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Overconfidence in Money Management: Balancing the Benefits and Costs

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Abstract

Individuals are overconfident, especially those in positions to influence outcomes. The impact of hiring an overconfident portfolio manager is studied here within the standard principal-agent framework. When compensation is endogenously determined, we find that investors can benefit from managerial overconfidence. Overconfidence induces a higher level of effort until the effects of restrictions on portfolio formation take over. Further, by increasing the incentive fee and sharing more risk the investor can curb excessive risk taking. However, excessive overconfidence is detrimental to the investor. We empirically test and confirm the effects of portfolio constraints and incentive fee on manager's self-attribution bias.

JEL classification: G23, M54

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Introduction

In modern financial markets, the role of a financial institution has increased a great deal. Looking at the 13F filings, Lewellen (2011) concludes that by the end of 2007 institutional investors held about 68%, up from 32% in 1980, of the overall market value of US common equity. Yet, in his presidential address Franklin Allen (see Allen (2001)) argues that there is very little discussion regarding the agency problems created in these institutions. Here, we analyze one such agency issue and its implications.

There is overwhelming evidence that points to the existence of behavioral biases and irrationality in investor's investment decisions (see Shleifer (2003), Barberis and Thaler (2003), and Subrahmanyam (2008) to survey the recent findings). One of the most well identified and widely studied behavioral biases is investor overconfidence (see Odean (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Barber and Odean (2001), and Gervais and Odean (2001)). Although well documented, most empirical studies and theoretical models limit their focus to overconfidence in individual investors who are not necessarily the marginal investor. In this paper we overcome this limitation and analyze the role of such biases in financial institutions where managerial compensation and investment constraints can address some of the agency concerns.

It is quite common for financial institutions to delegate the decision of portfolio formation to professional managers. Delegation is optimal because managers possess superior skills that allow them to collect and process information regarding the movement of security prices. The problem faced by an investment company in hiring an overconfident manager is studied here.[1] Bhattacharya and Pfleiderer (1985), in their seminal paper, consider the problem of delegation in portfolio management. They propose a compensation contract that screens agents based on their privately known ability. The above contract also elicits truthful revelation of their private signal from the manager. The economically relevant question in the current paper is whether screening overconfident manager is in the best interest of the investor or can hiring such a manager improve investor's welfare?

We study the above question within the standard principal-agent framework. A risk averse principal, who is aware of the manager's biases, sets the contract

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[1] In what follows the manager who makes the portfolio decision is also called the agent. The principal who hires the manager is often referred to as the investor.

parameters and offers this to the agent. If the contract is feasible to the risk averse manager, he accepts it and exerts effort. The manager then observes a signal and updates his beliefs about the distribution of the future states of world. An overconfident manager updates his beliefs in a biased way and wrongfully estimates the precision of the noise in his signal and hence also the precision of the ex-post distribution of the risky asset's returns. An overconfident manager, when compared to a rational manager, also believes that the mean return of the risky asset is much higher i.e., in the direction of the signal. These two effects closely reflect the notion of *overestimation* and *overprecision* discussed in Moore and Healy (2008). Based on his beliefs, the manager makes a decision about the riskiness of the portfolio. His decision is clearly going to be different from that of a rational person. We derive the comparative statics of hiring an overconfident manager within this framework.[2] In this article, we do not solve for the optimal contract conditional on hiring an overconfident manager. Instead, we evaluate the decision of hiring an overconfident manager within the standard compensation structure used in the industry.[3] Although the nature of the contract is assumed, the contract parameters are still determined endogenously.

In order to highlight the different effects of overconfidence, we study the problem in two distinct scenarios. First, we solve the problem in the case where there are no constraints on the portfolio holdings of the manager. Also, in order to isolate the effects of differences in risk preferences of the manager and the investor, we begin by assuming that they have identical utility functions including their risk aversion levels. In this case ("*unconstrained scenario*"), we find that the investor is always better off hiring an overconfident manager. Since an overconfident manager overestimates the extent to which his actions influence the final outcome, for any given level of compensation, he is always going to exert a higher level of effort. In spirit, this set up is very similar to that of Stoughton (1993). Similar to his findings, the optimal effort here is not a function of the incentives provided in the contract. Although a perverse result, the unconstrained scenario captures the essence of the gains from employing an overconfident manager as it eliminates all other effects. The principal gains from higher managerial effort. Moreover, the principal is able to use the incentive

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[2] The standard problem of moral hazard exists here as the investor cannot observe the manager's effort level. Therefore, the compensation contract cannot be dependent on the level of effort.

[3] Although most of the paper uses the case of linear performance incentive, analysis for convex structure is also presented.

parameter in the contract to choose the exact quantity of risky assets that he desires. Therefore, there is no sub-optimal risk taken on account of hiring an overconfident manager. Overall, the investor's expected utility is higher from hiring an overconfident manager than from hiring a rational one.

Second, we introduce an additional exogenous constraint to the manager's portfolio problem ("*constrained scenario*"). It is often the case that portfolio managers face restrictions on their portfolio choices. Almazan, Brown, Carlson and Chapman (2004, Table 1) extensively document the different kinds of restrictions and the percentage of funds that face these restrictions. These restrictions include constraints on short-selling, buying on margin, and on borrowing. A non trivial 91% of the funds face constraints relating to buying on margin and about 69% of the funds do not allow short selling. It is important to study this constrained problem as it has serious ramifications for effort choice and for overall expected utility. Dybvig, Farnsworth and Carpenter (2010) also stress on the importance of trade restrictions to the optimal contract design problem.

In the constrained case, the optimal effort level is an increasing function of the performance adjustment component. Earlier, in the first best case, the manager was able to undo the effects of incentives by changing his portfolio decision. That is why the incentives provided by investor did not matter for the manager's equilibrium effort choice (see Stoughton (1993) and Admati and Pfleiderer (1997)). However, the presence of portfolio constraints restricts the choice of the portfolio manager and thereby limits his ability to reverse the effects of incentives (see Gómez and Sharma (2006) for further explanation). Importantly, the equilibrium effort is not strictly an increasing function of the manager's overconfidence anymore. There are opposing forces at play here. First, given his perceived marginal benefit of effort, the manager is always going to put in more effort because according to his beliefs his signal's precision becomes sharper and hence also increases the expected utility. Second, the opposing effect comes from observing that the constraint on portfolio holdings is more binding on an overconfident manager than on a rational one. When there are no restrictions on the portfolio holdings, for any given level of effort and a given signal, an overconfident manager always demands a higher absolute quantity of risky assets. However, when constraints on portfolio formation are imposed, the set of signals for which the manager can demand his utility maximizing quantity shrinks. Hence he is at the boundary of allowed quantity more often, when compared to a rational manager. The manager does not derive any benefit from

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his additional effort and therefore is bound to reduce the optimal effort. As a result of these two effects the equilibrium effort increases in overconfidence until a point after which it decreases.

Since an overconfident manager perceives, in a biased way, the marginal benefit of his effort to be higher he is also likely to demand a higher reservation wealth in expectation. Even though the reservation utility of the managers are exogenously specified, the model allows for the required wealth to employ a manager to be increasing in his level of overconfidence. Overall, it is still beneficial for an investor to hire an overconfident manager, but only until a point. Beyond this level of overconfidence the investor's expected utility diminishes.

We also empirically test some the implications of our model. Choi and Lou (2010) already show empirical evidence of self-attribution bias, and hence overconfidence, among fund managers. Using the data of over 50,000 fund-quarter observations we first replicate their findings and show that managers adjust their portfolio holdings and deviate from their benchmarks differently to news confirming their private signal than to the one that discredits it. Given the role of investment constraints in our model, we follow Almazan et al. (2004) and create a composite index that quantifies the severity of constraints that managers face. In a multiple regression setting, we then show that funds having these constraints significantly mitigate the affect of the above discussed attribution bias. Obviously, this is after controlling for the direct affect of constraints on the deviation from portfolio benchmark. This evidence is in line with our model that shows that portfolio constraints are more binding on an overconfident manager and hence moderates his actions. In a separate specification we also test for the affect of managerial ownership in the fund, our measure of variable incentive parameter, on the above mentioned bias.[4] As overconfidence increases the principal has to increase the variable component of the contract in order to increase the variance in the agent's payoff and hence moderate his portfolio decisions. Consistent with this prior, we empirically find diminished evidence of attribution bias in funds having a higher percentage of managerial ownership.

The main contribution of the current paper is towards *understanding the implications of agent's overconfidence on the contract parameters and on the hiring decisions in a delegated portfolio management setting.* Empirical evidence of attribution bias clearly shows the presence of overconfident portfolio

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[4] Portfolio manager compensation contracts are private contracts and hence the contract details are not observable.

managers. The obvious question then is why are such managers not screened. It could either mean that screening mechanisms do not work or that there might to benefits to hiring a moderately overconfident manager in a contracting environment. Using a standard principal-agent model we present two potentially conflicting effects of overconfidence and find that, from an investor's perspective, there are gains to hiring an overconfident portfolio manager. Our empirical results substantiate our model and and show the mechanisms that are used to mitigate some of the agency problems in financial institutions and also explains why in equilibrium overconfident portfolio managers continue to exist.

The remainder of the paper is organized as follows. We begin by discussing with relevant literature in section 1. Section 2 presents the details of the model, the assumptions that have been made, and the solutions. Implications of the two scenarios are also discussed here. The proofs pertaining to the claims made are provided in the Appendix. Section 3 provides the details of our data and our empirical analysis. Finally, 4 has the concluding remarks.

1. Related literature

There is abundant evidence in psychology literature that individuals in different professions including clinical psychology, medicine, investment banking, entrepreneurship, and law exhibit overconfidence in their abilities and overestimate the precision of their knowledge[5]. According to Moore and Healy (2008) the manifestation of overconfidence happens in three distinct ways. First, *overestimation,* where the manager overestimates his ability. Second, *overplacement,* where the manager believes himself to be better than others. The emphasis here is on relativity. Third, *overprecision,* has to do with excessive certainty regarding the accuracy of the belief. One of the main factors for such bias is the illusion of control that managers have (see Weinstein (1980)). In other words, of the outcomes that managers can influence, they perceive that the level of their influence is higher than what is true in reality. In the current paper, most of these facts have been taken into account while modeling overconfidence.

Several empirical studies have also identified the existence of overconfidence and have highlighted its implications. Most of these studies focus on individual investors. Barber and Odean (2001) use differences in gender as a proxy for the extent of overconfidence and report that men, who have shown to be more

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[5]Odean (1998) provides an overview of overconfidence literature.

overconfident in areas such as finance, trade more often and also earn a lower net return than women. Barber and Odean (2002) find that once traders move from a traditional phone based system to a modern online trading system they trade more actively, more speculatively and earn a lower return. They attribute this to investor overconfidence. Grinblatt and Keloharju (2009) corroborate these results using personal characteristics and trade level data on individuals from Finland.

The literature pertaining to the problem of contracting in delegated portfolio management is also rich and relevant to the current work. Bhattacharya and Pfleiderer (1985) present a model to screen the managers by their ability. Since the focus of the current paper is to understand the effects of overconfidence, manager's ability is assumed to be common knowledge through out the paper. Heinkel and Stoughton (1994) study a dynamic model of portfolio management contracts. They present a model of adverse selection and moral hazard in which the hiring client does not know the quality or the skill of the manager being hired and they also cannot observe the effort exerted by the manager. In these circumstances they derive a contract which partially reveals the type of the manager initially but complete revelation and contract renegotiation happens only after subsequent performance evaluation. Although Heinkel and Stoughton (1994) have a tractable model, their results crucially depend on the simplifying assumption regarding the risk neutrality of the all the agents. More recently, Dybvig, Farnsworth and Carpenter (2010) derive that when the markets are complete and when there are trade restrictions in place a simple linear contract with benchmarking emerges as the optimal contract. This contract structure turns out to be the optimal in all cases except when the manager observes extreme signals. In such cases, additional incentives must be given to the manager in order to ensure that he does not undo the leverage effect of benchmarking by incorrectly reporting the observed signal. The work of Palomino and Sadrieh (2011) is probably the closest and most related to the results presented in this paper. Although they also solve a model of moral hazard where the portfolio manager is overconfident, the focus of their paper is to solve for the optimal contract. They design a contract in which the manager truthfully reveals his signal or in other words trades the quantity that is exactly desired by the principal. There are two main shortcomings of their paper. First, there are no trade restrictions implicit in their model (see Haugen and Taylor (1987),Gómez and Sharma (2006), and Dybvig et al. (2010) for the importance of having these restrictions in the contract). Second, the truth telling contract they propose are

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not commonly found in the mutual fund industry (see Ma, Tang and Gómez (2016)).

Although the current paper focuses on delegated portfolio management, the idea of an overconfident manager and a rational investor can be extended to an overconfident CEO representing the shareholders of the firm. In the corporate finance literature, the role of an overconfident CEO has been studied in many ways. Galasso and Simcoe (2011) focus on the influence of overconfidence on firm innovation. Malmendier, Tate and Yan (2011) study the role of overconfidence in the financing decisions of the firm while Billett and Qian (2008) and Malmendier and Tate (2008) do the same for acquisition decisions. Goel and Thakor (2008) analyze the impact of overconfidence in a corporate governance setting where there is a tournament for CEO selection. Amidst these findings, the direction that is most pertinent to the subject matter discussed in this paper would be the effect of overconfidence on CEO's investment decisions. Note, there exist a few distinctions between the delegated portfolio management and the corporate finance setting. When a CEO encounters a negative NPV project he cannot "short" the investment project. Also, often, the decision that the CEO makes is whether to accept or reject a project. There is no continuum of risk levels to choose from, as in the case of a portfolio manager.

Following Heaton (2002), Malmendier and Tate (2005a) present a model of an overconfident CEO making an investment decision[6]. The objective of their paper is to explain the widely observed investment-cash flow sensitivity. Manager's overconfidence is attributed as the main reason for the above phenomenon. An overconfident manager thinks that the mean return of the project is higher than what is rationally expected and so is bound to over-invest. However, he is also reluctant to raise capital from outside sources because he perceives the value of the company as undervalued by the market. Two distinct outcomes are realized from this set up. If the firm has sufficient internal funds then over-investment takes place. If the firm does not have sufficient internal funds then even the projects having a positive NPV are not undertaken. Overall, in the above model, CEO overconfidence leads to a sub-optimal outcome for the shareholders. The results of Malmendier and Tate (2005a) are in stark contrast to the results presented in this paper where hiring an overconfident manager

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[6] In addition to the model, the paper also provides an unique method to evaluate CEO overconfidence. Empirical results are also presented to support their claims. Malmendier and Tate (2005b) provides further empirical results in support.

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increases the investor's expected utility. Unlike Malmendier and Tate (2005a), Gervais, Heaton and Odean (2011) take the managerial compensation also into consideration while evaluating the CEO's investment decisions in the presence of overconfidence. They solve for the optimal contract and report the implications of hiring an overconfident manager. Gervais et al. (2011) find that the value of the firm is strictly increasing in CEO overconfidence (see Gervais et al. (2011, Proposition 2)). As overconfidence increases it "reduces" the affect of risk aversion and therefore high powered incentives can be given to engage the CEO in the contract. Further, when outside labor market is included in the model, they find that the overconfident managers also get a share of some of these benefits. The results of Gervais et al. (2011) are along the lines of the non-monotonic gains to overconfidence presented in this paper. The difference is that the focus of the current paper is on delegated portfolio management issues and it deals with the consequences of moral hazard and constraints on portfolio formation faced by the managers. Another important advantage of the the model presented here is that managerial effort is endogenously chosen as opposed to Gervais et al. (2011) where the manager's skill is exogenously specified. Overall, the channels through which the implications of overconfidence are presented here are vastly different.

2. Model

The model captures the contracting problem between an investor and a portfolio manager. In the interest of understanding the effects of overconfidence no additional layer of agency is modeled here.We abstract away from all other agency problems by assuming the investment adviser, the board of directors of the fund, and the individual investors as one unit. The model presented in this paper has borrowed a great deal from the one described in Gómez and Sharma (2006) and in spirit uses the same technology as in Ross (1973).

2.1. Problem description and preferences

The investor (principal) and the manager (agent) are both risk averse. They are assumed to have a negative exponential utility function where a and b are the absolute risk aversion coefficient of the manager and investor, respectively. Both a and b are non-negative real numbers. The contracting problem begins with the investor seeking to hire a manager who is to employ his skills and extract private signals about the future market prices. The investor strategically chooses the

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contract parameters. In this article we do not solve for the shape of the optimal contract. Instead, we take the contract form, commonly found in the mutual fund industry, as given and study the choice of contract parameters and evaluate the implications of the hiring decision.[7] The fees have two components a fixed flat fee, F and a performance adjustment fee which is governed by a parameter α. The investor has \$1 to begin with and requires the portfolio manager to invest this sum.

The manager has two assets to choose from. He has the option of investing in a risky asset which yields the net return of $\tilde{x}$ or investing in the risky free asset. The performance adjustment fee is paid when the returns are in excess of a benchmark. The performance fee is assumed to be benchmarked against the risk free bond, the net return of which is normalized to be zero. Once the contract parameters are offered to the manager, the manager decides to accept or reject the contract based on whether his unconditional expected utility meets the reservation utility. The game ends if the manager refuses to accept the contract. Competition in the managerial labor market is not explicitly modeled here. But, the reader could think that the reservation utility represents the utility from the equilibrium compensation. If the contract is accepted then the manager strategically chooses a level of effort, e, to be exerted. The effort expended allows the manager to observe a random signal , $\tilde{y}$, which is correlated with the future states of the world and hence the returns on the risky asset. After observing the signal the manager picks the level of risky assets, $\theta(y)$, in his portfolio. All the above decisions are made at the beginning of the period. After the portfolio is formed, the payoffs are realized at the end of the period. At this point the contract is settled. It is further assumed that there is no renegotiation that happens between the investor and the manager at any intermediate point.

Since both the investor and the manager are risk averse they maximize the expected utility of their respective terminal wealth. The terminal wealth of the manager depends on the level of risky assets in his portfolio. The manager is going to get a fixed compensation F, and also a share, α, in the difference

between the fund's value, $(1+\theta\tilde{x})$, and the \$1 invested in the risk free rate. The terminal wealth of the manager is given by

$$\tilde{W}_M(y) = F + \alpha\theta\tilde{x}. \tag{1}$$

Moral hazard in the model is motivated by the fact that unobservable effort is costly and is a source of disutility to the manager. The cost function, $V(a, e)$, is a convex increasing function in effort. Following cost function is assumed

$$V(a, e) = ae^2. \tag{2}$$

It is standard in this literature to assume a quadratic cost function as it is continuous, increasing, and is twice differentiable. The terminal wealth of the principal is the value of the portfolio at the end, net of the compensation to the manager. It should equal to $(1+\theta\tilde{x}) - F - \alpha\theta\tilde{x}$. Ignoring the initial capital, as it does not affect the maximization problem, following is the terminal wealth of the investor

$$\tilde{W}_I(y) = (1-\alpha)\theta\tilde{x} - F. \tag{3}$$

2.2. Rational and overconfident manager

The prior distribution of the net returns on the risky asset, $\tilde{x}$, is common knowledge and follows a standard normal distribution. The signal, $\tilde{y}$, is assumed to be a noisy indication of the future returns and is given as

$$\tilde{y} = \tilde{x} + \tilde{\xi} \tag{4}$$

where $\tilde{\xi}$ is the noise term. Obviously, the higher the noise in the signal the less precise it is about the future returns. It is further assumed that higher levels of effort helps in reducing the noise in the signal. In other words, the variance of the noise term is decreasing in the level of effort i.e., $\tilde{\xi} \sim N\left(0, \frac{1}{e}\right)$. Stoughton (1993) also shares a similar modeling assumption.[8] Based on these assumptions, for any chosen level of effort, the distribution of signal is $\tilde{y} \sim N\left(0, \frac{1+e}{e}\right)$. Note, the precision of the signal is increasing in manager's effort. In this model the the

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[8] It is probably fair to assume that the productivity of effort depends on skill of the manager. However, in order to make the larger point of the paper the skill level has been assumed to be cross-sectionally the same.

manager is assumed to be Bayesian. So, after observing the signal the manager updates his beliefs about the distribution of the risky asset's return[9] to

$$\tilde{x}|y \sim N\left(\frac{e}{1+e}y, \frac{1}{1+e}\right). \tag{5}$$

The above beliefs are that of a rational manager. An overconfident manager is going to believe that the marginal productivity of his effort is higher than what it truly is. An overconfident manager believes that for any level of effort that he chooses, following is the distribution of the noise in his signal

$$\tilde{\xi_\psi} \sim N(0, \frac{1}{\psi e}) \tag{6}$$

where $\psi \geq 1$ is the level of overconfidence. A higher ψ implies that the agent is more overconfident. In the case when $\psi = 1$ we are back to the rational world. An overconfident manager assumes that his effort reduces the variance in the noise term much more than a rational manager does.

One of the possible criticism of the above set up is that overconfidence is exogenously specified. To mitigate this concern, the reader should think of this game as one of the many periods in a multi-period game where nobody, including the manager himself, knows the true ability of the manager. They update their beliefs about his ability after every round of trading. The overconfident manager updates his belief in a biased way where undue amount of credit is taken by him in instances of success but proportional responsibility is not taken for failure. The investor, however, rationally updates his beliefs about the manager. Gervais and Odean (2001) show that this mechanism, often referred to as the self attribution bias, endogenously leads to overconfidence. Therefore, the model presented here is just the nested version of the above described framework. This abstraction is helpful as we use a simple model to present important effects of managerial overconfidence. What really matters for the analysis is that there is heterogeneity in beliefs; the source of it is less relevant. Given the above beliefs the overconfident manager is going to have the following as the conditional distribution for asset return

$$\tilde{x}|y \sim N\left(\frac{e\psi}{1+e\psi}y, \frac{1}{1+e\psi}\right). \tag{7}$$

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[9]This is the conditional normal distribution of the returns given effort and the signal.

In this model, there is heterogeneity in beliefs because the portfolio managers are assumed to be overconfident while the investors are rational. The focus of the paper is on their interactions while contracting. Another useful feature of the model is that it can also incorporate agent underconfidence. Moore and Healy (2008) detail studies that find underconfident agents. Although we don't explicitly tackle such a bias here, the model can address such a problem by extending the domain of the parameter ψ to zero . For values between zero and one the agent would be identified as underconfident.

2.3. Unconstrained problem

In order to solve her problem, the investor must first solve the manager's problem and understand the implications of overconfidence on the variables of the manager's choice. Here, in the first case, the manager strategically chooses a level of effort and also the quantity of risky asset in an unconstrained way. For any given level of effort and signal the manager is going to maximize his conditional utility by choosing an utility maximizing quantity. Solving the manager's utility maximization problem we get the following expression for the optimal quantity demanded[10]

$$\theta = \frac{e\psi}{a\alpha} y. \tag{8}$$

First, the proportion of wealth invested in the risky asset is an increasing function of the manager's overconfidence and effort and is, sensibly, decreasing in the level of his risk aversion. Second, as expected, a higher positive signal implies that a larger proportion of the wealth is invested in the risky asset.

The next step in this method of backward induction is to solve for the agent's equilibrium effort. The manager has to weigh the marginal benefit of effort, which is a higher signal precision, against the marginal cost of effort. The following equation represents the unconditional expected utility function of the manager.

$$E_m(U|e) = -\exp\{-aF + V(a,e)\}.g(e) \tag{9}$$

where $g(e) = \left(\frac{1}{1+e\psi}\right)^{\frac{1}{2}}$. Section A.2 of the appendix provides the detailed proof. It is evident from the above equation that manager's expected utility

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[10]See the appendix for proof

is increasing in the function $g(e)$. Also notice that in the above equation the unconditional expected utility of the manager is not a function of the incentive parameter α. In this context, Stoughton (1993) was the first to show that linear contracts cannot be used to induce a higher effort from the manager. Overall, the optimal effort that maximizes the manager's expected utility should solve the following first order condition

$$V'(a, e_{fb}) = \frac{\psi}{2}(\frac{1}{1 + \psi e_{fb}}). \tag{10}$$

Since the expected utility function was not dependent on α, the optimal effort is also not going to be a function of the incentive parameter. The question that is of interest to this paper is the response in effort choice to changes in level of overconfidence. Since the overconfident manager thinks that the precision of his signal is high, he is bound to overestimate the marginal benefit of his effort. Therefore the point of indifference between marginal utility of effort and marginal cost of effort is going to be at a higher effort level than what it is for a rational manager. The following Proposition states it.

Proposition 1. *Given any contract (α, F), the optimal effort, e_{fb}, of the manager is increasing in overconfidence, ψ.*

The proof to the proposition is provided in the Appendix.[11] The fact that the effort is increasing in overconfidence also has to do with the modeling assumptions of complementarity between effort and overconfidence. Although in some instances they could be thought of substitutes, there is overwhelming evidence of their complementarity. Bénabou and Tirole (2002) provide a summary of different findings in this regard. They argue that substitutability typically occurs when the reward for performance is of a "pass-fail" nature; which is not the case here.

After solving the manager's problem the investor gets to chose the contract parameters α and F. The investor chooses these parameters to maximize her unconditional expected utility which is dependent, obviously, on the manager's actions. The incentive compatibility constraint in equation (10) has to be satisfied. Further, we assume that the investor and the manager have the same

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[11] We can also solve the first order condition in equation (10) for the optimal level of effort. The optimal effort (e_{fb}) is equal to $\frac{-a + \sqrt{a\psi^2 + a^2}}{2\psi a}$. It is easy to show that this effort function is positive and increasing in ψ.

level of risk aversion a. This is done to study the effects of overconfidence in isolation and to exclude any confounding effects arising from the differences in agent's risk preferences. Since the investor in the model is rational, the distributions used in computing her expected utility is that of a rational person. While deriving the investor's expected utility function we define the following two functions

$$m(\alpha) = \frac{(1-\alpha)}{\alpha}\psi,$$

$$M(\alpha) = m(\alpha)(2 - m(\alpha)). \tag{11}$$

Following is the investor's unconditional expected utility and the steps to deriving it is provided in the appendix (see Section A.4)

$$E_i(U) = -\exp\{aF\}\left(\frac{1}{1 + eM(\alpha, \psi)}\right)^{1/2}. \tag{12}$$

The investor also has to ensure that the minimum reservation utility is paid in order to secure the manager's participation. The following is the participation constraint.

$$-\exp\{-aF + V(a, e_{fb})\}\left(\frac{1}{1 + e_{fb}\psi}\right)^{1/2} = -U_o. \tag{13}$$

Solution to the optimization problem of the investor is reported in the Lemma below.

Lemma 1. *In the unconstrained scenario, for a given level of managerial overconfidence ψ, the investor chooses*

$$\alpha_{fb} = \frac{\psi}{1 + \psi}, and$$

$$F = \left(\frac{1}{a}V(a, e_{fb}) + \frac{1}{2a}\log\left(\frac{1}{1 + e_{fb}\psi}\right) - \frac{1}{a}\log(U_o)\right)$$

as the contract parameters.

Having solved for the optimal contract parameters and knowing the expected utility function of the investor, it is natural to ask whether there is any benefit to hiring an overconfident manager?

Proposition 2. *The expected utility of a rational investor is always increasing in the level of managerial overconfidence.*

Intuitively since, for any given contract, an overconfident manager is always going to choose a higher equilibrium effort, there should be benefits from hiring an overconfident manager. But, an overconfident manager, due to his bias, will always pick a riskier portfolio for any given signal i.e., when compared to a rational person (see equation (8); $\theta(y)$ is increasing in ψ). Then why is it that the investor is always better off hiring an overconfident manager? The answer to this lies in the fact that the neither the manager's effort choice nor his expected utility is a function of the incentives in the contract. The only role that the parameter α plays is in picking the quantity. In the first best case, by picking an appropriate α the principal can implicitly choose the level of portfolio risk. To see this, compute the quantity of risky asset that the principal will demand in the event that she observes the signal himself. Given her expected utility function, the optimal quantity is the following

$$\theta_i(y) = \frac{e}{a(1-\alpha)} y. \tag{14}$$

Note, the above quantity is not a function of ψ since the manager is rational. Now when $\alpha = \frac{\psi}{1+\psi}$, like in Lemma 1, $\theta_i(y) = \theta$, the exact quantity that the manager will pick. Higher equilibrium effort and the ability to weigh in on the extent of portfolio risk, using the variable compensation parameter, ensures that it is always optimal for the investor to hire an overconfident investor in the unconstrained case.

What about the level of risk in the portfolio? Do overconfident managers invest a larger amount in risky asset? Odean (1998) and Daniel et al. (1998), through their model, argue that when individuals/traders are overconfident they trade more often and hold riskier positions. Barber and Odean (2001) and Grinblatt and Keloharju (2009) provide empirical support to these claims. Similar to their findings, even in the case of delegated portfolio management the equilibrium quantity of risky asset demanded by an overconfident manager is higher than that demanded by a rational manager.

Proposition 3. *The first best quantity of risky asset demanded by an overconfident agent is always higher than the quantity demanded by the rational manager.*

Whether, the additional risk in the portfolio generates higher returns is an

empirical question. But, it is important to note that the outcome of Proposition 3 is optimal from the principal's perspective.

2.4. Constrained problem

Portfolio managers often don't make decision in an unconstrained way, as was depicted in the unconstrained scenario. Using data from the Securities and Exchange Commission (SEC) filings, Almazan et al. (2004) report that a vast majority of funds have a variety of constraints on the portfolio holdings. Often there could be legislative reasons for such constraints for e.g., section 12(d) 1 of the Investment Company Act of 1940, restricts the ability of one investment company to invest in another. Regardless of the source of the constraint, imposing such constraints can have profound effects on the contracting decisions. Gómez and Sharma (2006) point to the importance of these constraints in resolving the "no-incentive" result found in the unconstrained case. Dybvig et al. (2010) also raise the importance of including trade restrictions in studying the contracts of delegated portfolio management. In their model they incorporate this idea by designing a contract, or mechanism, which would induce the manager to reveal the true signal.

The important question here is how do these investment constraints affect the overconfident manager? We follow Gómez and Sharma (2006), and introduce the constraint by restricting the absolute value of the level of risky asset demanded to a positive constant k in the following way

$$|\theta(y)| \leq k. \tag{15}$$

The value of k is exogenously specified and is used here just to illustrate a point. As k tends towards infinity we would be back to the case of no constraints. The demand function for quantity of the risky asset is no more a smooth function like in the unconstrained case. Instead we now have a piecewise function depending on the value of k

$$\theta(y) \begin{cases} k & y > \frac{ka\alpha}{\psi e} \\ \frac{\psi e}{a\alpha} y & |y| < \frac{ka\alpha}{\psi e} \\ -k & y < -\frac{ka\alpha}{\psi e}. \end{cases} \tag{16}$$

The manager can get the quantity of his choice as long as that quantity corresponds to the signal in $[-\frac{ka\alpha}{\psi e}, \frac{ka\alpha}{\psi e}]$. However, for any signal outside this

range i.e., $y < -\frac{ka\alpha}{\psi e}$ and $y > \frac{ka\alpha}{\psi e}$ the quantity demanded is restricted to $-k$ and k respectively.

2.4.1. Manager's Problem

From the manager's demand function, we can derive his unconditional expected utility function.

Lemma 2. *The unconditional expected utility function of the manager is given by*

$$E[\mathrm{U}_M] = -exp(-aF + V(a,e)).g(e, \psi | \alpha), \tag{17}$$

with

$$g(e, \psi | \alpha) = (\frac{1}{1 + \psi e})^{\frac{1}{2}} \Phi(\frac{(ka\alpha)^2}{\psi e}) + exp(\frac{(ka\alpha)^2}{2})(1 - \Phi(\frac{(ka\alpha)^2}{\psi e}(1 + \psi e)))$$

where Φ is the distribution function of a $\chi^2(1)$ random variable.

The function $g(e, \psi | \alpha)$, in the expected utility function, now has two distinct components. The first component corresponds to the set of signal within the bounds where the overconfident manager is not affected by this constraint. The second term relates to the those signals where the constraint is binding. Once the principal has solved for the manager's expected utility function she would like to understand the optimal effort level chosen. Following is the first order condition for effort choice

$$V'(a, e^*)g(e^*, \psi | \alpha) + g'(e^*, \psi | \alpha) = 0 \tag{18}$$

where

$$g'(e, \psi | \alpha) = \frac{-\psi}{2} \left(\frac{1}{1 + \psi e} \right)^{3/2} \Phi \left(\frac{(ka\alpha)^2}{\psi e} \right) \tag{19}$$

The key distinction here, from the first best case, is that now $g(e, \psi | \alpha)$ depends on α. This is one of the main contributions of Gómez and Sharma (2006). Moreover, it can be shown that equilibrium effort for any given level of overconfidence is increasing in α.

Lemma 3. *The manager's equilibrium second best effort is increasing in performance adjustment fee α.*

Credit for the proof of above lemma goes to (Gómez and Sharma, 2006, Lemma 1). The adaptation of their proof to the model presented in this paper is provided in the Appendix.

In the presence of a constraint on the quantity demanded, there are two opposing forces that influence the choice of effort for an overconfident manager. As shown in first best case, overconfidence leads to an increase in the amount of effort because the manager perceives the marginal benefit of his effort to be high. However, also note that the signal space for which the manager can choose the utility maximizing quantity is decreasing in the level of his overconfidence. This is evident from noticing that the measure of the set $[-\frac{ka\alpha}{\psi e}, \frac{ka\alpha}{\psi e}]$ decreases as overconfidence, ψ, increases. This implies that the unconditional probability of an overconfident manager, when compared to a rational manager, to be at the corner and be forced to pick $-k$ or k is higher. This is bound to decrease his expected utility. The intuitive response of the overconfident manager is to then reduce the effort ex-ante. The tradeoff between these two effects will determine the equilibrium level of effort. Following this line of thought also provides an economic explanation for Lemma 3. As α increases, the measure of set $[-\frac{ka\alpha}{\psi e}, \frac{ka\alpha}{\psi e}]$ also increases meaning that the set of possible signals for which the manager can pick his optimal quantity is increasing. This in turn raises his expected utility and hence induces higher effort.

Ideally, one would solve the first order condition in (18) to compute the optimal level of effort. Unfortunately, it is extremely hard to compute an analytical expression for the level of effort from (18). It is also not feasible to do any comparative statics given that we already expect a non-monotonic relationship between effort and overconfidence. Therefore, we present a numerical solution for the choice of effort.

Proposition 4. *Due to higher perceived precision by the manager, the second best optimal effort is increasing in overconfidence until a point. However, as overconfidence increases beyond this level it has a negative impact on the second best effort.*

Figure 1 plots of the optimal effort as a function of overconfidence. The plot is generated by assuming values of 1, 1.25, and 0.2 for k, a, and α respectively.[12]

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[12] The choice of these parameters have no bearing on the nature of relationship between these two variables. Multiple values for these parameters have been tried and they all qualitatively yield very similar results.
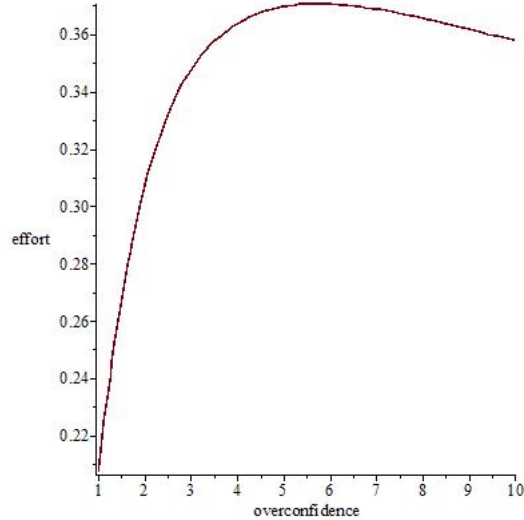
Figure 1: Optimal level of effort chosen by the manager as a function of the level of his overconfidence is presented here. Eqn (18) presents the first order condition for effort. It has been solved for effort by assuming values of 1, 1.25, and 0.2 for k, a, and α respectively at different levels of overconfidence.

The concave down relationship between effort and overconfidence meets the intuition presented earlier regarding the two opposing effects of overconfidence.

2.4.2. Investor's Problem and Numerical Results

Now, we turn our focus to the investor's problem. It is important to note that the above effort-overconfidence relationship is a function of the incentive parameter α, which is chosen by the investor. Lemma 3 asserts that the level of managerial effort is an increasing function of α. Therefore by changing this parameter investor can control the level of managerial effort and hence also the portfolio risk. This is an important distinction from the previous case.

The risk averse investor will maximize her expected utility by choosing the two parameters α and F, such that the reservation utility of the manager is met. Earlier, in the unconstrained case, it was assumed that all the managers have the exact same reservation utility. An argument could be made that if an overconfident manager perceives that the marginal benefit of his effort is high then he would also demand a higher compensation to be employed. In order to address this concern we allow the reservation utility of the manager to be an increasing function of the level of his overconfidence. Following is the new participation constraint

$$-exp(-aF + V(a,e)).g(e, \psi|\alpha) \geq -exp(-a \cdot r(\psi)), \tag{20}$$

where $r(\psi)$ is the reservation wealth of the agent. The exact form of this function is discussed below. For computing the contract parameters we have to specify the investor's objective function.

Lemma 4. *Investor's second best unconditional expected utility function is given by*

$$E(U_I) = -exp\{aF\} \cdot f(\alpha, e), \tag{21}$$

where,

$$f(\alpha, e) = \left(\frac{1}{1 + eM(\alpha)} \right)^{\frac{1}{2}} \Phi \left(\frac{(ka\alpha)^2}{e\psi^2} \frac{1 + eM(\alpha)}{1 + e} \right)$$

$$+$$

$$exp(\frac{(ka(1-\alpha))^2}{2}) \left(1 - \Phi \left(\frac{(ka\alpha)^2}{e\psi^2} \frac{(1 + em(\alpha))^2}{1 + e} \right) \right).$$

Φ in above is the distribution function of a $\chi^2(1)$ random variable. Given the nature of the above expression, there are no closed form solutions for the contract parameters α and F. However, it remains to ascertain whether it is beneficial for the investor to hire an overconfident manager. Therefore, we explore numerical solutions. The objective is to choose the contract parameters that maximizes the expected utility of investor subject to the participation constraint. Notice that the agent's choice of effort is not a function of the fixed compensation F; a standard result in most principal agent models. Additionally, from (21) we know that investor's expected utility is decreasing in F and from (20) that manager's expected utility is increasing in the same. This means that the participation constraint has to be binding at the optimum. So the investor's problem can be reduced further to make it a function of only one choice variable α in the following way

$$E(U_I) = -exp\{V(a,e) + a \cdot r(\psi)\} \cdot g(e, \psi|\alpha) \cdot f(\alpha, e). \tag{22}$$

In order to proceed further with the numerical calculations, assumptions regarding the values of a and k are to be made. Haubrich (1994) show that, using

a CARA utility function, relatively low levels of risk aversion is sufficient to explain the empirical pay-performance relationship in CEO compensation. Based on their findings we use the value of $a = 1.25$. However, we also report results using other levels of risk aversion. These results are qualitatively very similar to the base case. k is set equal to 1 for all further numerical computations.

The algorithm starts by creating a grid for the possible values of the incentive parameter α, i.e., between 0 and 1. In each iteration one of the possible hundred values of α is selected. Conditional on the chosen α, the next step involves solving the manager's problem and evaluating the optimal effort. Subsequently, for each pair of (α, e) and given level of overconfidence, investor's expected utility is computed using (22). Having evaluated the expected utility of the investor over all the possible values of α, the final step is to choose the α that provides the maximal expected utility. This procedure is then repeated for multiple levels of managerial overconfidence. The results of the numerical computations are reported in Table 1. Based on these findings the following proposition is in order.

Proposition 5. *Assuming a symmetric linear compensation structure for the fund manager*
a) It is always beneficial for the risk averse investor to hire a moderately overconfident manager in the second best case.
b) The level of portfolio risk is higher when an overconfident manager is hired.

Panel A - D of Table 1 present results to support the above proposition. The four different panels report results for each of the different assumptions regarding the reservation wealth. In Panel A it is assumed that the rational manager, $\psi = 1$, desires 1% of wealth in expectation. It is further assumed that it increases linearly in ψ. In a similar fashion, Panel B assumes that a rational manager expects to earn 3% of the wealth as fees. In Panel C risk aversion parameter used is changed. Finally in Panel D, we assume that the expected reservation wealth increases quadratically. The first row in each of these panels report the Investor's expected utility (IEU) from hiring managers of different overconfidence. The investor's expected utility is increasing in overconfidence until a point and then decreases as managerial overconfidence is higher than this point. There are two main effects of overconfidence. First, it increases the level of equilibrium effort. Therefore for any given level of incentive compensation the investor is better off hiring an overconfident manager because the mean and the precision of conditional return go up. Second, since the manager is over-

confident he is going to invest more in the risky stock than what is appropriate conditional on his signal. This decreases the investor's expected utility as it increases the variance of the portfolio. Traditionally, the compensation contract is used to trade-off between incentives and insurance. Therefore, the investor, in anticipation of manager's action, increases the incentive parameter and shares a higher percentage of the risk with the agent. The manager is willing to take on this risk because in his perception, although biased, this increases his expected utility. Hence, the manager's participation is ensured. Overall, in the equilibrium one expects to see α increase in the level of managerial overconfidence. Increasing α also marginally reduces the quantity of risky asset demanded (see eqn (16)).

The second row in all the panels of Table 1 detail the amount of money that is invested in the risky assets.[13] Consistent with the above intuition, we see that overconfident managers invest a higher proportion of wealth in the riskier asset. This result is qualitatively similar to the results of Palomino and Sadrieh (2011), who also predict that overconfident fund managers hold riskier portfolios, and to the empirical findings of Barber and Odean (2001), Barber and Odean (2002), and Grinblatt and Keloharju (2009), who find that overconfident individual investors hold riskier portfolios.

The role of increasing reservation utility is also important. As overconfidence increases it becomes increasingly expensive for the investor to ensure participation and so the shape of the expected reservation wealth function determines the extent of the benefits of hiring an overconfident manager. The final row in Table 1 reports the equilibrium level of effort chosen by the manager. The chosen effort level is increasing across the different levels of overconfidence. However, the reader should not construe this as a violation of Proposition 4, which holds only *ceteris paribus*. As the incentive parameter changes with each level of overconfidence, so does the effort level. Note, the first order condition for effort choice, eqn (18), is not a function of the reservation utility, $r(\psi)$. Therefore, the optimal effort is the same in Panel A, B, and D.

Table 1: Investor's Expected Utility in Second Best Case

Results from the numerical computations for the second best case are reported here. Investor's expected utility, IEU, is computed using eqn (22). Details of the exact algorithmic procedure is presented in the main text of the paper. $ in risky, is the expected % of initial capital that is invested in the risky asset by the manager. Performance adjustment fee, α, is the value of the optimal contract parameter chosen by the investor. Effort, e, is endogenously chosen by the manager given the contract parameters. The degree of portfolio constraints is uniformly set, $k = 1$. The values are reported for different levels of overconfidence parameter, ψ. Panel A reports the values assuming that the reservation wealth of the manager is 1% of assets under management and is linearly increasing in overconfidence. The values are reported under the assumption that the absolute risk aversion parameter for both the agents is 1.25. In Panel B reservation wealth of the manager is assumed to be 3% of assets under management. In Panel C values are reported assuming a risk aversion parameter of 2. In Panel D, it is assumed that the reservation wealth increases quadratically in overconfidence.

	Value of overconfidence (ψ)								
	1	1.5	2	2.5	3	3.5	4	4.5	5
Panel A: risk aversion parameter $a = 1.25$ - Linear 0.01									
IEU	-0.908	-0.875	-0.850	-0.830	-0.814	-0.800	-0.788	-0.779	-0.771
$ in risky	0.485	0.559	0.617	0.661	0.693	0.720	0.741	0.758	0.774
α	0.52	0.61	0.66	0.69	0.72	0.74	0.76	0.78	0.79
Effort	0.156	0.194	0.217	0.230	0.240	0.246	0.250	0.253	0.254
Panel B: risk aversion parameter $a = 1.25$ - Linear 0.03									
IEU	-0.931	-0.909	-0.894	-0.884	-0.877	-0.873	-0.872	-0.872	-0.874
$ in risky	0.485	0.559	0.617	0.662	0.694	0.720	0.741	0.758	0.774
α	0.52	0.61	0.66	0.69	0.72	0.74	0.76	0.78	0.79
Effort	0.156	0.194	0.217	0.230	0.240	0.246	0.250	0.253	0.254
Panel C: risk aversion parameter $a = 2$ - Linear 0.03									
IEU	-0.979	-0.965	-0.954	-0.947	-0.943	-0.942	-0.945	-0.949	-0.955
$ in risky	0.281	0.347	0.409	0.464	0.515	0.554	0.587	0.617	0.642
α	0.5	0.6	0.66	0.7	0.72	0.74	0.76	0.77	0.78
Effort	0.112	0.151	0.179	0.199	0.212	0.222	0.229	0.234	0.238
Panel D: risk aversion parameter $a = 1.25$ - Quadratic 0.01									
IEU	-0.908	-0.883	-0.872	-0.870	-0.877	-0.893	-0.916	-0.949	-0.990
$ in risky	0.485	0.559	0.617	0.662	0.694	0.720	0.741	0.758	0.774
α	0.52	0.61	0.66	0.69	0.72	0.74	0.76	0.78	0.79
Effort	0.156	0.194	0.217	0.230	0.240	0.246	0.250	0.253	0.254

Table 2: Investor's Expected Utility - Differences in Risk Aversion.

Results from the numerical computations for the second best case are reported here. Investor's expected utility, IEU, is computed using eqn (22). Details of the exact algorithmic procedure is presented in the main text of the paper. $ in risky, is the expected % of initial capital that is invested in the risky asset by the manager. Performance adjustment fee, α, is the value of the optimal contract parameter chosen by the investor. Effort, e, is endogenously chosen by the manager given the contract parameters. The degree of portfolio constraints is uniformly set, $k = 1$. The values are reported for different levels of overconfidence parameter, ψ. Panel A reports the values assuming that the reservation wealth of the manager is 3% of assets under management and it is linearly increasing in overconfidence. The values are reported under the assumption that the absolute risk aversion parameter for the investor is 1.25 and that for the manager is 2.5. In Panel B reservation wealth of the manager is assumed to be 1% of assets under management and that it increases quadratically in overconfidence. Like in Panel A, the absolute risk aversion parameter for the investor is assumed to be 1.25 and that for the manager to be 2.5.

	Value of overconfidence (ψ)								
	1	1.5	2	2.5	3	3.5	4	4.5	5
Panel A: risk aversion parameter $a = 2.5$, $b = 1.25$ - Linear 0.03									
IEU	-0.983	-0.975	-0.971	-0.969	-0.970	-0.973	-0.977	-0.983	-0.990
Effort	0.091	0.125	0.150	0.169	0.183	0.194	0.201	0.208	0.212
α	0.34	0.43	0.5	0.54	0.58	0.6	0.62	0.64	0.65
$ in risky	0.295	0.346	0.390	0.439	0.476	0.516	0.548	0.574	0.601
Panel B: risk aversion parameter $a = 2.5$, $b = 1.25$ - Quadratic 0.01									
IEU	-0.958	-0.948	-0.947	-0.954	-0.970	-0.994	-1.027	-1.069	-1.121
Effort	0.091	0.125	0.150	0.169	0.183	0.194	0.201	0.208	0.212
α	0.34	0.43	0.5	0.54	0.58	0.6	0.62	0.64	0.65
$ in risky	0.295	0.346	0.390	0.439	0.476	0.516	0.548	0.574	0.601

2.4.3. Role of risk aversion

In all the analysis above, we have assumed that the investor and the manager have the same risk aversion levels. In order to highlight the role of overconfidence and the heterogeneity in beliefs of the investor and the manager, it was imperative to eliminate the effects, if any, of the differences in risk aversion on effort choice and portfolio formation. Here, we explore the contracting problem by allowing their risk aversion levels to be different. It is standard to assume that the manager is more risk averse than the representative investor who represents the collection of investors (see Gómez and Sharma (2006) and Palomino and Sadrieh (2011)).

From Grossman and Hart (1983) we already know that when the agent has a CARA utility function, the loss to the principal on account of the moral hazard is increasing in the agent's degree of absolute risk aversion. The proof was provided for the case when there were two possible future states. Chade and de Serio (2002) generalize the above result and provide a proof for any finite number of states of the world. Therefore, a priori, the expectation is that investor's expected utility should go down as the manager's risk aversion increases. We follow the numerical procedure detailed in Section 2.4.2 and analyze the problem when there are differences in the risk aversion levels.

Having differences in the risk aversion levels is not going to affect the manager's problem. However, the investor has to solve the following equation instead of eqn (22)

$$E\left(U_I\right) = -exp\{V(a,e)\frac{b}{a} + b \cdot r(\psi)\} \cdot g(e,\psi|\alpha)^{\frac{b}{a}} \cdot f(\alpha,e), \qquad (23)$$

where,

$$m(\alpha) = \frac{b}{a}\frac{(1-\alpha)}{\alpha}\psi,$$

$$M(\alpha) = m(\alpha)(2 - m(\alpha)),$$

and,

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[13] Obviously, the proportion of wealth chosen to be invested in the risky asset is contingent on the signal. "$ in risky" reported here is in expectational terms and is equal to $2 * (\int_0^{\frac{ka\alpha}{\psi e}} \frac{\psi e}{a\alpha} y.f(y) + k \int_{\frac{ka\alpha}{\psi e}}^{\infty} f(y))$. $f(y)$ is the density function of the overconfident manager's signal which is distributed $N\left(0, \frac{1+\psi e}{\psi e}\right)$.

$$f(\alpha, e) = \left(\frac{1}{1 + eM(\alpha)} \right)^{\frac{1}{2}} \Phi \left(\frac{(ka\alpha)^2}{e\psi^2} \frac{1 + eM(\alpha)}{1 + e} \right)$$

$$+$$

$$exp(\frac{(ka\alpha m(\alpha))^2}{2\psi^2}) \left(1 - \Phi \left(\frac{(ka\alpha)^2}{e\psi^2} \frac{(1 + em(\alpha))^2}{1 + e} \right) \right).$$

Table 2 presents the results in a manner similar to that presented in Table 1. The results are presented in a way that facilitates easy comparison. The only way Panel A of Table 2 is different from Panel B of Table 1 is that it assumes that the manager's risk aversion coefficient, a, is 2.5 instead of the 1.25. Comparing these two tables one can observe that the investor's expected utility (row 1) is lower for all levels of overconfidence when the manager's risk aversion is higher. A similar conclusion can be drawn by comparing Panel D of Table 1 and Panel B of Table 2, where expected reservation wealth increases quadratically. Other values for manager's risk aversion have also been tried and the results are qualitatively similar. These results confirm our earlier intuition about the effects of differing risk aversion levels. Importantly, it still remains that the investor's expected utility increases from hiring a manager who is moderately overconfident.

2.4.4. Convex Contracts including Hedge Funds

A possible limitation of the above model is that it assumes linear contracts. In the mutual fund industry it is common practice to provide convex or asymmetric contracts to portfolio managers (see Ma, Tang and Gómez (2016)). This structure implies that incentive fees are paid when the fund earns a positive return but no money is deducted in the event of negative returns. Hedge funds also use a similar type of compensation contract. Elton et al. (2003) find that hedge funds never have the negative incentive fees and usually use zero as the reference benchmark.[14] Given this compensation structure is it still profitable to hire an overconfident manager?

We present a simple two state scenario where the manager has a convex

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[14] Hedge funds use a fee structure that is commonly referred as Two and Twenty fees. More specifically, the manager earns 2% of total asset value as a management fee and an additional 20% of any profits earned.

contract. Using this set up we show that the earlier arguments, made using linear contracts, continue to hold. Consider a two state economy where the risky asset could either return x_1 or $-x_1$, where $x_1 > 0$. The prior probability is that the two states are equally likely. Similar to the earlier set up, the manager now exerts effort and observes a signal regarding the future returns of the risky asset. If the manager observes the signal, s_1, then the probability of the future return being x_1 is given by $p(\psi, e)$. Assume that the posterior probability is given by

$$p(\psi, e) = \frac{1}{2} + \frac{1}{2} \frac{\psi}{1+\psi} \frac{e}{1+e}. \tag{24}$$

The posterior probability, $p(\psi, e)$, is increasing in overconfidence, ψ, and in effort, e, and is higher than the prior probability of 0.5. This would also imply that the probability of $-x_1$ given s_1 is less than 0.5. The wealth of the manager in the two states would be given by

$$\tilde{W}_M = \begin{cases} F + \beta x_1 \theta & with\, probability\, p \\ F & with\, probability\, (1-p) \end{cases}$$

where F is the fixed fee, β is the incentive fee, and θ is the quantity of risky asset demanded. The difference in the payoffs of the two states showcases the convexity in the compensation.

A portfolio manager with a CARA utility function (like the negative exponential) and normally distributed returns is similar to a mean-variance maximizer. Therefore, we assume that the the expected utility of the manager with terminal wealth, $\tilde{W}$, is given by

$$E(U_m) = E(\tilde{W}) - \frac{1}{2} a \, Var(\tilde{W}) - V(a, e). \tag{25}$$

The first step in solving the manager's problem is to compute the proportion of wealth invested in the risky asset. The optimal amount of risky portfolio is given by

$$\theta = \frac{1}{a \, (1-p)x \, \beta}. \tag{26}$$

The proof of the above follows much like the proof provided in Section A.1. The quantity of risky asset demanded is contingent on the signal that is ob-

served. $\frac{1}{a\,(1-p)x_1\,\beta}$ and $-\frac{1}{a\,(1-p)x_1\,\beta}$ are the units of risky asset demanded when the signals are s_1 and $-s_1$ respectively. Further, since $p(\psi, e)$ is increasing in both overconfidence and effort, it is easy to see that the amount invested in risky assets is also increasing in ψ, and e.

Having solved the investment problem, the expected utility of the manager (given effort) is going to be

$$E(U_m|e) = F + \frac{p}{2a\,(1-p)}. \tag{27}$$

Eqn (27) can be further used to compute the optimal effort expended by the manager. The first order condition for effort is the following

$$\frac{1}{a}\,\frac{\psi(1+\psi)}{(1+e+\psi)^2} - 2ae = 0. \tag{28}$$

The crucial relationship is the one between managerial overconfidence and the level of effort chosen. Define the left hand side of the above first order condition, eqn (28), as H. The partial derivatives of H with respect to ψ and e are given by

$$\frac{\partial H}{\partial \psi} = \frac{1 + \psi + e + 2\psi e}{(1 + \psi + e)^3} > 0,\ and$$

$$\frac{\partial H}{\partial e} = \frac{-2\psi(1+\psi)}{a(1+\psi+e)^3} - 2a < 0.$$

Using the implicit function theorem we can conclude that managerial overconfidence increases the endogenously chosen effort level ($\frac{\partial e}{\partial \psi} > 0$) in the current case of an unconstrained manager having a convex payoff. This result combined with the implications of equation (26) ensure that the results presented in the constrained case (Section 2.4) also hold for a manager with convex compensation.

3. Empirical Analysis

In this section we take our, above, insights to data and empirically test some of our predictions. It is very hard to get any data on the direct assessment of managerial overconfidence.[15] Gervais and Odean (2001), however, show that

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[15] Grinblatt and Keloharju (2009), to the best of our knowledge, is the first and only study to employ direct psychological assessment of overconfidence among traders. The data on

biased attribution of outcomes to one's ability leads to overconfidence. Biased attribution occurs when a Bayesian manager updates his priors about his private signal more aggressively after receiving evidence confirming his earlier signal than he does after receiving refuting subsequent evidence. Using a stylized model Choi and Lou (2010) show that biased attribution, on the part of portfolio managers, impacts subsequent portfolio choices. Within the framework of their model Choi and Lou (2010) establish that the number of positive/confirming signals the managers receive, measured by the sum of positive benchmark-adjusted return (SPR), is positively related to manager's confidence, measured as the sum of absolute deviations from one's benchmark index (*Active Share*).

We borrow the empirical design in Choi and Lou (2010) to test their predictions and confirm the existence of such a bias. Further, in section 2.4, we have clearly shown the importance of portfolio constraints and how they are more binding on overconfident managers. The presence of the portfolio constraints ex-ante lowers managerial effort, the precision of the signal, and hence the extent of portfolio deviation. Of course, this is after controlling for the direct effect of constraints on such deviations.

Hypothesis 1: The cumulative effect of confirming positive signals, the manager receives, on future portfolio deviations is diminished in the presence of more portfolio constraints.

Endogoneity of compensation and the ability to constraint managers are the main reasons that make it feasible to hire moderately overconfident managers. Lemma 1 and the results of Table 1 clearly show that as managerial overconfidence increases the variable compensation needs to increase with it. A higher variable compensation increase the risk sharing and also moderates the overconfident manager's portfolio decisions. Fund managers often have ownership in the fund that they manage and as this ownership increases it increases the variability of their payoff.

Hypothesis 2: The cumulative effect of confirming positive signals, the manager receives, on future portfolio deviations is diminished in the presence of higher managerial ownership in the fund.

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trader's psychological evaluation is available because standard psychological assessments are performed on all Finnish males at the time of their induction into mandatory military service.

Khorana, Servaes and Wedge (2007) also suggest that this ownership is intended to align the incentives of the manager with the shareholders, and managerial ownership is expected to curb the excessive deviation from the benchmark.

3.1. Mutual Fund and Benchmark Index Data

Data for the empirical analysis are from two main sources. First, we extract mutual fund holdings data from Thomson Reuters Mutual Funds Holdings database from 2000 to 2014. Most funds in this database file a quarterly holdings report. Moreover, the filing date (fdate) is often different from the report date for which the holdings are valid (rdate), and the reported number of shares in Thomson is split-adjusted as of the filing date. Following Choi and Lou (2010), we reverse the adjustment process done by Thompson, as we need to compute the number of shares held on the report date. In addition, to computing the shares held at the end of each quarter, we also adjust the reported number of shares for stock splits between the report date and quarter end.

Our second source of data is Center for Research in Security Prices (CRSP) Mutual Fund database, which includes fund characteristics, net asset values (NAVs), and returns for each share class. Although all these information is provided at the share class level, the underlying portfolio for the different share classes within a fund is the same. Therefore, to aggregate data at the fund level, we use the MFLINKS data provided by Wharton Research Data Services (WRDS). Fund's expense ratio and turnover ratio are the weighted averages of the ratios of its different share classes. The weights are based on the total net assets (TNA) of each share class at the beginning of the period. Finally, we merge Thompson Reuters Mutual Funds Holdings database with the CRSP Mutual Fund database by using the MFLINKS data as well. We remove index funds from the sample by removing funds that have *index*, *indx*, and *idx* in their names.

In our analysis, following Cremers and Petajisto (2009), we consider 19 widely used indices from three major U.S. index families. From the S&P indices, we pick S&P 500, S&P500/Barra Growth, S&P500/Barra Value, S&P MidCap 400, and S&P SmallCap 600. From the Russell indices, we pick Russell 2000, Russell 2000 Growth, Russell 2000 Value, Russell 1000, Russell 1000 Growth, Russell 1000 Value, Russell 3000, Russell 3000 Growth, Russell 3000 Value, Russell Midcap, Russell Midcap Growth, and Russell Midcap Value. Finally, from Wilshire indices, we pick Wilshire 4500 and Wilshire 5000. Index

constituents data are obtained directly from the companies that manage those indices.

3.2. Active Share and Delta Active Share

We are interested in documenting the effect of observing the public signal on changes in the deviation of the portfolio from its benchmark. According to Cremers and Petajisto (2009), Active Share is defined as one half of the sum of absolute deviations in portfolio weight of mutual fund from its benchmark index:

$$ActiveShare = \frac{1}{2} * \sum_{N=1}^{N} |w_n^{fund} - w_n^{index}| \tag{29}$$

where w_n^{fund} and w_n^{index} are the portfolio weight of stock n in the fund and that of each constituent in the fund's benchmark index, respectively. The sum is taken over the universe of all assets. Following Cremers and Petajisto (2009), we compute the Active Share of a fund with respect to nineteen indices and assign the index with the lowest Active Share as the fund's benchmark. This index has the largest overlap with the fund holdings.

However, one limitation of Active Share is that there can be an artificial variation in Active Share even if stock price change is not accompanied by the actual trading of fund holdings. Choi and Lou (2010) effectively circumvent such mechanical variation by using Delta Active Share (ΔAS). Delta Active Share is defined as Active Share of a fund at the end of quarter t minus Active Share of a hypothetical portfolio if a manager does not trade during the quarter t. Delta Active Share will be zero if the portfolio weights of funds holdings change due to the price effect. That is, Delta Active Share gauges the incremental changes in Active Share that is driven by actual trading in quarter t.

3.3. Investment Constraints and Ownership Variables

Almazan et al. (2004) discuss the six specific investment practices that are relevant to operations of equity funds: (i) borrowing of money, (ii) margin purchases, (iii) short selling, (iv) writing or investing in options on equities, (v) writing or investing in stock index futures, and (vi) investments in restricted securities. Fund managers are required to disclose information about (i) whether specific investment policies are permitted and (ii) (if permitted) whether they engage in these investment practices during the reporting period by responding "yes" or "no" in Form N-SAR. The first three practices are related to leverage

constraint, the fourth and fifth practices are related to derivatives constraint, and the last practice is related to illiquid assets constraint. Those practices will impose a binding constraint on managers' investment decisions, as these variables affect the extent to which a fund deviates from its benchmark. To measure the extent of constraints that fund managers face, following Almazan et al. (2004), we compute an aggregate score to summarize a fund's overall constraint. A constraint score approach of Almazan et al. (2004) places an equal weight across the three distinct constraint categories. More precisely,

$$Investment\ Constraints = \frac{1}{3} * (\frac{1}{3} * total\ leverage\ constraints) +$$
$$\frac{1}{3} * (\frac{1}{2} * total\ derivaties\ constraints) + \frac{1}{3} * (illiquid\ assets\ constraints).$$
$$(30)$$

The aggregate score varies between 0 and 1 and a higher score corresponds to a more constrained fund.[16] We also modify the original measure by focusing on the leverage constraint only, as this is a directly relevant constraint for altering the (Delta) Active Share.

The SEC required disclosure of ownership starting in 2005. The disclosure is in six categories ($0-$10,000; $10,000-$50,000; $50,000-$100,000; $100,000-$500,000; $500,000-$1M; above $1 million) and is for all portfolio managers of a fund. We obtained this data from Morningstar and created two variables: Sum of Maximum Ownership and Maximum Ownership. The first ownership variable is the sum of the upper bound of each manager's ownership interval. The second ownership variable is the maximum of the upper bound of each manager's ownership interval. We have this variable from 2007 until the end of our sample in 2013.

3.4. The Effects of Overconfidence

3.4.1. Regression Specification

To investigate the effect of overconfidence, following Choi and Lou (2010), we estimate the following regression model:

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[16]See section 2.2.1 and 2.2.2 of Almazan et al. (2004) for more detailed explanation

$$\Delta AS_{i,q,y} = \alpha + \beta_1 * SPR_{i,q-4:q-1} + \beta_2 * sumret_{i,q-4:q-1} + \beta_3 * Experience_{i,y}$$
$$+\beta_4 * Experience_{i,y} * SPR_{i,q-4:q-1} + \beta_5 * Investment\ Constraints_{i,y}$$
$$+\beta_6 * Investment\ Constraints_{i,y} * SPR_{i,q-4:q-1} + \beta_7 * Ownership_{i,y}$$
$$+\beta_8 * Ownership_{i,y} * SPRR_{i,q-4:q-1} + \gamma * Control, \tag{31}$$

where the dependent variable ($\Delta AS_{i,t}$) is the change in Active Share of fund i in quarter t that is purely attributable to the incremental trading activity. $\Delta AS_{i,t}$ is the difference between the Active Share constructed from the holdings reported at the end of quarter q and the the hypothetical Active Share constructed from the portfolio if the manager simply carry forward the position in quarter q-1 to q. The most important independent variable $SPR_{i,q-4:q-1}$ is defined as the sum of positive excess return relative to the benchmark in the previous twelve months. The intuition behind this variable is that managers become overconfident because of the bias in their learning. They update the precision of their signal in a significantly higher way when they get a positive feedback than the extent to which they reduce their precision when their signals are not corroborated by future outcomes. The higher the sum of positive returns, the more the manager has received confirming signals.

Important control variables include the benchmark adjusted return in quarter q-4 to q-1 (*past return*), which is one of the most important determinants of managers trading decision as suggested by most empirical mutual fund literature. In addition, we include the tracking error and turnover in the concurrent period to reflect the investment styles. Cremers and Petajisto (2009) argue that tracking error and Active Share are distinct active management measures in that one can choose tracking error as a proxy for factor bets and Active Share for stock selection.[17] Other control variables are fund flows from q-4 to q-1, expense ratio, dummies for fund size, fund age, and fund styles at the end of quarter q-1. The regression specifications include quarter-fixed effects, and standard errors are clustered at both the quarter and fund levels.

Table **3** provides the summary statistics of *Active Share, Delta Active Share,* and other fund characteristics used in the analyses. We are confident that the distribution of our *Active Share* measure is similar to that of Cremers and Peta-

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[17] Our results are qualitatively not sensitive to the exclusion of tracking error.

Table 3: Summary Statistics

This table provides the summary statistics for the sample of funds used in the paper. *Active Share* is defined as a half of the sum of absolute deviations in portfolio weights of a fund from its benchmark index during the quarter q. *Delta Active Share* is defined as Active Share of a fund at the end of quarter q minus Active Share of a hypothetical portfolio if a manager does not trade during the quarter q. *Sum of Positive Returns* (SPR) is defined as the sum of positive benchmark-adjusted returns during the past 12 months. *Past Return* is defined as the sum of benchmark-adjusted returns during the past 12 months. *Tracking Error* is defined as the standard deviation of the benchmark-adjusted returns during the past 12 months. *Turnover* is defined as minimum of aggregated sales or aggregated purchases of securities, divided by the average 12-month Total Net Assets of the fund. *Fund Age* is the number of years since the inception of a fund. *Investment Constrains* is an aggregate score to summarize a fund's overall investment constraint. The aggregate score varies between 0 and 1 and a higher score refers to a more constrained fund. Ownership in the fund is disclosed in six categories ($0-$10,000; $10,000-$50,000; $50,000-$100,000; $100,000-$500,000; $500,000-$1M; above $1 million) and is for all portfolio managers of a fund. *Sum of Maximum Ownership* is the sum of the upper bound of each manager's ownership interval. *Maximum Ownership* is the maximum of the upper bound of each manager's ownership interval.

	Mean	Standard Deviation	25th Pct	Median	75th Pct
Active Share (	77%	16%	67%	79%	90%
Delta Active Share (%)	-3%	10%	-3%	-1%	0%
Sum of Positive Returns (*SPR*) (	8%	8%	3%	6%	10%
Past Return (12 *months*) (%)	0%	8%	-4%	-1%	3%
Tracking Error (12 *months*) (%)	2%	1%	1%	1%	2%
Expense Ratio (%)	1.25%	0.43%	0.99%	1.20%	1.48%
Turnover (%)	89%	104%	36%	66%	110%
Fund Age	15.28	13	7	12	18
Investment Constraints(%)	22%	21%	11%	22%	28%
Sum of (*each*) *Maximum Ownership* ($)	0.65 mil	0.92 mil	0	0.27 mil	1 mil
Maximum Ownership ($)	0.39 mil	0.41 mil	0	0.1 mil	1 mil

jisto (2009). Our average *Active Share* of about 80%, with a standard deviation is 16%, is very close to the summary statistic present in Cremers and Petajisto (2009). On average, our *Delta Active Share* displays a mean (median) of -3% (-1%), which is comparable to Choi and Lou (2010). However, approximately 10% standard deviation of change in *Active Share* clearly indicates a substantial variation among mutual funds. On average, the annual benchmark-adjusted return is close to 0 with the median of about -100 basis points. Our main independent variable of interest, *SPR*, has a mean of 8% and a standard deviation of 8%. The 6% median indicates that the distribution of this variable is slightly right skewed. Briefly summarizing the distribution of other fund characteristics, the average fund age, turnover ratio, and expense ratio are 15 years, 89%, and 1.25%, respectively. Finally, the overall investment restriction proxied by the aggregate constraint score is 0.22 and the portfolio manager owns about 0.39 to 0.65 million dollars in the fund on average, depending on the definition of ownership.

3.4.2. Empirical Results

We fit a pooled OLS regression and the unit of our observation is at the fund-quarter level. Table 4 presents our first regression results. We begin by replicating the baseline results of Choi and Lou (2010) to test for the existence of self-attribution bias among fund managers. Consistent with their findings, $SPR_{i,q-4:q-1}$ is positive and statistically significant after including a host of control variables. Importantly, also in line with Choi and Lou (2010), the benchmark-adjusted return over the past 4 quarters (*past return)* has no effect on the on ΔAS. This is further evidence to show that managers do not decrease the perceived precision of their private signals upon receiving disconfirming feedback ex-post.

Managerial experience plays a crucial role in learning. With time managers learn about their own ability and exhibit lower bias (see Gervais and Odean (2001)). We use the number of years since fund's inception as our measure of manager's experience. We define a new dummy variable *Experience Proxy* which takes a value of one if the manager has above the median experience in that quarter. Column 2 of Table 4 displays the effect of managerial experience on the attribution bias. Consistent with the predictions, the more experienced managers show significantly lower bias.

The main variable of our focus is *Investment Constraints* and its effect on *SPR*. The computed portfolio constraints variable is a continuous variable be-

Table 4: Overconfidence and Fund Investment Constraints

This table presents the regression of *Delta Active Share* on *SPR*, *Past Return*, and other fund characterstics. Regression results include an interaction term of *SPR* with (i) *Experience Proxy* (fund age) and (ii) *Investment Constrains*, respectively. We use the number of years since fund's inception as a proxy for manager's experience. *Experience Proxy* takes a value of one if the manager has above median experience in that quarter. *Investment Constrains* is an aggregate score to summarize a fund's overall investment constraint. The aggregate score varies between 0 and 1 and a higher score refers to a more constrained fund. In addition to a constraint score measure (*Investment Constrains*) specified in *Eq.* (30), we modify the original measure by focusing on the leverage constraint only, (*Narrow Constraints*). *Investment Constrains* takes a value of one if the manager has above median constrains in that quarter. *Delta Active Share* is defined as Active Share of a fund at the end of quarter q minus Active Share of a hypothetical portfolio if a manager does not trade during the quarter q. *Sum of Positive Returns* (SPR) is defined as the sum of positive benchmark-adjusted returns during the past 12 months. *Past Return* is defined as the sum of benchmark-adjusted returns during the past 12 months. *Tracking Error* is defined as the standard deviation of the benchmark-adjusted returns during the past 12 months. *Turnover* is defined as minimum of aggregated sales or aggregated purchases of securities, divided by the average 12-month Total Net Assets of the fund. *Fund Age* is the number of years since the inception of a fund. *Style Code* 1-4 are investment objective code dummies (2 = aggressive growth, 3 = growth, 4 = growth and income). *Age Categories* 1-4 are the age (number of years since inception) quartile dummies (1 = youngest). *Size Categories* 1-4 are the fund size quartile dummies (1 = largest). The subscript q represents quarter q and the timing of the variables. Heteroskedasticity-consistent t-statistics are in parentheses. The significance levels are denoted by *, **, and *** and, indicate whether the results are statistically different from zero at the 10%, 5%, and 1% significance levels, respectively.

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
$SPR_{q-4:q-1}$ (%)	0.150***	0.176***	0.125**	0.155**	0.133***	0.163**
	(0.055)	(0.061)	(0.048)	(0.063)	(0.049)	(0.065)
Experience Proxy		0.002		0.003		0.003
		(0.005)		(0.005)		(0.005)
$SPR_{q-4:q-1}$*Experience Proxy		-0.047**		-0.056***		-0.055***
		(0.020)		(0.018)		(0.018)
Investment Constrains			0.008***	0.008***		
			(0.002)	(0.002)		
$SPR_{q-4:q-1}$*Investment Constrains			-0.029*	-0.030*		
			(0.015)	(0.015)		
Narrow Constraints					0.005***	0.006***
					(0.002)	(0.002)
$SPR_{q-4:q-1}$*Narrow Constraints					-0.028**	-0.031**
					(0.013)	(0.015)
$Tracking\ Error_{q-4:q-1}$(%)	-0.111	-0.016	-0.035	0.039	-0.034	0.039
	(0.224)	(0.241)	(0.200)	(0.258)	(0.200)	(0.257)
$Past\ Return_{q-4:q-1}$(%)	-0.032	-0.037	-0.016	-0.019	-0.017	-0.020
	(0.027)	(0.032)	(0.024)	(0.030)	(0.024)	(0.030)
$Expense\ Ratio_{q-4:q-1}$ (%)	1.102***	1.192***	0.555***	0.554**	0.576***	0.579**
	(0.329)	(0.390)	(0.207)	(0.236)	(0.208)	(0.237)
$Turnover\ Ratio_{q-4:q-1}$ (%)	-0.010***	-0.012***	-0.011***	-0.013***	-0.011***	-0.013***
	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)
$Fund\ Flows_{q-4:q-1}$ (%)	0.000***	0.000***	0.000***	0.000***	0.000***	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Style 1	0.002	0.002	-0.000	-0.000	-0.000	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Style 2	0.001	0.001	-0.001	-0.001	-0.001	-0.001
	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)
Style 3	0.009***	0.007**	0.003	0.002	0.003	0.001
	(0.003)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)
Size 1	-0.002	-0.000	-0.002	-0.001	-0.003	-0.002
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Size 2	-0.002	0.000	-0.001	0.000	-0.002	-0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Size 3	-0.003*	-0.001	-0.000	0.001	-0.001	0.000
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Age Category 1	0.003	0.002	0.001	0.001	0.002	0.001
	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)
Age Category 2	0.005**	0.005	0.004*	0.004	0.004**	0.004
	(0.002)	(0.004)	(0.002)	(0.005)	(0.002)	(0.005)
Age Category 3	0.005*	0.004	0.001	0.000	0.002	0.001
	(0.003)	(0.005)	(0.002)	(0.005)	(0.002)	(0.005)
Observations	73,372	56,370	54,971	44,916	54,971	44,916
Adjusted R-squared	0.023	0.030	0.029	0.036	0.028	0.035
Fixed Effect	Qtr	Qtr	Qtr	Qtr	Qtr	Qtr
Clustering	Qtr & Fund	Qtr & Fund	Qtr & Fund	Qtr & Fund	Qtr & Fund	Qtr & Fund

Table 5: Overconfidence and Manager Ownership

This table presents the regression of *Delta Active Share* on *SPR*, *Past Return*, and other fund characterstics. Regression results include an interaction term of *SPR* with (i) manager's ownership in the fund and (ii)*Experience Proxy*, respectively. *Delta Active Share* is defined as Active Share of a fund at the end of quarter q minus Active Share of a hypothetical portfolio if a manager does not trade during the quarter q. *Sum of Positive Returns* (SPR) is defined as the sum of positive benchmark-adjusted returns during the past 12 months. *Past Return* is defined as the sum of benchmark-adjusted returns during the past 12 months. *Tracking Error* is defined as the standard deviation of the benchmark-adjusted returns during the past 12 months. *Turnover* is defined as minimum of aggregated sales or aggregated purchases of securities, divided by the average 12-month Total Net Assets of the fund. *Fund Age* is the number of years since the inception of a fund. Ownership in the fund is disclosed in six categories ($0-$10,000; $10,000-$50,000; $50,000-$100,000; $100,000-$500,000; $500,000-$1M; above $1 million)and is for all portfolio managers of a fund. *Sum of Maximum Ownership* is the sum of the upper bound of each manager's ownership interval. *Maximum Ownership* is the maximum of the upper bound of each manager's ownership interval. Each ownership variable takes a value of one if the manager has above median ownership in the fund in that quarter. *Style Code* 1-4 are investment objective code dummies (2 = aggressive growth, 3 = growth, 4 = growth and income). *Age Categories* 1-4 are the age (number of years since inception) quartile dummies (1 = youngest). *Size Categories* 1-4 are the fund size quartile dummies (1 = largest). The subscript q represents quarter q and the timing of the variables. Heteroskedasticity-consistent t-statistics are in parentheses. The significance levels are denoted by *, **, and *** and, indicate whether the results are statistically different from zero at the 10%, 5%, and 1% significance levels, respectively.

VARIABLES	(1)	(2)	(3)	(4)
$SPR_{q-4:q-1}$ (%)	0.291**	0.301**	0.333**	0.341**
	(0.121)	(0.125)	(0.120)	(0.124)
Sum of Maximum Ownership	0.018***		0.018***	
	(0.004)		(0.004)	
$SPR_{q-4:q-1}$ (%) * Sum of Maximum Ownership	-0.131***		-0.133***	
	(0.043)		(0.044)	
Maximum Ownership		0.020***		0.020***
		(0.005)		(0.005)
$SPR_{q-4:q-1}$ (%) * Maximum Ownership		-0.142***		-0.145***
		(0.050)		(0.050)
Experience Proxy			0.008	0.007
			(0.005)	(0.005)
$SPR_{q-4:q-1}$ (%) * Experience Proxy			-0.050	-0.045
			(0.036)	(0.036)
Tracking Error$_{q-4:q-1}$(%)	-0.081	-0.066	-0.137	-0.120
	(0.406)	(0.404)	(0.402)	(0.400)
Past Return$_{q-4:q-1}$(%)	-0.072	-0.070	-0.077	-0.074
	(0.047)	(0.046)	(0.047)	(0.046)
Expense Ratio$_{q-4:q-1}$ (%)	0.816*	0.819**	0.802*	0.808*
	(0.398)	(0.397)	(0.413)	(0.412)
Turnover Ratio$_{q-4:q-1}$ (%)	-0.014***	-0.014***	-0.014***	-0.014***
	(0.003)	(0.003)	(0.003)	(0.003)
Fund Flows$_{q-4:q-1}$ (%)	0.000**	0.000**	0.000**	0.000**
	(0.000)	(0.000)	(0.000)	(0.000)
Style 1	0.002	0.002	0.002	0.002
	(0.003)	(0.003)	(0.003)	(0.003)
Style 2	0.001	0.001	0.001	0.001
	(0.003)	(0.003)	(0.003)	(0.003)
Style 3	0.005	0.004	0.004	0.004
	(0.003)	(0.003)	(0.003)	(0.003)
Size 1	-0.001	-0.001	-0.001	-0.001
	(0.003)	(0.003)	(0.003)	(0.003)
Size 2	0.001	0.002	0.001	0.001
	(0.003)	(0.003)	(0.003)	(0.003)
Size 3	-0.001	-0.000	-0.001	-0.001
	(0.002)	(0.002)	(0.002)	(0.002)
Age Category 1	-0.001	-0.001	-0.000	-0.000
	(0.004)	(0.004)	(0.004)	(0.004)
Age Category 2	0.001	0.001	-0.002	-0.003
	(0.003)	(0.003)	(0.006)	(0.006)
Age Category 3	-0.002	-0.002	-0.006	-0.006
	(0.004)	(0.004)	(0.005)	(0.005)
Observations	28,152	28,152	27,724	27,724
Adjusted R-squared	0.043	0.044	0.047	0.048
Fixed Effect	Qtr	Qtr	Qtr	Qtr
Clustering	Qtr &	Qtr &	Qtr &	Qtr &
	Fund	Fund	Fund	Fund

tween 0 and 1. In order to make sense of the interaction coefficient, we convert it into a dummy variable that takes a value of one if the manager has above the median constraints in that quarter. Column 3 presents our main findings. The coefficient of interaction between $Investment\,Constraints$ and SPR is negative and statistically significant, providing us ample support for hypothesis 1. In addition, focusing on leverage constraint makes more sense as the other two constraints are not likely to be related to the deviation from benchmark holdings. Therefore, we also interact SPR with $Narrow\,Constraints$, a dummy variable that takes a value of one if the manager has above median "*leverage*" constrains in that quarter. A strong negative result continues to emerge when we use a narrower measure of investment constraints the managers face. It is evident that the presence of portfolio constraints significantly inhibits the attribution bias the managers show, and hence overconfidence. We also acknowledge the endogeneity of the portfolio constraints the managers face. Nevertheless, our results show an association between investment constraints and the bias in manager's action as we analyze the managerial behavior in the quarter after observing the constraints. Importantly, $Investment\,Constraints$ has a direct and independent effect on ΔAS and the diminished effect of SPR on the deviation from the benchmark, in the presence of $Investment\,Constraints$, emerges after controlling for these effects. Also, contrary to reasonable expectation, having additional investment constraints does not directly reduce the magnitude of portfolio deviations.

We now focus on the manager's variable compensation and its effects on the attribution bias. As discussed above, the data on our proxy for variable compensation, managerial ownership, is presented as a range. We covert these variables to dollar values. Also, following our earlier treatment of the continuous interaction variable, we convert the ownership variables into dummy variables that takes a value of one if they are above the median for the quarter. Results for two different measures of ownership are displayed in Table 5. Consistent with hypothesis 2, if managers suffer from attribution bias, this effect is substantially diminished in the presence of managerial ownership. We make no claims of causality here. Clearly, the ownership share of the manager relies on the personal portfolio decision of the fund manager. However, we also have evidence of fund families requiring managers to invest in their own funds (see Laise (2006)).

Overall, we find a consistent pattern emerging from the empirical results which are in line with our theoretical model. Portfolio constraints and compensation parameters can be, and often are, designed to mitigate known agency

problems and induce higher effort while hiring an overconfident manager.

4. Conclusion

It is well established that individuals are overconfident. Therefore, when the principal chooses to align the interest of the agent to her interests she should take agent's traits into account. With the exception of a few papers, most of the literature ignores this aspect while designing the compensation contract. Similarly, while studying behavioral biases it is imperative that we include incentives in our analysis.

In this paper, we study the problem of a principal who wishes to delegate the responsibility of portfolio management to an agent. However, the principal is aware that the agent she hires is overconfident and so has to chose an appropriate compensation contract. Once the agent knows his incentives he chooses the appropriate effort and also the portfolio risk level. In this framework, we evaluate the implications of employment. The model shows that managerial effort as a function of overconfidence increases until a point. Thereafter, it decrease on account of restrictions on agent's portfolio choices. The investor can gain from commitment to such high effort as this increases the future conditional expected return of the portfolio. But, additional effort also leads to incremental risk taking. By designing appropriate incentives the investor can reduce the level of portfolio risk to some optimal level. These gains are not unbounded. As overconfidence increases it becomes increasingly expensive to hire such an agent until a point where the benefits outweigh the costs. The model clearly shows why, in the data, we find evidence of self-attribution among fund managers. Its because hiring a moderately overconfident fund manger is not sub-optimal.

We also empirically test some of the predictions of our model. Using data on portfolio formation constraints we show that these constraints bind managers to their benchmark and lead to lower attribution bias. Similarly, when the manager's wealth is tied to the variable fund performance managers show that they learn quicker and have a lower bias.

Finally, the current static model does not account for a few interesting practical phenomenons. For e.g., the size of the fund is exogenous in the current model. However, investors choose to direct their flows. Therefore it would also be important to understand how the decision to direct money, into and out of funds, interacts with managerial biases. Additionally, the assumption that the investor knows the exact level of agent overconfidence could be relaxed in

a model where the investor also learns about the agent type over time. These directions are interesting avenues for further research.

A. Appendix for Proofs

A.1. *Optimal level of risky assets*

The optimal quantity chosen is the solution to the following maximization problem

$$\max_\theta E_m(U(W_m)) = \max_\theta E_m(-\exp\{-aF - a\theta\tilde{x}|y\}),$$

where $\tilde{x}|y$ is the return distribution conditional on observing the signal y. Given the distribution of $\tilde{x}|y$

$$E_m(U(W_m)) = -\exp\{-aF\}\exp\left\{-a\alpha\theta y\frac{e\psi}{1+e\psi} + \tfrac{1}{2}(a\alpha\theta)^2\frac{1}{1+\psi e}\right\}$$

and the first order condition for the quantity demanded θ is going to be

$$a\alpha\theta\frac{1}{1+\psi e} - a\alpha y\frac{e\psi}{1+e\psi} = 0.$$

This implies that the optimal level of risky assets in the portfolio is given by

$$\theta = \frac{e\psi}{a\alpha}y. \tag{a.1}$$

A.2. *Expected Utility of the Manager*

Knowing the quantity demanded by the manager, his expected utility given the level of effort and the signal is given by

$$E_m(U|y) = -E[\exp\{-aF - a\alpha\theta(y)\tilde{x}|y + V(a,e)\}]$$

$$= -\int_{-\infty}^{\infty} \exp\{-aF - a\alpha\theta(y)\tilde{x}|y + V(a,e)\}f(x|y)dx.$$

$f(x|y)$ is the conditional return distribution. The above integral is over all the states that are possible after the portfolio has been picked. Simplifying the expression further we have

$$E_m(U|y) = -\int_{-\infty}^{\infty} \exp\left\{-aF - a\alpha\frac{e\psi}{a\alpha}y\tilde{x}|y + V(a,e)\right\}f(x)dx$$

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$$= -\exp\{-aF + V(a,e)\}\int_{-\infty}^{\infty}\exp\{-e\psi y\tilde{x}|y\}f(x)dx$$

$$= -\exp\{-aF + V(a,e)\}\exp\left\{-e\psi y\frac{e\psi}{1+e\psi}y + \frac{1}{2}\frac{(e\psi y)^2}{(1+e\psi)}\right\}$$

$$E_m(U|y) = -\exp\{-aF + V(a,e)\}\exp\left\{-\frac{1}{2}\frac{(e\psi y)^2}{(1+e\psi)}\right\}.$$

The unconditional expected utility of the manager, which is the integral of above with respect to all the possible signals is going to be

$$E_m(U|e) = -\exp\{-aF + V(a,e)\}E\left[\exp\left\{-\frac{1}{2}\frac{(e\psi\tilde{y})^2}{(1+e\psi)}\right\}\right]$$

$$= -\exp\{-aF+V(a,e)\}\sqrt{\frac{e\psi}{1+e\psi}}\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\frac{(e\psi y)^2}{(1+e\psi)}\right\}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{e\psi}{1+e\psi}\frac{y^2}{2}\right\}f(y)$$

$$= -\exp\{-aF+V(a,e)\}\sqrt{\frac{e\psi}{1+e\psi}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\frac{e\psi}{(1+e\psi)}(y^2 e\psi + y^2)\right\}f(y)$$

$$= -\exp\{-aF + V(a,e)\}\sqrt{\frac{e\psi}{1+e\psi}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}e\psi y^2\right\}.$$

For a random variable which is distributed $N\left(0,\frac{1}{e\psi}\right)$ the following is true

$$\int_{-\infty}^{\infty}f(y)dy = \int_{-\infty}^{\infty}\frac{1}{\sqrt{\frac{2\pi}{e\psi}}}\exp\left\{-\frac{1}{2}e\psi y^2\right\} = 2*\int_{0}^{\infty}\frac{1}{\sqrt{\frac{2\pi}{e\psi}}}\exp\left\{-\frac{1}{2}e\psi y^2\right\}dy.$$

Using the above expression we have

$$E_m(U|e) = -\exp\{-aF + V(a,e)\}2*\sqrt{\frac{1}{1+e\psi}}\int_{0}^{\infty}\frac{1}{\sqrt{\frac{2\pi}{e\psi}}}\exp\left\{-\frac{1}{2}e\psi y^2\right\}dy.$$

At this point substitute $s = e\psi y^2$. This substitution will give us $\frac{1}{2e\psi y}ds = dy$. Also if $y = 0$ then $s = 0$ and if $y = \infty$ then $s = \infty$. Since $y = \sqrt{\frac{s}{e\psi}}$ we have

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$$E_m(U|e) = -\exp\{-aF+V(a,e)\}2*\sqrt{\frac{1}{1+e\psi}} \int_0^\infty \frac{1}{\sqrt{\frac{2\pi}{e\psi}}} \exp\left\{-\frac{s}{2}\right\} \frac{1}{2e\psi\sqrt{\frac{s}{e\psi}}} ds.$$

$$E_m(U|e) = -\exp\{-aF+V(a,e)\}\sqrt{\frac{1}{1+e\psi}} \int_0^\infty \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{s}{2}\right\} s^{\frac{-1}{2}} ds.$$

The term in the integral is the distribution function of the $\chi^2(1)$ random variable so $\int_0^\infty \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{s}{2}\right\} s^{\frac{-1}{2}} ds \to 1$. Therefore the unconditinal expectation of the portfolio manager is given

$$E_m(U|e) = -\exp\{-aF+V(a,e)\} \left(\frac{1}{1+e\psi}\right)^{1/2}. \tag{a.2}$$

A.3. *Proof to Proposition 1*

Equation (10), the first order condition for effort, can be written as the following

$$V'(a,e_{fb}) - \frac{\psi}{2}(\frac{1}{1+\psi e_{fb}}) = 0$$

Let the function F be

$$F = V'(a,e_{fb}) - \frac{\psi}{2}(\frac{1}{1+\psi e_{fb}})$$

Based on the implicit function theorem we have that

$$\frac{\partial e}{\partial \psi} = -\frac{\frac{\partial F}{\partial \psi}}{\frac{\partial F}{\partial e}}$$

By definition of optimality we know that $\frac{\partial F}{\partial e_{fb}} < 0$. So, in order to prove the proposition need to show that $\frac{\partial F}{\partial \psi} > 0$. Differentiating F with respect to ψ we have

$$\frac{\partial F}{\partial \psi} = \frac{1}{2}\left(\frac{1}{1+e\psi}\right) - \frac{\psi e}{2}\left(\frac{1}{(1+e\psi)^2}\right) = \frac{1}{2}\left(\frac{1}{(1+e\psi)^2}\right) > 0.$$

Therefore we are done.

As mentioned in the main text,we are going to assume that the investor has the same preferences as the manager; including the level of risk aversion. Using the investor's conditional terminal wealth given in equation (3) the conditional expected utility of the investor is going to be

$$E_i(U|y) = -E\left[\exp\left\{-a(1-\alpha)\frac{e\psi}{a\alpha}y\tilde{x}|y + aF\right\}\right]$$

$$= -\exp\{aF\}\int_{-\infty}^{\infty}\exp\left\{-a(1-\alpha)\frac{e\psi}{a\alpha}y\tilde{x}|y\right\}f(x|y)dx$$

$$= -\exp\{aF\}\exp\left\{-\frac{(1-\alpha)e\psi}{\alpha}\frac{e}{1+e}y^2 + \frac{1}{2}\left(\frac{(1-\alpha)e\psi}{\alpha}\right)^2\frac{y^2}{1+e}\right\}$$

$$= -\exp\{aF\}\exp\left\{-\frac{(1-\alpha)\psi}{\alpha}\frac{e^2}{1+e}y^2\left(1-\frac{1}{2}\left(\frac{(1-\alpha)\psi}{\alpha}\right)\right)\right\}.$$

Define two new variables

$$m(\alpha) = \left(\frac{1-\alpha}{\alpha}\right)\psi, and$$

$$M(\alpha) = m(\alpha)(2 - m(\alpha)).$$

Substituting these variables in the above equation we have

$$E_i(U|y) = -\exp\{aF\}\exp\left\{-\frac{e^2}{2(1+e)}y^2M(\alpha)\right\}. \qquad (a.3)$$

We can now compute the unconditional expected utility of the investor by integrating over the range of possible signals y

$$E_i(U) = -\exp\{aF\}\sqrt{\frac{e}{1+e}}\int_{-\infty}^{\infty}\exp\left\{-\frac{e^2}{2(1+e)}y^2M(\alpha)\right\}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{e}{1+e}\frac{y^2}{2}\right\}dy$$

$$= -\exp\{aF\}\sqrt{\frac{e}{1+e}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{e^2}{2(1+e)}y^2M(\alpha) - \frac{e}{1+e}\frac{y^2}{2}\right\}dy$$

$$= -\exp\{aF\}\sqrt{\frac{e}{1+e}}\int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{ey^2}{2(1+e)}(eM(\alpha)+1)\right\}dy.$$

Substitute $s = \frac{ey^2}{2(1+e)}(eM(\alpha)+1)$ in the above equation. Simplifying it further leads to the following expression for the investor's unconditional expected utility

$$E_i(U) = -\exp\{aF\}\left(\frac{1}{1+eM(\alpha,\psi)}\right)^{1/2}. \tag{a.4}$$

A.5. *Proof to Lemma 1*

In order to compute the contract parameters the investor has to solve a constrained optimization problem where the constraint is on participation given by (13). The Lagrangian of the problem is the following

$$\mathcal{L} = -e^{aF}\left(\frac{1}{1+e_{fb}M(\alpha,\psi)}\right)^{\frac{1}{2}} + \lambda\left(-\exp\{-aF+V(a,e_{fb})\}\left(\frac{1}{1+e_{fb}\psi}\right)^{\frac{1}{2}} + U_0\right).$$

Notice that the participation constraint is not a function of α. We have the following first order condition with respect to α

$$\frac{\partial\mathcal{L}}{\partial\alpha} = -\exp\{aF\}(\frac{-1}{2})\left(\frac{1}{1+e_{fb}M(\alpha,\psi)}\right)^{3/2}e_{fb}M'(\alpha,\psi)=0.$$

Look in the proof of expected utility of the investor for definitions of $m(\alpha)$ and $M(\alpha)$. The above condition is equivalent to

$$\frac{\partial M(\alpha,\psi)}{\partial\alpha} = \frac{2\psi}{\alpha^2}(m(\alpha)-1) = 0.$$

Solving the above equation for α

$$\alpha_{\text{fb}} = \frac{\psi}{1+\psi}.$$

The other first order condition is with respect to F $(\frac{\partial\mathcal{L}}{\partial F})$ and is given by

$$-ae^{aF}\left(\frac{1}{1+e_{fb}M(\alpha,\psi)}\right)^{1/2} + \lambda\left(ae^{-aF+V(a,e_{fb})}\left(\frac{1}{1+e_{fb}\psi}\right)^{1/2}\right) = 0$$

$$\left(\frac{1+e_{fb}\psi}{1+e_{fb}M(\alpha,\psi)}\right)^{1/2} = \lambda e^{-2aF+V(a,e_{fb})}$$

taking the log of both sides we have

$$\tfrac{1}{2}log\left(\frac{1+e_{fb}\psi}{1+e_{fb}M(\alpha,\psi)}\right) = \log(\lambda) - 2aF + V(a,e_{fb}).$$

Notice that at the point of optimality

$$M(\alpha_{\text{fb}}) = \tfrac{1}{\psi}\psi\left(2 - \tfrac{1}{\psi}\psi\right) = 1.$$

Substituting this is previous equation and solving for F will gives us

$$F = \frac{1}{2a}\left[\log(\lambda) + V(a, e_{fb}) - \frac{1}{2}\log\left(\frac{1 + e_{fb}\psi}{1 + e_{fb}}\right)\right]. \qquad (a.5)$$

However, this is still an unknown function of λ. Since the investor does not gain from paying anything more than the reservation utility, the participation constraint will be binding at the optimum. So the following equality should hold

$$\exp\{-aF^* + V(a, e_{fb})\}\left(\tfrac{1}{1 + e_{fb}\psi}\right)^{1/2} = U_0.$$

Expanding this further and taking the log of both sides we get

$$\log(\lambda) = \frac{1}{2}\log\left(\frac{1 + e_{fb}\psi}{1 + e^*}\right) + V(a, e_{fb}) + \log\left(\frac{1}{1 + e_{fb}\psi}\right) - 2\log(U_o). \quad (a.6)$$

Substituting (a.6) in (a.5) we get

$$F = \left[\tfrac{1}{a}V(a, e_{fb}) + \tfrac{1}{2a}\log\left(\tfrac{1}{1 + e_{fb}\psi}\right) - \tfrac{1}{a}\log(U_o)\right].$$

A.6. $\underline{\textit{Proof to Proposition 2}}$

From Lemma 1 we know the optimal contract parameters. Substituting them in the expected utility function of the investor, equation (12), we get the following

$$E_I(U) = -e^{\left[V(a, e_{fb}) - \log(U_o) + \frac{1}{2}\log\left(\frac{1}{1 + e_{fb}\psi}\right)\right]}\left(\frac{1}{1 + e_{fb}M(\alpha,\psi)}\right)^{1/2}.$$

In order to determine if this function is increasing in overconfidence, differentiate the above equation with respect to ψ. Below is the expression

$$\frac{\partial}{\partial\psi}\left[-e^{V(a, e^*) + \frac{1}{2}\log\left(\frac{1}{1 + e^*\psi}\right)}\left(\frac{1}{1 + eM(\alpha,\psi)}\right)^{1/2}\right]$$

$$= -e^{V(a, e^*) + \log\left(\frac{1}{1 + e^*\psi}\right)^{\frac{1}{2}}}\frac{\partial}{\partial\psi}\left(\left(\frac{1}{1 + eM(\alpha,\psi)}\right)^{1/2}\right) -$$

$$\left(\frac{1}{1 + eM(\alpha,\psi)}\right)^{1/2}\frac{\partial}{\partial\psi}\left[e^{V(a, e^*) + \log\left(\frac{1}{1 + e^*\psi}\right)^{\frac{1}{2}}}\right].$$

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Remember, $M(\alpha_{\text{fb}}) = 1$. Lets focus on

$$\frac{\partial}{\partial \psi} \left[e^{V(a,e^*) + \frac{1}{2} \log\left(\frac{1}{1+e^*\psi}\right)} \right] = \frac{\partial}{\partial \psi} \left[e^{V(a,e^*)} \left(\frac{1}{1+e^*\psi} \right)^{\frac{1}{2}} \right]$$

$$= \left[\left(\frac{1}{1+e^*\psi} \right)^{\frac{1}{2}} e^{V(a,e^*)} \frac{\partial V(a,e^*)}{\partial e^*} \frac{\partial e^*}{\partial \psi} + e^{V(a,e^*)} \left(-\frac{1}{2} \right) \left(\frac{1}{1+e^*\psi} \right)^{\frac{3}{2}} \left(\frac{\partial e^*}{\partial \psi} \psi + e^* \right) \right]$$

$$= \left[\left(\frac{1}{1+e^*\psi} \right)^{\frac{1}{2}} e^{V(a,e^*)} \frac{\partial e^*}{\partial \psi} \left(\frac{\partial V(a,e^*)}{\partial e^*} - \frac{\Psi}{2} \left(\frac{1}{1+e^*\psi} \right) \right) - e^{V(a,e^*)} \frac{e^*}{2} \left(\frac{1}{1+e^*\psi} \right)^{\frac{3}{2}} \right].$$

Note, that the first order condition for effort

$$\left(\frac{\partial V(a,e^*)}{\partial e^*} - \frac{\Psi}{2} \left(\frac{1}{1+e^*\psi} \right) \right) = 0.$$

Therefore,

$$\frac{\partial}{\partial \psi} \left[e^{V(a,e^*) + \frac{1}{2} \log\left(\frac{1}{1+e^*\psi}\right)} \right] = -e^{V(a,e^*)} \frac{e^*}{2} \left(\frac{1}{1+e^*\psi} \right)^{\frac{3}{2}} < 0.$$

Also, since we already know that effort is increasing in overconfidence, it has to be that $\frac{\partial}{\partial \psi} \left[\left(\frac{1}{1+e^*} \right)^{1/2} \right] < 0$. Therefore $\frac{\partial E_I(U)}{\partial \psi} > 0 \; \forall \psi$.

A.7. *Proof to Proposition 3*

From Lemma 1 we already know that $\alpha_{fb} = \frac{\psi}{1+\psi}$. We also know that the quantity of risky asset demanded by the manager is given by $\theta(y) = \frac{e\psi}{a\alpha} y$. Substituting the value of α_{fb} in the demand function we get that

$$\theta(y) = \frac{ey}{a}(1+\psi).$$

From the above equation it is clear that the equilibrium risky quantity is increasing in ψ.

A.8. *Proof to Lemma 2*

Given the effort level and the signal observed the expected utility of the manager hinges on the quantity demanded. We have the following expression for the utility function

$$E_M(U|y) = -E[\exp\{-aF - a\alpha\theta(y)\tilde{x}|y + V(a,e)\}]$$

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$$= -e^{-aF+V(a,e)} \int_{-\infty}^{\infty} \exp\{-a\alpha\theta(y)\tilde{x}|y\} f(x|y) dx.$$

For signals below the bound $(y < \frac{-ka\alpha}{\psi e})$

$$\int_{-\infty}^{\infty} \exp\{a\alpha k\tilde{x}|y\} f(x|y) dx = \exp\left\{ \frac{\psi e(a\alpha ky)}{1+\psi e} + \frac{(ka\alpha)^2}{2} \frac{1}{1+\psi e} \right\}$$

$$= \exp\left\{ \frac{\psi e}{1+\psi e} ka\alpha \left(y + \frac{(ka\alpha)}{2\psi e} \right) \right\}. \tag{a.7}$$

For signals above the bound $(y > \frac{ka\alpha}{\psi e})$

$$\int_{-\infty}^{\infty} \exp\{-a\alpha k\tilde{x}|y\} f(x|y) dx = \exp\left\{ \frac{-\psi e(a\alpha ky)}{1+\psi e} + \frac{(ka\alpha)^2}{2} \frac{1}{1+\psi e} \right\}$$

$$= \exp\left\{ \frac{-\psi e}{1+\psi e} ka\alpha \left(y - \frac{(ka\alpha)}{2\psi e} \right) \right\}. \tag{a.8}$$

For signals within the bound $(|y| \le \frac{ka\alpha}{\psi e})$

$$\int_{-\infty}^{\infty} \exp\left\{ -a\alpha \frac{e\psi}{a\alpha} y\tilde{x}|y \right\} f(x|y) dx = \exp\left\{ \frac{-(\psi ey)^2}{1+\psi e} + \frac{1}{2} \frac{(\psi ey)^2}{1+\psi e} \right\} = \exp\left\{ -\frac{1}{2} \frac{(\psi ey)^2}{1+\psi e} \right\}.$$

Now lets integrate over all possible signals and solve for the unconditional expected utility of the manager. We still have to deal with the three regions separately. For the signals below the threshold we get the following expression as the share towards expected utility

$$-e^{-aF+V(a,e)} \int_{-\infty}^{-\frac{ka\alpha}{\psi e}} e^{\left\{ \frac{\psi e}{1+\psi e} ka\alpha \left(y + \frac{(ka\alpha)}{2\psi e} \right) \right\}} f(y) \, dy$$

where $f(y)$ is the distribution function of a random variable which is distributed $N\left(0, \frac{1+\psi e}{\psi e}\right)$. Applying the normal distribution's density function to the above equation we get

$$= -e^{-aF+V(a,e)} \int\limits_{-\infty}^{-\frac{ka\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{\psi e}{1+\psi e}\right)^{1/2} e^{\frac{-y^2}{2}\left(\frac{\psi e}{1+\psi e}\right)} e^{\left\{\frac{\psi e}{1+\psi e}ka\alpha\left(y+\frac{(ka\alpha)}{2\psi e}\right)\right\}} dy.$$

Using completion of squares we have

$$= -e^{-aF+V(a,e)} \int\limits_{-\infty}^{-\frac{ka\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{\psi e}{1+\psi e}\right)^{1/2} e^{\left\{\frac{-e\psi}{2(1+e\psi)}(y-ka\alpha)^2\right\}} e^{\frac{(ka\alpha)^2}{2}} dy.$$

Going to make a substitution $s = \frac{e\psi}{1+e\psi}(y-ka\alpha)^2$. It can be proved that after this substitution the share of expected utility from the signal below the threshold is given by

$$= -\frac{1}{2}(e^{-aF+V(a,e)})\left(e^{\frac{(ka\alpha)^2}{2}}\right) \int\limits_{\frac{(ka\alpha)^2}{\psi e}(1+\psi e)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{\left\{-\frac{s}{2}\right\}} \frac{1}{\sqrt{s}} ds.$$

The function in the integral is the probability density function of a $\chi^2(1)$ random variable. Let ϕ and Φ be the density density and the cumulative distribution function of $\chi^2(1)$ random variable. Also, one would get an exact same equation for the part above the threshold. For brevity, we don't show that proof here. Now, for the contribution of the part of the signal space which is within the bounds $(|y| \leq \frac{ka\alpha}{\psi e})$. The expression below represents that part of the expected utility.

$$-e^{-aF+V(a,e)} \int\limits_{-\frac{ka\alpha}{\psi e}}^{\frac{ka\alpha}{\psi e}} \exp\left\{-\frac{1}{2}\frac{(\psi e y)^2}{1+\psi e}\right\} f(y)\, dy$$

$$= -e^{-aF+V(a,e)} \int\limits_{-\frac{ka\alpha}{\psi e}}^{\frac{ka\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}} \left(\frac{\psi e}{1+\psi e}\right)^{1/2} e^{\frac{-y^2}{2}\left(\frac{\psi e}{1+\psi e}\right)-\frac{1}{2}\frac{(\psi e y)^2}{1+\psi e}} dy.$$

Substituting $s = \psi e y^2$ we get the following

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$$= -e^{-aF+V(a,e)} \left(\frac{1}{1+\psi e} \right)^{1/2} \int_0^{\frac{(ka\alpha)^2}{\psi e}} \frac{1}{\sqrt{2\pi}} e^{\frac{-s^2}{2}} \frac{1}{\sqrt{s}} ds.$$

Adding the three parts we get the following expression as the unconditional expected utility of the manager

$$EU_m = -e^{-aF+V(a,e)} \left[\left(\frac{1}{1+\psi e} \right)^{\frac{1}{2}} \Phi \left(\frac{(ka\alpha)^2}{\psi e} \right) \right] +$$
$$- e^{-aF+V(a,e)} \left[exp \left(\frac{(ka\alpha)^2}{2} \right) \left(1 - \Phi \left(\frac{(ka\alpha)^2}{\psi e} (1+\psi e) \right) \right) \right].$$

A.9. *Proof to Lemma 3*

Credit for the proof goes to Gómez and Sharma (2006). The result almost follows from the Lemma 1 and Corollary 2 in their paper. Equation (18) describes the first order condition for effort. Lets define a function M as follows

$$M := V'(a,e^*)g(e,\psi|\alpha) + g'(e^*,\psi|\alpha).$$

Then, using M and the implicit function theorem the proof would be complete if we can show that $\frac{\partial M}{\partial \alpha} < 0$. This is true because by definition, $\frac{\partial M}{\partial e^*} > 0$. Further, using Lemma 1 in Gómez and Sharma (2006) $\frac{\partial g(e^*,\psi|\alpha)}{\partial \alpha} < 0$. Also, from the definition of $g'(e^*,\psi|\alpha)$ in equation (19), we can see that $\frac{\partial g'(e^*,\psi|\alpha)}{\partial \alpha} < 0$. Moreover, by assumption the effort function, $V(a,e)$, is convex and increasing function for all levels of effort therefore $V'(a,e^*) > 0$. Using these facts, $\frac{\partial M}{\partial \alpha} = V'(a,e^*) \frac{\partial g(e^*,\psi|\alpha)}{\partial \alpha} + \frac{\partial g'(e^*,\psi|\alpha)}{\partial \alpha} < 0$.

This concludes the proof. On a related note, the proofs relating to the existence of a unique optimal second best effort, the continuity of the effort function with respect to α, and the differentiability of the effort function with respect to α are all applicable to the model here just as they were in Gómez and Sharma (2006).

A.10. *Proof to Lemma 4*

Note, the principal in this case is a rational person. Therefore, in evaluating investor's expected utility rational beliefs should be used. Expected utility of the investor given the effort level and the signal is

$$E\left(U_i \,|y\,,e\right) = -E\left[\exp\left(-a\left(1-\alpha\right)\theta\tilde{x} \,|y + aF\right)\right].$$

But the θ is dependent on the signal and on account of the constraints on holdings, like the proof of Lemma 2, there are three distinct cases to deal with.

Conditional Expectation

For signals within the bound $\left(|y| \leq \frac{ka\alpha}{\psi e}\right)$

$$E\left(U_i \,|y,e\right) = -E\left[\exp\left(-a\left(1-\alpha\right)\frac{e\psi}{a\alpha}y\tilde{x} \,|y + aF\right)\right]$$

$$= -\exp\left(aF\right)E\left[\exp\left(-a\left(1-\alpha\right)\frac{e\psi}{a\alpha}y\tilde{x} \,|y\right)\right].$$

Knowing the distribution of the $\tilde{x} \,|y$, the above expectation can be written as following

$$= -\exp\left(aF\right)\left[\exp\left(-\frac{(1-\alpha)}{\alpha}\frac{e^2 y^2 \psi}{1+e} + \frac{1}{2}\left(\frac{(1-\alpha)}{\alpha}\right)^2 \frac{e^2 \psi^2 y^2}{1+e}\right)\right].$$

Simplifying this further we have

$$= -\exp\left(aF\right)\left[\exp\left(-\frac{(1-\alpha)}{\alpha}\psi\frac{e^2 y^2}{1+e}\left(1 - \frac{1}{2}\left(\frac{(1-\alpha)}{\alpha}\right)\psi\right)\right)\right].$$

Like before, let us assume $m\left(\alpha\right) = \psi\frac{(1-\alpha)}{\alpha}$ and $M\left(\alpha\right) = m\left(a\right)\left(2 - m\left(a\right)\right)$. Then,

$$E\left(U_i \,|y,e\right) = -\exp\left(aF\right)\exp\left(-\frac{1}{2}\frac{e^2 y^2}{1+e}M\left(\alpha\right)\right).$$

For signals below the bound $\left(y < \frac{-ka\alpha}{\psi e}\right)$

$$E\left(U_i \,|y,e\right) = -\exp\left(aF\right)E\left[\exp\left(-a\left(1-\alpha\right)\left(-k\right)\tilde{x} \,|y\right)\right].$$

Evaluating the expectation we have the following

$$E\left(U_i \,|y,e\right) = -\exp\left(aF\right)\exp\left(\frac{ak(1-\alpha)}{1+e}\left(ye + \frac{1}{2}ak\left(1-\alpha\right)\right)\right).$$

For signals above the bound $\left(y > \frac{ka\alpha}{\psi e}\right)$

$$E\left(U_i \,|y,e\right) = -\exp\left(aF\right)E\left[\exp\left(-a\left(1-\alpha\right)k\tilde{x} \,|y\right)\right].$$

Evaluating the expectation we get the following expression

$$E\left(U_i \,|y,e\right) = -\exp\left(aF\right)\exp\left(\frac{-ak(1-\alpha)}{1+e}\left(ye - \frac{1}{2}ak\left(1-\alpha\right)\right)\right).$$

Unconditional Expectaion

Using the above computed conditional expected utility, now we are going to compute the unconditional expected utility, which is taking the expectation over all possible signals. Like before, there are going to be three different regions over which we need to integrate.

For signals within the bound $\left(|y| \leq \frac{ka\alpha}{\psi e}\right)$

$$E\left(U_i\,|e\right) = -\exp\left(aF\right) \int_{-\frac{\text{ka}\,\alpha}{\psi e}}^{\frac{\text{ka}\,\alpha}{\psi e}} \exp\left(-\tfrac{1}{2}\tfrac{e^2 y^2}{1+e} M\left(\alpha\right)\right) f\left(y\right) dy,$$

where $f\left(y\right)$ is the density function of the normal distribution given as $N\left(0, \frac{1+e}{e}\right)$. Using the distribution function of the gaussian random variable we get

$$= -\exp\left(aF\right) \int_{-\frac{\text{ka}\,\alpha}{\psi e}}^{\frac{\text{ka}\,\alpha}{\psi e}} \exp\left(-\tfrac{1}{2}\tfrac{e^2 y^2}{1+e} M\left(\alpha\right)\right) \tfrac{1}{\sqrt{2\pi}} \left(\tfrac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(-\tfrac{y^2}{2}\tfrac{e}{1+e}\right) dy.$$

Simplifying this further we get

$$= -\exp\left(aF\right) \int_{-\frac{\text{ka}\,\alpha}{\psi e}}^{\frac{\text{ka}\,\alpha}{\psi e}} \tfrac{1}{\sqrt{2\pi}} \left(\tfrac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(-\tfrac{1}{2}\tfrac{e y^2}{1+e}\left(eM\left(\alpha\right)+1\right)\right) dy.$$

Notice that we have the density function of a $\left(\frac{1}{1+eM(\alpha)}\right)^{\frac{1}{2}} N\left(0, \left(\frac{1+e}{e(1+eM(\alpha))}\right)\right)$ distributed random variable in the above integral. Using the symmetry of the Normal distribution we have

$$E\left(U_i\,|e\right) = -2\exp\left(aF\right) \int_{0}^{\frac{\text{ka}\,\alpha}{\psi e}} \tfrac{1}{\sqrt{2\pi}} \left(\tfrac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(-\tfrac{1}{2}\tfrac{e y^2}{1+e}\left(eM\left(\alpha\right)+1\right)\right) dy.$$

Substitute $s = \frac{e y^2}{1+e}\left(eM\left(\alpha\right)+1\right)$. Then, $ds = \frac{2e y}{1+e}\left(eM\left(\alpha\right)+1\right) dy$. For the limits of the integral: when $y = 0$ we have $s = 0$ and when $y = \frac{\text{ka}\,\alpha}{\psi e}$ we have $s = \frac{(eM(\alpha)+1)}{1+e}\frac{(\text{ka}\,\alpha)^2}{\psi^2 e}$. Using the above expression for s, we also get that $y = \pm\left(\frac{(1+e)s}{e(1+eM(\alpha))}\right)^{\frac{1}{2}}$. Since in the above integral y is strictly positive we can ignore the negative sign. Substituting this in the expectation we have

$$E\left(U_i\,|e\right)\bigg|_{|y| \leq \frac{ka\alpha}{\psi e}} = -e^{aF}\left(\frac{1}{1+eM\left(\alpha\right)}\right)^{\frac{1}{2}} \int_{0}^{\frac{(eM(\alpha)+1)}{1+e}\frac{(ka\alpha)^2}{\psi^2 e}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{s}{2}\right)\frac{1}{\sqrt{s}} ds.$$

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The function inside the integral is the density function of a $\chi^2(1)$ random variable. Let Φ represent the cumulative distribution of a $\chi^2(1)$ *variable*. So, for this part we get

$$E\left(U_i \,|e\right)\Big|_{|y| \le \frac{ka\alpha}{\psi e}} = -e^{aF}\left(\frac{1}{1 + eM\left(\alpha\right)}\right)^{\frac{1}{2}} \Phi\left(\frac{\left(eM\left(\alpha\right) + 1\right)}{1 + e}\frac{\left(ka\alpha\right)^2}{\psi^2 e}\right). \quad \text{(a.9)}$$

For signals below the bound $\left(y < \frac{-ka\alpha}{\psi e}\right)$

The expected utility of the investor in this region ignoring $-\exp\left(aF\right)$ is

$$= \int_{-\infty}^{-\frac{ka\,\alpha}{\psi e}} \exp\left(\frac{ak(1-\alpha)}{1+e}\left(ye + \tfrac{1}{2}ak\left(1-\alpha\right)\right)\right) \frac{1}{\sqrt{2\pi}}\left(\frac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(-\frac{y^2}{2}\frac{e}{1+e}\right)dy,$$

$$= \int_{-\infty}^{-\frac{ka\,\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}}\left(\frac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(\frac{-1}{2}\frac{e}{1+e}\left(y^2 - 2ayk\left(1-\alpha\right) - \frac{\left(ak(1-\alpha)\right)^2}{e}\right)\right)dy.$$

Multiply and divide the integral by $\exp\left(-\frac{1}{2}\frac{e}{1+e}\left(ak\left(1-\alpha\right)\right)^2\right)$. Then for the above equation we have

$$= \int_{-\infty}^{-\frac{ka\,\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}}\left(\frac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(\frac{-1}{2}\frac{e}{1+e}\left(\left(y - ak\left(1-\alpha\right)\right)^2 - \frac{1+e}{e}\left(ak\left(1-\alpha\right)\right)^2\right)\right)dy,$$

$$= \exp\left(\frac{\left(ak(1-\alpha)\right)^2}{2}\right) \int_{-\infty}^{-\frac{ka\,\alpha}{\psi e}} \frac{1}{\sqrt{2\pi}}\left(\frac{e}{1+e}\right)^{\frac{1}{2}} \exp\left(\frac{-1}{2}\frac{e}{1+e}\left(y - ak\left(1-\alpha\right)\right)^2\right)dy.$$

Now make the following substitution $s = \frac{e}{1+e}\left(y - ak\left(1-\alpha\right)\right)^2$. Then $ds = \frac{2e}{1+e}\left(y - ak\left(1-\alpha\right)\right)dy$. Based on the above equation $y = \pm\left(\frac{1+e}{e}s\right)^{\frac{1}{2}} + ak\left(1-\alpha\right)$. Since we are strictly restricting ourselves to the real line it has to be that $s > 0$. Note, in this case using the negative part of the expression of y is the only sensible thing to do since it is the only thing that will work when $y = -\infty$. For the limits of integral, when $y = -\infty$ $s = \infty$ and when $y = -\frac{ka\alpha}{\psi e}$ $s = \frac{e}{1+e}\left(\frac{ka\alpha}{\psi e} + ka\left(1-\alpha\right)\right)^2$. It can be seen that $\frac{e}{1+e}\left(\frac{ka\alpha}{\psi e} + ka\left(1-\alpha\right)\right)^2$ can be expressed as $\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1 + em(\alpha)\right)^2}{1+e}$. Making these substitutions we get the following

$$E\left(U_i \,|e\right)\Big|_{y < \frac{-ka\alpha}{\psi e}} = \frac{1}{2}\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right) \int_{\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em(\alpha)\right)^2}{1+e}}^{\infty} \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-s}{2}\right)\frac{\left(-ds\right)}{\sqrt{s}}.$$

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This gives the expected utility for this region

$$
E\left(U_i \,|e\right)\Big|_{y < \frac{-ka\alpha}{\psi e}} = -\frac{e^{aF}}{2}\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\left(1-\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em\left(\alpha\right)\right)^2}{1+e}\right)\right).
$$

(a.10)

For signals above the bound $\left(y > \frac{ka\alpha}{\psi e}\right)$

The expected utility of the investor in this region ignoring $-\exp\left(aF\right)$ is

$$
= \int_{\frac{ka\,\alpha}{\psi e}}^{\infty} \exp\left(\frac{-ak(1-\alpha)}{1+e}\left(ye - \frac{1}{2}ak\left(1-\alpha\right)\right)\right)\frac{1}{\sqrt{2\pi}}\left(\frac{e}{1+e}\right)^{\frac{1}{2}}\exp\left(-\frac{y^2}{2}\frac{e}{1+e}\right)dy
$$

$$
= \int_{\frac{ka\,\alpha}{\psi e}}^{\infty}\frac{1}{\sqrt{2\pi}}\left(\frac{e}{1+e}\right)^{\frac{1}{2}}\exp\left(-\frac{1}{2}\frac{e}{1+e}\left(y^2 + 2\,ya\,k\left(1-\alpha\right) - \frac{\left(ak(1-\alpha)\right)^2}{e}\right)\right)dy.
$$

Multiplying and dividing by $\exp\left(-\frac{1}{2}\frac{e}{1+e}\left(ak\left(1-\alpha\right)\right)^2\right)$ we get that above is

$$
= \exp\left(\frac{\left(ak(1-\alpha)\right)^2}{2}\right)\int_{\frac{ka\,\alpha}{\psi e}}^{\infty}\frac{1}{\sqrt{2\pi}}\left(\frac{e}{1+e}\right)^{\frac{1}{2}}\exp\left(-\frac{1}{2}\frac{e}{1+e}\left(y+ak\left(1-\alpha\right)\right)^2\right)dy.
$$

Now make the following substitution $s = \frac{e}{1+e}\left(y+ak\left(1-\alpha\right)\right)^2$. Then $ds = \frac{2e}{1+e}\left(y+ak\left(1-\alpha\right)\right)dy$. Based on the above equation $y = \pm\left(\frac{1+e}{e}s\right)^{\frac{1}{2}} - ak\left(1-\alpha\right)$. In this case using the positive part of the expression of y is the only sensible thing to do. For the limits of integral, when $y = \infty$ $s = \infty$ and when $y = \frac{ka\alpha}{\psi e}$, $s = \frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em(\alpha)\right)^2}{1+e}$. Substituting these in the equation for expected utility we have

$$
E\left(U_i\,|e\right)\Big|_{y > \frac{ka\alpha}{\psi e}} = \frac{1}{2}\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\int_{\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em(\alpha)\right)^2}{1+e}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{s}{2}\right)\frac{1}{2\sqrt{s}}ds.
$$

This gives the expected utility for this region

$$
E\left(U_i\,|e\right) = -\frac{e^{aF}}{2}\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\left(1-\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em\left(\alpha\right)\right)^2}{1+e}\right)\right)
$$

(a.11)

Adding these three parts up, eqn(a.9), eqn(a.10), and eqn(a.11), we have the following expression for the overall unconditional expected utility

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$$E\left(U_i\,|e\,\right) = -\exp\left(aF\right)f\left(\alpha,e\right),$$

where

$$f\left(\alpha,e\right) = \left(\frac{1}{1+eM\left(\alpha\right)}\right)^{\frac{1}{2}}\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(eM\left(\alpha\right)+1\right)}{1+e}\right)+$$
$$\exp\left(\frac{\left(ak\left(1-\alpha\right)\right)^2}{2}\right)\left(1-\Phi\left(\frac{\left(ka\alpha\right)^2}{\psi^2 e}\frac{\left(1+em\left(\alpha\right)\right)^2}{1+e}\right)\right).$$

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