to θ.*

Proof As defined in Lemma A.1, a PBE belongs to Case (c) if and only if its strategy profile satisfies

$$\forall i \in \{1, 2\} : 0 < \sigma_i(w) < 1 = \sigma_i(s). \tag{A.4}$$

Then Bayes's rule implies $p_i^A = 0$ and hence (by (2.6)) $q_i p_i^R = 1-\theta$ for each player i. The best response condition for (A.4) to constitute a PBE is that $V_i^R(w) - V_i^A(w) = 0$ and $V_i^R(s) - V_i^A(s) \geq 0$ for each player i. Since (A.4) is symmetric between the two players, let us assume without loss that

$$p_1^R \geq p_2^R. \tag{A.5}$$

Then (A.2) and (A.3) apply to the case $i = 1$ and hence

$$\begin{aligned}
V_1^R(w) - V_1^A(w) &= p_1^R - (1-\theta) - (1-q_2)x_1, \\
V_2^R(w) - V_2^A(w) &= (1-q_1)\left(p_2^R - p_1^A - x_2\right) = (1-q_1)(p_2^R - x_2),
\end{aligned}$$

with the last "=" due to $p_i^A = 0$. Thus, the condition $V_i^R(w) - V_i^A(w) = 0$ for both i becomes

$$p_1^R = 1 - \theta + (1 - q_2)x_1, \quad (A.6)$$

$$p_2^R = x_2. \quad (A.7)$$

Plug $q_2 = (1-\theta)/p_2^R$, $x_1 = 1 - x_2$ and (A.7) into (A.6) to have

$$p_1^R = 1 - \theta + \left(1 - \frac{1-\theta}{x_2}\right)(1 - x_2) = \frac{\theta + x_2(1-2\theta) - (1-x_2)^2}{x_2}. \quad (A.8)$$

Thus, Ineq. (A.5), $p_1^R \geq p_2^R$, is equivalent to

$$\begin{aligned} \theta + x_2(1-2\theta) - (1-x_2)^2) \geq x_2^2 &\iff \theta(1-2x_2) + x_2(1-x_2) - (1-x_2)^2 \geq 0 \\ &\iff \theta(1-2x_2) + (1-x_2)(x_2 - 1 + x_2) \geq 0 \\ &\iff (1-2x_2)(\theta - 1 + x_2) \geq 0. \end{aligned}$$

The last inequality in the multiline displayed above is equivalent to either (i) $1 - 2x_2 \geq 0$ and $\theta - 1 + x_2 \geq 0$, namely $1 - \theta \leq x_2 \leq 1/2$, or (ii) $1 - 2x_2 \leq 0$ and $\theta - 1 + x_2 \leq 0$, namely $1/2 \leq x_2 \leq 1 - \theta$, which is impossible due to (2.1). Thus,

$$p_1^R \geq p_2^R \iff 1 - \theta \leq x_2 \leq 1/2. \quad (A.9)$$

Denote S for the social surplus generated by the PBE. By Lemma 2.1, the WLOG condition $p_1^R \geq p_2^R$, and the fact $q_2 p_2^R = 1 - \theta$,

$$\begin{aligned} S &= 2\theta p_1^R + (q_2 - \theta)(p_1^R - p_2^R) \\ &= 2\theta p_1^R + \left(\frac{1-\theta}{p_2^R} - \theta\right)(p_1^R - p_2^R). \end{aligned}$$

Thus S is determined jointly by p_1^R and p_2^R, each a function of x_2 via (A.7) and (A.8). It follows that S is a function of x_2. Furthermore, we observe that S is strictly increasing in x_2:

$$\frac{\partial S}{\partial p_1^R} = \theta + q_2,$$

$$\frac{\partial S}{\partial p_2^R} = -\frac{1-\theta}{(p_2^R)^2}(p_1^R - p_2^R) + \theta - q_2 = -q_2\frac{p_1^R - p_2^R}{p_2^R} + \theta - q_2 = \theta - q_2\frac{p_1^R}{p_2^R}.$$

Then, by (A.7) and (A.8),

$$\begin{aligned} \frac{d}{dx_2}S &= \frac{\partial S}{\partial p_1^R}\frac{dp_1^R}{dx_2} + \frac{\partial S}{\partial p_2^R}\frac{dp_2^R}{dx_2} \\ &= \frac{\partial S}{\partial p_1^R} \cdot \frac{1-\theta-(x_2)^2}{(x_2)^2} + \frac{\partial S}{\partial p_2^R} \\ &= (\theta + q_2)\left(\frac{1-\theta}{(x_2)^2} - 1\right) + \theta - q_2\frac{p_1^R}{p_2^R} \\ &= (\theta + q_2)\left(\frac{q_2}{x_2} - 1\right) + \theta - q_2\frac{p_1^R}{x_2} \quad \text{(since } q_2 p_2^R = 1 - \theta \text{ and } p_2^R = x_2\text{)} \\ &= q_2\left(\frac{\theta - x_2 + q_2 - p_1^R}{x_2}\right) > 0. \end{aligned}$$

The inequality at the end holds because, by the fact $q_2 = (1-\theta)/p_2^R = (1-\theta)/x_2$ and (A.8),

$$\begin{aligned}\theta - x_2 + q_2 - p_1^R &= \theta - x_2 + \frac{1-\theta}{x_2} - \frac{\theta + x_2(1-2\theta) - (1-x_2)^2}{x_2} \\ &= \frac{3\theta x_2 + 2 - 2\theta - 3x_2}{x_2} = \frac{(1-\theta)(2-3x_2)}{x_2}\end{aligned}$$

is strictly positive whenever $x_2 < 2/3$, which is true because $x_2 \leq 1/2$ due to (A.5) and (A.9).

Now that S is strictly increasing in x_2 and $x_2 \leq 1/2$, S is maximized at $x_2 = 1/2$ among all the solutions $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ that belong to Case (c). It follows that the equal-split proposal, $x_1 = x_2 = 1/2$, attains the maximum of S among these solutions. Since it is easy to verify that the Case-(c) solution under this proposal does constitute a PBE, Claim (i) of the lemma is proved.

To prove Claim (ii) of the lemma, plug $x_1 = x_2 = 1/2$ into (A.6)–(A.8) to obtain $p_2^R = 1/2$, $q_2 = (1-\theta)/p_2^R = 2(1-\theta)$, and $p_1^R = 1/2$. By $p_1^R = p_2^R = 1/2$ and Lemma 2.1, the social surplus is equal to θ. Thus Claim (ii) follows. ■

By Lemma A.5, the largest social surplus that any Case-(c) solution can achieve is equal to θ. By contrast, the social surplus generated by the lopsided solution $(\theta, 1-\theta)$ is equal to $\theta(3 - 5\theta/2)$ by Lemma A.4. Our assumption $\theta \leq 3/4$ in the proposition implies the desired conclusion $\theta < \theta(3 - 5\theta/2)$.

A.6 Suboptimality of Case-(d) Solutions (Eq. (2.15))

In any PBE that belongs to the Case (d) described in Lemma A.1, each type of each player is totally mixing between Accept and Reject. That is,

$$0 < \sigma_i(w) < 1, \quad 0 < \sigma_i(s) < 1, \quad \forall i \in \{1,2\}. \tag{A.10}$$

This being symmetric between the two players, let us assume without loss that

$$p_2^R \geq p_1^R. \tag{A.11}$$

Lemma A.6 *A tuple $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ that satisfies (A.11) constitutes a Case-(d) solution if and only if it satisfies (A.10) and all the following:*

$$1 - p_1^R = q_2(1 - p_1^A), \tag{A.12}$$
$$1 - p_2^R = q_1(1 - p_2^A), \tag{A.13}$$
$$p_1^R = p_2^A + x_1, \tag{A.14}$$
$$p_2^R + \theta - 1 = (1 - q_1)x_2. \tag{A.15}$$

Proof The best response condition for (A.10) to constitute a PBE is that $V_i^R(w) - V_i^A(w) = V_i^R(s) - V_i^A(s) = 0$ for each player i. By (A.1), that is equivalent to simultaneous satisfaction of $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$, $\left(1 - p_1^R\right) = q_2\left(1 - p_1^A\right)$, and $1 - p_2^R = q_1(1 - p_2^A)$. To write the condition $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$ explicitly, note for each player i that $q_i < 1$ in this PBE and hence $p_i^A < 1 - \theta < p_i^R$ by Lemmas A.2. This combined with (A.11) implies that (A.2) and (A.3) apply to the case $i = 2$ and hence

$$\begin{aligned}V_2^R(w) - V_2^A(w) &= p_2^R - (1-\theta) - (1-q_1)x_2, \\ V_1^R(w) - V_1^A(w) &= (1-q_2)\left(p_1^R - p_2^A - x_1\right).\end{aligned}$$

Consequently, with $q_2 < 1$,

$$V_1^R(w) - V_1^A(w) = 0 \iff p_1^R = p_2^A + x_1,$$
$$V_2^R(w) - V_2^A(w) = 0 \iff p_2^R + \theta - 1 = (1 - q_1)x_2. \quad \blacksquare$$

Lemma A.7 *If $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ is a Case-(d) solution such that $p_2^R \geq p_1^R$, then*

$$\sigma_1(w) = \frac{\theta + x_1 - 1 + q_1(1 - 2x_1)}{\theta}, \tag{A.16}$$

$$\sigma_1(s) = \frac{q_1 - \theta\sigma_1(w)}{1 - \theta}, \tag{A.17}$$

$$\sigma_2(w) = 1 - \frac{x_2}{\theta}, \tag{A.18}$$

$$\sigma_2(s) = \frac{\theta - x_2}{1 - \theta} \cdot \frac{1 - \theta + x_2(1 - q_1)}{\theta + x_2(q_1 - 1)}, \tag{A.19}$$

$$x_2 < \theta, \quad \textit{and} \tag{A.20}$$

$$(q_1)^3 x_2 (1 - 2x_2) + (q_1)^2 x_2 (3x_2 - 1 - \theta) + q_1 (3x_2 - 1 - \theta)(\theta - x_2) + (\theta - x_2)^2 = 0. \tag{A.21}$$

Proof Eq. (A.17) follows trivially from $q_1 = \theta\sigma_1(w) + (1-\theta)\sigma_1(s)$. To prove the rest, first apply Bayes's rule to $1 - p_2^A$ and then to $1 - p_2^R$ to obtain

$$1 - p_2^A = \frac{\theta(1 - \sigma_2(w))}{1 - q_2} = \frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - (1 - p_2^R)q_2} = \frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - \theta\sigma_2(w)}.$$

Then

$$p_2^R - p_2^A = (1 - p_2^A) - (1 - p_2^R) = \frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - \theta\sigma_2(w)} - (1 - p_2^R) = \frac{(1 - p_2^R)(\theta + p_2^R - 1)}{1 - p_2^R - \theta\sigma_2(w)}.$$

By (A.13) we have $q_1 = (1 - p_2^R)/(1 - p_2^A)$. Plug this into (A.15) to obtain

$$(\theta + p_2^R - 1)(1 - p_2^A) = (p_2^R - p_2^A)x_2.$$

Plugging into this equation the formulas of $1 - p_2^A$ and $p_2^R - p_2^A$ obtained above, we have

$$(\theta + p_2^R - 1)\frac{(1 - p_2^R)\theta(1 - \sigma_2(w))}{1 - p_2^R - \theta\sigma_2(w)} = \frac{(1 - p_2^R)(\theta + p_2^R - 1)}{1 - p_2^R - \theta\sigma_2(w)}x_2,$$

namely,

$$\theta(1 - \sigma_2(w)) = x_2.$$

Thus (A.18) is true. Then Eq. (A.18) coupled with $\sigma_2(w) > 0$ implies (A.20).

Second, plug Eqs. (A.13) and (A.14) into Eq. (A.15) to obtain

$$1 - q_1(1 - p_1^R + x_1) = 1 - \theta + (1 - q_1)x_2.$$

eliminate $1 - \theta$ therein by Eq. (2.6) and cancel and combine terms to obtain

$$(1 - q_1)(1 - p_1^A) = x_2 - q_1(x_2 - x_1),$$

which, by Bayes's rule, is equivalent to

$$\theta(1 - \sigma_1(w)) = x_2 - q_1(x_2 - x_1), \tag{A.22}$$

which in turn is equivalent to Eq. (A.16).

Third, rewrite (A.13) as $q_1 = (1 - p_2^R)/(1 - p_2^A)$ and then rewrite the right-hand side by Bayes's rule to obtain

$$q_1 = \frac{\theta\sigma_2(w)}{\theta(1 - \sigma_2(w))} \cdot \frac{\theta(1 - \sigma_2(w)) + (1 - \theta)(1 - \sigma_2(s))}{\theta\sigma_2(w) + (1 - \theta)\sigma_2(s)} \overset{(A.18)}{=} \frac{\theta - x_2}{x_2} \cdot \frac{x_2 + (1 - \theta)(1 - \sigma_2(s))}{\theta - x_2 + (1 - \theta)\sigma_2(s)},$$

which implies Eq. (A.19).

Finally, we prove Eq. (A.21). Use Bayes's rule on player 2 and then use (A.18) to obtain

$$(1-q_2)(1-p_2^A)=\theta(1-\sigma_2(w))=x_2.$$

Eliminate the q_2 in this equation by (A.12), and p_2^A by (A.14), to rewrite the above equation as

$$\left(1-\frac{1-p_1^R}{1-p_1^A}\right)(1-p_1^R+x_1)=x_2,$$

namely,

$$(1-p_1^A)x_2=\left(p_1^R-p_1^A\right)\left(1-p_1^R+x_1\right). \tag{A.23}$$

Meanwhile, use Bayes's rule on player 1 and then use (A.22) to obtain

$$1-p_1^A=\frac{\theta(1-\sigma_1(w))}{1-q_1}=\frac{x_2-q_1\left(x_2-x_1\right)}{1-q_1}.$$

Analogously, use Bayes's rule on player 1 and then use Eq. (A.16) to obtain

$$1-p_1^R=\frac{\theta\sigma_1(w)}{q_1}=\frac{\theta+x_1-1+q_1(1-2x_1)}{q_1}.$$

From the two formulas we get

$$\begin{aligned}
p_1^R-p_1^A &= \frac{x_2-q_1\left(x_2-x_1\right)}{1-q_1}-\frac{\theta+x_1-1+q_1(x_2-x_1)}{q_1}\\
&= \frac{-\theta-x_1+1+q_1\theta+2q_1x_1-q}{q_1(1-q_1)}\\
&= \frac{x_2-\theta-q_1(x_2-x_1-\theta)}{q_1(1-q_1)} \quad (\text{by } x_1+x_2=1).
\end{aligned}$$

Replace the $1-p_1^A$, $1-p_1^R$ and $p_1^R-p_1^A$ in (A.23) with the above formulas to rewrite (A.23) as

$$\begin{aligned}
\frac{x_2-q_1\left(x_2-x_1\right)}{1-q_1}x_2 &= \left(\frac{x_2-\theta-q_1\left(x_2-x_1-\theta\right)}{(1-q_1)q_1}\right)\left(\frac{\theta+x_1-1+q_1(1-2x_1)}{q_1}+x_1\right)\\
&= \left(\frac{x_2-\theta-q_1\left(x_2-x_1-\theta\right)}{(1-q_1)q_1}\right)\left(\frac{q_1x_2+\theta-x_2}{q_1}\right),
\end{aligned}$$

with the second line due to $x_1+x_2=1$. Simplify the above equation into

$$x_2(x_2-q_1(x_2-x_1))=\frac{x_2-\theta-q_1(x_2-x_1-\theta)}{q_1}\cdot\frac{q_1x_2+\theta-x_2}{q_1},$$

namely,

$$(q_1)^2\,x_2\left(q_1x_1+(1-q_1)x_2\right)=\left(q_1x_1-(1-q_1)(\theta-x_2)\right)\left(q_1x_2+\theta-x_2\right).$$

Plug $x_2=1-x_1$ into the above displayed equation to obtain

$$\begin{aligned}
&(q_1)^2\,x_2\left(q_1(1-x_2)+(1-q_1)x_2\right)=\left(q_1(1-x_2)-(1-q_1)(\theta-x_2)\right)\cdot\left(q_1x_2+\theta-x_2\right)\\
\iff\ &(q_1)^2\,x_2\left(q_1(1-2x_2)+x_2\right)=\left(q_1(1+\theta-2x_2)+x_2-\theta\right)\cdot\left(q_1x_2+\theta-x_2\right),\\
\iff\ &(q_1)^3\,x_2(1-2x_2)+(q_1)^2\,(x_2)^2=(q_1)^2\,(1+\theta-2x_2)x_2+(\theta-x_2)q_1(1+\theta-3x_2)-(\theta-x_2)^2\\
\iff\ &(q_1)^3\,x_2(1-2x_2)+(q_1)^2\,x_2(3x_2-1-\theta)+q_1(\theta-x_2)(3x_2-1-\theta)+(\theta-x_2)^2=0.
\end{aligned}$$

Thus, Eq. (A.21) is true. ■

Lemma A.8 *If $(x_i,\sigma_i,p_i^A,p_i^R,q_i)_{i=1}^2$ is a Case-(d) solution such that $p_2^R\geq p_1^R$, $x_1\geq 1/2\geq x_2$.*

Proof By Bayes's rule,

$$1 - p_1^R = \frac{\theta\sigma_1(w)}{q_1} \overset{(A.16)}{=} \frac{\theta - x_2 + q_1(2x_2 - 1)}{q_1},$$

with the second "=" also using $x_1 + x_2 = 1$. Meanwhile, write (A.15) into

$$1 - p_2^R = \theta + x_2(q_1 - 1).$$

Thus,

$$\begin{aligned} p_2^R \geq p_1^R &\iff \frac{\theta - x_2 + q_1(2x_2 - 1)}{q_1} \geq \theta + x_2(q_1 - 1) \\ &\iff (3x_2 - 1 - \theta)\, q_1 + (\theta - x_2) \geq (q_1)^2\, x_2 \\ &\iff (3x_2 - 1 - \theta)\,(\theta - x_2)\, q_1 + (\theta - x_2)^2 \geq (q_1)^2\, x_2(\theta - x_2), \end{aligned} \tag{A.24}$$

with the last line due to $\theta - x_2 > 0$ (Ineq. (A.20)). Subtract Ineq. (A.24) by Eq. (A.21) and cancel some terms to see that Ineq. (A.24) is equivalent to

$$0 \quad \geq \quad (q_1)^3\, x_2\,(1 - 2x_2) + (q_1)^2\, x_2\,(3x_2 - 1 - \theta) + (q_1)^2\, x_2(\theta - x_2),$$

namely,

$$0 \geq (q_1)^2\, x_2\,(1 - q_1)\,(2x_2 - 1)\,.$$

Thus,

$$p_2^R \geq p_1^R \iff 0 \geq (q_1)^2\, x_2\,(1 - q_1)\,(2x_2 - 1) \iff 0 \geq 2x_2 - 1,$$

with the second " $\iff$ " due to the fact $q_1 < 1$ in all Case-(d) PBEs. Thus we have $2x_2 \leq 1$, which by $x_1 + x_2 = 1$ means $x_1 \geq 1/2 \geq x_2$, as claimed. ■

Lemma A.9 *If $\theta \geq 2/3$, then the peace proposal $(\theta, 1-\theta)$ (or $(1-\theta, \theta)$) admits a Case-(b) PBE that generates strictly larger social surplus than any Case-(d) solution does.*

Proof Consider any Case-(d) solution $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$. Without loss of generality, assume that $p_2^R \geq p_1^R$. Then Lemma 2.1 implies that the social surplus generated by this solution is equal to $2\theta p_2^R + (q_2 - \theta)\left(p_2^R - p_1^R\right)$. First, we observe that

$$x_2 \geq 1 - \theta \Rightarrow q_2 < \theta.$$

To see this, note from (A.18) that

$$q_2 = \theta\sigma_2(w) + (1 - \theta)\sigma_2(s) = \theta - x_2 + (1 - \theta)\sigma_2(s).$$

Thus, $q_2 < \theta$ if $\sigma_2(s) < \frac{x_2}{1-\theta}$. The latter inequality follows from $\sigma_2(s) < 1$ (part of the definition of Case (d)) and $1 - \theta \leq x_2$.

Thus, if $x_2 \geq 1 - \theta$ then the upper bound of social surplus admitted within Case-(d) PBEs is $2\theta p_2^R$. Since the social surplus generated by the Case-(b) solution under the proposal $(\theta, 1-\theta)$ is equal to $\theta(3 - 5\theta/2)$ (Lemma A.4), the proof is complete if $x_2 \geq 1 - \theta$ and $\theta(3 - 5\theta/2) \geq 2\theta p_2^R$.

To show $\theta(3 - 5\theta/2) \geq 2\theta p_2^R$, note from (A.15) that $p_2^R = 1 - \theta + (1 - q_1)x_2$. Thus, $\theta(3 - 5\theta/2) \geq 2\theta p_2^R$ is equivalent to $1 - \theta/2 \geq 2x_2(1 - q_1)$, which is true if $1 - \theta/2 \geq 1 - q_1$ (due to $x_2 \leq 1/2 \leq x_1$), namely, $q_1 > \theta/2$.

To prove $q_1 > \theta/2$, note from the definition of Case-(d) PBEs that $\sigma_2(s) < 1$, which by (A.19) is equivalent to

$$\frac{\theta - x_2}{1 - \theta} \cdot \frac{1 - \theta + x_2(1 - q_1)}{\theta - x_2 + x_2 q_1} < 1.$$

Since $\theta - x_2 > 0$ by (A.20), the above-displayed inequality can be simplified into

$$q_1 > \frac{\theta - x_2}{1 - x_2}.$$

Consequently, since $\frac{\theta - x_2}{1 - x_2}$ is strictly decreasing in x_2 and $x_2 \leq 1/2$, we have

$$q_1 > \frac{\theta - 1/2}{1 - 1/2} = 2\theta - 1 \geq \theta/2,$$

with the last inequality due to the assumption $\theta \geq 2/3$ in the lemma. Thus, $q_1 > \theta/2$ and hence $\theta(3 - 5\theta/2) \geq 2\theta p_2^R$.

Finally, we verify $x_2 \geq 1 - \theta$. By (A.16) and (A.17), the Case-(d) condition $\sigma_1(s) < 1$ implies

$$\frac{q_1 - \theta\left(\theta + x_1 - 1 + q_1(1 - 2x_1)\right)/\theta}{1 - \theta} < 1,$$

which simplifies to $q_1 < 1/2$. This, coupled with the previously proved $q_1 > \frac{\theta - x_2}{1 - x_2}$, implies

$$\frac{\theta - x_2}{1 - x_2} < \frac{1}{2},$$

namely, $x_2 > 2\theta - 1$. Since $\theta \geq 2/3$ by hypothesis, we have $2\theta - 1 \geq 1 - \theta$. Thus $x_2 > 1 - \theta$ follows. Both $x_2 \geq 1 - \theta$ and $\theta(3 - 5\theta/2) \geq 2\theta p_2^R$ established, the proof is complete. ■

A.7 Suboptimality of Case-(e) Solutions (Eqs. (2.12) or (2.13))

In any Case-(e) PBEs, exactly one of the two players is totally mixing between Accept and Reject for each type. Relabeling the players if necessary, assume without loss that in any Case-(e) PBEs it is player 1 who is totally mixing, i.e.,

$$0 < \sigma_1(w) < 1, \quad 0 < \sigma_1(s) < 1, \quad 0 < \sigma_2(w) < 1, \quad \sigma_2(s) = 1. \tag{A.25}$$

Call a Case-(e) solution *Case (e)-i* if $p_2^R \leq p_1^R$, and *Case (e)-ii* if $p_1^R < p_2^R$.

Lemma A.10 *A tuple $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ constitutes a Case-(e)-i solution if and only if it satisfies (A.25) and all the following:*

$$\begin{aligned} 1 - p_1^R &= q_2(1 - p_1^A), & \text{(A.26)}\\ 1 - p_2^R &\geq q_1, & \text{(A.27)}\\ p_2^R &\leq p_1^R, & \text{(A.28)}\\ p_1^R + \theta - 1 &= (1 - q_2)x_1, & \text{(A.29)}\\ p_2^R &= p_1^A + x_2. & \text{(A.30)} \end{aligned}$$

A tuple $(x_i, \sigma_i, p_i^A, p_i^R, q_i)_{i=1}^2$ constitutes a Case-(e)-ii solution if and only if it satisfies (A.25), (A.26), (A.27) and all the following:

$$\begin{aligned} p_1^R &< p_2^R, & \text{(A.31)}\\ p_1^R &= x_1, & \text{(A.32)}\\ p_2^R + \theta - 1 &= (1 - q_1)x_2. & \text{(A.33)} \end{aligned}$$

Proof The best response condition for (A.25) to constitute a PBE is that $V_1^R(w) - V_1^A(w) = V_1^R(s) - V_1^A(s) = 0$ for each player 1 and $V_1^R(w) - V_1^A(w) = 0 \leq V_1^R(s) - V_1^A(s)$. By (A.1), that is equivalent to simultaneous satisfaction of $V_1^R(w)-V_1^A(w) = V_2^R(w)-V_2^A(w) = 0$ and $\left(1-p_1^R\right) = q_2\left(1-p_1^A\right)$ and $1-p_2^R \geq q_1$ (the last inequality also uses the fact $p_2^A = 0$ implied by Bayes's rule with respect to $\sigma_2(s) = 1$). To write the condition $V_1^R(w) - V_1^A(w) = V_2^R(w) - V_2^A(w) = 0$ explicitly, note for each player i that $q_i < 1$ in this PBE and hence $p_i^A < 1-\theta < p_i^R$ by Lemmas A.2. If the solution belongs to Subcsae (i) of Case (e), $p_1^R \geq p_2^R$, then (A.2) and (A.3) apply to the case $i = 1$ and hence

$$\begin{aligned} V_1^R(w) - V_1^A(w) &= p_1^R - (1-\theta) - (1-q_2)x_1, \\ V_2^R(w) - V_2^A(w) &= (1-q_1)\left(p_2^R - p_1^A - x_2\right). \end{aligned}$$

Thus the condition $V_1^R(w) - V_1^A(w) = 0$ becomes (A.29), and the condition $V_2^R(w) - V_2^A(w) = 0$ becomes (A.30). Analogously, if it is Subcase (ii) of Case (e), $p_1^R \leq p_2^R$, then (A.2) and (A.3) apply to the case $i = 2$ and hence

$$\begin{aligned} V_2^R(w) - V_2^A(w) &= p_2^R - (1-\theta) - (1-q_1)x_2, \\ V_1^R(w) - V_1^A(w) &= (1-q_2)\left(p_1^R - p_2^A - x_1\right) = (1-q_2)\left(p_1^R - x_1\right), \end{aligned}$$

with the last "=" due to $p_2^A = 0$ (since $\sigma_2(s) = 1$). Thus, the condition $V_i^R(w) - V_i^A(w) = 0$ for both players i becomes (A.32) and (A.33). ■

A.7.1 Subcase (i): $p_1^R \geq p_2^R$ (Eq. (2.12))

Lemma A.11 *For any $x_2 \in [0,1]$ there is at most one tuple $\left(\sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ such that $\left(x_i, \sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ constitutes a Case-(e)-i solution, and for any such solution, $x_2 > 1-\theta$.*

Proof Let $x_2 \in [0,1]$ and $\left(x_i, \sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ be a Case-(e)-i solution. By Lemma A.10, the tuple satisfies Eqs. (A.26), (A.29) and (A.30). Combine (A.26), (A.29) and (A.30) with the fact that $q_2 = \theta\sigma_2(w)+1-\theta$, $p_2^R = (1-\theta)/q_2$ (Bayes's rule with respect to $\sigma_2(s) = 1$) and $x_1+x_2 = 1$ to obtain

$$\sigma_2(w) = 1 - \frac{1}{2\theta}. \tag{A.34}$$

Plug this back into the equation system to obtain a unique solution for all components of the tuple:

$$\begin{aligned} q_2 &= \theta\left(1-\frac{1}{2\theta}\right)+1-\theta = \frac{1}{2}, \\ p_2^R &= \frac{1-\theta}{q_2} = 2-2\theta, && \text{(A.35)} \\ p_1^R &= 1-\theta+(1-1/2)(1-x_2) = \frac{3-2\theta-x_2}{2}, && \text{(A.36)} \\ p_1^A &= p_2^R - x_2 = 2(1-\theta)-x_2, \\ q_1 &= \frac{1-\theta-p_1^A}{p_1^R-p_1^A} = \frac{2(\theta-1+x_2)}{2\theta+x_2-1}, && \text{(A.37)} \\ \sigma_1(w) &= \frac{\theta-1+x_2}{\theta}. && \text{(A.38)} \end{aligned}$$

In particular, (A.38) follows from

$$\begin{aligned}\theta\sigma_1(w) &= q_1 - (1-\theta)\sigma_1(s) \;=\; q_1 - p_1^R q_1 \\ &= \frac{2(\theta-1+x_2)}{2\theta+x_2-1}\left(1-\frac{3-2\theta-x_2}{2}\right) \\ &= \theta-1+x_2.\end{aligned}$$

Since $\sigma_1(w) > 0$ by definition of any Case-(e) solution, (A.38) implies $x_2 > 1-\theta$. ■

Lemma A.12 *If $\theta \le 3/4$, the Case-(b) PBE admitted by the proposal $(\theta, 1-\theta)$ generates larger social surplus than any Case-(e)-i solution.*

Proof By Lemma A.11, any Case-(e)-i solution is uniquely determined by the x_2 in the tuple, with 2 being the label for the player for whom $p_2^R \le p_1^R$. Thus, the social surplus generated by the solution is uniquely determined by x_2. Hence denote $S_e(x_2)$ for the social surplus generated by a Case-(e)-i solution that offers x_2 to the player $-i$ for whom $p_{-i}^R \le p_i^R$. Since Reject is a best reply for each type of each player in any Case-(e) solution, Lemma 2.1 implies

$$S_e(x_2) = 2\theta p_1^R + (q_1-\theta)(p_1^R - p_2^R). \tag{A.39}$$

By Lemma A.11, $x_2 > 1-\theta$. Taking the limit of (A.36) and (A.37) as x_2 converges to $1-\theta$ from above, we have

$$\begin{aligned}\lim_{x_2\downarrow 1-\theta} p_1^R &= \frac{2-\theta}{2}, \\ \lim_{x_2\downarrow 1-\theta} q_1 &= 0.\end{aligned}$$

Combine them with the above formula of $S_e(x_2)$ and (A.35) to obtain

$$\begin{aligned}\lim_{x_2\downarrow 1-\theta} S_e(x_2) &= 2\theta p_1^R - \theta(p_1^R - p_2^R) = \theta(p_1^R + p_2^R) \\ &= \theta\left(\frac{2-\theta}{2} + 2 - 2\theta\right) \\ &= \theta\left(3 - \frac{5}{2}\theta\right),\end{aligned}$$

which by Lemma A.4 is equal to the social surplus generated by the Case-(b) solution of proposal $(\theta, 1-\theta)$. Thus, it suffices to show that $S_e(x_2)$ is strictly decreasing in x_2.

To show $\frac{d}{dx_2}S_i(x_2) < 0$ for all $x_2 > 1-\theta$, use (A.39) and $dp_2^R/dx_2 = 0$ (Eq. (A.35)) to obtain

$$\begin{aligned}\frac{d}{dx_2}S_e(x_2) = \frac{\partial S_e}{\partial p_1^R}\frac{dp_1^R}{dx_2} + \frac{\partial S_e}{\partial q_1}\frac{dq_1}{dx_2} &= (q_1+\theta)\frac{dp_1^R}{dx_2} + \left(p_1^R - p_2^R\right)\frac{dq_1}{dx_2} \\ &= -\frac{q_1+\theta}{2} + \left(p_1^R - p_2^R\right)\frac{2\theta}{(2\theta+x_2-1)^2},\end{aligned} \tag{A.40}$$

with the last equality due to (A.36) and (A.37). Note that the expression (A.40) is strictly decreasing in x_2: By (A.35) and (A.36), $p_1^R - p_2^R = (2\theta - 1 - x_2)/2$, which is strictly decreasing in x_2; as can be seen above (due to (A.37)),

$$\frac{dq_1}{dx_2} = \frac{2\theta}{(2\theta+x_2-1)^2} > 0$$

and so $-\frac{q_1+\theta}{2}$ is strictly decreasing in x_2 as well. Thus, $\frac{d}{dx_2}S_e(x_2)$ is strictly decreasing in x_2.

Now that $\frac{d}{dx_2}S_e(x_2)$ is strictly decreasing in x_2 for all $x_2 > 1-\theta$, and $x_2 > 1-\theta$ for any

Case-(e)-i solution, to show that $S_e(x_2)$ is strictly decreasing in x_2, we need only

$$\lim_{x_2 \downarrow 1-\theta} \frac{d}{dx_2} S_e(x_2) < 0.$$

To show that, take the limit of (A.40) as x_2 converges to $1-\theta$ from above and use (A.35), (A.36), and (A.37) (so $\lim_{x_2 \downarrow 1-\theta} q_1 = 0$ and $\lim_{x_2 \downarrow 1-\theta} \left(p_1^R - p_2^R\right) = \frac{3\theta-2}{2}$) to obtain

$$\lim_{x_2 \downarrow 1-\theta} \frac{d}{dx_2} S_e(x_2) = -\frac{\theta}{2} + \frac{(3\theta-2)}{2}\frac{2}{\theta} = \frac{-\theta^2+6\theta-4}{2\theta} = -\frac{1}{2\theta}\left((\theta-3)^2-5\right),$$

which is negative because the condition $\theta \leq 3/4$ in the lemma implies $\theta < 3-\sqrt{5}$. Thus, the supremum of $\frac{d}{dx_2} S_e(x_2)$ is negative among all $x_2 > 1-\theta$, so $\lim_{x_2 \downarrow 1-\theta} S_e(x_2)$ is the supremum social surplus among all Case-(e)-i solutions. Since the supremum has been shown equal to the social surplus generated by the Case-(b) solution $(\theta, 1-\theta)$, the lemma is proved. ■

A.7.2 Subcase (ii): $p_2^R > p_1^R$ (Eq. (2.13))

Lemma A.13 *For any $x_2 \in [0,1]$ there is at most one tuple $\left(\sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ such that $\left(x_i, \sigma_i, p_i^A, p_i^R, q_i\right)_{i=1}^2$ constitutes a Case-(e)-ii solution, and for any such PBE, $2\theta-1 < x_2 < 1/2$.*

Proof By Lemma A.10, the tuple satisfies Eqs. (A.26), (A.32) and (A.33). Plug Bayes's rule $1-p_1^R = \theta\sigma_1(w)/q_1$ into Eq. (A.32) to obtain

$$\sigma_1(w) = \frac{(1-\theta)(1-x_1)}{\theta x_1}\sigma_1(s). \tag{A.41}$$

Eq. (A.32), combined with $1-p_1^R = \theta\sigma_1(w)/q_1$ and $x_1+x_2=1$, also implies

$$q_1 = \frac{\theta\sigma_1(w)}{x_2}. \tag{A.42}$$

Thus, from Bayes's rule we have

$$1-p_1^A = \frac{\theta(1-\sigma_1(w))}{1-q_1} = \frac{\theta-q_1x_2}{1-q_1}.$$

Plug this into (A.26), replace p_1^R via $p_1^R = x_1$ (Eq. (A.32)) and replace q_2 through $q_2 = (1-\theta)/p_2^R$ (due to (2.6) and $p_2^A = 0$, the latter due to $\sigma_2(s) = 1$), and eliminate p_2^R by (A.33). Then

$$x_2 = \frac{(1-\theta)}{1-\theta+(1-q_1)x_2}\frac{\theta-q_1x_2}{1-q_1},$$

which is simplified to a quadratic equation

$$(q_1)^2(x_2)^2 - 2q_1(x_2)^2 + x^2 + (1-\theta)(x-\theta) = 0,$$

namely,

$$x_2^2(q_1-1)^2 = (1-\theta)(\theta-x_2).$$

We claim that the right-hand side of this equation is strictly positive. To see that, note $p_1^A < p_1^R$ due to Lemma A.2 and $\sigma_1(w) < 1$ and hence $q_1 < 1$ in any Case-(e) PBE. Then the Bayesian plausibility condition (2.6) implies $p_1^R > 1-\theta$. This, combined with Bayes's rule $p_1^R = (1-\theta)\sigma_1/q_1$ and $1-p_1^R = \theta\sigma_1(w)/q_1$, implies $\sigma_1(w) < \sigma_1(s)$. Then (A.41 implies $1-x_1 < \theta$, namely,

$$\theta - x_2 > 0. \tag{A.43}$$

Thus, the quadratic equation implies $x_2(q_1 - 1) = -\sqrt{(1-\theta)(\theta - x_2)}$, namely,

$$q_1 = 1 - \frac{1}{x_2}\sqrt{(1-\theta)(\theta - x_2)}. \tag{A.44}$$

Thus, the Case-(e) solution is uniquely determined by x_2. In particular,

$$p_1^R \overset{(A.32)}{=} 1 - x_2, \tag{A.45}$$

$$p_2^R \overset{(A.33)}{=} 1 - \theta + \sqrt{(1-\theta)(\theta - x_2)}, \tag{A.46}$$

$$\sigma_1(w) \overset{(A.42)}{=} \frac{x_2 - \sqrt{(1-\theta)(\theta - x_2)}}{\theta}, \tag{A.47}$$

$$q_2 = \frac{1-\theta}{p_2^R}, \tag{A.48}$$

with (A.48) due to Bayes's rule with respect to $\sigma_2(s) = 1$.

Finally we verify that $2\theta - 1 < x_2 < 1/2$ in any Case-(e)-ii solution. Recall from the definition of Case-(e)-ii solutions that $p_2^R > p_1^R$. By (A.46) and (A.45),

$$\begin{aligned} p_2^R > p_1^R &\iff 1 - \theta + \sqrt{(1-\theta)(\theta - x_2)} > 1 - x_2 \\ &\iff \sqrt{(1-\theta)(\theta - x_2)} > \theta - x_2. \end{aligned} \tag{A.49}$$

By (A.43), the above inequality is equivalent to

$$\left(\sqrt{(1-\theta)(\theta - x_2)}\right)^2 > (\theta - x_2)^2,$$

namely, $1 - \theta > \theta - x_2$. Thus

$$x_2 > 2\theta - 1. \tag{A.50}$$

To prove $x_2 < 1/2$, recall that (A.27) holds for any Case-(e)-ii solution (Lemma A.10), namely, $q_1^R \leq 1 - p_2^R$. Plug (A.44) and (A.46) into this inequality to obtain

$$\left(\frac{1}{x_2} - 1\right)\sqrt{(1-\theta)(\theta - x_2)} \geq 1 - \theta,$$

namely,

$$(x_2)^2(1-\theta) \leq (\theta - x_2)(1 - x_2)^2.$$

This coupled with (A.50) implies

$$(x_2)^2(1-\theta) \leq (\theta - x_2)(1 - x_2)^2 < (1 - x_2)^2(1 - \theta)$$

and hence $x_2^2 < (1 - x_2)^2$. Thus $x_2 < 1/2$, as asserted. ■

Lemma A.14 *If $2/3 \leq \theta \leq 3/4$, then the Case-(b) PBE admitted by the proposal $(\theta, 1-\theta)$ generates larger social surplus than any Case-(e)-ii solution.*

Proof In any Case-(e)-ii solution, Reject is a best reply for each type of each player and hence Lemma 2.1 applies. Thus, since $p_2^R \geq p_1^R$ in Case-(e)-ii, the social surplus is equal to

$$S_e' := 2\theta p_2^R + (q_2 - \theta)(p_2^R - p_1^R).$$

To prove that S_e' is less than the social surplus generated by the Case-(b) solution $(\theta, 1-\theta)$, which is equal to $\theta(3 - 5\theta/2)$ by Lemma A.4, it suffices to prove $q_2 < \theta$ and $p_2^R < 2 - 2\theta$ for any Case-(e)-ii solution: If $q_2 < \theta$ then $S_e' < 2\theta p_2^R$ because $p_2^R - p_1^R > 0$ in any Case-(e)-ii solution; if, in addition, $p_2^R < 2 - 2\theta$, then

$$S_e' < 2\theta p_2^R < 2\theta(2 - 2\theta) \leq \theta(3 - 5\theta/2),$$

with the last inequality due to the condition $\theta \geq 2/3$ in the lemma.

Thus, we shall verify that $p_2^R < 2 - 2\theta$ and $q_2 < \theta$. First, note from (A.46) that $p_2^R < 2 - 2\theta$ is equivalent to

$$1 - \theta + \sqrt{(1-\theta)(\theta - x_2)} < 2 - 2\theta \iff \sqrt{(1-\theta)(\theta - x_2)} < 1 - \theta$$
$$\iff 1 - \theta < \theta - x_2 \iff 2\theta - 1 < x_2,$$

where $2\theta - 1 < x_2$ is true by Lemma A.13. Thus, $p_2^R < 2 - 2\theta$.

Second, to prove $q_2 < \theta$, note from (A.46) and (A.48).

$$q_2 < \theta \iff \frac{1-\theta}{1 - \theta + \sqrt{(1-\theta)(\theta - x_2)}} < \theta$$
$$\iff (1-\theta)^2 \leq \theta \sqrt{(1-\theta)(\theta - x_2)}$$
$$\iff x_2 \leq \theta - \frac{(1-\theta)^3}{\theta^2}.$$

Thus, since $x_2 < 1/2$ by Lemma A.13, it suffices to show $1/2 \leq \theta - (1-\theta)^3/\theta^2$, namely,

$$\frac{4\theta^3 - 7\theta^2 + 6\theta - 2}{2\theta^2} \geq 0.$$

Thus we are done if $4\theta^3 - 7\theta^2 + 6\theta - 2 \geq 0$. To show that, note

$$\frac{d}{d\theta}\left[4\theta^3 - 7\theta^2 + 6\theta - 2\right] = 12\theta^2 - 14\theta + 6 = 6\theta(2\theta - 1) + 2(3 - 4\theta) > 0$$

because $2\theta > 1$ by (2.1) and $\theta \leq 3/4$ by the hypothesis of lemma. Thus, the term $4\theta^3 - 7\theta^2 + 6\theta - 2$ is strictly increasing in θ. Since it is equal to $2/27$ at $\theta = 2/3$, it follows that $4\theta^3 - 7\theta^2 + 6\theta - 2 > 0$ for all $\theta \in [2/3, 3/4]$. This proves $q_2 < \theta$, as desired. ■

Appendix B

Appendices to Chapter 3

B.1 Categorizing All Kinds of Fully Participating PBEs

Lemma B.1 *For any $i \in \{1, 2\}$ and in any fully participating PBE, if player i's strategy $(\sigma_i(\nu_i; a), \sigma_i(\nu_i; 1))$ are specified by a row and column in the following table, then σ_i has the property stated in the corresponding cell:*

	$\sigma_i(\nu_i; 1) = 0$	$0 < \sigma_i(\nu_i; 1) < 1$	$\sigma_i(\nu_i; 1) = 1$
$\sigma_i(\nu_i; a) = 0$			
$0 < \sigma_i(\nu_i; a) < 1$	*impossible*		
$\sigma_i(\nu_i; a) = 1$	*impossible*	*impossible*	*always conflict*

Proof First, suppose $0 < \sigma_i(\nu_i; a) < 1$ and $\sigma_i(\nu_i; 1) = 0$. Then $\Delta_i(a) = 0$ and $\Delta_i(1) \leq 0$ by Ineqs. 3.15 and 3.16, $\pi_i^R = 1$ by definition, and $\pi_i^A = \theta/((\theta + (1-\theta)/(1-\sigma_i(\nu_i; a))) < \theta$ by Eq. (3.5). Thus, $0 \geq \Delta_i(1) - \Delta_i(a) = q_{-i}^A \pi_i^R + q_{-i}^R \left[\pi_i^R - \max\left\{\pi_i^A, \pi_{-i}^R\right\} + a\left(\pi_{-i}^R - \pi_i^A\right)^+\right] > 0$, where the last inequality can be readily verified for the two possible and exhaustive cases of $\pi_i^A \geq \pi_{-i}^R$ and $\pi_i^A \leq \pi_{-i}^R$ coupled with the fact that $\pi_i^A < \theta < \pi_i^R = 1$. The contradiction displayed above implies this case is impossible, as asserted in the cell.

Second, suppose $\sigma_i(\nu_i; a) = 1$ and $0 \leq \sigma_i(\nu_i; 1) < 1$, which corresponds to the cells of the third row and the first and second columns. Then $\Delta_i(a) \geq 0$ and $\Delta_i(1) \leq 0$ by Ineqs. 3.15 and 3.16, $\pi_i^A = 0$ by definition, and $\pi_i^R = \theta/(\theta + (1-\theta)\sigma_i(\nu_i; 1)) > \theta$ by Eq. (3.6). Thus,

$$0 \geq \Delta_i(1) - \Delta_i(a) = q_{-i}^A \left(\max\left\{\pi_i^R, \pi_{-i}^A\right\} - a\left(\pi_{-i}^A - \pi_i^R\right)^+\right) + q_{-i}^R \left[\max\left\{\pi_i^R, \pi_{-i}^R\right\} - \pi_{-i}^R - a\left(\pi_{-i}^R - \pi_i^R\right)^+ + a\pi_{-i}^R\right] > 0$$

where the last inequality can be readily verified for the four possible and exhaustive cases of $\pi_i^R \geq \pi_{-i}^R$, $\pi_i^R \leq \pi_{-i}^R$, $\pi_{-i}^A \geq \pi_{-i}^R$, and $\pi_{-i}^A \leq \pi_{-i}^R$ coupled with the fact that $\pi_i^A = 0 < \theta < \pi_i^R$. Hence this case is impossible, as asserted in the cells.

Finally, consider $\sigma_i(\nu_i; a) = \sigma_i(\nu_i; 1) = 1$, the cell of Row Three and Column Three. Then

$q_i^A = 0$, $q_i^R = 1$ and $\pi_i^R = \theta$ by definition. Apply Eq. (3.14) to the opponent $-i$ to obtain

$$\begin{bmatrix} \Delta_{-i}(1) \\ \Delta_{-i}(a) \end{bmatrix} = \begin{bmatrix} \max\left\{\pi_{-i}^R, \theta\right\} - \max\left\{\pi_{-i}^A, \theta\right\} \\ a\left(\theta - \pi_{-i}^R\right)^+ - a\left(\theta - \pi_{-i}^A\right)^+ \end{bmatrix}. \tag{B.1}$$

We claim that the posterior probability π_{-i} with which player $-i$'s type equals a is the same as the prior: $\pi_{-i} = \theta$. Suppose otherwise. We derive a contradiction for all possibilities:

1. $\sigma_{-i}(\nu_{-i}; a) = 0$. Then $\sigma_{-i}(\nu_{-i}; 1) > 0$, otherwise the claim $\pi_{-i} = \theta$ is true. Thus, $\pi_{-i}^A > \theta$ by Eq. (3.5), and $\Delta_{-i}(1) \geq 0$ by Ineq. 3.15. Then Eq. (B.1) implies $\pi_{-i}^R \geq \pi_{-i}^A > \theta$. But since $\sigma_{-i}(\nu_{-i}; a) = 0$ and $\sigma_{-i}(\nu_{-i}; 1) > 0$, $\pi_{-i}^R = 0$ by Bayes's rule: contradiction.

2. $\sigma_{-i}(\nu_{-i}; a) = 1$. Then $\sigma_{-i}(\nu_{-i}; 1) < 1$, otherwise the claim $\pi_{-i} = \theta$ is true. Thus, $\pi_{-i}^R > \theta$ by Eq. (3.6), and $\Delta_{-i}(a) \geq 0$ by Ineq 3.16. Then Eq. (B.1) implies $\pi_{-i}^A \geq \theta$. But since $\sigma_{-i}(\nu_{-i}; a) = 1$ and $\sigma_{-i}(\nu_{-i}; 1) < 1$, $\pi_{-i}^A = 0$ by Bayes's rule: contradiction.

3. $0 < \sigma_{-i}(\nu_{-i}; a) < 1$. Then Eq. (3.23) is applicable to player $-i$. Thus, either $\pi_{-i}^R < \theta < \pi_{-i}^A$ or $\pi_{-i}^R > \theta > \pi_{-i}^A$. Suppose $\pi_{-i}^R < \theta < \pi_{-i}^A$. Then Eq. (B.1) implies $\Delta_{-i}(1) < 0$ and $\Delta_{-i}(a) > 0$; hence $\sigma_{-i}(\nu_{-i}; a) = 1$ and $\sigma_{-i}(\nu_{-i}; 1) = 0$ (Ineqs. 3.15 and 3.16). But that is impossible according to the proved assertion in the cell of Row 3 and Column 1, with $-i$ playing the role of i in the table. Thus consider the only possibility, $\pi_{-i}^R > \theta > \pi_{-i}^A$. Then Eq. (B.1) implies $\Delta_{-i}(1) > 0$ and $\Delta_{-i}(a) < 0$; hence $\sigma_{-i}(\nu_{-i}; a) = 0$ and $\sigma_{-i}(\nu_{-i}; 1) = 1$ (Ineqs. 3.15 and 3.16). But that implies, $\pi_{-i}^R = 0$ and $\pi_{-i}^A = 1$, contradicting the condition $\pi_{-i}^R > \theta > \pi_{-i}^A$ assumed throughout this subcase.

All possible cases considered, I have derived a contradiction and proved $\pi_{-i} = \theta$. It follows that, in the conflict stage, which occurs for sure because $\sigma_i(\nu_i; a) = \sigma_i(\nu_i; 1) = 1$, the posteriors are $\pi_i = \pi_{-i} = \theta$. Hence it always conflict, asserted in the last cell of the table.

Lemma B.2 *In any fully participating PBE that is not always-conflict, for any $i \in \{1, 2\}$, it is impossible to have:*
(i) $\sigma_{-i}(\nu_{-i}; a) = \sigma_{-i}(\nu_{-i}; 1) = 0$, $\sigma_i(\nu_i; a) = 0$, *and* $0 < \sigma_i(\nu_i; 1) < 1$.
(ii) $\sigma_{-i}(\nu_{-i}; a) = \sigma_{-i}(\nu_{-i}; 1) = 0$, $0 < \sigma_i(\nu_i; a) < 1$, *and* $0 < \sigma_i(1) < 1$.

Proof To prove Case-(i) note that by Eqs. (3.3)-(3.6), $\sigma_i(\nu_i; a) = 0$ and $0 < \sigma_i(\nu_i; 1) < 1$ implies $\pi_i^R = 0$ and $\pi_i^A > \theta$. Analogously, $\sigma_{-i}(\nu_{-i}; a) = \sigma_{-i}(\nu_{-i}; 1) = 0$ implies $\pi_{-i}^A = \theta$, $q_{-i}^A = 1$, and off-path posterior belief π_{-i}^R is arbitrary and $\pi_{-i}^R \in [0, 1]$. Hence, by Eqs. (3.14) and Ineqs. (3.15) and (3.16),

$$0 < \sigma_i(\nu_i; 1) < 1 \Rightarrow \Delta_i(1) = 0 \iff x_i = \pi_{-i}^A = \theta,$$

$$\sigma_{-i}(\nu_{-i}; 1) = 0 \Rightarrow \Delta_{-i}(1) \leq 0 \iff q_i^A \max\left\{\pi_{-i}^R, \pi_i^A\right\} + q_i^R \pi_{-i}^R \leq q_i^A x_{-i} + q_i^R \pi_{-i}^A, \Rightarrow x_{-i} \geq \theta$$

where the last inequality in the above displayed set of equations can be readily verified for the two possible and exhaustive cases of $\pi_{-i}^R \leq \pi_i^A$ and $\pi_{-i}^R \geq \pi_i^A$. For both of these cases the term $q_i^A \max\left\{\pi_{-i}^R, \pi_i^A\right\} + q_i^R \pi_{-i}^R > q_i^A \pi_i^A = \theta$, where $q_i^A \pi_i^A = \theta$ is by by Eqs. (3.3)-(3.6), $\sigma_i(\nu_i; a) = 0$ and $0 < \sigma_i(\nu_i; 1) < 1$. Also, $\pi_{-i}^A = \theta$ by $\Delta_i(1) = 0$. Thus, one can verify $\Delta_{-i}(1) \leq 0$ implies $x_{-i} \geq \theta$. Hence, by the above displayed equations any Case-(i) PBE of this lemma should satisfy $x_i + x_{-i} = M \geq 2\theta$, which violates condition (3.22).

To prove Case-(ii) note that by Eqs. (3.3)-(3.6), $\pi_{-i}^A = \theta, q_{-i}^A = 1$, $\pi_i^R > 0$, and $\pi_i^A > 0$. Moreover, $0 < \sigma_i(\nu_i; a) < 1 \Rightarrow \Delta_i(a) = 0$ and $0 < \sigma_i(\nu_i; 1) < 1 \Rightarrow \Delta_i(1) = 0$. Hence, Eqs. (3.14) can be simplified further to $\max\left\{\pi_i^R, \pi_{-i}^A\right\} - x_i = 0$ and $a\left(\pi_{-i}^A - \pi_i^R\right)^+ - x_i = 0$. Consider the two exhaustive cases of $\pi_{-i}^A = \theta \geq \pi_i^R$ or $\pi_i^R \geq \theta = \pi_{-i}^A$. If $\pi_{-i}^A = \theta \geq \pi_i^R$, then the set of equations simplifies to $x_i = \theta$ and $x_i = a(\theta - \pi_i^R)$, which is impossible due to $\pi_i^R > 0$. Otherwise, if $\pi_{-i}^A = \theta \leq \pi_i^R$, then the set of equations simplifies $x_i = \pi_i^R$ and $x_i = 0$, which is impossible due to $\pi_i^R > 0$. Hence, the impossibility of this class of PBE. ■

Lemma B.3 *Any fully participating PBE that has $\sigma_i(\nu_i; 1) = 1$ for player i admits lower ex-ante probability of peace compared to a PBE where $\sigma_i(\nu_i; a) = 0, \sigma_i(\nu_i; 1) = 1$, and $q_{-i}^A = 1$.*

Proof By Eq. (3.3) at any fully participating PBE where $\sigma_i(\nu_i; 1) = 1$ and $0 \leq \sigma_i(\nu_i; a) \leq 1$ the ex-ante probability of accept is $q_i^A = \theta\left(1 - \sigma_i(\nu_i; a)\right) \leq \theta$. Hence, the ex-ante probability of peace, i.e., $q_i^A q_{-i}^A$, at any such PBE satisfies $q_i^A q_{-i}^A \leq \theta q_{-i}^A$. This ex-ante probability of peace is lower than one admitted by a fully participating PBE where $q_{-i}^A = 1$ and $\sigma_i(\nu_i; a) = 0,\ \sigma_i(\nu_i; 1) = 1$ rendering $q_i^A = \theta$. ■

Thus, by Lemmas B.1-B.3, there are at most four kinds of fully participating PBEs that are not always-conflict and not sub-optimal:

(i) For some $i \in \{1, 2\}$, $\sigma_{-i}(\nu_{-i}; a) = \sigma_{-i}(\nu_{-i}; 1) = 0 = \sigma_i(a) < 1 = \sigma_i(\nu_i; 1)$. Call this lopsided.

(ii) For each $i \in \{1, 2\}$, $\sigma_i(\nu_i; a) = 0$ and $\sigma_i(\nu_i; 1) \in (0, 1)$. Call this mutually partially mixed (MPM).

(iii) For some $i \in \{1, 2\}$, $\sigma_{-i}(\nu_{-i}; a) = 0$, $\sigma_{-i}(\nu_{-i}; 1) \in (0, 1)$, $\sigma_i(\nu_i; a) \in (0, 1)$, and $\sigma_i(\nu_i; 1) \in (0, 1)$. Call this hybrid.

(iv) For each $i \in \{1, 2\}$, $\sigma_i(\nu_i; a) \in (0, 1)$, and $\sigma_i(\nu_i; 1) \in (0, 1)$. Call this mutually totally mixed (MTM).

These results coupled with the fact peace cannot be guaranteed due to Ineq. (3.22) are summarized in the Table B.1, where I use the notations $\sigma_i(t_i)$ instead of $\sigma_i(\nu_i; t_i)$.

	$\sigma_{-i}(a) = 0$ $\sigma_{-i}(1) = 0$	$\sigma_{-i}(a) = 0$ $0 < \sigma_{-i}(1) < 1$	$\sigma_{-i}(a) = 0$ $\sigma_{-i}(1) = 1$	σ_{-i} is totally mixed	$0 < \sigma_{-i}(a) < 1$ $\sigma_{-i}(1) = 1$
$\sigma_i(a) = \sigma_i(1) = 0$	impossible				
$\sigma_i(a) = 0 < \sigma_i(1) < 1$	impossible	MPM			
$\sigma_i(a) = 0, \sigma_i(1) = 1$	Lopsided	suboptimal	suboptimal		
σ_i is totally mixed	impossible	Hybrid	suboptimal	MTM	
$0 < \sigma_i(a) < 1 = \sigma_i(1)$	suboptimal	suboptimal	suboptimal	suboptimal	suboptimal

Table B.1: All possible fully participating PBEs that are not always-conflict

Lemma B.4 *In any fully participating PBE that is not always-conflict, if $0 < q_i^R < 1$ for some player $i \in \{1, 2\}$, then mediation strategies are monotone in type, i.e., $\sigma_i\left(\nu_i; a\right) < \sigma_i\left(\nu_i; 1\right)$.*

Proof Table B.1 summarizes results of Lemmas B.1 and B.2 and provides all possible cases of PBEs that are not always-conflict. The empty cells are the symmetric cases of the filled ones. By this table, it suffices to prove the claim of the lemma for the MTM and Hybrid PBEs because in all other possible PBEs, by definition, the strategies are monotone in type.

First, at Hybrid PBEs, for each $i \in \{1,2\}$, $\sigma_{-i}(\nu_{-i};a) = 0$, $0 < \sigma_{-i}(\nu_{-i};1) < 1$, $0 < \sigma_i(\nu_i;a) < 1$, and $0 < \sigma_i(\nu_i;1) < 1$. By Eqs. (3.5) and (3.6), $\pi^R_{-i} = 0$ and $\pi^A_{-i} > \theta$. Moreover, $0 < \sigma_i(\nu_i;a) < 1$ implies that $\Delta_i(a) = 0$, where by Eq. (3.14), $\Delta_i(a) = q^A_{-i}\left(a\left(\pi^A_{-i} - \pi^R_i\right)^+ - x_i\right)$. By the definition of Hybrid PBEs $0 < q^A_{-i} < 1$, then $\Delta_i(a) = 0$ is equivalent to either: (i) $\pi^R_i < \pi^A_{-i}$ and $\pi^A_{-i} = \pi^R_i + x_i/a$ or (ii) $\pi^R_i \geq \pi^A_{-i}$ and $x_i = 0$. (ii) is impossible. Suppose it holds. Note that by plugging $x_i = 0$, $\pi^A_{-i} \leq \pi^R_i$, and $\pi^R_{-i} = 0$ in Eq. (3.14), one obtains $\Delta_i(1) = \pi^R_i - q^R_{-i}\pi^A_i$. Then, $0 < \sigma_i(\nu_i;1) < 1$ implies that $\Delta_i(1) = 0$, which coupled with the fact that $0 < q^R_{-i} = 1 - q^A_{-i} < 1$ implies that $\pi^R_i < \pi^A_i$, a contradiction with the assumption that I begin with. Hence, $\pi^R_i < \pi^A_{-i}$. Thus, by Eq. (3.23), $\sigma_i\left(\nu_i;a\right) < \sigma_i\left(\nu_i;1\right)$, as desired.

Second, at MTM PBEs, $0 < \sigma_i(\nu_i;a) < 1$, and $0 < \sigma_i(\nu_i;1) < 1$ for each $i \in \{1,2\}$. Hence, by Ineqs. (3.15) and (3.16), $\Delta_i(1) = \Delta_i(a) = 0$. Proof is by contradiction. Consider the two exhaustive cases. Case (i): both players follow non-monotone strategies; case (ii): player i follows non-monotone strategies and her opponent $-i$ follows monotone one.

Case (i): both players follow non-monotone strategies. Then, by implication of Eq. (3.23), $\pi^R_i \geq \theta \geq \pi^A_i$ for each i. Since the defining conditions of MTM PBEs are symmetric between the two players, let us assume, without loss of generality, that $\pi^R_i \geq \pi^R_{-i}$. Plug these posteriors in Eq. (3.14) to conclude that $\Delta_i(a) = 0 \iff 0 = q^A_{-i}x_i + q^R_{-i}a\left(\pi^R_{-i} - \pi^A_i\right)$, which by $0 < q^R_i < 1$ it can hold only if $x_i = 0$ and $\pi^R_{-i} - \pi^A_i$. Plug $\pi^R_i \geq \theta \geq \pi^A_i$ and $x_i = 0$ to Eq. (3.14) to observe $\Delta_i(1) = 0 \iff \pi^R_i = q^R_{-i}\pi^R_{-i}$, which is impossible by $\pi^R_i \geq \pi^R_{-i}$ and $0 < q^R_{-i} < 1$.

Case (ii): player i follows non-monotone strategies and her opponent $-i$ follows monotone one. Then, by implication of Eq. (3.23), $\pi^R_i \geq \theta \geq \pi^A_i$ and $\pi^R_{-i} < \theta < \pi^A_{-i}$. There will be different subcases. First, suppose that $\pi^A_{-i} < \pi^R_i$. Then, plug in these posteriors in Eq. (3.14) to conclude that $\Delta_i(a) = 0 \iff 0 = q^A_{-i}x_i + q^R_{-i}a\left(\pi^R_{-i} - \pi^A_i\right)$, which by $0 < q^R_i < 1$ it can only hold if $x_i = 0$ and $\pi^R_{-i} \leq \pi^A_i$. Plug in these posteriors in Eq. (3.14) to conclude that $\Delta_i(1) = 0 \iff q^A_{-i}\pi^A_{-i} = q^R_{-i}\left(\pi^A_i - \pi^R_i\right)$, which by $0 < q^A_i = 1 - q^R_i < 1$ and $\pi^R_i \geq \theta \geq \pi^A_i > 0$ is impossible. Second, suppose that $\pi^A_{-i} \geq \pi^R_i$ and $\pi^A_i \geq \pi^R_{-i}$. Then, plug these posteriors into Eq. (3.14) to conclude that $\Delta_i(1) - \Delta_i(a) = 0 \iff q^A_{-i}\left(\pi^A_{-i} - a\left(\pi^A_{-i} - \pi^R_i\right)\right) = q^R_{-i}\left(\pi^A_i - \pi^R_i\right)$, which is impossible because the left hand side of this equation is strictly positive and the right hand side is strictly negative by the assumption made at the top of this case. Lastly, suppose that $\pi^A_{-i} \geq \pi^R_i$ and $\pi^R_{-i} > \pi^A_i$. Then, plug in these posteriors into Eq. (3.14) to find that $0 = \Delta_i(1) - \Delta_i(a) = q^A_{-i}\left(\pi^A_{-i} - a\left(\pi^A_{-i} - \pi^R_i\right)\right) + q^R_{-i}\left(\pi^R_i - \pi^R_{-i}\right) + q^R_{-i}a\left(\pi^R_{-i} - \pi^A_i\right) > 0$. In brief, all possible subcases led to contradiction. Thus, Case (ii) is not possible either. ■

B.2 Characterization of Equilibria

B.2.1 Hybrid and MTM PBEs

Lemma B.5 *If $M \geq \theta + a$ then there does not exist any proposal that admits Hybrid PBEs.*

Proof At any Hybrid PBEs, for each $i \in \{1,2\}$, $\sigma_{-i}(\nu_{-i};a) = 0$, $0 < \sigma_{-i}(\nu_{-i};1) < 1$, $0 < \sigma_i(\nu_i;a) < 1$, and $0 < \sigma_i(\nu_i;1) < 1$. By Eqs. (3.5) and (3.6), $\pi_{-i}^R = 0$ and $\pi_{-i}^A > \theta$. The strategies are monotone (Lemma B.4) and by Ineq. (3.23), $\pi_i^R < \pi_{-i}^A$. By $0 < \sigma_i(\nu_i;a) < 1 \Rightarrow \Delta_i(a) = 0$, plug these posteriors into Eq. (3.14) to find that $\Delta_i(a) = 0$ is equivalent to $x_i = a\left(\pi_{-i}^A - \pi_i^R\right)$ which implies $x_i < a$. Analogously, $0 < \sigma_{-i}(\nu_{-i};1) < 1 \Rightarrow \Delta_{-i}(1) = 0$ which by Eq. (3.14) is

$$\Delta_{-i}(1) = 0 \iff \underbrace{q_i^A\pi_i^A + q_i^R\pi_i^R}_{=\theta \text{ By eqs. (3.5) and (3.6)}} - q_i^A x_{-i} - q_i^R\pi_{-i}^A = 0 \iff \theta = q_i^A x_{-i} + q_i^R\pi_{-i}^A,$$

The above set of displayed equations coupled with $1 > \pi_{-i}^A > \theta$, imply that $\theta = q_i^A x_{-i} + q_i^R\pi_{-i}^A > q_i^A x_{-i} + q_i^R\theta \Rightarrow x_{-i} < \theta$. Hence, one can conclude by $\Delta_i(a) = 0$ and $\Delta_{-i}(1) = 0$ that at any Hybrid PBEs $M = x_i + x_{-i} < \theta + a$. Hence, if $M \geq \theta + a$, then there does not exist any proposal that admits Hybrid PBEs. ■

Lemma B.6 *If $M \geq \theta + a$, then there does not exist any proposal that admits MTM PBEs.*

Proof The proof is analogous to that of Lemma B.5. At MTM PBEs, $0 < \sigma_i(\nu_i;a) < 1$, and $0 < \sigma_i(\nu_i;1) < 1$ for each $i \in \{1,2\}$. Hence, by Ineqs. (3.15) and (3.16), $\Delta_i(1) = \Delta_i(a) = 0$. Since the defining condition of this PBE is symmetric between the two players, let us assume, without loss of generality, that $\pi_i^R \geq \pi_{-i}^R$. The strategies are monotone (Lemma B.4) and by Ineq. (3.23), $\pi_i^A > \theta > \pi_i^R$ for each player. By Eq. (3.15) and (3.16), $0 < \sigma_i(w) < 1 \Rightarrow \Delta_i(a) = 0$ and $0 < \sigma_{-i}(1) < 1 \Rightarrow \Delta_{-i}(1) = 0$. Plug these posteriors into Eq. (3.14) to find that $\Delta_i(a) = 0 \iff x_i = a\left(\pi_{-i}^A - \pi_i^R\right) \Rightarrow x_i < a$ and $\Delta_{-i}(1) = 0 \iff \theta = q_i^A x_{-i} + q_i^R\pi_{-i}^A \Rightarrow x_{-i} < \theta$, where the last inequality is by $\pi_{-i}^A > \theta > \pi_i^R$. Hence, at any MTM PBEs $M = x_i + x_{-i} < \theta + a$. Thus, if $M \geq \theta + a$, then there does not exist any proposal that admits MTM PBEs. ■

B.2.2 Lopsided PBEs

Lemma B.7 *Suppose $a < \theta$. Conditional on participation, a peace proposal (x_i, x_{-i}), where $x_i < x_{-i}$ for some $i \in \{1,2\}$, admits Lopsided PBEs if and only if for some off-path belief $\tilde{\pi}_{-i}^R \in [0,1]$,*

$$a\theta \leq x_i \leq M - \left(\theta + \frac{(1-\theta)}{\theta}\tilde{\pi}_{-i}^R\right). \tag{B.2}$$

Such a peace proposal exists if and only if $\theta(1+a) + \frac{(1-\theta)}{\theta}\tilde{\pi}_{-i}^R \leq M < 2\theta$ and $\left(\theta, a, \tilde{\pi}_{-i}^R\right)$ satisfies either: (i) $\theta \geq \frac{-1+\sqrt{5-4a}}{2(1-a)}$, and $\tilde{\pi}_{-i}^R \in [0,1]$ or (ii) $a < \theta \leq \frac{-1+\sqrt{5-4a}}{2(1-a)}$ and $\tilde{\pi}_{-i}^R \in \left[0, \frac{\theta^2(1-a)}{1-\theta}\right]$.

Proof In any Lopsided PBE, conditional on participation and given peace proposal (x_i, x_{-i}), strategies are $\sigma_i(\nu_i;a) = 0$, $\sigma_i(\nu_i;1) = 1$, and $\sigma_{-i}(\nu_{-i};a) = \sigma_{-i}(\nu_{-i};1) = 0$. Thus, by Eqs. (3.3)-(3.6), $\pi_i^A = 1$, $\pi_i^R = 0$, $q_i^A = \theta$, $\pi_{-i}^A = \theta$, $q_{-i}^A = 1$, and and $\tilde{\pi}_{-i}^R$ is the off-path posterior belief and hence arbitrary. Couple these observations with Eq. (3.14) and Ineqs. (3.15) and (3.16) to obtain the best response conditions for these strategies to constitute a PBE for each player:

$$\Delta_i(a) \leq 0 \iff x_i \geq a\theta, \quad \Delta_i(1) \geq 0 \iff x_i \leq \theta, \tag{B.3}$$

$$\Delta_{-i}(a) \leq 0 \iff x_{-i} \geq a\left(1 - \tilde{\pi}_{-i}^R\right), \quad \Delta_{-i}(1) \leq 0 \iff x_{-i} \geq \theta + \frac{(1-\theta)}{\theta}\tilde{\pi}_{-i}^R. \tag{B.4}$$

Therefore, by $\Delta_i(1) \geq 0$ and $\Delta_{-i}(1) \leq 0$ it is necessary that

$$x_i < \theta < x_{-i}, \tag{B.5}$$

which coupled with $M < 2\theta$, by Ineq. (3.22), implies that $x_i < \frac{M}{2} < x_{-i}$. Moreover, by the assumption $\theta > a$ it is immediate that $\Delta_{-i}(a) \leq 0$ displayed above is redundant because it is implied by $\Delta_{-i}(1) \leq 0$. Thus, by Ineqs. (B.3), (B.4), (B.5), and $M < 2\theta$, a peace proposal (x_i, x_{-i}) admits a Lopsided PBE at the continuation game of mediation if and only if $a\theta \leq x_i \leq M - \left(\theta + \frac{(1-\theta)}{\theta}\tilde{\pi}^R_{-i}\right)$, as noted in the lemma.

Thus, conditional on participation, such a proposal that admits Lopsided PBEs exists if and only if there exists M that satisfies $\theta(1+a) + \frac{(1-\theta)}{\theta}\tilde{\pi}^R_{-i} \leq M$ and simultaneously satisfies $M < 2\theta$, where the latter inequality is by the condition (3.22). To ensure these conditions are simultaneously satisfied, pick the arbitrary off-path belief $\tilde{\pi}^R_{-i} \in [0,1]$ such that $\theta(1+a) + \frac{(1-\theta)}{\theta}\tilde{\pi}^R_{-i} < 2\theta \iff \tilde{\pi}^R_{-i} < \frac{\theta^2(1-a)}{1-\theta}$. Hence, the off path belief should be in $0 \leq \tilde{\pi}^R_{-i} \leq \min\left\{1, \frac{\theta^2(1-a)}{1-\theta}\right\}$. By $a \in (0,1)$ and $\theta > a$, one can readily verify that $\frac{\theta^2(1-a)}{1-\theta} \leq 1 \iff 0 < \theta \leq \frac{-1+\sqrt{5-4a}}{2(1-a)}$. One can readily verify that $\frac{-1+\sqrt{5-4a}}{2(1-a)} > a$ because:

$$\begin{aligned}\frac{-1+\sqrt{5-4a}}{2(1-a)} > a &\iff \sqrt{5-4a} > 1 + 2a(1-a)\\ &\iff 5-4a > (1-2a(1-a))^2 \iff 5-4a-(1-2a(1-a))^2 > 0\\ &\iff 4(1+a)(1-a)^3 > 0,\end{aligned}$$

where the last inequality in the above displayed set of inequalities always hold by $a \in (0,1)$. Thus, one can conclude the two subcases on $\left(\theta, a, \tilde{\pi}^R_{-i}\right)$ in the statement of this lemma. ■

Corollary B.1 *Lopsided proposals (x_i, x_{-i}) such that $x_i < x_{-i}$ admit Lopsided PBEs where $\pi^A_i = 1$, $\pi^R_i = 0$, $q^A_i = \theta$, $\pi^A_{-i} = \theta$, $q^A_{-i} = 1$, and the off-path posterior $\tilde{\pi}^R_{-i}$ can be any element of nonempty intervals characterized in Lemma B.7. Moreover, the ex-ante probability of conflict is $P(x_{-i}) = 1-\theta$.*

Proof The proof is an immediate implication of the definition of Lopsided PBEs where given (x_i, x_{-i}) then $\sigma_i(\nu_i; a) = 0, \sigma_i(\nu_i; 1) = 1, \sigma_{-i}(\nu_{-i}; a) = \sigma_{-i}(\nu_{-i}; 1) = 0$. Hence, by Eqs. (3.3)-(3.6) the posteriors and q^A_i for each i can be readily characterized. The off-path posterior intervals are given by Lemma B.7. Then, $P(x_{-i}) = 1 - q^A_i q^A_{-i} = 1-\theta$. ■

B.2.3 MPM PBEs

Lemma B.8 *Conditional on participation, a peace proposal (x_i, x_{-i}) admits an MPM PBE if and only if $x_i = x_{-i} = \frac{M}{2}$ and $\max\left\{2a\theta(2-a), \frac{2(2\theta-1)}{\theta}\right\} < M < 2\theta$. Furthermore, given an equal proposal,*

$$\sigma(\nu_i; a) = 0,\ \sigma(\nu_i; 1) = \frac{x-\theta+\sqrt{\theta^2 - x\theta}}{(1-\theta)x},\ \pi^R = 0, \pi^A > \theta,\ \text{and } q^A = \frac{\theta - \sqrt{\theta^2 - x\theta}}{x}. \quad \text{(B.6)}$$

Proof In any MPM PBEs, conditional on participation and given peace proposal (x_i, x_{-i}), the strategies are $\sigma_i(\nu_i; a) = 0$ and $0 < \sigma_i(\nu_i; 1) < 1$ for each $i \in \{1,2\}$. Hence, by Eqs. (3.5)-(3.6), $\pi^R_i = \pi^R_{-i} = 0$, $\pi^A_i > \theta$, $\pi^A_{-i} > \theta$, and $q^A_i \pi^A_i = \theta$ for each $i \in \{1,2\}$. Plug these observations into Eq. (3.14) coupled with Ineqs. (3.15) and (3.16) to obtain the best response conditions for these strategies to constitute a PBE for each player:

$$\sigma_i(\nu_i; a) = 0 \Rightarrow \Delta_i(a) \leq 0 \iff a\pi^A_{-i} \leq x_i, \quad \text{(B.7)}$$

$$0 < \sigma_i(\nu_i; 1) < 1 \Rightarrow \Delta_i(1) = 0 \iff \theta = q^A_{-i} x_i + q^R_{-i} \pi^A_i \quad \text{(B.8)}$$

Plug $q_{-i}^A = \frac{\theta}{\pi_{-i}^A}$ in Eq. (B.8) to get $\pi_{-i}^A = \theta\left(\frac{\pi_i^A - x_i}{\pi_i^A - \theta}\right)$. Plug π_{-i}^A in Eq. (B.8), with roles of i and $-i$ reversed, coupled with $q_i^A \pi_i^A = \theta$ to verify:

$$\begin{aligned}\theta = q_i^A x_{-i} + q_i^R \theta\left(\frac{\pi_i^A - x_i}{\pi_i^A - \theta}\right) &\iff \theta\left(\pi_i^A - \theta\right) = q_i^A x_{-i}\left(\pi_i^A - \theta\right) + q_i^R \theta\left(\pi_i^A - x_i\right) \\ &\iff \theta\left(\pi_i^A - \theta\right) = \theta x_{-i} - q_i^A x_{-i}\theta + (1 - q_i^A)\theta\left(\pi_i^A\right) - q_i^R \theta x_i \\ &\iff \theta\left(\pi_i^A - \theta\right) = \theta x_{-i} - q_i^A x_{-i}\theta + \theta\pi_i^A - \theta^2 - q_i^R \theta x_i \\ &\iff 0 = q_i^R \theta\left(x_{-i} - x_i\right).\end{aligned}$$

Note that by $0 < \sigma_i(\nu_i; 1) < 1$ then $0 < q_i^R < 1$. Hence, the latter equation holds if and only if $x_i = x_{-i} = \frac{M}{2}$. Thus, the equilibrium is symmetric and I drop the subscript i. Thus, one can solve for q^A by $\theta = q^A x + q^R \pi^A \iff 2\theta = q^A x + \pi^A = q^A x + \frac{\theta}{q^A}$ which is equivalent to $q^A = \frac{\theta \pm \sqrt{\theta^2 - x\theta}}{x}$. To have a well defined q^A, it is necessary that $x < \theta$ which is satisfied by $x = \frac{M}{2} < \theta$. This observation also implies that $\frac{\theta + \sqrt{\theta^2 - x\theta}}{x} > 1$. Hence, $q^A = \frac{\theta - \sqrt{\theta^2 - x\theta}}{x}$. Note that $q^A = \theta + (1-\theta)(1 - \sigma(\nu_i; 1))$. Hence, $\sigma(\nu_i; 1) = \frac{x - \theta + \sqrt{\theta^2 - x\theta}}{(1-\theta)x}$, where

$$0 < \sigma(\nu_i; 1) < 1 \iff \frac{2\theta - 1}{\theta} < x = \frac{M}{2} < \theta \tag{B.9}$$

By (B.7), a peace proposal admits MPM if $a\pi^A \leq x = \frac{M}{2}$. Plugging in for $\pi^A = \frac{\theta}{q^A}$:

$$a\pi^A \leq x \iff aq^A\pi^A \leq xq^A \iff a\theta \leq q^A x \tag{B.10}$$

Plugging in for q^A by Eq. (B.6) in the above displayed inequality one can obtain

$$a\theta \leq q^A x \iff a\theta \leq x\frac{\theta - \sqrt{\theta^2 - x\theta}}{x} \iff a\theta(2-a) \leq x. \tag{B.11}$$

In brief, an equal proposal admits a MPM PBE if and only if Ineqs. (B.11) and (B.9). Hence, given $x = \frac{M}{2}$, these necessary and sufficient conditions are $\max\left\{a\theta(2-a), \frac{2\theta-1}{\theta}\right\} < \frac{M}{2} < \theta$. ■

Corollary B.2 *Conditional on participation, MPM PBE exists if and only if either of the following hold: (i)* $\theta \geq \frac{1}{2-a}$ *and* $\frac{2(2\theta-1)}{\theta} < M < 2\theta$ *or (ii)* $\theta \leq \frac{1}{2-a}$ *and* $2a\theta(2-a) < M < 2\theta$.

Proof Proof is an immediate implication of Lemma B.8. An equal peace proposal admits MPM PBE if and only if $\max\left\{2a\theta(2-a), \frac{2(2\theta-1)}{\theta}\right\} < M < 2\theta$. Observe that $a\theta(2-a) \leq \frac{2\theta-1}{\theta} \iff \theta \geq \frac{1}{2-a}$ as stated in statement (i) of the Corollary. Statement (ii) is the complementary case of (i). It is easy to verify that the stated interval is not vacuous. ■

Lemma B.9 *Suppose Ineq. (3.21). Conditional on participation, MPM PBEs exists if and only if* (M, θ, a) *satisfies either of the following cases: (i)* $\theta < \frac{4-a-\sqrt{a^2-8a+8}}{2}$ *and* $\theta + a \leq M$ *or (ii)* $\theta \geq \frac{4-a-\sqrt{a^2-8a+8}}{2}$ *and* $\frac{2(2\theta-1)}{\theta} < M$.

Proof Ineq. (3.21) states that $M \geq \theta + a$ and $\theta > a$. Case (i) in the statement of Lemma B.9 is a direct implication of Corollary B.2, the upper branch of Claim B.1, coupled with easy to verify facts that $a < \frac{1}{2-a}$ and $\frac{1}{2-a} < \frac{4-a-\sqrt{a^2-8a+8}}{2} \iff 4(1-a)^2 > 0$:

$$\begin{aligned}
\frac{1}{2-a} < \frac{4-a-\sqrt{a^2-8a+8}}{2} &\iff (2-a)\sqrt{a^2-8a+8} < \underbrace{(4-a)(2-a)-2}_{>0 \text{ by } a\in(0,1)} \\
&\iff \left((2-a)\sqrt{a^2-8a+8}\right)^2 < \left((4-a)(2-a)-2\right)^2, \\
&\iff 4(1-a)^2 > 0.
\end{aligned}$$

Analogously, Case (ii) in the statement of of Lemma B.9 is a direct implication of Corollary B.2, the lower branch of Claim B.1, and the above mentioned fact that $a < \frac{1}{2-a} < \frac{4-a-\sqrt{a^2-8a+8}}{2}$ for all $a \in (0,1)$. ■

Claim B.1 *Suppose $a < \theta$. Then*

$$\max\left\{\theta + a, \frac{2(2\theta-1)}{\theta}, 2a\theta(2-a)\right\} = \begin{cases} \theta + a & \text{iff } \theta \leq \frac{4-a-\sqrt{a^2-8a+8}}{2}, \\ \frac{2(2\theta-1)}{\theta} & \text{iff } \theta \geq \frac{4-a-\sqrt{a^2-8a+8}}{2}. \end{cases}$$

Proof First note that,

$$\frac{2(2\theta-1)}{\theta} \leq 2a\theta(2-a) \iff \theta \leq \frac{1}{2-a}. \tag{B.12}$$

Second, $\theta + a \leq 2a\theta(2-a) \iff \frac{a}{2a(2-a)-1} \leq \theta$. Whenever this condition is binding it is implied by Ineq. (B.12) due to easily verifiable fact that $\frac{a}{2a(2-a)-1} \geq \frac{1}{2-a}$ for all $a \in (0,1)$. Hence, there does not exist (θ, a) such that $a < \theta < 1$ and $\max\left\{\theta + a, \frac{2(2\theta-1)}{\theta}, 2a\theta(2-a)\right\} = 2a\theta(2-a)$. Therefore, it suffices to verify (θ, a) such that the outcome of the max operator is either $\theta + a$ or $\frac{2(2\theta-1)}{\theta}$. Note that $\theta + a \leq \frac{2(2\theta-1)}{\theta} \iff \frac{4-a-\sqrt{a^2-8a+8}}{2} \leq \theta \leq 1$ where the equivalence is due to the $a \in (0,1)$, $a^2 - 8a + 8 > 0$, $\frac{4-a+\sqrt{a^2-8a+8}}{2} > 1$, and $a < \frac{4-a-\sqrt{a^2-8a+8}}{2} < 1$ (as shown in the proof of Lemma B.9). Hence, for any $a \in (0,1)$ and $\theta > a$,

$$\theta + a \leq \frac{2(2\theta-1)}{\theta} \iff \frac{4-a-\sqrt{a^2-8a+8}}{2} \leq \theta < 1. \quad ■ \tag{B.13}$$

B.3 Proof of Proposition 3.1

Conditional on participation in the mediation, as shown in Appendix B.1, and summarized in the Table B.1, there are at most four kinds of fully participating PBEs that are not always-conflict and not suboptimal. For each class of these PBEs, use Ineqs. (3.15) and (3.16) and definitions of each class to uniquely determines the strategy profile $(\sigma_i)_{i=1}^{i=2}$, on-path posterior beliefs $\left(\pi_i^A, \pi_i^R\right)_{i=1}^{i=2}$ by Bayes's rule whenever it applies, and, if needed, off-path posterior beliefs that rationalizes equilibrium. Moreover, use the aforementioned system of inequalities and definition of each class of PBE to find $x_{\min} = \min\{x_1, x_2\} \in [0, M]$ such that peace proposals $(M - x_{\min}, x_{\min})$ admits such PBEs. By Lemmas B.5 and B.6, if $\theta + a \leq M$ then two set of fully participating PBEs from the four previously mentioned ones are empty and only lopsided and MPM PBEs can possibly exist. ■

B.4 Conflict Minimization Details

Lemma B.10 *At MPM PBEs, ex-ante probability of Accept q^A, is strictly increasing in x.*

Proof By Eq. (B.6) $\frac{d}{dx}q^A = \frac{d}{dx}\left[\frac{\theta-\sqrt{\theta^2-x\theta}}{x}\right] = \frac{2\theta^2-x\theta-2\theta\sqrt{\theta^2-x\theta}}{2x^2\sqrt{\theta^2-x\theta}}$. By Lemma B.8 equal proposal $x=\frac{M}{2}$ admit MPM PBEs. By Ineq. (3.22), $x=\frac{M}{2}<\theta$. Hence, to verify $\frac{d}{dx}q^A>0$, it suffices to verify: $2\theta^2-x\theta-2\theta\sqrt{\theta^2-x\theta}>0 \iff \left(2\theta^2-x\theta\right)^2>4\theta^2\left(\theta^2-x\theta\right) \iff \theta^2x^2>0$, where $\theta^2x^2>0$ is always true by $\theta>0$ and $x>0$. ■

Lemma B.11 *Assume Ineq. (3.21). Suppose (M,θ,a) are such that $x=\frac{M}{2}=\frac{\theta+a}{2}$ admits MPM PBE. Then, ex-ante probability of conflict $P\left(\frac{\theta+a}{2}\right)\le 1-\theta$ if and only if $\theta\le 6-a-4\sqrt{2-a}$.*

Proof By Lemma B.8, the equal proposal admits MPM PBE, which generates an ex-ante probability of Accept q^A. Thus, given the equal proposal, ex-ante probability of conflict is $P(x)=1-\left(q^A\right)^2$. Hence, $1-\left(q^A\right)^2\le 1-\theta$ if and only if $\left(q^A\right)^2\ge\theta$. By Eq. (B.6), given $x=\frac{M}{2}=\frac{\theta+a}{2}$, we have $q^A=\frac{2\theta-\sqrt{2\theta(\theta-a)}}{\theta+a}$. Therefore, it suffices to verify that $\left(q^A\Big|_{x=\frac{\theta+a}{2}}\right)^2\ge\theta$ which one can readily show it is equivalent to $(\theta+a)^2\left[\theta^2+a^2+2a\theta-12\theta+4a+4\right]\ge 0$. For the latter inequality to hold it suffices to find (θ,a) such that the term in the parenthesis is positive and satisfy $\theta>a$ of (3.21). This in turn is equivalent to $a\le\theta\le 6-a-4\sqrt{2-a}$. ■

Proposition B.1 *Suppose Ineq. (3.21) and that players fully participate in the mediation:*

a. If $a<\theta<6-a-4\sqrt{2-a}$ then the equal split is the unique conflict-minimizing proposal;

b. If $6-a-4\sqrt{2-a}\le\theta<\frac{4-a-\sqrt{a^2-8a+8}}{2}$ then for some $\tilde{\pi}^R_{-i}\in[0,a\theta]$ there exists a unique $M^*\in[a+\theta,2\theta)$ such that the conflict-minimizing proposals are the lopsided proposal if $M\le M^*$ and the equal proposal if $M\ge M^*$;

c. If $\frac{4-a-\sqrt{a^2-8a+8}}{2}\le\theta<1$ then for some $\tilde{\pi}^R_{-i}\in\left[0,\min\left\{\frac{2(2\theta-1)-\theta^2(1+a)}{1-\theta},1\right\}\right]$ there exists a unique $M^*\in\left[\frac{2(2\theta-1)}{\theta},2\theta\right)$ such that the conflict-minimizing proposals are the lopsided proposal if $M\le M^*$ and the equal proposal if $M\ge M^*$.

Proof Conditional on participation, by Ineq. (3.21) and Lemmas B.5, B.6 it is immediate that given peace proposals (x_i,x_{-i}), there exist only two classes of equilibria: MPM and Lopsided PBEs. The objective of the mediator is to minimize the ex-ante probability of conflict defined as $P(x_{\min})=1-q_i^Aq_{-i}^A$ or equivalently maximize the ex-ante probability of peace denoted by $\Upsilon^m(x_{\min})=q_i^Aq_{-i}^A$, where $m\in\{\text{MPM},\text{LOP}\}$ denotes the class of PBEs. By Corollary B.1 and Lemma B.8, the ex-ante probability of peace in these two PBEs are:

$$\Upsilon^{\text{lop}}(x_{\min})=\theta \quad\text{and}\quad \Upsilon^{\text{MPM}}(x_{\min})=\left(q^A\right)^2. \tag{B.14}$$

By Lemma B.11, $\Upsilon^{MPM}(\frac{\theta+a}{2})>\theta$, if and only if $\theta\le 6-a-4\sqrt{2-a}$. Since by implication of Lemma B.10, $\Upsilon^{MPM}(\frac{M}{2})$ is strictly increasing in M, it is immediate that if and only if $\theta\le 6-a-4\sqrt{2-a}$ then for all $M\ge\theta+a$, $\Upsilon^{MPM}>\theta=\Upsilon^{lop}$. Hence, to prove the first claim of

Proposition B.1, by Lemma B.9, it suffices to show that $6-a-4\sqrt{2-a} < \frac{4-a-\sqrt{a^2-8a+8}}{2}$. This can be readily verified by the fact that for all $a \in (0,1)$, $6-a-4\sqrt{2-a} < \frac{1}{2-a} < \frac{4-a-\sqrt{a^2-8a+8}}{2}$. The fact that $\frac{1}{2-a} < \frac{4-a-\sqrt{a^2-8a+8}}{2}$ for all $a \in (0,1)$ is established in the proof of Lemma B.9. One can also show that:

$$\frac{1}{2-a} > 6-a-4\sqrt{2-a} \iff \left(4(2-a)\sqrt{(2-a)}\right)^2 > \left((2-a)(6-a)-1\right)^2$$
$$\iff \left((2-a)(a+2)+3-2a\right)(1-a)^2 > 0$$

where the latter inequality always holds for all $a \in (0,1)$.

To prove Proposition B.1-(ii), I use the fact $6-a-4\sqrt{2-a} < \frac{4-a-\sqrt{a^2-8a+8}}{2}$ coupled with implication of Lemma B.11 that if $\theta > 6-a-4\sqrt{2-a}$, then $\Upsilon^{MPM}(\frac{\theta+a}{2}) < \theta$. Hence, at $M=\theta+a$, $\Upsilon^{MPM}(\frac{\theta+a}{2}) < \theta$ and at $M=2\theta$, $\Upsilon^{MPM}(\theta)=1$. The latter equality is due to condition 3.22. By implication of Lemma B.10, $\Upsilon^{MPM}(\frac{M}{2})$ is strictly increasing in M. Therefore, by Intermediate Value Theorem there exists a unique $x^* = \frac{M^*}{2}$ such that at $x=x^*=M^*/2$, $\Upsilon^{\mathrm{MPM}}(x^*) = \left(q^A\right)^2 = \theta$ and Υ^{MPM} intersects Υ^{lop} from below. By Lemma B.7 the lower bound of M for the Lopsided PBEs is $\theta(1+a)+\frac{1-\theta}{\theta}\tilde{\pi}^R_{-i}$. Hence, it suffices to find some $\tilde{\pi}^R_{-i}$ such that $\theta(1+a)+\frac{1-\theta}{\theta}\tilde{\pi}^R_{-i} \le \theta+a$, which holds if and only if $\tilde{\pi}^R_{-i} \le a\theta$.

To prove Proposition B.1-(iii) note that by Lemma B.9-(ii) if $\theta \ge \frac{4-a-\sqrt{a^2-8a+8}}{2}$ and $\frac{2(2\theta-1)}{\theta} \le M$, then there exist an equal proposal that admits MPM PBEs and satisfies Ineq. (3.21). Moreover, by Eq. (B.6), at the limit when $x=\frac{M}{2}=\frac{2\theta-1}{\theta}$, $q^A=\theta$. Therefore, at such values of M, $\Upsilon^{\mathrm{MPM}} = \theta^2 < \theta$. Moreover, at $M=2\theta$, $\Upsilon^{MPM}(\theta)=1$. By Lemma B.10, $\Upsilon^{MPM}(\frac{M}{2})$ is strictly increasing in M. Therefore, by Intermediate Value Theorem there exists a unique $x^*=\frac{M^*}{2}$ such that at $x=x^*$, $\Upsilon^{\mathrm{MPM}} = \left(q^A\right)^2 = \theta$ and Υ^{MPM} intersects Υ^{lop} from below. By Lemma B.7 the lower bound of M for the Lopsided PBEs is $\theta(1+a)+\frac{1-\theta}{\theta}\tilde{\pi}^R_{-i}$. Hence, it suffices to find some $\tilde{\pi}^R_{-i}$ such that $\theta(1+a)+\frac{1-\theta}{\theta}\tilde{\pi}^R_{-i} \le \frac{2(2\theta-1)}{\theta}$, or equivalently $\tilde{\pi}^R_{-i} \le \frac{2(2\theta-1)-\theta^2(1+a)}{1-\theta}$. Couple this with $\tilde{\pi}^R_{-i} \in [0,1]$ to conclude $\tilde{\pi}^R_{-i} \in \left[0, \min\left\{\frac{2(2\theta-1)-\theta^2(1+a)}{1-\theta}, 1\right\}\right]$. Lastly, by $\theta > a$ and $\theta + a < \frac{2(2\theta-1)}{\theta}$ one can readily verify that $\theta^2(1+a) < 2(2\theta-1)$. ■

B.5 Proof of Propositions 3.2 and 3.3, and Theorem 3.2

Proof of Proposition 3.2 By Lemma B.8 MPM PBEs are symmetric and $\pi^R=0$, $\pi^A>\theta$, which by Eqs. (3.5)-(3.6) implies $q^A\pi^A=\theta$. Thus, by Eqs. (3.9) and (3.13), $V^R(a)=aq^A\pi^A=a\theta$ and by Eq. (3.18), $\tilde{V}^{np}(a)=a\left(\theta-\tilde{\pi}^{np}\right)^+$. By Eq. (3.16), $\sigma(a)=0 \Rightarrow V^A(x;a) \ge V^R(x;a)$. Note, $\tilde{V}^{np}(a) < a\theta < V^A(x;a)$ for all $\tilde{\pi}^{np}\in[0,1]$. Hence, the low type prefers participation.

Following similar steps for the type $t=1$, by Eq. (3.15), $0<\sigma(\nu_i;1)<1 \Rightarrow V^A(x;1) = V^R(x;1)$. Eqs. (3.8) and (3.13) coupled with $\pi^R=0$, $\pi^A>\theta$ and $q^A\pi^A=\theta$ implies that $V^R(1)=q^A\pi^A=\theta$. Therefore, $V^A(x;1)=\theta=V^R(x;1)$. By Eq. 3.17, type $t=1$ prefers participation if $\tilde{\pi}^{np}\le\theta$ and prefers nonparticipation if $\tilde{\pi}^{np}>\theta$. In summary, if $\tilde{\pi}^{np}\le\theta$ both types prefer participation and the equal proposal is admissible. If $\tilde{\pi}^{np}>\theta$, type $t=1$ of both players prefers nonparticipation. Since PBE is symmetric and $V^R(x,1)$ is equal for both players, no randomization device exists that can satisfy the full participation constraint. ■

Proof of Propositions 3.3 As before label the favored player as $-i$ and the less favored player as i. Corollary B.1 coupled with Eqs. (3.9) and (3.13) implies that $V_i^R(x_i,a) = aq_{-i}^A\pi_{-i}^A = a\theta$. By Eq. (3.18), $\tilde{V}_i^{np}(a) = a\,(\theta - \tilde{\pi}^{np})^+$. By Eq. (3.16), $\sigma_i(\nu_i;a) = 0 \Rightarrow V_i^A(x_i;a) \geq V_i^R(x_i;a)$. Thus, $\tilde{V}_i^{np}(a) < a\theta < V^A(x;a)$ for all $\tilde{\pi}^{np} \in [0,1]$. Hence, type $t = a$ of player i prefers participation. Analogously, for type $t = 1$ of player i, by Eq. (3.15), $\sigma_i(\nu_i;1) = 1 \Rightarrow V_i^A(x_i;1) < V_i^R(x_i;1)$. Corollary B.1 coupled with Eqs. (3.8) and (3.13) implies that $V_i^R(x_i;1) = \theta$. By Eq. (3.17), $\tilde{V}_i^{np}(1) = \max\{\theta, \tilde{\pi}^{np}\}$. Thus, type $t = 1$ of player i prefers participation if $\tilde{\pi}^{np} \leq \theta$ and prefers nonparticipation if $\tilde{\pi}^{np} > \theta$.

Corollary B.1 coupled with Eqs. (3.9) and (3.13), where roles of i and $-i$ is interchanged, implies that $V_{-i}^A(x_{-i};a) = \theta x_{-i}$. By Eq. (3.18), $\tilde{V}_{-i}^{np}(a) = a\,(\theta - \tilde{\pi}^{np})^+$. By Ineq. (B.5), at Lopsided PBEs, $x_{-i} > \theta$ and by Ineq. (3.21), $a < \theta$. Thus, $\tilde{V}_{-i}^{np}(a) = a\,(\theta - \tilde{\pi}^{np})^+ < x_{-i}\theta = V_{-i}^A(x_i;a)$ for all $\tilde{\pi}^{np} \in [0,1]$. Hence, type $t = a$ of player $-i$ prefers participation.

Following similar steps for the type $t = 1$ of player $-i$, by Eq. (3.15), $\sigma_i(\nu_i;1) = 0 \Rightarrow V_{-i}^A(x_{-i};1) > V_{-i}^R(x_{-i};1)$. By Corollary B.1, Eqs. (3.8), and (3.13), $V_{-i}^R(x_{-i};1) = \theta + (1-\theta)\tilde{\pi}_{-i}^R$. By Eq. (3.17), if $\tilde{\pi}^{np} \leq \theta$, then type $t = 1$ of player $-i$ prefers participation. In summary, if $\tilde{\pi}^{np} \leq \theta$ both players prefer participation and lopsided proposals are admissible. By Eq. (3.17), if $\tilde{\pi}^{np} > \theta$, then $\tilde{V}_{-i}^{np}(1) = \tilde{\pi}^{np}$. Hence, if one sets $\tilde{\pi}_{-i}^R = \tilde{\pi}^{np}$ and plug in the equation for $V_{-i}^R(x_{-i};1)$, then

$$V_{-i}^R(x_{-i};1) = \theta + (1-\theta)\tilde{\pi} > \tilde{\pi}^{np} = \tilde{V}_{-i}^{np}(1) \iff \theta\tilde{\pi}^{np} < \theta,$$

which holds for all $\tilde{\pi}^{np} \in [0,1]$. Thus, if $\tilde{\pi}^{np} > \theta$, then type $t = 1$ of player $-i$ prefers participation. ■

Proposition B.2 *Suppose $a < \theta$ and any $\tilde{\pi}^{np} \in (\theta, \frac{2\theta}{1+\theta}]$. Lopsided proposals augmented with equal probability randomization device satisfy participation constraint (3.20).*

Proof As before label favored player as $-i$ and the less favored one as i. Given $\tilde{\pi}^{np} > \theta$, by Proposition 3.3, only $t = 1$ of the less favored player does not participate. By Corollary B.1 coupled with Eqs. (3.9), (3.8), (3.12), and (3.13), the interim payoffs of $t = 1$ are:

$$V_i^R(x_i;1) := q_{-i}^A\pi_{-i}^A = \theta, \tag{B.15}$$

$$V_{-i}^A(x_{-i};1) = q_i^A x_{-i} + q_i^R \max\left\{\pi_{-i}^A, \pi_i^R\right\} = \theta x_{-i} + (1-\theta)\theta, \tag{B.16}$$

$$V_{-i}^R(x_{-i};1) = q_i^A \max\left\{\tilde{\pi}_{-i}^R, \pi_i^A\right\} + q_i^R \max\left\{\tilde{\pi}_{-i}^R, \pi_i^R\right\} = \theta + (1-\theta)\tilde{\pi}_{-i}^R. \tag{B.17}$$

By Eq. (3.15), $\sigma_i(\nu_i;1) = 1 \Rightarrow V_i^R(x_i;1) > V_i^A(x_i;1)$ and $\sigma_{-i}(\nu_{-i};1) = 0 \Rightarrow V_{-i}^A(x_{-i};1) > V_{-i}^R(x_{-i};1)$. Hence, to satisfy the participation constraint (3.20), it suffices that interim payoff of Reject, coupled with the randomization device, is larger than nonparticipation payoff.

Set $\tilde{\pi}_{-i}^R = \tilde{\pi}^{np}$ and plug it into Eqs. (B.15)-(B.17). Then by Eq. (3.19) the expected payoff each player's type $t = 1$ by participation and announcing Reject is $V(1) = 1/2\left(\theta + (1-\theta)\tilde{\pi}^{np}\right) + \theta/2$. By Eq. (3.17), if $\tilde{\pi}^{np} > \theta$, then $\tilde{V}_{-i}^{np}(1) = \tilde{\pi}^{np}$. Hence, if $V_{-i}(1) \geq \tilde{\pi}^{np} = \tilde{V}_{-i}^{np}(1)$, then type $t = 1$ of both players prefer participation. Note that $V_{-i}(1) \geq \tilde{\pi}^{np}$ is equivalent to $\tilde{\pi}^{np} \leq \frac{2\theta}{1+\theta}$, the hypothesis of this proposition. Note that $\theta < \frac{2\theta}{1+\theta}$ for all $\theta \in (0,1)$. ■

Proof of Theorem 3.2 By Ineq. (3.21) and lemmas B.5 and B.6, Hybrid and MTM PBEs are ruled out. Since $\tilde{\pi}^{np} > \theta$, then Proposition 3.2 rules out admissibility of the equal proposal.

Given lopsided proposals, by Proposition B.2, a fair coin satisfies participation constraint. The rest of this proof verifies that $\tilde{\pi}^{np} > \theta$ and $\theta + a < M$ have non-vacuous intersections with the sufficient conditions for deterministic Lopsided PBEs (Lemma B.7).

Lopsided PBEs are uniquely determined modulo the off-path posterior belief $\tilde{\pi}^R_{-i}$. Set this arbitrary off-path belief $\tilde{\pi}^R_{-i} = \tilde{\pi}^{np}$. By Lemma B.7, the lower bound of M is $\theta(1+a) + \frac{(1-\theta)}{\theta}\tilde{\pi}^R_{-i}$. Note that

$$\theta + a < \theta(1+a) + \frac{(1-\theta)}{\theta}\tilde{\pi}^R_{-i} \iff a\theta < \tilde{\pi}^R_{-i},$$

where $a\theta < \tilde{\pi}^R_{-i}$ is implied by $\tilde{\pi}^R_{-i} = \tilde{\pi}^{np} > \theta$. By Lemma B.7-(i) and (ii), off-path posterior beliefs should satisfy $\tilde{\pi}^R_{-i} \in \left[0, \min\left\{\frac{\theta^2(1-a)}{1-\theta}, 1\right\}\right]$. Couple these conditions on $\tilde{\pi}^R_{-i}$ with $\tilde{\pi}^{np} = \tilde{\pi}^R_{-i}$ such that $\tilde{\pi}^{np} \in \left(\theta, \frac{2\theta}{1+\theta}\right]$, it suffices to show that these two intervals have nonempty intersection. Note that $\theta \leq \frac{\theta^2(1-a)}{1-\theta} \iff \theta \geq \frac{1}{2-a}$. Also, it is easy to verify that

$$\frac{2\theta}{1+\theta} \leq \frac{\theta^2(1-a)}{1-\theta} \iff \theta \geq \frac{a-3+\sqrt{a^2-14a+17}}{2-2a.}.$$

Moreover, for all $a \in (0,1)$, $\theta \geq \frac{1}{2-a} \Rightarrow \theta \geq \frac{a-3+\sqrt{a^2-14a+17}}{2-2a.}$. This is true because

$$\frac{1}{2-a} \leq \frac{a-3+\sqrt{a^2-14a+17}}{2-2a.} \iff \frac{(2-a)\sqrt{a^2-14a+17}-(a^2-7a+8)}{(2-a)(2-2a)} \geq 0$$

where the last inequality in the above displayed inequalities is because

$$(2-a)\sqrt{a^2-14a+17} \geq (a^2-7a+8) \iff (2-a)^2(a^2-14a+17) \geq (a^2-7a+8)^2 \iff 4(1-a)^3 \geq 0.$$

Hence, if $\theta > \frac{1}{2-a}$ then $\tilde{\pi}^{np} = \tilde{\pi}^R_{-i} \in \left[\theta, \min\left\{\frac{\theta^2(1-a)}{1-\theta}, \frac{2\theta}{1+\theta}\right\}\right]$ is not vacuous. Therefore, the condition for existence of deterministic Lopsided proposals are satisfied. Lastly, $P(x_{\min}) = 1 - q_i^A q_{-i}^A = 1-\theta$, as characterized by Corollary B.1. ■

B.6 Proof of Theorem 3.1

By Proposition 3.1, it suffices to consider only two classes of MPM and Lopsided PBEs. Nonparticipation is off-path. Thus, $\tilde{\pi}^{np}$ is arbitrary. Pick any $\tilde{\pi}^{np} \leq \theta$. Then by Propositions 3.2 and 3.3, the equal proposal and lopsided proposals are admissible. Proposition B.1, conditional on participation, states conditions under which the equal proposal or lopsided proposals minimize probability of conflict. Thus, claim (a) of Theorem 3.1 follows Proposition B.1-(a)

To establish claim (b) of the theorem, one must verify that off-path beliefs that, conditional on participation, rationalize Lopsided PBEs and are specified in Proposition B.1, satisfy $\tilde{\pi}^{np} \leq \theta$ assumption. Given Lopsided proposals, the favored player, labeled as $-i$, always announces Accept while her rival i Accept if $t = a$ and Reject if $t = 1$. Hence, Lopsided PBEs are uniquely determined modulo the off-path belief $\tilde{\pi}^R_{-i}$. Set this arbitrary off-path belief $\tilde{\pi}^R_{-i} = \tilde{\pi}^{np}$. Then Proposition B.1-($b$) and ($c$) implies claim (b) of the theorem. Recall that for all $\theta^* \leq \theta < \frac{4-a-\sqrt{a^2-8a+8}}{2}$, Proposition B.1-($b$) states the off-path belief condition to be $\tilde{\pi}^{np} \in [0, a\theta]$, which is implied by the assumption $\tilde{\pi}^{np} \leq \theta$. If $\frac{4-a-\sqrt{a^2-8a+8}}{2} \leq \theta$, Proposition B.1-($c$) states the off-path belief condition to be $\tilde{\pi}^{np} \in \left[0, \frac{2(2\theta-1)-\theta^2(1+a)}{1-\theta}\right]$, which has none-empty intersection with $\tilde{\pi}^{np} \leq \theta$. The latter is true because of the two easy to verify facts: (i) $\theta \leq \frac{2(2\theta-1)-\theta^2(1+a)}{1-\theta} \iff \theta \geq \frac{2}{3-a}$

and (ii) $\frac{4-a-\sqrt{a^2-8a+8}}{2} \leq \frac{2}{3-a}$ for all $a \in (0,1)$. ■

B.7 Continuum of Types Details

B.7.1 The Equilibrium Condition During Mediation

Let $U_{mn}(t_i; F_i^m, F_{-i}^n)$ denote the payoff at the continuation game of conflict $\mathscr{G}(F_i^m, F_{-i}^n)$ for type t_i of player i who has announced $m \in \{A, R\}$ while facing player $-i$ that has announced $n \in \{\text{A, R}\}$ at the mediation. Conditional on participation, given the cutoff strategies $(\sigma_i)_{i=1}^{i=2}$ defined by Eq. (3.28), for each player i associated posterior probabilities F_i^m, ex-ante probabilities of Accept $F_i(\lambda_i)$, and thereby interim payoff from announcing $m \in \{A, R\}$ are determined. Let $V_i^A(\nu_i, t_i)$ denote the interim expected payoff of player i's type t_i who announces Accept. Then,

$$V_i^A(\nu_i, t_i) := F_{-i}(\lambda_{-i})\nu_1 + \left(1 - F_{-i}(\lambda_{-i})\right) U_{AR}(t_i; F_i^A, F_{-i}^R), \tag{B.18}$$

where $F_{-i}(\lambda_{-i})$ is the ex-ante probability of Accept by player $-i$, and $1 - F_{-i}(\lambda_{-i})$ is the ex-ante probability of Reject. The interim expected payoff of type $t_i \in [0,1]$ of player i who announces Reject is denoted by $V_i^R(\nu_i, t_i)$ and is characterized as

$$V_i^R(\nu_i, t_i) := F_{-i}(\lambda_{-i}) U_{RA}(t_i; F_i^R, F_{-i}^A) + \left(1 - F_{-i}(\lambda_{-i})\right) U_{RR}(t_i; F_i^R, F_{-i}^R). \tag{B.19}$$

Conditional on participation, the cutoff strategies $\sigma_i(\nu_i; t_i)$ defined by Eq. (3.28) constitutes an equilibrium if and only if

$$\forall i, t_i \in [0, \lambda_i] : V_i^A(\nu_i, t_i) \geq V_i^R(\nu_i, t_i), \tag{B.20}$$

and

$$\forall i, t_i \in [\lambda_i, 1] : V_i^R(\nu_i, t_i) \geq V_i^A(\nu_i, t_i), \tag{B.21}$$

where $V^A(\nu_i, t_i)$ and $V^A(\nu_i, t_i)$ are respectively defined by Eqs. (B.18) and (B.19).

Full participation condition translates itself into an interim participation constraint. Suppose player i unilaterally deviates to nonparticipation. In that event, by Bayes's rule posterior for player $-i$ is F_{-i} because she participates in the mechanism almost surely. $\tilde{F}_i^{np}$ is off-path and hence arbitrary. For each type t_i denote the nonparticipation payoff as $V_{np}(t_i)$.

$$V_{np}(t_i) = U_{np}\left(t_i, \tilde{F}_i^{np}, F_{-i}\right) \tag{B.22}$$

If the mechanism is stochastic, then the mediator would label player i as the favored player with probability 1/2 and offers her peace proposal $\nu_1 \geq \nu_2$ and with probability 1/2 she will be offered ν_2. Denote, the interim payoff of type t_i at the participation stage by $V(t_i)$:

$$V(t_i) = V_i^m(\nu_1, t_i)/2 + V_i^n(\nu_2, t_i)/2, \tag{B.23}$$

where $V^m(\nu_1, t_i)$ (resp. $V^n(\nu_2, t_i)$) is the interim payoff of type t_i of player i, conditional on participation, receiving proposal ν_1 (resp. ν_2) and announcing the equilibrium strategy $m \in \{A, R\}$ (resp. $n \in \{A, R\}$). Hence, to guarantee full participation, for each type t_i, the interim participation constraint should be satisfied which are

$$V(t_i) \geq V_{np}(t_i). \tag{B.24}$$

A tuple $\left(\nu_i, S - \nu_i, \sigma_i(\nu_i; t), F_i^A, F_i^R, \tilde{F}_i^{np}\right)_{i=1}^2$ constitutes a fully participating PBE if and only if it satisfies (3.28), (B.20), (B.21), and (B.24).

B.7.2 Payoffs at the Conflict

I use the pivotal type approach of Amann and Leininger (1996) and Kirkegaard (2008) to characterize the unique BNE and expected payoff at the continuation game of conflict. Whether this stage is entered because someone rejected the mediator's offer, or because one deviated and did not participate in the mediation, the game is the same. Each player knows their type, the history of the game, and has a posterior belief about the rival's types based on that history. Since, in this chapter I am interested in cutoff strategies, in this section, I shall characterize the conflict payoff for posterior beliefs that are truncated distributions of the prior with intervals as support. After announcements $m \in \{A, R\}$ and $n \in \{A, R\}$ by players i and $-i$, respectively, at the mediation stage, if at least one payer Reject the proposal the game enters the conflict stage $\mathscr{G}\left(F_i^m, F_{-i}^n\right)$.

By Theorem 6 of Zheng (2019), at any such continuation game of conflict, both players bid over a common support of $\left[0, \bar{b}\right]$, where $\bar{b}$ is the common maximal bid. Moreover, at most one player would bid zero with positive probability. Denote $\beta_i^{mn}(t)$ as the bidding strategy at the continuation game $\mathscr{G}\left(F_i^m, F_{-i}^n\right)$, where mn is AR, RA, or RR. Respectively, $\phi_i^{mn}(b)$ denote the inverse bid function of player $i \in \{1, 2\}$ at the continuation game $\mathscr{G}\left(F_i^m, F_{-i}^n\right)$. Hence, $\phi_i^{mn}(0)$ is the infimum of the set of bidder $i's$ type who would bid zero. For the moment, I assume that bidding strategies $\beta_i^{mn}(t)$, and respectively their inverse $\phi_i^{mn}(b)$, to be strictly increasing over $(0, \bar{b}]$, a requirement which will be satisfied in equilibrium.

Player i with valuation t_i is faced with the problem of selecting the bid b that maximizes: $\max_b t_i F_{-i}^n\left(\phi_{-i}^{nm}(b)\right) - b$. The equilibrium continuation payoff is then denoted by $U_{mn}(t_i, F_i^m, F_i^n)$. Hence, from the point of view of bidder i who has announced m at the mediation stage, $F_{-i}^n\left(\phi_{-i}^{nm}(b)\right)$ is the bidding distribution of the opponent $-i$ who has announced n at the mediation stage.

Symmetric continuation game of conflict: The following lemma characterizes the payoff at the continuation game $\mathscr{G}\left(F^R, F^R\right)$ where players have identical truncated distributions. This is a standard result in the literature and is just summarized here for further reference.

Lemma B.12 *Suppose a symmetric continuation game of conflict $\mathscr{G}\left(F^R, F^R\right)$. Suppose the truncated CDF $F^R = \frac{F(t)-F(\lambda)}{1-F(\lambda)}$ has the support $[\lambda, 1]$. Then there exist a BNE where type t of each player would get the equilibrium continuation payoff $U_{RR}(t, F^R, F^R) = \int_\lambda^1 \frac{F(v)-F(\lambda)}{1-F(\lambda)} dv$.*

Asymmetric continuation game of conflict: I characterize the payoffs for the continuation games where players follow asymmetric cutoff strategies defined by Eq. (3.28). Without loss of generality assume that player i announces A and player $-i$ announces R. The continuation game AR is denoted by $\mathscr{G}\left(F_i^A, F_{-i}^R\right)$, where F_i^A's support is $[0, \lambda_i]$ for all $\lambda_i \in (0, 1]$ and F_{-i}^R's support is $[\lambda_{-i}, 1]$ for all $\lambda_{-i} \in (0, 1)$ (Note that at $\lambda_i = 1$, we have $F_i^A = F$, the prior). Hence, player i's problem is $\max_b t_i F_{-i}^R\left(\phi_{-i}^{RA}(b)\right) - b$, and player $-i$'s problem is $\max_b t_{-i} F_i^A\left(\phi_i^{AR}(b)\right) - b$. The first order conditions are

$$t_i \frac{d}{db} F_{-i}^R\left(\phi_{-i}^{RA}(b)\right) - 1 = 0 \quad \text{and} \quad t_{-i} \frac{d}{db} F_i^A\left(\phi_i^{AR}(b)\right) - 1 = 0,$$

which can be rewritten as:

$$\frac{d\phi_{-i}^{RA}(b)}{db} = \frac{1}{\phi_i^{AR}(b) f_{-i}^R\left(\phi_{RA}^{-i}(b)\right)} \quad \text{and} \quad \frac{d\phi_i^{AR}(b)}{db} = \frac{1}{\phi_{-i}^{RA}(b) f_i^A\left(\phi_i^{AR}(b)\right)}.$$

These equations verify that the bidding functions are increasing in type and players are following pure strategies. By Theorem 6 of Zheng (2019), at equilibrium, both players bid over common support, and at most one player can have an atom at zero. Dividing the first order conditions yields:

$$\frac{d\phi_i^{AR}(b)}{d\phi_{-i}^{RA}(b)} = \frac{\phi_i^{AR}(b) f_{-i}^R\left(\phi_{-i}^{RA}(b)\right)}{\phi_{-i}^{RA}(b) f_i^A\left(\phi_i^{AR}(b)\right)}$$

If bidder $-i$'s type is t_{-i}, I define $k_{RA}(t_{-i};\lambda_i,\lambda_{-i})$ as the type of bidder i with whom type t_{-i} of bidder $-i$ would tie with as in Kirkegaard (2008). $\lambda_i \in (0,1]$ denote the cutoff type of player i, the upper bound of the support $[0,\lambda_i]$ of the truncated distribution F_i^A. $\lambda_{-i} \in (0,1)$ denote the cutoff type of player $-i$, the lower bound of the support $[\lambda_{-i},1]$ of the truncated distribution F_{-i}^R. Thus, $k_{RA}(t_{-i};\lambda_i,\lambda_{-i})$ is the pivotal type function and is the solution to the initial value problem

$$\frac{d}{dt}k_{RA}(t_{-i};\lambda_i,\lambda_{-i}) = \frac{k_{RA}(t_{-i};\lambda_i,\lambda_{-i}) f_{-i}^R(t)}{t f_i^A(k_{RA}(t_{-i};\lambda_i,\lambda_{-i}))} = \left(\frac{F(\lambda_i)}{1-F(\lambda_{-i})}\right)\left(\frac{k_{RA}(t_{-i};\lambda_i,\lambda_{-i}) f(t)}{t f(k_{RA}(t_{-i};\lambda_i,\lambda_{-i}))}\right) \quad \text{and}$$
$$k_{RA}(1,\lambda_i,\lambda_{-i}) = \lambda_i. \tag{B.25}$$

The boundary condition $k_{RA}(1,\lambda_i,\lambda_{-i}) = \lambda_i$ is because the bidders have a common maximal bid. This differential equation, along with the boundary condition, yields a unique solution to the pivotal type function $k_{RA}(t,\lambda_i,\lambda_{-i})$. Eq. (B.25) is a separable differential equation.

$$\frac{1}{F(\lambda_i)}\int_{k_{RA}(t_{-i};\lambda_i,\lambda_{-i})}^{\lambda_i} \frac{f(x)}{x}dx = \frac{1}{1-F(\lambda_{-i})}\int_t^1 \frac{f(x)}{x}dx. \tag{B.26}$$

By Eq. (B.25), $k_{RA}(t;\lambda_i,\lambda_{-i})$ is strictly increasing in t. Hence, its inverse $k_{AR}^{-1}(t_i;\lambda_i,\lambda_{-i})$ is well defined for $t_i \in \left(k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i}),\lambda_i\right]$. Characterization of pivotal type function allow us to determine $\phi_i^{AR}(0) = 0$ and $\phi_{-i}^{RA}(0) = k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i}) > 0$. Then one can couple the pivotal type function and first order conditions to characterize the bidding function and thereby fully characterize the unique BNE. Based on the characterization of the unique BNE of conflict, the following lemma characterizes the payoffs at $\mathscr{G}\left(F_i^A, F_{-i}^R\right)$ by using the pivotal type function and the envelope theorem.

Lemma B.13 *Suppose asymmetric continuation game of conflict $\mathscr{G}\left(F_i^A, F_{-i}^R\right)$, where F_i^A has the support $[0,\lambda_i]$ and $\lambda_i \in [0,1]$ and F_{-i}^R has the support $[\lambda_{-i},1]$ and $\lambda_{-i} \in (0,1)$. Then there exist a unique BNE where type t of each player gets the equilibrium continuation payoff*

$$U_{AR}\left(t_i; F_i^A, F_{-i}^R\right) = \begin{cases} 0, & \text{if } t_i \in \left[0, k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i})\right], \\ \int_{k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i})}^{t_i} \frac{F(k_{AR}^{-1}(t;\lambda_i,\lambda_{-i}))-F(\lambda_{-i})}{1-F(\lambda_{-i})}dt, & \text{if } t_i \in \left[k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i}),\lambda_i\right], \end{cases} \tag{B.27}$$

Furthermore,

$$U_{RA}\left(t_{-i}; F_{-i}^R, F_i^A\right) = \lambda_{-i}\frac{F(k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i}))}{F(\lambda_i)} + \int_{\lambda_{-i}}^{t_{-i}} \frac{F\left(k_{RA}(t;\lambda_i,\lambda_{-i})\right)dt}{F(\lambda_i)}, \ \text{if } t_{-i} \in [\lambda_{-i},1]. \tag{B.28}$$

The continuation payoffs for both players are weakly increasing and weakly convex in t.

Proof As stated above, existence and uniqueness of the equilibrium is by the unique pivotal type function that satisfies Eq. (B.26). Note that if $t_i > k_{RA}(t_{-i};\lambda_i,\lambda_{-i})$, then by the definition of pivotal type function and monotonicity of bidding functions in type, player i of type t_i bids higher than her opponent that has type t_{-i} and wins at the conflict. Thus, the pivotal type function determines who wins the conflict. Then the conflict payoffs as stated are an immediate application of the envelop theorem. Furthermore, by the strictly increasing property of the (inverse) pivotal function in t and the envelope theorem, it is immediate that continuation game payoffs are weakly increasing and weakly convex in t. ■

Corollary B.3 *Suppose the same hypothesis and notation of Lemma B.13. If type $t_i \in (\lambda_i, 1]$ deviates and announces Accept then her payoff at the continuation game $\mathscr{G}\left(F_i^A, F_{-i}^R\right)$ is*

$$U_{AR}\left(t_i; F_i^A, F_{-i}^R\right) = t_i - \lambda_i + \int_{k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i})}^{\lambda_i} \frac{F(k_{AR}^{-1}(t;\lambda_i,\lambda_{-i})) - F(\lambda_{-i})}{1-F(\lambda_{-i})} dt, \ \text{if}\ t_i \in [\lambda_i, 1]\,. \quad \text{(B.29)}$$

If $t_{-i} \in [0, \lambda_{-i})$ deviates and announces Reject then her payoff at the continuation game $\mathscr{G}\left(F_i^A, F_{-i}^R\right)$ is

$$U_{RA}\left(t_{-i}; F_{-i}^R, F_i^A\right) = t_{-i}\frac{F(k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i}))}{F(\lambda_i)}, \ \text{if}\ t_{-i} \in [0, \lambda_{-i}]\,. \quad \text{(B.30)}$$

The continuation payoffs for both players are increasing and weakly convex in t.

Proof Consider maximization problem of deviating type $t_{-i} \in [0, \lambda_{-i})$ that announces Reject:

$$\max_b t_{-i} F_i^A(\phi_{-i}^{RA}(b)) - b.$$

Since $t_{-i} < \lambda_{-i}$, t_{-i}'s rival overestimates her type, the first order condition is never satisfied, $\frac{d}{db}(t_{-i}F_i^A(\phi_{-i}^{RA}(b)) - b) = t_{-i} f^A(\phi_{-i}^{RA}(b))\frac{d\phi_{-i}^{RA}(b)}{db} - 1 < 0$. Thus, the optimal bid for these types is to mimic the λ_{-i} type at $\mathscr{G}\left(F_i^R, F_{-i}^A\right)$, the minimum of the support $[\lambda_{-i}, 1]$. Thus, the payoff for this deviating type follows Eq. (B.28) where this type bids zero and win with probability $\frac{F(k_{RA}(\lambda_{-i};\lambda_i,\lambda_{-i}))}{F(\lambda_i)}$. Thus, one can immediately obtain Eq. (B.30).

Analogously, the problem of the deviating type $t_i \in (\lambda_i, 1]$ is

$$\max_b t_i F_{-i}^R(\phi_i^{AR}(b)) - b.$$

Since $t_i > \lambda_i$, t_i's rival underestimates her type, the first order condition is never satisfied, $\frac{d}{db}(tF_{-i}^R(\phi_i^{AR}(b)) - b) = tf_i^R(\phi_i^{AR}(b))\frac{d\phi_i^{AR}(b)}{db} - 1 > 0$. Thus the optimal bid is to mimic the λ_i type at $\mathscr{G}\left(F_i^A, F_{-i}^F\right)$, the upper bound of support $[0, \lambda_i]$. Type λ_i bids the common maximal bid $\bar{b}$, wins with probability one, and gains the payoff $U_{AR}(\lambda_i; F_i^A, F_{-i}^R) = \lambda_i - \bar{b}$. Hence, the deviating type $t_i \in (\lambda_i, 1]$ bids the common maximal bid $\bar{b}$, wins with probability one, and gains payoff :

$$U_{AR}(t_i; F_i^A, F_{-i}^R) = t_i - \bar{b} = t_i - \lambda_i + U_{AR}(\lambda_i; F_i^A, F_{-i}^R),$$

where the second equality is because type λ_i bids $\bar{b}$ and win with probability one. Thus, one can use Eq. (B.27) to obtain Eq. (B.29) as stated in the Lemma. By Eqs. (B.30) and (B.29) it is immediate that payoffs are increasing and linear in t. ■

Next, I characterized payoffs at continuation game $\mathscr{G}\left(F_i^R, F_{-i}^R\right)$ where posteriors are truncation of the prior with supports $[\lambda_i, 1]$ and $[\lambda_{-i},]$. Without loss of generality assume $\lambda_i \leq \lambda_{-i}$, meaning player $-i$ is stochastically stronger than her rival. If bidder $-i$'s type is t_{-i}, I define $k_{RR}(t_{-i};\lambda_i,\lambda_{-i})$ as the type of bidder i with whom type t_{-i} of bidder $-i$ would tie with.

$k_{RR}(t_{-i};\lambda_i,\lambda_{-i})$ is the unique solution to the initial value problem

$$\frac{d}{dt}k_{RR}(t_{-i};\lambda_i,\lambda_{-i}) = \frac{k_{RR}(t_{-i};\lambda,\lambda_{-i})f^R_{-i}(t)}{tf^R_i(k_{RR}(t_{-i};\lambda_i,\lambda_{-i}))} = \left(\frac{1-F(\lambda_i)}{1-F(\lambda_{-i})}\right)\left(\frac{k_{RR}(t_{-i};\lambda_i,\lambda_{-i})f(t)}{tf(k^{RR}(t_{-i};\lambda_i,\lambda_{-i}))}\right) \text{ and}$$
$$k_{RR}(1;\lambda_i,\lambda_{-i}) = 1. \tag{B.31}$$

Eq. (B.31) is a separable differential equation. Hence, $k_{RR}(t_{-i},\lambda_{-i})$ is the unique solution to

$$\frac{1}{1-F(\lambda_i)}\int_{k_{RR}(t_{-i};\lambda_i,\lambda_{-i})}^{1}\frac{f(x)}{x}dx = \frac{1}{1-F(\lambda_{-i})}\int_t^1\frac{f(x)}{x}dx. \tag{B.32}$$

By Eq. (B.31), $k_{RR}(t;\lambda_i,\lambda_{-i})$ is strictly increasing in t and its inverse $k^{-1}_{RR}(t_i;\lambda_i,\lambda_{-i})$ is well defined for $t_i \in (k_{RR}(\lambda_{-i};\lambda_i,\lambda_{-i}),\lambda_i]$.

Lemma B.14 *Suppose asymmetric continuation game of conflict* $\mathscr{G}\left(F^R_i,F^R_{-i}\right)$*. Suppose* $0 < \lambda_i < \lambda_{-i} < 1$*. Then, there exist a unique BNE where type* t *of each player gets the payoff*

$$U_{RR}(t_i;F^R_i,F^R_{-i}) = \begin{cases} 0, & \text{if } t_i \in [\lambda_i, k_{RR}(\lambda_{-i};\lambda_i,\lambda_{-i})], \\ \int_{k_{RR}(\lambda_{-i};\lambda_i,\lambda_{-i})}^{t_i}\frac{F(k^{-1}_{RR}(t;\lambda_i,\lambda_{-i}))-F(\lambda_{-i})}{1-F(\lambda_{-i})}dt, & \text{if } t_i \in [k_{RR}(\lambda_{-i};\lambda_i,\lambda_{-i}),1]. \end{cases} \tag{B.33}$$

Furthermore,

$$U_{RR}(t_{-i};F^R_{-i},F^R_i) = \lambda_{-i}\frac{F(k_{RR}(\lambda_{-i};\lambda_i,\lambda_{-i}))-F(\lambda_i)}{1-F(\lambda_i)} + \int_{\lambda_{-i}}^{t_i}\frac{F\left(k_{RR}(t;\lambda_i,\lambda_{-i})\right)-F(\lambda_i)}{1-F(\lambda_i)}dt, \text{ if } t_{-i}\in[\lambda_{-i},1]. \tag{B.34}$$

The continuation payoffs for both players are weakly increasing and weakly convex in t*.*

Proof The characterization of payoffs is analogous to Lemma B.13. By the strictly increasing property of the (inverse) pivotal function in t and the envelope theorem, it is immediate that continuation game payoffs are weakly increasing and weakly convex in t. ■

Corollary B.4 *Suppose the same hypothesis and notation of Lemma B.14. If type* $t_i \in [0,\lambda_i)$ *deviates and announces Reject then her payoff at the continuation game* $\mathscr{G}\left(F^R_i,F^R_{-i}\right)$ *is*

$$U_{RR}\left(t_i;F^R_i,F^R_{-i}\right) = 0, \text{ if } t_i \in [0,\lambda_i). \tag{B.35}$$

If $t_{-i} \in [0,\lambda_{-i})$ *deviates and announces Reject then her payoff at the continuation game* $\mathscr{G}\left(F^R_i,F^R_{-i}\right)$ *is*

$$U_{RR}\left(t_{-i};F^R_{-i},F^R_i\right) = t_{-i}\frac{F(k_{RR}(\lambda_{-i};\lambda_i,\lambda_{-i}))-F(\lambda_i)}{1-F(\lambda_i)}, \text{ if } t_{-i}\in[0,\lambda_{-i}). \tag{B.36}$$

The continuation payoffs for players $-i$ *is increasing and weakly convex in* t*.*

Proof The problem of the deviating type $t_i \in [0,\lambda_i)$ is

$$\max_b t_iF^R_{-i}(\phi^{RR}_i(b)) - b$$

then since $t_i < \lambda_i$, t_i's opponent overestimated his type, the first order condition is never satisfied.Thus the optimal bid is to mimic that of $t_i = \lambda_i$, the minimum of the support $[\lambda_i, 1]$. Therefore, by Eq. (B.35) these types will get zero payoff. Analogously, the deviating type $t_{-i} \in [0,\lambda_{-i})$ mimic the bidding behavior of $t_{-i} = \lambda_{-i}$, bids zero and wins with the same probability as type $t_{-i} = \lambda_{-i}$. Thus, by Eq. (B.34) one can obtain the payoff for the deviating type as stated in Eq. (B.36). ■

B.7.3 Proof of Propositions 3.4–3.5 and Theorem 3.3

A tuple $\left(\nu_i, S-\nu_i, \sigma_i(\nu_i;t), F_i^A, F_i^R, \tilde{F}_i^{np}\right)_{i=1}^2$ constitutes a fully participating symmetric PBE if and only if it satisfies (3.29), (B.20), (B.21), and (B.24). Cutoff strategies (3.29) uniquely pin down ex-ante probability of Accept F_i^A and the associated posteriors F_i^A and F_i^R for each player i. Due to symmetry, I drop the subscript i. By continuity of conflict payoffs (Lemmas B.12 and B.13), conditions (B.20) and (B.21) should hold with strict inequality for types $[0,\lambda)$ and $(\lambda,1]$ and with equality for the cutoff type λ, which by symmetry the latter is equivalent to the equal proposal $\nu_i = \nu_{-i} = \frac{S}{2}$ and

$$V^A\left(S/2;\lambda\right) = V^R\left(S/2;\lambda\right).$$

In other words, $t=\lambda$ is indifferent between Accept and Reject. Otherwise there will be types immediately to the right or left of it that would have profitable deviations. Therefore, a tuple $\left(\nu_i, S-\nu_i, \sigma_i(\nu_i;t), F_i^A, F_i^R, \tilde{F}_i^{np}\right)_{i=1}^2$ constitutes a symmetric PBE if and only if it satisfies (3.29), $\nu_i = \nu_{-i} = S/2$, (B.20), (B.21), and (B.24).

Payoffs at $\mathscr{G}(F^R, F^R)$ are characterized in Lemma B.12. At $\mathscr{G}(F^A, F^R)$ the pivotal type function is characterized by Eqs. (B.25) or (B.26) after setting $\lambda_i = \lambda_{-i} = \lambda$. Hence, denote

$$k_{sym}(t;\lambda) := k_{RA}(t;\lambda,\lambda), \tag{B.37}$$

where $k_{RA}(t;\lambda_i,\lambda_{-i})$ is given by Eq. (B.26). Hence, set $\lambda_i = \lambda_{-i} = \lambda$ in Eqs. (B.25) and (B.26):

$$\frac{d}{dt}k_{sym}(t;\lambda) = \left(\frac{F(\lambda)}{1-F(\lambda)}\right)\left(\frac{k_{sym}(t;\lambda)f(t)}{tf(k_{sym}(t;\lambda))}\right), \tag{B.38}$$

with the boundary condition $k_{sym}(1)=\lambda$, or equivalently the separable differential equation

$$\frac{1}{F(\lambda)}\int_{k_{sym}(t;\lambda)}^{\lambda}\frac{f(x)}{x}dx = \frac{1}{1-F(\lambda)}\int_t^1\frac{f(x)}{x}dx. \tag{B.39}$$

Similarly, at $\mathscr{G}(F^A, F^R)$ payoffs by Lemma B.13, where $\lambda_i = \lambda_{-i} = \lambda$, are:

$$U_{AR}(t;F^A,F^R) = \begin{cases} 0, & \text{if } t \in \left[0, k_{sym}(\lambda;\lambda)\right], \\ \frac{\int_{k_{sym}(\lambda;\lambda)}^t \left(F(k_{sym}^{-1}(t;\lambda)) - F(\lambda)\right)dt}{1-F(\lambda)}, & \text{if } t \in \left[k_{sym}(\lambda;\lambda), \lambda\right]. \end{cases} \tag{B.40}$$

Furthermore,

$$U_{RA}(t;F^R,F^A) = \lambda\frac{F(k_{sym}(\lambda;\lambda))}{F(\lambda)} + \int_\lambda^1 \frac{F\left(k_{sym}(t;\lambda)\right)dt}{F(\lambda)} \text{ for all } t\in[\lambda,1] \tag{B.41}$$

By Lemma B.12, $U_{\mathrm{RR}}(\lambda;F^R,F^R) = 0$. Also, by Eq. (B.41) we have $U_{RA}(\lambda;F^R,F^A) = \lambda\frac{F(k_{sym}(\lambda;\lambda))}{F(\lambda)}$. Hence, the equilibrium condition $V^A\left(S/2;\lambda\right) = V^R\left(S/2;\lambda\right)$ for the cutoff is

$$F(\lambda)\frac{S}{2} + (1-F(\lambda))\,U_{\mathrm{AR}}(\lambda;F^A,F^R) = \lambda F(k_{sym}(\lambda;\lambda)). \tag{B.42}$$

Therefore, one can use this characterization to prove the following propositions.

Proof of Proposition 3.4 I shall show that, given hypothesis of this proposition, non-degenerate set of high types gains payoff less than nonparticipation. As shown above, the equal proposal admits symmetric strategies and thereby, admits equal interim payoffs. Thus, the participation constraint (B.24) for the high types, whom by cutoff strategies (3.29) announces Reject, is simplified to $V^R\left(S/2, t_i\right) \geq V_{np}(t_i)$. I shall show, given the hypothesis of this proposition, non-degenerate set of high types' payoff is less than nonparticipation.

First, given passive updating assumption, I show $t = 1$ is worse off by participation:

$$\underbrace{F(\lambda)U_{RA}(1; F^R, F^A) + (1 - F(\lambda))U_{RR}(1, F^R, F^R)}_{=V^R\left(\frac{s}{2},1\right)} < \underbrace{\int_0^1 F(t)dt = U_{np}(1, F, F)}_{=V_{np}(1)}, \tag{B.43}$$

where the equality in (B.43) is by passive updating $\tilde{F}_i^{np} = F$ and Lemma B.12 (by setting $\lambda = 0$). By Eqs. (B.40), (B.41), and the fact that at $\mathscr{G}\left(F^A, F^R\right)$ both $t = 1$ and $t = \lambda$ are the upper bounds of supports of F^R and F^A and bid the common maximal bid we have:

$$U_{RA}(1; F^R, F^A) = 1 - \lambda + U_{AR}(\lambda; F^A, F^R) = 1 - \lambda + \frac{\int_{k_{symm}(\lambda;\lambda)}^{\lambda}\left(F\left(k_{sym}^{-1}(t;\lambda)\right) - F(\lambda)\right)dt}{1 - F(\lambda)},$$

$$U_{RR}(1; F^R, F^R) = \frac{\int_{\lambda}^{1}\left(F(t) - F(\lambda)\right)dt}{1 - F(\lambda)}.$$

Plugging the above displayed equations in Ineq. (B.43) and rearrange its right hand side, it is equivalent to

$$F(\lambda)\left[1 - \lambda + \frac{\int_{k_{symm}(\lambda;\lambda)}^{\lambda}\left(F\left(k_{sym}^{-1}(t;\lambda)\right) - F(\lambda)\right)dt}{1 - F(\lambda)}\right] + (1 - F(\lambda))\left[\frac{\int_{\lambda}^{1}\left(F(t) - F(\lambda)\right)dt}{1 - F(\lambda)}\right] \leq$$

$$F(\lambda)\left[1 - \lambda + \frac{\int_0^{\lambda} F(t)dt}{F(\lambda)}\right] + (1 - F(\lambda))\left[\frac{\int_{\lambda}^{1}\left(F(t) - F(\lambda)\right)dt}{1 - F(\lambda)}\right],$$

which can be simplified further to

$$\frac{\int_{k_{symm}(\lambda;\lambda)}^{\lambda}\left(F\left(k_{sym}^{-1}(t;\lambda)\right) - F(\lambda)\right)dt}{1 - F(\lambda)} \leq \frac{\int_0^{\lambda} F(t)dt}{F(\lambda)}.$$

It suffices to verify for all $t \in [0, \lambda]$ the integrands satisfy: $\frac{\left(F\left(k_{sym}^{-1}(t;\lambda)\right) - F(\lambda)\right)}{1-F(\lambda)} < \frac{F(t)}{F(\lambda)}$. By definition $k_{sym}(t;\lambda) < t$ for all $t \in [\lambda, 1]$ because it maps these types to $[0, \lambda]$. Also, $k_{sym}^{-1}(t;\lambda) = \lambda$ for all $t \in \left[0, k_{sym}(\lambda;\lambda)\right]$ and $k_{sym}^{-1}(t;\lambda) > \lambda > t$ for all $t \in \left(k_{sym}(\lambda;\lambda), \lambda\right]$. Moreover, by Eq. (B.38) and its implication for inverse function's derivative, these functions are both strictly increasing in t. Hence, for all $t \in \left[0, k_{sym}(\lambda;\lambda)\right]$,

$$\frac{\left(F\left(k_{sym}^{-1}(t;\lambda)\right) - F(\lambda)\right)}{1 - F(\lambda)} = 0 < \frac{F(t)}{F(\lambda)}$$

and at $t = \lambda$ we have

$$\frac{\left(F\left(k_{sym}^{-1}(\lambda;\lambda)\right) - F(\lambda)\right)}{1 - F(\lambda)} = 1 = \frac{F(t)}{F(\lambda)}.$$

Hence, it suffices to show that for all $t \in \left[k_{sym}(\lambda;\lambda), \lambda\right]$ the term

$$\frac{F\left(k_{sym}^{-1}(t;\lambda)\right) - F(\lambda)}{1 - F(\lambda)} - \frac{F(t)}{F(\lambda)}$$

is strictly increasing in t.

Recall by Eq. (B.38) and properties of inverse functions differentiation

$$\frac{d}{dt}k_{sym}(t;\lambda) = \frac{F(\lambda)}{1-F(\lambda)}\frac{k_{sym}(t;\lambda)f(t)}{tf(k_{sym}(t;\lambda))} \implies \frac{d}{dt}k^{-1}_{sym}(t;\lambda) = \frac{1-F(\lambda)}{F(\lambda)}\frac{k^{-1}_{sym}(t;\lambda)f(t)}{tf(k^{-1}_{sym}(t;\lambda))}.$$

Thus, by the above displayed equations,

$$\frac{d}{dt}\left[\frac{F\left(k^{-1}_{sym}(t;\lambda)\right)-F(\lambda)}{1-F(\lambda)} - \frac{F(t)}{F(\lambda)}\right] = \frac{f(k^{-1}_{sym}(t;\lambda))\frac{dk^{-1}_{sym}(t,\lambda)}{dt}}{1-F(\lambda)} - \frac{f(t)}{F(\lambda)} = \left(k^{-1}_{sym}(t;\lambda)-t\right)\frac{f(t)}{F(\lambda)} > 0,$$

where the last inequality is due to $k^{-1}_{sym}(t;\lambda) > t$ for all $t \in \left(k_{sym}(\lambda;\lambda),\lambda\right]$.

Second, I show there exist a $\tilde{t} \in [0,1]$ such that for all $t > \tilde{t}$ we have $V^A(\frac{S}{2},t) > \int_0^{\tilde{t}} F(x)dx$. By Ineq. (B.20), $V^A_{on} = F(\lambda)\frac{S}{2}$. Also, $V_{np}(0) = 0$. As shown above, $V^R(\frac{S}{2},1) < V_{np}(1)$. The interim and nonparticipation payoff are continuous in t so by the Intermediate Value Theorem, there exist a $\tilde{t} < 1$, such that $V_{np}(t) = \int_0^t F(x)dx$ crosses $V(\frac{S}{2},t)$.

Finally, by Proposition 4 of Kirkegaard (2008), any truncated off-path distribution with the support $\left[0,\bar{t}\right]$ such that $\bar{t} < 1$ would lower the common maximal bid at the off-path continuation game of conflict compared to the game with passive updating. Type 1 bids the common maximal bid. Hence, larger payoff and more incentive to not participate. ■

Proof of Proposition 3.5 Upon nonparticipation the continuation game is $\mathscr{G}\left(\tilde{F},F\right)$ where $\tilde{F}$ is the off-path belief with the support $[\underline{t},1]$. Hence, payoff of the deviating player follows Eq. (B.28) in Lemma B.13 and Eq. (B.30) in Corollary B.3, where $\lambda_i = 1$ and $\lambda_{-i} = \underline{t}$, which can be summarized as

$$U_{np}(t;\tilde{F},F) = \begin{cases} tF(k_{RA}(\underline{t};1,\underline{t})) & \text{if } t \in [0,\underline{t}] \\ \underline{t}F(k_{RA}(\underline{t};1,\underline{t})) + \int_{\underline{t}}^t F(k_{RA}(t;1,\underline{t}))dt & \text{if } t \in [\underline{t},1], \end{cases} \tag{B.44}$$

where $k_{RA}(t;1,\underline{t})$ is defined by Eq. (B.26) where $\lambda_i = 1$ and $\lambda_{-i} = \underline{t}$. Replace λ with $\underline{t}$ and use the notation of Eq. (B.37) to define $k_{sym}(t;\underline{t}) = k_{RA}(t;\underline{t},\underline{t})$. Similarly by Eq. (B.45) define $k_{lop}(t;\underline{t}) = k_{RA}(t;1,\underline{t})$ by replacing λ with $\underline{t}$. Hence, Lemma B.18 applies, where by claim i of it $k_{RA}(\underline{t},\underline{t},\underline{t})$ is strictly increasing in $\underline{t}$, by claim (ii) of it $k_{RA}(\underline{t};1,\underline{t})$ is strictly increasing in $\underline{t}$, and by claim (iii) of it $k_{RA}(\underline{t},1,\underline{t}) = k_{RA}(\underline{t},\underline{t},\underline{t})$. Hence, for all $\underline{t} \in (\lambda,1]$, $k_{RA}(\underline{t},1,\underline{t}) = k_{RA}(\underline{t},\underline{t},\underline{t}) > k_{RA}(\lambda,\lambda,\lambda)$. By the upper branch of Eq. (B.44) for all $t \in [0,\underline{t}]$ the nonparticipation payoff is linear in t. Upon participation, for type $t = 0$ Eq. (B.18) coupled with Eqs. (B.40), (B.41), and (B.44) implies that the interim payoff of announcing Accept is equal to $V^A_{on}\left(\frac{S}{2},0\right) = \frac{F(\lambda)S}{2}$ which is bigger than nonparticipation payoff for this type is equal to $U_{np}(0,\tilde{F},F) = 0$. Also, at $t = \lambda$ by Eqs. (B.37) and (B.42) we have $V^R_{on}\left(\frac{S}{2},\lambda\right) = \lambda F(k_{RA}(\lambda;\lambda,\lambda))$ and by Eq. (B.44), $U_{np}(\lambda;\tilde{F},F) = \lambda F(k_{RA}(\underline{t};1,\underline{t}))$. Note that $V^R_{on}\left(\frac{S}{2},\lambda\right) = \lambda F(k_{RA}(\lambda;\lambda,\lambda)) < \lambda F(k_{RA}(\underline{t};1,\underline{t})) = \lambda F(k_{RA}(\underline{t},\underline{t},\underline{t})) = U_{np}(\lambda,\tilde{F},F)$, where the inequality is by strictly monotone property of $k_{RA}(\underline{t},\underline{t},\underline{t})$ in $\underline{t}$ (Lemma B.18-(i)) and $\underline{t} > \lambda$. Hence, type $t = \lambda$ is worse off by participation. Thus, by continuity of the payoff functions and the Intermediate Value Theorem one can conclude there are mid-range types in the neighborhood of $t = \lambda$ that do not have incentive to not participate. Since, the equilibrium is symmetric, randomization cannot help to secure participation. ■

Proof of Theorem 3.3 Proof is an immediate implication of Propositions 3.4 and 3.5. ■

B.7.4 Lopsided Equilibrium

A tuple $\left(\nu_i, S-\nu_i, \sigma_i(\nu_i;t), F_i^A, F_i^R, \tilde{F}_i^{np}\right)_{i=1}^2$ constitutes a fully participating Lopsided PBE if and only if it satisfies (3.31), (B.20), (B.21), and (B.24). Let (ν_1, ν_2) be any peace proposal. Relabeling the players if necessary, suppose $\nu_2 < \nu_1$. Hence, without loss of generality, I denote the favored player as 1 and the less favored one as 2. Strategies defined by Eq. (3.31) uniquely pin down ex-ante probability of Accept F_i^A and the associated posteriors F_i^A and F_i^R for each player i. The cutoff type of player 2, denoted by λ_{lop}, is indifferent between Accept and Reject. Thus, (B.20) and (B.21) hold with equality for her.

Since by Eq. (3.31), player 1 always announces Accept, conflict happens on the path of equilibrium only if player 2 announces Reject. At this event, $F_1^A = F$ and F_2^R has the support $[\lambda_{lop}, 1]$. At the off-path event that player 1 announces Reject, the off-path posterior is denoted by $\tilde{F}_1^R$. At the continuation game $\mathscr{G}\left(F, F_2^R\right)$ the pivotal type function is given Eq. (B.25), or Eq. (B.26), by setting $\lambda_i = 1$ and $\lambda_{-i} = \lambda$. Hence, denote

$$k_{lop}(t;\lambda) := k_{RA}(t;1,\lambda), \tag{B.45}$$

where $k_{RA}(t;1,\lambda)$ is characterized by (B.26). Setting $\lambda_i = 1$ and $\lambda_{-i} = \lambda$ in Eqs. (B.25) and (B.26):

$$\frac{d}{dt}k_{lop}(t;\lambda) = \left(\frac{1}{1-F(\lambda)}\right)\left(\frac{k_{lop}(t;\lambda)f(t)}{tf(k_{lop}(t;\lambda))}\right) \tag{B.46}$$

with the boundary condition $k_{lop}(1;\lambda) = 1$, or equivalently

$$\int_{k_{lop}(t;\lambda)}^1 \frac{f(x)}{x}dx = \frac{1}{1-F(\lambda)}\int_t^1 \frac{f(x)}{x}dx \tag{B.47}$$

The expected payoffs can be directly derived from Lemma B.13 by setting $\lambda_i = 1$ and $\lambda_{-i} = \lambda$:

$$U_{AR}(t;F,F_2^R) = \begin{cases} 0, & \text{if } t \in \left[0, k_{lop}(\lambda;\lambda)\right], \\ \frac{\int_{k_{lop}(\lambda;\lambda)}^1 F\left(k_{lop}^{-1}(t;\lambda) - F(\lambda)\right)dt}{1-F(\lambda)} & \text{if } t \in \left[k_{lop}(\lambda;\lambda), 1\right], \end{cases} \tag{B.48}$$

and

$$U_{RA}(t;F_2^R,F) = \lambda F(k_{lop}(\lambda;\lambda)) + \int_\lambda^t F(k_{lop}(t;\lambda))dt \text{ for all } t \in [\lambda,1], \tag{B.49}$$

Thus, the interim expected payoff of Accept and Reject by Eqs. (B.20) amd (B.21) are

$$V_1^A(\nu_1,t) = \begin{cases} F(\lambda)\nu_1 & \text{if } t \in \left[0, k_{lop}(\lambda;\lambda)\right] \\ F(\lambda)\nu_1 + (1-F(\lambda))\,U_{\text{AR}}(t_1,F,F_2^R) & \text{if } t \in \left[k_{lop}(\lambda;\lambda), \lambda\right], \end{cases} \tag{B.50}$$

$$V_2^A(\nu_2,t) = \nu_2 \quad \text{if} \quad t \in [0,\lambda], \tag{B.51}$$

$$V_2^R(\nu_2,t) = U_{RA}(t_2;F_2^R,F^A) \quad \text{if} \quad t \in [\lambda,1] \tag{B.52}$$

Thus equilibrium conditions (B.20) and (B.21) for all $t \in [0,1]$ can be rewritten as :

$$\underbrace{F(\lambda)\nu_1 + (1-F(\lambda))\,U_{\text{AR}}(t,F,F_2^R)}_{=V_1^A(\nu_1,t)} \geq \underbrace{F(\lambda)U_{RA}(t;\tilde{F}_1^R,F_2^A) + (1-F(\lambda))\,U_{RR}(t;\tilde{F}_1^R,F_2^R)}_{=V_1^R(\nu_1,t)}, \tag{B.53}$$

$$\nu_2 \geq U_{RA}(t;F_2^R,F_1^A), \quad \text{for all } t \in [0,\lambda], \tag{B.54}$$

$$U_{\text{RA}}(t;F_2^R,F_1^A) \geq \nu_2, \quad \text{for all } t \in [\lambda,1]. \tag{B.55}$$

The conflict payoff of the cutoff type by Eq. (B.49) is:

$$U_{RA}(t_2 = \lambda; F_2^R, F) = \lambda F(k_{lop}(\lambda; \lambda)). \quad \text{(B.56)}$$

Hence, the cutoff type of player 2 is indifferent between Accept and Reject if and only if

$$\nu_2 = U_{RA}(t_2 = \lambda; F_2^R, F) = \lambda F(k_{lop}(\lambda; \lambda)). \quad \text{(B.57)}$$

Therefore, given (ν_1, ν_2) strategies defined by Eq. (3.31) constitutes a fully participating Lopsided PBE if and only if conditions Ineqs. (B.53) - (B.55), (B.57), and full participation constraint (B.24) simultaneously hold.

I assume that if player 1 deviates to Reject, the off-path posterior distribution of player 1 is δ_1 with support $\{1\}$, i.e. $\tilde{F}_1^R = \delta_1$. Thus at such an event, the off-path continuation games are $\mathscr{G}(\delta_1, F_2^A)$ and $\mathscr{G}(\delta_1, F_2^R)$. By a direct implication of Corollary 3 of Zheng (2019):

$$C^*_{RA}(1; \lambda) := U_{RA}(1, \delta_1, F_2^A) = \inf\left\{ c \in [0,1] : \int_c^1 \frac{1}{F^{-1}\left(sF(\lambda)\right)} ds \leq 1 \right\}, \quad \text{(B.58)}$$

$$C^*_{RR}(1, \lambda) := U_{RR}(1, \delta_1, F_2^R) = \inf\left\{ c \in [0,1] : \int_c^1 \frac{1}{F^{-1}\Big(s\left(1 - F(\lambda)\right) + F(\lambda)\Big)} ds \leq 1 \right\}. \quad \text{(B.59)}$$

Hence, for any $\lambda \in [0,1]$, $C^*_{RA}(1; \lambda)$ and $C^*_{RR}(1; \lambda)$ are function of primitives. Denote,

$$S' = C^*_{RA}(1, \tilde{\lambda}) \quad \text{where} \quad \tilde{\lambda} \quad \text{satsifies} \quad F(\tilde{\lambda}) = \frac{1}{2}. \quad \text{(B.60)}$$

By Eq. (B.58), $C^*_{RA}(1; \lambda) \leq 1$ for all $\lambda \in [0,1]$. S' is function of the primitives and $S' \leq 1$.

Lemma B.15 *Suppose off-path belief δ_1. Conditional on participation, there exist lopsided proposal (ν_1, ν_2) such that $\nu_1 + \nu_2 = S$, $\nu_2 < \nu_1$, and it simultaneously satisfies:*

$$\nu_2 = \lambda F(k_{lop}(\lambda; \lambda)), \quad \text{(B.61)}$$

$$F(\lambda_{lop})\nu_1 + (1 - F(\lambda))U_{AR}(1; F, F_2^R) = F(\lambda)C^*_{RA}(1; \lambda) + (1 - F(\lambda_{lop}))C^*_{RA}(1; \lambda), \quad \text{(B.62)}$$

where $C^*_{RA}(1; \lambda)$ and $C^*_{RR}(1; \lambda)$ are defined by Eqs. (B.58) and (B.59). Moreover, if $S \geq S'$, where S' is defined by Eq. (B.60), lopsided proposal (ν_1, ν_2) that satisfies Eqs. (B.61) and (B.62) has the property that it admits a cutoff type λ such that $F(\lambda) > \frac{1}{2}$.

Proof By Lemma B.18-(ii), the right hand side of Eq. (B.61) is strictly increasing in λ. At $\lambda = 0$, it equals to zero and at $\lambda = 1$ to c^*. The latter is because $\lambda F\left(k_{lop}(\lambda, \lambda)\right)$ by Eq. (B.56) is the payoff of type λ of player 2 at $\mathscr{G}\left(F, F_2^R\right)$. As $\lambda \uparrow 1$ then $F^R \uparrow \delta_1$ and $\mathscr{G}\left(F, F_2^R\right)$ converges to $\mathscr{G}\left(F, \delta_1\right)$. Therefore, $\lim_{\lambda \uparrow 1} \lambda F\left(k_{lop}(\lambda, \lambda)\right) = c^* = U_{RA}(1; \delta_1, F)$, where $c^* = U_{RA}(1; \delta_1, F)$ is characterized by (3.27). Moreover, $\nu_2 < \frac{S}{2} < c^*$ (Ineq. (3.26)). Hence, any $\nu_2 < \frac{S}{2}$ would admit a unique λ that solves Eq. (B.61). To prove the first claim of lemma it suffices to show such a λ solves Eq. (B.62).

By Lemma B.18-(i), Eqs. (B.27), (B.58), and (B.59), both left and right hand side of Eq. (B.62) are continuous function in λ. At $\lambda = 0$, $F^R = F$. Also, at $\lambda = 0$, the left hand side of Eq. (B.62) equates $U_{AR}(1; F, F)$, and that of right hand side equals to $C^*_{RR}(1, 0)$ defined by Eq. (B.59). Observe that, $U_{AR}(1; F, F) > U_{RR}(1; \delta_1, F) = C^*_{RR}(1, 0)$, where the inequality is due to the observation that for type $t = 1$, δ_1 is the worst belief for type $t = 1$.

At $\lambda = 1$, we have $F^A = F$. Hence, the left hand side of Eq. (B.62) equates $\nu_1 = S - \nu_2$, and its right hand side is equal to $C^*_{RA}(1; 1)$ defined by Eq. (B.58). Note that $\lim_{\lambda \uparrow 1}(S - \nu_2) = S - \lim_{\lambda \uparrow 1} \lambda F\left(k_{lop}(\lambda, \lambda)\right) = S - c^* < c^* = C^*_{RA}(1; 1)$, where $\lim_{\lambda \uparrow 1} \lambda F\left(k_{lop}(\lambda, \lambda)\right) = c^* = U_{RA}(1; \delta_1, F)$ as shown above. The inequality is by Ineq. (3.26). In brief, at $\lambda = 0$, the left hand side of Eq. (B.62) is strictly greater than its right hand side and at $\lambda = 1$ the reverse holds. Both of these terms are continuous. Hence, by Intermediate Value Theorem there exists a cutoff type λ that solves this equation. Thus, by Eq. (B.61) and $\nu_1 + \nu_2 = S$, there exists a (ν_1, ν_2) that simultaneously satisfy Eqs. (B.61) and (B.62). This proves the first claim of this Lemma.

To prove the last claim of lemma, suppose $\tilde{\lambda}$ is such that $F(\tilde{\lambda}) = \frac{1}{2}$. As shown above, at $\lambda = 1$ the left hand side of Eq. (B.62) is strictly smaller than it right hand side. Hence, by Intermediate Value Theorem, it suffices to show at $\lambda = \tilde{\lambda}$ the reverse holds such that a λ that solves Eq. (B.62) is $\lambda > \tilde{\lambda}$ and as such $F(\lambda) > F(\tilde{\lambda}) > 1/2$ as claimed in this Lemma. In other words, by plugging in for $\nu_1 = S - \tilde{\lambda} F(k_{lop}(\tilde{\lambda}, \tilde{\lambda})$ and $F(\tilde{\lambda}) = \frac{1}{2}$ in (B.62) we want to show

$$\frac{1}{2}\left(S - \tilde{\lambda} F\left(k_{lop}(\tilde{\lambda}, \tilde{\lambda})\right)\right) + \frac{1}{2} U_{AR}(1; F, F_2^R) > \frac{1}{2} C^*_{RA}(1, \tilde{\lambda}) + \frac{1}{2} C^*_{RR}(1, \tilde{\lambda}),$$

which by plugging in for $U_{AR}(1; F, F_2^R) = U_{RA}(1; F_2^R, F) = \tilde{\lambda} F(k_{lop}(\tilde{\lambda}, \tilde{\lambda})) + \int_{\tilde{\lambda}}^1 F\left(k_{lop}(t; \tilde{\lambda})\right) dt$ in the previous inequality, the goal is to show

$$\int_{\tilde{\lambda}}^1 F\left(k_{lop}(t; \tilde{\lambda})\right) dt - C^*_{RR}(1; \tilde{\lambda}) > C^*_{RA}(1; \tilde{\lambda}) - S$$

By Eq. (B.60), $S' = C^*_{RA}(1; \tilde{\lambda})$. By assumption $S \geq S'$ in this lemma, then $C^*_{RA}(1; \tilde{\lambda}) - S \leq 0$. Hence, to show the inequality of interest, it suffices to prove

$$\int_{\tilde{\lambda}}^1 F\left(k_{lop}(t; \tilde{\lambda})\right) dt - C^*_{RR}(1; \tilde{\lambda}) > 0.$$

In fact, I show that it holds for all λ. First, δ_1 is the worst posterior belief for type $t = 1$ (Zheng, 2019), implying $C^*_{RR}(1; \lambda) = U_{RR}\left(1; \delta_1, F_2^R\right) < U_{RR}\left(1; F_2^R, F_2^R\right)$. Hence, suffices to show

$$\int_{\lambda}^1 F\left(k_{lop}(t; \lambda)\right) dt - U_{RR}\left(1; F_2^R, F_2^R\right) \geq 0.$$

Second, plugging in for $U_{RR}(1; F_2^R, F_2^R)$ by Lemma B.12 in the inequality to obtain

$$\int_{\lambda}^1 F\left(k_{lop}(t; \lambda)\right) dt - \int_{\lambda}^1 \frac{F(t) - F(\lambda)}{1 - F(\lambda)} dt = \int_{\lambda}^1 \frac{F(k_{lop}(t; \lambda)(1 - F(\lambda)) + F(\lambda) - F(t)}{1 - F(\lambda)} dt \geq 0.$$

It suffice to show the numerator of the integrand is positive. Its derivative with respect to t is

$$f\left(k_{lop}(t; \lambda)\right) \frac{d}{dt} k_{lop}(t; \lambda)(1 - F(\lambda)) - f(t) = \frac{\left(k_{lop}(t; \lambda) - t\right) f(t)}{t} < 0,$$

where the equality is by plugging in for $\frac{d}{dt} k_{lop}(t; \lambda)$ via Eq. (B.46) and the inequality is by $k_{lop}(t; \lambda) < t$ at $\mathscr{G}(F, F_2^R)$, $k_{lop}(t; \lambda) < t$ (because $k_{lop}(t; \lambda)$ is defined by Eq. (B.47), where the left hand side of it is strictly monotone in $k_{lop}(t; \lambda)$ and by $\frac{1}{1 - F(\lambda)} > 1$, then $k_{lop}(t; \lambda) < t$). Thus, the numerator of the integrand takes it minimum value at $t = 1$ and it is equal to $F(k_{lop}(1; \lambda)(1 - F(\lambda)) + F(\lambda) - F(1) = 0$, as desired. ■

Lemma B.16 *Suppose Assumption 3.4.1. Conditional on participation, given the lopsided proposals characterized in Lemma B.15, if $F(\lambda_{lop}) > 1 - c^*$, cutoff strategies (3.31) admits a Lopsided PBE.*

Proof Given proposal (ν_1, ν_2), strategies defined by Eq. (3.31) constitutes a Lopsided PBE if and only if Ineqs. (B.53)-(B.57) hold. In this proof λ refers to the equilibrium cutoff λ_{lop}. Player 2 does not deviate from Eq. (3.31). If she Reject her payoff $U_{RA}(t; F_2^R, F)$ is strictly increasing in t (by Lemma B.13), single crosses ν_2, her payoff if she Accept, at $t = \lambda$. Thus, Ineqs. (B.54)-(B.57) are satisfied for this player.

If player 1 deviates to Reject, given the off-path belief $\tilde{F}_1^R = \delta_1$, by Eqs. (B.58)-(B.59), and Lemmas (13) and (14) of Zheng (2019), her interim payoff from deviating to Reject is equal to

$$V_1^R(\nu_1, t) = t\left[F(\lambda)C_{RA}^*(1;\lambda) + (1-F(\lambda))C_{RR}^*(1;\lambda)\right]. \tag{B.63}$$

By Eq. (B.62) in Lemma B.15, $t = 1$ of player 1 does not have incentive to deviate to Reject, satisfying equilibrium condition (B.53) with equality. Also, the interim payoff of Accept function $V_1^A(\nu_1, t)$, by Eq. (B.50), is constant for all $t \in \left[0, k_{lop}(\lambda;\lambda)\right]$, strictly increasing and convex in t for all $t \in \left(k_{lop}(\lambda;\lambda), 1\right]$ (Lemma B.13), and differentiable almost everywhere. Thus, $\frac{d}{dt}V_1^A(\nu_1, t; \lambda) = F\left(k_{lop}^{-1}(t;\lambda)\right) - F(\lambda)$ if $t \in \left(k_{lop}(\lambda;\lambda), 1\right]$. Couple this with linearity of $V_1^R(\nu_1, t)$ in t (Eq. B.63); to verify equilibrium condition (B.53) is satisfied for player 1, i.e., $V_1^A(\nu_1, t) \geq V_1^R(\nu_1, t)$ for all $t \in [0, 1]$, it suffices to show that for all $t \in \left[k_{lop}(\lambda;\lambda), 1\right]$, $V_1^R(\nu_1, t) - V_1^A(\nu_1, t)$ is strictly increasing in t and single crosses zero at $t = 1$ as mentioned before. By previously mentioned facts, if at $t = 1$ the on-path payoff is less steeper than the off-path payoff then (B.53) is guaranteed. Hence, the incentive compatibility condition for player 1 is:

$$\frac{d}{dt}V_1^A(\nu_1, 1) = 1 - F(\lambda) < F(\lambda)C_{RA}^*(1;\lambda) + (1-F(\lambda))C_{RR}^*(1;\lambda) = \frac{d}{dt}V_1^R(\nu_1, 1). \tag{B.64}$$

Couple Ineq. (B.64) and Assumption 3.4.1, i.e,. $c^* < F(\lambda)C_{RA}^*(1;\lambda) + (1-F(\lambda))C_{RR}^*(1;\lambda)$, to observe the sufficient condition that implies (B.64) is $1 - F(\lambda) < c^*$, as stated in the lemma. ∎

Corollary B.5 *Suppose Assumption 3.4.1. For any* $c^* \in \left(\frac{1}{2}, 1\right)$*, and any* $S \in \left[S', 2c^*\right)$*, where* S' *is a function of primitives defined by Eq. (B.60), conditional on participation and given the lopsided proposals characterized by Lemma B.15, cutoff strategies (3.31) admits a Lopsided PBE..*

Proof By Lemma B.16, conditional on participation, the lopsided proposals characterized by Lemma B.15 admit a Lopsided PBE if $1 - F(\lambda) < c^*$. By Lemma B.15, we know that any expected peace surplus S such that $S' \leq S < 2c^*$ induces $F(\lambda_{lop}) > \frac{1}{2}$. By assumption $c^* > \frac{1}{2}$, of this Corollary, then $1 - F(\lambda_{lop}) < \frac{1}{2} < c^*$, as desired. ∎

B.7.5 Proof of Lemma 3.2 and Theorem 3.4

Proof of Lemma 3.2 In this proof λ refers to the equilibrium cutoff λ_{lop}. Label favored player as 1 and her rival as 2. By Corollary B.5, conditional on participation, the lopsided proposals characterized by Lemma B.15 admits a Lopsided PBE. Thus, at any such PBEs, neither type of any player has incentives to deviate from the cutoff strategies (3.31). Hence, player 1 always announces Accept implying $V_{on}^A(\nu_1, t) \geq V_{on}^R(\nu_1, t)$ for all types of player 1, where $V_{on}^R(\nu_1, t)$, given the off-path belief δ_1, is characterized by Eq. (B.63). If player 1 unilaterally deviates to noparticipation then her payoff by Lemma (14) of Zheng (2019) is

$$V_{np}(t) = t\tilde{U}(1, \delta_1, F) = tc^*. \tag{B.65}$$

At any Lopsided PBEs $\lambda > 0$. Thus, by Assumption 3.4.1, Eq. (B.63), and revealed preference argument, $V_{on}^A(\nu_1, t) \geq V_{on}^R(\nu_1, t) > tc^* = V_{np}(t)$. Hence, player 1 is better off by participation.

Player 2's interim payoffs is characterized by Eqs. (B.51) and (B.52). Observe: (i) for $t = 0$, $V_{on}^A(\nu_2, 0) = \nu_2 > 0 = V_{np}(0)$; (ii) at $t = \lambda$, $V_{on}^A(\nu_2, \lambda) = \nu_2 = \lambda F(k_{lop}(\lambda, \lambda)) < \lambda c^* = V_{np}(\lambda)$, where the inequality is by $\lambda F(k_{lop}(\lambda, \lambda)) < \lambda c^*$ (shown in the proof of Lemma B.15); (iii) by Zheng (2019) δ_1 is the worst belief for type $t = 1$ at $\mathscr{G}\left(\tilde{F}, F\right)$: $V_{on}^R(\nu_2, 1) = U_{RA}(1, F_2^R, F) > U(1, \delta_1, F) = c^* = V_{np}(1)$; (iv) $U_{RA}(t; F_2^R, F)$ is strictly increasing and stictly convex in t (Lemma B.13 and setting $\lambda_i = 1$ and $\lambda_{-i} = \lambda$ in Eq. (B.28)) and $V_{np}(t)$ is linear in t. Therefore, by the Intermediate Value theorem and these observations there exists $\underline{t} > 0$ such that all $t \leq \underline{t}$ are better off by participation. Also, there exists $\bar{t} < 1$ such that $\bar{t} > \underline{t}$ and $t \geq \bar{t}$ are better off by participation. Thus, $t \in (\underline{t}, \bar{t})$ are worse off by participation. ■

Lemma B.17 *Suppose the same hypothesis of Corollary B.5. If $F(\lambda_{lop}) > 2 - 2c^*$, then participation constraint (B.24) is satisfied.*

Proof In this proof , λ refers to the equilibrium cutoff λ_{lop}. By Corollary B.5, conditional on participation, lopsided proposals characterized by Lemma B.15 admit a Lopsided PBE. Couple the fact that player 1 always Accept with nonparticipation payoff characterized by Eq. (B.65), then participation constraint Ineq. (B.24) can be simplified further to $V(t) = V^A(\nu_1, t)/2 + V^m(\nu_2, t)/2 > tc^*$, where $V^m(\nu_2, t)$ in the interim payoff of player 2 for announcing some $m \in \{A, R\}$:

$$V(t) = \begin{cases} F(\lambda)\nu_1/2 + \nu_2/2, & \text{if } t \in \left[0, k_{lop}(\lambda;\lambda)\right] \\ \left[F(\lambda)\nu_1 + (1 - F(\lambda))\, U_{\text{AR}}(t_1, F, F_R)\right]/2 + \nu_2/2, & \text{if } t \in \left[k_{lop}(\lambda;\lambda), \lambda\right], \\ \left[F(\lambda)\nu_1 + (1 - F(\lambda))\, U_{\text{AR}}(t_1, F, F_R)\right]/2 + U_{\text{RA}}(t_2, F_R, F)/2, & \text{if } t \in [\lambda, 1]. \end{cases}$$

$V(t)$ is constant in t for all $t \in \left[0, k_{lop}(\lambda;\lambda)\right]$ and by direct implication of Lemma B.13 and Corollary B.3 it is strictly increasing and strictly convex in t for all $t \in \left[k_{lop}(\lambda;\lambda), 1\right]$. By Eq. (B.65), $V_{np} = tc^*$.

Using strictly increasing and strictly convex property of $V(t)$ in t for all $t \in \left[k_{lop}(\lambda;\lambda), 1\right]$ coupled with implication of Lemma 3.2 that $V(1) > c^* = V_{np}(1)$, to guarantee full participation it suffices to show $\lim_{t\uparrow 1} \frac{d}{dt}V(t) \leq c^* = \frac{d}{dt}(tc^*)$. If true, then it implies $V(t) - tc^*$ is strictly decreasing in t and always positive for all $t \in [0,1]$. By envelope theorem,

$$\frac{d}{dt}V(1) = (1 - F(\lambda))/2 + 1/2.$$

Hence,

$$(1 - F(\lambda))/2 + 1/2 \leq c^* \iff F(\lambda) \geq 2 - 2c^*,$$

as exposited in the Lemma. ■

Proof of Theorem 3.4 By Lemma B.17, if $F(\lambda_{lop}) \geq 2 - 2c^*$, then participation constraint Ineq. (B.24) is satisfied. Also, $F(\lambda_{lop}) \geq 2 - 2c^* \Rightarrow F(\lambda_{lop}) > 1 - c^*$, guaranteeing existence of Lopsided PBEs conditional on participation (Lemma B.16). By Lemma B.15, $S \geq S'$ then $F(\lambda_{lop}) > \frac{1}{2}$. Thus, to guarantee full participation, it suffices to have $2 - 2c^* \leq 1/2$, which is always true if $c^* \geq \frac{3}{4}$, as assumed in the hypothesis of this theorem. ■

B.7.6 Calculation Details

Lemma B.18 *The following properties for pivotal functions hold: (i) $\frac{dk_{sym}(\lambda;\lambda)}{d\lambda} > 0$, (ii) $\frac{dk_{lop}(\lambda;\lambda)}{d\lambda} > 0$, (iii) $\frac{dk_{lop}(t;\lambda)}{d\lambda} < 0$ for all $t \in (\lambda, 1]$ and $\frac{dk^{-1}_{lop}(t;\lambda)}{d\lambda} > 0$ for all $t \in [0, \lambda)$, and (iv) $k_{lop}(\lambda;\lambda) = k_{sym}(\lambda;\lambda)$ for all $\lambda \in [0,1]$.*

Proof Claim (i)-(ii): At $t = \lambda$, by implicit differentiation of Eq. (B.39) one can obtain:

$$\frac{dk_{sym}(\lambda;\lambda)}{d\lambda} = \frac{k_{sym}(\lambda;\lambda)f(\lambda)}{f\left(k_{sym}(\lambda;\lambda)\right)(1 - F(\lambda))}\left[\frac{1}{\lambda} - \frac{1}{1 - F(\lambda)}\int_{\lambda}^{1}\frac{f(x)}{x}dx\right].$$

The trem in the bracket is strictly positive by the strictly decreasing $\frac{1}{x}$. Thus, $\frac{dk_{sym}(\lambda;\lambda)}{d\lambda} > 0$. Analogously, at $t = \lambda$, by Eq. (B.47), one can verify $\frac{dk_{lop}(\lambda;\lambda)}{d\lambda} > 0$.

Claim (iii): By implicit differentiation of Eq. (B.47),

$$\frac{dk_{lop}(t;\lambda)}{d\lambda} = -\frac{k_{lop}(\lambda;\lambda)f(\lambda)}{f\left(k_{lop}(\lambda;\lambda)\right)\left(1-F(\lambda)\right)^2}\int_t^1 \frac{f(x)}{x}dx,$$

where all the terms are strictly positive and hence $\frac{dk_{lop}(t;\lambda)}{d\lambda} < 0$. $k_{lop}^{-1}(t;\lambda)$ is characterized analogous to Eq. (B.47), $\int_t^1 \frac{f(x)}{x}dx = \frac{1}{1-F(\lambda)}\int_{k_{lop}^{-1}(t;\lambda)}^1 \frac{f(x)}{x}dx$. Thus, by implicit differentiation

$$\frac{dk_{lop}^{-1}(t;\lambda)}{d\lambda} = \frac{k_{lop}^{-1}(t;\lambda)f(\lambda)}{f\left(k_{lop}^{-1}(t;\lambda)\right)\left(1-F(\lambda)\right)}\int_{k_{lop}^{-1}(t;\lambda)}^1 \frac{f(x)}{x}dx, \tag{B.66}$$

where all the terms are strictly positive and hence $\frac{dk_{lop}^{-1}(t;\lambda)}{d\lambda} > 0$.

Claim (iv): By Eq. (B.39) at $t = \lambda$,

$$\frac{1}{F(\lambda)}\int_{k_{sym}(\lambda;\lambda)}^{\lambda} \frac{f(x)}{x}dx = \frac{1}{1-F(\lambda)}\int_{\lambda}^1 \frac{f(x)}{x}dx.$$

Denote $G(x) := \int \frac{f(x)}{x}dx$. Then, $G(k_{sym}(\lambda;\lambda)) = \frac{G(\lambda)-F(\lambda)G(1)}{1-F(\lambda)}$. By Eq. (B.47) at $t = \lambda$,

$$\int_{k_{lop}(\lambda;\lambda)}^1 \frac{f(x)}{x}dx = \frac{1}{1-F(\lambda)}\int_{\lambda}^1 \frac{f(x)}{x}dx.$$

Then, $G(k_{lop}(\lambda;\lambda)) = \frac{G(\lambda)-F(\lambda)G(1)}{1-F(\lambda)}$ and $G(k_{sym}(\lambda;\lambda)) = G(k_{lop}(\lambda;\lambda))$. By positive density assumption, $\frac{dG(x)}{dx} = \frac{f(x)}{x} > 0$. Thus, $G(k_{sym}(\lambda;\lambda)) = G(k_{lop}(\lambda;\lambda)) \iff k_{sym}(\lambda;\lambda)) = k_{lop}(\lambda;\lambda))$ for all λ. ■

Lemma B.19 *Suppose the prior CDF $F(t) = t^\alpha$, where $\alpha \in (0,1)$ and $t \in [0,1]$. Then, $C_{RA}^*(1;\lambda) = \left(\frac{\alpha}{\alpha+(1-\alpha)\lambda}\right)^{\frac{\alpha}{1-\alpha}}$ and $C_{RR}^*(1;\lambda) = \frac{1}{1-F(\lambda)}\left(\frac{\alpha}{\alpha+(1-\alpha)\left(1-F(\lambda)\right)}\right)^{\frac{\alpha}{1-\alpha}} - \frac{F(\lambda)}{1-F(\lambda)}$.*

Proof $C_{RA}^*(1;\lambda)$ and $C_{RR}^*(1;\lambda)$, defined by Eqs. (B.58) and (B.59), are payoff of type $t = 1$ at $\mathscr{G}(\delta_1, \tilde{F}_{-i})$, where $\tilde{F}_{-i} \in \{F_{-i}^A, F_{-i}^R\}$. These payoffs by Lemmas (13) and (14) of Zheng (2019), equates to $c_{-i} \in [0,1]$, which is the solution to the set of equations $c_i c_{-i} = 0$ and

$$1 - c_i = \int_{c_{-i}}^1 \frac{1}{\tilde{F}_{-i}^{-1}(s)}ds,$$

where $\tilde{F}_{-i}^{-1}$ is the inverse CDF. First, I characterize $C_{RA}^*(1;\lambda)$. $\tilde{F}_{-i}^A = \frac{F(t)}{F(\lambda)}$ with support $[0,\lambda]$. Hence, the set of equations is $c_i c_{-i} = 0$ and

$$1 - c_i = \int_{c_{-i}}^1 \frac{1}{\tilde{F}_{-i}^{-1}(sF(\lambda))}ds \iff c_i = 1 - \frac{\alpha}{(1-\alpha)\lambda}\left(c_{-i}^{\frac{\alpha-1}{\alpha}} - 1\right). \tag{B.67}$$

By $c_i c_{-i} = 0$, we know at most one of c_i or c_{-i} could be strictly positive. $c_{-i} \neq 0$, otherwise, $c_i = 1 + \frac{\alpha}{(1-\alpha)\lambda} > 1$, a contradiction to $c_i \in [0,1]$. Hence, $c_i = 0$ and by Eq. (B.67) one can characterize

$$C_{RA}^*(1;\lambda) = c_{-i} = \left(\frac{\alpha}{\alpha+(1-\alpha)\lambda}\right)^{\frac{\alpha}{1-\alpha}},$$

as exposited in the Lemma.

Second, I characterizes payoff $C^*_{RR}(1;\lambda)$. $\tilde{F}^R_{-i} = \frac{F(t)-F(\lambda)}{1-F(\lambda)}$ with support $[\lambda,1]$. I initially assume $c_i = 0$, characterize c_{-i}, and later verify that $c_{-i} \in (0,1)$. Thus, c_{-i} solves

$$1 = \int_{c_{-i}}^{1} \frac{1}{\left(s(1-F(\lambda))+F(\lambda)\right)^{\frac{1}{\alpha}}} ds = \frac{\alpha}{1-\alpha}\left[\frac{\left(c_{-i}-c_{-i}\lambda^\alpha+\lambda^\alpha\right)^{\frac{1-\alpha}{\alpha}}-1}{\left(\lambda^\alpha-1\right)\left(c_{-i}-c_{-i}\lambda^\alpha+\lambda^\alpha\right)^{\frac{1-\alpha}{\alpha}}}\right].$$

Hence,

$$c_{-i} = \frac{1-\lambda^\alpha\left[1+\frac{(1-\alpha)}{\alpha}(1-\lambda^\alpha)\right]^{\frac{\alpha}{1-\alpha}}}{(1-\lambda^\alpha)\left[1+\frac{(1-\alpha)}{\alpha}(1-\lambda^\alpha)\right]^{\frac{\alpha}{1-\alpha}}}.$$

Next, I verify that $0 < c_{-i} < 1$. $c_{-i} < 1$ is immediate. To see that $c_{-i} > 0$, by $\lambda \in [0,1]$ and $\alpha \in (0,1)$, it suffices to show the numerator of its displayed equation is positive. One can easily verify this term is strictly decreasing in λ and take its minimum value at $\lambda = 1$ and equates to 0. Thus, for all $\lambda \in (0,1)$, we have $0 < c_{-i} < 1$, as desired. Hence, $C^*_{RR}(1;\lambda) = c_{-i}$, as stated in the lemma. ■

Lemma B.20 *Suppose the prior distribution has CDF $F(t) = t^\alpha$, where $\alpha \in (0,1)$ and $t \in [0,1]$. Then $F(\lambda)C^*_{RA}(1;\lambda) + (1-F(\lambda))C^*_{RR}(1;\lambda) \geq c^*$ for all $\lambda \in (0,1)$.*

Proof By Lemma B.19, the inequality of interest exposited in statement of this lemma is $\left(\frac{\alpha}{\alpha+(1-\alpha)(1-F(\lambda))}\right)^{\frac{\alpha}{1-\alpha}} - c^* > F(\lambda)(1-C^*_{RA}(1;\lambda))$, where c^* is defined by Eq. (3.27). Denote:

$$\zeta(\lambda) := \left(\frac{\alpha}{\alpha+(1-\alpha)(1-F(\lambda))}\right)^{\frac{\alpha}{1-\alpha}}. \tag{B.68}$$

Hence, denote

$$\mathscr{T}(\lambda) := \zeta(\lambda) - c^* - \left(F(\lambda)(1-C^*_{RA}(\lambda))\right).$$

The goal is to show that $\mathscr{T}(\lambda) \geq 0$. c^* can also can be characterized by $c^* = U(1;\delta_1,F) = \lim_{\lambda\uparrow 1} U(1;\delta_1,F^A) = \lim_{\lambda\uparrow 1} C^*_{RA}(1;\lambda) = \alpha^{\frac{\alpha}{1-\alpha}}$. This is because as $\lambda \uparrow 1$, $F_A = \frac{F(t)}{F(\lambda)}$ with the support $[0,\lambda]$ converges to F. So to prove this lemma, one need to check the slope and curvature of $\mathscr{T}(\lambda)$. It can be readily show,

$$\frac{d}{d\lambda}\zeta(\lambda) = f(\lambda)\zeta(\lambda)^{\frac{1}{\alpha}}, \tag{B.69}$$

$$\frac{d}{d\lambda}C^*_{RA}(\lambda) = -\left(C^*_{RA}(\lambda)\right)^{\frac{1}{\alpha}}, \tag{B.70}$$

Denote,

$$\eta(\lambda) := \zeta(\lambda)^{\frac{1}{\alpha}}$$

and

$$\omega(\lambda) := 1-(1-\lambda)\left(C^*_{RA}(\lambda)\right)^{\frac{1}{\alpha}}.$$

Thus,

$$\frac{d}{d\lambda}\mathscr{T}(\lambda) = f(\lambda)\left[\eta(\lambda)-\omega(\lambda)\right].$$

By Eq. (B.68), it is immediate that $\mathscr{T}(0) = 0$, $\mathscr{T}(1) = 0$, and $\mathscr{T}(\lambda)$ has positive slope at $\lambda = 0$ and zero at $\lambda = 1$. Hence, to verify $\mathscr{T}(\lambda) \geq 0$ for all $\lambda \in [0,1]$, I show $\mathscr{T}(\lambda)$ has a unique global max. To that end, by $f(t) > 0$ maintained in this chapter and observations (B.71)-(B.74), it suffices to show $\eta(\lambda) - \omega(\lambda) = 0$ has a unique root $\lambda^* \in (0,1)$. Note that,

$$\frac{d}{d\lambda}\eta(\lambda) = \frac{d}{d\lambda}(\zeta(\lambda)^{\frac{1}{\alpha}}) = \frac{f(\lambda)}{\alpha}(\zeta(\lambda))^{\frac{2-\alpha}{\alpha}} > 0, \tag{B.71}$$

$$\frac{d}{d\lambda}\omega(\lambda) = \frac{d}{d\lambda}\left(1-(1-\lambda)\left(C^*_{RA}(\lambda)\right)^{\frac{1}{\alpha}}\right) = \left(C^*_{RA}(\lambda)\right)^{\frac{1}{\alpha}}\left[\frac{1+\alpha(1-\lambda)}{\alpha+\lambda(1-\alpha)}\right] > 0, \tag{B.72}$$

$$\lim_{\lambda\downarrow 0}\frac{d}{d\lambda}\eta(\lambda) = +\infty > \frac{\alpha+1}{\alpha} = \lim_{\lambda\downarrow 0}\frac{d}{d\lambda}\omega(\lambda), \tag{B.73}$$

$$\lim_{\lambda\uparrow 1}\frac{d}{d\lambda}\eta(\lambda) = 1 > \alpha^{\frac{1}{1-\alpha}} = \lim_{\lambda\uparrow 1}\frac{d}{d\lambda}\omega(\lambda), \tag{B.74}$$

which coupled with the observations $\eta(0) = \alpha^{\frac{1}{1-\alpha}} > 0 = \omega(0)$ and $\eta(1) = 1 = \omega(1)$, implies that term $\omega(\lambda)$ starts from below the term $\eta(\lambda)$. Because they are both strictly increasing in λ, the slope of the latter at $\lambda = 1$ is higher than the former, and both have the same value at that $\lambda = 1$, it implies that $\eta(\lambda)$ approaches $\omega(\lambda)$ from below till they cross at $\lambda = 1$. Hence, there exist a neighborhood around $\lambda = 1$ where, $\eta(\lambda)$ is below $\omega(\lambda)$. Thus, by intermediate value theorem there exist $\lambda^* \in (0,1)$ at which these two functions crosses each other.

The goal is to show this root is unique. By Eq. (B.72), slope of $\omega(\lambda)$ is strictly positive and by Claim B.2 this slope is a strictly decreasing and convex function of λ. By Claim B.3, $\eta(\lambda)$ is strictly increasing in λ, it has a unique saddle point, and it's derivative has a unique global minimum. By Ineqs. (B.73) and (B.74), term $\eta(\lambda)$ has larger slope at both $\lambda = 0$ and $\lambda = 1$. At $\lambda = \lambda^*$ where is the first time that $\omega(\lambda)$ crosses $\eta(\lambda)$ from below by the strictly increasing property of these function, it is necessary that $\lim_{\lambda\to\lambda^*}\frac{d}{d\lambda}\omega(\lambda) > \lim_{\lambda\to\lambda^*}\frac{d}{d\lambda}\eta(\lambda)$. Hence, by the fact that $\frac{d}{d\lambda}\eta(\lambda)$ is U shape and $\frac{d}{d\lambda}\omega(\lambda)$ is strictly increasing and convex in λ it is necessary that these two function crosses each other exactly two times. Denote these points by λ_1 and λ_2. By the above mentioned observations on the curvature of the derivative functions, $0 < \lambda_1 < \lambda^*$ and $\lambda^* < \lambda_2 < 1$. Therefore, $\eta(\lambda) - \omega(\lambda)$ has one local maximum at $\lambda = \lambda_1$ and one local minimum at $\lambda = \lambda_2$. Hence, after the first time $\lambda = \lambda^*$ that $\omega(\lambda)$ crosses $\eta(\lambda)$, it remains above it till $\lambda = 1$ where they equates. Thus, λ^* is unique. ■

Claim B.2 *Denote* $\omega(\lambda) := 1-(1-\lambda)\left(C^*_{RA}(\lambda)\right)^{\frac{1}{\alpha}}$. *Then,* $\frac{d^2}{d\lambda^2}\omega(\lambda) < 0$ *and* $\frac{d^3}{d\lambda^3}\omega(\lambda) > 0$.

Proof By Eqs. (B.70) and (B.72),

$$\frac{d^2}{d\lambda^2}\omega(\lambda) = -\frac{\left(C^*_{RA}(\lambda)\right)^{\frac{2-\alpha}{\alpha}}}{\alpha}\left[\frac{1+\alpha(1-\lambda)}{\alpha+\lambda(1-\alpha)}\right] - \left(C^*_{RA}(\lambda)\right)^{\frac{1}{\alpha}}\frac{1}{(\alpha+(1-\alpha)\lambda)^2} < 0.$$

Moreover, using the above displayed equation coupled with Eqs. (B.70) and (B.72):

$$\frac{d^3}{d\lambda^3}\omega(\lambda) = -\frac{2-\alpha}{\alpha^2}\left(C^*_{RA}(\lambda)\right)^{\frac{2-2\alpha}{\alpha}}\frac{d}{d\lambda}(C^*_{RA}(\lambda))\left[\frac{1+\alpha(1-\lambda)}{\alpha+\lambda(1-\alpha)}\right] - \frac{\left(C^*_{RA}(\lambda)\right)^{\frac{2-\alpha}{\alpha}}}{\alpha^2}\left[\frac{-1}{(\alpha+\lambda(1-\alpha))^2}\right] -$$
$$\frac{\left(C^*_{RA}(\lambda)\right)^{\frac{1-\alpha}{\alpha}}}{\alpha}\frac{d}{d\lambda}(C^*_{RA}(\lambda))\frac{1}{(\alpha+(1-\alpha)\lambda)^2} + \left(C^*_{RA}(\lambda)\right)^{\frac{1}{\alpha}}\frac{2(1-\alpha)}{(\alpha+(1-\alpha)\lambda)^3} > 0.$$

The sign is by $C^*_{RA} > 0$ (Eq. B.58), $\alpha \in (0,1)$, $\lambda \in [0,1]$, and $\frac{d}{d\lambda}(C^*_{RA}(\lambda)) < 0$ (Eq. B.70). ■

Claim B.3 *Denote* $\eta(\lambda) := \zeta(\lambda)^{\frac{1}{\alpha}}$. $\eta(\lambda)$ *is strictly increasing in* λ*, it has a unique saddle point, and it's slope has a global minimum.*

Proof By Eq. (B.71), $\frac{d}{d\lambda}\eta(\lambda) > 0$. Also, by Eqs. (B.69) and (B.71), obtain

$$\frac{d^2}{d\lambda^2}\eta(\lambda) = \frac{1}{\alpha}\zeta(\lambda)^{\frac{3-2\alpha}{\alpha}}\lambda^{\alpha-2}(F(\lambda) + \alpha - 1)).$$

By $F(t) = t^\alpha$ and $\alpha \in (0, 1]$, there exist a unique $\tilde{\lambda}$ that solves $F(\tilde{\lambda}) = 1 - \alpha$, where for $\lambda < \tilde{\lambda}$, $\frac{d^2}{d\lambda^2}\eta(\lambda) < 0$ and for $\lambda \geq \tilde{\lambda}$ the reverse holds. Also,

$$\frac{d^3}{d\lambda^3}\eta(\lambda) = \frac{\lambda^{\alpha-2}}{\alpha}\zeta(\lambda)^{\frac{3-2\alpha}{\alpha}}\left[\frac{(2-\alpha)\left(\lambda^{2\alpha} + (\alpha^2 + \alpha - 2)\lambda^\alpha + 1 - \alpha\right)}{\alpha + (1-\alpha)(1 - F(\lambda))}\right] > 0,$$

where the sign is because $\alpha \in (0, 1)$, $\lambda \in (0, 1)$, $\eta(\lambda) > 0$ by (B.68), and the term

$$\lambda^{2\alpha} + (\alpha^2 + \alpha - 2)\lambda^\alpha + 1 - \alpha = \left(\lambda^\alpha - \frac{(1-\alpha)(\alpha+2)}{2}\right)^2 + \frac{(1-\alpha)\alpha^2(\alpha+3)}{4} > 0.$$

Hence, $\eta(\lambda)$ has the unique saddle point $\tilde{\lambda}$, and its slope is convex in λ and U shape. ■

Bibliography

Amann, E. and W. Leininger (1996). Asymmetric all-pay auctions with incomplete information: the two-player case. *Games and economic behavior 14*(1), 1–18.

Kirkegaard, R. (2008). Comparative statics and welfare in heterogeneous all-pay auctions: Bribes, caps, and performance thresholds. *The BE Journal of Theoretical Economics 8*(1), 1–32.

Zheng, C. Z. (2019). Necessary and sufficient conditions for peace: Implementability versus security. *Journal of Economic Theory 180*, 135–166.

Appendix C

Appendices to Chapter 4

C.1 Categorization of All Renege-Proof Equilibria

As mentioned in Section 4.3.1, in a renege-proof PBE, either both players ratify a proposal at the rectification stage, or a type that can renege on a proposal at the rectification stage has rejected the proposal, to begin with, at the mediation stage. Therefore all possible cases of renege-proof PBEs can be summarized as

a Both players ratify the peace proposal at the rectification stage.

b One player unilaterally rejects the peace proposal. Call this always-conflict.

c One player always ratifies the peace proposal while her opponent ratifies if she is the weak type and rejects the proposal if she is the strong type.

d One player always ratifies the peace proposal while her opponent ratifies if she is the strong type and rejects the proposal if she is the weak type.

e Both players always ratify if they are the weak type and reject the proposal if they are the strong type.

f Both players always ratify if they are the strong type and reject the proposal if they are the weak type.

Lemma C.1 *There does not exist any Case-(a) PBEs.*

Proof A Case-(a) PBE exists if both players ratify, which happens if and only if $x_i \geq \max\{\pi_i^D, \pi_{-i}^A\}$ and $x_i \geq \left(\pi_{-i}^A - \pi_i^D\right)^+$. Then player i's interim-expected-payoff differences between rejecting and accepting the proposal (4.16) will be simplified to

$$\begin{bmatrix} \Delta_i(s) \\ \Delta_i(w) \end{bmatrix} = q_{-i}^A \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^A\right\} - x_i \\ \left(\pi_{-i}^A - \pi_i^R\right)^+ - x_i \end{bmatrix} + q_{-i}^R \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^R\right\} - \max\left\{\pi_i^A, \pi_{-i}^R\right\} \\ \left(\pi_{-i}^R - \pi_i^R\right)^+ - \left(\pi_{-i}^R - \pi_i^A\right)^+ \end{bmatrix},$$

which is isomorphic to that of the Renege-Banning model of KZ, where they show in their Lemma 3 that for all possible classes of PBEs $\pi_i^R \leq \theta \leq \pi_i^A$. By $\max\{\pi_i^D, \pi_{-i}^A\} \geq \left(\pi_{-i}^A - \pi_i^D\right)^+$, that each type of each player ratifies if $x_i \geq \max\{\pi_i^D, \pi_{-i}^A\}$. Thus, given the observation by Lemma 3 of KZ that $\pi_{-i}^A \geq \theta$, each type of each player ratifies if $x_i \geq \pi_i^A \geq \theta$. This implies that, $x_i + x_{-i} = r \geq 2\theta$ violating the maintained assumption (4.15). Therefore, this class of PBE cannot exist.

Lemma C.2 *There always exists a renege-proof PBE where one player unilaterally rejects a peace proposal. In such always-conflict PBEs, the on-path posterior is equal to the prior for both players. Moreover, the social surplus admitted by the PBE is $2\theta(1-\theta)/r$.*

Proof Let $q_i^R = 1$ for some player i, such that player i unilaterally announces Reject. Then the on-path posterior about player i is $\pi_i^R = \theta$. For the convenience and without loss of generality denote $i := 2$. Then, since player 2 unilaterally rejects, we have $\sigma_2(s) = \sigma_2(w) = 1$, which imply that $\pi_2^R = \theta$, $q_2^R = 1$, and $q_2^A = 0$. Hence, Eq. (4.16) for player 2 would be simplified to:

$$\begin{bmatrix} \Delta_1(s) \\ \Delta_1(w) \end{bmatrix} = q_2^R \begin{bmatrix} \max\left\{\pi_1^R, \pi_2^R\right\} - \max\left\{\pi_1^A, \pi_2^R\right\} \\ \left(\pi_2^R - \pi_1^R\right)^+ - \left(\pi_2^R - \pi_1^A\right)^+ \end{bmatrix} = \begin{bmatrix} \max\left\{\pi_1^R, \theta\right\} - \max\left\{\pi_1^A, \theta\right\} \\ \left(\theta - \pi_1^R\right)^+ - \left(\theta - \pi_1^A\right)^+ \end{bmatrix}. \tag{C.1}$$

We claim that the posterior probability π_1 with which player 1's type is equal to w is the same as the prior: $\pi_1 = \theta$. Suppose otherwise. We derive a contradiction for all possibilities:

1. $\sigma_1(w) = 0$. Then $\sigma_1(s) > 0$, otherwise the claim $\pi_1 = \theta$ is true. Thus, $\pi_1^A > \theta$ by Eq. (4.5) applied to 1, and $\Delta_1(s) \geq 0$. Then Eq. (C.1) implies $\pi_1^R \geq \pi_1^A > \theta$. But since $\sigma_1(w) = 0$ and $\sigma_1(s) > 0$, $\pi_1^R = 0$ by Bayes's rule: contradiction.

2. $\sigma_1(w) = 1$. Then $\sigma_1(s) < 1$, otherwise the claim $\pi_1 = \theta$ is true. Thus, $\pi_1^R > \theta$ by Eq. (4.6) applied to 1, and $\Delta_1(w) \geq 0$. Then Eq. (C.1) implies $\pi_2^A \geq \theta$. But since $\sigma_1(w) = 1$ and $\sigma_1(s) < 1$, $\pi_1^A = 0$ by Bayes's rule: contradiction.

3. $0 < \sigma_1(w) < 1$. Then Eq. (4.8) is applicable to player 1. Thus, either $\pi_1^R < \theta < \pi_1^A$ or $\pi_1^R > \theta > \pi_1^A$. Suppose $\pi_1^R < \theta < \pi_1^A$. Then Eq. (C.1) implies $\Delta_1(s) < 0$ and $\Delta_1(w) > 0$; hence $\sigma_1(w) = 1$ and $\sigma_2(s) = 0$, implying $\pi_1^R = 1$ and $\pi_1^A = 0$, contradicting the condition $\pi_1^R < \theta < \pi_1^A$ assumed throughout this subcase. Thus consider the only possibility, $\pi_1^R > \theta > \pi_1^A$. Then Eq. (C.1) implies $\Delta_2(s) > 0$ and $\Delta_1(w) < 0$; hence $\sigma_1(w) = 0$ and $\sigma_1(s) = 1$, implying $\pi_1^R = 0$ and $\pi_1^A = 1$, contradicting the condition $\pi_1^R > \theta > \pi_1^A$ assumed throughout this subcase.

All possible cases considered, I have derived a contradiction. Thus, the claim $\pi_1 = \theta$ is true. It follows that for each $i \in \{1, 2\}$, in the conflict stage, which occurs for sure because $\sigma_i(w) = \sigma_i(s) = 1$, the posteriors are $\pi_i = \pi_{-i} = \theta$. Then each player's expected payoff is equal to θ/r if his type is high, and equal to zero if his type is low, hence the social surplus is equal to the always-conflict PBE.

Next, I caclulate the social surplus generated by any Case-(b) PBEs. I show above that any PBE that belongs to Case-(b) has the on-path posterior that is equal to the prior θ for each player. Since $q_i^R = 1$ for some player i, conflict takes place for sure and hence each player's ex-ante payoff from the PBE is equal to

$$\frac{\theta}{r}(\theta - \theta)^+ \frac{(1-\theta)}{r} \max\{\theta, \theta\} = \frac{\theta(1-\theta)}{r}.$$

Thus, the social surplus generated by the PBE is equal to $\frac{2\theta(1-\theta)}{r}$.

Lemma C.3 *There does not exist any Case-(d) and Case-(f) PBEs.*

Proof Suppose such PBEs exist. Strong type of one player ratify a proposal if and only if $x_i \geq \max\{\pi_i^D, \pi_{-i}^A\}$. By $\max\{\pi_i^D, \pi_{-i}^A\} \geq \left(\pi_{-i}^A - \pi_i^D\right)^+$, one can also conclude that $x_i \geq \left(\pi_{-i}^A - \pi_i^D\right)^+$. This implies that the weak type ratify the proposal too. Therefore, in these two classes of PBEs, both players ratify peace proposal. Then result of Lemma C.1 holds, which shows such class of PBEs are impossible. ■

Lemma C.4 *Within Case-(c) PBEs, the only possible equilibria are one where a player that ratifies a proposal also announces Accept in the mediation stage and the other player rejects the proposal if her type is strong and mixes between announcing Accept and Reject if her type is weak. Call this case Lopsided PBEs.*

Proof Proof of this proposition is based on two steps. In **Step 1**, I characterize all the possible PBEs that satisfy the Case-(c) definition. In **Step 2**, I explore these possible cases further and use equilibrium conditions to show that only one possible class of PBEs among them, called Lopsided PBEs, can exist that satisfies the Case-(c) definition. To that end, without loss of generality suppose player 1 is the player that always ratifies a proposal and player 2 is the player that always announces Reject if her type is strong and mixes between Reject and Accept if her type is weak.

Step 1- In this step, I show that given the Case-(c) definition, only three classes of equilibira are possible. I verify this observation via the following two claims.

Claim C.1 *In the Case-(c) PBEs, the renege-proof proposal admits a PBE where if player 1's strategy σ_1 is specified by a row and column in the following table then the equilibrium has the property in the corresponding cell provided that the cell contains a property.*

	$\sigma_1(s) = 0$	$0 < \sigma_1(s) < 1$	$\sigma_1(s) = 1$
$\sigma_1(w) = 0$		*impossible*	*impossible*
$0 < \sigma_1(w) < 1$	*impossible*		
$\sigma_1(w) = 1$	*impossible*	*impossible*	*always-conflict*

Proof By Case-(c) definition, player 1 always ratifies a proposal. Hence, $x_1 \geq \max\{\pi_1^D, \pi_2^A\}$ and $x_1 \geq \left(\pi_2^A - \pi_1^D\right)^+$. Plugging these inequalities into Eq. (4.16) for $i = 1$:

$$\begin{bmatrix} \Delta_1(s) \\ \Delta_1(w) \end{bmatrix} := q_2^A \begin{bmatrix} \max\left\{\pi_1^R, \pi_2^A\right\} - x_1 \\ \left(\pi_2^A - \pi_1^R\right)^+ - x_1 \end{bmatrix} + q_2^R \begin{bmatrix} \max\left\{\pi_1^R, \pi_2^R\right\} - \max\left\{\pi_1^A, \pi_2^R\right\} \\ \left(\pi_2^R - \pi_1^R\right)^+ - \left(\pi_2^R - \pi_1^A\right)^+ \end{bmatrix}.$$

First, suppose that $\sigma_1(w) = 0$ and $0 < \sigma_1(s) \leq 1$, the case corresponding to the first row and the second and third columns in the table. Then $\Delta_1(w) \leq 0$ and $\Delta_1(s) \geq 0$, and $\pi_1^R = 0$ and $\pi_1^A > \theta$ by Eqs. (4.5) and (4.6). Thus,

$$0 \leq \Delta_1(s) - \Delta_1(w) = \pi_1^R - q_2^R \pi_1^A = -q_2^R \pi_1^A.$$

Hence $0 \geq q_2^R \pi_1^A$. This, with $\pi_1^A > \theta > 0$, implies $q_2^R = 0$, i.e., $\sigma_2(s) = \sigma_2(w) = 0$, which is impossible since by Case-(b) definition $\sigma_2(s) = 1$.

Second, suppose $0 < \sigma_1(w) < 1$ and $\sigma_1(s) = 0$. Then $\Delta_1(w) = 0$ and $\Delta_1(w) \leq 0$, $\pi_1^R = 1$ by definition, and $\pi_1^A = \theta/((\theta + (1+\theta)/(1 - \sigma_1(w))) < \theta$ by Eq. (4.5). Thus,

$$0 \geq \Delta_1(s) - \Delta_i(w) = \pi_1^R - q_2^R \pi_1^A = 1 - q_2^R \pi_1^A > 0,$$

with the last inequality due to $\pi_1^A < \theta < 1$. The contradiction displayed above implies this case is also impossible, as asserted in the cell.

Third, suppose $\sigma_1(w) = 1$ and $0 \leq \sigma_1(s) < 1$, which corresponds to the cells of the third row and the first and second columns. Then $\Delta_1(w) \geq 0$ and $\Delta_1(s) \leq 0$, $\pi_1^A = 0$ by definition, and $\pi_1^R = \theta/(\theta + (1-\theta)\sigma_1(s)) > \theta$ by Eq. (4.6). Thus,

$$0 \geq \Delta_1(s) - \Delta_1(w) = \pi_1^R - q_2^R \pi_1^A = \pi_1^R > \theta > 0,$$

contradiction. Hence this case is impossible, as asserted in the cells.

Finally, consider the case $\sigma_1(w) = \sigma_1(s) = 1$, the cell of Row Three and Column Three. Then this is an always-conflict PBE where by Lemma C.2 posterior remains the same as the prior. Hence, the social-surplus equals to that of always-conflict equilibrium. ■

Claim C.2 *In the Case-(c) PBEs, it is impossible to have $\sigma_1(w) = \sigma_1(s) = 0 = \sigma_2(w)$.*

Proof Since $\sigma_1(w) = \sigma_1(s) = 0 = \sigma_2(w)$, we have $\sigma_2(s) = 1$. Then $q_2^R > 0$ and $q_2^A > 0$, and by Bayes's rule, $\pi_2^R = 0$ and $\pi_2^A = 1$. With $\sigma_1(w) = \sigma_1(s) = 0$, we have $q_1^R = 0$, $q_1^A = 1$ and, by Bayes's rule, $\pi_1^A = \theta$. For this (σ_1, σ_2) to constitute an equilibrium, the necessary and sufficient condition is that $\Delta_1(s) \leq 0$, $\Delta_1(w) \leq 0$ and $\Delta_2(s) \geq 0 \geq \Delta_2(w)$. By Case-(c) definition, weak type of player 2 ratifies and her strong type announces Reject. Thus, $\left(\pi_1^A - \pi_2^D\right)^+ \leq x_2 < \max\{\pi_1^A, \pi_2^D\}$. Eq. (4.16) applied to player 2 after plugging in these conditions on x_2 is simplified to:

$$\begin{aligned}
\begin{bmatrix} \Delta_2(s) \\ \Delta_2(w) \end{bmatrix} &= \begin{bmatrix} \max\left\{\pi_2^R, \pi_1^A\right\} - \left((1-\alpha)x_2 + \alpha \max\{\pi_2^D, \pi_1^A\}\right) \\ \left(\pi_1^A - \pi_2^R\right)^+ - ((1-\alpha)x_2 + \alpha x_2) \end{bmatrix} \\
&= \begin{bmatrix} \theta - (1-\alpha)x_2 - \alpha \max\{\pi_2^D, \pi_1^A\} \\ \theta - x_2 \end{bmatrix}.
\end{aligned}$$

Note, that $\Delta_2(s) \geq 0 \iff \theta - (1-\alpha)x_2 - \alpha \max\{\pi_2^D, \pi_1^A\} \geq 0$ which along with Case-(c) condition, i.e., $x_2 < \max\{\pi_2^D, \pi_1^A\}$, implies that $\theta - x_2 \geq 0$. Moreover, $\Delta_2(w) \leq 0 \iff \theta - x_2 \leq 0$. Thus, $\Delta_2(s) \geq 0 \geq \Delta_2(w)$ implies $\theta - x_2 = 0$ which implies that $x_1 = r - \theta$. However, this leads to a contradiction since by renege-proof conditions for player 1, $x_1 \geq \max\{\pi_2^A, \pi_1^D\} = 1$, which by $\theta > r/2$ and $r \in (1, 2)$ leads to the contradiction $x_1 = r - \theta < 1$. ■

Thus, by these two claims, all possible cases for any renege-proof equilibrium that satisfies the Case-(c) definitions are listed in the following table.

	$0 < \sigma_2(w) < 1 = \sigma_2(s)$
$\sigma_1(w) = \sigma_1(s) = 0$	lopsided
σ_1 is totally mixed	Case-(c)-ii
$0 < \sigma_1(w) < 1 = \sigma_1(s)$	Case-(c)-iii

Step 2. In this step, I show that there does not exist any Case-(c)-ii and Case-(c)-iii equilibria. By the definition of Case-(c) PBEs, player 1 always ratifies a proposal. Thus, $x_1 \geq \left(\pi_2^A - \pi_1^D\right)^+$ and $x_1 \geq \max\left\{\pi_1^D, \pi_2^A\right\}$. Therefore, player 1's interim-expected-payoff differences between rejecting and accepting the proposal equals to

$$\Delta_1(s) - \Delta_1(w) = \pi_1^R - q_2^R \pi_1^A.$$

By definition of Case-(c) PBEs, $0 < q_2^R < 1$. If player 1 does not always accept a proposal then by the above table Reject is always a best response for her, implying that $\Delta_1(s) - \Delta_1(w) \geq 0$. This coupled with the above mentioned equality implies $\pi_1^R < \pi_1^A$. Therefore, by definition of π_1^R and π_1^A, it can be easily verified that $\pi_1^R < \theta < \pi_1^A$. Moreover, since player 2 always announces Reject if she is strong and follows mixing strategy if she is weak, then $\pi_2^R < \theta < 1 = \pi_2^A$.

By definition of Case-(c) PBEs, $x_1 \geq \max\left\{\pi_1^D, \pi_2^A\right\} = 1$. Then, it is immediate that for this player the interim payoff of Reject is always less than that of Accept:

$$q_2^A x_1 + q_2^R \max\{\pi_1^A, \pi_2^R\} > q_2^A \max\{\pi_1^R, \pi_2^A\} + q_2^R \max\{\pi_1^R, \pi_2^R\},$$

where the inequality is by $x_1 \geq \max\left\{\pi_1^D, \pi_2^A\right\} = 1 = \max\{\pi_1^R, \pi_2^A\}$ and $\pi_1^A = \max\{\pi_1^A, \pi_2^R\} > \max\{\pi_1^R, \pi_2^R\}$. Thus, accept is always a best response for player 1, given $x_1 \geq \max\left\{\pi_1^D, \pi_2^A\right\} = 1$. Thus, Case-(c)-ii and Case-(c)-iii cannot exist. ■

Lemma C.5 *Within Case-(e) PBEs, the only possible PBE is one where both players always mix between accept and reject if their type is weak and reject the proposal if their type is strong. Call this case Mutually Partially Mixed (MPM) PBEs.*

Proof Within Case-(e) PBEs, by its definition, for each player i, $\left(\pi_{-i}^A - \pi_i^D\right)^+ \leq x_i < \max\{\pi_{-i}^A, \pi_i^D\}$.[1] First, given a proposal that satisfy the previously mentioned inequalities, one can readily show there does not exist any equilibrium such that $\sigma_i(w) = 0$ and $\sigma_i(s) = 1$. Plugging $\left(\pi_{-i}^A - \pi_i^D\right)^+ \leq x_i < \max\{\pi_{-i}^A, \pi_i^D\}$ into Eq. (4.16):

$$\begin{bmatrix} \Delta_i(s) \\ \Delta_i(w) \end{bmatrix} = q_{-i}^A \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^A\right\} - \left((1-\alpha)x_i + \alpha \max\{\pi_{-i}^A, \pi_i^D\}\right) \\ \left(\pi_{-i}^A - \pi_i^R\right)^+ - x_i \end{bmatrix}$$
$$+ \quad q_{-i}^R \begin{bmatrix} \max\left\{\pi_i^R, \pi_{-i}^R\right\} - \max\left\{\pi_i^A, \pi_{-i}^R\right\} \\ \left(\pi_{-i}^R - \pi_i^R\right)^+ - \left(\pi_{-i}^R - \pi_i^A\right)^+ \end{bmatrix}.$$

Suppose that $\sigma_i(w) = 0$ and $\sigma_i(s) = 1$. Then $\Delta_i(w) \leq 0$ and $\Delta_i(s) \geq 0$, and $\pi_i^R = 0$ and $\pi_i^A = 1$ by Eqs. (4.5) and (4.6). Also, $\sigma_{-i}(s) = 1 \Rightarrow \pi_{-i}^A = 1$. Thus,

$$0 \leq \Delta_i(s) - \Delta_i(w) = \pi_i^R - q_{-i}^R \pi_i^A - \alpha q_{-i}^A \left(\max\{\pi_{-i}^A, \pi_i^D\} - x_i\right) \leq \pi_i^R - q_{-i}^R \pi_i^A$$

Hence $0 \geq q_{-i}^R \pi_i^A$. This, with $\pi_i^A > \theta > 0$, implies $q_{-i}^R = 0$, i.e., $\sigma_{-i}(s) = \sigma_{-i}(w) = 0$, which is impossible since by definition of Case-(e) $\sigma_{-i}(s) = 1$.

Recall that by Lemma C.2, any unilateral reject, i.e., $\sigma_i(s) = \sigma_i(w) = 1$ for some $i \in \{1, 2\}$ admit always-conflict equilibrium. Consequently, the only remaining case such within Case-(e) PBEs that admit renege-proof equilibrium and it is not always-conflict is the one where $0 < \sigma_i(w) < 1$ and $\sigma_i(s) = 1$ for each $i \in \{1, 2\}$. ■

C.1.1 Proof of Lemma 4.2

Lemmas C.2-C.5 considers all possible cases of renege-proof PBEs. In section C.1, I categorize all possible renege-proof PBEs in Cases (a)-(f). Lemmas C.1 and C.3 rules out existence of Case-

[1] Note that there does not exist any Case-(e) PBE where $x_i \geq \max\{\pi_{-i}^A, \pi_i^D\}$ and the strong type of both players always Reject. This is due to the observation that given such proposals each type of each player would always ratifies the peace proposal. However, Lemma C.1 shows such a PBE does not exist.

(a), (d), and (f) PBEs. Lemmas C.2, states existence of an always-conflict PBE, i.e., Case-(b). Since in an always conflict PBE, a player unilaterally rejects a proposal and triggers conflict it is renege-proof. Lemmas C.4 states that within Case-(c) category only Lopsided PBEs are possible. Lemma C.5 states that within Case-(e) only MPM PBEs are possible. Thus, all cases are considered and the only two cases that are not renege-proof are Lopsided and MPM PBEs. ■

C.2 Proof of Proposition 4.2

Pick $1-r/2 \leq \pi_i^D \leq 1$ for each $i \in 1,2$. The characterization of equilibrium strategies and beliefs stated in the proposition would be the same as Eq. (4.20), where $x_1 = x_2 = r/2$ is plugged in:

$$\pi_i^R = \pi_{-i}^R = 1 - \frac{r}{2},$$
$$q_i^R = \frac{1-\theta}{\frac{r}{2}}$$

Thus, by definition of q_i^R in Eq. (4.4) and $\sigma_i(s) = 1$ we have

$$\theta\sigma_i(w) + (1-\theta) = \frac{1-\theta}{\frac{r}{2}} \iff \sigma_i(w) = \frac{(1-\theta)(2-r)}{\theta r}$$

Moreover, $\sigma_i(s) = 1$ implies $\Delta_i(s) \geq 0$ which is characterized by Ineqs. (4.21) and (4.22). Given the equal split, $\sigma_2(w) = \sigma_1(w) \iff q_2^A = q_1^A$. Thus, Ineqs. (4.21) and (4.22) are equivalent. Therefore, the necessary and sufficient conditions for the equal split to admit renege-proof MPM PBE are (4.20) and (4.21). By the characterization of the equilibrium exposited in the statement of the Proposition 4.2 and Ineq. (4.21) one can readily observe:

$$\begin{aligned}
\Delta_1(s) \geq 0 &\iff q_2^A(1-x_1)(1-\alpha) \geq (1-q_2^A)x_2 \\
&\iff q_2^A\,[1-\alpha]\left(1-\frac{r}{2}\right) - q_2^R\left[\frac{r}{2}\right] \geq 0 \\
&\iff \frac{(\alpha r^2 + 2\alpha r\theta - 4\alpha r - 4\alpha\theta - r^2 + 4\alpha + 2r + 4\theta - 4)}{2r} \geq 0 \\
&\iff \frac{\alpha(r-2)(r+2\theta-2) - \Big((r-1)^2 + 3 - 4\theta\Big)}{2r} \geq 0, \\
&\iff \alpha \leq \frac{4\theta - (r-1)^2 - 3}{(2-r)\,(r+2\theta-2)}.
\end{aligned}$$

It can be easily verified that $0 < \frac{4\theta-(r-1)^2-3}{(2-r)(r+2\theta-2)} < 1$ and hence such an $\alpha \in \left(0, \frac{4\theta-(r-1)^2-3}{(2-r)(r+2\theta-2)}\right]$ that satisfy the equilibrium condition exists. To that end one can verify $r/2 < \theta < 1$ (Ineq. 4.15):

$$\frac{4\theta-(r-1)^2-3}{(2-r)(r+2\theta-2)} \geq 0 \iff \theta \geq \frac{3+(r-1)^2}{4},$$
$$\frac{4\theta-(r-1)^2-3}{(2-r)(r+2\theta-2)} < 1 \iff (2-r)(r+2\theta-2) - \Big[4\theta-(r-1)^2-3\Big] > 0 \iff 2r(1-\theta) > 0,$$

where the first equivalence condition is verified by the assumption $\theta \geq \frac{3+(r-1)^2}{4}$ in the statement of this Propositions and the second chain of equivalence conditions is always true since $r/2 < \theta < 1$ by Ineq. (4.15). Hence, this class of PBEs exist if $\theta \geq \frac{3+(r-1)^2}{4}$ and $\alpha \leq \frac{4\theta-(r-1)^2-3}{(2-r)(r+2\theta-2)} = \Gamma(\theta, r)$.

It remains to show that the equal proposal, given the off-path posterior beliefs $1-r/2 \leq$

$\pi_i^D \leq 1$, satisfies renege-proof conditions. We have already shown that given these conditions strong type of both players reject the proposal at the mediation. Thus, it remains to show that the weak type of both players does not renege at the rectification. To see this, note that in the event that the proposal is accepted and the game is in the rectification stage, if player i of type w reneges she would get $(\pi_{-i}^A - \pi_i^D)$ which by $\pi_{-i}^A = 1$ and $1 - r/2 \leq \pi_i^D$ is less than the equal split: $(\pi_{-i}^A - \pi_i^D) \leq (1 - 1 + r/2) = r/2$. Therefore, she ratifies the equal proposal which brings her a higher payoff. ■

C.3 The Social Surplus Function

Lemma C.6 *Given any renege-proof peace proposal* (x_1, x_2) *and the associated PBE* (σ, q, π), *such that* $x_2 \leq x_1$, *then*
(i) *the social surplus admitted by Lopsided PBEs equals to* $1/r$ *times*

$$q_2^A x_1 + \theta(1-\theta)q_2^R + \theta\left(1 - \pi_2^R\right),$$

(ii) *the social surplus admitted by MPM PBEs equals to* $1/r$ *times*

$$\theta - \theta(\pi_1^R + \pi_2^R) + q_1^A \pi_1^A + q_1^R \pi_2^R$$

Proof Given any renege-proof peace proposal (x_1, x_2) and the associated PBE (σ, q, π), each player i's ex ante expected payoff from announcing Reject is equal to $1/r$ times

$$\theta\left(q_{-i}^A\left(\pi_{-i}^A - \pi_i^R\right)^+ + q_{-i}^R\left(\pi_{-i}^R - \pi_i^R\right)^+\right) + (1-\theta)\left(q_{-i}^A \max\left\{\pi_i^R, \pi_{-i}^A\right\} + q_{-i}^R \max\left\{\pi_i^R, \pi_{-i}^R\right\}\right),$$

and that from announcing Accept, equals to $1/r$ times

$$q_{-i}^A x_i + \theta q_{-i}^R\left(\pi_{-i}^R - \pi_i^A\right)^+ + (1-\theta)q_{-i}^R \max\left\{\pi_i^A, \pi_{-i}^R\right\}.$$

Pick any peace proposal (x_1, x_2) such that $x_2 \leq x_1$, i.e., $x_2 = \min\{x_1, x_2\} =: x_{\min}$. Let (σ, π) denote the associated equilibrium and q the associated probability system defined in (4.3)–(4.4). Recall the social surplus function $S(x_{\min})$ and its normalized version $\tilde{S}(x_{\min}) := rS(x_{\min})$ defined in Section 4.4.1.

First, when $x_2 = 0$, the equilibrium is always conflict and by Lemma C.2, $\tilde{S} = 2\theta(1-\theta)$. Second, let $x_2 \in (0, r-1]$. I shall show that $\tilde{S}(0) = 2\theta(1-\theta)$. Since $x_2 \in (0, r-1]$, the equilibrium is lopsided (Proposition 4.1) such that $\sigma_1(s) = \sigma_1(w) = 0 < \sigma_2(w) < 1 = \sigma_2(s)$. Player 1 always ratifies a proposal at the rectification stage. Then player 1's equilibrium surplus is equal to his expected payoff from Accept, and player 2's surplus equal to her expected payoff from Reject. Thus, by the above displayed formulas for interim payoff of announcing Accept and Reject, one can obtain:

$$\begin{aligned} rS(x_{\min}) &= \underbrace{q_2^A x_1 + \theta\left(\pi_2^R - \pi_1^A\right)^+ + (1-\theta)q_2^R \max\left\{\pi_1^A, \pi_2^R\right\}}_{\text{player 1}} \\ &\quad + \underbrace{\theta \cdot 1 \cdot \left(\pi_1^A - \pi_2^R\right)^+ + (1-\theta) \cdot 1 \cdot \max\left\{\pi_2^R, \pi_1^A\right\}}_{\text{player 2}} \\ &= q_2^A x_1 + (1-\theta)q_2^R\theta + \theta\left(\theta - \pi_2^R\right) + (1-\theta)\theta \\ &= q_2^A x_1 + \theta(1-\theta)q_2^R + \theta\left(1 - \pi_2^R\right), \end{aligned}$$

with the second equality due to the observation from Proposition 4.1 that $\pi_1^A = \theta > \theta - x_2 = \pi_2^R$. Thus, it verifies the social surplus function as exposited in the statement of the lemma for the lopsided PBEs. Note that $\tilde{S}(0) = 2\theta(1-\theta)$ as $q_2^A = 0$, $q_2^R = 1$, and $\pi_2^R = \theta$ when $x_2 = 0$ by Proposition 4.1, converging to the always-conflict PBE and its social surplus.

Third, by definition of MPM equilibrium, Reject is a best response for each type of each player. By Ineq. (4.23) in this class of PBEs, $x_2 \le x_1 \iff \pi_2^R \ge \pi_1^R$. Thus, by the above displayed formulas for interim payoff of announcing Reject, player 1's surplus is equal to $1/r$ times

$$\begin{aligned} & \theta\left(q_2^A\left(\pi_2^A - \pi_1^R\right)^+ + q_2^R\left(\pi_2^R - \pi_1^R\right)^+\right) + (1-\theta)\left(q_2^A \max\left\{\pi_1^R, \pi_2^A\right\} + q_2^R \max\left\{\pi_1^R, \pi_2^R\right\}\right) \\ = \; & \theta(q_2^A\pi_2^A + q_2^R\pi_2^R - \pi_1^R) + (1-\theta)(q_2^A\pi_2^A + q_2^R\pi_2^R) \\ = \; & \theta(\theta - \pi_1^R) + (1-\theta)\theta \\ = \; & \theta - \theta\pi_1^R, \end{aligned}$$

where the first equality is due to $\pi_i^A \ge \theta \ge \pi_i^R$ for each player i by Ineq. (4.8) and the hypothesis $\pi_2^R \ge \pi_1^R$, and the second equality due to $q_i^A\pi_i^A + q_i^R\pi_i^R = \theta$ by Eq. (4.7). Similarly, player 2's surplus is equal to $1/r$ times

$$\begin{aligned} & \theta\left(q_1^A\left(\pi_1^A - \pi_2^R\right)^+ + q_1^R\left(\pi_1^R - \pi_2^R\right)^+\right) + (1-\theta)\left(q_1^A \max\left\{\pi_2^R, \pi_1^A\right\} + q_2^R \max\left\{\pi_2^R, \pi_1^R\right\}\right) \\ = \; & \theta\left(q_1^A\left(\pi_1^A - \pi_2^R\right)\right) + (1-\theta)\left(q_1^A\pi_1^A + q_1^R\pi_2^R\right) \\ = \; & q_1^A\pi_1^A - \theta\pi_2^R + q_1^R\pi_1^R, \end{aligned}$$

where the first equality is due to $\pi_i^A \ge \theta \ge \pi_i^R$ for each player i by Ineq. (4.8) and the hypothesis $\pi_2^R \ge \pi_1^R$, and the second equality is by $q_1^A + q_1^R = 1$. Then sum the two components displayed above to obtain the social surplus function as exposited in the statement of the lemma for MPM PBEs. ■

Lemma C.7 *The equal split that admits MPM PBEs generates social surplus equals to θ.*

Proof By Proposition 4.2, the equal proposal that admits MPM PBEs, admits a symmetric PBE where

$$\pi_1^R = \pi_2^R = 1 - \frac{r}{2}, \text{ and } \pi_1^A = \pi_2^A = 1$$

Plug these posterior in the social surplus function characterized for the MPM PBEs in Lemma C.6 to obtain

$$rS(x_2) = \theta - \theta(2\pi_1^R) + q_1^A\pi_1^A + q_1^R\pi_1^R = \theta - \theta(2\pi_1^R) + \theta = 2\theta(1-\pi_1^R) = r\theta,$$

where the second equality above is due to $q_1^A\pi_1^A + q_1^R\pi_1^R = \theta$ by Eq, (4.7). ■

Lemma C.8 *The social surplus function admitted by Lopsided PBEs is strictly increasing in $x_{\min} := \min\{x_1, x_2\}$*

Proof I shall show that $\frac{d}{dx_2}\tilde{S}(x_2) > 0$ for all $x_2 \in (0, r-1)$. That can be verified easily by plugging into Eq. (4.25) the formula of π_2^R in Proposition 4.1 , and that of (q_2^A, q_2^R) based on the formula of $\sigma_2(w)$ in the same proposition.

$$\begin{aligned} \tilde{S}(x_2) &= \frac{x_2}{1-\theta+x_2}(r-x_2) + \frac{\theta(1-\theta)^2}{1-\theta+x_2} + \theta(1-\theta+x_2) \qquad \text{(C.2)} \\ &= -(1-\theta)\left(y + \frac{r+(1-\theta)^2}{y}\right) + r + 2(1-\theta), \end{aligned}$$

where $y := 1 - \theta + x_2$, so $x_2 = y - 1 + \theta$. Thus,

$$-\frac{1}{1-\theta}\frac{d}{dy}\left(rS_{\text{lop}}(x_2)\right) = 1 - \frac{r + (1-\theta)^2}{y^2} = 1 - \frac{r + (1-\theta)^2}{\left(1 - (\theta - x_2)\right)^2},$$

which is negative because $\left(1 - (\theta - x_{-i})\right)^2 = (1 - \pi_{-i}^R)^2 < 1 < r + (1-\theta)^2$, with the equality due to the formula of π_2^R in Proposition 4.1, and the last inequality due to $r > 1$ (Ineq. (4.15)). ■

Lemma C.9 *The social surplus function admitted by MPM PBEs is strictly increasing in* $x_{\min} := \min\{x_1, x_2\}$

Proof Since the equilibrium is MPM for any $x_2 \in (\gamma, r/2]$, combine Eq. (4.26) with Eqs. (4.3)–(4.5), as well as definition of MPM exposited in, to obtain

$$\frac{d}{dx_2} rS(x_2) = \theta - q_1^R - \theta\left(x_2 + \frac{\theta}{q_1^R}\left(1 - \pi_1^R\right)\right)\frac{d}{dx_2}\sigma_1(w)$$

for all splits that admit this class of equilibria. Plug into this equation the formulas of $\sigma_1(w)$ and π_1^R by Eq. (4.20), and the formula of q_1^R derived from that of σ_1^R and Eqs. (4.4). Then $\frac{d}{dx_2}\tilde{S}(x_2) > 0$ is equivalent to

$$r\theta M^2 > x_2^3\left(r(\theta + x_2 - 1) + (2x_2 - r)(1-\theta)\right),$$

where $M := x_2 - \left(\theta(x_2 - x_1) + x_1(1 - x_2)\right)$. Since $r/2 \geq x_2$, $x_2^2 \leq M$ and hence the left-hand side of the above-displayed inequality is no less than $r\theta x_2^4$; meanwhile, the right-hand side is no greater than $x_2^3 r(\theta + x_2 - 1)$. Since $1 > r/2 \geq x_2$, $r\theta x_2^4 > x_2^3 r(\theta + x_2 - 1)$, hence the above-displayed inequality is true, thus $\frac{d}{dx_2}\tilde{S}(x_2) > 0$, as desired. ■

C.4 The Probability of Conflict

Lemma C.10 *The probability of conflict admitted by Lopsided PBEs is strictly decreasing in* $x_{\min} := \min\{x_1, x_2\}$

Proof Let $x_2 = \min\{x_1, x_2\}$ for any peace proposal (x_1, x_2). Recall the conflict probability $P(x_2)$ defined in Section 4.4.2. We shall calculate $\frac{d}{dx_2}P$. For any $x_2 \in (0, r-1]$, the PBE is lopsided (Proposition 4.1), so $q_1^A = 1$ and $q_2^A = \theta(1 - \sigma_2(w)) = x_2/(1 - \theta + x_2)$. Thus $P(x_2) = (1-\theta)/(1 - \theta + x_2)$ is strictly decreasing in x_2. ■

Lemma C.11 *The probability of conflict admitted byMPM PBEs is strictly decreasing in* $x_{\min} := \min\{x_1, x_2\}$

Proof Recall the conflict probability $P(x_2)$ defined in Section 4.4.2. For any $x_2 \leq r/2$ that admit MPM PBEs by Eq. (4.20) $q_1^A = \frac{1 - \pi_1^R + \theta - 1}{1 - \pi_1^R}$, where π_1^R is also characterized by Eq. (4.20), and $q_2^A = (\theta - 1 + x_2)/x_2$. Since $\frac{d}{dx_2}P(x_2) = q_1^A \frac{d}{dx_2}q_2^R + q_2^A \frac{d}{dx_2}q_1^R$,

$$\begin{aligned}\frac{d}{dx_2}P(x_2) &= \theta \cdot \frac{\theta + x_2 - 1}{x_2}\frac{d}{dx_2}\sigma_1(w) - \frac{1-\theta}{x_2^2} \cdot \frac{(r - x_2)(\theta + x_2 - 1)}{M} \\ &= \frac{(\theta + x_2 - 1)(1-\theta)}{x_2 M}\left(\frac{x_2^2 + r(\theta - 1)}{M} - \frac{r - x_2}{x_2}\right),\end{aligned}$$

where M is defined by

$$M =: x_2 - \left(\theta(x_2 - x_1) + x_1(1 - x2)\right).$$

Since $r/2 \geq x_2$ (by Ineq. (4.23)) then one can readily verify that $x_2^2 \leq M$. In the above-displayed expression for $\frac{d}{dx_2}P(x_2)$, the first factor is positive because $\theta + x_2 - 1 > 0$ due to Ineq. (4.23); and the second factor, bracketed by $(\cdots)$, is negative: either $x_2^2 + r(\theta - 1) < 0$, or $x_2^2 + r(\theta - 1) \geq 0$ and so $\frac{x_2^2 + r(\theta-1)}{M} \leq \frac{x_2^2 + r(\theta-1)}{x^2}$ (since $M \geq x_2^2$), which is less than $\frac{r - x_2}{x_2}$. Thus $\frac{d}{dx_2}P(x_2) < 0$. ■

C.5 Proof of Theorem 4.1

By Lemma 4.2, only lopsided and MPM PBEs are renege-proof. Social-surplus function admitted by lopsided proposals is strictly increasing in $x_{min} = \min\{x_1, x_2\}$ (Lemma C.8). Without loss of generality assume $x_2 \leq x_1$. By Proposition 4.1, $x_2 = r - 1$ is the upper bound of proposal that admit renege-proof Lopsided PBEs. Hence, the lopsided split $\nu_2 = w/s$, or equivalently lopsided proposal $x_2 = r - 1$, is the one that admits the highest social surplus among the renege-proof lopsided proposals. Plugging q_{-i}^A and π_{-i}^R from Proposition (4.1) into Eq. (4.25) this split would admit

$$rS(r-1) = \frac{2\theta^3 - (2r+2)\theta^2 + (r^2+1)\theta + r - 1}{(r - \theta)}.$$

First, by an immediate implication of Proposition 4.2, if $\alpha \geq \Gamma(\theta, r)$ or $\theta < \frac{3+(r-1)^2}{4}$, then the equal proposal does not admit renege-proof PBEs. Thus, the only renege-proof proposal is $x_2 = r - 1$ and $x_1 = 1$, or respectively $\nu_2 = w/s$ and $\nu_1 = 1 - w/s$, as stated in the upper branch of the statement of the Theorem.

Second, consider the case where $\alpha < \Gamma(\theta, r)$. By Lemma C.9, social surplus function admitted by MPM PBEs is strictly increasing in x_2. By Ineq. (4.23), the equal proposal is the upper bound of x_2 that admit MPM PBEs. Moreover, Proposition 4.2, verifies existence and uniqueness of MPM PBEs admitted by the equal proposal. Hence, the equal proposal outperforms all other splits that admit MPM PBEs. Given the equal proposal, by Proposition (4.2), we know this class of equilibrium exists and is renege-proof if $\theta \geq \frac{3+(r-1)^2}{4}$ and $\alpha < \Gamma(\theta, r)$. By Lemma C.7, the social surplus admitted by the equal proposal is equal to:

$$rS(r/2) = r\theta.$$

It can be easily shown that:

$$rS(r-1) - rS^{MPM}(r/2) = \frac{(1-\theta)(r\theta - 2\theta^2 + r - 1)}{(r - \theta)}.$$

Thus,

$$\begin{aligned} rS(r-1) - rS(r/2) \geq 0 &\iff (r\theta - 2\theta^2 + r - 1) \geq 0 \\ &\iff \frac{r - \sqrt{r^2 + 8r - 8}}{4} \leq \theta \leq \frac{r + \sqrt{r^2 + 8r - 8}}{4}. \end{aligned}$$

By maintained assumption $\theta > r/2$ (Ineq. 4.15), the lower bound of the above displayed condition on θ is not binding. Therefore, if $r/2 \leq \theta \leq \frac{r+\sqrt{r^2+8r-8}}{4}$, the lopsided proposal $(r-1, 1)$ admits higher social surplus than the equal proposal. Moreover, by an immediate implication of Proposition 4.2, if $\theta \leq \frac{3+(r-1)^2}{4}$, then the equal proposal does not admit MPM PBEs. Combine these two observations to conclude that if $\theta \leq \max\left\{\frac{r+\sqrt{r^2+8r-8}}{4}, \frac{3+(r-1)^2}{4}\right\} = \phi(r)$ and $\alpha <$

$\Gamma(\theta, r)$, then the lopsided proposal is the optimal renege-proof proposal. This verifies the second and fourth branches in the statement of the Theorem. At $\theta = \phi(r)$ the switch is discontinuous. This verifies the third branch in the statement of the Theorem.

Note that $\max\left\{\frac{r+\sqrt{r^2+8r-8}}{4}, \frac{3+(r-1)^2}{4}\right\}$ is not a degenerate set. It is easy to verify that both of these term are strictly increasing in r and at $r \approx 1.23$ we have $\frac{r+\sqrt{r^2+8r-8}}{4} = \frac{3+(r-1)^2}{4}$. ■

C.6 Proof of Theorem 4.2

By Lemma 4.2, only lopsided and MPM PBEs are renege-proof. Relabeling the players if necessary, let $x_2 = \min\{x_1, x_2\}$ in any proposal (x_1, x_2). By Proposition (4.1) and (4.2) any $x_2 \in (0, r-1]$ and $x_2 = r/2$ determines a unique not-always-conflict and renege-proof PBE $(\sigma_i, \pi_i^A, \pi_i^R)_{i=1}^2$, with the associated $(q_i^A.q_i^R)$ defined by Eqs. (4.3)–(4.4). Recall the conflict probability $P(x_2)$ defined in Section 4.4.2. Thus, $P(x_2) = 1 - q_i^A q_{-i}^A$. By Lemmas C.10 and C.11 , the probability of conflict admitted by Lopsided and MPM PBEs is strictly decreasing in x_2. Thus, the probability of conflict is minimized within each class at the upper bound of x_2. By Ineq. 4.23, this amounts to $x_2 = r/2$ for the MPM PBEs, and by Proposition 4.1, it equals to $x_2 = r - 1$ for Lopsided PBEs.

First, by an immediate implication of Proposition 4.2, if $\alpha \geq \Gamma(\theta, r)$ or $\theta < \frac{3+(r-1)^2}{4}$, then the equal proposal does not admit renege-proof PBEs. Thus, the only renege-proof proposal is $x_2 = r - 1$ and $x_1 = 1$, or respectively $\nu_2 = w/s$ and $\nu_1 = 1 - w/s$, as stated in the upper branch of the statement of the Theorem.

Second, consider the cases where $\alpha < \Gamma(\theta, r)$. For MPM PBEs by Proposition (4.2), at the equal proposal $q_1^R = q_2^R = \frac{1-\theta}{x_2} = \frac{2-2\theta}{r}$. Hence,

$$P(r/2) = 1 - q_i^A q_{-i}^A = \frac{4(1-\theta)(r+\theta-1)}{r^2}$$

Similarly, by Proposition (4.1), for lopsided solutions $q_1^A = 1$ and $q_2^A = \frac{x_2}{1-\theta+x_2}$. Hence:

$$P(x_{\min}) = 1 - q_1^A q_2^A = 1 - q_2^A = \frac{1-\theta}{1-\theta+x_2}.$$

Hence, at the extreme lopsided proposal

$$P(r-1) = \frac{1-\theta}{r-\theta}.$$

Thus;

$$\begin{aligned} P(r/2) \geq P(r-1) &\iff \frac{4(1-\theta)(r+\theta-1)}{r^2} - \frac{1-\theta}{r-\theta} \geq 0, \\ &\iff \frac{(1-\theta)(3r^2 - 4\theta^2 - 4r + 4\theta)}{r^2(r-\theta)} \geq 0, \end{aligned}$$

Where the last inequality holds if and only if

$$3r^2 - 4\theta^2 - 4r + 4\theta \geq 0 \iff \frac{1 - \sqrt{3r^2 - 4r + 1}}{2} \leq \theta \leq \frac{1 + \sqrt{3r^2 - 4r + 1}}{2}$$

By maintained assumption $\theta > r/2$ (Ineq. 4.15), the lower bound of the above displayed condition on θ is not binding. The only binding inequality in the presence of $\theta > r/2$ is $\theta > \frac{1+\sqrt{(3r-1)(r-1)}}{2}$. Hence, If $\theta \leq \frac{1+\sqrt{3r^2-4r+1}}{2}$ then the lopsided proposal $(r-1, 1)$ admits lower probability of peace than the equal proposal. Combine these two observations to conclude that if $\theta \leq \max\left\{\frac{1+\sqrt{3r^2-4r+1}}{2}, \frac{3+(r-1)^2}{4}\right\} = \psi(r)$ and $\alpha < \Gamma(\theta, r)$, then the lopsided proposal is the conflict minimizing renege-proof proposal. This verifies the second and fourth branches in the statement of the Theorem. At $\theta = \psi(r)$ the switch is discontinuous. This verifies the third branch in the statement of the Theorem. ■

Curriculum Vitae

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