2007

2007-4 The Dynamics of Price and Advertising as Signals of Quality

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Working Paper # 2007-4 **November 2007**

RBC Financial Group
Economic Policy Research Institute
EPRI Working Paper Series

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Abstract

A monopolist introduces a new product of either low or high quality. It advertises to make consumers aware of the product and signals product quality using both price and advertising. When consumption does not reveal product quality, price is higher and advertising is lower than they would be if product quality is observable. Price rises and advertising falls as the fraction of aware consumers increases. When consumption reveals product quality, price is higher and advertising is lower than they would be if product quality is observable. Price declines as the fraction of aware consumers increases and advertising follows an inverted U shape. We find support for these empirical predictions from a data set on Direct-to-Consumer advertising on pharmaceutical drugs.

JEL: D82, L15, M37
Keywords: Quality, Signaling, Pricing, Advertising

1 Introduction

When a firm introduces a new product, it advertises to make consumers aware of the product and signals product quality using both price and advertising. Over time, as information about the product diffuses, more and more consumers become aware of the product. This paper examines the impact of increasing product awareness on advertising and on price. We study this issue in a static model under two kinds of information environments. For products like fire alarms and hair loss drugs, product quality is not easily verified since consumption is a highly imperfect signal of product quality. In these cases, consumers who are aware of the product remain uninformed about product quality. Hence, the value of signaling increases as more consumers become aware of the product. For other products like anti-histamine drugs and CD players, consumption reveals product quality. In these cases, there are likely to be three kinds of consumers: informed consumers who are

[1] I would like to thank Kenneth Hendricks for his input and advice, Max Stinchcombe, Randal Watson, Thomas Wiseman, Eugenio Miravete, Mihkel Tombak, and Paul Heidhues for their helpful comments. All errors are mine. Department of Economics, The University of Western Ontario, Social Science Centre, Room 4086 London, Ontario CANADA N6A 5C2; Phone: 519-661-2111; Ext: 85276; E-mail:mayar@uwo.ca.

aware of the product and know its quality, uninformed consumers who are aware of the product but do not know its quality, and unaware consumers who do not know about the product. We model this situation by assuming that consumers who are aware of the product at the beginning of the period are informed, but consumers who learn about the product from advertising during the period are uninformed. In this case, the value of signaling declines as more consumers become aware of the product and are informed.

In characterizing the predictions of the signaling model, we focus on the unique separating equilibrium that survives the Intuitive Criterion of Cho and Kreps (1987). In most cases, this is also the unique equilibrium. Our main findings are as follows. When product awareness does not lead to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable and, as the fraction of aware consumers increases, price rises and advertising decreases. Thus, the distortion on price gets larger and the distortion on advertising gets smaller. When awareness leads to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable. As the fraction of aware consumers increases, price declines and advertising follows an inverted U shape. Thus, the distortion on both price and advertising decreases as more consumers become aware and informed. We find support for these empirical implications from a data set on Direct-to-Consumer advertising on pharmaceutical drugs. After being approved by the FDA in December 1997, annual advertising for the hair loss prescription drug, Propecia, declined over the period 1998 to 2002. On the other hand, annual advertising for Singulair, an allergy prescription drug, over the same period follows an inverted U shape.

There is a voluminous theoretical literature on price and/or advertising as signals of product quality. The seminal paper is by Nelson (1974) and an excellent review of the literature can be found in Bagwell (2005). Cooper and Ross (1984), Bagwell and Riordan (1991), and Linnemer (2002) study signaling models in which all consumers are aware of the product but not all are informed about the quality of the product. Consequently, advertising can signal but not inform. Bagwell and Riordan show that the high quality firm will distort price upward and that the price will decline with the fraction of informed consumers. Linnemer allows the firm to use advertising as well as price to signal quality and characterizes conditions under which the firm will engage in dissipative advertising. He argues that advertising is zero during introductory and mature phases of the product cycle, but positive during the expansion phase. Our main contribution to this strand of the literature is to give advertising a positive role in making consumers aware of the product and to examine how price and advertising will change as more consumer become aware of the product.

Overgaard (1991), Zhao (2000), Orzach et al. (2002), and Bagwell and Over-

gaard (2005) study signaling models in which advertising enhances demand but product quality is not observable. In the language of this paper, advertising makes consumers aware of the product but they remain uninformed. These papers show that the high quality firm will distort price upward and advertising downward relative to the case in which product quality is observable. Our contribution to this literature is the comparative static result that price increases and advertising decreases with the fraction of aware consumers.

The empirical literature on advertising and product quality has failed to find a consistent relationship. For example, Caves and Green (1996) find that the correlation between quality and advertising varies across different markets and products. When testing the relationship, our model suggests that control should be made for two key variables: the age of the product and whether consumption reveals the product quality or not. Horstmann and MacDonald (2003) study data on advertising and price from the compact disc player market. They find that price falls at an accelerating pace and that advertising exhibits an inverted U shape. They were not able to reconcile these results with existing signaling models. However, the results are consistent with the model developed in this paper under the assumption that some of the consumers who are aware of the product are also informed, and that the fraction of informed consumers grows over time.

This paper is organized as follows. In Section 2, we describe the basic model. In Section 3, we study product markets in which consumers may and may not be aware of the product but are never informed about product quality. We characterize the separating equilibrium and obtain closed form solutions for advertising and price as functions of the fraction of aware consumers. In Section 4, we study product markets in which consumers who are aware of the product may also be informed about product quality. We characterize the separating equilibrium and solve for the solution numerically. In section 5, we document several advertising patterns for prescription drugs. Section 6 provides some concluding remarks, while the proofs are collected in the Appendix.

2 The Model

A monopolist manufactures a new product of uncertain quality. For simplicity, we will assume that product quality is either high or low: $q \in \{H, L\},\ H > L$. Let ρ_0 denote the ex ante probability of high quality. The monopolist knows product quality. Production costs of the high (low) quality product are constant and equal to c_H (c_L). We impose the following assumptions on product costs and quality: (i) $c_H > c_L$ and (ii) $c_H/H < c_L/L$. Condition (i) states that high quality product is more costly to produce and condition (ii) implies that cost per unit of quality is

lower for the high quality product i.e. efficiency condition.

There is a continuum of consumers for the new product, each with a potential demand for one unit. A consumer's utility for a product of quality q is given by

$$u(q, p) = \theta q - p$$

where p is the price of the product. Consumers are differentiated in their willingness to pay which is modelled by assuming θ is uniformly distributed on $[0, R]$. All consumers are willing to pay more for the higher quality good.

Some consumers are aware of the product while others are not. Let λ denote the fraction of consumers who are not aware of the product at the beginning of a period. The monopolist can increase the fraction of consumers who are aware of the product by advertising during a period. The probability of an unaware consumer learning about the product from advertising is given by $a/(1+a)$ where a denotes advertising expenditures. Thus, the fraction of potential consumers at the end of a period is

$$1 - \lambda + \frac{a\lambda}{1+a}.$$

In what follows, we distinguish two kinds of product markets. In the first case, we assume consumers who are aware of the product do not know its quality. This situation would apply to a product whose quality is not observable and cannot be learned, at least not until some time elapses. Examples would include hair-loss products or fire alarms. In the second case, we assume that the fraction of consumers who are aware of the product at the beginning of the period (i.e., $1-\lambda$) also know its quality but that the fraction of consumers who learn about the product during the period from advertising (i.e., $\lambda\frac{a}{1+a}$) do not know the quality of the product. This situation would apply to a product like an anti-histamines drug whose quality is not observable but is quickly learned from experience. We will refer to consumers who are aware of the product and know its quality as *informed* consumers and consumers who are aware of the product and do not know its quality as *uninformed.* The key difference between these two cases is in regard to how the monopolist responds to changes in the value of λ, which will be decreasing over time. In the first case, decreases in λ reduces the monopolist's incentive to advertise but increases its incentive to signal high quality; in the second case, decreases in λ reduces the monopolist's incentive to advertise and to signal quality.

Before analyzing these two cases, it will be useful to characterize the solution to the model when product quality is observable so all consumers who are aware of the product are informed. This benchmark case does not imply that all consumers are aware of the existence of the product, but whoever is aware of the product knows its quality. In this case, the monopolist who supplies product quality q chooses price and advertising to maximize

$$\Pi_q^o(p,a) = \left[1 - \lambda + \lambda \frac{a}{1+a}\right](R - \frac{p}{q})(p - c_q) - a,$$

where superscript o stands for the fact that product quality is *observable*. The high quality monopolist's (unique) profit-maximizing price is

$$p_H^o = \frac{RH + c_H}{2}$$

and advertising expenditure is

$$a_H^o = \begin{cases} \frac{\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}}{2\sqrt{H}} & \text{if} \quad \lambda \in (\overline{\lambda}_H, 1] \\ 0 & \text{if} \quad \lambda \in [0, \overline{\lambda}_H] \end{cases},$$

where

$$\overline{\lambda}_H = \left(\frac{2\sqrt{H}}{RH - c_H}\right)^2.$$

Similarly, the solution for the low quality monopolist is

$$p_L^o = \frac{RL + c_L}{2}$$

and

$$a_L^o = \begin{cases} \frac{\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}}{2\sqrt{L}} & \text{if} \quad \lambda \in (\overline{\lambda}_L, 1] \\ 0 & \text{if} \quad \lambda \in [0, \overline{\lambda}_L] \end{cases},$$

where

$$\overline{\lambda}_L = \left(\frac{2\sqrt{L}}{RL - c_L}\right)^2.$$

For each type of monopolist, optimal prices are independent of λ and advertising levels are nondecreasing in λ.

The following lemma compares the solutions of the high and low quality monopolists.

Lemma 1: (i) If $\lambda \in [\overline{\lambda}_H, 1]$, then $a_H^o > a_L^o$ (ii) $p_H^o > p_L^o$.

The Lemma states that the high quality monopolist advertises more and charges a higher price.

3 Case I: No Informed Consumers

In this section, we study the case where only a fraction of the consumers are aware of the product and they are not informed. The monopolist uses advertising to increase the fraction of potential consumers. The high quality monopolist wants to distinguish itself from the low quality monopolist and can use both advertising and price to do so.

Let the consumer assessment of the probability that the quality is H after observing some price and advertising pair, (p, a), be denoted by $\rho(p, a) \in [0, 1]$. How consumers make the inference requires an explanation at this point. First, as it is widely assumed in signaling literature, an unaware consumer who receives an advertisement observes all advertising spending[2]. Second, all aware consumers can observe advertising spending and price. Similarly, in Milgrom and Robert (1986), all consumers are aware of the product and they all observe advertising spending and price. With these assumptions, consumers who become aware of the product observe the firm's total advertising spending and price; thus, they hold the same inferences about the firm's quality, $\rho(p, a)$. The payoff of the monopolist who supplies quality q and chooses (p, a) is

$$\Pi_q(p, a; \rho) = D(p, a; \rho)(p - c_q) - a,$$

where

$$D(p, a; \rho) = \left[1 - \lambda + \lambda \frac{a}{1+a}\right] (R - \frac{p}{\rho H + (1-\rho)L}).$$

Two observations are in line at this point. First, the higher the consumer assessment of the probability that quality is high, the bigger is the payoff of the monopolist. In other words, for given (p, a), an increase in $\rho(p, a)$ increases the payoff of each type of monopolist. Hence, the low quality firm has an incentive to mimic the price and advertising selection of the high quality firm, if this fools potential costumers. Second, when quality is observable, consumers correctly form the belief of $\rho(p, a) = 1$ ($\rho(p, a) = 0$) for any pair of (p, a) for the high quality firm (the low quality firm). In case of an information environment where quality is not observable, we first define our equilibrium concept and present some basic characteristics of separating equilibria.

A *Perfect Bayesian Equilibrium* is a set of strategies $\{(p_L, a_L), (p_H, a_H)\}$ and beliefs $\rho(p, a)$, such that: (i) each strategy is optimal given the beliefs (i.e. (p_q, a_q)

[2]This assumption enables consumers to make the same inference for the product's quality. However, consumers do not have to observe all advertising spending. Only having a positive correlation between the firm's total advertising and consumer's observed advertising would be qualitatively sufficient.

maximizes $\Pi_q(p, a; \rho(p, a)))$, and (ii) the beliefs, derived from the equilibrium strategies, are consistent with Bayes' rule whenever possible. In a *separating equilibrium*, each type plays a different strategy (i.e., $(p_L, a_L) \neq (p_H, a_H)$); hence, uninformed consumers can infer quality from the strategy of the monopolist (i.e. $\rho(p_H, a_H) = 1 > 0 = \rho(p_L, a_L)$). In a *pooling equilibrium*, both types play the same strategy (i.e, $(p_L, a_L) = (p_H, a_H)$); hence, uninformed consumers can infer nothing from the strategy of the monopolist (i.e., $\rho(p_H, a_H) = \rho(p_L, a_L) = \rho_0$)

In a separating equilibrium, the low quality firm is revealed and acts as in observable quality benchmark case, (p_L^o, a_L^o), and earn the corresponding profit $\Pi_L(p_L^o, a_L^o; 0) = \Pi_L^o$. Therefore, to separate itself, the high quality firm must choose a pair (p_H, a_H) which the low quality firm has no incentive to mimic. Hence, (p_H, a_H) is incentive compatible for the low quality firm if

$$\Pi_L(p_H, a_H; 1) \leq \Pi_L(p_L^o, a_L^o; 0) = \Pi_L^o.$$

The following lemma shows that the low quality firm has an incentive to mimic the high quality firm's observable quality price and advertising pair, (p_H^o, a_H^o), if this fools potential costumers.

Lemma 2: $\Pi_L(p_H^o, a_H^o; 1) > \Pi_L(p_L^o, a_L^o; 0)$.

Thus, if the high quality firm is to separate, it must distort its selection (p_H, a_H), away from the observable quality maximizer, $(p_H^o, a_H^o\ )$.

Least-cost separating equilibrium
The only equilibrium outcome that survives Intuitive Criterion of Cho and Kreps (1987) is so-called *least-cost* separating outcome. In this equilibrium, the high quality firm chooses (p_H, a_H) to solve the following problem:

$$\begin{aligned}
\max_{p,a}\ \Pi_H(p, a; 1) &= \left[1 - \lambda + \lambda \frac{a}{1+a}\right](R - \frac{p}{H})(p - c_H) - a \\
&\textit{subject to} \\
\Pi_L(p, a; 1) &= \left[1 - \lambda + \lambda \frac{a}{1+a}\right](R - \frac{p}{H})(p - c_L) - a \leq \Pi_L^o
\end{aligned}$$

where

$$\Pi_L^o = \begin{cases} \frac{(RL-2\sqrt{L}-c_L)^2}{4L} + \frac{(1-\sqrt{\lambda})(RL-c_L)}{\sqrt{L}} & \text{if} \quad \lambda \in (\overline{\lambda}_L, 1] \\ \frac{(RL-c_L)^2}{4L} & \text{if} \quad \lambda \in [0, \overline{\lambda}_L] \end{cases}.$$

The following propositions and corollaries characterize the solution to the high quality firm's maximization problem.

Proposition 1. *In the unique separating equilibrium that satisfies the Intuitive Criteria,* $(p_L^s, a_L^s) = (p_L^o, a_L^o)$ *and*

$$(p_H^s, a_H^s) = \begin{cases} \left(\frac{(RH+c_L)\sqrt{\lambda}+\sqrt{\Delta}}{2\sqrt{\lambda}}, \frac{[\sqrt{\lambda}(RH-c_L)-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}}\right) & \textit{if} \quad \lambda \in (\overline{\lambda}_K, 1] \\ \left(\frac{RH+c_L+\sqrt{\Delta_1}}{2}, 0\right) & \textit{if} \quad \lambda \in (\overline{\lambda}_L, \overline{\lambda}_K] \\ \left(\frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-c_L{}^2)(H-L)}{4L}}, 0\right) & \textit{if} \quad \lambda \in [0, \overline{\lambda}_L] \end{cases}$$

where $\overline{\lambda}_K = \max\{\lambda : a_H^s(\lambda) = 0\}$, and $\overline{\lambda}_K > \overline{\lambda}_L$.

In the separating equilibrium, denoted by superscript s, the high quality monopolist employs advertising and/or price to separate itself depending on the fraction of unaware consumers. In the first region, the fraction of unaware consumers is high enough, (i.e., $\lambda \in (\overline{\lambda}_K, 1]$), that the high quality monopolist efficiently uses both advertising and price to separate itself. In the second region, the fraction of unaware consumers is in an intermediate range (i.e., $\lambda \in (\overline{\lambda}_L, \overline{\lambda}_K]$) so that the monopolist uses only price to separate. In the third region, the fraction of unaware consumers is so low that the monopolist charges a fixed price and does not advertise at all. In what follows, we explain the characteristics and the underlying intuition of the separating equilibrium as the fraction of aware consumers changes.

Corollary 1. p_H^s *is strictly decreasing in* λ *and greater than* p_H^o*.*

The high quality monopolist distorts price above monopoly price and distortion decreases with λ. Why does the high quality firm distort price upward? The answer can be seen by considering the mimicry incentive for the low quality firm. The low quality firm has a lower marginal cost and would like to set a price lower than p_H^o when consumers believe that it is of the high quality. Hence, in order to decrease mimicry incentive of the low quality, the distortion in price should be an increase from p_H^o. When consumers who are aware of the product remain uninformed about product quality, the value of signaling decreases with λ, which in turn decreases price distortion.

Corollary 2. ***(i)*** *If* $\lambda > \overline{\lambda}_K$*, then* a_H^s *is strictly increasing* ***(ii)*** *If* $\lambda > \overline{\lambda}_L$*, then* $a_H^s < a_L^o$*.*

The advertising is lower than it would be if product quality is observable and it falls as the fraction of aware consumers increases. More interestingly, the high quality firm advertises less than the low quality firm in the least-cost separating equilibrium. From Lemma 1, remember that when quality is observable, the high quality firm advertises more than the low quality firm. Why does low advertising expenditure signal product quality? When believed as the the high quality firm, the low quality firm enjoys an increase in profit margin since it can charge a higher price. Then the mimicry incentive of the low quality firm is to expand the market by increasing advertising. By doing this, the low quality firm takes advantage of high profit margin. As a result, the distortion should be a decrease in advertising not an increase. Moreover, advertising falls as the fraction of aware consumers increases for two reasons. First, the need for informative advertising decreases as the fraction of aware consumers increases. Second, the marginal cost of advertising is the same while the marginal benefit of advertising decreases as the fraction of aware consumers increases. Therefore, price becomes a more efficient signal compared to advertising, which in turn results in further decrease in advertising.

Finally, in the Proposition 1, consider the region where $\lambda \in [0, \overline{\lambda}_L)$. Why does the high quality firm set a constant high price and advertise at zero level? In this region, the marginal benefit of advertising is less than its marginal cost since the fraction of unaware consumers is so small. Hence, advertising expenditure is dissipative and can only be used as money burning. It turns out that price is a more efficient signal for the high quality firm compared to dissipative advertising. The cost of money burning is the same for both types of the monopolist while decreasing demand through price hurts the low quality monopolist more due to its higher price margin. That is why, higher quality product does not advertise and charges high and constant price.

Existence of the separating equilibrium

The least-cost separating equilibrium exists, when the high quality firm prefer the equilibrium pair of advertising and price (a_H^s, p_H^s) to any other choice of advertising and price where it is mistakenly considered as the low quality firm, that is,

$$\Pi_H(p_H^s, a_H^s; 1) \geq \max_{p,a} \Pi_H(p, a; 0).$$

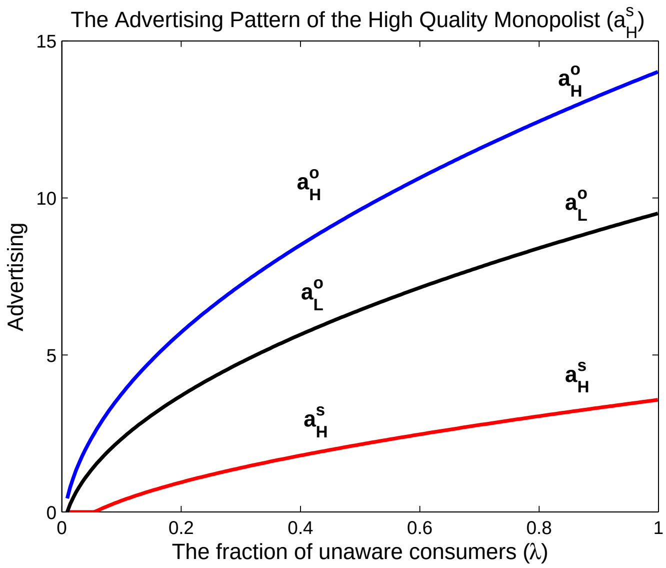
The following proposition characterizes conditions under which the separating equilibrium exists.

Proposition 2. *A separating equilibrium satisfying the Intuitive Criterion exists if (i)(H-L) is not too small and (ii)R is not too small.*

3.1 Numerical Example 1:

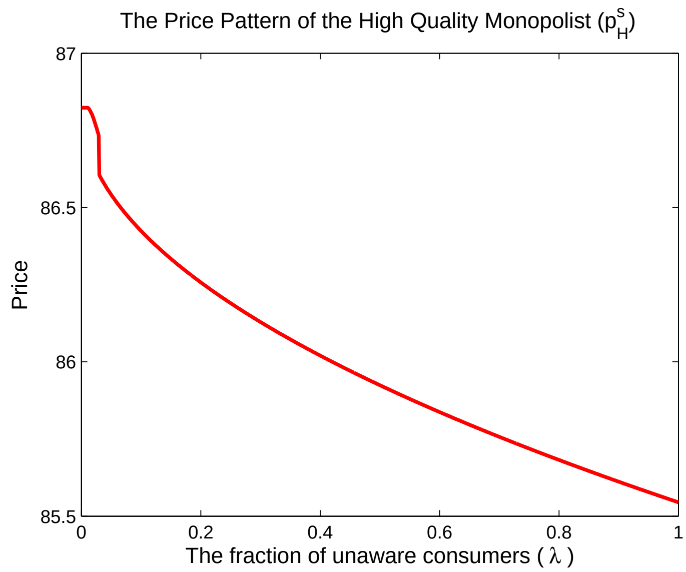
In this section, we propose a fully specified numerical example that give rise to above mentioned least-cost separating equilibrium. Assume that $R = 10, H = 10, c_H = 5, L = 5$, and $c_L = 3$ with which it is easy to check that both the efficiency condition, $(c_H/H < c_L/L\ )$, and the conditions required for the existence are satisfied.

The following figure presents advertising expenditure pattern when (i) quality is observable (ii) quality is not observable and aware consumers are uninformed about product quality. When product quality is not observable, the high quality firm advertises less than the low quality firm; which in turn implies that there is a negative relationship between product quality and advertising. Furthermore, advertising decreases as the fraction of unaware consumers decreases.



The following figure illustrates the unique least-cost separating equilibrium price pattern for the high quality firm.

When consumption does not reveal product quality, price rises as the fraction of unaware consumers decreases.



4 Case II: Informed Consumers

Consumption reveals product quality for some products like anti-histamine drugs and CD players. For such products, there are likely to be three kinds of consumers: informed consumers who are aware of the product and know its quality, uninformed consumers who are aware of the product but do not know its quality, and unaware consumers who do not know about the product. We assume that the fraction of consumers who are aware of the product at the beginning of the period (i.e., $1-\lambda$) also know its quality but that the fraction of consumers who learn about the product during the period from advertising (i.e., $\frac{a\lambda}{1+a}$) do not know the product quality.

When product quality is observable so all consumers who are aware of the product are informed, the profit maximizing price and advertising solutions of the monopolist are characterized in Section 2. To sum up, when quality is observable, the high quality monopolist advertises more and charges a higher price than the low quality firm (i.e., $a_H^o > a_L^o$ and $p_H^o > p_L^o$).

We now consider the case where quality is not observable. In a separating equilibrium, the low quality firm is revealed and acts as if quality is observable, (p_L^o, a_L^o) , and earn the corresponding profit $\Pi_L^o = \Pi_L(p_L^o, a_L^o; 0)$. Therefore, to separate itself, the high quality firm must choose a pair (p_H, a_H) which the low quality firm has no incentive to mimic. Hence, the price and advertising pair (p_H, a_H) is incentive compatible for the low quality firm if

$$\Pi_L(p_H, a_H; 1) = \lambda \frac{a}{1+a}(R - \frac{p}{H})(p - c_L) + (1-\lambda)(R - \frac{p}{L})(p - c_L) - a \leq \Pi_L^o$$

When some consumers have knowledge of the product's quality, the LHS of the inequality represents the low quality firm's mimicry profit ($\Pi_L(p_H, a_H; 1)$). By masquerading as high quality, the low quality monopolist could only deceive the uninformed consumers, represented by $\frac{\lambda a}{1+a}$, but not the informed consumers, represented by $(1-\lambda)$. Moreover, since the LHS of the inequality is increasing in λ, an increase in the fraction of informed consumers decreases the mimicry profit of the low quality monopolist.

In the separating equilibrium, the high quality firm chooses (p_H, a_H) to solve the following problem:

$$\begin{aligned}
\max_{p,a}\ \Pi_H(p,a;1) &= \left[1-\lambda+\lambda\frac{a}{1+a}\right](R-\frac{p}{H})(p-c_H)-a \\
&\textit{subject to} \\
\Pi_L(p,a;1) &= (1-\lambda)(R-\frac{p}{L})(p-c_L)+\lambda\frac{a}{1+a}(R-\frac{p}{H})(p-c_L)-a \leq \Pi_L^o
\end{aligned}$$

It is not possible to get a closed form solution easily, because the first order condition with respect to price (advertising) is a nonlinear function of advertising (price). Instead, in the following proposition, we characterize the properties of the solution to the high quality firm's maximization problem.

Proposition 3. *In the unique separating equilibrium that satisfies the Intuitive Criteria, $(p_L^{si}, a_L^{si}) = (p_L^o, a_L^o)$ and*

$$\begin{aligned}
&(i)\ \text{if}\ \lambda \in (\overline{\lambda}_I, 1], \text{then} && a_H^{si} < a_H^o\ \text{ with }\ \tfrac{d(a_H^o - a_H^{si})}{d\lambda} > 0 \text{ and} \\
& && p_H^{si} > p_H^o\ \text{ with }\ \tfrac{dp_H^{si}}{d\lambda} > 0. \\
&(ii)\ \text{if}\ \lambda \in [0, \overline{\lambda}_I], \text{then} && (p_H^{si}, a_H^{si}) = (p_H^o, a_H^o).
\end{aligned}$$

The intuition goes as follows. As the fraction of informed consumers increases, it becomes more costly for the low quality firm to masquerade as the high quality firm. Thus, it is optimal for the high quality firm to decrease the distortion in both price and advertising. When the fraction of informed consumers reaches a certain threshold, the high quality firm is able to charge its observable quality price and advertising pair while the low quality firm does not mimic and acts as if quality is observable.

Existence of the separating equilibrium

The least-cost separating equilibrium exists, when the high quality firm prefer the

equilibrium pair of advertising and price (a_H^{si}, p_H^{si}) to any other choice of advertising and price where it is mistakenly considered as the low quality firm, that is,

$$\Pi_H(p_H^{si}, a_H^{si}; 1) \geq \max_{p,a} \Pi_H(p, a; 0).$$

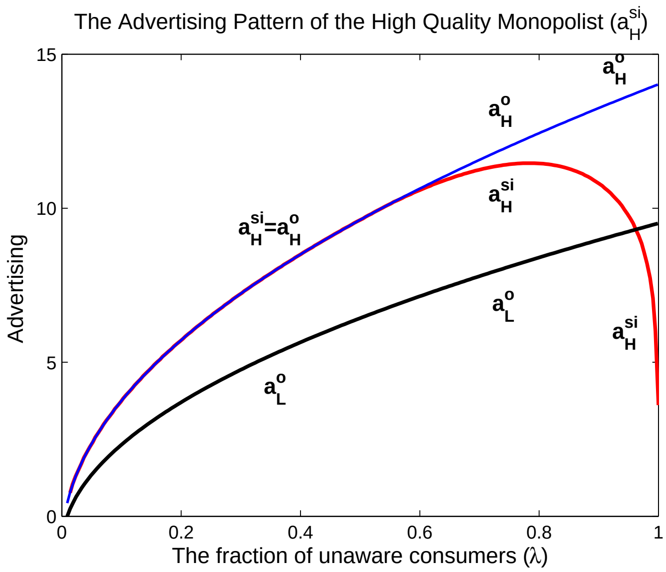
The following proposition characterizes conditions under which the separating equilibrium exists.

Proposition 4. *A separating equilibrium satisfying the Intuitive Criterion exists if (i)(H-L) is not too small and (ii)R is not too small.*

The following section numerically solves the high quality firm's maximization problem and characterizes the separating equilibrium price and advertising levels, (p_H^{si}, a_H^{si}) and $(p_L^{si}, a_L^{si}) = (p_L^o, a_L^o)$, where superscript *si* stands for *separating* when some consumers are *informed*.

4.1 Numerical Example 2:

We assume the same parametrization as in the numerical example of previous section i.e., that $R = 10, H = 10, c_H = 5, L = 5$, and $c_L = 3$. When awareness leads to knowledge of product quality, the following graph illustrates the advertising pattern of the high quality, (p_H^{si}, a_H^{si}), and the low quality firm, (p_L^o, a_L^o) in the separating equilibrium.

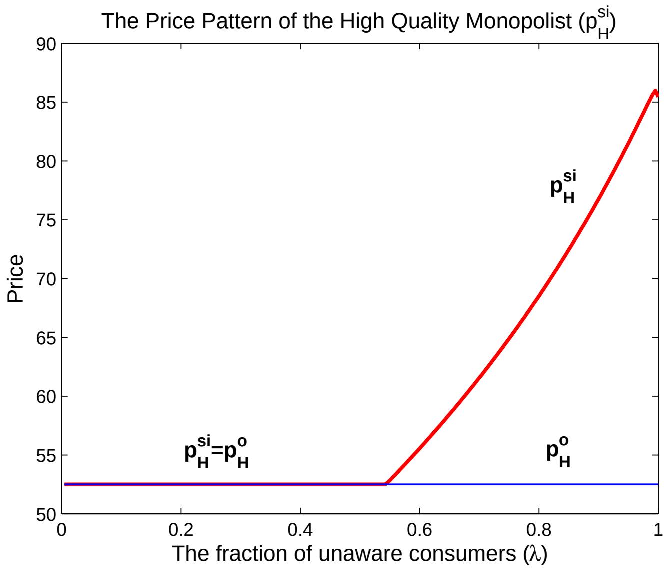


Advertising is lower than it would be if product quality is observable. As the fraction of informed consumers increases, advertising follows an inverted U shape.

The advertising of the high quality monopolist (i.e., a_H^{si}) first increases, but at a decreasing rate and then turns downward and converges to its observable quality advertising level (i.e., a_H^o). More importantly, as more consumers become aware and informed, the distortion in advertising decreases. The reason is simple. As the fraction of informed consumers increases, it becomes more costly for the low quality monopolist to signal a high quality falsely to uninformed consumers. As a result, the high quality monopolist can signal quality with a smaller advertising distortion.

From an empirical perspective, the quality-advertising correlation is generally weak in the early ages of the product, but it becomes stronger as the product matures. The experience with the product could affect not only the magnitude, but also the sign of the quality-advertising correlation. Tentatively, this may also explain the finding of an ambiguous correlation between advertising and quality in Caves and Greene (1996). Since they consider a cross-section of industries and do not account for experience with the product, it is likely that some products are in their early ages and the quality is signaled by modest advertising, while other products are in mature ages and the quality is signaled by extensive advertising. In testing the quality-advertising correlation, one has to control for the age of the product.

When awareness leads to knowledge of product quality, the following graph presents the price pattern in the separating equilibrium.



Price is higher than it would be if product quality is observable. The intuition is as follows. A low quality monopolist would lose more sales from informed consumers by charging a high price; hence, uninformed consumers rationally infer

higher quality from the higher price. As the fraction of aware consumers increases, price falls and converges to observable quality price. As more consumers become aware and informed, distortion in price decreases because it becomes more costly for the low quality monopolist to mimic a high quality and fool uninformed consumers.

Horstmann and MacDonald (2003) analyze a data set on advertising and price from the compact disc player market and find that advertising follows an inverted U shape and that price falls at an accelerating pace. They were not able to explain these results with existing signaling models while their results are consistent with the model developed in this paper.

5 Empirical Predictions

Some stylized facts on variation in advertising expenditure patterns can be found in a data set on Direct-to-Consumer advertising on pharmaceutical drugs. Direct to Consumer Advertising (DTCA) expenditure, obtained from TNS Media Intelligence, consists of individual brand-name drugs. TNS Media Intelligence monitors advertising expenditures for various media such as radio, newspaper, magazine, and TV. Their database includes all advertising expenditures for prescription drugs that appears in these media. We have total monthly advertising expenditure from 1996 to 2002. The FDA's Orange Book is used for each drug's approval date. We find some evidence that the advertising expenditure pattern differs for individual prescription drugs and seems to be affected by whether consumption reveals product quality or not.

The FDA approved Propecia, a hair loss prescription drug [3], in December 1997. Propecia is an example of a good for which consumption does not reveal product quality easily. Our model predicts that advertising decreases over the entire life cycle of the product. Figure 1 presents annual average advertising expenditures for Propecia, which is consistent with the empirical prediction of our model.

In contrast, Singulair, an allergy relief prescription drug, was approved by the FDA in February 1998. The average annual advertising pattern of Singulair is illustrated in the following figure 2. Consumption of Singulair is likely to reveal its quality. If the fraction of informed consumers grows over time, our model predicts that the advertising takes an inverted U shape, which is consistent with the advertising pattern of Singulair.

[3] From Merck's webpage "Propecia was developed to treat mild to moderate male pattern hair loss ...Remembering to take your pill each day is important...Most men see results 3 to 12 months after starting Propecia...If Propecia has not worked within 12 months, further treatment is not likely to help."

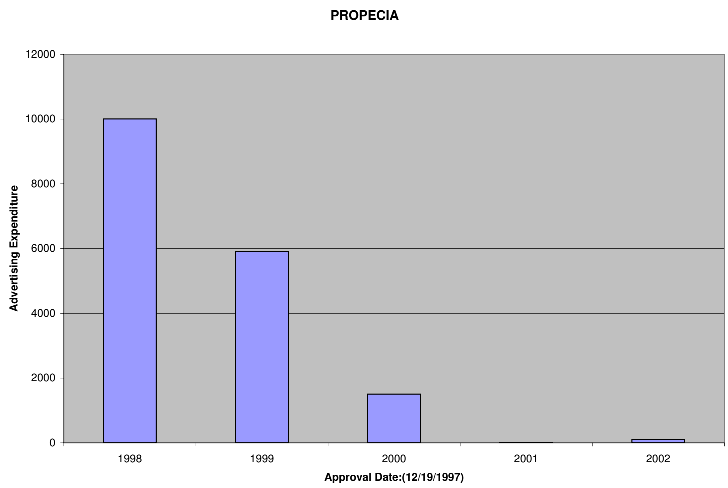


Figure 1:

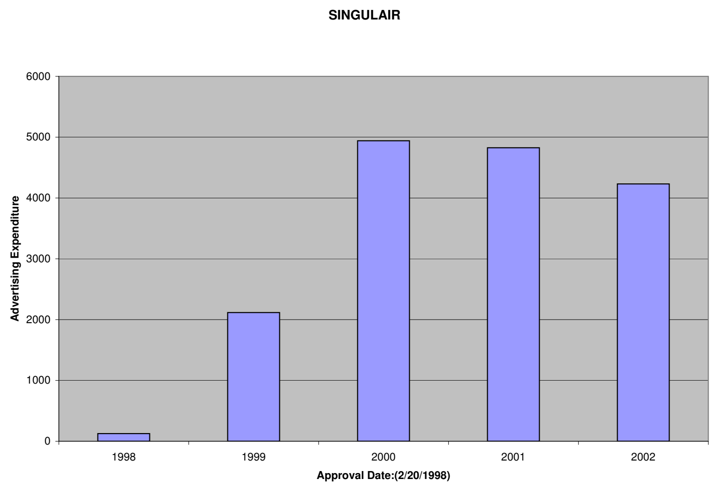


Figure 2:

At this point, the demonstration of average advertising expenditure pattern for a pool of prescription drugs would be an informative exercise. We consider the drugs which have approval dates between 1996 and 1998 and have stayed in the market for at least five years. In most of the cases, the approval date and the launch dates of the products coincide while a few have only a small difference. To calculate the age of the drug, we consider the approval date as a launch date of the drug. There are 25 brand-name drugs in this category. Figure 3 summarizes the average monthly advertising level of these drugs as a function of age of the drug.

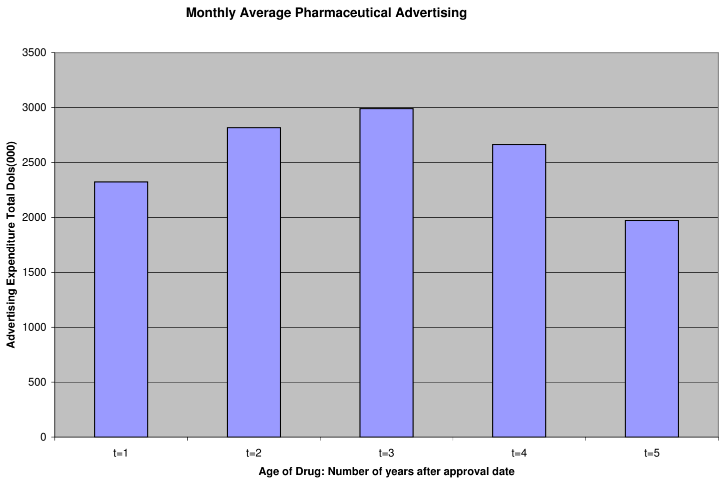


Figure 3:

Significant variation in advertising spending exists among prescription drugs and whether consumption reveals the product quality or not is a key product characteristic in explaining the variation. When testing the time series advertising behavior of experience goods, one should also control for this. With cross-sectional industry data, one has to be extra careful since it is likely that above mentioned key product characteristic may vary drastically across different industries.

6 Conclusion

This paper develops a model that gives advertising a positive role in making consumers aware of the product and examines the impact of increasing product awareness on advertising and on price. It considers two types of information environment depending on whether product awareness does lead to knowledge of product quality or not. When product awareness does not lead to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable and, as the fraction of aware consumers increases, price rises and advertising decreases. When awareness leads to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable. As the fraction of aware consumers increases, price declines and advertising follows an inverted U shape. We also find support for these advertising expenditure patterns from a data set on Direct-to-Consumer advertising on pharmaceutical drugs.

This paper also identifies two key variables in determining the quality-advertising relationship: age of the product and whether consumption reveals the quality of the product or not. From an empirical perspective, to test the quality-advertising relationship, control should be made for these key variables.

7 Appendix

Proof of Lemma 1:

(i) We first show that $\overline{\lambda}_L > \overline{\lambda}_H$

$$\overline{\lambda}_L = (\frac{2\sqrt{L}}{RL - c_L})^2 > \overline{\lambda}_H = (\frac{2\sqrt{H}}{RH - c_H})^2$$

$$\sqrt{L}(RH - c_H) > \sqrt{H}(RL - c_L) \Longleftrightarrow H\sqrt{L}(R - \frac{c_H}{H}) > L\sqrt{H}(R - \frac{c_L}{L})$$

In the last inequality, $H\sqrt{L} > L\sqrt{H}$ is always the case because of the assumption $H > L$. Also, observe that

$$(R - \frac{c_H}{H}) > (R - \frac{c_L}{L}) \Longleftrightarrow \frac{c_L}{L} > \frac{c_H}{H}$$

From the last inequality, we conclude that $\overline{\lambda}_L > \overline{\lambda}_H$ because $\frac{c_H}{H} < \frac{c_L}{L}$ is the efficiency assumption in this paper.

Now, consider the part (i) of Lemma 1. If $\lambda \in (\overline{\lambda}_H, 1]$, then

$$a_H^o > a_L^o \Longleftrightarrow a_H^o = \frac{\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}}{2\sqrt{H}} > a_L^o = \frac{\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}}{2\sqrt{L}}$$

$$2\sqrt{L}\sqrt{\lambda}(RH - c_H) > 2\sqrt{H}\sqrt{\lambda}(RL - c_L) \Longleftrightarrow \sqrt{L}(RH - c_H) > \sqrt{H}(RL - c_L)$$

$$H\sqrt{L}(R - \frac{c_H}{H}) > L\sqrt{H}(R - \frac{c_L}{L}) \tag{1}$$

We know that $H\sqrt{L} > L\sqrt{H}$ since $H > L$. Then, the inequality (**??**) is satisfied if

$$(R - \frac{c_H}{H}) > (R - \frac{c_L}{L}) \Longleftrightarrow \frac{H}{c_H} > \frac{L}{c_L}$$

The efficiency assumption of $\frac{c_L}{L} > \frac{c_H}{H}$ leads to $a_H^o > a_L^o$ for all $\lambda \in (\overline{\lambda}_H, 1]$

(ii)

$$p_H^c > p_H^c \Longleftrightarrow p_H^c = \frac{RH + c_H}{2} > p_H^c = \frac{RL + c_L}{2}$$

$$R(H - L) + c_H - c_L > 0$$

where $H > L$ and $c_H > c_L$. Therefore, the high quality monopolist charges higher prices compared to the low quality monopolist.

Proof of Lemma 2:
Observe first that

$$\Pi_L(p_H^o, a; 1) > \Pi_L(p_L^o, a; 0) \text{ for all } \lambda \in [0,1]$$

$$\Pi_L(p_H^o, a; 1) = [\frac{\lambda a}{1+a} + (1-\lambda)](\frac{RH - c_H}{2H})(\frac{RH + c_H - 2c_L}{2}) - a \tag{2}$$

$$\Pi_L(p_L^o, a; 0) = [\frac{\lambda a}{1+a} + (1-\lambda)](\frac{RL - c_L}{2L})(\frac{RL - c_L}{2}) - a \tag{3}$$

The payoff in the equation (**??**) is bigger than the payoff in the equation (**??**) if following inequality is satisfied

$$(\frac{RH - c_H}{2H})(\frac{RH + c_H - 2c_L}{2}) > (\frac{RL - c_L}{2L})(\frac{RL - c_L}{2}) \tag{4}$$

It is always the case that

$$(\frac{RH + c_H - 2c_L}{2}) > (\frac{RL - c_L}{2}) \Longleftrightarrow R(H-L) + c_H - c_L > 0$$

Moreover, it is also the case that

$$(\frac{RH - c_H}{2H}) > (\frac{RL - c_L}{2L}) \Longleftrightarrow \frac{c_L}{L} > \frac{c_H}{H}$$

As a result, in the inequality (**??**) both elements of the right hand side are bigger than the elements of the left hand side. Consequently, we can conclude that $\Pi_L(p_H^o, a; 1) > \Pi_L(p_L^o, a; 0)$ for all $\lambda \in [0,1]$

Now recall from Lemma 1 that if $\lambda \in (\overline{\lambda}_H, 1]$, then $a_H^o > a_L^o$ while if $\lambda \in [0, \overline{\lambda}_H]$, then $a_H^o = a_L^o = 0$

Start with the region where $\lambda \in [0, \overline{\lambda}_H]$. Since $a_H^o = a_L^o = 0$ (i.e. they are equal), the following inequality is the result of first step in this lemma.

$$\Pi_L(p_H^o, a_H^o; 1) > \Pi_L(p_L^o, a_L^o; 0)$$

Now, consider the region,$\lambda \in (\overline{\lambda}_H, 1]$. One can show that $\Pi_L(p_H^o, a_H^o; 1) > \Pi_L(p_L^o, a_L^o; 0)$ also holds with $a_H^o > a_L^o$ by the following

$$\frac{d\Pi_L(p_H^o, a; 1)}{da} |_{a=a_H^o} > 0$$

This last condition means that if the low quality firm can mimic the high quality firm, it prefers higher level of advertising to a_L^o (i.e. its profit is higher at a_H^o)

$$\frac{d\Pi_L(p_H^o, a; 1)}{da} |_{a=a_H^o} = \frac{\lambda}{(1 + \frac{\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}}{2\sqrt{H}})^2}(\frac{RH - c_H}{2H})(\frac{RH + c_H - 2c_L}{2}) - 1 > 0$$

After some straightforward calculations, the inequality reduces to the following

$$(RH - 2c_L + c_H) > RH - c_H \Longleftrightarrow c_H > c_L$$

Thus, the observable quality price and advertising spending (p_H^o, a_H^o) cannot be a separating equilibrium. Hence, if the high quality firm is to separate, it has to distort price and/or advertising from (p_H^o, a_H^o). In other words, signaling issue is relevant.

Proof of Proposition 1:

The Lagrangian for the maximization problem is the following;

$$\Lambda = [\lambda\frac{a}{1+a} + (1-\lambda)](R - \frac{p}{H})(p - c_H) - a]$$
$$+\mu[\Pi_L^o - [\lambda\frac{a}{1+a} + (1-\lambda)](R - \frac{p}{H})(p - c_L) + a]$$

The first order conditions are;

$$\frac{\partial\Lambda}{\partial p} = [R - \frac{2p}{H} + \frac{c_H}{H}] + \mu[R - \frac{2p}{H} + \frac{c_L}{H}] = 0 \tag{5}$$

$$\frac{\partial\Lambda}{\partial a} = [\frac{\lambda}{(1+a)^2}(R - \frac{p}{H})(p - c_H) - 1] + \mu[\frac{\lambda}{(1+a)^2}(R - \frac{p}{H})(p - c_L) - 1] = 0 \tag{6}$$

$$\frac{\partial\Lambda}{\partial\mu} = \Pi_L^o - [\lambda\frac{a}{1+a} + (1-\lambda)](R - \frac{p}{H})(p - c_L) + a = 0 \tag{7}$$

By solving (**??**) and (**??**)the optimal level of advertising and price in the equilibrium is

$$a = \frac{\sqrt{\lambda}(RH - p) - \sqrt{H}}{\sqrt{H}} \tag{8}$$

In order to find the equilibrium pair of p_H and a_H , we solve equation (**??**) and equation (**??**). The optimal advertising level, a_H, is the solution of the following equation

$$\sqrt{H}{a_H}^2 - [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]a_H - [(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1 - \lambda + a_L^o)^2 - (1-\lambda)\lambda\sqrt{H}] = 0$$

The next step is to find the available roots of this function. The roots are a_H^1 and a_H^2 where $a_H^1 < a_H^2$. Also, there is a corresponding price p_H^1 and p_H^2 for each level of advertising a_H^1 and a_H^2 respectively.

It should be shown that $(a_H^1, p_H^1) = (a_H^s, p_H^s)$ gives higher profit compared to (a_H^2, p_H^2) where $a_H^1 < a_H^2$ and $p_H^1 > p_H^2$ and both (a_H^1, p_H^1) and (a_H^2, p_H^2) yield the same for a mimicking low quality firm. Basically, it is required to show that $\Pi^H(a_H^1, p_H^1; 1) > \Pi_H(a_H^2, p_H^2; 1)$ when $\Pi_L(a_H^1, p_H^1; 1) = \Pi_L(a_H^2, p_H^2; 1)$. Then, we show that (a_H^1, p_H^1) is the least-cost separating equilibrium or the one survives by standard refinement (i.e., Cho and Kreps (1987)). Observe that

$\Pi_H(a_H^1, p_H^1; 1) - \Pi_H(a_H^2, p_H^2; 1)$
$= [\Pi_H(a_H^1, p_H^1; 1) - \Pi_H(a_H^2, p_H^2; 1)] - [\Pi_L(a_H^1, p_H^1; 1) - \Pi_L(a_H^2, p_H^2; 1)]$
$= [c_H - c_L][D(a_H^2, p_H^2; 1) - D(a_H^1, p_H^1; 1)]$
Since $c_H > c_L$, the high-quality firm gains more at (a_H^1, p_H^1) if demand is lower. It is easy to show that, however, demand at (a_H^1, p_H^1) is always lower than demand at (a_H^2, p_H^2) since $a_H^1 < a_H^2$ and $p_H^1 > p_H^2$. This equilibrium can also be called least-cost separating equilibrium. The only plausible root is;

$$a_H^s = a_H^1 = \frac{[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{\Delta}}{2\sqrt{H}}$$

where $\Delta = [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1 - \lambda + a_L^o)^2 - (1-\lambda)\lambda\sqrt{H}]$
Then by using the equality $p_H^s = RH - \frac{(1+a_H^s)\sqrt{H}}{\sqrt{\lambda}}$, the separating equilibrium price is;

$$p_H^s = p_H^1 = \frac{(RH + c_L)\sqrt{\lambda} + \sqrt{\Delta}}{2\sqrt{\lambda}}$$

In case of $\lambda \in [\overline{\lambda}_L, \overline{\lambda}_K)$, a_L^o is positive in the separating equilibrium with the profit $\Pi_L^o = \frac{(1+a_L^o-\lambda)^2}{\lambda} - (1-\lambda) = \frac{[(RL-c_L)^2+4L] - 4\sqrt{L}\sqrt{\lambda}(RL-c_L)}{4L}$. Then, the incentive compatibility condition for low-quality firm is satisfied if

$$(1-\lambda)(R - \frac{p}{H})(p - c_L) = \Pi_L^o = \frac{[(RL - c_L)^2 + 4L] - 4\sqrt{L}\sqrt{\lambda}(RL - c_L)}{4L} \tag{9}$$

The problem reduces to find price levels p_H^s that satisfies the equation (**??**). The only plausible root of this function is

$$p_s^H = \frac{RH + c_L + \sqrt{\Delta_1}}{2}$$

where $\Delta_1 = (RH + c_L)^2 - 4(RHc_L + \frac{H\Pi_L^o}{1-\lambda})$ Now, let's check the boundary values of p_H^s at $\overline{\lambda}_L$ and $\overline{\lambda}_K$. At $\lambda = \overline{\lambda}_L = (\frac{2\sqrt{L}}{RL - c_L})^2$, the incentive compatibility condition (**??**) reduces to

$$(R-\frac{p_H}{H})(p_H-c_L)=\frac{(RL-c_L)^2}{4L}$$

It reduces to

$$p_H^s(\overline{\lambda}_L)=\overline{p}=\frac{RH+c_L}{2}+\sqrt{\frac{(R^2HL-c_L{}^2)(H-L)}{4L}}$$

Next thing to solve is that what would be the value of p_H^s at $\overline{\lambda}_K$. First, remember that at $\overline{\lambda}_K=max\{\lambda : a_H^s(\lambda)=0\}$

$$a_H^s=\frac{[\sqrt{\lambda}(RH-c_L)-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}}=0$$

where $\Delta=[\sqrt{\lambda}(RH-c_L)-2\sqrt{H}]^2-\frac{H}{L}[\sqrt{\lambda}(RL-c_L)-2\sqrt{L}]+4(1-\lambda)\sqrt{\lambda}\sqrt{H}\sqrt{L}[\sqrt{L}(RH-c_L)-\sqrt{H}(RL-c_L)]$

By plugging Δ into $a_H^s=0$, we get the following equation:

$$H(RL-c_L)^2\overline{\lambda}_K+4LH=4\sqrt{\overline{\lambda}_K}\sqrt{H}\sqrt{L}[(1-\overline{\lambda}_K)\sqrt{L}(RH-c_L)+\overline{\lambda}_K\sqrt{H}(RL-c_L)] \tag{10}$$

Now, find the optimal price from the incentive compatibility (**??**) condition of the low quality firm

$(R-\frac{p}{H})(p-c_L)=\frac{[(RL-c_L)^2+4L]-4\sqrt{L}\sqrt{\overline{\lambda}_K}(RL-c_L)}{4L(1-\overline{\lambda}_K)}$ Now, lets use the equation (**??**) in incentive compatibility condition. $(R-\frac{p}{H})(p-c_L)=\sqrt{\overline{\lambda}_K}[\frac{RH-c_L}{\sqrt{H}}-\frac{RL-c_L}{\sqrt{L}}]$

Then, $p_H^s(\overline{\lambda}_K)=\frac{RH+c_L}{2}+\sqrt{\frac{(R^2HL-c_L{}^2)(H-L)}{4L}-H\sqrt{\overline{\lambda}_K}[\frac{RH-c_L}{\sqrt{H}}-\frac{RL-c_L}{\sqrt{L}}]}$

$p_H^s=\frac{RH+c_L}{2}+\sqrt{\frac{(R^2HL-c_L{}^2)(H-L)-4\sqrt{\overline{\lambda}_K}\sqrt{H}\sqrt{L}[\sqrt{L}(RH-c_L)-\sqrt{H}(RL-c_L)]}{4L}}$

This last equation is equal to the $p_H^s(\overline{\lambda}_K)$.

In the region of $\lambda\in[0,\overline{\lambda}_L]$, we start with analyzing the case where all consumers are aware of the product ($\lambda=0$). Consumers consist of only aware-type so that advertising spending only has the role of dissipative signaling (*money burning*) and does not directly enhance demand.

The payoff function of q-quality is

$$\Pi_q(p,a;\rho)=D(p,a;\rho)(p-c_q)-a$$

The profit maximizing equilibrium price and advertising are $p_L^o=\frac{RL+c_L}{2}$ and $a_L^o=0$ for the low-quality firm and $p_H^o=\frac{RH+c_H}{2}$ and $a_H^o=0$ for the high-quality firm and the profits are $\Pi_L^o=\frac{(RL-c_L)^2}{4L}$ and $\Pi_H^o=\frac{(RH-c_H)^2}{4L}$.

In the separating equilibrium, the low-quality firm would play the strategy $(p_L^o,a_L^o)=(\frac{RL+c_L}{2},0)$. The incentive compatibility condition for the low quality firm is as follows

$$\Pi_L(p,a;1) = (R-\tfrac{p}{H})(p-c_L) - a \leq \Pi_L^o = \tfrac{(RL-c_L)^2}{4L} \qquad (IC_L)$$
$$(R-\tfrac{p}{H})(p-c_L) - \tfrac{(RL-c_L)^2}{4L} \leq a \qquad (IC_L)$$

Then, by using IC_L, define a function $a(p)$ as the level of advertising required to deter imitation by a low quality firm for a given price p. Another way to think of the advertising decision is asking the question, how much advertising should the high-quality firm employ just to have the incentive compatibility condition of the low-quality firm satisfied?

$$a(p) = max\{0, (R-\tfrac{p}{H})(p-c_L) - \tfrac{(RL-c_L)^2}{4L}\}$$

It is easy to show that $\Pi_L^o = \frac{(RL-c_L)^2}{4L} = \Pi_L(\underline{p};1) = \Pi_L(\overline{p};1) < \Pi_L(p_L^o;1) < \Pi_L(p_H^o;1) < \Pi_L(p_L^H;1)$ where $\underline{p} < p_L^o < p_L^H < p_H^o < \overline{p}$ along with the values

$$\underline{p} = \tfrac{RH+c_L}{2} - \sqrt{\tfrac{(R^2HL-{c_L}^2)(H-L)}{4L}}$$
$$p_L^o = \tfrac{RL+c_L}{2}$$
$$p_L^H = \tfrac{RH+c_L}{2}$$
$$p_H^o = \tfrac{RH+c_H}{2}$$
$$\overline{p} = \tfrac{RH+c_L}{2} + \sqrt{\tfrac{(R^2HL-{c_L}^2)(H-L)}{4L}}$$

Even under the most favorable beliefs, the low quality firm does not mimic any price below $\underline{p}$ and above $\overline{p}$ since corresponding profit is less than its observable quality profit. Hence, $p \notin (\underline{p}, \overline{p})$, advertising spending is not required to ensure separation. However, given that price is in the region of $(\underline{p}, \overline{p})$, at least an amount $a(p)$ of advertising has to be spent to deter the mimicry of lower quality. Therefore, the maximization problem for the high quality firm could be written in the following form;

$$\begin{array}{rl} \max\limits_{p,a} & \Pi_H(p,a;1) = \frac{(RH-p)(p-c_H)}{H} - a \\ subject\ to & \\ (i) & a \geq a(p) \\ (ii) & p \ \epsilon \ [\underline{p}, \overline{p}] \end{array}$$

The firm will choose the lowest possible advertising, $a = a(p)$, to minimize the cost; then, its profit and the maximization problem reduce to

$$\Pi_H(p,a;1) = \frac{(RH-p)(p-c_H)}{H} - a(p) = \frac{(RH-p)(p-c_H)}{H} - [\frac{(RH-p)(p-c_L)}{H} - \frac{(RL-c_L)^2}{4L}]$$

$$\begin{aligned} \max_{p,a} \Pi_H(p,a;1) &= \frac{(c_H-c_L)p}{H} - \frac{(c_H-c_L)RH}{H} + \frac{(Rl-c_L)^2}{4L} \\ subject\ to & \\ & p \ \epsilon \ [\underline{p}, \overline{p}] \end{aligned}$$

Since the payoff function of the high-quality firm increases in price, it is optimal to increase the price to $\overline{p}$. Also, for the region $p\epsilon(0,\underline{p})\bigcup(\overline{p},\infty)$, the price itself is enough to ensure separation; therefore, it is again optimal to choose $\overline{p}$. To sum up, higher quality would price at $\overline{p}$ and does not advertise in the separating equilibrium.

For $\lambda \in (0, \overline{\lambda}_L]$, the idea of the proof is similar. The low quality firm does not mimic the high quality.

$$(1-\lambda)(R - \frac{p_H}{H})(p_H - c_L) = \Pi_L^o = (1-\lambda)\frac{(RL-c_L)^2}{4L} \qquad IC_L$$

Hence, the equilibrium price is $p_H^s = \overline{p} = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-{c_L}^2)(H-L)}{4L}}$

Proof of Corollary ??:

If $\lambda \in (\overline{\lambda}_K, 1]$, the separating equilibrium price is

$$p_H^s = \frac{(RH+c_L)\sqrt{\lambda}+\sqrt{\Delta}}{2\sqrt{\lambda}} = \frac{RH+c_L}{2} + \frac{\sqrt{\Delta}}{2\sqrt{\lambda}}$$

where $\Delta = \frac{\lambda[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}{L}$

$$p_H^s = \frac{RH+c_L}{2} + \frac{\sqrt{\frac{\lambda[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}{L}}}{2\sqrt{\lambda}}$$

$$p_H^s = \frac{RH+c_L}{2} + \frac{\sqrt{(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}}\sqrt{(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}}}{2\sqrt{L}}$$

Let's take the derivative of p_H^s with respect to λ

$$\frac{dp_H^s}{d\lambda} = \frac{\sqrt{(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}}}{2\sqrt{L}} \frac{1}{\sqrt{(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}}} \frac{-2\sqrt{H}\sqrt{L}}{\sqrt{\lambda}} < 0$$

As λ increases, the high quality firm's price p^s_H decreases.

Now, let's turn to the second part of the Corollary, $p^s_H > p^o_H$. The proof of $p^s_H > p^o_H$ follows immediately from the proof of $a^s_H < a^o_H$.

$$a^s_H - a^o_H = \frac{[(RH-c_L)\sqrt{\lambda}-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}} - \frac{(RH-c_H)\sqrt{\lambda}-2\sqrt{H}}{2\sqrt{H}}$$
$$a^s_H - a^o_H = \frac{(c_H-c_L)\sqrt{\lambda}-\sqrt{\Delta}}{2\sqrt{H}}$$

All we need is the sign of $(c_H - c_L)\sqrt{\lambda} - \sqrt{\Delta}$; hence, multiplying it with some other positive expression is not going to affect the sign.

$$[\sqrt{\Delta} - (c_H - c_L)\sqrt{\lambda}][\sqrt{\Delta} + (c_H - c_L)\sqrt{\lambda}] = \Delta - \lambda(c_H - c_L)^2$$

$$sign\{\Delta - \lambda(c_H - c_L)^2\} = -sign\{(c_H - c_L)\sqrt{\lambda} - \sqrt{\Delta}\}$$

We are interested in the sign of $\Delta - \lambda(c_H - c_L)^2$

$$\begin{aligned}
\Delta - \lambda(c_H - c_L)^2 = \; & [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1-\lambda)\sqrt{\lambda}(RH - c_L) - \sqrt{H}] \\
& -4H(1-\lambda+a^o_L)^2 - 4H(1-\lambda)\lambda - \lambda(c_H - c_L)^2 \\
= \; & [\sqrt{\lambda}(RH - c_H) - 2\sqrt{H} + (c_H - c_L)\sqrt{\lambda}]^2 + 4\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] \\
& -\sqrt{H}(1-\lambda+a^o_L)^2 - (1-\lambda)\lambda\sqrt{H}] - \lambda(c_H - c_L)^2 \\
= \; & [\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}]^2 + 2[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}](c_H - c_L)\sqrt{\lambda} \\
& 4\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1-\lambda+a^o_L)^2 - (1-\lambda)\lambda\sqrt{H}]
\end{aligned}$$

Let's plug a^o_L into the equation and multiply it by L.

$$\begin{aligned}
\Delta - \lambda(c_H - c_L)^2 = \; & L[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}]^2 + 2L[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}](c_H - c_L)\sqrt{\lambda} + \\
& 4L\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1-\lambda+\frac{\sqrt{\lambda}(RL-c_L)-2\sqrt{L}}{2\sqrt{L}})^2 - (1-\lambda)\lambda\sqrt{H}] \\
> \; & 4L\sqrt{H}(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - 4H\sqrt{L}[(1-\lambda)[\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}] \\
& -4HL(1-\lambda)^2 - 4HL(1-\lambda)\lambda \\
= \; & 4\sqrt{L}\sqrt{H}[(1-\lambda)\sqrt{\lambda}[\sqrt{L}(RH - c_H) - \sqrt{H}(RL - c_L) > 0
\end{aligned}$$

First inequality follows form the fact that

$$L[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}]^2 > H[\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}]^2 \quad .$$

The second inequality follows from the fact that

$$\frac{H}{c_H} > \frac{L}{c_L} \quad .$$

Hence $\Delta - \lambda(c_H - c_L)^2 > 0$ and $sign\{(c_H - c_L)\sqrt{\lambda} - \sqrt{\Delta}\} < 0$ so that $a_H^s < a_H^o$

$$p_H^s - p_H^o = \frac{(RH + c_L)\sqrt{\lambda} + \sqrt{\Delta}}{2\sqrt{\lambda}} - \frac{(RH + c_H)}{2}$$

$$p_H^s - p_H^o = \frac{\sqrt{\Delta} - (c_H - c_L)\sqrt{\lambda}}{2\sqrt{\lambda}}$$

In fact, we have just shown that

$$sign\{\sqrt{\Delta} - (c_H - c_L)\sqrt{\lambda}\} > 0$$

Hence, $p_H^s > p_H^o$.

Proof of Corollary ??:

(i) If $\lambda \in (\overline{\lambda}_K, 1]$, the separating equilibrium advertising level is

$$a_H^s = \frac{[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{\Delta}}{2\sqrt{H}}$$

$$a_H^s = \sqrt{\lambda}[\frac{(RH - c_L)}{2\sqrt{H}} - \frac{\sqrt{[(RH - c_L)\sqrt{L} - (RL - c_L)\sqrt{H}][(RH - c_L)\sqrt{L} + (RL - c_L)\sqrt{H} - 4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}}{2\sqrt{H}\sqrt{L}}] - 1$$

$$\frac{da_H^s}{d\lambda} = \frac{\lambda^{-0.5}}{2}[\frac{(RH - c_L)}{2\sqrt{H}} - \frac{\sqrt{[(RH - c_L)\sqrt{L} - (RL - c_L)\sqrt{H}][(RH - c_L)\sqrt{L} + (RL - c_L)\sqrt{H} - 4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}}{2\sqrt{H}\sqrt{L}}] +$$

$$\sqrt{\lambda}[-\frac{\left[((RH - c_L)\sqrt{L} - (RL - c_L)\sqrt{H})((RH - c_L)\sqrt{L} + (RL - c_L)\sqrt{H} - 4\sqrt{\lambda}\sqrt{H}\sqrt{L})\right]^{-0.5}}{2\sqrt{H}\sqrt{L}}(\frac{-2\lambda^{-0.5}\sqrt{H}\sqrt{L}}{2\sqrt{H}\sqrt{L}})] > 0$$

$$\frac{da_H^s}{d\lambda} > 0$$

(ii) If $\lambda \in (\overline{\lambda}_K, 1]$, $a_H^s - a_L^s$ can be written as follows

$$a_H^s - a_L^o = \frac{[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{\Delta} - 2\sqrt{H}a_L^o}{2\sqrt{H}} = \frac{A - B - \sqrt{\Delta}}{2\sqrt{H}}$$

where $A = \sqrt{\lambda}(RH - c_L) - 2\sqrt{H}$ and $B = 2\sqrt{H}a_L^o$.

Now, the task is to write down Δ in terms of A and B.

$$\Delta = [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1 - \lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1 - \lambda + a_L^o)^2 - (1 - \lambda)\lambda\sqrt{H}]$$

$$\Delta = [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1 - \lambda)\sqrt{\lambda}(RH - c_L) - 4H[(a_L^o)^2 + 2(1 - \lambda)a_L^o + 2(1 - \lambda)]]$$

We also know that $\quad a_L^o = \frac{(RL - c_L)\sqrt{\lambda} - 2\sqrt{L}}{2\sqrt{L}} \Leftrightarrow A = (RL - c_L) - 2\sqrt{L}$

$\Delta = A^2 + 4\sqrt{H}(1-\lambda)[A + 2\sqrt{H}] - 4H[(a_L^o)^2 + 2(1-\lambda)a_L] + 8H(1-\lambda)]$
$= A^2 + 4\sqrt{H}(1-\lambda)A - 4H[(a_L^o)^2 + 2(1-\lambda)a_L^o]$
$= [A + 2\sqrt{H}(1-\lambda)]^2 - 4H(1-\lambda)^2 - 4H[a_L + (1-\lambda)]^2 + 4H(1-\lambda)^2$
$= [A + 2\sqrt{H}(1-\lambda)]^2 - [2\sqrt{H}a_L + 2\sqrt{H}(1-\lambda)]^2$
$= [A + 2\sqrt{H}(1-\lambda)]^2 - [B + 2\sqrt{H}(1-\lambda)]^2 = (A-B)[A + B + 4\sqrt{H}(1-\lambda)]$

$$\Delta = (A-B)[A + B + 4\sqrt{H}(1-\lambda)]$$

Now, let's go back to our original problem and substitute for Δ

$$a_H^s - a_L^o = \frac{A-B-\sqrt{\Delta}}{2\sqrt{H}}$$

$$a_H^s - a_L^o = \frac{A-B-\sqrt{(A-B)[A+B+4\sqrt{H}(1-\lambda)]}}{2\sqrt{H}} = \frac{\sqrt{A-B}\left[\sqrt{A-B}-\sqrt{A+B+4\sqrt{H}(1-\lambda)}\right]}{2\sqrt{H}} < 0$$

Proof of Proposition ??

Let's first calculate the high quality firm's profit in the separating equilibrium
If $\lambda \in (\overline{\lambda}_L, 1]$, then $(p_H^s, a_H^s) = (\frac{(RH+c_L)\sqrt{\lambda}+\sqrt{\Delta}}{2\sqrt{\lambda}}, \frac{[(RH-c_L)\sqrt{\lambda}-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}})$ and

$$\Pi_H(p_H^s, a_H^s; 1) = \frac{[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}\lambda][(RH+c_L-2c_H)\sqrt{\lambda}+\sqrt{\Delta}]-2\sqrt{H}\lambda[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}]}{4H\lambda}$$

where $\Delta = \frac{\lambda[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}{L}$

And the H-quality firm's profit is in case of deviation from the separating equilibrium

$$\max_{p,a} \Pi_H(p, a; 0) = \frac{(RL-c_H)^2 - 4\sqrt{L}\sqrt{\lambda}(RL-c_H) + 4L}{4L}$$

The separating equilibrium exists if the H-quality prefers the separating equilibrium pair to any other pair where consumer mistakenly believes that it is of a low quality firm.

$$\Pi_H(p_H^s, a_H^s; 1) > \max_{p,a} \Pi_H(p, a; 0)$$

$$\Pi_H(p_H^s, a_H^s; 1) - \max_{p,a} \Pi_H(p, a; 0) > 0$$

$$\frac{[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}\lambda][(RH+c_L-2c_H)\sqrt{\lambda}+\sqrt{\Delta}]-2\sqrt{H}\lambda[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}]}{4H\lambda}$$
$$-\frac{(RL-c_H)^2 - 4\sqrt{L}\sqrt{\lambda}(RL-c_H) + 4L}{4L} > 0$$

After some algebra, the inequality reduces to the following;

$$\Pi_H(p_H^s,a_H^s;1)-\max\Pi_H(p,a;0)=\frac{(c_H-c_L)\lambda[2L(\frac{\sqrt{\Delta}}{\sqrt{\lambda}}+\lambda c_L+2\sqrt{H}\sqrt{\lambda})-H(c_H+c_L+4\sqrt{L}\sqrt{\lambda})]}{16LH\lambda}>0$$

The increasing marginal cost assumption (i.e., $c_H > c_L$) is necessary for the existence. The following part of the last inequality determines the conditions under which the separating equilibrium exists.

$$[2L(\frac{\sqrt{\Delta}}{\sqrt{\lambda}}+\lambda c_L+2\sqrt{H}\sqrt{\lambda})-H(c_H+c_L+4\sqrt{L}\sqrt{\lambda})]>0$$

$$\Longleftrightarrow$$

$$2L\frac{\sqrt{\Delta}}{\sqrt{\lambda}}>H(c_H+c_L+4\sqrt{L}\sqrt{\lambda})-2L(\lambda c_L+2\sqrt{H}\sqrt{\lambda})$$

Let's take the square of both sides

$$2L^2\frac{\Delta}{\lambda}>[H(c_H+c_L+4\sqrt{L}\sqrt{\lambda})-2L(\lambda c_L+2\sqrt{H}\sqrt{\lambda})]^2$$

Now substitute $\Delta=\frac{\lambda[(R^2LH-c_L{}^2)(H-L)-4\sqrt{\lambda LH}(R\sqrt{LH}-c_L)(\sqrt{H}-\sqrt{L})]}{L}$ into the equation;

$$4L(R^2LH-c_L{}^2)(H-L)>[H(c_H+c_L+4\sqrt{L}\sqrt{\lambda})-2L(\lambda c_L+2\sqrt{H}\sqrt{\lambda})]^2+16L\sqrt{\lambda LH}(R\sqrt{LH}-c_L)(\sqrt{H}-\sqrt{L})$$

This inequality is satisfied if (i) (H-L) is not too small and (2) R is not too small.

Let's now calculate the high quality firm's profit at the proposed separating equilibrium $(p_H^s,a_H^s)=(\overline{p},0)$ when $\lambda\in[0,\overline{\lambda}_L]$

$$\begin{aligned}\Pi_H(p_H^s,a_H^s;1)=\Pi_H(\overline{p},0;1)&=(1-\lambda)(R-\frac{\overline{p}}{H})(\overline{p}-c_H)\\&=(1-\lambda)(\frac{(RL-c_L)^2}{4L}-\frac{(RH-\overline{p})(c_H-c_L)}{H})\end{aligned}$$

The following is the high quality firm's profit when it deviates from the separating equilibrium. Realize that the high quality firm would not advertise in case of deviation because $\lambda\in[0,\overline{\lambda}_L]$ i.e., even the low quality firm with lower marginal cost does not advertise.

$$\max_p\Pi_H(p,0;0)=\max_p(1-\lambda)(R-\frac{p}{L})(p-c_H)=(1-\lambda)\frac{(RL-c_H)^2}{4L}$$

The separating equilibrium exists if the the following condition is satisfied;

$$\Pi_H(p_H^s,a_H^s;1)=(1-\lambda)(\frac{(RL-c_L)^2}{4L}-\frac{(RH-\overline{p})(c_H-c_L)}{H})>\max_p\Pi_H(p,0;0)=(1-\lambda)\frac{(RL-c_H)^2}{4L}$$

$$\frac{(c_H-c_L)[H(2RL-c_L-c_H)-4L(RH-\overline{p})]}{4LH}>0$$

In what follows, we show that the following equation is satisfied if (i) (H-L) is not too small and (2) R is not too small.

$$sign\{H(2RL - c_L - c_H) - 4L(RH - \overline{p})\} > 0$$

Let's substitute $\overline{p} = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL - c_L{}^2)(H-L)}{4L}}$ into the last equation.

$$sign\{-Hc_H - Hc_L + 2Lc_L + 2\sqrt{L}\sqrt{(R^2HL - c_L^2)(H-L)}\}$$

$$sign\{2\sqrt{L}\sqrt{(R^2HL - c_L^2)(H-L)} - [H(c_H + c_L) - 2Lc_L]\}$$

Let's multiply the last equation with the following positive equation

$$2\sqrt{L}\sqrt{(R^2HL - c_L^2)(H-L)} + [H(c_H + c_L) - 2Lc_L]$$

Then the inequality turns out to be;

$$sign\{4L(R^2HL - c_L^2)(H-L) - [H(c_H + c_L) - 2Lc_L]^2\}$$

After some algebra, this equality reduces to the following;

$$sign\{[2R\sqrt{H}L\sqrt{H-L}]^2 - [H(c_H + c_L)]^2\}$$

$$sign\{[2R\sqrt{H}L\sqrt{H-L} - H(c_H + c_L)][2R\sqrt{H}L\sqrt{H-L} + H(c_H + c_L)]\}$$

Finally, the separating equilibrium$(p_H^s, a_H^s) = (\overline{p}, 0)$ exists if

$$sign\{2R\sqrt{H}L\sqrt{H-L} - H(c_H + c_L)\} > 0$$

This inequality holds if (i)(H-L) is not too small (ii) R is not too small.

Finally, if $\lambda \in [\overline{\lambda}_L, \overline{\lambda}_K]$, then $(p_H^s, a_H^s) = (\frac{RH+c_L+\sqrt{\Delta_1}}{2}, 0)$ and $\Pi_H(p_H^s, a_H^s) = (1-\lambda)(R - \frac{p_H^s}{H})(p_H^s - c_H)$. In case of deviation, we have specified the off-the-equilibrium path beliefs such that consumers believe it is of low quality. In what follows, we describe the deviation profit of high quality

$$\Pi_H(p, a; 0) = (1-\lambda) + \lambda\frac{a}{1+a}(R - \frac{p}{L})(p - c_H) - a$$

$$(p_H^L, a_H^L) = (\frac{RL+c_H}{2}, \frac{\sqrt{\lambda}(RL-c_H) - 2\sqrt{L}}{2\sqrt{L}}) .$$

With $\lambda \in [\overline{\lambda}_L, \overline{\lambda}_K]$, there are two separate cases: (i)$a_H^s = 0 < a_H^L < a_L^o$ and (ii)$a_H^L = a_H^s = 0 < a_L^o$

Case(i): The H-quality firm's deviation advertising is positive, i.e. $a_H^s = 0 < a_H^L = \frac{\sqrt{\lambda}(RL-c_H) - 2\sqrt{L}}{2\sqrt{L}} < a_L^o$. Hence, the following inequality $\Pi_H(p_H^s, a_H^s; 1) >$

$\Pi_H(p_H^L, a_H^L; 0)$ where

$$\Pi_H(p_H^s, a_H^s; 1) = (1-\lambda)(R - \frac{p_H^s}{H})(p_H^s - c_H)$$
$$= (1-\lambda)(R - \frac{RH + c_L + \sqrt{\Delta_1}}{2H})(\frac{RH + c_L + \sqrt{\Delta_1}}{2} - c_H)$$
$$\Pi_H(p_H^L, a_H^L; 0) = \tfrac{(RL-c_H)^2 - 4\sqrt{L}\sqrt{\lambda}(RL-c_H)+4L}{4L}$$

After some algebra, the inequality reduces to

$$(c_H - c_L)[2RL - c_L - c_H - 4\sqrt{L}\sqrt{\lambda} - (1-\lambda)(RH - c_L - \sqrt{\Delta_1})] \geq 0$$

Case(ii): By using the previous case where $\lambda \in [0, \overline{\lambda}_L]$, it is easy to show that $\overline{p} > p_H^s > p_H^L$. The following is the incentive compatibility condition for H-quality firm

$$\Pi_H(p_H^s, a_H^s) = (1-\lambda)(R - \frac{p_H^s}{H})(p_H^s - c_H)$$
$$> (1-\lambda)(R - \frac{\overline{p}}{H})(\overline{p} - c_H)$$
$$> \Pi(p_H^L, a_H^L; 0) = (1-\lambda)\frac{(RL - c_H)^2}{4L}$$

However, this is the same inequality with the case where $\lambda \in [0, \overline{\lambda}_L]$; hence, the separating equilibrium exists if (i)(H-L) is not too small (ii) R is not too small.

No pooling equilibrium:

The proof is similar to Bagwell (2005). Before destabilization of pooling equilibria, we first introduce the method by Bagwell and Ramey (1988). Let's define the demand of any firm when the initial prior of being high-quality is ρ_0

$$D(p, a; \rho_0) = \lambda \tfrac{a}{1+a}(R - \tfrac{p}{\rho_0 H + (1-\rho_0)L}) + (1-\lambda)(R - \tfrac{p}{\rho_0 H + (1-\rho_0)L})$$

Let's define the following heuristic payoff function

$$\tilde{\Pi}(p, a; c, \rho) = (p - c)D(p, a; \rho) - a$$

where ρ represents the probability that the firm is of high quality. In fact, there are only two marginal cost levels: c_L and c_H while $\tilde{\Pi}(p, a; c)$ is heuristic payoff function with marginal costs c and demand $D(p, a; 1)$. Let's assume that for any given c, there exists a unique $p(c)$ and $a(c)$ that maximizes the payoff function which is concave in both p and a.

$$\gamma(c) = (p(c), a(c)) = \underset{p,a}{\text{argmax}}\, \tilde{\Pi}(p, a; c, 1)$$

As a special case, $\gamma(c_H) = (p_H^o(c_H), a_H^o(c_H))$ where p_H^o and a_H^o are observable quality price and advertising spending of a high-quality firm respectively. Furthermore, let's assume that, there exists $\overline{c} > c_L$ and $c_L > \underline{c}$ with the boundary condition that $\max\{\Pi_L(\gamma(\overline{c}); 1), \Pi_L(\gamma(\underline{c}); 1)\} < \Pi_L^o = \Pi(\gamma(c_L))$

In a candidate pure strategy pooling equilibrium such as $(\tilde{p}, \tilde{a})$, let's assume that both type of firms play this strategy with probability one and all exposed consumers believe that the firm is indeed a high-quality with probability ρ_0. In our case, under the condition that $c_L < c_H$ and the low-quality firm is indifferent, demand reducing changes shall make the high-quality better off as we have shown before. With $c_L < c_H$ and the boundary conditions , there exists $\dot{c} > c_L$ that gives the following *Indifference* equality

$$(\tilde{p} - c_L)D(\tilde{p}, \tilde{a}; \rho_0) - \tilde{a} - (p(\dot{c}) - c_L)D(p(\dot{c}), a(\dot{c}); 1) + a(\dot{c}) = 0$$

Here, there might be a $\ddot{c} < c_L$ that satisfies the last equality but we prefer $\dot{c}$ since it induces a profitable deviation by decreasing the demand for the high-quality firm while $\ddot{c}$ does the opposite. In order to destabilize the candidate pooling equilibrium all we need is another pair of price and advertising in which high-quality firm becomes better off while low-quality firm is indifferent(Cho and Kreps (1987) refinement).

We also have the following inequality by construction

$$(p(\dot{c}) - \dot{c})D(p(\dot{c}), a(\dot{c}); 1) - a(\dot{c}) - (\tilde{p} - \dot{c})D(\tilde{p}, \tilde{a}; \rho_0) + \tilde{a} > 0$$

By adding up last two equation, We drive the following inequality

$$(\dot{c} - c_L)[D(\tilde{p}, \tilde{a}; \rho_0) - D(p(\dot{c}), a(\dot{c}); 1)] > 0$$

Hence, it is a fact that $D(\tilde{p}, \tilde{a}; \rho_0) > D(p(\dot{c}), a(\dot{c}); 1)$ since $\dot{c} > c_L$

The next step is to show that this pair of strategies $(p(\dot{c}), a(\dot{c}))$ makes the high-quality firm better off compared to pooling strategy $(\tilde{p}, \tilde{a})$. The sign of the following equation determines whether deviation would be profitable for the high-quality firm;

$$(\tilde{p} - c_H)D(\tilde{p}, \tilde{a}; \rho_0) - \tilde{a} - (p(\dot{c}) - c_H)D(p(\dot{c}), a(\dot{c}); 1) + a(\dot{c})$$

Now, subtract the *indifference* equation to get

$$(c_L - c_H)[D(\tilde{p}, \tilde{a}; \rho_0) - D(p(\dot{c}), a(\dot{c}); 1)] < 0$$

Therefore, the high-quality firm has incentive to deviate from the candidate pooling equilibrium pair $(\tilde{p}, \tilde{a})$ to the pair $(p(\dot{c}), a(\dot{c}))$ and also consumers correctly believes that this deviation is an act of high quality firm with Cho and Kreps (1987). So, no pooling equilibria can survive under Cho and Kreps refinement.

Proof of Proposition ??
The incentive compatibility condition of the low quality firm (IC_L)is

$$\Pi_L(p, a; 1) = \lambda\frac{a}{1+a}(R - \frac{p}{H})(p - c_L) + (1-\lambda)(R - \frac{p}{L})(p - c_L) - a \leq \Pi_L^o.$$

If the IC_L is satisfied for the pair (p_H^o, a_H^o), the high quality firm prefers setting (p_H^o, a_H^o) to maximize its payoff for any value $\lambda \in [0,1]$. We characterize the properties of the (p_H^{si}, a_H^{si}) in three steps.

In the first step, we argue that at $\lambda = 1$, aforementioned maximization problem of the high quality firm perfectly coincides with the maximization problem in Section 3. When all consumers are unaware of the product (i.e., $\lambda = 1$), the fraction of aware consumers is zero so that whether aware consumers have the knowledge of product quality does not matter. Therefore, we conclude that the solutions at $\lambda = 1$ have following properties: $a_H^{si} < a_L^o < a_H^o$ and $p_H^{si} > p_H^o > p_L^o$.

In the second step, we show that Π_L^o (i.e., the RHS of IC_L) is decreasing in λ and $\Pi_L(p_H^o, a_H^o; 1)$ (i.e., the LHS of IC_L) is increasing in λ. The optimal observable quality profit for the low quality firm is

$$\Pi_L^o = \begin{cases} \frac{(RL-2\sqrt{L}-c_L)^2}{4L} + \frac{(1-\sqrt{\lambda})(RL-c_L)}{\sqrt{L}} & \text{if} \quad \lambda \in (\overline{\lambda}_L, 1] \\ \frac{(RL-c_L)^2}{4L} & \text{if} \quad \lambda \in [0, \overline{\lambda}_L] \end{cases}.$$

It is easy to show that if $\lambda \in (\overline{\lambda}_L, 1]$, then

$$\frac{d\Pi_L^o}{d\lambda} = -\frac{1}{2}\lambda^{-\frac{1}{2}}\frac{RL - c_L}{\sqrt{L}} < 0$$

and

$$\frac{d^2\Pi_L^o}{d\lambda^2} = \frac{1}{4}\lambda^{-\frac{3}{2}}\frac{RL - c_L}{\sqrt{L}} > 0.$$

Thus, Π_L^o is decreasing and convex in λ. Similarly,

$$\Pi_L(p_H^o, a_H^o; 1) = (\lambda(RH - c_H) - 2\sqrt{\lambda}\sqrt{H})(\frac{RH + c_H - 2c_L}{4H})$$
$$+(1-\lambda)(R(2L - H) - c_H)(\tfrac{RH+c_H-2c_L}{4L})$$

$$\frac{d\Pi_L(p_H^o, a_H^o; 1)}{d\lambda} = ((RH - c_H) - \lambda^{-\frac{1}{2}}\sqrt{H})(\frac{RH + c_H - 2c_L}{4H}) - (R(2L - H) - c_H)(\frac{RH + c_H - 2c_L}{4L})$$

After some simplification, this equality reduces to the following

$$\frac{d\Pi_L(p_H^o, a_H^o; 1)}{d\lambda} = (\frac{H - L}{L})(\frac{HR + c_H}{\sqrt{H}}) - \lambda^{-\frac{1}{2}}$$

At $\lambda = 1$, $\frac{d\Pi_L(p_H^o, a_H^o;1)}{d\lambda} > 0$. Moreover, the $\frac{d\Pi_L(p_H^o, a_H^o;1)}{d\lambda}$ takes the value of zero at

$$\lambda^* = \left[\frac{1}{(\frac{H-L}{L})(\frac{HR+c_H}{\sqrt{H}})}\right]^2$$

It is also easy to show that

$$\frac{d^2\Pi_L(p_H^o, a_H^o; 1)}{d\lambda^2} > 0$$

Hence, if $\lambda \in (\lambda^*, 1]$, $\Pi_L(p_H^o, a_H^o; 1)$ is increasing and convex in λ. To be able to argue that $\Pi_L(p_H^o, a_H^o; 1)$ decreasing in whole region of $\lambda \in [\overline{\lambda}_H, 1]$, we need to show that the value λ^* is less than $\overline{\lambda}_H$.

$$\overline{\lambda}_H = (\frac{2\sqrt{H}}{RH - c_H})^2 > \lambda^* = \left[\frac{1}{(\frac{H-L}{L})(\frac{RH+c_H}{\sqrt{H}})}\right]^2$$

$$\Longleftrightarrow$$

$$\left[\frac{2\sqrt{H}}{RH - c_H}\right] > \left[\frac{1}{(\frac{H-L}{L})(\frac{RH+c_H}{\sqrt{H}})}\right]$$

$$\Longleftrightarrow$$

$$2(\frac{H-L}{L})(HR + c_H) > (RH - c_H)$$

Thus, λ^* is always smaller than $\overline{\lambda}_H$.

In the third step, we argue that (i) at $\lambda = 1$, Π_L^o (i.e., the RHS of IC_L) is less than $\Pi_L(p_H^o, a_H^o; 1)$ (i.e., the LHS of IC_L) (ii) at $\lambda = \overline{\lambda}_H$, Π_L^o is bigger than $\Pi_L(p_H^o, a_H^o; 1)$. At $\lambda = 1$, the maximization problem in section 3 coincides with the one we are analyzing here. From Lemma 2, it is the case that $\Pi_L(p_H^o, a_H^o; 1) > \Pi_L^o$ at $\lambda = 1$. To show part (ii), remember that if $\lambda \leq \overline{\lambda}_H$, then $a_H^o = a_L^o = 0$. By definition of p_L^o, the following inequality is always satisfied at $\lambda = \overline{\lambda}_H$

$$\Pi_L(p_H^o, 0; 1) = (1-\lambda)(R - \frac{p_H^o}{L})(p_H^o - c_L) < \Pi_L^o = (1-\lambda)(R - \frac{p_L^o}{L})(p_L^o - c_L)$$

Now, by combining step two and three, we can conclude that there exists a unique $\overline{\lambda}_I$ such that

$$\begin{aligned} \Pi_L(p_H^o, a_H^o; 1) &< \Pi_L^o \quad \text{if} \quad \lambda \in [0, \overline{\lambda}_I) \\ \Pi_L(p_H^o, a_H^o; 1) &= \Pi_L^o \quad \text{if} \quad \lambda = \overline{\lambda}_I \\ \Pi_L(p_H^o, a_H^o; 1) &> \Pi_L^o \quad \text{if} \quad \lambda \in (\overline{\lambda}_I, 1] \end{aligned}$$

Thus, the high quality firm has to distort price and advertising from (p_H^o, a_H^o) if $\lambda \in (\overline{\lambda}_I, 1]$. For all other values of λ, it sets its optimal observable price and advertising (p_H^o, a_H^o) and there is no distortion.

Now, from second step, we know that $\Pi_L^o - \Pi_L(p_H^o, a_H^o; 1)$ is maximized at $\lambda = 1$ and decreases as λ decreases. Basically, the distortion in both price and advertising is highest when there is no informed consumer. Over time, as the fraction of informed consumers increases, the distortion in both price and advertising decreases.

Proof of Proposition ??

The nice property of the solution pair (p_H^{si}, a_H^{si}) is that the distortion decreases as λ decreases. In other words, if one can find the conditions under which

$$\Pi_H(p_H^{si}, a_H^{si}; 1) > \max_{p,a} \Pi_H(p, a; 0)$$

at $\lambda = 1$. Then, as the fraction of informed consumers increases, distortion decreases and $\Pi_H(p_H^{si}, a_H^{si}; 1)$ increases. As a result, the existence is satisfied for all other values of λ under the same conditions. However, the conditions under which this inequality is satisfied at $\lambda = 1$ is already characterized in Section 3. The separating equilibrium (p_H^{si}, a_H^{si}) exists if (i) (H-L) is not too small (ii) R is not too small.

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