Scaled Physical Modeling of the Dynamics of Granular Flow in Hopper Silos

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ABSTRACT

To understand the challenges on grain flow during handling in hoppers and silos require better understanding of the granular physical flow characteristics through experiments done at large scale. However, due to the complexity of the large, controlled tests, scaled models are often used to study the behaviour. Very few of these model scale studies had been done appropriately taking the effects of gravity on silo discharge into account. This subject has also been widely investigated in different areas of geotechnical engineering related to soil erosion, land slides, and dynamic liquefaction. Granular flow rate and the dynamics of the flow are also studied in fields like chemical, mining and material engineering. Two conditions for scaled model testing have been investigated in this study: the dynamics of the granular flow at natural gravity and enhanced gravities of 1.9, 3.8, 7.7 and 11.7 g conducted in a centrifuge. The granular outflow tests have covered the features of the granular shear flow, outflow rates and deformation patterns. The observed granular flow rates (W) for the novel half hopper model under natural and enhanced gravities were then estimated from the Beverloo et al. (1961) relations and were compared. Particle image velocimetry was used to study the vertical particle velocity profiles and the flow deformation patterns during the natural and enhanced gravity tests. The granular flow rate was found to be directly proportional to the ratio of orifice size and influenced by the soil particle size (W \propto \frac{D_o}{d}). However, this comparison was found to be more applicable for cohesionless soils compared to soils with small cohesions. The results in this study show that the flow mechanisms in a cylindrical hopper are quite complex and vary both temporally and spatially. Three distinct zones of behaviour were observed from the particle image velocimetry analysis: a) an upper inflow zone, b) a narrow vertical funnel flow zone and c) a near static stagnant zone. For the enhanced gravity tests, the mass flow direction inside and outside of the hopper was observed to be influenced by the resultant inclined vector of the enhanced gravity due to the combined effect of the centrifuge rotation and Earth’s gravity.

KEYWORDS: Granular flow, Granular flow rate, Outflow rate, Dynamics of outflow, Orifice size, Particle image velocimetry, Flow pattern, Centrifuge, Scaling laws, Enhanced gravity
SUMMARY FOR LAY AUDIENCE

To understand the challenges of the flow of particles during handling in hoppers and silos, we require better understanding of the physical flow characteristics through experiments done at full scale. However, due to the complexity of large-scale tests, reduced scale models are often used to study this behaviour. These scaled model tests of the outflow of particles can be completed using a geotechnical centrifuge. A centrifuge induces higher gravity levels than natural gravity thereby increases material self-weight and allowing small model tests to simulate large-scale prototype. Such behaviour is studied in fields like geotechnical, chemical, mining and material engineering.

Two conditions for the scaled model hopper testing have been investigated in this study: the behaviour of particle flow at natural gravity and at enhanced gravities of 1.9, 3.8, 7.7 and 11.7 g. The centrifuge particle outflow tests covered the important features such as rate of outflow, internal deformation patterns and material outflow behaviour as it leaves the opening of the hopper. The observed granular flow rates (W) for the novel half hopper model under natural and enhanced gravities were then estimated using the Beverloo et al. (1961) relations and were compared. Particle image velocimetry was used to study the vertical particle velocity profiles and the flow deformation patterns during the tests. The flow rate (W) was found to be directly proportional to the ratio of hopper orifice size (D_o) and influenced by the soil particle size (d). Three distinct zones of behaviour were observed from the particle image velocimetry analysis: a) an upper inflow zone, b) a narrow vertical funnel flow zone and c) a near static stagnant zone. For the enhanced gravity tests, the mass flow direction inside and outside of the hopper was observed to be influenced by the non-vertical resultant self-weight force of enhanced gravity due to the combined effect of the centrifuge rotation and Earth’s gravity.
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Finally, I would love to thank my sweet daughters Mary and Miriam, my parents Ngsti and Okbayes, and my whole family for their love, encouragement and patience throughout my research period.
DEDICATION

To my dear sister Letebrhan Okbayes, from the Santa Anna Sisters, who have dedicated her whole life to education and the growth of myself and others!
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_o$</td>
<td>Area of orifice</td>
</tr>
<tr>
<td>$C$</td>
<td>Discharge coefficient which depends on bulk density</td>
</tr>
<tr>
<td>$d$</td>
<td>Soil particle diameter</td>
</tr>
<tr>
<td>$D$</td>
<td>Hopper diameter</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Hydraulic diameter</td>
</tr>
<tr>
<td>$D_{h'}$</td>
<td>Effective hydraulic diameter</td>
</tr>
<tr>
<td>$D_o$</td>
<td>Orifice diameter</td>
</tr>
<tr>
<td>$D_{10}$</td>
<td>Grain size diameter for 10% passing</td>
</tr>
<tr>
<td>$D_{30}$</td>
<td>Grain size diameter for 30% passing</td>
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<td>$D_{50}$</td>
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<td>$e_{min}$</td>
<td>Minimum void ratio</td>
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<tr>
<td>$e_{max}$</td>
<td>Maximum void ratio</td>
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<tr>
<td>$h$</td>
<td>Slanted hopper height</td>
</tr>
<tr>
<td>$h_o$</td>
<td>Vertical hopper height</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration</td>
</tr>
<tr>
<td>$g^*$</td>
<td>Increased gravity value by N</td>
</tr>
<tr>
<td>$H$</td>
<td>Vertical silo height</td>
</tr>
<tr>
<td>$H_f$</td>
<td>Vertical silo fill height</td>
</tr>
<tr>
<td>$i_z$</td>
<td>Height ratio</td>
</tr>
<tr>
<td>$k$</td>
<td>Shape coefficient in Beverloo flow equation</td>
</tr>
<tr>
<td>$L^*$</td>
<td>Reduced model length by N</td>
</tr>
<tr>
<td>$N$, $n$</td>
<td>Scaling factor</td>
</tr>
<tr>
<td>$P_o$</td>
<td>Orifice perimeter</td>
</tr>
<tr>
<td>$r$</td>
<td>$1/N$ scaled model location (distance) from centrifuge center</td>
</tr>
<tr>
<td>$W$</td>
<td>Granular flow rate</td>
</tr>
<tr>
<td>$x^*$</td>
<td>Scale factor (in x, y and z direction)</td>
</tr>
<tr>
<td>$x_m$</td>
<td>Dimension of the model in the x − direction</td>
</tr>
</tbody>
</table>
\( x_p \) = Dimension of the prototype in the x-direction

\( Z \) = Particle shape factor

\( \alpha \) = Angle of repose

\( \alpha_s \) = Static angle of repose

\( \alpha_d \) = Dynamic angle of repose

\( \beta \) = Beta, proportional arch to aperture ratio

\( \gamma \) = Soil unit weight

\( \theta \) = Cone internal inclination angle

\( \rho_b, \rho \) = Bulk soil density

\( \rho_s \) = Particle density

\( \mu \) = Coefficient of friction

\( \phi_c \) = Critical angle of internal friction

\( \phi_p \) = Peak angle of internal friction

\( \Omega \) = Rotational speed
CHAPTER 1: INTRODUCTION

1.1 Overview

Granular materials are typically required to be stored and transported during their use. Common granular materials are sand, gravel, cement, seeds, and food grains (Meza-Diaz et al., 2003). Silos with conical hopper bases have been used for centuries to store and handle bulk materials in areas of food processing, mining, and chemical operations. They are usually filled from the top and emptied from a narrow opening from the bottom. One of the first stages of any silo design procedure is the choice of an appropriate outflow regime. For the emptying process, it is well known that the silos wall stiffness, roughness and geometry, and grain self-weight and shearing properties will control the flow regimes, which could be either mass flow or pipe (funnel) flow (Evgeny et al., 2020). In mass flow shown in Fig. 1.1 (a), the entire particulate material volume moves simultaneously during discharge. In funnel flow shown in Fig. 1.1 (b), the material in the central region of the silo exits first, followed by the material closest to the walls.

![Fig. 1.1: (a) mass flow (b) funnel flow](source: Evgeny et al., 2020)

The constant diameter upper section of the system is usually referred to as a ‘silo’ and the inclined lower contraction the ‘hopper’. Fig. 1.2 shows the geometric parameters of a typical cylindrical silo. A silo may contain only an inclined hopper part (‘A’ in Fig. 1.2) or a vertical walled body over a sloping hopper body (‘B’ and ‘A’ in Fig. 1.2). The literature review in Chapter 2 describes relevant research on the influence that the vertical fill height ($H_f$ and $h_o$) and the hopper or silo diameter ($D$) have on the granular flow rate. The out-flow orifice diameter ($D_0$) and the hopper
angle of inclination (θ) are also reported to affect the granular flow rate and the flow pattern (see Fig. 1.1) significantly.

![Diagram of a cylindrical silo and hopper]

Fig. 1.2: Schematic diagram of a cylindrical silo and hopper

The height of the hopper section (h₀), geometrically, depends on the size of the orifice diameter (D₀) and the hopper angle of inclination (θ). However, it is very important to define a reference point to study the granular flow within the body of the hopper. To study the flow behaviour in the hopper, horizontal (x) and vertical (y) reference coordinates are often considered. The normalized height ratio (iₗ) and width ratio (i₇) can be respectively used to represent a specific point or line in the hopper body. Equations 1.1 and 1.2 give the height ratio and width ratio for defining a point or line of study in the body of the hopper during out flow. The width ratio (i₇) provides a reference distance left (-ve) or right (+ve) of the vertical center line of the hopper due to symmetry. Height ratio (iₗ) gives the position of a point or a line upwards from a datum of y=0 units at the outflow orifice point as shown in Fig. 1.2.

\[ iₗ = \frac{h}{h₀} \tag{1.1} \]
This thesis investigates the behaviour of granular materials as they flow through an opening at the base of a conical hopper.

1.2 Granular Flow in Hoppers and Silos Under Natural and Enhanced Gravity

Granular materials can have multiphase behaviour since they can behave like solids, fluids or gases. The specific form of granular flow is controlled by the particular boundary or loading conditions. A good example of granular flow can be sand grains within a silo or a hopper. These grains are able to exert significant additional stresses perpendicular to the wall due to force-chains and shearing, whereas Newtonian fluids are only able to exert a hydrostatic pressure. Describing the flow of granular materials is phenomenological different from the flow of typical fluids due to, for instance, the presence of friction; and existing continuum models need modifications to accurately predict this flow behaviour and the applied forces. Therefore, granular mass flow is strongly related to the shearing motion of a collection of discrete solid particles, and this is commonly seen and widely utilized in various engineering and non engineering works.

In general, granular flow through an opening is encountered in many real-life problems in the fields of geotechnical engineering, food processing, chemical and material engineering. Some specific applications are: sand control in petroleum oil wells (Meza-Diaz et al., 2003), granular flow in hoppers and silos (Hilton and Cleary, 2011; Rao and Nott, 2008), sinkhole and ground surface collapse (Hermosilla, 2012), and soil loss into defective sewer pipes (Guo and Zhu, 2017). The underlying physical mechanisms during granular flow in these cases are particle free-fall and arch formation which produces specific characteristics in the granular flow, i.e. the flow rates and stresses are independent of the material height over the orifice, unlike fluid flow (Fowler and Glastonbury, 1959).

Many research works refer to the importance of experimental and analytical study of wall pressure distributions and flow analysis for different silo geometries (Nortje, 2002). In most cases, structural or operational failures occur due to inadequate design analysis of the dynamic behaviour of the bulk material during the discharge of the hopper (Yong, 1990). It should be noted that “dynamic” behaviour in the context of the movement of bulk material in silos or hoppers refers to

\[ i_D = \frac{\pm x}{D/2} \]  

(1.2)
the general properties and problems of granular materials as they flow due to gravity. Some common problems in hopper granular flow are arching, funnel flow with wide residence time and deterioration in product quality, ratholing, segregation, vibration, and noise (Schulze, 2007). For example, ratholing, in Fig. 1.3 (b), occurs during funnel-flow where the hopper may not empty completely under the force of gravity alone. Segregation, shown in Fig. 1.3 (c), is a common occurrence in funnel flow, where the grains or a powder show separating non-uniform fractions during discharge, as reported by Johanson (1978). Arching, as shown in Fig. 1.3 (a), can be defined but is not limited to, the formation of an inverted curve supported stagnant mass of bulk material in a bin or hopper upon opening of the orifice (outlet) or during a gravitational flow (see Chapter 2 for more details about arching during flow).

Fig. 1.3: Common problems of granular flow (source: Zuriguel et al., 2003)

The granular flow rate is the amount of flowing material passing through an orifice over a specific period of time. Many studies had developed different empirical formulas to calculate the granular flow rate for different silo and hopper geometries. It is important to understand interaction of the silo or hopper geometry, and the physical and mechanical properties of the granular materials, to understand how the granular outflow rates are influenced by the particle motions within the hopper body.

The granular flow of materials through an orifice has been widely studied for decades due to great interest in efficient and controllable outflow in industrial applications ranging from silos to
hoppers. Srivastava et. al (2003) showed that the discharge rate of granular materials from a bin is dependent on local conditions near the orifice, and it has been argued that the free-fall zone scales with the orifice diameter ($D_o$) or width. The free-fall surface defines the zone below which the granular material falls mostly due to the effect of gravity.

Many approaches have been used to better understand granular flow in these systems, such as analytical methods, numerical modeling (e.g., finite element and discrete element methods), and experimental full scale and reduced scale tests. However, measurements of flow patterns in full-scale silos are very rare and observations are typically based on changes in the top surface profile, outflow rate measurements or signs of abrasion and polishing of the hopper/silo walls (Ooi et al., 1998). Most internal velocity field or flow pattern studies have therefore used reduced full scale and scaled laboratory models. These approaches are often small enough to allow novel visualization techniques, such as spy-holes, transparent walls, radio transmitters, x-rays and positron emission, etc. (e.g., Bransby and Blair-Fish, 1974; Rotter et al., 1989). However, the effect of stress levels on the flow behaviour are extremely important and extrapolation of scaled model observations conducted under natural gravity to full-scale is still uncertain.

Large scale tests are currently necessary (and expensive) because the friction, density and flow properties of a granular material are related to self-weight, normal contact forces and tangential forces, and the stresses experienced in small scaled models are generally much lower than those experienced in full-scale silos. Some of these challenges can be addressed by using an enhanced gravity field for the model (e.g., placing it into a large centrifuge) and the criteria for modelling similarity has been defined previously by Nielsen and Askegaard (1977). A limited number of studies have been conducted using centrifuge-based silos (e.g., Mathews and Wu, 2016; Nielsen and Askegaard, 1977; Barbir and Mathews, 2016). However, this phenomenon has not been widely investigated in geotechnical centrifuges and this is undoubtedly due to the challenges associated with making quantitative observations of the internal flow. However, the recent developments of image techniques such as particle image velocimetry has provided greater opportunities. Uncertainties still exist about the complex velocity fields that occur within silos during material discharge, which makes many of the former analytical and numerical predictions somewhat speculative and design of these structures difficult.
The development of centrifuge based scaled models of hopper granular outflow would provide a very useful additional tool for development of the design and operation of silos and hoppers. In addition, the commissioning of the Western geotechnical drum centrifuge (in 2019) has made the investigation of outflow from small hoppers at enhanced ‘g’ levels necessary since this is the primary method of filling the drum with soil.

1.3 Research Objectives

The overall goal of this research thesis is to better understand the characteristics of flowing granular materials through hoppers and measure the granular outflow rate for model hoppers at natural gravity and at elevated gravity in a centrifuge. A half model hopper is used to study the flow mechanisms through the transparent plexiglass window with particle image velocimetry (PIV). The objectives of the research can be summarized as:

To investigate:

- The effect of particle size on outflow rate (W) of granular materials from a hopper;
- The effect of orifice size on outflow rate (W) of granular materials from a hopper;
- The effect of soil density and static angle of repose on the outflow rate (W) of granular materials from a hopper;
- The effect of ‘g’ level on the outflow rate (W) of granular materials from a hopper;
- The particle velocity magnitudes and understand the flow and deformation patterns of the granular material during outflow for different particle sizes, soil density, orifice sizes and ‘g’ level;
- Zones of dynamic arching and regions of free fall (through the location of the free fall surface) and any dynamic jamming events;
- Dimensionless parametric characteristics of the granular flow and identify and potentially modify flow equations for hoppers and silos to suit the scaled models used in this study.

1.4 Thesis Outline

The thesis after this chapter is divided into four chapters. Chapter 2 contains the literature review on the granular flow and granular flow rate, particle image velocimetry, and centrifuge testing.
This chapter discusses the silo or hopper parameters contributing to the change in outflow rates (W). For the general calculation of outflow rate, seven empirical formulas are reviewed from past studies. The literature review also covers past studies of the use of particle-based image analysis methods and the use of centrifuges for scaled model tests. The experimental design, methodology and the text matrix are discussed in Chapter 3. The experimental design presents the experimental apparatus used and the procedures used for testing. This chapter also presents the materials used in this study. To determine the physical and mechanical properties of these materials, this chapter also describes basic laboratory tests’ results for these materials. Chapter 4 contains the test results and observations for the mass accumulation graphs, outflow rate, PIV and centrifuge test results. This chapter discusses the general observations, discussions and analysis on the test results, theoretical results, and conclusions. This chapter provides a detailed analysis of the results and discusses the convenient outflow rate (W) equations for the hopper used in this study. This chapter also investigates the PIVlab output from the natural gravity outflow tests and the scaled centrifuge tests. Chapter 5 summarizes the findings from this study and this chapter also provides suggestions for future work based on the understanding developed from this study.
CHAPTER 2: LITERATURE REVIEW

This chapter reviews pertinent literature on granular flow from hoppers and silos, the use of imaging to track particle motions and scaled modelling in the centrifuge. The first topic in this chapter relates to granular flow rates from these structures. Many of experimental and analytical studies reviewed in this chapter show that the empirical approaches used to calculate the granular flow rates are dependent on the geometry of the storage body and the physical and mechanical properties of the flowing material. Several correlations have been proposed in the past to predict the outflow rate (W) results. Some of the more widely used ones are discussed in the following section.

The second topic in this chapter is particle image velocimetry (PIV). This reviews the general approach and procedures used for particle-based image analyses. This part explores the development of the particle image velocimetry (PIV). Some related research studies done using PIV are also reviewed to investigate the status of PIV, and some goals for future advances. The aim of the third topic in this chapter is to provide a technical discussion of the relevant centrifuge modeling laws. This part reviews general centrifuge scaling methods, and the past works for the use and application of the geotechnical centrifuge. Scaled models are often used to study the behaviour of silo discharge and the effects of gravity on silo discharge, and this section also reviews the effects of gravity on silo discharge and internal flow patterns.

2.1 Granular Flow Rates from Hoppers and Silos

2.1.1 Key parameters

This review will focus on the effects of the main geometric parameters on the granular flow rate, the flow pattern, and the effect of particle size distribution. Mamtani (2011) suggested that the key parameters affecting the granular flow rate from a silo hopper is as follows: hopper diameter (D), orifice diameter (Do), orifice shape, the vertical fill height (h_o), particle size, granular mass cohesion and coefficient of mass friction (μ), particle shape and angularity, cone angle of the base of the hopper (θ), particle size distribution, wall coefficient of friction and particle angle of repose (α).
2.1.1.1 Hopper fill height (H_f +h_o)

During the flow of the granular material, the pressure at the bottom saturates (tends not to increase) because the weight of the material is supported by the walls of the hopper. The saturation of the pressure is due to equilibrium state near the outflow orifice. This creates a “free flow” surface immediately above the orifice level where the discharge rate remains constant if the fill height is greater than the hopper diameter. Nedderman (1982) discussed that the height of the vertical cylinder only sizes the quantity of material in the hopper, however the effect on the flow rate is usually assumed to be small. The work further suggested that the stress analyses of Janssen (1895) and Shaxby and Evans (1923) showed that the weight of the material is supported significantly by ‘the walls of the hopper and the stress state near the orifice is effectively independent of H_f’. Most researchers have reported little or no dependence of the mass flow rate W on H_f, though Newton et al. (1945) reported that W is proportional to H_f^0.04. The results of Demming and Mehring (1929) showed that W is independent of H_f, provided H_f exceeds some critical value and their results suggested that the critical value to be 2.5D (i.e H_f > 2.5D, where D is the upper hopper diameter). Brown and Richards (1955) showed that during batch discharge, W remains constant until the material head in the hopper is less than the hopper diameter, i.e. H_f < D. At these low values of H, studies show that the top surface of the discharging material always shows a central depression. Rose and Tanaka (1959) suggested similar results that W is constant if H_f at the centre line is greater than the orifice diameter D_o. Thus, there seems to be a substantial agreement that W is independent of H_f until the hopper is almost empty, though the exact value of the critical height is not wholly clear in the literature.

2.1.1.2 Diameter of the hopper (D)

Previous research studies show that the outflow flux rate (W) is independent of the hopper or silo body diameter, provided that the value of D is not too small. Rose and Tanaka (1959) and Brown and Richards (1960) found that the discharge rate is independent of the container diameter if the diameter is greater than 2.6 times the orifice diameter (D > (2.5 - 2.6)*D_o). However, other research showed that the discharge rate does not vary with the hopper diameter if the difference between the hopper and the orifice diameters exceeds thirty times the particle diameter (d), (D - D_o) > 30d (Franklin and Johanson, 1955). It was found that for small values of D (i.e., as D gets
close to the value of \( D_o \) that the whole mass of the material flows quickly downwards, under the influence of the gravity.

2.1.1.3 Orifice diameter \((D_o)\)

The flow becomes intermittent and irreproducible when the orifice diameter is less than about six particle diameters, i.e. \( D_o < 6d \) (Nedderman et al., 1982). Brown and Richards (1959) suggested critical value was 2.5 for rectangular slots and 4.0 for circular orifices. Deming and Mehring (1929) noticed irregularity in the flow when orifice diameters ranged from 4 to 6.66 times the particle diameters depending on the particle shape. Anand et al. (2008) also made similar conclusions for a rectangular hopper; so long as the hopper diameter or width is sufficiently large, the hopper walls do not affect the flow regime near the outlet and hence the outflow rate.

2.1.1.4 Cone angle of the hopper \((\theta)\)

The research of Verghese and Nedderman (1995) showed that the discharge rate depends on cone angle only in mass flow from a hopper. However, Rose and Tanaka (1959) and Harmens (1963) suggested certain conditions occur where the discharge rate is not just affected by the cone angle \((\theta)\). The research done by Rose and Tanaka (1959) indicated that the discharge rates for bulk solids flowing from cylindrical hoppers with circular or non-circular apertures can be found using equation 2-2 (see section 2.1.2). According to Rose and Tanaka (1959), if \( \theta < 180 - 2\alpha \), then the discharge rate also depends on the material state (where \( \alpha \) is the static angle of repose).

2.1.1.5 Effect of particle size and distributions

Most studies assume uniform sized soil particles or particulate media, and therefore most analytical and numerical models follow this same assumption. In nature, dealing with uniform or binary size distributions of grains seems not to be reasonable, and particle size distribution (PSD) is believed to play an essential role in granular flow and has been ignored in most studies. Anand et al. (2008) investigated the effect of grain size experimentally and found that finer particles tend to have higher discharge rates than coarser ones and claimed that this is due to the empty annulus around
an orifice. Other investigations have shown similar results with higher flow rates for finer particles in comparison with coarser ones (e.g., Mohsen, 2019).

Another important aspect of grain size effects is the particle size distribution when different sizes of particles are mixed to form the material. Anand et al. (2008) studied the effect of PSD on granular flow numerically for 3D conditions (Discrete Element Method, DEM) and found that the granular flow rate increases due to the rise in the mass percentage of the finer particles. Dias et al. (2004) explained when the fine fraction of granular material increases, ‘the voids between coarse particles are filled, resulting an increase in flowing density and eventually increase in discharge rate’.

2.1.1.6 Angle of repose

One of the basic properties of cohesionless granular materials is the angle of repose; this is the maximum slope angle at which a loose packing of material is at rest (Lowe, 1976). Above this slope angle, the material starts to flow and below this angle, the material is stable (Kleinhans et al., 2011). Equation 2-2 from Rose and Tanaka (1959) shows that the combined limits of the cone angle (θ) and the static angle of repose (α) define whether a material will flow or not. Even though the equation was eventually modified for better flow results, some studies have indicated that angle of repose may be less accurate for estimating the flowability of cohesive and compacted materials (Bell, 1993; Ileleji and Zhou, 2008). It should be noted that the angle of repose is only likely to reflect the flowability of bulk solids in relatively unconsolidated loose state. Moreover, the angle of repose may not provide an accurate material description to apply in the design of silos when bulk solids are under high stress. This is because the angle of repose does not represent how the shear strength varies with its state of compaction (Ileleji and Zhou, 2008). Until now it has been assumed that the angles of repose are independent of gravitational acceleration, however, the study done by Kleinhans et al. (2011) found that for decreasing gravity, the static angle of repose appeared to increase, while the dynamic angle of repose decreased for all materials.

2.1.2 Granular flow rates: empirical correlations

Many granular discharge rate predictive equations have been developed through different research studies. Some of the most recent and commonly used correlations are discussed below.
Franklin and Johanson (1955) studied the discharge rate of glass beads, lead shot, cracking catalysts, ion exchange resins, puffed rice, coal and crushed olivine rock flowing in a cylindrical hopper with a circular orifice. These materials had a variety of shapes, both spherical and non-spherical. According to their study, the discharge rate (W) of the solid materials was adequately predicted by equation 2-1 below:

\[
W = \frac{\rho b^{2.93}}{((6.228\mu+23.16)(d+1.889)-44.90)}
\]

(2-1)

Where W is in lb/min, D and d in in. and \(\rho\) in lb/ft\(^3\). The particle size d ranged from 0.03 to 0.2 in. (0.08 cm to 0.51 cm), the particle density \(\rho_s\) from 7.3 to 676 lb/ft\(^3\) (0.12 g/cc to 10.83 g/cc) and orifice diameter \(D_o\) from 0.236 to 2.28 in. (0.6 cm to 5.79 cm). The study showed that the results predicted using equation 2-1 lie within 7% of the experimentally measured discharge rates on average.

Rose and Tanaka (1959) studied the effect of various parameters on the discharge rate from cylindrical hoppers provided with circular or non-circular apertures and proposed equation 2-2 to predict discharge rates for bulk solids. The range of particle sizes d used varied from 0.0112 to 0.318 cm, the orifice diameters \(D_o\) from 0.36 to 1.59 cm, half cone angles \(\theta/2\) from 15° to 90°, the wall coefficients of friction \(\mu\) from 0.71 to 1.25, and their measurements of discharge rates were found for silica sand, ground glass, steel balls and steel discs.

\[
W = 0.16D_o^{2.5}\rho_s\sqrt{g}\left(D_o/d - 3\right)^{0.3}f(\theta)(Z - 5)^{-0.5}
\]

(2-2)

Where:

\[
f(\theta) = \left(\tan\frac{\theta}{2}\right)^{-0.35} \quad \text{if} \quad \frac{\theta}{2} < 90 - \alpha
\]

(2-2-1)

\[
f(\theta) = \left[\tan (90 - \alpha)\right]^{-0.35} \quad \text{if} \quad \frac{\theta}{2} \geq 90 - \alpha
\]

(2-2-2)

and;
α – angle of repose of the flowing material
Z – particle shape factor; 6 for spherical materials (Heywood, 1933).
The term f(θ) gives the dependence of the hopper cone angle on the discharge rate.

Beverloo et al. (1961) proposed an equation to estimate the granular discharge rates from the outlet of a silo due to ‘shear flow’ from flat-bottomed hoppers with circular orifices. After later modification, equation 2-3 has become the most commonly used relationships for estimating the discharge rate of a granular material from a circular orifice of a silo or hopper body.

\[ W = C \rho \sqrt{g} (D_o - kd)^{5/2} \]  
(2-3)

Where: C = discharge coefficient which depends on bulk density, \( \rho = \) soil mass density, \( g = \) gravitational acceleration, \( D_o = \) orifice diameter, \( k = \) shape coefficient and \( d = \) soil particle diameter.

The constant k is, typically in the range of 1.3 < k < 2.9, (Beverloo et al., 1961). Beverloo et. al (1961) found a value of 2.9 for sands, however the most commonly used value of k is 1.4. The term ‘kd’ accounts for the wall effects, where the particles do not fully flow at the perimeter of the outlet. C is a parameter related to flow of particles through different orifice apertures thereby affecting the mass flow at the orifice level. C can be defined as:

\[ C = \sqrt{\frac{2\beta \pi}{4}} \]  
(2-3-1)

Where: \( \beta = 0.5 \) for a relative proportional arch to aperture ratio (\( H_f / D_o > 1 \)), C = 0.58 is a commonly used value for a circular opening (Bian, 2014). Linseed, spinach, watercress, rapeseed, kale, swede, turnip and sand fractions were used by Beverloo et al. (1961) as the testing materials. The experiments used granular particle sizes d varying from 0.045 to 0.30 cm, soil density \( \rho \) from 0.57 to 1.5 g/cc and orifice diameters from 0.26 to 3.02 cm.

Harmens (1963) proposed equation 2-4 to calculate the discharge rates for granular materials from flat-bottomed or conical hoppers provided with either circular or non-circular basal orifices.
\[ W = \rho_s A \sqrt{gD_h \left[ 0.505 - 0.160\mu - F_1 \left( \frac{d}{D_h} \right) \right]} F_2 (\mu, \theta) \]  \hspace{1cm} (2-4)

Where:

\[ F_1 \left( \frac{d}{D_h} \right) = \frac{0.38 \left( \frac{d}{D_h} \right)^{1.5}}{0.045 + \left( \frac{d}{D_h} \right)^{1.5}} \]  \hspace{1cm} (2-4-1)

\[ F_2 (\mu, \theta) = \left( \mu \tan \frac{\theta}{2} \right)^{-0.35} \quad \text{if} \quad \mu \tan \frac{\theta}{2} < 1 \]  \hspace{1cm} (2-4-2)

\[ F_2 (\mu, \theta) = 1 \quad \text{if} \quad \mu \tan \frac{\theta}{2} \geq 1 \]  \hspace{1cm} (2-4-3)

\[ D_h = \frac{4A_o}{P_o} \]  \hspace{1cm} (2-4-4)

With ‘A’ being the area of the circular hopper body and \(A_o\) and \(P_o\) are the area and perimeter of the orifice, respectively. This research proposed equation 2-4 for use based on a variety of spherical and non-spherical granules, such as glass beads, lead shot, wheat, rice, catalyst spheres and cylinders, sand, rapeseed, and watercress seed. He found that the predicted rates were within 6% of the measured ones when the particle size \(d\) ranged from 0.110 to 0.381 cm, soil density \(\rho\) from 0.118 to 10.83 g/cc, the coefficient of soil mass friction \(\mu\) from 0.34 to 0.79 and the orifice diameter from 0.5 to 5.79 cm.

Janda et al. (2012) found that the Beverloo correlation fit the flow rates well for big orifices. However, the study also confirmed that the Beverloo equation fails for smaller ones (\(D_o/d < 80\)) where clogging is possible. This was also experimentally observed by Mancok et al. (2007) who also proposed a new empirical expression (equation 2-5) to better fit the flow rates. This equation was further studied by Janda et al. (2012), and is presented below. ‘n’ in equation 2-5 was defined by Janda et al. (2012) as \(n=1\) for a 2D model and \(n=2\) for 3D.

\[ W = C'' \sqrt{g\rho \left[ 1 - \frac{1}{2} e^{-b(D-d)} \right]} (D - d)^{n+1/2} \]  \hspace{1cm} (2-5)

Where \(C''\) and \(b\) are fitting constants. Based on this study, this correlation suggested two important differences with respect to the Beverloo’s expression:

a. the flow of grains becomes zero when the outlet size equals the particle diameter,

b. an exponential factor is included to adjust the flow rate for a wide range of exit sizes. The study
suggests that this factor could be related to lower density of material near the orifice.

Janda et al. (2012) also presented equations for grain flow through an opening determine the velocity and density profiles. This study investigated grain flows through an aperture placed at the bottom of a 2D silo. In the equation below, \( R \) is the radius of the velocity contours above the orifice and \( x \) is the horizontal position left or right of the line of symmetry.

\[
v(x) = \sqrt{2gR \sqrt{1 - \left(\frac{x}{R}\right)^2}}
\]

\[
\Phi(x) = \Phi_\infty \left[1 - \alpha_1 e^{-R/\alpha_2}\right] \left(1 - \left(\frac{x}{R}\right)^2\right)^{1/v}
\]

Where \( v \) and \( \Phi \) are the velocity and density profiles respectively, \( \Phi_\infty \) is the asymptotic value of the granular volume fraction for big openings, and \( \alpha_1 \) and \( \alpha_2 \) are curve fitting parameters (\( \Phi_\infty = 0.83 \pm 0.01 \), \( \alpha_1 = 0.50 \pm 0.01 \) and \( \alpha_2 = 3.3 \pm 0.05 \)). Both profiles \([v(x) \text{ and } \Phi(x)]\) demonstrate the generality of the mechanisms for controlling the flow rate passing through the orifice and they do not show any difference between small orifices with clogging potential and large orifices with continuous flows. The exponent \( \frac{1}{v} \) is a velocity component for graph fitting and a value of 0.22 was used by Janda et al. (2012). The final expression for mass flow rate is therefore:

\[
W = C^* \sqrt{g \Phi_\infty \left[1 - \alpha_1 e^{-R/\alpha_2}\right]} R^{3/2}
\]

Where \( C^* = 4\beta \left(\frac{v^2+2}{2v} \right) / \pi d^2 \)

Rahman and Zhu (2012) conducted a numerical investigation of gravity flow of granular solids in a hopper through a circular orifice using the discrete element method (DEM). The research used a cylindrical hopper with a diameter of 5 cm, with different sizes and shapes of orifice of areas 0.785, 1.226 and 1.766 sq cm. This study provided a prediction method for the flow rate \( W \) of granular particles through a circular orifice as shown in equation 2-6.

\[
W = 45\rho A' \sqrt{g D' h}
\]
Where, $\rho$ is soil density of packing, $D_h$ is effective hydraulic diameter ($D'_h = D_h - 1.4d$; Beverloo et al., 1961) and $A'$ is the effective orifice area ($A' = \pi ((D'_h)/2)^2$). This approach was compared to Beverloo et al. (1961), and a similar $C$ coefficient of 45 was attained in both studies.

Srivastava et al. (2003) showed 2D DEM simulation results of the gravity discharge of particles from a bin. The simulations confirmed the ‘height-independent rate of discharge of particles from the bin, the volumetric dilation of the particle mass assembly near the exit orifice, the significant effect of the interstitial air on the discharge behavior of fine particles and the occurrence of a pressure deficit above the orifice.’ One mm diameter particles were allowed to flow through a bin of orifice, size of 1.4 cm in the DEM simulation. The outcome of this study also found that during the period of steady discharge, the height of material dropped almost 50% in only 3 seconds. This result suggested that the depth of material in the bin varied considerably during the period of steady flow. This research finding also inferred that the discharge rate is roughly independent of the height of the material in the bin. The flow rate, $W$ through 2D hoppers and bins was then proposed to be estimated with equation 2-7. The study also presented a frictional–kinetic constitutive model for particle phase stresses. Since the discharge rate of granular materials from a bin is dependent on conditions near the orifice, it has been argued that the free-fall surface (see section 2.1.2.1) scales with the orifice diameter or width ($D_0$). Ignoring the possible effects of particle diameter $d$, dimensional analysis by Srivastava et al. (2003) suggested that particle velocity $v$ at the orifice then scales with $(gD_0)^{1/2}$. Scaling for the discharge rate $W$ of material from a hopper or bin should therefore be:

$$ W \propto \begin{cases} \rho g^{1/2}D_0^{5/2} & 3 - \text{D bin or hopper} \\ \rho g^{1/2}D_0^{3/2}H & 2 - \text{D channel} \end{cases} $$

$$ W = C\rho g^{1/2}D_0^{3/2}H $$  \hspace{1cm} (2-7)

Where $\rho$ is the soil density achieved during the filling process and $C$ (C is Beverloo constant equivalent) is an empirical constant in the range $0.55 < C < 0.65$, and $H$ is the thickness channel or the diameter of the hopper.
2.1.3 Continuum constitutive models of granular flow

Many studies use continuum models for computationally predicting the flow of a consolidated granular material. This approach is often based on a continuous fluid representation of the granular solid, making no attempt to deal with the behaviour of individual particles, which is the reason why it is referred to as the “continuum” model. Harry (2007) discussed the efforts to understand particle dynamics during granular flow in part by adapting some continuum models. The continuum models are often based on hydrodynamic or elastoplastic descriptions. These models try to account for the differences between molecular fluids or solids and granular matter, such as the inhomogeneous character of particle contacts and complex stress transmission in a granular material. Bocquet et al. (2001) produced a detailed continuum model of granular flow based on hydrodynamic models which were motivated by Bocquet et al. (2001) pioneering theoretical and experimental work on shear forces in dense suspensions. The continuum model characterized the main features of granular shear flow through experimental measurements in a Couette geometry and a comparison to a locally Newtonian, continuum model of granular flow. The continuum model supported the findings of earlier hydrodynamic model studies, that the experimentally observed coupling between fluctuations in particle motion and mean-flow properties controls the continuous but fluctuating dynamics of the flow.

2.1.3.1 Dynamic arching theories

Arching can be defined (but is not limited to) as the formation of curve – like supported stagnant masses of bulk material in a bin or hopper. This flow state is created upon opening of the orifice (outlet) or during gravitational flow. The dynamic arch phenomenon (see Fig. 2.1) can be easily explained by the concept that if a soil loses part of its support, that portion of soil stress which was supported would then transfer to an adjoining soil mass which is still standing (Terzaghi, 1943). Published literature (e.g., Kudrolli and Samadani, 2001; Zuriguel et al., 2003; Zuriguel et al., 2005) indicate that a series of physical processes and phenomena affect the dynamics of grain flow during emptying of the silo. Those phenomena can be divided into: arching, ratholing, irregular flow, jamming, caking, and segregation (see Fig. 1.3). Arching can also possibly be due the effect of the bin or hopper geometry but is mainly due to the size of the hopper orifice.
Drescher, et al. (1995) provided a seminal study on the differences between various theories of arching and actual test results for hoppers. The work showed that predictions directly related to the assumed form of the instantaneous and effective yield loci, to the flow function derived, and to the material/wall interface friction angles ($\phi, \phi_w$) and the bulk unit weight ($\gamma$). Within these analytical approaches, the prediction of the arching was mainly explored using two theoretical solutions: (1) for a material assumed to behave like stacked structural members (or arches); and (2) as a single continuum mass that consolidates itself in the hopper and gives rise to its frictional strength. The structural member theory predicts that arching will occur if the strength of the isolated members is greater than the self weight induced stresses. Both approaches are based on the assumption that the soil behaves as a rigid hardening and softening plastic solid. The vertex radial stress field for the structural mechanics approach is shown in Fig. 2.1(a) and assumes that the major principal stress ($\sigma_1$) developed next to the walls varies with vertical distance. The critical radius ($r_{cr}$) is the point where arching occurs. Fig. 2.1(b) shows the continuum mechanism approach considers global equilibrium of the soil mass where the stresses satisfy a set of linear (Mohr-Coulomb) yield conditions for a static flow state.

![Fig. 2.1 (a) Assumptions in the structural mechanics approach; (b) assumptions in the continuum mechanics approach. (source: Drescher et al., 1995)](image)

These assumptions lead to the stresses acting normally to the curved slices, which changes from compressive to tensile minor principal stresses ($\sigma_3$) at a certain radius from the vertex ($r_{cr}$). In this case the critical distance for arching is determined from the condition $\sigma_3=0$. The existence of this stable soil arch implies that the soil mass is in equilibrium without any support from below. As the
hopper orifice opens, it creates greater vertical stresses ($\gamma$ and $\sigma_3$) and the combined effect creates a vertical unsupported soil mass at a radius, $r_{rc}$ from the position of the orifice (see Fig. 2.1(b)). In both theories, the material below $r_{rc}$ is in a ‘free fall state’ and is not involved in supporting the soil mass as it flows out of the hopper. Although both methods were found to provide good qualitative predictions of the hopper behaviour, the stress states and behaviour for different outlet sizes was found to deviate from reality. The study also outlined the difficulty of determining appropriate soil parameters from existing standard laboratory tests (eg. direct shear box).

2.1.3.2 Free fall arch

The kinetic energy of grains approaching an outflow orifice reach a minimum very close to the orifice, after which the boundary particles fall freely under their own weight (Brown, 1961). This arch formation during granular flow is affected by the presence of an orifice and is usually referred as a free fall arch (FFA) (Brown and Richards, 1965). A FFA is believed to form due to constant rates of flow, and the formation and subsequent breaking of the arch takes fractions of a second. The assumption of an FFA during studying the process of granular discharge can therefore be beneficial since it simplifies the analytical process. However, many studies confirm that direct experimental observation and visualization of a FFA can be difficult and complicated (Lin et al., 2015). Tian et al. (2015) studied free fall surfaces with a 3D simulation model assuming monosize steel spherical particles. They found that after forming the FFA, discharge flow becomes constant and the free fall surfaces have a parabolic shape rather than the simpler circular shape.
Le Pennec et al. (1996) claimed that they were able to identify structures in sand experiments, such as the FFA, using laser technology as shown in Fig. 2.2. The study showed that the boundary of the FFA occurred along the dotted line above the position of the orifice (at 0,0) as shown in Fig. 2.2. They also defined the FFA as a boundary separating regions where grains are in contact and regions where grains are not in contact and are falling freely, which may occur at the free fall velocity ($V_{FF}$). They explained that the stresses acting on the particles in the upper region (i.e. above the dotted boundary line) are from other particles in addition to gravity and hydrodynamic forces. In the second lowest region (i.e. inside and below the dotted boundary line), only gravity and hydrodynamic forces are acting. They explained that the existence of a well-defined sharp interface (assumed by theoretical ideas of the free-fall arch) is unclear. They also showed rapid density variation during intermittent sand flow and used a CCD (Charge Coupled Device) camera to capture the FFA zone.

Rubio-Largo et al. (2015) performed 3D numerical modeling (using the Discrete element method) to model granular flows in a silo. They observed that there is a distinct transition zone where the flow rate of the grains changed. In this zone, movement of grains is associated with macroscopic
flow. They also found that the shape of the soil arch followed a parabola. Janda et al. (2012) also considered the existence of an FFA in their investigations and their results showed that this boundary should be parabolic instead of the more commonly assumed circular arch.

2.1.3.3 Discontinuous velocity fields in gravity flows of granular materials through slots

The flow pattern (shown in Fig. 1.1 and 2.3) of gravity flow of granular materials through hopper outlets is often accompanied by the formation of a well-defined zone of relatively fast-moving material directly over the outlet. Material adjacent to the fast-moving zone towards the wall appears to move slower and approaches zero velocity near the wall. This situation suggests the presence of discontinuous velocity fields. Pariseau (1969) conducted experimental studies to determine the regions of discontinuity. Fig. 2.3 shows the velocity discontinuities which occurs during the flow process. In Fig. 2.3, the curved path passing through C to D traces a typical particle motion. Region B is a ‘core’ of fast-moving material relative to region ‘A’; the material in the dead zone near the wall is not moving.

![Fig. 2.3: Zones of flow around the outlet region of a deep flat-bottom bin (source: Pariseau, 1969)](image)

Pariseau (1969) also experimentally determined the particle paths during a gravity flow of a clean, dry sand in a wedge-shaped hopper with a slot outlet. Fig. 2.4 (a) and Fig. 2.4(b) show experimental velocity fields that were developed in the hopper during outflow. The numbers represent fractions
or multiples of the speed of the particle upper most in the hopper along the centerline. All particles along the same contour line move with approximately the same speed. Fig. 2.4(a) shows the existence of velocity discontinuities in the upper zones of the hopper. High velocity gradients are shown in Fig. 2.4(b).

![Figure 2.4: (a) Particle paths (b) Contours of relative particle speed](source: Pariseau, 1969)

Mollon and Zhao (2013) conducted 2D discrete element modelling of sand flow through a hopper using realistic grain shapes. The study found that radial waves originating from the lower centre of the hopper show flow fluctuations with different particle coordination number (contacts per particle), velocity magnitude and mean stress. These radial waves propagated in the opposite direction of the granular flow. The research suggested that rough angular particles have narrower and more intense funnel flow regions. The image results showed that the flow velocity demonstrated more fluctuations in the upper and lateral zones of the hopper and the areas where the velocity magnitude was low. Fig. 2.5 shows the sub-zone studied to identify the void ratio, the particle coordination number, the average magnitude of velocity and the mean stress. Based on Fig. 2.5, Mollon and Zhao (2013) found that ‘the fluctuations in velocity magnitude, mean stress and coordination number all take the form of radial waves propagating upwards from the lower part of the hopper’ with similar agreement to Pariseau (1969), the particle paths as shown in Fig. 2.4.
Fig. 2.5: Overview and zoom of 2D discrete element simulation of sand flow through a hopper. (source: Mollon and Zhao, 2013)

The vertical velocity magnitude down the hopper center line and the fluctuation of the velocity at one depth are presented in Fig. 2.6(a) and Fig. 2.6(b). The velocity field in Fig 2.6(a) indicate the average value of velocities cancelling the individual fluctuations the velocity showed down the center of the hopper. The velocity varied between 50 mm/s to 100 mm/s as shown in Fig. 2.6(b), and the study found that the stresses corresponding these velocity variations was 20 to 380 Pa.
Fig. 2.6 (a): Average fields and their standard deviations of the velocity magnitude, on the vertical axis of the hopper: instantaneous field at $t = t_1 + 32$ ms.

(b): Fluctuations in time of the velocity magnitude, for an observation window located at $y = 11$ mm (source: Mollon and Zhao, 2013)

2.2 Particle Image Velocimetry

The idea of image analysis of deformable or moving objects started a long time ago with our understanding of light and motion. The first investigators to achieve image analysis measurements used the laser speckle method. This method was originally developed in solid mechanics and was later applied to the measurement of fluid velocity (Adrian, 2005). Improvements over the last four decades have enabled the use of Particle Image Velocimetry (PIV) in many engineering and science fields. Even though PIV was initially developed to capture and depict the characteristics of flow in fluid mechanics, the improvements done with the precision and spatially varying displacement fields has enabled PIV to be used in a range of other disciplines. The detailed PIV analysis described by Adrian (1991) contributed greatly to the concept being adopted for studying soil deformation in geotechnical engineering.

2.2.1 Geotechnical PIV methodology

Particle Image Velocimetry (PIV) is an optical method of flow visualization used in education and research. It is a velocity-measuring procedure originally developed in the field of experimental
fluid mechanics and was reviewed by Adrian (1991). This thesis uses PIVlab to analyse and capture displacement data from soil deformation images during natural gravity and enhanced gravity in the centrifuge. PIVlab is a free image analysis module for MATLAB designed for geotechnical, material, chemical, water, structural engineering and more research applications. It is capable of sub-pixel measurement resolution for problems involving large displacements and deformations. The performance and developments of the software are discussed in White (2002), Take (2002) and by Thielicke and Stamhuis (2014).

Fig. 2.7 shows a typical single-camera system for PIV image capturing. The PIV setup commonly has the following components as shown in Fig. 2.7: tracer particles, light source, light sheet optics, and a camera. The overall velocity image capturing follows some measurement setting including interrogation and post-processing settings. Fig. 2.7 shows the PIV components where clear successive images are captured in a camera by the help of a light beam.

![Fig. 2.7: PIV commonly used components](image)

The main objective of the statistical evaluation of PIV recordings for medium image density is to analyse and quantify the displacement between two patterns of randomly distributed particle images, which are stored as a 2D distribution of gray levels (Markus et. al, 2017). A series of pictures is taken to capture the deformation of soil in geotechnical testing. PIV operates by tracking the texture within an image of soil throughout a series of images. PIV does not require coloring of
sand particles in order to track the particle motion. However, coloring of sand particles may be used for presentation purposes in order to show soil deformation patterns clearly both horizontally and vertically. Fig. 2.8 shows that the initial image used as a reference is to be divided into subsets made up of a mesh of PIV test patches. Each test patch within each subset has its corresponding reference coordinate. The displacement of this patch will be searched in the subsequent image series referred as image ‘n’. A search patch is extracted from the second image; this patch has a larger dimension than the test patch, defining the zone in which the test patch is to be searched. The cross-correlation between the test patch from the reference image at zero-time seconds and the search patch from second image at time \( t_2 \) seconds is evaluated. The location at which the highest correlation is found indicates the displaced location of the test patch (White et al., 2001; 2002; 2003). This process is repeated over the entire mesh of patches and over each subset within an image, and then repeated throughout a series of images, to produce a displacement field to represent the soil deformation.

Fig. 2.8: Principles of PIV Analysis (source: White et al., 2016)
2.2.1.1 PIVlab software overview

PIVlab was originally programmed by Thielicke (Thielicke and Stamhuis, 2014). PIVlab is a freeware which uses MATLAB interface and additionally requires the image processing toolbox to run. A DPIV analysis typically consists of three main steps namely image pre-processing, image evaluation, post-processing (see Fig. 2.9). Images captured using a camera are used as an input to the DPIV. PIVlab arranges those images in frames, i.e. two consecutive images as a single frame. The image pre-processing run some selected techniques that are implemented in PIVlab (see Fig. 2.10). These techniques are implemented to improve the measurement quality and to enhancement images before the actual image correlation takes place (Thielicke and Stamhuis, 2014).

![Diagram](source)

Fig. 2.9: Digital particle image velocimetry (DPIV) analyses in PIVlab (source: Thielicke and Stamhuis, 2014)

The contrast limited adaptive histogram equalization (CLAHE) was developed to increase the readability of the image data. CLAHE operates on small regions of the image known as tiles and in every tile, ‘the most frequent intensities of the image histogram are spread out to the full range of the data (from 0 to 255 in 8-bit images)’ (see Fig. 2.9). This improves the detection of valid vectors by $4.7 \pm 3.2\%$ (Thielicke and Stamhuis, 2014). The intensity high-pass applies a high pass filter to remove the low frequency background information caused by inhomogeneous lighting during the image capturing. This emphasizes the particle information in the image. The intensity capping filter neutralizes the effect of bright particles or bright spots in non-uniform flows. This
Intensity improves the probability of detecting valid vectors in experimental images by $5.2 \pm 2.5\%$ (Shavit et al., 2007).

![Fig. 2.10: The effect of several pre-processing techniques (source: Thielicke and Stamhuis, 2014)](image)

The image evaluation in PIVlab uses a cross-correlation algorithm in frames to derive the most probable particle displacement in the interrogation areas (tiles). Cross-correlation is a statistical pattern matching technique that tries to find the particle pattern from interrogation area A (subset in reference image on Fig. 2.9) back in interrogation area B (subpixel in target image in Fig. 2.9), where A and B are corresponding interrogation areas from image the reference image and the target image as shown in Fig. 2.9. The post-processing include data validation, data interpolation, data smoothing, data exploration and data quality control. This stage of PIVlab image DPIV analysis uses correlation matrices over the target subset. PIVlab allows to manually select the velocity limits in order to filter the outlier velocities [ this is a velocity which lies away from the visible and acceptable velocity limits].

### 2.2.2 PIV applications in geotechnical engineering

White et al. (2001) presented a non-contact measurement of soil deformation in physical models and element tests. The research captured soil displacements to a precision of $1/15000^{th}$ of the field of view. The study found that the intrinsic camera parameters of the digital still camera varied with g-level during the centrifuge testing. The study also added that the technique is equally applicable to triaxial testing with a resolution sufficient to measure pre-failure strains and the flexibility to capture non-homogenous deformations, which are invisible to conventional transducers. White et
al. (2001) investigated a dry sample under vacuum during a compression test on a 40mm ‘gauge length’ triaxial machine using PIV image analysis. 1100 vector field were captured during an uncalibrated displacement measurement and was obtained during a top platen movement of 50μm, as shown in Fig. 2.11 below. The study found that the boundary displacement matched closely with the vertical component of the displacement vectors at sample mid height which measure a nominal 24μm.

White et al. (2003) showed deformation measurements in other forms of geotechnical testing. The research emphasized that the performance of a measurement system can be assessed by considering the errors associated with accuracy, precision and resolution. Five tests were conducted with changes to the lighting, soil texture, camera resolution (pixel value) and pixel intensities on the soil image. The findings of the research using a 2-megapixel digital camera concluded that the availability of higher-resolution CCDs at lower cost will lead to corresponding increases in performance.

Fig. 2.11: The measurement of deformation in a triaxial test (source: White et al., 2001)
Iskander (2010) used PIV to compare the boundary soil displacement fields under a model footing in a transparent soil model with those from a natural soil model. The study found that transparent soil and similarly developed optical systems could be used to explore opportunities for more advanced nonintrusive three-dimensional deformation measurements for various soil–structure interaction problems. Amorphous silica powder was used as the transparent soil material in this study.

Take (2015) used a digital image correlation technique to visualize and analyze a geotechnical failure process such as plastic collapse of shallow foundation and failure evolution in landslides. The study confirmed that the use of digital image correlation by the ‘geotechnical engineering community over the past 15 years provided a crucial approach for failure visualization and the quantification of soil-structure interaction when physical models are used’. The research further implicated that the texture and size of the soil subset, camera noise, the subpixel interpolation scheme used, and the geometry of the camera used are important factors affecting the accuracy and precision of the outcomes.

Thielicke and Stamhuis (2014) described the freeware PIV MATLAB resource called PIVlab. This application gives accurate digital particle image velocimetry correlations. This study showed that PIVlab is suitable for micro-PIV analysis, which enables an easy approach to capture granular skeleton deformation properties. The results acquired from this analysis include shear rate (s⁻¹), strain rate (s⁻¹) and velocity magnitude (mm/s). Each result can be extracted inclusively across an area boundary or by drawing a line across a study target.

Li et al. (2021) used PIV in calculating the velocity of a fast-falling avalanche. Other studies in geotechnical and material engineering have also applied the PIV technique as a routine method to measure soil deformations in small-scale geotechnical models (Stanier and White, 2013). Pudasaini and Hutter (2006) also used the same concept to analyse granular flow experiments with large deformations.
2.3 Geotechnical Centrifuge Testing

Small scale physical model tests may be used to evaluate the performance of a particular prototype complex value problem or to study the effects of different parameters on a generic problem (Kramer, 1996). Model tests (i.e., small scale tests) usually attempt to reproduce the boundary conditions of a particular problem by subjecting a small-scale physical model of a full-scale prototype structure to appropriate loading or displacements. While model testing is very useful for identification of important phenomena and verification of predictive theories, it has not often used directly for the design of significant structures or facilities.

2.3.1 Centrifuge methodology review

The mechanical behavior of soils is sensitive to stress level and soil states. That is, soils can exhibit contractive shearing behavior under high normal stresses and may exhibit dilative behavior at lower stress levels. One of the most significant challenges for small scale models, therefore, is the problem of testing models whose stress dependency matches that of the full-scale prototype. Because this is quite difficult under the gravitational field of the earth, one common approach involves testing under enhanced gravitational fields in the centrifuge.

In a centrifuge test, a 1/N-scale model rotated at a distance, r, from the axis of a centrifuge a rotational velocity given by:

\[ \Omega = \sqrt{\frac{N}{r}} \]  

(2-8)

The rotational speed, \( \Omega \) is sufficient to increase the acceleration field of the model to \( N \) times the acceleration of Earth’s gravity. In principle, the stress conditions at any point in the model should then be identical to those at the corresponding point in the full-scale prototype due to the proportional increase in self-weight stresses. The overall behavior of the model in-terms of displacements, failure mechanisms, etc. should then be identical to that of the prototype. Similitude considerations are important in the planning and interpretation of centrifuge tests. The scaling factors also show how certain physical phenomena are speeded up in the centrifuge. Improved estimates of prototype behavior can also be obtained using the “modeling of model’s” technique which involves comparing the response of models of different sizes at the same prototype scale
i.e. under different enhanced gravitational fields, to check the scaling laws employed (Schofield, 1980).

2.3.1.1 Modelling factor and the scaling laws

Scaled models of prototype object have been used previously to study the behaviour of silo flows (Mathews, 2013) and granular flow stress-strain properties. Therefore, the same modelling approach will be used for modelling the hopper silo in the centrifuge tests in this thesis. Due to the small model geometry and the uniform and rigid drum centrifuge rotation, the radial acceleration will change through the model. Mathews (2013) provided initial steps towards better understanding of full-scale silos using instrumentation and investigation analysis of quasi-2D planar silo models. Equation 2-9 characterizes the geometric scaling factor between parameters in a model and a prototype.

\[ x^* = \frac{x_m}{x_p} \]  

Where:  
- \( x^* \) = scale factor (x, y and z direction) which
- \( x_m \) = dimension of the model in one - direction
- \( x_p \) = dimension of the prototype in the corresponding – direction

And,

\[ g^* = \frac{1}{L^*} \]  

Where  
- \( L^* \) = reduced model length (by N)
- \( g^* \) = enhanced gravity (by N)

Where N is the ratio of the centrifugal acceleration in the model to the acceleration due to gravity on the surface of the Earth, or simply referred as a gravity factor.

\[ N = \frac{g^*}{9.81 \text{ m/s}^2} \]  

This scaling law states that if lengths in the model are reduced by some factor ‘N’, then gravitational accelerations must be increased by the same factor ‘N’ to preserve equivalent stresses.
in the model and the prototype. If $W_1$, $v_1$ and $t_1$ are discharge rate, velocity of flow and the total period of flow in a full-scale silo or hopper flow test at normal gravitational acceleration, $g_1$, then comparing a similar model silo centrifuge test with mass discharge ($W_2$), period of flow ($t_2$) and velocity magnitude ($v_2$), the gravity scale factor ‘$N$’ at the different ‘$g_2$’ can be related as follows. Equations 2-12 up-to 2-14 are commonly known as Beverloo correlation (from Mathews and Barbir, 2016).

$$\frac{W_1}{W_2} = \sqrt{\frac{g_1}{g_2}} = \frac{1}{N}$$

(2-12)

$$\sqrt{\frac{v_2}{v_1}} = \sqrt{\frac{g_2}{g_1}} = N$$

(2-13)

$$t_2 = t_1 \sqrt{\frac{g_1}{g_2}}$$

(2-14)

Zhang et al. (2008) and Kim et al. (2009) investigated physical modeling of geotechnical systems using centrifuges. Comparing the stress ratios in the prototype and centrifuge scaled model, Zhang et al. (2008) suggested that although the ‘$g$’ level varied down a model that the error can be minimized by selecting the ‘target’ $N$ 2/3 down of the way down the model. It is shown that a model and prototype both experience similar stresses at two third of the height (2/3h) down the model as shown in Fig. 2.12. Similar conclusions were reached by Kim et al. (2009) based on the earlier studies done by Schofield (1980) and Taylor (1995).
Other common scaling laws for use with centrifuge models are shown in Table 2.1.

Table 2.1: Common scaling factors for centrifuge tests: parametric scale factors for model vs prototype (Zhejiang, 2014)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Scale Factor</th>
<th>Unit</th>
<th>Description</th>
<th>Scaling Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceleration</td>
<td>N</td>
<td></td>
<td>Linear dimension</td>
<td>1/N</td>
</tr>
<tr>
<td>Stress</td>
<td>1</td>
<td></td>
<td>Strain</td>
<td>1</td>
</tr>
<tr>
<td>Mass</td>
<td>1/n³</td>
<td></td>
<td>Density</td>
<td>1</td>
</tr>
<tr>
<td>Unit weight</td>
<td>n</td>
<td></td>
<td>Force</td>
<td>1/N²</td>
</tr>
<tr>
<td>Bending moment</td>
<td>1/n³</td>
<td></td>
<td>Bending moment/unit width</td>
<td>1/N²</td>
</tr>
<tr>
<td>Flexural stiffness</td>
<td>1/n⁴</td>
<td></td>
<td>Flexural stiffness/unit width</td>
<td>1/N³</td>
</tr>
<tr>
<td>Time (dynamic)</td>
<td>1/n</td>
<td></td>
<td>Time (consolidation/diffusion)</td>
<td>1/N²</td>
</tr>
<tr>
<td>Time (creep)</td>
<td>1</td>
<td></td>
<td>Pore fluid velocity</td>
<td>N</td>
</tr>
<tr>
<td>Velocity (dynamic)</td>
<td>1</td>
<td></td>
<td>Frequency</td>
<td>N</td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
<td>etc.</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2.12: Stress error along model depth in centrifuge modeling (source: Zhang et al., 2008)
2.3.2 Centrifuge testing of silos and hoppers

Grosstuck and Schwedes (2003) conducted centrifuge-based flow tests to investigate the critical outlet dimension for a silo with very fine and moist bulk solids. Two types of model silos were used with diameters of 10 cm. The first silo was a circular model made of stainless steel, with a height of 50 cm and equipped with various cone elements for changing the outlet diameters from 5 mm, 10 mm to 15 mm. Various wall inclinations of $10^0$, $15^0$ and $25^0$ to the axis symmetry were investigated. The study showed that the critical outlet diameter to prevent arching could be reduced by a factor of about two if soil creep (decreasing strains at constant stresses) is considered.

Mathews (2013) and Mathews and Barbir (2016) investigated the influence of gravity on granular flow using silo centrifuge models under increased gravity conditions. The study suggested that the mass flow rate as well as the local velocity of discharging is proportional to the square root of gravity. This study also found that the time required for a silo model to discharge cohesionless material scales with gravity. These findings compared discharge rates for each test from a combination of load cell and PIV results. Those results were compared with Beverloo’s correlation as presented in Fig. 2.13. The flow rate, $W$ was found out to 8% deviated from the Beverloo correlation (equation 2-3). Some of the discrepancy can be attributed to several simplifications that were made by proposing that $k$ has a value of 1, similar to other studies gained by analysing the Beverloo correlation (Mathews and Barbir, 2016). In general, the test results showed a strong correlation to the Beverloo equation.
Leonard et al. (2021) investigated the Coriolis-induced instabilities in centrifuge modeling of granular flows. The study defined “Coriolis” as an effect whereby a mass moving in a rotating system experiences a force (the Coriolis force) acting perpendicular to the direction of motion and to the axis of rotation. On the earth, the effect tends to deflect moving objects towards the earth gravitational direction (right in the Northern hemisphere and left in the Southern hemisphere). The study conducted a set of numerical experiments where the effects of the Coriolis acceleration were measured and analyzed. Three flow states were reported, namely dense, dilute, and unstable. It was found that flows generated under the influence of dilative Coriolis accelerations were more likely to become unstable. Nevertheless, the study also suggested that a steady dense flow can still be obtained if a large centrifuge is used. The study also suggested understanding the Coriolis-induced instabilities can ‘guide experimental designs and could help to avoid damage to the experimental apparatus and model instrumentation’.

2.4 Summary

This last section discusses various correlations proposed in the literature to predict discharge rates for granular materials. Though, most of these empirical correlations can be considered to be applicable for both spherical and non-spherical granular materials, the effect of particle size and distribution on the discharge rate is still not yet fully understood. The empirical formulae proposed
to calculate the granular flow rates from hoppers and silos have limitations in-terms of the description of geometry and the material properties. And hence, a proper analysis is required to establish a proper empirical formula for the specific model used in this paper.

The continuum model components in part 2.1.3 are studied using analytical and DEM approach in defining the velocity and the dynamic properties of the flow. Many of the studies used PIV to investigate the flow velocity profile and direction. These studies do not use PIV to clearly position the dynamic events of the flow, viz. dynamic arching, dynamic jamming, and free fall surface. The concept of free fall arch suggested the occurrence of nearly uniform velocity (with little fluctuations) below this arching soil mass. A close observation of the free fall velocity ($V_{FF}$) could be necessary using PIV to position and measure $V_{FF}$. These findings could also be related to geometry of hoppers and the soil property.

Mathews (2013) and Mathews and Barbir (2016) used a load cell and PIV to compare the flow rate based on Beverloo et al. (1961) correlations. Centrifuge enhanced gravity testing models operate under the influence of the Earth’s gravity and the enhanced gravities due to the centrifuge rotation. However, the previous studies did not specifically consider the soil reactions to these effects. Even though the effect of the resultant gravity (due to Earth’s and centrifuge gravities) on the flowing soil was covered (as Coriolis effect), however the effect of the self weight of the flowing soil due to Earth’s gravity was not fully discovered. Most of the centrifuge soil flow tests were conducted in a ‘beam centrifuge’. Even though the concept of gravities remain the same for conducting similar tests in a ‘drum centrifuge’, the instrumentation and the effects of the centrifuge structure to the overall flow testing have not been studied yet.
3.1 Overview of Methodology

The laboratory methodology was designed to investigate confined granular flow conditions in a model hopper under both natural gravity and enhanced gravity conditions. This was accomplished by studying and measuring the effect of the particle size of the materials, the orifice size of the hopper outlet, the material density and static or dynamic angles of repose on the granular flow rate. To explain any differences, these results were to be studied further by investigating the flow patterns. The dynamic flow behaviour was studied using particle image velocimetry (PIV) analysis. For flow conditions in a full body hopper, neither the flow pattern nor the material deformation conditions can be easily measured or observed. To preclude such difficulties, a half model hopper facing a camera for image analysis through a flat transparent face was used.

To assess the effect of the orifice size, a 2D scaled hopper (referred to here as the base hopper [BH]) see Fig. 3.1, has been built with uniform orifice extensions of 1cm, 1.5 cm and 2cm in diameter (further details shown in Fig. A1). The PIV analysis setup (Fig. 3-2) was adjusted to orientate it with the 2D hopper face. The PIV analysis was used to study the soil flow patterns, soil deformations and velocity components of the granular flows.

Fig. 3.1: Half model dimensions of the base hopper (BH), all dimensions in cm
Five materials were studied and used, viz. two Silica Sands (quartz) namely B-32, B-49; Toyoura (feldspar) and Mojave Martian Simulant (MMS, basalt). These different materials enabled the assessment of the effect of the material mass density, the particle size distribution and shape on the granular flow rate. The flow patterns, velocity component of the flow and the deformation conditions both at natural and enhanced gravity tests were studied. An investigation of the static angles of repose enabled an assessment of how the granular flow rate of the material relates to this material and state properties. A total of 46 granular flow tests were conducted to investigate the mass accumulation and granular outflow rate data. These tests were conducted with a soil state with relative density of $D_r=20\%$ and for all the materials. To investigate the flow properties of the materials at a single density, outflow tests were also conducted at a density of 1.58 g/cc for B-32, B-49 and Toyoura. To test the effect of increased density on the granular flow rate, additional flow tests were also conducted for B-49 at minimum soil density and at $D_r=60\%$.

The following sections describe the experimental setup, procedures followed, limited description of the measuring equipment, the measured data, test matrix and soil classification and basic characterization.

### 3.2 Experimental Apparatus and Procedures

The experimental apparatus setup includes the granular flow rate measurement test (Fig. 3.2), the PIV image capturing setup (Fig. 3.3) and the centrifuge testing setup (Fig. 3.3). The half model BH was created and 3D printed using the software Solidworks student version 2019. Table 3.1 shows the dimensions of the half model BH.

#### 3.2.1 ‘3D’ Printed hopper

The dimensions of the BH were selected to present a scaled geometric relationship to a prototype of 0.8 m depth at an enhanced centrifuge gravity 10g . Table 3.1 and Fig. 3.1 provide the detailed geometric dimensions of the BH. Fig. A-1 shows the 3D image of the BH with a $D_o=1$cm basal extension. The hopper orifice extensions were added to investigate the effect of the orifice size and contractions on outflow rate (W) and the deformation patterns as the soil flows.
Table 3.1: Geometric parameters of the 2D hopper (see Fig. A-1 for 3D orifice extension drawing)

<table>
<thead>
<tr>
<th>2D base hopper (BH)</th>
<th>Hopper orifice extensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclination angle, $\theta$</td>
<td>Extension height, $h_e$</td>
</tr>
<tr>
<td></td>
<td>30.7°</td>
</tr>
<tr>
<td>Hopper height, $h_o$</td>
<td>Extension diameters, $D_o$</td>
</tr>
<tr>
<td></td>
<td>18 cm</td>
</tr>
<tr>
<td>Hopper diameter, $D$</td>
<td>23.4 cm</td>
</tr>
<tr>
<td>Orifice diameter, $D_o$</td>
<td>2 cm</td>
</tr>
<tr>
<td></td>
<td>Extension height, $h_e$</td>
</tr>
<tr>
<td></td>
<td>4 cm</td>
</tr>
<tr>
<td></td>
<td>Extension diameters, $D_o$</td>
</tr>
<tr>
<td></td>
<td>1 cm</td>
</tr>
<tr>
<td></td>
<td>Inclination angle to the vertical</td>
</tr>
<tr>
<td></td>
<td>7.1°</td>
</tr>
<tr>
<td></td>
<td>3.6°</td>
</tr>
</tbody>
</table>

3.2.2 Granular flow rate actual measurements

A granular flow recording device shown in Fig. 3.2, based primarily on a Highland Portable Precision Balance (HCB 2200), was used with the BH, to measure the flowing material and data recording device. The HCB 2200 was used to measure the soil mass flowing from the hopper at a recording interval of 0.5 seconds. It has a measuring capacity from 120g to 6000g and readability precision of 0.001g to 0.1g However, a readability precision of 0.01g was used in this test for soil mass ranging from 1600g to 1950g.
3.2.3 PIV image capture

During the natural gravity granular flow tests, a series of images were taken throughout the hopper discharge period using a digital camera (GoPro Black 7), this was placed 10 cm from the experimental setup as shown in Fig. 3-2. The camera operates at 4K/60p, and this resolution is available with 10-bit color. The camera faces the flat plexiglass surface of the half model hopper for visualizing and capturing the flow patterns in this region. PIV image analysis techniques are reviewed in Chapter 2.
3.2.4 Drum centrifuge scaled tests

The half hopper body is designed to fit into an aluminum frame built in a centrifuge box. Fig. 3.4 shows the setup arrangement for the aluminum frame and the 2D hopper body. This frame arrangement is placed in the centrifuge box as shown in Fig. 3.4. This setup also includes a PIV camera system attached to the plexiglass face of the centrifuge box to enable the capturing of granular flow at any N times g. Fig. 3.4 shows the apparatus setup for the scaled flow tests done at Ng in the drum centrifuge. A GoPro7 camera class 10 UHS-I rating was attached to the camera stand position denoted by ‘B’ in Figure 3.4. The plexiglass face of the half body model was positioned facing 90° to the camera for a video recording during the test. An LED lighting system attached to the camera stand provides a bright image capture environment for best image resolution. A counterbalance mass was positioned opposite to this arrangement to enable the drum centrifuge to run in a mass equilibrium state.
Fig. 3.4: Testing model arrangement inside the drum centrifuge body

Fig. 3.5: The 2D granular flow rate measurement: BH and frame setup (see Appendix Fig. A-2 for detailed drawings and measurements)
3.2.4.1 Western University geotechnical drum centrifuge

There are two types of centrifuge facility available, namely beam and drum centrifuges. A drum centrifuge has a rotating annular channel (see Figs. 3.4 and 3.8) into which the model of a geotechnical structure is placed, and where the drum has a vertical axis. A beam centrifuge has a long arm with a swinging cradle at one end that rotates about a vertical axis (see Fig. 3.7). In this style of centrifuge, a model of a geotechnical structure is placed in a box or tub container that sits on the cradle. The cradle hangs vertically from its pivot pins when the beam is stationary and as the rotational speed increases, it swings up to a near-horizontal position.

The Western geotechnical drum centrifuge has a drum diameter of 2.2m and can accommodate a maximum of 450 rpm (revolutions per minute) which roughly gives 249g enhanced acceleration (g-gravitational acceleration=9.81 m/s^2). The centrifuge consists of a 360-degree drum channel with 0.4 m deep and 0.7 m wide and a diameter of 2.2 m. The drum is able to hold up to 3.5 tonnes of soil. The full centrifuge with motor system, covers and safety shrouds is shown in Fig. 3.6.

Fig. 3.6: The 2.2 m diameter drum channel geometry
Fig. 3.7: Schematic view of a beam centrifuge (source: Nater, 2005)

Fig. 3.8: The physical body structure of the Western geotechnical drum centrifuge (WGDC)

(source: https://www.eng.uwo.ca/grc/centrifuge/index.html)

The silo model in the drum centrifuge is placed inside a box in the orientation shown in Fig. 3.9. A balancing weight was placed in the opposite side of the drum for mass equilibrium during centrifuge operation. The centrifuge box with the model was placed where a camera can record the event during a test (see Fig. 3.3). The resultant gravity acting on the model is a result of the enhanced gravity input due to the centrifuge rotation and the Earth’s gravitational acceleration as shown in Fig. 3.9 and Fig. 3.10(d).
Protocols for centrifugation typically specify the amount of acceleration to be applied to the sample, rather than specifying a rotational speed such as revolutions per minute. During a circular motion the acceleration relative to "g" is traditionally named "relative centrifugal force" (RCF) (see equation 2.8). The acceleration is measured in multiples of "g" (or × "g") and is given by the product of the radius, R in m and the square of the angular velocity, ω similar to equation 2.8.

$$\text{RCF} = \frac{r \omega^2}{g}$$  \hspace{1cm} (3-1)

Equation 3-1 can further be expanded to equation 3-2 and 3-3 for ω=(2πRPM)².

$$\text{RCF} = 10^{-2} r_{cm} \frac{(2\pi\text{RPM})^2}{g}$$  \hspace{1cm} (3-2)

$$\text{RCF} = 1.118 \times 10^{-5} r_{cm}(\text{RPM})^2$$  \hspace{1cm} (3-3)

Where $r_{cm}$ - radius of sample from the center of the centrifuge in cm
RPM- revolution per minute
Equation 3-3 states that the centrifuge input acceleration (Ng) is in reference to the maximum radius of the centrifuge operation; the drum radius in a drum centrifuge. This equation helps to calculate the actual Ng-level on a body placed at a radius of \( r_{cm} \) from the center of the centrifuge.

### 3.2.4.2 Model preparation

During the acceleration, the force due to the Earth’s gravity acts parallel to the top surface of the model as shown in Fig. 3.9 and Fig. 3.10(d). The model was built to fit inside the centrifuge box such that the plexiglass of the model faces the transparent face of the centrifuge box as shown in Fig. 3.4 and Fig. 3.9. In general, the preparation of the models followed the following steps.

a) the model hopper is screwed into the aluminum frame (see Fig. 3.5 and Fig. 3.10(a)).
b) the model was filled with soil at the target density and securely held with a metallic cover from above and through a wireless operated electromagnet base cover over the basal orifice as shown in Fig. 3.10(b).
c) the model is then tilted 90° as in Fig. 3.10(c and d) through 90°
d) for its final positioning, the rotated box and model are placed in the drum channel (see Fig. 3.4 and Fig. 3.10(d)) and the target g-level is set, and the test commences.

---

**Fig. 3.10:** Process of model preparation outside and test inside the drum centrifuge for enhanced gravity outflow tests
3.2.4.3 Centrifuge g-level (Ng) test plan

The centrifuge operational acceleration was set by the control system outside the centrifuge. A steady target acceleration was usually reached after approximately 30 seconds of the order triggered from the control system. As well as the 1g tests, a range of enhanced g tests were conducted in the drum centrifuge for soil material B-49.

3.3 Basic Material Characteristics and Testing

3.3.1 Material mechanical properties

A detailed basic material property study was conducted for the Silica sand Barco-49 (B-49), Silica Sand B-32, Toyoura and Mojave Martian Simulant (MMS) soils. The mechanical tests included maximum and minimum densities ($\rho_{\text{max}}$ and $\rho_{\text{min}}$), maximum and minimum void ratios ($e_{\text{min}}$ and $e_{\text{max}}$), and specific gravity ($G_s$). These bulk density tests were done based on ASTM standard procedures (ASTM, 2006a, b, c) and the results are presented in Table 3.2. These results are compared with other findings in the literature and were consistent. A particle size distribution (PSD) and specific gravity ($G_s$) calculation study were also done for B-49. However the particle size distribution (PSD) and $G_s$ of the other material properties viz. Silica Sand B-32, Toyoura and Mojave Martian Simulant (MMS) were adopted from other studies (Peter et al., 2008; and Zaid and Sadrekarimi, 2015). Fig. 3.12 below shows the PSD of all the tested materials.

Table 3.2: Material state properties

<table>
<thead>
<tr>
<th>Sand Type</th>
<th>$\rho_{\text{min}}$</th>
<th>$\rho_{\text{max}}$</th>
<th>$G_s$</th>
<th>$e_{\text{min}}$</th>
<th>$e_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>1.519</td>
<td>1.743</td>
<td>2.65</td>
<td>0.52</td>
<td>0.745</td>
</tr>
<tr>
<td>B-49</td>
<td>1.577</td>
<td>1.791</td>
<td>2.65</td>
<td>0.48</td>
<td>0.68</td>
</tr>
<tr>
<td>Toyoura</td>
<td>1.349</td>
<td>1.634</td>
<td>2.65</td>
<td>0.61</td>
<td>0.95</td>
</tr>
<tr>
<td>MMS</td>
<td>1.187</td>
<td>1.517</td>
<td>2.67</td>
<td>0.76</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Table 3.2 shows that B-49, B-32 and Toyoura lie within a close range of maximum and minimum void ratios. However, MMS has a considerable variation between the maximum and minimum void ratios. This difference may affect the soil deformation patterns and the dilation as the soil
flows. The scanning electronic microscope (SEM) in Fig. 3.11 shows the Silica sands have sub-angular shape while Toyoura and MMS particles have a circular to semi-circular particle shape.

Fig. 3.11: Scanning Electronic Microscopic (SEM) images of materials
(source: SEM: for and Silica sand (Salfraj Ahmed, 2018); Toyoura and MMS (Western geotechnical team))

Fig. 3.12: Particle size distribution (PSD); (source: Toyoura and MMS from Peter et al., 2008 and Kobayashi et al., 2009; B-32 from Zaid and Sadrekarimi, 2015)
The $D_{50}$ size was used previously for correlation of the liquefaction potential of saturated granular soil during earthquakes (Das, 2002). Since this represents a similar ‘flow’ type behaviour, will be used as an effective mean-size particle diameter ($d$) for the granular flow calculations. Table 3-3 presents the average grain size ($D_{50}$) of all of the materials tested. The average particle size of the tested material ranges from 0.5 mm to 0.12 mm. The soil material uniformity and curvature coefficient of the materials is presented in Table 3.4. This test was done to better understand the materials’ grade distribution.

Table 3.3: Average particle size ($D_{50}$), mm

<table>
<thead>
<tr>
<th>Sand Type</th>
<th>Particle Size (d) mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>0.5</td>
</tr>
<tr>
<td>B-49</td>
<td>0.27</td>
</tr>
<tr>
<td>Toyoura</td>
<td>0.24</td>
</tr>
<tr>
<td>MMS</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Table 3.4: Uniformity and curvature coefficient of the materials

<table>
<thead>
<tr>
<th>Material type</th>
<th>Cu</th>
<th>Cc</th>
</tr>
</thead>
<tbody>
<tr>
<td>B - 32</td>
<td>1.833</td>
<td>0.990</td>
</tr>
<tr>
<td>B - 49</td>
<td>1.885</td>
<td>0.954</td>
</tr>
<tr>
<td>Toyoura</td>
<td>1.48</td>
<td>0.965</td>
</tr>
<tr>
<td>MMS</td>
<td>3.87</td>
<td>0.965</td>
</tr>
</tbody>
</table>

Where:

$$Cu = \frac{D_{60}}{D_{10}}$$  \hspace{1cm} Co-efficient of Uniformity  \hspace{1cm} (3-4)

$$Cc = \frac{(D_{30})^2}{D_{10} \times D_{60}}$$    \hspace{1cm} Co-efficient of Curvature  \hspace{1cm} (3-5)
ASTM D422 2007 specifies the value of $C_u<4$ and $C_c<(1-3)$ as poorly graded soils. Based on the Table 3.4, B-32, B-49 and Toyoura are therefore characterized as poorly graded soils. However, based on both $C_u$ and $C_c$ results, MMS is characterized as well graded soil.

### 3.3.2 Direct shear box test

The stress-strain behaviour of the sands was determined using the direct shear box test. In particular, the peak friction state, critical friction state and dilation angle were sought. Results for B-32, Toyoura and MMS was adopted from Deljoui (2012) and Fujikawa et al. (2019); see Table 3.4. Data for Barco B-49 was missing from these data set and was determined. The direct shear test results for B-49 were conducted based on ASTM D3080/D3080M-11 at different normal stresses of 25, 50, 75, 100 and 200 kPa respectively. The result presented in Fig. 3.13 shows the peak and critical state failure envelopes for the soil at a relative density of $D_r = 20\%$. This density was selected to be representative of typical filling situations for silo/hoppers. The peak dilation angle, as shown in Fig. 3.15, is seen to reduce with pressure from 25 kPa to 75 kPa, and is relatively constant above pressures of 75 kPa. Fig. 3.14 shows that, B-49 shows compression (-ve vertical displacement) during the staring of the test for 100 and 150 kPa. For the flow pattern tests, some of the B-49 soil was colored using food coloring, and retested to ensure no changes in the aforementioned properties. Even though PIV does not require such provisions to capture particle movement, coloring was used only to visualize the flow patterns for some presentation purposes.
Fig. 3.13: Shear stress vs horizontal displacement curve graph for B-49 sand

Fig. 3.14: Horizontal displacement and Vertical displacement for Ottawa Sand Barco-49 14 65
Table 3.5: Critical and peak internal friction angles of the different materials at $D_r = 20\%$ for pressures 25 to 200 kPa

<table>
<thead>
<tr>
<th>Material type</th>
<th>Friction angle, $\phi'_{\text{peak}}$</th>
<th>Friction angle, $\phi'_{\text{cs}}$</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>B - 32</td>
<td>40.4$^0$</td>
<td>35.5$^0$</td>
<td>Deljou, 2012</td>
</tr>
<tr>
<td>B - 49</td>
<td>36.9$^0$</td>
<td>32.6$^0$</td>
<td>Lab test result</td>
</tr>
<tr>
<td>Toyoura</td>
<td>31.6$^0$</td>
<td>29.8$^0$</td>
<td>Fujikawa et al., 2019</td>
</tr>
<tr>
<td>MMS</td>
<td>37.1$^0$</td>
<td>37.1$^0$</td>
<td>Fujikawa et al., 2019</td>
</tr>
</tbody>
</table>

Peak dilation angle $\psi$ for plane strain conditions in the direct shear tests can be estimated using Bolton’s (1986) empirical correlation [from Alessandro and Houlsby, 2006] as shown in equation 3-6. The dilation behavior of all four materials at different normal stress level is presented in Fig. 3.16 below.

$$\phi'_{\text{peak}} - \phi'_{cs} = 0.8\psi_{\text{peak}}$$  \hspace{1cm} (3-6)
3.3.3 Angles of repose

Although the angle of repose is well understood concept and has been identified as being similar to the critical state friction angle, there are few methods available to measure it. Hence, different values of the angle can be obtained for the same material. Hence, the static angle of repose for each material was studied by taking five replicates for the free-flow test for every soil material to ensure a statistically significant result.

3.3.3.1 Static angle of repose (\(\alpha_s\))

The static angle of repose may be related to capillary forces (no opposition to external forces like gravity) considering the microscopic fluid pockets between the particles. Fig. 3.17 shows the method adopted in assessing the static angle of repose. Five consecutive tests are assessed to calculate the static angle of repose. The average value adopted for this study are presented in Table 3-6. Kleinhans et al. (2011) details the factors that can affect an angle of repose for material flow. The study reported that the properties of particles, size and angularity, water content and the particle density affect the angle of repose of a soil material. If water is added to particles such as sand, water coating the grains would tend to bind them together by its surface tension This gives
rise to greater suction pore pressures and therefore higher shear strength. Kleinhans et al., 2011 also described some steps to determine the angle of repose.

The following procedure was adopted from Kleinhans et al., 2011 and follows the standard test method for measuring the angle of repose (ASTM C-1444)

1. For each type of sand, 200g was weighed.
2. It was poured through a conical funnel with diameter 0.95 cm.
3. The funnel was lift upwards close to the glass base, soil filling continues until the tip of the mold enters the funnel nozzle (ASTM C-1444) and the sands produce an intermediate slope on top of the rubber stopper with diameter of 6.4 cm.
4. The height and diameter of the peak was measured using a ruler.
5. Angle of repose was estimated using equation 3-7.

Fig. 3.17: Static angle of repose measurement procedure (photo: after full flow of material)

In this experiment, the static angle of repose of the four materials used in the granular flow tests was measured, viz. B-32, B-49, Toyoura and MMS.

The angle of repose ($\alpha$) was calculated, from the following equation:
\[ \tan(\alpha) = \frac{h}{0.5 \cdot \text{base } (D)} \] (3-7)

\( h_1 = \) height of rubber stopper = 4 cm

\( h_2 = \) height of rubber stopper + height of cone of sand

Height (h) = height of the cone of sand = \( h_2 - h_1 \)

\( D = 6.4 \) cm

Table 3.6: Static angle of repose, \( \alpha \) in degrees (from equation 3-5)

<table>
<thead>
<tr>
<th>Material</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
<th>Test 5</th>
<th>Average, in degrees</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>25.11</td>
<td>29.36</td>
<td>26.57</td>
<td>27.98</td>
<td>29.36</td>
<td>27.67</td>
<td>1.84</td>
</tr>
<tr>
<td>B-49</td>
<td>27.98</td>
<td>27.98</td>
<td>26.57</td>
<td>27.98</td>
<td>27.98</td>
<td>27.70</td>
<td>0.63</td>
</tr>
<tr>
<td>Toyoura</td>
<td>30.70</td>
<td>32.01</td>
<td>29.36</td>
<td>29.36</td>
<td>30.70</td>
<td>30.42</td>
<td>1.11</td>
</tr>
<tr>
<td>MMS</td>
<td>38.00</td>
<td>38.00</td>
<td>36.87</td>
<td>35.71</td>
<td>38.00</td>
<td>37.31</td>
<td>1.02</td>
</tr>
</tbody>
</table>

The experimental results in revealed that in contrary to the statements given in earlier studies, the angle of repose seems to increase as the size of particle of the material decreases. This study also shows that the more angular the granular material particles the higher the angle of repose. The averages for the angle of repose of the materials were: B-32 27.67⁰, B-49 27.70⁰, Toyoura 30.42⁰ and MMS being 37.31⁰. The angles of repose (\( \alpha \)) from this experiment (Table 3.6) can be compared to the critical state friction angles (\( \phi'_{cs} \)) in Table 3.5. Toyoura and MMS individually showed similar results of angle of repose and critical state friction angle. However, B-32 and B-49 showed slightly smaller angle of repose values compared to the critical state angle of friction. The MMS values may also be influence by the very small clay percent.
3.3.4 Sample preparation for the granular flows and PIV testing

3.3.4.1 Soil test groupings

The soil materials were put into different groups as shown in Table 3.7. This was done to determine aspects of interests as stated in the objectives. Chapters 4 and 5 in this thesis will follow this grouping with regard to the data presentation and analysis.

Table 3.7: Soil test groups for analysis and comparison

<table>
<thead>
<tr>
<th>Soil test group</th>
<th>Soil name</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-I B-32</td>
<td>To study and compare the effect on mass flow rate in g/s of:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• average particle size, $D_{50}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• the orifice (outlet) size, $D_o$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>To study and use these parameters to calculate mass flow rate</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Static angle of repose, $\alpha_s$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• difference in vertical velocity, Free flow velocity ($V_{FF}$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Evaluating:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• mass flow rate and therefore $C$ (discharge coefficient)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>and $k$ (particle shape coefficient)</td>
<td></td>
</tr>
<tr>
<td>S-II B-32</td>
<td>Exhibit similar flow patterns and therefore will be considered for:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Velocity magnitude profile scaling</td>
<td></td>
</tr>
<tr>
<td>S-III B-49</td>
<td>Deformation patterns</td>
<td></td>
</tr>
</tbody>
</table>

3.3.4.2 Target dry density (tapping density)

To compare the effect of density or relative density on the discharge flow rate ($W$), the materials’ flow was conducted at different dry densities. The dry density ($\gamma_d$) of the sands used for testing at different $D_r$ was calculated based on equation 3-8 from Budhu (2007) and Table 3.2. Where $\gamma_{max}$ and $\gamma_{min}$ are the maximum and minimum unit weight of the flowing material.
\[
D_t = \frac{\gamma_{max}(\gamma_d - \gamma_{min})}{\gamma_d(\gamma_{max} - \gamma_{min})}
\] (3-8)

The half model hopper has a diameter of 2.0 cm; and small extensions of 4 cm in height with diameters 1.5 and 1 cm were also added for different orifice size accommodations (see Fig. A-1). Based on the of the volume of the half body hopper and/or the extensions of Do=1.5 and 1 cm, the material mass for each soil was calculated as density multiplied by volume in grams (see Table 3.1 for dimensions). To attain the required dry density ($\gamma_d$), the hopper was filled carefully and volume and mass measured to ensure consistency. Table 3.8 gives the volume of the models used. This was calculated based on the dimensions presented in Table 3.1. These soil preparations apply to both natural gravity and enhanced gravity tests.

Table 3.8: Volume of model used.

<table>
<thead>
<tr>
<th>Model</th>
<th>Volume, cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half body hopper, Do=2cm</td>
<td>1409.835</td>
</tr>
<tr>
<td>Half body hopper and extension with Do=1.5cm</td>
<td>1414.678</td>
</tr>
<tr>
<td>Half body hopper and extension with Do=1cm</td>
<td>1413.5</td>
</tr>
</tbody>
</table>

### 3.3.5 Test Matrix

To avoid jamming, the individual orifice opening ‘Do’ was selected to be greater than six times the particle diameter ‘d’ (Beverloo et al, 1961). To ensure results independent of the ratio of Do to d, the granular flow experiments with four materials was repeated with three orifice sizes of diameters Do =2, 1.5 and 1 cm. A total of 36 granular flow tests were conducted, each tests repeated 3 times to assess the granular flow rate and data uniformity. To assess the effect of density on the granular flow, other additional 10 tests at a uniform density of 1.58 g/cc were conducted using Do=2 cm. Each test was repeated three times for statistical significance for B-32, B-49 and Toyoura. Table 3.9 details the test matrix conducted for the granular flow and PIV analysis during natural and enhanced gravity tests.
Table 3.9: Tests performed

<table>
<thead>
<tr>
<th>Material</th>
<th>Natural gravity tests (1g)</th>
<th>Enhance gravity tests (Ng) for B-49</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Balance mass flow, 36 tests</td>
<td>PIV analysis</td>
</tr>
<tr>
<td>B-32</td>
<td>Do=1 ✓</td>
<td>Do=1.5 ✓</td>
</tr>
<tr>
<td>B-49</td>
<td>✓ ✓ ✓</td>
<td>✓ ✓ ✓</td>
</tr>
<tr>
<td>Toyoura</td>
<td>✓ ✓ ✓</td>
<td>✓ ✓ ✓</td>
</tr>
<tr>
<td>MMS</td>
<td>✓ ✓ ✓</td>
<td>✓ ✓ ✓</td>
</tr>
</tbody>
</table>
CHAPTER 4: EXPERIMENTAL RESULTS AND DISCUSSIONS

4.1 Introduction

The results of the continuous mass flow measurements from model hoppers, the discharge rate (W) under natural and enhanced gravity along with the PIV analysis are presented in this chapter. The continuous mass flow measurements are presented first, followed by the comparison of the discharge rates of the different materials. The flow was investigated based on three different orifice sizes of 2.0, 1.5 and 1.0 cm. The first section of this chapter studies the effect of the orifice size, the material average particle size, mass density and static angle of repose on the discharge rate (W). The second section of the chapter will investigate the discharge flow tests conducted at enhanced gravities in the centrifuge. This discusses the enhanced flow results with reference to the commonly used Beverloo correlations and the resultant ‘g’ level direction created from a combination of the centrifuge and Earth’s gravity loads. The discharge rate under natural gravity is presented in relation to Beverloo literature comparisons, supplementary correlations, and empirical relations. PIV results show the flow patterns for each combination of material and orifice size, allowing comparison of the deformation patterns among different materials with the same orifice size. This section also investigates the PIV flow patterns for both natural and enhanced flows.

As an example, Fig. 4.1 shows a typical flow pattern using coloured B-49 sand with a half hopper of diameter 1.5 cm under natural gravity. The images show photos of the hopper sand material with PIV velocity vectors added to show areas of relatively high particle velocities. This analysis was done to investigate and find flow relationships and to determine the general flow patterns of the flow in this hopper models. Fig. 4.1 shows the soil deformation during the first 0.5 sec (Figs. 4.1(a to e) and at 1.8 secs (Fig. 4.1(f)). The first five images show an arch forming and moving upwards as the orifice opens and the soil starts to flow. Similar comparisons are made through this chapter for the various test variations. After detailed study of the flow patterns during the natural and enhanced gravity, the last section of this chapter presents general predictions of the flow
patterns for the half body hopper. These results will be compared to the natural gravity results in terms of scaling and flow behaviour.

Fig. 4.1: Funnel flow patterns of a coloured B-49 sand for D_o=1.5cm at natural gravity

4.2: Granular Flow Results Under Natural gravity

The granular flow tests conducted at natural gravity consisted a total of 45 individual tests as shown in Table 4.1, 4.2 and 4.3. Table 4.1 presents 36 individual tests at relative density of 20% (loose) for orifice sizes, D_o=1.0, 1.5 and 2.0 cm. Table 4.2 shows a second set of 9 granular flow tests conducted at a uniform density of 1.58 g/cc using D_o=2 cm. both tables show the total mass outflow collected in each test in grams and the total time of flow in seconds. Column 4 of Table 4.1 also gives the flow rate in g/s for each test and column 7 provides the standard deviation of the data from the measured average values shown in column 6.

4.2.1 Mass accumulation measurements (D_r=20%)

Chapter 3 discusses the use and operation of the HCB-2200 Highland portable precision balance. Presented below are the mass accumulation graphs for different materials with orifice sizes measured by the HCB-2200. From Figs. 4.2 to 4.4, it can be observed that the mass accumulation
and the period of flow are predominantly linearly and in have different slopes for the different materials and orifice sizes.

Fig. 4.2: Granular flow mass accumulation with $D_o=2\text{cm}$

Fig. 4.3: Granular flow mass accumulation with $D_o=1.5\text{cm}$
Tables 4.1 and 4.2 present a summary of the discharge results and the repeatability of the measurements. Columns 3 and 4 of Table 4.1 shows the total mass in grams and the period of flow in seconds measured for each test. Figs. 4.2 to 4.4 show the mass accumulation measured with the HCB-2200 every 0.5 secs to a precision of 0.01 grams for the three orifice diameters. Figs. 4.2 to 4.4 show a similar mass accumulation trends for B-32, B-49 and Toyoura sands. This is not unexpected due to the similar PSD graphs of these materials as shown in Fig. 3.12. Although these materials showed similar rate of mass accumulation, the variations of these trends seems to change as the ratio of the orifice diameter (D_o) and the average particle size (d) decreases (see Fig. 4.4).

The mass accumulation for the B-32 (with average particle diameter size of 0.5mm), B-49 (with average particle diameter size of 0.27mm) and Toyoura sands (with average particle diameter size of 0.24mm) increases for larger orifice diameters of 2.0 and 1.5 cm (see Figs. 4.2 and 4.3). However, B-32 showed a slightly different response for the 1.0 cm diameter flow as shown in Fig. 4.4. In addition to the non-linear trend, B-32 also showed a smaller slope of mass accumulation compared to B-49 and Toyoura for flows with orifice sizes of 1.5 and 1cm. The differences shown in the overall behaviours seems to relate to the ratio of the orifice size and the average particle size.
Based on these observations, a conclusion can be reached that the mass accumulation is directly related to $\frac{D_0}{d}$.

MMS (with average particle size of 0.12mm) in contrast showed a lower slope compared to B-32, B-49 and Toyoura in all cases (see Figs. 4.2 to 4.4). This seems to contradict the conclusion made above regarding the relations of $\frac{D_0}{d}$ and the mass accumulation. These results show that the mass accumulation may not depend only on the size of the orifice and the particle size distribution (PSD). MMS is known to have a small percentage of clay (< 5%) and is very angular compared to the other materials. Research studies have showed that MMS has a cohesion of 3.8 to 15 kPa measured with peak friction angles of 37.1° and 46.0° (Fujikawa et al., 2019). The effect of this cohesion and angularity not only affects the mass accumulation, but it was also observed to cause mass ratholing during the flow. This phenomenon will be discussed later in the thesis.

### 4.2.2 Granular flow rates

The discharge rate (W in g/s) for all of the tests at a relative density of 20% is given in column 6 of Table 4.1. Figs. 4.5 to 4.7 show the graphs of the average discharge rate for all of the materials and orifice sizes.

![Granular flow rates](image)

**Fig. 4.5:** Granular flow rates for MMS, B-32, B-49 and Toyoura, with $D_0$=2.0 cm, at frequency of 0.5 secs
The discontinuity of the outflow velocities (already noted in other studies discussed in Chapter 2) is also observed in the fluctuations during the measurements of the discharge rate over the full
period of flow. Figs. 4.5 to 4.7 show similar findings, and less fluctuations are shown for the cohesionless materials (B-32, B-49 and Toyoura) compared to the relatively cohesive material (MMS). The increased fluctuations for MMS in Figs. 4.5 to 4.7 appears to be the result of the cohesion of the soil. That is, the formation of ratholing during the flow causes large velocity variations across the body of the flow impacting the discharge rates (see PIV images analysis in section 4.5). The discharge rate measurements also hold true for \( \frac{D_o}{d} \geq 30 \), where graphs of discharge rate (W) vs period of flow show a direct relationship between the discharge rate (W) and \( \frac{D_o}{d} \) (See Fig. 4.5). Unlike the observations on the mass accumulation, all flowing materials showed nearly equal outflow rates for \( D_o=1.0 \) cm. Similar to the observations on the mass accumulation, decreases in discharge rate measurements were observed with B-32 after 50\%T (T-total period of flow) as the orifice size decreases to \( D_o=1.5 \) cm as shown in Figs. 4.6 and 4.7. The relative granular flows of B-49, Toyoura and MMS showed similar trends of discharge rate measurements for all orifice size as shown in Figs. 4.5 to 4.7. Similarly, as the orifice size further decreased to \( D_o=1.0 \) cm, the relative granular flow fluctuations for B-32 shows a relative decrease of discharge rate at 10\%T of the flow as shown in Fig. 4.6 and Fig. 4.7. This observation shows that the granular flow rate also started an inverse relationship to the particle size and orifice size for \( \frac{D_o}{d} \leq 30 \) as mentioned in the previous part.

4.2.3 Granular flow test result summary

Tables 4.1 presents a summary and measurement repeatability of flow tests for all the materials at 20\% relative density and with orifice sizes of 2.0, 1.5 and 1.0 cm.
Table 4.1: Repeatability of the granular flow measurements and variations (Dr=20%)

<table>
<thead>
<tr>
<th>Material</th>
<th>Orifice, Do</th>
<th>Observed total mass, g</th>
<th>Flow rate, W, g/s</th>
<th>Average, W g/s</th>
<th>Standard Deviation, g/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>1 cm</td>
<td>1697.26 64.50 26.31</td>
<td></td>
<td>26.98 0.72</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5 cm</td>
<td>2192.11 79.00 75.77</td>
<td></td>
<td>75.25 0.84</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 cm</td>
<td>2195.40 28.50 88.39</td>
<td></td>
<td>88.99 0.59</td>
<td></td>
</tr>
<tr>
<td>B-49</td>
<td>1 cm</td>
<td>2190.65 78.00 28.56</td>
<td></td>
<td>28.85 0.95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5 cm</td>
<td>2193.68 28.00 76.97</td>
<td></td>
<td>77.05 1.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 cm</td>
<td>2198.86 28.50 77.51</td>
<td></td>
<td>77.66 0.95</td>
<td></td>
</tr>
<tr>
<td>Toyoura</td>
<td>1 cm</td>
<td>1983.90 70.50 28.14</td>
<td></td>
<td>26.90 1.18</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5 cm</td>
<td>1989.95 29.50 66.33</td>
<td></td>
<td>67.01 1.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 cm</td>
<td>2094.12 29.50 70.99</td>
<td></td>
<td>70.74 0.52</td>
<td></td>
</tr>
<tr>
<td>MMS</td>
<td>1 cm</td>
<td>1775.33 92.38 19.22</td>
<td></td>
<td>18.62 0.54</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.5 cm</td>
<td>1582.65 74.00 21.24</td>
<td></td>
<td>22.18 0.94</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 cm</td>
<td>1721.02 78.50 21.92</td>
<td></td>
<td>21.36 0.57</td>
<td></td>
</tr>
</tbody>
</table>

4.2.3.1 Mass accumulation and granular flow rates at a density of ρ=1.58 g/cc

The fixed density granular flow tests were conducted to compare to and the behaviour of the low relative density tests (Figs. 4.2 to 4.7). Soil group S-II (B-32, B-49 and Toyoura) with Do=2cm
were studied for this purpose. Fig. 4.7 shows the mass accumulation per 0.5 seconds, and Fig. 4.8 shows the discharge rate measurements.

Fig. 4.8: Granular flow mass accumulation with \( D_o=2.0 \) cm for fixed density of \( \rho=1.58 \) g/cc

Fig. 4.9: Granular flow rates with \( D_o=2.0 \) cm for fixed density of \( \rho=1.58 \) g/cc, at frequency of 0.5 secs
The laboratory test results presented in Table 3.2, for the fixed 1.58 g/cc density was proposed because B-49 has a minimum density of approximately the same value. Based on equation 3-8 from Budhu (2007) and Table 3.2, a density of 1.58 g/cc gives relative densities for B-32 (D_r=28.9%) and Toyoura (D_r=83%). This shows that B-32 and Toyoura were tested at higher relative density. The mass accumulation measurements and discharge rate measurements at these relative densities as shown in Figs. 4.8 and 4.9 and were compared to the results in Fig. 4.2 and Fig. 4.5 (20% relative density tests). The comparison shows that, B-32 and Toyoura had less discharge rate fluctuation compared to the tests done at lower relative densities. However, the trend of flow for the 20% relative density and fixed density tests remains the same for D_o=2.0 cm; that is W increases as the particle size decreases. Less fluctuation in discharge rate suggests less velocity discontinuity. Hence, such result suggests that the granular materials flowing at higher relative density experience less “dynamic” stresses interacting with the walls of the hopper. Table 4.2 shows the repeatability of the measurement and the variations of these results.

Table 4.2: Repeatability of the granular flow measurements and variations at ρ=1.58 g/cc

<table>
<thead>
<tr>
<th>Material</th>
<th>Orifice size</th>
<th>Observed total mass</th>
<th>Observed total time</th>
<th>Observed flow rate</th>
<th>Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>Do=2cm</td>
<td>2038.26</td>
<td>24.50</td>
<td>84.00</td>
<td>89.06</td>
<td>7.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2038.39</td>
<td>20.50</td>
<td>97.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1894.29</td>
<td>21.50</td>
<td>86.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B-49</td>
<td>Do=2cm</td>
<td>1830.00</td>
<td>22.50</td>
<td>81.20</td>
<td>82.00</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1825.72</td>
<td>22.00</td>
<td>82.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1867.98</td>
<td>22.50</td>
<td>82.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Toyoura</td>
<td>Do=2cm</td>
<td>2012.07</td>
<td>26.50</td>
<td>74.52</td>
<td>74.51</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1998.56</td>
<td>26.50</td>
<td>74.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1987.05</td>
<td>26.00</td>
<td>74.98</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note that MMS showed no flow for relative densities greater than 20% under natural gravity flow tests

4.2.3.2 Discharge rates: the effect of densities for the B-49 flowing material

To investigate the effect of density on the flow rate, discharge rate tests were conducted on B-49 only at its minimum density (D_r=0%), and relative densities of 20% and 60% with an orifice diameter D_o = 2.0 cm. The minimum density of B-49 was found to be ρ=1.58 g/cc (see Table 3.2) and the densities at 20% and 60% from equation 3.6 (Budhu, 2007) and Table 3.2 give ρ=1.616
g/cc and ρ=1.699 g/cc respectively. Figs. 4.10 and 4.11, and Table 4.3 show the results of these tests. The discharge rate results in column 6 of Table 4.3 indicate that the discharge rate at relative densities of 20% and 60% showed a decrease by -5% and -10% compared to the discharge rate at minimum density. These, -5% and -10% differences in discharge rates correspond to an increase in density of 0.036 g/cc and 0.119 g/cc respectively. However, the difference in discharge rate compared to the percentage difference and density difference are relatively small and could be neglected in the case of general use. This indicates that, changing the density across the same material with this relative orifice size may not significantly contribute to variations in the discharge rate measurements.

Fig. 4.10: Granular flow mass accumulation with D_o=2.0 cm at different densities for B-49 sand
Fig. 4.11: Granular flow rate for with \( D_o = 2.0 \) cm at different densities of B-49 sand, at frequency of 0.5 sec

Table 4.3: The effect of density on granular flow rate test results for B-49 with \( D_o = 2.0 \) cm for \( \text{Dr} = 0\% \), 20\% and 60\%

<table>
<thead>
<tr>
<th>Material</th>
<th>Orifice size</th>
<th>Material state</th>
<th>Observed total mass</th>
<th>flow rate</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>( D_o = 2 \text{cm} )</td>
<td>( \text{Dr} = 0% )</td>
<td>1825.72</td>
<td>82.76</td>
<td>Table 4.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \text{Dr} = 20% )</td>
<td>2163.44</td>
<td>78.67</td>
<td>Table 4.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \text{Dr} = 60% )</td>
<td>2175.92</td>
<td>75.03</td>
<td>Fig. 4.10 and 4.11</td>
</tr>
</tbody>
</table>

4.2.4 **Summary: mass flow and discharge rates under natural gravity**

The average granular flow rate and the granular flow rate relative fluctuations increased for \( 82 > \frac{D_o}{d} \geq 30 \). A closer study of the discharge fluctuations, shown in Fig. 4.12 below (global flow can be seen in Fig. 4.5) and show however, that the relative granular flow rate fluctuation decreases from 10\%T (T-total flow period, see Table 4.1). These results show that, the granular flow empirical correlations can not efficiently be used for all conditions of \( \frac{D_o}{d} \). This comparison also
suggests that $\frac{D_o}{d}$ could relatively be a main factor to calculate the granular flow rate for cohesionless materials with $82 < \frac{D_o}{d} \geq 30$ and under natural gravity.

The literature review on continuum mechanics covered the fact that these flow fluctuations do not affect the uniform flow rate in general. However, the granular flow rate relative fluctuations may not necessarily affect the average granular flow rate, but the effect of decreasing or increasing granular flow rate relative fluctuations can cause a significant discharge rate fluctuation which in turn may cause a significant measurement error for granular flow at higher scale silos or hoppers.

As discussed in Chapter 2, Mollon and Zhao (2013) showed the presence of vertical velocity fluctuation. The observation on the granular flow rate of the tested materials also shows the fluctuations of the measured granular flow. Fig. 4.12 shows a fluctuations by around ±2%, in every 1 second period for all the materials. This effect can also be observed on videos recorded during the flow, where the soil falling from the orifice was seen to flow in a spreading and smaller areas.

![Graph showing granular flow rate fluctuations](image)

**Fig. 4.12:** Granular flow rate fluctuations from 2%T to 100%T of the flow, $D_o=2cm$ (see Table 4.1 for Total periods of flow, T-values)
4.3 Surface depress

This section studies the surface depression sloes as the materials flows in the half body hopper. These slope angles are commonly referred as the dynamic angle of repose ($\alpha_d$) and relates to the frictional response of the materials. Mobilization of the material into a moving avalanche necessarily involves dilation, which reduces the number of contacts and potentially the contact forces. Once a granular material is moving, the momentum and reduced friction can cause it to flow below the static angle of repose, and rest at the dynamic angle of repose (Hungr, 1995; Walton et al., 2007; Mangeney et al., 2010 from Kleinhans et al., 2011).

In the hopper model tests, the soil materials appeared to flow at two different angles, steepest at the bottom of the surface flow and slightly less steep at the top (see Fig. 4.14). Cheng (2016) quantified the differences for dynamic angles of repose in different situations. The study conducted a series experiments with a rotating drum to measure the dynamic angle of repose describing the inclination of a surface layer of a continuous sediment motion. When the drum was rotated, initially at a very low rotating speed, the sediment grains moved together with the drum, demonstrating a rigid body motion until the slopes reached an upper angle, $\alpha_u$ (see Fig. 4.13(a)). Then, for a further increase in the slope angle - an avalanche, transporting grains down the slope. At the end of the avalanche, a new slope formed at a lower angle, $\alpha_l$ (see Fig. 4.13(b)). Cheng (2016) found that the difference between these angles of repose was 5 to 11 degrees and may represent upper and lower bounds on these types of flow.
The dynamic angle of repose for the granular flowing materials in the half body hopper with $D_o=1.5\text{cm}$ was measured at different stages of the flow (15\%T, 30\%T, 45\%T, 60\%T and 80\%T; where T is total period of material flow). The image analysis seems to be in agreement with Cheng (2016) who showed a steep angle ($\alpha_U$) near the central funnel flow followed by a slightly less steep angle ($\alpha_L$). Fig. 4.15 shows an example of such a finding for the B-49 sand. The height (H, cm) and the length (L, cm) of the vertically falling and flowing material were measured with of an e-ruler. Dynamic angles of repose ($\alpha_U$ and $\alpha_L$) were then calculated using equation 4-1.

$$\tan(\alpha_d) = \frac{H}{L}$$ (4-1)
Fig. 4.14: The dynamic angles of repose in different zones of the surface depression flow (Flow at $T=11.3$ secs, $30\%T$, $D_o=1.5\text{cm}$)

It was found that $\alpha_U$ (lower slope angle) was only slightly greater than $\alpha_L$ from the image analysis (between $3^0$ to $7^0$). Therefore, in this thesis the dynamic angle will be estimated from the whole slope angle as shown in Fig. 4.15.

Fig. 4.15: Measurement of slope angle using e-ruler
Table 4.4: Dynamic angle of repose calculation results in degrees (equation 3-7)

<table>
<thead>
<tr>
<th>Material</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>Average angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>32.47</td>
<td>32.54</td>
<td>31.76</td>
<td>32.50</td>
<td>32.97</td>
<td>32.45</td>
</tr>
<tr>
<td>B-49</td>
<td>28.07</td>
<td>28.67</td>
<td>34.35</td>
<td>32.01</td>
<td>31.50</td>
<td>30.92</td>
</tr>
<tr>
<td>Toyoura</td>
<td>22.78</td>
<td>29.36</td>
<td>33.33</td>
<td>35.22</td>
<td>33.69</td>
<td>30.87</td>
</tr>
<tr>
<td>MMS</td>
<td>46.47</td>
<td>53.75</td>
<td>45.00</td>
<td>56.48</td>
<td>49.32</td>
<td>50.20</td>
</tr>
</tbody>
</table>

Table 4.5: Static and dynamic (1g) average angle of repose in degrees

<table>
<thead>
<tr>
<th>Material</th>
<th>Static angle of repose</th>
<th>Dynamic angle of repose</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>27.67</td>
<td>32.45</td>
</tr>
<tr>
<td>B-49</td>
<td>27.70</td>
<td>30.92</td>
</tr>
<tr>
<td>Toyoura</td>
<td>30.42</td>
<td>30.87</td>
</tr>
<tr>
<td>MMS</td>
<td>37.31</td>
<td>50.20</td>
</tr>
</tbody>
</table>

Table 4.5 shows that the dynamic angle of repose with the static values (see also Table 3.6). Generally the dynamic angles are higher than the static angles, although there is quite significant variation between the materials (from 74% to 90%). The study done by Hamzah et al. (2018) showed that granular particles can flow at higher angles of repose. The dynamic angle of repose for MMS is much larger than the static angle of repose (see Table 4.5). This may be caused due to the cohesive and angular nature of the soil. These findings will be further discussed in the coming sections.

4.4 Enhanced Gravity Discharge Rates

This part presents the results of the enhanced gravity flow tests conducted in the Western geotechnical drum centrifuge (see section 3.2.4 for the instrumentation and procedures). The same half model hopper used for the natural gravity tests was also used for the enhanced gravity flow tests (see section 3.2.1 for details on the model). This model represents a prototype with dimensions of ‘N times’ the model dimensions for each enhanced gravity (N-g) test. The discharge rate tests were conducted for centrifuge drum base gravities of 2.5, 5, 10 and 15 g. Fig. 4.16 shows the position of the hopper inside the centrifuge and the radius upon which the gravity acts, \( r_{cm} \) (see also Figs. 3.4 and 3.9 for 3D visualization).
Based on the review on Chapter 2, and many studies done in the past, the target ‘g’ level on the model can be taken at 2/3 of the height from the top (Schofield, 1980; Taylor, 1995; Zhang et al., 2008; and Kim et al., 2009). Table 4.6 shows the ‘average’ enhanced gravity calculated based on equation 2-8 and 3-3 and acting at \( r_{cm}=84.7 \) (see Fig. 4.16).

Table 4.6: The enhanced gravity (N*g) on the hopper body, at radius \( r_{cm}=84.7 \) cm from the center

<table>
<thead>
<tr>
<th>Centrifuge input acceleration (Ng) at R=110cm</th>
<th>Enhanced gravity acting 2/3 down length of hopper</th>
<th>( r_{cm}=84.7 ) cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>45</td>
<td>1.9</td>
</tr>
<tr>
<td>5</td>
<td>64</td>
<td>3.9</td>
</tr>
<tr>
<td>10</td>
<td>90</td>
<td>7.7</td>
</tr>
<tr>
<td>15.2</td>
<td>111</td>
<td>11.7</td>
</tr>
</tbody>
</table>

* \( N^* = \) enhanced gravity factor at \( r_{cm} \) acting on the model hopper

Equations 2-9 to 2-11 present the appropriate scaling laws, and states that if lengths in the model were being reduced by some factor ‘N’, then gravitational accelerations must be increased by the same factor ‘N’ to preserve equivalent stresses in the model and the prototype. Based on these scaling principles and Table 4.6, Table 4.7 gives the equivalent prototype dimensions represented in these enhanced gravity flow tests.

Table 4.7: Model hopper at natural gravity and prototype dimensions at enhanced gravity

<table>
<thead>
<tr>
<th>Gravity level</th>
<th>Model hopper</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>1g</td>
<td>18</td>
<td>70</td>
</tr>
<tr>
<td>1.9g</td>
<td>35</td>
<td>138</td>
</tr>
<tr>
<td>3.8g</td>
<td>70</td>
<td>179</td>
</tr>
<tr>
<td>7.7g</td>
<td>91</td>
<td>273</td>
</tr>
<tr>
<td>11.7g</td>
<td>210</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4.16: Hopper model and N-g gravity radius ($r_{cm}$)

### 4.4.1 Coriolis effect

The review by Leonard et al. (2021) defined the Coriolis effect as an effect whereby a mass moving in a rotating system experiences a force (the Coriolis force) acting perpendicular to the direction of motion and to the axis of rotation (see Chapter 2). Figs. 3.8 and 3.9 show the directions of the enhanced gravity acting on the model (N-g) from the centrifuge, the Earth’s gravity and a vector resultant of the two. Similarly, the flow tests conducted on the centrifuge also experienced this effect, where the materials appear to flow in an inclined direction, in the direction of the resultant gravity as shown in Fig. 4.17, for enhanced gravities below 11.7g.
The measurement of the inclination angle due to this Coriolis effect ($\theta_C$) during the enhanced flow tests is shown in Fig. 4.19. This effect was investigated under the actual enhanced gravity acting on the model at a radius $r_{cm}$ from the center of the centrifuge ($N^*-g$) and with the centrifuge input enhanced gravity ($N-g$) acting at the radius of the drum, $R=110$cm. The relationships show a decrease of the effect as the enhanced gravity increases. The Coriolis effect becomes relatively negligible at $N^*-g$ of 11.7g (see Figure 4.19). Fig. 4.18 demonstrates that, the resultant vector angle of Earth’s gravity ($N=1$) and the enhanced gravity ($N>1$), decreases as the enhanced gravity increases. Fig. 4.18 gives a polynomial relationship with $R^2=0.997$ between the enhanced gravity and the inclination angle of the flow due to Coriolis effect. For general use purposes equation 4.2 (due to gravity at $r_{cm}$) can be used to predict the vector angle due to the Coriolis effect.

$$\theta_C = -0.1101(g)^2 + 0.0895(g) + 20.618 \quad R^2 = 0.9975$$ (4-2)

The Coriolis force is shown to cause the flow of material (below the outlet) to fall at an angle under gravity and should be taken into account if the target landing zone is important (e.g., For a soil
It is anticipated that the flow in the hopper will also be influenced, but this is discussed later in the Chapter.

Fig. 4.18: Inclination angle due Coriolis effect ($\theta_C$) in degrees vs enhanced gravities
Fig. 4.19: Coriolis effect on the flowing material (below the outlet) due to centrifuge rotation and angle of inclination, $\theta_C$ in degrees

### 4.4.2 Discharge rate measurements at enhanced gravity ($N^*-g$ level)

The centrifuge flow tests were predominantly conducted using B-49 as a flowing material. This material was tested at enhanced centrifuge input gravities of 1.9, 3.8, 7.7 and 11.7 g. The discharge rate measurements, for B-49 sand, were done based on the total flowing mass filled in the hoppers versus the total period of flow and checked with the natural gravity flow based on Beverloo correlations. The total mass of the sand was measured before being filled into the hopper, and the total period of flow was recorded, and outflow rate was calculated as the ratio of the total mass in grams to the total period of flow. This was because, the measuring balance was not possible to fit and used within the centrifuge. However, video observations and the PIV results (see part 4.6.2) indicated that, a uniform flow was attained immediately as the orifice was opened for flow to start.
The effects of changing the relative density were studied and discussed in parts 4.1.3 and 4.1.4, for the natural gravity tests. These results showed a discharge rate difference less than 5% due to the relative density differences of 20%. This showed that, small density differences seem to have relatively modest effects on the discharge rate measurements. Since the centrifuge takes a few seconds (2 to 15 seconds) until it reaches the target N-g level, this may have created some vibratory effect on the soil within the hopper (see Fig. 3.10(d)), thus densifying the looser samples. The studies by Carr (1965) and Hausner (1967) showed similar effects where the structure of bulk solids would collapse or densify significantly with smaller particles filling the inter-granular spaces due to vibration created by the centrifuge rotation to acquire to a specific g-level. To reduce such an effect (density increase due to vibration), a relative density of 40 percentage was used during soil preparation for the centrifuge tests.

Mathews and Barbir (2016) investigated the Beverloo correlations and showed that discharge rate (W) and periods of flow (t) in centrifuge flow tests scale directly with the square root of the gravity levels. Figs. 4.20 and 4.21 compare the measured granular flow rates (W) and periods of low (t), and the expected granular flow rates and periods of flow based on the Beverloo correlations (equations 2-12 and 2-13).

\[
\frac{W_1}{W_2} = \sqrt{\frac{g_1}{g_2}} = \frac{1}{N} \quad \text{and} \quad \sqrt{\frac{t_1}{t_2}} = \sqrt{\frac{g_2}{g_1}} = N \quad \text{(ibid: equations 2-12 and 2-13)}
\]

Beverloo correlations were calculated based on the flow measurements at natural gravity (1g) and equations 2-12 and 2-13. Figs. 4.20 and 4.21 present these predictions and those measured in the testing. The Beverloo correlations showed an agreement of 78%. The review in Chapter 2 presented a similar study with an agreement of 82% (Mathews and Barbir, 2016). The correlations from Figs. 4.20 and 4.21 can be fitted and equation 4-5 gives the calculated discharge rates and period of flow scaled to gravity. These relations scale at square root to gravity, \( \sqrt{g} \) with relatively good fitting, \( R^2 = 0.96 \). Equation 4-4 can be used for general purpose, in the use of half body hopper, to predict the discharge rates and periods of flow for different g-level up to 11.7g.
\[ W(\text{g/s}) = 69.376g^{0.4039} \quad R^2 = 0.9701; \quad t(\text{sec}) = 30.583g^{-0.427} \quad R^2 = 0.9499 \quad (4-4) \]

The agreement between the measured and Beverloo correlations show that (78\%), the discharge rate and other scaling factors may be predicted with less error using the empirical correlations derived based on the experimental flow measurements for the half hopper used in this thesis.

Fig. 4.20: Measured and calculated discharge rates in g/s against enhanced gravity at radius \( r_{cm} \)

Fig. 4.21: Measured and calculated periods of flow in secs against enhanced gravity at radius \( r_{cm} \)
4.5 Granular Flow Rate: Empirical Comparisons with Natural Gravity Flows

Dimensional analysis using the Buckingham Pi Theorem was utilized to investigate the relationship between the significant parameters contributing to the discharge rate (W) from hoppers or silos (see review on significant parameters on Chapter 2). In accordance with the objectives stated in this thesis: the effects of the orifice size, the effect of average particle size, and state of the soil on the discharge rate was investigated in accordance with the results of this dimensional analysis, namely dimensional groups. This section studies the discharge rate results from the natural gravity flow tests to investigate those effects, and to compare the laboratory test results with existing empirical correlations reviewed in the literature chapter of this thesis.

4.5.1 Buckingham Pi Theorem

The Buckingham method uses a ‘π’ group equations ($\pi_1, \pi_2, \pi_3 \ldots \ldots \pi_{n-m}$) such that if there are n variables in a problem and these variables have m primary dimensions (for example Mass[M], Length[L], Time[T]) the equation relating all of the variables will have (n-m) dimensionless groups. The final equation generated may be a function of the π group and is defined as: $\pi_1 = f(\pi_2, \pi_3 \ldots \ldots \pi_{n-m})$. The Buckingham Pi Theorem assumes some conditions where: a) each of the fundamental dimensions must appear in at least one of the m variables and b) it must not be possible to form a dimensionless group from one of the variables within a recurring set. A recurring set is a group of variables forming a dimensionless group.

The granular flow rate, W from a hopper/silo through an orifice of diameter $D_o$ can be expressed as a function of the depth of the material in the hopper $h_o$, the diameter of the hopper D, the particle diameter d, the gravitational acceleration g, the density of the material $\rho$ and the coefficient of friction $\mu$. Hence $W = f(D_o, D, h_o, d, g, \rho \text{ and } \mu)$ and excluding $D, h_o$ and d our repeating variables become $D_o, g, \rho$ and $\mu$, which gives $(n = 4)$ and $\mu$ is a dimensionless parameter. The repeating variables consist of all three reference dimensions: [M], [L] and [T], which gives $(m = 3)$. This gives the number of π group $= n - m = 4 - 3 = 1$. The cancelling method is found a useful approach to derive the relations of the dimensionless groups (n-m). Table 4.8 presents the dimensional analysis results, and this reduction gives the relations 4-5. In this thesis, relation 4-5 was investigated for the dimensional groups $W = f(\pi_1, \pi_2, \pi_3)$ for different materials. $W = f(\pi_1, \pi_3)$
were studies to compare the effect of the orifice size of the hopper and the average particle size on the granular flow. \( W = f (\phi) \) was used to investigate the effect of the soil state (soil density) on the discharge rate of the materials from the half body hopper.

\[
\frac{W \sqrt{D_0}}{\rho(D_0)^{3/2} \sqrt{g}} \propto 1 \quad \text{and} \quad W \propto \rho D_0^{5/2} \sqrt{g^{1/2}}
\]  

(4-5)

\[ J_1 = \frac{W \sqrt{D_0}}{\rho(D_0)^{3/2} \sqrt{g}} \quad J_2 = \mu \quad J_3 = \frac{D_0}{d} \]

Table 4.8: Buckingham Pi Theorem (cancelling method)

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Cancelling ‘L’</th>
<th>Cancelling ‘M’</th>
<th>Cancelling ‘T’</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_0 )</td>
<td>( L )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( g )</td>
<td>( \frac{L}{T^2} )</td>
<td>( \frac{g}{D_0} )</td>
<td>( \frac{1}{T^2} )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>( \frac{M}{L^3} )</td>
<td>( \rho(D_0)^3 )</td>
<td>( M )</td>
</tr>
<tr>
<td>( W )</td>
<td>( \frac{M}{T} )</td>
<td>( \frac{W}{\rho(D_0)^3} )</td>
<td>( \frac{1}{T} )</td>
</tr>
</tbody>
</table>

(Does not have dimension of \([L]\))

4.5.2 Empirical comparisons

The dimensional analysis based on the Buckingham \( \pi \) theorem shows an agreement with the dimensional groups used by Beverloo et al. (1961) and Srivastava et al. (2003). Based on these findings, this section presents a comparison of measured mass flows with empirical correlations proposed in the literature review viz. equation 2-3 from Beverloo et al. (1961) and equation 2-7 from Srivastava et al. (2003). Note that the Beverloo et al. (1961) empirical results relate to a full body hopper. However, based on the calculated results and observed laboratory tests, this empirical formula can be transformed into one for a half body hopper by multiplying it by 0.5, which also corresponds to \( W^{2/3} \) (Srivastava et al., 2003). Table 4.9 presents the hopper and soil material parameters used for comparison with Beverloo et al. (1961) and Srivastava et al. (2003). Based on earlier chapter findings, this section investigated the cohesionless materials, in soil group S-II.
Table 4.9: Comparison of flowing material properties and flowing conditions (see part 2.1.2)

<table>
<thead>
<tr>
<th>Notes</th>
<th>Materials</th>
<th>Orifice</th>
<th>Particle screen size</th>
<th>Bulk density, $\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thesis</td>
<td>Silica sand B-32 and B-49, Toyoura and MMS</td>
<td>Circular: 2cm, 1.5cm and 1cm;</td>
<td>0.12 – 0.5 mm</td>
<td>1.19 – 1.52 g/cc</td>
</tr>
<tr>
<td>Experimental, half model hopper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beverloo (1961)</td>
<td>Sand, linseed, spinach, watercress, rapeseed, kale, swede, and turnip</td>
<td>Circular and non-circular: 0.26 cm to 3.02 cm</td>
<td>0.45 – 1.7 mm</td>
<td>0.68 – 1.5 g/cc</td>
</tr>
<tr>
<td>Experimental, full body hopper</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Srivastava et. al (2003)</td>
<td>Cohesionless material and the effect of air bubble</td>
<td>1.4 cm, circular</td>
<td>1 mm</td>
<td>2.9 g/cc</td>
</tr>
<tr>
<td>DEM, half model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.5.2.1 Discussion on comparison of laboratory results: Beverloo et al. (1961) and Srivastava et. al (2003)

In general, the balance readings of the granular flow results agree with the commonly used Beverloo et al. (1961) equations, except for those for $D_o=1.5$cm as shown in Fig. 4.23. Beverloo et al. (1961) predictions for $D_o=2.0$ cm in Fig. 4.22 agree within 72% to 98% to the balance readings. This agreement reflects the similarity of the material properties and the orifice sizes used in both experiments, as shown in Table 4.9. Compared to the Beverloo et al (1961), however, Srivastava et. al (2003) predictions showed less agreement. Srivastava et. al (2003) agrees 80% to 96% to the balance reading for $D_o=1$cm compared to 57% to 60% for Beverloo et al. (1961) (see Fig. 4.24). In general, the percentages of agreement observed reflect the similarity and difference between the parameters considered in each study, for example the material property and orifice size (see Table 4.9) as well as the ‘g’ level variations that occur across the centrifuge tests. The following section investigated the effects of the orifice and particle size ratio for natural gravity flow results [$W = f(\lambda_1, \lambda_3)$].
Fig. 4.22: Granular flow rate for $D_0=2\text{cm}$

Fig. 4.23: Granular flow rate for $D_0=1.5\text{cm}$
Fig. 4.24: Granular flow rate for $D_o=1$ cm

4.5.2.2 The effect of particle and orifice size of the hopper on 1g granular flow rate

The effect of average particle size and orifice size on the granular flow rate ($W$) can be observed in graphs of $W$ against $\frac{D_o}{d}$. S-I soil group (B-32, B-49 and Toyoura, also see Table 3.7) will be used to present this study. As discussed in previous chapters, this soil group have close particle size distribution pattern as shown in Fig. 3.12.
As shown in Fig. 4.25 the flow rate increases as the orifice size increases such that $82 > \frac{D_o}{d} \geq 40$ as stated in section 4.1.1. The effect of the particle size seems to be negligible beyond $\frac{D_o}{d} = 70$. Beverloo et al. (1961) proposed a value of $\frac{D_o}{d} = 20$ where the effect particle size effect becomes negligible for orifices with higher diameter.

The literature review in Chapter 2 shows an empirical correlation for sand after Beverloo et al. (1961). This study found that equation 2-3 can be used to calculate the granular flow rate of granular materials flowing in a full body hopper with a circular or non-circular orifice with $C=38.8$. Based on the analysis given in this thesis in previous section, equation 2-3 can be modified to suit the 2D hopper flow condition and $C=19.4$.

$$W = C \rho \sqrt{g} (D_o - kd)^{5/2}$$  \hspace{1cm} \text{(ibid: equation 2-3)}

Earlier studies showed a zone which is useless or less fit for use in the outflow, along the margins of the orifice. The size of that zone was showed to be proportional to $d$ and therefore the effective outflow orifice diameter would reduce to ‘$D_o – d$’ (Beverloo et al., 1961). The studies also showed
that the average particle size ‘d’ primarily depends on the shape of the individual particles. Therefore, investigations on the effects of the shape factor introduced a dimensionless parameter ‘k’ as shown in equation 2-3 (Beverloo et al., 1961). For best results however, the correlative value of these dimensionless parameters in equation 2-3 should be determined based on the experimental test results for each hopper or silo case. Therefore, the following section follows a common approach used in many research to iterate the best fit experimental values for ‘C, discharge coefficient’ and ‘k, particle shape factor’, in equation 2-3.

4.5.2.2.1 Evaluating k (shape coefficient)

The relations in relation 4-5 may be plotted to examine the pattern and the effect of the particle size ‘d’ on the outflow rate. Fig. 4.26 shows the graph of relation 4-5 for the testing materials of B-32, B-49 and Toyoura, and a polynomial relation is observed. From Fig. 2.25, it is observed that the outflow rate (W) is inversely proportional to the particle size ‘d’. However, Fig. 4.26 shows that the outflow (W, where the effect of particle size is not considered) have relatively equal values for smaller orifice sizes of 1.5 and 1.0 cm. These observations may predict the effect of particle size and particle shape to the outflow rate (W). Similar findings were also described by Beverloo et al. (1961). Many studies used the slope of these two-dimensional parameters to get the value of k (Beverloo et al. 1961). This relation, \( (W/d)^{2/5} \) vs \( D_o \) is plotted in Fig. 4.27 below. This section aims to find the best fit k value for the half model hopper used in this thesis. For best fit data in Fig. 4.27, the granular flow rate is normalized by the average particle size.

\[
\frac{W \sqrt{D_o}}{\rho (D_o)^{3/2} g^{1/2}} \propto 1 \quad W \propto \rho^* D_o^{5/2} g^{1/2} \quad \text{(ibid: relation 4-5)}
\]
Beverloo et al. (1961) measured value of $k$ of 1.9 for sands for a full body hopper. The slope of the lines $(W/d)^{2/5}$ vs $D_o$, from Fig. 4.25, shows average value of $k=2.1$ for soil group S-II (B-32,
B-49 and Toyoura) and 0.3 for MMS. For a best fit result, k=2.1 and k=0.3 will be adopted in this thesis for S-I and MMS respectively for the half body hopper. Sphericity (S), roundness (R) and regularity was identified by Joshi (2021) for B-49, Toyoura and MMS, based on the approach of Cho et al. (2006). These results showed ranges of sphericity (0.71 – 0.58), roundness (0.36 – 0.63) and regularity (0.47 – 0.67). The parameter k is also known to be influenced by the particle size. Thus, the combined effect of shape and size are seen to influence k: the angularity of the MMS material reduces the value, in combination with the smaller size, Mamtani (2011). The value of k for the non-cohesive materials are relatively similar due to their similar shape and size characteristics, although k is a little higher than the usually adopted value of 9 (Beverloo et al., 1961). Further analysis of these values could be conducted using the aspect ratio approach of Mamtani (2011).

4.5.2.2 Evaluating C (discharge coefficient)

The dimensional analysis result in section 4.4.1 gives a dimensional group, \( W = f (\lambda_1) \), where

\[
\lambda_1 = \frac{W \sqrt{D_0}}{\rho (D_0)^{3/2}}.
\]

This dimensional group defines the discharge coefficient ‘C’ and shows that \( W \) is directly proportional to \( \rho (D_0)^{5/2} \sqrt{g} \) (also see Fig. 4.26). Based on this relation, Table 4.10 shows the best fit results for ‘C’ using k values from the previous section.

<table>
<thead>
<tr>
<th>Material</th>
<th>( D_o=2\text{cm} )</th>
<th>( D_o=1.5 )</th>
<th>( D_o=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-32</td>
<td>( 1.2 \rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
<td>( 2.1\rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
<td>( 2.2\rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
</tr>
<tr>
<td>B-49</td>
<td>( 0.9\rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
<td>( 1.9\rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
<td>( 2.1\rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
</tr>
<tr>
<td>Toyoura</td>
<td>( \rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
<td>( 1.9\rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
<td>( 2.2\rho \sqrt{g} (D_o - 2.1d)^{5/2} )</td>
</tr>
<tr>
<td>MMS</td>
<td>( 0.3\rho \sqrt{g} (D_o - 0.3d)^{5/2} )</td>
<td>( 0.6\rho \sqrt{g} (D_o - 0.3d)^{5/2} )</td>
<td>( 1.5\rho \sqrt{g} (D_o - 0.3d)^{5/2} )</td>
</tr>
</tbody>
</table>

The results in Table 4.10 predict that, the coefficients in the respective equations are sufficient to be used for all materials flowing on the same orifice size for the respective soil groups. Based on the results presented in Table 4.10, equation 2-3 could be modified to reflect the results and the
half model hopper for 1g flow tests. For general use using the hopper in this thesis and other similar geometry hoppers, equation 4-6 and equation 4-7 may be used for purely cohesionless materials (soil group S-I) and for soils with small cohesive behaviour (MMS) respectively. The agreement between the measured flow measurement at 1g and these modified equations is 100%.

\[
W = 1.7 \sqrt{\rho g} (D_o - 2.1d)^{5/2} \quad (4-6)
\]

\[
W = 0.8 \sqrt{\rho g} (D_o - 0.3d)^{5/2} \quad (4-7)
\]

4.6 Particle Image Velocimetry (PIV): Results and Discussion

This section investigated PIV analysis data for natural gravity and enhanced gravity granular flows, using the half model hopper. PIVlab was used to study the deformation patterns and the velocity magnitudes during the flow. The contours of velocity magnitude (Figs. 4.28 to 4.30) and graphs of velocity magnitude across a vertical (Fig. 4.32) and horizontal (Figs. 4.33 to 4.35) reference lines are presented to analyse the general behaviour of the materials during outflow. A uniform flow (uniform discharge) occurred within a few seconds of the flow starting. This occurred at flow positions of greater than 16cm above the orifice (total hopper height is 18cm for the base hopper and 22cm for hopper with extensions). To capture typical flow behaviour, a study was done at 14cm flow height, over periods of flow between 7 to 10 seconds, from the orifice position (see Table 4.1 for total periods of flow).

4.6.1 Contours of velocity magnitude and velocity magnitude profiles under natural gravity

This section investigates the PIVlab analysis results: contour maps of velocity magnitude and the velocity vector profiles. Figs. 4.28 to 4.30 shows the velocity contours and the velocity vectors during a flow with a soil height of 14 cm. This was thought to be close to ‘steady state’ flow for all of the materials. The different colors indicate the velocity of the particles, and the arrows indicate the velocity vectors. Dark yellow to light yellow in zones close to the center of symmetry show a region of active material flow (with high velocity magnitude 0.33 to 0.45 m/s). The dark blue to light blue color between the center of symmetry and towards the wall of the hopper represent a region of less active material flow (with velocity magnitude 0.05 to 0.33 m/s). A dark
indigo color close to the walls of the hopper show dead zone (stagnant region) with velocity magnitudes close to zero. The dead zone was observed above 5 cm ($i_z = 0.3$) from the left and right end of orifice position and extending towards the walls of the hopper (very light green to indigo colour) vertically and horizontally. The central region shows a funnel (pipe) flow with a relative higher velocity of 0.33 to 0.45 m/s. Soil group - II (B-32, B-49 and Toyoura) showed similar outflow pattern. However, MMS showed a pattern with high disturbance and velocity discontinue. The outflow contour maps for the hopper with extensions (4 cm height extension to accommodate 1.5 and 1.0 cm orifice sizes) showed less velocity difference and therefore less outflow disturbance compared to the hopper without extension. A detailed investigation of such observations is discussed in the following paragraphs.
Fig. 4.28: Velocity magnitude contours and flow velocity vectors at flow height of $i_z=0.78$ (14 cm above orifice position) with $D_o=2.0$ cm

*see Fig. B-5 for the velocity vector presentation

* PIV: less efficient in capturing the flow of MMS, which showed ratholing, internal body funnel flow and MMS particles sticking to the plexiglass making it difficult for PIV analysis.
Fig. 4.29: Velocity magnitude contours and flow velocity vectors at flow height of \( z = 0.78 \) (14 cm above orifice position) with \( D_o = 1.5 \) cm

* Hopper with 4 cm height of orifice extension
Fig. 4.30: Velocity magnitude contours and flow velocity vector profile at flow height of \( i_z = 0.78 \) (14cm above orifice position) with \( D_o = 1\text{cm} \)

* Hopper with 4cm height of orifice extension

The ratio orifice to particle size \( \left( \frac{D_o}{d} \right) \) vs velocity magnitudes were investigated for the same material and different the orifice sizes. For these studies, the maximum velocity magnitude of the materials increased as the orifice size increased. This can also be observed where the high velocity zones (yellow colour in the images) for each material contour maps with 2.0, 1.5 and 1.0 cm orifice sizes (see velocity contours of the same material across different orifices; Figs. 4.28 to 4.30). Figs. 4.32 and 4.32 show the graphs of vertical velocity magnitude down the hopper center line and different horizontal positions, respectively as shown in Figs. 4.33 to 3.35 below. As shown in Fig. 4.31, section C-C’ shows the position of the vertical velocity magnitude, and Sections A-A’, B-B’ and D-D’ are the positions of the velocity magnitudes across a horizontal line.
Fig. 4.31: Vertical and horizontal positions of velocity magnitudes

The maximum magnitude of the velocities in the Tables of Figs. 4.32 and 4.35 show increased velocity magnitude measurements for the same material with different orifice sizes. For example, in Fig. 4.32, B-32 showed velocity magnitudes of 0.39, 0.41 and 0.45 m/s for orifice sizes of 1.0, 1.5 and 2.0 cm. Similar results were observed for velocity magnitude measured across the horizontal lines ($i_z = 0.5$, 0.23 and 0.1) in Figs. 4.33 to 4.35.

The vertical velocity magnitudes show a decrease as the average particle size of the flowing material decreases. This is also shown by a decrease in size and sharpness of the yellow colour across the materials’ velocity contour images for flows with the same orifice. B-32, B-49 and Toyoura in this investigation are cohesionless (see Chapter 3) and have similar trends in the PSD as shown in Fig. 3.12. These materials also showed a similar response of flow in-terms of flow patterns, vertical velocity magnitude and deformation.
The velocity profile down the vertical centerline and across the horizontal lines show discontinuities (see Figs. 4.32 to 4.35). The vertical velocity magnitudes along the centerline show similar trends for all materials across all orifice sizes, where the particle velocity increases vertically downwards up to a vertical position of 4.14 cm (i.e., \( i_z = 0.23 \); section B-B’) above the orifice. Higher vertical velocities were observed at 9 cm above the orifice (i.e., \( i_z = 0.5 \); section A-A’). The vertical velocities then show nearly a uniform vertical velocity below 1.8 cm above the orifice (i.e., \( i_z = 0.1 \); section D-D’). These observations show the complexity of the flow. However, these profiles clearly define the patterns of flow for the different materials show at different orifice sizes. MMS showed a reduced vertical velocity compared to the other soil materials used in this thesis. Observations of the velocity and contour profiles for MMS in Figs. 4.28 to 4.30 and Fig. 4.32 to 4.35, suggest high formations of stagnation, arching and subsequent column failures leading to high dynamic responses and, creating disorderly vectors. Though extra research may be needed on this profile discontinuity, the contours of the velocity magnitude and the velocity profiles are likely due to the material cohesive and angular nature.

The dynamic angle of repose is the product of a granular flow (see sections 4.2 and Table 4.5). The static and dynamic angles of repose were studied in section 3.3.3 and 4.2. Table 4.5 compares these results, and it was found that the dynamic angles of repose were slightly greater than the static angles of repose, for B-32, B-49 and Toyoura. However, the dynamic angle of repose for MMS was measured to be greater than the static angle of repose by 13\(^\circ\). As stated in the sections above, this difference corresponds to the cohesive property of the MMS soil. The studies of Figs. 4.28 to 4.30 showed a stagnant zone through out the hopper depth above 3 cm from the orifice. These observations show the angles at which the stagnant zone and intercepts the hopper wall fluctuates between the static and dynamic angles of repose.
Fig. 4.32: Vertical velocity magnitude: down centerline of the hopper and maximum velocity magnitudes (C - C’ in Fig. 4.27)
Fig. 4.33: Vertical velocity magnitude across a horizontal line at $i_z=0.5$ (9cm above the orifice exit) and maximum velocity magnitude (A - A’ in Fig. 4.27)
Fig. 4.34: Vertical velocity magnitude at $z=0.23$ (4.14cm above the orifice exit) and maximum velocity magnitude (B - B’ in Fig. 4.27)
Fig. 4.35: Vertical velocity magnitude at \( z = 0.1 \) (1.8 cm above the orifice exit) and maximum velocity magnitude (D - D’ in Fig. 4.27)
4.6.1.1 Arching, free flow surface and free flow velocity

Three consecutive images at 0.1 second frame rate were used to study the position of the free flow surface. The observations on the contour maps of the material outflow showed relatively small dynamic arch forming above the free flow surface (A in Fig. 4.36). However, in agreement with Fig. 2.1(b), below this arching shows a zone of relatively uniform and reduced vertical velocity of outflow shown the free flow surface. Fig. 4.36 suggests the position of the free flow to be between points ‘A’ and ‘B’ after which the particles seem to have relatively the same velocity magnitude from ‘B’ to ‘C’. The velocity from ‘B’ to ‘C’ is where the particles flow freely, except due particle-particle collision. The region from A to C is the free flow surface and this velocity may be referred as free flow velocity (see Chapter 2). The measured vertical velocities of the particles falling freely at a relatively equal velocity is shown in Table 4.11.

Table 4.11: Maximum vertical velocity magnitudes under the free fall surface (free fall velocity)

<table>
<thead>
<tr>
<th>Orifice size</th>
<th>B-32</th>
<th>B-49</th>
<th>TOYOURA</th>
<th>MMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{FF}$, m/s</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D_o = 1$ cm</td>
<td>0.07</td>
<td>0.067</td>
<td>0.07</td>
<td>0.06</td>
</tr>
<tr>
<td>$D_o = 1.5$ cm</td>
<td>0.165</td>
<td>0.17</td>
<td>0.165</td>
<td>0.06</td>
</tr>
<tr>
<td>$D_o = 2$ cm</td>
<td>0.15</td>
<td>0.12</td>
<td>0.12</td>
<td>0.043</td>
</tr>
</tbody>
</table>
Fig. 4.36: Free fall surface (D₀=2cm, B-49, flow during 7 to 7.2 seconds, total T=27 seconds)
4.6.2 Enhanced gravity ‘centrifuge outflow tests’

This section presents velocity contours and velocity magnitude profiles for the enhanced centrifuge gravities of 1.9, 3.8, 7.7 and 11.7 g. The discharge rates and the Coriolis effect due to the enhanced gravity during flow were discussed in section 4.3. Fig. 37 presents the contours of the vertical velocity profiles for the different enhanced gravity flows and Figs. 4.38 and 4.39 show the velocity magnitude profiles down the centerline (section C-C’ in Fig. 4.31) of the hopper and across the same horizontal positions in the hopper (sections A-A’, B-B’ and D-D’ in Fig. 4.31). In Fig. 4.37, each enhanced gravity flow is presented by two frames separated by a 0.1 second period and at flowing height of 14 cm above the orifice level; similar flow heights were adopted for the velocity contours and velocity magnitude profiles with natural gravity flow in the previous section. The velocity magnitude contour maps in Fig. 4.37 show how the soil behaves for the different g levels. The observations from these contours show that the velocity of the material flowing from the hopper top towards the outlet increases with increase in gravity. These observations may also be noted in Fig. 4.38, where the maximum value of the velocity magnitude increases with increasing order of g-level.

The contour maps in Fig. 4.37 are positioned parallel to the vertical axis for presentation and comparison purposes. Due to the inclined angle of the resultant gravity, the stagnant zones appeared to be shifted to the wall of the hopper in the direction of the natural gravity. This zone is positioned on the left side of the contour maps in Fig. 4.37 and is most likely due to the resulting differential frictional forces on the walls of the hopper. For flows with gravity less than 11.7g, the active flow zone appeared to be on the right side of the contour maps in Fig. 4.37, which is towards the upper walls of the hopper during the enhanced tests. This pattern of flow appeared to be close to perpendicular to the direction of the resultant gravity. However, during the 11.7g test, the soil appeared to have funnel flow along the symmetric center of the hopper. Similar flow patterns to 11.7g flows were observed during the natural gravity flow tests presented in Figs. 4.28 to 4.30.
Fig. 4.37: Contours of vertical velocity magnitude for 1.9, 3.8, 7.7 and 11.7 g; and flow height of 14 cm above the orifice position; $B=49$ and $D_o = 2.0$ cm
Fig. 4.38: Vertical velocity down centerline of hopper under natural gravity (1g), and 1.9, 3.9, 7.7 and 11.7 g; and flow height 14 cm from the orifice position; B=49 and D_o=2.0 cm

<table>
<thead>
<tr>
<th>Position</th>
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<th>5g</th>
<th>10g</th>
<th>15g</th>
</tr>
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<tbody>
<tr>
<td>iz=0.5</td>
<td>0.50</td>
<td>0.37</td>
<td>0.69</td>
<td>0.52</td>
</tr>
<tr>
<td>iz=0.23</td>
<td>0.40</td>
<td>0.73</td>
<td>0.39</td>
<td>0.45</td>
</tr>
<tr>
<td>iz=0.1</td>
<td>0.07</td>
<td>0.12</td>
<td>0.44</td>
<td>0.41</td>
</tr>
</tbody>
</table>

Fig. 4.39: Vertical velocity across horizontal lines under natural gravity (1g), and 1.9, 3.9, 7.7 and 11.7 g; and flow height at 14 cm from the orifice position; B=49 and D_o=2.0 cm
A higher velocity discontinuity was shown during the enhanced flow tests compared to the natural gravity ones. This can be noted by locating the high velocity zones (sharp yellow colour bar) in consecutive frames for each gravity contour map as presented in Fig. 4.37 above. From the contour maps of the 1.9g and 3.8g flows, the sand particles appear to flow at a maximum velocity above and towards the right of the orifice. A uniform pattern of flow continued below this region similar to the free flow surfaces observed during the natural gravity flow (see Fig. 4.36 for the free fall surface during natural gravity flow). In contrary to the observation for 1.9 and 3.8g flows, a region of high velocity occurred above the orifice and along the horizontal center of the hopper during the 7.7 and 11.7g flows. Based on the observations during the natural gravity flows, high velocity zones appeared to occur in regions of susceptible column failures along the formation of the soil depression lines (see section 4.2). Based on these results, a general pattern of flow for the natural and enhanced gravity cases is presented in the next section in Fig. 4.42.

The flow patterns under different gravity levels could also understood better by studying the velocity magnitude vertically at the symmetric center of the hopper and across a horizontal distance over the area of flow. Table 4.12 presents the average maximum velocity magnitude values from Fig. 4.38 and 4.39. As discussed in the previous paragraphs, these results appear to agree with the commonly used Beverloo correlation. The review of centrifuge flow in Chapter 2 discussed the Beverloo correlation (from Mathews and Barbir, 2016), comparing two similar silo centrifuge tests on mass discharge (W), velocity magnitude (v), the gravity scale factor ‘N’ and discharge time ‘t’ at different ‘g’ value. Based on Beverloo correlations, equation 2-14 (also provided below) was experimentally used to predict the velocity of flow under different g-level flows.

\[
\frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2}} = \sqrt{N}
\]  

(ibid: equations 2-14)

The magnitude of the velocity profiles presented in Table 4.12 were graphed against the Beveloo correlation as shown in Fig. 4.40. Beverloo correlation results were calculated based on equation 2-14 and the magnitude of the velocity at 1.9g. The enhanced flow results compared to the Beverloo correlation results show an agreement by 80%.
Table 4.12: Average maximum velocity magnitude from Fig. 3.32, m/s

<table>
<thead>
<tr>
<th>Natural gravity, 1g</th>
<th>Enhanced gravities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.9g</td>
</tr>
<tr>
<td>0.25 m/s</td>
<td>0.32 m/s</td>
</tr>
</tbody>
</table>

Fig. 4.40: Average maximum vertical velocities measured by PIV and from Beverloo correlation (equation 2-14)

4.7 Summary

The particle image velocimetry (PIV) method can be used as an effective optical technique to evaluate the flow patterns and quantify the particle velocity of flowing granular materials by processing successive digital images. Though outflow behaviour of the soil mass inside the hopper cannot be traced by using this method, PIV helps to visualize the surface movement characteristics. The study of the velocity magnitude profiles of the granular flow in hoppers and silos can be helpful to understand the variability of the flows. Understanding this variability of the flow may give clear indications that the design of silos and hoppers should consider the dynamic stresses created during the material flow. Theories providing hopper wall pressure estimates were first presented by Walker (1966) followed by Jenike (1961). Jenike (1961 and 1973) proposed equations for the static and dynamic stresses during storage and during material flow respectively.
These theories showed that the dynamic stress created during a material flow is by far greater than the static stress created due to a material stored in the silo or hopper. The PIV results clearly show that the dynamic stresses created non uniform and discontinuous flows.

Materials with adequate texture for the PIV analysis were B-32, B-49 and Toyoura. MMS gave encouraging results but could be improved by using more magnifying devices, like x-ray or laser capturing. It was also observed that B-32, B-49 and Toyoura exhibit similar flow pattern at all levels temporarily and down the hopper body. Comparing results with findings from other studies helps to understand and compare the significance of the results. Fig. 4.40 presented the scaling of the velocity magnitude at different g-level with the Beverloo correlations. Similarly, a study done by Srivastava et. al (2003) under natural gravity showed that the velocities measured under natural gravity around the orifice scale to \((gD_o)^{1/2}\). To investigate the effect of this scaling, a graph of velocity, normalized by the suggested scaling \((gD_o)^{1/2}\) were plotted against the orifice size. Average of maximum velocities presented in Figs. 4.32 and 4.35 were plotted and investigated to study this prediction. These results are presented in Fig. 3.41 below. The measured velocity magnitudes in Fig. 3.41 showed a different pattern for B-49 (i.e., an increasing curve) for flow position of \(i_z=0.23\). However, this variation only shows the velocity discontinuity and fluctuation discussed in above sections. However, this does not affect the overall pattern of the flow. The graphs of \(v/(gD_o)^{0.5}\) and \(D_o\) at different positions \((i_z=0.5, 0.23\) and \(0.1\) were studied and it was observed, all the relations were correlated by a uniform relation of degree 2. The uniformity of these data therefore suggests that the velocities measured using PIV also show a similar scaling.
A closer observation of the patterns of flow under natural and enhanced gravities indicate that, a general suggestion could be made to predict the general pattern of the flow in both cases. Based on the above observations and discussions (see sections 4.5.2 and 4.5.3), Fig. 4.42 can be used to understand the patterns of flow in a half hopper body subjected to natural and enhanced gravities. The velocity colour bars in figure (Fig. 4.42) were chosen to reflect a similar velocity bar presented in the contour maps as shown in Figs. 4.28 to 4.30 (natural gravity flow) and Fig. 4.37 (enhanced gravity flow). Fig. 3.42 shows a general prediction of zones of different velocity ranges during a natural gravity flow. A region of high velocity (sharp yellow colour) appears to occur where a high and continuous failures of soil columns take place during the flow (point A in Fig. 3.42). Based on the physical observation of the flow (see Coriolis effect in section 4.3) and the PIV results discussed in the previous sections, it can be concluded that, understanding the flow pattern would help for a better design and operation of granular flow under the influence of gravity.
A. General flow pattern under Earth’s gravity (natural gravity)

B. General flow patterns for 1.9 and 3.8 g

C. General flow patterns for 7.7 and 11.7 g

Fig. 4.42: General flow patterns under natural and enhanced gravity for B-49, D₀=2.0cm
CHAPTER 5: SUMMARY AND CONCLUSIONS

5.1 Overview

In this thesis, a detailed study of model hopper surface and internal flow patterns, and material discharge rate was done under natural and enhanced gravities. However, to investigate the testing materials discharge rates and the internal flow patterns in the hopper, a detailed study was required in a controlled environment, that is under natural gravity. This required a proper instrumentation to be in place and a detail characterization of the testing materials be worked on before the physical testing was to be done. The granular flow in hoppers and silos under natural and enhanced gravities, and the thesis objectives were discussed in Chapter 1. Important aspects of this studies were reviewed in Chapter 2. These covered the important findings on the natural and enhanced gravity outflow studies, PIV flow pattern characterization, and enhanced centrifuge flow tests and model/prototype scaling laws for the half body hopper. Chapter 3 discussed the methodology and the experimental design used in this thesis. The experimental design in this chapter discussed the instrumentations used for outflow and PIV tests under natural and enhanced gravities. The results of the laboratory tests were presented in Chapter 4, followed by discussions and conclusion. The focus of this chapter was to investigate the effects of orifice size, soil mass density and average particle size on mass accumulation and outflow rates (W) under natural gravity. Based on the natural gravity outflow rate for B-49 sand, the enhanced gravities outflow test results for gravities of 1.9, 3.8, 7.7 and 11.7 g were graphed and compared to Beverloo correlations. This chapter also investigated the outflow patterns and the velocity profiles across a vertical and horizontal positions in the hopper body using a particle image velocimetry (PIV), both under natural and enhanced gravities.

5.2 Discussion of Major Findings

- The outflow rate (W) for cohesionless soils is directly proportional to the ratio of orifice and average particle sizes \( \frac{D_o}{d} \).
- However, the outflow rate for soils with some cohesion behaviour, should be predicted separately. Continuous and uninterrupted mass outflow accumulations can be attained regardless of the internal fluctuations during flow.
Increasing the flow density in cohesionless materials showed slight decrease in discharge rate measurements.

The discharges of cohesionless granular materials at higher relative density showed less periodic fluctuations in discharge rate and reduced velocity discontinuity suggesting that this material state may have created dynamic stresses no the walls of hoppers.

Beverloo correlations indicated that the discharge rate (W) was proportional to the square root of gravity (g) and similar results were demonstrated using experimental results from a novel silo centrifuge model with good agreement.

Even though the hopper model used in this thesis showed a funnel outflow pattern at natural gravity, it was observed that, such uniformity of pattern may not occur for the enhanced gravity outflow tests up to 14g.

The pattern in the centrifuge was observed to be distorted due to the combined effect of the different gravities acting on the hopper and the soil.

The particle image velocimetry (PIV) method can be used as an effective optical technique to evaluate the flow pattern and the magnitude of velocity of granular materials by processing successive digital image analysis.

The study of the vertical velocity magnitude profile of the granular flow in hoppers and silos can be helpful to understand the variability of the flow.

The PIV results clearly shows that the dynamic stresses could possibly be created due to the non uniform and discontinuous flow.

Deformation patterns captured during the granular flow clearly show the stagnant zones within the hopper body. Three distinct zones of behaviour were observed: a) an upper inflow zone, b) a narrow vertical funnel flow zone and c) a near static stagnant zone.

The characteristics of the different flow zones are influenced by the material and geometric properties of the silo.

The PIV also showed that this technique can locate the position of the dynamic arches.
5.3 Future Work

The work completed in this thesis had been restricted to a simplified axisymmetric model. For better understanding of the overall behaviour of the hopper and soil outflow, it is recommended to extend the work in future studies based on the recommendations below.

5.3.1 Discrete element method (DEM) simulation

Recently, many studies have used DEM for the estimation of outflow rates and to study the particle interaction inside the body of silo or hopper (Srivastava et al., 2003; Janda et al., 2012; Rahman and Zhu, 2012; Mollon and Zhao, 2013 and Rubio-Largo et al., 2015). Even though PIV was able to evaluate the outflow patterns, the magnitude of velocity of granular materials and the soil mass deformation patterns; the particle-particle interaction and the stresses inside the soil mass and on the walls of the hopper cannot be determined with this method. In the future, DEM simulation using proper soil material calibration could be developed to provide a deeper understanding of the hopper outflow process and the soil-structure interaction. It could also be used to better understand the differences between 2D and 3D material outflow behaviours.

5.3.2 Transparent soil

Transparent soils have been commonly used to investigate soil-structure interactions (Iskander and Liu, 2010; Liu et al., 2010; Beckett and Augarde, 2011 and Kelly and Black, 2012;). In the future, transparent soils with close-range photogrammetry or using a laser source could be used to examine the spatial particle displacements beyond the plexiglass interface, towards the back of the hopper.

5.3.3 Dynamic pressure

Classical research has studied pressures on the walls of silos in terms of static and dynamic pressures (Janssen, 1895; Walker, 1966 and Jenike, 1961; Jenike et al., 1973 and Wilms, 1985). Static pressure occurs during the filling stage of a silo or hopper, while dynamic pressure occurs during discharging of a silo or hopper. The PIV results in this thesis suggested that the velocity
discontinuity, soil mass dynamic arching and subsequent formation and falling of columns could cause dynamic stress on the walls and the overall structure of silo. In the future, proper instrumentation could be added to the hopper walls to measure the amount of dynamic stress on the walls of the silo due to the discharging granular material during natural and enhanced gravity outflow tests.
References


Appendix A: Model arrangement dimensions

Fig. A-1: $D_0=1\text{cm}$ and BH assembly

Fig. A-1 shows a half hopper model geometry with an extension of $4.0\text{ cm}$ orifice extension, $D_0 = 2.0\text{ cm}$. The half model hopper referred as the base hopper (BH) has an orifice size of $2.0\text{ cm}$; however, the orifices sizes of $1.0$ and $1.5\text{ cm}$ were provided as an extension of $4.0\text{ cm}$ down the BH orifice position. All individual parts are 3D printed for consistency.
Fig. A-2: 2D dimensions of Aluminum frame and 2D BH

Fig. A-2 shows the geometric details of the hopper model and the aluminum bar as shown in Fig. 3.5. The aluminum bars were cut to fit into the centrifuge box (see Fig. 3.4). For structural support and quasi static state of all the body parts, the base hopper was attached and screwed to the aluminum bar as shown in Fig. 3.5.
Appendix B: Granular flow tests’ graph data figures

Fig. B-1: Silica Sand B-32 mass accumulation test, g per time in seconds, ($D_0 = 2$ cm)

Fig. B-2: Silica Sand B-49 mass accumulation test, g per time in seconds, ($D_0 = 2$ cm)
Fig. B-3: Toyoura mass accumulation test, g per time in seconds, (D_o = 2 cm)

Fig. B-4: MMS mass accumulation test, g per time in seconds, (D_o = 2 cm)
<table>
<thead>
<tr>
<th>Sample</th>
<th>B-32</th>
<th>B-49</th>
<th>Toyoura</th>
<th>MMS</th>
</tr>
</thead>
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<td><img src="image1.png" alt="Image" /></td>
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<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Fig. B-5 Velocity magnitude flow vector presentation during natural gravity outflow
Appendix C: Tables showing correlations for mass accumulation graphs (Figs. 4.2 to 4.4) and dynamic angle of repose data

Table C-1: Correlations for data in Fig. 4.2 at 0.5 seconds frequency

<table>
<thead>
<tr>
<th>Material</th>
<th>Measurement</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
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<td>3.7</td>
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Table C-2: Correlations for data in Fig. 4.3 at 0.5 seconds frequency

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<th>M2</th>
<th>M3</th>
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</thead>
<tbody>
<tr>
<td>Toyora</td>
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<tr>
<td>B-49</td>
<td>L</td>
<td>18.443x + 17.776</td>
<td>R² = 0.9989</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>MMS</td>
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<td>77.189x + 29.535</td>
<td>R² = 0.9996</td>
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</table>

Table C-3: Correlations for data in Fig. 4.4 at 0.5 seconds frequency

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<td>R² = 0.9989</td>
<td></td>
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<tr>
<td>MMS</td>
<td></td>
<td>77.189x + 29.535</td>
<td>R² = 0.9996</td>
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Table C-4: Dynamic angle of repose: Height, H and Length, L in cm measurements

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<tr>
<th>Material</th>
<th>Measurement</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
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<td>H</td>
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<td>3.7</td>
<td>3.9</td>
<td>3.95</td>
<td>2.4</td>
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<tr>
<td></td>
<td>L</td>
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<td>5.8</td>
<td>6.3</td>
<td>6.2</td>
<td>3.7</td>
</tr>
<tr>
<td>B-49</td>
<td>H</td>
<td>3.2</td>
<td>3.5</td>
<td>4.1</td>
<td>4</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>6</td>
<td>6.4</td>
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<td>3.1</td>
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<tr>
<td>Toyoura</td>
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<td>3.6</td>
<td>4.8</td>
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<tr>
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<td>L</td>
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<td>6.4</td>
<td>7.3</td>
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<td>3</td>
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<tr>
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<td>H</td>
<td>2</td>
<td>3</td>
<td>4.5</td>
<td>7.7</td>
<td>5.7</td>
</tr>
<tr>
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<td>L</td>
<td>1.9</td>
<td>2.2</td>
<td>4.5</td>
<td>5.1</td>
<td>4.9</td>
</tr>
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</table>
CURRICULUM VITAE

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Research Assistant.

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