The Dynamical Evolution of Classical Be Stars

Keegan Marr, The University of Western Ontario

Supervisor: Jones, Carol E., The University of Western Ontario

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Abstract

This thesis focuses on the evolution of the disks of two classical B-emission (Be) stars, 66 Ophiuchi and Pleione, and on the thermal structure for disks tilted out of the star’s equatorial plane.

We used a hydrodynamic code to model the disk of the Be star 66 Ophiuchi. Observations from 1957 to 2020 were compiled to follow the growth and subsequent dissipation of the disk. Our models are constrained by new and archival photometry, spectroscopy and polarization observations. Using Markov chain Monte Carlo methods, we confirm that 66 Oph is a B2Ve star. We constrain the density profile of the disk before dissipation using a grid of disk models. At the onset of dissipation, the disk has an equatorial density of $\rho(R) = 2.5 \times 10^{-11} (R/R_\star)^{-2.6}$ g cm$^{-3}$. After 21 years of disk dissipation, our work shows that 66 Oph’s outer disk remains bright in the radio. We find an isothermal disk with constant viscosity with an $\alpha = 0.4$ and an outer disk radius of $\sim 115$ stellar radii best reproduces the dissipation. We determined the interstellar polarization in the direction of the star in the V-band is $p = 0.63 \pm 0.02\%$ with a polarization position angle of $\theta_{IS} \approx 85.7 \pm 0.7^\circ$. Using the Stokes QU diagram, we find the intrinsic polarization position angle of 66 Oph’s disk is $\theta_{int} \approx 98 \pm 3^\circ$.

We acquired H$\alpha$ spectroscopy from 2005 to 2019 that shows Pleione has transitioned from a Be phase to a Be-shell phase. We created disk models which successfully reproduce the transition from Be to Be-shell with a disk model that varies in inclination while maintaining a constant, equatorial density of $\rho(R) = 3 \times 10^{-11} (R/R_\star)^{-2.7}$ g cm$^{-3}$, and an H$\alpha$ emitting region extending to $R_{out} = 15 R_{eq}$. We use a precessing disk model to follow variability in disk inclination over 120 years. The best-fit disk model precesses with an inclination between $\sim 25^\circ$ and $\sim 144^\circ$ with a period of $\sim 80.5$ years. Our precessing models match some of the observed variability but fail to reproduce all of the historical data available. Therefore, we propose an ad-hoc model based on our precessing model and recent disk tearing simulations of similar systems. In this model, a single disk is slowly tilted to an angle of $30^\circ$ from the stellar equator over 34 years. Then, the disk is torn by the companion’s tidal torque, with the outer region separating from the innermost disk. The inner disk returns to the stellar equator as mass injection remains constant. The outer disk precesses for $\sim 15$ years before gradually dissipating. This model reproduces all the variability trends, repeating every 34 years.

Our research on Pleione led to a detailed investigation of the thermal structure of tilted disks. For this research, we modelled the radiative transfer in tilted disks self-consistently. We constructed disk models for a range of spectral types, rotation rates and disk densities. We find as the tilt angle increases to $60^\circ$, the minimum disk temperature of our B0 V star model, with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$, can increase up to $\sim 114\%$, while the maximum disk temperature decreases by up to $\sim 8\%$. When $W = 0.7$, the changes in disk temperature for the same model are smaller, and at lower density the disk temperature increases globally. In the B2 V model, both the disk temperature and ionization fraction globally increase. In the B5 V and B8 V models, the disk temperature globally decreases, but increases around $\sim 10 R_{eq}$. The ionization fraction increases as modest changes to the disk temperature allow it to exceed the hydrogen ionization temperature. Overall, we find that the trends in the disk temperature and ionization fraction with the disk tilt angle greatly depend upon the stellar spectral type.

Keywords: B stars, emission line stars, circumstellar disks, disk evolution, disk precession, 66 Oph, 28 Tau, Pleione, radiative transfer, hydrodynamics
Summary for Lay Audience

There are many different types of astrophysical disks observed in the Universe. For example, there are disks of material swirling around black holes, our own Milky Way Galaxy is in the shape of a flattened disk, even our Solar System formed from within a disk of material leftover when the Sun formed, and there are many more examples. As such, it is important to understand the physical conditions in astrophysical disks and the processes that operate in them.

My work involves a particular type of massive star that is surrounded by a disk of material sometimes extending to hundreds of the star’s radius from its surface. They are called classical B-emission stars, Be stars for short. These stars are bright and numerous making them ideal candidates to help understand disks. The starlight interacts with disk material, and when a telescope or orbiting satellite observes it, signatures of both the disk and star are collected together which provides lots of information. Despite decades of study, we still do not know for sure what triggers the formation of a disk in these stars. Certainly, rapid rotation of the star helps to launch material into orbit and form a disk but something else must also be operating since we think they rotate below their break-up speed.

I construct detailed computer models that follow disk evolution over time so that I can try to match all of the observed signatures in the light to understand how these systems work. My thesis research focuses on two particular Be stars. One star, called 66 Oph, completely lost its disk during my study. With the other star, called Pleione, the disk seems to tilt, become unstable, then tears into two pieces and eventually the cycle repeats. Finally, I investigate the temperature distribution within these tilted disks. As the disk tilts, it interacts with light coming from the star’s surface which has different temperatures. Getting the disk temperatures correct is essential because it directly affects the state of the gas in the disk, which in turn, allows us to predict observables. If the gas temperature is wrong, then the interpretation of observations will be wrong too.

My research is making significant advances to understand these systems and it has the potential to help understand other types of astrophysical disks too.
Co-Authorship Statement

The following authors contributed to Chapter 2, entitled “The Be Star 66 Ophiuchi: 60 Years of Disk Evolution”. This article was published in the Astrophysical Journal, Volume 912, Issue 1, id.76, 15pp in May 2021.

Keegan Marr (KCM) took primary responsibility in the preparation of the article comprising Chapter 2. KCM completed the background literature search, and was the primary correspondent with the journal referees. KCM developed and tested the line profile broadening MCMC code used for fitting line profiles. KCM modified the singlebe routine to have variable temperature with radius. KCM ran each of the Be star simulation routines to create the Be star models. KCM interpreted the results of the simulation data and created all figures found in Chapter 2. KCM acquired and prepared all archival data found in the enclosed work, and processed observations acquired from ACC as needed. As an estimate, KCM completed ~ 85% of the work comprising this thesis chapter.

Prof. Carol Jones (CEJ) assisted with the background literature search, deciding the appropriate ranges of model input parameters, and interpreting the simulation data. CEJ contributed to formulating the models described in Chapter 2. CEJ had input to the writing of the article and suggested revisions in preparation for publication.

Prof. Alex Carciofi (ACC) created the hdust routine and gave guidance for its use. ACC gave consistent feedback on the projects. ACC provided reduced polarimetric observations of the Be star 66 Oph. ACC suggested revisions to published articles comprising Chapters 2 and 3.

Dr. Bruno Mota (BM) developed the bemcee routine and provided instruction on its use.

Amanda Rubio (AR) assisted BM in the development of the bemcee routine. AR gave feedback on interpreting the results of the bemcee routine, and insight on improving the model results.

Dr. Maziar Ghoreyshi assisted in using the singlebe routine and provided insight on interpreting the simulation data.

Dr. Leandro Rimulo modified the singlebe routine to implement variable disk viscosity with radius.

The following authors contributed to Chapter 3, entitled “The Role of Disk Tearing and Precession in the Observed Variability of Pleione”. This article was submitted to the Astrophysical Journal, in August 2021, and is currently in the refereeing process at the time of submission for this thesis.

KCM took primary responsibility in preparation of the article comprising Chapter 3. KCM completed the background literature search, and is the primary correspondent with the journal
referees. KCM ran each of the Be star simulation routines to create the Be star models. KCM interpreted the results of the simulation data and created all of the figures found in Chapter 3. KCM acquired and prepared all archival data found in the enclosed work, and modified observations acquired from ACC and CT as needed. As an estimate, KCM completed 80% of the work comprising this chapter.

CEJ’s contributions to this work extended to the same responsibilities listed for Chapter 2.

Prof. Chris Tycner (CT) provided reduced Hα observations of the Be star Pleione. CT gave feedback on interpreting the results found in Chapter 3. CT suggested revisions to the published article comprising Chapter 3.

ACC’s contributions to this work extended to the same responsibilities listed for Chapter 2.

Ariane Fonseca Silva provided reduced polarimetric observations of the Be star Pleione.

The following researchers contributed to Chapter 4 entitled “The Thermal Structure of Tilted Be Star Disks”. This chapter has not been submitted to a refereed journal at the time of submission for this thesis.

KCM took primary responsibility in preparation of the article comprising Chapter 4. KCM ran each of the Be star simulation routines to create the Be star models. KCM interpreted the results of the simulation data and created all of the figures found in Chapter 4. As an estimate, KCM completed 80% of the work comprising this chapter.

CEJ’s contributions to this work extended to the same responsibilities listed for Chapters 2 and 3.

Mark Suffak created a routine which generates the grid that comprises the disk, and computes the necessary grid positions and disk velocities at each grid location for the tilted disk.

ACC and Tajan Amori updated the hdust code to accept velocity derivatives in three-dimensions.
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Dedication

To Mom, Dad, Nicole, Colin and Kelsey.
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Chapter 1

Introduction

1.1 Classical Be Stars

Stars are engines of creation, forging the heavy elements required for life. The fates of these complex systems are strictly regulated by energy; determining how a star evolves, the length of its life, and the process by which it dies. The most massive stars shape the chemistry and dynamics of their local environments through extreme luminosity and their stellar winds. Unlike low-mass stars, massive stars continue nuclear burning beyond hydrogen and deuterium, producing much of the heavy elements found throughout the Universe. Successive cycles of stellar birth and death of massive stars, often in the form of supernovae, influence how galaxies form and evolve.

Astrophysical objects that exhibit unusual observational characteristics provide opportunities to study particular aspects of physics. One such characteristic is the presence of Balmer emission lines within stellar spectra, a phenomenon first noted by Secchi (1866). With time, it became clear that a variety of massive emission-line stars exist, each with an extended region of excess gas or dust that orbits beyond the stellar photosphere. The presence of circumstellar material causes the emission lines to form through radiative interaction (e.g. recombination) with the light incident from the central star. This thesis focuses on one particular type of massive, emission-line star called the classical Be star (hereafter Be stars).

The conventional definition of a Be star was first suggested by Jaschek et al. (1981) and later refined by Collins (1987):

“A non-supergiant B star whose spectrum has, or had at some time, one or more Balmer lines in emission.”

However, this definition is too broad as it includes all B-type stars that have a circumstellar envelope with a density greater than $1.0 \times 10^{-13}$ g cm$^{-3}$ (Rivinius et al., 2013). To remove any uncertainty, we now list taxonomically similar objects that fall under this definition but are not Be stars. They include:

- B Supergiants: Rapidly rotating evolved stars that exhibit Balmer emission lines which originate from angular momentum conserving wind-like outflows (Kaufer et al., 2006; Puls et al., 2008). These objects can be misidentified as non-supergiant objects since

\[90x628\]
their UV and visible spectra can appear very similar, particularly if the supergiants have a high rotation rate.

- P-Cygni stars: Massive evolved stars with either powerful stellar winds or an expanding shell of gas. These stars exhibit a specific line profile shape, called a P-Cygni profile, which is characterized by a strong emission component in the line originating from an expanding shell, superimposed with blue-shifted absorption from the part of the shell moving towards the observer (Israelian & de Groot, 1999). This produces emission lines with different shapes from those of Be stars.

- B[e] stars: B spectral type stars with forbidden lines present in their spectra (Lamers et al., 1998). The B[e] group includes B[e] supergiants, Herbig B[e] stars, symbiotic binaries, compact planetary nebulae, and FS CMa type objects (Miroshnichenko, 2007). Forbidden lines are not present in the spectra of Be stars.

- Herbig Ae/Be stars: Young massive stars with late-stage protoplanetary accretion disks (Waters & Waelkens, 1998). Early-type Herbig stars are spectroscopically differentiated from Be stars by the presence of P-Cygni or inverse P-Cygni profiles. Late-type Herbig stars lack these profiles; however, the dust in their disks leads to infrared and radio continuum excess, which distinguishes them from the dust-free Be star disks (Waters et al., 1988).

- Mass-transferring binaries: Binary systems which contain a B-type primary star in an eclipsing contact or semi-detached system. Once the secondary star fills its Roche lobe, gaseous material accretes onto the primary, leading to line emission. These systems include Algol variables (semi-detached binary system) and W UMa type variables (contact binary system) (Harmanec & Krýz, 1976). No Be star is currently known to have a Roche lobe-filling companion (Harmanec et al., 2002).

- Stars with emission line magnetospheres: Magnetic B stars can show emission lines that form from gaseous material trapped in the stellar magnetosphere (Petit et al., 2013). The variability of these lines is consistent with the rotational period of the photosphere, with little or no long-term non-periodic variance, making them distinct from Be stars. Surveys have also shown no evidence of large-scale magnetic fields in Be stars (Grunhut et al., 2011; Rivinius et al., 2013).

In an attempt to differentiate Be stars from other systems, Struve (1931) omitted stars with P Cyg profiles and spectra with features due to binarity. What remained were objects that he deemed as rotationally unstable stars, and specifically

“lens shaped bodies which eject matter at the equator, thus forming a nebulous ring which revolves around the star and give rise to emission lines.”

This model necessitates that Be stars are a group of rapidly rotating stars with dense, gaseous, circumstellar disks that orbit in Keplerian fashion (see Subsection 1.3.1 for a discussion on the rotation of Be star disks). Behr (1959) further supported Struve’s model with polarimetric observations, which confirmed that the gaseous envelope has a preferred orientation and is not
spherically symmetric. The disk-shaped geometry was further confirmed through radio and optical interferometric methods, for example, in the works of Dougherty & Taylor (1992), Quirrenbach et al. (1994), Stee (1995) and Quirrenbach et al. (1997).

Several other physical models have been proposed for Be stars. Early models relied heavily on stellar winds and used one-dimensional approximations for the equatorial flow versus the polar flow. The mass loss rates were consistently too low, or the rotational velocity required was too high. More sophisticated two-dimensional models followed, notably including:

- The bi-stability mechanism: This model was initially proposed for P-Cygni (HD 193237). Lamers & Pauldrach (1991) adapted the model to explain the outward flow of Be star disks as a result of radiation (or line) driven winds that depend on stellar latitude. The lower wind speeds around the stellar equator allow for disk formation in the much less dense B[e] stars, although no such mechanism exists to meet the mass loss rates needed to build Be star disks.

- Wind-compressed disk model: In this model, proposed by Bjorkman & Cassinelli (1993), the high-density disk of a Be star is caused by shock compression of polar winds as they meet in the equatorial region compressing the disk into a geometrically thin and confined region. The inclusion of non-radial line-driving led to the demise of this model (Owocki et al., 1996). Moreover, the wind-compressed disk model predicted the disk rotation to follow an angular momentum conserving disk, which was ruled out as the disks were found to be Keplerian (see Subsection 1.4.4).

Other early models of Be stars include the rotationally enhanced stellar wind model (Marlborough, 1987), Be stars as interacting binaries (Harmanec, 1987), Be stars as non-radial pulsators (Baade, 1987; Gies, 1994; Saio, 1994), the spheroidal/ellipsoidal variable mass loss decelerated model (Doazan, 1987a), the magnetical loop model (Doazan, 1987b; Smith, 1994), and the rotational modulation model (Balona, 1990; Baade & Balona, 1994).

It is now widely accepted that Be stars are best described by the viscous decretion disk (VDD) model proposed by Lee et al. (1991), and later developed by Papaloizou et al. (1992); Bjorkman & Carciofi (2005); Klement et al. (2015) among others, which builds on Struve’s model and includes viscosity dominated dynamics. In this model, mass is ejected from the central star, and while the majority of the ejected gas falls back onto the stellar surface (~ 99% according to Okazaki et al. (2002)), the remaining gas diffuses outward from the star due to viscous forces, thus forming the circumstellar disk. The VDD model adequately explains the linear polarization, IR excess, and emission-line formation in Be stars while also following a Keplerian velocity law and effectively distributing angular momentum (Haubois et al., 2012; Rı́mulo et al., 2018; Ghoreyshi et al., 2021; Marr et al., 2021).

The central star is most often B spectral type, although early O-type and late A-type stars can exhibit the same characteristics. Surveys have shown that 15-20% of all B-type stars are Be stars, with a greater number of early-types peaking at B2 (see Figure 1.1). This percentage is much greater in young stellar clusters, where ~40% of B stars exhibit the Be phenomenon (Tarasov, 2017). Be stars are typically on the main sequence (luminosity class V) or are slightly evolved (luminosity class IV). They have masses ranging from 3 to 20 M⊙, radii from 2.5 to 8 R⊙, effective temperatures from 10,000 to 30,000 K, and luminosities from 100 to 10,000 L⊙ (Cox, 2000). The rapid rotation of the central star is often estimated to be from 70 to 80% of the
critical velocity (Porter & Rivinius, 2003) but could be underestimated due to a phenomenon called gravity darkening (see Subsection 1.3.1).

With the development of Be star disk theory, our understanding of the physical and kinematic nature has improved. However, many uncertainties remain about the phenomena of Be stars, including phase variation, the origin of mass loss, the geometry and dynamics of the circumstellar disk, irregular and quasi-irregular variations of disk structure, rotational and radial motions of the envelope, and finally, the relation between the circumstellar disk and the properties of the central stars. Be stars also provide a testbed for learning about the impact of stellar rotation, pulsation and mass loss on the evolution of massive stars. Other areas of astrophysics also benefit from a better understanding of the Be phenomenon and the physics of astrophysical disks and gaseous envelopes.

1.2 Observational Characteristics of Be Stars

Be stars show unique observational features due to the presence of the circumstellar disk. These features provide insight into the physical nature of the circumstellar environment and the central star. In this section, we discuss the observational features of Be stars and their disks.

1.2.1 Spectroscopy

The spectrum of a Be star contains many absorption line profiles which originate from the rapidly rotating stellar photosphere. The rapid stellar rotation leads to significant Doppler broadening which causes the line profiles to become very broad and flat. To a lesser degree, these line profiles are also affected by gravity darkening (see Subsection 1.3.1), which causes
the lines to appear narrower. By understanding the compounding effects of each of these processes, the photospheric lines can be used to learn about the physical state of the central star.

The disks of Be stars also significantly affect the observed spectrum: in particular, they are the source of the emission lines. The emission spectrum is superimposed onto the absorption spectrum, causing an excess of light beyond the stellar flux. The brightest emission lines are those in the Balmer series, with the Hα profile being the strongest as it is the most probable transition, and it emits from a large volume within the disk. The Hβ, Hγ, and subsequent lines, appear relatively weaker as they are less probable and are restricted to smaller volumes. Strong spectroscopic features are also found in other hydrogen series, including the Paschen and Pfund series (Hony et al., 2000), which are most prominent in early-type Be stars (Jaschek & Jaschek, 1987). In cases where the disk does not emit very brightly (for example, in late-type Be stars, or if the disk is diffuse), Be stars can have deep absorption lines in these series instead of emission. Emission lines from atomic species other than H i are also present within the spectra, including He i, Fe ii, Si ii, and Mg ii for spectral types ranging between B0 and B5 (Jaschek & Jaschek, 1987).

Spectroscopic observations show that emission lines in Be stars can be singly- or doubly-peaked. The appearance of the line is highly dependant on the inclination at which the system is viewed, although changes in disk density structure can also affect the line shape (Silaj et al., 2010; Arcos et al., 2017). A singly-peaked line is a rare case, observed only when the system is viewed pole on (i = 0°). Be stars are more commonly inclined with respect to the line of sight, therefore showing doubly-peaked emission lines. The two peaks result from Doppler shifting due to the Keplerian rotation of the disk, with one blue shifted (V) and one red-shifted (R). The ratio of these two components, called the violet-to-red (V/R) ratio, is often used to characterize doubly-peaked lines. The center of a doubly-peaked line drops due to the non-Doppler shifted regions of the disk appearing optically thick, causing the emitting surface area to be smaller along the line of sight (Bjorkman, 2012). In many Be stars, this effect can cause the center of the doubly-peaked line to drop below the continuum flux. Stars exhibiting such lines are classified as shell stars, and their unique lines as shell lines. Stars with shell lines are modernly referred to as Be-shell stars, and are widely accepted to be Be stars with near edge-on inclinations (i = 90°). Recent findings have shown that shell profiles can occur at disk inclinations as low as 40° if the disk density falls off more slowly with radius (Silaj et al., 2014). In Chapter 3, we show that shell lines also occur at relatively low inclinations for Be star disks that have been tilted away from the stellar equator. Figure 1.2 summarizes the viewing angles and the resulting lines observed at different inclinations. Figure 1.3 illustrates where each part of a doubly-peaked line forms within the disk.

Most doubly-peaked emission profiles are symmetric about their centers; however approximately one-third show cyclic variability in the V/R ratio (Hanuschik et al., 1996). The variability of doubly-peaked emission lines is discussed in greater detail in the context of Be star disk evolution in Subsection 1.4.5.

Shell line profiles may also contain central quasi emission (CQE) peaks, where an emission peak can form within the drop in flux at line center between the V and R peaks. In Be-shell stars, CQE peaks are absorption features resulting from a greater amount of scattered and absorbed radiation having some radial velocity different from zero (Rivinius et al., 1999). CQE peaks in Be stars are evidence that the disks are supported by rotation and not dominated by radial outflow (Sigut et al., 2015) (See Subsection 1.4.4).
Figure 1.2: Illustration of the correlation between observed the angle at which a Be star is observed and the expected emission line shape. The example emission lines include singly-peaked emission (A), doubly-peaked emission (B, C) and shell emission (D). Figure from Rivinius et al. (2013), adapted from the original figure by Slettebak (1979).

Another defining spectroscopic feature of Be stars is an excess of infrared flux. The origin of the excess was originally uncertain, as the emission could originate from scattering of light by free electrons in the circumstellar environment or from the re-radiation of light by a circumstellar dust cloud (Woolf et al., 1970). Gehrz et al. (1974) ruled out the circumstellar dust model, as previously discussed, by showing that, in the wavelength range from 2.3\( \mu \text{m} \) to 19.5\( \mu \text{m} \), the excess must originate from free-free emission in gaseous environments at hotter than 10,000 K. Cote & Waters (1987) confirmed the source of the infrared excess is free electrons, and that the disk is dust-free with the observations made by the Infrared Astronomical Satellite (IRAS) in 1983.

The UV spectra of Be stars do not differ significantly from B-type stars (Slettebak, 1994). Photospheric absorption lines do show a greater degree of broadening due to the higher rotational velocities. The stellar winds of Be stars are another source of ultraviolet absorption lines with greater excitation energies than photospheric lines and large terminal velocities (Kogure & Leung, 2007). These broad lines characterize the stellar winds as low density and relatively hot gas with high outflow velocities. See Subsection 1.4.1 for a discussion on the polar winds that originate from the polar regions of Be stars. The central star is the source of the majority of UV flux in the system, however the presence of the disk can affect the UV observations. For example, the UV flux increases for a pole-on system due to scattering off the disk, while an edge-on system attenuates some of the light and re-emits the rest of it at IR wavelengths (Mota, 2019).

Be stars also exhibit X-ray emission that originates from stellar winds. Lucy & White (1980) and Lucy (1982) found that when stellar winds become unstable, shock heating can result in super-heated winds that produce the observed X-ray emission. While this is the most
1.2. **Observational Characteristics of Be Stars**

commonly accepted mechanism, several other mechanisms have been suggested, including inverse-Compton emission and coronal heating (Cassinelli et al., 1994). Another source of X-ray emission from Be stars is the presence of a compact companion star which accretes matter from the Be star; such systems are referred to as Be/X-ray binaries. The X-ray luminosity of Be stars extends from $10^{28}$ erg s$^{-1}$ to $10^{32}$ erg s$^{-1}$, where early-types tend to show higher values than late-types. This range of X-ray luminosities only differs modestly from that of normal B-type stars (Kogure & Leung, 2007; Cohen, 2000). Some X-ray outbursts, called Type II events, are luminous enough to destroy the Be star disk and occur seemingly at random in the companion’s orbit (Reig, 2011). Type I outbursts occur at the periastron and are typically far weaker so that the disk survives. $\gamma$ Cas is one such star that emits hard X-rays (Peters, 1982), and the subset of Be stars which also emit hard X-rays are called $\gamma$ Cas stars (Smith et al., 2016). Recently, Labadie-Bartz et al. (2021) confirmed that $\gamma$ Cas exhibits low-order non-radial pulsations (see Subsection 1.3.2) using photometry from the TESS satellite. They conclude that

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**Figure 1.3:** Doubly-peaked H$\alpha$ line formation in a circumstellar disk inclined at 45°. The outer images are surface brightness maps as a function of Doppler shift; their correlation to each part of the spectral line is shown by the coloured lines. Figure from Bjorkman (2012).
the presence of hard X-ray flux remains the only major property which distinguishes $\gamma$ Cas stars from Be stars.

The outermost regions of Be star disks are dim in the radio regime. This flux results from outflowing gas accelerated to escape velocity which exists at large radial distances from the central star (Dougherty et al., 1991). The radio emissions arise from thermal free-free emission, the same process that leads to the observed far-IR and millimetre emissions (Taylor et al., 1990). This similarity provides further evidence that these types of emission must originate from the same contiguous structure.

### 1.2.2 Interferometry

The brightness, proximity, and presence of circumstellar disks make Be stars prime candidates to observe with radio and optical/IR interferometers. Using optical long baseline interferometry (OLBI; also LBOI), the disks of the closest Be stars and their central stars have now been resolved with sub-milliarcsecond resolution (Vakili et al., 1994). While radio interferometry has been commonly used for decades, interferometry in optical and IR wavelengths is a newer technology that provides new challenges, including mechanical stability with nanometer precision and accounting for changes in the Earth’s atmosphere (Oudmaijer et al., 2012). Below, we provide an overview of recent studies of Be stars which use interferometric observations.

Be stars are challenging to observe with the angular resolutions available with radio interferometers because of their low radio flux. The first radial upper limit placed on a Be star disk was reported by Dougherty & Taylor (1992), who observed $\psi$ Per (HD 22192), the brightest Be star in radio, with the Very Large Array (VLA) radio interferometer. They noted a flattened, disk-like distribution of gas that extends to a radial distance of $\sim 400 R_\star$. This result was later contested by Klement et al. (2017), who found their best fit disk model for $\psi$ Per is too small to be resolved by the VLA, instead finding an outer radius of $\sim 100 R_\star$. Other Be stars in their sample were found to have outer disk radii ranging from 30 to 150 $R_\star$. In their follow up work, Klement et al. (2019) found the radio emission from 26 Be stars in their sample of 57 to show evidence of disk truncation at large radii, causing a systematic lower flux than models predict.

Meilland et al. (2008) resolved the disk of the Be star $\delta$ Cen and its binary companion using the VLTI/AMBER instrument operating in the H- and K-bands. Meilland et al. (2009) then resolved seven more stars ($p$ Car, $\zeta$ Tau, $\kappa$ CMa, $\alpha$ Col, $\delta$ Cen, $\beta$ CMi, and $\alpha$ Ara) using the VLTI/MIDI instrument in the N-band. Gies et al. (2007) observed $\gamma$ Cas, $\phi$ Per, $\zeta$ Tau, and $\kappa$ Dra using the CHARA Array, and were able to determine the geometrical and physical properties of their disks.

Using the Navy Prototype Optical Interferometer (NPOI), Tycner et al. (2004, 2005, 2006,
2008) followed on this analysis to resolve the circumstellar disks of γ Cas, Alycone, Pleione, ζ Tau, φ Per, β CMi, and χ Oph in Hα, and reported on the angular sizes of their Hα emitting regions. They found φ Per’s disk is likely truncated as it extends to approximately the same size as its companion’s periastron distance. Their observations showed that γ Cas’s disk is smaller than its companion’s periastron distance and therefore is not truncated, however, the recent radio observations by Klement et al. (2017) showed evidence of truncation (see Subsection 1.4.6 for a discussion on disk truncation).

Jones et al. (2008b) computed theoretical disk models for the Be stars κ Dra, β Psc, ν Cyg. Models were constructed using a non-LTE radiative transfer code developed by Sigut & Jones (2007). These models were constrained by direct comparison with long-baseline optical interferometric observation of the Be stars’ emitting regions and by their Hα profiles. By comparing their Hα interferometry and spectroscopy, they determined the density distribution of each star’s disk.

The circumstellar disk of δ Sco was resolved in Hα and Br γ by Meilland et al. (2011). They found the disk size to be 4.8 ± 1.5 mas in Hα, and 2.9 ± 0.5 mas in Br γ. Kraus et al. (2012) observed β CMi and ζ Tau using CHARA and VLTI near-infrared spectro-interferometry, and modelled the H- and K- continuum and Br γ line profile to constrain the physical extent of these bright star disks.

Observations of a sample of 24 northern sky Be stars by Touhami et al. (2013) obtained in the K-band using the CHARA Array revealed that the average ratio of the K-band emitting region’s diameter to the stellar diameter is 4.4. This value is notably smaller than the same ratio observed in Hα emission. Furthermore, they found that approximately half of the stars in
their sample rotate at or near their critical velocity, which was also confirmed by Grzenia et al. (2013) with the Palomar Testbed Interferometer.

1.2.3 Polarimetry

Most Be stars emit linearly polarized continuous light. Hall & Mikesell (1950) and Behr (1959) are among the first reports on the polarized light of Be stars. This provided the first evidence that the circumstellar environment of Be stars is non-spherical, as linearly polarized light cannot result from circularly symmetric projections on the sky (Waters & Marlborough, 1992). Since then, it has become an active area of research for Be stars due to the unique information that polarimetric methods provide. The polarization signature accounts for up to $\sim 2\%$ of the total light emitted (Wood et al., 1997), although Yudin (2001) established that the total polarized light ranges from 0 to $1.5\%$ for 95\% of Be stars. The total polarization of some Be stars is variable over time (Coyne & Kruszewski, 1969; ?). Free electrons scatter light from the central star as it interacts with the circumstellar envelope. In this process, the light is linearly polarized perpendicular to the plane containing the incident and scattered radiation (Quirrenbach et al., 1997).

Waters & Marlborough (1992) determined that polarization measurements are indirect measurements of the electron density of the disk; in general, a stronger polarization signal is acquired from a disk with a greater electron density, as more scattering of photons can occur along the line of sight. In the same work, Waters & Marlborough (1992) were able to constrain the density structure of the disk by using linear polarization in conjunction with IR excess measurements from IRAS and a simple disk model (see Section 1.4.3 for a discussion on density distributions of Be star disks). Waters & Marlborough (1992) concluded that the vast majority of the polarized light is created in the first two to three stellar radii of the disk, where the ionizing radiation is most likely to scatter.

The observed level of polarization is highly dependent on the inclination angle of the system. For a pole-on system, an inclination of $i = 0^\circ$, the disk is circularly symmetric and no polarized light is observed. When the system is inclined edge-on, $i = 90^\circ$, a greater portion of the disk appears optically thick and each photon undergoes many scattering events causing the polarized light to be attenuated. Wood et al. (1996) showed that the strongest polarization signal occurs when the system is inclined at $i = 70 - 80^\circ$ with respect to the line of sight. This was supported by Monte Carlo simulations of Halonen & Jones (2013a) (see Figure 1.5).

The polarized light of a Be star provides information on the geometry and physical nature (e.g. chemical composition, ionization levels, opacity) of the disk without resolving the system. However, there is a complex interaction between the scattering and absorption processes in the disk, which imparts a wavelength dependence on the polarization signature due to hydrogen opacity in a unique saw-tooth shape (seen in the left panel of Figure 1.5). The rapid “jumps” in the total polarized light observed correspond to the discontinuities at each hydrogen spectral series limit. Each jump is characterized by the difference in electron scattering optical depth, a product of hydrogen opacity, at wavelengths shortward and longward of the jump. Before the jump (shorter wavelengths), the effect of the hydrogen opacity is very strong, so a large number of polarized photons are attenuated. After the jump (longer wavelengths), the hydrogen opacity is weaker, so few polarized photons are scattered. In the continuum, the effect of the hydrogen opacity is relatively small near the shorter wavelength jump; however, it increases as the wave-
1.3. The Central Star

In this section, we provide a summary of the physical nature of the central star, including rapid rotation, non-radial pulsations, and the lack of detectable surface magnetic fields.

1.3.1 Rotation

Be stars have rotation rates up to 150 km/s faster than regular B-type stars (Slettebak, 1949), making them the fastest rotating non-degenerate stars (see Figure 1.6) (Townsend et al., 2004). The precise distribution of rotational velocities is unknown, which raises the question: what is the upper limit on Be star rotation? The physical limit is the critical velocity, \( v_{\text{crit}} \), which is the rotational velocity at which the outward centripetal force matches the star’s inward gravitational force. It is defined as:

\[
v_{\text{crit}} = \sqrt{\frac{GM}{R_{\text{eq}}}}.
\]  

Figure 1.5: Simulated amount of polarized light observed from a Be star, demonstrating the contributions from multiple scattering. The solid line represents the total polarized light, the dashed line represents the polarization due to single scattering, and the dotted line represents the polarization resulting from multiple scattering events. Notably, the left panel shows the saw-tooth structure resulting from the Balmer and Paschen jumps, and the right panel shows the maximum polarization observed at inclinations \( i = 70 - 80^\circ \). Figure adapted from Halonen et al. (2013).
Figure 1.6: Largest observed rotational velocities for main sequence and slightly evolved stars of spectral types O9.5 to F0, and the corresponding computed equatorial break-up velocities. The highest rotational velocities peak in B-type stars $\sim B2$. The relative frequency of Be stars is included in the sub-panel. Figure from Slettebak (1966).

Whether or not Be stars are critical rotators has been debated for a long time. In a survey of 233 Be stars, Zorec et al. (2016) concluded that the Be phenomenon occurs for a wide range of rotational velocities, implying that Be stars do not rotate critically, but do rotate very rapidly. Some Be stars have been found to rotate near their critical limit, with an extreme example being Achernar ($\alpha$ Eri, HD 10144) which rotates at 0.96 $v_{\text{crit}}$ (Domiciano de Souza et al., 2012). Due to rapid rotation, Be stars remain on the main sequence for 40% longer than normal B-type stars as the rotation counters the gravity needed for fusion (Maeder & Meynet, 2010). Rapid rotation also induces mixing, bringing more hydrogen into the core for fusion, so clearly the effects of rotation are not one sided.

Rapid stellar rotation also causes short-term variability in the photospheric line shapes and photometry on timescales between 0.5 and 2 days (Percy et al., 1994), in $\sim$90% of early-type Be stars and less commonly in later-types (Cuypers et al., 1989; Balona et al., 1992). Localized mass injections in the inner disk can also lead to these short term variations (See Subsection 1.3.2 for the short-term variation caused by non-radial pulsations).

As mentioned above, the spectral lines of Be stars are broadened by hundreds of km/s due to the rapid rotation. However, the rotational velocities observed through broadening are not measurements of the true stellar rotation rate but the $v_{\text{sin}i}$ values, as the rotational velocity is projected after being inclined along the line of sight. This explains why some Be stars show rotation rates of order 50 km s$^{-1}$ (Yudin, 2001); statistically, some stars are expected to be observed at small inclination.

The traditional method of finding stellar rotation rates involves measuring the line broadening in photospheric lines, and using it to determine the $v_{\text{sin}i}$. However, determining the
equatorial rotational velocity from values of $v_{\sin i}$ requires independent determination of the stellar inclination. Studies have addressed this problem by observing a statistically large sample of objects. In doing this, Yudin (2001) found that for dwarf stars, on average, late-type Be stars rotate faster than early-types; this relation is not as apparent in slightly evolved stars. They also report that, independent of luminosity class, the ratio of $v_{\sin i}/v_{\text{crit}}$ is larger for late-type stars than for early-type stars, meaning late-type stars rotate closer to their critical velocities. In general, $v_{\sin i}/v_{\text{crit}}$ ranges from 0.5 to 0.8 in Be stars, with the ratio increasing for later spectral types (Slettebak, 1982; Porter, 1996; Yudin, 2001; Huang et al., 2010). An additional challenge is faced when determining $v_{\text{crit}}$, requiring each star’s stellar mass which is also uncertain since the stellar spectral type relies on prior knowledge of the rotation rate (see below for a discussion on the effects of gravity darkening). Presently, the oblateness can be measured accurately for nearby stars through OLBI techniques (see Subsection 1.2.2), which provides a more direct way to determine rotation rates (van Belle, 2012).

Even the earliest models of Be stars invoked rapid rotation. Struve (1931)’s model used the critical rotation velocity as a way to explain mass transfer from star to disk. Slettebak (1979) showed that these stars do not rotate at critical, and the consensus shifted towards Be stars rotating sub-critically when Porter (1996) showed that the statistical peak of rotation occurs sharply at 70% to 80% of the critical. Chauville et al. (2001) showed that in a survey of 116 Be stars, on average, $v_{\sin i}/v_{\text{crit}} \approx 0.8$. The currently accepted view is that Be stars rotate at 75% to 100% of the critical velocity (Rivinius et al., 2013).

The actual mechanism for mass transfer is still enigmatic, as the observed rotational velocities are too slow to eject matter on their own. However, in describing this process it is useful to express the rotation rate of Be stars as

$$W = \frac{v_{\text{eq}}}{v_{\text{orb}}},$$  

where $v_{\text{orb}}$ is the Keplerian rotational velocity just above the stellar surface where the gas orbits at the critical velocity, and $v_{\text{eq}}$ is the velocity of the star at its equator. This value is useful as the difference $v_{\text{orb}} - v_{\text{eq}}$ indicates the amount of velocity required to lift material from the stellar surface. Owocki (2004) suggested that a combination of rapid rotation and another “weak” process (such as non-radial pulsations; see Subsection 1.3.2) could be sufficient to push the matter out, although these processes would still require the rotational velocity to be increased to 95% of $v_{\text{crit}}$ before mass ejection occurs (Townsend et al., 2004).

One possible disk formation mechanism stems from the idea that the rotational velocities are systematically underestimated (Stoeckley, 1968; Townsend et al., 2004) due to the effects of gravity darkening. In addition to line broadening, rapid rotation affects the physical properties of the stellar photosphere in two ways: (1) the deformation of the star such that the equatorial radius is larger than the polar radius; (2) the effective temperature becomes a function of latitude with the cooler material at the equator, which is called gravity darkening (von Zeipel, 1924).

von Zeipel’s theorem proposes that the local flux is proportional to the surface gravity and to the fourth power of the effective temperature in all stars that are barotropic and in radiative equilibrium. The Roche model is a treatment of the effect of rotation on stellar structure, often used in conjunction with von Zeipel’s theorem. According to the Roche model, critical rotation places a theoretical limit on the oblateness of the central star of $R_e/R_p = 3/2$ (see Chapter 2 of
Maeder (2009), and Figure 1.7).

Espinosa Lara & Rieutord (2011) proposed a refined version of von Zeipel’s theorem. The differences between their model and von Zeipel (1924)’s lie in the order that the model assumptions are placed. The assumption applied in von Zeipel’s theorem is the Wavre-Poincaré theorem (Tassoul, 1978), followed by setting the divergence of the flux in the radial direction to zero ($\nabla \cdot F_{\text{rad}} = 0$), which is impossible. Espinosa Lara & Rieutord (2011)’s treatment reverses the order of these restrictions, showing that the Wavre-Poincaré theorem cannot be consistent with a model with $\nabla \cdot F_{\text{rad}} = 0$. Ideally, the order of the restrictions should not matter, however Espinosa Lara & Rieutord (2011) showed this is not the case in von Zeipel’s theorem. There is no consensus on which model is better, although Espinosa Lara & Rieutord (2011)’s model appears closer to that which is observed.

In a gravity darkened Be star, the light from the cool equator appears dimmed and is possibly outside of the detection limit. The resulting line profiles appear narrower, leading to an underestimation of the true rotational velocity. Townsend et al. (2004) showed that when the effects of gravity darkening are accounted for, most stars are found to be at 95% $v_{\text{crit}}$, at which point additional physical phenomena such as pulsations or gas pressure are sufficient to overcome the stellar gravity, and eject material into the circumstellar environment.

In Subsection 1.4.6, we discuss the role which a binary companion star can have in spinning up a Be star through mass transfer, which provides another explanation for the onset of disk formation.

1.3.2 Non-Radial Pulsations

Be stars are classified as a unique subset of pulsating stars, with most of them showing pulsational periods (Rivinius et al., 2013). Several studies have shown that non-radial pulsations
in addition to rapid rotation may provide the necessary energy to act as the mechanism for mass loss and disk formation. The most complete works on the subject are the series of papers “Short-term variability and mass loss in Be stars” (Baade et al., 2016; Rivinius et al., 2016; Baade et al., 2018a,b). As non-radial pulsations have been found for all Be stars that have been observed with sufficient precision, there is evidence in favour of non-radial pulsations playing a role in the Be phenomenon that leads to Be star disks.

The velocity amplitudes caused by non-radial pulsation are on the same order as the sound speed and therefore are generally not high enough to eject matter into orbit (Owocki, 2006). However, the combination of multiple pulsational periods undergoing constructive interference can periodically enhance the velocity amplitude. These effects can create flows with velocities on the order of the sound speed, \( V_{\text{sound}} = 12.85 \text{ km s}^{-1} \sqrt{T_{\text{eff}}/10000\text{K}} \), providing enough of a boost for some gas to reach Keplerian velocity and escape the stellar surface (Rivinius et al., 1998). So far, this effect has only been evidenced in the star \( \mu \) Cen, which has unique pulsational properties (Baade et al., 2016).

The variations observed in the photospheric line shapes, designated line profile variations, are produced by non-radial pulsations (Baade, 1982). Initially, it was unclear if the effect of rotational modulation due to starspots (Balona, 1990) could be another possible mechanism, however, further investigation showed that the starspots would have to be too large and too cool to match with the observed variations (Balona, 1995).

The line profile variations which result from non-radial pulsations appear as rapidly changing bumps or dips throughout the line profile. Kogure & Leung (2007) describe the features of non-radial pulsations as follows: Over an oscillation period of 0.1 to 3 days, the amplitude varies from 3 km s\(^{-1}\) to 20 km s\(^{-1}\), the amplitude of radial motion varies within ± 0.02 stellar radii, and the local surface temperatures vary by ± 2000 to 3000 K. Rivinius et al. (2003) showed that non-radial pulsations can explain line profile variations for up to 80% of early-type Be stars.

### 1.3.3 Magnetism

Be stars are generally recognized as non-magnetic objects, although previously magnetic fields were suggested as a possible mechanism for disk formation (Friend & MacGregor, 1984; Poe & Friend, 1986; Ignace et al., 1996). Cassinelli et al. (2002) proposed that a B2V star with a 300 G field would be sufficient to eject gas from the stellar equator. Subsequently, such a disk could be sustained with a field of only 10 G (Maheswaran, 2003). Nonetheless, the existence of a large-scale, global magnetic field has never been observed in a Be star (Grunhut et al., 2012).

A report from Neiner et al. (2003) claimed the presence of a 530 ± 230 G field in \( \omega \) Ori, but these findings were rejected following the Magnetism in Massive Stars (MiMeS) survey at the Canada-France-Hawaii Telescope (CFHT) (Grunhut et al., 2011; Rivinius et al., 2013). As part of the MiMeS survey, Wade et al. (2014) reported a complete absence of large-scale, longitudinal magnetic fields in 85 Be stars. The surveys published by Yudin et al. (2007, 2009) found magnetic fields in Be stars ranging from 100 G to 150 G, although these were disputed by Silvester et al. (2009) who observed the same stars and found no evidence of magnetic fields.

Localized, small-scale magnetic fields in Be stars have not yet been found and would be
difficult to detect, although their existence has not yet been disproved. Should they exist, it is possible that magnetic flaring can account for the ultra-rapid variability of Be stars which occurs on the timescale of tens of minutes. In this phenomenon, the stellar magnetic field and the circumstellar disk could interact and accelerate particles from the disk into the stellar surface, creating hard X-ray flares observed in some Be stars (Smith & Robinson, 1998; Puls et al., 2008). However, it is commonly believed that localized shocks and mass ejection from the stellar surface into the near circumstellar environment can also lead to these short-term variations (Penrod, 1986).

1.4 The Circumstellar Envelope

The environment surrounding the central star is comprised of two components: the hot, diffuse, polar wind originating from the poles, and the denser, cooler, gaseous component that surrounds the star and is largely confined to the equatorial plane. In this section we describe each in detail, and discuss the effects which a binary companion star may have on the gaseous equatorial disk.

1.4.1 Stellar Polar Winds

The polar winds of Be stars do not differ from those of normal B-type stars and are described by CAK theory (Castor et al., 1975). The model described in CAK theory parameterized stellar winds with two distinct factors: (1) \( k \), a measure of the number of lines driving the wind, and (2) \( \alpha \), an index which represents the combination of optically thin and optically thick lines.

The polar winds of Be stars are relatively weak in comparison to the O-type star winds that CAK theory was originally developed for. Because of this, the matter they eject from the stellar surface is typically on the order of 1000-times less dense than Be star disks (e.g. Bogomazov 2005). The winds are also much hotter than Be star disks, with the radiative shocks they produce having temperatures on the order of \( 10^5 \) K. At these densities and temperatures usually only UV lines are observed, with typical species observed including C IV, Si III, and Si IV (Lamers & Waters, 1987; Bjorkman & Cassinelli, 1993).

The current understanding of stellar winds in Be stars is a modified version of CAK theory, called m-CAK theory. The modifications notably include the addition of the parameter \( \delta \), which represents how the ionization of the wind changes (Abbott, 1982), and a finite disk correction factor which changed the description of the star from a point source to a uniform bright disk (Friend & Abbott, 1986; Pauldrach et al., 1986). These modifications have made this description successful in matching the expected mass loss rates and wind terminal velocities.

A promising addition to this model was reported by Curé (2004), who found that for stars rotating faster than 75% of the critical velocity, a different physical solution exists. The so-called \( \Omega \)-slow solution applies to a one-dimensional, nonlinear m-CAK hydrodynamic equation, and offers higher mass loss rates while predicting wind terminal velocities which are approximately one-third of the strength of those previously predicted. This solution naturally fits current models of Be stars, wherein the non-rotational poles produce very rapid winds, while the winds at the equator are much slower such that the circumstellar disk is not disrupted.
Silaj et al. (2014) investigated the emission line profiles that result from the Ω-slow solution, and found that the resulting density structure can explain the structure of a Be star disk. However, unphysically large values of the line-force parameter \( k \) were required to reproduce the emission line profiles. In the followup work by Araya et al. (2017), it was further confirmed that the polar winds are unable to explain the observed H\( \alpha \) emission in Be stars alone.

### 1.4.2 Disk Geometry

As discussed in Subsection 1.1, Struve was the first to claim the source of the unique emissions of Be stars is a disk confined to the equatorial plane. Other configurations have been proposed, although interferometric and polarimetric observations have proven that Struve’s model is correct. Through OLBI and polarimetric techniques, the disk-shaped geometry of the circumstellar envelope has been confirmed (Quirrenbach et al., 1997; Stee, 1995; Tycner et al., 2004).

Various parameters have been used to characterize the geometry of Be star disks. One such parameter is the opening angle, which is a measurement of a disk’s geometric height. It is generally thought that the disks flare outwards with increasing distance from the star. Numerous reports have attempted to determine opening angles, although the results are difficult to compare because each type of observing method focuses on a unique part of the disk. In a statistical comparison of the ratio of Be-shell stars to all Be stars, Porter (1996) and Hanuschik (1996) independently found opening angle values of 5° and 13°, respectively. For the star \( \zeta \) Tau, the opening angle was measured through spectroscopic and interferometric techniques by both Quirrenbach et al. (1997) and Wood et al. (1997), who found opposing values of 20° and 2.5°, respectively. Collectively these results do show consensus that the circumstellar disks are geometrically thin.

The inclinations of Be star disks have been determined with spectroscopy (see Figure 1.2), interferometry and polarimetry. In the case of interferometry, by approximating the disk as infinitely thin, a lower limit on the inclination angle may be found from:

\[
i_{\text{min}} \leq \arccos(r),
\]

where \( r \) is the ratio of the minor axis to the major axis (i.e. the projected axis ratio).

Using polarimetry, Quirrenbach et al. (1997) was able to measure the inclination angle independent of the \( v \sin i \), IR excess, or interferometry. They estimated the inclination angle from the polarization level due to single scattering, which are related as:

\[
P \propto \sin^2(i).
\]

By comparing the polarization levels determined through polarimetry to the polarization levels found by means of a triangle diagram (which relates the IR excess and disk inclination to the polarization; Cote & Waters 1987), Quirrenbach et al. (1997) determined that the inclination angles from both data sets agreed.

Quirrenbach et al. (1997) was also able to confirm Brown & McLean (1977)’s prediction that the polarization position angle of an axisymmetric disk is oriented perpendicular to the plane of the disk. This is because the polarization position angle indicates the vibrational
direction of the light which must, from theory, vibrate perpendicular to a geometrically and optically thin disk. This result ruled out other possible disk geometries which were geometrically thick, optically thick or ellipsoidal, which would produce polarization parallel to the plane of the disk.

1.4.3 Disk Density and Temperature Structure

The density structure of Be star disks proposed by Waters (1986) is a prescription that is commonly used in disk modelling. An expanded form of Waters’ power-law relation may be written as

\[ \rho(r, z) = \rho_0 \left( \frac{r}{R_*} \right)^{-n} e^{-|z/H|^2} \]  

(1.5)

where \( \rho_0 \) is the density of the inner boundary of the disk, \( r \) is the radial distance measured from the stellar surface, \( z \) is the distance above the disk’s midplane, \( R_* \) is the stellar equatorial radius, \( n \) an index called the radial density exponent, and \( H \) is the disk scale height.

According to Waters et al. (1987) the value for \( n \) should lie in the range of 2 to 3.5 based on comparison with IR data from IRAS. There have been many reports on the ranges of values expected for \( n \); for example Tycner et al. (2008) found acceptable fits for \( n = 2 \) to 4.0 for \( \chi \) Oph. Gies et al. (2007) determined values of \( n = 2.70, 1.80, 3.14 \) and \( \rho_0 = 10^{-10.14}, 10^{-10.92}, 10^{-9.71} \) g cm\(^{-3} \) respectively for \( \gamma \) Cas, \( \phi \) Per, and \( \zeta \) Tau. Jones et al. (2008a) reported \( n = 2.5, 4.2, 2.1 \) and \( \rho_0 = 2 \times 10^{-11}, 1.5 \times 10^{-10}, 3 \times 10^{-12} \) g cm\(^{-3} \) for \( \kappa \) Dra, \( \beta \) Psc, and \( \nu \) Cyg, respectively. Recently, Vieira et al. (2017) showed in a report on the SEDs of 80 Be stars, that values of \( \rho_0 = 10^{-12} \) g cm\(^{-3} \) are most probable, and can be as dense as \( 10^{-10} \) g cm\(^{-3} \).

The distribution of continuum emission using VDD theory was summarized by Vieira et al. (2015) with the pseudo-photosphere model, a formulation capable of calculating the disk flux emission and spectral slope. In this model, the disk is split into two parts: the inner optically thick region (the pseudo-photosphere) and the outer optically thin region. In their followup work, Vieira et al. (2017) revisited the results of Waters et al. (1987) using the VDD model, and from their sample of 80 Be stars found the disk density exponent lies between \( n = 1.5 \) and 3.5, with the most common base density of \( \rho_0 = 10^{-12} \) g cm\(^{-3} \) and a maximum value of \( 10^{-10} \) g cm\(^{-3} \). By computing the evolutionary tracks for their models, they concluded that stars with \( n < 3.0 \) are dissipating their disks, from \( n = 3 \) to \( n = 3.5 \) are steady-state, and \( n > 3.5 \) are forming their disks.

Other studies have shown, however, that stable late-type Be stars seem to trend towards value of \( n \leq 3 \), as is the case for \( \beta \) CMi with \( n = 2.9 \) (Klement et al., 2017), \( \alpha \) Col with \( n = 2.5 \) (Rubio, 2019), and \( \omicron \) Aqr with \( n = 3 \) (de Almeida et al., 2020). Furthermore, Granada et al. (2021) studied the highly stable disk of 1 Del with a non-isothermal temperature and variable viscosity with radius, finding best fit values of \( n = 2.75 \) and \( n = 3.25 \) fit to their observations.

Determining the temperature structure of circumstellar disks is a non-trivial task; while it is appropriate to assume the disk is isothermal as a first order approximation, in reality the disks are non-isothermal especially for dense disks where the optical depths vary substantially. Using the disk models of Poeckert & Marlborough (1978), Millar & Marlborough (1998) were able to determine a self-consistent solution of the electron temperature structure, by calculating the energy balance within a pure hydrogen disk. The solution showed that while an isothermal
structure is sufficient as a first-order approximation, there is significant dependence on the position within the disk, particularly in the midplane near the inner boundary of the disk where many observables originate. Refinements to this model were reported in Jones et al. (2004), in which iron line cooling was added, having the effect of cooling the disk overall while retaining the positional dependence of temperature.

Carciofi & Bjorkman (2006) developed a Monte Carlo non-LTE radiative transfer code, which was used to study the temperature structure of Be star disks. The code uses a pure hydrogen composition to determine a self-consistent temperature distribution. They reported that in the innermost region of the disk, where the density is greatest, the disk is relatively cool due to the optical depth.

Sigut & Jones (2007) presented a solar composition calculation of the disk temperature structure (see Figure 1.8). The authors reported that in less dense disks the metals are responsible for the disk cooling, while in denser disks the cooling due to metals is balanced by photoionization heating, so there is little variance from a pure hydrogen disk.

The effects of disk tilting (due to binary companion influence; see Section 1.4.6) must affect the thermal structure of the disk. As the disk tilts towards the stellar poles, it is exposed to hotter regions of the stellar surface which could increase the ionization fraction in the disk. In Chapter 4 we explore how these changes influence the thermal structure and the corresponding effect on the observables.

1.4.4 Disk Rotation

Circumstellar disks are governed by the laws of rotation, for which there are three possible cases which describe the kinematics of the disk. The azimuthal velocity \( v_\phi(r) \) may be expressed in each case to be of the form

\[
v_\phi(r) \propto \begin{cases} 
  r^{-1} & \text{for angular momentum conserving outflow} \\
  r & \text{for rigid rotation} \\
  r^{-1/2} & \text{for Keplerian rotation.} 
\end{cases}
\]
An angular momentum conserving disk implies that the disks would be supported by polar stellar winds. A rigidly rotating disk would imply the disk is held in place by a large-scale, global magnetic field. As established in Subsection 1.3.3, Be stars have no detectable magnetic fields, so rigid rotation of the disk may be quickly ruled out. Finally, a disk that rotates at Keplerian velocities requires continuous injection of angular momentum to keep the disk stable over long timescales.

The Keplerian velocity structure was first supported by the work of Hummel & Vrancken (2000), whose best-fit models indicated a velocity structure with $r^{-0.65}$, which is relatively consistent with Keplerian rotation. Over the past decade, interferometric techniques like OLBI have been able to resolve circumstellar disks, and along with spectroastrometry of emission lines, the Keplerian velocity structure has been confirmed (Meilland et al., 2007; Delaa et al., 2011; Kraus et al., 2012; Wheelwright et al., 2012).

Another confirmation of Keplerian rotation comes from the global disk oscillations which Okazaki (1991) suggests leads to the formation of one-armed density waves precessing in the disk which cause the observed $V/R$ variability. They find that the spiral shape of the density wave can only persist if the radial outflow within the disk is small, otherwise the wave would not form. Hanuschik (2000) showed that optically thin shell lines of Fe II exhibit no signature of radial motions in the disk.

### 1.4.5 Disk Evolution

The best way to study the evolution of Be star disks is to observe the variability of the light which is produced over various timescales. Short-term hour-long variations tell us about mass ejection events and changes in the stellar photosphere (see Subsection 1.3.2), while mid-term variability on days to months arises from density waves in the disk. The longest variations occur on decadal timescales, and are attributed to disk growth and dissipation events.

In the VDD model (previously described in Subsection 1.1), Be star disks form through the ejection of gas from the stellar surface into the circumstellar environment. The ejected gas then forms a disk from the inside-out as viscosity facilitates the movement of gas outward. The physics of Be star decretion disks does not differ greatly from that of accretion disks (e.g. those in Herbig Be stars) aside from the direction in which that matter flows. Shakura & Sunyaev (1973) proposed that the viscosity in the disk originates from turbulence and eddies in the gas, and parameterized the viscosity $\nu$ as:

$$\nu = \alpha c_s H,$$

where $\alpha$ is the viscosity parameter related to the scale height of the eddies (and is different from the $\alpha$ parameter used in CAK theory on stellar winds), $c_s$ is the sound speed, and $H$ is the disk scale height. While $\alpha$ has been found to range from 0.1 to 1.0 in different systems (Rímulo et al., 2018; Brown et al., 2019), recent works indicate $\alpha$ to be towards the lower end of this range (Ghoreyshi et al., 2021; Granada et al., 2021).

Some Be stars have been observed to have gas continuously supplied to their circumstellar environment, allowing the disk to be stable for many decades (e.g. 1 Del (HD 195325) (Marlborough & Cowley, 1974)). However, the variability of other Be stars suggests the outflowing mass loss can be “turned off”, and the circumstellar matter then reaccretes back onto the stellar surface. Over decades a disk can completely dissipate, causing all the unique emission features
1.4. The Circumstellar Envelope

Figure 1.9: The observed V/R variations in ζ Tau, from 1995 to 2009 (open circles) and the modelled Hα and Brγ emissions. Figure from Carciofi et al. (2009); Rivinius et al. (2013).

of an active Be star to disappear as the star enters the B-phase. This major change has been observed in many stars, including 66 Oph (HD 164284), for which we model the dissipation in Chapter 2. The diskless phases can be quite useful in research, as the absence of the disk can provide an excellent opportunity to parameterize the central star without obstruction by the disk.

In other Be stars, the disk density becomes enhanced as mass ejection “turns on” before the disk has fully dissipated. ω CMa (HD 56139) is one such star; showing cyclic phases of disk growth for ~ 2.5 to 4 years, followed by disk dissipation for ~ 4.5 to 6.5 years (Štefl et al., 2003; Ghoreyshi & Carciofi, 2017). Another example is the Be star Pleione (HD 23862), for which it is suggested that its disk has become misaligned by tidal torque from its companion, and a new disk has formed while the old disk persists at a different inclination (Tanaka et al., 2007; Nemravová et al., 2010). In Chapter 3 we investigate the dynamics and evolution of Pleione’s disk in detail.

Intermediate variations which occur over months to years are characterized by the strength of the emission lines, measured with the V/R ratio of doubly-peaked lines (Hanuschik et al., 1996; Rivinius et al., 2006) (see Subsection 1.2.1). The timescale of these variations suggests that they are independent of the rotation of the Hα emitting region, as they are much longer than the orbital period. Progressively more evidence indicates that these variations result from one-armed density waves that precess through the disk (Okazaki, 1991, 1996). This model is referred to as the global oscillation model.

Carciofi et al. (2009) confirmed the presence of a spiral density arm in the disk of the Be star ζ Tau (HD 37202) by reproducing its observed variability with a 3D non-LTE Be star model. They were also able to show that the oscillation mode must extend to the outer regions of the disk to fit the large amplitude of the V/R variations properly. This suggests that long period variations could result from mass ejection into the disk. However, Carciofi et al. (2009)’s model also exhibited inconsistencies with particular observations, such as large variation in the polarization signature over the V/R cycle, which suggests that the innermost disk may not follow the density structure outlined in Equation 1.5. Figure 1.9 shows the V/R variations reported by Carciofi et al. (2009).
1.4.6 Effects of Binarity

A significant fraction of Be stars are thought to have at least one companion. Oudmaijer & Parr (2010) found that 30% of Be stars are in binary systems, which is approximately the same fraction for regular B-type stars. The possibility that all Be stars have binary companions was first posed by Kriz & Harmanec (1975). Recently, this idea has gained more support from Klement et al. (2019), who reported that all stars in their sample of 23 showed strong signs of having a companion. These authors also showed that the radio emission observed from each star is systematically less than model predictions, suggesting the disks are truncated by their companion. Bodensteiner et al. (2020) proposed that the lack of Be star binaries with normal B-type companions further supports binary mass transfer as a catalyst for Be star disk formation events, while Hastings et al. (2021) finds binary interactions can account for an upper limit of one-third of all main-sequence B stars being Be stars.

With so many Be stars being found in binary systems, it is interesting to consider what role companion stars may have in the formation of Be star disks. One possible catalyst for disk formation was first proposed by Gies et al. (1998), who found that for intermediate-mass stars in close binary systems, an evolved companion can overflow its Roche lobe, donating gas to a potential Be star. The transfer of mass is accompanied by a transfer of angular momentum, causing the potential Be star to spin-up. As the Be star rotates faster, it can shed gas and angular momentum through the formation of a circumstellar disk. UV observations of the Be star $\phi$ Per by Gies et al. (1998) provided evidence for the mass-transfer process, which leaves the companion as a small and hot sub-dwarf star. Recently, Wang et al. (2021) detected the presence of sub-dwarf companions in 10 of the 13 stars in their sample. They emphasize that only one of the 13 objects were previously known to have companions, indicating the difficulty in detecting them through spectroscopic observations.

Binary companions have also been found to affect the evolution of Be star disks. In some Be stars, companions have been found to orbit close enough to the Be star that the disk becomes truncated. In this process, gas is prevented from moving further out, causing an increase in density inward of the truncation radius (Waters et al., 1991; Okazaki et al., 2002). Panoglou et al. (2016) reported on the effects of binarity on coplanar orbits, while Cyr et al. (2017) focused on tilted orbits. Evidence of disk truncation and two-armed density waves were confirmed, and the truncation radius for coplanar orbits was found to relate to the orbital separation of the system as $R_{\text{orb}} \approx R_D/0.8$.

Companions have also been reported to perturb disks, making them warped or tilted from the stellar equatorial plane, as first suggested by Hummel (1998) for the stars $\gamma$ Cas (HD 5394) and 59 Cyg (HD 200120). So far, disk tilting has been reported in relatively few Be stars, however increasing evidence has shown that it should occur in any system where the companion’s orbital plane is misaligned from the stellar equatorial plane (Cyr et al., 2017; ?). Warped and tilted disks have unique observational signatures which should make them easily identifiable with a large sample of high time-resolution photometric and polarimetric observations as the disk rotates (Marr et al., 2018). Furthermore, studies by Hirata (2007) and Martayan et al. (2011) have proposed that Be star disks can precess over long timescales due to the companion’s tidal torque. In Chapter 3 we investigate disk precession in the Be star, Pleione.

Several Be stars have been observed to have a companion with a highly eccentric orbit,
for example δ Sco (\(e \approx 0.94\) (Tycner et al., 2011)) and Pleione (\(e \approx 0.7\) (Nemravová et al., 2010)). These systems allow us to study the structure of the circumstellar disks as the companion progresses along its orbit. Štefl et al. (2012) showed that the interaction of δ Sco’s disk with the companion object was relatively weak. Through analysis of interferometric phase signatures, δ Sco’s disk and its companion were found to be in the same plane, although rotating in opposite directions, effectively lowering the interaction timescale. In contrast, Pleione’s disk is highly perturbed by its eccentric companion (Hirata, 2007; Tanaka et al., 2007), which has a shorter orbital period of 218 days, compared to δ Sco’s \(\sim 11\) years.

Some Be stars have also been found to have companions which orbit at large distances where any tidal effects on the disk are negligible. For example, the Be star ο And (HD 217675) was found to be part of a four component system (Hill et al., 1988), and 66 Oph (HD 164284) is a Be star in a triple system (Štefl et al., 2004). Both systems have persisted for decades.

Many Be stars are known to be in spectroscopic binaries through periodic radial velocity variations in their spectral lines. Several cases have shown that for systems with a well-established orbital period, the period of the V/R variations matches (e.g. Pollmann (2012)). There is increasing evidence in these cases that the companion object interacts with the circumstellar disk through tidal interactions, causing regions of enhanced density. These density waves extend to large radii and are observable in many different wavelengths. Further modelling shows that density waves can form as two-armed spiral waves (e.g. Halonen & Jones (2013b); Panoglou et al. (2016); Cyr et al. (2017)).

### 1.5 Classical Be Star Research

Be stars are a unique set of massive stars whose main distinguishing features include spectral line emission, excess continuum emission, linear polarization, and a range of variability. These features arise from the presence of a circumstellar envelope which is a geometrically-thin, gaseous, decretion disk. The physical and dynamical nature of these disks is largely agreed upon. Despite this, some dynamical aspects of Be stars remain enigmatic, including:

- What is the interplay between stellar rotation and evolution?
- Why do Be stars spin up and form a disk?
- What is the mechanism that ejects the gas from the stellar surface?
- What is the source of the viscosity in Be star disks?
- Why are some Be star disks variable while others stay stable for long periods?
- How do binary companions affect the disk formation, dynamics, and geometry?

The focus of this thesis is to tackle some of these long standing questions and important unknowns in Be star research. Each of the projects presented in Chapters 2 through 4 contribute to the understanding of these questions by modelling the dynamical evolution of Be stars.

We first examine the effects that disk growth and dissipation has on the observed signatures from the star 66 Oph in Chapter 2. In Chapter 3 we address how binary companions can
change the large-scale structure of Be star disks by modelling the observed variability of the
star Pleione. In particular, we model the effects of disk tilting, precession, and ultimately
disk tearing on the star’s observables. It is expected that as a companion star tilts a Be disk,
the disk’s temperature structure would change as it is exposed to the different latitudes of the
stellar surface. In Chapter 4, we quantify how the temperature changes with tilt angle and how
disk tilting impacts the resulting observables. Finally, in Chapter 5, we summarize the major
results of these studies and discuss their present-day relevance to research on Be stars.


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Chapter 2

The Be Star 66 Ophiuchi: 60 Years of Disk Evolution


2.1 Introduction

Classical B-emission (Be) stars are rapidly rotating, main sequence, B-type stars that form outwardly diffusing disks of gas that have been ejected from the stellar surface. These geometrically thin disks are comprised of almost fully-ionized hydrogen gas and rotate around the stellar equator in Keplerian fashion (Rivinius et al., 2013). The rapid rotation of these stars (Slettebak, 1982) is believed to lead to the disk formation, which is likely assisted by non-radial pulsations (Rivinius et al., 2003; Baade et al., 2016). The disks are characterized by infrared and radio excesses in the star’s spectrum, polarized light, and the presence of hydrogen series emission lines (Rivinius et al., 2013).

Many Be stars show variability over timescales ranging from minutes (Goraya, 2008) to decades (Okazaki, 1997). The rapid variability observed from these systems is commonly associated with localized mass ejections (Balona, 1990), β-Cephei type pulsations (Balona & Rozowsky, 1991) and non-radial pulsations (Baade, 1982; Rivinius et al., 2003). Some disks exhibit cycles of growth and dissipation, which are dependent on the viscosity of the gas as outlined in the viscous decretion disk (VDD) model (Lee et al., 1991; Papaloizou et al., 1992; Klement et al., 2015). This type of variability occurs typically over periods of decades (e.g. Wisniewski et al., 2010).

66 Ophiuchi (HR 6712, HD 164284) is a bright ($m_V \sim 4.8$ mag), multiple star system (Štefl et al., 2004) containing a classical Be star of spectral type B2Ve (Floquet et al., 2002), at a distance of $\sim$199.6 pc$^1$. Since 1957, observations indicate that 66 Oph built a large disk which it subsequently lost over a period of dissipation that started around 1990 and finished about 20 years later. These events make 66 Oph an ideal system for studying the physics of disk dissipation.

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$^1$Based on Hipparcos parallaxes (van Leeuwen, 2007)
Many observational campaigns have focused on this star because of the variability of its disk. In 1957, the transition of the Hα line into emission signalled the onset of disk formation. (Rakotoarijimy & Herman, 1958). Grady et al. (1987), Percy & Attard (1992) and Percy & Bakos (2001) found that metal lines at UV wavelengths showed evidence of recurrent episodic mass-loss from 1982 to 1985, and again from 1989 to 1999 using UBV photometry. Hanuschik (1996) reported that the ratio of the violet (V) and red (R) peaks of the Hα profile began to vary in 1988, and continued through 1995 when it had a period of five years. Štefl et al. (2004) showed that the V/R ratio was constant (i.e., V = R) at the onset of dissipation, suggesting the disk was axisymmetric. Floquet et al. (2002) gave a detailed history of the pulsation period of 66 Oph before the period of disk dissipation, which they claim began in 1992. Since then, there has been a slow decline of the Hα emission strength with a transition to an absorption line in 2010 (Sabogal et al., 2017). The line profile has remained unchanged since then.

In this study, we investigate the evolution of 66 Oph’s disk at the onset and throughout the dissipation period. We characterized the physical state of the disk before dissipation and the dynamics of the disk during dissipation. This includes identifying the density structure and radial extent of the disk, as well as the viscosity and temperature distribution. This was accomplished by finding the best model to reproduce observations of the spectral energy distribution (SED), Hα spectral line, V-band polarization and V-band photometry during dissipation. By including observations from previous works in addition to presenting new observations, our unique data set allows us to model the complete dissipation of 66 Oph’s disk for the first time.

The models presented in this work were created following similar methods to Haubois et al. (2012), Carciofi (2012), Rímulo et al. (2018) and Ghoreyshi et al. (2018). The 1D dynamical code SINGLEBE was used to compute the disk surface density of an axisymmetric disk as a function of time, given a disk viscosity. The resulting disk density distributions were then input to the 3D radiative transfer code HDUST to calculate the emergent spectrum. In Section 2.2, we describe the observations compiled over the period of 1957 to 2020. Section 2.3 gives a detailed description of our method to determine the parameters of the star. Section 2.4 describes our modelling routines along with the results of the modelling. Section 2.5 provides a discussion and summary of our work including a comparison to other findings in the literature.

### 2.2 Observations

We compiled observations from a variety of different sources to investigate the disk’s evolution over many decades. Figure 2.1 shows the V-magnitude photometry, Hα equivalent width (henceforth, EW) and continuum V-band polarization from the onset of disk growth and subsequent dissipation from 1957 to 2020. We adopt the convention that negative EW means flux above the continuum.

Most of the observations used in this work were from the literature. We used V-magnitude photometry from the works of Haupt & Schroll (1974), Pavlovski et al. (1997), Percy et al. (1997), and Floquet et al. (2002). We also acquired Hα EWs from the works of Slettebak & Reynolds (1978), Ghosh (1988), Krishnamurthy (1999), Hanuschik et al. (1995), Floquet et al. (2002), and from the Be Star Spectra Database (BeSS)

2http://basebe.obspm.fr/basebe/
Figure 2.1: Observations of 66 Oph from 1957 to 2020. Top: V-band photometry. Middle: Hα EW. Bottom: V-band polarization. The sources of the data are listed in the legend; those without dates are previously unpublished. The solid grey vertical line indicates the time for the onset of disk dissipation and the dashed grey vertical line indicates when the Hα line transitioned to absorption. The solid and dashed grey lines also approximately correspond to the periods of observation for IRAS and WISE, respectively.

EW and V-band polarization were acquired from the archive for the Lyot Spectropolarimeter and the Halfwave Spectropolarimeter (HPOL) at the University of Wisconsin-Madison Pine Bluff Observatory, which were reduced in the work of Draper et al. (2014). We also used observations of the V-band polarization published by ? and Hayes (1983).

A number of previously unreported observations of 66 Oph are also presented. We used measurements of the Hα EW determined from observations of the Hα spectra made using the fibre-fed échelle spectrograph attached to the 1.1 meter John S. Hall telescope at the Lowell Observatory in Flagstaff, Arizona. Observations from this instrument were obtained between 2005 and 2007, with a resolving power of $R = 10000$. The reduction process of these observations follows that previously described in Jones et al. (2017).

Our models are constrained by observations of the Hα EW and V-band polarization, ac-

\footnote{http://www.sal.wisc.edu/PBO/LYOT/}
Table 2.1: OPD/LNA observations of 66 Oph used in this work as an estimate of the interstellar polarization.

<table>
<thead>
<tr>
<th>Modified Julian Date</th>
<th>Filter</th>
<th>P[%]</th>
<th>$\theta[^{\circ}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>57615.04</td>
<td>I</td>
<td>0.54 ± 0.03</td>
<td>85.1 ± 1.6</td>
</tr>
<tr>
<td>57615.05</td>
<td>R</td>
<td>0.61 ± 0.01</td>
<td>86.3 ± 0.5</td>
</tr>
<tr>
<td>57615.05</td>
<td>V</td>
<td>0.63 ± 0.02</td>
<td>85.7 ± 0.7</td>
</tr>
<tr>
<td>57615.06</td>
<td>B</td>
<td>0.60 ± 0.01</td>
<td>83.9 ± 0.2</td>
</tr>
<tr>
<td>57623.12</td>
<td>I</td>
<td>0.51 ± 0.01</td>
<td>86.6 ± 0.5</td>
</tr>
</tbody>
</table>

**2.2. Observations**

quired respectively using the MUSICOS spectrograph and the IAGPOL polarimeter at the Pico dos Dias Observatory (OPD), operated by the National Astrophysical Laboratory of Brazil (LNA) in Minas Gerais, Brazil. These observations were reduced with packages developed by the BEACON group

$^4$

and described in Magalhaes et al. (1984, 1996) and Carciofi et al. (2007).

The most recent Hα EW observation was obtained by the NRES spectrograph at the Las Cumbres Observatory LCOGT network. Details about the instrument and reduction process of this observation can be found in Brown et al. (2013). Since this observation was acquired while 66 Oph was diskless, it was used for comparison to our diskless models. This observation is later presented in Subsection 2.4.2.

Over a 63 year period, the V-band photometry, $m_\nu$, was observed to range between $4.5 < m_\nu < 4.9$ mag. During dissipation, the episodic variability continued with nine outbursts of between 1991 and 2008, while the overall brightness asymptotically dimmed.

As the V-band photometry available to us is sparse during the disk building phase, the variation of the light could not be used to model the evolution of the disk as was done in the studies of Ghoreyshi et al. (2018) and Rímulo et al. (2018). Photometric observations in other bands (UV, IR, etc.) are also sparse, with only snapshots available. Here, we use the V-band photometry along with observations of the entire SED, the Hα line profile and the percentage of polarized light over time.

The vertical, solid grey line in Figure 2.1 indicates the onset of dissipation (and is further discussed in Subsection 2.4.1). As the dissipation event begins, the continuum flux drops and the inner disk re-accretes causing the Hα EW to increase. This is seen in Figure 2.1 around 1995 as shown by the observations from Krishnamurthy (1999) and Hanuschik et al. (1995). After this time, the EW began to steadily decrease until the line went into absorption in 2010 (indicated by the vertical, dashed grey line). Our most recent observation, obtained by NRES/LCOGT in March 2020, indicates that 66 Oph continues to be diskless.

The bottom panel of Figure 2.1 shows the change in the observed polarization, $p_\nu$, with time for 66 Oph. Since 1980, the percent polarization slowly decayed until it approached the interstellar base level. From the base level polarimetric observations obtained by OPD/LNA in 2017 (listed in Table 2.1), the V-band interstellar polarization is $\sim 0.63\%$ with a polarization position angle of $\theta \approx 85.7[^{\circ}]$. This average value was subtracted from each of the observations using the scheme outlined by Quirrenbach et al. (1997). The intrinsic polarization of 66 Oph obtained is discussed further in Section 2.4.2.2.

The star’s SED was also used to refine our models. We selected 112 observations of the UV

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$^4$http://beacon.iag.usp.br/
portion of 66 Oph’s SED observed by the International Ultraviolet Explorer (IUE) telescope from the INES database (González-Riestra et al., 2001) following the selection procedure described by Freire Ferrero et al. (2012). The data selected were observed using the large aperture and high dispersion modes on the LWR and SWP cameras to ensure proper flux calibration and high spectral resolution. The observations were obtained over the period of March 1980 to September 1995, during which the observed flux showed no significant changes. We chose to remove the IUE data beyond 0.3 \( \mu m \) due to instrumental limitations which cause significant uncertainty (González-Riestra et al., 2001).

Visible and IR wavelength observations were obtained from the CDS Portal application from the Université de Strasbourg. Observations at these wavelengths were acquired while the disk was present (e.g. IRAS in 1989) and absent (e.g. AKARI in 2006-07 and WISE in 2010), providing upper and lower limits for modelling the SED. In addition to indicating the onset of dissipation and transition to absorption, the grey lines in Figure 2.1 also lie at the approximate dates of observation of IRAS (solid grey line) and WISE (dashed grey line) data. We also collected radio observations from VLA and ATCA (Apparao et al., 1990; Clark et al., 1998) and JVLA (Klement et al., 2019). These observations are presented alongside our models in Subsection 2.4.1.
2.2.1 The Interstellar Polarization

Figure 2.2 shows a Stokes QU diagram of the V-band polarization obtained by HPOL and by OPD/LNA. The observed path on the QU diagram during the disk dissipation phase is upward and to the right. As discussed by Draper et al. (2014) and more recently by Ghoreyshi et al. (2021), the process of formation and/or dissipation of a Be star disk is associated with a linear trend in the QU diagram, during which the polarization level rises or lowers in response to changes in the disk density, but the polarization angle remains steady. By fitting a linear regression to the HPOL observations, and fixing it to the assumed diskless 2016 observation, we determined the polarization position angle of the disk to be $\sim 98 \pm 3^\circ$. This was also confirmed by fitting a Serkowski law (Serkowski, 1973) to the OPD/LNA observations in Table 2.1, using

$$P(\lambda)/P(\lambda_{\text{max}}) = \exp[-1.15 \ln^2(\lambda_{\text{max}}/\lambda)].$$

We determined this law to best fit the observations in Table 2.1 when $P(\lambda_{\text{max}}) = 0.62 \pm 0.02\%$ at $0.61 \pm 0.04 \mu m$. Subtracting this spectrum from the HPOL observation, we accurately modelled the polarization of the disk (shown later in Subsection 2.4.2.2). From this spectrum, the wavelength averaged polarization position angle of the disk was computed to be $96 \pm 4^\circ$, in perfect agreement with the estimate made using the QU diagram.

2.3 Stellar Parameters

The use of Markov chain Monte Carlo (MCMC) routines along with Bayesian statistics has recently found success in the modelling of Be stars (e.g. Rímulo et al., 2018; Mota, 2019; Suffak et al., 2020; Mota et al., in prep.). In this Section, we use these methods to determine which stellar parameters best reproduce observations from the IUE, previously described in Section 2.2. The UV spectrum is not strongly affected by the disk, so we used it for our fitting procedure to find the stellar parameters. We determine values for the stellar mass $M$, critical fraction of rotation $W$ (as defined in equation 6 of Rivinius et al., 2013), age $t/t_{\text{ms}}$ (where $t_{\text{ms}}$ is the main sequence lifetime), inclination $i$, distance $d$ and the degree of interstellar reddening $E(B-V)$, and from these compute the derived parameters listed in Table 2.2.

We use a grid of 770 diskless Be star models, called BeAtlas, to create a parameter space for evaluating the observed spectrum from IUE. The BeAtlas grid was originally computed by Mota (2019) using a 3D non-local thermodynamic equilibrium (non-LTE) Monte Carlo radiative transfer code called $\text{hdust}$ (Carciofi & Bjorkman, 2006). This code simulates Be stars with or without disks by solving the coupled problems of radiative equilibrium, statistical equilibrium, and radiative transfer to compute synthetic observables. Table 2.3 summarizes the ranges of the stellar parameters for the BeAtlas models; the masses are consistent with those computed by Georgy et al. (2013).

The parameter space was sampled using $\text{emcee}$ (Foreman-Mackey et al., 2013), a Python code of the Affine Invariant MCMC Ensemble sampler (Goodman & Weare, 2010). We defined Gaussian prior distributions with mean and variance taken from literature values (see Table 2.4). We used the parallax from Hipparcos (van Leeuwen, 2007) since the parallaxes from GAIA’s DR2 catalogue (Gaia Collaboration et al., 2018) contained large errors for bright stars, including 66 Oph. Recently, we found recomputing the stellar parameters using the parallax
reported in GAIA’s eDR3 catalogue (Gaia Collaboration et al., 2020) (∼ 4.90 ± 0.37 mas, or ∼ 204±17 pc) produced a consistent set of parameters.

Before fitting, each model was also artificially reddened using the standard Fitzpatrick (1999) parameterization, with $E(B-V)$ as a free parameter in determining each model’s goodness of fit. We used a $\log(\chi^2)$ likelihood function to evaluate the fit of the models to the observations.

<table>
<thead>
<tr>
<th>Best Fit Parameters</th>
<th>Values</th>
<th>Derived Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ [M$_\odot$]</td>
<td>11.03$^{+0.55}_{-0.53}$</td>
<td>$L$ [L$_\odot$]</td>
<td>8200$^{+1600}_{-1300}$</td>
</tr>
<tr>
<td>$W$</td>
<td>0.52$^{+0.05}_{-0.05}$</td>
<td>$T_{eff}$ [K]</td>
<td>25940$^{+800}_{-750}$</td>
</tr>
<tr>
<td>$t/t_{ms}$</td>
<td>0.33$^{+0.10}_{-0.08}$</td>
<td>log $g$</td>
<td>4.17$^{+0.06}_{-0.05}$</td>
</tr>
<tr>
<td>$i$ [$^\circ$]</td>
<td>57.6$^{+6.8}_{-6.8}$</td>
<td>$R_{pole}$ [R$_\odot$]</td>
<td>4.50$^{+0.32}_{-0.23}$</td>
</tr>
<tr>
<td>$d$ [pc]</td>
<td>208.6$^{+8.9}_{-9.0}$</td>
<td>$R_{eq}$ [R$_\odot$]</td>
<td>5.11$^{+0.37}_{-0.26}$</td>
</tr>
<tr>
<td>$E(B-V)$</td>
<td>0.22$^{+0.01}_{-0.01}$</td>
<td>$v_{\sin(i)}$</td>
<td>290$^{+11}_{-9}$</td>
</tr>
</tbody>
</table>

Table 2.2: The best fitting stellar parameters for 66 Oph computed with emcee, and derived from the computed values using the models of Georgy et al. (2013).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Grid Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ [M$_\odot$]</td>
<td>1.7, 2, 2.5, 3, 4, 5, 7, 9, 12, 15, 20</td>
</tr>
<tr>
<td>$W$</td>
<td>0.00, 0.33, 0.47, 0.57, 0.66, 0.74, 0.81, 0.93, 0.99</td>
</tr>
<tr>
<td>$t/t_{ms}$</td>
<td>0, 0.25, 0.5, 0.75, 1, 1.01, 1.02</td>
</tr>
</tbody>
</table>

Table 2.3: The mass, critical fraction of rotation and age used for the BeAtlas grid of models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallax [mas]</td>
<td>5.01 ± 0.26</td>
<td>(van Leeuwen, 2007)</td>
</tr>
<tr>
<td>$v_{\sin(i)}$ [km/s]</td>
<td>280 ± 17</td>
<td>(Granada et al., 2010)</td>
</tr>
<tr>
<td>$i$ [$^\circ$]</td>
<td>43 ± 8</td>
<td>(Floquet et al., 2002)</td>
</tr>
</tbody>
</table>

Table 2.4: The values used as prior distributions in emcee.

Our emcee computation used 30 walkers and 50000 steps with a burn-in of 5000 steps, which were chosen by following the guidelines discussed by Foreman-Mackey et al. (2013). Approximately 27% of the steps proposed by the MCMC were accepted, which is consistent with the range recommended by Foreman-Mackey et al. (2013), when using a walker step size of 3.0.

Figure 2.3 shows the resulting probability density functions (PDFs) for the stellar parameters and correlation between these parameters as predicted by emcee. The top right corner of Figure 2.3 shows UV data along with a model computed with the best fit parameters in red. The grey lines show the model predictions corresponding to the last step of each walker. The residuals between the data and the model are shown directly below this panel.
Figure 2.3: The best fitting stellar parameters to 66 Oph’s UV spectrum. The probability density functions of each parameter are shown on the main diagonal axis while the intersection for each parameter shows the correlation map. The six parameters included in the fitting procedure are the stellar mass \( M \), the critical fraction of rotation \( W \), time of life on the main sequence \( t/t_{\text{ms}} \), stellar inclination \( i \), distance \( d \), and interstellar reddening \( E(B - V) \). The subfigure in the top-right corner shows the UV data, the model corresponding to the best fit parameters and the predictions from the last step of each walker. The residuals between the data and model are shown directly below this subfigure.

The stellar parameters predicted by \texttt{emcee} are used to compute the mean stellar luminosity \( L \), mean effective temperature \( T_{\text{eff}} \), mean surface gravity \( g \), polar radius \( R_{\text{pole}} \), equatorial radius \( R_{\text{eq}} \), and \( v \sin(i) \) by interpolating the Geneva stellar evolutionary models from Georgy.
et al. (2013) (more details are available in Mota, 2019). Table 2.2 lists the most probable stellar parameters and the additional parameters derived from the Geneva models. The mass, temperature and radius are consistent with standard values for B0 to B2 stars (Cox, 2000), and are in agreement with Waters et al. (1987), Floquet et al. (2002), Frémat et al. (2006) and Vieira et al. (2017). Values of $v \sin(i)$ reported in the literature range from 200 km/s (Bernacca & Perinotto, 1970) to 292 km/s (Floquet et al., 2002); our derived value agrees more closely with the upper limit. Our stellar parameters are further confirmed in Subsection 2.4.1, by comparison with the observed diskless SED.

2.4 Disk Structure

In Subsection 2.4.1 we determine the quasi-steady state of the disk by SED fitting and Hα line modelling prior to the dissipation. We model the Hα observation obtained by the HPOL Be star campaign on June 28, 1989. This observation (previously discussed by Draper et al., 2014) corresponds to our determination of the onset of disk dissipation in March 1989. This observation was obtained at a key time when 66 Oph’s disk was neither growing nor dissipating. The evolution of the disk before and after the quasi-steady state is modelled in Subsection 2.4.2. These models consider four different scenarios: whether the disk is isothermal or non-isothermal, and whether the disk builds to a steady state then dissipates, or dissipates from the steady state. These scenarios are presented as follows; Subsection 2.4.2.1 – isothermal disks that build then dissipate, Subsection 2.4.2.2 – isothermal disks that dissipate only, Subsection 2.4.2.3 – non-isothermal disks that build then dissipate, and Subsection 2.4.2.4 – non-isothermal disks that dissipate only. Models for each scenario were constrained by SED and Hα line fitting, and further compared to polarization when appropriate.

2.4.1 Modelling the Quasi-Steady State Phase

In the VDD model, gas is injected into the inner disk and diffuses outward via viscous forces (Porter, 1999). VDDs were initially modelled by Porter (1999) and later by Okazaki (2001), Bjorkman & Carciofi (2005), Jones et al. (2008) and Lee (2013). These studies showed that with a constant mass injection rate the disk’s density profile becomes relatively constant as a steady state is reached over months, years or longer, since gas that is moving outward is replenished at the innermost boundary. However, Haubois et al. (2012) claims that a true steady state is never realized but that a quasi-steady state phase is observed in many Be stars (e.g. Klement et al., 2015).

The Hα EW did not vary significantly between 1985 and 1995 (as seen in the middle panel in Figure 2.1). During this time, 66 Oph’s disk was approximately quasi-steady, however the drop in V-band magnitude and polarization, which form closer to the star (Faes et al., 2013), suggest that the dissipation began between 1989 and 1990 (indicated by the vertical, solid grey line in Figure 2.1).

Our initial goal is to quantify the density structure of 66 Oph’s disk while in this quasi-steady state so that we can use this model as a baseline to follow subsequent dissipation.

Often in the literature, a power-law density distribution is used to represent a snapshot of the disk in time. We use the density structure originally adopted by Waters (1986) for IR
continuum observations,

$$\rho(r) = \rho_0 \left( \frac{R_{eq}}{r} \right)^n.$$  \hspace{1cm} (2.2)

Here, $\rho_0$ is the density at the innermost disk radius in the equatorial plane, $r$ is the radial distance from the central star, and $n$ is the power law exponent.

We computed a grid of 61568 models with densities from $\rho_0 = 10^{-10}$ to $10^{-13}$ g cm$^{-3}$ for every quarter magnitude, with $n = 2.0$ to $3.5$ in steps of $0.1$, inclination from $i = 0$ to $90^\circ$ in steps of $2.5^\circ$, and outer disk radii of $R_{out} = 20, 30, 40, 50, 60, 80, 100$ and $200$ R$_{eq}$. For each model, \texttt{hdust} (previously described in Section 2.3) was used to compute the SED and H$\alpha$ line profile.

Non-coherent electron scattering within the disk also affects the shape of line profiles by broadening the wings of the line (Hummel & Dachs, 1992). This process is not accounted for in \texttt{hdust}, so to approximate the effect, we assume that a fraction $f$ of the H$\alpha$ flux is scattered by electrons with thermal velocities ranging from $v_e = 300$ to $800$ km/s, and that the scattered flux has a Gaussian profile. Thus, a new line profile $F_{new}$ is obtained by

$$F_{new}(\lambda) = (1 - f)F_{nc}(\lambda) + f \times F_e(\lambda),$$  \hspace{1cm} (2.3)

where $F_{nc}$ is the non-convolved line profile predicted by \texttt{hdust} and $F_e$ is the convolution of $F_{nc}$ with a Gaussian with FWHM of $v_e$. The convolved line profiles were then fit to the HPOL 1989 spectrum using \texttt{emcee} (distinct from the routine which used \texttt{emcee} previously described in Section 2.3). The fits were evaluated by interpolating the model fluxes to the wavelengths of the observed data and then calculating $\chi^2$ values.

After convolving, we found a subset of models within $1\sigma$ of the observed H$\alpha$ EW ($-58$ Å). The best fit model according to $\chi^2$ testing has a density of $\rho_0 = 2.5 \times 10^{-11}$ g cm$^{-3}$, with $n = 2.6$, 

Figure 2.4: A comparison of the HPOL 1989 H$\alpha$ profile to the best fit unconvolved model (dotted, grey) and final convolved spectra (dashed, blue). The vertical dashed lines show the range over which the fit was evaluated.
at $i = 57.5^\circ$, and $R_{\text{out}} = 100 \ R_{\text{eq}}$, with a fit of $\chi^2_v = 1.17$ when using $F_c = 0.22$, $v_e = 700 \ \text{km s}^{-1}$. Figure 2.4 shows the observed profile (black line), and the best fit model before (dotted line) and after (dashed line) convolving. These profiles all have EWs of $-58 \ \text{Å}$. The thermal velocity used for the convolution corresponds to a kinetic temperature of $T_K = 10100 \ \text{K}$. This agrees with the disk temperature of $\sim 10500 \ \text{K}$ at the edge of the optically thick H$\alpha$ emitting region at $\sim 10.1 \ R_{\text{eq}}$ (computed by following Appendix D of Vieira et al., 2017). This process of convolving lines was previously used by Klement et al. (2015), without MCMC fitting, for the Be star $\beta$ CMi. $\beta$ CMi is a late-type star with a spectral type of B8Ve and is expected to have a cooler disk. We note that these authors obtained values of $f = 0.6$ and a smaller value of $v_e = 300 \ \text{km s}^{-1}$, consistent with its disk temperature.

In the case of a disk with $n = 3.5$, the H$\alpha$ emitting volume probes a significant portion of the disk (Tycner et al., 2005) typically thought to be the innermost 20 $R_{\text{eq}}$ (Faes et al., 2013). However, we found these regions become increasingly extended as $n$ decreases. This means that for an early-type star like 66 Oph, lower values of $n$ result in more ionized material at larger distances. Figure 2.5 shows the H$\alpha$ EW and corresponding $\chi^2_v$ value of the best fit model with increasing $R_{\text{out}}$. Here, with $n = 2.6$, we find the EW increases until the disk reaches a
radius of \( \sim 100 \, R_{\text{eq}} \), beyond which there is no significant increase. Similarly, \( \chi^2 \) improves as \( R_{\text{out}} \) increases and the H\( \alpha \) flux increases until \( \sim 100 \, R_{\text{eq}} \), with no appreciable change at larger radii. This provides an upper limit on the size of the H\( \alpha \) emitting region.

The star’s SED (Figure 2.6) was also used to independently constrain our model values of \( \rho_0 \) and \( n \). The SED upper bound, prior to dissipation, is set by IRAS (1989) in the IR, as well as radio observations from VLA (1988). Conveniently, the upper bound from IRAS was observed just before the onset of dissipation, providing important observational constraints for the quasi-steady state phase. Data from ATCA (1997) and JVLA (2010) are also on the upper bound, as the disk dissipation had not significantly changed the disk’s radio emission which comes from the outermost disk. The SED’s lower bound is constrained by observations from WISE (2010) and AKARI (2006-07). Observations of the lower bound were obtained after dissipation and likely correspond to a diskless state. Using the diskless model of Section 3, we find a very reasonable fit to the observations at visible to radio wavelengths observed after 1989 with \( \chi^2 = 1.12 \).

We fit the upper bound SED with the same grid of models used above for H\( \alpha \) fitting. We find that a model of \( \rho_0 = 10^{-11} \, \text{g cm}^{-3} \), \( n = 2.4 \), \( i = 57.5^\circ \) and \( R_{\text{out}} = 100 \, R_{\text{eq}} \) (shown as the solid black line in Figure 2.6, and the black dot in Figure 2.7) produced the best fit to the visible, IR and radio photometry observed before 1989 with \( \chi^2_v = 1.09 \). Increasing the outer disk radius to 200 \( R_{\text{eq}} \) marginally improved the fit to \( \chi^2_v = 1.07 \). The model which best fit the H\( \alpha \) line profile (\( \rho_0 = 2.5 \times 10^{-11} \, \text{g cm}^{-3} \) and \( n = 2.6 \)) fit the SED observations before 1989 with \( \chi^2_v = 1.15 \) for \( R_{\text{out}} = 100 \, R_{\text{eq}} \), or \( \chi^2_v = 1.14 \) for \( R_{\text{out}} = 200 \, R_{\text{eq}} \).

Figure 2.7 shows a comparison of the best fit models to the H\( \alpha \) line profile and SED accord-
Figure 2.7: Summary of the best fitting disk densities. Contours corresponding to the 1σ confidence level were determined by interpolation of the $\chi^2_r$ values over the model grid. The black contours show the SED fitting and the grey contour shows the H\(\alpha\) fitting, with corresponding coloured dots indicating the best fit model.

The contours show the models which fit best to within 1σ. The overlap has a range of $\rho_0 = 8.5 \times 10^{-12}$ to $3 \times 10^{-11}$ g cm\(^{-3}\), and $n = 2.4$ to 2.7. Within the same overlapping region, only the $\rho_0 = 2.5 \times 10^{-11}$ g cm\(^{-3}\) and $n = 2.6$ model satisfies both the SED and H\(\alpha\) line profile fitting. The model which best fit the SED produced a relatively poor fit to the H\(\alpha\) line profile, with $\chi^2_r = 3.2$.

The $\rho_0 = 2.5 \times 10^{-11}$ g cm\(^{-3}\) and $n = 2.6$ model predicts a V-band magnitude of 4.72 mag, which is consistent with the range of 4.76 to 4.55 mag observed in 1989 (Floquet et al., 2002). The same best model predicts the V-band polarization to be $\sim 0.72\%$, while observations obtained 5 years prior to dissipation show the V-band polarization to range from 0.58% to 0.65% (after subtracting the interstellar polarization, as described in Section 2.2).

Since the $\rho_0 = 2.5 \times 10^{-11}$ g cm\(^{-3}\) and $n = 2.6$ model with $i = 57.5\degree$ and $R_{\text{out}} = 100 R_{\text{eq}}$ is clearly successful in reproducing the observed H\(\alpha\) line profile, SED, V-band magnitude and V-band polarization, we conclude it is the best representation of the quasi-steady state phase for 66 Oph’s disk.

### 2.4.2 Hydrodynamic Calculations

In this section, we model the formation of 66 Oph’s disk and the subsequent 21 years of dissipation until the H\(\alpha\) emission went into absorption. We use the best fit quasi-steady model from Section 2.4.1 as a mid-point between the building and dissipation. To simulate the disk evolution, we use the 1D dynamical code SINGLEBE, developed by Okazaki et al. (2002) and later used by Carciofi (2012), Haubois et al. (2012), Rímulo et al. (2018) and Ghoreyshi et al.
2.4. Disk Structure

SINGLEBE solves the 1D fluid hydrodynamic equations (Pringle, 1981) for the surface density of a viscous disk using the thin disk approximation. The disk is assumed to be axisymmetric, Keplerian, and in vertical hydrostatic equilibrium. See Okazaki et al. (2002) and Rímulo et al. (2018) for more details on SINGLEBE.

The evolution of VDDs is controlled by kinematic viscous forces within the disk. SINGLEBE adopts the commonly used $\alpha$-prescription for viscosity, defined by Shakura & Sunyaev (1973) as

$$\nu(r) = \frac{2}{3} \alpha(r) c_s(r) H(r),$$

where $\nu$ is the disk viscosity, $r$ is the radius in the disk, and $H$ is the disk scale height. The sound speed is given by

$$c_s(r) = \left[ \frac{k_B T(r)}{(\mu m_a)} \right]^{1/2},$$

where $k_B$ is the Boltzmann constant, $T$ is the gas kinetic temperature, $\mu$ is the mean molecular weight, and $m_a$ is the atomic mass unit. Note, for a non-isothermal disk, the radial dependence of the temperature can change the sound speed, which in turn changes the viscosity in Equation 2.4. In this case, $\alpha(r)$ and $T(r)$ become degenerate and indistinguishable. If one parameter were to be held constant, the other parameter may be adjusted to produce the same viscosity. Therefore, we distinguish isothermal disks from non-isothermal disks based on constant or variable $\alpha T$ with radius, respectively.

Viscosity affects disk evolution by influencing the outflow rate of the gas (Lee et al., 1991). Given $\alpha(r)$, a target inner surface density $\Sigma(r = R_{eq})$ (i.e. $\Sigma_0$) or inner volume density $\rho_0$, the outer disk radius $R_{out}$, and a value for the mass injection rate at the base of the disk $\dot{M}_{inj}$, SINGLEBE computes the surface density as a function of time and distance from the star, $\Sigma(r,t)$. Details about the computational procedure and assumed boundary conditions can be found in Rímulo et al. (2018).

Using equation 3.6 of Okazaki (1991), the mass density can be computed as

$$\rho(r,t) = \frac{\Sigma(r,t)}{\sqrt{2 \pi H(r,t)}}.$$  

We note that, assuming a constant $\alpha T$, hydrodynamic theory for an axisymmetric disk with Keplerian rotation in a quasi-steady state predicts that the mass density will have approximately a radial power law (Equation 2.2) with $n = 3.5$ (Bjorkman & Carciofi, 2005).

We input the stellar parameters determined in Section 2.3 (see Table 2.2), $\rho_0$, $n$, and $R_{out}$ of the best fit model from Subsection 2.4.1, and the duration of the growth and dissipation events to SINGLEBE. We use $\alpha = 1.0$ to ensure rapid evolution (Haubois et al., 2012), however as $\alpha$ can be scaled by changing the length of the evolutionary epoch (as illustrated in figure 1 of Haubois et al., 2012) our models can explore all values of $\alpha$. For each model, the disk’s innermost radius was set to 1 $R_{eq}$ and the radial grid points were spaced logarithmically from the inner radius. Following Rímulo et al. (2018), we define the onset of dissipation to occur at 0 years for all models. The density profiles from SINGLEBE were input to H\textsc{dust} to compute the SED, H$\alpha$ line profile, and polarization at times corresponding with the observed data.
2.4.2.1 Building and Dissipating an Isothermal Disk

Here, we investigate an isothermal disk which builds to a steady state and then dissipates, which we refer to as the constant $\alpha T$, $n = 3.5$ scenario. The isothermal assumption also makes our models similar to the mixed models used in Carciofi & Bjorkman (2008), where the disk is isothermal in the fluid equations and variable in temperature when computing observables using $hdust$.

To match the base density found in Section 2.4.1, this scenario requires a mass injection rate of $\dot{M} = 8.5 \times 10^{18}$ g/s (or $\sim 1.3 \times 10^{-7} M_\odot$/yr) at $R = 1.02 R_{eq}$, of which $\sim 99\%$ falls back onto the star. We chose a constant disk temperature at 60\% of $T_{eff}$ (from Table 2.2), following Carciofi & Bjorkman (2006).

Panel a) of Figure 2.8 shows density profiles of this scenario after the building period during the dissipation. The quasi-steady state is indicated by the thick, black line on the panel. We note that it has the expected value of $n = 3.5$ which differs from the $n = 2.6$ of the best fitting quasi-steady model. During dissipation, the disk empties in an “inside-out” fashion, which is expected for Be stars as discussed by Poeckert et al. (1982), Rivinius et al. (2001) and Clark et al. (2003). After the dissipation, the maximum density of the disk is $\sim 5 \times 10^{-13}$ g cm$^{-3}$.

Figure 2.9 shows the H$\alpha$ line profiles, the disk temperature profiles, and the visible, IR and radio SEDs compared to observed data (previously shown in Figure 2.6), at different times during dissipation. The SEDs in the three right panels show that this scenario is unable to reproduce the observed flux before dissipation. This is most notable at IR wavelengths where the flux is $\sim 1$ to 2 orders of magnitude fainter. In the top left panel of Figure 2.9, the H$\alpha$ line at 0 years shows a peak flux of $\sim 1.5$ times the continuum flux. In contrast, the HPOL 1989 observation (previously shown in Figure 2.4) shows a peak flux of $\sim 11$ times the continuum flux.

The $n = 3.5$ disk also has a local maximum of the peak H$\alpha$ flux at 3 years after the dissipation started. This is due to an increase of stellar ionizing radiation (and corresponding increase in H$\alpha$ flux) reaching larger radial distances of the inner disk due to the reaccretion of material closest to the star. (See the bottom left panel of Figure 2.9.) After this, the disk continues to dissipate and the H$\alpha$ flux drops smoothly. This phenomenon was also observed for disk models with $\alpha = 0.1$ and $\alpha = 0.5$, with the flux increase occurring at later times in the dissipation.

In Figure 2.10 we show the SED for this scenario after 21 years of dissipation (black line), and also the best quasi-steady state model (grey line). As the disk dissipates, the model approaches the theoretical diskless SED. This model closely fits the UV and IR observations, however produces a weaker radio excess than the 2010 JVLA observations.

Figure 2.11 shows the H$\alpha$ line profile (black line) for this scenario after 21 years of dissipation. The line profile shows no emission and is consistent with the absorption profile of the NRES/LCOGT 2020 observation. Note, the NRES/LCOGT observation closely matches with the observations from BESS obtained in 2010.

Figure 2.12 shows the H$\alpha$ EW during dissipation for this scenario (black line). Here it is apparent that the disk could only build to approximately half of the observed EW of the quasi-steady state and dissipates at a much faster rate than the observations.

In conclusion, the constant $\alpha T$, $n = 3.5$ scenario fails to reproduce the two main observables chosen to constrain the models, namely the SED and H$\alpha$ line emission. This is further illustrated in Figure 2.8, panel a), which shows that this model produces a disk far less massive
Figure 2.8: Disk density profiles at different times from the start of dissipation for the following scenarios: Panel a) constant $\alpha T$, $n = 3.5$, b) constant $\alpha T$, $n = 2.6$, c) variable $\alpha T$, $n = 2.7$, d) variable $\alpha T$, $n = 2.6$. A grey line in each panel indicates the best quasi-steady model.
Figure 2.9: Changes in the Hα line (top left), the disk temperature structure (bottom left) and the visible (top right), IR (middle right) and radio (bottom right) SEDs during the disk dissipation of the constant $\alpha T$, $n = 3.5$ model. The Hα spectra are not continuum normalized to clearly show the increase in peak flux during dissipation. The SEDs include observations made before 1989 (grey triangles) and after 1989 (black circles) from Figure 2.6.

than what is required. This will be further discussed below.

2.4.2.2 Dissipating an Isothermal Disk from a Defined Density

Here we use an ad hoc scenario that starts to dissipate from the quasi-steady state with $n = 2.6$ in order to bypass the building phase. This constant $\alpha T$, $n = 2.6$ scenario, uses the same $\alpha$ and $T_{\text{disk}}$ as the constant $\alpha T$, $n = 3.5$ scenario.

Figure 2.8, panel b) shows that as dissipation begins, this scenario quickly moves toward a steeper density slope. During the first five years, the gas from the outer disk moves to the inner disk, preventing the density of the disk between $\sim 5$ and $10$ $R_{\text{eq}}$ from depleting. As dissipation continues, the entire disk appears to dissipate at a steady rate across all radii.

Figure 2.10 shows the SED for this scenario (blue dashed line) closely matches the visible
2.4. Disk Structure

Figure 2.10: SEDs for each of the dynamical scenarios after dissipation. For comparison the best fitting initial steady state is shown. The observed data is the same as that presented in Figure 2.6. The two models from the variable $\alpha T$ scenarios lie on top of each other. The colours for each model are consistent with Figure 2.8.

and IR observations after 21 years of dissipation. An excess of radio flux is maintained as gas remains in the outer disk.

Figure 2.11 shows no H$\alpha$ emission (blue dashed line) remains after dissipation, consistent with the diskless NRES/LCOGT 2020 observation. In Figure 2.12, the H$\alpha$ EW shows that this scenario (blue dashed line) requires $\sim$ 17 years to reproduce the diskless profile when $\alpha = 1$.

Since $\alpha$ and $R_{out}$ both affect the rate of disk dissipation, they must be determined simultaneously. Figure 2.13 shows an interpolated grid of models for the this scenario computed over the ranges of $0.2 < \alpha < 1.5$ and $100 < R_{out} < 1000 R_{eq}$. The black line indicates which models dissipate from the initial state to having no H$\alpha$ emission in 21 years. The combination of $\alpha = 0.4$ and $R_{out} = 115 R_{eq}$ was determined to best reproduce the observations acquired after 2, 5, 8, 9, 12, 14, 17, 19 and 21 years of dissipation. Figure 2.12 shows that this model closely reproduces the H$\alpha$ EW curve. This model is further compared to the data in Figures 2.14, 2.15, and 2.16.

For each stage of dissipation in Figure 2.14, non-coherent electron scattering was accounted using the method described in Subsection 2.4.1. As previously mentioned, the 0 year model required the electron thermal velocity in the disk to be 500 km s$^{-1}$ when 22% of the light is scattered by the electrons. The electron thermal velocity required to broaden the line at all times during the dissipation is $\sim 560 \pm 10$ km s$^{-1}$ (which corresponds to a kinetic temperature of $\sim 12400 \pm 500$ K). The fraction, $f$, of scattered light drops during dissipation, going from $f = 20\%$ at 5 years to $f = 14\%$ at 12 years, and $f = 4\%$ at 19 years. This is expected, as $f$
should decrease with decreasing disk density.

Figure 2.15 shows the observed polarization from HPOL near the onset of dissipation. The constant $\alpha T$, $n = 2.6$ scenario with $\alpha = 0.4$ and $R_{out} = 115 R_{eq}$ is overplotted for comparison at the same time intervals as in Figure 2.14.

In Figure 2.16, we compare to each of the observables for $\alpha = 0.1$, 0.4 and 1.0. The errors for each model were computed using a $1\sigma$ deviation of ten unique simulations run for each model. The EW begins to change most rapidly nine years after the onset of dissipation, and decreases steadily for the following ten years until the line drops into absorption. For both the V-band magnitude and the continuum polarization the most rapid change occurs between zero and five years. During this time, the $\alpha = 0.4$ disk dims by $\sim 0.1$ mag. The percent polarization intrinsic to the $\alpha = 0.4$ disk drops steadily by $\sim 0.2$ % every five years. For comparison, the bottom panel shows the observed polarization position angle intrinsic to the disk, computed from the percent polarization. We find polarization position angles range between $\sim 94^\circ$ and $\sim 104^\circ$ and appear to remain constant through the dissipation event. These polarization position angles are consistent with the angle of 98° determined in Section 2.2.
### 2.4. Disk Structure

#### 2.4.2.3 Building and Dissipating a Non-Isothermal Disk

Here we investigate both the building and dissipation phases using the assumption of a non-isothermal disk. For reasons made apparent below, we refer to these models as the variable $\alpha T$, $n = 2.6$ scenario. In this case, the fluid equations no longer limit the density profile to $n = 3.5$, however we require knowledge of the now inseparable $\alpha T$ parameter. We assume a power-law relation between the product $\alpha T$ and the disk radius given by,

$$\alpha(r)T(r) = \alpha_0 T_0 \left( \frac{R_{eq}}{r} \right)^C$$

where $\alpha(r)$ and $T(r)$ are the disk viscosity and disk temperature at radius $r$, $\alpha_0$ and $T_0$ are the disk viscosity and disk temperature at $r = R_{eq}$, and $C$ is a constant power-law index. We chose $\alpha_0 = 1$ and $T_0 = 60\%$ of $T_{eff}$.

We tested values of $C$ ranging from 0.1 to 100 and found $C = 2$ to most closely match the $n = 2.6$ density slope. As shown in panel c) of Figure 2.8 the disk was unable to build to $n = 2.6$, with the majority of the outer disk having $n \approx 2.7$. The timescales required to build the outer disk to $n = 2.6$ were on the order of 1000 years, significantly greater than time period that 66 Oph built its disk. The long viscous timescales at the outer disk are explained by the low viscosity inferred from Equation 2.7. For instance, assuming a constant temperature and $C = 2$, then $\alpha(r = 100 R_{eq})/\alpha_0 = 0.0001$.

**Figure 2.12:** The same as Figure 2.10 for the Hα EW. The solid vertical grey line indicates the onset of dissipation, and the dashed grey line indicates when the Hα line was observed to transition to absorption. While the model parameters differ, the constant $\alpha T$, $n = 2.6$ scenario with $\alpha = 0.4$ and $R_{out} = 115 R_{eq}$ is shown for comparison. The colours for each model are consistent with Figure 2.8.
Figure 2.13: Combinations of $\alpha$ and $R_{out}$ that allow the constant $\alpha T$, $n = 2.6$ scenario to lose all H$\alpha$ emission in 21 years. The grid of models has been interpolated. The vertical dotted line indicates the transonic radius (Okazaki, 2001). The models which best match the observed H$\alpha$ emission line dissipation are indicated by the point at $\alpha = 0.4$ and $R_{out} = 115 R_{eq}$.

Figure 2.10 shows that the SED for this scenario (yellow line) reproduces the observed visible spectrum and radio excess after 21 years of dissipation, however it remains too bright at IR wavelengths. The H$\alpha$ line profile is shown in Figure 2.11 (yellow line) after accounting for non-coherent electron scattering using the process outlined in Subsection 2.4.1. We see that the emission remains too strong, evidence that the disk depletes too slowly, which is also seen in the H$\alpha$ EW in Figure 2.12 (yellow line).

2.4.2.4 Dissipating a Non-Isothermal Disk from a Defined Density

For our final dynamical scenario, we again adopt an ad hoc solution starting from the best quasi-steady model of Section 2.4.2.1 to study the dissipation phase using the non-isothermal assumption. We refer to these models as the variable $\alpha T$, $n = 2.6$ scenario. In this case, we assume that the outer disk could have built through past (undocumented) disk events, since Be stars remain on the main sequence for most of their lifetime (Georgy et al., 2013). We chose to use the $\alpha T$ profile described in Equation 2.7, with $C = 2$.

Panel d) of Figure 2.8 shows that the disk clears from the “inside-out”. The most rapid change to the disk’s density occurs within the first five years, beyond which the density decreases more slowly. The final five years of dissipation produces a change in density at 10 $R_{eq}$ of $\sim 0.5\%$. As before, we find the viscous timescales of the outer disk are too large since $\alpha T$ decreases too rapidly with radius.

Figure 2.10 shows this scenario (red line) does not match the diskless SED observations as closely as the isothermal models. After dissipation, this scenario more closely matches the diskless model from the UV to the near IR, and more closely matches the quasi-steady state in
Figure 2.14: Hα emission line dissipation modelled with the constant $\alpha T$, $n = 2.6$ scenario. The thick, dark lines correspond to observations. The thinner, lighter lines are models using constant $\alpha$ and isothermal disks with $\alpha = 0.4$ and $R_{out} = 115 \, R_{eq}$. The 0 year model corresponds to $\rho_0 = 2.5 \times 10^{-11} \, \text{g cm}^{-3}$ and $n = 2.6$.

Figure 2.11 shows Hα emission (red line) remains too strong after 21 years of dissipation. Again, the line was adjusted to account for non-coherent electron scattering. During dissipation the peak flux of the line profile is reduced from $\sim$ eleven to nine times the continuum flux, but does not go into absorption. Dissipating for an additional 1000 years, further decreases the peak flux of the line profile by a negligible $\sim 0.1\%$. Figure 2.12 shows the Hα EW dissipates too slowly to match observations in this scenario (red line).
Figure 2.15: Comparison of the polarization spectrum observed by HPOL on June 28th, 1989 and the models presented in Figure 2.16. The interstellar polarization, as determined in Section 2.2, was subtracted from the HPOL spectrum.

2.5 Discussion and Summary

The goal of this work is to use VDD theory to model the quasi-steady state of 66 Oph’s disk, and then follow the subsequent 21 year dissipation until the Hα line transitions from emission to absorption. Using thousands of models constrained by photometric, polarimetric and Hα observations, we determined the physical properties of the disk as it evolved.

We determined a set of stellar parameters (Table 2.2) using MCMC fitting of the UV spectrum. These parameters are consistent with those reported in the literature and were also confirmed by comparison to observations of 66 Oph obtained after dissipation (see Figure 2.10).

We used diskless polarimetric observations obtained in 2016 to determine the interstellar polarization in the direction of 66 Oph more accurately than previous studies using nearby field stars (see the work by Draper et al., 2014). We modelled the interstellar polarization by fitting a Serkowski law to these diskless observations. The interstellar polarization was then subtracted from HPOL observations, acquired when the disk was present, to determine the polarization level and position angle of the disk (Figure 2.16). Using the QU diagram (Figure 2.2) we also determined the disk’s polarization position angle to be $\theta_{\text{int}} = 98 \pm 3^\circ$. Our two methods of determining the polarization position angle were found to agree. This confirms our interstellar polarization correction using the 2016 LNA/OPD observations.

In this investigation, we begin by studying in detail the quasi-steady state phase of the disk, prior to the dissipation that started in 1989. Then, we used four different hydrodynamic scenarios to explore disk evolution. These scenarios assumed the disk was either isothermal or non-isothermal, and built to the quasi-steady state before dissipating or dissipated from a defined density distribution.
Figure 2.16: Comparison of the observables shown in Figure 2.1 (black crosses) with the models presented in Figure 2.14 (coloured circles) over the period of disk dissipation. Additional simulated observables for $\alpha = 0.1$ and $\alpha = 1$, and the observed polarization position angle intrinsic to the disk, are presented for comparison. The error bars show the 1$\sigma$ deviation of 10 simulations computed for each model. The color of each model corresponds with the same colors used in Figure 2.14. The vertical grey lines are the same as in Figure 2.1.
Our best fit disk model to the quasi-steady state, constrained by observations of the Hα line profile and SED, and verified by the polarization, has a density of $\rho_0 = 2.5 \times 10^{-11} \text{ g cm}^{-3}$, $n = 2.6$, and $i = 57.5^\circ$ with the outer disk radius at the lower-limit of $R_{\text{out}} = 100 \, R_{\text{eq}}$. The density slope, $n$, is consistent with the results of Waters et al. (1987) who reported $n = 2.5$ and Vieira et al. (2017) who found $n = 2.6$, however, both of these studies found disk base densities at the innermost disk approximately one order of magnitude less dense. The close agreement of the density slope suggests it has been well constrained, however further study into the base density is necessary.

The scenario assuming an isothermal disk that builds and then dissipates (the constant $\alpha T$, $n = 3.5$ scenario) reached a density slope of only $n = 3.5$, much steeper than the best fit quasi-steady state model with $n = 2.6$. As a result, the disk was not bright enough and dissipated too quickly due to the rapid density fall-off with radial distance. Interestingly, this scenario predicted a brightening of the Hα flux shortly after the onset of dissipation. We attribute this to the heating of cool regions of the disk at greater radial distance from the star as the innermost disk accretes. This phenomenon was not observed in disks with smaller values of $n$.

Next, we explored isothermal disks that dissipated from our best fit quasi-steady state (the constant $\alpha T$, $n = 2.6$ scenario). We were able to constrain $R_{\text{out}}$ simultaneously with $\alpha$ as shown in Figure 2.13. We find $R_{\text{out}} = 115 \, R_{\text{eq}}$ and $\alpha = 0.4$. This scenario and these values successfully reproduced the rate of dissipation for all observables considered (Hα, V-band polarization and magnitude). A remarkable fit of Hα line profile was obtained at selected epochs of disk dissipation (Figure 2.14).

Krtička et al. (2011) estimated the outer radius of Be star disks. Using their equation 5 and estimating the rate of change in the moment of inertia using the prescription from Clark et al. (1998) along with our determination of $\dot{M}$ from SINGLEBE, we obtain an outer disk radius of $\sim 125 \, R_{\text{eq}}$, which is in relatively good agreement with our value determined above.

For the scenario with a non-isothermal disk that builds and dissipates (the variable $\alpha T$, $n = 2.7$ scenario) we use a power-law $\alpha T$ that varies with radius as $r^{-2}$. The viscous timescales of this scenario was found to be too large to reproduce the rate of dissipation. After 21 years of dissipation much of the disk remained, and while the visible and near IR flux match the diskless star, the Hα and far IR remained too bright. However, the outer disk dissipated very little, and the predicted radio emission closely matched the JVLA observations from 2010.

Finally, we considered a non-isothermal disk and followed dissipation again from our best fit quasi-steady state (the variable $\alpha T$, $n = 2.6$ scenario). We use the same procedure as the constant $\alpha T$, $n = 2.6$ scenario, and the power-law $\alpha T$ from the variable $\alpha T$, $n = 2.7$ scenario. This scenario dissipated similar to the other variable $\alpha T$ scenario, and the predicted observables reveal that the disk was still present after 21 years.

Our models show that the excess radio emission observed in 2010 (Figure 2.6) can only be reproduced if 66 Oph’s outermost disk is unaffected by the dissipation. This suggests that a large mass reservoir was built in the outer disk, as discussed in Rímulo et al. (2018) and Ghoreyshi et al. (2018).

Overall, we find that the constant $\alpha T$, $n = 2.6$ (isothermal) scenario, using $\alpha = 0.4$ and $R_{\text{out}} = 115 \, R_{\text{eq}}$, best reproduces the observed dissipation of 66 Oph’s disk. However, this scenario fails to explain how the disk could build to $n = 2.6$. These smaller values of $n$ can be reached with a non-isothermal assumption, however all of our non-isothermal scenarios failed to match the timescale of dissipation. It is possible that other prescriptions for $\alpha T$, other than
a power-law, might help this issue.

Another explanation for low values of $n$ might due to the accumulation effect described by Panoglou et al. (2016) and by Cyr et al. (2017). Basically, the disk density becomes shallower due to the build up of material inward of a binary companion. This effect was not explored here, but we point out that our results indicate that the disk around 66 Oph is quite large, suggesting that the existence of a close by companion is unlikely.

In future work, we aim to better understand the structures of $\alpha$ and $T$ in the disk. It may be possible to empirically model $\alpha$ as a function of radius using a set of other time-series emission lines during the dissipation phase, as different lines form within different radial locations in the disk.
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Chapter 3

The Role of Disk Tearing and Precession in the Observed Variability of Pleione

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3.1 Introduction

Over the past 130 years, the Be star Pleione (HD23862, 28 Tau) has exhibited remarkable spectroscopic and photometric variations. Frost (1906) first realized the extreme nature of its variability in 1905 and 1906 when he discovered that its previously bright emission lines had disappeared in only a few years. Since this time, Pleione’s hydrogen and metal line profiles have exhibited a wide range of shapes, including absorption lines (diskless phase) (McLaughlin, 1938), doubly-peaked emission lines (Be phase) (Doazan & Underhill, 1986) and emission with narrow absorption cores (Be-shell phase) (Delplace & Hubert, 1973). The reader can find a historical account of the changes in Pleione’s spectrum by Gulliver (1977) and Hirata (1995), with recent activity described by Nemravová et al. (2010).

The spectral type of B8 Vpe for Pleione was first determined by observations taken during a diskless phase by Lindblad (1922), and was later confirmed by Frost et al. (1926) and Merrill & Burwell (1933). Other details about this system, such as the nature of its companion stars and the interpretation of its variability are much less clear. Katahira et al. (1996a,b) found the near companion to have a 218 day orbital period with an eccentricity of 0.6 by analyzing the variation of radial velocities in Hα emission from two consecutive shell phases. By similar means, Nemravová et al. (2010) confirmed the 218 day orbit and found that the eccentricity is likely > 0.7. The inclination angle of the companion’s orbital plane has not been reported in the literature.

Long-term variability has also been observed in Pleione since the end of its last diskless phase in 1937. Since then, its Balmer emission lines have continuously transitioned between Be and Be-shell phases with a ~ 34 year period (Katahira et al., 1996a; Hummel, 1998). Using lunar occultation methods, Gies et al. (1990) detected asymmetry in the disk that was consistent with previously known long-term V/R variability, and speculated that the repeating Be to Be-shell phase transitions are a product of the companion’s periastron passages. Iliev et al. (2007)
and Iliev (2019) found that the size of the Hα and Hβ emitting regions are synchronized with the ~34 year period.

The most recent transition from a Be phase to a Be-shell phase in Hα (which, hereafter, will be referred to as the Be phase and the Be-shell phase) from 2006 to 2007 has caused uncertainty about the physical conditions of Pleione’s disk causing its variability. Before this transition occurred, Hirata (2007) found the polarization position angle to steadily change over long time-scales. They argued this was the result of a uniformly precessing disk that had been tilted off-axis by tidal interactions with the companion. They asserted that this model explains the repeating Be-shell to Be phase changes, with the shell lines appearing (1973) and disappearing (1988) around a disk inclination of 60°. However, their model was unable to match the rapid drop in magnitude that accompanied the 2006 to 2007 Be to Be-shell transition.

Studying the photometric and spectroscopic variability of Pleione from 2005 November to 2007 April, Tanaka et al. (2007) proposed that the precessing disk suggested by Hirata (2007) had been partially re-accreted before a new disk began building. With this two-disk model, they claim that the newly forming disk explains the formation of new components to the Hα and Hβ lines, while the old tilted disk is evidenced by the weakening Balmer line profiles at that time.

Using Hα observations spanning from 1994 August to 2009 February, Nemravová et al. (2010) argue that the changes in the spectrum cannot be attributed to disk precession, but rather result from physical changes to the disk’s structure. Their assertion is that the full width at half maximum (FWHM) of the Hα line should increase as the precessing disk becomes more highly inclined. However, they find the FWHM of their Hα observations decreases over a period of 15 years before the recent Be to Be-shell transition occurred, which they believe indicates physical changes to the circumstellar disk instead of a geometrical effect such as disk precession. Between 2006 and 2007 the FWHM of their Hα observations rapidly increased, and they suggest that a new disk has formed.

The goal of this work is to find a Be star disk model that describes the physical and geometrical state of Pleione’s disk and evaluate a precessing disk model as constrained by Hα spectra, V-band photometry and optical polarimetry. These observations are described along with archival UV through IR continuum observations, as well as photometry and Balmer series spectra corresponding to a diskless phase in Section 3.2. Section 3.3 outlines our method for determining the star’s parameters. In Section 3.4, we describe our modelling procedures and the best-fit disk models. We also evaluate a precessing disk model and then propose an ad-hoc model that fits all observables based on our findings. Section 3.5 provides a comparison of our results with the literature and a discussion of future work.

### 3.2 Observations

#### 3.2.1 Spectroscopy

##### 3.2.1.1 Be and Be-shell Phase Spectra

Hα spectra were obtained at the Lowell Observatory, in Flagstaff, AZ, USA, using a fiber-fed echelle spectrograph connected to the 1.1 m John S. Hall telescope. The properties of the spec-
3.2. Observations

Figure 3.1: Hα line profile of Pleione observed at Lowell Observatory between 2005 August and 2019 December. The three epochs are differentiated by colour: the Be phase (blue), the transitioning phase (yellow), and the Be-shell phase (red).

trograph are outlined in Hall & Lockwood (1995) and the spectroscopic reduction steps used to extract the spectra from echelle orders are described in Hall et al. (1994). We have acquired 31 spectra with this instrument from 2005 to 2019 at a resolving power of 10,000 in the Hα region. These observations are available online in a machine readable format (Table 3.1).

The Hα spectra shown in Figure 3.1 begin during Pleione’s most recent Be phase in 2005 and follow the transition to a Be-shell phase in 2006 and 2007. From 2008 to 2015, the line brightened as the ratio of the peak flux over the continuum flux \( F/F_c \) increased from \( \sim 2 \) to \( 2.5 \). During 2015, the system’s Hα equivalent width (EW) dropped by \( \sim 20 \% \). From 2016 until our latest observations in 2019, the line profile has continued to brighten to a maximum of \( F/F_c \approx 3.5 \).

Table 3.2 summarizes the characteristics of each spectrum. In particular, the peak flux of the Hα profile was on average \( F/F_c \approx 5.2 \) from 2005 August to 2006 March. After the transition from the Be to Be shell phase between 2006 March and 2006 December, the peak flux had decreased to a minimum of \( \sim 1.8 \). By 2013, the peak flux increased to \( \sim 2.8 \), and, despite a small decrease in 2015, continued to increase to \( \sim 3.5 \) by 2019 December.

The violet (V) and red (R) peaks, measured as the maximum flux values within each peak, had the greatest difference during the Be phase, between 2005 August and 2006 February, with an average ratio of \( V/R \approx 0.87 \). Afterward, during the shell phase, the \( V/R \) ratio remained closer to unity.

The EW was relatively constant between 2005 August and 2006 March. Between 2006 March
Table 3.1: Hα observations of Pleione from the Lowell Observatory. The 31 spectra are identified chronologically by spectrum ID number. The full table is available online in machine-readable form in the published version of this manuscript.

<table>
<thead>
<tr>
<th>Spectrum ID Number</th>
<th>MJD (+2400000.5)</th>
<th>λ [nm]</th>
<th>F/F_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>53604</td>
<td>648.183655</td>
<td>1.005251</td>
</tr>
<tr>
<td>1</td>
<td>53604</td>
<td>648.207947</td>
<td>1.036522</td>
</tr>
<tr>
<td>1</td>
<td>53604</td>
<td>648.232300</td>
<td>0.971167</td>
</tr>
</tbody>
</table>

and 2006 December, during the transition from the Be to Be-shell phase, the Hα EW decreased by ~ 1.1 nm. The EW increased by ~ 0.5 nm between 2008 and 2013, and then decreased by ~ 0.3 nm by the end of 2015. Since then, the EW has increased by ~ 0.9 nm to ~ 1.7 nm.

Archival Hα spectra of Pleione from the Be Star Spectra Database were also acquired to track the change in Hα EW over time. We used 419 spectra from 2007 to 2021 observed by amateur astronomers Ernst Pollmann (Pollmann, 2020) and Joan Guarro i Fló. We also used Hα EW observations from Hirata & Kogure (1976); Hirata (1995, 2007), which collectively span from 1953 to 2004. Each of these measurements are presented alongside our own for comparison to our precessing disk model in Section 3.4.

We obtained 70 ultraviolet (UV) spectra, observed by the International Ultraviolet Explorer (IUE) between 1979 July and 1995 March, from the INES database (González-Riestra et al., 2001) following the selection procedure described by Freire Ferrero et al. (2012). These observations were taken with the large aperture and high dispersion settings on both the short-wavelength (1,150 – 2,000 Å) and long-wavelength (1,850 – 3,300 Å) spectrographs to ensure proper flux calibration and a high spectral resolution of approximately 0.2 Å (ESA, 2000). We chose to remove the IUE data beyond 0.3 µm due to instrumental limitations that cause significant uncertainty (González-Riestra et al., 2001). Figure 3.2 illustrates how the UV flux steadily increased by a factor of ~ 3 to 4 during the time-frame of the IUE observations. How this affects our determination of the stellar parameters is discussed in Section 3.3.

3.2.1.2 Digitization of Diskless Spectra

We acquired four Balmer series spectra of Pleione during its last diskless phase (1906 to 1938 (Frost, 1906; McLaughlin, 1938)) from the Harvard Astronomical Plate Collection. These spectra were recorded on photographic plates in 1927 at the Cambridge Observatory, and contain only photospheric flux unaffected by the presence of a circumstellar disk. These spectra were obtained using low-resolution dispersion objective prisms on a 24-inch Clark reflector, with a 0.6 m aperture at a scale of 59.6 arcsec/mm. These photographic plates were previously unavailable in a digital format.

High-resolution photographs were taken of the emulsion side of plates designated as H02957, H02958, and H02959, according to classification system of the Harvard Astronomical Plate Collection. Figure 3.3 shows the averaged spectra acquired from each plate, with two spectra labelled (a) and (b) coming from plate H02957. Table 3.3 provides the observational details.
Table 3.2: Hα emission line profile characteristics of Pleione.

<table>
<thead>
<tr>
<th>Date</th>
<th>MJD (+2400000.5)</th>
<th>EW [nm]</th>
<th>Peak Flux</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005 Aug 22</td>
<td>53604</td>
<td>-2.571</td>
<td>5.081</td>
<td>0.861</td>
</tr>
<tr>
<td>2005 Sep 16</td>
<td>53629</td>
<td>-2.691</td>
<td>5.296</td>
<td>0.867</td>
</tr>
<tr>
<td>2005 Sep 17</td>
<td>53630</td>
<td>-2.691</td>
<td>5.353</td>
<td>0.850</td>
</tr>
<tr>
<td>2005 Oct 11</td>
<td>53654</td>
<td>-2.658</td>
<td>5.291</td>
<td>0.871</td>
</tr>
<tr>
<td>2005 Nov 18</td>
<td>53692</td>
<td>-2.619</td>
<td>5.298</td>
<td>0.871</td>
</tr>
<tr>
<td>2005 Dec 20</td>
<td>53724</td>
<td>-2.620</td>
<td>5.215</td>
<td>0.896</td>
</tr>
<tr>
<td>2006 Jan 14</td>
<td>53749</td>
<td>-2.630</td>
<td>5.308</td>
<td>0.878</td>
</tr>
<tr>
<td>2006 Jan 24</td>
<td>53759</td>
<td>-2.562</td>
<td>5.122</td>
<td>0.874</td>
</tr>
<tr>
<td>2006 Feb 07</td>
<td>53773</td>
<td>-2.650</td>
<td>5.355</td>
<td>0.871</td>
</tr>
<tr>
<td>2006 Mar 17</td>
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<td>-2.563</td>
<td>5.198</td>
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<tr>
<td>2006 Dec 09</td>
<td>54078</td>
<td>-1.458</td>
<td>3.484</td>
<td>0.979</td>
</tr>
<tr>
<td>2007 Jan 26</td>
<td>54126</td>
<td>-1.209</td>
<td>3.021</td>
<td>1.050</td>
</tr>
<tr>
<td>2007 Feb 03</td>
<td>54134</td>
<td>-1.160</td>
<td>2.870</td>
<td>1.012</td>
</tr>
<tr>
<td>2007 Dec 18</td>
<td>54452</td>
<td>-0.621</td>
<td>1.833</td>
<td>0.952</td>
</tr>
<tr>
<td>2008 Nov 14</td>
<td>54784</td>
<td>-0.676</td>
<td>1.955</td>
<td>0.994</td>
</tr>
<tr>
<td>2008 Nov 15</td>
<td>54785</td>
<td>-0.704</td>
<td>1.985</td>
<td>0.985</td>
</tr>
<tr>
<td>2009 Dec 06</td>
<td>54994</td>
<td>-0.825</td>
<td>2.180</td>
<td>1.005</td>
</tr>
<tr>
<td>2013 Dec 10</td>
<td>56636</td>
<td>-1.174</td>
<td>2.804</td>
<td>0.916</td>
</tr>
<tr>
<td>2013 Dec 14</td>
<td>56636</td>
<td>-1.194</td>
<td>2.811</td>
<td>0.913</td>
</tr>
<tr>
<td>2013 Dec 17</td>
<td>56637</td>
<td>-1.160</td>
<td>2.824</td>
<td>0.900</td>
</tr>
<tr>
<td>2015 Apr 01</td>
<td>57113</td>
<td>-0.941</td>
<td>2.725</td>
<td>1.052</td>
</tr>
<tr>
<td>2015 Apr 01</td>
<td>57113</td>
<td>-0.928</td>
<td>2.721</td>
<td>1.055</td>
</tr>
<tr>
<td>2015 Dec 13</td>
<td>57369</td>
<td>-0.860</td>
<td>2.656</td>
<td>1.083</td>
</tr>
<tr>
<td>2015 Dec 14</td>
<td>57370</td>
<td>-0.862</td>
<td>2.661</td>
<td>1.084</td>
</tr>
<tr>
<td>2017 Dec 20</td>
<td>58107</td>
<td>-1.097</td>
<td>2.842</td>
<td>1.064</td>
</tr>
<tr>
<td>2017 Dec 20</td>
<td>58107</td>
<td>-1.100</td>
<td>2.854</td>
<td>1.067</td>
</tr>
<tr>
<td>2017 Dec 21</td>
<td>58107</td>
<td>-1.097</td>
<td>2.854</td>
<td>1.072</td>
</tr>
<tr>
<td>2018 Dec 19</td>
<td>58471</td>
<td>-1.325</td>
<td>3.053</td>
<td>1.044</td>
</tr>
<tr>
<td>2018 Dec 20</td>
<td>58472</td>
<td>-1.381</td>
<td>3.132</td>
<td>1.058</td>
</tr>
<tr>
<td>2019 Dec 18</td>
<td>58835</td>
<td>-1.748</td>
<td>3.560</td>
<td>0.982</td>
</tr>
<tr>
<td>2019 Dec 18</td>
<td>58835</td>
<td>-1.795</td>
<td>3.582</td>
<td>0.982</td>
</tr>
<tr>
<td>2019 Dec 19</td>
<td>58836</td>
<td>-1.723</td>
<td>3.517</td>
<td>0.986</td>
</tr>
<tr>
<td>2019 Dec 19</td>
<td>58836</td>
<td>-1.718</td>
<td>3.506</td>
<td>0.991</td>
</tr>
</tbody>
</table>
The Role of Disk Tearing and Precession in the Observed Variability of Pleione

The spectra were extracted from the photographs by the following method. After correcting for tilt on the plate, we averaged the logarithmic intensity at each wavelength across the vertical axis of the spectrum, effectively creating a time-averaged spectrum across the duration of the exposure. The spectra were wavelength-calibrated at the center of the H\(\alpha\) and H\(\delta\) lines, and sampled at every 1 Å from 7000 Å to 3990 Å. The H\(\epsilon\) line was not included in our data set due to low signal-to-noise ratio. Figure 3.4 shows the final H\(\alpha\), H\(\beta\), H\(\gamma\) and H\(\delta\) spectra, where each Balmer series line has been averaged across the four observations.

To acquire the relative fluxes from each photographic plate we required the non-linear response curve of each emulsion as a function of wavelength. Since the photographic plates also have other Pleiades cluster stars, we were able to extract the response curve from the star Atlas (HD 23850, 27 Tau), a B8III spectral-type star with no reports of H\(\alpha\) variability in the literature over the past 100 years. Atlas lies near Pleione on the plane of the sky, and at a similar...
Figure 3.3: Photographs of Balmer series observations of Pleione taken on photographic plates in 1927 at the Cambridge Observatory. Each spectrum contains $\text{H}\alpha$, $\text{H}\beta$, $\text{H}\gamma$, and $\text{H}\delta$ lines in order from left to right. These line profiles were obtained during Pleione’s last diskless phase. The plates are labelled for each spectrum by their designation in the Harvard Astronomical Plate Collection.
Figure 3.4: Hydrogen Balmer series line profiles digitized from photographic observations shown in Figure 3.3. These line profiles are an average of the matching profiles across each of the four photographic plate spectra. The process used for combining the different exposures of the observed spectra allows for high signal-to-noise ratio, leaving relatively little noise in the line profiles. The flux of the Hβ, Hγ and Hδ lines has been offset for ease of viewing.

distance (387 ly for Pleione, 422 ly for Atlas, van Leeuwen 2007). The response curve was then reconstructed by equating Atlas’ spectrum from each plate to a 2004 February spectrum acquired from the ELODIE archive (Moultaka et al., 2004). The spectra for Pleione were then extracted, and transformed as a function of wavelength according to the response curve. Figure 3.4 shows our final Hα, Hβ, Hγ and Hδ spectra which have been averaged across the four plates. We note that Atlas is brighter than Pleione (3.5 mag vs 5.0 mag in the V-band, respectively, Ducati 2002). Therefore, for our method of obtaining the fluxes, we are assuming that both stars are appropriately exposed such that the main spectral features have photographic densities in the linear part of the response curve.

3.2.2 Photometry

Pleione has been known to be photometrically variable since 1936 (Binnendijk, 1949). V-band photometric observations from 1980 to 2010 were compiled from the following publications: Sharov & Lyutyi (1988); Hirata & Kogure (1976, 1977); Hopp & Witzigmann (1980); Hopp et al. (1982); Dapergolas et al. (1981); Bohme (1984, 1985, 1986); Boehme (1988); Sharov
3.3. Stellar Parameters

Archival observations were acquired from the Hipparcos (van Leeuwen, 2007) mission and the ASAS-3 (Pojmanski, 1997) telescope archive. Visible and infrared (IR) flux observations were obtained from the CDS Portal application from the Université de Strasbourg. The observations were compiled from catalogs which listed target objects within 0.5 arcsec of Pleione’s position. The catalogs included observations from 1990 to 2010, covering both the most recent Be and Be-shell phases. This data was sourced from Stauffer et al. (2007), Egan et al. (2003) and the following missions: GAIA (Gaia Collaboration et al., 2020), 2MASS (Skrutskie et al., 2006), WISE (Wright et al., 2010), AKARI (Murakami et al., 2007), IRAS (Neugebauer et al., 1984), and Spitzer (Werner et al., 2004). These observations and the V-band photometry are presented for comparison against our precessing disk model in Section 3.4.

Photographic magnitudes obtained during Pleione’s last diskless epoch were also obtained. In 1918, Parsons (1918) reported that Pleione had a photo-visual magnitude of 5.15 mag, and a Müller and Kempf visual magnitude of 5.08 mag. In 1922, Lindblad (1922) also reported a photo-visual magnitude of 5.15 mag. These magnitudes were used along with UV spectra to constrain Pleione’s stellar parameters in Section 3.3.

3.2.3 Polarimetry

We acquired observations of the optical (BVRI) linear polarization from 2010 to 2021 using the IAGPOL polarimeter at the Pico dos Dias Observatory (OPD), operated by the National Astrophysical Laboratory of Brazil (LNA) in Minas Gerais, Brazil. These observations were reduced with packages developed by the Beacon group\(^2\), and described in Magalhaes et al. (1984, 1996) and Carciofi et al. (2007).

Archival polarization data of Pleione in the V-band were acquired from Hirata (2007), who regularly observed the star from 1975 to 2004. Observations of the V-band polarization were also acquired from the archive for the Lyot Spectropolarimeter\(^3\) and the Halfwave Spectropolarimeter (HPOL) at the University of Wisconsin-Madison Pine Bluff Observatory, which were reduced by Draper et al. (2014). Additional polarimetric observations were extracted from Hirata (2007) using the WebPlotDigitizer tool from Rohatgi (2020). These archival data and our observations are presented in comparison to the percent polarization and polarization position angles of our precessing disk model in our results (Section 3.4).

3.3 Stellar Parameters

We used IUE UV continuum observations and diskless photographic and photo-visual band photometry from Parsons (1918) and Lindblad (1922) to determine Pleione’s stellar parameters using Monte Carlo methods, using the same procedure as Marr et al. (2021). We determined the stellar mass \(M\), critical fraction of rotation \(W\) (as defined in equation 6 of Rivinius et al., 2013), age \(t/t_{\text{ms}}\) (where \(t_{\text{ms}}\) is the main sequence lifetime, and is related to the fraction of hydrogen

\(^2\)http://beacon.iag.usp.br/

\(^3\)http://www.sal.wisc.edu/PBO/LYOT/
Table 3.4: The best-fitting stellar parameters for Pleione computed with emcee.

<table>
<thead>
<tr>
<th>Best-Fit Parameters</th>
<th>Values</th>
<th>Derived Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ [M$_\odot$]</td>
<td>4.1$^{+0.2}_{-0.2}$</td>
<td>$L$ [L$_\odot$]</td>
<td>380$^{+50}_{-50}$</td>
</tr>
<tr>
<td>$W$</td>
<td>0.8$^{+0.1}_{-0.1}$</td>
<td>$T_{\text{eff}}$ [K]</td>
<td>14000$^{+100}_{-100}$</td>
</tr>
<tr>
<td>$t/t_{\text{ms}}$</td>
<td>0.7$^{+0.1}_{-0.1}$</td>
<td>log $g$</td>
<td>4.0$^{+0.1}_{-0.1}$</td>
</tr>
<tr>
<td>$i$ [°]</td>
<td>61$^{+9}_{-8}$</td>
<td>$R_{\text{pole}}$ [R$_\odot$]</td>
<td>3.3$^{+0.3}_{-0.3}$</td>
</tr>
<tr>
<td>$d$ [pc]</td>
<td>134$^{+5}_{-5}$</td>
<td>$R_{\text{eq}}$ [R$_\odot$]</td>
<td>4.4$^{+0.6}_{-0.5}$</td>
</tr>
<tr>
<td>$E(B - V)$</td>
<td>0.09$^{+0.02}_{-0.02}$</td>
<td>$R_{\text{eq}}/R_{\text{pole}}$</td>
<td>1.3$^{+0.1}_{-0.1}$</td>
</tr>
</tbody>
</table>

Table 3.5: The mass, critical fraction of rotation and age used for the BeAtlas grid of models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Grid Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$ [M$_\odot$]</td>
<td>1.7, 2, 2.5, 3, 4, 5, 7, 9, 12, 15, 20</td>
</tr>
<tr>
<td>$W$</td>
<td>0.00, 0.33, 0.47, 0.57, 0.66, 0.74, 0.81, 0.93, 0.99</td>
</tr>
<tr>
<td>$t/t_{\text{ms}}$</td>
<td>0.25, 0.5, 0.75, 1, 1.01, 1.02</td>
</tr>
</tbody>
</table>

Table 3.6: Adopted stellar parameters used as priors in emcee when fitting to the BeAtlas grid.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallax [mas]</td>
<td>7.24 ± 0.13</td>
<td>Gaia Collaboration et al. (2020)</td>
</tr>
<tr>
<td>$v\sin(i)$ [km/s]</td>
<td>286 ± 16</td>
<td>Frémat et al. (2005)</td>
</tr>
<tr>
<td>$i$ [°]</td>
<td>60 ± 10</td>
<td>Hirata (2007)</td>
</tr>
</tbody>
</table>

remaining in the stellar core), inclination $i$, distance $d$, and the degree of interstellar reddening $E(B - V)$. Additional parameters, listed in Table 3.4, were self-consistently derived.

Overall, a grid of 770 diskless Be star model spectra, called BeAtlas (Mota, 2019), were fit to the observations. The models were computed using the 3D non-LTE radiative transfer code hdust (Carciofi & Bjorkman, 2006). hdust computes model spectra from given stellar parameters, including $M$ and $W$, as well as the polar radius ($R_{\text{pole}}$), luminosity ($L$) and gravity darkening exponent ($\beta$). The values of $R_{\text{pole}}$, $L$ and $\beta$ are correlated to $M$ and $W$ through the stellar evolutionary models of Georgy et al. (2013). Table 3.5 lists the grid size and spacing. The excess flux due to interstellar reddening, $E(B - V)$, was also included as a free parameter, with its effect on the simulated spectrum being determined using the Fitzpatrick (1999) prescription.

We used the Markov Chain Monte Carlo (MCMC) routine emcee by Foreman-Mackey et al. (2013) to determine which models from the BeAtlas grid best reproduce the observed UV spectrum using hdust. This MCMC routine was used to generate a list of stellar parameters inside pre-determined ranges that were weighted using literature values for parallax, $v\sin(i)$, and $i$ (Table 3.6), and then the goodness of fit for each model was computed based on the fit of the resulting continuum to the observation using a log($\chi^2$) likelihood function. The fitting procedure reached convergence using 30 walkers, and 50,000 steps, with a burn-in of 5,000 steps, which were chosen following the guidelines of Foreman-Mackey et al. (2013). Further details on the emcee fitting process are given in Mota (2019).
3.3. Stellar Parameters

The probability density functions of each parameter are shown on the main diagonal axis while the intersection for each parameter shows the correlation map. The six parameters used in the fitting procedure are the stellar mass \( M \), the rotation rate \( W \), time of life on the main sequence \( t/t_{\text{ms}} \), stellar inclination \( i \), distance \( d \), and interstellar reddening \( E(B-V) \). The subfigure in the top-right corner shows the most probable fit model to the UV spectra and diskless visible magnitudes, and the residuals below.

We quantified the impact of the variable UV flux on the stellar parameters by separately fitting the low (1979 July) and high (1995 March) extremes (recall Figure 3.2), along with the diskless visible photometry. We found \( M \) in the range 3.98 to 4.23 \( M_\odot \) and \( E(B-V) \) from 0.07 to 0.10 mag, while the other parameters did not change. By averaging the UV observations we
find a set of stellar parameters which reproduce the UV and visible spectrum with $\chi^2_\nu = 1.65$. The errors on these parameters include the ranges of $M$ and $E(B-V)$ from the UV extremes.

Figure 3.5 depicts the stellar parameters when fitting to the averaged UV spectra and diskless visible photometry. The probability density functions (PDF) for each stellar parameter are shown along the main diagonal as histograms, and the intersections of the parameters show the corresponding correlation maps. The width of the PDF indicates how well the parameter is constrained, while the center dashed line on each histogram shows the most probable value, and the dashed lines on the left and right of center indicate the first and third quartiles. In the upper right corner of Figure 3.5 the SED is shown along with the best-fit model. Below the SED, the residuals between the observations and model are shown. The best-fit stellar parameters are summarized in Table 3.4.

As a final check of our best-fit stellar characteristics, we compared the model stellar absorption profiles to what would be expected for late B-type stars, since in the literature the spectral type of B8 V is often applied to Pleione (e.g., Gulliver, 1977; Abt & Levato, 1978). Specifically, we compared the synthetic hydrogen Balmer series spectra to model profiles calculated with hdust based on standard B7 V, B8 V and B9 V star parameters from Cox (2000). Figure 3.6 shows our best-fit diskless model most closely resembles the B8 V hydrogen line profiles.
3.4 Disk Modelling

3.4.1 A Precessing Disk Model

We computed a grid of 94,720 Be star and disk models using hdust, to model the changes observed in the Hα emission. This grid consists of axisymmetric, non-isothermal, thin-disk envelopes. The vertical disk density distributions are Gaussian, while the radial distributions follow a power-law of the form $\rho(r) \propto \rho_0 r^{-n}$, where $\rho_0$ is the density at the base of the disk ($r = R_{eq}$), and $n$ is the density falloff exponent. Values of $\rho_0$ were explored in the range of $1 \times 10^{-13}$ to $1 \times 10^{-10}$ g cm$^{-3}$ for every tenth of a magnitude, and the density falloff exponent $n$ over the range of 2.0 to 3.5 in steps of 0.1. Each model was computed with the outer disk radius $R_{out}$ at 5, 10, 15, 20, 25, 50, 75 and 100 stellar equatorial radii ($R_{eq}$), and using inclinations from
Figure 3.8: Change in the observed V-band apparent magnitude for different system inclinations, using the models with the best-fit stellar and disk parameters. Shown are a diskless star at different inclinations (triangles), a star and untilted disk which incline together (open circles), and a star fixed at 60° while the disk inclination changes (filled circles).

0 to 90° in steps of 2.5°.

The line broadening effect due to non-coherent electron scattering within the disk (Auer & Mihalas, 1968; Poeckert & Marlborough, 1979) was approximated as it is not accounted for in H\textsc{dust} simulations. For each synthetic profile, a fraction ($F_e$) of the synthetic H\textalpha{} flux (where $F_e$ ranged from 0.2 to 0.4) was convolved with a Gaussian of width equal to the electron velocity $v_e$ (where $v_e$ ranged from 400 to 800 km s$^{-1}$). The remaining $1 - F_e$ of the flux was left unBroadened. Through MCMC fitting with \texttt{emcee}, $F_e$ and $v_e$ were treated as free parameters within the given ranges, to determine the best possible fit for each model to the observed H\textalpha{} spectra (see Marr et al., 2021, for more details on this procedure).

We find that the observed trend of the H\textalpha{} profiles from 2005 to 2019 (recall Figure 3.1) can be reproduced by a single disk model by varying the inclination. Our models were limited to axisymmetric disks in the stellar equatorial plane, which when viewed at different inclinations require the star to be inclined at the same angle as the disk. To account for tilting, the absolute flux of the diskless model at the appropriate $i$ was subtracted from the star and disk model at the same $i$ to obtain flux for the inclined disk. The inclined disk flux was then added to the flux of the diskless model at $i_{\text{star}} = 60°$. We believe this first-order correction captures the most
### Table 3.7: Best-fit disk parameters obtained for each H\(\alpha\) line emission profile.

<table>
<thead>
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<th>Date</th>
<th>MJD</th>
<th>(i) [°]</th>
<th>(R_{\text{out}}) [(R_{\text{eq}})]</th>
<th>(F_e)</th>
<th>(v_e) [km s(^{-1})]</th>
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significant effect on the change in flux by the tilting of the disk. This correction was applied across the entire spectrum, and affected models at low $i$ values most. For example, at a disk inclination of $i = 0^\circ$, correcting for the stellar component to be at $i = 60^\circ$ (instead of $i = 0^\circ$) caused an increase in normalized Hα peak flux of less than 40% for all models, and at $i = 30^\circ$ the effect was less than 20%. In the V-band, this correction decreased the continuum level by $\sim 0.2$ mag at $i = 0^\circ$ models, $\sim 0.15$ mag at $i = 30^\circ$, and increased by $\sim 0.05$ at $i = 75^\circ$ (see Figure 3.8).

Our best-fit model to the Hα observations has density $\rho_0 = 3 \times 10^{-11}$ g cm$^{-3}$ and $n = 2.7$, an outer disk radius of $R_{\text{out}} = 100 R_{\text{eq}}$, and a range of inclinations from 30$^\circ$ to 80$^\circ$. This single density model suggests that the mass-loss rate from the star was constant during this period. We note that for the models with radii of 15 $R_{\text{eq}}$ and larger, the improvements to the $\chi^2$ fit were not statistically significant. Therefore, we quote a lower limit of 15 $R_{\text{eq}}$ for the Hα emitting region, while its real size remains undetermined.

Figure 3.9 shows a sample of three Hα profiles, chosen to illustrate how well the best-fit models match the data. The other fits to the observations that are not shown were of similar quality. The non-coherent electron scattering in the best-fit models has values of $F_e = 0.33 \pm 0.04$ and $v_e = 635\pm25$ km s$^{-1}$. This electron velocity corresponds to a temperature of $\sim 8900$ K, which is similar to the disk temperature of $\sim 9000$ K at the outer edge of the Hα emitting region given by $h\text{Dust}$ models, and the isothermal disk temperature of $\sim 8300$ K (60% of $T_{\text{eff}}$, Carciofi & Bjorkman 2006). Our method of accounting for the broadening due to electron scattering more strongly affects the width of Hα profiles formed at lower disk inclinations. As a result, profiles at different inclinations can have the same width, while having different strengths. The models fit with reduced $\chi^2$ values ranging from 1.33 to 3.82. Table 3.7 summarizes the best-fit model disk parameters obtained from each Hα emission line observation.

During the recent Be to Be-shell transition, Pleione exhibited a rapid drop in the visible and IR continuum flux (Tanaka et al., 2007). In Figure 3.10, we show that our best-fit model
reproduces this drop in IR flux when considering the minimum (30°) and maximum (80°) inclinations found. The best-fit model to the 2005 September 16 Hα profile fits the Be phase SED with reduced $\chi^2 = 1.93$, while the best-fit model to the 2015 December 14 observation fits the Be-shell SED with a reduced $\chi^2 = 1.65$. The UV continuum changed by less than 2% between the phases (i.e., within the error ranges). The predicted radio spectra of the simulations are shown for reference, however no radio continuum observations were available for comparison.

The inclinations of our best-fit model are shown in Figure 3.11 at the time of each observation. We find that over the course of our observations the disk’s inclination changes at a rate of $\sim 3.7°$ per year, and was once again viewed edge-on in 2021 November.

To investigate the scenario of a precessing disk as proposed by Hirata (2007), we created a disk model by adopting equation 1 of Dunn et al. (2006), which describes the motion of a precessing galactic disk. This equation determines the inclination of the disk when provided with two values: $\gamma$, the angle between the precession axis and the observer’s line of sight, and $\delta$, the angle between the disk’s normal and the precession axis. We used MCMC fitting with emcee to determine the values of $\gamma$ and $\delta$ that best reproduced the inclinations determined through Hα fitting. We note that $\gamma$ is not fixed to the stellar spin-axis. This is a reasonable assumption since the dynamical simulations of show that the precession axis of a misaligned circumstellar disk lies normal to the orbital plane of the companion and is not coupled to the stellar spin axis. Since we do not know the value of the orbital inclination for Pleione, it is reasonable to regard the precession axis as a free parameter. The timing of the minimum inclination and the period of the precession were also fitted parameters. We find the best-fit precession model has $\gamma = 84 \pm 3°$ and $\delta = 59 \pm 3°$, with the minimum inclination of 25° occurring in 2001 May, the maximum inclination of 144° in 2040 September, and the precession period of 29400 ± 100 days, or $\sim 80.5$ years. This precessing disk model is shown in Figure 3.11, with the grey band indicating the uncertainty. However, since only a portion of the precession period is sampled by our observed Hα profiles, the errors we find through this MCMC analysis are likely a lower bound.

The variation in Hα EW with disk inclination is shown for the precessing disk model in Figure 3.12. Here, the Hα EW is greatest at $-2.9$ Å when the disk inclination is 0°. A secondary peak at $-2.6$ Å occurs around 90°. Dramatic changes to the EW occur around particular inclinations. From 35° to 40°, the EW decreases from $-2.5$ Å to $-0.6$ Å as the line transitions to a Be-shell profile. Beyond 40°, the line has a shell profile and the peak flux increases with the inclination while also becoming narrower. At higher inclinations as the disk becomes edge-on, the stellar continuum emission falls more rapidly, resulting in increases in our normalized Hα spectra and the corresponding EW. This effect is illustrated in Figure 3.13. The Figure shows the Hα spectrum for different inclinations with the star fixed at 60°, prior to normalization and the convolution process. Here, we can see the continuum flux around Hα in absolute units decreases more rapidly at high disk inclinations.

We extrapolated the precessing disk model inclinations over 120 years, from 1930 to 2050. The left side of Figure 3.14 shows the inclinations of the extrapolated model, along with the Hα EW, V-band photometry, V-band polarization level, and the V-band polarization position angle of our best-fit model at the inclinations of the precessing disk model (the right side of this Figure is discussed in Section 3.4.2). The observations previously described in Section 3.2 are overplotted for comparison, as well as the inclinations that Hirata (2007) derived from their model. As our disk models are axisymmetric, the observables produced are symmetric about
In the second panel on the left side of Figure 3.14, we show the trend of the Hα EW of our precessing disk model, which is limited to the inclinations determined from the line profile fitting. As the inclination range is > 25°, the maximum EW observed is ~2.8 Å. Note that in this Figure the grey shaded region is the same as what is shown in Figure 3.12 to the right of the vertical dashed line. Overall, the trends in the Hα EW, are mirrored about 90° due to the symmetry of the precessing disk.

From 2005 to 2019, the best-fit tilted disk model fits our Hα observations with reduced $\chi^2 = 1.6$. Outside of this period, our models cannot reproduce the observed trend using the ~ 80.5 year precession period. We note that while a shorter period would improve the timing of the fit to the previous Be-shell phase in the 1970s, it also would make the model progress through its Hα EW trend faster than what is observed.

In the third panel on the left side of Figure 3.14, we see our precessing disk model is able to reproduce the gradually increasing V-band magnitude, from 1988 to 2004. In 2004, the model reaches an average observed magnitude of ~ 5.02 mag, before the model and observations diverge in 2007 when the magnitude was observed to rapidly drop by 0.3 mag. During this time, our precessing disk model takes ~ 14 years to decline to this value. The minimum brightness of our model is ~ 5.38 mag, which occurs in 2021. As the rapid Be to Be-shell transition occurs every ~ 34 years while half of the precession period is ~ 40 years, we see that the previous minimum in brightness occurs in 1982, just ~ 8 years after the observed minimum.

The polarization level of our best-fit model is shown in the fourth panel of the left side of Figure 3.14. The maximum polarization level of ~ 0.53% is observed at ~ 70°. Projecting our best-fit models forward in time along inclination curve produces the polarization signature of a tilted disk, similar to those shown in Marr et al. (2018). The polarization from the model fits the observations with reduced $\chi^2 = 1.7$.

To get the polarization position angle of our precessing disk model, we used equation 2 from Dunn et al. (2006) to first extract the angle between the precession axis of the star projected onto the sky and the normal of the disk projected onto the sky. This angle has a constant offset from the polarization position angle, which is the angle between north on the sky and the projected precession axis of the star. We find this angle is ~ 115° to align our best-fit precession model with the observed, intrinsic polarization position angle. The bottom panel of Figure 3.14 shows the V-band polarization position angle of our precessing disk model compared to the observed polarization position angle after correction for interstellar polarization (following the same method as described in Marr et al. 2021). The minimum polarization position angle of our model is ~ 55.8° in 1983, and the maximum is 174.2° in 2021.

Like the Hα EW and V-band photometry, the polarization position angle was also observed to rapidly drop in 2007. Prior to this, our precessing disk model closely follows the observed trend, and fits with a reduced $\chi^2 = 1.4$. After the rapid drop our model is unable to reproduce the observations.

### 3.4.2 An Ad-hoc Disk Tearing Model

We find that at some times Pleione’s Hα emitting region must be at a different inclination than the V-band emitting region. For example, the precessing disk model shows that while a single
disk can reproduce the rapid drop in Hα EW in 2007, the V-band photometry and polarization position angle do not drop at the same time. Based on recent results of Suffak et al. (2022), we propose an ad-hoc model to reproduce the observed trends. These authors investigate the effects that companion stars have on the dynamics of circumstellar disks. They show that in a Be star-binary system with a 30-day orbital period, where the companion is misaligned by 40° from the Be star equatorial plane, the disk can separate into two parts with different inclinations while mass is constantly ejected from the stellar surface. In their simulations, the disk periodically tears and merges back into one disk every 30 orbital periods. While their models consider different mass-loss rates, they find that disk tearing only occurs in active phases, i.e., when mass-loss is on. Their results also suggest that disk tearing can occur in a variety of different Be star-binary systems with different masses or orbital periods. Previously, a similar phenomenon was modelled in protoplanetary disks of triple-star systems by Kraus et al. (2020), who showed the disk around the primary can break into multiple precessing rings aligned with the companion’s orbital planes.

We define our ad-hoc model with two distinct stages which occur in each cycle from a Be-shell phase to Be phase and back. Figure 3.15 illustrates these stages, with a) and b) corresponding to the first stage, and c) and d) to the second stage. In essence, the first stage is a single disk model which tilts off-axis, and the second stage is a two-disk model with one disk anchored to the stellar equator while the other disk is free to precess.

In the first stage, the disk is whole and fixed to the stellar equator. Owing to the companion’s tidal torque, it becomes tilted away from the midplane by $i \approx 30°$ (this tilting has also been shown to occur in the simulations of Suffak et al. 2022 and previously by Cyr et al. 2017). Therefore, in this phase the Hα emitting region – which corresponds to the entire disk, see Carciofi (2011) – and the inner disk – where the V-band flux excess and the optical polarimetry originate – are approximately aligned.

The second stage begins when the disk is torn into two parts. Now, as shown by the simulations of Suffak et al. (2022) and previously outlined by Okazaki (2017), a gap appears between the small inner disk, which is still anchored to the star, and the outer disk, which starts precessing. Therefore, in this phase the Hα emitting region has an orientation that gradually varies in time. Because in this phase the outer disk is no longer fed by the star, it gradually dissipates as the inner disk grows. As a result, the disk slowly transitions to the configuration of the first stage.

The right side of Figure 3.14 illustrates our ad-hoc model in detail. In the first stage of our model (indicated by dotted lines in the right panel of Figure 3.14), the innermost disk and Hα emitting region have the same inclination as they gradually change from 60° to 30°. The disk tearing events (1938, 1972, 2007) correspond to the phases immediately before the vertical lines in the figure and mark the beginning of the second stage. Observationally, this stage is related to the Be-shell phase, as the outer disk, now detached from the inner disk, starts precessing around the star. The innermost disk is rebuilt at the original inclination of 60° over ~ 1.5 years, with constant mass-injection during this period as suggested from our Hα modelling in Subsection 3.4.1. The Hα emitting region begins to precess from 30° to 90°, with a period of ~ 80.5 years following our best-fit precession model. The precession of the Hα region becomes unstable after ~ 15 years, and the outer disk then gradually dissipates as gas is lost to the interstellar medium or recombines with the innermost disk. As the inner disk grows, it slowly tilts back to $i \approx 30°$. This marks the phase of the first stage again.
It is important to stress that this ad-hoc model of Pleione’s disk addresses the most important observational features seen in the right side of Figure 3.14, namely

- In the first stage, the gradual change in inclination from 60° to 30° of the inner disk explains the rate of change in brightness, the drop in polarization level and the change in the polarization position angle qualitatively.

- Likewise, the sudden change in the inner disk orientation from 60° to 30° once disk tearing has occurred (during the transition from the first stage to the second) also explains the sudden drop in V-band brightness well, and the dramatic change in the polarization position angle (from ~ 120° in 2007 to ~ 50° in 2010).

- The precession model developed in Section 3.4.1 remains valid, as in the beginning of the second stage (e.g., the shaded area in the right side of Figure 3.14) the outer disk is allowed to freely precess with a period of 80.5 yrs.

Our ad hoc model, however, is unable to reproduce the strong increase in Hα EW seen in the second half of the first stage (e.g., between 1990 and 2007 in the right side of Figure 3.14). At this stage the disk becomes gradually more tilted away from the equator and likely also warped (Suffak et al., 2022). The increase in Hα EW might be due to the upper part of the disk, which is closest to the hot stellar poles, becoming hotter and more ionized. This effect, if relevant, is not captured by our simple models that consider axisymmetric and flat disks.

Furthermore, since the density of our disk model was determined solely through Hα line profile fitting, the disk density distribution remains constant through the disk tearing period. Perhaps with this scenario, disk gas could be redistributed within the disk which may alter the value of n at particular snapshots in time at certain radial distances from the central star.

In the models of Suffak et al. (2022), the disk tearing events happen every ~ 30 orbital cycles for a model with an equal-mass binary in a 30 day orbit. Since Pleione’s less massive companion has a 218 day orbital period, this means that more than 57 orbital periods occur before disk tearing every ~ 34 years.

3.5 Discussion and Conclusions

We began our work by modelling the large-scale structure of Pleione’s disk constrained by Hα spectroscopy acquired between 2005 to 2019. During this time, the Hα emission shows that Pleione underwent a transition from a Be phase to a Be-shell phase. We find that one single disk model, with fixed density and size, can reproduce the Hα observations from the above period by simply varying the disk inclination from 30° to 80° at a rate of ~ 3.7°/year. This best-fit disk model is axisymmetric with a base density of \( \rho_0 = 3 \times 10^{-11} \text{ g cm}^{-3} \), density power-law exponent of \( n = 2.7 \), and a minimum outer disk radius of \( R_{\text{out}} = 15 R_{\text{eq}} \).

Our Hα disk size agrees with Nemravová et al. (2010) who found that an upper limit on Pleione’s outer disk radius is set by the periastron separation of 53 \( R_\odot \) (12.1 \( R_{\text{eq}} \) using the value of \( R_{\text{eq}} \) in Table 3.4). However, the outer dimensions of the disk could be restricted by the companion’s Roche radius. We note that the lower limit on the outer disk radius, if it is set by the Roche radius, is ~ 7 \( R_{\text{eq}} \). We note that in the SPH simulations reported by Cyr et al.
(2017), they show in tables 3 and 4 that the truncation radius is larger in systems where the disk and companion’s orbital plane are not aligned. A full dynamic simulation will be required to determine whether the disk could be further truncated by the companion’s Roche lobe.

We fit a precessing disk model to the inclinations determined through Hα fitting, and find the best-fit model has a minimum inclination of 25°, a maximum inclination of 145°, and a period of ~ 80.5 years. We extrapolated this model over 120 years to evaluate the fit to archival observations.

We find that this model reproduces the observed EW from 2001 to 2021, including the rapid drop in brightness in 2007. Our model’s peak Hα EW is ~ −2.8 Å while the maximum observed value is ~ −3.8 Å in ~ 1960 and again in ~ 1997. The observed data from ~ 1970 to ~ 1985 could also be matched if the precessing model were shifted by ~ 8 years, so that i = 30° in 1971.

Our precessing model is unable to follow the gradual rise in Hα EW from ~ 1980 to ~ 1995 leading to the peak value, and the decrease from ~ 1997 to ~ 2001. The V-band magnitude matches from ~ 1945 to ~ 1970 and again from ~ 1990 to ~ 2007, but fits poorly outside of these times because of the rapid drops in 1971 and 2007. Before 1938 Pleione was diskless, so the model is not expected to reproduce the observed trend before then. The V-band polarization position angle fits from ~ 1974 to ~ 2005, and like the V-band magnitude is unable to capture the rapid drop in 2007. We note that the model would reproduce the data if it were to reset to i = 60° in 2007. We also find the V-band percent polarization fits the observations at all times, but the degree of variation is relatively small.

To align the precessing disk model’s polarization position angle to observations, we find that the precession axis of the star must lie at 115° east of north when projected onto the sky. This is in approximate agreement with the value of ~ 122° determined by Hirata (2007) with their model based on observations of the polarization position angle.

We further compared our observed polarization position angles to the interferometric disk normal position angle inferred from interferometric observations. Our values were consistent with those of Touhami et al. (2013) who found a value equivalent to 69° in 2008, and with Cochetti et al. (2019) who found a value equivalent to 63° in 2014 (recall the red dots in the bottom left and right panels of Figure 3.14).

Overall, the precessing model (in which the whole disk precesses with the same rate and orientation) failed. We find this model is unable to reproduce most of Pleione’s observational features.

Inspired by the recent models of Kraus et al. (2020) and Suffak et al. (2022), we created an ad-hoc disk tearing model which incorporates our precessing disk model and explains the large-scale variations of Pleione’s observables, and also seamlessly incorporates the results of Hirata (2007), Tanaka et al. (2007) and Nemravová et al. (2010). In our ad-hoc model, the disk initially is whole and tilted away from the stellar equator. At some point (likely once the disk becomes sufficiently massive), the disk is torn into two parts: an inner part still anchored to the star, having therefore an inclination angle of 60°, and an outer part which is free to precess around the star. What follows is a slow increase in size and density of the inner disk and a gradual dissipation of the outer disk, which is no longer fed by the central star.

Previously, Hirata (2007) and Tanaka et al. (2007) also found that Pleione’s disk inclination changes over time. Using a precessing disk model, Hirata (2007) reproduced the change in polarization position angle across the Be-shell to Be transition in 1989. Using the times between edge-on events, they also found the period of precession to be 80.5 years, the same
value we determined. Building on Hirata (2007)’s model, Tanaka et al. (2007) claimed that the precessing disk had partially re-accreted leading to the rapid drop in brightness in 2007. They also proposed that the sudden appearance of shell lines alongside the drop could be explained by a secondary disk forming in the stellar equatorial plane, which is consistent with our ad-hoc disk tearing model. Nemravová et al. (2010) also explained the appearance of shell lines with the formation of a new disk, and we expand on this below.

Our best-fit disk model shows that during the Be to Be-shell transition, that shell lines appear at inclinations near 40° and greater. We acknowledge that these inclinations are smaller than typically assumed for the appearance of Hα shell lines. Hanuschik (1996) studied the geometry of Be star disks including the range of inclinations typical for shell lines. They find that shell lines most frequently occur for disk inclinations of 75° ± 5°. However, they focused only on changes in geometry and did not consider disk temperature and the associated changes in ionization fraction nor did they consider the density law. Our best-fit model has a disk density slope of \( n = 2.7 \), meaning that the density falls more gradually with increasing radius, allowing for shell lines to appear at lower inclinations. Also, viscous disks flare with radial distance from the central star (see, e.g., Carciofi 2011) resulting in more material further from the equatorial plane with increasing radial distance. Silaj et al. (2014) also showed that shell lines could form at lower inclinations. For example, for the B8 Ve star, 4 Aql, they modelled shell lines at inclinations of 46° with \( n = 2.5 \), and at 43° with \( n = 3.0 \) and 3.5. Furthermore, in Pleione’s tilted disk, the path length through the disk along the line of sight changes over time as the disk inclination varies as well. We also believe that the disk may be warped which could also affect shape of the emission lines although our models only include tilted disks and not warped disks.

We see that when the disk is aligned at the stellar inclination of 61 ± 9°, the model agrees with each of the observables, suggesting that our disk model is well constrained. Hirata (2007) noted that at the onset of a shell phase in 1973, the disk’s inclination was known to be 60° through polarimetric means. From this, they inferred the star’s inclination is also 60°.

We find that our ad-hoc disk tearing model largely reproduces the observed trends of Pleione’s circumstellar environment, and agrees with previous descriptions of the disk from the literature. Some details of our model should be examined in further detail, such as the changing thermal structure with a tilting disk, and the dynamical evolution of the disk considering the companion’s influence. We also noted a number of other minor discrepancies with work in the literature that we now discuss.

While our model reproduces observations by changing inclination only, we see that small changes to the physical parameters and geometry may help the disk model fit the observables better. In particular, our model suffers from the drawback that the disk is axisymmetric and flat, while the geometry of a precessing disk is more likely curved or warped, with possible density enhancements (Martin et al., 2009). Because of this, we did not attempt to reproduce the excess Hα flux at high velocities (> 180 km/s) observed from 2006 December to 2007 February. These bumps (recall the middle panel in Figure 3.9), which were also noted by Nemravová et al. (2010), cause an increase in the EW of the line in the transition to the Be-shell phase. At this time, there is a clear transition where the core of the profile decreases while the flux in the wings of the profile increases. This is strongly indicative of a new disk forming at the equator of the star, while at larger radii the disk continues to dissipate. We expect a fully dynamical computation following our ad-hoc disk tearing model would reproduce these features, as after
the disk tears the inner disk would continue to contribute to the high-velocity component of the Hα line profile.

Silaj et al. (2014) modelled Pleione’s disk using the same 2007 Dec 18 Hα observation used in this work, and found the disk’s density to be $\rho_0 = 6.2 \times 10^{-12}$ g cm$^{-3}$ and $n = 2.5$, at an inclination of 76°. However, we note that our fit is constrained by a series of Hα observations and confirmed by V-band photometric and polarimetric observations which show a clear trend, providing a stronger constraint (Klement et al., 2015).

Pleione is presently in a Be-shell phase which began in 2007. As the inclination of the Hα emitting region is once again nearly edge-on, this marks the half-way point to the next rapid drop in brightness. We highly encourage astronomers to continue collecting high time-resolution data on this unique system, to motivate not only the study of Pleione, but other Be stars, such as γ Cas and 59 Cyg (Hummel, 1998), that have been found to have variable disk inclinations. In addition to studying the large-scale variations caused by the influence of companions on the disk, the impact of smaller scale dynamical changes on the disk, such as those found by Wang et al. (2017), should be investigated as well. Future work should also investigate whether SPH simulations using Pleione’s orbital parameters are able to explain both the disk tearing period and the outer disk precession period. In our follow-up work, we will determine the effect of disk tilting on the thermal structure of Pleione’s disk.
Figure 3.10: SED of Pleione showing the best-fit disk and diskless models in comparison to observations in the UV (top), visible (upper middle), IR (lower middle), and radio (bottom). The observations have been separated into those taken during the Be phase (black) and during the Be-shell phase (grey). The IUE observations from the Be phase are in dark grey for ease of viewing. The radio SED is shown for the best-fit disk and diskless models despite no radio observations being available for comparison.
Figure 3.11: Change in the stellar inclination with time determined from our best-fit models to the Hα observations, along with our precessing disk model. The horizontal dashed line indicates 90° inclination. Inclinations greater than 90° occur when viewing the lower part of the disk.
Figure 3.12: Variation in the EW of the H\(\alpha\) line profile with disk inclination, with the star fixed at 60° for our precessing disk model. All predicted inclinations from 0° to 90° are shown, however the lower limit of the inclination range determined from H\(\alpha\) profile fitting is indicated by the vertical dashed line.
Figure 3.13: Subset of the best-fit predicted Hα profiles in absolute flux units, before convolution. The stellar inclination is fixed at 60°. The disk inclinations for each model spectrum are given in the legend.
Figure 3.14: **Left:** Comparison of the precessing disk model (black, solid line) with an 80.5 year period to archival observations (red), and to the inclinations from Hirata (2007)’s model (blue) and from our best-fit Hα model (black). The light grey vertical band indicates the region over which our Hα observations were modelled. In the top panel, the grey horizontal dashed line shows the inclination of 83.6° that the precession is centered about. **Right:** The same as the left side but instead showing the ad-hoc disk tearing disk model. The precessing Hα emitting region (black, solid line) precesses for the first 15 years of the 34 year Be-shell to Be phase cycle. In the same period, the innermost disk (black, dotted line) gradually transitions from the stellar equator at 60° to 30°. The thin, vertical grey lines indicate each disk tearing event. Hirata (2007)’s inclinations are not included as they follow the precession of a disk in V-band polarization, not Hα emission.
Figure 3.15: Schematic of the disk tilting and tearing model. The red region of the disk corresponds to the innermost disk, and the blue region corresponds to the outer region which eventually separates. Stage one of our model is represented by a) and b), where the disk is whole and becomes progressively more tilted away from the equator from a) to b). Stage two is represented by c) and d), where the outer region tears, precesses, and eventually dissipates. In stage two, the innermost disk gradually returns to the original inclination at the stellar equator while continuing to build with constant mass injection prior to the cycle repeating. The sizes of the coloured sections are for illustrative purposes only and do not represent actual dimensions.
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Chapter 4

The Thermal Structure of Tilted Be Star Disks

4.1 Introduction

The disks of Be stars were historically modelled under the assumption that they are approximately isothermal. Determining the distribution of temperature throughout the disk proved to be more difficult than identifying changes in the disk density (Waters, 1986). Currently, the temperature structure of Be star disks has been predicted through theoretical disk models; see Millar & Marlborough (1998), Jones et al. (2004), Carciofi & Bjorkman (2006), Sigut & Jones (2007) and Carciofi & Bjorkman (2008) for an overview on this topic. Each of these studies show that the maximum and minimum temperatures in Be star disks differ significantly from an isothermal disk. Jones et al. (2004) established the importance of including metals in the disk, which act to cool the disk. Collectively, these studies established the importance of correctly determining the disk temperature structure when modelling Be star observables. This is done by solving the problem of radiative transfer under the assumptions of radiative and statistical equilibrium.

The temperature structure of Be star disks is largely dependent on the availability of ionizing radiation, the disk density structure, and in particular, the proximity of the disk material to the stellar surface. As the density increases, the material deviates more significantly from the isothermal temperature. This is most evident at small radii near the stellar surface where the disk is densest, and optically thick to a large portion of the stellar radiation, causing a cool region to form (Millar & Marlborough, 1998; Sigut & Jones, 2007). At larger radii, the hydrogen opacity decreases due to lower density. In early-type Be stars, the temperature rises to ~ 60% of the effective temperature where it stays constant (Carciofi & Bjorkman, 2006).

As previously mentioned in Chapter 1, the surface temperatures of Be stars are also known to be non-isothermal. Due to rapid rotation, gravity is reduced near the stellar equator causing a larger equatorial radius and smaller polar radius than a non-rotating star. This effect, called gravity darkening (Espinosa Lara & Rieutord, 2011; McGill, 2013), causes the surface temperature and flux to vary with stellar latitude.

Figure 4.1 shows the relation between stellar rotation rate $W$ (defined in Equation 1.2) and the effective temperature for a B2 V star. Here, we see that B2 V type stars, which rotate at
~ 75 to 80% of the critical velocity (Townsend et al., 2004), have a difference of ~ 3000 K between the equator and pole. Some B2 V stars have been observed to rotate at 95% of the critical velocity (Domiciano de Souza et al., 2012), which causes a ~ 8500 K difference from equator to pole.

This is further illustrated in Figure 4.2, where the surface temperature of a non-rotating B0 V star is compared to a one that rotates at 95% of its critical velocity. While the surface of the non-rotating star is clearly isothermal, the rapidly rotating star has a difference of ~ 10000 K between equator and pole.

Since the main source of energy to the disk is the central star, it is key to firmly establish how the thermal structure of the disk is affected by the disk geometry and orientation. Of course, the variation of stellar flux with latitude of these rapidly rotating stars is also required for a full understanding.

Recent models have shown that Be star disks can be tilted about the central star, such that the disk might be in direct line of sight with higher stellar latitudes. In Chapter 3, this phenomenon was shown to occur for the Be star, Pleione. Other possible examples of tilted Be star disks are γ Cas and 59 Cyg, which Hummel (1998) noted have both been recorded to have two successive shell phases while also exhibiting variation in emission-line width, and the double disk of X Per reported by Clark et al. (2001). It is expected that as the disk is tilted
4.2 Tilted Disk Models

We created a grid of 84 Be star disk models using the 3D non-LTE Monte Carlo radiative transfer code H{	extsc{dust}} (previously described in Subsection 2.3), with pure hydrogen disk models. The grid explores a set of standard stars including B0 V, B2 V, B5 V, and B8 V spectral types, and stars which have been previously modelled in the literature including γ Cas (Sigut & Jones, 2007), a B3 IV star (Carciofi & Bjorkman, 2006), and Pleione using the parameters determined in Chapter 3. For each star, we computed a range of critical rotation fractions $W$, disk tilt angles $\theta_{tilt}$, and disk base densities $\rho_0$. Table 4.1 summarizes the stellar and disk parameters used for the models. The models of γ Cas, a B3 IV star, and Pleione were computed with the reported values of stellar rotation rate and disk density, while the standard star models (B0 V, B2 V, B5 V, and B8 V) were computed with $W = 0.7$ and $\rho_0 = 1 \times 10^{-11}$ g cm$^{-3}$. For sake of comparison, all models were also computed with a higher critical rotation fraction of $W = 0.95$, and a lower base disk density that is one order of magnitude less than the reported value. Each model was computed for $\theta_{tilt} = 0^\circ$, $20^\circ$, and $60^\circ$, with respect to the equatorial plane.
Table 4.1: The full range of stellar and disk parameters for the models in the grid. Stellar parameters were acquired from the following sources: \textsuperscript{a} Cox (2000), \textsuperscript{b} Sigut & Jones (2007), \textsuperscript{c} Carciofi & Bjorkman (2006), \textsuperscript{d} This work; Ch. 3. Two values of both $W$ and $\rho_0$ are listed; both the lower and higher values of the respective quantities were used to explore a range on these parameters.

<table>
<thead>
<tr>
<th>Star or Spec. Type</th>
<th>$M$ [M$_\odot$]</th>
<th>$R_{\text{pole}}$ [R$_\odot$]</th>
<th>$L$ [L$_\odot$]</th>
<th>$T_{\text{eff}}$ [K]</th>
<th>$W$</th>
<th>$W$</th>
<th>$\rho_0$ [g cm$^{-3}$]</th>
<th>$\rho_0$ [g cm$^{-3}$]</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B0 V $^a$</td>
<td>17.5</td>
<td>7.4</td>
<td>39900</td>
<td>30000</td>
<td>0.7</td>
<td>0.95</td>
<td>1e-11</td>
<td>1e-12</td>
<td>2.5</td>
</tr>
<tr>
<td>$\gamma$ Cas (B0.5 IV) $^b$</td>
<td>17.0</td>
<td>9.5</td>
<td>34000</td>
<td>25000</td>
<td>0.7</td>
<td>0.95</td>
<td>5e-11</td>
<td>5e-12</td>
<td>2.5</td>
</tr>
<tr>
<td>B2 V $^a$</td>
<td>7.6</td>
<td>4.32</td>
<td>2600</td>
<td>18800</td>
<td>0.7</td>
<td>0.95</td>
<td>1e-11</td>
<td>1e-12</td>
<td>2.5</td>
</tr>
<tr>
<td>B3 IV $^c$</td>
<td>9.2</td>
<td>5.2</td>
<td>2000</td>
<td>19000</td>
<td>0.65</td>
<td>0.95</td>
<td>1.66e-11</td>
<td>1.66e-12</td>
<td>3.5</td>
</tr>
<tr>
<td>B5 V $^a$</td>
<td>5.9</td>
<td>3.51</td>
<td>730</td>
<td>15200</td>
<td>0.7</td>
<td>0.95</td>
<td>1e-11</td>
<td>1e-12</td>
<td>2.5</td>
</tr>
<tr>
<td>B8 V $^a$</td>
<td>3.8</td>
<td>2.45</td>
<td>140</td>
<td>11400</td>
<td>0.7</td>
<td>0.95</td>
<td>1e-11</td>
<td>1e-12</td>
<td>2.5</td>
</tr>
<tr>
<td>Pleione (B8 V) $^d$</td>
<td>4.11</td>
<td>3.33</td>
<td>380</td>
<td>13900</td>
<td>0.79</td>
<td>0.95</td>
<td>3e-11</td>
<td>3e-12</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Figure 4.3 illustrates the spatial volume grid of the disk models, over-plotted on the density structure of the B0 V model with $W = 0.7$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$. The grid is divided into 125,000 cells, which are binned as follows: 50 radial bins spaced exponentially from the stellar surface, 50 $\mu$ bins spaced exponentially from the densest plane in the disk, and 50 $\phi$ bins which circle the disk at each radial bin and are spaced equally. With this configuration, the disk grid provides high resolution in the densest parts of the disk for all angles of $\theta_{\text{tilt}}$. In this work we refer to the disk midplane, which is the densest plane of the disk that extends from the stellar surface. Each disk model was computed with an outer disk radius of 100 $R_{\text{eq}}$, however the stars taken from the literature may have differently sized disks. For those disks which are reported to

Figure 4.3: The spatial grid (white lines) used for the disk models, with the $\mu$ and $r$ cells shown in the side view (left), and the $\phi$ and $r$ cells shown in the top view (right). The disk density structure of the B0 V disk model with $W = 0.7$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$ is shown along the grid. See Figure 4.4 for reference values of the $\phi$ angles. The color range is the same for both panels.
Figure 4.4: Schematic representation of our tilted disk model with the untilted disk shown for reference, shown from a perspective view (top) and from side-on (bottom). The tilted disk is rotated about the axis extending through $\phi = 0^\circ$ and $180^\circ$, and therefore has the same position at these $\phi$ angles as the untilted disk. The angle at which the tilted disk lies from the untilted disk is $\theta_{\text{tilt}}$. 
be truncated, it would be further necessary to evaluate how this might affect the disk’s thermal structure.

A schematic representation of our tilted disk model is shown in Figure 4.4, as well as an untilted disk for comparison. Here it is evident that $\phi = 0^\circ$ and $180^\circ$ are the same between the tilted and untilted disks since the flat disk is anchored at these angles. However, for all other values of $\phi$, the position of the disk around the star is affected by tilting. The greatest distance from the stellar equatorial plane for a tilted disk occurs at $\phi = 90^\circ$ and $270^\circ$, which are therefore the most tilted parts of the disk.

Figure 4.5 shows a side-on cross-section of the B0 V model with $W = 0.7$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$ in the untilted state ($\theta_{\text{tilt}} = 0^\circ$) and in each tilted state ($\theta_{\text{tilt}} = 20^\circ$ and $60^\circ$), with the colour axis representing the disk density. The model grid is simulated for angular thickness of $20^\circ$; $10^\circ$ above and below the disk midplane. The disk density, however, falls-off according to Equation 1.5, so the disk is geometrically thin despite the thickness of the sampling grid. In this work we often describe the disk in terms of small $r (< 10 \, R_{\text{eq}})$ and large $r (> 10 \, R_{\text{eq}})$. At this radius, the disk density in the midplane is $\sim 10^{-14}$ g cm$^{-3}$. At a radius of $50 \, R_{\text{eq}}$, the midplane density is $\sim 10^{-17}$ g cm$^{-3}$.

We note, that as a result of tilting a flat disk where the location of the first grid point is dependent on the stellar equatorial radius, a small gap forms between the base of disk and the stellar surface. The radial size of the gap increases for larger $\theta_{\text{tilt}}$, and is maximum for $\phi = 90^\circ$ and $270^\circ$. This could potentially affect the temperature of the inner cells especially for models with the greatest tilt.

In combination with the surface temperature variation shown in Figures 4.1 and 4.2, Figure 4.5 also illustrates how the disk aligns with hotter regions of the stellar surface as $\theta_{\text{tilt}}$ increases. This prompts the key question for this study: how significantly does disk tilting impact the temperature structure of Be star disks?
4.3 Temperature Structure

In this Subsection, we compare the disk temperature structure of our tilted and untilted disk models. The variation of temperature across different slices of the disk are shown below as functions of $r$, $\mu$ and $\phi$. In what follows, we discuss and compare the B0 V and B8 V models with $W = 0.95$ and $\rho_0 = 10^{-11} \text{ g cm}^{-3}$ where the greatest effects of disk tilting are expected. We mention the trends of the other models we investigated at appropriate places in this chapter, or in our summary at the end of this Subsection.

Figure 4.6: Variation of the disk temperature with radial distance at the disk midplane ($\mu = 0$) for the B0 V and B8 V models, with $W = 0.95$ and $\rho_0 = 10^{-11} \text{ g cm}^{-3}$. Specific locations in the $\phi$ direction are shown as indicated in the legend. The left panels are for the stars with $\theta_{\text{tilt}} = 0^\circ$, and the right panels show disks with $\theta_{\text{tilt}} = 60^\circ$.

Figure 4.6 shows the disk temperature structure as a function of radius in the disk’s mid-
plane when $\theta_{\text{tilt}} = 0^\circ$ and $60^\circ$ for the B0 V and B8 V models. The models shown have $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$, with the models for $W = 0.7$ described below. The untilted disk temperatures appear as typical Be star temperature profiles, similar to those presented by Jones et al. (2004), Jones et al. (2008), and Carciofi & Bjorkman (2006).

In the top panels of Figure 4.6, we see when the B0 V model is tilted by $60^\circ$, at small $r (< 10$ R$_{\text{eq}}$) and at $\phi = 90^\circ$ and $270^\circ$ (the most tilted regions of the disk, where $270^\circ$ is not shown to reduce redundancy; see Figure 4.4) the minimum disk temperature increases by $\sim 7000$ K, or 114%. This can be directly attributed to the increased stellar temperature and UV photons that the disk is exposed to at the higher stellar latitudes. At $\phi = 0^\circ$ and $180^\circ$ (where the disk is anchored at the stellar equator) the disk temperature does not change, as expected.

At radial disk distances of about 10 to 20 R$_{\text{eq}}$ from the central star, and at $\phi = 0^\circ$ and $180^\circ$, the maximum disk temperature is seen to decrease modestly by $\sim 1000$ K as $\theta_{\text{tilt}}$ increases to $60^\circ$. At $\phi = 90^\circ$ and $270^\circ$, the maximum temperature further decreases from $\sim 25000$ to $\sim 23000$ K, making the tilted regions of the disk colder than the regions at the $\phi = 0^\circ$. This cooling occurs due to the regions of the disk below the midplane ($\mu < 0$; see Figure 4.3) being exposed to cooler latitudes of the star. Above the midplane ($\mu > 0$), we find that the disk does not change in temperature, so the net effect is that the disk becomes colder (see, also, the discussion of temperature varying with $\mu$ below, and the accompanying Figure).

At large $r$ (beyond 10 R$_{\text{eq}}$), the tilted B0 V model with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$, and with $\theta_{\text{tilt}} = 60^\circ$ is more consistently isothermal with increasing $r$ (around $\sim 23000$ K for the B0 V model) than the untitled disk model. The tilted disk is, however, modestly cooler than the untitled disk by $\sim 1000$ K between 10 and 40 R$_{\text{eq}}$.

For the B0 V model with $W = 0.7$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$, at small $r$ (< 10 R$_{\text{eq}}$) the minimum disk temperature increases from $\sim 9900$ at $\theta_{\text{tilt}} = 0^\circ$ to $\sim 14000$ K at $\theta_{\text{tilt}} = 60^\circ$ (an increase of 41%). This change in temperature is less than half of the previously mentioned change of 114% when $W = 0.95$. At large $r$ the temperature increases from 20000 at $\theta_{\text{tilt}} = 0^\circ$ to 20900 K at $\theta_{\text{tilt}} = 60^\circ$ (an increase of just 5%), which is more significant than temperature change with $W = 0.95$ of < 5%. In comparison to the $W = 0.95$ B0 V models, the $W = 0.7$ models have hotter stellar effective temperatures at the equator, and so the regions of the disk at $\mu < 0$ do not cool as significantly as at large $r$. Overall, the disk temperature varies with $\theta_{\text{tilt}}$ more significantly at small $r$ when $W = 0.95$, and at larger $r$ the disk is hotter when $W = 0.7$ than when $W = 0.95$.

Looking at later spectral type stars, we find the changes in disk temperature caused by tilting are reduced from those seen in the B0 V models. As an example, for the B2 V model with $W = 0.7$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$, at small $r$ and $\phi = 90^\circ$ and $270^\circ$, the disk temperature increases by $\sim 7\%$ when tilted by $\theta_{\text{tilt}} = 60^\circ$, and by $\sim 15\%$ when $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$. By comparison, the same metric in the B0 V models shows an increase of $\sim 41\%$ and $88\%$, respectively. For extra comparison, at large $r$ the B2 V models with $W = 0.7$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$ have an increase in disk temperature by $\sim 19\%$ when $\theta_{\text{tilt}} = 60^\circ$, and by $\sim 16\%$ with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$. For the B0 V model with $\rho_0 = 10^{-11}$ g cm$^{-3}$, at large $r$ the temperature increases with tilting by $\sim 5\%$ for the $W = 0.7$ model, and by $< 2\%$ when $W = 0.95$.

The bottom panels of Figure 4.6 show the change in the disk temperature of the B8 V model due to disk tilting. We find the disk temperature at small $r$ does not significantly change when the disk is tilted. Around 10 to 20 R$_{\text{eq}}$, the tilted disk becomes hotter at $\phi = 90^\circ$ and $270^\circ$.
than at $\phi = 0^\circ$ and $180^\circ$, opposite to what is seen in the early spectral type models. With this temperature increase the disk is $\sim 10000$ K in this region, so it may increase the ionization and, in turn, the strength of the H$\alpha$ emission line (as was predicted in Chapter 3). Of course, the disk viewing angle also will also play a role in the shape and strength of spectral lines. Finally, at large $r$ the tilted disk becomes colder at $\phi = 90^\circ$ and $270^\circ$ than at $\phi = 0^\circ$ and $180^\circ$. This is due to the same effect mentioned above (i.e., the disk becoming colder for regions of the disk at $\mu < 0$ which face away from the stellar pole) for the B0 V model with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$, however for the B8 V star with the same $W$, $\rho_0$ and lower luminosity, its outer disk cools relatively more.

Similar to Figure 4.6, Figure 4.7 provides the disk temperature as a function of radius, but here all $\phi$ angles are shown in this 2D plot. The disk temperature is given on the colour axis. This figure allows the trends in the midplane disk temperature to be compared with changes in $\phi$. We emphasize that the temperature range in the colour axis for the B0 V and B8 V models are different so that details in each can be distinguished.

In the B0 V model, the coolest region in the tilted disk remains at the same radial location as the untilted disk. Conversely, the location of the maximum disk temperature gradually changes from 10 $R_{\text{eq}}$ at $\phi = 0^\circ$ and $180^\circ$, to 5 $R_{\text{eq}}$ at $\phi = 90^\circ$ and $270^\circ$. At large $r$, the untilted disk becomes colder with increasing $r$, but the tilted disk is approximately isothermal. These isothermal regions are hottest at $\phi = 0^\circ$ and $180^\circ$, second hottest at $\phi = 90^\circ$ and $270^\circ$, and coldest between these $\phi$ angles.

When tilted, the B8 V model’s disk temperature at small $r$ shows a dependence on $\phi$, but overall does not vary significantly from $\sim 6000$ K. The cool region of the tilted disk remains at larger $r$ than the B0 V model. The maximum disk temperature does not vary with $\phi$ as much as the B0 V model, and remains between 8 and 10 $R_{\text{eq}}$. Unlike the B0 V model, the B8 V model’s disk temperature continues to increase with radius at large $r$ once tilted.

In Figure 4.8, the disk temperature profiles at $\phi = 90^\circ$ and $270^\circ$ and for different stellar critical rotation fractions are compared for each $\theta_{\text{tilt}}$. For the B0 V model, we find that the disk temperature at the stellar surface ($r = R_{\text{eq}}$) increases as the $\theta_{\text{tilt}}$ increases. Our models show this is also true for the $\gamma$ Cas, B2 V, and B3 IV models, however the B5 V, B8 V and Pleione models showed the disk temperature at $r = R_{\text{eq}}$ decreases with increasing $\theta_{\text{tilt}}$. We expand on this point in Section 4.4.

For the B0 V model, with both simulated densities, at small $r$, the minimum disk temperature becomes colder as the critical rotation fraction increases because the stellar equator is colder. For progressively later spectral types, the change in minimum disk temperature with increasing critical rotation fractions is reduced. This is consistent with later spectral types emitting fewer UV photons, causing a reduction of temperature in the dense region. For the B8 V model, and again for both simulated densities, there is no change in minimum temperature above the level of uncertainty.

In the B0 V and B2 V models, and for all values of $W$ and $\rho_0$, the maximum disk temperature occurs between 10 and 20 $R_{\text{eq}}$. As $\theta_{\text{tilt}}$ increases, the maximum temperature decreases as the disk is no longer heated by both poles of the star, but instead one pole and the cooler stellar equator. This effect influences the disk out to its outer edge at 100 $R_{\text{eq}}$. While the maximum temperature occurs at larger $r$ for all of the later spectral type models, regardless of $W$ and $\rho_0$, each of these models also show that regions of the disk at large $r$ become colder as $\theta_{\text{tilt}}$ increases.
Figure 4.7: Variation of the disk midplane temperature with radius for varying $\phi$ angle, from $0^\circ \leq \phi \leq 90^\circ$, for B0 V and B8 V stars with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$ and $\theta_{tilt} = 0^\circ$ and $60^\circ$. Note that the temperature range on the colour bar differs between the B0 V and B8 V models.

Figure 4.9 shows how the disk temperature changes with $\phi$ at a given $r$ and $\mu$, for the B0 V model with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$. Essentially, these profiles trace a circle in the disk along the disk midplane and at one scale height above the midplane (i.e. $H(r) = 1$). While many of the features shown here were seen and discussed with previous Figures, this Figure highlights two key findings, which are consistent across all models in our grid including all spectral types, $W$, and $\rho_0$. 
4.3. Temperature Structure

Figure 4.8: Temperature structure of the B0 V and B8 V models with \( \rho_0 = 10^{-11} \text{ g cm}^{-3} \) as a function of radius, comparing for different critical rotation fractions and disk tilt angles.

First, when the disk is untilted, the temperature dependence on \( r \) is stronger in the midplane than at \( H(r) = 1 \). This is consistent with previous disk temperature models, such as those of Jones et al. (2004) and Carciofi & Bjorkman (2006). However, once tilted, the range of temperatures in the midplane is the same as at \( H(r) = 1 \) for large \( r \) (as was previously mentioned when Figure 4.6 was discussed). This indicates that the disk’s temperature dependence on \( \mu \) has changed. We find that for large \( r \) at \( \phi = 90^\circ \) and \( \mu > 0 \) (the half of the disk that is closer to stellar pole), the changes in temperature are roughly constant in \( \mu \) when the disk is tilted as the the UV ionizing radiation which is coming from regions of hotter stellar effective temperature can penetrate further into the disk. This is because the light enters the disk at a higher angle and has less material to pass through to get to the midplane. In the other half of the disk (\( \mu < 0 \)) that faces away from the stellar pole, the disk becomes cooler toward \( H(r) = 1 \) below the midplane.

The second notable effect is the change in the strength of the \( \phi \) dependence of the temper-
Figure 4.9: Variation of disk temperature for grid cells at given \( r \) and \( \mu \) as a function of \( \phi \) for the B0 V disk model, with \( W = 0.95 \) and \( \rho_0 = 10^{-11} \) g cm\(^{-3} \). The two left panels show disk temperatures at one scale height above the disk midplane, while the two right panels show the same for the disk midplane. The profiles are shown for radial cells 1, 5, 10, 20, 30 and 40 of our grid; the corresponding values in \( R_{\text{eq}} \) are given in the legend.

Table 4.2 lists the density-weighted temperatures averaged across the entire disk for our entire grid of models. Following the prescription outlined by Sigut & Jones (2007), the density-weighted temperature between the disk midplane and at \( H(r) = 1 \). For the tilted disk, at \( H(r) = 1 \) and for small \( r \), the temperature does not vary significantly with the \( \phi \) angle. In the midplane, the changes in temperature with \( \phi \) are more significant because the disk is already cooler and therefore more sensitive to heating. At large \( r \), it is expected that the temperature also varies with \( \phi \), however in the midplane the scatter of the data is too large to resolve this.

Table 4.2 lists the density-weighted temperatures averaged across the entire disk for our entire grid of models. Following the prescription outlined by Sigut & Jones (2007), the density-
Table 4.2: Density-weighted average disk temperatures. The middle two columns correspond to the models lower values of $W$, and the right two columns the higher rotation rates. The values of $W$ and $\rho_0$ for each model are listed in Table 4.1.

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Table 4.3: The same as Table 4.2, but for a simple globally averaged (not weighted) disk temperatures. The values of $T_{\text{eff}}$ correspond to those given in Table 4.1.

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4.3. Temperature Structure

Figure 4.10: Variation of the disk ionization fraction with radius for the B0 V and B8 V models, with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$. Specific locations in the $\phi$ direction are shown as indicated in the legend. The upper two panels are for $\theta_{\text{tilt}} = 0^\circ$, and the bottom panels for $\theta_{\text{tilt}} = 60^\circ$. A value of 1 indicates all of the hydrogen is ionized, and 0 is neutral.

The density-weighted temperature is defined as

$$\bar{T}_\rho = \frac{1}{M_{\text{disk}}} \int T(r, \mu, \phi) \rho(r, \mu, \phi) dV.$$  \hfill (4.1)

The density-weighted temperature is a useful diagnostic of the disk temperature in the densest parts the disk.

We find that the disks of all the models, including all spectral types, $W$ and $\rho_0$, are overall colder when tilted at $\theta_{\text{tilt}} = 20^\circ$ than when untilted. The effect which causes this is best seen in Figure 4.8, for both the low and high critical rotation fractions. There, we see that for the B0 V model, the first 20-degrees of tilt does not cause a significant change in temperature for
Figure 4.11: The same as Figure 4.10, but showing all $\phi$ angles as a function of radius and with the ionization fraction indicated by the colour axis. Note that the temperature range on the colour bar differs between the B0 V and B8 V models.

$r < 10 \ R_{\text{eq}}$, while at larger $r$ the temperature of the disk uniformly decreases by $\sim 2000$ to $4000$ K. As previously mentioned, this is a result of the disk no longer being heated by both poles. Since the densest regions of the disk stay relatively constant in temperature, the density-weighted temperature in Table 4.2 simply reflects the disk cooling at large $r$ for $\theta_{\text{tilt}} = 20^\circ$.

When the disks are tilted by $\theta_{\text{tilt}} = 60^\circ$, the trends observed are dependent on the spectral type of the star. For the early-type models, including the B0 V, $\gamma$ Cas, and B2 V models for both values of $W = 0.7$ and both $\rho_0$, the density-weighted disk temperature is hottest at this $\theta_{\text{tilt}}$. Again this can be seen in Figure 4.8, as at small $r$ the temperature increases significantly due
4.3. Temperature Structure

to being at higher stellar latitudes. In the B3 IV and B5 V models, the increase in temperature at small $r$ due to disk tilting is lessened to where the $W = 0.7$ models have the same density-weighted temperature when tilted and not tilted, and the $W = 0.95$ models have only a marginal increase by tilting. In the lower density $\rho_0 = 10^{-12} \text{ g cm}^{-3}$ model, the temperature of the disk globally increases, and the changes in temperature with $\theta_{\text{tilt}}$ are larger as well. Finally, for the B8 V and Pleione with both values of $W = 0.95$ and $\rho_0 = 10^{-11} \text{ g cm}^{-3}$, further disk tilting continues to decrease the density-weighted temperature as at small $r$ the disk does not change in temperature, and at large $r$ it becomes cooler. As was previously mentioned, for these late-type models, only a small radial region of the disk is heated at $\phi = 90^\circ$ and $270^\circ$, around 10 to 20 $R_{\text{eq}}$, which is in sufficient to be reflected in the density-weighted temperature. The B8 V and Pleione models also show that when $\rho_0 = 10^{-11} \text{ g cm}^{-3}$, for either $W$, the disk temperature increases again when $\theta_{\text{tilt}} = 60^\circ$.

In Table 4.3, we list the globally averaged disk temperatures for each model. Many of the trends mirror those of the density-weighted temperature; however the globally averaged temperature is a reflection of the disk temperature across the entire grid. The globally averaged temperature is a good diagnostic for how well the disk satisfies the isothermal approximation that $T_{\text{iso}} = 60\% T_{\text{eff}}$ determined by Carciofi & Bjorkman (2006). In the rightmost column of Table 4.3, we list $T_{\text{iso}}$ for each model based on 60\% of the $T_{\text{eff}}$ listed in Table 4.1. Overall, we find that the B0 V and B5 V models are satisfied by this prediction, while the $\gamma$ Cas, B2 V and B3 IV models are overestimated by this approximation, and the B8 V and Pleione models are underestimated. However, it should be noted that these temperatures depend on the spacing and size of the grid; most notably the exponentially increasing spacing in the $r$ and $\mu$ directions. Furthermore, these values are not useful for comparison to truncated Be star disks (as tilted disk may be made by a companion star), since the simple average is grid dependent.

Below, we investigate the fraction of hydrogen in the disk that is ionized throughout the tilted and untilted disk models. The ionization fraction is a useful check of our temperature values provided previously and also for future work, will be important for interpreting observations from these systems.

Figure 4.10 shows the hydrogen ionization fraction in the disk’s midplane, for the B0 V and B8 V models with the value of $W = 0.95$ and $\rho_0 = 10^{-11} \text{ g cm}^{-3}$ found in Table 4.1. We see that the B0 V star emits enough UV photons to essentially ionize the hydrogen in the disk for all $\theta_{\text{tilt}}$. At the outer edge of the disk at 100 $R_{\text{eq}}$, the ionization fraction decreases by $\sim 1\%$, again at all $\theta_{\text{tilt}}$. The innermost regions of the disk show a decrease of a fraction of a percent; this is discussed in further detail below.

The B8 V star has a much stronger dependence on radius than the B0 V star, as it is comparatively much dimmer in the UV. As expected, the ionization fraction reaches its minimum value region around 2 $R_{\text{eq}}$. This location corresponds to the coldest region shown earlier in the bottom panels of Figure 4.7. At larger $r$ the ionization fraction increases as the disk density decreases with radius and is heated by UV photons emitted from stellar latitudes above and below the equatorial regions. The tilted disk shows a different dependence on radius with $\phi$, which varies with radius less than the untilted disk, and has a minimum of 0.9 in comparison to the untilted disk’s minimum of 0.63. For both the B0 V and B8 V models, for all values of $W$ and $\rho_0$, we see variation in ionization fraction with $\phi$ when the disk is tilted. Figure 4.11 illustrates this for the same models as in Figure 4.10. The left panels show that, as expected, the ionization fraction has no $\phi$ dependence for the untilted models. Once the disk is tilted, it
becomes exposed to different stellar latitudes with different temperatures and with tilting the UV photons will be able to more easily penetrate the disk. The B0 V star’s disk is almost completely ionized, however there is a small decrease of \( \sim 0.002\% \) at small \( r \) in the cool region. Once tilted, the regions of the disk which are exposed to higher stellar latitudes \((\phi = 90^\circ \text{ and } 270^\circ)\) become completely ionized. The regions which are anchored at stellar equator \((\phi = 0^\circ \text{ and } 180^\circ)\) behave the same as the untilted disk, as expected.

The exact same dependence on \( \phi \) appears in the B8 V star, however there is more variation in the ionization fraction. Interestingly, the ionization fractions have less variation with \( \phi \) around 5 \( R_{\text{eq}} \), and then varies a greater amount again around 10 to 20 \( R_{\text{eq}} \). Beyond 30 \( R_{\text{eq}} \), the disk is fully ionized due to lower disk density and the availability of UV radiation.

Overall, we find the change in the trends of the temperature and ionization fraction with \( \theta_{\text{tilt}} \) differs between spectral types. The trends we find do not change for different values of \( W \) and \( \rho_0 \), only the magnitude of the changes in the trends. The disks of the B0 V and \( \gamma \) Cas models have unique trends, with temperature and ionization fraction increasing for larger \( \theta_{\text{tilt}} \) at small \( r \), and relatively the same temperature and ionization fraction at large \( r \). The B2 V and B3 IV models have a smaller increase in temperature and ionization fraction at small \( r \) than the B0 V, but at large \( r \) the temperature and ionization fraction increase with larger \( \theta_{\text{tilt}} \). The B5 V, B8 V and Pleione models do not change or marginally change with \( \theta_{\text{tilt}} \) at small \( r \), and at large \( r \) the temperature and ionization fractions decrease with increasing \( \theta_{\text{tilt}} \).

### 4.4 Discussion and Conclusions

In this work, we established that the temperature structure of tilted Be star disks changes significantly when the disk is tilted from the stellar equatorial plane. Our models indicate that differences in temperature result from the surface effective temperature changing with stellar latitude due to rapid rotation. We find that the change in disk temperature that results from disk tilting is not a global increase or decrease, but is nuanced on the spectral type, stellar critical rotation fraction, disk tilt angle, disk density, and most importantly the location within the disk.

We find that for a B0 V spectral type star with \( W = 0.95 \) and \( \rho_0 = 10^{-11} \text{ g cm}^{-3} \), where the greatest effects due to disk tilting are expected to be seen, disk tilting can increase the minimum disk temperature by up to 7000 K, or \( \sim 114\% \) when \( \theta_{\text{tilt}} = 60^\circ \). The maximum disk temperature, however, decreases with disk tilting by \( \sim 2000 \) K, or 8% when \( \theta_{\text{tilt}} = 60^\circ \). At small \( r \) (< 10 \( R_{\text{eq}} \)) the disk becomes hotter, while at large \( r \) (> 10 \( R_{\text{eq}} \)) the disk becomes more consistently isothermal in both the \( r \) and \( \mu \) direction when \( \theta_{\text{tilt}} \) is large. This is due to the disk being primarily heated by the UV light from the stellar poles at this angle. When the B0 V model has a lower critical rotation fraction of \( W = 0.7 \), the trends in the disk temperature with increasing \( \theta_{\text{tilt}} \) are the same but changes in temperature are smaller. We also computed the B0 V model with a lower density disk of \( \rho_0 = 10^{-11} \text{ g cm}^{-3} \), and found that again the trends with increasing \( \theta_{\text{tilt}} \) did not change, while the disk temperature globally increased.

The disk of the B0 V star is nearly 100% ionized when untilted, where at small \( r \) the disk is only 0.01% neutral. Beyond 30 \( R_{\text{eq}} \), the ionization fraction begins to decrease with \( r \) to a minimum of 0.03% at the outer disk radius \( r = 100 \text{ R}_{\text{eq}} \). We find that the ionization fraction increases with disk tilting most significantly at \( \phi = 90^\circ \) and 270°, but note that the ionization fraction at \( \phi = 0^\circ \) and 180° increases to a lesser degree due to the heating of the disk around it.
We find that our model $\gamma$ Cas follows the same trends in the disk temperature and ionization fraction with increasing $\theta_{\text{tilt}}$ as the B0 V model. For later spectral type stars, the changes we find in these trends are different than those for the B0 V and $\gamma$ Cas models. In the B2 V and B3 IV models, for both values of $W = 0.7$ and $W = 0.95$, and for $\rho_0 = 10^{-11}$ g cm$^{-3}$ and $10^{-12}$ g cm$^{-3}$, the minimum and maximum disk temperatures increase with $\theta_{\text{tilt}}$, and the ionization fraction follows the trends of the globally increasing disk temperature.

In our B8 V model, with $W = 0.95$ and $\rho_0 = 10^{-11}$ g cm$^{-3}$, we find that disk tilting causes the disk to become globally colder for increasing $\theta_{\text{tilt}}$. At small $r$, the temperature decrease is $<5\%$, while at large $r$ the temperature decreases by $\sim 2000$ K, or 14%. However, one region of the disk around $10 R_{\text{eq}}$ is seen to get hotter from $\sim 8000$ K to $10000$ K, which significantly increases the ionization fraction throughout the H$\alpha$ emitting region. We expect that the heating of this particular region would cause the equivalent width of the H$\alpha$ profile to increase. Overall, the changes with $\theta_{\text{tilt}}$ we find in the B8 V model are mirrored by the Pleione model, which is consistent with predictions made in Chapter 3. Furthermore, because these models were simulated to $100 R_{\text{eq}}$, and the disk of Pleione was determined to be truncated at $15 R_{\text{eq}}$ in Chapter 3, it will be interesting to see how a truncated disk further modifies the thermal structure. These model simulations are currently being computed.

We recreated the B3 IV model from Carciofi & Bjorkman (2008). These authors use hDUST to predict self-consistent disk temperatures. Using the same stellar and disk parameters and when the disk is untilted, we find the temperature profile of our disk model is consistent with their findings of a minimum disk temperature of $\sim 7000$ K, and at large $r$ the disk becomes approximately isothermal at $\sim 11000$ K. When tilting the disk to $\theta_{\text{tilt}} = 60^\circ$, we find the minimum disk temperature at $\phi = 90^\circ$ and $270^\circ$ increases to $\sim 9000$ K, and at large $r$ the disk remains approximately isothermal but cools slightly to $\sim 9500$ K.

Our model of Pleione shows that the disk on average becomes colder as $\theta_{\text{tilt}}$ increases, which is reflected in both the density-weighted disk temperature and the globally averaged disk temperature. The cooling of the disk is seen to occur at all radii, except around $10 R_{\text{eq}}$ where the disk increases from $\sim 8000$ K to $\sim 10000$ K. Recalling our disk tearing and precessing models of Pleione presented in Chapter 3, we predicted that the H$\alpha$ EW should increase as the disk is tilted from the stellar equatorial plane. This increase in temperature, and the corresponding increase in hydrogen ionization, is consistent with our previous predictions.

To understand the evolution of Be stars and their disks, it is necessary to correctly interpret observations and to be able to follow disk structure over time. Getting the disk temperature distribution correct is a key first step. The observables, our direct link to these systems, depend on the the state of the gas which is dependent on the thermal structure. Observations cannot be predicted with certainty unless the thermal structure is correct. Furthermore, the temperature plays a critical role in the disk density distribution through the scale height and also affects the sound speed. In turn, the sound speed affects the viscosity, a critical component of the successful viscous disk theory which was described earlier in Subsections 1.1 and 1.4.5.

In this Chapter, we have made important steps in understanding how the disk thermal structure is affected by disk tilting. This work will be especially important to interpreting the disk dynamics in misaligned binary systems where the disk is tilted.

The next step in this work will be to predict observations from our tilted disks. Then, in future work, we plan to expand our range of geometry to include disk warping which represents another major study since the number of free parameters would increase substantially. Finally,
it will be interesting to expand this work on thermal structure to investigate disk precession and disk tearing that was discussed in 2 to eventually predict self-consistent time dependent changes in observations.
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Chapter 5

Conclusions

5.1 Introduction

The variability observed in classical Be stars is key to understanding why their disks form, and how they change over time. Perhaps the most striking long term variability seen in Be stars are those associated with major changes in disk density distributions and geometry. The observational features of these star and disk systems change with transitions between the normal B, Be and Be-shell phases on timescales from months to decades. Changes even indicate that these disks can be built and completely disappear on these timescales. It is these variations that I am passionate about studying and understanding.

Overall, my investigations of these long term changes of Be star disks over time, I have contributed substantial findings to field of Be star research. Furthermore, my work on the thermal structure of tilted disks at particular times is vital to interpreting observations correctly from tilted systems and will likely be particularly impactful in the study of binary systems. Highlights of my contributions include:

• A fully realized dynamical solution which reproduces the complete dissipation of a Be star disk.

• An independent determination of the $\alpha$ viscosity parameter for a particular star.

• An analysis of how a binary companion star can influence the evolution of a Be star disk, leading to disk tearing and precession.

• Predicting the expected trends of Be star observables when the disk is tilted and precessing.

• Determining the temperature and ionization distributions of tilted Be star disks.

In this concluding Chapter, I begin in Section 5.2 by reiterating the important findings from the three major research studies of my work and emphasize the impact of this research on my field. In Section 5.3, I discuss topics that I believe are important to build on this body of work in future. Finally, I conclude my thesis with a summary of the key accomplishments of my work.
5.2 Summary

I began my research by studying how the dynamics and evolution of Be star disks can describe the variability of their observables. In each major research study, I created disk models using a Monte Carlo radiative transfer code to compute the disk temperatures, level populations, and ionization fractions which formed the basis of my disk models. When required, I also predicted the disk observables sometimes over a large range of wavelength from the UV to radio. With these steps, I investigated, in detail, relations between the dynamical evolution of Be star disks and their observed long term variability.

In Chapter 2, I studied the growth and dissipation of the Be star, 66 Oph. 66 Oph was historically known to start building its disk in 1957, and continued to do so until approximately 1989. After this, the star began a period of disk loss when the mechanism for mass ejection turned off and the disk began to re-accrete onto the central star with material at larger radial distances, lost to the system. By 2010, all of the Hα emission had disappeared. Using new observations of the remaining diskless star, I determined the star’s intrinsic polarization position angle to be $\theta_{\text{int}} = 98 \pm 3^\circ$. Using observations of the star’s SED and Hα profile, I successfully determined that at the onset of dissipation in 1989, 66 Oph’s disk had a base density of $\rho_0 = 2.5 \times 10^{-11}$ g cm$^{-3}$, and a density slope of $n = 2.5$, with a disk inclination of $i = 57.5^\circ$. Then I evaluated different hydrodynamical scenarios with various disk temperature and viscosity in efforts to best reproduce the rate of dissipation observed for the Hα emitting region. I find that 66 Oph’s 21 year dissipation event is best reproduced using an $\alpha$ disk viscosity prescription that is constant with radius and has a value of $\alpha = 0.4$, with an isothermal disk that is set to 60% of the stellar effective temperature, and has an outer disk radius of 100 R$_{\text{eq}}$. This model matched the rate at which the Hα equivalent width, V-band magnitude, and V-band polarization decreased over the period of disk dissipation.

In Chapter 3, I determined that the disk of the Be star Pleione is periodically tilted. Archival observations of Pleione’s spectrum in the Balmer series during Pleione’s last diskless phase in 1937 allowed an accurate determination of the physical parameters of the central star. Using a novel set of Hα spectroscopic observations, I confirmed that Pleione had transitioned to a new Be-shell phase in 2007. The physical parameters of Pleione’s disk were then determined from a grid of disk models which were fit to the Hα profiles at the end of Pleione’s last Be phase. I determined that the Hα equivalent width could be followed through this transition into the Be-shell phase and up to 2021 by simply changing the inclination of the system as seen by an observer, with a disk of base density equal to $3 \times 10^{-11}$ g cm$^{-3}$, a density slope of $n = 2.7$, and an Hα emitting region extending to 115 R$_{\text{eq}}$. However, this disk model is unable to reproduce the trends observed in Hα in the earlier Be and Be-shell phases, but was able to follow V-band photometry and V-band polarization in the earlier Be phase. At this stage, and with inspiration from recent smoothed particle hydrodynamics simulations of Be star disks in binary star systems in the literature, I established that Pleione’s disk periodically undergoes a series of disk warping, tilting, and tearing events. The disk consists of an innermost region that emits in V-band and a region extending to larger radius that emits brightly in Hα. I find that the disk tearing process is followed by a $\sim$ 15 year period of disk precession for the Hα emitting region, while the innermost disk is slowly rebuilt and flows outward from the stellar equator.

Lastly, in Chapter 4, I studied the change in the temperature structure of Be star tilted disks.
I find that the temperature structure of the disk changes significantly as the disk tilts from the equatorial plane. For B0 V stars with $W = 0.95$ and $\rho_0 = 10^{-11} \text{ g cm}^{-3}$, I find the minimum disk temperature can increase substantially, by up to 114% at specific locations in the disk for a tilt angle of 60°. For the same system, the maximum disk temperature decreases by $\sim 8\%$, while at large $r$ the disk temperature stays relatively constant. For a lower critical rotation fraction of $W = 0.7$, the minimum disk temperature increases by up to $\sim 41\%$ when tilted by $\theta_{\text{tilt}} = 60^\circ$. When the disk density is reduced by an order of magnitude, the disk temperature increases globally. I find that the ionization fraction follows the same trends as the temperature structure, and when temperature increases above the ionization potential of hydrogen, the ionization fraction increases, as expected. The trends observed in the γ Cas model mirror those of the B0 V model, but with lower values of temperature and ionization fraction. For all later spectral types, I find that the trends of the disk temperature are different from the B0 V model as follows. For the B2 V and B3 IV models, the disk temperature and ionization fraction at small $r$ increases with larger $\theta_{\text{tilt}}$ similar to the B0 V model, but at large $r$ the temperature and ionization fraction increase as well for both $W = 0.7$ and $W = 0.95$, and for $\rho_0 = 10^{-11}$ and $10^{-12} \text{ g cm}^{-3}$. In the B5 V, B8 V and Pleione models, the disk temperature and ionization fraction do not change with disk tilting at small $r$, but at $\sim 10 \, R_{\text{eq}}$ the temperature is seen to increase in all of our models, including for both values of $W$ and $\rho_0$ listed above. Overall, since a significant fraction of Be stars are reported to exist in binary systems, it is critical to understand how the tidal effects of close binary interactions may lead to disk tilting which impacts the disk temperature structure and, in turn, will affect the interpretation of observables.

5.3 Future Directions

My thesis research has contributed to new results and understanding about Be star disks. My work has also prompted questions which should be addressed in follow-up. For example:

- What is the nature of the disk viscosity, and is it appropriate to assume a single value for the entire disk at all times?

  One possible way this could be investigated would be by modelling disk growth and dissipation with observables that form at different locations in the disk. For example, this could be probed with a set of emission lines that form near the stellar photosphere, then compared with the Hα region that forms out an intermediate disk radius, and finally with IR or radio band photometry to sample at large radii.

- How common is disk tearing and disk precession in Be/binary systems?

  The current understanding of disk tearing and the circumstances when this occurs, is novel and deserves further study. The same is true for disk precession. Observational evidence indicates that there are at least four possible targets; Pleione, 59 Cyg, γ Cas, and χ Oph where this phenomena may be operating. I strongly encourage researchers in this field to continue observing and modelling these systems. Modelling the variability of Be star disks can also assist in establishing the observational patterns associated with disk tearing and disk precession, from which more potential targets can be found.
5.4 Final Remarks

- How significantly do the observables of a tilted disk change as the tilt angle increases?

Now that the temperature structure of the tilted disk has been established, the next logical step is to compute the observables associated with a tilted Be star disk. Then, the next step, would be to consider a warped disk. In this case, the disk would always be anchored at the stellar equator, and could have a greater tilt angle at large radii depending on the tidal influence of a potential orbiting companion. Perhaps, this configuration may be more physically realistic and would be consistent with the disk tearing and precession model that was proposed for Pleione in Chapter 3. This could be achieved through smoothed particle hydrodynamic modelling of Pleione’s system, using the star and disk parameters determined in my work.

The answer to these and many more questions could be more firmly addressed by regular and continuous monitoring of Be stars in all wavelengths from X-ray to radio regimes. Photometric, interferometric, and polarimetric studies of Be stars will also be valuable to understanding how these systems form and evolve. In addition, further development of sophisticated radiative transfer computations combined with Markov Chain Monte Carlo fitting as well as machine learning will contribute to new understanding when tightly constrained by observations.

5.4 Final Remarks

My thesis work has demonstrated that understanding the dynamical changes which occur in Be star systems is an important step to address unsolved puzzles in this field of research. Using the latest generation of computational codes, I created a series of Be star disk models that are more physically complete than ever before. Also, I have made the first steps to understanding the temperature structure of Be stars tilted by the influence of a binary companion star.

I have endeavoured to show how the dynamical changes are reflected in the observational properties of Be star disks. I have advanced the understanding of the diagnostic potential of variability in the Hα profile, photometry and polarimetry. It is clear that it is necessary to use multiple observations of Be stars to fully constrain a system. The understanding that comes by constraining models with multiple observations cannot be understated.

Be stars and their disks offer the potential to help us understand astrophysical disks and stellar evolution with the light they emit, and ultimately help to tackle mysteries in the Universe.
Curriculum Vitae

Name: Keegan C. Marr

Post-Secondary Education and Degrees:
The University of Western Ontario
London, ON
2017 - 2021 Ph.D. Astronomy

The University of Western Ontario
London, ON
2015 - 2017 M.Sc. Astronomy

The University of Prince Edward Island
Charlottetown, PE
2011 - 2015 B.Sc. Honours Physics

Honours and Awards:
Mitacs-JSPS Globalink Research Award
2020

Science International Engagement Fund Graduate Award
2020

Regis and Joan Duffy Science Fund
2014, 2015

Summer Program for Undergraduate Research Award
2014

Dr. Wenonah Foster Memorial Prize in Astronomy
2013

Inspiring Excellence Award
2011
5.4. Final Remarks

**Related Work Experience:**
Teaching Assistant  
The University of Western Ontario  
2015 - 2021

Research Assistant  
The University of Western Ontario  
2015 - 2021

Summer Research Assistant  
The University of Prince Edward Island  
2014 - 2015

Teaching Assistant  
The University of Prince Edward Island  
2013 - 2015

**Publications:**

**Refereed Publications**


**Conference Proceedings**


**Further Contributions**


