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This paper contains preliminary findings from research still in progress and should not be quoted without prior approval of the author.

THE ROLE OF FUTURES MARKETS IN INTERNATIONAL TRADE:
A GENERAL EQUILIBRIUM APPROACH

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Second Draft

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ABSTRACT

Futures trading, which has mainly been studied in the partial equilibrium framework, is incorporated into a general equilibrium trade model under uncertainty. We demonstrate that the opening of a futures market may harm a country, and that two stylized facts, "good harvest poverty" and "short hedging," are equivalent. The relationship between agricultural production risks and the spot price variability of agricultural products is considered by the comparative statics method. Interpretations are given to our results in relation to models of trade between two developed countries, between developing and developed countries, and between two sectors in a country.
In their important work, Helpman and Razin (1978) incorporate uncertainty in production processes into a general equilibrium model and consider international trade in an economy with security markets, extending Diamond's model (1967). Following their work, a number of studies have been concerned with the structure of production in economies with markets for risk sharing. However, very little is yet known about the determination of the terms of trade in such an economy, even though the terms of trade are one of the most important subjects in trade theory. In this study, we focus upon this somewhat neglected issue, considering a simple general equilibrium trade model with a market for risk sharing, more specifically, with a futures market.

The consideration of a futures market from the risk sharing viewpoint originates from Keynes (1930) and Kaldor (1939, 1940). Since then, theoretical studies in the literature of futures trading have mainly been carried out in a partial equilibrium framework. Extending the work of Keynes and Kaldor, we build a general equilibrium model with futures markets in an international trade framework. We consider a two-good model with manufacturing and agricultural countries. (Of course, our model may also be considered as a two-sector model of a closed economy.)

This study has two major purposes. One is to characterize the nature of a rational expectations equilibrium in our model. The other is to consider the effect of a country's entry into a futures market for an agricultural good on the welfare of countries and on the variability of the spot price of the good. The latter subject in particular is closely related to development economics. Since organized futures markets exist primarily in developed countries and since producers in developing countries seldom engage in futures transactions, the consideration of the effect of a country's entry into
a futures market is especially important for developing countries.

This research consists of three parts. The first part shows that a country's starting futures transactions may harm some countries, including the country itself. Two situations are considered. One is that in which a futures market is introduced into an economy with only spot markets; the other is that in which a country which has not formerly engaged in futures transactions enters into the existing futures markets in foreign countries. We conclude that introducing a futures market may harm a country and that a country's entry into an existing market may harm that country or some other countries.

The second part demonstrates that the agricultural country sells contracts for future delivery of its products if and only if agricultural income in the spot market is lower after a good harvest than after a bad harvest. This result is extremely interesting since it shows the equivalence of two well known stylized facts. First, it has been long recognized in the literature of real trade that a good harvest decreases agricultural income ("good harvest poverty")\(^3\); second, it has been widely accepted in the literature of futures trading that traders who hedge against risks by engaging in a futures market (agricultural producers in our model) sell in the futures market (short hedging).\(^4\) Although the phenomena of both good harvest poverty and short hedging have been known for a long time, their equivalence has never been pointed out.

In the third part, our main concern is whether or not opening a futures market for a good reduces the variability of the spot price of the good in an economy which formerly had only spot markets. Of course, it is technically impossible to answer this question directly given the generality of our model, since it would be necessary to find equilibrium distributions of the spot market...
prices both in the futures - spot market system and in the pure spot market system. We intend to approach the question by means of comparative statics, considering a small increase in agricultural production risk.

Since, for the sake of simplicity, the agricultural good is randomly associated with only two output levels, those of a good harvest and a bad harvest, an increase in production risk is associated with an increase in the level of a good harvest and a decrease in the level of a bad harvest. Since the price of a good is negatively correlated with the level of its output in an economy with only spot markets, an increase in agricultural production risk raises the price of the agricultural good after the bad harvest, and reduces the price after the good harvest. As a result, the variability of the spot price of the good increases. If there is a futures market for the good, the spot price and its variability respond to the same increase in agricultural production risk in a different manner. The difference between the responses of the spot price variability in the futures - spot market system and in the pure spot market system is regarded as the result of opening a futures market.

It is shown that in an economy with a futures market, the spot price
of an agricultural good may respond to a change in its output level in a
perverse way. In other words, an increase in agricultural production risk
may raise the spot price of the agricultural good after the good harvest and
reduce that after the bad harvest, even though the increase in risk expands
the size of a harvest after the good harvest state is realized and contracts
that after the bad harvest state is realized.

We show that the main role of a futures market is to reallocate purchasing
power between the two countries by means of capital gains and losses in each
spot market. In a spot market, this reallocation appears similar to a transfer
in the deterministic model. As is emphasized by Samuelson (1952), Jones
(1970, 1975), and Yano (1981, 1983), the difference in tastes (or in marginal
propensity to consume a good) between the two countries plays a crucial role
in the transfer problem. We demonstrate that if the two countries have the
same tastes, the effects of an increase in agricultural production risk in
an economy with a futures market are similar to those in the economy without
one. Thus, the perverse response of a spot price discussed above is more
likely to appear if the taste difference is larger.
I. Model

The model is essentially a single-period exchange model with uncertainty. The period starts at time 0 and ends at time 1. There are two countries in the world: the home country (country H) and the foreign country (country F). Each country is completely specialized. The manufactured good (good M) is produced in country H; the agricultural good (good A) in country F. Random factors such as weather are assumed to affect only agricultural production. There are only two states of nature: the good harvest state (state 1) and the bad harvest state (state 2). State i occurs with probability $\phi_i$. Output becomes available at time 1. $X_M$ units of good M are produced by country H regardless of states of nature. Country F produces $X_{A1}^*$ units of good A after state 1 is realized and $X_{A2}^*$ units after state 2. Note that $X_{A1}^* > X_{A2}^*$.

The spot market where actual goods are traded opens and clears at time 1. Good M is the numeraire good whose price is unity regardless of states of nature. The price of good A, $\bar{p}$, is a stochastic variable; $p_i$ is that after state i is realized. In the futures market which opens and clears at time 0, futures contracts of good A are traded. One contract stipulates that one unit of good A is delivered at time 1 regardless of states of nature. $q$ is the price of futures in terms of the numeraire good (good M). Payment for the contracts purchased at time 0 is made at time 1.

Suppose that state i is realized during the period. Then, $X_M$ is the income of the manufacturing sector in country H and $p_i X_{A_i}^*$ is that of the agricultural sector in country F. Suppose that in the futures market at time 0 country F sold $X^*$ units of good A futures and country H bought D units
(D = X^* > 0). Then, country H pays qD = qX^* units of good M to country F since payment is made upon delivery. Since the spot price of good A is p_i,

(p_i - q)D = (p_i - q)X^* is country H's capital gain (loss) and country F's capital loss (gain) if it is positive (negative). Thus, the total incomes of countries H and F are X_M + (p_i - q)D and p_i X_A^* + (q - p_i)X^*, respectively. They are spent in order to consume goods A and M. A_i and M_i (A_i^* and M_i^*) are the total consumption of the goods in country H(F). Therefore, if we define

B_i = (p_i - q)D and B_i^* = (q - p_i)X, they are surpluses (deficits) in the trade account if they are positive (negative). The tastes of consumers in a country are represented by a single utility function: u(A_i, M_i) for country H and u*(A_i^*, M_i^*) for country F. The optimization behavior in the spot market is summarized as follows. For \( i = 1, 2 \),

\[
\begin{align*}
\max & \quad u(A_i, M_i) \\
\text{s.t.} & \quad p_i A_i + M_i = X_M + B_i
\end{align*}
\]

\[
\begin{align*}
\max u^*(A_i^*, M_i^*) \\
\text{s.t.} & \quad p_i A_i^* + M_i^* = p_i X_A^* + B_i^*
\end{align*}
\]

The spot market clears at time 1 instantaneously, i.e., for \( i = 1, 2 \),

\[
\begin{align*}
& \quad A_i + A_i^* = X_A^* \\
& \quad M_i + M_i^* = X_M
\end{align*}
\]

We assume, for the sake of simplicity, that all consumers trade in the futures market. Since we consider only complete information rational expectations equilibria, all agents are assumed to know the correct (equilibrium) spot price of good A after each state is realized, the correct probability attached to each state, and their incomes from production. \( v(Y_i, p_i) \) and
\( v^*(Y_i, p_i) \) are the indirect utility functions derived from \( u \) and \( u^* \), respectively; \( Y_i \) and \( Y_i^* \) are the incomes of countries H and F in terms of good M in the spot market after state i. If country H (F) buys (sells) \( D(X^*) \) units of good A futures, and if state i is realized, its income is anticipated to be \( X_M + (p_i - q)D \quad (p_i X_{Ai}^* + (q - p_i) X^*) \) in the spot market. Thus, in the futures market, country H chooses \( D \) so as to maximize expected utility

\[
\sum_{i=1}^{2} \phi_i v(Y_i, p_i) \quad \text{(and similarly for F)}.
\]

Therefore, the optimization behavior in the futures market is summarized as follows.

\[
\begin{align*}
&\max \sum_{i=1}^{2} \phi_i v(Y_i, p_i) \\
&\text{s.t. } Y_i = X_M + (p_i - q)D \quad \text{and} \\
&\quad \text{s.t. } Y_i^* = p_i X_{Ai}^* + (q - p_i) X^* \\
&\quad i = 1, 2
\end{align*}
\]

The futures market establishes the price of futures contracts, \( q \), so as to clear the market, i.e.,

\[
(4) \quad D = X^*.
\]

We call the equilibrium system (1) - (4) the futures-spot market system.

As we noted above, an equilibrium in the system is that of complete information rational expectations. Since we consider only two states of nature, our market system will in general be complete; if equilibrium spot prices are different across states of nature, an equilibrium allocation is a Pareto optimum with respect to state contingent commodities.\(^5\)

We consider a single period model, ignoring storable goods. The problem of inventories is an important subject in the literature of futures trading. However, since the main concern of this study is behavior in the futures market for an agricultural good, and since we envisage the length of a time period to be one
crop cycle, often one year, which is long enough to worsen the quality of a product significantly, the single period model is reasonable.\textsuperscript{6} We assume that payment for futures contracts is made upon delivery. This is reasonable since in actual practice only between 5 and 10 percent of the value of a contract is required to be deposited at the time of purchase and the balance is paid on delivery.

We make no distinction between futures contracts traded in organized markets and actuals contracts for future delivery made between specific individuals. This is natural since we consider a rational expectations equilibrium. Moreover, it is clear in reality that arbitrage considerations tend to keep prices for forward sales of a commodity closely in line with the futures price. Although developing countries seem seldom to engage in organized futures markets, they are known often to sell forward.\textsuperscript{7} Therefore, our model can be interpreted as one of developing and developed countries, or a north-south model.
II. Gains from Opening Futures Markets

We first show that opening a futures market in an economy with only spot markets may harm a country.

Suppose that $p_1$ and $p_2$ are equilibrium spot prices of good A after states 1 and 2 are realized, respectively. Given $p_1$ and $p_2$, the optimization of transactors in each country, (3), implies the following first order condition.

$$\frac{\phi_1 v_{Y1}}{\phi_2 v_{Y2}} = \frac{\phi_1 v_{Y1}^*}{\phi_2 v_{Y2}^*} = \frac{p_2 - q}{q - p_1},$$

where $v_{Y1} = v_Y(Y_1, p_1) = \frac{3}{\theta Y} v(Y_1, p_1)$ and $v_{Y1}^*$ is defined similarly.

Let us see Figure 1, where the left (right) diagram shows the economy after the good (bad) harvest. Points $H_1$ and $H_2$ ($F_1$ and $F_2$) are the origins of country $H$ ($F$). $E_1$ and $E_1'$ ($E_2$ and $E_2'$) are spot equilibria in the economy without the futures market after state 1 (2) is realized. Note that $p_2 > p_1$ if $p_2' > p_1'$. Suppose that each state occurs with probability 1/2. We are free to assume that each country has the same marginal utility of income at $E_1'$ as at $E_2'$, and that each has different marginal utilities of income between $E_1$ and $E_2$. Note that $E_1$ and $E_2$ indicate a rational expectations equilibrium in our futures - spot market system; that is, if we choose $q = (p_2' - p_1')/2$, the first order condition (5) is satisfied. Suppose that in the economy without the futures market, the economy is at $E_1$ or $E_2$, depending upon states. Since under our assumption $E_1$ and $E_2$ do not constitute a rational expectations equilibrium in our futures - spot system, the introduction of the futures market shifts the allocation to $E_1'$ and $E_2'$. Country $F$ is unambiguously worse off after the change.
Our model may also be considered as that of a closed economy of a developing country which has as yet no futures market. Our example is important for such a country, especially because the partial equilibrium results of Keynes and Kaldor suggest that opening a futures market benefits all agents by providing a risk sharing device.

Our example is similar to that of Hart (1975, example 4), who shows that the introduction of a security market may harm an agent. His example is based upon a complicated three period model; whereas our market system is complete, his is incomplete; the nature of his securities are different from that of our futures contracts. Because of these reasons, we can not use Hart's example in order to prove our result above. A merit of our example in comparison with his is its simplicity; it is clear in our diagram that the terms of trade deterioration causes the paradox.

The examples of both ours and Hart's are concerned with introducing a new ex-ante market into the economy with only ex-post (spot) markets. In the next example, we consider that there is a futures market in country H which now has both manufacturing and agricultural sectors, that country F has only an agricultural sector as before, and that the two countries trade in the spot markets but not in the futures market.

It is shown that country F may become worse off after joining the futures market of country H even if it has the correct information about the future. This result is important since futures markets exist primarily in developed countries and since producers in developing countries seldom engage in futures transactions.

Now, country H also has an agricultural sector, of which the optimization behavior is parallel to that of country F. $O_1C_1$ and $O_2C_2$ indicate the agricultural output of country H, and $F_1O_1$ and $F_2O_2$ that of country F.
Indifference curves of country F are now indicated by the dotted curves through $A_1$, $A_1'$, $A_2$, and $A_2'$; those of country H's agricultural sector are depicted by the dotted curves through $B_1$, $B_1'$, $B_2$, and $B_2'$, where the origins of indifference curves are $O_1$ and $O_2$; those of country H's manufacturing sector are the same as before. $p_1$, $p_1'$, $p_2$, and $p_2'$ are again equilibrium spot prices in an economy with only spot markets. (When $p_1$ is, for example, the market price, country F consumes at $A_1$, country H's agricultural sector at $B_1$, and its manufacturing sector at $E_1$; thus, vectors $F_1A_1$ and $O_1B_1$ add up to vector $F_1E_1$.) Again, we assume that each agent has the same marginal utility of income when the market price is $p_2'$ as when it is $p_1'$. Thus, the allocation which corresponds to $p_1'$ and $p_2'$ is also a rational expectations equilibrium when all countries trade in the futures market. Assume that the agricultural and manufacturing sectors of country H, each, have the same marginal utility of income when the market price is $p_2$ as when it is $p_1$, but that country F has different marginal utilities of income between $p_1$ and $p_2$. Then, the allocation which corresponds to $p_1$ and $p_2$ is a rational expectations equilibrium when futures contracts are traded only within country H. Now, it is clear that if country F joins the futures market, it unambiguously becomes worse off.\textsuperscript{8}
III. Trade Patterns and the Theory of Normal Backwardation

(a) Trade pattern reversals.

Using the first order condition (5), we may derive the demand and supply functions for good A futures, given $p_1$ and $p_2$. In Figure 2, curve D is the demand curve of home transactors and curve $X^*$ the supply curve of foreign transactors. We consider it reasonable to assume that the spot relative price of a good A in state 2, $p_2$, is higher than that in state 1, $p_1$, since State 2 is the bad harvest state (or $x_{A2}^* < x_{A1}^*$). 9

Since demand and supply curves D and $X^*$ each intersect the vertical axis only once at $\bar{q}$ and $\bar{q}^*$, respectively, a routine argument in trade theory establishes

(6) $D \geq 0$ if and only if $\bar{q} \geq \bar{q}^*$, by using the property of the demand and supply curves. 10 If futures contracts are sold by country H to country F, the trade patterns of each country are reversed between the futures and spot markets. It is interesting to ask if there is any presumption as to the likelihood of a trade pattern reversal.

In this study, we assume that the marginal utility of income depends only upon income. Although this is a somewhat restrictive assumption, it is often
used in the literature of economics under uncertainty (see Newbery and Stiglitz (1981, p. 116), for example. Under this assumption, conditions (5) and (6) imply that $D > 0$ if and only if $v_Y(p_1 X^*_A) > v_Y(p_2 X^*_A)$. Since the countries are risk averse, this implies

$$(7) \quad D > 0 \quad \text{if and only if} \quad p_1 X^*_A < p_2 X^*_A.$$ 

It is widely accepted that agricultural income in the spot market is lower after a good harvest than after a bad harvest; i.e., $p_1 X^*_A < p_2 X^*_A$. Thus by condition (7) we have a presumption in favor of trade pattern non-reversals.

(b) long run imbalance of trade account

$B_1$ and $B_1^*$ are deficits in the trade accounts of countries H and F in the spot market after state i is realized. Thus, if we consider that our single period economy repeats over periods, the expected value $E(B_i)$ reflects the long run balance of trade in each country. $E(B_i)$ may also be regarded as an expected capital gain of country H, or

$$(8) \quad E(B_i) = (\phi_1(p_1 - q) + \phi_2(p_2 - q)) D.$$ 

Since country H is risk averse and has non-risky income, it is clear that $E(B_i)$ has to be positive for the country to take risks in its income, i.e., $E(B_i)$ is a risk premium. In other words, the long run (expected) balance of trade of the manufacturing country is always in deficit and that of the agricultural country in surplus.

An imbalance in a trade account, in general, reflects two motives consumers' behavior in a country. One is to allocate purchasing power inter-temporally; borrowing and lending correspond to deficits and surpluses. The other is to allocate risk across states of nature; capital gains and losses due to uncertainty correspond to deficits and surpluses. Although monetary aspects of the balance of payments have been emphasized in the traditional literature,
recently there have been many studies which attempt to explain trade balances from the side of real trade, concentrating particularly upon the implications of intertemporal consumption behavior. However, not much is yet known about trade balances in relation to the allocation of risks in the real sectors of the economy. Our model provides a simple structure where we may study this aspect of the balance of payments. As simple as it is, the long run imbalance of trade accounts in this study is noteworthy, especially because it has been pointed out from the viewpoint of intertemporal allocation that trade accounts are in general unbalanced in the long run.12

Since \( E(B_t) \) is always positive, equation (8) implies

\[
(9) \quad \sum_{i=1}^{2} \phi_i p_i > q \quad \text{if and only if} \quad D < 0.
\]

Thus, condition (7) implies that the futures price is biased downwards (upwards) as a predictor of the spot price if and only if agricultural income is lower (higher) after the good (bad) harvest. Our discussion above therefore indicates that we have a presumption that the futures price is set lower than the expected spot price.

(b) the theory of normal backwardation

In the literature on futures trading, the transactors who reduce risks in their income by engaging in futures trading are called hedgers, and those who assume risks by trading with hedgers are called speculators.13 In our context, it is clear that home traders are speculators and foreign transactors hedgers. Under this terminology condition (9) coincides with the result well known as the theory of normal backwardation, which is first discussed by Keynes (1930, p. 142) and then refined by Kaldor (1939, 1940). The theory, which is based upon the assumption that hedgers are in general short (or sell futures contracts)
in the net sense, concludes "the forward price must always fall short of the expected [spot] price (for the same date) by the amount of the marginal risk premium" (see Kaldor (1939, p. 5)\(^*\). The assumption that hedgers are short is widely accepted as "characteristic of most futures market at most times" (Goss and Yamey (1976, p. 27)\(^15\).

Our result on the equivalence of short hedging and good harvest poverty is highly important in two respects. First, it provides a justification for the short hedging assumption of the theory of normal backwardation by appealing to the relationship between real trade in the spot market and financial transactions in the futures market. Although various attempts have been made to explain this phenomenon (Telser (1967), McKinnon (1967), Houthakker (1968), Yamey (1971), and Goss and Yamey (1976)), the crucial relationship between agricultural income levels in spot markets and short hedging has been overlooked. Second and more importantly, the equivalence of the two stylized facts, good harvest poverty and short hedging reinforces our presumption that trade patterns tend not to be reversed between the futures market and the spot market.

The assumption that the marginal utility of income does not depend upon spot prices is crucial to establish the results above. Without this assumption, the equivalence of short hedging and good harvest poverty, the positivity of a risk premium paid to speculators, and the downward biasedness of the future price relative to the expected spot price are all in general not true. Speculators may participate in futures markets even if they lose money in the expected value sense;\(^{15}\) even if hedgers are short, the futures price may exceed the expected spot price.\(^{15}\)
IV. Effects of an Increase in Risk

(a) spot market

Foreign agricultural production \( \tilde{X}_A \) is a random variable. For the moment, an increase in agricultural risk is regarded simply as an increase in the good harvest \( (dX^*_{A1} > 0) \) accompanied by a decrease in the bad harvest \( (dX^*_{A2} < 0) \). The spot market equilibrium condition, equations (2), after state \( i \) is realized during period 0 is expressed as \( p_i D_{Ai} = D^{*}_{Mi} - B^*_i \), by using the foreign budget constraint. When agricultural risk is increased, the behavior of consumers in the spot market as well as that of transactors in the futures market changes. Due to the spot market equilibrium condition, this change has to satisfy

\[
(10) \quad \hat{p}_i + \hat{D}_{Ai} = \frac{D^*_{Mi}}{p_i D^*_{Ai}} \hat{D}^*_{Mi} - \frac{B^*_i}{p_i D^*_{Ai}} \hat{B}^*_i.
\]

Define the real income change of countries \( H \) and \( F \) in the spot market after state \( i \) (state \( i \) real income) as \( dy_i = p_i dD_{Ai} + dD_{Mi} \) and \( dy^*_i = p_i dD^*_{Ai} + dD^*_{Mi} \). Then, by the budget constraints in equations (1),

\[
(11) \quad dy^*_i = p_i dD^*_{Ai} \hat{p}_i + B^*_i \hat{B}^*_i + p_i dX^*_{A1}.
\]

It is well known that the relative change in demand is decomposed as follows (see Caves and Jones (1981)); \( \hat{D}_{Ai} = -s_{Ai} \hat{p}_i + \frac{m_{Ai}}{p_i D^*_{Ai}} dy_i \) and

\[
\hat{D}^*_{Mi} = \frac{p_i D^*_{Ai}}{D^*_{Mi}} s^*_{Mi} \hat{p}_i + \frac{m_{Mi}}{D^*_{Mi}} dy^*_i,
\]

where \( s_{Ai} \) and \( s^*_{Mi} \) are the substitution elasticities of demand of countries \( H \) and \( F \) for goods \( A \) and \( M \), respectively, after state \( i \) is realized, and
\( m_{Ai} \) and \( m_{Mi} (m^{*}_{Ai} \) and \( m^{*}_{Mi} \) are the marginal propensities to consume goods A and M of country H (F), respectively. We assume that all goods are normal, i.e., \( m_{Ai} > 0, m_{Mi} > 0, m^{*}_{Ai} > 0, \) and \( m^{*}_{Mi} > 0, \) \( i = 1, 2. \) Using these two equations together with equations (10), (11), and \( B_i + B^{*}_i = 0, \) we obtain

\[
\hat{p}_1 = -\frac{B^*_1}{p_1 D_{A1\Delta_1}} (m_{Ai} - m^{*}_{Ai}) B_1 - \frac{m_{Mi}}{D_{A1\Delta_1}} dX^*_{A1}
\]

(12)

\[
\hat{p}_2 = \frac{B^*_2 (m_{A2} - m^{*}_{A2})}{p_2 D_{A2\Delta_2}} B_2 - \frac{m^{*}_{M2}}{D_{A2\Delta_2}} dX^*_{A2}
\]

\[
\Delta_i = e_{Ai} + e^{*}_{Mi} - 1,
\]

where \( e_{Ai} = s_{Ai} + m_{Ai} \) and \( e^{*}_{Mi} = s^{*}_{Mi} + m^{*}_{Mi} \) are the elasticities of demand for goods A and M. We assume that the spot market is stable given a futures price; i.e., \( \Delta_i > 0 \) (the Marshall-Lerner condition). By using equations (11) and (12), a change in state i real income is expressed as

\[
dy_1 = -\frac{B^*_1}{\Delta_1} s_1 B^*_1 + \frac{p_1 m_{Mi}}{\Delta_1} dX^*_{A1}
\]

(13)

\[
dy_2 = \frac{B^*_2}{\Delta_2} s_2 B^*_2 + \frac{p_2 m^{*}_{M2}}{\Delta_2} dX^*_{A2},
\]

where \( s_i = s_{Ai} + s^{*}_{Mi} > 0 \) is the total elasticity of substitution in demand.

Equations (12) and (13) show the effects of an increase in agricultural risk on an equilibrium in the spot market after each state is realized. When the variability in agricultural production changes, there are two forces which shift a spot equilibrium. One is the direct effect of a change in output. In each spot market, a change in output affects the spot price and spot real income. The other is the indirect effect of a change in futures trading. An increase
in agricultural risk, affecting futures transactions, changes capital gains
and losses of the countries in each spot market. In a spot market, capital
gains of each country, \( B_2 \) and \( B_1^* (= -B_1) \), play the role of a unilateral transfer
between the countries. This transfer affects a spot equilibrium. In equations
(12) and (13), therefore, the changes in spot prices and spot real incomes are
each decomposed into two parts. The coefficients of \( B_1^* \) and \( B_2 \) represent indi-
direct effects; they are well known terms in the literature on unilateral
transfers. The coefficients of \( dX_{A1}^* \) and \( dX_{A2}^* \) represent direct effects; they
are also well known as terms showing the effects of growth. (See Caves and
Jones (1981, Supplement to Chapter 4.)

(b) Futures market

Let \( r = (p_2 - q)/(q - p_1) \). \( r \) is the capital gain/loss ratio of the
country which buys futures. Condition (5) implies that the marginal rate of
substitution between states 1 and 2 expenditures is equated to \( r \). Thus, \( r \)
may be considered as the (implicit) relative price of states 1 and 2
expenditures. Define \( dy = r dy_1 + dy_2 \) and \( dy^* = r dy_1^* + dy_2^* \). The
optimization of countries in the futures market implies that \( dy \) and \( dy^* \) are
positive (negative) if and only if the expected utilities,
\[ \sum_{i=1}^{2} \phi_i v(Y_i, p_i) \]
and
\[ \sum_{i=1}^{2} \phi_i v^*(Y_i^*, p_i) \]
of countries H and F are increased (decreased) by
a change in parameters. In this sense, we may consider \( y \) and \( y^* \) as the
expected real income of the countries and \( dy \) and \( dy^* \) as their changes. Since
from the definitions of \( B_i \) and \( B_i^* \), \( rB_1 + B_2 = 0 \) and \( r B_1^* + B_2^* = 0 \), the
definition of expected real income together with (11) implies

\[
\begin{align*}
dy &= B_2 \hat{r} - r p_1 D_{A1} \hat{p}_1 - p_2 D_{A2} \hat{p}_2 \\
dy^* &= -r B_1^* \hat{r} - r p_1 (D_{A1}^* - X_{A1}^*) \hat{p}_1 - p_2 (D_{A2}^* - X_{A2}^*) \hat{p}_2 \\
&\quad + r p_1 dX_{A1}^* + p_2 dX_{A2}^* .
\end{align*}
\]
The only increases in agricultural risks we consider are those which keep world expected real income constant; by equation (14), such a change satisfies $dX^*_{A1} > 0$ and $r p_1 dX^*_{A1} + p_2 dX^*_{A2} = 0$. We consider that this change is a pure magnification of an agricultural risk. The two countries' expected utility levels at the initial equilibrium lie on the utility possibility frontier associated with the initial production activities. A change in production pattern shifts the equilibrium expected utility levels. Under our assumption of pure risk magnification, this shift is in the same direction as the line tangent to the initial utility possibility frontier at the equilibrium; in other words, a small change in risk shifts the equilibrium utility levels along the utility possibility frontier. In contrast, a general change in risk may expand or contract the frontier. This suggests that our way of increasing a risk is most appropriate for isolating the effects of an increase in risk from those of a change in world resources. By equations (14), this assumption implies

(15) \[ dy^* = -r B^* \hat{r} - r p_1 (D^*_{A1} - X^*_{A1}) \hat{p}_1 - p_2 (D^*_{A2} - X^*_{A2}) \hat{p}_2. \]

$Y_i = p_i D_{Ai} + D_{Mi}$ and $Y_i^* = p_i D_{Ai}^* + D_{Mi}^*$ may be considered as the demands for state $i$ expenditure. Since $dy = r dy_1 + dy_2$, we have

(16) \[ r dy_1 + dy_2 = dy + r p_1 D_{Ai} \hat{p}_1 + p_2 D_{A2} \hat{p}_2. \]

Note that the relative change in marginal utility of income is expressed as

(17) \[ \hat{v}_{Y_i} = -\rho_i dY_i + (p_i D_{Ai} \rho_i - m_{Ai}) \hat{p}_i, \]

where $\rho_i$ is the degree of absolute risk aversion. In the previous section, we assumed that the spot relative price does not affect the marginal utility of income. Equation (18) shows that this assumption is satisfied if $p_i D_{Ai} \rho = m_{Ai}$. This condition means that the degree of relative risk aversion, $Y_i$,
is equal to the income elasticity of demand for good A. Under this assumption, equation (17) implies

(18) \[ \rho_1 \, dY_1 - \rho_2 \, dY_2 = -\hat{r}, \]

since the first order condition (6) implies \( \hat{v}_{Y_1} - \hat{v}_{Y_2} = \hat{r} \). Equations (16) and (18) imply

(19) \[
\begin{align*}
    dY_1 &= -\frac{1}{\rho} \hat{r} + r \, p_1 \, D_{A1} \hat{p}_1 + p_2 \, D_{A2} \hat{p}_2 + \frac{\rho_2}{\rho} dy \\
    dY_2 &= \frac{r}{\rho} \hat{r} + r \, p_1 \, D_{A1} \hat{p}_1 + p_2 \, D_{A2} \hat{p}_2 + \frac{\rho_1}{\rho} dy
\end{align*}
\]

where \( \rho = \rho_1 + r \rho_2 \).

Similar expressions may be obtained for \( dY_1^* \) and \( dY_2^* \) by adding superscript * to all arguments (except \( r, p_1, \) and \( p_2 \)).

In equations (19) the coefficients of \( dy \) show income effects on expenditure in each state. Since we are concerned with von Neumann-Morgenstern utility functions, the income effects are positive. The coefficients of \( \hat{r}, \hat{p}_1, \) and \( \hat{p}_2 \) show substitution effects. When the implicit price of state 1 expenditure \( r \) increases, state 2 expenditure is substituted for that of state 1; the coefficient of \( \hat{r} \) is negative for \( dY_1 \) and positive for \( dY_2 \).

The interpretation of the coefficients of \( \hat{p}_1 \) and \( \hat{p}_2 \) is somewhat more complicated. Since \( dY_i = dy_i + p_i \, D_{Ai} \hat{p}_i \), equation (17) implies

\[ \hat{v}_{Y_i} = -\rho_i \, dy_i - m_{Ai} \, \hat{p}_i. \]

Thus, if state i real income, \( dy_i \), is compensated, the marginal utility of income is negatively correlated with the state i relative price of good A, \( p_i \). Suppose that state 1 real income, \( dy_1 \), as well as total expected real income, \( dy \), is compensated after \( p_1 \) increases. Then, by the
negative correlation above, the marginal utility of income in the spot market after state 1 is lower than it should be in order to satisfy the first order condition (6). Consequently state 2 real income has to be increased and that of state 1 has to be decreased. Thus, state 2 expenditure is increased, i.e., equations (19) implies \( \frac{\partial Y_2}{\partial p_1} = rD_{A1} \frac{p_1}{\rho} > 0 \). There are two effects on state 1 real income. First, the decrease in state 1 real income reduces state 1 expenditure. Second, the increase in the price of good A in state 1 directly increases the value of state 1 expenditure. Under our assumption that the relative price does not affect \( v_y \), the second effect exceeds the first one. Thus, \( \frac{\partial Y_1}{\partial p_1} = rD_{A1} \frac{p_2}{\rho} \) is positive.

As we have seen in the previous section, there is a strong presumption in favor for trade pattern non-reversals. Thus, for the rest of this study we assume country H buys futures (the other case can be analyzed in a similar way). Then, the definitions of \( B_i \) and \( B_i^* \) and the assumption that \( p_2 > p_1 \) imply that \( B_2 \) and \( B_1^* \) are positive. The budget constraints of condition (1) imply that \( B_2 = Y_2 - X_M \) and \( B_1^* = Y_1^* - p_1 X_{A1}^* \). Note that \( Y_2(Y_1^*) \) is the demand for expenditure of country H (F) after state 2 (1) and that country H's (F's) income from its production sector after the state is \( X_M (p_1 X_{A1}^*) \). Therefore \( B_2 (B_1^*) \) may be thought of as country H's (F's) import demand for state 2 (1) expenditure. Equation (4) implies the following trade balance equation.

\[
(20) \quad r B_1^* = B_2 .
\]

Therefore, we have

\[
(20') \quad r + B_1^* = B_2 .
\]
Using equations (14), (15), (19), the expression for $dY_1^*$ similar to (19), and $\hat{B}_2 = \frac{dY_2}{B_2}$, equation (16) can be expressed as

$$\hat{B}_2 = \epsilon_2 \hat{r}$$

(21)

$$\hat{B}_1 = -\epsilon_1 \hat{r} - \epsilon_A^* \hat{p}_1 + \epsilon_A^* \hat{p}_2 - \frac{p_1}{B_1} dX_{A1}^* .$$

$$\epsilon_2 = (\rho + \frac{r}{B_2})/\rho$$

$$\epsilon_A = (\frac{1}{B_1} + r \rho_2^*)/\rho$$

$$\epsilon_A^* = \frac{p_1 X_{A1}^*}{B_1^*} \frac{\rho_1^*}{\rho}$$

$$\epsilon_A^* = \frac{p_2 X_{A2}^*}{B_1^*} \frac{\rho_2^*}{\rho} .$$

$\epsilon_A^*$ and $\epsilon_A^*$ are the foreign elasticities of demand for capital gains with respect to the spot prices $p_1$ and $p_2$. Under the assumption that $v_y$ is independent of $p_1$, the corresponding domestic elasticities are zero. $\epsilon_2$ and $\epsilon_1^*$ are the elasticities of import for state contingent incomes with respect to the implicit relative price of state contingent incomes. As in the case of trading actual goods, $\epsilon_2$ ($\epsilon_1^*$) is expressed as the sum of the substitution elasticity $r/\rho B_2 (1/\rho B_1^*) > 0$ and the marginal propensity to import state contingent income $\rho_1/\rho$ ($r \rho_2^*/\rho^*$).

(c) rational expectations equilibrium and terms of trade

We assume that the futures market clears instantaneously given the anticipated spot prices of good A for the two states, $p_1$ and $p_2$. The response of the futures price $q$ when the anticipated spot prices change is implicitly captured by a
change in the capital gain/loss ratio \( r = \frac{p_2 - q}{q - p_1} \). This latter change is specified by equations (20') and (21). That is to say,

\[
\hat{r} = \frac{1}{\delta} (- \varepsilon_{A1} \hat{p}_1 + \varepsilon_{A2} \hat{p}_2 - \frac{p_1}{B_1} dX_{A1}^*).
\]

where \( \delta = \varepsilon_2 + \varepsilon_1^* - 1 \). We assume that the futures market is stable given anticipated spot prices of good A, \( p_1 \) and \( p_2 \); i.e., \( \delta > 0 \) (the Marshall-Lerner condition). By equations (21'), and (22), we have

\[
\hat{B}_2 = \frac{\varepsilon_2}{\delta} (- \varepsilon_{A1} \hat{p}_1 + \varepsilon_{A2} \hat{p}_2 - \frac{p_1}{B_1} dX_{A1}^*)
\]

(21')

\[
\hat{B}_1 = \frac{\varepsilon_2 - 1}{\delta} (- \varepsilon_{A1} \hat{p}_1 + \varepsilon_{A2} \hat{p}_2 - \frac{p_1}{B_1} dX_{A1}^*)
\]

Using equations (12) and (21'), we may specify the changes in anticipated spot prices when risk increases. Using the definitions of \( \varepsilon_{A1}^* \) and \( \varepsilon_{A2}^* \), we have

\[
(12')
\begin{bmatrix}
\varepsilon_{A1}^* \Delta_1 \delta - (\varepsilon_2 - 1)(m_{A1} - m_{A1}^*) \\
\varepsilon_2(m_{A2} - m_{A2}^*)
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{A1}^* \hat{p}_1 \\
\varepsilon_{A2}^* \hat{p}_2
\end{bmatrix}
= \begin{bmatrix}
(-\delta') \frac{p_1}{B_1} dX_{A1}^*
\end{bmatrix}
\]

\[
\delta' = (\varepsilon_2 - 1)m_{M1} + \varepsilon_{A1}^* m_{M1}^*
\]

\[
\delta'' = \varepsilon_2 m_{M2} + (\varepsilon_{A2}^* - 1)m_{M2}^*
\]

where \( \zeta_1^* = \frac{D_{A1}^*}{X_{A1}^*} \) and \( \zeta_2^* = \frac{D_{A2}^*}{X_{A2}^*} \) are the ratios of country F's export coefficients, \( D_{A1}^*/X_{A1}^* \) and \( D_{A2}^*/X_{A2}^* \), and its marginal propensities to spend, \( \frac{p_1^*}{\rho^*} \) and \( \frac{r_{p_2}^*}{\rho^*} \), in the two states.
Solving \((12')\) we obtain

\[
\hat{p}_1 = (-\delta' \xi_2^* \Delta_2 + \alpha) \frac{p_1^*}{B_{1e} A_1|A|} d\chi^*_{A_1}
\]

\[
\hat{p}_2 = (\delta'' \xi_1^* \Delta_1 + \alpha) \frac{p_1^*}{B_{1e} A_2|A|} d\chi^*_{A_1}
\]

(23)

\[
\alpha = \delta' \frac{\epsilon_2}{\delta} (m_{A_2} - m_{A_2}^*) - \delta'' \frac{\epsilon_2 - 1}{\delta} (m_{A_1} - m_{A_1}^*),
\]

where \(|A|\) is the determinant of the coefficient matrix of equation \((12')\). Since the futures market clears instantaneously, the two spot prices affect each other. We assume that the spot markets are stable in the overall sense, taking this interaction into account; i.e., \(|A| > 0\).

Equations (23) capture the simultaneous adjustment in the futures and spot markets after risk increases. To understand this adjustment fully, we have to go back to equations (21), (22), and (21'). When risk increases, there are two different changes which feed through to change the anticipated spot prices, \(p_1\) and \(p_2\). First, since the magnified risk is accompanied by an increase in the good harvest, it decreases country F's demand for capital gains after the good harvest (state 1) (see equations (21)). In other words, an excess supply of state 1 income is created. Since the futures market clears instantaneously, this leads to a reduction in the implicit relative price of state 1 income, \(r\), caused by an increase in the price of futures, \(q\) (see equation (22)). This implies a reduction in country H's capital gains and a change in country F's capital gains (see equation (21')). These two changes play the role of an income transfer in each spot market. Thus, depending upon the difference between the two countries' marginal propensities to consume good
A, they create either an excess supply or an excess demand for good A in each spot market. Recall that the anticipated spot prices are still kept at the initial levels. The other effect of a risk magnification is simply a direct change in the supply of good A in each spot market. The sum of these changes in excess demand in each spot market is captured by $\delta'$ and $-\delta''$ above. The stability condition of the futures market, $\delta = \varepsilon_2 + \varepsilon_1^* - 1 > 0$, implies that $\delta' = (\varepsilon_2 - 1) m_{M1} + c_1^* m_{M1}$, and $\delta'' = \varepsilon_2 m_{M2} + (\varepsilon_1^* - 1) m_{M2}$ are likely to be positive. This implies that additional supply and additional demand are likely to be created in the spot market after state 1 and state 2, respectively, before the spot prices change. This is summarized by the right hand side term of equation (12').

In order to absorb the market excess demand changes described above, the spot prices have to change. Effects of changes in spot prices on excess demands for actual goods are somewhat complicated. An increase in a spot price, say, $p_1$ has two types of effects. First, it directly reduces the demand for good A in the state 1 spot market. In order for this direct effect to absorb the excess supply created by the risk magnification, $p_1$ has to fall; $\varepsilon_1^* \delta$ of the coefficient, matrix A of (12') essentially captures this. The other type of effect is an indirect one, working through the impact of changes in capital gains and losses in each state. An increase in $p_1$ reduces country F's demand for capital gains in the state 1 spot market (see equation (21)). This reduces the implicit relative price of state 1 income, $r$, (see equation (22)). For the sake of simplicity, suppose that $\varepsilon_2 - 1$ is positive. Then, this reduction in $r$ decreases both country F's capital gains in the state 1 spot market. This implies that additional transfers are paid by country H in the state 1 spot market and by country F in the state 2 spot market. Depending upon the two countries' taste patterns (or marginal propensities to
consume good A), the transfers create either an excess demand or an excess supply in both state 1 and state 2 markets. These two effects of an increase in $p_1$ are captured by the three terms of the first column of the coefficient matrix $A$ of equation (12'). An increase in $p_2$ also has the similar two types of effect, which are captured by the second column of $A$. The indirect effects of increases in $p_1$ and $p_2$ affect the demands for good A in both spot markets simultaneously. The changes in $p_1$ and $p_2$ required to absorb the excess supply and demand created in the two spot markets by the risk magnification are specified by equations (23). Terms $-\delta' \zeta_2^* \Delta_2$ and $\delta'' \zeta_1^* \Delta_1$ reflect the direct effect of changes in $p_1$ and $p_2$ on demand, and term $\alpha$ reflects the indirect effect through the futures market.

It is important to note that the direct effect of an increase in risk on spot prices is likely to work in the normal direction whereas the indirect effect may not. As equations (23) show, the direct effect works towards raising the spot price of good A after the bad harvest the size of which is reduced by the increase in risk, and towards lowering the spot price of good A after the good harvest the size of which is expanded. It is also important to note that the indirect effect working in an abnormal direction may exceed the direct effect, resulting in a perverse reaction of a spot price. Since the indirect effect works towards changing the two spot prices of good A in the same direction, the two spot prices never react in a perverse way at the same time.

Term $\alpha$, which captures the indirect effect, can be expressed as

$$\alpha = (m_{A1}^* - m_{A1})m_{M2}^* - \varepsilon_2 (m_{M1}m_{M2} - m_{M1}^*m_{M2}^*).$$

This shows that the larger the difference in taste (marginal propensity to consume a good) between the two countries, the larger the indirect effects.
Equation (24) shows that there is an asymmetry between the two states of nature in the indirect effect. This is due to the asymmetry between the responses of $B_2^*$ and $B_1^*$ to a spot price change shown in equations (21).

The relative change in the relative price of state 1 expenditure, $\hat{r}$, is specified by equations (22) and (23); i.e.,

\begin{equation}
\hat{r} = \left( m_{M2}^* \zeta_1^* \Delta_1 + m_{M1}^* \zeta_2^* 2^2 - \zeta_1^* \zeta_2^* \Delta_1 \Delta_2 \right) \frac{p_{11}^*}{B_1^*|A|} \, dX_{A1}^*.
\end{equation}

Note that the indirect effects on spot prices discussed above cancel each other and do not appear in equation (25). The direct effects on spot prices, reducing $p_1$ and raising $p_2$, work towards increasing the foreign import demand for state 1 income (see equations (21)) and, thus, raising the price of state 1 income $r$. This is captured by $m_{M2}^* \zeta_1^* \Delta_1 + m_{M1}^* \zeta_2^* 2^2$ in equation (25). An increase in agricultural output after state 1 reduces the foreign import demand for state 1 income (equation (21)) and in turn $r$. This is shown by $-\zeta_1^* \zeta_2^* \Delta_1 \Delta_2$ in equation (25).

(d) role of futures market in transmitting a production risk

We now consider the effect on the variability of spot prices of introducing a futures market into the pure spot market economy. The spot prices after the bad and good harvests react differently to a given increase in risk depending upon whether the economy has a futures market or not. We will compare these two different reactions of spot prices. This comparison is difficult since given the same structure of tastes and production, the pure spot market economy and the economy with the futures market establish different spot equilibria after each harvest.
In order to circumvent this difficulty we propose the following method of analysis. We observe first that an equilibrium with futures trading is equivalent to one in which the two countries have a suitable (bilateral) transfer agreement and at the same time trade only in the spot market. The transfers must be set exactly equal to the capital gains or losses which would have been incurred had there been a futures market. In these circumstances spot incomes, relative prices and the allocation of commodities will be identical in the two equilibria.

We then introduce a pure magnification of risk, as defined above. In the spot market economy with transfers, we hold the level of transfers constant and examine the resulting impact on spot price variability. We then compare this effect with that which we have already calculated, in which the level of futures trading, and the resulting capital gains and losses are allowed freely to adjust to the change in riskiness of agricultural output. We argue that this comparison enables us to identify the factors which will be important in determining the impact of futures trading on spot price variability.

We consider the ratio of the spot price of good A after the bad harvest and that after the good harvest, \( p_2/p_1 \), as the index of spot price variability. Note that the coefficient of variation, \( \sqrt{\frac{\text{var}(\hat{p})}{\text{E}(\hat{p})}} \) is monotonically related to \( p_2/p_1 \) for \( p_2/p_1 > 1 \).

Suppose that agricultural risk increases keeping expected world real income constant. In the pure spot market economy with the transfer agreement discussed above, \( B_1^* \) and \( B_2 \) amounts of transfers are paid by country H after the good harvest and by country F after the bad harvest, respectively, at the initial equilibrium. These transfers are kept unchanged after the increase in
agricultural risk. Effects of risk magnification on each spot price are shown by equations (12) after setting \( \hat{B}_1^* = \hat{B}_2^* = 0 \). That is to say,

\[
\hat{p}_1 |_{\hat{B}_1^* = 0} = -\frac{m_{M1}}{D_{A1} \Delta_1} \, dX_{A1}^* \quad \text{and} \quad \hat{p}_2 |_{\hat{B}_2^* = 0} = -\frac{rp_1 m_{M2}^*}{p_2 D_{A2} \Delta_2} \, dX_{A1}^*. \]

Therefore, we have

\[
(26) \quad (\hat{p}_2 - \hat{p}_1) |_{\hat{B}_1^* = \hat{B}_2^* = 0} = \left( \frac{rp_1 m_{M2}^*}{p_2 D_{A2} \Delta_2} + \frac{m_{M1}}{D_{A1} \Delta_1} \right) dX_{A1}^*.
\]

This shows that if there is no futures market, an increase in agricultural risk always magnifies the risk incorporated in the random spot price. If there is a futures market, equations (23) specify the spot price variability, i.e.,

\[
(27) \quad \hat{p}_2 - \hat{p}_1 = \left( \frac{\delta^* \xi_{1A1}}{\varepsilon_{A2}} + \frac{\delta^* \xi_{2A2}}{\varepsilon_{A1}} + \alpha \left( \frac{1}{\varepsilon_{A2}} - \frac{1}{\varepsilon_{A1}} \right) \right) \frac{p_1 \xi^*}{B_1 |A|} \, dX_{A1}^*.
\]

This shows that even if there is a futures market, an increase in agricultural risk is more likely to increase the variability of the spot price. It is however possible that spot price variability decreases after an increase in the variability of agricultural production, if term \( \alpha \left( \frac{1}{\varepsilon_{A2}} - \frac{1}{\varepsilon_{A1}} \right) \) is negative. The effect of introducing the futures market into the pure spot market economy (with the transfer agreement described above) can be measured by \( \nu = (\hat{p}_2 - \hat{p}_1) - (\hat{p}_2 - \hat{p}_1) |_{\hat{B}_1^* = \hat{B}_2^* = 0} \).

Since the rest of our analysis is highly complicated in its full generality, we make the following simplifying assumption. That is, some of the characteristics of the demands for a good are similar between the states of nature; more precisely,

\[
m_{Mi} = m_{M}, \quad m_{M}^{*} = m_{Mi}^{*} = m_{M}^{*}, \quad e_{Ai} = e_{A}, \quad e_{Mi}^{*} = e_{Mi}^{*}, \quad B_2/p_2 D_{A2} = B_1/p_1 D_{A1} = \xi \quad \text{and}
\]
\( \hat{\zeta}_i = \frac{D_{Ai}}{X_{Ai}} / \mu_i^* = \zeta^* \), where \( \mu_i^* = \frac{r \rho^*_2}{p^*} \) and \( \nu_2 = 1 - \mu_1^* \). We denote \( m_{Ai} = m_A(=1 - m_M) \) and \( m_{Ai}^* = m_A^*(=1 - m_M^*) \). In the first two equalities, the marginal propensity to consume is independent of states; in the next two equalities, so is the elasticity of import demand; in the next equation, so is the ratio between capital gains and the values of imports in country H; in the last equation, so is the ratio between the export coefficient \( (P_{Ai}/X_{Ai}^*) \) and the marginal propensity to spend in state i (see equation (19)).

This assumption is appropriate for our analysis. The characteristics of demand for a good may vary between the countries and between the states of nature. A polar case is that they differ only between the states of nature. Since \( m_{A1} = m_A^* \) and \( m_{A2} = m_A^* \) in this case, as equations (12) show, the changes in spot prices are independent of whether or not the futures market exists.

The other polar case, which we will consider below, is that the characteristics of demand differ only between the countries. General results of our comparative statics are expected to lie between these polar cases. In international economics, consideration of the difference between countries in such parameters as taste, production, endowments, etc., is of the most importance. If the fluctuation of the amount of agricultural output in country F over the states of nature is not large, all the variables which we assume to be independent of states do not vary much across the states, and the comparative statics results are therefore mainly governed by the taste difference between the countries. All these considerations justify our assumption.

By equations (20') and (21), we have \( \hat{\beta}_2 = \varepsilon_2 \hat{r} \) and \( \hat{\beta}_1^* = (\varepsilon_2 - 1)\hat{r} \). Therefore, under our state independence assumption, equation (12) implies the extra change in spot price variability after introducing the futures market, \( \nu \), may be expressed as follows.
(28) \[ v = \frac{\delta}{\Delta} (m_A^* - m_A^*) (2\varepsilon_2 - 1) \hat{r}, \]

where \( \Delta = \Delta_1 \). (\( \Delta_1 \) is independent of states under the assumption above.) This expression is important since it captures the basic role of the futures market. The effect of introducing the futures market on spot price variability is a result of the reallocation of purchasing power in each state between the two countries which is due to changes in capital gains and losses. This reallocation works as a transfer in each state; this is captured by term \((m_A^* - m_A^*)\), which is known to play the critical role in determining the effect of a transfer on the terms of trade in the deterministic trade model (see Samuelson (1952) and Jones (1970)). The volume of a change in the transfer of each state is determined by the elasticities, \( \varepsilon_2 \) and \( \varepsilon_2 - 1 \), of import demand for state contingent incomes. This is captured by term \((2\varepsilon_2 - 1)\). (Note that our assumption of price independent marginal utility of income works in a crucial way to simplify our results.)\(^{22}\) Finally, the direction of a change in the transfer, or in the capital gain and loss, is determined by the change in the capital gain loss ratio, \( \hat{r} \).

We consider that it is more likely that \( 2 \varepsilon_2 > 1 \). This is because the stability of the futures market requires \( \delta = \varepsilon_2 + \varepsilon_1^* - 1 > 0 \), as is seen above. If, for example, the elasticities of import demand for state contingent income, \( \varepsilon_2 \) and \( \varepsilon_1^* \), are similar between the two countries, or if the elasticity is higher in country H, it holds that \( 2 \varepsilon_2 > 1 \). Since \( \hat{B}_2 = \varepsilon_2 \hat{r} \), an increase in the relative price of state 1 expenditure, \( r \), increases country H's import demand for state 2 expenditure. Thus, according to equations (12), an additional transfer is paid in state 2 by country F to country H. Since \( \hat{B}_1^* = (\varepsilon_2 - 1) \hat{r} \), the movement of a transfer in state 1 is ambiguous. However, the
assumption that $2 \varepsilon_2 > 1$ implies that the impact on price variability is dominated by the movement of a transfer in state 2. This relationship is captured by term $(2 \varepsilon_2 - 1)\hat{r}$ in equation (28). The final impact on price variability, $\nu$, is determined by this transfer movement together with the sign of $(m_A - m_A^*)$.

As is seen above, the direction of a change in the relative price of state contingent expenditures is important. Under our state independence assumption, equations (25) and (28) imply

\begin{equation}
\nu = -(2 \varepsilon_2 - 1) (m_A - m_A^*) (\xi^* \Delta - 2m_M^*) \frac{\xi^* p_1}{B_1 A} \Delta x_{A_1}^*
\end{equation}

Since $p_2 > p_1$, if $\nu$ is negative (positive), the introduction of the futures market into an economy only with spot markets decreases (increases) spot price variability in the economy.

(e) change in expected real income

Since $\hat{B}_2 = \varepsilon_2 \hat{r}$ and $\hat{B}_1^* = (\varepsilon_2 - 1)\hat{r}$ by equations (22) and (21'), and since $dy = r dy_1 + dy_2$, the effect of an increase in risk (keeping world expected
real income constant) on country H's real income is given by equations (13) as follows.

\[(29) \quad dy = r B_1^* \left( - (e_2 - 1) \frac{s_1}{\Delta_1} + e_2 \frac{s_2}{2 \Delta_2} \right) \hat{r} \]

\[+ r \left[ m_{11}^* \frac{m_{11}}{\Delta_1} - m_{22}^* \frac{m_{12}}{\Delta_2} \right] dX_{A1}^*. \]

Since we consider an increase in agricultural risk keeping world expected real income constant, the changes in real income for each country are equal in absolute value and of opposite sign. The change in expected real income in the pure spot market economy (with the transfer agreement discussed above) is shown by the second term of the right hand side of equation (29), since \( B_1^* = B_2^* = 0 \) in this case. Thus, whether or not the introduction of the futures market increases a country's expected real income depends solely upon the first term of (29).

Since the general expression of \( dy \) with respect to \( dX_{A1}^* \) is fairly complicated, we keep our state independence assumption. Then, since the second term of the right hand side of equation (29) is zero, the effect of an increase in agricultural risk on expected real income produced by opening a futures market is

\[(30) \quad dy = - (s_A^* + s_M^*) (\zeta^* \Delta - 2 m_M^*) \frac{r p_1 \zeta^*}{|A|} dX_{A1}^*. \]
Footnotes

1. Dumas (1980), Anderson (1981), and Grinols (1983) are mainly concerned with the structure of production. Also, see an extensive survey by Pomeroy (1982).

1'. Few exceptions are Cass and Shell (1983) and Fries (1983).


Recently, some theorists consider futures markets from the viewpoint of simple general equilibrium models; see Hirshleifer (1975), Feiger (1976), Salant (1976), and Anderson and Danthine (1983a). Whereas Hirshleifer, Feiger, and Salant consider transactions of goods in futures markets, Anderson and Danthine as well as this study consider futures contracts which are closer to those used in the actual institutions. We consider a model simpler than that of Anderson and Danthine, and study issues which are not treated by them.

number of studies which consider futures markets in a general equilibrium framework. However, they are mainly concerned with the existence and efficiency of an equilibrium in highly abstract structures, whereas we are interested in considering the nature of an equilibrium in a simpler model.

3. This observation has been made in terms of price elasticities of demand in the literature; Marshall (1920, p. 106) considers that the demand for agricultural goods is inelastic; Samuelson (1970, p. 392) states, "Because the demand for farm products is generally inelastic, limiting total output will actually raise the total revenues received by farmers..." Moreover, many empirical studies indicate that agricultural income falls after a good harvest, that the elasticity of demand for an agricultural good is less than one; according to Newbery and Stiglitz (1981, p. 293), for example, the price elasticities of such major agricultural products as cocoa, coffee, cotton, jute, rubber, and sugar are all between 0.4 and 0.8.

The expression "good harvest poverty" is a direct translation of a Japanese word "hosakubinbou" which means poverty caused by a good harvest.

4. Keynes (1930, p. 142) considers that producers who face risks in the price of their output sell futures. Kaldor (1939, p. 6) states, "...the 'hedgers' are forward sellers... This is probably true in the majority of markets." Moreover, Goss and Yamey (1976, p. 27) argue that short hedging is "characteristic of most futures markets at most times."

5. Suppose that \( p_2 > p_1 \) at an equilibrium. Then, \( p_2 > q > p_1 \) has to hold. Thus, the optimization problems of each country in conditions (1) and (3) may be rewritten

\[
\begin{align*}
\text{(f-1) } & \max \sum_i \phi_i u(D_{Ai}, D_{Mi}) \\
\text{s.t. } & \sum_j y_{ji} d_{ji} = y_{M1} x_m \\
\text{(f-2) } & \max \sum_i \phi_i u^*(D_{Ai}^*, D_{Mi}^*) \\
\text{s.t. } & \sum_j y_{ji}^* d_{ji} = y_{M1}^* x_m^* \\
\end{align*}
\]

where \( y_{M1} = 1/(q-p_1) \), \( y_{M2} = 1/(p_2-q) \), \( y_{A1} = p_1/(q-p_1) \), and \( y_{A2} = p_2/(p_2-q) \).
Thus, if \( y_{ji} \) is interpreted as the price of good \( j \) contingent upon state \( i \), condition (f-1) and (2) imply that a rational expectations equilibrium satisfying conditions (1) - (4) is also an equilibrium in a state contingent commodity market system. Thus, our futures-spot system is complete. This is similar to the result of Arrow (1963), who considers a security-spot market system different from ours.

Our result may be extended into a \( m \)-good, \( n \)-state model; if the number of goods traded in futures markets is equal to \( (n-1) \), and if the patterns of capital gains and losses for futures contracts are linearly independent, our futures-spot market system is complete.

6. In the assumption of a single period, it is also implicit that the manufactured good is non-storable. This is purely for the sake of simplicity; since we choose to assume production in the manufacturing sector to be independent of the state of nature and consider a rational expectations equilibrium, consideration of a storage decision would add little to our analysis.

7. It is reported, for example, that in December 1965 Ghana sold forward almost the whole of its anticipated 1965-66 main crop of cocoa and Nigeria the greater part (see Barclays Bank (1966, p. 192)).

8. Hart has a diagram which looks like ours. However, in the diagram he merely considers an economy with spot markets.

8'. It is easy to produce an example where someone else becomes worse off instead of country \( F \).

9. The assumption that \( X^*_{A2} < X^*_{A1} \) does not necessarily imply that \( p_2 < p_1 \). The possible existence of multiple equilibria makes the task of establishing general conditions under which this would hold problematic.

10. See Caves and Jones (1981, Chs. 3 and 4), for example. Also, note that \( \bar{q} \) and \( \bar{q}^* \) are not autarky equilibria in our model. If the countries can not trade futures, the spot prices \( p_1 \) and \( p_2 \) have to adjust to a new equilibrium.

11. It is known that the so-called Stone-Geary utility function generates this type of indirect utility function.

12. See footnote 3.

13. See, for example, Yano (1984) for such a result.
14. See Kaldor (1939), for example.

15. See footnote 4.

15'. This is similar to the result of Johnson (1976), who shows that Friedman's observation that destabilizing speculators must lose money is in general not true in a general equilibrium framework (see Friedman (1953)).

15". If the marginal utility of income does not depend upon spot prices, the agricultural country (a hedger) under-hedges, or it does not offset income risks completely. If it does, the country may over-hedge (more than offset income risks), or it may even become a speculator.

16. The notation "\(^-\)" on an argument means the relative change in the argument, i.e., \(x = dx/x\).

17. This definition is often used in trade literature. See Caves and Jones (1981, equation 3, S. 3).

18. More precisely, for \(D_{Ai} = D_{Ai}(p_i, y_i)\) and \(D_{Mi}^* = D_{Mi}^*(p_i, y_i^*)\), \(S_{Ai} = -\frac{p_i}{\partial D_{Ai}/\partial p_i}\), \(S_{Mi}^* = \frac{p_i}{\partial D_{Mi}^*/\partial p_i}\), \(m_{Ai} = \frac{3}{\partial D_{Ai}/\partial y_i} y_i\), and \(m_{Mi}^* = \frac{3}{\partial D_{Mi}^*/\partial y_i} y_i^*\), where \(D_{Ai}(p_i, y_i)\) and \(D_{Mi}^*(p_i, y_i^*)\) are compensated demand functions.

19. This can be seen in the following way. \(du_i = u_{Ai}dD_{Ai} + u_{Mi}dD_{Mi} = v_{Y_i}(p_dD_{Ai} + dD_{Mi}) = v_{Y_i}dY_i\) where \(u_{Gi} = \frac{3}{\partial D_{Gi}/\partial y_i} y_i\) for \(G = A, M\).

\((v_{Y_i}\) is defined above). By using this relationship, \(d(\sum \phi_i u_i) = \sum \phi_i du_i = \sum \phi_i v_{Y_i}dY_i = \phi_1 v_{Y_1}dy_1 + dy_2 = \phi_2 v_{Y_2}(r dy_1 + dy_2) = \phi_2 v_{Y_2}dy_2\),

where the fourth equality follows from the first order condition (5). Since \(\phi_2 v_{Y_2}\) is positive, the above statement follows.

20. Note that our equilibrium is Pareto efficient, as is discussed in Section I.

21. This may be seen in the following way. \(\hat{v}_i = v_{Y_i} + \frac{p_i V_{Y_i} p_i}{\hat{v}_{Y_i}}\), where \(v_{Y_i} = \frac{3}{\partial Y_i} v_Y(Y_i, p_i)\) and \(v_{Y_i} p_i = \frac{3}{\partial p_i} v_Y(Y_i, p_i)\). Since \(du_i = v_{Y_i}dY_i = v_{Y_i}(dy_i - D_{Ai} dp_i)\) (see footnote 19),

\(v_{pi} = \frac{3}{\partial p_i} v(Y_i, p_i) = -D_{Ai} v_{Y_i}\). Thus, \(p_i V_{Y_i} p_i = -v_{Y_i} m_{Ai} - p_i D_{Ai} r_{Yi}.\) This together with the first equation above implies equation (17).
22 Without this assumption, $\hat{B}_2$ depends upon $\hat{p}_1$ and $\hat{p}_2$ as well (see equation (19)). This will complicate our results.

23 $s_{A_i}^*$ and $s_{M_i}^*$ are independent of states under our state independence assumption, and denoted as $s_A^*$ and $s_M^*$, respectively.

24 The term in \( \{ \) is unambiguously positive if $m_A^* - m_A^* > 0$. Even if $m_A - m_A^* < 0$, if $S_A + S_M$ is small, $\Delta = S_A + S_M^* + (m_A^* - m_A^*) > 0$ is also small. This implies that the first term in \( \{ \) is ignorable.

25 The Jones presumption plays important roles in various studies of the transfer problem. (See Jones (1975), Yano (1983).)
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Figure 1
Figure 2
1981


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