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by

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Abstract

Research has shown that children's numeracy skills in Kindergarten are predictive of their math skills and overall academic achievement later in life. Using data collected from 155 Senior Kindergarten students (74 males; *M* = 70.10 months), the purpose of the current study was to investigate the relations between cognitive predictors and early numeracy. The predictors examined in this study, as identified by the Pathways to Mathematics model, are quantitative, linguistic and working memory abilities, while the control variables are age and processing speed (LeFevre et al., 2010). It was hypothesized that quantitative, linguistic, and working memory abilities would each significantly predict early numeracy. Quantitative abilities were measured using a subitizing task, along with symbolic and non-symbolic magnitude comparison tasks. Linguistic abilities were measured using receptive vocabulary and phonological awareness tasks. Working memory abilities were measured by verbal and visuospatial span tasks. A multiple regression revealed that both linguistic and working memory abilities predicted early numeracy skills, but quantitative abilities did not. These findings suggest that domain general abilities play a pivotal role in early numeracy and indicate that more research is needed to understand the quantitative precursors of early numeracy.

## Exploring Cognitive Predictors Through the Pathways to Mathematics Model

Of all the skills that children develop throughout their education, few are as important as numeracy. Not only does numeracy contribute to decisions about how people spend their money and their time, it has also been shown to play a key role in determining employment outcomes and income levels (Bynner & Parsons, 1997; Parsons & Bynner, 1997; Ritchie & Bates, 2013). For these reasons, it is vital that educators understand the factors that contribute to below-average numeracy skills among students.

To date, one of the key findings in this field is that early numeracy skills are a key predictor of later numeracy skills (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Locuniak & Jordan, 2008; Morgan, Farkas, & Wu, 2009). This is an important finding, and it clearly demonstrates the value of developing strong early numeracy skills. However, there is still considerable uncertainty around the cognitive abilities that contribute to early numeracy skills (Cirino, 2011; Fuchs et al., 2010; LeFevre et al., 2010; Passolunghi, Vercelloni, & Schadee, 2007). Without a clearer understanding of these cognitive precursors, it may be difficult to develop interventions that effectively address deficiencies in early numeracy.

In order to understand the cognitive underpinnings of early numeracy, it is important to explore the relations between various cognitive predictors and early numeracy. To develop a better understanding of these relations, the current study was designed to test LeFevre et al.'s (2010) Pathways to Mathematics model. The pathways model, combined with subsequent research, suggests that quantitative, linguistic and working memory abilities are unique predictors of early numeracy. In the current study, it is hypothesized that quantitative, linguistic and working memory abilities will each uniquely predict early numeracy skills in Kindergarten students.

LeFevre et al.'s Pathways to Mathematics model is a useful framework for exploring the cognitive predictors of early numeracy because it incorporates two theories that have guided much of the research in this field. The first theory is that domain-specific cognitive abilities, those that are specialized for processing numerical information, are the main determinants of numeracy skills. In support of this theory, many studies have found that domain-specific numerical processing abilities are significant predictors of early numeracy skills (De Smedt, Verschaffel, & Ghesquière, 2009; Jordan et al., 2009; Krajewski & Schneider, 2009; Locuniak & Jordan, 2008; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Piazza et al., 2010). For example, Locuniak and Jordan (2008) found that kindergarten students' basic addition and subtraction skills significantly predicted their calculation abilities in second grade, even when controlling for age, memory, reading abilities and verbal and spatial cognition. Similarly, Lyons et al. (2014) found that performance on magnitude comparison and number ordering tasks uniquely contributed to students' arithmetic performance in grades one to six. The second theory is that domain-general cognitive abilities, those that are involved in many cognitive tasks, are the key cognitive predictors of early numeracy. There is also extensive research that supports this theory by demonstrating that domain-general abilities related to language, working memory and processing speed significantly predict early numeracy (Bull, Espy, & Wiebe, 2008; Bull & Johnston, 1997; Cirino, 2011; Geary, 2011; Hornung, Schiltz, Brunner, & Martin, 2014; Passolunghi, & Lanfranchi, 2012; Purpura, & Ganley, 2014; Sowinski et al., 2015). For example, Bull et al. (2008) found that preschool children's performance on memory span and problem solving tasks predicted their math achievement at seven years old. In addition, Purpura and

Ganley (2014) found that both vocabulary and word recall abilities were key predictors of preschool and kindergarten students' numeracy skills.

LeFevre et al. (2010) expanded upon existing research by developing and validating a model that aimed to explain how both domain-general and domain-specific cognitive abilities contributed to early numeracy. Drawing on the work of Dehaene and Butterworth (Butterworth, 2005; Castelli, Glaser, & Butterworth, 2006; Dehaene, Piazza, Pinel, & Cohen, 2003), their model included three separate types of cognitive predictors: quantitative, linguistic and spatial attention. The model's quantitative predictors measure the domain-specific ability to evaluate quantities, its linguistic predictors measure the domain-general ability to acquire language about number systems and its spatial attention predictors measure domain-general visual working memory abilities (LeFevre et al., 2010). The model uses these three types of cognitive abilities to predict early numeracy and is referred to as Pathways to Mathematics.

Dehaene, Piazza, and Pinel's (2003) work suggested that the parietal lobe processes numerical information in three separate neural circuits. First, quantitative tasks activate the horizontal segment of the intraparietal sulcus. Second, linguistic tasks activate a portion of the left angular gyrus. Third, tasks involving spatial attention activate the posterior superior parietal lobule. Similarly, Butterworth and colleagues showed that the horizontal segment of the intraparietal sulcus is specialized in the processing of numerical information (Butterworth et al. 2005; Castelli, Glaser, & Butterworth, 2006). Although Dehaene et al. (2003) and Butterworth et al. (2005; Castelli et al., 2006) disagree about how quantitative information is processed, they both found that the quantitative circuit is specialized for processing numerical information. In other words, it is domain-specific. The linguistic and spatial attention circuits, on the other hand, are involved in many cognitive tasks. In other words, they are domain-general. Based on this evidence, in addition to past research suggesting that the development of numeracy skills involves several distinct cognitive systems (Halberda, Feigenson, & Mazzocco, 2008), LeFevre et al. (2010) developed the Pathways to Mathematics model.

Using this model, LeFevre et al. (2010) found support that quantitative, linguistic and spatial attention abilities all formed separate pathways that predicted numeracy skill two years later. In addition, they found that the predictive value of each pathway depended on how closely it was linked to the outcome measure. For example, an outcome measure that tested geometry and measurement skills did not involve quantitative abilities and therefore it was not predicted by the quantitative pathway (LeFevre et al., 2010). These findings offered new insights into the cognitive determinants of early numeracy, because they demonstrated that each cognitive pathway contributed uniquely to children's early numeracy skills. As a result, LeFevre et al.'s (2010) study established the Pathways to Mathematics model as a valid framework for exploring the cognitive predictors of early numeracy.

While LeFevre et al.'s (2010) study is an important validation of the Pathways model on its own, it is equally important to note that the broader body of early numeracy research supports the model. Past research provides further evidence that each of the model's pathways contribute to early numeracy, and several subsequent studies have supported LeFevre et al.'s (2010) findings. In addition, a close examination of related research reveals opportunities to further explore the relations between cognitive abilities and early numeracy by making additions to the model.

**Quantitative Predictors**

There is extensive evidence to support the existence of a domain-specific quantitative pathway that contributes to early numeracy skills (Cirion, 2011; De Smedt et al., 2009; Hornung et al., 2014; Jordan et al., 2009; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; Locuniak, & Jordan, 2008; Passolunghi & Lanfranchi, 2012; Sowinski et al., 2015). Research suggests that certain basic quantitative abilities are key to understanding more complex numerical concepts (Gersten, Jordan, & Flojo, 2005; Krajewski & Schneider, 2009; Kroesbergen et al., 2009;). This idea is supported by studies that have found domain-specific quantitative abilities contribute to early numeracy, even when controlling for domain-general abilities that are commonly cited as predictors (De Smedt et al., 2009; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Locuniak, & Jordan, 2008; Passolunghi & Lanfranchi, 2012). For example, Geary et al. (2007) found that children with below-average numeracy skills performed worse than their peers on tests of their addition, counting and number recognition skills, even after accounting for differences in performance on memory span and processing speed tasks. (Geary et al., 2007). In addition, studies that have tested the Pathways to Mathematics model validated LeFevre et al.'s finding that the quantitative pathway uniquely predicted numeracy skills (Cirino, 2011; Hornung et al., 2014; Sowinski et al., 2015). Overall, there is strong evidence that domain-specific quantitative abilities play an important role in early numeracy development.

Early numeracy research also suggests that a variety of quantitative measures predict early numeracy. The quantitative measure used by LeFevre et al. (2010) was a subitizing task. Subitizing is the ability to rapidly identify the quantity of small sets of objects and several studies have found it to be a strong predictor of early numeracy skills (Geary, 2011; Kroesbergen et al., 2009; Landerl, 2013; Reeve, Reynolds, Humberstone, & Butterworth, 2012). A second measure that several studies identified as a significant predictor of early numeracy was symbolic magnitude comparison, where children are shown two numbers and asked to determine which is greater (De Smedt et al., 2009; Lyons et al., 2014; Sowinski et al., 2015). A third measure that several studies found to predict early numeracy was non-symbolic magnitude comparison, where children are shown two sets of dots and asked to determine which set has more dots (De Smedt, van der Schoot, & van Lieshout, 2013; Halberda et al., 2008; Hornung et al., 2014; Libertus, Feigenson, & Halberda, 2011; Xenidou-Dervou, ). In studies that tested the Pathways to Mathematics model, Cirino (2011) and Hornung et al. (2014) added symbolic and non-symbolic magnitude comparison to the quantitative pathway and found that it remained a valid predictor of early numeracy.

**Linguistic Predictors**

A number of studies have validated LeFevre et al.'s (2010) inclusion of a domain-general linguistic pathway by identifying linguistic abilities as a key predictor of early numeracy (Hornung er al., 2014; Krajewski & Schneider, 2009; Kroesbergen et al., 2009; Locuniak & Jordan, 2008; Purpura & Ganley, 2014; Simmons, Singleton, & Horne, 2008; Sowinski et al., 2015). The most common explanation for linguistic skills' contribution to early numeracy is that they set the foundation for numeracy development by allowing children to understand and articulate numerical symbols (Krajewski & Schneider, 2009; Purpura & Ganley, 2014). The idea that linguistic skills play a foundational role in numeracy development has been supported by several studies that found linguistic skills broadly predicted performance on various early numeracy measures (Cirino, 2011; Purpura, & Ganley, 2014; Simmons et al., 2008; Sowinski et al., 2015). As a result, early numeracy research supports the inclusion of linguistic skills as a predictor of early numeracy.

Studies examining the link between language skills and early numeracy have used a wide variety of measures (LeFevre et al., 2010; Purpura, & Ganley, 2014; Simmons et al., 2008). Two of the most common measures were tests of phonological awareness and receptive vocabulary. Phonological awareness is the ability to identify and manipulate different components of oral language, and receptive vocabulary is all of the words that a person understands. Despite differing on the precise tests that they used, a number of studies linking linguistic skills to early numeracy included phonological awareness and receptive vocabulary measures (Cirino, 2011; Hornung et al., 2014; Krajewski & Schneider, 2009; Simmons et al., 2008; Sowinski et al., 2015). Drawing on those studies, the linguistic measures in the current study are phonological awareness and receptive vocabulary tasks.

**Working Memory Predictors**

LeFevre et al.'s (2010) use of spatial attention as a predictor of early numeracy is supported by a number of studies (Bull et al., 2008; Geary, 2011; Hornung et al., 2014; Krajewski & Schneider, 2009; Sowinski et al., 2015). However, spatial attention is essentially one of three components of working memory. Working memory is the system responsible for active maintenance and temporary storage of task-relevant information (Miyake & Shaw, 1999) and it has been found to be predictive of early numeracy skills (Bull et al., 2008; Geary, 2011; Mazzocco & Kover, 2007; Passolunghi & Lanfranchi, 2012; Purpura, & Ganley, 2014). Based on Baddeley's model of working memory (2001), research also suggests that all three aspects of working memory, the visual-spatial sketchpad, the central executive, and the phonological loop, contribute to early numeracy skills (Bull et al., 2008). Based on this research, working memory, not just spatial attention, should be tested as a predictor of early numeracy.

Incorporating this research, both Hornung et al. (2014) and Sowinski et al. (2015) included a broader set of working memory tasks when they tested the Pathways model and found that the expanded pathway uniquely predicted early numeracy. To expand the spatial attention pathway, Hornung and Sowinski selected tasks that measured the three components of Baddeley's (2001) model of working memory. Both used a forward digit span task, a reverse digit span task, and a visual-spatial span task (Hornung et al., 2014; Sowinski et al., 2015).

**Control Variables**

Past early numeracy research has included a wide variety of control variables. One of the most commonly used controls was age, which Jordan et al. (2009) found was associated with quantitative skills in Kindergarten. Many early numeracy studies chose to account for variations in age (Bull et al., 2008; Cirino, 2011; Geary et al., 2007; Jordan et al., 2009; Reeve et al., 2012). Another common control in early numeracy studies is processing speed (De Smedt et al., 2009; Geary, 2011; Passolunghi, & Lanfranchi, 2012; Sowinski et al., 2015). Processing speed is the time that it tasks for basic cognitive process to be completed (Salthouse, 1996) and it has been found to be a predictor of early numeracy skills (Geary 2011; Passolunghi, & Lanfranchi, 2012).

**Current Study**

The aim of the current study is to develop a better understanding of the cognitive predictors of early numeracy by testing the Pathways to Mathematics model. With its inclusion of three types of cognitive predictors, the Pathways model is a useful framework for exploring how different cognitive abilities contribute to early numeracy. In addition, the model has been supported by several subsequent studies and the broader body of early numeracy research suggests that each pathway is a valid predictor of early numeracy. The design of the present study will, however, deviate from LeFevre et al.'s original study by including additional measures and controls identified by other early numeracy studies. In the current study, the spatial attention pathway includes a verbal working memory measure, the quantitative pathway includes symbolic and non-symbolic magnitude comparison measures, and age and processing speed are the control variables. By testing the Pathways to Mathematics model with this unique set of measures and controls, the current study is an expansion of the existing research into the relations between cognitive abilities and early numeracy. Overall, it is hypothesized that quantitative, linguistic, and working memory abilities will each significantly predict early numeracy.

# Methods

## Participants

Participants were 155 children (74 male) tested in the spring of their Senior Kindergarten year in 14 elementary schools in the Brant Haldimand Norfolk Catholic District School Board. The children's average age was 70.10 months (*SD* = 3.48 months, range = 64 - 77 months). Each student in a participating classroom received a fancy pencil on the day that consent forms were due, whether parental consent was given or not. At the end of each session, children that agreed to participate and had parental consent received a sticker.

## Measures

**Subitizing.** Subitizing was measured using the Count Dots task ($\alpha$ = .95, Lyons et al., 2014). In this task, children were presented a set of 1-8 dots on an iPad screen and asked, "how many are there?". Children responded verbally to 24 trials (i.e., 3 trials at each set size). The task concluded if no input was recorded for four consecutive trials. Subitizing slopes were created using response times for the arrays containing 1-3 dots. Children's response time slopes were calculated and used as the measure of subitizing. A lower slope suggests that the children were subitizing the dot array, as opposed to counting. Lower response times are associated with subitizing because it is a quick pattern-matching process, whereas counting is a slower process (LeFevre et al., 2010).

**Magnitude Comparison**. Magnitude comparison was measured in two separate tasks, one symbolic and the other non-symbolic. In symbolic magnitude comparison, children viewed two single-digit numbers, with one on each side of the iPad screen. Children were asked to "touch the number that is more", but were reminded to be as accurate and fast as possible (α = .98, Lyons et al., 2014). Similarly, for non-symbolic magnitude comparison, children viewed two different single-digit dot arrays on each side of the iPad screen. Children were asked "which side is more?" and told to be as fast and accurate as possible (α= .96, Lyons et al., 2014). There were 18 trials for both symbolic and non-symbolic magnitude comparison. The tasks were concluded if no input was recorded for 5 consecutive trials. The children's total error rates were recorded as the measures of symbolic and non-symbolic magnitude comparison.

**Visuospatial Working Memory**. Visuospatial working memory was measured using a spatial span task. A computerized version of the Corsi Block task was used (α = .70, LeFevre et al., 2010). Children were shown a set of nine lily pads and asked to watch the frog's jumping sequence. The minimum span of the frog's jumping sequence was two, and the maximum span was seven. The length of the span increased after the children completed two trials at each length. In the forward task, the child was asked to copy the frog's path in the same order that the frog jumped. In the reverse task, children were asked to copy the frog's path backwards. The experimenter did one demonstration in both task types and each path was presented twice. The task was concluded if the child gave incorrect answers for two spans of the same length. Children's maximum span, which is the largest sequence of jumps they correctly remembered, was recorded as the measure of visuospatial working memory.

**Verbal Working Memory.** Verbal working memory was measured using a digit span task. Children were read a series of numbers (e.g., 5,8,2) and were asked to repeat the sequence back either forward or backwards. The Children were given one practice trial at the start of the forward and backward trials. The spans began with a minimum of two numbers and increased by one digit to a maximum of nine. The length of the span increased after the children completed two trials at each length. The task concluded when the child incorrectly recited both spans of the same length. Children's maximum span, which is the largest sequence of digits they correctly remembered, was recorded as the measure of verbal working memory.

**Phonological Awareness**. Phonological awareness was measured using the Elision subtest of the Comprehensive Test of Phonological Processing II (CTOPP 2; Wagner, Torgesen, Rashotte, & Pearson, 2013). The task is 34-items long ($\alpha$ = .90, LeFevre et al., 2010). Children were asked to hear a word and say the word again, but omit a sound. An example of this would be asking a child to say the word "brat" without the /r/. The correct answer to this is "bat". The task concluded after three consecutive errors. Children's total raw scores were used as the measure of phonological awareness.

**Receptive Vocabulary**. Children's Receptive Vocabulary was measured using the Peabody Picture Vocabulary Test – Revised, Form B (PPVT, Dunn & Dunn, 1997). Children viewed four images and were asked to select the picture that corresponds to the verbally presented word. The verbally presented words increased in difficulty as the participant continued. The task concluded after six incorrect answers in eight consecutive questions. Children's total raw scores were used as the measure of receptive vocabulary ($\alpha$= .94, LeFevre et al., 2010).

**Processing Speed**. Processing speed was measured using a simple choice reaction time task ($\alpha$ = .89, Le Fevre, 2010). Two types of stimuli (an X or an O) were displayed for 1 second. Children were instructed to press a button corresponding to the target letter shown on the screen.

There were 24 trials, and the task was terminated if there was no input recorded for three consecutive trials. Children's average reaction times for the 24 trials were used as the measure of processing speed.

**Mathematics Achievement.** Mathematics Achievement was measured using the Numeration subtest of the multi-domain math achievement test, the KeyMath Test-Revised ($\alpha$ = .70, Connolly, 2000). It covers concepts such as quantity, order, and place value over 49 items. An example of a question in the Kindergarten range is, "count to 4". The test concluded after four consecutive incorrect answers. Children's KeyMath raw scores were used as the outcome variable.

**Procedure**

Ethics approval was obtained from the School Board, who then provided a list of principals that were interested in having their schools participate in the study. Principals provided a list of interested teachers, who were contacted by the experimenter. A total of 14 schools agreed to participate.

In Spring of Senior Kindergarten, students completed two 30-minute testing sessions administered by trained research assistants. The children verbally assented before each session, and their parents gave written consent prior to testing. Sessions were completed individually in a quiet room next to the class room or in the library. One set of tasks was completed using pencil and paper and a second set was completed using an iPad. Within each session, the order of task was consistent for each student. In the pencil and paper session, students completed the KeyMath numeration test, verbal working memory task, and the Peabody Picture Vocabulary Test. In the iPad session, children completed the visual working memory task, magnitude comparison tasks, processing speed, subitizing, and the Comprehensive Test of Phonological Processing II.

# Results

## Principle Components Analysis

A factor analysis was conducted to create a factor for each cognitive predictor. The quantitative factor is comprised of subitizing, symbolic magnitude comparison and non-symbolic magnitude comparison, which loaded on a single factor, together explaining 55.48% of the variance. The linguistic factor is comprised of phonological awareness and receptive vocabulary, which loaded on a single factor, together explaining 68.87% of the variance. The working memory factor is comprised of visuospatial and verbal working memory both forward and reverse, which loaded on a single factor, together explaining 46.46% of the variance.

## Descriptive statistics and correlations

Descriptive statistics for all measures are reported in Table 1. Correlations among factors, control measures and the outcome variable are reported in Table 2. Pearson's bivariate correlation was used to analyze the relations between linguistic abilities, quantitative abilities, working memory abilities, KeyMath scores, processing speed and age. The linguistic factor significantly correlated with KeyMath, $r(152) = .55$, $p < .001$. This correlation shows that higher receptive vocabulary and phonological awareness scores were associated with higher scores on the math achievement test. Next, the quantitative factor significantly correlated with KeyMath scores, $r(152) = -.35$, $p < .001$. The correlation indicates that faster response times in the subitizing task and fewer errors in the magnitude comparison tasks were associated with higher scores on the math achievement test. Finally, working memory significantly correlated with KeyMath, $r(152) = .52$, $p < .001$. This correlation indicates that higher scores on the digit and spatial span tasks were associated with higher scores on the math achievement test.

Table 1
Descriptive Statistics (*N*= 154)

| | Mean | SD |
|---|---|---|
| Age (Months) | 70.10 | 3.48 |
| Processing Speed (ms) | 855.30 | 222.47 |
| Subitizing (ms) | 181.70 | 160.16 |
| Symbolic Comparison (% error) | .07 | .11 |
| Non-Symbolic Comparison (% error) | .09 | .11 |
| Receptive Vocabulary | 112.60 | 14.04 |
| Phonological Awareness | 11.24 | 5.55 |
| Digit Span Reverse | 2.28 | 1.02 |
| Digit Span Forward | 4.43 | .88 |
| Corsi Block Forward | 2.94 | 1.48 |
| Corsi Block Reverse | 2.28 | 1.47 |
| KeyMath | 6.96 | 2.69 |

Table 2
Correlation Among Measures (*N*= 154)

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1. Age (Months) | | | | | | |
| 2. Processing Speed (RT) | -.027 | | | | | |
| 3. Linguistic Factor | .183* | -.089 | | | | |
| 4. Working Memory Factor | .234** | .003 | .562** | | | |
| 5. Quantitative Factor | -.094 | .023 | -.401** | -.413 | | |
| 6. KeyMath | .122 | -.097 | .551** | .520** | -.350** | |

*$p$<.05, **$p$<.01

**Predicting Math Achievement**

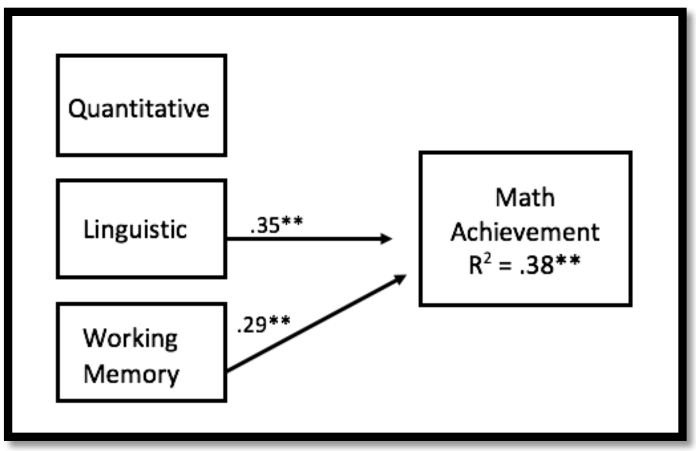
A multiple regression was used to determine how quantitative, linguistic and working memory factors predict KeyMath scores. It was entered in a two block design with processing speed and age in the first block and quantitative, linguistic and working memory factors in the second block. The results of the multiple regression show that the model significantly predicted math achievement over and above the control variables, $R^2 = .38$, $F(5, 148) = 18.08$, $p < .001$(see Table 3). Linguistic scores significantly predicted math achievement; participants with higher linguistic scores typically answered more math questions correctly, $\beta = .35$, $p < .001$ (see Figure 1). Working memory scores significantly predicted math achievement; participants with higher working memory scores typically answered more math questions correctly, $\beta = .29$, $p = .001$. Quantitative scores did not significantly predict math achievement. There was, however, a trend such that participants who made fewer comparison errors and had lower subitizing slopes typically answered more math questions correctly, $\beta = -.25$, *ns*.

Table 3
Regression analysis predicting KeyMath scores from cognitive abilities and control variables (*N*= 154)

| | β |
|---|---|
| Age (Months) | -.001 |
| Processing Speed (RT) | -.02 |
| Quantitative Factor | -.25 |
| Working Memory Factor | .77** |
| Linguistic Factor | .92** |
| Total *R* | .38 |

**$p$<.01

*Figure 1.* Regression model predicting KeyMath scores. Standard regression coefficients shown for significant pathways only. ** indicates $p$<.01



## Discussion

The purpose of the current study was to examine and clarify the relations between certain cognitive abilities and early numeracy skills. Drawing on the Pathways to Mathematics Model, this study focused on quantitative, linguistic, and working memory abilities (LeFevre et al., 2010). Based on previous studies that support their contribution to early numeracy, it was hypothesized that quantitative, linguistic and working memory abilities would each significantly predict early numeracy. Similar to past tests of the pathways model, it was found that the model was predictive of early numeracy. Additionally, consistent with past research, the current study's findings supported the predictability of both linguistic and working memory abilities for early numeracy. However, contrary to past research, quantitative abilities were not found to be significant predictors of early numeracy. In addition, the effects that age and processing speed had on early numeracy in the current study were inconsistent with past research as well.

Consistent with past tests of the Pathways model, linguistic and working memory abilities were both significant predictors of early numeracy. The finding that linguistic skills significantly predicted early numeracy skills supports previous studies that found language skills were key to children's understanding of symbolic number systems (Krajewski & Schneider, 2009; Purpura, & Ganley, 2014). This finding also supports past research that found phonological awareness and receptive vocabulary, which were the linguistic abilities measured in the current study, uniquely contribute to numeracy skills in children (Krajewski & Schneider, 2009; Lefevre et al, 2010). The current study's finding that working memory significantly predicted early numeracy is also in line with past research (Bull et al., 2008; Geary, 2011; Passolunghi & Lanfranchi, 2012). As a result, it also supports the idea that all aspects of working memory, not just spatial attention, contribute to early numeracy (Hornung et al., 2014; Sowinski et al., 2015).

Similar to Lefevre et al. (2010), Cirino (2011), Hornung et al. (2014) and Sowinski et al.'s (2015) studies, both of the domain-general predictors uniquely predicted children's numeracy skills. This finding is consistent with past studies that suggested various domain-general abilities, including working memory and language skills, predicted early numeracy (Passolunghi, & Lanfranchi, 2012; Purpura & Ganley, 2014). It is contrary, however, to past research that indicated domain-specific abilities related to numerical processing were the key predictors of early numeracy (Jordan et al. 2009). Therefore, the current study offers further evidence that domain-general cognitive abilities play an important role in numeracy development.

In the current study, the domain-specific, or quantitative, factor significantly correlated with early numeracy skills, but it did not uniquely predict. This finding contradicts much of the previous research in this field (Jordan et al., 2009; Krajewski & Schneider, 2009; Lyons et al., 2014). It also contradicts studies that previously tested the Pathways model and found the current study's quantitative measures to be predictive of early numeracy (Cirino, 2011; Hornung et al., 2014; Sowinski et al., 2015).

One possible explanation for this finding is that the choice of outcome measure could have impacted the predictive value of the measures used to test quantitative abilities. LeFevre et al. (2010) noted that the predictability of their cognitive precursors varied across the different outcome measures that they used. In addition, Sowinski et al. (2015) found that their quantitative measures were more predictive of certain early numeracy skills, but less predictive of others. While Lefevre et al. (2010) did find that quantitative abilities uniquely predicted performance on the outcome measure used in the current study, the Numeration subtest of the KeyMath Test – Revised, the current study used different tasks to measure quantitative abilities. Therefore, it is possible that the quantitative measures used in the current study are simply not as predictive of the specific numeracy skills tested by the Numeration subtest of the KeyMath Test – Revised.

A second possible explanation for the quantitative factor's inability to uniquely predict early numeracy is the inclusion of non-symbolic magnitude comparison as a quantitative measure. Contrary to several studies that found non-symbolic magnitude comparison predicted early numeracy development (Hornung et al., 2014; Libertus et al., 2011; Xenidou et al., 2013;), some research has shown that it may not be a strong predictor (Holloway & Ansari, 2009; Rousselle & Noël, 2007). More specifically, some research has found no significant difference between the non-symbolic magnitude comparison scores of children with a mathematical learning disability and the scores of children with typical numeracy skills (Holloway & Ansari, 2009; Rousselle & Noël, 2007). As a result, it is possible that the non-symbolic magnitude comparison task is not a strong predictor of children's numeracy skills.

In light of the uncertainty around non-symbolic magnitude comparison's predictive value, a new quantitative factor was created using only subitizing and symbolic magnitude comparison. A multiple regression was conducted with processing speed and age in the first block and quantitative, linguistic and working memory factors in the second block. The results indicated that the model still significantly predicted early numeracy, and that linguistic and working memory scores still significantly predicted early numeracy. However, without non-symbolic magnitude comparison, quantitative scores significantly predicted early numeracy skills; participants with fewer symbolic comparison errors and lower subitizing slopes typically answer more math questions correctly, $\beta = -.39$, $p = .039$. This finding suggests that non-symbolic magnitude comparison limited the predictability of the quantitative factor. Thus, the choice of quantitative measures used impacts the predictive value of the quantitative pathway.

In the current study, age significantly correlated with language and working memory. This finding is not consistent with Jordan et al.'s (2009) finding that age significantly correlated with quantitative skills in Kindergarten. It also contradicts Cirino's (2011) finding that age was modestly related to performance across all measures. However, taken together, these findings indicate that age may influence performance on tasks that have been shown to predict early numeracy. As such, the age of participants should not be discounted in studies related to early numeracy.

The current study's results also showed that processing speed did not predict early numeracy. This finding contradicts past research, which has typically shown that processing speed contributes to early numeracy skills (Geary, 2011; Passolunghi & Lanfranchi, 2012). One possible explanation for this disparity is that past studies suggesting processing speed predicted early numeracy were designed differently than the current study. Whereas the current study tested Kindergarten students' processing speed and numeracy skills concurrently, it appears other early numeracy studies that included processing speed were either longitudinal or they sampled older students (Bull & Johnston, 1997; Geary, 2011; Passolunghi & Lanfranchi, 2012; Sowinski et al., 2015). Therefore, differences in the current study's design may have affected processing speed's ability to predict numeracy.

Though this study's findings provide a useful look at the relations between cognitive predictors and early numeracy, it is not without its limitations. First, the data was collected concurrently, whereas research in developmental fields is typically conducted using a longitudinal design. Longitudinal designs are important in developmental research because they allow researchers to understand the temporal development of skills and understand the direction of causality. In addition, many of the studies that the current study was based upon were longitudinal. As a result, the use of concurrent data collection may cause the current study's results to vary from past research. A second limitation of the current study is the inclusion of the non-symbolic magnitude comparison task. As demonstrated by the secondary analysis that excluded its results, the inclusion of the non-symbolic magnitude comparison task in the quantitative factor reduced its predictive value. These results, combined with the inconsistent findings of past research, suggest that the non-symbolic magnitude comparison task's contribution to early numeracy should be explored further.

The current study's findings indicate that future researchers should examine whether non-symbolic magnitude comparison is a reliable predictor of early numeracy. To better understand the current study's results, future studies could specifically consider whether the task's predictability is affected by concurrent data collection, impacted by the choice of outcome measure, or mediated by the effects of other well-supported predictors. More broadly, considering the inconsistency of existing research, future studies could test non-symbolic magnitude comparison's predictability across a wide range of ages and skills levels to try and identify cases where it is and is not predictive of early numeracy.

In conclusion, the current study's hypothesis was not fully supported. Though linguistic and working memory abilities significantly predicted early numeracy, the current study's quantitative measures did not. Despite this, the current study contributes to early numeracy research by reaffirming the importance of domain-general cognitive abilities and highlighting potential predictability issues with the non-symbolic magnitude comparison task. Going forward, this research can help identify children who are struggling to develop early numeracy skills and inform the development of effective numeracy intervention tools.

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