Photonic Crystal Edge Couplers for Sensing Applications

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Abstract

Microelectromechanical systems (MEMS) have not only enabled the development of inexpensive sensors but have also improved their performance by lowering their mass thus enabling faster sensor response. However, there are limitations regarding the mass-scaling of conventional MEMS sensors that prevent further miniaturization. This makes the measurement of distributed forces with high spatiotemporal resolution challenging. Optical-based sensors provide low-volume confinement of electromagnetic energy and enable further mass-scaling. This thesis investigates the application of an optical deflection sensing mechanism that relies on the position dependent coupling between dielectric-like edge states on nearby photonic crystal slabs for the purposes of acoustic pressure, wall shear stress, mass and magnetic field sensing.

The performance of these sensors for these different applications developed using this same deflection sensing mechanism are presented in this thesis. The primary element in all these sensors is a suspended photonic crystal (PC) membrane with edge defect waveguides which was fabricated on silicon-on-insulator using a combination of optical chip foundry surface micromachining and post-foundry etching process to selectively remove the buried oxide.

The different dynamic mechanical modes of the suspended membrane, namely in-plane and vertical modes, have been exploited in different sensors. The vertical mode of the PC membrane with a vertical offset was utilized for acoustic pressure sensing while the horizontal in-plane mode was used to develop a wall shear sensor. Thermal fluctuations in the vibrational modes of the membrane position were used to measure the noise floor and calibrate the sensors. The detection limit for the ultrasonic pressure sensor was measured to be 12.5 mPa/√Hz. A wall shear stress sensor was demonstrated using a novel experimental setup consisting of a thermoacoustic emitter at the end of an acoustic waveguide. The sensor exhibited a noise floor of 80 µPa/√Hz, resonance at 627 kHz, and 0.12% full-scale/Pa across 1497–1531 nm. By introducing microbeam arrays, vertical misalignment of PC edges could be reduced leading to lower noise and crosstalk with dynamic pressure. The sensor area is over three-orders of magnitude smaller than sensors with similar sensitivity and can find application in fluid dynamics research where such high resolutions are required.

In the final study, PC directional coupler based suspended membrane sensors as small as 30 µm x 20 µm were metallized to form magnetic sensors relying on the Lorentz force to excite the vertical vibrational mode. The magnetic sensors were tested in ambient conditions and exhibited a noise floor of 130 nT/√Hz with flat mechanical response up to 1.64 MHz resonance. To the best of our knowledge, this is one of the first integrated photonic magnetic sensors to be demonstrated on the SOI platform. Overall, PCDC-based sensors enable low-mass, broad optical bandwidth, high spatiotemporal resolution measurements and compatibility with standardized silicon photonic foundry process.
**Keywords:** Silicon photonics, photonic crystals, directional coupler, silicon-on-insulator, ultrasonic sensors, ultrasonic transducers, wall shear stress sensors, microelectromechanical systems, magnetic field sensors
Summary for Lay Audience

This thesis explores a new optical sensor technology based on specially patterned materials with the ability to slow light down and enhance light-matter interactions for microdeflection sensing. The devices were created using the same manufacturing methods used to make inexpensive silicon computer chips and their performance as pressure, wall shear stress, mass, and magnetic field sensors was investigated. Using a hand-built custom experimental apparatus, shear force sensors—thousands of times smaller that state-of-the-art sensors of similar sensitivity—were demonstrated and capable of resolving details of microflows. Such sensors may be important for the detection of boundary layer separation in aircraft, fuel injection sensors, and fundamental fluids research while finding potential application in the growing multi-billion dollar unmanned aerial vehicle market. In another study, miniature magnetic sensors were created based on the Lorentz force. The Lorentz force arises whenever a wire is placed in a magnetic field at certain angles while conducting electricity and was used to vibrate microwires fitted with the new sensor technology. Devices as small as a human hair could easily detect the Earth’s magnetic field and is one of the first demonstrations of integrated photonic magnetic sensors developed on silicon photonic platforms. Potential applications of the sensors include wear-free automobile and aerospace sensors, electrical current sensors, and non-destructive testing. Overall, the sensors developed in this thesis enable low-mass, small size, high-speed measurements and can be readily integrated with other devices into single-chip.
Co-Authorship Statement

This doctoral thesis has been prepared according to the regulations for an integrated-article format thesis stipulated by the School of Graduate and Postdoctoral Studies at The University of Western Ontario. Michael Zylstra was the primary author of all the papers presented in chapters 2-8 and was responsible for the setup, design and acquisition of the experimental work including writing the manuscript. Dr. Jayshri Sabarinathan was responsible for the supervision of Michael Zylstra over the course of his doctoral studies and contributed to the discussion and analysis of the results, provided comments, editing and revision of the manuscripts. Chapters 2 and 3 have been published. The role of other co-authors for each chapter are as described below.


Chapter 3 is based on the published conference proceedings “Silicon photonic crystal membrane ultrasonic sensor” Michael Zylstra, Brett Poulsen, Jayshri Sabarinathan, ”Silicon photonic crystal membrane ultrasonic sensor,” Proc. SPIE 11354, Optical Sensing and Detection VI, 113540Y (13 April 2020). Michael Zylstra designed the experiment; created custom measurement software.

Chapter 4 is based on the paper in preparation for submission “Demonstration of high-resolution air-coupled silicon photonic crystal shear sensor” with co-author Jayshri Sabarinathan. Michael Zylstra designed and constructed the experiment; performed all simulations and measurements.

Chapter 5 is based on the paper in preparation for submission “Noise and pressure crosstalk of vertically misaligned coupled-waveguide shear stress sensors” with co-author Jayshri Sabarinathan. Michael Zylstra designed the experiment, performed measurements.

Chapter 6 is based on the paper in preparation for submission “Mitigation of vertical misalignment of SOI coupled-waveguide shear sensors” with co-authors Aref Bakhtazad and Jayshri Sabarinathan. Michael Zylstra designed the experiment; performed all simulations, post-foundry fabrication, and measurements. Photonic layout was performed jointly by Michael Zylstra and Aref Bakhtazad.

Chapter 7 is based on the paper in preparation for submission “Photonic crystal edge-coupled added-mass sensor” with co-author Jayshri Sabarinathan. Michael Zylstra designed the experiment; performed all post-foundry fabrication and measurements.
Chapter 8 is based on the paper in preparation for submission “Minature Lorentz force magnetometer with 2D PCDC optical read-out” with co-authors Brett Poulsen and Jayshri Sabarinathan. Michael Zylstra designed the experiment; performed all simulations, post-foundry fabrication, and measurements. Photonic layout design was performed jointly by Michael Zylstra and Brett Poulsen.
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<td>Atomic Force Microscope</td>
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<tr>
<td>AMF</td>
<td>Advanced Microfoundry</td>
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<tr>
<td>AMR</td>
<td>Anisotropic Magnetoresistance</td>
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<td>AW</td>
<td>Acoustic Waveguide</td>
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<td>BHF</td>
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<td>Finite Element Method</td>
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<td>GMR</td>
<td>Giant Magnetoresistance</td>
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<tr>
<td>IMEC</td>
<td>Interuniversity Microelectronics Centre</td>
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<tr>
<td>IMF</td>
<td>Institute of Microelectronics</td>
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<tr>
<td>JJ</td>
<td>Josephson Junction</td>
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<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
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<tr>
<td>MMI</td>
<td>Multimode Interference</td>
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<tr>
<td>MRI</td>
<td>Magnetic Resonance Imaging</td>
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<td>MZI</td>
<td>Mach-Zehnder Interferometer</td>
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<td>NMR</td>
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<td>OMR</td>
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<td>PBG</td>
<td>Photonic Band Gap</td>
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<td>PC</td>
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<td>SEM</td>
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<td>SGC</td>
<td>Surface Grating Coupler</td>
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<tr>
<td>SiPh-MIP</td>
<td>Silicon Photonic Microsystems Integration Platform</td>
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<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
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<td>SOI</td>
<td>Silicon-on-Insulator</td>
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<tr>
<td>SQUID</td>
<td>Superconducting Quantum Interference Device</td>
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<td>TAE</td>
<td>Thermoacoustic Emitter</td>
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<td>TMR</td>
<td>Tunneling Magnetoresistance</td>
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Chapter 1

Introduction

The most common microsensor technologies are illustrated in Fig. 1.1 which includes piezoresistive, capacitive, piezoelectric, thermal, ray optical, optical dissipative, interferometric, and cavity optomechanical methods. With the exception of thermal sensors, this literature review focuses on deflection-based sensors, with an emphasis on optical sensors since they are generally used where precision, high speed, remote sensing, and immunity to electromagnetic interference (EMI) are desirable [1]. Applications of microsensors are presented with a focus on the measurement of distributed forces, such as pressure and wall shear stress, as well as mass and magnetic field sensing. This literature review is by no means an exhaustive list of microsensor technologies and, depending on the application, non-displacement based transduction technology may exist. In the case of magnetic sensors, these alternative technologies are summarized later in Section 1.2 on microsensor applications. Finally, the motivation for photonic crystal directional coupler based sensors is presented along with the thesis objectives and outline.

1.1 Microsensor Technologies

Microsensors have had a profound impact on modern society appearing nearly everywhere alongside electronics in smart phones, automobiles, avionics, biomedical devices, robotics, and industrial control systems. Using the same established silicon fabrication technology used to make computer microprocessors, relatively inexpensive sensors based on the deflection of microstructures can be created allowing direct integration with electronics while benefitting from an economy of scale.
Figure 1.1: Illustrations of common microsensor transduction technologies are shown. (a) Cross-section of a piezoresistive-based sensor depicting doped regions where the highest stress is expected. (b) Capacitive-based MEMS sensors detect the changes in capacitance based on microplate spacing. (c) Piezoelectric sensors rely on electrical charges that accumulate in response to mechanical stress. (d) Thermal anemometer sensors rely on convective heat transfer. (e) Ray optical sensors rely on the deflection or blocking of light. (f) Optical dissipative sensors rely on position-dependent substrate-coupled losses. (g) Top view of an integrated optical interference device where light is split into a sensor and reference path and recombined to form interference fringes. (h) Cavity optomechanical sensors rely on changes in optical resonance.

1.1.1 Piezoresistive

Piezoresistive sensors rely on materials that have stress-dependent resistance [2] commonly achieved using p-doped $10^{15} - 10^{22}$ cm$^{-3}$ silicon [3]. The piezoresistive elements are typically placed in regions subject to high stress during deformation as shown in Fig. 1.1(a), and typically probed using an integrated Wheatstone bridge. A disadvantage of piezoresistive sensors is the requirement of a test current flowing through the transduction element that can heat the sensor region. The heating thermalizes the carriers present in the band structure of the piezoresistive material which diminishes its sensitivity [4, 5]. Moreover, the heating also leads to a decrease in performance due to Johnson-Nyquist noise with voltage spectrum density given by
where \( T, R, \) and \( k_B \) are temperature in K, piezoresistance, and Boltzmann’s constant respectively. Piezoresistive sensors therefore require signal conditioning to offset thermal effects, although temperatures as high as 600°C have been achieved with isolation packaging [6]. Thermal effects may be mitigated by increasing doping levels, but this inevitably decreases sensitivity. The performance of piezoresistive sensors might also be improved by enhancing the strain felt by the sensor region through narrowing of the piezoresistor region. However, this approach leads to higher resistances, more heating, and reduces the operating frequency. Therefore, further advances in piezoresistive sensors lie in material science and there is active research using graphene [7], carbon nanotubes [8], and SiC [9] as piezoresistive elements.

1.1.2 Capacitive

Capacitive MEMS sensors are well suited for deflection sensing as changes in capacitance can be directly related to changes in microplate spacing as shown in Fig. 1.1(b). The capacitance of a parallel plate capacitor is given by Eq. (1.2).

\[
C = \frac{\epsilon A}{d}
\]  

(1.2)

where \( C, \epsilon, A, \) and \( d \) are capacitance, dielectric constant, area, and plate separation. Capacitive devices can be used for both sensing or actuation using the vertical spacing of microplates or the horizontal deflection of comb drives. An advantage of capacitive sensors is their low power consumption; however, they suffer from non-linearities and are vulnerable to electromagnetic interference. Since the ability to detect capacitance is limited to about 0.5 fF [1, 10] and, due to limitations arising from the breakdown voltage of air [11], the sensitivity of capacitive MEMS sensors declines with decreasing microplate area and increasing plate separation. Therefore, improving the spatial resolution of capacitive sensors is challenging. Efforts to improve the frequency response by using stiffer plate supports introduce a trade-off between frequency response and sensitivity. Additionally, thin substrate-coupled membranes with high aspect ratio are prone to stiction due to the presence of undesired adhesion forces between closely separated microstructures [12].

1.1.3 Piezoelectric

Piezoelectric devices form another important class of MEMS that rely on strain-dependent accumulation of electric charge as shown in Fig. 1.1(c). Piezoelectric MEMS are capable of
actuation as well as sensing, possess high energy density, and scale favourable upon miniatur-
ization [13] with lead-based zirconate titanate [14], aluminum nitride [15], barium titanate [16],
and strontium titanate [17] being commonly used materials. Despite the challenges associated
with microfabrication such as crystallographic orientation and lattice matching [18], the mini-
aturization of piezoelectric MEMS has benefitted from thin-film deposition techniques [19],
particularly on silicon substrates [20]. Through use of strain engineering, piezoelectric prop-
erties can be enhanced leading to stronger electromechanical coupling and lower drive volt-
ages [17, 21].

The high Young’s modulus and strong electromechanical coupling of make piezoelectric
materials well suited for ultrasonic transduction. Ultrasonic MEMS transducers can operate
at frequencies 50 MHz–1 GHz [22, 23] which benefit from proximity to integrated cir-
cuity. Piezoelectric resonators have been successfully implemented as an “electric-nose”,
where changes in mass due to selectively molecular adsorption alter the electromechanical
resonance [24]. However, since piezoelectric materials are based on ferroelectric phenomenon,
they suffer from hysteresis making open-loop measurement of steady-state deflections chal-
 lenging [25] and face issues regarding impedance matching with air [26]. These effects can be
mitigated with integrated electronics but add to the fabrication complexity and make remote
sensing challenging.

1.1.4 Thermal Anemometers and Accelerometers

Thermal devices form a broad and unique class of microsystems. Due to the interplay between
resistive heating, thermal-dependent resistivity, and thermal expansion, thermal microsys-
tems may be used as either sensors or actuators [27]. The sensor configuration in Fig. 1.1(d) is
widely used in the automotive and aerospace industries for flow sensing and accelerome-
ters [28]. The operating principle is based on the balance between the supplied electrical input
power and the power lost due to convective heat transfer:

\[ Q = I_w^2 R_w = h(v) A(T_w - T_0) \]  \hspace{1cm} (1.3)

where \( I_w, R_w, h(v), A, T_w, \) and \( T_0, \) are the electrical current, wire resistance, heat transfer coef-
ficient, heating area, temperature of the wire, and temperature of the flowing fluid, respectively.
The temperature of the wire can be determined using the thermal-dependent resistor

\[ R_w = R_0(1 + \alpha(T_w - T_0)) \]  \hspace{1cm} (1.4)

where both \( R_0 \) and \( T_0 \) are a reference resistance and temperature, respectively. The sensor ele-
1.1. Microsensor Technologies

Microsensor Technologies are typically isolated to avoid parasitic loss of heat. In the absence of external flow, these sensors can be used as highly shock-resistant accelerometers by measuring the rising air [29]. The main drawback from thermal flow sensors is the necessity of an empirical model linking the resistive heating to the convective air flow that can only be determined under certain assumptions about flow conditions [30]. In laminar flow, the dissipated heat is related to cubic of wall shear stress; however, this simple relationship breaks down in turbulent flow and requires a complicated calibration process [31]. Additionally, these sensors exhibit a trade-off between sensitivity and frequency response due to thermal inertia [29, 32].

1.1.5 Ray Optical Sensors

Ray optical sensors are based on either the deflection an optical beam from a reflective surface, as shown in Fig. 1.1(e), or the shadow generated by a movable structure. In weakly coupled optomechanical systems, since very little momentum is carried by light, displacements can be passively interrogated by an optical probe, allowing mechanical and optical aspects to be designed independently. By growing or etching a tip onto the underside of a cantilever, atomic force microscopes (AFM) can be created that can resolve individual atoms and are used extensively in surface characterization, molecular metrology, biological sciences [33], and the manipulation of individual atoms [34]. In a similar way, magnetic field sensors can be formed by using the Lorentz force generated by an electrical current flowing across a microstructure [35–40].

Using optical shadowing effects, wall shear stress sensors based on the partial blockage of light from suspended-element structures have been fabricated [41]. The lateral deflection of the suspended microstructure changes the amount of transmitted light captured by photodetectors buried beneath the structure. Variations on this technique using Moiré interferometry have been successfully implemented [42, 43]. However, in many setups, either the optical source or detection is done off-chip, making further integration challenging.

1.1.6 Dissipative Optical Sensors

Dissipative optical sensors are based on the position-dependent absorption of optical power that occurs along the length of a waveguide as shown in Fig. 1.1(f). The intensity of light at the output is given by

\[ I = I_0 e^{-\mu L}, \]  

where \( I_0 \), \( \mu \), and \( L \) are the optical intensity at the input, absorption coefficient, and length of
the waveguide, respectively. Position-dependent absorption can be achieved using substrate-coupled waveguides, where light propagating along a suspended waveguide is coupled to the radiative modes of the wafer handle layer, forming a readily integrated sensor platform. Strong dissipative coupling due to the evanescent leakage of constricted waveguides placed near a silicon nitride diaphragm has been observed [44]. For transverse electric modes, the sensitivity can be enhanced by thinning the waveguide [45] or using photonic crystal line defects [46]. However, as with capacitive sensors, the sensitivity of substrate-coupled optical sensors is non-linearly related to the distance between the waveguide and substrate, therefore stiction is a risk along with the same trade-offs associated with aerodynamic drag and squeeze dampening. Also, optical power coupled to the substrate cannot be recovered and diminishes the power efficiency of the device.

1.1.7 Interferometric

Interferometry is a powerful tool to probe subatomic distances having been recently used to detect gravitational waves billions of light years away, earning scientists a Nobel Prize in 2017 [47]. Due to its wave-like nature, light can be made to interfere with itself either constructively or deconstructively, forming a series of fringes in the spectral and spatial domains. Mach-Zehnder interferometers (MZI), as shown in Fig. 1.1(g), are a popular configuration found in integrated photonics [48] and widely used as electro-optical modulators (EOMs) not only in telecommunications [49], but also in biosensing [50], and deflection sensing [51].

For MZIs, light is divided into two paths and recombined after having traversed different optical path lengths. Assuming ideal splitters and combiners, and that the coherence length of the optical source is much longer than the optical path length difference, the intensity of light observed at the output of a MZI interferometer is given by

\[ I = I_1 + I_2 + 2 \sqrt{I_1 I_2} \cos \Delta \phi, \]

where \( I_1, I_2, \) and \( \Delta \phi \) are the intensities of light entering each input of the combiner and the optical phase difference, respectively. If a waveguide segment is inserted into one of the optical paths which are otherwise equal, the phase difference can be expressed in terms of the effective index of the waveguide

\[ \Delta \phi = \frac{2\pi n_{\text{eff}} L}{\lambda}, \]

where \( n_{\text{eff}}, \lambda, \) and \( L \) are the effective index, free-space wavelength, and length of the waveguide segment. Thus, any change to the effective index of the waveguide segment will cause a shift
in fringes observable in the optical spectrum. Shorter designs can be achieved by using either slotted waveguides that confine the electromagnetic (EM) field to an air gap [52] or slow-light effects that arise in periodic structures [53–55]. However, additional integrated photonic components are required to realize MZI sensors for intensity based optical read-out and require coherent optical sources.

### 1.1.8 Cavity Optomechanical

Using optical microcavities or interferometric schemes, deflections of the order of femtometers can be measured routinely [56] whose noise floor can then be related to statistical mechanical models that provide a means of calibration [57]. By cleaving, fusing, and wet-etching, a sealed optical cavity can be formed at the tip of an optical fiber, as shown in Fig. 1.1(h), creating an Fabry-Pérot (FP) optical resonator. The spectrum of light that is reflected from such a cavity is a series of fringes separated by the free-spectral range (FSR) given by

\[ \Delta \omega_{\text{FSR}} = \frac{\pi c}{L}, \tag{1.8} \]

where \( \Delta \omega_{\text{FSR}} \), \( c \), and \( L \) are the frequency separation between two longitudinal modes, the speed of light, and the cavity length, respectively. By exerting pressure, the cavity length is compressed causing an observable spectral shift of wavelength minima [58]. FP sensors are very robust with some being able to withstand temperatures as high as 710°C [59] and offer the best compromise in terms of affordability, pressure range, sensitivity, and miniaturization that are suitable for many applications including disposable medical devices [60]. Similar FP effects are present in the stretching of fiber Bragg gratings (FBG) [61] and the deflection of ring resonators [62]. In an alternative arrangement, the refractive index of the air itself is detected, as opposed to measuring changes in the cavity length, forming sensitive pressure sensors with excellent frequency response in the MHz [63].

Optical microcavities have enabled not only ultrasensitive position measurements, but also the manipulation of microstructures through a sufficient build up of optical energy. Optical backaction has been used to amplify the influence of mechanical perturbations [64] and enhance the mechanical quality factor of micro-discs [65, 66], ring resonators [67], photonic crystal waveguides [68, 69], and under-etched vertical cavities [70]. Strong optomechanical coupling has also been observed in optical cavities formed by microtoroids where mechanical vibrations modulate the optical resonance and can be used as mass sensors with sub-picogram resolution [71]. Similar arrangements using microspheres have been used as label-free biosensors to detect a single proteins [72] or viruses [73]. Using semiconductor discs, the Brownian motion in liquid can be observed as a broadening in transmission spectrum, even in heavily dis-
sipative liquids, that can assist in rheological studies [74]. Using dual cavity ring-resonators, the mass of chromosomes has been measured [75]. Optomechanical coupling has also been reported in carbon nanotubes which could be made to actuate 1.1 μm under 3 mW illumination [76]. However, high optical quality factors do not have a direct influence on the theoretical quantum limited displacement sensitivity [77]. Although these sensors can achieve high sensitivity, they often require demanding fabrication tolerances, tunable instrumentation, and trade-offs between optical bandwidth and sensitivity [1].

1.2 Applications

In this section, some important applications of microsensors are presented along with a summary of the state-of-the-art in microsensor technologies reviewed in the previous section.

1.2.1 Pressure Acoustics and Wall Shear Stress Sensing

Pressure and wall shear stress sensors are important to many applications such as the measurement of fuel injection rates and engine timing in automobiles [28], arterial flow for medical diagnosis [78], or boundary layer separation in aircraft [79]. Viscous drag, measurable by wall shear stress sensors, account for nearly 50% of total drag in commercial aircraft; a small reduction in viscous forces would have a large impact on fuel economy saving millions of dollars and CO$_2$ emissions [80]. The market for unmanned aerial vehicles (UAVs) is expected to grow considerably from 22.7 to 49.0 billion USD over the next five years [81], and further miniaturization can lead to lighter and safer aircraft with increased maneuverability. The National Aeronautics and Space Administration (NASA) has identified real-time adaptive morphing aircraft as a key technology of future aircraft [82] which has been recently demonstrated [83]. However, there remains a need for robust, high-speed sensors [84].

Despite fluid dynamics being fully described by the Navier-Stokes equations, they form a complicated set of partial differential equations that are difficult to solve analytically without underlying assumptions. A one-million-dollar Millennium Prize offered by Clay Mathematics Institute is available to anyone who can prove (or disprove) the existence and smoothness of the Navier Stokes equation [85]. Since approximations are commonly made using computer models, improved sensors can be used to validate numerical models and assist in advancing fluid research.

There is great interest, both theoretically and experimentally, to understand turbulent phenomena since it is encountered frequently in nature in the form of weather fronts and ocean currents, or arising from the flow around human-made objects such as cars, buildings, aero-
1.2. Applications

planes, ships, and pipes. Essentially, turbulence is characterized by the appearance of eddies of varying scale that exchange energy through a cascade process where the kinetic energy at larger scales is transferred to the lower scales [86]. While there is no precise definition of turbulence, it is generally agreed to have the following properties: eddies at varying scales; increased thermal diffusivity and skin friction; large Reynolds numbers; three dimensional (3D); thermal dissipation; and continuum [87]. In many cases, the type of flow can be characterized by the Reynolds number which expresses the ratio between inertial fluid forces and viscous forces and is given by [86]

\[ Re \equiv \frac{ul}{\nu} \] (1.9)

where \( u, l, \) and \( \nu \) are the free-stream velocity, characteristic length scale of the system, and kinematic viscosity of the medium, which is about \( 1.5 \times 10^{-5} \text{ m}^2/\text{s} \) for air. The kinematic viscosity is related to the dynamic viscosity by the density (i.e., \( \mu/\rho = \nu \)). Higher Reynolds numbers result in a transition from laminar to turbulent flow occurring around \( Re > 500,000 \) for a plate [86] and \( Re > 2000 - 4000 \) for a pipe [88].

At smallest scales, skin friction in the form of viscous forces and kinetic energy are dissipated into random thermal motion. To better understand the expected spatiotemporal requirements of microsensors in the context of turbulence, it is possible to obtain a relation between the small and large scales of the flow [87, 89] that widen considerably as the Reynolds number increases [30, 31] and given by

\[ u_0 = Re^{-1/4}u \] (1.10a)

\[ l_0 = Re^{-3/4}l \] (1.10b)

\[ t_0 = Re^{-1/2} (l/u) \] (1.10c)

where \( u_0, l_0, \) and \( t_0 \) are the Kolmogorov velocity, length, and time scales named after the Russian mathematician Andrey Kolmogorov [86]. For example, the Reynolds number for a 5 m car traveling at 100 km/h is about \( 8 \times 10^6 \). If it is considered approximately to be a plate, this results in turbulent flow with Kolmogorov length, time, and velocity scales of 33 \( \mu \text{m} \), 70 \( \mu \text{s} \), and 0.47 m/s, respectively. Ideally, pressure and wall shear stress sensors would be small enough to resolve these Kolmogorov fluctuations, and not an integrated average over the sensor area.

The performance of state-of-the-art pressure and wall shear stress sensors are summarized in Figs. 1.2(a) and (b) in terms of sensor area and response time, respectively, showing a lack of sensors in the mPa range with spatiotemporal resolution smaller than the Komogorov scales. To

Gain insight as to why achieving high-spatiotemporal resolution is challenging, we can examine the detection limit of deflection-based sensors using the dissipation fluctuation theorem (DFT). Using the DFT, the smallest measurable force over a given bandwidth is given by

\[ \delta F = \sqrt{4k_B T \gamma}, \]  \hspace{1cm} (1.11)

where \( \gamma \) is the drag coefficient [108]. In the case of wall shear stress sensors, the damping forces acting on a suspended membrane arise primarily from a combination of Couette-flow between the membrane and wafer-handle, or the Stokes-flow on the upper surface [109]. In this case, the viscous drag force is expected to have the form

\[ F_d \equiv \gamma \dot{x} = \mu \frac{A}{\delta_d} \ddot{x}, \]  \hspace{1cm} (1.12)

where \( A \) is the area of the membrane, \( \dot{x} \) is the horizontal velocity of the membrane, and \( \delta_d \) is a distance referring to the separation between the membrane and wafer handle, in the case of
Couette-flow, or the Stokes boundary layer given by \( \delta_d = \sqrt{2\mu/(\rho\omega)} \) [109]. In this case, the detection limit to wall shear stress can be determined

\[
\delta_t \equiv \frac{\delta_F}{A} = \sqrt{\frac{4k_BT\mu}{\delta_dA}}
\]  

(1.13)

From this equation, the detection limit of deflection-based wall shear stress sensors is inversely proportional to the square root of the sensor area. At higher frequencies, the Stokes boundary layer diminishes slowly, as can be seen in Fig. 1.3(a). Therefore, at high frequencies, control over \( \delta_d \) is rather limited and the detection limit of wall shear stress sensors depends primarily on the area. This result is independent of the transduction method provided it is sensitivity enough to resolve fluctuations in the membrane position.

Using Eq. (1.13), the detection limit of wall shear stress sensors as a function of length scale (i.e., the square root of the sensor area) is plotted in Fig. 1.3(b). Below a 10 \( \mu \text{m} \) length scale range, there is a drop-off in sensor performance as the detection limit increases rapidly. In contrast, larger area devices see a comparatively little performance boost. From Figs. 1.2(a) and (b), it is apparent that the theoretical limit on wall shear stress sensors has not yet been achieved and what is needed are sensors that can resolve the thermal fluctuations of membranes with dimensions on the order of the Kolmogorov length scale.

![Figure 1.3](image-url)

Figure 1.3: (a) Stokes boundary layer in air as a function of frequency. (b) Detection limit of wall shear stress at different length scales.

### 1.2.2 Magnetic Field Sensing

Magnetic sensors have numerous applications ranging from navigation systems using the Earth’s magnetic field [110]; inexpensive and wear-free sensors for automobiles [111]; electrical current sensors [112]; non-destructive evaluation using eddy currents [113]; the monitoring of cardiovascular [114, 115], muscular [116], and neural [117, 118] activity; and microsatellite applications [119, 120] for attitude control, planetary surveys, and the monitoring of space weather.
In automobile and aerospace industries, the size, weight, power, and cost are (SWaP-C) are the key criteria when designing sensors. Increasingly, magnetometers with optical read-out, including atomic magnetometers, quantum defects in diamonds, and optomechanical systems, are considered to be promising candidates to achieve low SWaP-C [121].

There are numerous examples of magnetic field sensors based on the deformation of microstructures using piezoresistive [122–125], piezoelectric [126], capacitive [127], ray optical [35, 36, 38, 39], and optical cavity [128, 129] read-out. In this case, deflection of the microstructure can be achieved using either the Lorentz force, magnetostrictive materials, or permanent magnets bonded to the microstructure. However, in contrast to pressure and wall shear stress sensors, solid-state measurement of magnetic fields can be achieved using the Hall effect [130–132], magnetoresistive materials [133–136], superconducting quantum interference devices (SQUIDs) [137–139], spin-exchange relaxation-free magnetometry (SERF) [140, 141], and nitrogen-vacancy defects in diamond [142].

Although there are a wide variety of magnetic transduction technologies, they can be generally categorized in terms of their detection limit that restricts their application [143]. Search coils exhibit high sensitivity below 1 pT and are one of the oldest and most well-known types of magnetic sensors. Since these sensors are based on Faraday’s law of induction, only AC magnetic fields can be measured with due to frequency-dependent output and face difficulties with miniaturization [144]. Fluxgate sensors measure down to 10 pT measuring both DC and AC magnetic fields where a soft magnetic material is periodically saturated with an excitation field and pick-up coil is used to measure the higher harmonics due to saturation [145]. Fluxgate sensors are difficult to miniaturized since the magnetic noise rapidly increases with decreasing sensor length [146]. Hall effect sensors are compact and low cost are sensitive to the noise electrodes, calibration, however the detection limit is typically above 1 uT [143]. Giant magnetoresistance (GMR) sensors rely on the magnetic field-dependent resistance arising in a thin conductor sandwiched between ferromagnetic materials [143]. GMR sensors are compact and sensitive to fields as low as 1nT. However, since they are based on ferromagnetic materials, they suffer from hysteresis, signal offsets, and temperature dependent output that can reduce measurement accuracy [147] in addition to Johnson noise arising from the test current. The most sensitive magnetometers are superconducting quantum interference devices (SQUIDs) that are capable of measuring fields lower than 0.01 fT [148]. However, the main drawback from these sensors are the cryogenic temperatures required to maintain superconductivity and their incompatibility with high-field applications that destroy the delicate quantum state. Since biomagnetic fields generated in the human body are in the 1 fT–1 pT range [143], efforts to improve magnetic sensor detection limit without the use of cryogenics is an active area of research which have led to the development of newer technologies such as spin-exchange relaxation-
1.2. Applications

free (SERF) magnetometer which uses alkali metal measuring the precession of alkali metal vapour [149].

The performance metrics for magnetic sensors commonly include the sensitivity and detection limit across a given bandwidth. However, the energy resolution, power consumption, dynamic range, and bandwidth are also important, in addition to whether full vector measurement of the field is possible. Since a magnetic field can be input coupled to the sensor region using pick-up coils, the magnetic energy resolution is a more fundamental metric for comparison that takes into account the volume of the sensor [150]. While the dynamic range and linearity of the bare sensor are important, these can be compensated through closed-loop read-out system designs [143]. Thermal response, calibration, system costs, and ease of integration with existing nanofabrication platforms are also important criteria to consider.

The detection limit and volume of various miniature magnetic sensors are summarized in Fig. 1.4. Since magnetic sensors can be input coupled using flux transformers, the energy resolution of a sensor is considered to be a more fundamental performance metric that takes into account the sensor volume and given by [150]

\[ \delta \epsilon = \frac{\delta_b^2}{2\mu_0}V, \]  

(1.14)

where \( \delta_b \) is the detection limit of the sensor (T/\( \sqrt{\text{Hz}} \)) in terms of the mean magnetic field over a volume \( V \) and \( \mu_0 \) is the vacuum permeability.

Due to their simplicity, operation at room temperature, enhanced sensitivity at mechanical resonance, wide dynamic range, no hysteresis, and low power consumption [151], ray optical MEMS form a versatile class of magnetometers. Detection limits of the order nT with sensor areas of the order millimeters enabling low SWaP-C designs that are currently considered used for space missions [37]. For relatively low drive currents, the detection limit of ray optical magnetic sensors depends directly on the electrical current flowing through the microstructure and is typically expressed in terms of T·A/\( \sqrt{\text{Hz}} \). However, since the detection limit of deflection-based sensors is ultimately determined by temperature-dependent fluctuations in the oscillator position given by Eq. (1.11), there are diminishing returns with using larger electrical currents that heat the microstructure. This presents a trade-off since long microstructures are required to achieve adequate transduction sensitivity yet simultaneously increase the electrical and thermal resistance. Driving the sensor at resonance can enhance the sensitivity; however, this limits the operational bandwidth. A key drawback of current demonstrations of ray optical designs is that the read-out is typically performed off-chip which cannot be readily integrated into monolithic fabrication process.

Cavity optomechanical sensors are theoretically able to achieve the same energy resolution
as SQUIDs [77] and have been implemented as magnetic sensors using magnetostrictive materials operating at room temperature. In the presence of an external magnetic field, expansion of the magnetostrictive alters the mechanical resonance and can be detected in the transmission spectrum [129, 152]. However, these structures relied on non-standardized fabrication methods and currently incompatible monolithic planar fabrication. While numerical analysis silicon photonic crystal cavity infiltrated with a magnetic fluid has been proposed [153], there are few demonstrations of fully integrated photonic magnetometers.

Figure 1.4: Detection limit and sensor volume of various magnetic microsensor technology. Piezoresistive (black diamonds), capacitive (black circles), piezoelectric (black boxes), ray optical (black asterisks), optical cavity (black plus), hall effect (blue circles), SQUID (blue asterisks), SERF (blue plus), and magnetoresistive (blue diamond). Volume was computed using $A^{3/2}$ with the sensor area $A$ found in article. Black markers depict magnetic sensors based on mechanical deflection of a test structure and blue markers indicate sensors without moving parts. (1) Herrera-May et al. (2008) [122], (2) Herrera-May et al. (2015) [123], (3) Mehdizadeh et al. (2014) [124], (4) Kumar et al. (2016) [125], (5) Okada et al. (2018) [126], (6) Kadar et al. (1998) [127], (7) Dennis et al. (2015) [154], (8) Park et al. (2016) [38], (9) Givens et al. (1996) [35], (10) Wickenden et al. (1999) [36], (11) Forstner et al. (2012) [128], (12) Colombano et al. (2020) [155], (13) Kirtley et al. (1995) [137], (14) Shibata et al. (2015) [138], (15) Chatraphorn et al. (2000) [139], (16) Grosz et al. (2016) [130], (17) Petridis et al. (2009) [131], (18-20) Sandhu et al. (2004) [132], (21) Gusrarov et al. (2009) [140], (22) Wang et al. (2018) [141], (23) Shirotori et al. (2021) [133], (24) Quynh et al. (2019) [134], (25) Zhang et al. (2021) [135], (26) Luong et al. (2017) [136],
1.3 Motivation for Optical Microsensors

In all the microsensor technologies and applications reviewed so far, we need to understand and model the forces acting on the microstructure. If the deflection sensor is treated as a damped harmonic oscillator, the smallest resolvable force is fundamentally limited by the statistical thermal fluctuations in oscillator’s position. Using the fluctuation dissipation theorem (FDT), the smallest measurable force over a given bandwidth is given by [108]

\[ \delta F = \sqrt{\frac{2k_B T m \omega_0}{Q}}, \]  

(1.15)

where \( \delta F \), \( k_B \), \( T \), \( m \), \( \omega_0 \), and \( Q \) are force fluctuations in units \( \text{N}/\sqrt{\text{Hz}} \), Boltzmann’s constant \( (1.38 \times 10^{-23} \text{ J}/\text{K}) \), ambient temperature in K, mass in kg, resonant frequency in rad/s, and the quality factor that describes the fraction of energy lost per oscillation of the mechanical oscillator, respectively.

In order to achieve higher signal-to-noise ratios (SNRs), it is desirable for \( \delta F \) to be as small as possible. This can be achieved by lowering either the ambient temperature, resonant frequency, or mass of the microstructure, or by increasing the quality factor. Since lowering of the resonant frequency directly affects the reaction speed of the mechanical oscillator and, in many cases, it is impractical to use cryogenic temperatures, the development of both sensitive and high-speed microsensors is best achieved with smaller masses and larger quality factors. However, there are limitations regarding the mass-scaling of conventional microsensor technology, such as piezoresistive and capacitive transducers, making the measurement of distributed forces, such as pressure, wall shear stress, or magnetic flux density, with high spatiotemporal resolution challenging. As an alternative, optical sensors have shown great potential for scalable, high-precision measurement.

Using single-crystal silicon, which is transparent to infrared light, photonic integrated circuits (PICs) analogous to electrical circuits can be fabricated. Having a higher refractive index than the cladding that surrounds it, light can be confined to regions of silicon defined by surface micromachining using established fabrication methods. In this way, low-loss guiding of light can be achieved either two dimensionally (2D) inside thin silicon films, one dimensionally (1D) along silicon nanowires, or confined to small volumes (0D) using point-like microcavities. Sensors can then be formed by probing the intensity, phase, or polarization of light that is affected by changes to either the core material or its immediate vicinity.

A special type of interferometric device is a directional coupler that is able to transfer optical energy between two adjacent waveguides. Consider the case of two identical and parallel waveguides that are single-mode when spaced far apart. As the waveguides approach one an-
other, they will support two modes, differentiated by their even or odd parity with respect to the symmetry presented by the system. If the field profile of the isolated waveguide can be reconstructed using a superposition of these even and odd modes, then direction coupling will occur and power will be exchanged between the two waveguides due to the difference between their respective effective indices, as shown in Fig. 1.5. The field amplitude in each waveguide labeled WG1 and WG2 can be described by using the isolated field profiles \[ \Psi(z) = A(z)\phi_1 + B(z)\phi_2. \] (1.16)

where \( A(z) \) and \( B(z) \) are the amplitudes of the normalized mode profiles \( \phi_1 \) and \( \phi_2 \) of each waveguide in isolation. The evolution of these complex coefficients are described using coupled mode theory and given by

\[
\frac{A(z)}{dz} = -j\kappa B(z)e^{-j(\beta_1 - \beta_2 - 2\pi m/a)z} \tag{1.17a}
\]

\[
\frac{B(z)}{dz} = -j\kappa A(z)e^{-j(\beta_1 - \beta_2 - 2\pi m/a)z} \tag{1.17b}
\]

where \( \beta_1 \) and \( \beta_2 \) are the propagation constants associated with their respective \( \phi_1 \) and \( \phi_2 \) fields. The strength of the directional coupler is described by the coupling coefficient given by \( \kappa \) in rad/m. To account for possible scattering with periodic structure, the propagation constants can differ by a possible integer multiple \( m \) of the reciprocal lattice vector [157, 158]. In this case, the solution can be obtained through integration of Eqs. (1.17b) and (1.17b)

\[
A(z) = [a_+ e^{-j\beta_1 z} + a_- e^{j\beta_2 z}]e^{-j\Delta z} \tag{1.18a}
\]
1.3. Motivation for Optical Microsensors

\[ B(z) = \left[ b_e \frac{\kappa}{\beta_c - \Delta} e^{-j\beta_c z} - b_o \frac{\kappa}{\beta_c - \Delta} e^{j\beta_c z} \right] e^{-j\Delta z} \]  
(1.18b)

where \( a_{e,o} \) and \( b_{e,o} \) are the complex coefficients determined by the boundary conditions. The parameters in Eqs. (1.17b) and (1.17b) are defined as follows

\[ \beta_c \equiv \sqrt{\kappa^2 + \Delta^2} \]  
(1.19a)

\[ \Delta \equiv \beta_2 - \beta_1 - \frac{2\pi}{a} \]  
(1.19b)

\[ \beta_e = \frac{\beta_1 + \beta_2}{2} + \beta_c \]  
(1.19c)

\[ \beta_o \equiv \frac{\beta_1 + \beta_2}{2} - \beta_c \]  
(1.19d)

If the waveguides are symmetric (i.e. \( \beta_1 = \beta_2 \)), then complete power transfer is possible which can be expressed as

\[ \left| \frac{A(z)}{A(0)} \right|^2 = \cos^2(\kappa z) \]  
(1.20a)

\[ \left| \frac{B(z)}{B(0)} \right|^2 = \sin^2(\kappa z) \]  
(1.20b)

In this case, for the lowest order mode \( (m = 0) \), phase matching is automatically satisfied \( (\Delta = 0) \) and the coupling coefficient can be determined by

\[ \kappa = \frac{1}{2(\beta_e - \beta_o)}. \]  
(1.21)

When the propagation constants of the two waveguides are equal, the beat length \( L_c \) is given by

\[ L_c = \frac{\pi}{2\kappa}. \]  
(1.22)

Thus, under suitable conditions, light initially launched into one of the waveguides, is transferred to the other after some distance known as the beat-length. Since the beat-length depends on the waveguide separation, any change in the separation can be observed directly from the exchange of optical power between the two outputs of a coupler of finite length. This allows direct intensity-based read-out without extra components such as combiners and splitters.

Only recently have directional coupler deflection sensors been studied experimentally, with our work in Chapter 2 being perhaps the first [160] which was later followed up by another study using directional couplers based on conventional index-guided waveguides on SOI [159]. Using conventional waveguides, high deflection sensitivities can be achieved by narrowing of
the waveguides. However, for coupling to distributed forces, the waveguides need to be coupled to a central area. While this can be achieved using a partially etched waveguide cladding to attach the waveguide to a central sensor area, this negates the strategy of waveguide narrowing as more of the EM field is drawn into the cladding, as shown in Fig. 1.6. In order to maintain low beat-lengths, a fully etched cladding is therefore required on both sides of the narrowed waveguides. Alternatively, if the cladding is provided instead by the photonic band gap of a photonic crystal, light can be simultaneously guided along the edge of a silicon membrane without sacrificing the area required for a fully etched cladding. Moreover, directional coupling based on PC edges can take advantage of slow-light dispersion effects that enhance optomechanical sensitivity and enable smaller design footprints. In the next section we will review the silicon photonics fabrication technology followed by photonic crystal directional coupler which was the focus of this thesis work.

1.4 Silicon Photonics Fabrication Platform

Due to the multi-billion-dollar electronics industry [161], silicon is perhaps the most well studied material on the planet with countless studies performed to characterize doping with impurities, annealing, thermal oxidization, and surface micromachining while being widely available as single-crystal silicon-on-insulator (SOI). Additionally, surface micromachined SOI can be undercut with post-foundry selective etching of the buried oxide using hydrofluoric acid to create suspended-element microstructures [162]. Silicon photonic platforms are seen as a promising platform for SWaP-c reductions compared to existing III-V or purely electronics circuits for use in national security applications [163].

Open-access silicon photonics foundries are widely available [164] with device layer thickness of 220 nm being common due to improved performance, simplified fabrication processes,
and relaxed tolerances in the SCL-band (1460–1610 nm) devices [165]. This bandwidth is of particular interest since it experiences minimal attenuation in commercial silica-based optical fibers. Most foundries commonly allow access to multiple silicon etch depths, as illustrated in Fig. 1.7, enabling the design of efficient surface grating couplers and rapid prototyping. A summary of silicon etches available at various silicon photonic foundries is shown in Table 1.1.

![Cross section of various low loss 220 nm silicon photonic fabrication features](image)

Figure 1.7: Cross section of various low loss 220 nm silicon photonic fabrication features offered.

<table>
<thead>
<tr>
<th>Silicon height (post etch)</th>
<th>OpSIS-IME</th>
<th>CEA-LETI</th>
<th>IMEC</th>
<th>IHP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unetched silicon (nm)</td>
<td>220</td>
<td>220</td>
<td>220</td>
<td>220</td>
</tr>
<tr>
<td>Grating coupler layer (nm)</td>
<td>160</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Slab layer (nm)</td>
<td>90</td>
<td>100</td>
<td>60</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 1.1: Etch profiles of common silicon photonic foundries [166].

While silicon photonics allows for the fabrication of inexpensive layouts, one of the main drawbacks of silicon photonics is the lack of efficient integrated lasing sources. This is due to the fact that silicon has an indirect band gap and resists epitaxial bonding with direct band gap materials. Despite recent advances integrating Er-related light sources, Ge-on-Si lasers, and III-V-based lasers onto silicon, more research is needed to lower threshold currents or improve bonding techniques [167]. However, there has been recent progress toward integrating III-V PC membrane lasers using quantum dots [168]. Alternatively, different wafer materials may be used altogether, with indium phosphide being a direct bandgap III-V material widely used for integrated photonics that allows efficient optical sources and detectors to be integrated [169] suitable for MEMS [170].

In contrast to optical sources, efficient Ge-based detectors exist for silicon photonic integrated circuits (PICs) [171, 172] and widely available in the process design kits (PDKs) among fabrication foundries. On one hand, optical microsensor technology can benefit from the integration of photodetectors. On the other hand, it may be desirable to maintain optical fibre coupling to and from the chip since high-density wire-bonding on the surface of the chip could interfere with sensitive measurements such as wall shear stress [30, 31] and are susceptible to
EMI. Coupling light to silicon PICs is generally achieved using either surface grating couplers (SGC) or edge couplers; however, more novel approaches do exist [173].

1.5 PCDC Optomechanical Systems

To form a compact, low-loss, broadband, optical deflection sensor, a directional coupler (DC) can be used where two identical waveguides are brought near to each other. Sensors based on coupled waveguides do not depend on buried oxide thickness, thereby eliminating the risk of stiction that affect capacitive and substrate-coupled optical sensors. Additionally, differential sensing modality mode is enabled by measuring the difference between both output ports of a DC that can remove common-mode noise components. This provides an advantage over dissipative optical sensors since optical energy can potentially be recovered. Since the sensitivity of such a deflection sensor is related to the length of the waveguide, it scales more favourably for the purposes of measuring distributed forces, in contrast with capacitive MEMS, that scale with area.

A common way of enhancing the performance of integrated photonic devices is by etching a periodic pattern into material, forming a photonic crystal (PC). If the wavelength of light matches the repetition distance of the PC, a resonance can occur where light is effectively slowed down due to the multiple coherent scattering interactions. It is a remarkable fact of PCs that it is possible to pattern transparent media such that the propagation of light is forbidden in any number of directions, effectively acting as a mirror. The continuous range of frequencies where propagation is forbidden is known as the photonic band gap (PBG). Using the PBG, a repeating pattern of holes can be introduced into a thin silicon membrane and light can be confined to propagate along its edge while the PC membrane provides a surface to couple to distributed forces. Previous studies indicate pairs of closely spaced PC line defects can form low-loss directional couplers with reduced coupling-lengths [174–177]. Therefore, it is expected the directional coupling between mechanically isolated PC edges will form a compact optical deflection platform.

Photonic crystal directional coupler (PCDC) deflection sensors can enjoy the benefits of all-optical systems such as size, sensitivity, bandwidth, and immunity to electromagnetic interference (EMI). As a weakly coupled system, the optical and mechanical properties of the PC membranes can be engineered independently as the sensor stiffness can be altered without sacrificing displacement sensitivity, in contrast to piezoresistive sensors. Moreover, optical read-out can be performed using low impedance photodetectors, thereby reducing Johnson noise. Photonic nanosensors based on thin film patterning of single-crystal silicon are compatible with established silicon photonic foundries, enabling further photonic microsystems inte-
1.6 Thesis Objectives

The overall goal of this thesis was to design, fabricate, and test PC edge-coupled sensors capable of outperforming state-of-the-art deflection, fluid, mass, and magnetic field based microsensors in terms of their detection limit and spatiotemporal response. This overall goal was divided into objectives aimed at motivating and characterizing key performance characteristics of the PCDC sensors within a particular sensor context. The objectives are summarized as follows:

- Motivate PC edge-coupled sensors (Chapter 1)
- Design, fabricate, and characterize PCDC sensors using static displacements (Chapter 2)
- Demonstrate broadband optical response (Chapter 2)
- Test PCDC sensors using air-coupled ultrasound (Chapter 3)
- Acquire spatial response of PCDC-based wall shear stress sensors (Chapter 4)
- Calibrate wall shear stress sensor using thermal noise floor (Chapter 4)
- Characterize crosstalk between wall shear stress and dynamic pressure (Chapter 5)
- Characterize and mitigate the effects of PC-edge misalignment (Chapter 6)
- Determine added-mass sensitivity of PC edge-coupled membranes (Chapter 7)
- Design, fabricate, and test of PCDC magnetic field sensors (Chapter 8)

1.7 Thesis Structure

The results of this thesis are organized into an integrated-article format where Chapters 2–8 are presented as publishable stand-alone documents addressing the performance of PC edge-coupled sensors within a particular measurement context. Due to the nature of the integrated-article thesis format, there may be some repetition of topics and discussion points.
Chapter 2 is the first of the integrated-articles and describes the design, fabrication, and testing of PC edge-coupled deflection sensors. Both plane wave expansion (PWE) and finite-difference time-domain (FDTD) numerical simulation techniques were used to model the PC edge-coupled sensors. Sensors with varying coupler lengths were fabricated and the optical transmission spectrum was measured for static horizontal and vertical displacements and found to validate directional coupling between the two PC edges.

Chapter 3 presents the measured optical response of the PCDC sensors driven by coiled ultrasonic speaker. By measuring both outputs of the PCDC with a balanced photodetector, noise common to both outputs could be reduced while the sensitivity was enhanced.

Chapter 4 presents the spatial response of the PCDC-sensor to wall shear stress using a novel thermoacoustic integrated photonics testbench constructed by combining a thermoacoustic emitter (TAE) and acoustic waveguide (AW). Using finite element method (FEM), the mechanical response of the sensor was analyzed using normal mode decomposition and the sensor was calibrated using the acquired thermal noise floor with basic assumptions regarding the mass and effective area of the sensor.

Chapter 5 investigates the effect of vertical misalignment of PC edges on the crosstalk between the dynamic pressure and wall shear stress using the newly developed TAE testbench.

Chapter 6 demonstrates the mitigation of vertical misalignment of PC-edges using microbeam arrays that allow the relaxation of compressive stress built-up in the SOI wafer that leads to buckling.

Chapter 7 models the PC membrane and supports as a simple spring-mass system and compares the effect of varying the mass of the suspended membrane. Changes in the mass of the PC membrane were introduced by fabricating identical structures differing only by their PC hole diameters and observed through shifts in the horizontal mechanical resonance.

Chapter 8 is the last of the integrated-articles and describes the design, fabrication, and testing of PCDC magnetic field sensors based on the Lorentz force. In the presence of a magnetic field, electrical current injected across metallized PCDC microstructures exerted a force on the membrane whose deflection was measured using the optical read-out provided by coupled PC edges. Sensor performance was modeled using lumped-elements that took into account the optical, magnetic, mechanical, thermal, and fluid damping effects that were found to be in close agreement with the measurements.

Finally, Chapter 9 relates the separate studies to each other and emphasizes the relevance of PC edge-coupled sensors in the context of microsensor technology and identifies potential applications and future work. While the chapters of the integrated-article are included as they appear in publication without revision, connecting text and figures are included between the chapters to provide logical bridges, integration of information, and may include additional
information not available at the time of the publication.

Bibliography


Chapter 1. Introduction


Chapter 2

Photonic Crystal Slab Edge Directional Coupler for Deflection Sensing

This article introduces the design, fabrication, and static transmission measurements of a photonic crystal directional coupler (PCDC) for submicron deflection sensing. Both plane wave expansion (PWE), finite-difference time-domain (FDTD) simulation were used to determine the coupling coefficient and beat-length of the PCDC. Static displacements were introduced using either (1) fabrication-defined horizontal separations determined during the layout design stage or (2) vertical misalignment due to the buckling of fully released microstructures due to compressive strain present in the SOI. Optical transmission measurements were performed using a silicon photonics microsystems integration platform (SiPh-MIP) that allowed automatic optical alignment to surface grating couplers and found to agree with simulation results.

2.1 Introduction

Micro-optomechanical systems (MOMS) are relevant where precision, high speed, remote sensing, and immunity to electromagnetic (EM) interference are desirable [1]. An important class of MOMS is one which makes use of the observed changes in optical transmission associated with near-field coupling to measure the separation between a waveguide and a nearby substrate, membrane or ancillary waveguide. In such systems, the optical sensitivity to perturbations in element separation depends on the coupling strength which can be enhanced through manipulation of the evanescent field by waveguide thinning [2, 3], or by the use of dispersive media such as suspended photonic crystal (PC) membrane line defects [4]. However, thin

\[1\] A version of this chapter has been published. Michael Zylstra, Aref Bakhtazad, and Jayshri Sabarinathan, published in *Optics Express* 27, 38509-38520 (2019). Reprinted (adapted) with permission from the authors and OSA. ©2019 OSA. (see Appendix A for copyright permission)
substrate-coupled membranes with high aspect ratio are prone to stiction due to the presence of undesired adhesion forces between closely separated microstructures [5]. This stiction can be mitigated by considering lateral coupling between closely spaced waveguides, instead of vertical coupling to the wafer substrate. In such a configuration the thickness of the buried oxide (BOX) is no longer an issue.

Slotted waveguide structures have been demonstrated by several groups, where the air-like modes (with even transverse electric parity) are particularly sensitive to changes in refractive index of the air gap. Enhancement of the air gap field has been demonstrated using periodic structures [6–8], or ring-assisted structures [9] for biochemical sensing applications. However, measuring changes in effective index requires expensive tunable lasers to observe changes in cavity resonances, or integrated interferometric methods that increase measurement complexity and layout footprints. In this work we instead examine the dielectric-like modes of photonic crystal edges such that both even and odd transverse electric (TE) modes overlap within the same transmission band allowing for the design of a directional coupler. By using a directional coupler as a sensor element, we reduce the measurement complexity as relative changes in the effective index between the even and odd modes are directly observable as changes in intensity at the output ports.

In a symmetrical directional coupler, power can be completely cycled between the two waveguides along the length of the coupler [10]. For directional couplers fabricated on silicon-on-insulator (SOI) with 220 nm top silicon, broadband coupler designs for 50:50 power splitting can be achieved using coupling lengths of 100–200 µm [11] while state-of-the-art designs have achieved 50:50 splitting around 20 µm (40 µm beat-length) [12]; however, this can be further reduced using a pair of PC line defects with low insertion losses [13] and beat-lengths as low as 4 µm [14]. Directional couplers assisted by periodic structures also exhibit beat-lengths that can be desensitized to variations in wavelengths [15, 16]. The key advantage to using a directional coupler sensor assisted by a periodic structure is the reduced device footprint size while maintaining compatibility with inexpensive broadband light sources. The higher mechanical resonance enabled by smaller devices also results in flatter dynamic response; an important consideration for sensor array applications.

In this paper, the design, fabrication, and transmission of a directional coupler formed by silicon photonic crystal slab edges is presented. In section 2, a model for a directional coupler-based sensor is developed and numerically simulated. In section 3, the fabrication of PCDC sensors on SOI is described. In section 4, the optical transmission measurements obtained from PCDCs with different horizontal and vertical separation distances between the PC slab edges is presented followed by a conclusion in section 5. The obtained measurements validate the operating principle of the proposed sensor and its suitability as a broadband deflection sensor.
element.

2.2 Design and Analysis

2.2.1 Analytical model for directional coupler based sensor

![Diagram](image)

Figure 2.1: (a) PCDC with a fundamental TE mode excitation input, through output, and coupled output ports labelled black, red, and blue, respectively. (b) Cross-section showing PC edges with horizontal separation $s$ and vertical deflection $h$ (c) PCDC junction as viewed from top. All units in nm.

The directional coupler sensor geometry consists of two parallel waveguides, as shown in Fig. 2.1(a), separated by a horizontal separation $s$ and vertical separation $h$, as shown in Figs. 2.1(b) and 2.1(c). The coupling is considered uniform along an interaction length $L$ and negligible elsewhere. The excitation of a single waveguide creates a superposition of even and odd modes and power is completely exchanged between the waveguides along the length of the coupler [10]. The input coupling to the coupler is assumed sufficient to ignore Fabry Pérot cavity effects. In this paper we define the through port to be on the same half as the excitation and the coupled port on the opposite side as can be seen in Fig. 2.1(a).

The strength of a directional coupler can be described by the coupling coefficient $\kappa$ which is related to the difference between the odd $k_o$ and even $k_e$ wavevectors [17] and is given by

$$\kappa = \frac{1}{2} (k_o - k_e) \quad (2.1)$$

Alternatively, the length required for power to be completely transferred from one waveguide to the other, also known as the beat-length, can be expressed as the reciprocal of the coupling coefficient given by

$$L_b = \frac{\pi}{2\kappa} \quad (2.2)$$
Ignoring propagation losses, the transmission at the through output $P$ of a four-port directional coupler [18] is given by

$$P = P_0 \cos^2 \theta$$ (2.3)

where

$$\theta = \kappa L$$ (2.4)

is the accumulated phase difference between the even and odd modes and $P_0$ is the power seen at the through output in the limit of large separations. The change in optical power with respect to small changes in accumulated phase can be obtained by differentiating Eq. (2.3) which gives:

$$\Delta P = -P_0 \Delta \theta \sin 2\theta$$ (2.5)

A general conclusion of this model is that the sensitivity of $P$ with respect to $\theta$ is maximized whenever the absolute value of the sine term in Eq. (2.5) attains a value of 1 and power is equally shared at the outputs. This occurs at select $\theta$ values given by

$$\theta_n = \left(\frac{n}{2} + \frac{1}{4}\right) \pi$$ (2.6)

where $n$ is an integer.

We consider the case where the two waveguides remain parallel even when one of the waveguides is perturbed by $\Delta x$ in the horizontal direction ($x$) and by $\Delta y$ the vertical direction ($y$). In this case, the change in accumulated phase difference is given by

$$\Delta \theta = L \left[ \frac{\partial \kappa}{\partial x} \Delta x + \frac{\partial \kappa}{\partial y} \Delta y \right]$$ (2.7)

Since the coupling strength is related to the evanescent field overlap, we expect it to have an exponential relationship with respect to separation. Hence, we define a sensitivity parameters $\alpha_x$ and $\alpha_y$ to be given by the logarithmic partial derivatives of $\kappa$ that describes the spatial decay rate of the coupling strength in each direction given by

$$\alpha_x = \frac{1}{\frac{\kappa}{\partial x}}$$ (2.8a)

$$\alpha_y = \frac{1}{\frac{\kappa}{\partial y}}$$ (2.8b)

By adopting this convention, the relative change in optical power for small displacements along
each direction is given by

\[ \frac{\partial P}{\partial x} = P_0 \alpha_x \theta \sin 2\theta \]  
(2.9a)

\[ \frac{\partial P}{\partial y} = P_0 \alpha_y \theta \sin 2\theta \]  
(2.9b)

The expressions given in Eqs. (2.9a) and (2.9b) are referred to in this manuscript as the horizontal and vertical sensitivities of the designed PCDC.

### 2.2.2 Bulk PC Band Structure

The PCDC sensor was designed to operate in the C-band (1530–1565 nm) which corresponds to the minimally attenuated spectral range of silica based optical fibres. To confine light within the PC defect, the photonic band gap (PBG) must be engineered to span the desired operating wavelengths. For sufficient dielectric contrast, large PBGs are known to exist for TE modes in a triangular PC lattice of holes-in-slab [19]. The thickness of the top silicon layer was set to 220 nm by the fabrication process and the PBG was computed with plane wave expansion (PWE) using RSoft across a range of lattice pitches \( a \) and hole diameters \( d \). A pitch of 450 nm and hole diameter 270 nm was found to generate a suitable band structure that satisfies the PBG requirements, which is shown in Fig. 2.2(a).

### 2.2.3 PCDC Band Structure - Edge Mode Determination

The projected band structure of a PCDC consisting of two edge regions of width \( w = 540 \text{ nm} \) and separated by an air gap of width \( s = 200 \text{ nm} \) is shown in Fig. 2.2(b). It features two pairs of defect modes within the PBG. The pair of defect modes near the upper PBG edge have their field concentrated within the air gap. Although the even air-mode is most sensitive to changes in gap geometry, the odd air-mode is strongly blue-shifted due to its TE node in the gap centre [6], and insufficient overlap exists for directional coupling to occur.

The pair of defect modes appearing near the lower PBG have a portion of their field concentrated in the dielectric region as shown in Fig. 2.2(c), and their properties are predominantly determined by the extended PC edge width. The width of the extended edge region was tuned to 540 nm such that the dielectric modes of the projected band structure are centred on the C-band.

The dielectric modes were sufficiently unaffected by the air gap such that directional coupling could occur across a range of frequencies wherever both modes were supported inside the light line. However, enough field energy remains in the air gap for the even mode, as shown in
Figure 2.2: PWE simulation results for (a) TE photonic band structure for a hole-in-slab photonic crystal of thickness 220 nm, pitch 450 nm, hole diameter 270 nm, and $n = 3.47$ [20]. (b) The projected TE band structure for PCDC with $w=540$ nm and $s=200$ nm. (c) $E_x$ field profile of the even and odd dielectric directional coupling modes at the band edge ($k_z a/2\pi = 0.5$). (d) Coupling-lengths at various separations.

Fig. 2.2(c), resulting in optomechanical coupling that pulls down the dispersion for decreasing separation while the dispersion of the odd mode remains stable. For the dielectric-like directional coupling modes, the even mode with the nonzero air gap field has the higher effective index than the odd mode. This seems counter intuitive but is consistent as the gap separation goes to zero. Here, the even mode transitions to the fundamental TE mode while the odd mode transitions to the second order TE mode. In this case, the fundamental TE mode should have the higher refractive index than the second order mode, as is typical for conventional waveguides.

The post-processed coupling-lengths were computed using Eq. (2.1) and are shown in Fig. 2.2(d) for various horizontal separations $s$. The curves in Fig. 2.2(d) are only defined where...
the even and odd modes are both supported simultaneously. A nonlinear relationship between $L_b$ and wavelength can be observed, however the curves are reasonably flat near wavelength of 1550 nm allowing $\kappa$ to be insensitive to the change in excitation wavelength around that region. The beat-length of the PCDC is less than 15 $\mu$m. For comparison, the TE beat-length for a pair of silicon waveguides measuring 220 nm by 500 nm that are horizontally separated by 200 nm is 87 $\mu$m computed using Lumerical MODE solver. This shows that at least 5 to 6 fold reduction in coupler length can be obtained in PCDC’s.

### 2.2.4 3D FDTD Transmission and Coupling Strength of PCDC

Three-dimensional finite-difference time-domain (3D FDTD) calculations with Lumerical was used to simulate the transmission from a PCDC of finite length. The PCDC is interfaced asymmetrically with rib and strip waveguides as shown in Fig. 2.1(c). On the through side of the directional coupler, a 150 nm partial silicon etch defines the cladding of a rib waveguide intended to protect the BOX during selective etching to release the membrane. The coupled side, which forms the edge of the released PC membrane, was interfaced with strip waveguides that support the membrane after selective etching. The fundamental TE mode of the interfacing rib waveguide was used as the FDTD excitation source.

The through transmission from coplanar ($h=0$) PC edges of length $54a$ is shown in Fig. 2.3(a) for varying air gap separations. A broadband regime can be identified centred around 1540 nm, where the $\kappa$ dependence on wavelength is minimized. The fringes seen are due to Fabry Pérot effects arising from reflections at the input and output junctions. At 1540 nm, the through transmission increases from zero to its maximum value as the horizontal separation $s$ increases from 130 nm to 200 nm and the directional coupler transitions from $L = 3L_b$ to $2L_b$. Figure 2.3(b) shows the exchange of power between the output ports of the coupler as the PC edges are separated. As the coupler and waveguide losses have been subtracted from the transmission loss, the observed 2.2 dB loss is due to the conversion of the fundamental TE mode of the rib waveguide to the even and odd DC modes, indicating a loss of 1.1 dB per junction. In the flat band below 1515 nm, the even mode becomes radiative and only the odd mode of the directional coupler is supported. This is consistent with the PWE simulation. Therefore, the phase difference between the even and odd mode is no longer observable in the transmission.

The change in the coupling coefficient $\kappa$ with air gap separation $s$ is plotted in Fig. 2.3(c) and was evaluated using Eq. (2.2). The beat-lengths $L_b$ required in Eq. (2.2) were determined from the simulated field profiles along the propagation length of the directional coupler. The field profiles for two separations, $s=150$ nm and 200 nm respectively are shown in Figs. 2.4(a) and 2.4(b) from which the beat-lengths of $20a$ and $27a$, respectively are obtained (i.e. the dis-
tance where the first complete exchange of power occurs). The oscillation of the field profiles is due to the periodicity of the PC which arises as per the Bloch theorem. Similarly, the through transmission from PC edges separated by 200 nm, 350 nm, and 510 nm for varying vertical displacements is shown in Fig. 2.5.

Figure 2.3: FDTD simulation results for coplanar PC edges \((h = 0)\) of length \(54a\), (a) the through transmission spectra at varying air gap separations. (b) At 1540 nm, power is exchanged between the through and coupled outputs as the PC edges are separated. (c) The coupling coefficient \(\kappa\) computed using Eq. (2.2) and the beat-lengths determined by the mode profiles.

Figure 2.4: FDTD simulation results for the \(|E_x|^2\) field profile along the propagation length of a PCDC with (a) \(s = 150\) nm and (b) \(s = 200\) nm.

2.2.5 PCDC Sensitivity

The change in optical power due to small changes in horizontal (x-direction) and vertical spacing (y-direction) is measured by the horizontal and vertical sensitivities which can be computed
Figure 2.5: FDTD simulation results for PC edges separated by 200 nm and length 54a. (a) the through transmission spectra at varying vertical displacements. (b) At 1540 nm, power is exchanged between the through and coupled outputs as the PC edges are separated. (c) The coupling coefficient $\kappa$ was computed using Eq. (2.2) and the beat-lengths determined by the mode profiles.

using Eqs. (2.9a) and (2.9b) respectively. Both the sensitivities are dependent on the product of respective $\alpha$ and $\theta$ values. To evaluate the sensitivity parameter of the PCDC, the logarithmic derivative of $\kappa$ in each direction namely $\alpha_x$ and $\alpha_y$ were computed using Eq. (8). The computed values of $\alpha_x$ and $\alpha_y$ for different $s$ and $h$ values are shown in Figs. 2.6(a) and 2.6(b) respectively.

For the coplanar case ($h=0$), from Fig. 2.6(a) we notice that a smaller $s$ value results in a larger $\alpha_x$ value and likewise from Fig. 2.3(c) a smaller value of $s$ ensures a larger $\kappa$ and thereby also of $\theta$ via their relationship expressed in Eq. (2.4). This ensures that a smaller $s$ value results in a larger value of $\alpha_x \theta$, thereby achieving larger horizontal sensitivity in the coplanar case. In addition, due to the symmetry that exists about the mid-slab plane, $\alpha_y$ remains zero regardless of the value of horizontal separation $s$ as seen in Fig. 2.6(b). Therefore, to capture vertical deflections using the PCDC one requires an initial vertical displacement of the PC edges, i.e, non-zero value for $h$ at any value of $s$.

For the non-coplanar case ($h > 0$), we notice from Fig. 2.5(c) that at any given fixed value of $h$, $\kappa$ and thereby $\theta$ via Eq. (2.4) decreases with increasing $s$, and likewise from Fig. 2.6(b) we notice a similar trend for $\alpha_y$. Hence the vertical sensitivity also decreases with increasing $s$. In contrast, at a given value of $s$, increasing $h$ decreases the value of $\kappa$ and thereby $\theta$ with values approaching zero at larger values of $h$ (see Fig. 2.5(c)), while increasing $h$ increases the value of $\alpha_y$ (see Fig. 2.6(b)) with the rate of increase reducing to zero at larger values. This ensures that their product reaches an optimum value at some intermediate value of $h$, which was computed to occur around $h=300$ nm.
2.3 Fabrication

The PCDC structures were surface micromachined using IMEC ePIXfab silicon photonic process supported through CMC Microsystems. Selective etching of the BOX was done at the University of Western Ontario Nanofabrication Facility. PCDCs of varying lengths were fabricated to measure the power output behaviour corresponding to various multiples of the beat-length.

The fabricated structure, as shown in Fig. 2.7(a), consists of a central PC membrane whose two parallel edges form the inside half of two separate PCDCs. The strip waveguides of the PCDC also act as mechanical supports to the membrane. Both output ports of one of the two PCDCs were accessed and used to validate the directional coupling of the PCDC. The PC edge that forms the outer half of a PCDC is interfaced with rib waveguides that remain rigidly connected to the substrate after BOX etching whereas the PC edge of the inner half located on the membrane is free to move.

Figure 2.7: SEM micrograph of PC membrane before BOX etching showing (a) overall view, and PCDC with air gap separations (b) 191 nm and (c) 278 nm

Scanning electron microscopy (SEM) images were taken before releasing the PCDC membranes and they are shown in Figs. 2.7(b) and 2.7(c). The PC pitch, hole diameter, and edge
width shown in the images have values of 450 nm, 275 nm and 545 nm respectively. To evaluate the horizontal sensitivity, structures with air gap widths of 190 nm and 280 nm were fabricated. Selective etching of the BOX was done using a buffered hydrofluoric acid wet-etch at room temperature followed by critical point CO$_2$ drying.

The wet-etch was done using either a "partial" or a "full" photoresist mask. Compressive strain known to exist in flip-bonded SOI can lead to buckling of the membrane. The extent of the buckling depends on the support structure length, geometry, and imperfections [21, 22]. By using a partial mask on one set of devices to reduce the length of the support structures, buckling was avoided, and coplanar PC edges were obtained (corresponding to $h=0$). Additionally, using a full photoresist mask, the PC membrane was fully released from a second set of devices and the subsequent buckling introduced a vertical displacement of 300 nm between the PC edges. The vertical sensitivity was obtained by comparing the transmission between the partially etched and fully etched structures, as shown in Figs. 2.8(a) and 2.8(b) respectively.

![Figure 2.8: Optical images of the PC membrane after (a) partial BOX etch and (b) full BOX etch.](image)

### 2.4 Measurement Results and Discussion

The devices were excited with a polarization-maintaining fibre optic array positioned near the chip surface and aligned to grating couplers (GC). A tunable laser source providing 1 mW excitation was swept across a spectrum range from 1460–1610 nm while a power meter recorded the transmission at each optical frequency. Transmission was normalized using the spectrum acquired from a blank device included on the chip that allowed for the removal of the GC artefacts.

#### 2.4.1 Dependence on Horizontal Separation $s$

For devices with the partial BOX etch corresponding to $h=0$, the through transmission from PCDCs of varying lengths was measured. The results from the measurements are shown in
2.4. Measurement Results and Discussion

Fig. 2.9 for two different horizontal separations of 190 nm and 280 nm respectively. The measured transmission spectra when compared with FDTD simulations (Fig. 2.3) show an overall 40 nm shift which can be attributed to deviations from design geometry in the fabricated devices.

In Figs. 2.9(a) and 2.9(b), a 30 nm wide band in the transmission centred at 1495 nm can be observed with a peak value which changes with the length of the PCDC. From these measured results, the beat-length at each air gap separation $s$ was extracted by fitting a curve using Eq. (2.3) for the through transmission values of PCDCs with different fabricated lengths $L$ at a wavelength of 1495 nm. This is illustrated in Figs. 2.10(a) and 2.10(b) for air gap separations $s$ of 190 nm and 280 nm respectively. Here, the $P_0$ required was determined from the transmission measured through an isolated PC edge. The beat-lengths were computed to be $22a$ and $33a$ for PCDC separations 190 nm and 280 nm respectively. The coupling coefficients $\kappa=0.16 \text{ rad}/\mu\text{m}$ and $\kappa=0.11 \text{ rad}/\mu\text{m}$ respectively were computed using Eq. (2.2).

The horizontal sensitivity parameter $\alpha_x$ was computed using Eq. (2.8a). Since measurements were obtained for only two values of the horizontal separation, the derivative was replaced with a two-point finite-difference approximation. The coupling coefficient $\kappa$ used in Eq. (2.8a) was determined by taking the geometric mean ($\kappa = \sqrt{\kappa_1 \kappa_2}$, where $\kappa_1$ and $\kappa_2$ are the coupling coefficients at horizontal separations 190 nm and 280 nm respectively). This resulted in an $\alpha_x$ of 4.2 $\mu\text{m}^{-1}$.

For the specific device with a PCDC length of $54a$, horizontal separation of $s=190$ nm, and vertical separation $h=0$ nm, $\theta$ was computed to be $1.23\pi$ using Eq. (2.4) and the horizontal sensitivity of this specific device was computed to be 1.6% of $P_0$ per nanometer ($%/\text{nm}$), using Eq. (2.9a).

![Figure 2.9](image)

Figure 2.9: The measured through PCDC transmission spectra with partially etched BOX and fixed PC edge width 545 nm and air gap width measured to be (a) 190 nm and (b) 280 nm, respectively.
Figure 2.10: The through PCDC transmission at 1495 nm (blue dots) for PCDCs fabricated with different lengths fit to Eq. (2.3) (dashed line) for a separation of (a) $s=190$ nm and (b) $s=280$ nm.

### 2.4.2 Dependence on Vertical Separation $h$

The transmission from both the through and coupled output ports of a PCDC of length $28a$ and horizontal separation $s=190$ nm with partial and full BOX etch are shown in Figs. 2.11(a) and 2.11(b) respectively. An exchange of power can be seen between the coupler outputs, thereby validating the operating principle of the sensor. We observed that in the buckled position the fully released PC membrane had a displacement of 300 nm (corresponding to $h=300$ nm). This value was obtained with an optical microscope using a focus variation method.

A similar broadband change in transmission was observed at both outputs centred at 1495 nm where the beat-length is maximized, as shown in Figs. 2.11(a) and 2.11(b). For both vertical positions, both directional coupling modes are supported between 1485–1534 nm. At wavelengths longer than 1534 nm only the even mode exists and the outputs become equal. Additionally, as can be seen in Figs. 2.11(a) and 2.11(b), the cut-off frequency of the even mode changes from 1550 nm at $h=0$ nm to 1540 nm at $h=300$ nm which can be attributed to the increase in PC edge separation.

The vertical sensitivity is computed similarly to the horizontal case. The vertical sensitivity parameter $\alpha_y$ was computed using Eq. (2.8b). Since measurements were obtained for only two values of the vertical separation, the derivative was replaced with a two-point finite-difference approximation. The coupling coefficient $\kappa$ used in Eq. (2.8b) was determined by taking the geometric mean ($\kappa = \sqrt{\kappa_1 \kappa_2}$, where $\kappa_1$ and $\kappa_2$ are the coupling coefficients at vertical separations 0 nm and 300 nm respectively). However, in this case we compute the coupling coefficient at $h=300$ nm to be 0.054 rad/µm using Eq. (2.3) where $P_0$ was taken to be the sum of the coupled and through transmission outputs. This resulted in an $\alpha_y$ of 3.8 µm$^{-1}$.

For the specific device with a PCDC length of $28a$, horizontal separation of $s=190$ nm, and vertical separation $h=300$ nm, $\theta$ was computed to be $0.22\pi$ using Eq. (2.4) and the vertical...
2.4. Measurement Results and Discussion

The measured through and coupled transmission spectra for a PCDC with air gap width \( s = 190 \) nm and length \( 28a \) (a) with partial BOX etch \((h=0)\) and (b) with full BOX etch \((h=300\) nm).

Sensitivity of this specific device was computed to be 0.25 \%/nm, using Eq. (2.9b).

2.4.3 Discussion

The values of coupling coefficient \( \kappa \) computed from measurements show good agreement to the ones computed from FDTD simulations and this comparison is shown in Table 6.1. We observed a reasonable agreement between the measured and simulated values with \( \alpha_{x,\text{meas}} = 4.2 \) \( \mu m^{-1} \) and \( \alpha_{x,\text{FDTD}} = 5.0 \) \( \mu m^{-1} \) taken at the midpoint \( s = 235 \) nm and with \( \alpha_{y,\text{meas}} = 3.8 \) \( \mu m^{-1} \) and \( \alpha_{y,\text{FDTD}} = 1.4 \) \( \mu m^{-1} \) taken at the midpoint \( h = 150 \) nm. Since only two-point derivatives were used to evaluate \( \alpha_x \) and \( \alpha_y \), there is some uncertainty in how these sensitivity parameters evolve between the different separations, particularly in the vertical direction near \( h = 0 \).

We attribute the 40 nm blue-shift observed between the simulated and measured transmission spectrum to variations in the dimensions of the finer geometric features in the fabricated membrane which can include larger PC hole diameters, smaller edge widths and larger air gap separations, or possibly a thinner device layer, compared to the computational model. Uncertainties in the SEM measurement due to charging effects, which can be significant in these membrane structures, also contribute to the differences. The shift can be accounted for within the tolerance uncertainties in fabrication and SEM measurements. However, the response of the device is as expected and the working principle of the PCDC design has been validated.

<table>
<thead>
<tr>
<th>( h ) [nm]</th>
<th>( s ) [nm]</th>
<th>( \kappa_{\text{meas}} ) [rad/( \mu m )]</th>
<th>( \kappa_{\text{FDTD}} ) [rad/( \mu m )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>0.16</td>
<td>0.14</td>
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<tr>
<td>0</td>
<td>280</td>
<td>0.11</td>
<td>0.087</td>
</tr>
<tr>
<td>300</td>
<td>190</td>
<td>0.054</td>
<td>0.082</td>
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</table>
2.5 Conclusion

A photonic crystal directional coupler utilizing two adjacent parallel PC edge modes was designed, fabricated, and demonstrated for use in optomechanical sensing applications. The devices were fabricated using industry standard silicon photonic fabrication processes followed by selective wet-etching of the BOX. The vertical sensitivity in this work is comparable to our previous work on vertical substrate-coupled PC sensors that had reported a peak sensitivity of 0.5 %/nm for a PC line-defect of length $24a$ when suspended 160 nm from the substrate [4]. Crucially, the new designs presented in this paper do not depend on the membrane-substrate distance and therefore reduce the probability of stiction. The designs presented here also have the added capability of measuring horizontal deflections.

Directional coupler-based sensors integrate an interferometric measurement platform into a single sensor element since the phase difference between even and odd DC modes is directly observed at the coupler outputs. Having demonstrated this sensing mechanism, future work will include testing the displacement sensor under a range of dynamic conditions - driven by acoustic or by thermo-mechanical means. The small size and compatibility with broadband sources offer a potential solution to chip-scale submicron deflection measurement and the future development of dense sensor arrays with potential uses in industrial control systems, automotive sensors, avionics, and environmental monitoring.

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2.6 Supplementary Information - FDTD Fit

Provided here is supplementary information not included in the original publication. The experimental results could be fit to FDTD simulation by varying the PC hole diameter, the PC edge width, vertical separation, and horizontal separation. The lattice pitch and silicon thickness were assumed to be 450 nm and 220 nm. To reduce the number of fitting parameters initially, the transmission from an isolated PC edge was fitted first as the transmission in this case does not depend on vertical and horizontal separation. The isolated edges were formed wherever the membranes had collapsed during post-foundry undercut and found to agree with FDTD simulation with a PC hole diameter of 292 nm and edge width of 513 nm as shown in Fig. 2.12(a). Next, it was assumed the sum of the two edge widths plus the horizontal separation was stable against any possible fabrication biases. This assumption can be expressed mathematically as

\[ s_{\text{fit}} = 2 \times w_{\text{CAD}} + s_{\text{CAD}} - 2 \times w_{\text{fit}}, \]  

(2.10)

which determines the horizontal separation of 254 nm. Finally, the vertical separation between the PC edges was fit for both the partial and fully etched PC membranes and shown in Fig. 2.12(b) and (c), respectively. Vertical separation of 100 nm and 500 nm was determined for the partial and fully etched membranes, respectively, and shown later in Chapter 6 to be in close agreement with optical profilometry measurements.

Using updated fitted data parameters, this brings the measured \( \alpha_{x,\text{meas}} \) to 3.8 um\(^{-1} \) (assuming \( s=254 \) and 354nm savg=304nm) and \( \alpha_{y,\text{meas}} \) to 2.3 um\(^{-1} \) (assuming \( s=254 \) and \( h=250 \)).
Bibliography


Chapter 3

Silicon Photonic Crystal Membrane Ultrasonic Sensor

In the previous chapter, the design, fabrication, and optical transmission of PCDC-based deflection sensors was presented. By comparing the transmission spectra between the partially-released and fully-released post-fabricated PC membranes, the fully-released PC membranes exhibited changes in the transmission spectra consistent with the buckling of the microstructure arising from compressive strain built up in the device layer of silicon-on-insulator wafer. The buckling of fully-etched structures pushes the steady-state position of the PC membranes out-of-plane thereby sensitizing the PCDC sensor to vertical motion.

In this chapter, a PC membrane was driven harmonically using an ultrasonic acoustic source. For the ultrasonic frequencies used, the wavelength of sound in air was considered to be much larger than the dimensions of the membrane, and the dynamic pressure across the membrane was assumed to be uniform and thereby driving vertical motion of the sensor.

3.1 Introduction

Currently, there is a need for high-performance integrated acoustic pressure and shear stress microsensors for use in aerospace [1, 2], turbulence research [3–5], ultrasonic sound localization [6, 7], robotics navigation [8, 9], and non-destructive evaluation [10] and biomedical [11]. A key motivation for miniaturized pressure sensors is the increased resonant frequency that results in faster sensor response. In the context of fluid dynamics, it is possible to obtain a relation between the small and large scales of flow that widens considerably for increasing

1A version of this chapter has been published. Michael Zylstra, Brett Poulsen, and Jayshri Sabarinathan, published in Proc. SPIE 11354, Optical Sensing and Detection VI, 113540Y (13 April 2020). Reprinted (adapted) with permission from the authors and SPIE. ©2020 SPIE. (see Appendix B for copyright permission)
Reynolds number associated with turbulent phenomena [12, 13]. Moreover, the flatter transducer response accompanied by smaller sensors not only improve the spectra and statistical moments of turbulent flow measurements, but also minimize phase delays that otherwise add complexity to the design of sensor arrays [14].

Photonic ultrasonic sensors offer smaller sizes, fast response, immunity to electromagnetic interference (EMI) and are well suited for harsh environments. Many photonic pressure acoustic sensor designs rely on the probing of a micromachined membrane using light [15, 16]. By leveraging the dispersive properties of periodic dielectric structures, many efforts have been undertaken to enhance photonic sensor sensitivity through the use of gratings [17–20] and fiber Bragg gratings [21]. However, the fabrication and testing of photonic sensors typically require demanding fabrication tolerances and expensive probing equipment such as tunable lasers at specific wavelengths or spectrometers to detect small changes in optical cavity resonances [22].

To overcome these challenges, we have previously demonstrated a suspended-element deflection sensor based on the position-dependent directional coupling between dielectric-like edge states of neighbouring photonic crystal slabs [23]. The dispersive properties of the PC edges enable wavelength-independent directional coupling and smaller sensor footprints can be achieved when compared to directional couplers based on conventional index-guided waveguides. Additionally, since the insertion loss to the single PC edge remains low, regardless of the separation, there is greater freedom in the mechanical design. Also, being edge-based, the design does not depend on the depth of the buried oxide therefore avoiding stiction and trade-offs associated with the frequency response. Finally, using a directional coupler-based design, differential sensing modality is possible that allows the suppression of common noise sources without the use of additional photonic components.

The work presented here focuses on driving a suspended photonic crystal membrane featuring the photonic crystal directional coupler (PCDC) sensor element using free-space ultrasonic forces. Driven by a 7-Watt coiled speaker, we observed the acoustic signal from a device with a membrane dimensions measuring 15 µm by 12 µm and that common noise components could be suppressed using a differential photodetection scheme. Following this introduction, Sect. 3.2 describes the device under testing. In Sect. 3.3, the measurement system and differential detection scheme are introduced. In Sect. 3.4 and Sect. 3.5, the static optical transmission and driven acoustic response are presented, respectively. In Sect. 3.6, the measurement results are summarized and discussed followed by the conclusion in Sect. 3.7.
Figure 3.1: (a) Top view of the device depicting the input rib waveguide on the left, central PC membrane, and the through (blue) and coupled (red) outputs on the right. (b) SEM image of PCDC junction and coupling region (indicated by dashed box). (c) The beat-length shown for varying separation [23].

Figure 3.2: Cross section of the membrane with one of the PCDC sensor elements outlined by the green box. A coiled speaker was used to generate an ultrasonic pressure front that causes the membrane to oscillate vertically around its steady-state position.

3.2 Photonic Crystal Directional Coupler Acoustic Sensor

The design of the sensor is described in our previous work [23] and based on a transverse electric (TE) photonic band gap that confines light to the edges of a photonic crystal (PC) membrane. By extending the dielectric region of the PC edge, dielectric-like edge states can be supported in the SCL band (1460–1625 nm) suitable for directional coupling [23]. The device was fabricated on silicon-on-insulator and consisted of a central silicon photonic crystal membrane, as shown in Fig. 3.1(a), whose two sides form the inside half of two separate PCDC elements. Fully etched regions in the top silicon layer form vias that expose the buried oxide (BOX) which is then removed by selective etching with buffered hydrofluoric acid (BHF).

The design is such that four strip waveguides support the PC after BOX etching. By using
strip waveguides as mechanical supports, both output ports were accessible and were used to monitor the coupling between one of the PCDCs. Although this is not strictly necessary for operation of the device, this design enabled the differential measurement scheme described later. The edges forming the outside halves of the PCDCs were rigidly fixed to the substrate and interfaced with rib waveguides whose partially etched silicon cladding protected the BOX during BHF etching. A scanning electron microscope (SEM) image of the PCDC junction is shown in Fig. 3.1(b).

Light enters one of the PC edges from one side and the transmission from both PCDC outputs was measured. From here on, we refer to the transmission from the same PC edge as the input to be the through transmission, indicated by blue. The transmission from the neighbouring PC edge, we refer to as the coupled transmission indicated by red. The beat-lengths for the PCDC for different separations are shown in Fig. 3.1(c) showing a wavelength-independent regime centred on the C-band. These PCDC device structures were dynamically driven using an acoustic pressure source as described in Fig. 3.2. The acoustic pressure front causes the PC membrane to oscillate about its steady-state position and the resulting oscillations in the position-dependent beat-length can be observed as an exchange of power between the two PCDC outputs.

### 3.3 Measurement System

![Block diagram of the experiment](image)

Figure 3.3: Block diagram of the experiment showing a tunable laser source (TL), the device under test (DUT), balanced photodetectors, and signal analyzer (SA). A function generator (FG) generating sinusoidal tones is fed to a speaker after amplification. Coupling to the DUT, done using an optical fiber array, is modeled by a relative noise intensity source $N_{FA}(f)$.
The PCDC membranes were tested using the setup described in the block diagram shown in Fig. 3.3. Using surface grating couplers, a tunable laser source (TL) was coupled to the input of the device under test (DUT) and the transmission at the through and coupled outputs were sent to a balanced photodetector as a differential measurement. The photodiode voltage signal was then fed to a signal analyzer (SA) which displays the photodetector voltage spectrum density. Here, fluctuations due to the laser and vibration of the measurement equipment were added as relative intensity noise sources \( N_{RIN} \) and \( N_{FA} \), respectively. Acoustic testing was performed in an anechoic chamber where a function generator producing sinusoidal tones was sent to a 7 W pre-amp that generated 1 volt peak to peak that was fed to speaker positioned 0.5 meters on-axis with the DUT. The entire measurement apparatus sits atop a pneumatic vibration isolation table.

### 3.3.1 Differential Photodetection Scheme

Using the voltage signal difference between the through and coupled PCDC outputs, noise common to both directional coupler outputs, such as relative intensity noise of the laser or vibration of optical probes, can be suppressed. To see this, the photodiode voltage spectrum density measured by the spectrum analyzer at the through and coupled photodetector outputs, defined by \( v_{THRU} \) and \( v_{COUP} \) respectively, was described by Eqs. (3.1a) and (3.1b).

\[
\begin{align*}
v_{THRU} &= \langle v_{THRU} \rangle + (N_{RIN}[f] + N_{FA}[f])\langle v_{THRU} \rangle + \alpha_{THRU} P[f] \\
v_{COUP} &= \langle v_{COUP} \rangle + (N_{RIN}[f] + N_{FA}[f])\langle v_{COUP} \rangle + \alpha_{COUP} P[f]
\end{align*}
\]

Here, the time-averaged values for each voltage are described by a bracket notation \( \langle \cdot \rangle \). The spectrum density contributions from the relative intensity noise sources \( N_{RIN} \) and \( N_{FA} \) are proportional to the time-averaged value of each voltage. The root-mean-square (rms) amplitude of the incoming acoustic pressure at frequency \( f \) is described by \( P[f] \) and is related to the photodiode voltage signal by sensitivity coefficients \( \alpha_{THRU} \) and \( \alpha_{COUP} \).

If photodiodes are balanced (i.e. \( \langle v_{THRU} \rangle = \langle v_{COUP} \rangle \)), the common noise sources can be suppressed by taking the difference between the through and coupled photodiode signals, as shown by Eq. (3.2a)

\[
v_{DIFF} \equiv v_{THRU} - v_{COUP} = (\alpha_{THRU} - \alpha_{COUP})P[f]
\]

\[
\alpha_{DIFF} \equiv \frac{v_{DIFF}}{P[f]}
\]

Here, we have defined in Eq. (3.2b) a sensitivity term \( \alpha_{DIFF} \) that corresponds to the voltage
difference.

3.4 Static Optical Transmission Measurement

![Graph showing through (blue) and coupled (red) transmission spectra for the undriven device normalized with respect to the blank device. Inset shows exchange of power between the through and coupled ports near 1533.55 nm where measurements were taken.](image)

Figure 3.4: The through (blue) and coupled (red) transmission spectra for the undriven device normalized with respect to the blank device. Inset shows exchange of power between the through and coupled ports near 1533.55 nm where measurements were taken.

For the undriven PC membrane, the optical transmission spectrum at the through and coupled outputs are shown in Fig. 7.4 for a DUT 28 periods long. The transmission has been normalized with respect to a control device, consisting of only a pair of surface grating couplers where the device has been replaced with a shunt waveguide. The wavelength was tuned to 1533.55 nm where the power was balanced as shown in the inset of Fig. 7.4. With the TL set to 10 dBm, the through and coupled outputs of the DUT were balanced at 1533.55 nm where the transmission was measured to be -13.00 dBm (50 µW (optical)). For an ideal directional coupler, the change in power with respect to wavelength would be equal for both outputs. However, Fabry Perot effects arising from reflections at the PCDC junctions lead to an imbalance in the exchanged power. Since the time-averaged power remains the same, the differential photodetection scheme can still be applied.

3.5 Driven Acoustic Sensor Measurements

For the driven acoustic measurements, the drive voltage to the speaker was set to 1 Vpp and driven acoustic tones were generated from 10 kHz to 80 kHz in 10 kHz increments. The voltage spectrum densities for each driven acoustic frequency are shown stacked on top of one another for the through, coupled, and difference signals of the DUT and are shown in Figs. 3.5(a), 3.5(b) and 3.5(c), respectively. We observed that all driven frequency tones were present at the through and coupled outputs ports regardless of FG drive frequency.
In addition to driven acoustic tones, a common noise tone associated with the tunable laser could be detected at 62.5 kHz. We associate this tone with the laser relative intensity noise since it also appears when the laser is fed directly into the photodetector. Using the differential photodetection scheme described in Sect. 3.3.1, this common noise tone was suppressed while simultaneously enhancing the driven acoustic tones as can be seen in the difference plots shown in Fig. 3.5(c). We observed that the noise floor was determined by the thermal motion of membrane and was also boosted by the differential detection scheme. Although it is not shown, the thermally driven motion of the vertical vibrational mode could be resolved in the signal analyzer with a peak centred at 560 kHz and agreed with finite element simulation results.
3.6 Discussion

The driven acoustic measurement results are summarized in Fig. 3.6. In Fig. 3.6(a), the spectrum density peaks corresponding to each drive frequency are shown for the through, coupled, and difference signals. The photodetector has a built-in high pass filter with cutoff frequency at 30 kHz therefore the tones below this cutoff were suppressed. Above the cutoff frequency, the difference signal was enhanced by a factor ranging from 1.42–1.74 with respect to the average of the through and coupled voltage peaks at each drive frequency.

Figure 3.6: For each drive frequency, (a) the voltage spectrum density peaks observed at the drive frequency is plotted for the through (blue), coupled (red), and difference (black) signals. (b) For each drive frequency, the voltage spectrum density peak for the common noise tone at 62.5 kHz tone is plotted for the through (blue), coupled (red), and difference (black) signals.

The largest voltage difference signal of 8 µV/√Hz was found to occur at 40 kHz and observed to diminish with increasing drive frequency. Since the drive frequencies were well below vertical vibrational resonance of the membrane, we attribute this to a combination of resonances due to the anechoic chamber geometry and the diminishing mechanical response of the drive speaker. Using the speaker datasheet, an rms pressure of 200 mPa at 40 kHz was estimated using the measured 1 Vpp drive voltage and distance from device. This pressure was then used to determine the sensitivity of the voltage difference to the pressure to be 40 µV/Pa using Eq. (3.2b).

In Fig. 3.6(b), the voltage spectrum density peak at 62.5 kHz present in voltage spectrum density for each drive frequency are shown for the through, coupled, and difference signals. We observed that using the differential detection scheme, this common noise component could be suppressed to below the noise floor measured at 0.5 µV/√Hz. Using the sensitivity and the measured noise floor level, the equivalent noise pressure level of 12.5 mPa/√Hz was determined. This pressure level was converted to an acoustic decibel level of 56 dB$_{SPL}$ using the standard reference sound pressure of 20 µPa.

A comparison of our work to existing devices can be found in Table 3.1. For sensors
Table 3.1: Comparison of microphone designs [24]

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Transducer</th>
<th>Diaphragm area mm x mm</th>
<th>Upper frequency [kHz]</th>
<th>Sensitivity [mV/Pa]</th>
<th>Equivalent level [dB_{SPL}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schellin and Hess[25]</td>
<td>Piezoresistive</td>
<td>1 x 1</td>
<td>10</td>
<td>0.025</td>
<td>–</td>
</tr>
<tr>
<td>Bourouina et al[26]</td>
<td>Capacitive</td>
<td>1 x 1</td>
<td>2.5</td>
<td>3.5</td>
<td>35</td>
</tr>
<tr>
<td>Ried et al[27]</td>
<td>Piezoelectric</td>
<td>2.5 x 2.5</td>
<td>18</td>
<td>0.92</td>
<td>57</td>
</tr>
<tr>
<td>Kalvesten et al[28]</td>
<td>Piezoresistive</td>
<td>0.1 x 0.1</td>
<td>&gt;25</td>
<td>0.0009</td>
<td>90</td>
</tr>
<tr>
<td>Scheeper et al[29]</td>
<td>Capacitive</td>
<td>2 x 2</td>
<td>14</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>Bergqvist[30]</td>
<td>Capacitive</td>
<td>2 x 2</td>
<td>&gt;17</td>
<td>11</td>
<td>30</td>
</tr>
<tr>
<td>Schellin et al[31]</td>
<td>Piezoelectric</td>
<td>1 x 1</td>
<td>10</td>
<td>0.1</td>
<td>60</td>
</tr>
<tr>
<td>Huang et al[32]</td>
<td>Piezoresistive</td>
<td>0.71 x 0.71</td>
<td>10</td>
<td>0.0011</td>
<td>–</td>
</tr>
<tr>
<td>Song[18]</td>
<td>Optical</td>
<td>0.8 x 0.8</td>
<td>3</td>
<td>1000</td>
<td>–</td>
</tr>
<tr>
<td>White et al[4]</td>
<td>Capacitive</td>
<td>0.6 (diameter)</td>
<td>&gt;40</td>
<td>0.25</td>
<td>85</td>
</tr>
<tr>
<td>Kim and Hall[19]</td>
<td>Optical</td>
<td>1 x 1</td>
<td>10</td>
<td>298</td>
<td>23</td>
</tr>
<tr>
<td>Bakhoum and Cheng[33]</td>
<td>Optical</td>
<td>10 (diameter)</td>
<td>16</td>
<td>–</td>
<td>20</td>
</tr>
<tr>
<td>Bandutunga et al[21]</td>
<td>Optical</td>
<td>8.5 (diameter)</td>
<td>&gt;20</td>
<td>298</td>
<td>11</td>
</tr>
<tr>
<td>Xarion[34]</td>
<td>Optical</td>
<td>2 x 0.3</td>
<td>1000</td>
<td>10</td>
<td>-12</td>
</tr>
<tr>
<td><strong>This work</strong></td>
<td><strong>Optical (PCDC)</strong></td>
<td><strong>0.015 x 0.012</strong></td>
<td><strong>&gt;80</strong></td>
<td><strong>0.04</strong></td>
<td><strong>56</strong></td>
</tr>
</tbody>
</table>

whose upper frequency was theoretically expected to extend beyond the experimental testing, a greater-than sign was used. Using the PCDC sensor mechanism, a two-order reduction of magnitude in sensor diaphragm area was achieved. Due to the limitations in measurement system, we have not tested the upper frequency limit of the sensor but we expect good frequency response up to the fundamental vertical vibration resonance at 560 kHz. Efforts to extend the frequency range of the measurement system are ongoing. The sensitivity was lower when compared to existing devices, however, the device has not been yet been optimized for this application and serves only as a proof-of-concept at this moment. We reiterate that with this design, there is a lot of freedom with mechanical and acoustic design space, such as vent sizes and the volume of the buried oxide undercut cavity.

### 3.7 Conclusion

We have outlined the need for miniaturized pressure sensors, especially for applications involving high Reynolds number phenomena associated with turbulence. By forming a directional coupler using PC edges, we created an acoustic sensor with a size measuring 12 µm by 15
µm. By tuning the wavelength such that power is equally shared between the PCDC outputs, we found that the signal could be enhanced using a differential detection technique, which simplified testing in the presence of common noise sources. For the driven acoustic measurements, tones as high as 80 kHz could be detected and we report a sensitivity of 40 µV/Pa at 40 kHz. Ultrasonic pressure acoustic sensors based on PCDC exhibit broad Band compatibility and achieve small device footprints suitable for sensor arrays and a potential technology in aerospace and medical sciences.

Bibliography


[34] Xarion. *Datasheet - ETA250 Ultra*. Xarion Laser Acoustics.
Chapter 4

Demonstration of High-Resolution Air-Coupled Silicon Photonic Crystal Shear Sensor

In the previous chapters, microstructures were released from the substrate oxide using a wet-etch post-foundry fabrication process that selectively etched away the buried oxide to form suspended silicon PC membranes. Although critical point CO₂ was used to reduce liquid surface tension during drying, only a few of the smallest membranes successfully survived the wet-etching process without collapsing. For the PC membranes that did survive, optical transmission measurements indicated the PC membranes had buckled out-of-plane with respect to the surrounding substrate, thereby sensitizing the PCDC to vertical deflection.

To reduce the probability of membranes collapsing during post-fabrication, a vapour-phase etchant was used instead to remove the buried oxide, thereby eliminating the effects of surface tension and the need for critical point drying. By using the vapour-etch process, nearly all devices were successfully released from the buried oxide without collapsing. However, these newly fabricated devices were not nearly as sensitive to the same ultrasonic testing performed in the previous chapter. It was therefore hypothesized that the new devices, having undergone the vapour-etching, had much smaller vertical misalignment between the PC edges. In this steady-state position, the PCDC sensitivity to vertical motion would be reduced while the sensitivity to horizontal motion would be enhanced, making the new batch of devices more suitable as wall shear stress sensors.

To test this hypothesis, a thermoacoustic emitter and acoustic waveguide apparatus were constructed to generate distinct pressure and wall shear stress spatial profiles. The design of

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1 A version of this chapter is in preparation for publication: Michael Zylstra and Jayshri Sabarinathan, ‘Demonstration of High-Resolution Air-Coupled Silicon Photonic Crystal Shear Sensor’
experimental apparatus was based on mass conservation which dictates the flow velocity of an incompressible fluid increases in the presence of a constriction. Since wall shear stress is related to the free-stream velocity, any constriction in flow therefore generates a distinct spatial stress profile. In this chapter, the results of this experiment as performed on the newly fabricated devices is presented.

4.1 Introduction

Wall shear stress sensors are important to many applications such as the evaluation viscous drag in automobiles and aircraft [1], detection of boundary layer separation in aircraft [2], study of heat transfer in impinging jets [3], fuel injection sensors [4], and measurement of arterial flow for medical diagnosis [5]. Based on the Kolmogorov scales, it is possible to obtain a relation between the small and large scales of the flow [6, 7] that widens considerably as the Reynolds number increases [8, 9]. Therefore, high spatiotemporal resolution of wall shear stress sensors is crucial to characterizing the type of flow being measured. Smaller sensors not only improve the spectra and statistical moments of measurements, but also reduce phase delays affecting correlated sensor array measurements [9]. However, the measurement of wall shear stress in air with high spatiotemporal resolution is difficult due to the scaling of transduction methods and calibration of the sensor.

The measurement of wall shear stress sensors using microsystems can be achieved either indirectly, as is the case with thermal anemometers [10], or directly, by probing the deflection of a microstructure subject to viscous forces using either piezoresistive [11, 12], piezoelectric [13, 14], capacitive [15–19], or optical [20–24] methods. Thermal anemometers require a complicated calibration process with assumptions regarding the flow type [8] and exhibit trade-offs between the frequency response and sensitivity [10]. Direct measurement of wall shear stress is considered more accurate than thermal sensors since it minimizes the disturbance on the flow being measured [8].

Direct wall shear stress measurement using optical-based sensors have the added advantages of immunity to electromagnetic interference [25] and do not require surface wire-bonding that can interfere with measurements [9]. Micro-optical wall shear stress sensors relying on the shadowing effects of a suspended element have been demonstrated [20]; however, these devices require an external optical source positioned above the device and cannot be used outside of highly controlled research environments. This sensor also required built-in photodetectors (PD) underneath the suspended element that required wire-bonding and susceptible to EMI. An improvement to this type of design was made by Horowitz et al by using Moire interferometry [23, 24]. However, the detection-limit was limited by read-out electronics. Using
optical microcavities or interferometric schemes, deflections of the order of femtometers can be measured routinely [26] whose noise floor can then be related to statistical mechanical models that provide a means of calibration [27]. Examples of shear stress sensors using changes in optical resonances have been previously demonstrated using Fabry Pérot (FP) microcavities [21] and the whispering gallery modes (WGM) of microspheres [22]. However, these rely on non-standard fabrication methods and are not easily integrated into sensor arrays.

Due to the predominance of silicon photonic foundries with submicon SOI device layers, undercut SOI is therefore a promising platform for creating suspended-element wall shear stress sensors that also enable photonic microsystems integration [28]. Previously, we demonstrated an optical deflection sensor using a suspended photonic crystal (PC) membrane. Optical read-out was provided by the position-dependent directional coupling between dielectric-like edge states of neighbouring PC membranes [29]. However, dynamic testing of the sensor to horizontal deflections induced by wall shear stress has been unexplored.

To demonstrate the spatiotemporal response of the microsensor, a novel thermoacoustic apparatus was constructed where an acoustic waveguide was made to impinge onto the SOI die containing the sensor. In contrast to conventional acoustic generation, there were no moving parts and the measurement frequency range was not restricted by mechanical resonances associated with the acoustic source. While thermoacoustic emitters have been previously used to evaluate ultrasonic optical microphones [30], piezoelectric microphones [31], and the non-destructive testing of composite structures for use in aerospace [32], this is the first publication to use the thermoacoustic effect in this way to evaluate the performance of wall shear stress sensors.

In this paper, we demonstrate over three orders of magnitude reduction in sensor area when compared to state-of-the-art wall shear stress sensors of similar detection limit by probing the deflection of a suspended PC membrane using a photonic crystal directional coupler (PCDC). Following this introduction, Section 4.2 describes the physical operating principle of the sensor. Section 4.3 describes the sensor fabrication using post-foundy vapour hydrofluoric acid undercut of SOI. Section 4.4 describes the construction and modelling of the thermoacoustic experimental apparatus. Section 4.5 presents optical transmission spectra, sensor calibration, and acquired wall shear stress profiles of the impinging acoustic waveguide. Section 4.6 compares the results with existing sensors found in literature. Finally, the conclusion and final remarks are presented in Section 4.7.
4.2 Physical Sensor Operating Principle

The wall shear stress sensor design consisted of a suspended silicon PC membrane with four parallel supports at each corner as shown in Fig. 4.1(a). To satisfy the no-slip boundary conditions, the fluid velocity must be zero at the surface of the membrane and reach its free-stream value after the boundary layer distance above the membrane as shown in Fig. 4.1(b). The wall shear stress is proportional to the gradient of the flow velocity at the wall and can be written as [33]

\[ \tau = \mu \frac{\partial U}{\partial z} \]  

(4.1)

where \( \tau \), \( U \), and \( \mu \) are wall shear stress in the x-direction, the fluid velocity in the x-direction, and the dynamic viscosity of the fluid which about \( 1.8 \times 10^{-5} \) Pa·s for air at 1 atm. One of the PC edges belongs to the suspended membrane while the other is rigidly fixed to the substrate and together the PC edges form the PCDC line defect shown in Fig. 4.1(c). Viscous forces exerted on PC membrane perturb the separation between the PC edges and the power exchanged between them is observable at the output. If no initial vertical misalignment exists between the PC edges, the optical sensitivity to small changes in vertical displacement of the PC edges vanishes due to the symmetry presented across the mid-slab plane [29]. Since pressure is coupled mainly to the vertical mode of the membrane, the crosstalk with pressure is expected to be minimized in this case, while the sensitivity to horizontal deflection arising from wall shear stress is maximized.
4.2.1 Optical Read-Out

Optical read-out of the sensor was based on the PCDC deflection sensor recently demonstrated on undercut 220 nm device layer SOI operating in the S-band (1460–1530 nm) [29]. Light was confined to the edges of a PC using a combination total internal reflection and the transverse electric (TE) photonic band gap generated by a triangular lattice of holes of diameter \( d = 270 \text{ nm} \) and pitch \( a = 450 \text{ nm} \). Adding \( b = 540 \text{ nm} \) of dielectric material to the PC edges enabled low-loss coupling to directional coupling modes with beat-lengths as low as 10 \( \mu \text{m} \). Since the coupling strength along the PCDC was assumed to be much stronger than the routing waveguides, directional coupling was restricted to only the PCDC region. In this case, the photodiode current generated by a detector at the through output from a directional coupler can be described by

\[
I = I_0 \cos^2(\kappa L_{pc})
\]

where \( I_0, \kappa, \) and \( L_{pc} \), are the photodiode current from an isolated PC edge, the PCDC coupling coefficient, and length of the PC membrane. This model can be linearized by separating steady-state changes in the coupling coefficient from small fluctuations

\[
\kappa = \bar{\kappa} + \frac{\partial \kappa}{\partial q} q.
\]

where \( \bar{\kappa} \) is the time-averaged coupling coefficient, and \( q \) is a small perturbation in the PC edge separation. If we restrict the analysis to the special case where the time-averaged coupling coefficient has been tuned such that

\[
2\bar{\kappa}L_{pc} = \frac{\pi}{2} + n\pi,
\]

then power is equally shared between the PCDC outputs. In this case, the optical sensitivity to small changes affecting the separation of PC edges is maximized and can be related to the photocurrent taking the derivative of Eq.(4.2)

\[
|\Delta I| = I_0 L_{pc} \frac{\partial \kappa}{\partial q} q \equiv \alpha q.
\]

Here, a conversion factor \( \alpha \) has been defined and is later determined experimentally using thermal fluctuations in the membrane position in Section 4.5.

4.2.2 Finite Element Modelling

The membrane response to the wall shear stress was modelled by analyzing the eigenfrequency of the lateral vibrational mode of the PC membrane. The displacement profiles, eigenfrequen-
Figure 4.2: Finite-element model showing (a) eigenfrequency analysis and (b) steady-state deflection of the PC membrane using 10 mPa shear stress boundary load.

cies, and effective mass of this mode are shown in Fig. 4.2(c) for a device with $L_{pc}$, $L_{tot}$, $W_{pc}$ of 30 μm (66a), 75 μm, and 16 μm, respectively. Since the membrane supports were much narrower than the membrane itself, deformation of the structure is primarily limited to the four supports. In the following analysis, we normalized the mode profiles $r(x) = [u, v, w]$ such that the maximum value of $|r(x)|$ was unity. The membrane dynamics can be modelled using a generalized coordinate $q$ (i.e., $r(x, t) = q(t)r(x)$) that is governed by Newton’s second law

$$m\ddot{q} + c\dot{q} + kq = F.$$  \hspace{1cm} (4.6)

Here, $m$, $c = m\omega_0/Q$, $k$, and $F$ are the effective mass, damping coefficient, spring coefficient, and external force, respectively. The spring coefficient is related to the mass and the resonant frequency by

$$k = \omega_0^2 m.$$  \hspace{1cm} (4.7)

The effective mass of the sensor was defined using the normalized mode profiles and given by

$$m = \int \rho |r(x)|^2 dV,$$  \hspace{1cm} (4.8)

where, $\rho$ represents the density of the material. The $F$ source term describes the coupling of wall shear stress to the lateral mode and given by

$$F = \tau A.$$  \hspace{1cm} (4.9)

If the wall shear stress is uniform across the membrane, an effective sensor area can be defined

$$A \equiv \int u dA.$$  \hspace{1cm} (4.10)
where \( u \) is the x-component of the normalized lateral mode profile. For the dimensions previously described, this results in an effective area of 363 \( \mu m^2 \).

In the low frequency limit, the sensor deflection is governed by Hooke’s law (i.e., \( F = -kq \)) with the spring coefficient of 2.77 N/m determined by Eq. (4.7). Using Hooke’s law, and combining Eqs. (4.5) and (4.9), variations in the photocurrent can be related to the wall shear stress

\[
|\Delta I| = \frac{\alpha \tau A}{k}.
\]

### 4.3 Sensor Fabrication

The sensor was fabricated on single-crystal silicon-on-insulator (SOI) followed by selective etching of the buried oxide to create suspended microstructures using the process described in Fig. 4.3. The designs were surface micromachined on SOI using imec ePIXfab silicon photonics process as part of a multiproject wafer (MPW). The SOI featured a 220 nm device layer thickness and a 2 \( \mu m \) buried oxide thickness. A scanning electron microscope (SEM) image of the fabricated structure before selective etching is shown in Fig. 4.4(a). The darkest regions of the SEM indicate where silicon has been fully etched away, exposing the buried oxide to subsequent VHF. Partially etched regions of silicon not only provided cladding for the interfacing waveguides but protected the buried oxide from post-process etching. A close-up of the PCDC junction is shown in Fig. 4.4(b) that shows the annotated PC hole diameter, air gap, and PC edge width of 280 nm, 193 nm, and 543 nm, respectively. Interfacing waveguides were directly coupled to the fixed PC edge allowing the transmission from the PCDC to be measured.

![Fabrication Process Diagram](image)

**Figure 4.3:** Post-process fabrication steps for the hard-mask VHF process.

For the post-process, an etch mask for the vapour-phase hydrofluoric acid (VHF) etch was
4.4 Shear Stress Measurement System Design

required to avoid the uncontrolled release of microstructures present on the MPW. Since photopolymer photoresists were not compatible with VHF, a 100 nm amorphous silicon (a-Si) hard-mask layer was deposited on the chip using a bilayer lift-off resist and electron beam evaporation. VHF was done using a 1 hr etch using Memsstar Orbis Alpha with a pressure and chuck temperature of 11 Torr and 21°C, respectively. The flow rates of HF, vaporizer N₂, and H₂O were 40 sccm and 20 sccm, and 5 mg/min, respectively. All post-process steps were conducted at the University of Alberta NanoFAB Centre. An optical image of the overall fabricated structure after post-processing is shown in Fig. 4.4(c) that indicates the VHF etch in regions where the buried oxide was exposed.

![Figure 4.4](image)

Figure 4.4: (a) SEM image of the full device under test before VHF post-processing. (b) SEM image of the inset showing the PCDC input junction before VHF post-processing. (c) Optical image of the full device under test after VHF-processing.

4.4 Shear Stress Measurement System Design

To investigate the the device in the context of wall shear stress sensing, an experimental apparatus was constructed consisting of an acoustic waveguide (AW) butt-coupled to a thermoacoustic emitter (TAE), as shown in Fig. 4.5(a). The AW consisted of a brass tube, polished on both ends, and made to impinge upon the chip die, as shown in Fig. 4.5(b). In the annular region directly beneath the edge of the AW, a microchannel was formed creating a localized increase in the free-stream flow velocity. Since the wall shear stress is related to the stream velocity by Eq (4.1), the shear stress under the edge of the AW was expected to generate distinct stress
profiles. Using position controllers, the location of the AW edge over the sensor could be controlled and while the sensor signal was recorded.

The length of the AW was 60 mm with an inside diameter of 2.40 mm and an outside diameter 2.80 mm. For an AW of circular cross-section, the cut-off frequency for the first higher-order mode is \( f_{\text{cut-off}} = \frac{1.84v_s}{\pi D} \) where \( v_s \) is the speed of sound of the fluid and \( D \) is the inside diameter of the AW [34]. Based on the dimensions of the AW, it is expected to be single mode up to 84 kHz, assuming the speed of sound in air is \( v_s \) is 343 m/s. Therefore, an acoustic frequency of 40 kHz was used for the experiment to avoid excitation of higher order modes within the AW.

To drive the AW, a thermoacoustic emitter was constructed using of a thin conductive film of indium tin oxide (ITO) (Sigma-Aldrich 70-100 Ω/sq, 15–30 nm). The transducer film was selected considering the film thickness, density, heat capacity, hardness, and chemical stability and based previous analysis done by Daschweski et al. [35, 36]. The ITO film was mounted to a custom temperature-resistant polytetrafluoroethylene holder and connected to copper electrodes using conductive paint (Ted Pella, Pelco colloidal silver) leaving a 7 mm by 7 mm region of ITO exposed which was then butt-coupled to the AW.

Using thermoviscous acoustics physics in the frequency domain (COMSOL), a 2D axisymmetric model of the AW and chip surface was constructed and the acoustic pressure at 40 kHz is shown in the revolved domain in Fig. 4.6(a). The TAE was modeled using a heat flux source of 80 mW/mm² located at the top boundary. A close-up of the impingement region

![Figure 4.5: Shear stress measurement system is shown. (a) Thermoacoustic emitter module with electrical contacts and ITO coated glass slide shown. (b) Experimental setup showing the TAE, acoustic waveguide, photonic integrated circuit, and optical fiber array. Front view of the acoustic waveguide (c) 10 μm and (d) 500 μm from the chip surface.](image-url)
is shown in Fig. 4.6(b) indicating the chip surface and AW edge. A close-up of the simulated pressure and velocity profiles in the constriction for the AW 10 µm above the chip die are shown in Figs. 4.6(c) and (d), respectively. Due to Knudsen number approaching the slip flow regime at this distance, slip boundary conditions were used on the impinging edge of the AW.

The simulated spatial profiles of the acoustic pressure and wall shear stress at the chip surface are shown in Figs. 4.6(e) and (f), respectively, for different AW distances from the chip. If the wavelength of sound is much larger than the diameter of the AW, the surface of the chip directly below the centre of the AW experiences maximum acoustic pressure that decreases monotonically away from the centre of the AW. As the AW gets closer to the chip, the wall shear stress directly beneath the AW edge increases until the air gap distance approaches the Stokes oscillating boundary layer. The $1/e$ Stokes boundary layer thickness for an oscillating plane (or stationary plane in oscillating fluid) is given by [37]

$$
\delta_v = \sqrt{\frac{\mu}{\pi f \rho_0}},
$$

where $\rho_0$ is the fluid density which is 1.225 kg/m$^3$ for 1 atm air. At 40 kHz, the velocity boundary layer is approximately 10.8 µm in air. Since the viscous boundary layer is much larger than the PC hole diameter and PCDC air gap, it was considered hydraulically smooth.

4.5 Measurement Results and Discussion

4.5.1 Static Optical Transmission Measurements

Static optical measurements were preformed using an optical power meter (PM, Agilent N7744) and a tunable laser source (TL, Anritsu Tunics Reference Tunable Laser SCL-band). Optical interfacing to the device under test (DUT) was achieved by automatic alignment of a TE polarization-maintaining optical fiber array to surface grating couplers (SGC) integrated onto the PIC. The measured transmission spectra are shown in Fig. 4.7(a) for devices of varying PCDC length. Transmission was normalized with respect to a control structure consisting of a loop-back waveguide without the sensor to remove the effects of the SGC. A broadband change in optical response across 1500–1535 nm was observed between the structures which was consistent with our previous work indicating a non-linear wavelength dependent beat-length [29]. Further measurements were performed on the structure of length $66a$ due to satisfying the condition in Eq (4.4) in its broadband optical regime.
Figure 4.6: (a) Revolved thermoviscous axis-symmetric simulation domain is shown at 40 kHz with the rms of the acoustic pressure indicated by the colour map. (b) A close-up of the impingement region. The RMS (c) pressure and (d) acoustic velocity for an AW edge 10 µm above the chip die. (e) The RMS wall shear stress for an AW edge 10 µm above the chip die. The surface acoustic pressure and (e) the wall shear stress (computed using Eq. (4.1)) are plotted at different AW heights above the chip from 10 µm (red/blue) to 100 µm (black) in 10 µm increments. The grey region in the plot indicates the region directly beneath the AW edge.

4.5.2 Sensitivity and Detection Limit

The noise floor of the undriven DUT was acquired in order to calibrate the device. For this measurement, the TL was tuned to 1550.0 nm and the optical output from the DUT was fed to a reverse bias photodetector (Thorlabs, PDB450C 1 A/mW gain) and signal analyzer (Agilent, N9030A PXA). The amplitude spectrum density (ASD) was acquired using 100 sample averages by the using a SA and is shown in Fig. 4.7(b). The observed resonance in the sensor noise floor at 627.8 kHz was close the FEM model and no other resonances could be observed, indicating minimal vertical misalignment between the PC edges.

A general procedure for thermal mechanical calibration of nano- and micromechanical resonators was outlined by B.D. Hauer et al [27]. If the system described by Eq. (4.6) is under-
Figure 4.7: Undriven measurements for devices are (a) Transmission spectra for PCDC defects of varying length. (b) (blue line) The ASD of the photocurrent at 1550 nm for the device of length 66a showing resonance at 627.8 kHz and (red line) fitted ASD. Y-axis on left-hand side shows the photocurrent amplitude density as measured by the signal analyzer. Y-axis on left-hand side shows the vibrational amplitude computed by fitting to the statistical mechanical model. (c) The sensitivity with respect to wavelength is shown.

damped and thermally driven, then the single-sided thermal noise spectral density $S_{x x}$ of the generalized coordinate $q$ is given by [27].

$$S_{x x}^{th}(f) = \frac{k_B T f_0}{2\pi^3 m Q[(f^2 - f_0^2)^2 + (f f_0/Q)^2]} \tag{4.13}$$

where $k_B$, $T$ and $Q$ are Boltzmann’s constant, ambient temperature, and the quality factor, respectively. When the effective mass $m$ is computed using Eq. (4.8), this is consistent with the equipartition theorem [27]. If the other noise sources, such as dark current and shot noise, are assumed to be white noise, the power spectrum density photocurrent signal received by the signal analyzer from the undriven device can be modeled by
\[ S_H(f) = S_H^w + \alpha^2 S_{xx}^{th}(f), \]  

where the previously defined \( \alpha \) has units A/m and provides a conversion factor between the generalized coordinate and the measured photocurrent.

The measured data was fit to Eq. (4.14) which determined \( \alpha, Q, \) and \( f \) to be 1.93\( \times 10^6 \) A/m, 25.6, and 627.8 kHz, respectively. Assuming the FEM generated mode mass of 1.646 \( \times 10^{-13} \) kg, a spring coefficient of 2.56 N/m was computed using Eq. (4.7). Assuming the FEM generated effective area of 363 \( \mu m^2 \), the PD photocurrent sensitivity to the wall shear stress was determined to be 247 \( \mu A/\text{Pa} \) (12.35 mV/Pa) using Eq. (4.11) and the fitted \( \alpha \). For the low frequency response, the noise floor of the SA was observed to be 0.0221 uA/\( \sqrt{Hz} \), which corresponds to a displacement detection limit of 11.4 fm/\( \sqrt{Hz} \) using the fitted \( \alpha \), or 80.7 uPa/\( \sqrt{Hz} \) in terms of the equivalent shear stress.

### 4.5.3 Thermoacoustic Spatial Response Measurements

![Figure 4.8](image-url)

Figure 4.8: (a) Cross-section of the experiment showing the position of the acoustic waveguide with respect to the device and (b) the corresponding simulation and sensor response. (c) Cross-section of the experiment varying the air gap between the acoustic waveguide and (d) the corresponding simulation and sensor response. (e) Cross-section of the experiment where the acoustic waveguide is offset in the y-direction (into and out of the page) (f) the corresponding sensor response.
To excite the AW, the TAE was fed a sinusoidal varying current without DC offset thereby generating an ultrasonic signal at the second harmonic of the drive frequency. The resistance across the TAE was measured to be 100 Ω. A 20 Vrms voltage supplied by a power amplifier was fed into the TAE and a steady-state substrate temperature of 115°C was produced using a 20 kHz sinusoidal signal with no DC offset. This corresponds to a 40 kHz acoustic signal with 4.0 W of power distributed over the 7 mm by 7 mm area yielding 82 mW/mm². The photocurrent signal from the sensor at 40 kHz was recorded by the signal analyzer while the position of the AW was moved along the X, Y, and Z axis as shown in Fig. 4.8. The x coordinate refers to the distance between the sensor and the centre of the AW. The y coordinate refers to distance between the sensor and the lateral offset of the AW. The z coordinate refers to the height of the AW above the chip. The acoustic wavelength was assumed to be much longer than the spatial variations performed in the experiment, therefore detuning of the longitudinal AW resonances were not expected to contribute significantly to the overall sensor spatial response.

Figure 4.8(a) shows the AW axis of motion in the x-direction with the corresponding sensor signal shown in Fig. 4.8(b) centred at y = 0 at three different z distances. Using the previously determined 247 µA/Pa conversion factor, an equivalent shear stress was computed. While the thermoviscous model of the AW predicts the pressure inside the AW to be over an order of magnitude larger than the wall shear stress, the sensor signal was observed to be very weak inside the AW (i.e., x < 1.2 mm) indicating low crosstalk with the pressure. For the smaller values of z, a noticeable cusp appeared near the outside edge of the AW at x = 1.4 mm that was much smaller than the 8.5 mm acoustic wavelength. Based on the thermoviscous simulation, the cusp arises at the transition between slip flow and non-slip flow. However, the cusp in the thermoviscous simulation smaller in magnitude and appears on both the inside and outside edge of the tube. This could be due to a number of factors such as small tilt in the AW, interference with the optical fiber array, or unaccounted convective effects related to the thermal boundary being thicker than inertial boundary layer (Prandtl number of 0.71).

Figure 4.8(c) shows the AW centred at y = 0 and swept in the z-direction. The respective spatial response from the sensor directly below the mid-point of the AW edge at x = 1.3 mm was recorded and shown in Fig. 4.8(d). Both the magnitude of the wall shear stress and shape of the measured sensor response agrees with the thermoviscous FEM simulation. An initial z position of 10 µm was determined based on optical images of the front of the AW shown in Figs. 4.5 (c) and (d). Due to the possible uncertainty in this initial value, there may be a small z-offset (±5 µm) in the spatial response data that was measured with respect to this initial position. To avoid inadvertently colliding the AW with the chip and damaging the sensor, measurements of z below 10 µm were not taken.

Figure 4.8(e) shows the AW swept in the y-direction parallel to the optical fiber array facet.
with the respective sensor response shown in Fig. 4.8(f) for \( z \) and \( x \) of 10 \( \mu \text{m} \), and 1.3 mm, respectively. The asymmetric sensor response suggests that the cusp may be due to possible tilt in the AW. We note that none of the measurement signals could be generated on a blank control device consisting of a surface grating couplers and a loop-back waveguide. This confirmed the acquired measurements originated from the sensor itself and not due to electrical interference or the fibre array.

### 4.5.4 Sensitivity Dependence on Wavelength

In the previous sections, the measurements were performed at a specific wavelength. To determine the sensitivity for all wavelengths, the AW edge was positioned above DUT and the TAE was driven at 40 kHz (acoustic) which generated a wall shear stress whose magnitude was determined to be 12.5 mPa using the previous calibration step. Next, the photocurrent signal at the acoustic frequency was recorded as the TL wavelength was swept. The effects of the surface grating couplers were removed by dividing by the transmission from a blank control device that consisted of a loop-back waveguide. This procedure can be summarized by the following equation:

\[
S = \frac{1}{\tau} \frac{R}{G} \frac{I(\lambda)}{T_{SGC}(\lambda)},
\]

where \( I(\lambda) \), \( R \), \( G \), and \( T_{SGC}(\lambda) \) were the measured photocurrent signal, SA input impedance (50 \( \Omega \)), and the PD gain (\( 0.5 \times 10^5 \) V/W) and measured optical transmission from the control structure. Sensitivity therefore represents the fraction of full-scale input optical power that is converted to the first harmonic of the acoustic signal per Pa of shear stress (i.e., \% f.s./Pa) and is shown in Fig. 4.7(c). Due the non-linearities in the coupling coefficient, the sensitivity exhibits an average and peak broadband sensitivity across 1497–1531 nm of 0.12\% f.s./Pa and 0.21\% f.s./Pa, respectively, that is consistent with the optical transmission spectra measurements and the directional coupler sensor model.

### 4.6 Sensor Comparison

The results of this paper, along with recent works wall shear stress sensor, are presented in Table 8.5. The detection limit reported here is lower than most entries and only outperformed by the capacitive wall shear stress sensors developed by Chandrasekharan [17] and Freidkes [19]. The suspended element area reported here is several orders of magnitude smaller than all sensors listed. The advantage of smaller designs is the higher achievable mechanical resonant
4.7 Conclusion

An optical broadband PCDC sensor was demonstrated as a wall shear stress sensor using an integrated photonic thermoacoustic test bench. Static optical transmission results indicate a non-linear coupling coefficient that could be desensitized to wavelength with an integrated average sensitivity of 0.120% full-scale power per Pa was observed across 1497–1531 nm. By measuring the amplitude spectrum density of optical signal from the device, the horizontal vibrational mode could be observed at 627 kHz with a quality factor of 25. Using a statistical mechanical model, the sensor was calibrated resulting in a detection limit, effective sensor area, and mode mass of 80 μPa/√Hz, 363 μm², 1.65×10⁻¹³ kg, respectively.

To demonstrate the device, a novel thermoacoustic test bench was constructed to generate distinct shear stress profiles. Wall shear stress measurements were consistent with the FEM thermoviscous simulation exhibiting slip regime flow with discrepancies possibly due tilt in the AW, thermal boundary layer effects, or interference from the optical fibre array. Despite the acoustic pressure that was predicted to be several times larger than the wall shear stress, there was little to no measurable pressure signal from the device during spatial response measure-
ments indicating low crosstalk with pressure. The results of this paper not only demonstrate a high-resolution air-coupled wall shear stress sensor suitable for photonic microsystems integration, but also introduces a new method of experimental validation of future acoustic pressure and wall shear stress sensors.

**Bibliography**


Chapter 5

Noise and Crosstalk of Vertically Misaligned Coupled-Waveguide Shear Stress Sensors

In the previous chapter, PC membranes having undergone the vapour-etch post-fabrication process were driven using a thermoacoustic waveguide apparatus that was constructed to generate distinct pressure and wall shear stress spatial profiles. Since the vapour-etched PC membranes exhibited relatively low sensitivity to dynamic pressure, it was hypothesized that these membranes were not significantly buckled, with the PC edges comprising the coupled-sensor element having only small steady-state vertical offset. This is in contrast to the PC membranes that were previously processed using the wet-etch which exhibited a higher sensitivity to dynamic pressure and believed to be highly buckled.

In this chapter, identical structures, after having undergone either wet- or vapour-etch post-foundry processing, were compared. Using optical profilometry, it was confirmed that the wet-etched membranes had larger vertical misalignment between the PC edges when compared to the vapour-etched membranes. In the context of wall shear stress sensors that rely on the coupling of symmetric waveguides, vertical misalignment is unwanted since it (1) diminishes the sensitivity of the coupling coefficient to horizontal motion of the membrane and (2) introduces a sensitivity of the coupling coefficient to vertical motion of the membrane. The effects due to (2) are particularly important since this not only leads to crosstalk between pressure and wall shear stress measurements in the optical read-out, but also raises the overall noise floor as thermal fluctuations of the more heavily damped vertical vibration mode are much larger than fluctuations of the horizontal vibrational mode.

1 A version of this chapter is in preparation for publication: Michael Zylstra, Aref Bakhtazad, and Jayshri Sabarinathan, ‘Noise and Crosstalk of Vertically Misaligned Coupled-Waveguide Shear Stress Sensors’
5.1 Introduction

The accurate measurement of wall shear stress is important to many industrial and aerospace applications such as characterizing surface flows, improving performance of vehicles through the reduction of drag [1, 2], separation detection and flow control applications [3], or validation of computation fluid dynamics models [4]. As arrays, smart materials which can change shape according external stimulus are important elements for turbulent and laminar flow research [5] and are considered a future technology for advanced aircrafts [6].

An important consideration facing suspended-element wall shear stress sensor design is the misalignment between sensor elements [7, 8]. Silicon-on-insulator (SOI) is a popular material used in the fabrication of microelectromechanical systems (MEMS) and increasingly used for integrated photonics [9]. However, compressive strain built up in the device layer of SOI is known to exist, possibly due to mismatches in thermal expansivity during wafer fabrication process in which the bonding of silicon and thermal oxide occurs at high temperature (Smartcut by Soitec [10]). Axial compression on released microstructures is known to lead to buckling [11, 12] and can cause shifts in eigenfrequencies and vibrational mode shapes [13].

In the context of coupled-waveguide wall shear stress sensors based on the horizontal separation between two waveguides, any vertical misalignment will break the symmetry presented by perfectly coplanar waveguides. This not only decreases the sensitivity to changes in horizontal separation of the waveguides, but sensitizes the optical read-out to changes in vertical separation [14]. This leads to a mixture of shear and pressure measurement signals in the optical read-out and the emergence of additional vibrational modes in the sensor noise floor. In the low frequency limit, where the time scale of forces acting upon the structure are much longer than the mechanical relaxation time, thermal fluctuations of all vibrational modes contribute incoherently to the overall noise floor of the device and thereby increase the detection limit.

In our previous work, we had demonstrated a suspended-element deflection sensors based on the position-dependent directional coupling between dielectric-like edge states of neighbouring silicon photonic crystal slabs [14]. Recently, these devices were demonstrated as ultrasonic dynamic pressure sensors [15] and wall shear stress sensors [16]. However, the sensor response to a mixture of pressure and wall shear stress was unexplored. In this paper, we compare the crosstalk and noise floor between highly-buckled (HB) and lightly-buckled (LB) PC edge-coupled wall shear stress sensors with different vertical misalignments. Following this introduction, Section 5.2 describes the sensor description, crosstalk, and noise floor model. In Section 5.3, the fabrication, optical profilometry, and scanning electron microscope images (SEM) are presented. In Section 5.4, the noise floor measurements and spatial response of each sensor are presented and discussed. The conclusion and final remarks are presented in the
5.2 SOI Sensor Description

A cross section of the membrane is shown in Fig. 5.1(a). To satisfy the no-slip boundary conditions, the fluid velocity must be zero at the surface of the membrane and reach its free-stream value after the boundary layer distance above the membrane. The wall shear stress arises from the gradient of the flow velocity at the wall and can be written as [17]

$$
\tau_x = \mu \frac{\partial U}{\partial z}
$$

(5.1)

where $\tau_x$, $U$, and $\mu$ are wall shear stress in the x-direction, the fluid velocity in the x-direction, and the dynamic viscosity of the fluid which about $1.8 \times 10^{-5} \text{ Pa}\cdot\text{s}$ for air at 1 atm.

To readout the membrane position, the PCDC line defect depicted in Fig. 5.1(b) was used with the target dimensions outlined in our previous work [14] and described in the figure caption. Using the photonic band gap of the PC, light was confined to the membrane edges whose widths have been tuned to support dielectric-like modes that enable directional coupling. By interfacing with one of the PC edges, changes in the PC edge separation that affect the PCDC coupling strength could be observed in the transmission. Since nonlinear effects were not observed, we assumed an adiabatic optomechanical regime where optical backaction was ignored which is ideal for direct read-out of the position of the membrane [18].
The HB and LB wall shear stress sensors studied in this paper were identical before post-processing chemical etching of the buried oxide and were surface-micromachined on SOI with 220 nm device layer. The HB and LB devices consisted of a silicon photonic crystal (PC) membrane suspended by four parallel supports at each corner as shown in Fig. 5.1(c). Due to compressive stress in the device layer arising from a number factors such as wafer processing conditions, die location, and post-processing method, these structures were susceptible to buckling after selective etching of the buried oxide that leads to uncontrolled vertical misalignment as shown in Fig. 5.1(d).

5.2.1 Crosstalk

In the measurement setup, the optical signal exiting the device was coupled to a reverse-bias photodetector and the photocurrent was measured by the signal analyzer. The power spectrum density of the photocurrent $S_{II}$ measured by the signal analyzer was assumed to have the following relationship

$$S_{II} = \alpha_p^2 \rho^2 + \alpha_r^2 \tau^2 + N$$

(5.2)

where $\alpha_p$, $\alpha_r$, $N$ are the photocurrent sensitivity to acoustic pressure, sensitivity to wall shear stress, and noise floor, respectively. The crosstalk was defined as

$$\eta = \frac{\alpha_p}{\alpha_p + \alpha_r}$$

(5.3)

If the wavelength of sound is much longer than the dimensions of the microstructure, the mechanical response to pressure and wall shear stress will depend mainly on the properties of the fundamental horizontal and vertical vibrational modes in the x- and z-directions, respectively. For frequencies much lower than the mechanical resonances of the microstructure, the mechanical response of the sensor is governed by Hooke’s law and the sensitivity to wall shear stress can be expressed in terms of the sensitivity to horizontal deflection

$$\alpha_r = \alpha_x A / k.$$  

(5.4)

where $\alpha_x$ is the photocurrent sensitivity to deflection of the membrane in the horizontal direction and $k = 4\pi^2 f_x^2 m_x$ is the spring coefficient of the membrane in the horizontal direction. The effective area was defined as

$$A = \int u \, dA.$$  

(5.5)
where \( \mathbf{r}(x) = [u, v, w] \) is the displacement profile of the fundamental lateral vibrational mode of the membrane that is normalized to unity with respect to the point of maximum deflection which can be obtained using the finite-element method (FEM) solid mechanics eigenfrequency analysis.

### 5.2.2 Noise Floor

If the system is underdamped and thermally driven, then the single-sided thermal noise spectral density related to the displacement amplitude of an arbitrary vibrational mode is given by [19, 20]

\[
S_{nn}(f) = \frac{k_B T f_n}{2\pi^3 m_n Q_n (f^2 - f_n^2)^2 + (ff_n/Q_n)^2},
\]

where \( k_B, T, f_n \) and \( Q_n \) are Boltzmann’s constant, ambient temperature, resonant frequency, and the quality factor of the \( n \)th mode, respectively. To satisfy the equipartition theorem, the modal mass \( m \) was defined using the normalized mode profiles defined by the volume integral

\[
m = \int \rho|\mathbf{r}(x)|^2 \, dV,
\]

where \( \rho \) represents the density of the material. If the noise floor of the sensor is mainly due to the thermally induced motion of membrane, Eq. (5.6) can be used to determine the photocurrent sensitivity to the amplitude of each vibrational mode and is given by [19].

\[
N = \alpha_x^2 S_{xx} + \alpha_z^2 S_{zz} + N_0
\]

where, \( \alpha_x \) and \( \alpha_z \) are the photocurrent sensitivity to the amplitude of the lateral vibrational mode and horizontal vibrational mode, respectively. An additional noise source term \( N_0 \) was included to model other noise sources. Minimizing the noise floor is important as it determines the equivalent detection limit of the sensor which is given by

\[
\tau_{\text{min}} = \sqrt{N/\alpha_z}.
\]

From Eq. (5.8), the ideal situation is where \( \alpha_z \) is zero since the vertical vibrational mode does not couple strongly to the wall shear stress yet still contributes to the noise floor in the low frequency limit. Thus, the crosstalk introduced by vertical misalignment of the PC edges not only diminishes the overall sensitivity of the sensor but increases the detection limit.
5.3 Fabrication

To study the effects of vertical misalignment, wet- and vapour-phase selective etching was used to induce different buckling states of the PC membrane. The surface tension forces arising from the wet chemical etching process tend to draw the membranes toward the substrate. While vapour phase etching eliminates the effects of liquid surface tension, it does not entirely guarantee buckling will not occur as stress built up in the SOI device layer may still be present. The fabrication of the HB and LB devices followed different post-processes that are summarized in Fig. 5.2.

Both the HB and LB devices were surface-micromachined on single crystal SOI using IMEC ePIXfab silicon photonics foundry arranged through CMC Microsystems as multi-project wafers. For all devices, the device layer thickness and buried oxide thickness of the SOI were 220 nm and 2 μm, respectively. All wet-etch post-process steps for the HB device were performed at Western University Nanofab while all vapour-etch steps for the LB device was conducted at the University of Alberta nanoFAB Centre.

For the wet-etch post-process used for HB device, exposure of the SOI chip to the wet etchant was restricted using a patterned photoresist. To pattern the chip, the chip was first cleaned using acetone and 5 min of ultrasonication (Branson 200, 46 kHz, 30 W), 1 min rinse with isopropanol, 1 min rinse H₂O, and N₂ air dry. The cleaned chips were then coated with hexamethyldisilazane adhesion promoter using YES-310TA at 150°C. Photoresist (S1805) was spin-coated at 3000 rpm for 45 s followed by soft bake at 115°C for 1 min. Using a mask aligner (NXQ 4006 Neutronix-Quintel), the photoresist was exposed to 38 mW/cm² of 405 nm for 2.2 s and then developed using 75 s of MF-319 followed by 1 min H₂O rinse. To remove residual photoresist in the patterned openings, oxygen plasma descum was used at 200 mTorr for 180 s (Trion Phantom III). For the wet-etch process, buffered hydrofluoric acid etch (BHF) (Ammonium Hydrogen Difluoride Solution, 8 (6.1), PGII, Transene) for 30 mins with an etch rate of about 100 nm/min. After selective etching, the sample was transferred to H₂O for 15 min and acetone for 30 min to remove the photoresist pattern. During these steps, the sample was kept inundated to avoid premature drying and collapse of the membranes due to surface tension. After stripping the resist, the chip was transfered to ethanol and loaded into the critical point CO₂ drying chamber (EMS850 Critical Point Drier).

For the vapour-etch post-process used for the LB device, a vapour-phase hydrofluoric acid (VHF) etch was used as the surface tension created by liquid-phase etchant can potentially collapse the microstructures during the subsequent drying step. VHF was done using a 1 hr etch using memsstar Orbis Alpha with a pressure and chuck temperature of 11 Torr and 21°C. The flow rates of HF, vaporizer N₂, and H₂O were 40 sccm and 20 sccm, and 5 mg/min,
respectively.

Figure 5.2: Post-foundry processing of highly-buckled and lightly-buckled PCDC wall shear sensors. After surface-micromachining the SOI, selective removal of the buried oxide (BOX) was done using either buffered hydrofluoric acid wet-etch (BHF) or vapour hydrofluoric acid etch (VHF). To prevent collapse of the structures, critical point CO₂ drying (CPD) was used as a final step for the wet-etch.

To characterize the vertical misalignment of the HB and LB devices, optical profilometry was used (Bruker Contour GT-K 3D Optical Profiler and Zygo Newview 9000) with the 3D topography of the LB device shown in Fig. 5.3(a). The extracted profiles across the membrane indicate a 220 nm vertical misalignment for the LB device and 550 nm for the HB device and are shown in Figs. 5.3(b) and (c), respectively.

Figure 5.3: (a) Optical profilometry of non-bucked device. Extracted surface profile of (b) the highly-buckled and (c) lightly-buckled device taken along the path depicted.

5.4 Results and Discussion

Optical transmission measurements on the fabricated devices were performed using setup described in Fig. 5.4(a) which consisted of a tunable laser source (TL, Anritsu Tunics Reference
SCL band), photodetector (PD, PBD450C Thorlabs), band-pass filter (BPF, 1 kHz–4 MHz), and signal analyzer (SA, Keysight N9030A PXA). The devices under test (DUT) were interfaced using an optical fiber array (FA) that coupled light to surface grating couplers on the photonic integrated circuit (PIC). For the driven spatial response measurements, a function generator (FG, Keysight 33120A) and power amplifier (PA, Siglent SPA1010) delivered a sinusoidal voltage signal to a thermoacoustic emitter (TA) consisting of an indium tin oxide (ITO) thin film that was described in more detail in our previous work [15]. The TA was coupled to an acoustic waveguide (AW) 60 mm long with an inner and outer radius of 1.2 mm and 1.4 mm, respectively, and could be positioned above the chip using an X,Y,Z translation stage.

5.4.1 Undriven sensor noise-floor

Before driven thermoacoustic testing, the noise floor of the devices was measured by tuning the wavelength of the TL such that the noise floor could be resolved by the SA. The measured noise floor for the HB and LB devices are shown in Fig. 5.4(b) and (c), respectively. From Fig. 5.4(b), three distinct vibrational modes could be observed. The sharpest peak was identified as the first horizontal vibrational mode since the reduced aspect ratio along its axis of motion was expected to result in the lowest fluid dampening. The other two modes, which were absent in the spectrum density of the LB device, we associated with vibrational modes having displacement profiles lying primarily in the vertical direction, as this was consistent with the optical profilometry data.

For each device, the acquired amplitude spectrum density was fitted to Eqs. (5.6) and (5.8) using non-linear least squares method by varying resonant frequency $f_n$, quality factor $Q_n$, and sensitivity $\alpha_n$ and the results are summarized in Table 5.4.1. Using the fitted sensitivity to the first horizontal vibrational mode $\alpha_i$, the measured photocurrent could be related to the in-plane displacement of the membrane. The low frequency response of the sensor, characterized by the spring coefficient $k_i$, was determined using the fitted resonant frequency and FEM generated effective mass. This allowed horizontal deflections observed in the photocurrent to be converted to a shear force which was then related to a shear stress by Eq. (5.4) using the FEM generated effective area. Using wall shear stress sensitivity and the measure noise floor, the detection limit was computed using Eq. (5.9). The reduction in vertical misalignment of the PC edges led to an improvement in the detection limit by a factor of 4.5 times.

5.4.2 Spatial Response and Crosstalk

To determine the crosstalk between the wall shear stress measurement with the pressure, spatial response measurements were performed with the AW positioned above the DUT as shown in
Figure 5.4: (a) The optical measurement system showing the tunable laser (TL), device under test (DUT), photodetector (PD), band-pass filter (BPF), and signal analyzer (SA). A function generator (FG) and power amplifier (PA) were used to excite a thermoacoustic emitter (TA) that coupled an ultrasonic signal to the DUT using an acoustic waveguide (AW). The AW was positioned above the chip with an X,Y,Z stage. The undriven thermal noise floor measured by the SA for the (b) HB and (c) LB devices.

Table 5.1: Summary of sensor noise floor measurements

<table>
<thead>
<tr>
<th>Device</th>
<th>$f_{i,\text{sim}}$</th>
<th>$m_{i,\text{sim}}$</th>
<th>$A_{i,\text{sim}}$</th>
<th>$f_{i,\text{sim}}$</th>
<th>$k_i$</th>
<th>$Q_i$</th>
<th>$\alpha_r$</th>
<th>$\tau_{\text{min}}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(HB)</td>
<td>899</td>
<td>84.9</td>
<td>171</td>
<td>778</td>
<td>2.03</td>
<td>27.7</td>
<td>36.1</td>
<td>806</td>
<td>2.8</td>
</tr>
<tr>
<td>(LB)</td>
<td>899</td>
<td>84.9</td>
<td>171</td>
<td>853</td>
<td>2.44</td>
<td>27.7</td>
<td>134</td>
<td>181</td>
<td>0.35</td>
</tr>
</tbody>
</table>
surface was fixed and the AW was moved parallel to the surface of the PIC. The LB structure showed good fit with thermoviscous simulated wall shear stress generated when the AW was 30 µm above the PIC; however, a noticeable cusp in the measured spatial response of all devices appeared near the outside edge of the AW at \( r_{\text{out}} = 1.4 \) mm that did not appear in the thermoviscous model. This could be due to a number of factors related to the thermal boundary layer being thicker than the inertial boundary layer (\( Pr = 0.71 \)), convective effects, or interference with the optical fiber array. The response of the sensor while the edge of the AW tube was centred above the DUT at \( r = 1.3 \) mm while its height increased is shown in Fig. 5.5(c) with the LB device showing good agreement with the thermoviscous model.

The crosstalk was determined using the sensor response inside the AW where the wall shear stress was expected to be low and the pressure highest. At 40 kHz, the thermoviscous simulation indicates the acoustic pressure at the surface of the PIC monotonically decreases from the centre of the AW and is shown in Fig. 5.5(d). Assuming the photocurrent signal measured by the DUT inside the AW was entirely due to the acoustic pressure, the sensitivity to pressure was computed to be 1.02 and 0.465 \( \text{uA/Pa} \) for the HB and LB devices, respectively. This allowed the computation of the crosstalk for each device to be computed using Eq. (5.3) to be 2.8% and 0.35% for the HB and LB devices, respectively.

![Figure 5.5: (a) Cross section of the AW, PIC, and FA, with respect to the location of the DUT. (b) The simulated and measured wall shear stress profiles from the HB and LB devices for an AW 30 µm above the DUT. (d) The measured sensor response and simulated wall shear stress directly under the edge of the AW wall for \( r = 1.3 \) µm as the air gap separation \( z \) is varied. (d) The simulated acoustic pressure for a AW height of 30 µm.](image)

### 5.5 Conclusion

In this paper, the noise and crosstalk of vertically misaligned PC edge-coupled shear stress sensors were investigated. Identical PC membranes were surface micromachined on SOI then released from the buried oxide using different post-foundry etch processes to achieve buckled states with different vertical misalignment. The vertical misalignment of the coupled PC-edges
was characterized using optical profilometry that determined them to be 550 and 220 nm for the HB and LB devices, respectively. Thermal fluctuations related to the vibrational modes of the structures were observable in the optical read-out with the mode with highest quality factor identified as the horizontal vibrational mode which is most strongly coupled to shear stress. By modeling thermal fluctuations in the horizontal vibrational mode, the photocurrent was converted to an equivalent shear stress. It was found that the larger vertical misalignment of the HB device sensitized the optical read-out to thermal fluctuations of additional vertical vibrational modes leading to a 4.5 times increase in noise floor at low frequencies.

The crosstalk was determined by exciting a single-mode acoustic waveguide at 40 kHz made to impinge upon the SOI samples and generate distinct pressure and shear stress spatial profiles. The measured photocurrents were converted to an equivalent shear stress and found to agree with thermoviscous simulation results. Within the inner radius of the tube, where the shear stress is much smaller than the pressure, the sensor signal was found to be 8 times larger for the HB device and attributed to a mixing with the dynamic pressure signal. The results of this paper demonstrate the importance of managing vertical misalignment of coupled-waveguide shear stress sensors and its impact on crosstalk and noise floor.

5.6 Supplementary Information - Frequency Reponse of the Tube

![Figure 5.6: Frequency response of the acoustic resonator 60 mm and inner radius 1.2 mm measured using PCDC sensor located within the inner radius with the shaded red region indicating the single-mode regime. Below are the first four acoustic mode profiles of a circular duct.](image)

The frequency response of the 60 mm tube was acquired by the highly buckled sensor and shown in Fig. 5.6. Since the sensor was positioned within the inner radius of the AW where the wall shear stress was low, the signal was assumed to be primarily from the acoustic
pressure. In the single-mode regime of the tube, longitudinal resonances of the tube can be observed separated by 2.85 kHz. Assuming the pressure to be maximum at the boundaries of the tube, this is consistent with the length of the tube using the speed of sound \( v \) of 343 m/s (i.e., \( L = \frac{v}{2\Delta f} \)). For frequencies above the single-mode, multimode interference between the acoustic modes occurs and marked by a distinct change in the frequency response of the tube. In Fig. 5.6, the first four acoustic mode profiles of the circular duct are shown with the cut-off frequencies determined by \( \frac{v}{1.7063D} \), \( \frac{v}{1.0286D} \), or \( \frac{v}{0.8199D} \) using the waveguide inside diameter \( D \) for modes \((1,0), (2,0), \) and \((0,1)\), respectively [21].

Bibliography


Chapter 6

Mitigation of Misalignment in
Silicon-on-Insulator Coupled-Waveguide
Shear Sensors\(^1\)

In the previous chapter, it was determined that vertical misalignment of coupled-waveguide sensors was detrimental to wall shear stress measurement since it (1) diminishes the sensitivity, (2) raises the sensor noise floor, and (3) introduces pressure crosstalk. In this chapter, microbeam arrays were integrated into the PC membrane supports that allow compressive strain present in the device layer of SOI to relax and prevent buckling and vertical misalignment of the PC edges. Vertical misalignment of buckle-compensated and non-compensated sensors was compared using optical profilometry and the crosstalk with pressure was determined using the thermoacoustic integrated photonics testbench.

6.1 Introduction

Silicon-on-insulator (SOI) fabrication techniques are now mature to enable commercially viable photonic integrated circuits (PICs) \(^1\). Single-crystal SOI wafers with 220 nm top silicon layer are widely available among global foundries and are well suited for C-band (1530 to 1565 nm) to create low loss integrated photonic devices such as waveguides, splitters, and resonators for transverse electric (TE) modes \(^2\). Furthermore, post-foundry undercutting of patterned SOI, achieved through selective etching of the buried oxide using liquid \(^3\) or vapour \(^4\) phase hydrofluoric acid (HF), is commonly used to create suspended microelectromechanical and micro-optomechanical devices. However, SOI generally

\(^1\) A version of this chapter is in preparation for publication: Michael Zylstra, Aref Bakhtazad, and Jayshri Sabarinathan, ‘Mitigation of Misalignment in SOI Coupled-Waveguide Shear Sensors’
has compressive strain in the device layer, possibly due to mismatches in thermal expansivity built up during the flip-bonding process, that lead to buckling after sacrificial etching of the buried oxide. Compressive strains present in SOI wafers can lead to buckling [5] and shifts in resonant frequencies of microstructures [5].

Previously, we have demonstrated PC edge coupled sensors as deflection sensors [6], pressure sensors [7], and wall shear stress sensors [8] with high spatiotemporal resolution using coupled silicon photonic crystal edge waveguides. However, the vertical misalignment due to buckling was previously shown to hinder sensor performance by raising the noise floor and introducing crosstalk with acoustic pressure signals [9]. Microbeam arrays, previously investigated by Iwase et al., were shown to reduce vertical misalignment on membranes fabricated on SOI substrates [10]. The objective of this work was to integrate these microbeam arrays for coupled waveguide shear sensing.

### 6.2 Sensor Design Theory

![Figure 6.1: (a) SEM image of a non-compensated structure and (b) a buckle-compensated structure. (c) Displacement of compensated structure under 10 mPa boundary load at the surface. (d) PC coupled edge defect.](image)

Oftentimes, it is desirable to measure a distributed force acting upon a surface. For the case of shear stress arising from the tangential flow of a viscous fluid across a membrane, the wall shear stress is given by [11]

\[
\tau = \mu \frac{\partial U}{\partial z},
\]

where \( \mu \) is the dynamic viscosity of air taken to be 18.1 \( \mu \)Pa·s and \( U(z) \) is the tangential fluid velocity at the surface. The noise floor of a microstructure in terms of its equivalent shear stress while operating under ambient atmospheric conditions is fundamentally limited by the thermal fluctuations present in the membrane position and is described by the dissipation fluctuation theorem and given by [12]
Here, $k_B$, $T$, $m$, $\omega_0$, $Q$, and $A$ are Boltzmann’s constant, ambient temperature, mass, resonant frequency, quality factor, and effective area of the sensor, respectively. Since the mass is proportional to the area (i.e., $m = \rho A$), the ideal material for measuring distributed forces, such as shear stress, with high spatiotemporal resolution (i.e., small $A$ and large $\omega$) is to use geometries that are as thin as possible (i.e, low $h$). On one hand, this makes high aspect ratio of undercut SOI ideally suited for the transduction of distributed forces. On the other hand, thinner geometries are more susceptible to buckling due to compressive strain known to exist in the device layer.

As patterned structures on the device layer are released from the buried oxide, they may find a new equilibrium point and buckling happens. Figure 6.1(a) shows the scanning electron microscope (SEM) image of a non-compensated structure that has buckled after undercutting. For any specific geometry, one can find a critical residual strain that causes the beam to start buckling using linear buckling analysis. In our case, where the thin silicon device layer is 220 nm thick, the residual compressive strain was expected to lie between $5 \times 10^{-5}$ to $5 \times 10^{-4}$ [10]. Using COMSOL linear buckling finite element modelling, the critical buckling strain of the non-compensated structure was determined to be $7.1 \times 10^{-5}$, which was close to the lower limit of the expected strain and therefore expected to buckle. Figure 6.1(b) shows the SEM image of a buckle-compensated structure with curvilinear microbeam arrays (CMBA) that allow relaxation of wafer strain. To avoid buckling, we designed the CMBA to achieve a critical buckling strain approximately twice that of the highest expected strain present in the wafer. Using COMSOL, the critical buckling strain of the compensated structure was found to be $1.0 \times 10^{-3}$ and therefore not expected to buckle in the worst case scenario.

In figure 6.1(c), COMSOL was used to numerically model the steady-state displacement of the compensated structure under 10 mPa of wall shear stress, leading to a 6.37 picometers of lateral displacement of the membrane. The displacement was strongly coupled to the lateral vibrational mode with the resonant frequency, effective area, and modal mass determined using eigenfrequency analysis and summarized in table 6.1. To satisfy (6.2), the modal mass was determined by integrating the vibration displacement profiles that have been normalized such that their maximum displacement was unity (i.e., $m = \rho \int |r|^2dV$) [13].

The optical readout was based on the coupling of adjacent PC slab edges described in our previous work [6] and shown in figure 6.1(d). In order to maximize the coupling both edge PC waveguides were assumed identical. Any change in the geometry (i.e. bridge deflection or air gap changes) alters the coupling that changes the transmitted power. It is crucial that

\[ \tau_{th} = \sqrt{\frac{2k_B T m \omega_0}{QA^2}}. \]
the operating wavelength of the sensor falls inside the photonic band gap (PBG) of the slab photonic crystal, where light is forbidden to propagate in any lateral direction.

6.3 Optical Profilometry

In order to quantify the buckling, we used white light interferometric technique using WYKO NT1100 optical surface profiler from Veeco. The non-compensated device shown in figure 6.2(a) had 654 nm of downward vertical misalignment with the profiles of fixed and membrane PC edges shown in figure 6.2(b). The compensated device featuring the microbeam arrays is shown in figure 6.2(c) had a 68 nm vertical misalignment with the profiles of the PC edges shown in figure 6.2(d).

![Figure 6.2: (a) Microscope image of the non-buckled compensated structure and (b) optical profilometry of the fixed (black) and membrane (blue) PC edges. (c) Microscope image of the buckle compensated structure featuring curilinear microbeam arrays and (d) the its corresponding optical profilometry.](image)

6.4 Dynamic Measurements

The optical readout of the buckle-compensated device was probed using the experimental setup summarized in figure 6.3(a) which shows the block diagram of the experimental apparatus depicting the sinusoidal function generator (FG), power amplifier (PA), thermoacoustic emitter (TAE), and acoustic waveguide (AW) whose position was controlled using a translation stage (X,Z). The device under test (DUT) was interfaced with a tunable laser (TL) and measured
using a photodetector (PD). The photocurrent was filtered using a bandpass filter (BPF), and fed to a signal analyzer (SA).

The TAE consisted of an indium tin oxide (ITO) thin film selected due to its thermodynamic properties, chemical inertness, and scratch resistance [14]. The ITO film was fed with 40 kW/m² of power using a pure sinusoidal signal at 20 kHz. This generated an acoustic tone at 40 kHz along with heating of the ITO substrate. The AW consisted of a 6 cm long cylindrical brass tube polished on both sides with an inside diameter of 2.4 mm and outside diameter 2.8 mm. These dimensions allowed single mode operation up to 83 kHz [15]. One end of the the AW was butt-coupled to the ITO thin film while the opposite end was made to impinge on the photonic integrated circuit (PIC) as shown in figure 6.3(b). A close-up of the constriction formed between the edge of the AW and the PIC is shown in figure 6.3(c). The constriction creates a localized increase the fluid velocity flow directly under the edge of the AW. In this experiment, the AW position was scanned across the sensor and the photocurrent amplitude at 40 kHz was recorded.

Before scanning the AW position, the noise floor of the sensor photocurrent was recorded and shown in figure 6.3(d). The peak corresponds to thermal fluctuations in the horizontal vibrational mode of the microstructure that can be modelled using the mechanical susceptibility \( \chi(\omega) = [m(\omega_0^2 - \omega^2) + i m \omega_0 Q]^{-1} \) and (6.2) (i.e., \( X_{th}(\omega) = \tau_{th} A \chi(\omega) \)). By fitting the quality factor and the resonant frequency of the structure, the conversion factor between photodiode current and horizontal deflection of the membrane was determined and summarized in table 6.1.

To demonstrate the shear sensor and determine the crosstalk with pressure, the height of the AW was fixed at 30 µm, and it was swept across the DUT while the measured photocurrent at 40 kHz was converted to an equivalent shear stress and shown in figure 6.3(e). The constriction was simulated in COMSOL using an axis-symmetric thermoviscous model resulting in harmonic shear stress and acoustic pressure profiles at the PIC surface. The sensor photodiode measurements were fit to the simulation by allowing a mixture of simulated pressure at the surface of the PIC. The measurement results agree with the thermoviscous simulation when the sensitivity to pressure is 0.3% of the sensitivity to shear stress. A noticeable cusp that appears directly beneath the outer edge of the AW at \( x = 1.4 \text{ mm} \) is possibly due to a transition between slip-flow and non-slip flow regime.

Next, the edge of the AW was centred above the DUT at \( x = 1.3 \text{ mm} \) and the height was swept. As before, the photocurrent at 40 kHz was converted to an equivalent shear stress and shown in figure 6.3(f). As the height increases, the free stream velocity in the constriction decreases leading to a decrease of shear stress that is good agreement with the COMSOL model.
6.5 Conclusion

Curvilinear microbeam arrays were successfully integrated into a wall shear sensor based on PC edge coupled waveguides. The sensors were fabricated on SOI using the foundry IMF passives silicon phontonic process in addition post-foundry VHF process to release the membranes. Optical profilometry was used to confirmed the integration of CMBA reduced the vertical misalignment. The noise floor of the fabricated devices photocurrent, due to stochastic vibrations of the membrane at thermal equilibrium, could be used to convert the measured photocurrent to horizontal displacement and shear stress acting on the membrane. By impinging an acoustic waveguide driven by a thermoacoustic emission, the shear sensor was demonstrated with low pressure crosstalk. Since it was previously shown that vertical misalignment of coupled-waveguide sensors was crucial to improving the SNR and reducing crosstalk with

Figure 6.3: (a) Block diagram of the experimental setup. (b) Cross-section of the acoustic waveguide impinging on the PIC. (c) Zoom of constriction formed by the acoustic waveguide edge and the PIC surface showing the velocity flow profile. (d) The measured (red) and fit (black) noise floor of the device under test. (e) Response of the device by scanning the acoustic waveguide across the PIC surface. (f) Response of the device by scanning the height of the acoustic waveguide positioned directly above the device.

<table>
<thead>
<tr>
<th>( f_{\text{sim}} ) (kHz)</th>
<th>( m_{\text{sim}} ) (pg)</th>
<th>( A_{\text{sim}} ) (( \mu \text{m}^2 ))</th>
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pressure, the adaptation of microbeam arrays is considered to be an important design technique in future coupled-waveguide shear sensors fabricated on prestrained substrates.

Bibliography


Chapter 7

Photonic Crystal Edge-Coupled Mass Sensor

In the previous chapters, PC membranes underwent varying post-foundry fabrication processes to evaluate different performance aspects of the PCDC-based sensors. In Chapter 3, partially and fully released the PC membranes were used to evaluate the sensitivity of the PCDC coupling coefficient to horizontal and vertical deflections the membrane. In Chapter 6, wet- and vapour-etch were used to achieve different vertical positions of the membrane and its effect on sensor performance in the context of wall shear stress sensing.

In this chapter, the effect of varying the mass of the PC membrane was examined. Changes in mass were implemented by through variations in the PC hole diameter of otherwise identical suspended PC membranes. Using the thermal fluctuations of the membrane position, changes in the mechanical resonance of the PC membrane were observed in the spectrum density of the photocurrent generated and related to changes in the membrane mass.

7.1 Introduction

By observing changes in the resonant frequency of a vibrating structure, mass sensors have been used to detect biomolecules [1, 2], cells [3], microdroplet evaporation [4], proteins [5], and viruses [6, 7]. The major advantages of mass sensors based on this method are the simplicity of their construction and operation, low weight, and small power requirements [8].

There exist numerous techniques to probe the resonant frequency of microstructures including piezoelectric [9], piezoresistive [10], atomic force microscopy [11], interferometric [12],

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1A version of this chapter is in preparation for publication: Michael Zylstra and Jayshri Sabarinathan, ‘Photonic Crystal Edge-Coupled Mass Sensor’
and coupled cavity optomechanical systems [5–7, 13]. In general, the sensitivity and detection limit of added-mass sensors benefit from miniaturization and smaller masses [14], along with higher mechanical quality factors [2]. However, the implementation and characterization of integrated photonic sensors often requires either demanding fabrication tolerances, or expensive probing equipment such as optical amplifiers, tunable narrow-linewidth lasers, or high-resolution spectrometers to detect changes in optical cavity resonances [15].

A common approach to reducing the size of photonic sensors is through the use of periodic dielectric structures that allow engineering of waveguide dispersion that enhance light-matter interactions. By engineering the dispersion of subwavelength waveguides (SWG), interference-based devices that rely on the relative phase differences between SWGs have been conceived as a way to achieve wavelength-independent designs and robustness against fabrication variations and have been demonstrated for use as phase-shifters [16] and directional couplers [17, 18].

In our previous work, we demonstrated a suspended-element deflection sensor based on the position-dependent directional coupling between dielectric edge modes of neighbouring photonic crystal slabs [19]. The dispersion of the photonic crystal (PC) edges enabled shorter beat-lengths when compared to directional couplers based on conventional index-guided waveguides, while exhibiting broadband changes in optical transmission under static displacements. Being edge-based, the design does not rely on the membrane-substrate distance and thereby reduced the probability of stiction and allowed direct read-out of the in-plane position. This is important in the context of added-mass sensors as the in-plane vibration modes can substantially decrease the liquid drag force when compared to conventional out-of-plane resonance-modes and thereby achieve a much higher Q-factor [20].

In this paper, we now present the measurements of the suspended-element photonic crystal directional coupler (PCDC) device for the purposes of mass sensing. By measuring the photodiode current amplitude spectrum density from the PCDC, the thermal fluctuations of the PC membrane in ambient conditions were observed and the lowest vibration modes could be identified. Prescribed changes of mass, implemented by varying the PC hole diameter, allowed characterization of masses of the order of picograms through observable changes to the fundamental in-plane resonance.

7.2 Optical Design and Fabrication

The design of the microstructure is described in our previous work and was based on a transverse electric (TE) photonic band gap that creates single-mode dielectric-like edge states in the SCL-band (1460–1625 nm) suitable for directional coupling [19]. The microstructure was fab-
Fabricated on silicon-on-insulator (SOI) and consisted of a central silicon PC membrane, as shown in Fig. 7.1, whose two sides form two separate PCDC elements. Fully etched regions in the top silicon layer exposed the buried oxide (BOX) so it could be removed by selective etching with buffered hydrofluoric acid (BHF). The design is such that four strip waveguides support the PC after BOX etching. By using strip waveguides as mechanical supports, both output ports were accessible and used to monitor the coupling between PC edges. In this work, we assumed an adiabatic regime, with weak optomechanical coupling and a fast optical cavity decay, which is ideally suited for precise readout of the instantaneous oscillator position [21].

![Figure 7.1](image-url)

Figure 7.1: (a) The SOI fabricated photonic crystal membrane directional coupler showing the input (black), through (blue), and coupled (red) output ports. (b) Top view of PCDC defect structure. (c) Cross section view of the PCDC.

### 7.3 Mechanical Model

A general procedure for thermal mechanical calibration of nano and micro mechanical resonators using the fluctuation-dissipation theorem is outlined by B.D. Hauer et al [22]. Solutions to the undamped linearized model yield a set of eigenfrequencies and mode displacement profiles that may be solved for using finite element methods (FEM). Assuming the modes are uncoupled, the generalized coordinates are governed by Newton’s second law.

\[
\ddot{q}_n + \frac{\Omega_n}{Q_n} \dot{q}_n + \Omega_n^2 q_n = \frac{F(t)}{m_n}
\]  

(7.1)
Here, $\Omega_n$, $Q_n$, and $m_n$ are the angular eigenfrequency, quality factor, and effective mass for the $n$th mode, and $F(t)$ is the sum of all external forces acting on the mechanical oscillator.

Following Hauer et al, we normalized the mode displacement profiles such that the absolute value of the maximum displacement was unity. To ensure consistency with the equipartition theorem, the effective mass $m_n$ was computed by integrating the normalized mode profiles over the membrane volume

$$m_n = \rho \int dV |\mathbf{r}_n(x)|^2$$

where $\rho$ represents the density of the material. The spring coefficients for each vibration mode can be computed using their respective effective masses and eigenfrequencies.

$$k_n = m_n \Omega_n^2$$

Since the PC region is much wider than the narrow support structures, we assumed structural deformation was limited to only the four support structures and deformation of the PC membrane was negligible. Under this assumption, an added-mass restricted to only the PC membrane region will therefore have a negligible affect the spring coefficient. Using Eq. (7.3), the altered resonant frequency is therefore given by

$$\Omega_n + \Delta \Omega_n = \sqrt{\frac{k_n}{m_n + \Delta m}}$$

For the fundamental in-plane and out-of-plane vibration modes, maximum deflection occurs homogeneously across the PC region, therefore the added-mass contribution can be directly added to the effective mass for these modes. Note, that this is in contrast to higher-order and torsional modes, where changes to the effective mass would depend on the specific displacement profile membrane. For this study, the added-mass was achieved by varying the PC hole radius and is given by

$$\Delta m = \frac{1}{4} N \rho_{Si} h \pi (d_1^2 - d_2^2)$$

where $N$ is the number of PC holes, $\rho_{Si}$ is the density of silicon taken to be 2330 kg/m$^3$, $h$ is the thickness of membrane, $d_1$ is the nominal PC hole diameter, and $d_2$ is the altered PC hole diameter.
7.3.1 Finite Element Analysis

A finite element model (FEM) was created using COMSOL with nonlinear structural mechanics using a Young’s modulus $E_{Si}$ of 169.8 GPa and Poisson’s ratio of 0.361 [23]. Compressive strain built up in the device layer of the SOI, possibly due to mismatches in thermal expansivity, is known to exist in SOI samples [24] and was included in the FEM model as a compressive strain $\sigma_0$. Axial compression on released microstructures is known to lead to buckling [25] and cause shifts in eigenfrequencies and mode shapes [26].

Applying fixed-fixed boundary conditions to all supports, the effect of $\sigma_0$ was introduced by using prescribed displacements $\Delta L = \sigma_0 L_{TOT} / E_{Si}$. Above the critical buckling load of 12 MPa, a bifurcation occurs and the membrane no longer lies in-plane with the surrounding device layer, introducing a static displacement component. The structure thereafter was assumed to conform to the shape of the first buckling mode.

![Figure 7.2: Eigenfrequencies of supported PC membrane under axial strain, with inset showing the shapes of the displacement mode profiles. A is the fundamental out-of-plane mode, B is a higher-order out-of-plane mode antisymmetric about the z-axis, C is the fundamental in-plane mode, and D is a higher-order out-of-plane mode antisymmetric about the x-axis](image)

The FEM-generated eigenfrequencies of the first four modes were observed to depend on the SOI strain, as shown in Fig. 7.2. Near the critical buckling load, a sharp change in the eigenfrequencies was observed, corresponding to modes with displacement profiles predominately in the out-of-plane direction (identified as mode A, B, and D). For the fundamental in-plane mode (identified as mode C in Fig. 7.2), very little change with respect to intrinsic SOI strain
was observed. This is important, since the changes in eigenfrequency brought about by an added mass must be distinguished from changes brought about by temperature changes that alter the $\sigma_0$ at the boundary conditions of the supports.

### 7.4 Measurements

![Measurement setup diagram](image)

Figure 7.3: Measurement setup featuring a tunable laser, PCDC membrane, and photodetectors. For static testing, a four-channel optical power meter measured both the through (blue) and coupled (red) outputs. For thermal mechanical testing, the through output is fed to a reverse-bias photodiode and signal analyzer.

The measurement setup, shown in Fig. 7.3, consisted of a tunable laser and two photodetection options. For static measurements, a multichannel power meter recorded the through and coupled PCDC outputs. For dynamic added-mass measurements, the photocurrent from a reverse-bias photodetector was fed to a signal analyzer. The devices were interfaced using a polarization-maintaining fibre optic array positioned near the chip surface, and aligned to surface grating couplers (SGC) that couple the TE light to single-mode waveguides.

#### 7.4.1 Static Measurements

For static optical measurements, a 1 mW excitation was used and the optical power was recorded at the through and coupled outputs of baseline PCDC device of length $32a$ and PC hole diameter 292 nm, and shown in Fig. 7.4. An exchange of power could be observed between the output ports across 1485–1534 nm, where both even and odd PCDC modes were supported. The ripples present in the transmission spectrum are due to the Fabry-Perot cavity formed by the reflections at the waveguide/PCDC junction.
7.4.2 Dynamic Added-Mass Measurements

For dynamic added-mass measurements, the laser was set to 1 mW and tuned near the transmission cross-over points. This was done as the sensitivity of a directional coupler-based sensor is maximized wherever power is shared equally between the two outputs [19]. The amplitude spectrum densities (ASD) of the through output from three different PCDC devices that varied only by their PC hole diameter are shown in Fig. 7.5. We identified the ASD peak with highest quality factor ($Q = 25$) to be the fundamental in-plane vibration mode due to reduced aspect ratio of the membrane along this vibration axis. For increasing added-mass (i.e. smaller PC hole diameters), a decrease in the resonant frequency could be clearly observed. We note that the ASD peak diminished for increasing mass as the PC hole diameters necessarily altered the photonic band structure.

To characterize the optical broadband response of the baseline PCDC device with the 292 nm PC hole diameter, the ASD amplitude of the strongest peak located at 777 kHz associated with the in-plane vibration mode was recorded as the wavelength was swept and shown in Fig. 8.14. We observed the ASD peak could be detected across the measurement bandwidth, and was maximized where power was equally shared between the outputs and minimized wherever a large imbalance existed.
Figure 7.5: The amplitude spectrum density of the photodiode current for structures with varying PC hole diameter. The vibration mode A is the fundamental out-of-plane mode, B is a higher-order out-of-plane mode antisymmetric about the z-axis, C is the fundamental in-plane mode.

Figure 7.6: The amplitude spectrum density peak of the fundamental in-plane mode C at 777 kHz was measured as the wavelength is swept.
7.5 Discussion

The results of the added-mass measurements are summarized in Table 7.1 showing the PC hole diameters, added-mass, and corresponding frequencies of the fundamental in-plane vibration mode. The added-mass was calculated using Eq. (7.5). A total of 1229 PC holes were present on each device with their diameters determined using SEM. Using these added-masses and the resulting changes in frequency, we report a sensor responsivity of 0.185 fg/Hz.

Table 7.1: Summary of PC membrane, hole diameter, added-mass, and measured horizontal resonance

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<th>$d$</th>
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Based the frequency position of the ASD peaks in the baseline PCDC device, we estimate the SOI strain to be 31 MPa which is in line with previously reported strains found in literature [27]. Using the computed FEM in-plane modal mass and eigenfrequency of 75.3 pg and 747 kHz respectively, the spring coefficient was computed to be 1.66 N/m using Eq. (7.3). The spring coefficient was also computed experimentally by using a linear fit of $\Delta m$ vs $\Omega^{-2}$ and calculating the slope which was found to be 1.53 N/m. We attribute this 8.5% difference between these spring coefficients to discrepancies in the geometry of the narrow support structures between the FEM model and measured structure. Also, uncertainty in the SEM measurements of the PC hole diameters can lead to errors in calculating the add-mass contributions.

7.6 Conclusion

Operating in the adiabatic optomechanical regime, the optical and mechanical quality factors in this paper are lower when compared to the optomechanical mass sensors found in literature that rely on optomechanical actuation or optical feedback mechanisms. This was compensated by the small size of the structure presented here that could easily resolve added-mass changes on the order of a picogram. Using an interferometric detection approach based on the even and odd PCDC modes, compatibility with broadband optical sources was demonstrated. The designs presented here do not rely on high-quality factor optical cavities therefore only a few rows of the PC are required for optical confinement along the PC edge and the centre of membrane can be functionalized for selective adsorption of biomolecules without seriously affecting the
performance of the device. Due to the planar fabrication, these designs can be readily adapted as sensor arrays for future use in environmental monitoring and biomedical sciences.

**Bibliography**


Chapter 8

Miniature Lorentz Force Magnetometer With 2D Photonic Crystal Edge-Coupled Optical Read-Out

In the previous chapters, the PC membrane designs consisted of the same basic design featuring a central membrane with four parallel supports. These devices were then evaluated in the context of pressure, wall shear stress, and mass sensing. Due to the small mass of the devices, these sensors were found to have relatively high spatiotemporal resolution while maintaining low detection limits when compared to existing state-of-the-art sensors. It is therefore natural to ask how coupled PC edges could be utilized to measure other forces. In the context of magnetic field sensing, the energy resolution is believed to be more important performance metric that factors the volume of the sensor.

In this chapter, new designs, fabrication methods, and experimental apparatus were used to integrate coupled PC edges to allow measurement of magnetic fields. The designs were based on the Lorentz force where an electrical current, in the presence of a magnetic field, experiences a force given by the right-hand-rule. By adding a post-foundry metallization step, an electrically conductive path was patterned across the microstructures that incorporated a coupled PC edge. Since the electrical current heats PC edge, a lumped element model was developed to model joule-heating, thermal dissipation, thermal-optical effects, and thermal expansion along with a semi-empirical fluid damping of the microstructures that determined the detection limit of sensors of varying length. The sensors were evaluated using an electromagnet testbench and compared with existing sensor technologies.

\(^1\)A version of this chapter is in preparation for publication: Michael Zylstra, Brett Poulsen, and Jayshri Sabarinathan, "Miniature Lorentz Force Magnetometer With 2D Photonic Crystal Edge-Coupled Optical Read-Out"
8.1 Introduction

Magnetic sensors have numerous applications ranging from navigation systems that use the Earth’s magnetic field [1], inexpensive and wear-free sensors for automobiles [2], electrical current sensors [3], and non-destructive evaluation of high performance components through the detection of eddy currents [4]. In biomedical imaging, distortions can be entirely removed in magnetic resonance imaging (MRI) if the time-shape of the gradient fields can be acquired with high accuracy [5]. The most sensitive magnetic sensors have been used for monitoring cardiovascular [6, 7], muscular [8], and neural [9, 10] activity which assist in medical research and diagnosis. For low-field MRI, which is less expensive and bulky than high-field MRI, there is a need for miniature and sensitive magnetic sensors as the lower polarization fields result in a slower Larmor precession making pick-up coil sensors less effective [11]. Moreover, there is emerging demand for smaller magnetic sensors for micro- and nanosatellite applications [12][13] for attitude control, geographical surveys, and monitoring of space weather.

Due to their inexpensive fabrication and small size, an important class of magnetic sensors are based on microelectromechanical systems (MEMS) that use the deflection of a microstructure in the presence of a magnetic field. MEMS sensors have detection limits typically in the 1–100 nT range and commonly rely on the Lorentz force, where an alternating current (AC) can be coupled to the mechanical modes of a microstructure that are usually driven to resonance to enhance the sensitivity. By adjusting the excitation current, a range of sensitivities are possible as well as closed-loop measurements. The deflection of the microstructure can be probed by adapting established MEMS transduction techniques such as piezoresistive [14–17], capacitive [18–21], or optical [22–27] methods. In a different arrangement, MEMS magnetic sensors have been fabricated using magnetostrictive materials by relating changes in microstructure resonance to a redistribution of its mass [28]. If the sensor transduction is sensitive enough to measure the random thermal vibrations of the microstructure, the generated noise spectrum can be used to calibrate the device using the fluctuation dissipation theorem (FDT) [29]. In this way, the detection limit of MEMS sensors is fundamentally related to the damping and the sensor performance is commonly enhanced using vacuum packaging that reduces fluid damping.

Optical MEMS sensors are generally more sensitive than other MEMS transduction techniques while also providing immunity to electromagnetic interference (EMI). It has been previously shown that cavity optomechanical sensors can theoretically achieve a detection limit per unit volume as low as SQUIDs [30]. However, optical MEMS sensors often require expensive equipment and face challenges with miniaturization and integration. In most optical setups, read-out was done off-chip by probing a reflected laser beam focused on the MEMS device [22–27] and very few integrated optical MEMS magnetic sensors have been realized.
While efforts have been made to integrate optical fibers to probe magnetic MEMS by Keplinger et al., near perfect alignment of the optical fibers was required along with low facet surface roughness [31].

By leveraging established surface micromachining techniques, increasing standardization of silicon photonic fabrication has created the possibility of fully integrated optical read-out MEMS deflection sensors [32]. From our previous work, we demonstrated optical deflection sensing based on coupling of two-dimensional (2D) photonic crystal (PC) edge states [33]. By specifically using a 2D photonic crystal structure, low-group velocity effects present in 2D PC line defects can arise from distinct scattering processes that are not possible in 1D PCs [34]. By balancing these low-group velocity effects, the coupling strength between the two PC edges was enhanced when compared to conventional index guided directional couplers while maintaining low insertion loss and broad optical bandwidth, enabling miniaturization of the sensors to less than 100 µm [33, 35]. The fact that the sensor relies on propagating modes rather than high optical quality factor does not have a direct influence on the theoretical quantum limited displacement sensitivity [30].

In this paper, we realize an integrated silicon photonic Lorentz force MEMS magnetic field sensor with optical read-out in the S-band (1460–1530 nm) achieved using integrated coupled PC edges. Following this introduction, Section 8.2 presents the overall sensor design along with the mechanical, optical, damping, and thermal lumped-element models. The expected performance metrics were evaluated for three sensors designs of varying lengths. In Section 8.3, the fabrication of the sensors using a combination of silicon-photonic foundry micromachining and post-process metallization with buffered hydrofluoric acid release is provided. The optical, thermal, and driven magnetic measurement results of the fabricated sensors under ambient atmospheric conditions are contained in Section 8.4. The measurements agreed with the theoretical and numerical modelling and summarized in Section 8.5 along with concluding remarks in Section 8.6. The measured detection limits of the devices were determined to be 0.70–1.31 nT·A/Hz\(^{0.5}\) which could be measured across the full mechanical bandwidth. Due to their small size, the sensors exhibit a low energy resolution of 8.17\(^{-23}\) J/Hz, that can further lower through vacuum packaging, making these designs a inexpensive, low-power technology relevant to non-destructive testing, aerospace, and biomedical applications.

8.2 Sensor Design

The overall design of the sensor is shown in Fig. 8.1(a) consisting of a suspended, double-clamped, microstructure fabricated on SOI wafer using a combination of foundry surface-micromachining followed by post-process metallization and isotropic wet-etching steps. In
Figure 8.1: (a) Perspective view of the sensor fabricated on metalized SOI with oxide undercut. In the presence of the magnetic field $B$, the current $i_m$ injected into the microstructure experiences the Lorentz force $f_m$ in the direction given by the right-hand-rule. (b) Transverse cross-section view of the sensor. (c) Longitudinal cross section view of the sensor. Due to differences in thermal expansivity between the gold and the silicon, a thermal mechanical force $f_{tm}$ is also exerted on the microstructure.

the presence of a magnetic field, an electrical current travelling along the metal experiences the Lorentz force $f_m$ that deflects the structure as shown in Fig. 8.1(b). Along one of the edges of the microstructure, a PC edge defect was introduced, and near-field coupled to a neighbouring PC edge that remained rigidly fixed to the substrate. Together, the PC edges form a directional coupler (PCDC) and the optical read-out was provided by the transmitted optical intensity that was affected by the separation-dependent coupling between the PC edges.

Due to the finite electrical resistance, heat generated by the electrical current leads to changes in the material refractive index (i.e., thermo-optical effect) and changes in the position of the PC edge due to the mismatch in thermal expansivities at the metal-silicon interface. These effects were respectively introduced as a linear temperature-dependent coupling strength and a force $f_{tm}$, respectively, and shown in Fig. 8.1(c). In the remainder of this section, a lumped model was developed to assess the performance of sensors with three different PC lengths 44$a$, 112$a$, 224$a$. A summary of the default design parameters used in this paper are contained in Table 8.1.

### 8.2.1 Optical model

The optical read-out of the oscillator position was based on a PCDC deflection sensor recently demonstrated on 220 nm device layer SOI operating in the SCL-band (1460–1610 nm). Using coupled mode theory, a pair of symmetrical waveguides brought near to each other can
### Table 8.1: Design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Default Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>silicon thickness</td>
<td>$H_s$</td>
<td>220 nm</td>
</tr>
<tr>
<td>buried oxide thickness</td>
<td>$H_b$</td>
<td>2 µm</td>
</tr>
<tr>
<td>PC lattice pitch</td>
<td>$a$</td>
<td>450 nm</td>
</tr>
<tr>
<td>PC hole diameter</td>
<td>$d$</td>
<td>270 nm</td>
</tr>
<tr>
<td>PC edge defect width</td>
<td>$b$</td>
<td>540 nm</td>
</tr>
<tr>
<td>PC edge separation in x-direc.</td>
<td>$s_x$</td>
<td>200 nm</td>
</tr>
<tr>
<td>No. periods in y-direc.</td>
<td>$N_y$</td>
<td>4</td>
</tr>
<tr>
<td>No. periods in x-direc.</td>
<td>$N_x$</td>
<td>4, 112, 224</td>
</tr>
<tr>
<td>PC length</td>
<td>$L_{pc}$</td>
<td>$N_x a$</td>
</tr>
<tr>
<td>taper length</td>
<td>$L_t$</td>
<td>4 µm</td>
</tr>
<tr>
<td>metal width</td>
<td>$W_m$</td>
<td>5 µm</td>
</tr>
<tr>
<td>metal thickness</td>
<td>$H_m$</td>
<td>250 nm</td>
</tr>
<tr>
<td>total length</td>
<td>$L_m$</td>
<td>30.0 µm, 60.6 µm, 111 µm</td>
</tr>
<tr>
<td>total width</td>
<td>$W_{tot}$</td>
<td>9.66 µm</td>
</tr>
</tbody>
</table>

Support a pair of travelling modes with even and odd parity [36]. Launching light into one of the waveguides excites a superposition of these modes and optical power can be exchanged between the two waveguides along the length of the coupler. The coupling strength of the directional coupler can be described by

$$\kappa = \frac{1}{2}(\beta_o - \beta_e),$$  \hspace{1cm} (8.1)

where $\kappa$ is the coupling strength of the directional coupler, and $\beta_o$ and $\beta_e$ are the propagation constants of the even and odd mode, respectively. Alternatively, the beat-length, or the length required for power to be completely transferred from one waveguide to the other, is given by

$$l_b = \frac{\pi}{2\kappa},$$  \hspace{1cm} (8.2)

Precise read-out of the oscillator position in an optomechanical system may be done in the adiabatic optomechanical regime consisting of a fast optical cavity decay rate $\Gamma_{opt}$ and weak optomechanical coupling $g = g_0 \sqrt{n_{cav}}$ with respect to the mechanical resonance frequency $f_0$ (i.e., $\Gamma_{opt} \gg f_0 \gg g$) [37]. For the mechanical frequencies encountered in this paper, ranging up to the low MHz, the fast decay rate assumption was considered valid since an exceptionally high optical quality factor of several million would be required. Since the optomechanical coupling coefficient is related to the number of photons in the PCDC ($n_{cav}$), the weak coupling assumption was considered valid if a sufficiently low laser power is used during measurements.
Therefore, in the ensuing analysis, we did not consider the effects of optical backaction and $\kappa$ was considered to be an integrated average of the instantaneous coupling coefficient at each point along the length of the PC edges.

Since the coupling strength along the PCDC was assumed to be much stronger than the routing waveguides, directional coupling was restricted to only the PCDC region. In this case, the photodiode current generated by a detector at the through output from a directional coupler can be described by

$$I = I_0 \cos^2(\kappa L_{pc})$$

where $I_0$ is the photodiode current from an isolated PC edge and therefore models the insertion loss to structure. Since Eq. (8.3) is non-linear with respect to $\kappa$, it was linearized by assuming $\kappa$ is of the form

$$\kappa = \kappa_0 + \Delta \kappa,$$

where $\Delta \kappa L_{pc} \ll \pi$. Expanding Eq. (8.3) as a Taylor series around $\kappa = \kappa_0$ yields

$$I = \frac{1}{2} I_0 (1 + \cos 2\kappa_0 L_{pc}) - (\Delta \kappa) I_0 L_{pc} \sin 2\kappa_0 L_{pc} - (\Delta \kappa)^2 I_0 L_{pc}^2 \cos 2\kappa_0 L_{pc} + ...$$

(8.5)

We restrict the analysis to the special case where the coupling coefficient has been tuned such that $\cos(2\kappa_0 L_{pc}) = 0$. Therefore,

$$2\kappa_0 L_{pc} = \frac{\pi}{2} + n\pi$$

(8.6)

where $n$ is an integer. In this case, the non-linear term vanishes, and the sine term oscillates between $-1$ and $+1$ depending on the value of $n$ and the resulting photocurrent is given by

$$I = I_0 \left[ \frac{1}{2} + (-1)^{n+1} L_{pc} \Delta \kappa \right].$$

(8.7)

Small changes in the coupling coefficient can be related to the membrane position, thermo-optical effects, and wavelength-dependence

$$\Delta \kappa = \frac{\partial \kappa}{\partial q} q + \frac{\partial \kappa}{\partial T} \Delta T + \frac{\partial \kappa}{\partial \lambda} \Delta \lambda.$$  

(8.8)

Here, $q$ is the amplitude of oscillator vibration mode of interest and $\Delta T$ is the difference between the PC edge and the ambient temperature. Substituting Eq. (8.8) into Eq. (8.7) yields expected photocurrent signal.
\[ I = I_0 \left[ \frac{1}{2} + (-1)^{n+1} L_{pc} \left( \frac{\partial \kappa}{\partial q} q + \frac{\partial \kappa}{\partial T} \Delta T + \frac{\partial \kappa}{\partial \lambda} \Delta \lambda \right) \right]. \quad (8.9) \]

If the temperature difference is considered constant in time, the wavelength may be tuned to cancel this effect such that the condition described by Eq. (8.6) is re-established. For this case, the coupling coefficient dependency on the wavelength and temperature can be ignored and the position of the oscillator can be directly related to the measured photocurrent a conversion factor \( \alpha \) can be defined

\[ \left| \frac{I}{q} \right| \equiv \alpha = I_0 L_{pc} \frac{\partial \kappa}{\partial q}. \quad (8.10) \]

Later, it will be shown that \( \alpha \) can be determined experimentally using FDT and, by using Eq. (8.10) and the measured \( I_0 \), the sensitivity of the coupling coefficient to \( q \) can be determined experimentally.

**Photonic Crystal Enhancement of Direction Coupling**

To enhance directional coupling, light was confined to the edges of a photonic crystal slab along the K-direction using a transverse electric (TE) photonic band gap. Together, the two PC edges support a pair of even and odd dielectric-like modes in the SCL-band (1460–1625 nm) suitable for directional coupling. Based on our previous work, the PCDC defect shown in Fig. 8.2(a) and (b) was simulated with the default parameters using plane wave expansion (PWE) method using RSOFT. The projected band structure along the x-direction is shown in Fig. 8.2(c) with a horizontal separation \( s_x \) of 200 nm and vertical separation \( s_y \) of zero.

Using the interpolated propagation constants and Eqs. (8.1) and (8.2), the beat length as a function of wavelength was computed at different vertical displacements and shown in Fig. 8.2(d). As the vertical displacement increased, the PCDC coupling strength diminished monotonically and is shown in Fig. 8.2(e) at different wavelengths. The sensitivity of the coupling coefficient to the vertical position was found to be maximized at vertical deflection 300 nm. Here, the vertical sensitivity of the coupling coefficient is maximized to -0.223 rad/\( \mu \)m\(^2\), -0.258 rad/\( \mu \)m\(^2\), and -0.482 rad/\( \mu \)m\(^2\), at 1550, 1570, and 1590 nm, respectively. In the PWE model, the side-wall angle of the silicon etch was considered perfectly 90 degrees. Due to the symmetry presented, the sensitivity of \( \kappa \) to changes in vertical separation is exactly zero in the absence of any steady-state vertical displacement. However, a steady-state non-zero vertical separation was introduced between the PC edges using a combination of compressive strain already present in the flip-bonded SOI and deformation originating from difference between operating temperature and metal deposition temperature.
Figure 8.2: (a) Top view of the PCDC PWE simulation domain with horizontal separation $s_x$ of 200 nm. (b) Cross-section view of the PCDC defect depicting the vertical separation $s_z$ that was varied. (c) PWE generated projected band structure along the x-direction for $s_x$ of 200 nm and $s_z$ of 0. (d) The beat-length dependence on excitation wavelength at various vertical separations. (e) The coupling coefficient of the PCDC at 1550, 1570, and 1590 nm for varying vertical separation.

### 8.2.2 Mechanical Model

The vibrational modes of any object can be described using normal mode expansion calculated using linear elastic theory under suitable boundary conditions determined by the geometry [38]. Here, we assumed a combination of inertial and stiffness effects with damping added after the fact. Solutions to the undamped, linearized model yield a set of eigenfrequencies $\omega_n$ and normal mode displacement profiles $r_n = [u_n, v_n, w_n]$ that may be solved for using finite element methods (FEM). Following Hauer et al, we normalize $r_n$ such that the maximum value of $|r_n|$ is unity which makes subsequent analysis consistent with the equipartition theorem [39].

The displacement of each point on the membrane in time can be described by a superposition of independent eigenmodes using normal mode coordinates $q_n$ that represent the amplitude of the $n$th mode profile

$$R(x,t) = R_0 + \sum_n q_n(t)r_n.$$  \hspace{1cm} (8.11)

Here, $R_0$ is a possible steady-state displacement of the structure. If the modes are uncoupled, then the normal mode coordinates can be described by Newton’s second law

$$m \ddot{q} + c \dot{q} + m\omega_0^2 q = f(t),$$  \hspace{1cm} (8.12)
where \( m \) is the mode mass, \( c \) is the damping coefficient, \( \omega \) is the resonant frequency, and \( f(t) \) is a body load coupled to the mode of interest. In this paper, the current injected through the microstructure was aligned in the x-direction and subject to a uniform magnetic field \( \mathbf{B} \) prepared in the y-direction. By the right-hand-rule, a Lorentz force is generated in the z-direction and will be strongly coupled to the symmetric vibrational mode associated with vertical displacement of the microstructure and assumed to have the form \( \mathbf{r} = [0, 0, \phi(x)] \). In Eq. (8.12) and subsequent expressions involving \( q \) and \( \mathbf{r} \), the \( n \) subscript was dropped and was assumed to refer to this specific displacement mode. Using the normalized mode profiles, the mode mass and mode-coupled source term can be computed using the equations

\[
m = \iiint \rho |\mathbf{r}|^2 \, dV \quad (8.13)
\]

and

\[
f(t) = \iiint \mathbf{f} \cdot \mathbf{r} \, dV, \quad (8.14)
\]

respectively, where \( \rho \) represents the density of the material and \( \mathbf{f} \) is a body load.

If \( f(t) = f[\omega]e^{-i\omega t} \) and \( q(t) = q[\omega]e^{-i\omega t} \), then the system is linear, and the force and oscillator position can be related to each other using the equation

\[
q[\omega] = \chi[\omega] f[\omega], \quad (8.15)
\]

where the mechanical susceptibility \( \chi[\omega] \) is defined as

\[
\chi[\omega] \equiv \frac{1}{m(\omega_0^2 - \omega^2) - ic\omega}. \quad (8.16)
\]

When driven at resonance, \( \omega = \omega_0 \), the mechanical response of the system is enhanced by the quality factor

\[
|\chi[\omega_0]| = \frac{Q}{k} \quad (8.17)
\]

where \( k = m\omega_0^2 \) is the spring coefficient and \( Q = m\omega_0/c \) is the quality factor of the mechanical resonator.

Due to varying wafer processing and post-process temperatures, mismatches in material thermal expansivity can lead to buckling and deformation of the structure. To determine the resulting steady-state microstructure position \( \mathbf{R}_0 \) and resonant frequencies \( f_0 \) arising from these effects, a prestrained model was created using COMSOL FEM software where solid mechanics and heat transfer physics were coupled together using thermal expansion multiphysics with ge-
8.2. Sensor Design

Figure 8.3: The FEM generated steady-state profiles $R_0$ are shown for the devices with default dimensions of length (a) 44a, (b) 112a, and (c) 224a. The inset shows the vertical displacement along the transverse direction at the mid-point of the structure. (d) Shows the integrated average vertical displacement of the PC edge $\bar{s}_z$ as the strain was increased. The steady-state position of the structure depends on the ambient temperature. The solid line indicates an ambient temperature of 20°C and the dashed line indicates an ambient temperature of 25°C.

The resulting steady-state displacement profiles $R_0$ are shown in Figs. 8.3(a), (b), and (c) for devices of PC length 44a, 112a, and 224a, respectively. Since gold has a much larger coefficient of thermal expansion than silicon, the beam is curved downward along the transverse direction (y-axis) as shown in the inset. The introduced curvature affects the second moment of area of the beam thereby increasing its resistance to bending to forces in the vertical direction. This results in uniform steady state deflection of the PC edge that becomes more apparent for the longer structures. For each structure, the average displacement of the PC edge is shown in Fig. 8.3(d) as the strain was increased from zero to $5 \times 10^{-4}$ corresponding roughly to the expected strain found in SOI [40].

For eigenfrequency analysis, since the exact value of strain present in SOI was somewhat unpredictable, it was assumed to be $\epsilon$ of $1 \times 10^{-4}$, which was then used to compute the mode
profiles of the first symmetric vertical vibrational mode and shown in Fig. 8.4(a), (b), and (c) for devices of length 44a, 112a, 224a, respectively. As with the steady-state displacement profiles, the mode profiles for the two longer devices are nearly uniform away from the supports and therefore expected to have good coupling to uniform loads along the x-axis. As the strain parameter was increased, the eigenfrequency and mode mass as defined by Eq. (8.13) was computed and shown in Figs. 8.4(d) and (e), respectively. The frequency and mode mass of the microstructures are summarized in Table 8.3.

Table 8.2: FEM material properties

<table>
<thead>
<tr>
<th>Material</th>
<th>Silicon</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal expansion coeff.</td>
<td>2.6×10^{-6} 1/K</td>
<td>14.2×10^{-6} 1/K</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>700 J/(kg·K)</td>
<td>129 J/(kg·K)</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>2329 kg/m³</td>
<td>19300 kg/m³</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>170 GPa</td>
<td>70 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.28</td>
<td>0.44</td>
</tr>
<tr>
<td>Electrical conductivity</td>
<td>–</td>
<td>45.6×10^{-6} S/m</td>
</tr>
<tr>
<td>Refractive index</td>
<td>3.4767</td>
<td>–</td>
</tr>
</tbody>
</table>
8.2.3 Magnetic Coupling

We now consider the case where a body load is generated due to the Lorentz force arising from a current $i_m$ injected along x-axis through the metal in the presence of a magnetic field. For continuous nonmagnetic media, the Lorentz force is described as a body load given by [41]

$$ f = \rho E + J \times B, \quad (8.18) $$

where $\rho$ is the free charge density and $J$ is current density that can be described by $J = i_m/(W_mH_m)$. For a current carrying conductor, the electrical force is counteracted by random scattering described by Ohm’s law ($J = \sigma E$) that leads to heating. Since the charge carriers are bound to the conductor, the magnetic force is coupled to the vibrational mode by Eq. (8.19)

$$ f_m = \iiint (J \times B) \cdot r \ dV. \quad (8.19) $$

Due to the presence of the cross-product, only the y-component of the magnetic field is strongly coupled to vibrational modes whose displacement lies primarily along z-axis. If the normalized vibrational mode profile can be described by $r = [0, 0, \phi(x)]$, then the magnetic force coupled to mode is given by

$$ f_b = \int_{-L/2}^{L/2} B_y i_m \phi(x) \ dx. \quad (8.20) $$

If the magnetic field and current are considered uniform across the device, they may be taken out of the integral yielding

$$ f_b = B_y i_m L_{\text{eff}}, \quad (8.21) $$

where an effective length was defined as

$$ L_{\text{eff}} = \int_{-L/2}^{L/2} \phi(x) \ dx. \quad (8.22) $$

Since Eq. (8.21) describes the magnetic coupling to the first vertical vibrational mode, the effective length is ideally equal to the total length. The effective lengths of the devices were evaluated to be 14.1 $\mu$m, 38.5 $\mu$m, 88.9 $\mu$m for the devices of total length 30.0 $\mu$m, 60.6 $\mu$m, 111 $\mu$m, respectively.
8.2.4 Thermal Model

To generate the Lorentz force, an electrical current must flow across the microstructure which inevitably leads to heating of the device. To model the heating of the microstructure, a lumped-element thermal model was devised. The total heat generated by the device was related to the instantaneous current through the device by the Joule-heating law

\[ Q_{el} = I^2 R_{el}, \]  

(8.23)

where \( Q_{el} \) is the heat generated by the electrical resistance \( R_{el} \). With \( Q_{el} \) as an equivalent current source for the thermal circuit, the temperature was modeled as the RC electrical circuit shown in Fig. 8.5(a) and governed by the following first-order differential equation

\[ Q_{el} = \frac{\Delta T}{R_{th}} + C_{th} \frac{dT}{dt}. \]  

(8.24)

Here, \( R_{th} \) is the thermal resistance that models the resistance of heat flow away from the microstructure while \( C_{th} \) is the thermal capacitance that models the thermal energy stored in the microstructure. The temperature \( \Delta T \) was considered to be the average temperature difference across the PC edge with respect to the ambient temperature. To account for the junction resis-
To model the response of the microstructure to a time-varying thermal source, the thermal transfer function was determined by taking the Laplace transform of Eq. (8.24) and rearranging to yield

\[ H(s) = \frac{\Delta T[s]}{Q_{el}[s]} = \frac{R_{th}}{1 + \tau s}, \]  

(8.25)

where \( \tau = R_{th}C_{th} \) is the thermal time constant. Using the substitution \( s = i\omega \), the temperature of the PC edge can be expressed in the frequency domain in terms of the injected current by substituting the Fourier transformed version of Eq. (8.23) into Eq. (8.25) yielding

\[ \Delta T[\omega] = H[\omega]Q_{el}[\omega] = \frac{R_{th}R_{el}}{1 + i\tau \omega} I_m * I_m. \]  

(8.26)

Here, \( I_m \) is the Fourier transform of the time-domain current \( i_m \), and convolution was used.

Using FEM, the thermal resistance, electrical resistance, and thermal time constant were computed. Using steady-state analysis, the second term of Eq. (8.24) becomes zero and \( R_{th} \) was determined by finding the slope \( \Delta T \) vs \( Q_{el} \) generated by varying \( i_m \). The results are shown in Figs. 8.5(c) and (d) for the default model parameters under varying metal thickness and metal width, respectively. The electrical resistance was computed and shown in Figs. 8.5(e) and (f) under varying metal thickness and metal width, respectively. The time constant was determined using transient analysis by fitting the time-domain temperature of the PC edge to a first-order response governed by Eq. (8.25) with a step input current. The extracted time constants are shown in Fig. 8.5(b) and the results are summarized in Table 8.3.

### 8.2.5 Thermal Mechanical Coupling

Electrical current injected through the device leads to heating and deformation of the structure due to mismatches in thermal expansivity between the metal and silicon. This was modeled using the same prestrained model as before but with an elevated steady-state temperature of 25°C. The resulting position of the PC edge shown in Fig. 8.3(d). For a strain of \( 1 \times 10^{-4} \), the change in average vertical displacement of the PC edge was determined to be 0.53 nm/K, 2.53 nm/K, and 5.22 nm/K for a device of length 44a, 112a, 224a, respectively. Since this analysis was done using steady-state FEM, the temperature related displacements were converted to an equivalent force \( f_m \) (applied in the downward direction for \( T > T_{dep} \)). Since the temperature-dependent displacement profiles were nearly equal to the normal mode profiles, \( f_m \) was determined using the spring coefficient and is given by
In this way, the position of the oscillator could be extended to time-varying temperature differences since the response to mechanical deformation will also be determined by the mechanical susceptibility. The resulting $\gamma$ for each structure is summarized in Table 8.3

### 8.2.6 Fluid Damping Model

![Figure 8.6: The damping model for an infinitesimal cross-section of the microstructure along the x-direction. Per unit length, the air displaced by the structure $Q_0$ is balanced by the output flows due to squeeze flow $Q_{sq}$, the photonic crystal holes $Q_{pc}$, and the PC edge gap $Q_s$. The pressure profile, shown by the red curve, was determined using the Reynold’s equation and combining the effective resistances due to each outflow.](image)

It is important to consider fluid damping effects of the oscillating microstructure as the dissipation of energy can be directly related to the detection limit of the device. Fluid damping analysis was performed by assigning flow resistances arising from a combination of squeeze flow effects, PC holes, and the gap between the PC edges. For this analysis, the dynamic viscosity $\mu$, pressure $P$, and density $\rho_a$ of $1.8 \times 10^{-5}$ Pa·s, 101.3 kPa, and 1.225 kg/m$^3$, respectively, and based on ambient atmospheric conditions.

The type of flow experienced by the air can be characterized by the Reynolds number which was estimated with some basic assumptions about the dimensions of the microstructure. Assuming an upper operating frequency of 10 MHz, vibration amplitude of 1 μm, and total beam width of 10 μm ($W_{tot} = W_{pc} + W_m$), this resulted in an estimated speed of $v=1$ m/s and Reynolds number of 6.8 ($Re = \rho_a v W_{tot}/\mu$) which is far below the onset of turbulence ($Re \ll 1000$). The flow was therefore assumed to be laminar and described by the Reynolds equation [42]

$$
\frac{\partial}{\partial x} \left( h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( h^3 \frac{\partial p}{\partial y} \right) = 12\mu \frac{\partial h}{\partial t}. 
$$

(8.28)
Here, \( h \) is the gap distance between the microstructure and the silicon wafer handle layer the created by the BOX undercut and \( p \) is the dynamic pressure. Assuming the length of the structure in the x-direction is much longer than the total width \( (L \gg W_{\text{tot}}) \), then \( \partial P/\partial y \gg \partial P/\partial x \) and there is negligible volume flow along the x-direction. This leads to the situation described by Fig. 8.6 and the total air flow can be determined by integrating the infinitesimal flow contributions along the beam. The input flow generated by the motion of the structure per unit length can be expressed in terms of the normal mode coordinate, mode profile, and width of the microstructure

\[
Q_0 = \phi(x)W_{\text{tot}} \dot{q}.
\]

(8.29)

The output flow per unit length along the microstructure was assumed to arise from a combination of squeeze flow \( Q_{sq} \), PC holes \( Q_{pc} \), and the gap between the PC edges \( Q_s \). Assuming incompressibility and mass conservation, the flows are related by

\[
Q_0 = Q_{sq} + Q_{pc} + Q_s
\]

(8.30)

which can be rewritten in terms of air flow resistances per unit length. Next, it was assumed each outflow could be related to the undercut cavity pressure \( p_0 \) their corresponding flow resistance

\[
\dot{q} \phi(x)W_{\text{tot}} = P_0 \left( R_{sq}^{-1} + R_{pc}^{-1} + R_s^{-1} \right) \equiv \frac{p_0}{R_c}.
\]

(8.31)

Here, \( R_{sq} \) is the air flow resistance due to the squeeze film effects per unit length, \( R_{pc} \) is the air flow resistance due to the PC holes present per unit length, \( R_s \) is the air flow resistance of the PC gap across per unit length. An equivalent air resistance \( R_c \) has been defined using parallel resistor rule.

**Squeeze Flow Resistance**

To determine the air resistance due to the squeeze film, we temporarily ignore the effect of the PC holes and PC air gap. If it is assumed that the initial gap \( H_b \) is uniform and the vibration amplitude in the z-direction is much smaller than the gap \( (q \ll H_b) \), then the Reynolds equation can be rewritten as

\[
H_b^2 \frac{\partial^2 P}{\partial y^2} = 12\mu \dot{q} \phi(x)
\]

(8.32)

where \( h = H_b + \dot{q} \phi(x) \) was used. The equation can be solved directly through integration using appropriate boundary conditions. At \( y = W_{\text{tot}}/2 \) the dynamic pressure was assumed to be zero.
since the flow is exposed to directly ambient atmospheric pressure. Since cavity dimensions under the fixed PC edge region \( y < -W_{\text{tot}}/2 \) were small compared to wavelength of sound, the pressure was assumed constant here yielding Neumann boundary condition \( \partial P/\partial y = 0 \) at \( y = -W_{\text{tot}}/2 \). With these boundary conditions, Eq. (8.32) leads to

\[
p(y) = \frac{P_0 W_{\text{tot}}^2}{2} \left( y - \frac{W_{\text{tot}}}{2} \right) \left( y + \frac{3W_{\text{tot}}}{2} \right),
\]

where the maximum pressure occurring at \( y = -W_{\text{tot}}/2 \) is given by

\[
p_0 = 6\mu \frac{W_{\text{tot}}^2}{H_{b}^3} \dot{q} \phi(x).
\]

The squeeze flow air resistance can then be defined by taking the ratio between cavity pressure and the change in volume flow rate in the infinitesimal segment defined in Eq. (8.29)

\[
R_{sq} \equiv \frac{p_0}{Q_0} = \frac{p_0}{6\mu W_{\text{tot}}}{H_{b}^3}
\]

For the total width \( W_{\text{tot}} \) of 9.66 µm and gap \( H_{b} \) of 2 µm, this results in a squeeze resistance of 1.30×10⁸ Pa·s/m².

### Flow Resistance Due to PC Gap

Perforations are commonly encountered in MEMS devices to allow etchant to reach the sacrificial layer and release microstructures. However, since the dimensions of the PC holes and edge gap are close to the mean free path of air, special considerations are required to model these flow resistances. The gas flow regime can be identified using the Knudsen number defined as

\[
\text{Kn} = \frac{\lambda}{l_c},
\]

where \( l_c \) is the characteristic length scale of the system and \( \lambda \) is the mean free path of gas molecules which was taken to be 65 nm under atmospheric conditions [43]. Using the PC gap of 200 nm, this results in a Knudsen number around 0.32 which can be characterized as a transition flow regime (i.e., \( \text{Kn}>10 \) molecular flow, \( 10>\text{Kn}>0.1 \) transitional flow (slip-flow), \( 0.1>\text{Kn} \) continuum flow (non-slip) [43]).

Empirical models for rarefied gas flow in the transition regime exist for simple geometries [44] and the mass flow rate through the PC gap per unit length was modeled as flow through a slit of width \( s \) and infinitesimal thickness by defining a reduced flow rate \( G^s \)
\[ \dot{M}^s \equiv \frac{\dot{M}^s}{sP} \left( \frac{2\pi k_B T}{m_0} \right)^{1/2}. \] (8.37)

Here, \( \dot{M}^s \) is a quantity of mass passed through the PC gap per unit of time and per unit length of the PC gap in \( \text{kg/(s \cdot m)} \) and \( m_0 \) is the average mass per molecule taken to be \( 4.811 \times 10^{-26} \text{ kg} \).

For the isothermal case and small pressure differences \( (\Delta P/P \ll 1) \), the reduced flow rate was related to the relative pressure difference and an empirically determined coefficient \( G_p^s \)

\[ G^s = G_p^s \frac{\Delta P}{P}. \] (8.38)

The value of \( G_p^s \) was determined using a look-up table based on a rarefaction parameter \( \delta_s = \sqrt{\pi s/(2\lambda)} \). Taking the PC edge separation of 200 nm for \( s \) yielded a rarefaction parameter of 2.73 and resulted in an empirically determined \( G_p^s \) of 2.05 [44]. Using Eqs. (8.37) and (8.38), a relationship between the mass flow rate and pressure difference was determined

\[ \frac{\Delta P}{\dot{M}^s} = \frac{G_p^s}{s} \left( \frac{2\pi k_B T}{m_0} \right)^{1/2}. \] (8.39)

If the density of the gas is assumed constant, then the equivalent volume flow rate per unit length is given by \( \dot{M}^s = \rho a Q_g \) and the flow resistance per unit length along the PC gap is

\[ R_g \equiv \frac{\Delta P}{Q_g} = \frac{\rho_a}{\dot{M}^s} \left( \frac{2\pi k_B T}{m_0} \right)^{1/2}. \] (8.40)

Using the PC edge separation of 200 nm resulted in an air flow resistance of \( 2.128 \times 10^9 \text{ Pa} \cdot \text{s/m}^2 \).

**Flow Resistance Due to PC Holes**

For the PC holes, a similar approach to modeling the flow through the PC edge separation was used. The reduced flow rate per hole was modeled as circular orifice of negligible thickness and defined as [44].

\[ G_{pc} = \frac{4M_{pc}}{d^2P} \left( \frac{2k_B T}{\pi m_0} \right)^{1/2} \] (8.41)

where \( \dot{M}_{pc} \) is the mass flow rate \( \text{kg/s} \) for a single PC hole of diameter \( d \). Again, a rarefaction parameter was defined \( \delta_{pc} = \sqrt{\pi d/(4\lambda)} \). For the PC hole diameter of 270 nm, this resulted in a \( \delta_{pc} \) of 1.84. For the circular orifice, the following formula is applicable for \( \delta_{pc} < 50 \) [44]

\[ G_{pc} = 1 + 0.342\delta_{pc} \] (8.42)
Using the previously determined $\delta_{pc}$ of 1.84, this results in a $G_{pc}$ of 1.63. As before with the PC gap, a relationship between the pressure difference and mass flow rate can be determined

$$\frac{\Delta P}{M_{pc}} = \frac{4}{G_{pc}d^2} \left( \frac{2k_BT}{\pi m_0} \right)^{1/2}. \quad (8.43)$$

As before, if the density of the gas is constant, rearranging Eq. (8.43) gives the air flow resistance per PC hole

$$r_{pc} \equiv \frac{\Delta P}{Q_{pc}} = \frac{4\rho_a}{G_{pc}d^2} \left( \frac{2k_BT}{\pi m_0} \right)^{1/2}. \quad (8.44)$$

For the 270 nm PC hole diameter, this results in an air flow resistance of $9.35 \times 10^{15}$ Pa·s/m$^3$ per hole. If there are $N$ holes per lattice pitch in the x-direction, the PC hole air flow resistance per unit length is given by

$$R_{pc} = \frac{4a\rho_a}{G_{pc}d^2N} \left( \frac{2k_BT}{\pi m_0} \right)^{1/2}. \quad (8.45)$$

For the default design, $N = 16$ (i.e., 8 holes on each side of gap per PC lattice distance) and $a = 450$ nm which resulted in a flow resistance of $2.63 \times 10^8$ Pa·s/m$^2$. Considering the results, the flow resistance due to the PC holes and squeeze flow are nearly the same magnitude while the flow resistance due the air gap is about an order of magnitude larger.

**Damping Coefficient**

To determine the damping coefficient appearing in Eq. (8.12), the fluid pressure exerted on the microstructure was expressed in terms of the normal coordinate velocity $\dot{q}$. This was evaluated by first assuming the dynamic pressure beneath the fixed PC edge $p_0$ could be determined using Eq. (8.31) using the equivalent flow resistance $R_c$. For the default structure, this was determined to be $8.36 \times 10^7$ Pa·s/m$^2$ using the parallel resistor rule. Next, the pressure profile beneath the microstructure was assumed to have the same shape as Eq. (8.33) but with the reduced $p_0$ determined using the equivalent flow resistance instead. With this pressure profile, the force per unit length in the x-direction was obtained by integrating the pressure profile across the beam width.

$$F = \int_{-W_{tot}/2}^{W_{tot}/2} p \, dy = -\frac{2}{3} W_{tot}^2 R_c \dot{q} \phi(x) \quad (8.46)$$

Using normal decomposition for the first vertical vibrational mode given by Eq. (8.14), the force can be defined as a velocity dependent damping force.


\[ c \ddot{q} \equiv \int_{-L/2}^{L/2} F \cdot \phi(x) \, dx. \] (8.47)

After substitution of Eq. (8.46) into Eq. (8.47) and cancellation of the \( \dot{q} \) terms, the damping coefficient was determined to be

\[ c = \frac{2}{3} W_{t0} R_c \int_{-L/2}^{L/2} \phi^2 \, dx. \] (8.48)

The damping of a system can sometimes be more conveniently be described by the quality factor given by

\[ Q = \frac{m\omega_0}{c}, \] (8.49)

where \( \omega_0 = 2\pi f_0 \) is the resonance of the oscillator in angular units. Using FEM, the mode mass, resonant frequency, and normal mode profiles, \( \phi(x) \) can be easily evaluated the computed quality factors for the default structures are summarized in Table 8.3.

### 8.2.7 Thermal Mechanical Detection Limit

If a harmonic oscillator is in thermal equilibrium with its surroundings, then by the equipartition theorem, fluctuations in the oscillator can be modeled as a spectrum density that is independent of frequency [45]. If the primary noise source is due to the thermal motion of the microstructure, then the smallest possible force that can be resolved is given by

\[ 2k_B T c = |\delta_f[v]|^2, \] (8.50)

where \( \delta_f \) is the equivalent force noise floor in N/\( \sqrt{\text{Hz}} \) that is coupled to the vibrational mode. Using Eq. (8.21), this force can be related to an equivalent magnetic field noise floor \( \delta_b \) given by

\[ \delta_b = \frac{\sqrt{2k_B T c}}{i_m L_{\text{eff}}} = \frac{\sqrt{2k_B T m\omega_0/Q}}{i_m L_{\text{eff}}} \] (8.51)

that determines the detection limit of the device. If the excitation current does not lead to significant heating of microstructure (\( \Delta T \ll T \)), then the detection limit can be normalized with respect to the injected current

\[ \bar{\delta}_b \equiv \delta_b i_m. \] (8.52)

Using the FEM models, the computed \( \bar{\delta}_b \) for each of the default structures is shown in Table 8.3.
If the current leads to significant heating of the structure, it may be more appropriate to express the detection limit in terms of an acceptable temperature difference. Under steady-state DC conditions, the current can be expressed in terms of a temperature difference using the thermal and electrical resistances and the detection limit can be re-expressed as

$$\delta_{b} = \sqrt{\frac{2k_B T m \omega_0 R_{el} R_{th}}{Q \Delta T \omega_0 R_{el} R_{th}}} \frac{1}{L_{eff}}.$$  \hspace{1cm} (8.53)

Using FEM and the semi-analytic damping model, the magnetic field noise floor was computed with a $1^\circ C$ allowable temperature difference in the PC edge and is shown in Figs. 8.7(a) and (b) for the default structure under varying metal width and metal height, respectively. Wider metal widths provide more ampacity; however, any potential gain in sensor performance is counteracted by an increase in fluid damping leading to minimal improvement overall beyond default metal width of 5 $\mu m$ in ambient conditions. Since the damping coefficient does not depend on the metal thickness, there is potential for improvement in the sensor design using thicker metal depositions. The default thickness of 250 nm was selected based on other criteria such as the deposition adhesion, stability, and cost of materials.

**Figure 8.7:** The theoretical magnetic noise floor of the sensor under ambient conditions for the default devices with a $1^\circ C$ allowable temperature difference of the PC edge under varying (a) metal width and (b) varying metal height.

### 8.2.8 Thermal Calibration

A general procedure for thermal mechanical calibration of nano- and micro-mechanical resonators was outlined by B.D. Hauer et al [29]. If the system described by Eq. (8.12) is under-damped and thermally driven, then the single-sided thermal noise spectral density $S_{qq}$ of the normal mode coordinate $q$ is given by [29].
8.2. Sensor Design

Table 8.3: FEM summary

<table>
<thead>
<tr>
<th>$N_s$</th>
<th>$R_{el}$</th>
<th>$R_{th}$</th>
<th>$\tau$</th>
<th>$f_0$</th>
<th>$m$</th>
<th>$\gamma$</th>
<th>$Q$</th>
<th>$\delta_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>$\Omega$</td>
<td>K/mW</td>
<td>$\mu$s</td>
<td>MHz</td>
<td>ng</td>
<td>nN/K</td>
<td>-</td>
<td>nT·A</td>
</tr>
<tr>
<td>44</td>
<td>0.8188</td>
<td>7.667</td>
<td>2.48</td>
<td>1.56</td>
<td>0.263</td>
<td>13.4</td>
<td>51.9</td>
<td>1.42</td>
</tr>
<tr>
<td>112</td>
<td>1.356</td>
<td>12.160</td>
<td>6.55</td>
<td>0.547</td>
<td>0.898</td>
<td>26.8</td>
<td>19.0</td>
<td>0.942</td>
</tr>
<tr>
<td>224</td>
<td>2.240</td>
<td>19.080</td>
<td>17.8</td>
<td>0.289</td>
<td>2.33</td>
<td>40.1</td>
<td>10.0</td>
<td>0.658</td>
</tr>
</tbody>
</table>

$$S_{qq}^{th}(f) = \frac{k_B T f_0}{2\pi^3 m Q [(f^2 - f_0^2)^2 + (f f_0/Q)^2]}$$ (8.54)

When the effective mass $m_n$ is computed using Eq. (8.13), this is consistent with the equipartition theorem [29].

If the other noise sources, such as dark current and shot noise, are assumed to be white noise, the power spectrum density photocurrent signal received by the signal analyzer from the undriven device can be modeled by

$$S_{II}(f) = S_{II}^w + \alpha^2 S_{qq}^{th}(f),$$ (8.55)

where $\alpha$ has units A/m and provides a conversion factor between the normal mode coordinate and the measured photocurrent. If the thermal noise spectrum can be resolved, it can be fit to Eqs. (8.54) and (8.55). By using the FEM generated mode mass, the fitted parameters can be used to determine $\alpha$, thereby calibrating the sensor.

8.2.9 Driven Harmonic Response

To test the sensor, an AC current was injected through the device in the presence of a non-varying magnetic field. The resulting spectrum density of photocurrent due to a time-harmonic force can be found by substituting the mechanical response function given by Eq. (8.16) into the linearized optical model given by Eq. (8.9)

$$I[\omega] = I_0 \left[ \frac{1}{2} + (-1)^{n+1} L \left( \frac{\partial k}{\partial q} \chi[\omega] f[\omega] + \frac{\partial k}{\partial T} \Delta T[\omega] \right) \right].$$ (8.56)

Here, $f$ is the sum of the magnetic and thermal mechanical effects. Here, the thermal fluctuations were considered negligible. Substituting Eqs. (8.27) and (8.21) into Eq. (8.57) and gathering the temperature-dependent terms leads to
\[ I[\omega] = I_0 \left[ \frac{1}{2} + (-1)^{n+1} L_{pc} \left( \frac{\partial \kappa}{\partial q} \chi[\omega] B_y L_{eff} I_m[\omega] + \left( \frac{\partial \kappa}{\partial q} \chi[\omega] \gamma + \frac{\partial \kappa}{\partial T} \right) \Delta T[\omega] \right) \right]. \quad (8.57) \]

In the frequency domain, a purely sinusoidal current at frequency \( \omega' \) can be expressed as

\[ I_m[\omega'] = \frac{1}{2} (\delta(\omega + \omega') + \delta(\omega - \omega')), \quad (8.58) \]

where \( \delta(\omega) \) is the Dirac delta function. The temperature difference can be determined using convolution of the current given by Eq. (8.25) which results in DC and 2nd harmonic component. Therefore, for small enough current, only the magnetic field interaction appears at the drive frequency \( \omega' \) in the photocurrent spectrum density

\[ I[\omega'] = (-1)^{n+1} I_0 L_{pc} \frac{\partial \kappa}{\partial q} \chi[\omega'] B_y L_{eff} I_m[\omega']. \quad (8.59) \]

An “intrinsic” sensitivity of the device can be defined at resonance, as a fraction of the full-scale photodiode swing at the fundamental harmonic is given by

\[ \beta \equiv \frac{I[\omega_0]}{I_0} \frac{1}{B_y i_0} = L_{pc} \frac{\partial \kappa}{\partial q} \frac{Q}{k} L_{eff}. \quad (8.60) \]

which is independent of the measurement parameters.

### 8.3 Fabrication

The sensor fabrication process is outlined in Fig. 8.8 beginning with surface-micromachining of SOI followed by metallization and selective etching of the buried oxide. The devices were surface micromachined on \(<100>\) oriented SOI wafers using the Advanced Microfoundry (AFM) silicon photonics process technology as a multiproject wafer (MPW). All subsequent post-processing of the batch-fabricated chips were done at Western Nanofabrication Facility.

#### 8.3.1 Metallization

The chips were first cleaned to remove the protective resist using a 5 min ultrasonic bath (Branston 200, 30 W) in acetone, 1 min isopropanol alcohol bath, 1 min deionized (DI) water rinse, \( N_2 \) air dry, then dehydrated on a hot-plate at 200\(^\circ\)C for 5 min. Next, photolithography was used to define the metallized regions using a bilayer lift-off resist process beginning with spin-coating LOR 5A (500 rpm ramp-up 5 s, 2000 rpm 45 s) followed by a 180\(^\circ\)C bake on a hot-plate for 5 min. The sample was then spin-coated with photoresist S1827 (500 rpm ramp-up 5 s,
4000 rpm 45 s) followed by 115°C pre-exposure bake for 1 min. After pre-bake, the sample was exposed to 38 mW/cm² 405 nm line for 12 s using a mask aligner (Neutronix-Quintel NXQ 4006), developed in M-319 for 90 s, rinsed in DI water for 5 min, N₂ air dry, then post-baked at 120°C for 5 min on a hot-plate.

Oxygen plasma descum was performed using reactive ion etching (Trion RIE) with 200 mTorr O₂ pressure at 25 W for 180 s to clean the chip surface before the metal deposition. Electron-beam thermal evaporation (Angstom Evaporator) was used to deposit 5 nm of chromium for adhesion followed by 250 nm of gold at a rate of 0.5 A/s at 1.81×10⁻⁵ mBar. After metallization, the lift-off resist was stripped using successive ultrasonic baths in Remover PG until there was no visible debris.

### 8.3.2 Buried Oxide Etch

To release the metallized structures from the substrate, buffered hydrofluoric acid (BHF) was used to selectively remove the buried oxide. To protect the metal and prevent the uncontrolled release of structures on the MPW, photolithographically defined openings were created to control where the chip surface would be exposed to BHF. For this post-process step, the metallized chips were cleaned using a 5 min ultrasonic bath in acetone, 1 min isopropanol alcohol bath,
1 min DI water rinse, and N\textsubscript{2} air dry. Before spin-coating the photoresist, an adhesion promoter hexamethyldisilazane (HMDS) was deposited on the samples using vapour prime oven (YES-310TA) at 150\textdegree C for 15 min. The sample was then spin-coated with photoresist S1805 (500 rpm 5 s ramp-up, 3500 rpm) followed by 115\textdegree C pre-exposure bake for 1 min. After pre-bake, the sample was exposed to 38 mW/cm\textsuperscript{2} 405 nm line for 2.2 s, developed in MF-319 for 75 s, rinsed in DI water for 5 min, and N\textsubscript{2} air dried. Oxygen plasma descum was used for 120 s to clean the samples before being submerged in BHF (Buffered HF Improved UN2817, Transene Company Inc.) for 35 min at room temperature. After BHF etching, the chips were transferred to DI water to soak for 5 min then transferred to acetone for 30 min to remove the photoresist. Next, the samples were transferred to a large beaker of ethanol and loaded into a critical point CO\textsubscript{2} drying chamber (EMS 850). The completed fabricated devices are shown in Fig. 8.9.

![Figure 8.9: Microscope images of the fabricated structures after the BHF and metallization steps showing the deposited gold for \( L_{pc} \) of (a) 44\( \mu \)m, (b) 112\( \mu \)m, and (c) 224\( \mu \)m. The waveguides and BHF undercut can be seen around the perimeter of the exposed buried oxide.](image)

### 8.4 Measurement Results

#### 8.4.1 Measurement Setup

The measurement testbench is shown in Fig. 8.10(a) and consisted of the photonic integrated circuit (PIC) mounted to an aluminum ground plane. The experiment was performed under ambient conditions and the PIC was interfaced with a polarization maintaining optical fibre array (FA) that coupled TE light to surface grating couplers and electrical probes that injected current \( i_m \) to the device. An electromagnet and core assembly directed the magnetic field to the PIC which was controlled using a DC excitation current \( I_{ex} \). For a 1 A DC excitation current, the resulting \( B_y \) field profile of the stage was measured using an magnetometer (AlphaLab GM1-ST) and is shown in Fig. 8.10(b). In terms of the excitation current, the \( B_y \) flux density at the location of the devices was found to be 36 mT/A×\( I_{ex} \) experimentally.

A block diagram of the setup is shown in Fig. 8.10(c). A function generator (FG Keysight 33210A) supplied current to the device under test (DUT). Since the electrical resistance of the
DUT was expected to be relatively low (below 5 Ω), a 50 Ω shunt resistor was used to help stabilize the current against possible fluctuations in the probe contact resistances. The total electrical resistance was measured using a digital multimeter. The voltage from the FG was monitored using channel 1 (CH1) of the oscilloscope (OS, Tektronix DPO 7054) and was used to set the electrical current \( i_m \) according to the Ohm’s law.

A tuneable SCL-band laser source (TL, Anritsu Tunics Reference) was set to 0 dBm and coupled to the DUT using the FA. Optical detection was done using a reverse bias photodetector (DET, Thorlabs PBA450). The photodetector responsivity and transimpedance gain were taken to be 1 A/W, \( 5 \times 10^4 \) V/A with a DC–4.5 MHz low-pass filter. Depending on the measurement being performed, the output electrical signal was monitored using either channel 2 (CH2) of the OS or an electrical spectrum analyzer (SA, Keysight N9030A) with input impedances of 1 MΩ and 50 Ω, respectively. The measurements were divided into either magnetic or non-magnetic measurements. For the non-magnetic measurements, the electromagnet was removed.

8.4.2 Non-Magnetic Measurements

Transmission Spectrum

The optical transmission spectra of all three devices were acquired while different DC currents \( i_m \) were injected in the device using the electrical probes and shown in Figs. 8.11(a), (b), and (c) for devices with PC length 44\( a \), 112\( a \), and 224\( a \), respectively. The transmission was normalized with respect to the optical transmission of a control structure with the device absent.
thereby enabling the removal of measurement artifacts related to the surface grating coupler and routing. Since only the steady-state optical power was measured, a power meter (N7744 Keysight) was used to measure the optical power from the device. While the structures were design to operate near the 1550 nm wavelength, there was a blue-shift in transmission spectrum that was attributed to visible etching of the silicon after BHF etching.

![Graph](image)

Figure 8.11: Transmission spectrum with no magnetic field present for device for $L_{pc}$ of (a) 44a, (b) 112a, and (c) 224a. Different DC current injected into the device induces thermo-optical and thermal mechanical effects that shift the spectrum.

### Noise Spectrum and Sensor Calibration

After tuning the wavelength of the laser to a slope in the transmission spectrum such that the condition described by Eq. (8.6) was satisfied, the optical output was then sent to the SA in the configuration shown in Fig. 8.10(c). The measured noise spectrum densities centred on their first resonant frequency for devices with PC length 44a, 112a, and 224a in Figs. 8.12(a), (b), and (c), respectively. Using nonlinear regression, the acquired noise spectrum density for each
device was fitted to Eq. (8.54) and the resonant frequency and quality factor of each device could be determined and are summarized in Table 8.6.

Since no current was injected in the device during this measurement, the device was considered at ambient temperature. Assuming only small changes in temperature of the device, the detection limit in terms of the input current was computed using Eq. (8.51) and summarized in Table 8.4. Using the fitted parameters along with the FEM computed mode mass, the calibration factor $\alpha$ between photodiode current and oscillator position was determined and an equivalent noise spectrum density of the vibration amplitude $q$ is shown on the right-side y-axis in Figs. 8.12(a), (b), and (c). Using Eq. (8.10) and the determined calibration factor $\alpha$, the sensitivity of the coupling strength to the oscillator position was determined and summarized in Table 8.4. Here, $I_0$ was taken to be the photocurrent measured from the control device at the specified wavelength. With the fitted $Q$, $f_0$, FEM mass $m$, and $d\kappa/dq$, Eq. (8.60) was used to determine the intrinsic sensitivity $\beta$ and the results are summarized in Table 8.4 for all the devices.

![Figure 8.12](image)

**Figure 8.12:** Measured (red) and fit (black) noise spectrum density with no magnetic field present for device for $L_{pc}$ of (a) 44$a$, (b) 112$a$, and (c) 224$a$.

<table>
<thead>
<tr>
<th>$L_{pc}$</th>
<th>$\delta_b$</th>
<th>$\alpha$</th>
<th>$I_0$</th>
<th>$d\kappa/dq$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[a]$</td>
<td>[nT·A/Hz]</td>
<td>[A/µm]</td>
<td>[mA]</td>
<td>[rad/µm$^2$]</td>
<td>[1/(A·T)]</td>
</tr>
<tr>
<td>44$a$</td>
<td>1.31</td>
<td>0.106</td>
<td>15.2</td>
<td>0.352</td>
<td>326</td>
</tr>
<tr>
<td>112$a$</td>
<td>0.88</td>
<td>0.188</td>
<td>22.3</td>
<td>0.167</td>
<td>853</td>
</tr>
<tr>
<td>224$a$</td>
<td>0.70</td>
<td>0.322</td>
<td>24.0</td>
<td>0.133</td>
<td>3210</td>
</tr>
</tbody>
</table>

**Table 8.4:** Thermal calibration results
8.4.3 Magnetic Driven Measurements

For the driven magnetic response, the electromagnet core was placed around the device and the excitation current $I_{ex}$ was set to 1 A that generated a measured $B_y$ of 36 mT. For these measurements, the driven harmonic response, resonant field response, and step-response of the device were performed.

**Driven Harmonic Response**

To acquire the frequency response of the devices, the FG generated a purely sinusoidal current that was injected across the microstructure while the amplitude of the fundamental harmonic of the photodiode was recorded using the SA while the FG frequency was swept. This led to heating and detuning of the device which was then compensated by re-tuning the TL wavelength to re-establish the condition in Eq. (8.6). Since the measured photocurrent signal depended on the input optical power, injected AC current, and field strength, the recorded photodiode amplitudes were normalized against all three parameters using Eq. (8.60) which yielded the intrinsic sensitivity $\beta$ that is shown in Fig. 8.13(a) for each device. This sensitivity represents the fraction of input power that is coupled to the fundamental harmonic of the output per unit Tesla per Ampere of injected current.

**Resonant Field Response**

For the resonant field response, a purely sinusoidal current was injected across the microstructure at a frequency tuned to the mechanical resonance of the respective devices. The amplitude of the fundamental harmonic was recorded by the SA as the field strength was varied using the electromagnet excitation current $I_{ex}$. As before, the measured amplitudes were normalized and shown in Fig. 8.13(a). The resulting data was fitted to a line with the slope $\beta$ that may be compared with the thermal calibration data. For the 112$a$ long device, nonlinearity was observed.

**Sensitivity and wavelength dependence**

To better understand the wavelength dependence on the magnetic field sensitivity, a sinusoidal current at the structure resonance was injected across the microstructure and the amplitude of the fundamental harmonic amplitude was recorded as the wavelength was swept. At each wavelength, $\beta$ was computed and shown in Fig. 8.14. The insets show the input FG voltage and the corresponding AC-coupled output voltage from the DET at wavelengths corresponding to peak sensitivity. Since the sinusoidal current was driven at the mechanical resonance, the
8.4. Measurement Results

Figure 8.13: (b) the black lines are the best fit slope with 215 (mT·mA)$^{-1}$, 876 mT·mA$^{-1}$, and 2365 mT·mA$^{-1}$

normal coordinate response has a $\pm 90$ degree phase difference that was whose sign depends on the $n$ and the orientation of the magnetic field component.

Figure 8.14: The fraction of input optical power that is coupled to the fundamental harmonic per amplitude per Tesla rms Amp as the wavelength is swept for the 112$a$ device. The insets show the input FG voltage and the corresponding AC coupled output voltage from the DET at each wavelength with $\pm 90$ degree lag.

Step Response

To determine the thermal time constant of the device, current was injected into the device based on a step function. In this case, the generated electrical heat is also described by a step response
and photodiode voltage can be effectively described by a linear differential equation having a solution of the following form

\[
v = v_0 + c_1(1 - e^{-t/\tau}) + c_2[1 - e^{-\zeta \omega_0 t} \cos(\omega_0 t)] + c_3 e^{-\zeta \omega_0 t} \sin(\omega_0 t),
\]  

(8.61)

where \(v_0\), \(c_1\), \(c_2\), and \(c_3\) are fitting coefficients. The step-response was acquired both when the magnetic field was on and off and each response was fitted to Eq. (8.61) using non-linear regression. The quality factor was computed using \(Q = 1/(2\zeta)\) and the fitted resonant frequencies and quality factors for each device are summarized in Table 8.6. The fitted thermal time constants \(\tau\) for each device are shown in Table 8.5 and agree with FEM simulation results.

Figure 8.15: The photocurrent response to \(i_{ac}\) the step with and without a magnetic field. 10 mA for 44a, 5 mA for 112a, and 1 mA for 224a

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<tr>
<th>Method</th>
<th>(N_i)</th>
<th>(\tau) ((\mu s))</th>
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</thead>
<tbody>
<tr>
<td>FEM</td>
<td>44</td>
<td>2.48 (\mu s)</td>
</tr>
<tr>
<td>Step Resp. w/o B</td>
<td>44</td>
<td>3.29 (\mu s)</td>
</tr>
<tr>
<td>Step Resp. w/ B</td>
<td>44</td>
<td>3.35 (\mu s)</td>
</tr>
<tr>
<td>FEM</td>
<td>112</td>
<td>6.55 (\mu s)</td>
</tr>
<tr>
<td>Step Resp. w/o B</td>
<td>112</td>
<td>7.26 (\mu s)</td>
</tr>
<tr>
<td>Step Resp. w/ B</td>
<td>112</td>
<td>7.12 (\mu s)</td>
</tr>
<tr>
<td>FEM</td>
<td>224</td>
<td>17.8 (\mu s)</td>
</tr>
<tr>
<td>Step Resp. w/o B</td>
<td>224</td>
<td>15.4 (\mu s)</td>
</tr>
<tr>
<td>Step Resp. w/ B</td>
<td>224</td>
<td>13.9 (\mu s)</td>
</tr>
</tbody>
</table>
8.5 Discussion

Since the resonant frequency and mechanical quality factors were determined using different methodologies, they were summarized in Table 8.6. The resonant frequency and semi-analytically determined quality factors of the devices determined using FEM found good agreement with the measurement results. There appeared to be larger discrepancies between the FEM and measurement results for the longer devices where prestraining effects were more unpredictable. Additionally, the resonant frequencies for the two longer devices tended to increase by about 6–8% during magnetic measurements compared to non-magnetic measurements.

The devices fabricated here were compared with MEMS magnetic sensors found in literature and summarized in Table 8.5. The energy resolution was determined using \( \delta_E \approx \delta_b^2 V/(2\mu_0) \) [46] where the volume was determined using \( V = (L_{\text{tot}} \times W_{\text{tot}})^{1.5} \). Based on the smallest structure that achieved a detection limit of 130 nT/\( \sqrt{\text{Hz}} \), the energy resolution was computed to be 8.17\( \times 10^{-23} \) J·s. Based on the thermal time constants being close to the FEM generated values, it was expected that electrical resistances of the structures to be close to the FEM generated as well. For the injected currents used in the experiments, this results in temperature differences of about 0.5 K. Since this is much less than the ambient temperature, even lower detection limits can be achieved by increasing the injected current.

The experiment was aligned with the magnetic north and able to measure the Earth’s magnetic field which generated a 2.75 uA photocurrent signal for the 44a device at resonance with injected current of 10 mA. Using the calibration results in Table 8.4 and Eq. (8.60), this resulted in 55.5 \( \mu \text{T} \) which is within 5% to the World Magnetic Model dataset value of 53.3 \( \mu \text{T} \).

Table 8.6: Mechanical comparison

<table>
<thead>
<tr>
<th>Method</th>
<th>( N_x )</th>
<th>( f_0 )</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
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<td>FEM/( \text{theory} )</td>
<td>44</td>
<td>1.56 MHz</td>
<td>51.9</td>
</tr>
<tr>
<td>Thermal Noise</td>
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<td>1.65 MHz</td>
<td>57.8</td>
</tr>
<tr>
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<td>53.8</td>
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<tr>
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<tr>
<td>Step Resp. w/ B</td>
<td>44</td>
<td>1.64 MHz</td>
<td>59.0</td>
</tr>
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<td>FEM/( \text{theory} )</td>
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<td>547 kHz</td>
<td>19.0</td>
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<tr>
<td>Thermal Noise</td>
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<td>554 kHz</td>
<td>22.2</td>
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<td>Frequency Resp.</td>
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<td>605 kHz</td>
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<td>Step Resp. w/o B</td>
<td>112</td>
<td>556 kHz</td>
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</tr>
<tr>
<td>Step Resp. w/ B</td>
<td>112</td>
<td>609 kHz</td>
<td>22.8</td>
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<tr>
<td>FEM/( \text{theory} )</td>
<td>224</td>
<td>289 kHz</td>
<td>10.0</td>
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<td>Thermal Noise</td>
<td>224</td>
<td>217 kHz</td>
<td>9.01</td>
</tr>
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<td>Frequency Resp.</td>
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<td>231 kHz</td>
<td>9.35</td>
</tr>
<tr>
<td>Step Resp. w/o B</td>
<td>224</td>
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<td>14.3</td>
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<tr>
<td>Step Resp. w/ B</td>
<td>224</td>
<td>232 kHz</td>
<td>9.58</td>
</tr>
</tbody>
</table>
## 8.6 Conclusion

Using a combination of standard silicon photonics foundry process along with metallization and wet-etching, miniature Lorentz force magnetometers with 2D photonic crystal edge-coupled optical read-out were created of varying lengths. Devices as small as 30 μm long were fabricated and calibrated using thermal fluctuations that were modeled using the dissipation fluctuation theorem resulting in an equivalent magnetic noise floor of 130 nT/Hz\(^{0.5}\). While the longer devices were more to be sensitive that the shorter devices (i.e., larger \(\beta\)), they also exhibited higher noise levels due to its larger mass and smaller quality factors. Additionally, the longer devices larger resistance leading to more heating. Further research is needed to evaluate the stability of the sensor and establish the upper temperature limit.

The devices characterized in this paper were done under ambient environment conditions, without the use of cryogenics and high vacuum. Detection of magnetic fields could be done without the need to operate near the mechanical resonance enabling the acquisition of complicated time-varying magnetic signals without a frequency-dependent sensitivity that is present in search coils. Moreover, these designs presented in this paper are expected to be compatible with high magnetic fields. Thus, the measurement of small time-varying magnetic fields superimposed with a large static background field simply shifts the quiescence point of the sensor. This contrasts with SQUID and SERF-based magnetometry that have fundamentally determined upper limit to magnetic fields.

It is worth noting that most optical-based MEMS magnetometers found in literature use off-chip optical read-out that are not easily integrated with optical fibres. While the optical source and photodetection in this paper were also off-chip, these designs can be more easily be integrated to optical fibres using integrated waveguides and established optical fibre bonding techniques and therefore seen as an important step towards highly sensitive and inexpensive
MEMS-based magnetometry.

8.7 Acknowledgements

The authors would like to acknowledge CMC Microsystems for funding and technical support including access to design software, foundries, and select measurement equipment. Funding for this research was also provided by the CMC Microsystems MNT grant and Ontario Graduate Scholarship (OGS). The devices were fabricated through IMEC ePIXfab foundry and post processed at the University of Western Ontario Nanofabrication Facility.

Bibliography


Chapter 9

Conclusion

In this chapter, the key results of the previous chapters were compared with existing microsensor technologies. Based on the results of this thesis, future work and research directions are presented. Finally, in the closing remarks, the thesis objectives are re-stated along with a summary of how each objective was addressed by the integrated articles.

9.1 Comparison and to Existing Sensor Technology

The PCDC sensors investigated in this thesis were compared to the same sensor technologies reviewed in Chapter 1. Figs. 9.1(a) and (b) show the spatial and temporal resolutions of state-of-the-art pressure and wall shear stress sensors along with the updated results of this thesis. For the highly buckled PCDC ultrasonic pressure sensor with vertical misalignment, the sensitivity of 12.5 mPa/$\sqrt{\text{Hz}}$ reported in Chapter 3 was used along with a membrane area of 15 $\mu$m x 12 $\mu$m. The response time of 1.8 $\mu$s was estimated based on the measured resonance frequency of the vertical mode, occurring at 560 kHz (i.e., $\tau \approx 1/f$).

For the PCDC wall shear stress sensor, the sensitivity of 80.7 $\mu$Pa/$\sqrt{\text{Hz}}$ reported in Chapter 4 was used along with a membrane area of 30 $\mu$m x 16 $\mu$m. The response time of 1.6 $\mu$s was estimated based on the measured resonance frequency of the horizontal mode occurring at 628 kHz. The results are particularly remarkable for the wall shear stress sensor as they are several orders of magnitude smaller than existing sensors while maintaining a detection limit below 100 $\mu$Pa/$\sqrt{\text{Hz}}$.

The performances of state-of-the-art magnetic sensors from the literature review in Chapter 1 are shown in Fig. 9.2 along with the updated performance of the PCDC magnetic sensor. For the PCDC magnetic sensors based on the vertical vibrational resonance, the measured detection limit of 130 nT/$\sqrt{\text{Hz}}$ and effective volume of $2.05 \times 10^{-14}$ m$^3$ was used. The effective volume was determined by taking the area $A^{1.5}$ and where the area 30 $\mu$m x 25 $\mu$m was used.
9.2. Future Research Topics

There are many aspects of PC edge coupled sensors that can be further explored. A suggested list of possible designs and experiments are listed below:

- PCDC heterostructures
- Optical interfacing
- Wall jet measurements
- Multimode analysis
- Integration with magnetostriction materials
- Optomechanical coupling
Further modifications in the PCDC design can be explored for added capability. Although conventional rib and channel waveguides exhibit low insertion loss to the dielectric-like modes of the PC edge, heterostructures might be used to improve input coupling, particularly near the band edge where the group index mismatch is greater and Fabry Pérot ripples are observed in the optical spectrum. While apodization can be used to couple light to more efficiently to photonic crystals, the PC edges have two low group velocity points in the band structure making simultaneous high efficiency coupling to both cavity modes more challenging.

For the thermoacoustic experimental apparatus, larger clearances between the optical fibre array and the test structure are recommended for future experiments. This can be done at the layout stage by elongating the waveguides leading from the surface grating couplers to the PC membrane. Alternatively, edge facet coupling may be more suitable for further experimentation and optical fibres can be bonded to the chip for use in field testing.

The testing of PCDC sensors in this thesis was performed at ultrasonic frequencies and their performance under steady flows was unexplored. The measurement system described in this thesis can easily be adapted to generate steady state flow. Using actuated microfluidic syringes, controlled microvolumes of fluid can be made to impinge upon the chip die using a
tube, forming wall jet system. Wall jets are common to numerous applications involving heat and mass transfer [1].

The observation of other mechanical modes using the PCDC magnetic field sensor can give insight on the vector components of the magnetic field and potentially improve the measurement precision. Antiresonances that can form wherever multiple mechanical modes are excited coherently, can give specific information about how external forces are coupled to the microstructure. These effects were not included in the integrated article in order to limit the scope of this thesis, however they can be predicted using an eigenvalue formulation when given a forcing function, mass, and stiffness matrices [2].

There were observed instabilities in the resonant frequencies of the PCDC magnetic sensors investigated, particularly for the longer structures. It is possible that an additional heat treatment step after metallization could enhance bonding with the silicon however, this can lead to a build up in interfacial stress between the metal and silicon [3]. Alternatively, by depositing ferromagnetic and magnetostrictive materials onto PC membranes, magnetic sensors can also be formed without the need for an injected excitation current as required by the Lorentz force.

Although it was not explored in this thesis, coupled PC edges exhibit strong optomechanical coupling, particularly for even modes where the field is enhanced inside the air gap. In the low group velocity regime near the band edge, optical confinement is enhanced due to resonance with the PC and the frequency of optical cavities formed in this regime are strongly dependent on the PC edge separation $G \equiv \partial \omega / \partial x = 10^{20} - 10^{21}$ Hz/m. In nonlinear quantum optomechanical experiments, it is desirable to have a high optomechanical coupling and low optical decay rate (i.e., high photon life-time) [4], making coupled PC edges a potential platform for preparing quantum states of macroscopic objects.
9.3 Closing Remarks

In the closing remarks, we revisit the objectives of this thesis and outline how each has been achieved. The objectives listed in Chapter 1 are restated below:

- Motivate PC edge-coupled sensors (Chapter 1)
- Design, fabricate, and characterize PCDC sensors using static displacements (Chapter 2)
- Demonstrate broadband optical response (Chapter 2)
- Demonstrate PCDC sensors using air-coupled ultrasound (Chapter 3)
- Acquire spatial response of PCDC-based wall shear stress sensors (Chapter 4)
- Calibrate wall shear stress sensor using thermal noise floor (Chapter 4)
- Characterize crosstalk between wall shear stress and dynamic pressure (Chapter 5)
- Mitigate the effects of PC-edge misalignment (Chapter 6)
- Determine mass sensitivity of PC edge-coupled membranes (Chapter 7)
- Design, fabricate, and test of PCDC magnetic field sensors (Chapter 8)

In Chapter 1, a comparison of microsensor technology was presented which focused on deflection based micromechanical sensors and the challenges facing the miniaturization of common transduction techniques. It was shown that integrated optical systems are capable of leveraging mass-scaling effects allowing the improvement of spatiotemporal performance. Key applications of miniature sensors were identified which include resolving the Kolmogorov scales associated with high Reynolds number phenomena. Since magnetic sensors can be coupled using coils to larger sensing areas, a need for high-resolution magnetic sensors was identified based on the energy resolution that takes into account the sensor volume. Standardized silicon photonic foundries based on 220 nm SOI were identified as an inexpensive platform for rapid prototyping as multiproject wafers, with excellent optical and mechanical properties, allowing the integration of waveguides and surface grating couplers. By using a directional coupler based sensor, fewer optical components are required, enabling lower footprints.

In Chapter 2, photonic crystal directional couplers were designed, fabricated, and characterized using static horizontal and vertical displacements. Light was confined to the edges of a PC membrane using the photonic band gap generated by a suitable selection of photonic crystal lattice pitch and hole diameter using plane wave expansion. Finite-difference time-domain numerical simulation was used to model structures of finite length which demonstrated broadband optical response. Due to the symmetry presented by perfectly aligned PC edges, it was determined that a non-zero vertical misalignment between the PC edges is required to sensitize the PCDC coupling coefficient to vertical displacements. The foundry-generated chips were subject to a wet-etch post-foundry process to selectively remove the buried oxide and release the
PC membranes. Horizontal displacement between the PC edges was defined during the layout design stage while vertical displacements were achieved through buckling of PC membranes. The experimental results were in good agreement with the FDTD generated transmission with a discrepancy later attributed to variations in the design dimensions.

Since the wet-etched samples were buckled, it was believed that they would be sensitive to dynamic pressure. In Chapter 3, ultrasonic testing on the sensors was performed using a coiled speaker up to 80 kHz. Due to the complementary nature of outputs provided by the directional coupler, a balanced photodetector was used to reduce noise components common to each output when the optical wavelength was tuned such that power was equally shared between the coupler outputs. In order to extend the measurement frequency without introducing frequency artifacts related to the mechanical elements of the coiled speaker, a thermoacoustic emitter was designed and built.

In Chapter 4, a new batch of devices was released from the buried oxide using a post-foundry vapour-etch process and tested as wall shear stress sensors. The newly fabricated vapour-etched sensors were significantly less sensitive than the wet-etched sensors to dynamic pressure and therefore presumed to have smaller vertical misalignment. To test this hypothesis, a thermoacoustic waveguide apparatus was made to impinge on the microchip and generated distinct pressure and wall shear stress profiles. Based on the acquired thermal noise floor, a membrane area measure 30 µm x 16 µm was found to have a horizontal resonance of 628 kHz with the optical read-out which could be calibrated to 80.7 µPa/√Hz using the FEM generated effective area and mass. The device exhibited an integrated average sensitivity of 0.120% full scale optical power per Pascal across a 35 nm optical bandwidth.

In Chapter 5, the thermoacoustic apparatus was used to investigate the effect of vertical misalignment of PC edges on the crosstalk and noise floor in the context of wall shear stress sensing. Optical profilometry performed on identical devices having undergone either wet- or vapour-etch post-foundry processing confirmed the wet-etched devices having a larger misalignment of 550 nm when compared to 100 nm for the vapour-etched device. Testing both devices using the thermoacoustic waveguide apparatus showed reduction in misalignment led to decrease in the fraction of the total signal associated with pressure (i.e. crosstalk) going from 2.8% for the wet-etched devices to 0.35% for the vapour-etched devices. The vapour-etched devices also demonstrated a 4.5 reduction in the noise floor due to the reduced sensitivity of the PCDC to thermal fluctuations of the membrane in the vertical direction. In Chapter 6, by introducing microbeam arrays into the PC membrane supports, vertical misalignment could be avoided as compressive strain present in the SOI could relax and prevent buckling.

In Chapter 7, the added-mass sensitivity of the PC membranes based on the shift in horizontal resonant frequency that could be observed using the thermal fluctuations that comprise
the noise floor. Using changes in the PC hole radius defined at the design layout stage, the change in mass could be estimated resulting in a mass sensitivity of 0.185 fg/Hz. Due the quality factor of the order picograms.

In Chapter 8, magnetic sensors based on the Lorentz force and PCDC optical read-out were fabricated. Three different microstructures with total lengths of 30 µm, 60 µm, and 110 µm were evaluated with the measured resonant frequencies 1.66 MHz, 555 kHz, 231 kHz found to be in close agreement with a prestrained FEM model. The quality factors of devices were measured to be 58, 22, and 9 and were found to be in close agreement with a semi-analytical transition flow regime fluid damping model in ambient conditions. By injecting a step current through the microstructure, the thermal time constants of 3.3 µs, 7.2 µs, and 15 µs were measured for each device and found to agree with FEM modelling. The temperature difference due to joule-heating is much less than the ambient temperature, the resulting sensors had a detection limits of could be expressed in terms of the excitation current and found to be 1.31, 0.88, and 0.70 nT·A/√Hz for the 30, 60, and 110 µm devices, respectively.

All thesis objectives have been accomplished and the PCDCs have been successfully demonstrated within various sensor contexts. Using standardized silicon photonic foundry processes, inexpensive sensors can now be fabricated with superior spatiotemporal resolution to existing sensor technologies due to the favourable mass-scaling of optomechanical systems. The development of custom nanofabrication processes and novel experimental apparatus have enabled the demonstration of PCDC-based microsensor technology, and can potentially find applications in aerospace and biomedicine.

Bibliography


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Publications:


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<th><strong>Name:</strong></th>
<th>Michael Zylstra</th>
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<tr>
<td><strong>Post-Secondary Education and Degrees:</strong></td>
<td>McMaster University Hamilton, ON 2003 - 2008 B.Sc.</td>
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<tr>
<td></td>
<td>University of Western Ontario London, ON 2010 - 2014 B.Eng.</td>
</tr>
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<td><strong>Honours and Awards:</strong></td>
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