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The Role of Production Sharing and Trade in the Transmission of the Great Recession

By JACOB WIBE*

The great recession of 2008-2009 resulted in a large fall in trade relative to output. Real trade fell roughly three times more than real GDP in the U.S. and Mexico, and by a factor of five in Canada. The decline in trade and output was particularly large in sectors with high levels of production sharing (goods produced in multiple, sequential stages in more than one country). Motivated by these observations, this paper asks two quantitative questions: 1) What was the role of trade in the transmission of the recession in North America? 2) What was the contribution of production sharing to the large fall in trade? To answer these questions this paper develops a quantitative open economy model of production sharing. The benchmark calibration can account for 72% of the fall in output in Canada, 19% of the fall in output in Mexico, and about two-thirds of the fall in trade for both countries. In the quantitative exercises production sharing can account for 40% of the fall in trade.

JEL: F4; F1

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I. Introduction

During the 2008-2009 recession, real trade fell roughly three times more than real GDP in the U.S. and Mexico, and by a factor of five in Canada. The fall in output and trade was largely accounted for by manufacturing, and the decline was particularly large in sectors with high levels of production sharing. The sudden and synchronized nature of the fall in output and trade suggest that international linkages played an important role in the transmission of the recession across countries.

Motivated by these observations, I develop a quantitative small open economy model to study the role of trade in the transmission of the recession in North America. A key feature of my model is production sharing. Production sharing, or vertical specialization, refers to the production of goods in multiple, sequential stages where value added is provided by two or more countries. In NAFTA, the production sharing intensity of intra-region manufacturing trade is about 50 percent, and production sharing is particularly prevalent in the auto industry and the Mexican Maquiladoras trade (Burstein, Kurz and Tesar (2008)).¹

Production sharing may have played a significant role in the transmission of the US economic slowdown. International supply chains in manufacturing are generally very specialized, and there is little scope to substitute inputs at each production stage. This makes the supply chains vulnerable to demand shocks and interruptions caused by external events, because a fall in demand at any stage can cause a fall in demand across the whole chain.² Then, due to the high level of specialization at each production stage, such interruptions often lead to idling of productive factors, as full production shifts can be laid off and the capital may go underutilized. International supply chains therefore increase the interdependence of manufacturing sectors across countries.

The large fall in trade relative to output during the recession may also be related to production sharing. At each step in a supply chain, some value added is produced before the intermediate good is shipped to the next location for further processing or sale at its final destination. Because trade flows are measured on a gross value basis, imported intermediate goods are double counted when they are re-exported as part of later stage intermediate goods or final goods. This double counting generates a larger fall in trade relative to output for production sharing goods than for standard

¹The Maquiladoras Trade in Mexico consists of mostly US owned assembly plants that import intermediate goods and raw materials to produce goods that are re-exported to the US.

²E.g. the recent earthquake in Japan and flooding in Thailand.

traded goods, and the effect would be exacerbated for international supply chains crossing multiple national borders.

To quantify the contribution of trade and production sharing in the transmission of the recession I develop a small open economy model that nests the production structure of Backus, Kehoe and Kydland (1994). The economy produces two tradable and one non-tradable intermediate good. The first tradable intermediate is combined with imported intermediate goods to produce a generic tradable composite good. The second tradable intermediate is combined with imported intermediates to produce the production sharing composite. The production sharing composite good is only demanded abroad, representing goods produced by Canada and Mexico for the US market. The generic tradable composite is combined with the non-tradable intermediate to produce the final good which is used for consumption and investment. Lastly, I add convex adjustment costs to capital.

The quantitative experiments focus on the role of trade in transmitting the US slowdown to Canada and Mexico. This is modeled as shocks to foreign import demand. I calibrate the demand shocks such that the model matches the observed terms-of-trade movements exactly for Canada and Mexico. I calibrate production sharing using OECD Input-Output tables and bilateral trade data.³ By assuming that the share of imported intermediates used in producing export goods is proportional to industry output, the I-O tables provide weights to convert gross trade into value added measures. The benchmark calibration takes the stance that Canadian exports of auto parts and finished light vehicles, and Mexican exports from the Maquiladoras industry are *production sharing exports*.

The results indicate that trade was an important factor in transmitting the recession to Canada and Mexico. In the benchmark calibration the model can account for 72% of the fall in output in Canada, 19% of the fall in output in Mexico, and about two-thirds of the fall in trade for both countries. The tradable sector accounts for about three quarters of the fall in output. Intuitively, since the shock hits the economy's exports, the fall in output is larger in the tradable sectors than in the non-tradable sector. Output falls more in the production sharing sector because the shock can only be absorbed by reallocating productive factors. In the generic tradable sector the shock can be absorbed by either reallocating productive inputs or changing the household's consumption allocation, and output therefore falls less rel-

³Several different measures are available to calibrate the degree of production sharing in trade. See for example Hummels, Ishii and Yi (2001), Yi (2003), or Chen, Kondratowicz and Yi (2005)

ative to the production sharing sector. Following shocks to foreign import demand, the capital adjustment costs act as a friction to the reallocation of productive factors across sectors. The interaction between the capital adjustment costs and the share of production sharing in the tradable sector generate the transmission dynamics in the model.

In the counterfactual experiments I quantitatively assess the contribution of production sharing to transmission. By comparing the model with zero production sharing to the benchmark (holding the share of value added exports to GDP constant) I find that production sharing can account for 40% of the fall in trade and 12% of the fall in output. Production sharing has a bigger impact on trade than output because of the relatively larger share of production sharing goods in the composition of trade. This suggests that production sharing was a contributing factor to the large fall in trade relative to output.

My work contributes to three main bodies of literature. First, my paper contributes to the relatively recent literature investigating the impact of international production sharing on comovement. Di Giovanni and Levchenko (2010) use industry level data and find that international production linkages explain 32% of the impact of bilateral trade on aggregate comovement. Burstein, Kurz and Tesar (2008) use data on US multinationals and find that manufacturing sectors with higher levels of production sharing experience greater comovement in trade flows and output. Their results also suggest that the production sharing intensity is at least as important as trade volume in accounting for bilateral manufacturing output correlations. In Arkolakis and Ramanarayanan (2009) the authors study a model based on Eaton and Kortum (2002) where the degree of production sharing varies with trade barriers. With imperfect competition their model generates a positive link between trade intensity and output comovement. In my model I highlight how production sharing in North America is characterized by Canada and Mexico importing intermediate goods and producing for the US market. I model production sharing as a separate tradable sector producing a composite good that is exclusively exported. I argue that it is important to consider the location of production plants and the direction of trade flows when studying the impact of production sharing on comovement.

This paper is also closely related to recent work on the post-Lehman fall in world trade and how it contributed to the transmission of the 2008-2009 recession. The empirical work in this literature generally agrees with the conclusion of my paper; for example, Eaton et al. (2011), Levchenko, Lewis and Tesar (2010), and Bems, Johnson

and Yi (2010) all argue that trade linkages were important in the propagation of the global recession. Eaton et al. (2011) use a multi-sector model based on Eaton and Kortum (2002) and Alvarez and Lucas (2007), and argue that the fall in global trade and output was largely accounted for by a fall in demand for manufacturing goods. Bems, Johnson and Yi (2010) use a global Input-Output framework and study how changes in final demand in the US and Europe was transmitted to other countries. Their estimates suggest that 27% of the fall in US demand was borne by foreign countries. Levchenko, Lewis and Tesar (2010) find that the fall in US trade relative to GDP was larger than in previous recessions and argue that sectors producing intermediate inputs experienced larger falls in imports and exports. In addition, James (2009) analyze data from the US International Trade Commission and finds that US trade with preferential trade partners contracted faster than trade with the rest of the world. He suggests that the transmission of the recession in North America was principally through international trade. Chor and Manova (2011) argue that credit conditions were important for transmission of the trade shock. They find that countries with relatively tighter credit markets exported less to the US during the recession. In this paper I restrict my attention to North America, and I focus on the impact of production sharing on trade transmission. I abstract from credit market and trade barrier frictions.

Lastly, my paper contributes to the literature on international transmission of domestic shocks. A key challenge in this literature has been to account for comovement in international business cycle models. Schmitt-Grohe (1998) studies open economy models and finds that interest rate and terms-of-trade variations cannot explain US/Canadian output comovement. Baxter and Crucini (1995) develop a two-country model and study the importance of financial market linkages for the behaviour of business cycles. They find that the degree of financial integration is only important if shocks are highly persistent or are not transmitted internationally. Stockman and Tesar (1995) allow for non-traded goods in a two-country model. They find that technology shocks alone are insufficient to match the data, and include taste shocks to get predictions more consistent with measurements of comovement. Kose and Yi (2006) use a three-country framework with transportation costs to study the impact of trade linkages on comovement. The authors find a positive correlation between trade and comovement, but the model still falls short of matching empirical findings.

In this literature, my work is most closely related to Burstein, Kurz and Tesar (2008). The foremost difference between our work is that my paper examines the

2008-2009 recession, whereas the aim of Burstein et al. is to evaluate the importance of production sharing as a mechanism to generate comovement. Structurally, our frameworks are similar as we both extend Backus, Kehoe and Kydland (1994) and model production sharing as producing a composite good only consumed by one country. The main difference between our frameworks is that I develop a small open economy model where the production sharing good is traded, while in their two-country model only intermediate goods are traded. A second difference is that their model only has one intermediate good for each country, compared to my model which has two tradable goods and one non-tradable intermediate good. The number of sectors and which goods are traded are important distinctions because I include capital adjustment costs which impact the transmission dynamics in response to shocks. In the counterfactual experiments I carefully analyze the effects of the capital adjustment costs.

This paper is organized as follows: Section 2 gives a brief review of evidence on output, trade and production sharing during the recession. Section 3 describes the model. In section 4 I describe the model parameters and calibration strategy. The benchmark results and quantitative exercises are described in section 5. Section 6 concludes.

II. Key Facts from the North American Recession

In this section I present three key facts on trade and the great recession in NAFTA: (i) the timing of the decline in output and trade, (ii) the magnitude of the fall in trade relative to output, and finally, (iii) production sharing and the composition of the fall in output and trade.

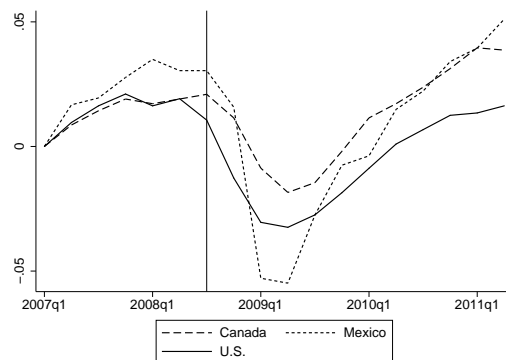
Timing

Several authors, including Baldwin and Evenett (2009) and Bems, Johnson and Yi (2010), have pointed out the synchronised nature of the fall in output and trade during the global recession.

Figures 1 and 2 show the logarithm of real GDP and real trade for Canada, Mexico, and the US from Q1 2007 to Q2 2011. In Figure 1, the fall in US output leads Canada and Mexico by a quarter, indicating that the recession started earlier in the US. Figure 2 shows how the fall in real trade is more synchronized than the fall in output. Note that the fall in output in Canada and Mexico coincides with the fall in trade

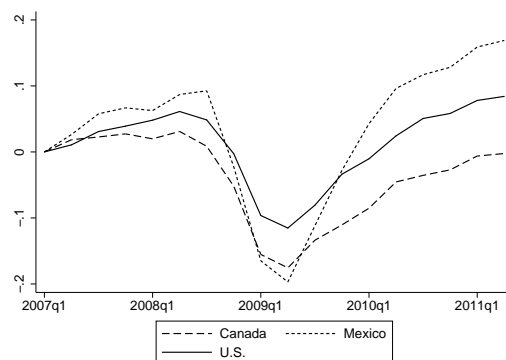
across all three countries. This suggests that trade played a role in the transmission of the recession.

FIGURE 1. NATURAL LOGARITHM OF REAL GDP, SEASONALLY ADJUSTED



Note: Vertical line approximately marks the fall of the Lehman Brothers which filed for Chapter 11 bankruptcy protection on September 15, 2008. *Source:* OECD Statistics - Quarterly National Accounts

FIGURE 2. NATURAL LOGARITHM OF REAL TRADE, SEASONALLY ADJUSTED



Source: OECD Statistics - Quarterly National Accounts

The quantitative exercises in this paper focus on Q2 2008 to Q2 2009. This period roughly coincides with the peak to trough of US real GDP per capita. As shown in Figure 1, there is a small dip in US GDP (solid line) from Q4 2007 to Q1 2008 before it reaches a local peak at Q2 2008, and then declines until Q2 2009.

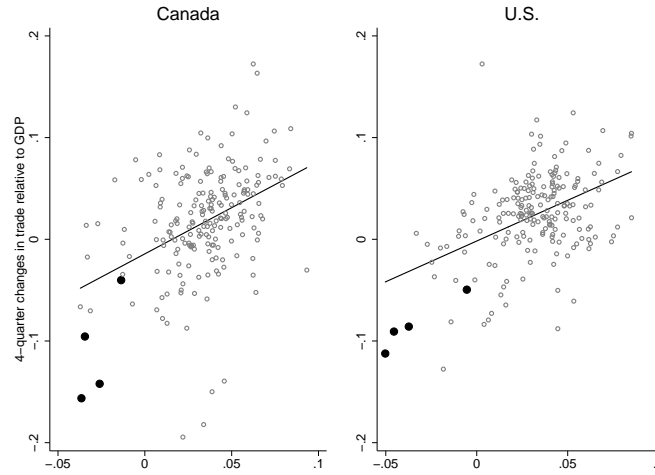
The Fall in Trade Relative to Output

Table 1 displays the change in real GDP and real trade over Q2 2008 to Q2 2009. Real GDP fell 5% in the US, 3.7% in Canada, and 9.9% in Mexico. The declines in trade are more striking, as trade falls roughly three times more than real GDP in the U.S. and Mexico, and by a factor of five in Canada.

TABLE 1—REAL GDP AND REAL TRADE - US, CANADA, AND MEXICO

	U.S.	Canada	Mexico
Real GDP	-5.0%	-3.7%	-9.9%
Real Trade	-15.7%	-18.7%	-26.9%

FIGURE 3. TRADE RELATIVE TO GDP, 4 QUARTER CHANGES



Note: Four-quarter changes in trade relative to GDP against the change in real GDP, 1960-2010. A similar plot appears in Eaton et al. (2011). *Source:* IMF International Financial Statistics

For Canada and the US the fall in trade relative to output during the recession was large compared to previous episodes. Figure 3 plots four-quarter changes in trade relative to GDP against the change in real GDP from Q1 1960 to Q4 2010 for Canada (left panel) and the US (right panel). The smaller gray dots and the regression line is based on the observations prior to the 2008-2009 recession, and the four

solid black dots represents the observations for the recession period. For Canada, the solid black dots appear to the far left, indicating the severity of the recession, and three of the four dots are well below the regression line representing a deviation from earlier episodes. The US shows a similar but less pronounced pattern.

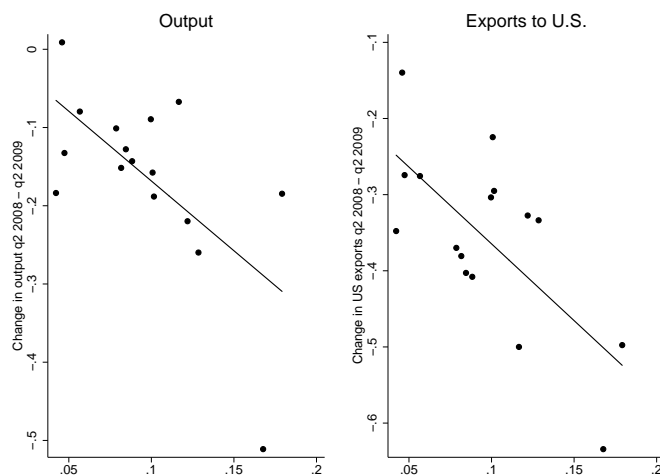
Production Sharing and the Fall in Output and Trade

Table 2 presents a decomposition of GDP, and shows the contribution of each sector to the fall in GDP from Q2 2008 to Q2 2009.

TABLE 2—DECOMPOSITION OF GDP - US, CANADA, AND MEXICO

	Share of GDP Average 2006 - 2010			% Δ Q2 2008 - Q2 2009			Contribution to fall in GDP		
	U.S.	Can	Mex	U.S.	Can	Mex	U.S.	Can	Mex
Mining, oil, gas	1%	5%	5%	-39%	-7%	-2%	13%	9%	1%
Manufacturing	13%	14%	18%	-15%	-14%	-14%	47%	58%	30%
Other tradable	9%	19%	27%	-12%	-6%	-14%	25%	31%	45%
Non-tradable	77%	62%	50%	-1%	0%	-4%	15%	3%	24%

FIGURE 4. PRODUCTION SHARING AND THE FALL IN OUTPUT AND TRADE



Note: Fall in output (left) and exports to U.S. (right) for manufacturing sectors in Canada against the level of imported intermediates relative to industry output in U.S, Q2/2008-Q2/2009. *Source:* Statistics Canada, US International Trade Commission, OECD Input-Output Tables

The sectoral impact of the recession in Canada and the US is similar. The tradable sector (mainly manufacturing) largely accounts for the fall in output. The picture is less clear for Mexico, where manufacturing accounts for a third and the non-tradable sector a quarter of the fall in output, but transportation, retail and wholesale trade also experienced significant declines.

During the recession, Canadian manufacturing sectors with production linkages to the US experienced greater declines in output and exports. Figure 4 shows scatter plots of the fall in output (left panel) and exports to the US (right panel) for Canadian manufacturing sectors plotted against imported intermediates relative to industry output in the US. The regression lines show a negative relationship, suggesting that production sharing was important in transmitting the recession to Canada.

As an example, consider the impact on the Canadian automotive industry following the closure of several North American assembly plants during 2009. Most of the closures were temporary, although GM's Oshawa Truck plant and six US plants shut down for good. The effect of the assembly plant closures was felt by the Canadian auto parts industry. According to Industry Canada (2006), Canadian auto parts and component manufacturing consists of about 900 establishments which on average export 61% of their production value. The recession led to large scale layoffs at several major parts manufacturers, including about 400 workers at Magna International and 700 workers at Linmar Corp.

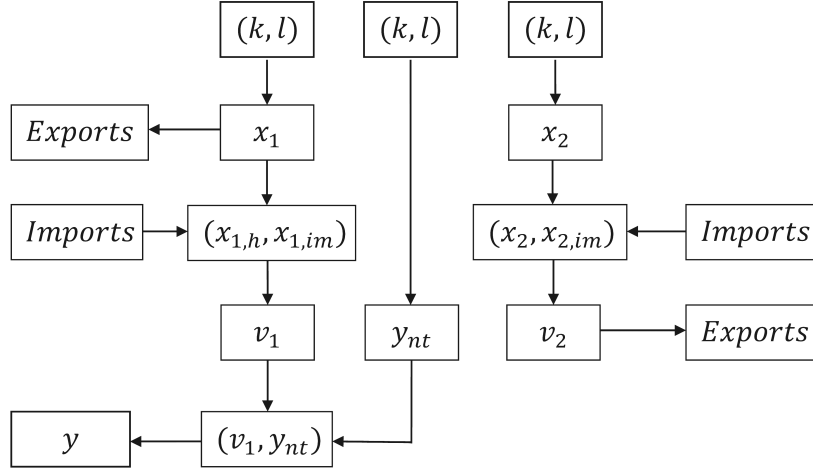
III. Model

To quantify the contribution of trade and production sharing in the transmission of the recession, I develop a real business cycle framework that incorporates production sharing. The model is a small open economy that nests the production structure of Backus, Kehoe and Kydland (1994).⁴ The economy produces two tradable intermediate goods and one non-tradable intermediate good. The first tradable intermediate good is exported and combined with an imported intermediate good to produce a tradable composite good. The second tradable intermediate good is combined with an imported intermediate good to produce the production-sharing composite good. This good is only demanded abroad, and all of its production is exported. The production sharing composite represents goods produced by Canada and Mexico for the US market. Lastly, the first composite good is combined with the non-tradable inter-

⁴By setting the production sharing sector and the non-tradable sector to zero, my model collapse to an open economy version of the Backus, Kehoe and Kydland (1994) framework.

mediate to produce the (non-traded) final good which is used for consumption and investment. A flowchart describing the model is included in Figure 5.

FIGURE 5. MODEL FLOWCHART



Note: x_1 is an intermediate good that is exported and used in producing the generic tradable composite good, v_1 . x_2 is the production sharing intermediate good, aggregated with the imported intermediate $x_{2,im}$ to produce the production sharing composite good, v_2 , which is exclusively exported. The final good y is produced by aggregating the non-traded good y_{nt} and the tradable composite good, v_1 . The (non-traded) final good is used for consumption and investment.

To avoid excess volatility of investment in response to foreign demand shocks I include capital adjustment costs. The adjustment costs limit the investment response to shocks and change the transmission dynamics in the model. The financial market is represented by a one-period, non-contingent bond. Unless otherwise stated, all variables are denoted in per capita quantities.

The Representative Household

The economy is populated by a representative household that chooses consumption, leisure, investment, and foreign debt to maximize:

$$(1) \quad E_0 \left(\sum_{t=0}^{\infty} \frac{\beta^t (c_t^\mu (1 - n_t)^{1-\mu})^{1-\sigma}}{1 - \sigma} \right), \quad 0 < \mu < 1, 0 < \beta < 1, 0 < \sigma$$

where c_t is consumption and n_t is the amount of labour supplied in period t . β is the discount factor, μ is the intratemporal share parameter for consumption and leisure, and σ pins down the intertemporal elasticity of substitution. The household's time endowment is normalized to 1.

The household supplies labour services and rents capital to the firms. The law of motion for gross investment in sector j (two tradable and one non-tradable) is:

$$(2) \quad i_{j,t} = k_{j,t+1} - (1 - \delta)k_{j,t} + \Phi_k(k_{j,t+1}, k_{j,t}), \quad j = 1, 2, nt$$

Φ_k is the capital adjustment cost function which follows Cogley and Nason (1995). The functional form implies that the marginal cost of adjusting the capital stock is a linear function of the rate of net investment:⁵

$$(3) \quad \Phi(k_{i,t+1}, k_{i,t}) = \frac{\psi_k}{2} \left(\frac{k_{i,t+1} - k_{i,t}}{k_{i,t}} \right)^2, \quad 0 < \psi_k, \quad i = 1, 2, nt$$

Here, ψ_k is a constant parameter defining the capital adjustment cost function.

The household can borrow or lend in the international financial market by a risk-free bond. In this paper, since Canada and Mexico are net debtors, I refer to the asset d_t as the household's debt. The household's debt evolves according to:

$$(4) \quad d_{t+1} = d_t(1 + r_{d,t}) - tb_t$$

where $tb_t = exports_t - imports_t$ is the trade balance.

To avoid a unit root in the log-linearized system, I introduce portfolio adjustment costs following Schmitt-Grohe and Uribe (2003). The representative household faces quadratic costs of holding debt quantities that deviate from the steady state level:

$$(5) \quad \Phi(d_{t+1}) = \frac{\psi_d}{2} (d_{t+1} - \bar{d})^2, \quad 0 < \psi_d$$

where d_t is the current debt level, \bar{d} is the steady state debt level, and ψ_d is a constant

⁵Mendoza (1991) uses a related specification where the marginal cost is a linear function of net investment.

parameter defining the portfolio adjustment cost function.⁶

The household's budget constraint is:

$$(6) \quad c_t + \sum_j i_{j,t} + (1 + r_t^d) d_t + \Phi_d(d_{t+1}) \leq \sum_j \left(r_{j,t}^k k_{j,t} + w_{j,t} n_{j,t} \right) + d_{t+1}$$

where $i_{j,t}$, $k_{j,t}$, $n_{j,t}$ is investment, capital, and labour supplied to sector j in period t respectively, d_t is the current period's debt, $r_{d,t}$ is the risk-free interest rate, and Φ_k and Φ_d are the adjustment cost functions for capital and external debt.

Technology

In the model, representative firms produce two tradable intermediate goods, the non-tradable intermediate good, two tradable composite goods, and the (non-traded) final good.

INTERMEDIATE GOOD PRODUCTION

The two tradable intermediate goods are produced by competitive firms. Each firm has a Cobb-Douglas production technology and takes capital and labour as inputs.

$$(7) \quad x_j = k_j^\alpha n_j^{1-\alpha}, \quad 0 < \alpha < 1, \quad j = 1, 2$$

where k_j is the amount of capital rented, n_j is the amount of labour hired, and x_j is the amount of intermediate goods produced in sector j . α is capital's share in output. Each period, firms maximize profits:

$$(8) \quad \max_{k_j, n_j} q_j^x x_j - r_j k_j - w_j n_j \quad \text{s.t.} \quad k_j, n_j > 0$$

where w_j is the wage rate, r_j the rental rate for capital, and q_j^x is the relative price of intermediate good j in terms of the final good.

⁶The portfolio adjustment cost function is a technical detail to make the model stationary for simulation purposes. Any impact on the quantitative results is negligible. See Schmitt-Grohe and Uribe (2003).

The non-tradable intermediate good is produced from capital and labour by a Cobb-Douglas production technology:

$$(9) \quad y_{nt} = k_{nt}^\alpha n_{nt}^{(1-\alpha)}, \quad 0 < \alpha < 1$$

where α is capital's share in output for the non-tradable sector. Each period the representative firm producing the non-tradable intermediate maximizes profits:

$$(10) \quad \max_{k_{nt}, n_{nt}} q_{nt} y_{nt} - r_{nt} k_{nt} - w_{nt} n_{nt}$$

Here, q_{nt} is the price of the non-tradable good in terms of the final good.

COMPOSITE GOOD AGGREGATION

In each tradable sector j , a composite good is produced by a representative firm combining domestic and imported intermediates in an Armington aggregator:

$$(11) \quad v_j = \left(\omega_j x_{j,h}^{\eta_j} + (1 - \omega) x_{j,im}^{\eta_j} \right)^{1/\eta_j}, \quad 0 \leq \omega \leq 1, \eta_j \leq 1, \quad j = 1, 2$$

where $x_{j,h}$ is the domestic intermediate and $x_{j,im}$ the imported intermediate used in producing the composite good v_j . Note that, for $j = 2$, in the production sharing sector, $x_{2,h} = x_2$. ω is the CES share parameter representing the home-bias, and $1/(1 - \eta_j)$ is the elasticity of substitution for the domestic and imported inputs. The perfectly competitive composite goods producers maximize profits each period:

$$(12) \quad \max_{x_{j,h}, x_{j,im}} q_j^v v_j - q_j^x x_{j,h} - q_j^{x^*} x_{j,im}$$

where q_j^v is the price of composite good j and $q_j^{x^*}$ is the price of the imported intermediate good, both in terms of the final good.

FINAL GOOD AGGREGATION AND MARKET CLEARING

The final good is produced by a representative firm taking the tradable composite from sector 1 and the non-tradable intermediate good as inputs in an Armington aggregator:

$$(13) \quad y = \left(\gamma v_1^\theta + (1 - \gamma) y_{nt}^\theta \right)^{1/\theta}, \quad 0 \leq \gamma \leq 1, \theta \leq 1,$$

where γ is the CES share parameter for the home-bias and $1/(1 - \theta)$ is the elasticity of substitution for the tradable composite and the non-tradable good. Each period the perfectly competitive firm producing the final good maximizes profits:

$$(14) \quad \max_{v_1, y_{nt}} y - q_1^v v_1 - q_{nt} y_{nt}$$

The price of the final good has been normalized to 1. The resource constraint for the final good is:⁷

$$(15) \quad c_t + \sum_j i_{j,t} + \Phi_d(d_{t+1}) \leq y_t$$

In the labour and capital markets, the quantities supplied by the household must equal the quantities demanded by the firms each period:

$$(16) \quad n^s = n_1^d + n_2^d + n_{nt}^d \quad \text{and} \quad k^s = k_1^d + k_2^d + k_{nt}^d$$

Market clearing for intermediate goods in sector 1 implies:

$$(17) \quad x_1 = x_{1,h} + x_{1,ex}$$

where x_1 is the quantity of intermediate good 1 produced, $x_{1,h}$ the quantity con-

⁷By substituting for the value of the final good you can show that the resource constraint is equivalent to the household's budget constraint. See appendix for details.

sumed at home, and $x_{1,ex}$ the quantity exported. The intermediate good produced in sector 2 is only used to produce the composite good in sector 2, and is not exported.

Foreign Import Demand Equations

The intermediate goods from sector 1 not consumed domestically, and all of the composite goods produced in sector 2 (the production sharing sector) are exported. The foreign demand for goods 1 and 2 is modeled as CES import demand equations:

$$(18) \quad \frac{q_1^{x*}}{q_1^x} = \left(\frac{\omega_1^*}{1 - \omega_1^*} \right) \left(\frac{x_{1,im}^*}{e^z x_1^* - x_{1,ex}^*} \right)^{1-\eta_1^*}, \quad 0 \leq \omega_1^* \leq 1, \eta_1^* \leq 1$$

$$(19) \quad \frac{q_2^{v*}}{q_2^v} = \left(\frac{\pi^*}{1 - \pi^*} \right) \left(\frac{v_{2,im}^*}{e^z v_2^*} \right)^{1-\phi^*}, \quad 0 \leq \pi^* \leq 1, \phi^* \leq 1$$

Here, from the perspective of the foreign economy, ω_1^* and π^* are the CES share parameters, while $1/(1-\eta_1^*)$ and $1/(1-\phi^*)$ are the elasticities of substitution between domestic and imported goods respectively. The prices q_1^{x*} and q_2^{v*} , and the size of the sectors x_1^* , v_2^* are given exogenously. z represents the foreign demand shock, and follows an AR(1) process:

$$(20) \quad z_{t+1} = \rho z_t + \epsilon_t, \quad 0 < \rho < 1$$

where ρ is the persistence parameter and ϵ_t is a normally distributed random variable with mean 0 and variance σ_ϵ^2 .

Equilibrium & Solving the Model

An equilibrium in this model is a sequence of prices and quantities such that the first order conditions to the firms' and the household's maximization problems, and the market clearing conditions are satisfied in every period. The household maximizes (1) with respect to (6), (4), and (2).

To solve the model I use the linearization method now common in the international

business cycle literature (e.g. see Uhlig (1995)). To linearize and simulate the model I use *Dynare*.⁸

IV. Parameterization & Calibration Strategy

This section describes the model parameter values and the calibration strategy employed in the paper. First, I describe the choice of typical international business cycle parameters and the parameters specific to my model; second, I explain the calibration exercise used to match a set of observable moments.

I calibrate the model to Canada and Mexico. Each period corresponds to a quarter.

International Business Cycle Parameters

For parameters typically found in international business cycle models I take common parameter values from the literature. Table 3 lists the benchmark values for the parameters. Each parameter falls within the range of values used in the literature.

TABLE 3—INTERNATIONAL BUSINESS CYCLE PARAMETERS

Parameter	Value	Description
α	0.32	Capital share in output
β	0.99	Discount factor
δ	0.025	Depreciation rate
μ	0.36	Share parameter for consumption and leisure
σ	2.0	Risk aversion parameter
γ	0.50	CES share parameter, tradable and non-tradable goods
ψ_d	0.00074	Portfolio adjustment cost
ρ	0.95	AR(1) persistence parameter
$1/(1 - \eta_1)$	3.0	E_s domestic and imported intermediate 1
$1/(1 - \eta_2)$	3.0	E_s domestic and imported intermediate 2
$1/(1 - \eta_1^*)$	1.5	E_s foreign import demand intermediate 1
$1/(1 - \phi^*)$	1.5	E_s foreign import demand composite 2
$1/(1 - \theta)$	2.0	E_s tradable and non-tradable goods

The portfolio adjustment cost parameter, ψ_d , from Schmitt-Grohe and Uribe (2003), is calibrated in their small open economy model to match observed volatility in the Canadian current-account-to-GDP ratio. α , capital's share of output is set to 0.32, μ , the share parameter for consumption and leisure is set to 0.36, and σ , the coefficient of relative risk aversion is set to 2.0. $\beta = 0.99$ implies an annual risk-free interest rate

⁸Dynare is a software package developed at Cepremap. See Adjemian et al. (2011).

of 4%. Similarly $\delta = 0.025$ implies an annual depreciation rate of 10%. ρ , the AR(1) persistence parameter, is set to 0.95 because business cycle models generally need shocks to be very persistent in order to match observed quantity movements.⁹ The Armington elasticity parameters are set to target the relative volatility of exports to output in the domestic sectors. The model matches the data better when the elasticities in the domestic sectors are higher relative to the foreign import demand equations. In the benchmark model, $1/(1 - \eta_1)$ and $1/(1 - \eta_2)$ are set to 3.0, and $1/(1 - \eta_1^*)$ and $1/(1 - \phi^*)$ are set to 1.5.

TABLE 4—MODEL SPECIFIC PARAMETERS

Parameter	Canada	Mexico	Target Moment
\bar{d}	0.64	0.19	Net external debt share of GDP (\bar{d}/\bar{y})
x_1^*	0.25	0.23	Relative sector size of tradable sector (x_1/x_1^*)
v_2^*	0.12	0.10	Relative sector size of manufacturing (v_2/v_2^*)
ψ_k	1.46	1.82	Relative volatility of investment and GDP (cv_i/cv_y)

Table 4 lists the parameters I choose to target specific moments for Canada and Mexico. The steady state debt-level, \bar{d} , targets the net external debt as a share of GDP. x_1^* and v_2^* , the parameters representing the size of the foreign sectors for intermediate good 1 and composite good 2, are set to match the size of the Canadian and Mexican manufacturing and tradable sectors relative to the US. ψ_k , the capital adjustment cost parameter, is set to match the volatility of investment relative to GDP.

Calibration of Production Sharing

To calibrate production sharing in the model I use the CES share parameters from the domestic Armington aggregators and foreign import demand equations. I target the four moments listed in Table 5.

I use data on services, construction, and utilities to calculate the non-tradable share of GDP. The value added share of exports is calculated by subtracting the weighted average of imported intermediates used in production from gross exports. I assume that the content of imported intermediates used in the production of exports is proportional to the average for each sector. The share of the type 2 composite good

⁹See for example King, Plosser and Rebelo (1988)

in exports is the production sharing content of exports. For the benchmark calibration I assume that auto parts and light vehicles represents Canadian production sharing exports, and that the Maquiladoras sector represents production sharing exports for Mexico. To calculate the value added in the production sharing sector I subtract the weighted average of imported intermediates used in production in the respective sectors.

To implement the calibration I add four additional restrictions to the system of equations characterizing the steady state in the model. I solve for the CES share parameters from the domestic composite good aggregation and the foreign import demand equations simultaneously with the steady state. The calibrated CES share parameters are listed in Table 6.

TABLE 5—CALIBRATION MOMENTS

Benchmark Model Moments	Canada	Mexico
Non-tradable share of GDP	61%	50%
Value added export share of GDP	30%	23%
Type 2 composite share in exports	26%	52%
Value added in type 2 composite	56%	61%

TABLE 6—CALIBRATED PARAMETERS

Parameter	Canada	Mexico	Description
ω_1	0.54	0.51	CES share parameter, home-bias, intermediate 1
ω_2	0.38	0.35	CES share parameter, home-bias, intermediate 2
ω_1^*	0.61	0.89	CES share parameter, foreign home-bias, intermediate 1
π_1^*	0.76	0.80	CES share parameter, foreign home-bias, composite 2

V. Model Results

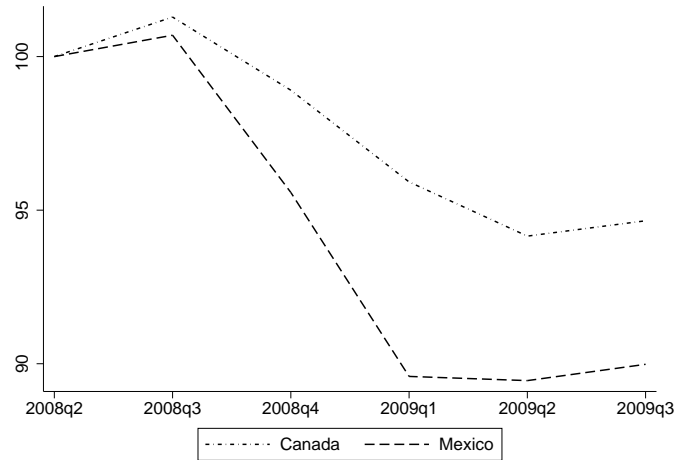
This section uses the model to quantitatively assess the role of trade and production sharing in transmitting the 2008-2009 recession from the US to Canada and Mexico. I restrict my attention to North America because of the region’s strong production and trade linkages.

I first present the benchmark results to quantify the total impact of trade on the transmission process. I then present counterfactual experiments to measure the contribution of production sharing to transmission, and the model’s sensitivity to the

capital adjustment costs. In the first experiment I vary the share of the production sharing export good in total exports, holding the capital adjustment costs constant. In the second experiment I vary the capital adjustment costs while holding the share of production sharing exports constant.

In the quantitative exercises I introduce a shock to the foreign import demand equations. For the benchmark, the shock is calibrated to match the observed terms of trade movements for Canada and Mexico. In the counterfactual experiments I restrict the analysis to Canada. The respective terms of trade shocks are displayed in Figure 6. For the simulations, I focus on the period from Q2 2008 to Q2 2009, and measure the impact of the shock on GDP, trade, investment, and hours.

FIGURE 6. TERMS OF TRADE - CANADA AND MEXICO



Note: Bilateral terms of trade with US, manufactured goods.

Source: US Bureau of Labor Statistics

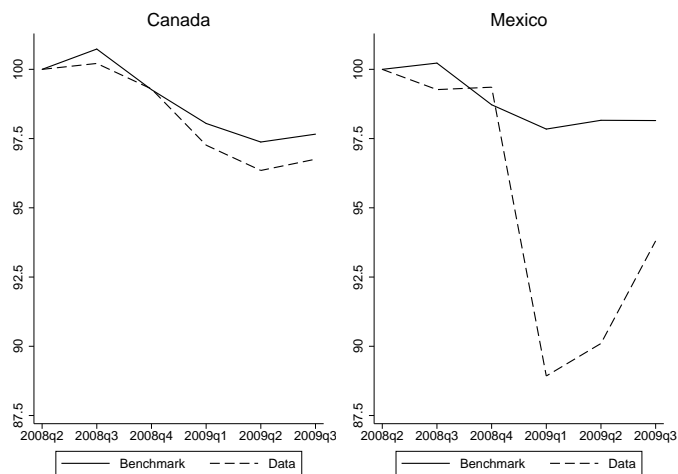
Benchmark Model Results

Canada

The benchmark results are displayed in Figures 7 - 11. For Canada, the model predictions account for 72% of the fall in GDP, 65% of the fall in trade, 54% of the fall in investment, and 20% of the fall in hours worked.

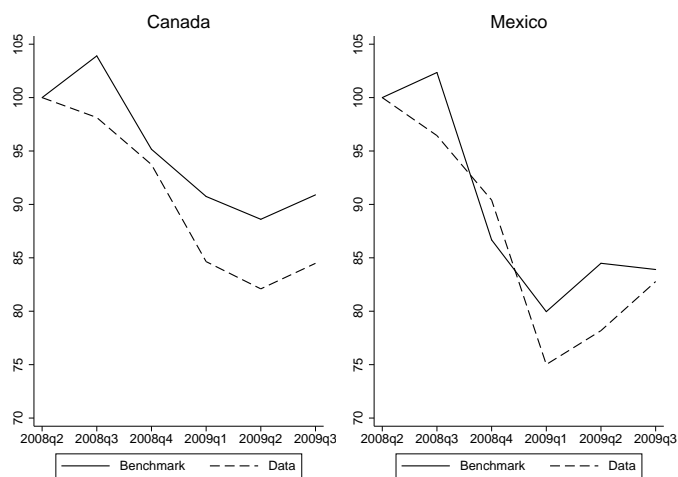
The left panel of Figure 7 displays real GDP for Canada and the model's prediction. The only shock in the model is the import demand shock, which is calibrated to

FIGURE 7. REAL GDP - BENCHMARK MODEL RESULTS AND DATA



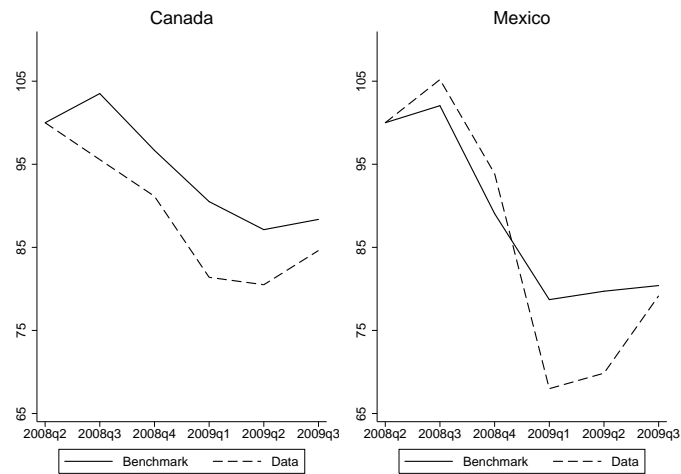
Source: IMF IFS.

FIGURE 8. REAL EXPORTS - BENCHMARK MODEL RESULTS AND DATA



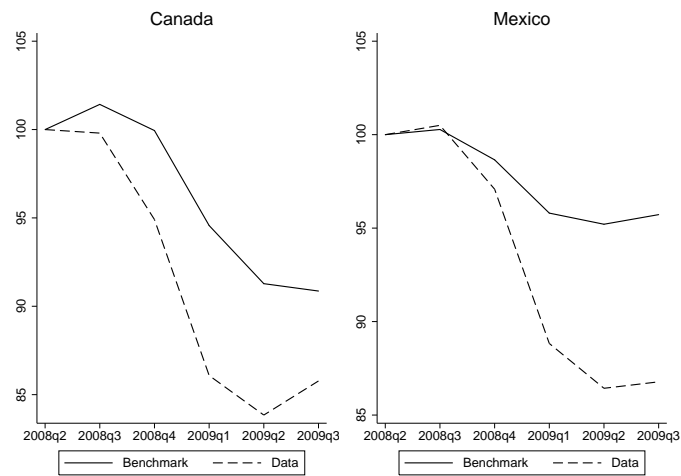
Source: IMF IFS, BLS, and INEGI-BIE.

FIGURE 9. REAL IMPORTS - BENCHMARK MODEL RESULTS AND DATA



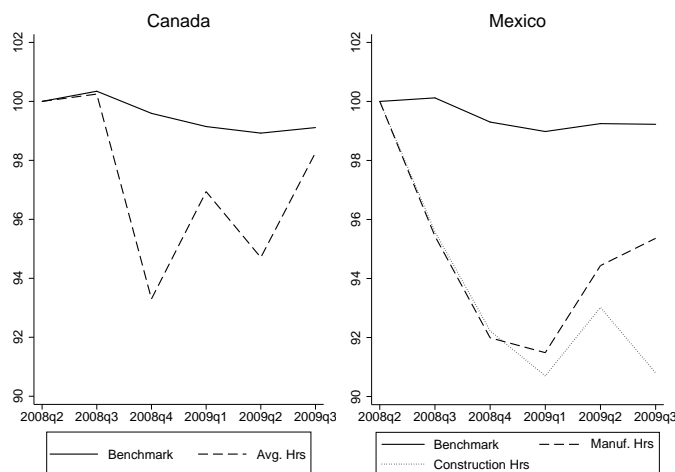
Source: IMF IFS, BLS, and INEGI-BIE.

FIGURE 10. INVESTMENT - BENCHMARK MODEL RESULTS AND DATA



Source: OECD StatExtracts.

FIGURE 11. HOURS WORKED - BENCHMARK MODEL RESULTS AND DATA



Source: Statistics Canada and INEGI-BIE.

match the observed terms-of-trade movement, and the simulated variables in the model inherit this shape. Therefore, the predicted path for GDP has an initial peak at Q3 2008, and then declines until the trough in Q2 2009. 74% of the decline in GDP is from the tradable sector, and the production sharing sector accounts for almost 30% of that decline. In the data, the tradable sector accounts for 97% of the fall in output (counting wholesale and retail trade as tradable sectors), and transportation equipment manufacturing accounts for about 20% of the decline. The fall in output in the non-tradable sector is negligible as moderate declines in output for construction and utilities are offset by a small increase in output for services.

The results for Canadian exports are presented with the data in Figure 8. The shape is from the terms-of-trade, but the initial increase and subsequent fall are more exaggerated than GDP. 38% of the fall in exports is accounted for by the production sharing composite good, and the remainder is accounted for by the generic tradable sector. Exports are more responsive to the demand shock because its composition includes a larger share of the production sharing sector relative to GDP. The shock has a greater impact in the production sharing sector because the domestic economy can only respond to the shock by reallocating productive factors. For the traded intermediate in sector 1, the domestic economy can reallocate productive factors *and* adjust its consumption allocation between the tradable and the non-tradable

composites. The magnitude of this effect depends on the severity of the capital adjustment costs.

Figure 9 shows real imports in the data and model. Imports experience a relatively large decline because the demand for imported intermediates falls following the foreign demand shock. The impact on imports is also affected by the relative size of the production sharing sector as intermediates used in the production sharing sector are more responsive to the demand shock compared to intermediates used in the sector 1 composite.

The model can account for roughly half of the fall in investment for Canada. Figure 10 shows the path for investment and the model prediction. The capital adjustment cost parameter was set such that the model matches the observed volatility of investment relative to output (as measured by the ratio of the coefficients of variation). The benchmark results explain about half of the fall in investment during the recession.

Figure 11 shows hours worked for the model and data. The model falls short in explaining the fall in hours worked, as there is no labour friction in the model. Following a shock to the tradable sectors, there is a moderate fall in aggregate hours worked, and some labour is reallocated into the non-tradable sector. Hours worked in the production sharing sector fall by 11%, in the other tradable sector they fall by 2%, while hours increase by 0.5% in the non-tradable sector. Aggregate hours worked fall by about 1%.

Mexico

For Mexico, the model predictions account for 19% of the fall in GDP, 69% of the fall in trade, 35% of the fall in investment, and 13% of the fall in hours worked.

The calibration for Mexico has a larger production sharing component in exports, but a smaller value added share of exports in GDP. Because of the larger production sharing share in exports, Mexican exports are more responsive to the demand shock than Canadian exports (Figure 8). However, because of the lower value added share of exports in GDP, Mexican GDP is less responsive to a demand shock than Canadian GDP (Figure 7).

71% of the decline in GDP is from the tradable sector, where the production sharing sector accounts for about 68% the decline. The production sharing sector also accounts for 83% of the decline in Mexican exports. These findings suggest that production sharing was more important in transmitting the trade shock to Mexico than

to Canada.

In my model, these results are due to the larger share of production sharing exports in the benchmark calibration for Mexico. The results are consistent with the empirical findings of Di Giovanni and Levchenko (2010). According to their results, the bilateral trade intensity is more important for the impact of trade on comovement for North-North country pairs, while production sharing is more important for North-South pairs.¹⁰ They estimate that vertical linkages can account for 73% of the overall impact of trade on comovement for North-South pairs, but only 17% for North-North pairs.

Overall, the model falls short in explaining the fall in output for Mexico. However, this is actually a positive sign since the Mexican economy experienced additional shocks that are not accounted for by my model. Remittance transfers from migrant workers and tourism receipts fell about 16% over the same period, and the H1N1 flu pandemic which broke out in March 2009 likely exacerbated the recession in Mexico. In addition, Mexico has a large informal sector without a social security system to absorb shocks to the economy. All these factors likely contributed to the much larger fall in GDP experienced by Mexico relative to Canada and the US.

Experiment 1 - The Role of Production Sharing in Transmission

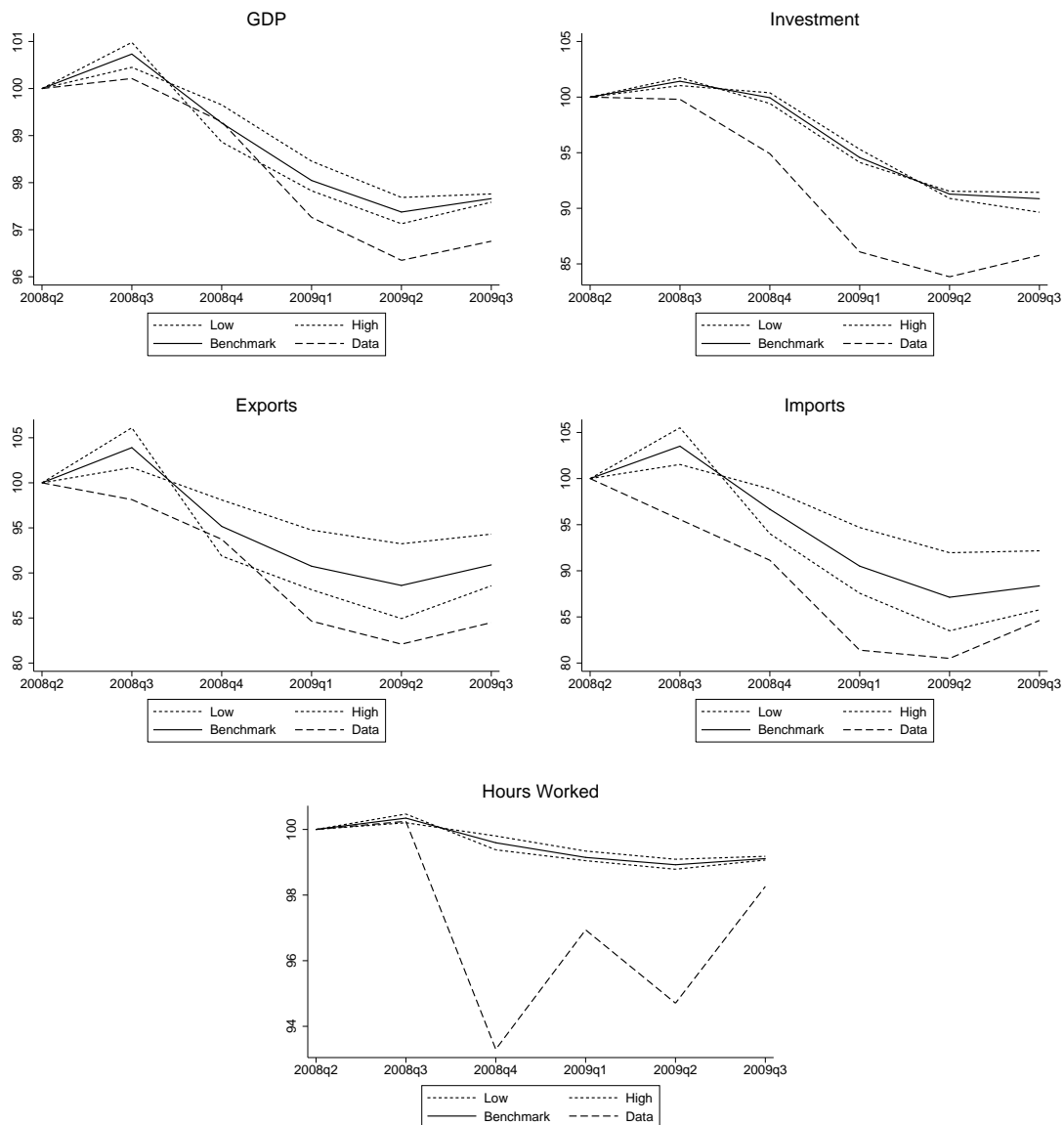
Production sharing may have been a contributing factor to the large fall in trade and the transmission of the US economic slowdown. Di Giovanni and Levchenko (2010) estimate that vertical linkages can account for about 30% of the impact of bilateral trade on aggregate comovement, and Burstein, Kurz and Tesar (2008) suggest that the production sharing intensity is at least as important as trade volume in accounting for bilateral manufacturing output correlations. In this experiment I quantify the relative contribution of production sharing in trade transmission for Canada. I use the Canadian calibration and vary the share of production sharing exports in total exports. I recalibrate the model when setting the production sharing share of exports to zero, and to 39%, a 50% increase relative to the benchmark.

The results are displayed in Figure 12. Comparing the zero production sharing case (labeled 'low') to the benchmark the results suggest that production sharing can

¹⁰Here North refers to OECD countries and South refers to non-OECD countries. Their sample spans the period 1970-1999. Mexico became an OECD member in 1994 and is therefore counted as a non-OECD country in their estimations.

account for 12% of the fall in GDP, and about 40% of the fall in trade in Canada. The impact on investment and hours worked is negligible.

FIGURE 12. EXPERIMENT 1 - GDP, EXPORTS, IMPORTS, INVESTMENT, AND HOURS WORKED



My benchmark results suggest that international production sharing is less important in explaining comovement in output, but more important for trade. This finding indicates that production sharing can be part of the explanation for the large fall in trade relative to output during the recession. However, my results are sensitive to the calibration of production sharing, capital adjustment costs, as well as the choices of Armington elasticity parameters.

In the model, production sharing and the capital adjustment costs amplify the effect of the demand shock because capital becomes 'stuck' in the production sharing sector. As explained in the previous section, in the production sharing sector the shock can only be absorbed by reallocating productive factors. The capital adjustment costs restrict capital movement across sectors, and therefore the efficiency loss to the adjustment costs is greater when the production sharing sector is bigger.

When the production sharing share in exports is increased from 26% to 39% the trade channel explains 79% of the fall in GDP and 84% of the fall in trade for Canada.

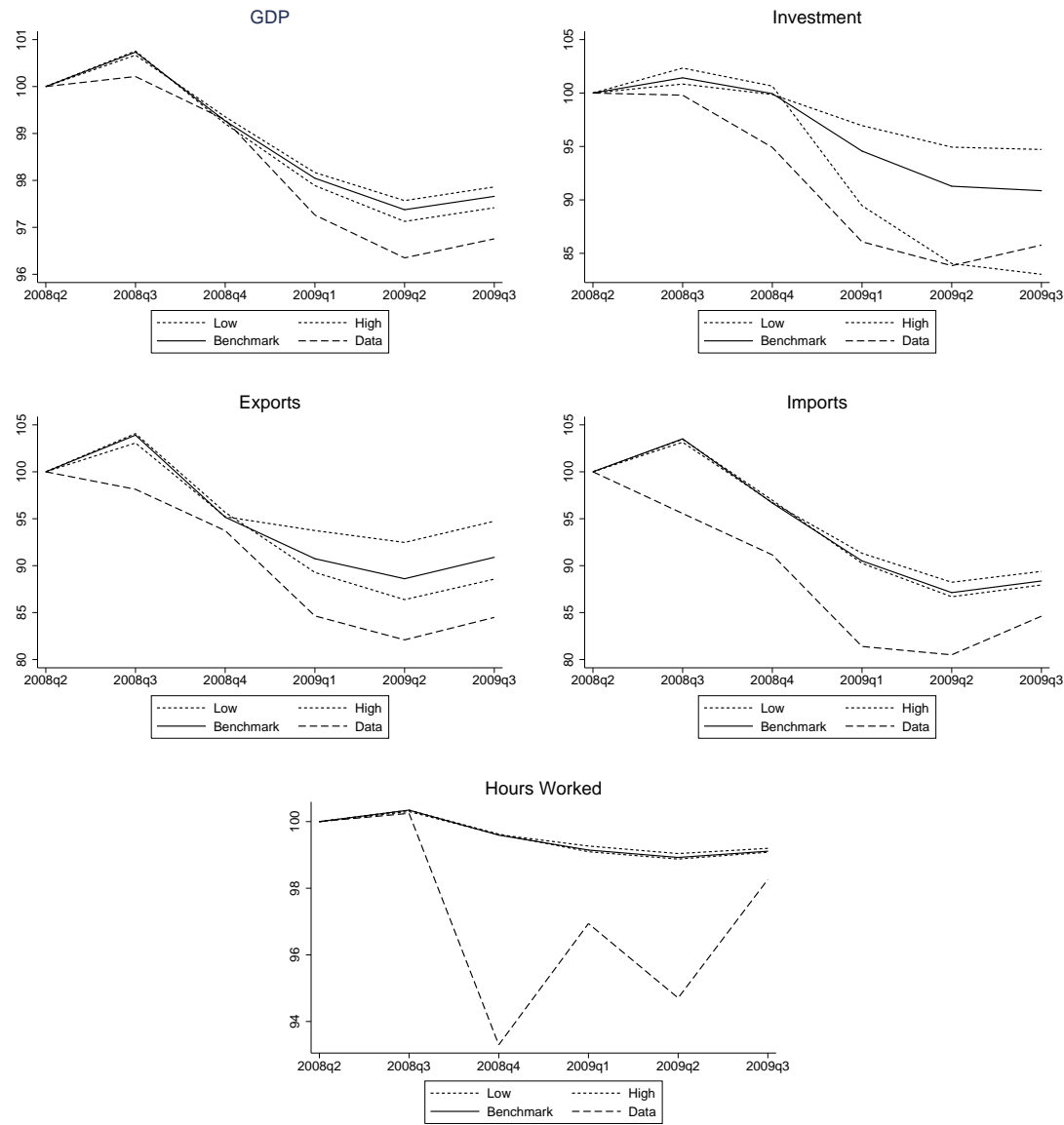
Experiment 2 - Capital Adjustment Costs

In my model, the link between the capital adjustment costs and the production sharing sector plays an important role in generating the transmission dynamics. As all the goods produced in the production sharing sector are exported, the model can only absorb shocks to this sector by reallocating productive factors. The capital adjustment costs slow the reallocation of capital, and the impact of external shocks is exacerbated. In this experiment I quantify the impact of the capital adjustment costs on the transmission of the demand shock in the model. In the Canadian calibration I vary the capital adjustment costs while holding the production sharing share of exports constant at the benchmark level. I recalibrate the model for capital adjustment costs reduced to half, and double that of the benchmark value. This implies volatilities for Investment relative to GDP of 1.98 and 1.03 respectively, compared to the benchmark value of 1.46.

The results are displayed in Figure 13. The results show relatively small changes in GDP, aggregate hours worked, and imports in response to changing the capital adjustment costs. Exports on the other hand experience larger movements as the responsiveness of production sharing exports is directly linked to the mobility of productive factors.

With higher capital adjustment costs, capital movement is more restricted and the

FIGURE 13. EXPERIMENT 2 - GDP, EXPORTS, IMPORTS, INVESTMENT, AND HOURS WORKED



changes in hours worked across sectors are larger. That is, the labour allocation moves more across sectors in order to compensate for the less mobile capital input. The reallocation of labour results in a larger drop in production sharing output and exports.

With lower capital adjustment costs, capital has more freedom to reallocate and investment in the tradable sectors falls more relative to the benchmark. In response to the shock there is less forced reallocation of labour across sectors, and output in the production sharing sector and exports fall less.

VI. Conclusion

The 2008-2009 recession had a large impact on GDP and trade in North America. The results of this paper suggest that trade linkages played a significant role in the transmission of the US recession to its regional trading partners. In the benchmark calibration the model predictions can account for 72% of the fall in output for Canada, 19% for Mexico, and almost two-thirds of the fall in trade. The quantitative experiments suggest that production sharing accounts for about 40% of the fall in trade, but only 12% of the fall in output. Together these results indicate that production sharing may be an important factor in explaining why trade fell so much relative to output during the great recession, and in explaining trade comovement in general.

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MATHEMATICAL APPENDIX

A1. Derivation of resource constraint from household's budget constraint

Let $c_t + \sum_j i_{j,t} + \Phi_d(d_{t+1}) = \Gamma_t$.

For each period t , the budget constraint holds with equality in equilibrium:

$$\Gamma_t + (1 + r_t^d)d_t - d_{t+1} = \sum_j \left(r_{j,t}^k k_{j,t} + w_{j,t} n_{j,t} \right) \quad , \text{ for } j = 1, 2, nt$$

$$\Gamma_t + (1 + r_t^d)d_t - d_{t+1} = q_{1,t}^x x_{1,t} + q_{2,t}^x x_{2,t} + q_{nt,t} y_{nt,t}$$

Substitute for the trade balance, $d_{t+1} = d_t(1 + r_{d,t}) - tb_t$, and drop time subscripts:

$$\Gamma + tb = q_1^x x_1 + q_2^x x_2 + q_{nt} y_{nt}$$

Substitute for *exports* $= q_1^x x_{1,ex} + q_2^v v_{2,ex}$ and *imports* $= q_1^{x*} x_{1,im} + q_2^{x*} x_{2,im}$:

$$\Gamma = q_1^x x_1 + q_2^x x_2 + q_{nt} y_{nt} - q_1^x x_{1,ex} - q_2^v v_{2,ex} + q_1^{x*} x_{1,im} + q_2^{v*} x_{2,im}$$

Note that:

$$q_2^v v_{2,ex} = q_2^x x_2 + q_2^{x*} x_{2,im}$$

$$q_1^x x_{1,h} = q_1^x x_1 - q_1^x x_{1,ex}$$

$$q_1^v v_1 = q_1^x x_1 + q_1^{x*} x_{1,im}$$

Cancel terms and substitute for $q_1^x x_1 + q_1^{x*} x_{1,im}$:

$$\Gamma = q_1^v v_{1,ex} + q_{nt} y_{nt} = y$$

Thus, the period t resource constraint is:

$$c_t + \sum_j i_{j,t} + \Phi_d(d_{t+1}) = y_t$$