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Applied General Equilibrium Models of Taxation and International Trade

John B. Shoven
John Whalley

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John B. Shoven
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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.
Preliminary draft
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Comments Welcome

APPLIED GENERAL EQUILIBRIUM MODELS OF
TAXATION AND INTERNATIONAL TRADE

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Stanford University and
National Bureau of Economic Research

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University of Western Ontario

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</tbody>
</table>
1. Introduction

The concern has been repeatedly expressed in recent years that as formal economic theory develops, it becomes even further removed from the policy process because of the difficulties of communication between academics and policy makers. Equally, the relative lack of numerical work on policy issues carried out in a clearly specified theoretical framework acceptable to economic theorists has been lamented by others. The body of research discussed in this paper is part of a wider series of recent developments, the explicit aim of which is to convert the Walrasian general equilibrium structure formalized in the 1950s by Kenneth Arrow, Gerard Debreu, and others from an abstract representation of a hypothetical economy into realistic models of actual economies. The idea is to use these models to evaluate alternative policy options. The approach is to specify production and demand parameters drawing on econometric and other literature, incorporate policy parameters and assess the impacts of alternative policy changes. The Walrasian model provides an ideal framework for appraising the effects of policy changes on resource allocation, and for assessing who gains and loses; these are exactly those policy impacts not covered by empirical macro models. In the text, we discuss a number of ways in which these models are already providing fresh insights into long standing policy controversies, and we anticipate further contributions in future years as this field develops. In order to contain our paper, we limit discussion to recent modeling efforts in the fields of taxation and international trade.

The applied general equilibrium models already in use are numerical analogues of the traditional analytic two-sector general equilibrium models. James Meade, Harry Johnson, Arnold Harberger and others popularized them in
the 1950s and 1960s. Earlier analytic work with these models has examined
the distortionary effects of trade and other policies, along with functional
incidence questions. The recent applied computational models go a stage
further in providing numerical estimates of efficiency effects, and functional
and personal distributional impacts of policies within the same framework.

A further important development is that using a computer removes the
need to work in low dimensions and the necessity to limit severely model
structure in order to generate qualitative results. As a result, much more
detail and complexity can be incorporated than in simple theoretical models.
Tax policy models, for instance, can simultaneously accommodate several taxes
in one model. This is important since even when evaluating changes in only
one tax, taxes compound in effect with other taxes. Models involving 30 or
more sectors and industries are commonly employed in this area, providing
substantial detail for policy makers concerned with feedback effects of policy
initiatives directed only at specified products or sector groups.

The earlier work of two people provides the background for much of
the recent activity in this area. One is Arnold Harberger who in 1962 was
the first author to investigate numerically tax policy questions in a two
sector general equilibrium framework. An equally important source of stimulus
has been an ingenious computer algorithm for the numerical determination of
the equilibrium of a Walrasian system developed by Herbert Scarf in 1967.
In spite of important extensions to the original algorithm, and more recently
the use of alternative solution techniques, Scarf's work remains a major
driving force in persuading some of the latest generations of mathematically
trained economists to approach general equilibrium from a computational
and, ultimately, practical perspective.
Recent general equilibrium tax models have been used to analyze such policy initiatives as integrating personal and corporate taxes, introducing value added taxes, and indexing the tax system. International trade area customs union issues, international trade negotiations under the GATT, and North-South trade questions have all been analyzed.

The plan of our paper is as follows. Section 2 presents a simple numerical example designed to illustrate the applied general equilibrium approach. Section 3 discusses how the methods illustrated by the numerical example can be implemented. This section discusses the choice of parameter values and functional forms, the use of data, solution methods, and how policy conclusions are formulated. Sections 4 and 6 highlight the main features of recent tax and trade models, and Sections 5 and 7 review their policy implications. We close with a discussion of weaknesses of the approach and an outline of what we believe could be useful future directions for this work.
2. What is Applied General Equilibrium Analysis?

By "applied general equilibrium" we mean a numerically specified general equilibrium model applied to policy analysis. In spite of the widespread use of the term "general equilibrium" in modern economics, it is surprising how large is the ambiguity both in the literature and in the profession at large as to what constitutes a general equilibrium model. Everyone seems to agree that a general equilibrium model is one in which all markets clear in equilibrium; there seems to be less agreement as to what are the elements of structure which underlie the equilibrium formulation.

Our understanding of the term corresponds to the well-known Arrow-Debreu model elaborated on in Arrow and Hahn (1971). On the demand side modelers give initial allocations of commodities to each consumer and specific market demand functions for each commodity. Commodity demands depend on all prices, are continuous, non-negative, homogeneous of degree zero and satisfy Walras' Law. On the production side, constant returns to scale activities or non-increasing returns to scale production functions describe technology. Equilibrium is characterized by a set of prices and levels of industry operation such that demands equal supplies for all commodities (including disposal if the commodity in question is a free good), and producers maximize profits. In the case of constant returns to scale, this implies that no activity (or cost minimizing techniques for production functions) does any better than break even at the equilibrium price.

The zero homogeneity of demand functions and the linear homogeneity of profits in prices implies that only relative prices are of any significance in the model; the absolute price level has no impact on the equilibrium outcome.
Market demands are represented as the sum of individual household demand functions, each of which may or may not be derived from utility maximization subject to a budget constraint. Walras' Law states that the value of market demands equal the value of endowments (income). This property holds at any set of prices whether or not they clear all markets simultaneously.

Constant returns to scale production functions are widely used to describe production possibilities in such models, although more abstract theoretical work in general equilibrium often uses activity analysis or production sets. Production functions are more convenient to use than activities in applied work since they can be parameterized using econometric literature on substitution elasticities and other parameters. Decreasing returns to scale can be considered, although they are typically not employed in applied models. Increasing returns to scale models are not widely used due to the absence of clearly defined theoretical structures incorporating scale economies.

It may clarify matters if we lay out the algebra of a simple N commodity general equilibrium model. Market demand functions for each commodity are denoted as $\xi_i(\pi)$, $i = 1, \ldots, N$, where the vector $\pi$ represents the N market prices. The non-negative, economywide endowments are given by the vector $w = (w_1, \ldots, w_N)$. Because of the homogeneity of the market demand functions, an arbitrary normalization of prices can be chosen; a common treatment is to consider prices which sum to unity, i.e., lie on a unit price simplex ($\pi_i > 0; \sum_i \pi_i = 1$).

The matrix $A$ defines $J$ constant returns to scale activities; $a_{ij}$ denotes the use of good $i$ in activity $j$ when operated at unit intensity.
If $a_{ij}$ is negative, an input is involved. The matrix of activities includes $N$ disposal activities to allow for costless disposal of each commodity. The activity coefficients $a_{ij}$ are thus arranged in the $(N \times J)$ matrix $A$, the first $N$ columns of which are usually the disposal activities. The number of activities is assumed to be finite and joint products are possible. Appealing to the so-called "no free lunch" assumption, the matrix $A$ is usually restricted to satisfy the boundedness condition, that the set of non-negative activity levels $x$ such that $Ax + w \geq 0$, is bounded. The interpretation of this condition is that if endowments are finite, infinite production of any of the commodities is excluded.

Equilibrium in this model is characterized by a vector of prices and activity levels $(\pi^*, x^*)$ such that

\begin{equation}
\xi_i(\pi^*) = \sum_{j=1}^{J} a_{ij} x_j + w_i \quad \text{for } i = 1, \ldots, N,
\end{equation}

and

\begin{equation}
\sum_{i=1}^{N} \pi_i^* a_{ij} \leq 0 \quad (= 0 \text{ if } x_j^* > 0) \quad \text{for } j = 1, \ldots, J.
\end{equation}

To illustrate this formulation further, we present a simplified numerical example representative of those actually used to analyze policy issues. We consider a model with two final goods (manufacturing and non-manufacturing), two factors of production (capital and labor), and two classes of consumers. Consumers have initial endowments of factors but have no initial endowments of goods. A "rich" consumer group owns
all the capital, while a "poor" group owns all the labor. Production of each
good takes place according to a constant elasticity of substitution (CES)
production function, and each consumer class has demands derived from maximizing
a CES utility function subject to its budget constraint.

Although a CES production function can only be approximated by a finite
list of activities, this departure from the structure outlined above is not
of any major significance. In order to solve numerically models with continuous
functions, cost minimizing activities are generated during the equilibrium
computations and only a finite number of such activities are examined. The
two consumer/two producer nature of this example means that it is similar to
the Harberger (1962) tax model and could be solved analytically. The solution
techniques used here, however, are applicable to much larger and more
sophisticated models.

The production functions for the sample are given by

$$Q_i = \phi_i \left[ \frac{\sigma_i^{-1}}{\delta_i} L_i + (1-\delta_i)K_i \right]^{\sigma_i-1}$$

where $Q_i$ denotes output of the $i^{th}$ industry, $\phi_i$ is the scale or units parameter,
$\delta_i$ is the distribution parameter, $K_i$ and $L_i$ are the factor inputs, and $\sigma_i$ is
the elasticity of factor substitution.

The CES utility functions are given by

$$U^q = \left[ \sum_{i=1}^{q} \alpha_i^q \cdot (X_i^q)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{1}{\sigma_i-1}}$$

where $X_i^q$ is the quantity of good $i$ demanded by the $q^{th}$ consumer, $\alpha_i^q$ are share
parameters, and $\sigma_i$ is the substitution elasticity in consumer class $q$'s CES
utility function. If we maximize this utility function subject to the constraint that the consumer cannot spend more than his income,

\[(P_1X_1^q + P_2X_2^q < P_LW_L^q + P_KW_K^q)\]

we get the demand functions:

\[X_i^q = \frac{a_{1i}^q}{\frac{\sigma}{P_1^q(1-\sigma)} + \frac{\sigma}{P_2^q(1-\sigma)}}\]

\[\text{where } I^q \text{ is individual } q\text{'s income (derived from selling factor endowments at factor prices) and the } P_i \text{ are market prices.}

Once we have specified the parameters of these production and demand functions, plus the individual endowments, we have a complete general equilibrium model. Tax and other policy variables can then be added as desired. A sample numerical specification for the no tax case is given in Table 1. This example has been solved using Merrill's (1972) general equilibrium algorithm, a refinement of the Scarf algorithm. The algorithm finds a set of market clearing prices for goods and factors providing a solution to the simultaneous nonlinear equations or correspondences given by the demand supply and zero profit equilibrium conditions.

For this example, the solution is shown in Table 2. Since only relative prices matter in general equilibrium models such as this, we adopt the normalization that the price of labor is unity. Note that at the computed set of equilibrium prices, total demand for each output exactly matches the amount produced, and producer revenues equal consumer expenditures. To a very close approximation, labor and capital endowments are fully employed, and consumer factor incomes equal producer factor costs. The cost per unit in each sector matches the selling price, meaning that economic profits are zero.
TABLE 1
SPECIFICATION OF PRODUCTION PARAMETERS, DEMAND PARAMETERS, AND ENDOWMENTS FOR A SIMPLE GENERAL EQUILIBRIUM ECONOMY

<table>
<thead>
<tr>
<th>Sector</th>
<th>Production Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\phi_i$  $\delta_i$  $\sigma_i$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>1.5      .6      2.0</td>
</tr>
<tr>
<td>Non-manufacturing</td>
<td>2.0      .7      .5</td>
</tr>
</tbody>
</table>

**Demand Parameters**

<table>
<thead>
<tr>
<th>Rich Consumers</th>
<th>Poor Consumers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^{q}_{MFG}$</td>
<td>$\alpha^{q}_{MFG}$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>1.5</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**Endowments**

<table>
<thead>
<tr>
<th>Rich Households</th>
<th>Poor Households</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>L</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
</tr>
</tbody>
</table>
### TABLE 2

**EQUILIBRIUM SOLUTION FOR ILLUSTRATIVE SIMPLE NO TAX GENERAL EQUILIBRIUM MODEL**
*(specified in Table 1)*

**Equilibrium Prices**

<table>
<thead>
<tr>
<th>Output Type</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing Output</td>
<td>1.399</td>
</tr>
<tr>
<td>Non-manufacturing Output</td>
<td>1.093</td>
</tr>
<tr>
<td>Capital</td>
<td>1.373</td>
</tr>
<tr>
<td>Labor</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**Production**

<table>
<thead>
<tr>
<th></th>
<th>Quantity</th>
<th>Revenue</th>
<th>Capital</th>
<th>Capital Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>24.942</td>
<td>34.89</td>
<td>6.212</td>
<td>8.532</td>
</tr>
<tr>
<td>Non-manufacturing</td>
<td>54.378</td>
<td>59.43</td>
<td>18.788</td>
<td>25.805</td>
</tr>
<tr>
<td>Total</td>
<td>94.33</td>
<td>94.33</td>
<td>25.000</td>
<td>34.337</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Labor</th>
<th>Labor Cost</th>
<th>Total Cost</th>
<th>Cost Per Unit Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>26.366</td>
<td>26.366</td>
<td>34.898</td>
<td>1.399</td>
</tr>
<tr>
<td>Non-manufacturing</td>
<td>33.634</td>
<td>33.634</td>
<td>59.439</td>
<td>1.093</td>
</tr>
<tr>
<td>Total</td>
<td>60.000</td>
<td>60.000</td>
<td>94.337</td>
<td></td>
</tr>
</tbody>
</table>

**Demands**

<table>
<thead>
<tr>
<th>Household Type</th>
<th>Manufacturing</th>
<th>Non-manufacturing</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich Households</td>
<td>11.514</td>
<td>16.674</td>
<td>34.337</td>
</tr>
<tr>
<td>Poor Households</td>
<td>13.428</td>
<td>37.704</td>
<td>60.000</td>
</tr>
<tr>
<td>Total</td>
<td>24.942</td>
<td>54.378</td>
<td>94.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Labor Income</th>
<th>Capital Income</th>
<th>Total Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich Households</td>
<td>0</td>
<td>34.337</td>
<td>34.337</td>
</tr>
<tr>
<td>Poor Households</td>
<td>60.000</td>
<td>0</td>
<td>60.000</td>
</tr>
<tr>
<td>Total</td>
<td>60.000</td>
<td>34.337</td>
<td>94.337</td>
</tr>
</tbody>
</table>
The expenditures of each household exhausts its income. Thus, the solution has all of the properties of an equilibrium for this economy. The closeness of the approximation can be enhanced by increasing the amount of computation for the solution algorithm.

To illustrate how these models can be used for policy evaluation work, we can consider the same numerical example as above, but now with a tax policy regime added. The methods through which taxes can be added to a computational general equilibrium model are shown in Shoven and Whalley (1973). The fundamental theoretical difficulty created by taxes is the simultaneity of demands and tax revenues. Although, for a given tax program, tax revenues will be determined once demands, production levels, and factor employments are known, demands also depend on tax proceeds since these provide income to one or more of the agents in the economy. The solution suggested by Shoven and Whalley is to solve not only for equilibrium prices as in the example above but also for equilibrium tax revenues. In the process of searching for an equilibrium, the individual consumers and producers take the "announced" prices and revenue as given and formulate their responses accordingly. The basic dimensionality of the equilibrium problem is increased by one, and the equilibrium conditions now include that the announced revenues equal tax collections as well as the other demand-supply and zero profit equilibrium conditions.

To illustrate how these methods can be used, we have computed an additional equilibrium for the example of Table 1, in which in addition to the parameters given above, a 50 percent tax has been imposed on capital used in the manufacturing sector.
TABLE 3
EQUILIBRIUM SOLUTION FOR ILLUSTRATIVE SIMPLE
GENERAL EQUILIBRIUM MODEL WITH 50%
TAX ON MANUFACTURING CAPITAL

**Equilibrium Prices**

<table>
<thead>
<tr>
<th>Output Type</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing Output</td>
<td>1.467</td>
</tr>
<tr>
<td>Non-manufacturing Output</td>
<td>1.006</td>
</tr>
<tr>
<td>Capital</td>
<td>1.128</td>
</tr>
<tr>
<td>Labor</td>
<td>1.000</td>
</tr>
</tbody>
</table>

**Production**

<table>
<thead>
<tr>
<th>Output Type</th>
<th>Quantity</th>
<th>Revenue</th>
<th>Capital</th>
<th>Capital Cost (including tax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>22.387</td>
<td>32.830</td>
<td>4.039</td>
<td>6.832</td>
</tr>
<tr>
<td>Non-manufacturing</td>
<td>57.307</td>
<td>57.639</td>
<td>20.961</td>
<td>23.637</td>
</tr>
<tr>
<td>Total</td>
<td>90.469</td>
<td>25.000</td>
<td></td>
<td>30.469</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output Type</th>
<th>Labor Cost</th>
<th>Total Cost</th>
<th>Per Unit Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>25.999</td>
<td>32.831</td>
<td>1.467</td>
</tr>
<tr>
<td>Non-manufacturing</td>
<td>34.001</td>
<td>57.638</td>
<td>1.006</td>
</tr>
<tr>
<td>Total</td>
<td>60.000</td>
<td>90.469</td>
<td></td>
</tr>
</tbody>
</table>

**Demands**

<table>
<thead>
<tr>
<th>Household Type</th>
<th>Manufacturing</th>
<th>Non-manufacturing</th>
<th>Expenditure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich Households</td>
<td>8.989</td>
<td>15.827</td>
<td>29.102</td>
</tr>
<tr>
<td>Poor Households</td>
<td>13.398</td>
<td>41.480</td>
<td>61.367</td>
</tr>
<tr>
<td>Total</td>
<td>22.387</td>
<td>57.307</td>
<td>90.469</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income Type</th>
<th>Labor Income</th>
<th>Capital Income</th>
<th>Transfers</th>
<th>Total Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich Households</td>
<td>0</td>
<td>28.191</td>
<td>.911</td>
<td>29.102</td>
</tr>
<tr>
<td>Poor Households</td>
<td>60.000</td>
<td>0</td>
<td>1.367</td>
<td>61.367</td>
</tr>
<tr>
<td>Total</td>
<td>60.000</td>
<td>28.191</td>
<td>2.278</td>
<td>90.469</td>
</tr>
</tbody>
</table>
The new equilibrium solution is shown in Table 3. The 50 percent tax on capital produces an equilibrium in which, compared to the no tax equilibrium in Table 2, the relative price of manufacturing to non-manufacturing output rises, while the net-of-tax price of capital falls. The gross-of-tax user cost of capital in manufacturing, however, increases. Less manufacturing and more non-manufacturing output is produced. Expenditures of "poor" (labor owning) households increase, while those of "rich" (capital owning) households fall. Tax revenues are transferred back to households with 60 percent going to the poor and 40 percent to the rich households.

Although Table 3 provides details on the characteristics of the new equilibrium in the presence of the tax, policy appraisal using these techniques usually relies

comparison between equilibria. Because the underlying theoretical structure of these models is so firmly rooted in traditional micro theory, a common procedure is to construct welfare measures of the gain or loss involved, both individually by household and in aggregate.

The measures most widely employed are Hicksian compensating and equivalent variations associated with the equilibrium comparison. The compensating variation (CV) is the amount of money needed to compensate a household for a change which has occurred; the equivalent variation (EV) is the amount of money that a change is equivalent to. Put another way, the CV takes the new equilibrium incomes and prices, and asks how much income must be taken away or added to return households to their pre-change utility level; the EV takes the old equilibrium incomes and prices and computes the change needed to achieve new equilibrium utilities. For a welfare improving change, the CV is negative and the EV is positive, although it is quite common to
employ a sign convention that a positive value for either measure indicates a welfare improvement. For an aggregate measure, the arithmetic sum of CVs or EVs is used as the welfare criterion.  

In Table 4 we display the Hicksian compensating and equivalent variations for this hypothetical numerical example. Here, we compare a no-tax (Pareto optimal) equilibrium to an equilibrium with tax distortions and we get aggregate losses. The redistribution caused by the factor price change means, however, that poor households gain and rich households lose. The aggregate welfare cost of the tax is around 0.6 percent of national income, a number similar to Harberger's (1966) estimates of the cost of the corporate tax in the U.S. where the tax discriminant was not that dissimilar to that used in this example. Although 0.6 percent of GNP may not seem to be a large number, when measured against revenues, the welfare cost is much larger. The deadweight loss is around one-fourth of revenues, suggesting that taxes in this case are an inefficient mechanism for raising revenues. Also, as stressed by Browning (1976) and Usher (1982), the marginal welfare costs of raising an extra dollar in revenues will significantly exceed these average cost estimates; in this case we find the marginal deadweight loss figure to be 79 cents for each dollar raised.

This numerical example suggests that researchers could use a similar approach for larger, more realistic models of actual economies, with the hope that they could obtain insights into key policy issues. The approach would be to parameterize models similar to that in the example above, but for actual economies using realistic data and estimated parameters. Comparing alternative equilibria resulting from different economic (tax or tariff) policies would then lead to an evaluation of various policy initiatives.
TABLE 4
WELFARE MEASURES OF THE IMPACT OF A 50% TAX ON CAPITAL ON MANUFACTURING IN TABLE 1

<table>
<thead>
<tr>
<th></th>
<th>Hicksian Compensating Variations</th>
<th>Hicksian Equivalent Variations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rich Households</td>
<td>-4.55</td>
<td>-4.45</td>
</tr>
<tr>
<td>Poor Households</td>
<td>+3.99</td>
<td>+3.83</td>
</tr>
<tr>
<td>Total</td>
<td>-0.56</td>
<td>-0.62</td>
</tr>
</tbody>
</table>

Welfare Loss as a Percent of National Income

|                      | 0.62%                            | 0.66%                         |

Welfare Loss as a Percent of Tax Revenues

|                      | 24.59%                           | 27.23%                        |

Decline in Welfare as a Percent of Marginal Dollar Raised and Returned through Transfers

|                      | 79.3%                            | 79.3%                         |
Table 5 lists the applied models described in more detail later in this paper in the areas of tax and trade policies. Models are listed alphabetically by author of the paper or book chosen as representative of the modeling effort. The fundamental differences between these models taken as a group and the numerical example above lie in their dimensionality, their parameter specification procedures based on empirical estimates, and their inclusion of more complex policy regimes than a simple tax on one factor in one sector. No major differences in structure separate large scale applied models from the simple example above. The tax models we survey cover a number of countries. They vary in the degree to which they specify the whole tax system; some incorporate the entire tax structure of the country, while others include only those portions of the tax system relevant to the issues being directly examined. In the trade models, a key difference is between those models which are multi-country or global in orientation, and those which examine how trade with the rest of the world affects individual countries.

Applied general equilibrium analyses, then, are attempts to assemble and use "theoretically pure" models for policy evaluation with a claim that the data and analysis is representative of conditions in an actual economy. In the tax models, for instance, a proposal may be for the corporate tax to be abolished and replaced by a value added tax. In trade models, multi-lateral tariff cuts proposed in a set of international negotiations could be the issue. Using general equilibrium techniques, it is possible to compute alternative equilibria for different policy regimes and to assess impacts of the change.

One point frequently made is that this approach would not be particularly instructive if the equilibrium solution in any of these models was not unique
<table>
<thead>
<tr>
<th>Model</th>
<th>Country(ies)</th>
<th>Base Years for Data Specification</th>
<th>Areas of Policy Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballard, Fullerton, Shoven, Whalley</td>
<td>United States</td>
<td>1973</td>
<td>Taxation: Various U.S. tax policy changes including integrating personal and corporate taxes and adoption of a consumption tax in place of income taxes</td>
</tr>
<tr>
<td>(forthcoming)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ballentine and Thirk (1979)</td>
<td>Canada</td>
<td>1969</td>
<td>Taxation: Incidence analysis of changes in taxes and expenditures</td>
</tr>
<tr>
<td>Broadway, Treddick (1978)</td>
<td>Canada</td>
<td>1966</td>
<td>Trade: Elimination of Canadian tariffs and taxes</td>
</tr>
<tr>
<td>Carrin, Gunning, Waelbroeck (1980)</td>
<td>9 geographic groups of</td>
<td>1978</td>
<td>Trade: Experiments with impacts of higher world oil prices on developing countries.</td>
</tr>
<tr>
<td></td>
<td>of developing countries and the</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>rest of the world</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deardorff, Stern (1981)</td>
<td>18 major industrialized</td>
<td>1976</td>
<td>Trade: Tokyo Round changes in tariffs and non-tariff barriers</td>
</tr>
<tr>
<td></td>
<td>countries, 16 developing</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>countries and the rest of the</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>world</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dervis, deMelo, Robinson (1982)</td>
<td>Turkey</td>
<td>1973</td>
<td>Trade: Applications include experiments with tariff changes and examination of 1977 Turkish foreign exchange crisis</td>
</tr>
<tr>
<td>Keller (1979)</td>
<td>The Netherlands</td>
<td>1973</td>
<td>Taxation: Changes in marginal tax rates in various production and consumption sectors</td>
</tr>
<tr>
<td>Reference</td>
<td>Geography Description</td>
<td>Year(s)</td>
<td>Type of Analysis</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>----------------------------------------------------------------------------------------</td>
<td>---------------</td>
<td>----------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Manne (1980)</td>
<td>3 regions: industrialized countries, oil-exporting developing countries, oil-importing developing countries</td>
<td>1978</td>
<td>Trade: scenarios involving varying assumptions on world energy supply and demand, and on capital flows</td>
</tr>
<tr>
<td>Miller, Spencer (1977)</td>
<td>U.K., EEC, Commonwealth (Australia and New Zealand) and the rest of the world</td>
<td>1960 (production), 1968 (demand)</td>
<td>Trade: Changes accompanying the U.K.'s entry into the EEC</td>
</tr>
<tr>
<td>Piggott, Whalley (forthcoming)</td>
<td>United Kingdom</td>
<td>1973</td>
<td>Taxation: Variations in U.K. taxes and subsidies</td>
</tr>
<tr>
<td>Shoven, Whalley (1972)</td>
<td>United States</td>
<td>1953-1959 (average)</td>
<td>Taxation: Imposition and removal of taxes on income from capital under various model parameter specifications</td>
</tr>
<tr>
<td>Whalley (1982)</td>
<td>U.S., EEC, Japan, Other Developed Countries, OPEC, newly industrialized countries, Less Developed Countries</td>
<td>1977</td>
<td>Trade: Terms of trade issues in the North-South debate</td>
</tr>
</tbody>
</table>
for any particular tax or tariff policy. Uniqueness, or the lack of it, has been a long standing interest of general equilibrium theorists (see Kehoe, 1980). There is, however, no theoretical argument that guarantees uniqueness in the applied models referred to in Table 5. With some of the models, researchers have conducted *ad hoc* numerical experimentation (approaching equilibria from different directions and at different speeds), but have yet to reveal a case of non-uniqueness. In the case of the Ballard, Fullerton, Shoven, Whalley tax model of the U.S., uniqueness has been numerically demonstrated by Kehoe and Whalley (1982). The current working hypothesis is that uniqueness can be presumed for all of the models discussed here until a clear case of non-uniqueness is found.

Many other problems beyond the possibility of non-uniqueness are also encountered. What type of model is to be used? Should it be, for instance, a traditional fixed factor static model or should it have dynamic features? Once the model form is determined, how are functions and parameter values to be chosen? How are foreign trade, investment, government expenditures, and a range of other complicating features to be treated? How is the model to be solved? And, finally, even after the model has been solved, how are equilibria to be compared; that is, which summary statistics are to be used in evaluating the policy change? These questions apply equally to all applied general equilibrium modeling efforts whether or not they are directed towards tax and trade issues. We now turn to techniques for modeling.
3. Implementing Applied General Equilibrium Analysis

How Do You Choose the Model?

Although the appropriate general equilibrium model for tax or trade policy analysis depends in part on the focus of the model, most models currently in use have a similar form. Most are variants of the static, two fixed factor models which have long been employed in public finance and international trade, and are associated with the work of James Meade, Harry Johnson, Arnold Harberger, and others. Most computational models involve more than two goods, while aggregating the factors of production into two broad types, capital and labor. In some models, these composite factors are sometimes disaggregated into sub-groups (e.g., labor distinguished by skilled and non-skilled). Intermediate transactions are also usually incorporated into the applied models either through fixed or flexible coefficient input-output matrices.

In some cases, static models have been extended to dynamic equilibrium models, which have been applied to intertemporal issues in taxation analysis. Under this approach, static equilibria are sequenced through savings decisions which change the capital stock of the economy through time. In each period a market clearing equilibrium is computed, with characteristics of the equilibria changing through time as the capital stock grows.

A further feature in the applied models concerns the treatment of external sector transactions. This can be especially important in the tax models since the effects of tax policies for an economy which is a taker of rental rates on world capital markets will be significantly different from those for a closed economy. While international capital mobility is usually ignored, Goulder, Shoven, and Whalley (1981) have shown how its incorporation
can change the analysis of policy options quite substantially compared to a case with immobile capital. In the multi-country international trade models, a common treatment is to use the so-called "Armington" formulation, which treats similar products produced in different countries as different goods. This differs from theoretical Hecksher-Ohlin models in which homogeneous products across countries are assumed, and is adopted both to accommodate the statistical phenomenon of countries both importing and exporting the same good, and to allow for model parameterization to estimated import and export demand elasticities.

Other key features of models include the treatment of investment and government expenditures. Investment is usually financed by household savings (broadly defined to include corporate retentions). Household savings are based either on constant expenditure shares in static models or intertemporal utility maximization in dynamic formulations. Constant expenditure shares are usually assumed for the government. In some work, however, models have been used with public goods in household utility functions, and public goods equilibria have been computed.

Perhaps a natural question to ask is why models have evolved in this way when it is possible to use more general specifications, possibly involving activities in place of production functions, along with joint production, and more alternative inputs than capital and labor composite factors. Although it is possible that in future work these features will gradually appear, at the present, three reasons seem to account for the popularity of the basic two sector structure in applied work.

First, many policy issues have already been analyzed theoretically in the two sector framework. If the major contribution of numerical work is to
advance to quantitative from qualitative analysis, it is clearly natural to retain the same basic theoretical structures. This way researchers can use the intuition gleaned from theoretical work to guide numerical investigations of policy alternatives.

Second, most of the data on which the numerical specifications are based come in a form consistent with two sector-type models. National accounts data identify wages and salaries and operating surpluses as major cost components. This suggests a model with capital and labor as inputs. Input-output data provide intermediate transaction data, with value added broken down in a similar way.

Finally, the partition between goods and factors is used in these models in ways similar to theoretical work so as to simplify the effective dimensionality of the model. This is done by using factor prices to generate cost covering goods prices. This dimension reduction greatly reduces execution costs in the computer solution of models, making feasible the incorporation of a fair amount of elaborate detail in the treatment of households and goods.

How Do You Choose Functional Forms for the Model?

The major constraints on the selection of demand and production functions in all the applied models, whether they are oriented to tax, trade or other issues, is that they be both consistent with the theoretical approach and analytically tractable. The first constraint involves choosing functions that satisfy the restrictions listed in the algebraic presentation of a general equilibrium model above (such as Walras' Law for demand functions). The second requires that the demand and supply responses of the economy be reasonably easy to evaluate for any price vector considered
as a candidate equilibrium solution for the economy. This largely explains
why the functional forms used are so often restricted to the family of
"convenient" forms (Cobb-Douglas; Constant Elasticity of Substitution (CES);
Linear Expenditure System (LES); CRESH (Constant Ratios of Elasticities of
Substitution, Homothetic); Translog, and others).

The choice of a specific functional form depends on the objectives
which are set for the model. This point is best illustrated by considering
the demand side of these models. Demands derived from Cobb-Douglas utility
functions are easy to work with but have the restrictions of unitary income
and uncompensated own price elasticities, and zero cross price elasticities.
These restrictions may be implausible given empirical estimates of elasticities
applicable to a particular model, but can only be relaxed
by using more general functional forms. With CES functions, unitary own price
elasticities no longer apply. However, if all expenditure shares are small,
the compensated price elasticities equal the elasticity of substitution in
the preferences, and it may be unacceptable to model all commodities as having
essentially the same compensated own price elasticities. A response to this
difficulty is to use hierarchical or nested CES functions, adding further
complexity in structure. The unitary income elasticities in the Cobb-Douglas
functions can also be relaxed. One way is to use LES functions with a displaced
origin, but then the origin displacements need to be specified.

On the production side, CES value added functions are usually used to
allow for substitution between primary factors. Unlike Cobb-
Douglas functions, these allow for substitution elasticities between factors
different from unity. If more than two factors are used, hierarchical CES
functions are again used. Intermediate production is sometimes modeled as
fixed coefficient; on other occasions, limited intermediate substitutability is present. In the international trade models one possible specification would be to have fixed coefficients in terms of composite goods but with substitution possible among the components of the composite. By way of example, a fixed steel requirement per car may be specified but substitution between imported and domestic steel may be represented by CES functions. This may be necessary because of the large amount of trade in intermediate products and the unrealistically low import price elasticities which fixed coefficient intermediate production would imply if the Armington treatment of country subscripted products is used.

How are Parameter Values Selected?

Parameter values for the functional forms are often crucial in determining results of policy simulations generated by the applied models. The procedure most commonly used at the present time to select parameter values has come to be labeled "calibration" (see Mansur and Whalley, 1983). This procedure is schematically outlined in Figure 1. It makes the strong assumption that the economy under consideration is in equilibrium in the presence of existing policies. This state is usually referred to as the "benchmark" equilibrium. In practice, benchmark equilibria are constructed from national accounts and related data sets. In this process, a number of adjustments are required to the basic data to ensure that equilibrium conditions hold. The construction of data sets of this type is described in St. Hilaire
Figure 1

FLOW CHART OUTLINING CALIBRATION PROCEDURES AND MODEL USE IN TYPICAL APPLIED GENERAL EQUILIBRIUM MODEL

Basic Data for economy for single year or average of years (national accounts, household income and expenditure, input-output tables, tax data, trade and balance of payments)

Adjustments for mutual consistency BENCHMARK EQUILIBRIUM DATA SET

Replication Check

Choice of functional form and CALIBRATION to Benchmark Equilibrium

Policy Change Specified

"Counterfactual" Equilibrium computed for new policy regime

Policy Appraisal based on pairwise comparison between counterfactual and benchmark

Further policy changes to be evaluated?

Extraneous Specification of Elasticities
and Whalley (1980), Piggott and Whalley (forthcoming), and Ballard, Fullerton, Shoven and Whalley (forthcoming). Since the benchmark data are usually in value terms, units must be chosen for goods and factors so that separate price and quantity observations are obtained. A commonly used units convention, originally adopted by Harberger, is to choose units for both goods and factors so that they have a price of unity in the benchmark equilibrium.

With the benchmark observation at hand, parameters are then chosen so that the solution to the model will replicate the benchmark data. Parameter values thus generated can then be used to solve for the alternative equilibrium associated with any changed policy regime. These are usually termed "counterfactual" or "policy replacement" equilibrium.

Typically, calibration involves only one year's data or a single observation represented as an average over a number of years. Since equilibrium observations are in value terms, the separate price and quantity observations generated rely on a time dependent units convention. However, the sequencing of equilibrium observations into consistent time series has not been explored thus far in calibrating models. Because of the reliance on a single observation, the benchmark data may not uniquely identify the parameters.

With Cobb-Douglas functions, a single benchmark observation uniquely identifies parameter values since expenditure shares by household and factor shares by sector are known. With other more complex functions, it is usually the case that an infinite number of combinations of parameters can replicate the data in the required manner. In such cases, extraneously specified elasticities are used as identifying restrictions. Once specified, these allow the other parameters to be determined uniquely from the equilibrium observation.
The extraneous specification of elasticities is most easily thought of as determining the curvature of isoquants and indifference surfaces, with their position given by the benchmark equilibrium data. For Cobb-Douglas demand or production functions, a single price and quantity observation is sufficient to uniquely determine the parameters of the whole function if unity or profit maximization is assumed. For CES functions, extraneous values of substitution elasticities are required since the curvature of indifference curves and isoquants (given by the single elasticity parameter), is not contained in benchmark data. Similarly, for LES demand functions income elasticities are determined once the origin coordinates for utility measurement are known.

The current procedure in selecting these additional parameters beyond the benchmark equilibrium data is to scan empirical literature and select appropriate point estimates of substitution elasticities for the underlying utility and production functions. This places a lot of reliance on literature surveys of elasticities, and as many of the modelers have commented in discussing their procedures, it is surprising how sparse (and sometimes contradictory) the literature is on some elasticity values.

Calibration thus determines share and unit parameters in utility and production functions, once elasticities are known. No statistical test of the chosen model specification is involved since a deterministic procedure of calculating parameter values from the equilibrium observation is employed.² This entire procedure is dependent both on the accuracy of the assembled data and the assumption that it represents an equilibrium. Also, elasticity specifications play a key role in this procedure.
Once the calibration procedure is completed, a fully specified numerical model is available which can be used for policy analysis. As indicated in Figure 1, any policy change can be considered and a counterfactual equilibrium computed for a new policy regime. Policy appraisal then proceeds on the basis of pairwise comparisons of counterfactual and benchmark equilibria. If further policy changes are to be evaluated, the new policy is incorporated and the resulting counterfactual equilibrium is also compared to the benchmark.

It is perhaps worth outlining some of the reasons why this calibration approach rather than a more direct econometric approach is so widely used in parameterizing applied models, since the use of deterministic calibration rather than stochastic estimation in these models is often troubling to econometricians. First, in some of these models many thousands of parameters are involved, and to estimate simultaneously all of the parameters of the model using time series methods would require either unrealistically large numbers of observations or overly severe identifying restrictions. Although partitioning models into submodels (such as a demand and production system) may reduce or overcome this problem, partitioning does not fully incorporate all the restrictions from overall equilibrium which are emphasized in calibration. Second, as mentioned, benchmark data sets are in value terms, and the decomposition into separate price and quantity observations makes it difficult to sequence equilibrium observations with consistent units through time as would be required for time series estimation.

These problems, combined with the difficulty of incorporating equilibrium restrictions into a satisfactory estimation procedure, have thus far largely excluded complete econometric estimation of general equilibrium systems, although
some progress in this direction has been made in work by Clements (1979) and Mansur (1980). Mansur, for instance, notes the difficulties in writing down a likelihood function for a maximum likelihood procedure incorporating full equilibrium restrictions. He suggests a partitioning approach using segmented production and demand systems with a third segment incorporating their equilibrium interdependence. Other attempts to estimate econometrically complete general equilibrium systems have been made by Allingham (1974) and Jorgenson (1983), although these are not for the same types of models which appear in Table 5.

How are Models Solved for a Counterfactual Equilibrium?

In order to solve models for counterfactual equilibria, early applied models used Scarf's algorithm (1967, 1973). Several current models use faster variants of Scarf's algorithm due to Merrill (1971), Kuhn and MacKinnon (1973), Eaves (1974) and van der Laan and Talman (1978) as the computational procedure. As work has developed on applied models, however, it has become apparent that a Newton-type method or other local linearization technique often works as quickly as the advanced simplical subdivision methods just listed, although these methods do not necessarily guarantee convergence. Kimbell and Harrison (1983a), for instance, have developed a procedure based on a factor price revision rule which corresponds closely to a method for analytic solution of models in which all substitution elasticities are identical. Their experience indicates very rapid convergence to a solution. There is, therefore, currently some difference across the applied models in choice of computational method. Newton and other localization methods seem to be faster, though precise comparisons depend crucially on the features of particular models.
Where Newton methods are used, Newton steps usually follow a Jacobian matrix of excess demand functions. These are sometimes calculated in a reduced dimension (such as factor space) to conserve on execution costs. Another device (adopted in the so-called Johansen models, after Johansen, 1964), is to use a linearized system of excess demands to solve for a first Newton step as an approximation to an equilibrium. Where this is done, however, the accuracy of the approximation remains in doubt as long as the full equilibrium is not computed. Where simplicial subdivision methods are used to compute the new equilibrium, Merrill's algorithm seems the most widely used. Execution costs for the models using these techniques currently seem manageable, even on a production run basis. No standard off-the-shelf computer routine has yet emerged for the complete sequence of data adjustment, calibration, and equilibrium computation due to the complexities involved in each application of these methods. What seems fairly clear from recent literature, however, is that it is no longer the technology required to solve numerically general equilibrium models that constrains their applications, but the availability of data and the ability of modelers to specify accurately key parameters.

How are Policy Conclusions Reached?

As has already been noted in discussing the numerical example above, theoretical literature in applied welfare economics is usually followed in making comparisons between equilibria in order to arrive at policy conclusions from the tax and trade models. As is widely recognized in this literature, there are many problems in choosing summary measures by which to compare equilibria. Since these models provide a detailed evaluation of who gains,
who loses and by how much as a result of a policy change, no single summary measure needs to be chosen.

For welfare impacts, the most commonly used summary measures are the Hicksian compensating (CV) and equivalent variations (EV) discussed above. In tax models where a differential approach (replacing one set of taxes by a yield preserving alternative) is followed, welfare effects for government can be excluded. Where government revenues change, however, the welfare impact on government needs to be added to the economywide measure.

In addition to welfare impacts, other differences between equilibria are also evaluated. Income distribution effects are highlighted by examining the Lorenz curve (or the Gini coefficient or some other measure) for the distribution of alternative income concepts. Changes in relative prices are examined, and in the international trade models, changes computed in each country's terms of trade. Changes in the factors of production across industries and the product composition of consumer demands are important in some policy evaluations and can also be extracted from the equilibrium computations. The focus of many (but not all) of the applied models in Table 5 is on the welfare impacts of policy changes with particular emphasis on aggregate efficiency impacts. Although distributional effects are highlighted, the bottom line in most policy evaluations is whether any given policy change is a welfare worsening or welfare improving proposition. In the trade models, the difference between national and global welfare can be important. A tariff, for instance, may improve the national terms of trade and raise national welfare even though a global loss may be involved. As a result, global and national welfare considerations can lead to quite different policy conclusions.
4. Characteristics of Applied General Equilibrium Tax Models

In this section we outline some of the main features of the recent applied general equilibrium tax models. Table 6 details the structure of the models, and Table 7 outlines the data used.

These models all derive in one way or another from the work of Harberger (1959, 1962, 1966) on U.S. corporate and capital income taxes. In Harberger's 1962 paper, two sectors of production are identified: the corporate and the non-corporate sector. The corporate tax is assumed to be a partial factor tax, a tax on capital in the corporate sector. Using linearization and approximation techniques, Harberger is able to generate an algebraic expression for the change in the net rental price of capital which would result from the introduction of a corporate income tax. He selects substitution elasticities in production functions, demand elasticities, and uses U.S. data for the mid-1950s on factor and expenditure shares. His main conclusion is that the reduction in the net return to capital is approximately equal to the tax revenues raised, and therefore that capital fully bears the burden of the corporate tax.

In this paper, Harberger implicitly outlines the calibration procedure mentioned above. He chooses units for factors of production as those amounts which sell for one dollar in the presence of the tax, i.e., in a benchmark equilibrium. His counterfactual experiment involves removing the corporate tax and replacing it by a nondistorting alternative.

Harberger's work provided the motivation for further work on distortionary capital income taxes in the U.S. Shoven and Whalley (1972) and Shoven (1976) examined this issue in depth. Although Shoven and Whalley incorporated tax distortions in their general equilibrium model, they did not use the revenue
<table>
<thead>
<tr>
<th>Model</th>
<th>Country(ies)</th>
<th>Demand Side</th>
<th>Production Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballard, Fullerton, Shoven, Whalley (forthcoming)</td>
<td>U.S.</td>
<td>Derived from nested CES/ Cobb-Douglas utility functions</td>
<td>12 consumer income groups</td>
</tr>
<tr>
<td>Ballentine, Thirsk (1979)</td>
<td>Canada</td>
<td>Differential equations giving quantity changes in terms of elasticities</td>
<td>12 income classes</td>
</tr>
<tr>
<td>Keller (1979)</td>
<td>The Netherlands</td>
<td>Derived from nested CES utility functions</td>
<td>4 demand sectors: skilled and unskilled labor, public, foreign</td>
</tr>
<tr>
<td>Piggott (1979)</td>
<td>Australia</td>
<td>Derived from nested CES utility functions</td>
<td>12 socio-economic household groups, plus government, foreign and corporate sectors</td>
</tr>
<tr>
<td>Piggott, Whalley (forthcoming)</td>
<td>United Kingdom</td>
<td>Derived from nested CES utility functions</td>
<td>100 socio-economic household groups, plus public, investment and external sectors</td>
</tr>
<tr>
<td>Serra-Puche (1981)</td>
<td>Mexico</td>
<td>Derived from Cobb-Douglas utility functions</td>
<td>10 rural/urban income groups, plus government, plus the rest of the world</td>
</tr>
<tr>
<td>Study</td>
<td>Country</td>
<td>Method</td>
<td>Income Groups</td>
</tr>
<tr>
<td>------------------</td>
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<td>---------------</td>
</tr>
<tr>
<td>Shoven, Whalley</td>
<td>U.S.</td>
<td>Derived from Cobb-Douglas utility functions</td>
<td>2</td>
</tr>
<tr>
<td>(1972)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silenrod (1981)</td>
<td>U.S.</td>
<td>Derived from Cobb-Douglas utility functions</td>
<td>9</td>
</tr>
<tr>
<td>Whalley (1975)</td>
<td>United Kingdom</td>
<td>Derived from CES utility functions</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>Base Year for Data</td>
<td>Extraneous Use of Elasticities</td>
<td>Production Data</td>
</tr>
<tr>
<td>------------------------------</td>
<td>--------------------</td>
<td>--------------------------------</td>
<td>------------------------------------------------------</td>
</tr>
<tr>
<td>Ballard, Fullerton, Shoven, Whalley (forthcoming)</td>
<td>1973</td>
<td>Labor supply, savings (literature search) production elasticities of substitution between capital and labor (literature search)</td>
<td>National Accounts, Input-output tables</td>
</tr>
<tr>
<td>Ballentine, Thirsk</td>
<td>1969</td>
<td>Factor substitution, price and income demand elasticities (literature search)</td>
<td>Input-output tables, plus National Accounts data</td>
</tr>
<tr>
<td>Source</td>
<td>Year</td>
<td>Type of Elasticity</td>
<td>Source</td>
</tr>
<tr>
<td>-------------------------------</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>(literature search)</td>
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<tr>
<td>Shoven, Whalley (1972)</td>
<td>1953-1959 (average)</td>
<td>Elasticities of substitution in production (various specifications)</td>
<td>Literature source</td>
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augmented unit price simplex subsequently developed in Shoven and Whalley (1973). This paper was the first to analyze taxes through a full general equilibrium computational procedure. In Shoven and Whalley (1972), an artificial commodity called capital tax tickets is used to incorporate the tax distortions. This feature limits the applicability of the analysis to, effectively, one tax at a time. In Shoven and Whalley (1973), a procedure is developed to deal with several simultaneous tax distortions without involving artificial commodities. Scarf's algorithm enables existence of a tax equilibrium to be shown and also provides a method through which such equilibria can be computed. Shoven (1976) presents a further reexamination of the Harberger calculations of efficiency costs of distortionary capital income taxes using a more disaggregated approach.

This method of simultaneously incorporating several tax distortions was also used by Whalley (1975) to examine the impact of 1973 tax changes in the U.K. Income, corporate, property, sales (value added), and social security taxes are all incorporated in the model, and packages of tax changes in which one or more of these are altered together are able to be evaluated. This model was further developed by Piggott and Whalley (1976 and forthcoming) into a 33 product and 100 household-type model, and has been used to evaluate structural characteristics of the tax/subsidy system. They produce estimates of the welfare gains and losses for household groups classified by income, occupation, and family size from the operation of the whole U.K. tax/subsidy system. Following Musgrave (1959), they concentrate on differential incidence calculations in which the entire tax system is replaced by a yield preserving "neutral" alternative (typically a single rate comprehensive sales tax).
The more recent general equilibrium model of the U.S. due to Ballard, Fullerton, Shoven, Whalley (BFSW) (forthcoming) also incorporates all major distorting taxes, but differs from the Piggott-Whalley model through the incorporation of time via dynamic sequencing of single period equilibria. In the BFSW model, savings decisions are made by households on the basis of the anticipated rate of return on savings. This contrasts with the Piggott-Whalley model in which fixed savings propensities are assumed. Saving decisions are based on myopic expectations regarding the future rate of return to capital, allowing each period's equilibrium to be computed without requiring information on future period's prices. Saving results in an increase in the capital stock and affects intertemporal behavior through changed consumption possibilities in future periods. Calibration is made to an assumed growth path in the presence of existing tax policies rather than to a single benchmark equilibrium. A change in policy displaces the economy from this path. After a transition period, the economy settles on a new growth path with an alternative capital/labor ratio. The pairwise comparison between equilibria in static models is replaced by a pairwise comparison between the equilibrium sequences under the alternative policy regimes.

Two further models closely related to the Shoven-Whalley work are those by Piggott (1979) on Australia and Serra-Puche (1981) on Mexico. Piggott's model differs from the other tax models in using two stage CES production functions with differing types of capital and labor. In subsequent work (Piggott, 1982), this model has been used to examine the effects of interaction between inflation and the tax system, an area as yet not covered by the Shoven-Whalley models.
Serra-Puche analyzes tax incidence in Mexico in a model with three factors, 14 production sectors, three non-consumption demand categories, and 10 household income groups. Subsequent work by Kehoe and Serra (forthcoming) has used a similar model to analyze the 1980 fiscal reform in Mexico incorporating unemployment generated by a downward rigid real wage.

Keller's (1978) tax model of Holland differs from the Shoven-Whalley work in using a local linearization procedure to solve for tax change equilibria. Four groups of agents on the demand side are incorporated. Government and the foreign sector are separately identified, along with low income/unskilled labor and high income/skilled labor groups. His incidence analysis concentrates on distributional effects between these two latter groups. Major Dutch taxes (payroll, value added, corporate, and income taxes) are all considered in model equivalent form. Data for 1973 are used to give a consistent base year data set.

Ballentine and Thirsk (1980) also use a local linearization approach in their general equilibrium tax work on Canada. Their main concern is incidence analysis of changes in financing arrangements for local government expenditures, such as increases in federal, personal or corporate taxes to finance increased municipal expenditures. No explicit functional forms for demand and production functions are required since total differentials through the equilibrium conditions yield approximate estimates of changes between equilibria in elasticities form. On the demand side, however, they are careful to ensure that Engel and Slutsky aggregation conditions are satisfied by the elasticities shown. An especially interesting departure in this model is the attempt to incorporate a degree of factor mobility both domestically between regions and internationally.
Slemrod's (1980) tax model of the U.S. differs from all of the above models in incorporating endogenous financial behavior of firms into the general equilibrium approach. His work is motivated by the extensive literature in recent years which stresses the corporate tax as a tax on equity returns only rather than as a tax on all capital income originating in the corporate sector. Slemrod introduces a risk aversion parameter into the preference functions in the model, which are defined over expected consumption and the variance of income. Both risky and riskless assets define claims on capital income, resulting in a portfolio allocation problem for households in addition to the usual budget problem yielding consumption demands. Household commodity and asset demands are based on maximization of a two stage preference function, the first stage incorporating the risk aversion parameter. Market clearing for all goods and assets is incorporated with a supply response in financial assets based on an extraneous elasticity of financial response of firm debt equity ratios with respect to relative tax costs of debt and equity. The model is parameterized to represent a "stylized" 1977 economy rather than calibrated to an exact benchmark equilibrium as in the other models.
5. Policy Findings from Tax Models

Although the ability to apply general equilibrium techniques to policy questions may strike some readers in itself as an accomplishment worth noting, ultimately the strengths and weaknesses of the applied models lie in their results. The two great virtues of the applied general equilibrium tax models are first, their ability to capture simultaneously resource allocation and redistributive effects of taxes, and, second, the broad view they take of whole tax systems. The compounding effects of tax distortions is perhaps one of the strongest of the results to emerge thus far.

For several years following the original Harberger calculations on the resource allocation effects of capital income taxation in the U.S., public finance economists argued that deadweight losses from taxes were small (perhaps one percent of GNP per year). When combined with the results of incidence studies, such as Pechman and Okner (1974) suggesting little redistribution in the tax system, this naturally led to a policy stance emphasizing redistributive tax reform with only limited attention focused on allocative efficiency. A striking feature of the results from the general equilibrium tax models surveyed here is their suggestion of considerably larger deadweight losses from tax distortions, especially at the margin and through the compounding of many distortions. In addition, these models' results indicate that tax policies have more redistributive power when their general equilibrium effects are taken into account.

Table 8 summarizes some of the results of the tax models. Ballard, Fullerton, Shoven, and Whalley (BFSW) (forthcoming) study various possible
tax changes in the U.S. For a plan to integrate personal and corporate
taxes, results indicate efficiency gains whose discounted present value
amounts to approximately $500 billion in 1973 prices. The distributional
impacts associated with integration depend on the nature of the replacement
taxes considered to preserve government revenue. However, for the replacement
taxes investigated, the results suggest the change is progressive. These
authors have also applied the same model to an analysis of a replacement
of the existing U.S. income tax by a progressive consumption tax. Table 8
reports dynamic welfare gains whose discounted present value on an annualized
equivalent basis is a little more than one percent of the discounted present
value of the gross national product.

In other applications of the model, Henderson and Fullerton (1982)
have analyzed changes in depreciation tax rules. Fullerton (1981) has
examined the possibility of an inverse relationship between tax rates and
revenues, and Ballard, Shoven, and Whalley (1983) have calculated the marginal
deadweight loss of U.S. taxes. Their estimate for the entire U.S. tax system
at 46 cents deadweight loss for an extra $1.00 of revenues further emphasizes
the importance of resource allocation costs of taxes in the model results.

Ballentine and Thirsk (1980) have used their model of Canada to
evaluate alternative packages of increases in municipal expenditures financed
by various tax changes. They conclude that the incidence effects of benefit
side changes are small and concentrate most heavily on tax incidence. In the
process, they analyze the incidence of some of the major Canadian taxes. They
show that income tax to be markedly progressive while property and corporate
taxes have a more mixed incidence pattern. The latter two taxes are borne by
<table>
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<tr>
<th>Model</th>
<th>Policy Interventions Incorporated</th>
<th>Policy Data Used</th>
<th>Policy Conclusions</th>
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<tbody>
<tr>
<td>Ballard, Fullerton, Shoven, Whalley</td>
<td>Integration analysis: four alternative plans for corporate and personal income tax integrations. Consumption tax alternatives: change in the tax treatment of savings</td>
<td>U.S. personal and corporate income taxes</td>
<td>Total integration of personal and corporate income taxes yields gains whose discounted present value is $500 bil. or 1% of national income. Total integration with scaling to preserve tax yields leads to a progression change income distribution even though every class is better off. Consumption tax alternatives yield gains of $650 bil. in present value terms.</td>
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<td>(forthcoming)</td>
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<tr>
<td>Ballentine, Thirlk (1980)</td>
<td>Changes in local government expenditures, corporate and property income taxes, and federal income taxes</td>
<td>Tax and expenditure data</td>
<td>Personal income taxes markedly progressive while property and corporate income taxes have a more mixed incidence pattern. Incidence effects of different expenditure programs small.</td>
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<tr>
<td>Keller (1979)</td>
<td>Changes in marginal tax rates in various production and consumption sectors</td>
<td>Major taxes in the Netherlands (value-added, corporate, social security and income)</td>
<td>Efficiency effects of taxes generally small (excepting corporate income tax); only small amounts of tax shifting.</td>
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<tr>
<td>Piggott (1979)</td>
<td>Total and sectoral abolition of taxes and subsidies under various model parameter specifications</td>
<td>Existing Australian sectoral taxes and subsidies</td>
<td>Replacing all taxes and subsidies with an equal-yield replacement tax leads to decrease in total domestic final demand: demand for imports rises modestly and world demand for Australian exports rises more. Replacing all taxes and subsidies with an equal-yield export tax leads to total welfare gain of 3.5% of Australian NDP.</td>
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<td>Author</td>
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<tr>
<td>Piggott, Whalley (forthcoming)</td>
<td>Variations in U.K. taxes and subsidies</td>
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<td>Serra-Puche (1981)</td>
<td>Replacement of indirect turnover taxes with consumption value-added tax (as instituted in Mexico in 1981)</td>
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<tr>
<td>Shoven, Whalley (1972)</td>
<td>Imposition and removal of existing taxes on income from capital under various model parameter specifications</td>
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<tr>
<td>Slemrod (1981)</td>
<td>Complete indexing of U.S. tax system for inflation</td>
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<td>Whalley (1975)</td>
<td>1973 U.K. tax reform</td>
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<td></td>
<td><strong>Existing U.K. taxes and subsidies</strong></td>
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<td></td>
<td>Existing U.K. tax system yields distorting losses of 6-9% of NNP per year. Subsidies to local authority housing area a significant source of welfare loss. Significant redistributive effects of taxes.</td>
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<td>Resource allocation moved in favor of the government target sectors (agriculture and foodstuffs); income distribution improved, reducing differentials between urban and rural households.</td>
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<td>In 6 of the 12 cases examined, capital bears more than the full burden of the surtax, while in the remaining 6 cases, labor shares in the burden.</td>
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<td>Indexing the U.S. tax system leads to aggregate efficiency gains with the lowest income groups experiencing slight losses and the highest income groups receiving substantial gains.</td>
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<td>Welfare gain from 1973 U.K. tax changes found to be small and in some cases may be negative. Replacement of purchase tax and SET by VAT appears to yield welfare losses, while changes made to income tax systems may yield gains.</td>
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\*Other applications of this model are mentioned in the text.*
owners of reproducible capital and land, and the distribution of the tax burden depends crucially on the distribution of the ownership of capital by income class. An interesting conclusion is the strength of tax exporting by Canada, a result somewhat similar to those reported by Whalley (1980). Their estimate of the foreign burden of an increase in Canadian corporate taxes ranges from 25-50 percent of the increase in government revenues. Their inter-regional results are also interesting, indicating that regional tax differences within Canada may well work against a regional distribution of output based on comparative advantage. The one tax with no marked impact on regional location is the provincial sales tax.

Keller (1979) has applied his model of Holland to incidence analysis of tax changes in various production and consumption sectors. Alternative changes in taxes are considered, including a one percent increase in all taxes, and a change in corporate taxes yielding enough revenues to increase public consumption by one percent. In general, Keller finds limited amounts of tax shifting occurs. Notable exceptions are found for the tax on imports (of which 40 percent is shifted) and the labor tax where nearly 30 percent is shifted by skilled labor. Negative shifting occurs with unskilled labor for whom the net-of-tax wage falls by more than one percent if a one percent tax is removed. Efficiency effects of taxes as measured by the excess burden are generally around ten percent of the induced changes in public consumption (extra revenues collected). In the case of the corporate tax, excess burdens are estimated at twice the resulting change in public consumption. As an estimate of marginal deadweight loss of the Dutch corporate tax, this is considerably higher than comparable estimates for the United States obtained by Ballard, Fullerton, Shoven, and Whalley.
Piggott (1979) has used his model to evaluate the structure of the Australian tax system. He looks at total and component abolition of taxes and subsidies under various model parameter specifications. His main result is that replacing all taxes and subsidies in Australia with an equal yield tax, along with a terms of trade-neutralizing tax, leads to a total welfare gain of 3.5 percent of Australian net domestic product, around 13 percent of tax collections. Sensitivity analyses on this estimate are presented along with detailed analyses of components of the Australian tax system.

Piggott and Whalley (forthcoming) also take a systemwide approach to their analysis of the U.K. taxes and subsidies. They estimate that the 1973 U.K. tax/subsidy system yields distorting losses of between six to nine percent of net national product per year, with subsidies to local authority housing identified as a significant source of welfare loss. They suggest that around one-quarter of net revenues raised by government each year are foregone through the deadweight loss associated with the tax subsidy system. Sharp distributional gains and losses occur through replacing the existing tax system by a yield-preserving neutral sales tax. The welfare gain to the top ten percent of households is around 25 percent of disposable income; the loss to the bottom ten percent is around 20 percent of disposable income. This, of course, differs from the well-known Pechman-Okner conclusion that tax systems have only limited distributional impacts. The tax system is shown to penalize manufacturing, but substantially protect and promote housing. Additional welfare costs are calculated from savings and labor supply distortions and are shown to be small or modest. The costs of distortions from the tax subsidy system appear to be heavily concentrated in three areas: capital taxes, public sector housing subsidies, and excise taxes. Distortionary costs for the most part are shown to be additive with the notable exceptions of corporate and property taxes.
Serra-Puche (1981) uses his model for Mexico to analyze the effect of the introduction of a consumption value added tax in place of existing turnover taxes. Income distribution and resource allocation effects are reported. His two major conclusions are that resource allocation moves in favor of tax favored sectors (agriculture and foodstuffs) and that income distribution improves, reducing the differentials between urban and rural groups. Results indicate that the tax switch is consistent with one of the main priorities of government policy in Mexico; namely, the relative improvement of incomes of rural groups.

Slemrod's (1979) endogenous financial behavior tax model has been applied to the analysis of indexing capital income for tax purposes in the U.S. His results suggest that such a change is efficiency improving for the U.S. In terms of equity, the lowest income groups experience slight losses, and the highest income groups receive substantial gains. A market shift in the allocation of private risk bearing occurs, indicating that there would be significant adjustments in financial markets to indexation. A slight reallocation of the capital stock away from owner-occupied housing toward other industries also occurs.

Whalley (1975) considers the effects of tax changes introduced in the U.K. in 1973. He finds that welfare gains from his tax change appear to be small and in some cases may be negative. An interesting result is that removal of the selective employment tax, a payroll tax on labor in service industries only, appears to be a welfare losing change. Although this seems counter-intuitive since the tax is a distorting tax, the tax can be shown to be a partial offset to distortions elsewhere in the tax system; a clear example of a second best result not apparent at first sight.
In general, then, these tax models are being applied to a range of policy issues and yielding important insights not readily apparent. In this way, the results are helping to frame positions on policy issues, especially in the quantitative directions of suggesting which effects of taxes are large and which are small.
6. **Characteristics of Applied General Equilibrium Trade Models**

The trade models differ from the tax models both in their multi-country orientation and also in the wider differences among the models involved. The main features along with data sources are outlined in Tables 9 and 10. For ease of exposition, the models are partitioned into the two groups of multi-country and single country models.

In the Carrin, Gunning, and Waelbroeck (1980) ten-region model, which follows from earlier work by Ginsburgh and Waelbroeck (1975), the main focus is an assessment of the impacts of higher world oil prices on less developed countries. Demands are based on an extended linear expenditure system for savings and commodity composites, which themselves are aggregations over comparable domestic and imported products. Two consumer groups are considered for each of the nine regions of developing countries plus a rudimentary rest of the world. Cobb-Douglas value added functions along with fixed coefficient intermediate production appear in the model for each of the five sectors in each region. The model is based on 1978 data underlying the World Bank Development Report, also used by Gupta et al. (1979). Special features of this model include the treatment of migration from rural to urban areas in each region, and its ability to solve both fixed and flex price versions of the model. The former involves a calculation of excess demands and supplies generated by exogenous changes in world oil prices.

The Deardorff-Stern model (1981) was originally built to evaluate the effects of GATT trade policy reductions on major developed countries, although the model has subsequently been used to study the effects of exogenous exchange rate shifts and changes in trade on unemployment. The model uses data for 1976 with extraneous elasticities incorporated. They consider 18 major developed
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<th>Model</th>
<th>Country(ies)</th>
<th>Demand Side</th>
<th>Production Side</th>
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<td><strong>Multi-Country Models</strong></td>
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<tr>
<td>Carrin, Gunning, Waelbroeck (1980)</td>
<td>9 geographic groups of developing countries and the rest of the world</td>
<td>Demand for CES import-export composite goods modeled as Extended Linear Expenditure System (ELES) for each consumer group</td>
<td>Cobb-Douglas value-added functions plus fixed coefficient intermediate use of composite goods</td>
</tr>
<tr>
<td>Deardorff, Stern (1979)</td>
<td>18 major industrialized countries, 16 major developing countries, and the rest of the world</td>
<td>Cobb-Douglas utility functions; CES between home and imported goods in the same industry</td>
<td>CES value-added functions; fixed coefficient intermediate use of CES composite of home and imported goods</td>
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<tr>
<td>Manne (1980)</td>
<td>3 regions: industrialized countries, oil-exporting developing countries, oil importing developing countries</td>
<td>Demand for energy and non-energy imports derived from CES production functions</td>
<td>Nested CES</td>
</tr>
<tr>
<td>Miller, Spencer (1977)</td>
<td>4 &quot;countries&quot;: U.K., (6 member) EEC, Australia and New Zealand, rest of the world</td>
<td>Derived from two-stage CES utility functions</td>
<td>Cobb-Douglas</td>
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<tr>
<td>Model</td>
<td>Description</td>
<td>Commodity Characteristics</td>
<td>Country/Region</td>
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<tr>
<td>Whalley (1980)</td>
<td>EEC, U.S., Japan, rest of the world</td>
<td>Derived from nested CES utility functions</td>
<td>41 consuming groups comprising households, government, investment (stratified by income in U.S. and Japan and by region in EEC)</td>
</tr>
<tr>
<td>Whalley (1982)</td>
<td>7 trade blocs; U.S., EEC, Japan, Other Developed Countries, OPEC, newly industrialized countries, Less Developed Countries</td>
<td>Derived from nested CES-LES utility functions</td>
<td>7 trade blocs</td>
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<tr>
<td>Single Country Models</td>
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<tr>
<td>Broadway, Treddenick, (1978)</td>
<td>Canada</td>
<td>Domestic final demand for each CES composite good is unit price and income elastic; import supply and export demand own price-dependent only (constant elasticity)</td>
<td>Domestic demand for final goods, world demand for domestic exports</td>
</tr>
<tr>
<td>Dervis, deMelo, Robinson (1982)</td>
<td>Turkey</td>
<td>Constant expenditure proportions for import-export composite goods</td>
<td>One aggregate household</td>
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<tr>
<td>Model</td>
<td>Country(ies)</td>
<td>Demand Side</td>
<td>Production Side</td>
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<td>Dixon, Parmenter, Sutton, Vincent (1981)</td>
<td>Australia</td>
<td>Derived from Klein-Rubin utility functions with CES aggregation of comparable imported and domestic goods</td>
<td>Effectively 1 household</td>
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<td>Model</td>
<td>Year</td>
<td>Base Year Data</td>
<td>Extraneous Use of Elasticities</td>
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<tr>
<td>Multi-Country Models</td>
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<tr>
<td>Deardorff, Stern (1979)</td>
<td>1976</td>
<td>Data derived from various sources</td>
<td>Import demand, production function elasticities, elasticity of substitution between home and imported goods (literature search)</td>
</tr>
<tr>
<td>Miller, Spencer (1977)</td>
<td>1960 (Production) 1968 (Demand)</td>
<td>Based on data from various sources</td>
<td>Production elasticities (literature search) Elasticities of substitution between domestic and imported goods and between goods from each production sector</td>
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<tr>
<td>Source</td>
<td>Year</td>
<td>Type of Data</td>
<td>Description</td>
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<tr>
<td>Whalley (1980)</td>
<td>1973</td>
<td>Constructed international benchmark data set</td>
<td>Elasticities of substitution between home and imported goods and within product categories (best guess) elasticity of substitution in production (literature search)</td>
</tr>
<tr>
<td>Whalley (1982)</td>
<td>1977</td>
<td>Constructed benchmark data set</td>
<td>Production elasticities of substitution, demand elasticities of substitution within good categories, between categories, between imported and domestic goods (literature search (central case) and various specifications)</td>
</tr>
<tr>
<td>Single Country Models</td>
<td></td>
<td>1966 Input-output table</td>
<td>Production elasticities of substitution (literature search) world elasticities of supply of imports and demand for exports, domestic elasticities of substitution between domestic and imported goods (various specifications)</td>
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<td>Model</td>
<td>Year</td>
<td>Base Year Date</td>
<td>Extraneous Use of Elasticities</td>
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and 16 major developing countries. Cobb-Douglas utility functions are used for demands for composite products (aggregations over comparable domestic and imported products) with constant elasticity of substitution functions determining the disaggregation between home and imported products. The general equilibrium features of the model only involve goods market equilibrium conditions and as a result only incomplete equilibria are analyzed. Labor markets do not clear and trade balance does not hold.

Manne (1980) presents a global model different from either of the above and oriented towards analysis of the effects of different world oil pricing scenarios on world trade and income distribution. Three regions are identified with data for each being based on 1978 World Bank sources, and oil industry supply forecasts. Derived factor demand functions are used, based on nested CES production functions for a single non-energy commodity in each region. In equilibrium, all markets clear and trade balance conditions hold. Manne highlights the importance of the extraneous elasticities of substitution between domestic and imported products for his results. He also stresses that the model is narrowly focused and not designed to deal with trade issues outside the energy sector. This model would be inappropriate, for instance, for evaluation of trade policy issues such as tariff changes.

In contrast, Miller and Spencer (1977) present a four region model incorporating the U.K., the six-member EEC, Australia and New Zealand, and a residual rest of the world to analyze the impacts on the U.K. of entry into the EEC. Most of the work with this model was performed prior to British entry in 1973; the issues at stake involved the realignment of trade policies in the U.K. as entry occurred and the budget transfers by the U.K. to Brussels. Trade barriers were lowered between the U.K. and the EEC, while barriers between
the U.K. and other Commonwealth countries were raised through the common external EEC tariff. Demand functions in each region are based on two stage CES utility functions with Cobb-Douglas production functions for each product in each region. No intermediate production is incorporated. Two commodities, agriculture and non-agriculture, are identified for each region with data being based on a number of different literature sources. Production and demand elasticities are also based on a literature search.

Whalley (forthcoming) reports on a four-region general equilibrium model of world trade closely related to an earlier model used by Brown and Whalley (1980) and similar in structure to that used by Miller and Spencer. The modeling effort is focused on an evaluation of the effects of trade policy negotiations under the GATT, but unlike the Deardorff and Stern work, the model incorporates only the major participants in the GATT: the EEC, U.S., Japan and a residual rest of the world. Full global equilibria are considered in which all goods and factor markets clear and external sector balance conditions hold for each region. Factors are immobile between regions. The demand side uses nested CES utility functions. CES value added functions are used on the production side. Substitution also occurs between intermediate products but is limited to that between comparable domestic and imported goods. A worldwide benchmark equilibrium data set constructed for 1973 is used, with extraneous elasticities specified. In Brown and Whalley, a simplified version of the model involving five products is used, in contrast to the 33 product variant in Whalley (forthcoming). In Whalley (1982), this same approach has been used for a seven region model involving disaggregation of the rest of the world into Other Developed Countries, OPEC, Newly Industrialized Countries, and Less Developed Countries. He analyzes North-South trade issues, although
using only six rather than 33 commodities for each of the seven regions.

Among single country models, Boadway and Treddenick (1978) have analyzed the effects of changes in trade policies in Canada. Cobb-Douglas demands are assumed across CES composite products, with extraneously specified import and export elasticities. Three alternative specifications are used on the production side: fixed intermediate coefficients with Cobb-Douglas functions for capital and labor; fixed intermediate coefficients with CES functions for capital and labor; and variable coefficients using Cobb-Douglas functions. Two alternative levels of aggregation, involving 16 and 56 product types, are used. The model is calibrated to published Input-Output tables and other data. Since Canada is not modeled as a small open price taking economy, terms of trade effects occur and are important in their results. Boadway and Treddenick also discuss alternative ways of specifying external sector behavior which Canada faces. In their external sector closure rule, Canada faces export demand and import supply functions of constant price elasticity.

Dervis, de Melo, and Robinson (1982) in their work on trade policy in developing countries use a single country general equilibrium formulation for Turkey due to Dervis and Robinson (1978). The model has been used to analyze foreign exchange rationing and tariff changes. Cobb-Douglas demands for composite goods with CES aggregation functions over components of composites are used. One household is assumed. On the production side, two level CES value added functions, along with fixed coefficient intermediate production appear. An external sector formulation similar to Boadway and Treddenick is used with specified foreign export demand and import supply functions. External sector balance conditions hold in equilibrium.
Finally, Dixon, Parmenter, Sutton and Vincent (1981), in their work on the IMPACT project, have constructed an elaborate model of Australia designed for multi-purpose analysis beyond trade policy questions, although changes in Australia's trade policies are included in the analyses performed with the model. On the demand side, (effectively) one household is identified. This household has demands for product types derived from LES utility functions, with CES aggregation over comparable import and domestic products. The model uses a complex production structure that involves a combination of four functions: CES, CRESH, Leontief input functions, and CRETH output functions. One hundred-fourteen products are considered, giving large amounts of detail among the items in the model. Data are largely centered on 1968 and 1969 and involves an input-output data base derived from government and agricultural statistics. An extensive elasticities file is constructed for use along with the model, based both on literature search and separate estimation. It should perhaps be emphasized that this is one of the most ambitious of the applied modeling efforts generating a large amount of new data, estimation, and policy analysis, and only those portions of the work relevant to trade policy evaluation are referred to here.
7. **A Summary of Policy Findings from the Trade Models**

As with the tax models, the applied trade models are also being applied to a range of policy issues and, consequently, generating fresh insights into various policies. A less uniform view emerges from these results than with the tax models. Some results appear to suggest the possible significance of terms of trade and third country effects, usually neglected in partial equilibrium literature. Another result is the small size of welfare and other impacts of trade policy changes, although this may reflect the use of constant returns to scale in production. In a model of U.S.-Canadian trade, not included here for space reasons, Harris (1982) has produced considerably larger welfare effects from trade policy changes in a model incorporating increasing returns to scale.

The major policy findings from the trade models listed earlier are summarized in Table 11. Carrin, Gunning, and Waelbroeck (1980) have used their model to simulate the effects of high oil prices on groups of developing countries. World oil prices are exogenous to the model; in one variant of the model, prices of domestic energy resources (substitutes for oil) are flexible and in the other, they are fixed. In their base model run of their flex price variant, world oil prices are assumed constant in real terms from 1980 to 1990; the counterfactual experiment is to allow for a four percent real increase each year over the period. The effects of the four percent annual increase in real oil prices on income and investment are generally surprisingly small in the flexible price case. An average reduction in real GDP for all developing country regions of less than one percent per year occurs. Effects are weaker for Africa than for Latin America and the Caribbean (excluding Mexico and Venezuela). Larger losses to energy importing regions occur in the fixed domestic resource price variant.
<table>
<thead>
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<th>Model</th>
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<th>Policy Conclusions</th>
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<td>Multi-Country Models</td>
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<tr>
<td>Carrin, Cunning,</td>
<td>Simulation experiments on the impact of higher oil prices under both fixed and flexible domestic</td>
<td>Simulation experiments, no data required</td>
<td>Effects of a 4% annual increase in real oil prices on income and investment</td>
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<tr>
<td>Waibelroek (1980)</td>
<td>resource prices</td>
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<td>generally small in the fixed-price case; substantial losses to importing regions</td>
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<td></td>
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<td>in the fixed-price case.</td>
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<tr>
<td>Deardorff, Stern</td>
<td>Tokyo Round changes in tariff and non-tariff barriers (agricultural quota concessions, government</td>
<td>Post-Kennedy Round base rate tariffs, Tokyo Round offer rate tariffs, quantification of non-tariff barriers</td>
<td>Economic welfare will increase in all industrialized countries except</td>
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<td>(1979)</td>
<td>procurement liberalization)</td>
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<td>Australia, New Zealand and the Netherlands. Welfare will decrease in most of</td>
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<td>the developing countries.</td>
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<td>Manne (1980)</td>
<td>Alternative scenarios incorporating: 2 assumptions on energy supply; 2 assumptions on energy</td>
<td>Alternative energy supply and demand scenarios</td>
<td>Increases in world oil prices have very little effect on GDP growth in industrial-</td>
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<td></td>
<td>demand; 2 assumptions on capital flows</td>
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<td>ized countries, but could have a major (negative) impact on the terms of trade of</td>
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<td>oil importing developing countries, inducing GDP growth substantially.</td>
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<td>(1977)</td>
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<td>increases U.K. income by only 1/6 of 1%; with transfer to EEC of 1.5 percent of</td>
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<td>income, U.K. net loss is 1.8% of national income.</td>
</tr>
<tr>
<td>Author/Year</td>
<td>Type of Model</td>
<td>Methodology/Model Description</td>
<td>Welfare Gain/Outcomes</td>
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<tr>
<td>Whalley (1980)</td>
<td>Changes in tariffs, non-tariff barriers and taxes in the EEC, U.S. and Japan</td>
<td>Tokyo Round tariff formulae and ad valorem equivalent of non-tariff barriers; domestic taxes</td>
<td>World welfare gain from tariff cuts no greater than .1% of world GNP; EEC and Japan gain proportionally more than U.S. or rest of the world, but this could be offset by proposed changes in non-tariff barriers.</td>
</tr>
<tr>
<td>Whalley (1982)</td>
<td>Abolition of tariff and non-tariff barriers in the North, the South, and in both regions simultaneously</td>
<td>All tariff and non-tariff barriers (represented in ad valorem form) in each of the seven blocs</td>
<td>Abolition of tariff and non-tariff barriers in: 1) the North results in annual welfare gains of $21 billion, the majority of which accrues to the LDCs and NICs; 2) the South leads to annual gains of $17 billion, but with a $65 billion gain to the North and a $48 billion loss to the South; 3) both North and South gains world welfare gains of $30 billion, with gains accruing to the North and losses to the South.</td>
</tr>
<tr>
<td>Single Country Models</td>
<td>Elimination of tariffs, elimination of tariffs along with taxes (commodity and capital income)</td>
<td>Canadian tariff and taxation rates</td>
<td>Similar results for all cases studied (excepting variations in export demand elasticity): when tariffs are removed, the welfare index falls by 1.16% (when export demand elasticity = 1) and rises by .06% when export demand elasticity = 25; when taxes and tariffs are removed, the welfare index falls by 2.63% with unit export demand elasticity and rises by .27% with export demand elasticity of 25.</td>
</tr>
<tr>
<td>Model</td>
<td>Policy Interventions Incorporated</td>
<td>Policy Data Used</td>
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<tr>
<td>Dervis, de Melo, Robinson (1982)</td>
<td>Setting a 50% tariff on imports; giving a 50% subsidy to exports; examining 1977 Turkish foreign exchange crisis</td>
<td>Simulation experiments and actual events; no data required</td>
<td>Imposing a 50% tariff in one sector at a time produces small short-run allocational effects with no sector experiencing more than a 5% change in output. A 50% export subsidy has greater effect on domestic output than does the 50% tariff: the home country is more sensitive to export-side than import-side disturbances. Causes of the 1977 foreign exchange crisis in Turkey were principally differential domestic inflation and increases in oil prices.</td>
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<tr>
<td>Dixon, Farmenter, Sutton, Vincent (1981)</td>
<td>A 25% across-the-board increase in Australian import tariffs</td>
<td>Simulation experiment; no data required</td>
<td>25% increase in all protection rates leads to a .21% fall in total employment, an increased deficit in the balance of trade, and increases in consumer and capital goods prices.</td>
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</table>
Deardorff and Stern (1981) analyze the effect of Tokyo Round tariff and quantifiable non-tariff barrier changes under a flexible exchange rates variant of their model. Their results indicate that the Tokyo Round trade policy changes increase welfare in all major industrialized countries except Australia, the Netherlands, and New Zealand. The total annual welfare gain is estimated at between $1.1 and $4.3 billion, less than one-tenth of one percent of the combined gross domestic product. Most developing countries experience a welfare loss. World trade increases by over $13 billion over 1976 levels, around a 1.8 percent increase, but all of this is accounted for by increased exports of major industrialized countries (exports of developing countries fall slightly). Employment effects are small. Total employment increases in seven of the 18 industrialized countries, including the U.S., but the increase in the U.S. is only 11,000 workers (less than one hundredth of one percent of the work force).

Manne (1980) considers a number of alternative scenarios incorporating assumptions on energy supply, energy demand and trade elasticities. Despite allowing for a wide range of possible energy prices, his main finding is that GDP growth rates of industrialized countries do not vary widely. Imported energy remains a small fraction of GDP, and growth rates largely reflect productivity gains. Even a 100 percent difference in energy prices over the period 1978 to 1990 only has at most a 0.5 percent annual impact on growth rates. Manne does find, however, that the lower the energy supply growth rate, the more OPEC nations gain at the expense of all importing countries. Among oil importers, the LDC loss is relatively more than that of the OECD nations, suggesting that LDCs are the major sufferers in proportional terms from high oil prices.
Miller and Spencer (1977) evaluate the effects of Britain joining the Common Market in 1973. Their results indicate that the major effects with British entry are the budgetary transfers from the U.K. to the EEC, rather than the trade creation-trade diversion effects identified in the customs union literature. Gains from trade creation are estimated to be small. Even though U.K. entry into the EEC leads to a 50 percent increase in EEC imports of manufactures from the U.K., this gives rise to a gain equivalent to only one-sixth of one percent of income for the U.K. Against the trade creation gains (the main benefit to the U.K. of entry) are to be set trade diversion losses, and losses from transfer of tariff revenues to the EEC. The overall loss to the U.K. on entry to the EEC with an assumed "high" transfer of 1.5 percent of national income is estimated at 1.8 percent of national income.

Whalley (forthcoming), and Brown and Whalley (1980) use their four region trade model to evaluate the effects of Tokyo Round tariff cutting formulae and consider similar changes in ad valorem equivalents for tariff and non-tariff barriers to those analyzed by Deardorff and Stern. The results indicate that worldwide welfare gains from tariff cuts are generally small. The EEC and Japan gain proportionally more than the U.S. and the rest of the world, although this may be partially offset in the case of the Tokyo Round trade agreement by changes in non-tariff barriers. In analyzing the various multilateral tariff cutting formulae proposed at various stages in the Tokyo Round, Brown and Whalley conclude that the EEC is in most cases the main gainer while the rest of the world is the main loser. Interestingly, the U.S. gains more from the Japanese proposal than its own, and Japan and the EEC gain least from their own proposals. This suggests a view of multi-
lateral trade policy negotiations as one in which countries alternatively promote their own proposals ignorant of the fact that other country proposals for multilateral tariff cuts are closer to their own interests. Brown and Whalley comment on and investigate this feature of their results.

Whalley (1982) has used a similar approach to investigate the impact of trade protection in the North on the North-South terms of trade in a seven region version of the same model. An interesting finding is the size of the Northern gains from the terms of trade improvement from protection against LDC imports. The annual loss to the South is estimated at around $20 billion per year, approximately equaling the annual aid flow from North to South. A further finding is that despite its relatively smaller size in GNP terms, the South loses from unilateral trade liberalization. Given the current popularity of unilateral trade liberalization as a policy option for LDCs, this result suggests the importance of differentiating between unilateral liberalization by one country where no other LDC's liberalize, and blocwide liberalization. Put another way, strong growth experience through trade liberalization in isolated cases may well have been made possible by the absence of liberalization elsewhere in the developing world.

Among the single country models, Broadway and Treddennick (1978) consider the elimination of tariffs in Canada along with various kinds of tax distortions. They obtain similar results in most of their cases. When tariffs are removed, the welfare index in Canada falls approximately one percent because of the worsening in national terms of trade. This indicates existing Canadian tariffs are below optimal tariffs, reflecting the trade elasticities
used in the model. Larger welfare losses occur when taxes and tariffs are simultaneously removed. Broadway and Treddenick also stress the influences of protection on resource allocation. Tariffs tend to encourage production of non-traded goods and discourage production of traded goods. Both primary and manufacturing industry are reduced in size and tertiary service related industries expand, contrary to what one might conclude if only the tariff schedule were used as a way of evaluating protection for any industry.

Dixon, Parmenter, Sutton and Vincent (1982) consider a wide range of policy applications. In the trade field, they examine the effect of a 25 percent across-the-board increase in Australian tariffs. They show that the increase in protection leads to a reduction in total employment, an increase in the trade deficit, and an increase in consumer and capital goods prices. Regional effects are uneven; manufacturing areas gain while three largely non-manufacturing states in Australia bear the major portion of the costs of protection.

Dervis, de Melo, and Robinson (1982) examine a range of policy alternatives using the Dervis-Robinson data set for Turkey. They show that the percentage change in the corresponding domestic price for a given change in a tariff is lower the larger is the ratio of exports to domestic use, and higher the foreign elasticity of demand. They find that imposing a 50 percent tariff on each sector in their model, taken a sector at a time, results in only small short-run allocational effects, with no sector experiencing more than a five percent change in output. A 50 percent export subsidy has greater effects on domestic output than a 50 percent tariff.
These applied trade models can thus claim to have contributed to policy debate through their model findings. While in some isolated cases it can perhaps be argued that the conclusions are obvious implications of model assumptions with the quantification adding little to the final conclusion, in most cases this is not so. The quantification allows for the relative strength of opposing effects to be evaluated, yielding conclusions which cannot be obtained merely from a priori reasoning.

In the models, the relatively small size of the effects involved, along with the potential role of terms of trade effects, come through in a number of model calculations. While the smallness of estimates may be misleading due to ignored scale economy effects (see especially Wonnacott and Wonnacott's (1967) and Harris's (1982) work on U.S.-Canadian trade mentioned earlier), the terms of trade effects have important implications. The importance of these effects suggests large countries have more to gain from protection than small countries since their optimal tariffs are higher. This seems contrary to current world trade arrangements in which small countries usually have more protection than large countries, and multilateral trade negotiations are initiated and effectively conducted by the large countries which have least to gain (or most to lose) from reductions in protection. Broad ranging conclusions such as these which suggest a fresh perspective on policy issues in these areas are clearly a major contribution of these models, and may well prove to be more enduring than specific features of results.
8. Issues and Difficulties in Applied General Equilibrium Models

While the applied general equilibrium models described in this paper can reasonably claim to have advanced from the simple numerical examples of ten to fifteen years ago to a stage where quasi-realistic larger dimensional models are yielding new insights into policy debates, it should be obvious to readers that there are difficulties which arise with the approach.

Robustness of Results

A key issue with the models described earlier is how robust are the results to alternative parameter values. Because of the use of the calibration procedure to select parameter values, no meaningful statistical test of any model specification is possible, and users of model results are often left with little sense of whether any given result will disappear, or even change sign, if there is a relatively small change in a parameter value.

There seems widespread agreement among the modelers that once policy parameters are specified (which themselves are sometimes contentious), the elasticity values are the single most important set of parameter values in determining results. Because of the reliance on empirical estimates, one response to the robustness issue is to say that model results are only as robust as elasticity estimates appearing in the literature. The problem, however, is more severe than this for two reasons. First, elasticity values combine in both offsetting and compounding ways in these models and the robustness of any single elasticity value in the literature may mean little when used in conjunction with other elasticities in a large model. Second, the robustness issue cannot be discussed independently of the particular features of results one has in mind. Some features of results may be very
sensitive, while others hardly change. Changing trade elasticities, for instance, may make a big difference to projected changes in trade flows from a policy change while leaving welfare impacts largely unaffected.

The response thus far in the applied models has been to take alternative elasticity values to those used in a "central case" specification, displacing the key values by what seems to the particular modeler as "large" changes. Most modelers appear to claim a reasonable degree of robustness for their results, while admitting the limited sensitivity tests performed. Usually elasticities are only varied singly and not in combination, for the understandable reason that the volume of results generated is difficult to digest. Perhaps the best hope for further insight into this issue are the systematic sensitivity analyses being carried out by Kimbell and Harrison (1983b, c), who have computed over a million solutions to their model in exploring robustness. The main difficulty in this work appears to be synthesizing the results into a form where a clear judgment on robustness can be made. Some things are little affected, others more so. Also, how these results translate to other models is unclear.

A related difficulty also stemming from calibration is the absence of any notion of statistical tests of the numerical specification used in the applied models. Audiences digesting results are largely left to decide as to whether they trust the integrity and judgment of the modelers, but receive no clear guidance as to how "reasonable" any particular specification is. As already mentioned, the large dimensionalities in the applied models, along with the problems of data generation with consistent units, appear to rule out systemwide econometric estimation for large scale models, although a partitioning approach has been followed by some modelers. Estimates for a
small scale two-sector model using time series methods by Mansur indicate that welfare costs of distortions using an econometrically estimated general equilibrium system are not too different from the welfare cost estimates obtained from calibration for some functional forms, but for others, significant differences are involved.

Model Preselection

A further difficulty with the applied models is the question of model pre-selection, or: the necessity to decide on a particular model structure before the policy analysis proceeds. A good way of illustrating this problem is to consider the classic Harberger analysis of the impacts of the corporate tax. Using the standard assumption in static models of a closed economy with a fixed amount of capital, Harberger concludes from his numerical analysis that capital bears the burden of the corporate tax. Clearly, if the economy in question is viewed as a participant in an international capital market such that it is a taker of rental rates on world capital markets, the policy conclusions would obviously change. In this case, it is impossible for capital to bear the burden of a capital tax, because the effect of a tax would simply be to cause capital to leave the country until the net-of-tax return is equal to that prevailing on world markets. Model pre-selection can thus powerfully affect the conclusions which are reached from numerical analysis.

The fundamental difficulty is that there are many alternative models in the literature, each applicable to the policy question at hand, and each yielding different policy implications. Applied general equilibrium analysis does not provide a way of discriminating between alternative models since no form of hypothesis testing is involved. Thus, a broadly based single rate
income tax which is non-distorting in a static fixed factor model becomes
distorting in a dynamic model due to the double taxation of savings. The
effects of a tariff are different in models with or without international
factor mobility, with or without a downward rigid real wage. The "noise"
in economic theories is not removed merely by putting numerical values on
parameters in specified functional forms, and some degree of summary judgment
by modelers in selecting the theoretical structure to be used is inevitable.
Our preferred position on this issue is to emphasize that applied models of
form described here cannot settle disputes between rival models. Computations
are only to be used and interpreted conditional on the belief of users in the
virtues of the underlying model.

A related difficulty which all modelers have come up against is that
in choosing their particular model structure they have found that there is
no single all purpose general equilibrium model that can be used. Unless
one truly believes that the Lord, in his infinite wisdom, used the seventh
day to create a complete set of Arrow-Debreu markets which have predetermined
all history, some form of "closure" with respect to time and space is
inevitably involved in all applied general equilibrium exercises. In reality,
most models are not so much general equilibrium models as "less partial"
equilibrium models. In order to work on a particular policy issue in a
particular country, modelers have to find some simple way of closing the model
with respect to time (savings and investment), space (foreign trade and factor
mobility), and other issues such as government expenditures, taxes, and
regulatory activity of government. The need to close models in this way
differentiates models, makes comparison difficult, and presents challenges to
theorists to work out the implications of some of the closure rules which
have not always been fully thought through.
Elasticities

Not only are elasticity values key to sensitivity analysis, the choice of values used in central case specifications also poses problems. The most common procedure is a literature search, but it is widely agreed by modelers that the literature on elasticities is both incomplete and in places contradictory. Great difficulty is encountered in selecting widely agreed elasticity values on the basis of literature alone. A cynic viewing these models could argue that the arbitrariness in the choice of elasticities produces a corresponding arbitrariness in the determination of final results. Under this view, current modeling efforts may simply be attempting the unattainable (until literature estimates of elasticities improve).

Elasticities seem surprisingly difficult to obtain from literature for the demand side of models. The last complete survey of price elasticity estimates by commodity seems to be that by Hirsch in 1951. While recent applied econometric work on demand systems is clearly helpful, much of it seems to be primarily concerned with estimation difficulties rather than with providing parameter estimates usable for modelers.

The elasticity problem is also severe on the production side where there is even significant doubt as to what the value of the elasticity of substitution between capital and labor would be in an aggregate production function, let alone individual industries. As emphasized by Berndt (1976), differences between time series and cross section estimates of a factor of two are involved for the aggregate parameter. Also, little evidence is available on substitution elasticities in intermediate production.

In the trade area, many people have commented on the seemingly low values of import and export demand elasticities. While specification bias
has been raised as an issue, no set of alternative estimates have emerged
to those both widely used and simultaneously criticized.

A related point which non-modelers might keep in mind is that there
appear to be surprisingly few compendia of elasticity estimates to which
modelers can refer to select their elasticity values. For production functions
Caddy's (1976) survey is widely used, and for trade elasticities the compendium
by Stern, Francis, and Schumacher (1976) is often cited. Perhaps one of the
notable outputs from the applied models is to generate a "demand for
elasticities" which as a profession economists have seemed to somewhat neglect.
In modeling, a lesson appears to be that unglamorous activities can still be
very important.

"Theoretical" Pedigree

A further issue with the applied models derives from the attempt to
develop models which are consistent with the theoretical general equilibrium
literature developed in the 1950s and 1960s and thus allow for welfare
statements on policy issues. Because of the difficulties of accommodating a
wide range of empirical phenomenon in model building, there is often a
tendency to depart from the essential structure and graft on "ad hoc" portions
of the model not rooted in traditional theory. The problem here is that models
which make major departures from known theoretical structures unfortunately
become uninterpretable. For example, if the choice of the absolute price level
affects resource allocation and distribution, as has been true in some applied
models in the past, the applied models may yield results that are difficult
to relate to the theoretical literature. Some of the closure rules used in
the models have their own elements of "ad hocery" which depart from theoretically
pure models and influence results. Theoretical pedigree needs to be maintained in future development of these models to allow for meaningful interpretation of results.

To our taste, the conflict between modelers' desires to build realistic models which seek to capture real features of the policy issue at hand, and the difficulties of running ahead of developed economic theory is something which seems to be becoming increasingly apparent in some of the more recent models.

**Issue Specific versus General Purpose Models**

Another issue being raised by more recent modeling efforts in this area is that of the design issue of large scale multi-purpose versus smaller scale issue specific models. The models developed in the early 1970s have, over time, become larger in scale and now provide a multi-purpose modeling capability, but in analyzing any one issue a significant portion of the modeling is often unimportant and can complicate the process of model adaptation to the policy question. With smaller scale models, it is clearly much easier to identify key parameters which affect results, to work with those parameter values and subsequently trace through the main effects of the policy change being evaluated; but equally, it seems counterproductive to repeatedly reformulate models. Also, excessive use of small scale models naturally raise the issue of whether or not the crude level of aggregation in the models results in systematic biases.

Perhaps the main point to be borne in mind relates not so much to the use of existing models as to the strategy to be followed in developing new models. Existing models represent sunk costs and even if they are overly
elaborate for a particular issue at hand, it still seems worthwhile to see what a particular model has to say, since the marginal costs of model use are so small relative to model development. For new models, however, experience gained so far does suggest that it is well worthwhile to consider carefully exactly what the model will be used for and how much detail makes sense before embarking on model construction. Often, half an hour's thought before model construction can save many months of work on model construction, which with hindsight, is found either not to change fundamentally results or be a largely unnecessary embellishment.

Issues with the Tax Models

In the tax modeling area, besides the more general modeling questions raised above, several more specific issues are now coming to the fore. An important question which has recently been raised by Fullerton and Gordon (1981) concerns the calculation of model equivalent tax rates before the general equilibrium calculations are made. Most models continue to use the Harberger approach of calculating average rates by dividing tax payments by the calculated tax base in the benchmark equilibrium data set. This procedure assures an equality between average and marginal tax rates. Fullerton and Gordon adopt a different approach and calculate marginal tax rates directly emphasizing, in the case of capital taxes, the Hall-Jorgenson (1967) cost of capital approach. The difference in calculated tax rates from the traditional approach is marked, highlighting the importance of this issue.

Further issues currently receiving attention are the relative neglect of the expenditure side of government activity (see Piggott and Whalley (1982)), the limited confidence which can be attached to distributional results in the
absence of explicit life cycle modeling, and the appropriate model treatment of individual taxes in light of literature disagreements as to what the appropriate treatment of each tax is. With the corporate tax Stiglitz (1973), for instance, has suggested that the corporate tax is a lump sum tax with no marginal distorting effects. This contrasts sharply with the Harberger treatment followed in most applied models. Social security taxes are treated in some of the tax models as a partial payroll tax with different rates by industry, neglecting the intertemporal effects on savings and retirement decisions stressed by Feldstein (1974) and others.

**Issues with the Trade Models**

With the trade models, issues involve the limited model treatment of, and lack of data and information on, non-tariff barriers. The key role played by trade elasticities (and disagreements on their values), and the lack of attention given thus far to factor mobility, investment, and dynamic issues is a concern.

Many policy people in the trade area seem to feel that currently non-tariff barriers are increasing, and are substantially more important than visible barriers such as tariffs. However, the lack of documentation as to how many of these barriers actually operate, combined with a sparcity of data, makes it difficult for the applied models to analyze. Also, how these barriers operate at the margin is often difficult to determine and model. Voluntary export restraints, for instance, are quite different in their protective effects than, say, government valuation procedures for tariffs.

On the elasticity issue, values currently used are often in the neighborhood of -1 to -2 for both import and export price elasticities. There
is little doubt that it is these relatively low values which lead to significant terms of trade effects in the trade models. The strength of these effects has led some researchers to suggest that these values may be too low. An especially important source of these estimates is the compendium by Stern, Francis, and Schumacher (1976). Lower export price elasticities for smaller countries such as Canada than for larger countries such as the United States in their literature survey central tendency values are evidence of the discomfort some modelers have in the literature values currently being used. The specification bias problem in this area, first raised by Orcutt (1950), also still remains as an important issue.

As regards factor mobility, investment, and dynamic modeling, little work has been done incorporating extensions in these directions into the trade models. With the increasing role played by international capital markets, and the significance attached to technology transfer in policy circles, these are likely to be important areas for future modeling efforts.

In spite of all these problems, however, both the tax and trade models are undoubtedly making significant contributions to policy debates by providing more refined calculations of efficiency costs and distributional impacts of policy changes than previously existed. The point to be emphasized is not that these models are either right or wrong, but that policy decisions have to be made and that these models are capable of providing fresh insights on policy options not available from any other source. Sometimes results will be dismissed as unconvincing, on other occasions policy makers may stop and think, and it is more than likely that occasions will arise when a policy maker's prior position will be changed. We would never advocate slavish mechanistic use of any of these models in policy making, but used in the right way and at the right time, their potential contribution seems to be very large.
9. **Directions for Future Research**

Because of the clear demonstration of the capability of existing applied general equilibrium models to provide fresh insights into policy issues, we believe that this field will continue to develop in the years ahead. The directions that seem fruitful for future research partially reflect our comments on difficulties and problems in the preceding section and partially our experience thus far with models.

Work in the last few years seems to be moving more strongly in an issues-oriented direction. Most major policy issues, of course, have elements which lend themselves to a general equilibrium treatment and applied equilibrium analysis should be able to focus on these. Examples of policy issues which could be taken up through these techniques are: (i) the long run equilibrium effects of social security financing options on the wage rentals ratio (since demographic changes not only affect the ratio of beneficiaries to contributors, but also the capital/work force ratio), (ii) equilibrium effects of reform of welfare systems, moving eventually to variants of a negative income tax, (iii) the effects of alternative proposals for a "flat tax," (iv) the effects of trade policy changes of the form being debated currently in the U.S., such as "reciprocity" and "domestic content" rules, (v) impacts of modifying existing immigration restrictions on labor mobility, (vi) the possibilities for and effects of geographically discriminatory trade arrangements which go against the multilateralism doctrines of the GATT (such as the North American Free Trade Area).

Beyond policy analysis, however, a range of questions remain concerning the reliability and quality of these models. While computing equilibria is no longer the technical difficulty it seemed fifteen years ago, specifying the
model for which the equilibrium is to be computed remains a challenge. Better data, and especially more and better elasticity estimates seem to be crucial to advancement of the field. One of us in the past has gone so far as to argue for the establishment of an "elasticity bank" in which elasticity estimates would be archived, evaluated by groups of experts with a quality rating produced, and an on-file compendium of these values maintained. While this may be overly ambitious, the general direction is one that is sorely needed.

Equally, the robustness and estimation issues are both worthy of considerably more attention. Years ago in the debate on central planning in the 1930s, Lionel Robbins referred to the difficulty of solving the millions of equations characterizing a Pareto optimal allocation. No one in this debate seems to have raised the prior question as to how do you know how to write down the equations even before you worry about solution? The robustness and estimation issues are precisely these questions; what is the most reasonable numerical specification of a general equilibrium model suitable for analyzing the policy issue under discussion, and how reliable are results from this model? The debate in the 1930s was in many ways the inspiration for the work in the 1960s on general equilibrium computation. The experiences of modelers in the 1970s may prove to be the impetus for a new genre of work on specification of general equilibrium systems.

Other questions arise from the experience of modelers with model design. One of the most common problems encountered by modelers is the necessity to be simultaneously a "jack of all trades." Modelers must know general equilibrium theory so that their models are theoretically pure; they must know how to solve their models; they need to be able to program (or at least communicate with
programmers); they must understand the policy issues on which they work; they have to know about data sources and all their associated problems; and, they have to be conversant with relevant literature, especially that on elasticities. Not surprisingly, modelers can at times feel a sense of inadequacy when faced with colleagues specializing in just one of these topics. This need to do several things well can also inhibit graduate students from doing thesis work in the area. Perhaps a future direction is for more teams of modelers, each with different skills, run on the lines of research teams in natural sciences. While a "Manhattan Project" for general equilibrium modeling may be going too far, a team built on complementary modeling, computing, and data skills would almost certainly be able to make outstanding contributions to the field.

A fruitful direction as yet unexplored is to develop further the implications of the applied work of the last decade for theoretical work. Joseph Schumpeter labeled Walrasian general equilibrium as the "Magna Carta" of economics, and others subsequently argued that general equilibrium analysis has no operational content. The theorists might want to make a judgment as to whether the experience with applied models supports or denies this claim and how they might redirect their work. The experience of modelers in finding they need closing rules, simplified treatment of various features and the like, seems to indicate a need for more specificity rather than generality in equilibrium modeling. What are the properties of these simplified treatments, and how do they affect results?

Finally, it is worth raising the issue of data organization and their use in applied equilibrium models. Since the work of Simon Kuznets and the early Keynesian macro models, our national accounting procedures have been
heavily oriented towards calculating macro aggregates rather than sub-aggregate microeconomic detail. As a result, full Walrasian accounts do not appear in the publications of statistical agencies; we cannot, for instance, open a statistical publication and identify separate demand and supply accounts. In constructing their benchmark equilibria to which they calibrate their models, this is implicitly what the new generation of general equilibrium modelers were doing. Perhaps the next ten years might see further progress in this direction, possibly through an expansion of the social accounting matrix approach developed by Richard Stone and others.
10. **Conclusion**

In this paper we have surveyed some recent applied general equilibrium modeling efforts in the areas of public finance and international trade. These modeling efforts are most easily understood as attempts to quantify the impacts of alternative tax and trade policies within the traditional general equilibrium framework. Use of the computer permits the quantitative analysis of large dimensional models. The basic structure of production and preference functions in these models is the same as in their theoretical counterparts. Qualitative analysis of an issue in a general equilibrium framework can often only identify potentially offsetting effects, and this new quantification offers a way to determine the size of the net effect. Qualitative analysis is also frequently unsatisfactory for a policy maker who wants quantitative orders of magnitude to tell him which policy changes are significant and worth pursuing. The applied general equilibrium models in these fields seek to help out in both of these directions.

A number of policy findings have thus far been generated by these models. In the tax models, a general theme seems to be that efficiency costs (deadweight losses) of taxes may be more severe than had previously been supposed. This is especially the case with marginal deadweight losses from taxes. A further finding suggests that tax systems may be significantly progressive in their distributional impact, rather than proportional as often supposed. In the trade models, the role of terms of trade effects and the difference between national and global interests is an important theme.

Like all such modeling efforts, policy statements generated by these models have to be treated with an appropriate degree of caution. However, our view is that these models have already contributed to policy debate, and
if used sensibly can make further important contributions. This is especially so with estimates of combined efficiency and distributional effects of policies, where prior to these models, no wholly satisfactory way of quantifying these effects existed.
Footnotes

1. Two points on the use of this criterion are that, firstly, as noted by Kay (1980), the sum of EVs is a more easily interpreted measure if repeated pairwise comparisons are being performed. This is because "old" incomes and prices are used and are typically the same in the sequence of pairwise comparisons. The second point is that aggregation difficulties may arise in using an arithmetic sum of EVs or CVs as a welfare criterion, even though this is widely used elsewhere, including cost benefit analysis. Some of these difficulties are raised in Broadway (1974).

2. An important further feature of this procedure is that once calibration is complete, it should be possible to reproduce the benchmark equilibrium data set as an equilibrium solution of the model. This is the replication check referred to in Figure 1, which serves as an important test of the accuracy of computer code. If the replication check fails, then a programming error has been discovered and the coding must be investigated further.

3. The acronyms CRESH and CRETH refer to constant ratios of elasticities of substitution that are homothetic, and constant ratios of elasticities of transformation that are homothetic.

4. A useful survey of the way in which large scale energy models have been used in the Energy Modeling Forum at Stanford (in ways which are not dissimilar to what we have in mind) is contained in "Modeling for Insights, Not Numbers" by Huntington, Weyant and Sweeney (1982).
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