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Do We Know What We Know? Investigating the Validity of Number Representation Indices

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by

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Abstract

Predicting mathematical performance in individuals is critical in the development of evidence-based interventions. Thus, it is important to use valid measures when measuring mental number representation. Using symbolic and non-symbolic magnitude comparison tasks is the current method for measuring an individual's mental representation of numbers. However, recent research questions the validity of the current indices of number representation (Inglis & Gilmore, 2014). This study examined the relations of the number representation indices in non-symbolic and symbolic number formats separately in adults and children. Participants for the current study include adults (*n* = 51) and senior kindergarten children (*n* = 159). The current study is an investigation of the number representation indices and their predictive relation with mathematical skill. Results showed three of five indices related across presentation formats for children, only overall response time related across formats for adults. Results suggest the related indices are more linked across presentation formats in children. Additionally, predictive validity of the indices is not shared across adults and children.

Keywords: non-symbolic magnitude comparison, symbolic magnitude comparison

Do We Know What We Know? Investigating the Validity of Number Representation Indices

Some people excel in learning math while others struggle with mathematical concepts. Research indicates a strong, positive association between math performance and success in education, career, and financial stability (Parsons & Bynner, 2006). Higher number competency in early childhood is tied to being able to solve complex calculations later (Jordan, Kaplan, Ramineni, & Locuniak, 2009). There are also other real-world disadvantages like having difficulties with budgeting household income (Jordan, Kaplan, Ramineni, & Locuniak, 2009; Parsons & Bynner, 2006). Thus, it is critical that research investigates why some individuals struggle with math and others do not. Being able to predict early mathematical difficulties in individuals is important to informing best practices for interventions. Our current understanding of how well individuals think of numbers centres around a system that mentally processes numbers automatically.

The Approximate Number System (ANS) is an individual's mental representation of numbers without relying on symbolic numbers and language. Evidence of this system is found in human adults, babies, and non-human animals (Dehaene, Dehaene-Lambertz, & Cohen, 1998). Research associates the ANS with how we think about symbolic numbers, such as Arabic numerals. This suggests that the ANS is activated whenever we use symbolic representation of numbers (Dehaene, 1998; Dehaene, Piazza, Pinel, & Cohen, 2003). However, more recent research found that the non-symbolic and symbolic number systems may not be related in the way previously believed (Inglis & Gilmore, 2011, Leibovich & Ansari, 2016). Because of this, the field is left with more questions than answers. The current study proposed an investigation of the relation between the indices of the number representation strength and math ability to address the mixed findings.

Research is somewhat divided on the role of the ANS with mental representations of symbolic numerals. Some believe the ANS is the underpinning of symbolic digit representation and is automatically activated in the presence of Arabic numeral digits (Dehaene, Dehaene – Lambertz, & Cohen, 1998; Dehaene, Piazza, Pinel, & Cohen, 2003), while others question this assumption (Leibovich & Ansari, 2016; Lyons, Nuerk & Ansari, 2015; Inglis & Gilmore, 2014). Early investigation of brain imaging data suggested a shared region of interest between non-symbolic and symbolic number processing in the right inferior parietal sulcus (IPS), however the technological limitations of that era were significant and are in need of updating (Dehaene, Dehaene-Lambertz, & Cohen, 1998). More recent brain imaging studies differentiated areas of activation of magnitude comparison tasks within either presentation formats (digits and dots) (Bluthé, deSmedt, & Op de Beeck, 2014). These studies did not find overlap between non-symbolic and symbolic representations (Darmla & Just, 2013). Since the current brain imaging data suggests a lack of relation between the non-symbolic and symbolic number system, behavioural data using valid measures will be required for a meaningful investigation of the relation between the two number systems.

Measurement of individual number representation currently involves gathering of behavioural data from non-symbolic and symbolic numerical comparison tasks. The comparison tasks consist of two sets of non-symbolic (dots) or symbolic (digits) presented simultaneously or side-by side, and participants are asked to quickly choose “which is more”. Data from the behavioural tasks are broken down into the following measures: overall accuracy, Weber fraction, and numerical ratio effect. Weber fraction is the ‘just noticeable difference’ needed to discern the difference between two magnitudes. Numerical ratio effect is the decrease in accuracy, or increase in response time when the ratio between the magnitude of two numbers approaches one. Currently, the use of dot magnitude comparison (non-symbolic) tasks allow researchers to gather information on an individual’s mental representation of magnitude. This is assumed to measure number representation. The digit magnitude comparison (symbolic) task is used to assess how well an individual processes symbolic number representations (Halberda, Mazzocco, & Feigenson, 2008).

The non-symbolic magnitude comparison task is a set of two dot arrays that are either presented simultaneously (Inglis, Attridge, & Gilmore, 2011; Inglis & Gilmore, 2014), intermixed (Halberda, Mazzocco, & Feigenson, 2008), or sequentially (Dehaene, Dehaene-Lambertz, & Cohen, 1998). Participants are asked to quickly and accurately identify which set contains more dots. Typically, the sets are presented for a brief length of time (7800ms) to prevent participants from counting the dots. The number of dots in non-symbolic dot arrays may range from five to twenty-five, but do not generally display one to four dots. Individuals are typically able to subitize small collection of items in a brief period (Krajcsi, Szabó, & Mórocz, 2013). In other words, individuals can ‘know’ if there are one to three items on a table without having to count the items one at a time. Since non-symbolic magnitude comparison tasks are interested in measuring a person’s accurate mental representation of numbers and not their ability to subitize smaller numbers, some researchers use tasks that omit dot arrays with one-four items for adults (Krakcsi, 2016).

The symbolic magnitude comparison task is similar to the non-symbolic task in choices of presentation order and length of time. Participants are also asked to quickly and accurately choose which one is greater. The types of digits presented in this task is varied because of the age of participants. Since adults have more experience with symbolic number representations the symbolic magnitude comparison task may be presented in two formats: single and double digit comparison tasks. Research suggests that people process triple digit, and higher numbers differently, possibly measuring a different construct (Hinrichs, Berie, & Mosell, 1982). Measuring symbolic number representation in children requires special consideration to their developing understanding of symbolic numerosity, and some studies present different ranges of digit comparison tasks across younger and older children (Lyons, Nuerk, & Ansari, 2015).

Number representation strength is currently measured through one or more of the following indices: overall accuracy, Weber fraction, and numerical ratio effect. Accuracy is typically reported as the proportion of correct responses in a magnitude comparison task, or the amount of correct responses (Inglis & Gilmore, 2014). This particular method is not grounded in theoretical support, however there is an awareness of the usefulness of accuracy in assessing number strength (Inglis & Gilmore, 2014). The Weber Fraction (*w*-score) is a calculated score that represents the precision of the individual's mental number representation. Lower *w*-scores are associated with less overlap between the two representations which translates to a more accurate mental representation of numbers. Higher *w*-scores are tied to greater overlap between the two representations and are associated with poorer mental representation of numbers. The *w*-scores for numerical representation have predicted mathematical performance in 14-year-old adolescents (Halberda, Mazzocco, & Feigenson, 2008; Halberda & Feigenson, 2008). The numerical ratio effect is the decrease in either accuracy or increase of response time of an individual's responses when the ratio between two numbers gets closer to one. For example, it is easier for an individual to choose between two dots and nine dots, than it is to choose between eight dots and nine dots. The ratio is calculated through the two presented numbers ($n1/n2<1$) (Inglis & Gilmore, 2014).

The predictive relation between the ratio effect and math outcome has more recently been called into question (Lyons, Nuerk, & Ansari, 2015; Leibovich & Ansari, 2016). Lyons, Nuerk, and Ansari's investigation into the numerical ratio effect, effect size, indicated that the variability of the numerical ratio effect on an individual level was a greater predictor of math performance than the ratio itself in children (2015). Moreover, the ratio effects between non-symbolic and symbolic formats did not share a relation. In other words, the numerical ratio effect as traditionally used did not seem to be a valid measure when predicting math performance (Lyons, Nuerk, & Ansari, 2015). Inglis and Gilmore also investigated non-symbolic number comparison and were unable to find a relation between an individual's *w*-score and numerical ratio effect, and thus suggest that non-symbolic number processes are not related to the Weber fraction (2014).

Research in adults that investigated the predictive nature of performance on the symbolic and non-symbolic tasks suggests that symbolic representation is a predictor of math achievement in adults, not non-symbolic representation (Newton, Waring, & Penner-Wilger, 2014). Additionally, similar results for symbolic comparison task performance were found in children with the mean response times being predictive but not ratio effects (Lyons, Nuerk, & Ansari, 2015). This suggests the need for further validity testing of indices for the number representation in adults and children. Results such as the ones discussed above call into question whether the current indices of number representation are valid, as well as their predictive ability for math performance in adults and children.

This study addresses lingering questions about the construct and predictive validity with symbolic magnitude comparison tasks and non-symbolic magnitude comparison tasks. Questions such as: 1) Are the number representation indices measuring the same constructs within both number systems? 2) Are symbolic magnitude and non-symbolic magnitude systems related? 3) Which indices are a predictor of math achievement in either symbolic or non-symbolic magnitude system? This study hypothesizes the following: 1) If the number representation indices are related, then the indices should be related within non-symbolic and symbolic comparison tasks, 2) If the non-symbolic indices are related to the symbolic indices, then the number representation indices should be related between digit and dot comparison tasks, and 3) If the indices measure the number representation, then the indices should predict math performance.

We assessed non-symbolic and symbolic number comparison abilities along with age appropriate math ability in adult university students (n = 51), and senior kindergarten children (n = 159). The advantage in looking at two different age groups is the ability to see the possible differences of relations between the two number systems, non-symbolic and symbolic, in children and adults. Measurements will consist of performance on non-symbolic and symbolic magnitude comparison tasks and age appropriate standardized math assessments.

General Method

Overview

In the experiments reported here, participants were assessed on their symbolic and non-symbolic number representation through magnitude comparison tasks. Both experiments investigated the correlations within and between overall accuracy, overall response time, numerical ratio effect accuracy, numerical ratio effect response time, and Weber fraction in non-symbolic and symbolic number comparison tasks. Age-appropriate math outcomes were also measured in both experiments.

Experiment 1

Participants

The participants for the study consisted of 51 students (Male = 27, Female = 24, M_{age} = 19.8 years, SD = 1.0, Range = 18-23 years), from a local university college. All participants completed their elementary and secondary education in Canada. Participation in this study was on a voluntary basis.

Materials

Magnitude comparison task. Participants were presented with two single digit numbers (ranging from 1 to 9) on an iPad screen, and instructed to choose the numerically larger number as quickly as possible without making any errors. Magnitude comparisons appeared in two different formats: symbolic (digits) and non-symbolic (dots). For non-symbolic tasks, the surface area of the dots was were presented in one of three equally likely configurations, congruent (larger number with a larger surface area than the smaller number), non-congruent (larger number with a smaller surface area than the smaller number) or matched (both numbers take up the same surface area). The stimuli remained on the screen for 7800ms or until the participant made a choice, and the time between trials was 1000ms. Participants performed two blocks of 54 trials (one symbolic, one non-symbolic for a total of 108 trials) and the presentation order of these blocks was counterbalanced based on participant number. The order of the problems presented in each block was randomized.

Math ability. Adult participants completed the addition and subtraction-multiplication subtests of the Kit of Factor-Referenced Cognitive Tests (French, Ekstrom & Price, 1963). Each subtest of this paper-and-pencil task consisted of two-pages of multi-digit arithmetic problems (two pages containing 3 digit addition problems, and two pages containing both 2 digit subtraction problems and 2 digit multiplication problems). Participants were instructed to solve the problems as quickly and accurately as possible and were given two minutes per page.

Procedure

Participants were tested in a quiet room and consent was obtained prior to testing. After the iPad tasks, the iPad was removed and participants completed the Kit of Factor-Referenced Cognitive Test. These tasks were completed in one session, along with other tasks as part of a larger study, lasting approximately one hour. After completion of the above tasks, participants were debriefed and thanked for their participation.

Design

The current study is a correlational design with two presentation formats: symbolic, non-symbolic, and five indices of number representation for each presentation format: overall accuracy, overall response time, NRE accuracy, NRE response time, and Weber fraction. To use number representation indices as predictors for math outcomes calculation fluency was measured as the total number of correct solutions on both tests, and reflected an individual's ability to quickly and accurately execute simple arithmetic procedures on multi-digit problems. Performance on Kit of Factor-Referenced Cognitive Test was used as an outcome measure for mathematical ability.

Results and Discussion

The means and standard deviations for adult symbolic and non-symbolic number representation indices are found in Table 1.

Are the number representation indices measuring the same thing within symbolic and non-symbolic formats? For more information on the number representation indices within presentation formats, an exploratory factor analysis was conducted on the five number representation indices separately for non-symbolic and symbolic formats. The non-symbolic factor analysis revealed two factors, as shown in Table 2. Factor 1 accounted for 52.05% of the variance and factor two accounted for 32.77% of the variance. Factors 1 and 2 combined accounted for 84.82% of the the variance. The non-symbolic indices that loaded on factor 1 were overall accuracy at -.974 and Weber fraction at .976. The non-symbolic indices that loaded on factor 2 were overall response time at .955 and numerical ratio effect-response time at .950. Numerical ratio effect - accuracy loaded on factor 1 at .531 and factor 2 at .476.

The symbolic factor analysis showed all five number representation indices loaded to one factor, as shown in Table 3. This factor accounted for 58.68% of the variance. With a cutoff of .45 the rotated component matrix showed overall response time at .64, overall accuracy at .81, numerical ratio effect-response time at .51, numerical ratio effect-accuracy at .94, and Weber fraction at -.85.

Table 1

Means of Adult Symbolic and Non-Symbolic Number Representation Indices

	Symbolic	Non-Symbolic
Number Representation Indices	M (SD)	M (SD)
Overall Response Time	594.86ms (97.48)	939.13ms (305.57)
Overall Accuracy	.99 (.01)	.99 (.03)
NRE- Response Time	146.93ms (91.45)	1055.76ms (735.95)
NRE- Accuracy	-.04 (.05)	-.02 (11)
Weber fraction	.05 (.05)	.06 (.11)

Note. n = 51

Table 2

Factor Loadings of Rotated Component Matrix for Non-Symbolic Number Representation Indices in Adults

Number Representation Indices	Factor 1	Factor 2
Overall Response Time	.101	**.955**
Overall Accuracy	**-.974**	-.050
NRE-RT	.033	**.950**
NRE-ACC	**.531**	**.476**
Weber fraction	**.976**	.048

Table 3

Factor Loadings of Component Matrix for Symbolic Number Representation Indices in Adults

Number Representation Indices	Factor 1
Overall Response Time	**0.637**
Overall Accuracy	**0.81**
NRE-RT	**0.505**
NRE-ACC	**0.943**
Weber fraction	**-0.853**

Results indicated number representation indices do not measure the same construct within non-symbolic presentation format. However, number representation indices did measure a single construct for symbolic presentation format.

Are the non-symbolic and symbolic number representation systems related? In order to investigate the relation between non-symbolic number representation indices and their symbolic counterpart in adults a correlational analysis was conducted on the five number representation indices. The positive relation between non-symbolic overall response time and symbolic overall response time was moderate, r (49) = .61, $p < .001$, shown in Table 4. The remaining four relations were not significant.

Results indicated that the only overall response time related across the non-symbolic and symbolic systems. This suggests that the two systems do not share a strong relation.

Do the number representation indices predict math ability? A correlation matrix indicated weak relations between math ability and two indices. Symbolic overall response time was negative and weakly correlated with math ability (M = 53.75, SD = 20.00), r(49) = -.32, p = .021. Symbolic numerical ratio effect-response time was also negative and weakly correlated with math ability (M = 53.75, SD = 20.00), r(49) = -.36, p = .009. The relations with response time data were expected to be negative as lower response times indicates more accurate number representation.

A stepwise multiple regression analysis was conducted in order to observe the strongest possible predictors of math ability with the number representation indices across the two presentation formats. Raw score from math ability was entered as the criterion variable. All symbolic and non-symbolic number representation indices were entered as predictor variables.

Results indicated that symbolic numerical ratio effect - response time had a positive weak correlation with a raw score on math ability (*M* = 53.75, *SD* = 20.00), $R(49) = .36$, $p < .01$. R^2 indicated this relation accounted for 13% of the variance. Results, shown in fig.1, also indicated that symbolic numerical ratio effect-response time was the only predictor of mathematical math ability (*M* = 53.75, *SD* = 20.00), $\beta = -.36$, $t(49) = -2.70$, $p < .01$.

The correlation matrix between non-symbolic and symbolic number representation indices revealed low relations. The stepwise multiple regression indicated that symbolic numerical ratio effect-response time was the strongest predictor when all the indices are present. Findings are in agreement with previous research indicating that symbolic indices are predictor of math skill in adults (Newton, Waring, & Penner-Wilger, 2014).

Table 4

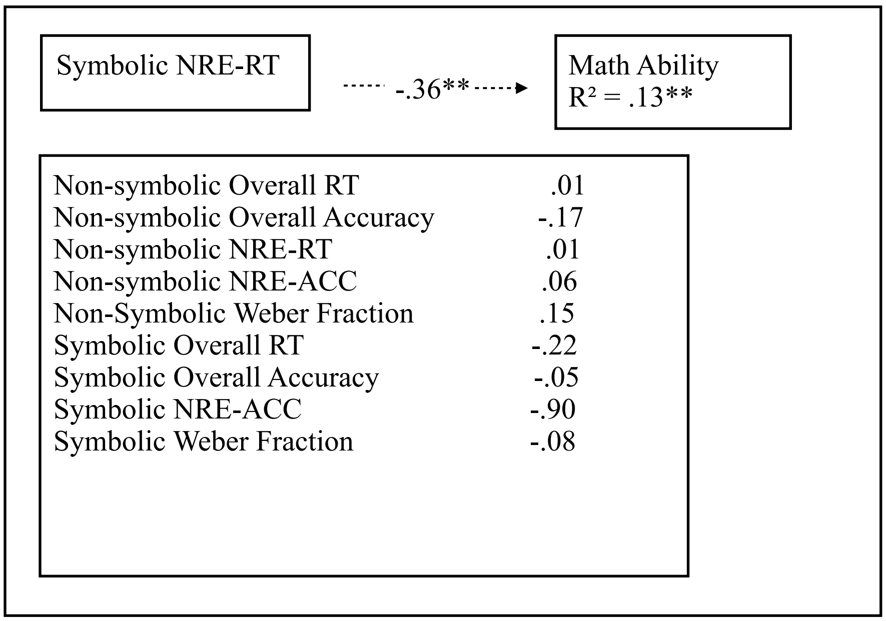
Correlation Matrix for Relations Between Non-Symbolic Number Representation Indices and Their Symbolic Counterparts in Adults

	Non-Symbolic Overall RT	Non-Symbolic Overall Acc	Non-Symbolic NRE-RT	Non-Symbolic NRE-ACC	Non-Symbolic Weber fraction
Symbolic Overall RT	.614**				
Symbolic Overall Accuracy	.381**	.019			
Symbolic NRE-RT	.034	-.214	.057		
Symbolic NRE-ACC	.289	-.069	.247	.16	
Symbolic Weber fraction	-.147	.08	-.158	-.104	-.067

Note: **indicates p < .01

Fig.1

Stepwise Multiple Regression of Number Representation Indices as Predictor of Math Performance in Adults



Note: Standardized Beta shown. **indicates $p<.01$

Experiment 2

Participants

Participants were Senior Kindergarten students (N = 165, Female = 91, Male = 74). Participant ages ranged from 64 months to 77 months (*M* = 70.06, *SD* = 3.47). Testing occurred at fourteen different schools. Inclusion criteria required participants with complete data, and performance of better than chance on magnitude comparison, thus six participants were excluded from subsequent analysis. Remaining participant (*N* = 159, Female = 89 Male = 70) ages ranged from 64 months to 77 months (*M* = 70.20, *SD* = 3.49). All parents returned consent forms and assent from the child was obtained for each session. Children were compensated with stickers and pencils.

Materials

Magnitude comparison task. Two separate tasks were conducted, one using symbolic representations of numbers (*a* = .98) and the other using non-symbolic representations of numbers (*a* = .96, Lyons *et al.,* 2014). In the tasks, the child was presented with two numbers or dot clusters on the screen and asked to touch "which is more" as quickly as possible without making mistakes. For non-symbolic tasks, the surface area of the dots was were presented in one of three equally likely configurations, congruent (larger number with a larger surface area than the smaller number), non-congruent (larger number with a smaller surface area than the smaller number) or matched (both numbers take up the same surface area). The stimuli remained on the screen for 7800ms or until the participant made a choice, and the time between trials was

1000ms. There were a total of 18 trials in each task, for a total of 36 trials. The child completed all trials, however, if there was no input for five consecutive trials, the task was terminated.

Math ability. Children's understanding of whole and rational numbers was assessed using the Key Math III Numeration subtest (a = .70, Skwarchuk, Sowinski, & LeFevre, 2014; Connolly, 2000). In this task, children were presented with numerical questions and asked to solve each one. The numerical questions began at a basic level and became increasingly difficult. The subtest has 49 items. After four consecutive incorrect answers, the task was terminated.

Procedure

Prior to testing the lead researcher contacted members of the school board, principals and senior kindergarten educators, within the school district, to send out consent forms to parents of children attending the schools. Parents returned consent forms to the teachers and were collected by research assistants prior to testing. During the spring of 2016, children were tested during regular school hours by trained research assistants. The testing for each child took place in two 30-minute testing sessions over the course of two separate school days. Each testing session took place in a quiet room in the child's school. To minimize distractions only the child and researcher were in the room during the time of testing. Each child was informed verbally about the study and gave verbal assent. In one session, the child completed Key Math III Numeration subtest and other tasks as a part of the larger study. In the other session, the child completed the Magnitude task as well as other tasks on an iPad that were also part of the larger study. The sessions occurred in randomized order based on research assistant and material availability. After each session, the child was thanked for their participation and given either a math pencil or sticker as compensation.

Design

The current study is a correlational design with two presentation formats: symbolic, non-symbolic, and five indices of number representation for each presentation format: overall accuracy, overall response time, NRE accuracy, NRE response time, and Weber fraction. Performance on Key Math III Numeration Subtest was used as mathematical ability. The magnitude task was used to assess the representation of numeracy, both symbolic and non-symbolic and was calculated as a raw score. Dependent variables of overall accuracy, overall response time, numerical ratio effect accuracy, numerical ratio effect-response time, and Weber fraction were calculated from the behavioural data. Symbolic number knowledge was measured as the total raw score from Key Math III Numeration subtest.

Results and Discussion

The means and standard deviations for child symbolic and non-symbolic number representation indices are found in Table 5.

Are the number representation indices measuring the same thing within the symbolic and non symbolic number systems? For more information on the number representation indices within presentation formats, an exploratory factor analysis was conducted on the five number representation indices separately for non-symbolic and symbolic formats. The non-symbolic factor analysis for the children also revealed two factors for the five number representation indices, shown in Table 6. Factor 1 accounted for 44.30% of the variance, and factor two accounted for 23.98% of the variance. A total of 68.28% of the variance was explained by factors 1 and 2. Using a loading cutoff of .45 on the rotated component matrix the non-symbolic indices that loaded on factor 1 were overall accuracy at .96, numerical ratio effect-accuracy at .54, and Weber fraction at -.95.

Table 5

Means of Child Symbolic and Non-Symbolic Number Representation Indices

	Symbolic	Non-Symbolic
Number Representation Indices	M (SD)	M (SD)
Overall Response Time	1620.57ms (494.39)	1592.96ms (464.28)
Overall Accuracy	.92 (.09)	.95 (.09)
NRE- Response Time	597.50ms (937.28)	1120.98ms (1000.78)
NRE- Accuracy	-.18 (.25)	-.11 (.24)
Weber fraction	.21 (.26)	.16 (.24)

Note. n = 159

Table 6

Factor Loadings of Rotated Component Matrix for Non-Symbolic Number Representation Indices in Children

Number Representation Indice	Factor 1	Factor 2
Overall Response Time	-.026	**0.835**
Overall Accuracy	**0.957**	.083
NRE-RT	.182	**0.742**
NRE-ACC	**0.539**	.117
Weber fraction	**-0.951**	-.002

The indices that loaded on factor 2 were overall response time at .84 and numerical ration effect-response time at .74. As shown in Table 7, the symbolic factor analysis for the children revealed two factors for the five number representation indices. Factor 1 accounted for 44.16% of the variance and factor 2 accounted for 22.98% of the variance. A total of 67.14% of the variance was explained between the factors 1 and 2. Using a loading cutoff of .45 on the rotated component matrix the symbolic indices that loaded on factor 1 were overall accuracy at .95, numerical ratio effect-Accuracy at .62, and Weber fraction at -.94. The symbolic indices that loaded on factor 2 were overall response time at .74, and numerical ratio effect-response time at .76.

Results indicated that the number representation indices loaded on two factors. This suggests that the indices do not measure the same construct within non-symbolic and symbolic presentation formats for children.

Are the symbolic and non-symbolic number systems related? In order to investigate the relation between non-symbolic number representation indices and their symbolic counterpart in children. A correlational analysis was conducted on the five number representation, shown in Table 8. Results revealed a moderate, positive relation between symbolic overall response time and non-symbolic overall response time, r (157) = .52, p <.001. The relation between symbolic overall accuracy and non-symbolic overall accuracy was positive and moderate, r (157) = .48, p <.001. A positive and moderate relation was found between symbolic Weber fraction and non-symbolic Weber fraction, r (157) = .45. p < .001.

Three number representation indices share a relation across presentation formats. Results suggest some relation between the non-symbolic and symbolic number systems in children.

Table 7

Factor Loadings of Rotated Component Matrix for Symbolic Number Representation Indices in Children

Number Representation Indices	Factor 1	Factor 2
Overall Response Time	.125	**0.740**
Overall Accuracy	**0.948**	.077
NRE-RT	-.166	**0.761**
NRE-ACC	**0.623**	-.123
Weber fraction	**-0.937**	-.035

Table 8

Correlation Matrix for Relationships Between Non-Symbolic Number Representation Indices and Their Symbolic Counterparts in Children

	Non-Symbolic Overall RT	Non-Symbolic Overall Acc	Non-Symbolic NRE-RT	Non-Symbolic NRE-ACC	Non-Symbolic Weber fraction
Symbolic Overall RT	.519**				
Symbolic Overall Accuracy	.036	.475**			
Symbolic NRE-RT	.168*	.169**	-.012		
Symbolic NRE-ACC	-.019	.122	.019	.09	
Symbolic Weber fraction	-.001	-.444**	.043	.006	.45**

Note: **indicates p < .01

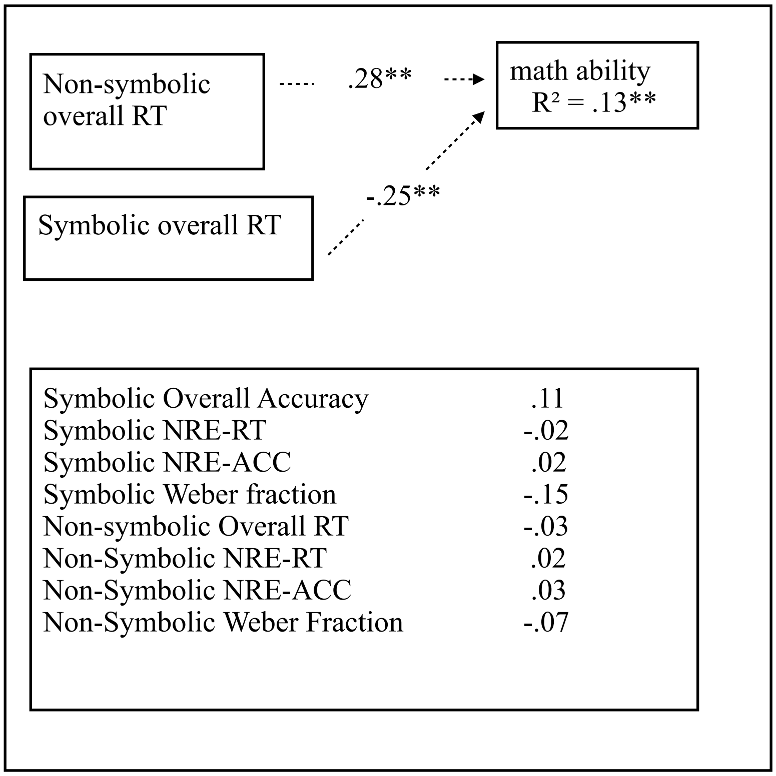
Do the number representation indices predict math ability? A correlation matrix indicated weak relations between math ability and several indices. There was a weak relation between math ability symbolic overall response time, $r(157) = -.24$, $p = .002$. Symbolic overall accuracy weakly correlated with math ability ($M = 7.00$, $SD = 2.66$), $r(157) = .19$, $p = .015$. Symbolic Weber fraction weakly correlated with math ability ($M = 7.00$, $SD = 2.66$), $r(157) = -.23$, $p = .004$. Non-symbolic overall accuracy weakly correlated with math ability ($M = 7.00$, $SD = 2.66$), $r(157) = .27$, $p = .001$. Non-symbolic Weber fraction weakly correlated with math ability ($M = 7.00$, $SD = 2.66$), $r(157) = -.27$, $p = .001$.

Also, a stepwise multiple regression analysis, shown in fig.2, was conducted in order to observe the strongest possible predictors of math ability when all of the the symbolic and non-symbolic number representation indices are present. Performance on Key Math III assessment was entered as a criterion variable. All symbolic and non-symbolic number representation indices were entered as predictor variables. Initial results showed non-symbolic overall accuracy as a weak positive correlation with math ability ($M = 7.00$, $SD = 2.66$), $r(157) = .266$, p = .001. This relation explained 7.1% of the variance. However, the second model showed non-symbolic overall accuracy and symbolic overall response time with a stronger positive, weak correlation, $R(157) = .37$, $p < .001$. This relation explains 13.3% of the variance. Results from the stepwise multiple regression also indicated that non-symbolic overall accuracy ($\beta = .28$) was a significant predictor of math ability ($M = 7.00$, $SD = 2.66$), t(157) = 3.68, p <.001. Symbolic overall response time ($\beta = -.25$) was also a significant predictor of math ability assessment, t(157) = -3.36, p=.001.

Results suggest low relations between the non-symbolic and symbolic number representation indices with math ability. A stepwise multiple regression revealed two predictors of math ability. However, evidence for strong predictors of math ability is lacking.

Fig.2

Stepwise Multiple Regression of Number Representation Indices as Predictor of Math Performance in Children



Note: Betas shown. **indicates p<.01

Summary and Concluding Discussion

The current study investigated three research questions about the construct and predictive validity of the indices used to measure the strength of number representation in the non-symbolic and symbolic magnitude comparison tasks. We looked at two groups, adults and senior kindergarten children. The three questions were: 1) Do the number representation indices measure the same thing within presentation formats? 2) Do the number representation indices correlate across presentation format? 3) Do the number representation indices predict math outcome?

Results indicated support for recent research that suggested the non-symbolic and symbolic mental number representation systems are not related in adults (Leibovich & Ansari, 2016). Our results also support the findings that the number representation indices may not be measuring the same constructs within non-symbolic and symbolic number systems (Inglis & Gilmore, 2014). However, a difference between the two age groups was revealed with more number representation indices relating across presentation formats for children than adults. For adults, only overall response time correlated across presentation formats. For children, overall response time, overall accuracy, and Weber fraction correlated across non-symbolic and symbolic presentation formats. In addition, symbolic factor analysis of number representation indices in adults revealed single factor loading, while the same analysis in children revealed two loading factors. There were no single factor loadings for non-symbolic factor analysis in either group. Lastly, children had more number representation indices that predicted math skills than adults.

Do the number representation indices measure the same construct within presentation formats? To test this question the current study hypothesized that if the indices measured the same construct within the non-symbolic and symbolic presentation formats, then the indices would load onto a single factor. However, our results showed that in the case of both non-symbolic and symbolic system in children, and the symbolic system in adults the indices loaded onto two factors. This result is in support of current literature suggesting that the number representation indices may not measure the same construct (Inglis & Gilmore, 2014).

With one exception, in both children and adults the non-symbolic number representation indices that measured accuracy (overall accuracy, numerical ratio effect-accuracy, Weber fraction) loaded on one factor, while the indices that measured response time (overall response time, numerical ratio effect-response time) loaded on a different factor. The one exception to this divide was the non-symbolic numerical ratio effect-accuracy, in adults, that loaded on both response time and accuracy factors, but at a much lower loading than the other indices. Child symbolic number representation indices also loaded on two factors, with accuracy and response time number representation indices split similarly. In contrast, adult symbolic number representation indices loaded on a single factor. This result suggests that the number representation indices may be a valid measure for adult symbolic number representation.

While these indices may be valid for measuring symbolic number representation in adults, they appear to lack the validity in all other cases. Results from the exploratory factor analysis revealed the weak two-factor loading non-symbolic numerical ratio effect-accuracy may be measuring an underlying construct, in adults, that is common to both accuracy and response time. All other number representation indices measured two constructs for non-symbolic (adult and child) and symbolic (child) presentation formats. This suggests that number representation indices may not be measuring the same construct within presentation formats.

Do the indices correlate across presentation format? In order to test this question, the current study conducted a correlation matrix between the non-symbolic and symbolic indices. We hypothesized, if the non-symbolic and symbolic number systems are related, we should find that the indices are related across both presentation formats.

In adults, the results showed that overall accuracy, numerical ratio effect-response time, numerical ratio effect-accuracy, and Weber fraction did not correlate between the non-symbolic and symbolic presentation formats. Only overall response time correlated across the presentation formats for adults. The correlation in overall response time was moderate and this may suggest that it was the participant's processing speed instead of a link between the two number systems. The results for adults are in agreement with recent research that questioned the relation between non-symbolic and symbolic number representation systems (Leibovich & Ansari, 2016; Krajcsi, 2016). Low correlations across the non-symbolic and symbolic number representation indices were present in prior research (Gilmore, Attridge, & Inglis, 2011). This lack of strong correlation across number systems for most of the number strength indices suggests that one process does not support all of numerical representation for adults.

In children overall response time, overall accuracy, and Weber fraction correlated across non-symbolic and symbolic presentation formats for children. Numerical ratio effect-accuracy and numerical ratio effect-response were not related across the two presentation formats. Although, the correlations across the two presentation formats, for the other indices, were moderate at best. This level of correlation between the two number representations suggests to us that the correlation had more to do with the similarities in the number representation indices themselves than any connection between non-symbolic and symbolic number systems.

Do the indices predict math skill? In order to answer this question the current study hypothesized that if the number representation indices measured the participants' mental number representation strength, then the indices should be predictive of math skill. To test this hypothesis, a correlation matrix was conducted to look at the overall predictors of math skill, as well as a stepwise multiple regression in order to observe the strongest possible predictors when all of the indices are present.

The correlation matrix for adults revealed significant, but weak, negative correlations between math ability and symbolic overall response time, and a weak, but significant correlation was present between math ability and numerical ratio effect-response time. The lack of predictors among non-symbolic number representation indices in adults is consistent with previous research. Price, Palmer, Battista & Ansari were also unable to find non-symbolic predictors among the numerical ratio indices and the Weber fraction (2012). Additionally, the weak correlations between math skill and symbolic overall response time we found is similarly supported by prior research, suggesting the symbolic representation system as a predictor for math skill in adults (Newton, Waring, & Penner-Wilger, 2014).

The correlation matrix for children revealed more number representation indices predicted math skill. Symbolic overall response time, symbolic overall accuracy, symbolic Weber fraction, non-symbolic overall accuracy, and non-symbolic Weber fraction correlated with math ability. Although, the correlations with the child predictors of math skill were weaker than the correlations in adults. This difference in correlated indices suggests a group difference between adults and children when it comes to any given index's ability to predict math skill. One possible explanation for this difference might be that an adult's symbolic number representation system is largely matured, while children are still experiencing drastic developmental changes.

A stepwise multiple regression was conducted to see which indices best predicted math skill when all of the indices were present. We found that non-symbolic numerical ratio effect-response time predicted math skill in adults; meanwhile, non-symbolic overall accuracy and symbolic overall response time predicted math skill in children. Both model fits for adult and child predictors were poor, suggesting that the current number representation indices for children and adults are not as valid predictors of math skill as previously thought.

Results from our current study highlight some areas where additional research is needed. While some number representation indices have statistically significant correlations with each age group, the indices that show the correlations are not the same between adult and child groups. Since our current study was limited to only two age groups, the reasons for this change, and when this change happens is not known. Studying this transition in greater detail may offer insight into the underlying systems used in number representation as individuals pass through developmental stages. A second factor our study was not setup to differentiate was the possibility that the difficulty of magnitude comparison trials may have influenced the behavioural data. Calculating split-half reliabilities on the magnitude comparison trials might provide clarification as to whether difficulty was a factor.

Investigations into the construct and predictive validity of number representation indices are important when researching what cognitive systems underlie number representation. A more precise understanding, and ability to measure this number representation, will allow for a more complex understanding of the relation between number representation and math skill. This understanding in turn is important in informing best practices in math education. Because good math skills are associated with several positive life outcomes, having a better measure of what underlies the symbolic number system and how it predicts math skill in children, may help researchers develop better real world interventions.

These factors offer some possibilities for future research. Since no single measure can be universally applied as a number representation index, more work is required to understand why index correlations differ with age, whether this change a function of development, and if this difference is based on physiological differences or socially developed heuristics. If this change turns out to be developmental, it would be instructive to investigate why, and at what stage, the change in index correlation occurs. Future research should potentially investigate the nature of developmental changes in mental number representation through a longitudinal study of children.

The current study combined data from non-symbolic and symbolic magnitude comparison tasks, performed by children and adults, in order to systematically investigate the construct and predictive validity of commonly used indices. Our findings found no strong evidence that number representation indices measured the same construct in most cases. Within non-symbolic (adults and children) and symbolic presentation formats (children) two factor loading suggests the indices measured different constructs. This finding supported recent research that involved testing the validity of non-symbolic number representation indices (Inglis & Gilmore, 2014). Our findings suggested that number representation indices measured the same construct in the symbolic presentation format in adults.

The indices that correlated across presentation formats for children and adults differed, suggesting that number representation may be constructed differently in adults and children. The differences in adult and child results may imply a developmental change in mental number representation as an individual ages. Predictive validity of number representation indices was not shared across children and adults. More research is required to correctly identify the underpinnings of number representation, how, when, and if it changes as we age, and what indices correctly measure an individual's mental number representation system.

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