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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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INTERNATIONAL BORROWING AND TIME-CONSISTENT FISCAL POLICY

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1. Introduction

Recently the dynamic consistency of government policies has attracted a great deal of attention. Since the influential papers of Kydland and Prescott (1977) and Calvo (1978), it has become widely recognized that in a framework of rational expectations optimal policies are often not time-consistent. That is, a government that has chosen an optimal policy at some date will not find it optimal to continue following it, when the policy is reconsidered by maximizing the same objective function at some later date.

It has also become recognized that some sort of commitments might improve matters above the situation when the government reoptimizes as time elapses and the public bases its actions on the understanding of the government's objectives. Some writers have used that insight to advocate the use of totally precommitted policies in the form of fixed rules.

A recent penetrating study by Lucas and Stokey (1983) sheds some new light on these issues. The (major part of the) paper deals with the following problem: suppose there is a given path of government spending that can be financed either by distortionary taxes on labor income, or by borrowing. As is well-known from earlier studies by Barro (1979) and Brock and Turnovsky (1980)
Suppose that the government in each period is allowed to choose its own tax rates, but must honor the outstanding debt obligations held by the private sector. Then Lucas and Stokey show that (contrary to the assertions of Brock and Turnovsky), the optimal policy can indeed be made time-consistent, provided that there exists debt of sufficiently rich maturity (and contingency in the case of uncertainty). Then the government can induce its successors to continue the optimal policy by a, uniquely defined, restructuring of its debt.

This result is important for two reasons. First, it shows that although some sort of commitments are needed to make the optimal policy time-consistent, this need not involve precommitting all the actual policy instruments. Second, in the words of Fischer (1983), it provides an example of an anti-Modigliani-Miller theorem, where the maturity structure of the debt matters.

In this paper we extend Lucas and Stokey's analysis of optimal fiscal policy to open economics, thereby relaxing their assumption that all debt is a "debt to ourselves." We look both at a small open economy and at an economy large enough to affect the intertemporal terms of trade at which it can borrow and lend. The question we address is whether an optimal fiscal policy still is time-consistent, and if so whether that requires any systematic management of the maturity structure of the, domestic and foreign, debt.

Section 2 presents the model we use, which is a deterministic open-economy version of Lucas and Stokey's intertemporal general equilibrium model. In the first part of the paper we make the simplifying assumption that only the government, but not the private sector, can engage in foreign borrowing and
lending. We derive the optimal fiscal policy, as seen from an initial period, and discuss the properties of the resulting allocation. Then we go on to discuss under what conditions the optimal policy is in fact time consistent.

In Section 3 we show that time-consistency requires the government's domestic debt obligations be restructured over time and discuss the properties of the unique restructuring scheme. That section also provides an alternative interpretation of Lucas and Stokey's results, which relies heavily on the concept of government "cash-flow"; see also Svensson and Persson (1984).

Section 4 discusses the requirements on the government's (and the country's) foreign debt. We show that in a small open economy there are no requirements (except solvency) at all on the foreign debt. In the large open economy, on the other hand, there is a unique maturity structure that sustains the optimal policy.

In Section 5 we relax the assumption that all agents can not trade on the international capital market. Time consistent policies still exist but the earlier results are considerably modified, both in the small and the large economy.

Section 6 offers some concluding remarks.

2. **Optimal Policy with Commitments**

We look at an open-economy, one-good, deterministic version of the model in Lucas and Stokey (1983). As mentioned in the introduction, we start by considering an economy where the government, but not the private sector, can trade on the international capital market. We may therefore think of all
imports as carried out by the government.

The economy's production technology is linear: one unit of labor results in one unit of output. There is one representative consumer with a constant endowment of labor, which we take to be unity, in each period. Private consumption of goods and leisure in period \( t \) is denoted by \( c_t \) and \( x_t \), while the given amount of government consumption is \( g_t \). With net imports denoted by \( m_t \), the economy's resource constraint is thus:

\[
(2.1) \quad c_t + g_t + x_t < 1 + m_t, \quad t = 0, 1, 2, \ldots
\]

Our representative consumer has additively separable preferences

\[
(2.2) \quad \Sigma_0^\infty \beta^t U(c_t, x_t), \quad 0 < \beta < 1,
\]

where \( U(\cdot) \) is strictly concave in \( c_t \) and \( x_t \).

There are proportional taxes on labor income, where \( t_t \) denotes the tax rate in period \( t \). At the outset, government debt obligations to the consumer are predetermined and described by the vector \( 0^b = (0^b_0, 0^b_1, \ldots) \), where \( 0^b_t \) is the consumer's claim to goods in period \( t \) -- the sum of interest and repayment of maturing domestic debt. Denoting the domestic present value prices (interest rate factors), which do not have to equal the international prices, by \( p = (p_0, p_1, \ldots) \), we can then express the consumer's intertemporal budget constraint as

\[
(2.3) \quad \Sigma_0^\infty p_t c_t - \Sigma_0^\infty p_t (1-t_t)(1-x_t) < \Sigma_0^\infty p_t 0^b_t.
\]

The consumer maximizes \( (2.2) \) subject to \( (2.3) \), which gives the following first order conditions
\[(2.4a) \quad (1-\tau_t)U_c(x_t) = U_x(c_t, x_t), \quad t = 0, 1, 2, \ldots,\]

\[(2.4b) \quad \beta^t U_c(x_t) = p_t, \quad t = 0, 1, 2, \ldots.\]

The government's given sequence of consumption \(g = (g_0, g_1, \ldots)\), has to be
financed either with the proportional taxes on labor or by borrowing. The
borrowing can be done at home or abroad. On the domestic capital market the
government trades at prices \(p\), while at the international market the present
value prices are \(p^* = (p^*_0, p^*_1, \ldots)\). The time 0 government faces the intertemporal
budget constraints

\[(2.5a) \quad \Sigma^\infty_0 p_t y_{0t} > 0,\]

\[(2.5b) \quad \Sigma^\infty_0 p^*_t z_{0t} > 0.\]

Here, \(z_{0t}\) is defined by

\[(2.5c) \quad z_{0t} = -m_t - b^*_t, \quad t = 0, 1, 2, \ldots,\]

where \(b^*_t\) is the foreign country's claims to goods in period \(t\) -- the sum of
interest and repayment of the government's (and the economy's) foreign debt --
and the sum of \(y_{0t}\) and \(z_{0t}\) is the excess of tax income over the sum of government
consumption and total debt obligation in period \(t\), namely

\[(2.5d) \quad y_{0t} + z_{0t} = \tau_t (1-x_t) - \tau_t - b^*_t - b^*_t, \quad t = 0, 1, 2, \ldots,\]

Following the same terminology as in Svensson and Persson (1984), we shall
call \(y_{0t} + z_{0t}\) the cash-flow of the government (at \(t = 0\)),\(^1\) and we shall refer to
\(y_{0t}\) and \(z_{0t}\) as the domestic and foreign cash-flow, respectively.

It follows from (2.5c) and (2.5d) that the domestic cash-flow
fulfills

(2.5e) \[ y_{ot} = r_{t}(1-x_{t}) + m_{t} - s_{t} - o_{b_{t}}, \quad t = 0,1,2, \ldots. \]

Before going into the governments' decision problem, we have to say something about the behavior of the rest of the world, which we shall refer to as the foreign country. In principle, we model the foreign country in the same way as the home country, although we make a few simplifying assumptions. First, we assume that foreign consumption of leisure (supply of labor) \( x^* \) is totally inelastic and constant over time. Then we can write the foreign representative consumer's utility as

(2.6) \[ E_{0}^{*}B^{*}u^{*}(c^{*}_{t}), \quad 0 < \beta^{*} < 1. \]

Second, we abstract from all government activity in the foreign country. Since foreign imports are the negative of home imports, we can then write the foreign resource constraint as

(2.7) \[ c^{*}_{t} < 1 - x^{*} - m_{t}, \quad t = 0,1,2, \ldots. \]

The foreign country's intertemporal budget constraint may be expressed as

(2.8a) \[ E_{0}^{*}p^{*}(m_{t} + o_{b^{*}_{t}}) > 0. \]

Maximizing (2.6) subject to (2.7) and (2.8a), we get the relation

(2.8b) \[ p^{*}_{t} = B^{*}_{c}(1-x^{*}-m_{t}) \equiv p^{*}_{t}(m_{t}), \quad t = 0,1,2, \ldots, \]

which defines the foreign country's demand-price functions \( p^{*}_{t}(m_{t}) \). Equation (2.8) (where (2.8a) is fulfilled with equality) summarizes what we need to know about the foreign country's behavior.

In equilibrium any decisions of the government must be consistent with
private maximizing behavior at home and abroad. Using (2.4) through (2.8) we may therefore rewrite (2.5) as

\[(2.9a) \quad \mathbb{E}_0^\pi \beta^t U_c(c_t, x_t) y_{0t} \geq 0, \]

\[(2.9b) \quad \mathbb{E}_0^\pi \beta^t (m_{-t} - m_t^*) \geq 0, \]

\[(2.9c) \quad U_c(c_t, x_t) (1 - x_t - g_t - y_{0t} - \beta_t m_t) + U_x(c_t, x_t) (x_t - 1) = 0 \]

The government's objective is to choose \( c_t, x_t \) and \( m_t \), so as to maximize the representative consumer's welfare given by (2.2), subject to its budget constraints (2.9) and the economy's resource constraint (2.1). Carrying out the maximization, one obtains the first-order conditions

\[(2.10a) \quad \beta_t U_{ct} + \lambda_0 \beta_t [U_{cct}(1 - \tau_t)(1 - x_t) + U_{xct}(x_t - 1)]
\quad + \lambda_0 \beta_t U_{cct} y_{0t} - \beta_t \mu_{0t} = 0, \quad t = 0, 1, 2, \ldots, \]

\[(2.10b) \quad \beta_t U_{xt} + \beta_t \lambda_0 [U_{xt} - U_{ct} + U_{cxt}(1 - \tau_t)(1 - x_t) + U_{xxt}(x_t - 1)]
\quad + \lambda_0 \beta_t U_{cxt} y_{0t} - \beta_t \mu_{0t} = 0, \quad t = 0, 1, 2, \ldots, \]

\[(2.10c) \quad \lambda_0 \beta_t U_{ct} - y_0 (\pi^* - \pi^* z_{0t}) + \beta_t \mu_{0t} = 0, \quad t = 0, 1, 2, \ldots \]

where we have employed the shorthand \( U_{ct} = U_c(c_t, x_t) \), etc., where \( \mu_{0t}, \lambda_0, y_0 \) are the Lagrange multipliers associated with (2.1), (2.9a) and (2.9b), and where we have used that \( 1 - x_t - g_t - y_{0t} = \beta_t m_t + m_t = (1 - \tau_t)(1 - x_t) \).

To understand (2.10), let us multiply (2.10a), (2.10b) and (2.10c) by \( dc_t, dx_t \) and \( dm_t \), respectively, and add the resulting equations to get

\[2 \]
\[ (2.11) \quad g^t(U_{ct} dc_t + U_{xt} dx_t) + \lambda_0 p_t d[\tau_t(1-x_t)] + \lambda_0 y_{0t} dp_t \\
+ \lambda_0 p_t d m_t - \gamma_0 (p^*_t - p^*_m z_{0t}) dm_t \\
- \beta^t u_{0t} (dc_t + dx_t - dm_t) = 0, \quad t = 0, 1, 2, \ldots. \]

The left-hand side of expression (2.11) shows the overall effect on welfare of an arbitrary change in the allocation in period \( t \) and each term has a clearcut interpretation.

First, we have the "direct" effect on utility of the change in consumption and leisure, \( \beta^t dU_t = \beta^t (U_{ct} dc_t + U_{xt} dx_t) \).

The changes in \( c_t \) and \( x_t \) also change tax revenue in period \( t \) by \( d[\tau_t(1-x_t)] \).

The effect on utility of the tax change is equal to \( \lambda_0 p_t d[\tau_t(1-x_t)] \); the second term in (2.9). Here, \( \lambda_0 \) measures the distortionary effect of proportional taxation, more precisely the marginal effect of switching from proportional to lump-sum taxes, at constant government expenditure. In the sequel, we shall refer to \( \lambda \) as the (level of) "tax distortion."

Next, the changes in consumption and leisure in period \( t \) imply a change in that period's domestic present value price (interest rate factor) \( dp_t \). As a consequence, government "domestic net wealth" \( \sum_0^\infty p_t y_{0t} \) changes by \( y_{0t} dp_t \). If government wealth increases (decreases) taxes can be lowered (must be raised) in other periods. In effect then, such a wealth revaluation is identical to a switch to lump-sum taxes and consequently its effect on utility is \( \lambda_0 y_{0t} dp_t \); the third term in (2.9).

An increase in imports \( dm_t \) at given tax rates and government spending by
(2.5c) increases domestic cash-flow and decreases foreign cash-flow by the same amount. The increase in domestic cash-flow, by itself, increases utility by \( \lambda_0 p_t \) \( dm_t \), the fourth term in (2.11). (Conversely, the decrease in foreign cash-flow decreases utility; as we shall see, this appears in the next term.)

Then we have a fifth term reflecting the cost of imports. The bracketed expression is the country's true international shadow price of goods in period \( t \), where \( p_t^* \) is adjusted for the fact that a change in imports alters intertemporal prices and hence the country's "foreign net wealth" by \( z_{0t} dp_t^* = z_{0t} p_t^* \) \( dm_t \). This "effective price" is multiplied by a Lagrange multiplier \( \gamma_0 \) -- the marginal cost of imports -- the value of which recognizes that an increase in imports involves foreign borrowing that ultimately must be repaid by raising the distortionary taxes (cf. above).

Finally, the sixth term measures the resource cost of changing the allocation in period \( t \). For all reallocations that obey the economy's resource constraint (2.1), this term is zero of course.

In an optimum, the sum of these six terms must be zero for any reallocations.

The government at time 0 inherited the domestic and foreign debt obligations \( (0_{01} b_{10}, b_{11}, \ldots) \) and \( (0_{01} b_{11}^*, b_{11}^*, \ldots) \) and chooses now in its turn new debt structures \( 1b_t = (1b_{11}^*, 1b_{12}, \ldots), 1b_t^* = (1b_{11}^*, 1b_{12}^*, \ldots) \) that the government at \( t = 1 \) will inherit. For the foreign country to accept the new debt structure, the value of the net issue of foreign debt must satisfy
(2.12a) \[ \sum_{t=0}^{\infty} \left( b_t^* - b_t^0 \right) / p_t^0 = -z_0^* \]

Analogously, the net issue of domestic debt must fulfill

(2.12b) \[ \sum_{t=0}^{\infty} \left( b_t - b_t^0 \right) / p_0 = -v_{00}^* \]

If the government at \( t = 1 \) would be committed to set the future tax rates in accordance with the optimal policy as seen from \( t = 0 \), any debt structure that fulfills (2.12) would do. If, however, the next government is allowed to set its own tax rates, but must honor the debt obligations it inherits, the restructuring of the government debt becomes a more delicate matter. Before turning to that problem, we shall close the present section with a few remarks on the character of the optimal allocation.

It can readily be verified from (2.10a) and (2.10c) that, in general \( p_t = \beta U_{ct} \) is not proportional to \( p_t^* \); that is, it is not optimal to set domestic prices equal to the foreign prices. This is true even for an economy that is small enough to take world market prices for given, so that \( p_t^* = 0 \) in (2.10c), where the wedge between the domestic and foreign price vector arises as a result of taxes being distortionary and of the possibility to change the real value of the domestic debt. If the economy is large enough to affect \( p_t^* \) -- its intertemporal terms of trade -- there will be an "optimal intertemporal tariff" on top of this, implicitly defined in the optimal allocation.

The optimal allocation will smooth both labor supply and consumption. In a closed economy an optimal pattern of taxes financing a given sequence of government consumption should smooth tax distortions over time, which involves
government borrowing (lending) in periods with high (low) government consumption; see the discussion and examples in Barro (1979) and in Lucas and Stokey (1983). Essentially, this result comes about because it is desirable to stabilize the wedge $U_{ct} - U_{xt}$ and hence keep the marginal rate of substitution between consumption and leisure $U_{xt}/U_{ct}$ as close as possible to the marginal rate of transformation, which is 1 by definition.

In an economy that can trade at the world capital market, it also becomes desirable to stabilize $U_{ct}$ over time. This will involve further smoothing of consumption and leisure than in a closed economy, where the (domestic) resource constraint forces government consumption to completely "crowd out" private consumption and leisure. From another angle, in periods when government consumption is low there will be a high private endowment and the country will have a "comparative advantage" and engage in net exports. Conversely, there will be net imports in periods when government consumption is high. The smaller the country in relation to the world economy, the more it can smooth out the adjustment.

This general line of argument will be valid also in the present model (although it has to be slightly modified to allow for the possibility of devaluing government debt). Consequently, in periods with positive government borrowing, the country will borrow abroad and the optimal allocation implies a positive correlation between the deficits in the government budget and the country's current account.
3. **Time Consistency and Domestic Government Debt**

We now turn to the question under what circumstances the optimal fiscal policy is time-consistent. That is, we want to describe the conditions, if any, under which succeeding governments find it optimal to choose the same allocation as did the government at \( t = 0 \). As we shall show, time consistency requires a successive restructuring of the government's debt obligations at home as well as abroad.

Since the problem turns out to be recursive, we may proceed in two steps. In this section, then, we describe how the government's domestic debt should be restructured over time. The management of foreign debt that maintains time consistency is treated in the next section.

The reader can verify from (2.10) that \( y_{0t} \) (and thereby \( b_{0t} \)) only appears in the first-order conditions for \( c_t \) and \( x_t \). As a preliminary step, let us add (2.10a) and (2.10b), and divide the resulting expression by \( (U_{cct} - U_{cxt}) \) to obtain

\[
(3.1a) \quad \lambda_0 y_{0t} + \lambda_0 A_t = B_t, \quad t = 0, 1, \ldots,
\]

where \( A_t \) and \( B_t \) are given by

\[
(3.1b) \quad A_t = A(c_t, x_t) = \left[ U_{ct} - U_{xt} + (U_{cct} - U_{cxt})(1 - \tau_t)(1 - x_t) + (U_{xct} - U_{xxt})(x_t - 1) \right] / (U_{cct} - U_{cxt})
\]

and

\[
(3.1c) \quad B_t = B(c_t, x_t) = -(U_{ct} - U_{xt}) / (U_{cct} - U_{cxt}),
\]

and where \( B_t \) is positive as long as taxes are positive and consumption and leisure are both normal. From equation (3.1), we can obtain an explicit
solution for $\lambda_0$ for future reference. Multiplying (3.1a) with $p_t$ and adding for $t = 0, 1, 2, \ldots$, we get

$$\lambda_0 = \sum_0^\infty p_t B_t / \sum_0^\infty p_t A_t$$

(3.2)

Now, $B_t$ measures the wedge between marginal rates of transformation and substitution in period $t$ caused by the distortional taxes, while it can be shown that $A_t$ is the derivative of government tax income with respect to $c_t$ and $x_t$ (at constant intertemporal prices). Therefore $\lambda_0$ provides a natural measure of the level of the tax distortion in the economy (at $t = 0$).

Let us now turn to the decision problem of the government at $t = 1$. It maximizes $\sum_t^\infty r^t U(c_t, x_t)$ subject to (2.1) and the $t = 1$ analog to (2.9), which yields first-order conditions for $c_t, x_t$ and $m_t$ on the same form as in (2.10).

(3.3a) \[ \begin{align*}
\beta^t U_{ct} + \lambda^t & \left[U_{cct} (1-\tau_t) (1-x_t) + U_{xct} (x_t-1)\right] \\
& + \lambda^t U_{cxt} y_{lt} - \beta^t u_{lt} = 0, \quad t = 1, 2, \ldots,
\end{align*} \]

(3.3b) \[ \begin{align*}
\beta^t U_{xt} + \lambda^t & \left[U_{xt} - U_{ct} + U_{xxt} (1-\tau_t) (1-x_t) + U_{xxt} (x_t-1)\right] \\
& + \lambda^t U_{xxt} y_{lt} - \beta^t u_{lt} = 0, \quad t = 1, 2, \ldots,
\end{align*} \]

and

(3.3c) \[ \begin{align*}
\lambda^t & \beta^t U_{ct} - \gamma^t (p^t - p^* \tau_t z_{lt}) + \beta^t u_{lt} = 0, \quad t = 1, 2, \ldots
\end{align*} \]

Here $\lambda^t$, $\mu_{lt}$ and $\gamma_{lt}$ are the Lagrange multipliers of the analogs to (2.9a), (2.1) and (2.9b) for the government at $t = 1$. The domestic and foreign
cash-flows of the government at \( t = 1 \), \( y_{1t} \) and \( z_{1t} \) fulfill

\[(3.4a)\quad y_{1t} = r_t (1-x_t) + m_t - e_t - \lambda b_t^*, \quad t = 1, 2, \ldots, \text{ and} \]

\[(3.4b)\quad z_{1t} = -m_t - \lambda b_t^*, \quad t = 1, 2, \ldots, \]

where \( \lambda b_t \) and \( \lambda b_t^* \) are home and foreign debt inherited from the government at \( t = 0 \).

Subtracting (3.3b) from (3.3a) and manipulating, we get

\[(3.5)\quad \lambda y_{1t} + \lambda A_t = B_t, \quad t = 1, 2, \ldots, \]

Also, we may solve here for the tax distortion, as

\[(3.6)\quad \lambda = \sum_{t=0}^{\infty} p_t B_t / \sum_{t=0}^{\infty} p_t A_t. \]

For time consistency, the government at \( t = 1 \) must obviously choose the same \( c_t \) and \( x_t \) for each \( t = 1, 2, \ldots \) as did the government at \( t = 0 \). Since \( A_t \) and \( B_t \) in equations (3.1) and (3.5) depend only on \( c_t \) and \( x_t \), they must then be the same. Combining the two equations, we get

\[(3.7)\quad \lambda y_{1t} = \lambda y_{0t} - (\lambda - \lambda_0) A_t, \quad t = 1, 2, \ldots \]

Equations (2.5) and (3.4) together with (3.7) yield

\[(3.8)\quad \lambda b_t - \lambda b_t^* = -(y_{1t} - y_{0t}) = (1 - \lambda_0 / \lambda) (y_{0t} + A_t), \quad t = 1, 2, \ldots \]

In other words, if the government at \( t = 0 \) restructures its domestic debt according to (3.7) it will give its predecessor an appropriate incentive to choose the same \( c_t \) and \( x_t \), which is necessary, but in general not sufficient, for a continuation of the optimal tax policy (the government at \( t = 1 \) must also have
incentives to choose the same \( m_t \); see further below).

To understand what this rule involves, consider the change in the tax distortion from \( t = 0 \) to \( t = 1 \). From (3.2) and (3.6) it is clear that \( \lambda_1 \) is greater than \( \lambda_0 \), whenever \( B_t < \lambda_0 A_t \). We see from (3.1a) that such an increase in the tax distortion must be associated with \( y_{00} < 0 \), that is a negative domestic cash-flow at \( t = 0 \). Indeed, solving explicitly for \( \lambda_1 - \lambda_0 \), we get

\[
\lambda_1 - \lambda_0 = -\lambda_0 p_0 y_{00} / \sum_p A_t.
\]

So we may deduce

\[
(3.10) \quad \lambda_1 \geq \lambda_0 \text{ if and only if } y_{00} \leq 0.
\]

Also, from (3.1) and (3.3)

\[
(3.11) \quad \lambda_0 (y_{0t} + A_t) = \lambda_1 (y_{1t} + A_t) = B_t, \quad t = 1, 2, \ldots,
\]

so that

\[
(3.12) \quad y_{1t} \geq y_{0t}, \text{ if and only if } \lambda_1 \geq \lambda_0, \quad t = 1, 2, \ldots
\]

Hence, by (3.8), (3.10) and (3.12), a negative domestic cash-flow in period 0 is distributed over all future periods so that the cash-flow \( y_{1t} \) is smaller than \( y_{0t} \) in all \( t \). With tax revenue and government spending given, this corresponds to an increase in interest payments and/or debt maturing in period \( t \) \( (b_t > 0 b_t) \). A negative (positive) cash-flow in period 0 means that domestic debt should be restructured so that obligations to the consumer are increased (decreased) in all future periods; and (3.8) states the precise way that this should be done.
As long as $y_{00}$ is non-zero, the government at $t = 1$ has a different tax distortion than the government at $t = 0$ and therefore a different incentive to change taxes in all periods. The logic behind the debt restructuring is to change the domestic cash-flow, and therefore the base for domestic debt devaluations, in all periods in such a way that it exactly matches the different tax distortion. By choosing the maturity structure of its domestic debt according to the principles we have just described, the government at time 0 can indeed "bind the hands" of its successor so that it has incentives to continue choosing the same $c_t$ and $x_t$ and hence the same tax rates.

Exactly the same reasoning can be applied to any pair of governments. A complete description of the sequence of debt restructurings that are necessary for time consistency is therefore obtained just by changing the subscripts 0 and 1 to $t$ and $t+1$, respectively, in equations (3.1) through (3.12).

If the economy were closed, these necessary conditions would also be sufficient for time consistency of the optimal policy. In fact, the restructuring scheme we have just derived is exactly the scheme derived in the closed-economy model of Lucas and Stokey (1983), although cast in a different form (for a further comparison with Lucas and Stokey's results the reader is referred to Svensson and Persson (1984)).

Since we deal with an open economy, however, we must also find out whether succeeding governments have incentives to continue choosing the same $m_t$ as the government at $t = 0$. If so, the optimal policy is indeed time-consistent.
4. **Foreign Debt**

It remains to examine whether the government at \( t = 1 \) has incentive to choose the same import levels \( m_t, t = 1, 2, \ldots \), as the government at time 0. Let us first analyze the case where the country is small and cannot affect world market present value prices \( p^*_t \), that is, where the import derivative of the foreign demand-price function is zero,

\[
(4.1) \quad p^*_{t m} = 0, \ t = 0, 1, \ldots
\]

Under this assumption (2.10c) simplifies to

\[
(4.2) \quad \lambda_0 \beta^t u_{ct} - \gamma_0 p^*_t + \beta^t u_{bt} = 0, \quad t = 0, 1, \ldots
\]

from which it follows that \( \gamma_0 \) fulfills

\[
(4.3) \quad \gamma_0 = (\lambda_0 \beta^t u_{ct} + \beta^t u_{bt}) / p^*_t, \quad t = 0, 1, \ldots
\]

The decision problem for the government at \( t = 1 \) implies the corresponding first-order condition

\[
(4.4) \quad \lambda_1 \beta^t u_{ct} - \gamma_1 p^*_t + \beta^t u_{lt} = 0, \quad t = 1, 2, \ldots
\]

which can be solved for

\[
(4.5) \quad \gamma_1 = (\lambda_1 \beta^t u_{ct} + \beta^t u_{lt}) / p^*_t, \quad t = 1, 2, \ldots
\]

In Section 3 we have seen that a necessary condition for the government at \( t = 1 \) to choose the same \( c_t \) and \( x_t \) as the government at \( t = 0 \) is that \( \lambda_1 \) is given by (3.6) and \( \gamma_{lt} \) in turn by (3.7). Then \( \mu_{lt} \) is given by (3.3a) and (3.3b), and we have a unique \( \gamma_1 \) that satisfies (4.5). Since the international cash-flows \( z_{lt} \) do not enter (4.5), the cash-flows that are necessary in time-consistency are not unique.
Indeed, any cash-flows that satisfy (2.12a) and (3.4b) will do, and the maturity structure of the foreign debt does not matter. Intuitively, since for the small economy world market interest rates are given, there is no way the government can manipulate those interest rates and devalue the foreign debt.

Let us next consider the large country case, when world market interest rates can be affected, that is when the demand-price derivative $P_{tm}^*$ is no longer equal to zero. The relevant first-order conditions are now (2.10c) and (3.3c) rather than (4.2) and (4.3). Multiplying (3.3c) by $p_t^*$, adding for $t = 1, 2, \ldots$, and using $\Sigma_{1}^{\infty} P_{1t}^* z_{1t} = 0$, we can solve for $\gamma_1$ and get

$$
(4.6) \quad \gamma_1 = \Sigma_{1}^{\infty} \left( (\lambda_1 \bar{B}_t^U c_t + m_t) P_{t}^* / P_{tm}^* \right) / \Sigma_{1}^{\infty} P_{1t}^* / P_{tm}^* .
$$

Hence there is a unique $\gamma_1$ that fulfills (4.6) for given $c_t, x_t, m_t, \lambda_1$ and $\mu_{it}$. From (3.3c) it then follows that there is a unique sequence of foreign cash-flows $z_{1t}, t = 1, 2, \ldots$ consistent with (3.3c). Put differently, there is a unique foreign debt structure $b_t^*, t = 1, 2, \ldots$ given by (3.4b) that gives the government at $t = 1$ incentive to choose the same sequence of $c_t, x_t, m_t, t = 1, 2, \ldots$, as the government at $t = 1$.

Let us try to characterize the time-consistent sequence of cash-flows. First, by subtracting (2.10c) from (9.3c) for $t = 1, 2, \ldots$, dividing by $P_{tm}^*$, multiplying by $p_t^*$ and adding for $t = 1, 2, \ldots$, and using $\Sigma_{1}^{\infty} P_{1t}^* z_{1t} = 0$ and $\Sigma_{1}^{\infty} P_{t}^* z_{0t} = P_{0}^* 00$, we get

$$
(4.7) \quad \gamma_1 - \gamma_0 = \gamma_0 P_{00}^* / \Sigma_{1}^{\infty} (p_t^*/P_{tm}^*) + (\lambda_1 - \lambda_0) \Sigma_{1}^{\infty} (p_t^* D_{t}/P_{tm}^*) / \Sigma_{1}^{\infty} (p_t^*/P_{tm}^*),
$$

where we also have used (2.10a) and (3.3a) to substitute for $\mu_{1t} - \mu_{0t}$, and where $D_t$ is given by
\[ D_t = D_t(c_t, x_t) = \beta^t[U_{ct} + U_{cct}(1-T_t)(1-x_t) + I_{xct}(x_t - 1) - U_{cct}A_t]. \]

It can be shown that \( D_t \) is positive for \( t = 1, 2, \ldots \), if goods and leisure are normal goods.

With some algebra one can then derive from (2.10c), (3.3c) and (4.7),

\[
\gamma_1(z_{1t} - z_{0t}) = (p_t^*/p_{tm}^* - z_{0t})[\gamma_0 p_0^*/\Sigma_1(p_t^*/p_{tm}^*)]z_{00} + [(p_t^*/p_{tm}^* - z_{0t})\Sigma_1(p_t^*/p_{tm}^*)/\Sigma_1(p_t^*/p_{tm}^*) - D_t](\lambda_1 - \lambda_0)
\]

Let us first consider the case when \( \lambda_1 = \lambda_0 \), that is when \( y_{00} = 0 \). Since \( p_t^*/p_{tm}^* - z_{0t} \) is positive, so is the term multiplying \( z_{00} \) in (4.8). Therefore, when \( \lambda_1 = \lambda_0 \), we have

\[
z_{1t} \geq z_{0t}, \quad t = 1, 2, \ldots, \text{if and only if} \ z_{00} \geq 0.
\]

That is, a positive foreign cash-flow in period 0 means that the government at \( t = 1 \) has all its foreign cash-flows for \( t = 1, 2, \ldots \) larger than the government at \( t = 0 \). In terms of the debt structure, this implies that the level of foreign debt the government at \( t = 1 \) inherits is lower for all maturities, \( 1b_t^* < 0b_t^* \), \( t = 1, 2, \ldots \)

To understand this result, recall that when \( \lambda_1 = \lambda_0 \), \( y_{00} = 0 \), and by

\[
y_{1t} = y_{0t}, \quad t = 1, 2, \ldots \text{ Therefore, by (2.10a) and (3.3a) } \mu_{0t} = \mu_{1t}, \quad t = 1, 2, \ldots \text{ But then it follows from (2.10c) and (3.3c) that}
\]

\[
\gamma_0(p_t^*/p_{tm}^*z_{0t}) = \gamma_1(p_t^*/p_{tm}^*z_{1t}), \quad t = 1, 2, \ldots
\]

that is, the product of the multiplier \( \gamma \) and the effective price of imports should be the same in each period for both governments. (Note that the effective price
of imports is always positive.) Now, when \( z_{00} \) is positive, the remaining cash-flows \( z_{0t}, t = 1, 2, \ldots \), are on average negative, since then
\[
\sum_{t} p^* t^* z_{0t} = -p^* z_{00} < 0.
\]
Hence, the effective price of imports of the government at \( t = 1, p^* - p^*_t z_{0t} \), are on average lower than the effective price of imports for the government at \( t = 0, p^* - p^*_t z_{0t} \). It then follows from (4.10) that the multiplier \( \gamma_1 \) exceeds \( \gamma_0 \). This is indeed an intuitive explanation to (4.7).

Anyhow, if \( \gamma_1 \) is greater than \( \gamma_0 \), it follows from (4.10) that for time-consistency, the effective price of imports for the government at \( t = 1 \) should not only be lower on average, but lower in each period, which requires its foreign cash-flow \( z_{1t} \) to exceed \( z_{0t} \) in each period.

The situation becomes more complex whenever \( \lambda_1 \neq \lambda_0 \), that is, whenever \( y_{00} \neq 0 \). This is apparent from (4.8), since the term multiplying \((\lambda_1 - \lambda_0)\) can be of any sign. If \( y_{00} > 0 \) and \( \lambda_1 < \lambda_0 \), say, we have by (3.12) \( y_{1t} < y_{0t} \), and it can be shown that \( \lambda_1 \beta^t u_{ct} + \beta^t \mu_{1t} < \lambda_0 \beta^t u_{ct} + \beta^t \mu_{0t} \) and from (2.10c) and (3.3c) we get

\[
(4.11) \quad \gamma_1 (p^*_t - p^*_{tm} z_{1t}) < \gamma_0 (p^*_t - p^*_{tm} z_{0t}).
\]

Intuitively, with lower tax distortion and less private debt, the benefit \( \lambda_1 \beta^t u_{ct} + \beta^t \mu_{1t} = \lambda_1 p_t + \beta^t \mu_{1t} \) of additional imports for the government at \( t = 1 \) is lower, hence in equilibrium the cost of imports \( \gamma_1 (p^*_t - p^*_{tm} z_{1t}) \) must be lower. Furthermore, from (4.7) we see that \( \lambda_1 \) < \( \lambda_0 \) contributes to make \( \gamma_1 \) less than \( \gamma_0 \). Intuitively, we can understand that by observing that the multiplier \( \gamma \) incorporates that increased imports must eventually be paid by increasing taxes, the costs of which are higher, the higher the tax distortion \( \lambda \). Hence, with
\( \lambda_1 < \lambda_0 \), we have \( \gamma_1 < \gamma_0 \). We then realize that (4.11) may hold for effective prices for imports for the government at \( t = 1 \), being either greater or smaller than those for the government at \( t = 0 \). And therefore, we do not get an unambiguous relation between the two governments' foreign cash-flows when \( \lambda_1 \neq \lambda_0 \) and \( \gamma_{00} \neq 0 \). However, if the difference between \( \lambda_1 \) and \( \lambda_0 \) is sufficiently small relative to \( z_{00} \), that is \( \gamma_{00} \) is sufficiently small relative to \( z_{00} \), the "pure" effect of \( z_{00} \) will dominate and (4.9) still holds.

5. **Private Borrowing**

Let us now consider the situation where private foreign borrowing is allowed. Since taxes on international capital flows are excluded, home present value prices must be proportional to international prices, that is

\[
(5.1) \quad p_t = b_t U_t = \alpha \pi^*(m_t), \quad t = 0, 1, \ldots,
\]

for some \( \alpha > 0 \). Furthermore, we must distinguish government foreign cash-flows and import, \( z_{0t} \) and \( \tilde{m}_t \), from the economy's total foreign cash-flows and import, \( z_{0t} \) and \( m_t \). Also, we should distinguish government foreign debt \( \hat{o}_t \) from total foreign debt \( o_t \), the difference \( o_t - \hat{o}_t \) being private foreign debt. We have the identities

\[
(5.2a) \quad z_{0t} = -m_t - o_t \quad \text{and}
\]

\[
(5.2b) \quad \tilde{z}_{0t} = -\tilde{m}_t - \hat{o}_t
\]

The constraints (2.9) can now be replaced by
(5.3a) \[ \sum^\infty_0 \delta^t U_{ct}(y_{0t} + \tilde{z}_{0t}) = \sum^\infty_0 \delta^t U_{ct}y_{0t} + \sum^\infty_1 \alpha^p_t (m_t)(-\tilde{m}_t - \delta^*_t) > 0, \]

(5.3b) \[ \sum^\infty_0 \alpha^p_t (m_t)(-\tilde{m}_t - \delta^*_t) > 0, \]

and

(5.3c) \[ U_{ct}(1-x_t - b_t - y_{0t} + \tilde{m}_t) + U_{ct}(x_t - 1) = 0. \]

The first constraint is the overall government budget constraint (rewriting it using domestic and foreign prices separately turns out to be convenient when comparing with the previous first-order conditions). The second is the economy's budget constraint relative to the foreign country. The third is the constraint expressing private maximizing behavior which follows since total government cash-flow fulfills

(5.4) \[ y_{0t} + \tilde{z}_{0t} = r_t (1-x_t) - \delta_t - \delta^*_t \]

and thus

(5.5) \[ (1-r_t)(1-x_t) = 1 - x_t - \delta_t - \delta^*_t - y_{0t} + \tilde{m}_t, \]

which can be substituted into (2.4a) to give (5.3c).

The first-order conditions for the government at \( t = 0 \) are

(5.6a) \[ \beta^t U_{ct} + \lambda_t^0 \beta^t [U_{cct}(1-r_t)(1-x_t) + U_{xct}(x_t - 1)] + \lambda_t^0 \beta^t U_{cct}y_{0t} + \tau_t^0 \beta^t U_{cct} - \beta_t^t \delta_t^0 = 0, \]

(5.6b) \[ \beta^t U_{xt} + \lambda_t^0 \beta^t [U_{xt} - U_{ct} + U_{cxt}(1-r_t)(1-x_t) + U_{xxt}(x_t - 1)] + \lambda_t^0 \beta^t U_{cxt}y_{0t} + \tau_t^0 \beta^t U_{cxt} - \beta_t^t \delta_t^0 = 0, \]
(5.6a) \[ \beta^t U_{ct} + \lambda_0 \beta^t [U_{cct}(1-\tau_t)(1-x_t) + U_{xct}(x_t-1)] + \lambda_0 \beta^t U_{cct} y_{0t} + \pi_{0t} \beta^t u_{ct} - \beta^t u_{0t} = 0, \]

(5.6b) \[ \beta^t U_{xt} + \lambda_0 \beta^t [U_{xt} - U_{ct} + U_{cxt}(1-\tau_t)(1-x_t) + U_{xxt}(x_t-1)] + \lambda_0 \beta^t U_{cxt} y_{0t} + \pi_{0t} \beta^t U_{cxt} - \beta^t u_{0t} = 0, \]

(5.6c) \[ \lambda_0 a_{p^*} \hat{z}_{0t} - \gamma_0 a_{p^*-p^*} \hat{z}_{0t} - \pi_{0t} a_{p^*} + \beta^t u_{0t} = 0, \]

(5.6d) \[ \lambda_0 \hat{m}_{0t} \hat{m}_{0t} - \pi_{0t} a_{p^*} = 0, \]

where \( \lambda_0, \mu_{0t}, \gamma_0 \) and \( \pi_{0t} \) are the Lagrange multipliers corresponding to the constraints (5.3a), (2.1), (5.3b) and (5.1), respectively. The last condition follows from choosing the unique \( \alpha \) that fulfills (5.1).

We assume that the optimal solution is unique with respect to \( c_t, x_t, m_t, \lambda_0, \mu_{0t} \) and \( \gamma_0 \). However, one may easily check that the solution need not be unique with respect so \( y_{0t}, \hat{m}_t \) and \( \pi_{0t} \). This follows since only the sum \( y_{0t} + \hat{z}_{0t} \) and the difference \( y_{0t} - \hat{m}_t \) enter in the constraints (5.3a) and (5.3b). Indeed, what is unique is the difference between \( y_{0t} \) and \( \hat{m}_t \), or alternatively, the sum of \( y_{0t} + \hat{z}_{0t} \), which amounts to total government cash-flow as given in (5.4), but not the government's private and foreign cash-flows separately.

This result can also be seen by multiplying (5.6a) with \( dc_t \), (5.6b) with \( dx_t \) and (5.6c) with \( dm_t \), and adding the three equations. Restricting the changes in \( dc_t, dx_t \) and \( dm_t \) to those that fulfill the price constraint (5.1) and the resource constraint (2.1), one gets
(5.7) \[ \beta^t dU_t + \lambda_0 \alpha_t d[\tau_t (1-x_t)] + \lambda_0 (y_{0t} + \tilde{z}_{0t}) dP_{tm}^* - \gamma_0 (p_t^* - P_t^* z_{0t}) d\mu_t = 0, \]

in analogy with (2.11). Here we see that \( z_{0t} \) (and hence \( m_t \)) enters separately (in the effective price of imports), but only total government cash-flow, \( y_{0t} + \tilde{z}_{0t} \) enters, not private and foreign government cash-flow separately.

Intuitively, what is relevant for the government wealth revaluation effect is total government cash-flow, not its composition.

Let us now investigate whether a time-consistent policy exists. The decision problem for the government at \( t = 1 \) results in first-order conditions completely analogous to (5.4), only that \( \lambda_1, \pi_{1t}, \mu_{1t}, \gamma_1, y_{1t} + \tilde{z}_{1t}, \) and \( z_{1t} \) enter. We first consider the small economy case, when

(5.8) \[ P_{tm}^* = 0, \ t = 0, 1, \ldots, \]

and \( p_t^* \) are given. Not surprisingly, fiscal policy is time-consistent in that case. Then (5.7) is simplified to

(5.9) \[ \beta^t dU_t + \lambda_0 \alpha_t d[\tau_t (1-x_t)] - \gamma_0 \alpha_t^* d\mu_t = 0 \]

for the government at \( t = 0 \), with an analogous expression for the government at \( t = 1 \). Since no cash-flow terms enter in (5.9), there are no wealth revaluation effects, and the debt structure no longer matters. This is of course quite obvious, since the whole time-consistency problem arises only when intertemporal prices can be affected and wealth revaluation effects are relevant.

Let us then look at the large economy problem. Consider the analogy of (5.6) for the government at \( t = 1 \). Plug in the sequence of \( c_t, x_t \) and \( m_t, t = 1, 2, \ldots, \) that is a solution to (5.6) for the government at time 0. This
results in unique $\lambda_1$, $\mu_{1t}$ and $\gamma_1$. Then one can indeed find unique total
government cash-flows $y_{1t} + \hat{z}_{1t}$ and total foreign cash-flows $z_{1t}$, and non-unique
cash-flows $y_{1t}$ and $\hat{z}_{1t}$ and multipliers $w_{0t}$ such that the $t = 1$ analog of
(5.6) together with the constraints (2.1), (5.1) and (5.3) are fulfilled.
Hence, there is a time-consistent policy.

This time-consistent policy requires the government to choose the new total
foreign debt structure according to

\[(5.10)\quad l_{b_t}^* = -z_{1t} - m_t, \quad t = 1,2,\ldots,\]

and the total government debt structure

\[(5.11)\quad l_{b_t} + \hat{b}_t^* = r_t(1 - x_t) - g_t - (y_{1t} + \hat{z}_{1t}), \quad t = 1,2,\ldots\]

We have so far not been able to obtain simple characteristics of the
relations between foreign cash-flows $z_{0t}$ and $z_{1t}$ (that is, the relation between
total foreign debt $0b_t^*$ and $1b_t^*$) or of the relations between government total
cash-flows $y_{0t} + \hat{z}_{0t}$ and $y_{1t} + \hat{z}_{0t}$ (that is, the relation between the
total government debt structure $0b_t + \hat{b}_t^*$ and $1b_t + \hat{b}_t$).

Let us finally make the following observation. We have seen that government
foreign cash-flow $\hat{z}_{0t}$ is not unique. Then it can be chosen equal to $z_{0t}$, which
means that private foreign cash-flow is zero, while maintaining the same level of
utility. What this means is that a restriction to zero private foreign cash-
flows is not binding. This in turn suggests (assuming private initial foreign debt
$0b_t^* - \hat{b}_t$ equal to zero) that we can get the same solution by starting in the
situation we considered in previous sections, namely when private international
borrowing is forbidden and home (relative) prices are not restricted to equal
foreign (relative) prices, and then adding the constraint that home and
foreign prices are equal. Clearly, this implies that the level of utility cannot be higher with private international borrowing, and if the constraints (5.1) are binding, the level of utility is lower with private borrowing. In the distorted world we are considering here, it is better to forbid private international borrowing, separate the home and foreign credit market, and allow home interest rates to differ from world interest rates.

6. **Concluding Remarks**

We have discussed the design of optimal fiscal policy in open economies. Apart from smoothing out the tax distortions associated with financing a given sequence of government consumption over time, an optimal allocation also smooths out private consumption of goods and leisure by borrowing (lending) on the international capital market in periods of high (low) government consumption.

The bulk of the paper dealt with if and how the optimal policy could be made time consistent, when successive governments are allowed to reoptimize with respect to current and future tax rates, but must honor the government debt obligations they inherit. We first treated the case when the government, but not the private sector, is allowed to trade (over time) at international markets. It was then necessary that governments were able to issue domestic debt of sufficiently rich maturity. We found, and were able to characterize, a unique restructuring scheme of the debt that was necessary to give succeeding governments incentives to continue following the optimal policy. In a small open economy this scheme for domestic debt was sufficient for time consistency. For a large economy, however, it was also necessary to follow a unique
restructuring scheme for the government's (and the country's) foreign debt.

When the private sector was allowed to borrow abroad the results changed considerably. For a small open economy, with no possibility to change world market prices, time inconsistency of the optimal policy was no longer a problem and, consequently, no particular pattern of debt obligations was called for. In a large economy what mattered was total government debt (rather than its composition) and total foreign debt. We showed that there was a unique maturity structure, for total government as well as foreign debt, necessary and sufficient for time consistency. Unfortunately, we were not able to characterize the implied restructuring scheme, however.

An interesting observation is that welfare turned out to be higher when private capital movements were not allowed. Whether this case for capital controls continues to hold under more general circumstances remains an area for further research.

We should hasten to point out some serious limitations of the analysis. Central to our results that each government can bind the hands of its successors is the assumption that the governments debt obligations are always honored. As a consequence, we could not allow any taxation of interest income. That would immediately raise problems of time inconsistency since it would always be optimal to tax that income in the short-run. In the same way, it would be optimal in the short-run to tax income from pre-existing capital. The absence of capital is thus central to our results. Likewise is the absence of money, since in a monetary economy, governments would have short-run incentives to engage in capital levies on the outstanding money stock via "surprise" inflations. These issues are further discussed in Lucas and Stokey (1983).
There are also some drawbacks that are specific to the international aspects of the analysis. Our assumption of no government activity abroad could have been relaxed to allow for passive policies in the foreign country. Although that assumption might be standard, it is still not very reasonable. Strategic considerations abroad would lead directly to a full-fledged game-theoretic analysis of conflicting government policies.

Finally, we have ignored by assumption a question very much at issue today, namely the possible repudiation of foreign debt by sovereign borrowers. As shown by the recent literature on LDC borrowing, opening up the possibility of default lends the problem of optimal foreign borrowing entirely new dimensions. Choosing the maturity structure of the foreign debt, for instance, then also involves influencing the expectations of foreign creditors in a proper way, (as discussed by Sachs and Cohen (1982)).
Footnotes

1. Government cash-flow so defined is not equal to the budget surplus, as it is conventionally defined, neither is it equal to net government saving; see Svensson and Persson (1984), for a further discussion.

2. The second term in (2.11) comes from the second term in (2.10a) and (2.10b) since, \( \lambda_0 \beta^t [(U_{xt}-U_{ct})dx_t + (U_{cxt}dc_t + U_{xt}dx_t \tau_t)(1-x_t) + (U_{cxt}dc_t + U_{xt}dx_t)(x_t-1)] \)
\( = \lambda_0 \rho([U_{xt}-U_{ct})dx_t/U_{ct} - dU_{xt}(1-x_t)/U_{ct} + dU_{cxt}U_{xt}(1-x_t)/(U_{ct})^2] = \lambda_0 \rho_t [d(\tau_t(1-x_t)]. \)

3. This is qualitatively the same result as that obtained by Razin and Svensson (1983), who look at optimal taxation in a small open economy subject to productivity shocks.

4. We have \( U_{ct} - U_{xt} = \tau_t U_{ct} > 0 \), if \( 0 < \tau_t < 1 \). If goods are normal, \( (\partial/\partial c)(U_c/U_x) = (U_{cc}U_{cc} - U_{c}U_{cx})(U_x)^2 < 0 \) for all \((c, x)\), which implies \( U_{cc} - U_{cx} < 0 \).


6. Evaluating \( D_t \) we get \( D_t = [((U_{cc}U_{cc} - U_{c}U_{cx}) + (U_{cc}U_{xx} - U_{cx}^2)(x_t-1))(U_{cc} - U_{cx}^2]. \)
The first term within brackets and the denominator are negative when goods and leisure are normal, and \( U_{cc}U_{xx} - U_{cx}^2 \) is positive by the concavity of \( U(\cdot). \)

7. We have \( \lambda_1 \beta^t U_{ct} + \beta^t U_{lt} = (\lambda_0 \beta^t U_{ct} - \beta^t U_{0t}) = (\lambda_1 - \lambda_0) D_t \), by (2.10a), (2.10b) and the definition of \( D_t \).

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