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Water Resources Decision Making Under Uncertainty

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WATER RESOURCES DECISION MAKING UNDER UNCERTAINTY

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ABSTRACT

Uncertainty is in part about variability in relation to the physical characteristics of water resources systems. But uncertainty is also about ambiguity (Simonovic, 2009). Both variability and ambiguity are associated with a lack of clarity because of the behaviour of all system components, a lack of data, a lack of detail, a lack of structure to consider water resources management problems, working and framing assumptions being used to consider the problems, known and unknown sources of bias, and ignorance about how much effort it is worth expending to clarify the management situation. Climate change, addressed in this research project (CFCAS, 2008), is another important source of uncertainty that contributes to the variability in the input variables for water resources management.

This report presents a set of examples that illustrate (a) probabilistic and (b) fuzzy set approaches for solving various water resources management problems. The main goal of this report is to demonstrate how information provided to water resources decision makers can be improved by using the tools that incorporate risk and uncertainty. The uncertainty associated with water resources decision making problems is quantified using probabilistic and fuzzy set approaches. A set of selected examples are presented to illustrate the application of probabilistic and fuzzy simulation, optimization, and multi-objective analysis to water resources design, planning and operations. Selected examples include dike design, sewer pipe design, optimal operations of a single purpose reservoir, and planning of a multi-purpose reservoir system. Demonstrated probabilistic and fuzzy tools can be easily adapted to many other water resources decision making problems.

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1. Introduction

When dealing with water resources infrastructure design and management the decision process is subject to many uncertainties. It is then of great importance to provide decision makers with tools that incorporate risk and uncertainty in decisions. The goal of this report is to demonstrate how information provided to decision makers can be improved through the use of probabilistic and fuzzy approach to deal with risk and uncertainty in water resources management. The inclusion of such information can lead to more informed decisions.

1.1 The decision making process

The water resources decision making is a complex process that involves management of risk that may arise from various sources of uncertainty. Furthermore, the decision making process is subject to participation of multiple or single decision makers from various disciplines and responsibilities resulting in conflicting goals and decision attitudes. The decision making process offers a framework for making decisions in systematic and rational ways (Simonović, 2009).

The decision making process is an iterative process. The decision making process used for the implementation in water resource systems management consists of 7 practical steps adopted from Jewell (1986). They consist of:

1. Definition of the problem;
2. Gathering data;
3. Development of criteria for evaluating alternatives;
4. Formulation of alternatives;
5. Evaluation of alternatives;
6. Choosing the best alternative;
7. Final design/plan implementation.

The decision making process sometimes has several stages simultaneously being considered, facilitating feedback and allowing a natural progression of the problem solving process.

The **problem definition** should be as general as possible in order to allow for largest scope of solutions or alternatives to be considered. A key part of the problem definition is identifying the systems or subsystems that the problem is a part of, known as the environment of the problem. The factors considered in analyzing the problem are limited by the environment. Furthermore, the problem over which there is a reasonable chance of maintaining control should be the problem defined. The problem definition may require careful investigation and iterations as more information as a result of the decision process becomes available.

Gathering data for water resources systems management may be required in several stages of the decision making process. Some background data is required in order to be able to formulate a problem and the additional data gathering continue all the way to the final stage of the decision making process - the final design or plan implementation. When feedback is required, the data previously acquired can assist in redefining the problem.

Development of criteria for evaluating alternatives is required to measure the degree of attainment of system objectives. The criteria developed facilitate the rational choice of an alternative (from a wide range of feasible alternatives) that will accomplish the established objectives. Economic criteria such as cost-benefit can be used in this process. In reality water problems are of complex nature typically with multiple objectives. In some cases the objectives can be formulated as constraints and the optimal solution can be obtained in accordance to remaining objectives. In most water resources problems, cost effectiveness is still considered as the primary criteria.

The **formulation of alternatives** essentially involves the development of system model that will be used in decision-making, in conjunction with the criteria for evaluation of the outcomes. If possible these models should be mathematical in nature. Where mathematical quantification is not appropriate a more subjective models could be constructed.

Evaluation of alternatives is done using various mathematical techniques. They include the simplex method for linear programming(LP) optimization models, the various methods for solving ordinary and partial differential equations or systems of differential equations, matrix algebra, various economic analyses and deterministic or stochastic computer simulation.

Subjective analysis techniques may be used for the subjective analysis of intangibles. The appropriate analysis procedures for a particular problem will generate a set of solutions for the alternatives which can be tested according to the established evaluation criteria.

The choice of the **best alternative** from among those analyzed must be made in the context of the objectives and evaluation criteria previously established. It must take into account non-quantifiable aspects of the problem such as aesthetics and political considerations. The chosen alternative will greatly influence the development of the final plan/design, and will determine in large part the implementation of the suggested solution.

The **final plan/design/operation strategy** are technical steps which are conducted within the constraints and specifications developed in the earlier stages of the decision making process. The result is a report with clean and concise recommendations for the problem solution.

Decision making process in water resources management is a very broad. Let us consider a problem of selecting an appropriate dike height in the design of a flood protection system. It should be noted that this is just one decision that needs to be made by decision makers out of many needed to finalize a dike design. Where to build the dike? How high? What slope, width and material should be used? These are just examples of other questions that the decision making process will have to deal with.

Going back to the problem definition of selecting an appropriate dike height, the decision maker must be able to identify the problem environment, factors that can be used to develop a set of decision making criteria. For example, the economic concerns may include benefits from reduced inundation; the environment implications may include negative effects such as downstream flooding; the soil condition (poor soil may result in decrease of the dike height). The alternatives are formulated based on the specific criteria like costs, benefits, settlement (soil condition), environmental impacts, etc. A series of either continuous or discrete alternatives is developed and evaluated. The selection of an optimal solution is made from a set of feasible solutions that maximizes/minimizes a set of objective functions representing selected criteria. For example as the dike height increases flood protection increases and so thus the potential benefit from flood damage reduction. However, as the dike height increases the construction cost also increases. Similarly as the dike height increases, the more significant are the environmental impacts due to downstream flooding. As can be seen, multiple criteria govern a problem solution, and they may be of conflicting nature. Various toolsets are used to aid the decision makers in the selection of the best alternative.

1.2 UNCERTAINTY IN THE WATER RESOURCES DECISION MAKING PROCESS

Uncertainty is in all stages of the decision making process. To understand the uncertainties requires understanding of the sources of uncertainty. Uncertainty in water resource management can be divided into two basic forms: uncertainty caused by inherent hydrologic variability and uncertainty caused by a fundamental lack of knowledge (Simonović, 2009). The first form is described as stochastic variability, and the second one as ambiguity. The variability is caused by the inherent fluctuations in the quantity of interest (hydrological variables). The three main sources of variability are temporal, spatial and individual heterogeneity. Temporal variability occurs when values fluctuate over time. Spatial variability occurs when values are dependent on the location of an area. The third category encompasses all other sources of variability, not mentioned. In water resource management variability is mainly associated with the spatial and temporal variation of hydrological variables (precipitation, river flow, water quality, etc.).

The more elusive type of uncertainty is ambiguity. It occurs when the particular values that are of interest cannot be assessed with complete confidence because of a lack of understanding or limitations of knowledge. Three sources of ambiguity are from model and structural uncertainty, parameter uncertainty and decision uncertainty. Model and structural uncertainty arise due to an attempt to form a simplified expression of a real world process which as a result introduces uncertainty though oversimplification, approximation and failure to capture the true characteristics of the process under investigation. Parameter uncertainty involves the fine tuning of a model, and thus cannot cause the large variations as in model uncertainty. Common example of parameter uncertainty is random direct measurement error due to imprecise instruments and systematic error - error as a result of subjective judgment.

The final category of ambiguity is decision uncertainty which arises when there is controversy concerning how to compare and weigh social objectives. The first source of decision uncertainty is due to risk measurement (measure must be technically correct, measurable and meaningful). Second source of decision uncertainty deals with deciding the social cost of risk (transforming risk measures into comparable quantities). The difficulties in this process are clearly illustrated in the concept of developing a monetary equivalent for the value of life in flood control analysis. The quantification of social values is the third source of uncertainty. Once a risk measure and the cost of risk are generated, controversy still remains over what level of risk is acceptable. This level is dependent upon the attitude of society to risk.

The decision making process is subject to uncertainty coming from both sources, ambiguity and variability. Table 1.1 illustrates an attempt to identify the sources of uncertainty associated with each stage of the decision making process. For clarity a graphical representation of Table 1.1 is presented in Figure 1.1.

TABLE 1.1- UNCERTAINTY SOURCES IN WATER RESOURCES DECISION MAKING

1. **Definition of the problem**;→ ambiguity, more precisely decision uncertainty as risk measure and cost of risk are fundamental in problem recognition; the problem existence may be an area of controversy depending on decision maker's attitude; accuracy and completeness of data.
2. **Gathering data**;→ variability due to stochastic nature of physical variables (temporal, spatial, etc.); ambiguity due to direct measurements or imprecise instruments.
3. **Development of criteria for evaluating alternatives**;→ ambiguity (or more precisely decision uncertainty); attitude of society and decision makers; risk perception; quantification of social values.
4. **Formulation of alternatives**;→ model and structural uncertainty (ambiguity); accuracy and completeness of data.
5. **Evaluation of alternatives**; → variability from stochastic nature of real world problem;

	decision ambiguity from criteria development; model and structural uncertainty from the formulation of alternatives.
6.	**Choosing the best alternative**;→ parameter , model and structural ambiguity due to the fact that the accuracy of the toolset used for selecting the optimal alternative is based on the best available technique; decision uncertainty; risk perception; quantification of social values.
7.	**Final design/plan implementation;** → accuracy and completeness of data; model and structural ambiguity; decision uncertainty.

Different stages of the decision making process may directly be subject to only one source of uncertainty. However, indirectly many additional sources of uncertainty are introduced due to the nature of the process allowing feedback relationships between various stages. . Thus each decision making process stage may be subject to multiple sources of uncertainty. Initial complexity of the decision making process is challenging enough for most decision makers. Combining all the sources of uncertainty makes the process even more difficult. All decisions have to be made based on partial information with uncertainty.

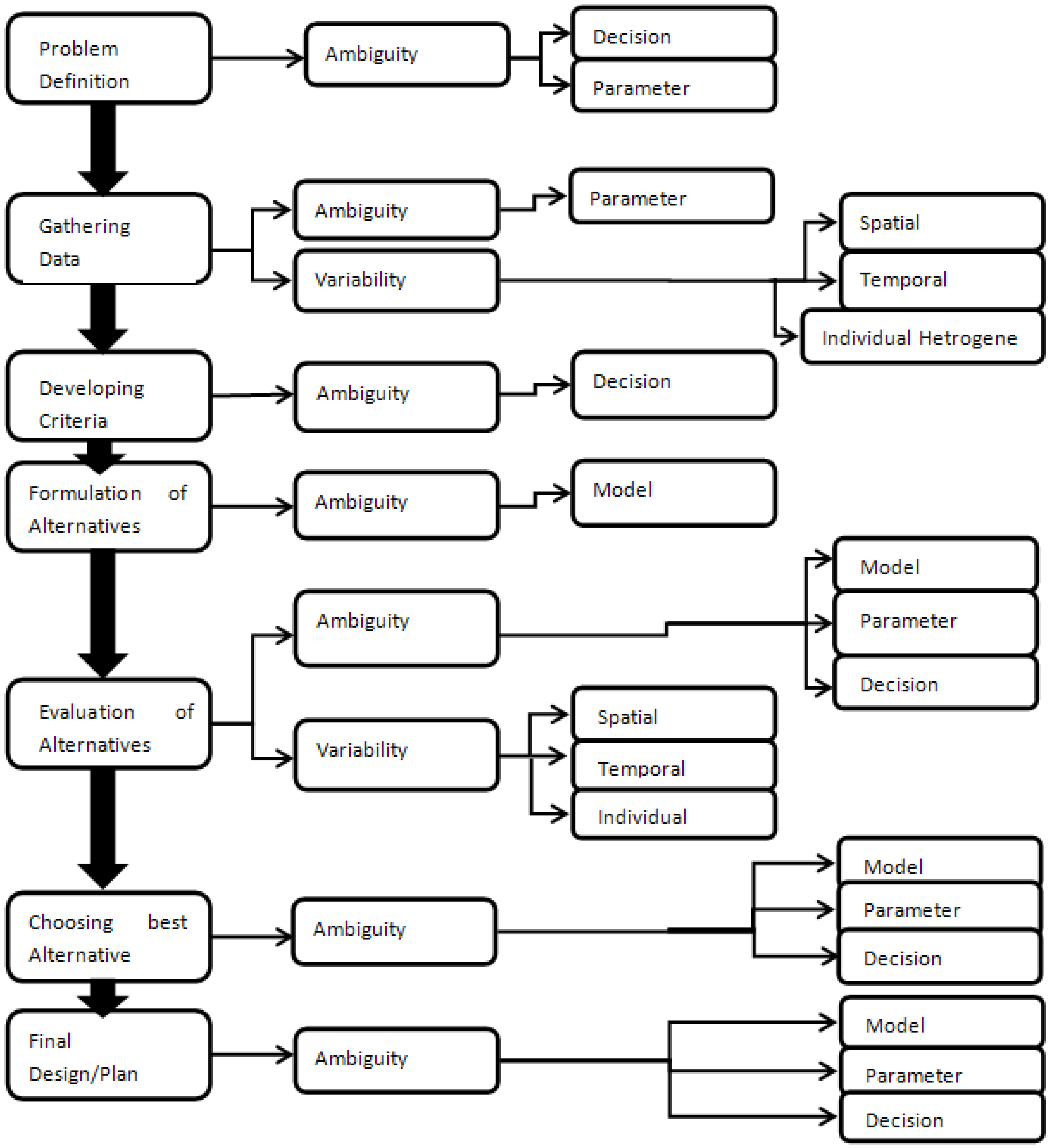


FIGURE 1.1- UNCERTAINTY SOURCES IN WATER RESOURCES DECISION MAKING

Continuing with the dike design example (introduced in the previous section), to formulate an alternative a great deal of uncertainty has to be considered. The alternatives may results in dike heights corresponding to various flow return periods. In this way, the dike height is determined using past information while the design is for the future. As the variables involved (such as flows and water levels) are subject to inherent stochastic variability and ambiguity, it can be concluded that significant sources of uncertainties are present in the determination of a dike height. These uncertainties are additionally transferred to other stages of the decision making process, resulting in the uncertain final decision that may prevent the future action.

1.3 RISK

Risk can be viewed as the quantification of uncertainties that may cause unwelcome effect from the water resources system performance. Perhaps the most expressive definition of risk is the one that conveys its multidimensional character by framing risk as the set of answers to three questions: What can happen? How likely is it to happen? If it does happen, what are the consequences? (Simonovic, 2009 after Kaplan and Garrick, 1981). The answers to these questions emphasize the notion that risk is a prediction or expectation which involves a hazard (the source of danger), uncertainty of occurrence and outcomes (the chance of occurrence), adverse consequences (the possible outcomes), a timeframe for evaluation, and the perspectives of those affected about what is important to them. The answers to these questions also form the basis of conventional quantitative risk analysis methodologies.

Here a general definition of risk based on the concept of load (L) and resistance(R) coming from structural engineering is presented. Load is a variable reflecting the behavior of the system under certain external conditions of stress or loading. Resistance is a characteristic variable which describes the capacity of the system to overcome an external load (Ganoulis, 1994). When the load exceeds the resistance (L>R) there should be a failure or an incident. Safety or reliability state is obtained if the resistance exceeds or is equal to the load (R≥L).

Continuing with the dike example introduced in section 1.1, the level of flood protection provided by a dike is not certain, it is subject to a risk of dike failure (overtopping, sliding, or breach). The consequences of incident or failure would mean loss of property and human lives caused by flooding. In this case the flood level (water level) is representing a load and the dike height resistance. In this case risk is a result of hydrologic variability and ambiguity as discussed in the previous section. Risk is one way for quantifying uncertainty. In the scope of decision making process, communication of risk of failure is important so that the informed decisions can be made. There are two basic approaches to risk and uncertainty management: (1) the probabilistic approach, in which risk is defined as the probability of failure and, (2) the fuzzy set approach, in which characteristic measures are introduced to define risk.

1.4 Approaches for dealing with uncertainty

The sources of uncertainty in water resources management are diverse and many. The following discussion provides the basic concepts of both, probabilistic and fuzzy, approaches.

1.4.1 Probabilistic approach

Probability theory has a long history of application in the field of water resources management. Hydrologic processes are random and thus the uncertainty as a result of variability may be appropriately quantified using the probabilistic approach. The basic mathematical concept of sets is fundamental in probability operations; sets are collections of elements, each with some specific characteristics. These sets are evaluated through use of Boolean algebra. In probability theory, the elements that comprise a set are outcomes of an experiment. The sample space of an experiment is the mutually exclusive listing of all possible outcomes of the experiment which is represented by the universal set Ω. In probability theory a subset of the sample space is the event.

Associated with any event E of a sample space S is a probability, P(E), that may be obtained as the number of elements in the event E divided by the number of elements in the sample Space S (classical interpretation of probability – equally likely concept). Continuing from the general definition of risk, in the probabilistic framework, L (load) and R (resistance) are taken as random or stochastic variables. In probabilistic terms, the risk is defined as the chance of failure or the likelihood of failure:

$$RISK = probability\ of\ failure = P\ (L > R) \tag{1.1}$$

A prerequisite for using the probabilistic approach is the requirement of a prior knowledge of the probability density functions of both resistance and load, and their joint probability distribution function. In practice, data is usually lacking to provide such information and where available, approximations still need to be made to estimate appropriate distributions.

1.4.2 Fuzzy set approach

Fuzzy set theory was intentionally developed to try to capture judgmental belief, or the uncertainty that is caused by the lack of knowledge or ambiguity. The concept of a fuzzy set can be described as a "class" (set) with a continuum of grades of membership (Zadeh, 1965). Each object within a fuzzy set is graded in the interval [0, 1]. For example, in the class of animals, rocks may be said to have 0 degree of membership in the set of animals that is they do not belong, while cats may have full membership and belong. These definitions are common to traditional ordinary sets, where the values are crisp either belonging or not with no partial degree of belonging (Zadeh, 1965). Fuzzy sets extend the ordinary sets, consider in the set animals starfish have an ambiguous status and thus hold degree of membership in the interval [0, 1] that is partial membership. Therefore, starfish can be properly represented without the need to classify them as either belonging or not to the set (class). Fuzziness thus measures the degree to which an event occurs, not whether it occurs, a contrast to probability theory.

In the application of fuzzy approach L and R are considered as fuzzy numbers. Then risk may be defined by means of appropriate fuzzy measures such as linguistic rules.

1.4.3 Comparison of approaches

The probabilistic and fuzzy approaches each have benefits and limitations when it comes to quantifying uncertainty in water resources management. The probabilistic approach for quantifying uncertainty addresses the uncertainty as a result of stochastic variability. However, the probabilistic approach has limitations in addressing the problem of uncertainty which goes along with human input, subjectivity, a lack of history and records. Furthermore, the results using the probabilistic approach may show potentially misleading levels of precision due to the full dependency on the underlying appropriateness of the selected probability distribution. Therefore, in areas where the probabilistic approach is limited, there is a need for an alternative approach. The fuzzy set approach can be used for the representation of perceived qualitative ambiguity sources of uncertainty that may not be measurable, giving results with some precision. Neither fuzziness nor probability can successfully quantify all sources of uncertainty in the water resources decision making process alone, thus, these concepts must be utilized together.

1.5 Organization of the report

Water resources management decision making process is subject to many challenges from risk and uncertainty. In the past, imprecise safety factors were used to address uncertainty and risk. There is a need for providing water resources decision makers with formal decision support tools that accurately incorporate risk and uncertainty. The goal of this report is to demonstrate how information provided to decision makers can be improved through the use of probabilistic and fuzzy set approaches for quantifying risk and uncertainty in water resources management. Probabilistic and fuzzy set approaches are used to expand on existing decision making procedures and toolsets to account for uncertainty and risk. Toolsets like simplex linear programming optimization, multi-objective analysis, and simulation of mathematical models can modified for use in the probabilistic and fuzzy domains. The methodologies for simulation, optimization, and multi-objective analysis under uncertainty are detailed in this report. In order to demonstrate how uncertainty and risk may be quantified using the probabilistic and fuzzy toolsets a set of generic problems is presented in the report. It should be noted that the tools detailed in the report may find wide application beyond the problems discussed here.

Two water resources engineering cases, the design of a dike height and the sewer pipe sizing, demonstrate design under uncertainty. The deterministic procedure is modified to demonstrate how variability and ambiguity uncertainties may be quantified using fuzzy and probability based simulation tools.

Two cases relating to water resources planning and operations problems are presented too. The first one demonstrates the optimization of reservoir operations. The second one deals with the multipurpose reservoir planning. These two cases demonstrate the use of fuzzy and probabilistic based optimization and multi-objective analysis techniques under uncertainty.

2. METHODOLOGY

The following sections present methodological background of water resources management tools for quantifying uncertainty using the probabilistic and fuzzy approach. The presentation includes simulation, optimization, and multi-objective analysis tools under uncertainty. The tools are used later for solving the selected case study examples for illustrative purposes. The implementation of presented tools is certainly not limited to those presented in the report.

2.1 PROBABILISTIC APPROACH

The probabilistic approach is often used in water resources management to address various sources of uncertainty. The following discussion includes probabilistic simulation, optimization and multi-objective analysis.

2.1.1 SIMULATION

Simulation models describe how a system operates, and are used to predict what changes will result from a specific course of action. Alternatively, simulation models are called cause-and-effect models. They describe the state of the system in response to various inputs, but give no direct measure what decision should be taken to improve the performance of the system. The probabilistic simulation modifies the existing deterministic simulation models through the use of probability density functions to represent the random variables.

The probabilistic simulation has two forms: (a) the **implicit probabilistic approach** which uses simulation in order to generate random numbers based on underlying distributions, and (b) the **explicit probabilistic approach** which directly uses the probability equations and their analytical solutions. The latter method includes the following steps:

Step 1. Approximation of a statistical distribution using the appropriate statistical parameters such as population mean and standard deviation.

Step 2. Determination of expected value using the probability density function:

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx \tag{2.1}$$

The above probabilistic explicit steps can be applied also with the implicit approach where simulation is used instead of using distributions to directly solve for, for example expected value. The random numbers are generated based on underlying distribution, the mean of which represents the expected value. These generated random values may then be used as direct input into the deterministic model, yielding stochastic simulation.

Consider that each random variable within the modified deterministic model is subject to some uncertainty and this uncertainty is fitted with an appropriate continuous probability distribution function that is randomly sampled to produce hundred or even thousands of scenarios or iterations. The distribution of the values calculated for the model outcome therefore reflects the probability of the values that could occur. This technique is known as the Monte Carlo Simulation (MCS). MCS creates an artificial model that will hopefully reproduce the distribution of input variables.

The Monte Carlo sampling method starts with looking at a cumulative distribution function F(x), which gives the probability P that the variable X will be smaller than or equal to the distribution of an uncertain input variable x, i.e.

$$F(x) = P(X \leq x) \tag{2.2}$$

where F(x) ranges from zero to one. The next step is looking at the inverse function G(F(x)) written as:

$$G(F(x)) = x \tag{2.3}$$

The inverse function is used in the generation of random samples from each distribution. Thus to generate a sample from an input probability distribution fitted to the uncertain variable, a random number (r) is generated between zero and one. This value is substituted into Eq. (2.3) where F(x) is equal to (r). The random number r is generated from the Uniform (0, 1) distribution to provide equal opportunity of an x value being generated in any percentile range. The Monte Carlo simulation process is automated with the use of a computer and a software package like MATLAB. The output of the simulations can be studied for the statistical properties and to answer what if questions of the decision maker.

2.1.2 Optimization

An example of the probabilistic optimization approach known as the Chance Constrained Programming is presented here. It has been conceptualized by Charnes and Cooper (1959) and implemented by them and others to deal with linear programming optimization under uncertainty. The approach expands the linear programming optimization model by adding probabilistic constraints that allow for violation. With the Chance Constrained Programming, when knowing or approximating the distribution of the random variable, we are able to evaluate the probability of the constraint violation. The reliability, α [0, 1] of not violating a constraint is specified by the decision maker, thus it allows for decision maker to directly control the level of risk he/she finds acceptable.

The classical linear programming formulation, based on the simplex method is given as

$$Max(or\ Min)\ x_0 = \sum_{j=1}^{n} c_j x_j$$

Subject to:

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad for\ i = 1,2,\dots,m \tag{2.4}$$

$$x_j \ge 0 \quad for\ j = 1,2,\dots,n$$

where c_j represents the objective function(x_0) coefficients, x_j is the decision variable, a_{ij} is the coefficient of the constraint, b_i is the right-hand side of the constraint, n is the total number of decision variables, and m is the total number of constraints.

The transformation to stochastic optimization using the Chance Constrained approach is done through the introduction of an additional probabilistic constraint, shown below.

$$P\left\{\sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i\right\} \ge 1 - \propto_r \qquad for\ i = 1,2,\dots,r \tag{2.5}$$

where $\tilde{b}_i$ represents the random variable with known historical data for approximating its probability distribution, r is the number of chance constraints , and α_r is the decision maker specified reliability of not violating the constraint (0 to 1).

The expression in Eq. (2.5) may also be presented in distribution function form as

$$F_{\tilde{b}_i}\left(\sum_{j=1}^{n} a_{ij}x_j\right) \ge 1 - \propto_r \qquad for\ i = 1,2,\dots,r \tag{2.6}$$

A linear deterministic equivalent of Eq. (2.6) is obtained by inversion and rearrangement

$$\sum_{j=1}^{n} a_{ij}x_j \ge F_{\tilde{b}_i}^{-1}(1 - \propto_r) \quad for\ i = 1,2,\dots,r \tag{2.7}$$

where $F_{\tilde{b}_i}^{-1}(\mu)$ is the inverse of the distribution function evaluated at (1-α_r). The value of $F_{\tilde{b}_i}^{-1}(1 - \propto_r)$ is replaced by $b_i^{1-\propto_r}$ such that the linear deterministic equivalent can then be rewritten as

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i^{1-\alpha_r} \quad for\ i = 1,2,\dots,r \tag{2.8}$$

Once the probabilistic constraints are converted into linear deterministic equivalents, the optimization problem can be solved using classical linear programming optimization algorithm.

2.1.3 Multi-objective analysis

Goicoechea et al. (1982) developed a stochastic multi-objective analysis method known as the Probabilistic Tradeoff Development (PROTRADE). This method is used to deal with problems involving the decision makers preferences and is capable of handling risk. The PROTRADE method consists of the formulation of surrogate and multiple attribute utility functions. The construction of these utility functions leads to their direct translation into the fitness function. The PROTRADE method is presented by the 12 step procedure below.

Step 1. A vector of objective functions is defined using the expected values of the objective functions coefficients:

$$Z(\boldsymbol{x}) = [Z_1(\boldsymbol{x}), Z_2(\boldsymbol{x}), \dots\dots, Z_p(\boldsymbol{x})]$$

$$g_q(\boldsymbol{x}) \leq 0\ where\ q \in I[1, Q]$$

$$\boldsymbol{x} > 0, \tag{2.9}$$

$$\tilde{Z}_i(\boldsymbol{x}) = \sum_{j=1}^{n} \tilde{c}_{ij}x_j, Z_i(\boldsymbol{x}) = E[\tilde{Z}_i(\boldsymbol{x})]$$

Step 2. Vectors $\mathbf{U_1}$ and **M** are defined, having the maximum and minimum values of the objective functions respectively:

$$Z_i(\boldsymbol{x}_i^*) = max \quad Z_i(\boldsymbol{x}),\ \ i \epsilon I[1,p]$$

$$U_1 = \begin{bmatrix} Z_1(\boldsymbol{x}_1^*) \\ Z_2(\boldsymbol{x}_2^*) \\ \cdot \\ \cdot \\ \cdot \\ Z_p(\boldsymbol{x}_p^*) \end{bmatrix} \tag{2.10}$$

$$M = \begin{bmatrix} Z_{1min} \\ Z_{2min} \\ \cdot \\ \cdot \\ \cdot \\ Z_{pmin} \end{bmatrix}$$

To find the maximum and minimum values it is necessary to perform optimization of each objective function separately, subject to the set of constraints $g_q(\boldsymbol{x}) \leq 0$.

Step 3. An initial surrogate function is formulated:

$$F(\boldsymbol{x}) = \sum_{i=1}^{p} G_i(\boldsymbol{x}) \tag{2.11}$$

Where

$$G_i = \frac{Z_i(\boldsymbol{x}) - Z_{imin}}{Z_i(\boldsymbol{x}_i^*) - Z_{imin}} \tag{2.12}$$

where $Z_i(\boldsymbol{x})$ is the value of objective function i, i= 1,2,….,n; Z_{imin} is the minimum value obtained when objective i is subjected to the constraints; and $Z_i(\boldsymbol{x}_i^*)$ is the maximum value obtained when objective i is subjected to the constraints.

Step 4. An initial solution $\mathbf{x_1}$ is obtained by maximizing F(**x**), subject to constraints $g_q(\boldsymbol{x}) \leq 0$. This solution is used to generate a goal vector $\mathbf{G_1}$:

$$G_1 = \begin{bmatrix} G_1(\boldsymbol{x_1}) \\ G_2(\boldsymbol{x_1}) \\ . \\ . \\ . \\ G_p(\boldsymbol{x_1}) \end{bmatrix} \tag{2.13}$$

Step 5. A multidimensional utility function is defined; in this case Giocoechea et al. (1982) proposed a multiplicative form (Keeney and Raiffa, 1976):

$$1 + ku(\boldsymbol{G}) = \prod_{i=1}^{p} [1 + kk_i u_i(G_i)] \tag{2.14}$$

This function is used to reflect the DM's goal utility assessment, where k and k_i are constants which are determined by questions posed to the DM. The procedure for determining the parameters of the above function is discussed in Keeney and Raiffa (1976) and Krzysztofowicz and Duckstein (1979).

Step 6. A new surrogate objective function is defined:

$$S_1(\boldsymbol{x}) = \sum_{i=1}^{p} w_i G_i(x) \tag{2.15}$$

where,

$$w_i = 1 + \frac{r}{G_i(x_1)} \frac{\partial u(\boldsymbol{G})}{\partial G_i} \bigg|_{G_1} \tag{2.16}$$

Step 7. An alternative solution is generated maximizing the surrogate solution S_1 finding a solution called $\mathbf{x_2}$ used to generate $\mathbf{G_2}$ and $\mathbf{U_2}$:

$$\boldsymbol{U_2} = \begin{bmatrix} Z_1(\boldsymbol{x_2}) \\ Z_2(\boldsymbol{x_2}) \\ . \\ . \\ . \\ Z_p(\boldsymbol{x_2}) \end{bmatrix} \quad \boldsymbol{G_2} = \begin{bmatrix} G_1(\boldsymbol{x_2}) \\ G_2(\boldsymbol{x_2}) \\ . \\ . \\ . \\ G_p(\boldsymbol{x_2}) \end{bmatrix} \tag{2.17}$$

Step 8. A vector $\mathbf{V_1}$ that expresses the tradeoff between the goal value and its probability of achievement is generated:

$$\boldsymbol{V_1} = \begin{bmatrix} G_1(\boldsymbol{x_2}), & 1-\alpha_1 \\ G_2(\boldsymbol{x_2}), & 1-\alpha_2 \\ & . \\ & . \\ & . \\ G_p(\boldsymbol{x_2}), & 1-\alpha_p \end{bmatrix} \tag{2.18}$$

where $1 - \alpha_i$ is such that,

$$prob[Z_i(\boldsymbol{x}) \geq Z_i(\boldsymbol{x_2})] \geq 1 - \alpha_i \tag{2.19}$$

Step 9. The DM has to answer the following question: "Are all the $Z_i(\mathbf{x_2})$ values satisfactory?" If the answer is affirmative, the vector $\mathbf{U_2}$ is a solution, if not go to step 10.

Step 10. The $Z_k(\mathbf{x})$ with the least satisfactory pair of ($G_k(\mathbf{x_2})$, $1-\alpha_k$) is selected and the DM specifies a new probability for that pair.

Step 11. The solution space is redefined creating a new x-space.

Step 12. A new surrogate objective function is generated and a sequential search for a satisfactory solution is performed going back to step 7 as many times as necessary.

2.2 Fuzzy set approach

The following presents a set of generalized tools for water resource management based on the use of fuzzy set theory. In addition, some of the techniques for generating fuzzy membership functions are explained.

2.2.1 Membership function concept

A fuzzy set (class) is characterized by a membership (characteristic) function which associates each member of the fuzzy set with a real number in the interval [0, 1] (Zadeh, 1965;Ross, 2004). The membership function essentially embodies all fuzziness for a particular fuzzy set; its description is the essence of a fuzzy property or operation. There are numerous ways to assign membership values or functions to fuzzy variables; more ways than there are to assign probability density functions to random variables. In the following sections a sample of the available methods for assigning membership values or functions are summarized. For further details the reader is directed to the textbook by Ross (2004).

2.2.1.1 Intuition

This method is derived simply from the capacity of humans to develop membership functions through their own innate intelligence and understanding (Ross, 2004). In order to utilize intuition, contextual and semantic knowledge about an issue is essential. Thus, the membership function development is dependent on the subjectivity of the individual or individuals consulted in its development. A single fuzzy variable may have more than one membership function, that is, there may be many partitions. An important characteristic for the purposes of use in fuzzy operations is that these partitions overlap.

2.2.1.2 Inference

The inference method comes from our ability to perform deductive reasoning. When given a body of facts or knowledge we are able to deduce or infer a conclusion. The inference method can take many forms; consider an example of identifying a triangle when we possess a formal knowledge of geometry and geometric shapes, Ross (2004). In identifying a triangle, let A, B and C be the inner angles of a triangle in the order $A \geq B \geq C \geq 0$ and let U be the universe of triangles, such that,

$$U = \{(A, B, C) | A \geq B \geq C \geq 0; A + B + C = 180°\} \quad (2.20)$$

We can infer membership of different triangle types, because we possess knowledge of geometry. We can determine if a triangle is approximately isosceles by developing an algorithm for the membership meeting the constraints of Eq. (2.20) we have:

$$\mu_1(A, B, C) = 1 - \frac{1}{60} \min(A - B, B - C) \quad (2.21)$$

So, for example if A=B or B=C the membership value of isosceles triangle is μ_1=1 however if A=120°, B=60°, C=0° then μ_1=0. In the first case we thus have full membership or belonging of the fuzzy variable in the fuzzy set for an approximate isosceles triangle while the second case is a total contrast.

2.2.1.3 Rank Ordering

The approach arises from assessing preferences by a single individual, a committee, a poll and other opinion methods that can be used to assign membership values to a fuzzy variable (Ross, 2004). Preferences are determined by pairwise comparisons, and these determine the ordering of the membership. This method is similar to finding relative preferences through a questionnaire and developing membership functions as a result.

2.2.1.4 Neural Networks

Neural network is a technique that seeks to build an intelligent program using models that try to recreate the working of neurons in the human brain. Neurons are believed to be responsible for humans ability to learn, thus the goal is to implement this to machine language to use for generating membership functions. Neural networks use in membership function generation is centered on a training process (learning as a result of available data for input) and an unsupervised clustering process (Ross, 2004). After training, degree of membership function for a given input value may be estimated through the network computation. That is, each input value has a certain estimated degree of belonging to a cluster which is equivalent to the degree of the membership function represented by the cluster.

2.2.1.5 Genetic Algorithms

Genetic algorithms use the concept of Darwin's theory of evolution in searching for the best solution of a given set based on the principle of "survival of the fittest" (Ross, 2004). Among all possible solutions, a fraction of the good solutions is selected, and the others are eliminated. The selected solutions undergo a process of reproduction, crossover, and mutation to create a new generation of possible solution. The process continues until there is a convergence within a generation. The genetic algorithms can be used in the derivation of membership functions. The process starts by assuming some functional mapping for a system (membership functions and their shapes for fuzzy variable/s). The membership functions are then converted to a code familiar to the algorithm, bit strings (zeros and ones) which can then be connected together to make a longer chain of code for manipulation in the genetic algorithm (i.e. crossover, elimination, reproduction). An evaluation function is used to evaluate the fitness of each set of membership functions (parameters that define the functional mapping). Based on the fitness value, unsatisfactory strings are eliminated and reproduction of satisfactory strings proceeds for the next generation. This process of generating and evaluating strings is continued until the membership functions with the best fitness value are obtained.

2.2.1.6 Inductive Reasoning

This approach utilizes the inductive reasoning to generate the membership functions by deriving a general consensus from the particular (Ross, 2004). Inductive reasoning assumes availability of no information other than a set of data (Russell & Kim, 1993). The approach is to partition a set of data into classes based on minimizing the entropy. The entropy, S, where only one outcome is true is the expected value of the information contained in the data set and is given by

$$S = -k \sum_{i=1}^{N} [p_i lnp_i + (1 - p_i)\ln(1 - p_i)] \tag{2.22}$$

where the probability of the *i*th sample to be true is p_i and N is the number of samples. The minus sign in front of the parameter k in Eq. (2.22) ensures that entropy will be a positive value greater than or equal to zero. Through iteratively partitioning, the segmented data calculation of an estimate for entropy is possible. The result is a solution of points in the region of data interval used to define the membership function. The choice of shape of membership functions is arbitrary as long as some overlap is present between membership functions, therefore simple shapes like triangles, which exhibit some degree of overlap is often sensible.

2.2.2 Fuzzy simulation

The fuzzy approach used for simulation is derived from utilizing the fuzzy inference method, based on the representation of human knowledge in IF-THEN rule-based form, such that we are able to infer a conclusion or fact (consequent) given an initial known fact (premise, hypothesis, antecedent) (Ross, 2004).

A typical form of the IF-THEN rule-based form also referred to as a deductive form is shown in the expression below:

$$IF\ premise\,(antecedent), THEN\ conclusion\ (consequent) \tag{2.23}$$

The fuzzy simulation (rule-based system) is the most useful in modeling complex systems that can be observed by humans. The linguistic variables are used as antecedents and consequents. These linguistic variables can be naturally represented by fuzzy sets and logical connectives of these sets.

Mamdani's fuzzy inference method is the most commonly seen fuzzy simulation methodology, and is the methodology presented in this report (Ross, 2004). The method was originally proposed as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. The Mamdani inference method is a graphical technique that follows five main steps: (1) development of fuzzy sets and linguistic rules, (2) fuzzification of inputs, (3) application of fuzzy operators, (4) aggregation of all outputs, and (5) defuzzification of aggregated output.

Step 1. Development of fuzzy sets and linguistic rules

To begin, the Mamdani form rules may be described by the collection of r linguistic IF-THEN expressions. Equation (2.24) shows the expression for a fuzzy system with two non-interactive inputs x_1 and x_2 (antecedents) and a single output (consequent) y. The concept holds for any number of antecedents (inputs) and consequents (outputs).

$$IF\ \ x_1\ is\ \underset{\sim}{A}_1^k\ and(or)\ x_2\ is\ \underset{\sim}{A}_2^k\ THEN\ y^k\ is\ \underset{\sim}{B}^k\quad for\ k = 1,2,\dots,r \tag{2.24}$$

where $\underset{\sim}{A}_1^k$ and $\underset{\sim}{A}_2^k$ are the fuzzy sets representing the *k*th antecedent pairs, and $\underset{\sim}{B}^k$ is the fuzzy set representing the *k*th consequent. The membership functions for the fuzzy sets may be generated with one of the methods discussed in section 2.2.1.

Step 2. Fuzzification of Inputs

The inputs to the system x_1 and x_2 are scalar values. In order to proceed with the inference method the corresponding degree to which the inputs belong to the appropriate fuzzy sets via membership functions need to be found. Fuzzification of the input thus requires the membership function of the fuzzy linguistic set to be known and through function evaluation the corresponding degree of membership for the scalar input belonging to the universe of discourse is found. Figure 2.1 outlines the procedure in a graphical form.

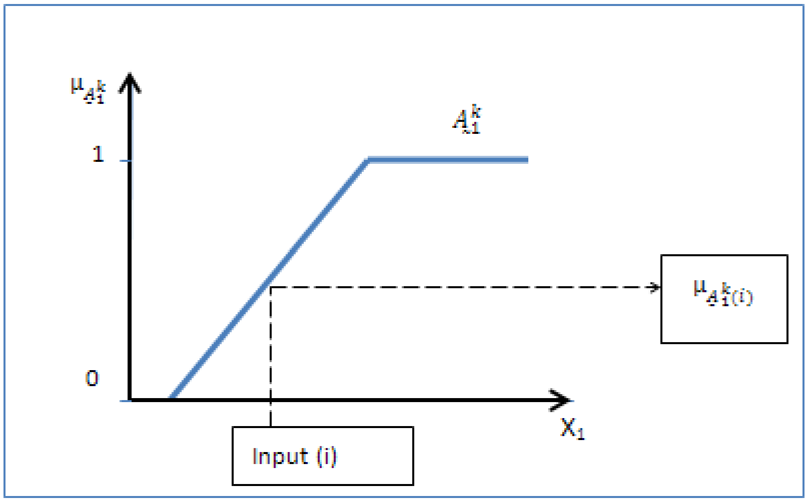


FIGURE 2.1- FUZZIFICATION OF SCALAR INPUT FROM KNOWN MEMBERSHIP FUNCTION.

It should be noted that inputs to any fuzzy system can be a membership function, such as for example gauge reading that has been fuzzified already. Either way, the methodology is the same as one that employs fuzzy singletons (scalar values) as the input.

Step 3. Application of fuzzy operators

Once the inputs are fuzzified, the degree by which each condition of the antecedent is satisfied is known for each rule. If there are multiple antecedent conditions for each rule, as in the case of expression (2.24) then a fuzzy operator is used to obtain one number that represents the antecedent for that rule. This number is applied to the output function producing a single truth value for the rule.

The logical operators commonly employed are described.

The expression in (2.24) has conjunctive antecedents and in brackets for illustration shows disjunctive antecedents.

For conjunctive antecedents, assuming a new fuzzy subset $\underset{\sim}{A}_s$ as

$$\underset{\sim}{A}_s^k = \underset{\sim}{A}_1^k \cap \underset{\sim}{A}_2^k \qquad for\ k = 1,2,\ldots,r \tag{2.25}$$

expressed by means of membership function, shown in Figure 2.2

$$\mu_{\underset{\sim}{A}_s^k}(x) = \min\left[\mu_{\underset{\sim}{A}_1^k}, \mu_{\underset{\sim}{A}_2^k}\right] \qquad for\ k = 1,2, \dots, r\ . \tag{2.26}$$

For disjunctive antecedent a similar procedure follows. This time fuzzy set $\underset{\sim}{A}_s$ is defined as

$$\underset{\sim}{A}_s^k = \underset{\sim}{A}_1^k \cup \underset{\sim}{A}_2^k \qquad for\ k = 1,2, \dots, r \tag{2.27}$$

expressed by means of membership function, shown in Figure 2.2

$$\mu_{\underset{\sim}{A}_s^k}(x) = \max\left[\mu_{\underset{\sim}{A}_1^k}, \mu_{\underset{\sim}{A}_2^k}\right] \qquad for\ k = 1,2, \dots, r\ . \tag{2.28}$$

Given the above, the compound rule may be rewritten as

$$IF\ \underset{\sim}{A}_s^k\ THEN\ \underset{\sim}{B}_s^k \qquad for\ k = 1,2, \dots, r \tag{2.29}$$

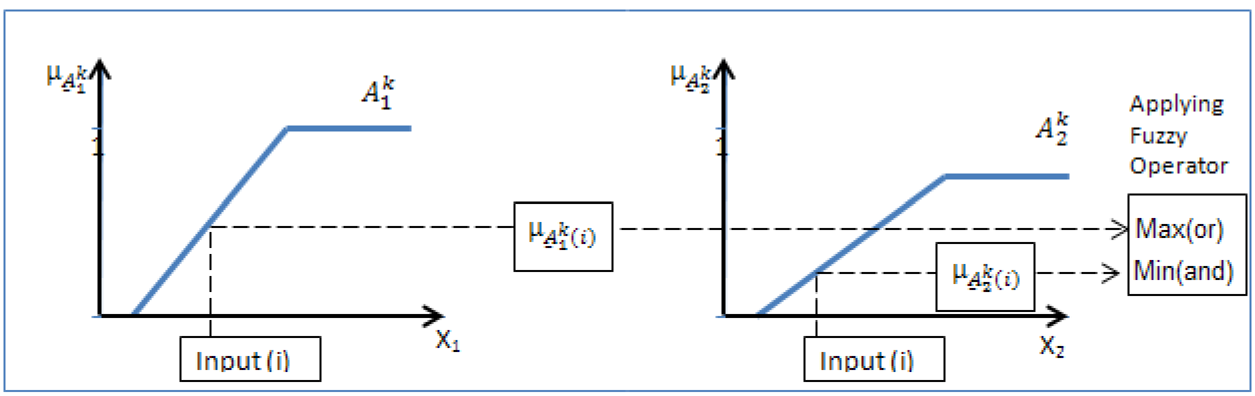


FIGURE 2.2- FUZZY OPERATOR USE FOR THE GENERALIZED EXPRESSION (2.24) OF A RULE

Step 4. Aggregation of outputs

It is common for a rule-based system to involve more than one rule. As such, in order to reach a decision or overall conclusion aggregation of individual consequents or outputs contributed by each rule is required, so that all the outputs are combined into a single fuzzy set, which may be defuzzified in the final step to obtain a scalar solution.

The aggregation of outputs may be achieved in two ways (1) max-min truncation, (2) max-product scaling. Only the first case will be discussed in this report. In the max-min case aggregation is achieved by the minimum or maximum membership function value from the antecedents (depending on the logical operator used in the rule) propagating through to the consequent and in doing so truncating the membership function for the consequent of each rule. This procedure is done for each rule. The truncated membership functions of each rule will need to be combined. This may be achieved through use of disjunctive rules, or conjunctive rules, using the same fuzzy operators as in step 3.

If the system of rules needs to be jointly satisfied the truncated outputs should be aggregated as a conjunctive system - the rules are connected by "and" connectives. In the case where the objective is for at least one rule to be satisfied, the aggregation of outputs may be treated by the definition of disjunctive system - the rules are connected by "or" connectives. Figure 2.3 illustrates the aggregation of outputs into a single fuzzy membership function. Each antecedent is treated as conjunctive and the aggregation of outputs of each rule is treated as a disjunctive system.

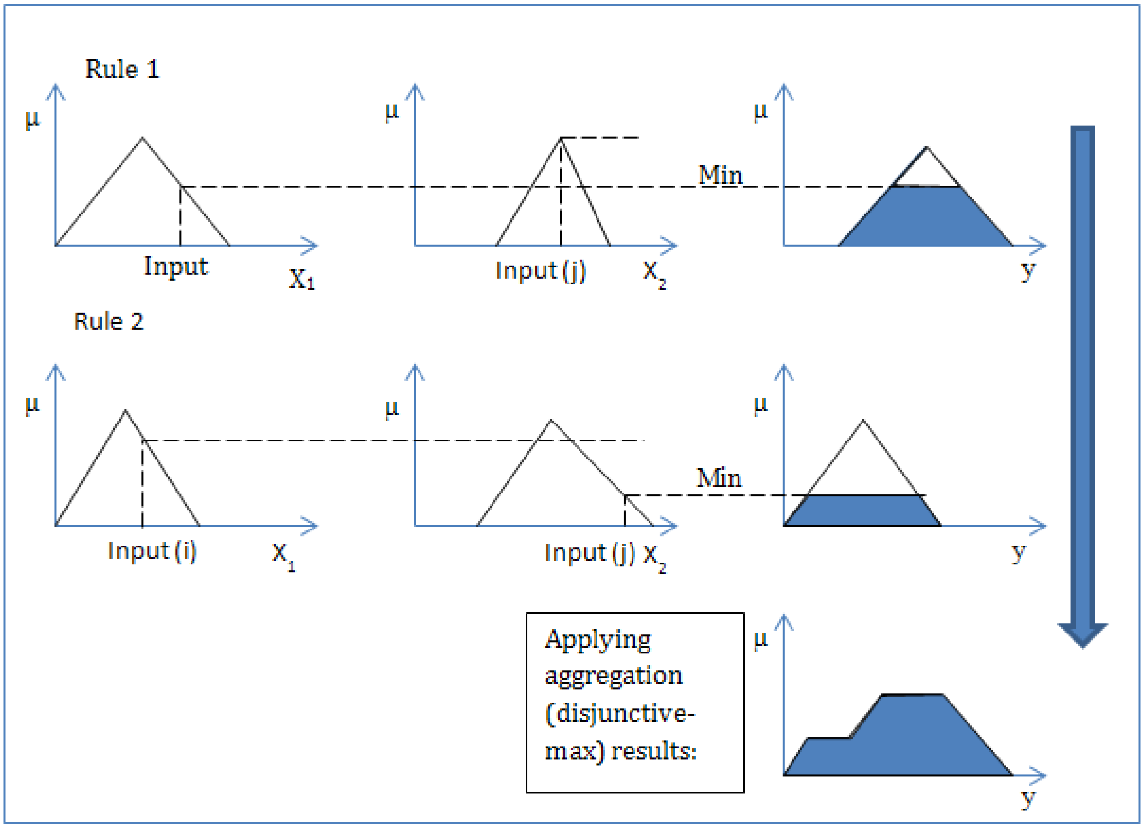


FIGURE 2.3- AGGREGATION OF RULE OUTPUTS INTO A SINGLE FUZZY MEMBERSHIP FUNCTION

Step 5. Defuzzification of aggregated result

The final objective of the rule-based system simulation is typically a single value obtained from the defuzzification of the aggregated fuzzy set of all outputs. Many defuzzification methods are available in the literature: max membership principle, centroid method, weighted average method, and numerous other methods. There is no one most suitable defuzzification method. Selection of the best method for defuzzification is context or problem-dependent. For the purpose of this report the centroid method will be used, because it is well established and physically appealing among all the defuzzification methods (Ross, 2004). The centroid method shown in Figure 2.4, may also be referred to as the center of gravity or center of an area. Its expression is given as,

$$y^* = \frac{\int \mu_c(y)dy}{\int \mu_c(y)dy} \tag{2.30}$$

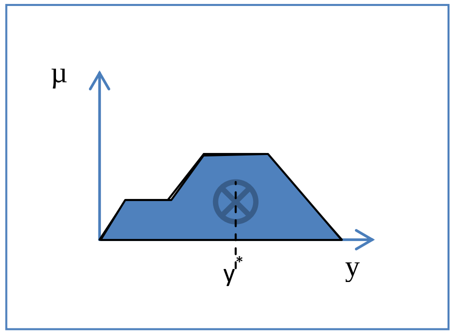


FIGURE 2.4- CENTROID METHOD FOR DEFUZZIFICATION

2.2.3 Fuzzy optimization

The optimization tool selected for presentation in this report is the fuzzy linear programming approach. The fuzzy linear programming approach departs from the classical assumptions that all coefficients of the constraints need to be crisp numbers and that the objective function must be minimized or maximized (Zimmermann, 1996). Fuzzy optimization allows for certain aspirations to be targeted in the objective function and for constraints to be loose accounting for uncertainty or imprecision. In this way decision makers are no longer required to give exact crisp constraints, where uncertainty exists and are further able to target a range of accepted aspiration values for the objective function.

The fuzzy version of the traditional linear programming optimization problem presented in Eq. (2.31) is:

$$\begin{aligned} cx &\lesssim z_0 \\ Ax &\lesssim b \\ x &\geq 0 \end{aligned} \tag{2.31}$$

where the symbol "$\lesssim$" denotes a relaxed or fuzzy version of the ordinary inequality "≤". The fuzzy inequalities represent the decision maker's fuzzy goal and fuzzy constraints and mean that "the objective function cx should be essentially smaller than or equal to an aspiration level z_0 of the decision maker" and "the constraints Ax should be essentially smaller than or equal to b," respectively. Furthermore, the fuzzy constraints and goal are viewed as equally important with respect to the fuzzy decision.

Zimmermann (1978) expressed the problem in simplified form for the fully symmetric objective and constraints.

$$Bx \quad \lesssim d \tag{2.32}$$

$$x \quad \geq 0$$

Where,

$$B = \begin{bmatrix} c \\ A \end{bmatrix}, \quad d = \begin{bmatrix} z_0 \\ b \end{bmatrix} \tag{2.33}$$

The following expression for the (monotonically decreasing) linear membership function illustrated in Figure 2.5 was proposed by Zimmerman for the ith fuzzy inequality $(Bx)_i \lesssim d_i$.

$$\mu_i((Bx)_i) = \begin{cases} 1 & ; \quad (Bx)_i \leq d_i \\ 1 - \dfrac{(Bx)_i - d_i}{p_i} & ; \quad d_i \leq (Bx)_i \leq d_i + p_i \\ 0 & ; \quad (Bx)_i \geq d_i + p_i \end{cases} \tag{2.34}$$

where, each d_i and p_i are the subjectively chosen constant values corresponding to the aspiration level and the violation tolerance of the ith inequality, respectively. If the constraints (including objective function) are well satisfied the ith membership function value should be 1. If the constraint is violated beyond the limit of tolerance, p_i than the value will be 0 and between 0 and 1 will be linear.

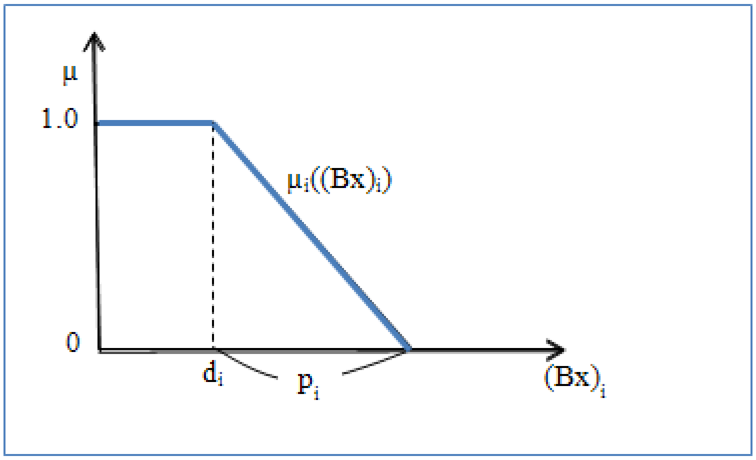


FIGURE 2.5- LINEAR MEMBERSHIP FUNCTION

The membership function of the fuzzy set "decision" of model in Eq. (2.32) including the linear membership functions is shown below. The problem of finding the maximum decision is to choose x* such that

$$\mu_D(x^*) = max_{x\geq 0} min_{i=0,\dots,m}\{\mu_i((Bx)_i)\} \tag{2.35}$$

In other words, the problem is to find the $x^*\geq 0$ which maximizes the minimum membership function value. This value satisfies the fuzzy inequalities, $(Bx)_i \lesssim d_i$ with the degree of x* (Sakawa, 1993).

Substituting the expression (2.34) for linear membership function into Eq. (2.35) yields

$$\mu_D(x^*) = max_{x\geq 0} min_{i=0,\dots,m}\{1 + \frac{d_i}{p_i} - \frac{(Bx)_i}{p_i}\} \tag{2.36}$$

The fuzzy set for decision can be transformed to an equivalent conventional linear programming problem by introducing the auxiliary variable λ:

$$\begin{aligned} &maximize && \lambda \\ &subject\ to && \lambda \leq 1 + \frac{d_i}{p_i} - \frac{(Bx)_i}{p_i}, i = 0, \dots, m \end{aligned} \tag{2.37}$$

$$x \geq 0$$

It should be emphasized that the above formulation is for a minimization of the objective function and less than constraints, thus should be modified appropriately for other conditions.

2.2.4 Fuzzy multi-objective analysis

The methodology detailed for optimization using fuzzy linear programming can be extended to multi-objective analysis (optimization) problems (Sakawa, 1993). The multi-objective linear programming problem with k linear objective functions may be stated:

$$z_i(x) = c_i x, i = 1, \dots, k: \tag{2.38}$$

$$minimize\ z(x) \triangleq (z_1(x), z_2(x), \dots, z_k(x))^T$$

$$subject\ to\ Ax \leq b, x \geq 0$$

where $c_i=(c_{i1},\dots,c_{in})$, $i=1,\dots,k$, $x=(x_1,\dots,x_n)^T$, $b=(b_1,\dots,b_m)^T$ and $A=[a_{ij}]$ is an m x n matrix.

For each of the objective functions $z_i(x) = c_i x, i = 1, \dots, k$, of this problem, assume that the decision maker (DM) has a fuzzy goal such as "the objective function $z_i(x)$ should be substantially less than or equal to some value". Then the corresponding linear membership function $\mu_i^L(z_i(x))$ is defined as

$$\mu_i^L(z_i(x)) = \begin{cases} 0 & ;\ z_i(x) \geq z_i^0(x) \\ \dfrac{z_i(x) - z_i^0}{z_i^1 - z_i^0} & ;\ z_i^0(x) \geq z_i(x) \geq z_i^1(x) \\ 1 & ;\ z_i(x) \leq z_i^1(x) \end{cases} \tag{2.39}$$

where z_i^0 or z_i^1 denotes the value of the objective function $z_i(x)$ such that the degree of membership function is 0 or 1 respectively (Sakawa, 1993). Zimmermann (1978) suggested a way to determine the parameters z_i^0 and z_i^1 by solving the individual objective functions with respect to the non-fuzzy constraints for both maximum and minimum values of the objective, thus establishing a range of valid goal values. To be more specific, assuming the existence of the optimal solution x^{io},

$$z_i^{max} = z_i(x^{io}) = \max z_i(x), i = 1, \dots, k, \qquad (2.40)$$

$$z_i^{min} = z_i(x^{io}) = \min z_i(x),, i = 1, \dots, k, \qquad (2.41)$$

where for the decreasing membership function shown in Eq. (2.39), the parameter z_i^0 may be chosen as z_i^{max} and the parameter z_i^1 chosen as z_i^{min}.

Figure 2.6 illustrates the possible shape of the decreasing linear membership function, for the minimizing objectives.

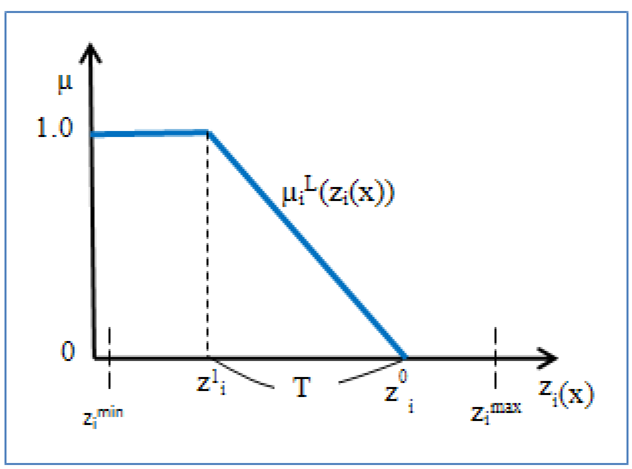


FIGURE 2.6- DECREASING LINEAR MEMBERSHIP FUNCTION, FOR MINIMIZATION OBJECTIVE FUNCTION

Using such linear membership functions $\mu_i^L(z_i(x))$, 1,…,k, with the original multi-objective linear programming problem the fuzzy set “decision” can be formulated as

$$\mu_D(x^*) = max_{x\geq 0} min_{i=1,\dots,k}\{\mu_i^L(z_i(x))\} \tag{2.42}$$
$$subject\ to\ Ax \leq b, x \geq 0$$

By introducing the auxiliary variable λ, the problem can be interpreted in the following conventional linear programming form

$$\begin{aligned} &maximize && \lambda \\ &subject\ to && \lambda \leq \mu_i^L(z_i(x)), i = 1,2,\dots,k \\ & && Ax \geq b, x \geq 0 \end{aligned} \tag{2.43}$$

or substituting the membership function $\mu_i^L(z_i(x))$,

$$\begin{aligned} &maximize && \lambda \\ &subject\ to && \lambda \leq \frac{z_i^0}{T_i} - \frac{z_i(x)}{T_i}, i = 1,2,\dots,k \\ & && Ax \geq b, x \geq 0 \end{aligned} \tag{2.44}$$

where T_i represents the absolute difference between z_i^0 and z_i^1 and the variable λ represents the maximum degree of overall satisfaction for all the fuzzy objectives and constraints.

The constraint Ax≥ b can be converted into fuzzy form as shown in the discussion of fuzzy linear programming methodology (section 2.2.3). The presented formulation is for minimization of objectives and thus the linear membership function Eq. (2.39) needs to be slightly modified to represent the maximization objectives as shown in Fig. 2.7.

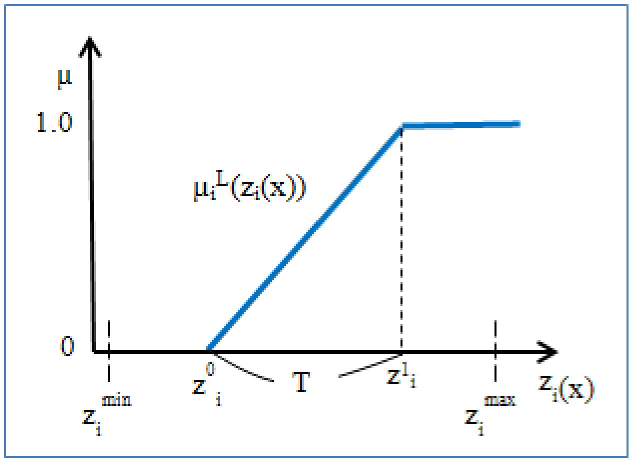


FIGURE 2.7- INCREASING LINEAR MEMBERSHIP FUNCTION, FOR MAXIMIZATION OBJECTIVE FUNCTION

3. Implementation of water resource management tools under uncertainty

The following set of selected cases is chosen to demonstrate how uncertainty and risk may be quantified to aid the decision process using techniques discussed in the previous chapter. The selected cases include dike height design, storm water sewer pipe design, single reservoir planning, and management problems. The cases will showcase the modification of traditional deterministic approaches in order to address various sources of uncertainty.

3.1 Dike height design

Dike is the oldest, most common and often most economical structural measure used for management of floods. Dike is a barrier usually erected at a location that provides the greatest net benefit and roughly parallel to a river or a coast. A dike is commonly made of earthen materials which can fail from overtopping (flood or wind induced) and seepage/piping. One of the main hazards involved with a diking system is that it provides a community with full protection up to a certain flood stage and none after, which leads communities to continue further development in the flood prone regions unaware of the risk.

The height of a dike is the key variable in the decision of the level of protection from floods. The greater the dike height the greater the potential level of protection of the region behind the structure. Traditionally there is no one single method for dike height design. Various design principles exist for height determination and their choice depends on local preferences. Different methods are used to address the uncertainty in dike height design. Uncertainty arises due to errors in sampling, measurements, estimation, forecasting and modeling (Debo & Reese, 2003). For dike design, the water level (stage) and discharge are of prime importance. Uncertainty in discharge is due to a short or nonexistent flood records, inaccurate rainfall-runoff modeling and inaccuracy in known flood flow regulation (Debo & Reese, 2003). Stage uncertainty comes from errors and unknowns associated with roughness, geometry, debris accumulation, sediment impacts and others factors (Debo & Reese, 2003).

Self-learning dike height design strategy comes from Netherlands and it suggests that dike height adjustment be made immediately following the actual extreme flood event. The height of a dike is determined by applying a safety margin on top of the highest recorded water level (Kok & Hoekstra, 2008). Gui et al, 1998 showed a strategy of dike height design for the simultaneous occurrence of flood and wind caused waves. The height of waves is used for determining the freeboard. The FEMA certification guidelines in 2007 state that “the freeboard must be established at one foot above the height of the 1% wave or the maximum wave run-up (whichever is greater) associated with the 100-year still water surge elevation at the site” (Van Ledden et al, 2007). These guidelines proved to be insufficient for the hydraulic design of dikes in the New Orleans area.

The freeboard allowance strategy dike height design method is based on historical stream gauge data and preselected return period in order to determine a probabilistic flood stage level. An increase of freeboard of 0.3-1m depending on the location is usually provided. Various other design strategies are available in the literature but the main objective of the design remains to account for uncertainty in choosing the appropriate dike height level in order to provide with confidence the desired protection level.

3.1.1 Problem identification

The limitation of most currently available dike height design strategies is that they rely on limited past historical hydraulic conditions data to predict the future ones. This means that the current deterministic strategies have a great deal of uncertainty that they usually try to deal with by selecting a freeboard value.

The implementation of a probabilistic approach instead of the deterministic strategies requires addition of a probability density function for each estimated parameter. Additionally simulation can be used to generate synthetic data series based on the predefined statistical distributions, which may be used for dike height design and lead to better understanding of the uncertainty in hydrologic processes associated with the dike height design. Through the implementation of the fuzzy approach, uncertainty as a result of partial or missing data may be subjectively alleviated allowing for a solution to be reached.

3.1.2 Mathematical formulation

The mathematical problem formulation for the selection of the appropriate design height of a dike is based on the traditional deterministic methodology (freeboard allowance strategy) expanded with the probabilistic simulation approach (both implicit, based on the Monte Carlo simulations, and explicit) and fuzzy simulation approach.

3.1.2.1 Deterministic approach

The traditional deterministic procedure for dike design is as follows:

Step 1 Data must be gathered to develop discharge-frequency and stage-discharge (also known as rating curve) curves for the dike design location.

Step 2. Find the flood stage with decision maker specified annual exceedance probability.

Step 3. Find the stage from the rating curve corresponding to the discharge found in step 2.

Step 4. Add the freeboard to account for uncertainty; this in equation form is shown below

$$H_t = H + H_f \tag{3.1}$$

where H_t is the total dike height, H is the flood stage and H_f is the allowance of freeboard. The units used must be kept consistent.

3.1.2.2 Probabilistic approach

The development of the probabilistic mathematical formulation is based on the methodology of probabilistic simulation discussed in section 2.1.1. In the probabilistic approach each point of the discharge-frequency and the stage-discharge curve is represented by a probability density function. The probabilistic approach has two forms: (a) implicit probabilistic approach which uses simulation in order to generate random numbers based on the underlying distributions, and (b) explicit probabilistic approach which directly uses the probability equations for solving analytically the dike height design problem. The latter method is presented below.

Step 1. Find the flood stage with specified annual exceedance probability from a discharge-probability function that for each point has a corresponding probability density function. For a single return period (exceedance probability) a discharge statistical distribution corresponding to the appropriate statistical parameters (such as population mean and standard deviation) is found.

Step 2. The discharge probability density function is used to find the expected value of stage corresponding to the given return period, given by Eq. (2.1), where f(x) is the probability density function that best describes the hydraulic characteristics of the site.

Step 3. The expected value of discharge is then used to find the probabilistic discharge from the discharge-frequency curve.

Step 4. The expected value of flood stage is determined from the distribution of stage (Step 3) that corresponds to the selected exeedance probability.

Step 5. Finally the addition of freeboard is selected.

The above probabilistic explicit steps can be applied also for the implicit case that utilizes the Monte Carlo Simulation approach (presented in section 2.1.1).

3.1.2.3 Fuzzy approach

The deterministic problem of dike height design can be transformed using a fuzzy set approach and solved using the fuzzy rule-based Mamdani inference method (presented in section 2.2.2). The fuzzy mathematical model formulation for the dike height design problem is based on the three simple linguistic rules for estimating the dike height as shown in Table 3.1. Each of the rules comes with two disjunctive antecedents and a single consequent safety corresponding to the designed dike height. The inputs are based on the design flows and the rules return an output of the appropriate dike safety, or rather height, for design depending on the frequency and stage of the flow used for input. The rules are used to represent some inherent knowledge possessed to infer appropriate dike height levels for design. As an example the first rule states that for flows that are frequent or for flows that are associated with shallow depths a low dike height safety level is required. That is, the rules separate the subjectively and ambiguously defined ranges of potential frequency of occurrence (frequent, infrequent, rare) and the water depth (shallow, average, deep) with respect to flow quantity in establishing required dike safety level (low, medium, high).

TABLE 3.1- THREE SIMPLE RULES FOR SIMULATING DIKE HEIGHT FOR DESIGN.

1. If (Flow is frequent) or (Flow* is shallow) then (Safety is low)
2. If (Flow is infrequent) or (Flow* is average) then (Safety is medium)
3. If (Flow is rare) or (Flow* is deep) then (Safety is high)

3.1.3 Numerical example

The following demonstrates the deterministic procedure for dike height design and its modification for the implementation in the probabilistic and fuzzy domains.

3.1.3.1 Problem

Determine the height of a dike for 100 year return period flood protection with the discharge frequency curve in Table 3.2 and the stage discharge curve in Table 3.3. The freeboard value is 1 m. In the design problem use:

TABLE 3.2- THE DISCHARGE FREQUENCY DATA

T_r (years)	Exceedance probability	Discharge(m^3/s)
500	0.002	898.8
200	0.005	676.1
100	0.010	538.5
50	0.020	423.0
20	0.050	298.8

10	0.100	222.5
5	0.200	158.4

TABLE 3.3- THE STAGE DISCHARGE DATA

Discharge (m^3/s)	Stage (m)
898.8	8.32
676.1	7.57
538.5	6.70
423.0	5.80
298.8	4.76
222.5	4.00
158.4	3.24

a. Deterministic procedure
b. Explicit probabilistic simulation approach with normal distribution and given population properties for 100 year return period in Table 3.4.
    i. Expected value for dike height
    ii. Percentile (The height of the dike which will account for the flood stage value at or below which 90 percent of units lie.)
c. Implicit probabilistic simulation procedure with log-normal distribution and given population properties for 100 year return period in Table 3.4 and simulation program in MATLAB given in Appendix A.
    i. Expected value for dike height
    ii. Percentile (The height of the dike which will account for the flood stage value at or below which 90 percent of units lie.)

TABLE 3.4- MONTE CARLO SIMULATION INPUT DATA FOR LOG-NORMAL AND NORMAL DISTRIBUTION

	Mean	Standard deviation
100 year Discharge(m^3/s)	μ=538.5	σ =100 m^3/s
Stage(m)	$\mu= -6E\text{-}06x^2 + 0.0134x +$	σ =0.3

	1.2903 (where x is discharge)	

d. Fuzzy simulation procedure with the rule-based approach. Assume that 580 m^3/s design discharge is representative of a 100 year return period flow. Assume triangular membership functions for the linguistic variables. Use Table 3.2 and Table 3.3 as an aid for the membership function development.

3.1.3.2 Solution using deterministic dike design procedure

The deterministic design procedure follows the steps used in the description of the methodology.

Step 1. The values in Table 3.2 are graphed as discharge-frequency curve(Figure 3.1) and values in Table 3.3 are graphed as stage-discharge curve(Figure 3.2).

Step 2. Following deterministic procedure, the coresponding discharge is first found for the 100 year return period flood protection. Figure 3.1 shows how is the discharge found graphically to be 538.5 m^3/s.

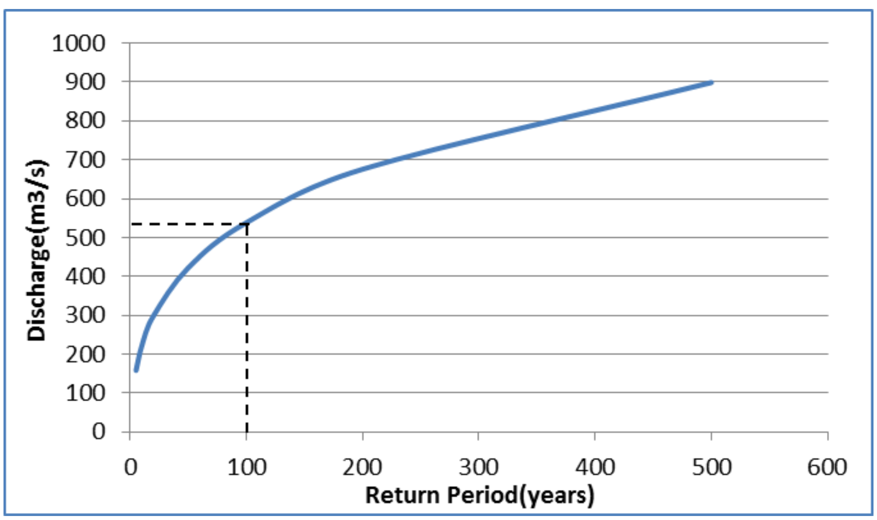


FIGURE 3.1- DISCHARGE-FREQUENCY CURVE (DOTTED LINE SHOWING DISCHARGE CORRESPONDING TO 100 YEAR RETURN PERIOD)

Step 3. Using the rating curve in Figure 3.2 or using the equation for the curve and solving with respect to the discharge of 538.5 m^3/s, a coresponding stage of 6.7 m is found.

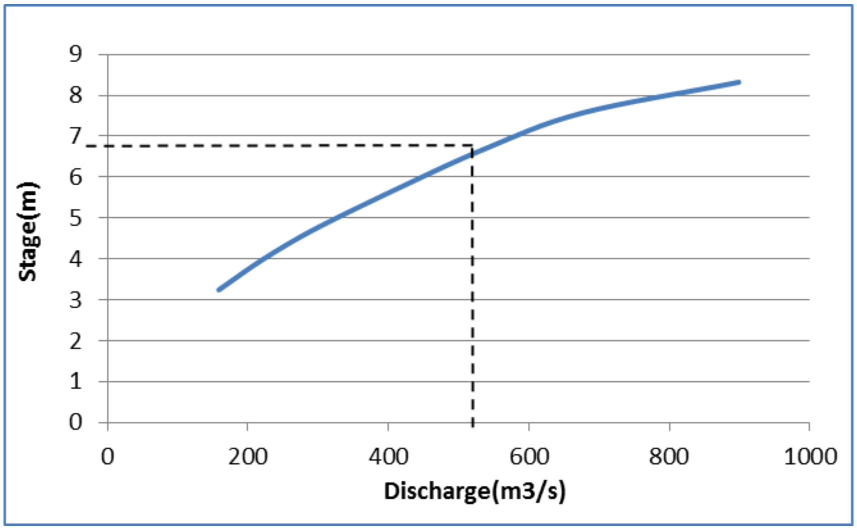


FIGURE 3.2- RATING CURVE

Step 4. Using the equation (3.1) the total design dike height is found:

The freeboard was given as 1 m and flood stage was solved earlier as 6.7 m thus:

$$H_t = H + H_f = 6.7 + 1 = 7.7\ m \tag{3.2}$$

Therefore it can be concluded that using the deterministic approach the appropriate total dike height for the 100 year flood is 7.7 m.

3.1.3.3. Solution using explicit probabilistic procedure

The probabilistic approach is applied to find the expected value of dike height as well as the 90% percentile value.

i) Expected dike height value

Step 1. Given expression (3.3) here for expected value of a normal distribution and Table 3.4, we first find the expected discharge.

$$E(X) = \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}\, dx \tag{3.3}$$

substitute z=(x-μ)/σ

$$E(X) = \int_{-\infty}^{\infty} (\sigma z + \mu) \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \tag{3.4}$$

$$E(X) = \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-z^2/2} dz \;\; + \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \tag{3.5}$$

substituting for first integral $V = -\frac{z^2}{2}$ $\quad dV = -z$ dz

$$E(X) = -e^{\frac{-z^2}{2}} \frac{\sigma}{\sqrt{2\pi}} + \;\; \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \tag{3.6}$$

$$E(X) = 0 + \mu(1) = \mu \tag{3.7}$$

Expression (3.3) simplifies to μ as shown above, a property of a normal distribution. Thus for a 100 year return period the expected value of discharge is:

$$E(Q) = 538.5 \text{ m}^3/s \tag{3.8}$$

Step 2. Using the data in Table 3.4 and the expected value equation (3.7) we find the expected value of flood stage corresponding to the expected value of discharge (538.5 m^3/s) is:

$$E(H) = \mu = -6.0\,(10^{-6})(538.5)^2 + 0.0134(538.5) + 1.2903 = 6.7\,m \tag{3.9}$$

6.7

Step 3. Using value from equation (3.9) and adding freeboard of 1 m the final value of the dike height is found.

$$E(H_t) = E(H) + H_f = 6.7 + 1 = 7.7 \text{ m} \tag{3.10}$$

The explicit probabilistic method yields 7.7 m as the solution for which the dike should be built the same as the solution from the deterministic method.

ii) The 90% percentile value of dike height

Step 1. The discharge is solved corresponding to the stated percentile and 100 year return period as shown in the equation:

$$0.9 = F(x) = \Pr[X \leq x] = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-(y-\mu)^2/2\sigma^2}\, dy \tag{3.11}$$

The integral becomes difficult to solve due to the error functions. Alternatively the transformation equation can be solved for x using the normal deviate z corresponding to the stated percentile:

$$z = \frac{x - \mu}{\sigma} \tag{3.12}$$

Normal deviate z is equal to 1.28 which is read in reverse order from the standard cumulative normal distribution table corresponding to 90th percentile. In addition the discharge normal distribution population mean and standard deviation is given in Table 3.4, rearranging Eq. (3.12) and substituting givens we get:

$$Q = X = (538.5) + (100)1.28 = 666.5\ m^3/s \tag{3.13}$$

The 90th percentile discharge (Q) is found to be 666.5 m^3/s.

Step 2. For the discharge of 666.5 m^3/s the corresponding stage population mean and standard deviation are found in Table 3.4.

The stage mean is found using the rating function given in Table 3.4 and substituting the discharge found in previous step

$$\mu = -6.0\ (10^{-6})(666.5)^{\ 2} + 0.0134(666.5) + 1.2903 = 7.556\ \mathrm{m} \tag{3.14}$$

The stage population standard deviation determined from Table 3.4 is

$$\sigma\ = 0.3\ m \tag{3.15}$$

Thus again using the normal deviate Eq. (3.12) for 90th percentile (z is equal to 1.28), with values from Eq. (3.14; 3.15) and rearranging the flood stage is found to be:

$$\text{Flood Stage, H} = \ x = (7.556) + 1.28(0.3) = 7.94\ m \tag{3.16}$$

Step 3. The flood stage from Eq.(3.16) of 7.94 m is significantly different from the value determined using the deterministic approach, especially after adding the additional freeboard of 1 m using equation (3.1).

$$H_t = \mathrm{H} + \mathrm{H}_f = 7.94 + 1 = 8.94\ \mathrm{m} \tag{3.17}$$

The design height for the dike is thus 8.94 m which is at 90 percent confidence for the 100 year return period. This is a much more conservative solution and may not be financially feasible.

3.1.3.4 Solution using implicit probabilistic procedure

Implicit probabilistic approach is used to find the expected and 90% percentile value of the dike height based on the provided data.

i) Expected dike height value

The implicit probabilistic simulation approach in a way will replicate the deterministic approach. Using the expected value of the log-normal distribution we first find the expected value of discharge for the 100 year return period and then the corresponding expected value of the stage. The equations below correspond to the expected value of the log-normal distribution. These equations are implemented with the Monte Carlo Simulation as discussed in section 2.1.1.

$$f(X) = \int_0^{\infty} \frac{1}{x\sqrt{2\pi\sigma_1^2}} e^{-(\ln x-\mu_1)^2/2\sigma_1^2}\, dx \tag{3.18}$$

$$E(X) = \int_0^{\infty} x\frac{1}{x\sqrt{2\pi\sigma_1^2}} e^{-(\ln x-\mu_1)^2/2\sigma_1^2}\, dx \tag{3.19}$$

Where,

$$\mu_1 = \ln\left(\frac{\mu^2}{\sqrt{\sigma^2 + \mu^2}}\right) \tag{3.20}$$

$$\sigma_1 = \sqrt{\ln\left(\frac{\mu^2 + \sigma^2}{\mu^2}\right)} \tag{3.21}$$

where, μ and σ correspond to the normal distribution mean and standard deviation respectively.

$$E(X) = e^{\mu_1 + \frac{\sigma_1^2}{2}} \tag{3.22}$$

Or

$$E(X) = \mu \tag{3.23}$$

In this example 2000 trails of input combinations are evaluated through the use of random number generator in an automated process. In this example the MATLAB software package was used to evaluate the expected value. The program code used for this example is included in Appendix A.

Step 1. For a 100 year return period using discharge parameters for the log-normal distribution provided in Table 3.4 for inputs into the MATLAB MCS program the expected discharge is determined to be:

$$E(Q) = 538.1\, m^3/s \tag{3.24}$$

Figure 6.1 in Appendix A shows the output from the MATLAB MCS program.

Step 2. Using the stage parameters for the log-normal distribution provided in Table 3.4 the corresponding expected stage is found. The mean value of stage is first solved in Eq. (3.25) using the mean stage-discharge function provided in Table 3.4.

$$\mu = -6.0\,(10^{-6})x^{2} + 0.0134x + 1.2903 \tag{3.25}$$

where x is the expected discharge from Eq. (3.24).

$$\mu = -6.0\,(10^{-6})(538.1)^{2} + 0.0134(538.1) + 1.2903 = 6.763\text{ m} \tag{3.26}$$

The mean of stage found in Eq. (3.26) and standard deviation given in Table 3.4 is used as inputs into the MATLAB MCS program yielding an expected flood stage value of:

$$E(H) = 6.773\,m \tag{3.27}$$

Figure 6.2 in Appendix A shows the output from the MATLAB MCS program.

Step 3. Using the equation (3.1) with freeboard of 1 m and expected flood stage of 6.773 m the total height the dike is found:

$$H_t = \mathrm{H} + \mathrm{H}_f = 6.773 + 1 = 7.773\text{ m} \tag{3.28}$$

The result of 7.773 m will vary from simulation to simulation as it is based on random generated values.

ii) The 90% percentile value of dike height

Step 1. Use the Monte Carlo simulation and find 90^{th} percent quartile of the log-normal distribution. Given the input discharge values in Table 3.4, MCS yields a 90^{th} percentile discharge of 667.3 m^3/s. Figure 6.1 in Appendix A shows the MATLAB MCS program output.

Step 2. Use the discharge value of 667.3 m^3/s and substitute into the mean stage equation (3.21). The result is a mean value of stage of 7.56 m in addition to the standard deviation of 0.3 m provided in Table 3.4.

The values of mean and standard deviation are used as input for the 90th percentile stage to be determined with MCS. Figure 6.2 in Appendix A shows the MATLAB MCS program output for 90th percentile flood stage to be

$$H = 7.96\ m \tag{3.29}$$

Step 3. The design dike height that flood level will be at or below 90 percent of the time given the addition of 1 m freeboard and using Eq. (3.1) is

$$H_t = \mathrm{H} + \mathrm{H}_f = 7.96 + 1 = \ 8.96\ \mathrm{m} \tag{3.30}$$

The dike height of 8.96 meters is conservative and provides a high safety level that may not be economically feasible.

3.1.3.5 Solution using fuzzy simulation procedure

The deterministic problem of dike design is transformed into a fuzzy domain and solved using the fuzzy rule-based Mamdani inference method presented in section 3.1.2.3.

Step 1. Development of fuzzy membership functions. We will start with partitioning the flow input space into three linguistic partitions within the interval of [0 m^3/s, 1000 m^3/s],"frequent", "infrequent", and "rare". Similarly we will partition stage input space according to flow into three fuzzy membership functions described linguistically within the interval of [0 m^3/s, 1000 m^3/s] as "shallow", "average", and "deep". The output variable safety that describes the required safety level of dike is represented with a fuzzy set with three linguistic partitions of "low", "medium" and "high" within the interval of [1m, 10m]. The fuzzy membership functions are assumed triangular for illustrative simplicity. The range for each partition and value which has the greatest membership in each fuzzy set (full membership is 1) governs the triangular membership function shape. These parameters were subjectively chosen by the authors. The fuzzy sets and their triangular membership functions are illustrated in Figs. 3.3, 3.4 and 3.5.

In Figure 3.3 the flow input space membership function shapes are selected based on the subjective belief that frequent flow is most appropriately represented by 280 m^3/s, infrequent flow is most appropriately represented by 520 m^3/s and rare flow is most appropriately represented by 1000 m^3/s. Each of the partitions have an ambiguous range surrounding the value representing the full degree of membership in the fuzzy set. In the case of the rare flow event the ambiguous range is one sided unlike the other partitions due to the subjective assumption that there is nothing rarer (no ambiguity) in terms of occurrence then the flow event of 1000 m^3/s. Similarly the parameters that govern the shapes of triangular membership functions in Fig. 3.4 and Fig. 3.5 are determined.

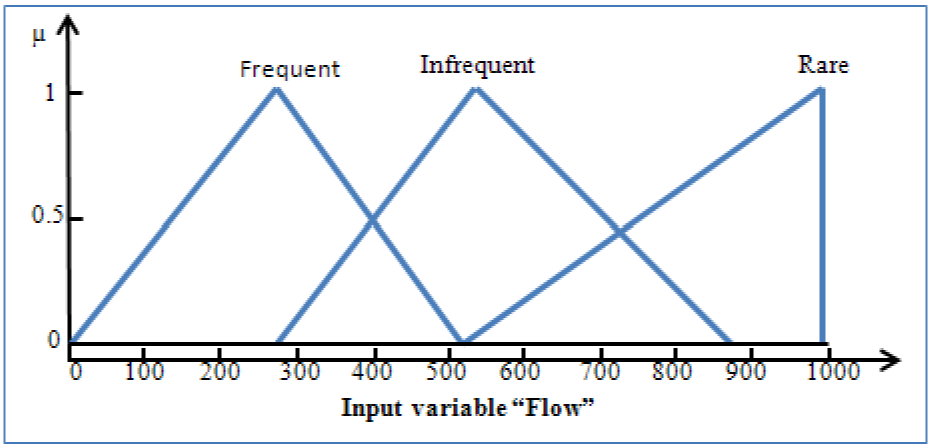


FIGURE 3.3- TRIANGULAR FUZZY MEMBERSHIP FUNCTIONS FOR FLOW, FLOW[M^3/S]

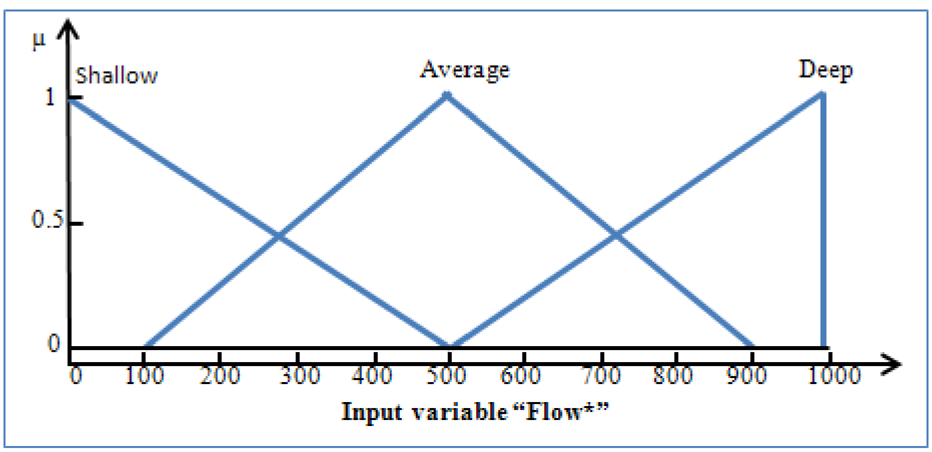


FIGURE 3.4- TRIANGULAR FUZZY MEMBERSHIP FUNCTIONS FOR FLOW*, FLOW*[M^3/S]

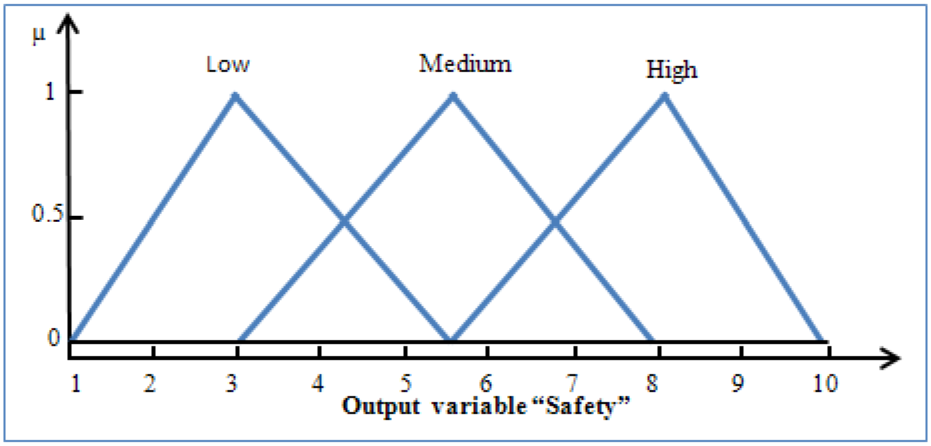


FIGURE 3.5- TRIANGULAR FUZZY MEMBERSHIP FUNCTIONS FOR SAFETY, SAFETY [M]

Step 2. Input fuzzification. The design flow input of 580 m^3/s is fuzzified in order for the fuzzy inference procedure to proceed. Using the appropriate membership functions, the scalar inputs are fuzzified and their results (results of rule 2 and 3 firing) are shown in Fig. 3.6.

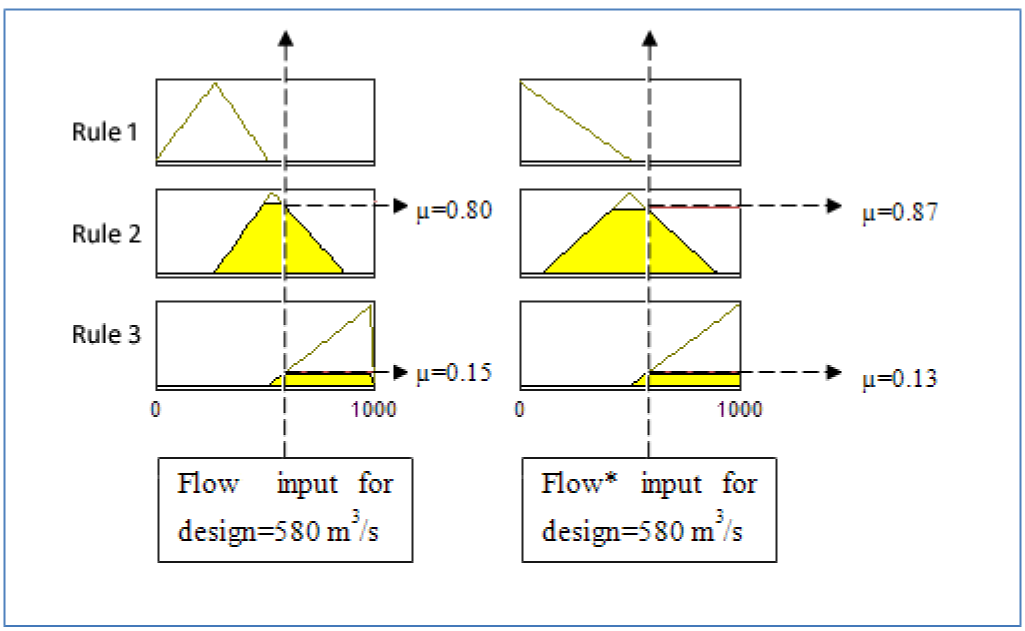


FIGURE 3.6- FUZZIFICATION OF THE DESIGN FLOW INPUT

Step 3. Application of fuzzy operators. As the antecedents are disjunctive the max operator is used. The antecedents for each rule the are represented by a single membership value

$$\mu_1 = max[0,0] = 0 \tag{3.31}$$

$$\mu_2 = max[0.8,0.87] = 0.87$$

$$\mu_3 = max[0.15,0.13] = 0.15$$

where μ_1, μ_2 and μ_3 are fuzzy membership values corresponding to rule 1, 2 and 3 respectively.

Step 4. Aggregation of outputs. The fuzzy membership functions corresponding to the output for each rule are truncated with respect to the membership values found in the previous step. These memberships are further aggregated using a disjunctive rule (max) system definition. The aggregation of outputs for dike height is illustrated in Figure 3.7.

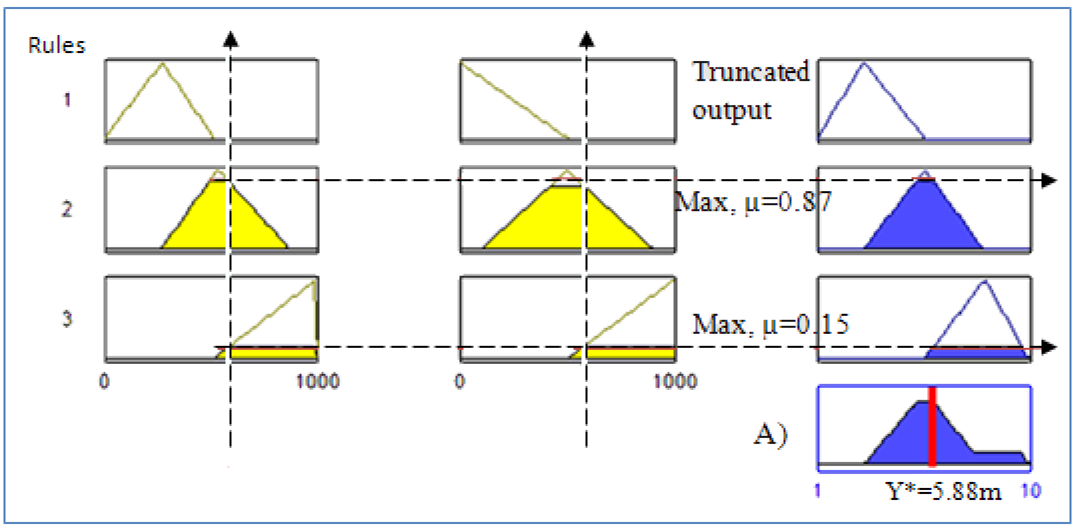


FIGURE 3.7- OUTLINE OF AGGREGATION PROCEDURE FOR DIKE HEIGHT DESIGN A) DISJUNCTIVE AGGREGATION OF RULES

Step 5. Deffuzification of the aggregated output. Finally, the aggregated output is defuzzified using the centroid method given in Eq. (2.30). The deffuzified value location is shown in Fig.3.7 as Y*.

$$H = Y^* = 5.88\ m. \tag{3.32}$$

This is the final value of the height designed to account for the flood stage. Eq. (3.1) and freeboard of 1 m is used for additional safety yielding

$$H_t = \mathrm{H} + \mathrm{H}_f = 5.88 + 1 = \ 6.88\ \mathrm{m} \tag{3.33}$$

The dike height is 6.88 meters based on the fuzzy simulation approach. This value is smaller than the one obtained by the deterministic approach, indicating more risk prone design.

3.2 Stormwater sewer pipe design

Stormwater is conveyed by buried pipes that carry it to a point where it is discharged to a stream, lake or ocean (Akan & Houghtalen, 2003). In reality the storm sewer system is not limited to just the sewer pipe but includes various structural components including inlets, manholes, junction chambers, transition structures, flow splitters and siphons(Akan & Houghtalen, 2003). A well designed, functional storm sewer system is an important part of any stormwater drainage system and is prerequisite for good storm water management. The right hydraulic design gives the proper diameter, slope and depth for a storm sewer line, so that it will drain storm water and not allow it to back up.

The sewer pipe design problem addressed here includes the selection of appropriate pipe diameter to carry the design stormwater runoff. The stormwater pipe size is determined by three main parameters; (1) the flow of water, (2) the grade the pipe will be placed at and (3) the pipes surface roughness. The pipe grade is dependent on the level of the pipe outlet to achieve drainage, the grade of the surface, avoiding obstacles and other pipes, and cover requirements. The pipe material affects the roughness. The pipe may be made from concrete, PVC, or of other material depending on what the decision maker feels is most appropriate application. The selection of these parameters for the design is dependent on the rainfall intensity of the design storm. The relationship of the parameters with pipe diameter is such that more flow, flatter grade and a rougher pipe internal surface all result in larger pipe size requirements.

Sewer pipe design can be done according to different available methods. Two common methods available for pipe sizing are the Manning's and Darcy-Weisbach / Colebrook-White equations. In order to use such equations the design flow or peak discharge must be known. The peak discharge is traditionally found using the Rational Method.

3.2.1 Problem identification

Pipe sizing design is dependent on natural variability of the amount of rainfall from the design storm that determines the flow that must be carried by the sewer pipe. The presence of natural variability in the data used for design is the main source of uncertainty. There is a need to quantify this uncertainty by modifying existing deterministic methods to show the risk of failure and in turn reliability of chosen design.

3.2.2 Mathematical formulation

The mathematical formulation of the stormwater sewer pipe design problem starts with the common deterministic procedure, and then its transformation into a probabilistic or fuzzy domain such that uncertainty may be quantified. The transformation is done through the probabilistic and fuzzy simulation approaches detailed in sections 2.1.1 and 2.2.2, respectively.

The modified approaches hope to model the inherent uncertainty with the hydrologic variables and bring more certainty to decision makers. The mathematical formulation will be followed up with a numerical example of the application of the deterministic, probabilistic ((a) implicit using Monte Carlo simulation and (b) explicit analytically solving with probability equations), and fuzzy approach.

The deterministic approach uses region specified hydrologic data from Intensity Duration Frequency (IDF) curves. This differs from the probabilistic design approach that is based on assigning probability density functions to intensity corresponding to each duration and frequency (return period). Simulation is carried out using the Monte Carlo simulation method because solving the problem analytically becomes too complex. Where data for IDF curves development is unavailable or only partially available, the fuzzy approach may be used to subjectively arrive at a potential solution with adequate precision.

3.2.2.1 Deterministic approach

The deterministic approach of sizing a sewer pipe is summarized in the 4 step procedure below:

Step 1. Find the time of concentration. The time of concentration is defined as the time required for storm water to flow from the hydrologically most remote point in the basin to the pipe inlet structure. It is sometimes referred to as the hydraulic length. The peak discharge under a constant rate of effective rainfall will be reached if the effective rain duration is equal to the time of concentration.

$$T_c = t_0 + t_f \tag{3.34}$$

where t_0 (inlet time) is the time required for storm water to reach an inlet from the hydrologically most remote point, t_f is the flow time in the pipes upstream of the design point and T_c is the time of concentration.

The flow time in the pipes upstream of the design point can be determined using:

$$t_f = \sum_{j=1}^{N} \frac{L_j}{V_j} \tag{3.35}$$

where L_j is the length of the *j*th pipe, V_j is the average velocity in the *j*th pipe and N is the number of pipes upstream along the flow path considered.

The inlet time is calculated by (a) use of Table 3.5 below; (b) by the well documented and widely used Soil Conservation Service Time of concentration method; or (c) one of other many available methods. These methods are beyond the scope of this report and can be followed up in the textbook by Akan & Houghtalen (2003).

TABLE 3.5- INLET TIME COMMON VALUES (AKAN & HOUGHTALEN, 2003).

Densely developed impervious surfaces directly connected to drainage system	5 minutes
Well-developed districts with relatively flat slopes	10-15 minutes
Flat residential areas with widely spaced street inlets	20-30 minutes

Step 2. For the selected return period, the intensity of the design rainfall is obtained from the IDF curves, assuming the storm duration equals the time of concentration.

Frequency analysis methods are used to develop the IDF curves. First, the annual maximum rainfall depths corresponding to various durations are extracted from the local historical rainfall data. Then a frequency analysis of annual maximum depths is performed for each duration. Frequency analysis of rainfall aims to determine the return periods associated with different magnitudes of the annual maximum rainfall depth (intensity) for a particular duration. A probability distribution is fit to the annual maximum series. Experience shows that most rainfall data fit well the Extreme Value Type I Gumbel distribution. In practice this distribution is often used for frequency analysis of rainfall data.

Step 3. Once the IDF curves are developed and intensity is obtained the design discharge can be found by Rational Method:

$$Q_p = i \sum_{j=1}^{M} C_j A_j \tag{3.36}$$

where i is the design rainfall intensity from IDF curve, M the number of subareas above the storm water pipe, A the drainage area of subarea j, C the runoff coefficient and Q_p the design peak discharge.

Step 4. Finally, once the design discharge is determined, Manning's equation can be used to find the required pipe size. For circular pipes the formula is,

$$Dr = \left[\frac{nQ_p}{0.31 k_n \sqrt{S_0}}\right]^{\frac{3}{8}} \tag{3.37}$$

where Dr is the minimum diameter of pipe (actual size is next standard pipe larger size available), K_n is the conversion (1.0 $m^{1/3}$/s for Si units and 1.49 $ft^{1/3}$/s for U.S. customary units), S_0 is the bottom slope of sewer and n the manning roughness factor.

The above formula is only valid under the assumption that the flow is full at the design discharge in the pipe. In addition there is a minimum velocity requirement for the flow in the pipe of 0.6-0.9 m/s to prevent the deposition of suspended materials and a maximum velocity of 3-4.5 m/s to prevent scouring (Chin, 2006).

3.2.2.2 Probabilistic approach

The probabilistic approach follows much the same procedure as the deterministic approach with alteration in how the values of rainfall intensity are modified to represent uncertainty and risk. Modifications are done to step 3 and step 4 of deterministic method to be probability based. Furthermore, the probabilistic approach can be in implicit or explicit form. The implicit approach assumes an underlying distribution and based on that distribution generates random numbers. The explicit procedure follows direct use of probability distribution equations, when the distribution can be solved analytically.

Consider the explicit procedure first. The rainfall intensity is subject to a source of uncertainty and this uncertainty is fitted with an appropriate continuous probability distribution with appropriate statistical parameters (population mean and standard deviation). It should be noted that the population mean and standard deviation in practice are not known and are usually replaced by sample mean and standard deviation that are based on finite number of historical observations. The probability distribution, once known, can be used to analytically solve for the expected value and percentiles (value at or below which the stated percentage of units lie) of the probabilistic variable which in this case would be the rainfall intensity.

The intensity ($\tilde{\imath}$) probabilistic variable would replace the deterministic intensity variable in Eq. (3.36) resulting in a probabilistic discharge value ($\widetilde{Q_p}$).

$$\widetilde{Q_p} = \tilde{\imath} \sum_{j=1}^{M} C_j A_j \tag{3.38}$$

For expected discharge value E(Q_p), given the expected intensity E(i), the above equation can be rewritten as:

$$E(Q_p) = E(i) \sum_{j=1}^{M} C_j A_j \tag{3.39}$$

Similarly equation (3.37) as a result of the probabilistic discharge variable ($\widetilde{Q_p}$) would result in a probabilistic sizing of sewer pipe ($\widetilde{Dr}$).

$$\widetilde{Dr} = \left[\frac{n\widetilde{Q_p}}{0.31k_n\sqrt{S_0}}\right]^{\frac{3}{8}} \tag{3.40}$$

For expected value of sewer pipe diameter,$E(Dr)$ given expected value for discharge, the above equation can be rewritten as:

$$E(Dr) = \left[\frac{nE(Q_p)}{0.31k_n\sqrt{S_0}}\right]^{\frac{3}{8}} \tag{3.41}$$

The implicit procedure of using simulation relates closely to the explicit formulation. The implicit procedure accounts for uncertainty in intensity by fitting it with an appropriate continuous probability distribution function that is randomly sampled to produce hundred or even thousands of scenarios or iterations. The distribution of the values calculated for the model outcome therefore reflects the probability of the values that could occur. The aforementioned technique is known as Monte Carlo Simulation (MCS), and is discussed in section 2.1.1.

The output of the simulations can be studied for the statistical properties and to answer what if questions of the decision maker. These outputs would be substituted for intensity in the modified rational method equation (3.42). Where the random generated intensity is denoted by $\hat{\imath}$ which subsequently makes the discharge a random number $\widetilde{Q_p}$, using the modified Manning equation (3.43), the diameter of pipe also becomes a random variable $\widetilde{D_r}$.

$$\widetilde{Q_p} = \hat{\imath} \sum_{j=1}^{M} C_j A_j \tag{3.42}$$

$$\widetilde{D_r} = \left[\frac{n\widetilde{Q_p}}{0.31 k_n \sqrt{S_0}}\right]^{\frac{3}{8}} \tag{3.43}$$

3.2.2.3 Fuzzy approach

The fuzzy approach used to simulate approximate pipe size follows the fuzzy inference rule-based approach (presented in section 2.2.2). The mathematical model formulation for the stormwater sewer pipe design problem utilizing the fuzzy simulation approach will be based on five simple linguistic rules, listed in Table 3.6, each with a single antecedent of flow and a single consequent pipe size (diameter). These rules are subjective and ambiguous, developed using the knowledge of the complex form that is available. For example, the rules are developed with some knowledge of hydraulics or empirical evidence of increasing flow requiring incrementally larger pipe sizes. The rules in Table 3.6 are used to represent this knowledge by using linguistic variables to separate range of flows and pipe sizes. Obviously a deterministic model already exists that gives exact solutions in the form of the Manning equation for pipe size. However, assuming such a relationship was not made the rule-based approach would be best utilized to give some precision where none existed.

TABLE 3.6- FIVE SIMPLE RULES FOR DESCRIBING SIZING A PIPE.

1.	If (flow is zero) then (pipe size is zero)
2.	If (flow is small) then (pipe size is small)
3.	If (flow is medium) then (pipe size is medium)
4.	If (flow is large) then (pipe size is large)
5.	If (flow is extra large) then (pipe size is extra large)

3.2.3 NUMERICAL EXAMPLE

This problem demonstrates the existing deterministic procedure for sewer pipe design and its modification for the implementation in the probabilistic and the fuzzy domains.

3.2.3.1 Problem description

The design problem considers a basin with an area of 2 hectares and runoff coefficient of 0.6 where a concrete (n=0.013) sewer pipe will be installed at a slope of 0.5%. The preliminary basin investigations determined the longest flow path time to the proposed pipe location to be 15 minutes. Determine the appropriate pipe size for the data shown in Table 3.7.

The design problem is to be addressed using:

a. Deterministic approach(given the IDF curve in Figure 3.8)
b. Explicit probabilistic approach
    i. Find the expected value of pipe size
    ii. Find the percentile (The size of pipe which will account for the intensity value at or below which 90 percent of units lie.)
c. Implicit probabilistic simulation approach(MATLAB program for simulation given in Appendix B)
    i. Find the expected value of pipe size
    ii. Find the percentile (The sizing of pipe which will account for the intensity value at or below which 90 percent of units lie.)
d. Fuzzy approach(using fuzzy simulation and rule-based inference)

The population distribution is assumed to be normal for the explicit probabilistic approach and log-normal for the implicit probabilistic approach. The properties of the distributions are assumed based on the sample mean and the standard deviation statistics shown in Table 3.7. For simplicity, assume full flow and omit scouring and deposition checks. For the fuzzy approach assume that the pipe size is a result of the simulation valid only for the basin under consideration; assuming no previous knowledge of a deterministic model for sizing or IDF curves.

TABLE 3.7- STATISTICAL PROPERTIES FOR THE 5 YEAR DESIGN STORM

Intensity / Duration	Mean	Std.
(Minutes)	μ(mm/min)	σ(mm/min)
15	3	2
30	2	2
60	1	2
120	0.5	2

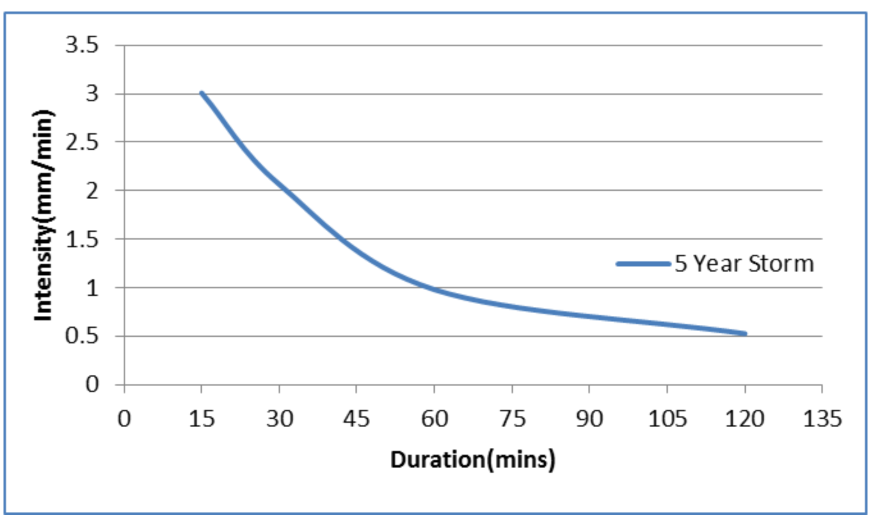


FIGURE 3.8- INTENSITY DURATION FREQUENCY (IDF) CURVE

3.2.3.2 Solution using deterministic pipe design approach

Step 1. Determine the time of concentration. Time of concentration is given as 15 minutes and there is no upstream pipe to contribute to a longer flow time. Therefore using equation (3.34):

$$T_c = t_0 + t_f = 15 + 0 \quad (3.44)$$

$$T_c = 15\ minutes$$

Step 2. Determine the peak flow rate, given that the runoff coefficient is 0.6 and drainage area is 20 000 m^2.

The intensity is found to be 3mm/min (as shown in Figure 3.9) based on the IDF curve in Figure 3.8 and the known 15 minute time of concentration for the critical duration of storm.

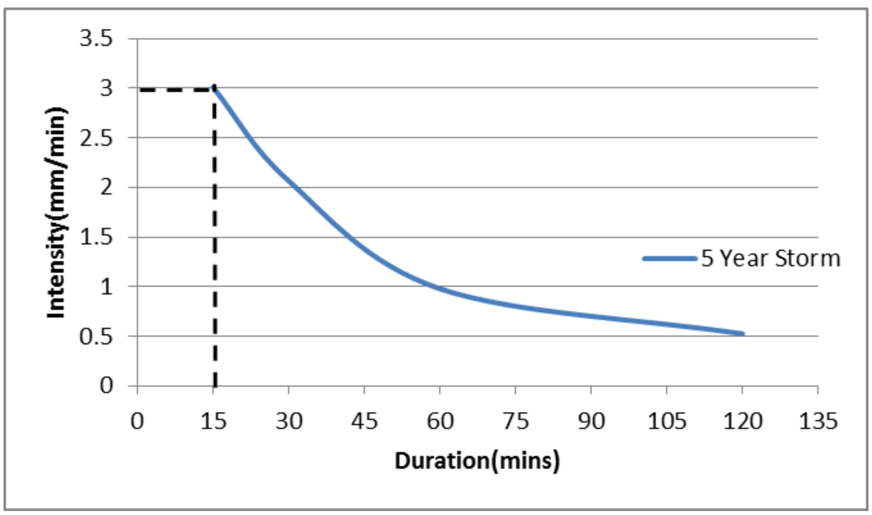


FIGURE 3.9- THE 5 YEAR DESIGN STORM IDF CURVE AND DETERMINATION OF INTENSITY FOR 15 MINUTE STORM

Finally using the rational method, equation (3.36) and substituting for the known variables the peak flow is determined.

$$Q_p = 0.6 * 20000 * \frac{3}{(60 * 1000)} = 0.6\ m^3/s \tag{3.45}$$

Step 3. The Manning equation (3.36) for sizing of the pipe provides the diameter of sewer pipe:

$$Dr = \left[\frac{(0.013)(0.6\ )}{0.31(1)\sqrt{(\frac{0.5}{100})}}\right]^{\frac{3}{8}} \tag{3.46}$$

$$Dr = 0.6787\ m$$

The diameter of the pipe for design is to be at minimum 678.7 mm, the next largest standard size available is 700 mm.

3.2.3.3 Solution using explicit probabilistic approach

i) Expected value

The same example from the deterministic case is addressed using the probabilistic expected value method.

Step 1. The distribution as stated in the problem is normal with the properties as assumed in Table 3.7. For the time of concentration of 15 minutes and the same critical storm duration, the modified rational method (Eq. 3.39) is used to determine the expected peak flow. The expected value E(i) is determined using expression (3.7), with the population mean and population standard deviation corresponding to the assumed values for the given duration originating from Table 3.7, where μ is the population mean, E(i) or μ is 3mm/min corresponding to 15 minutes design storm duration. Substituting this value into the rational method equation (3.39) the expected flow peak value becomes:

$$E(Q_p) = 0.6 * 20000 * \frac{3}{(60 * 1000)} = 0.6\ m^3/s \tag{3.47}$$

Step 2. The sewer pipe is size is obtained using equation (3.41).

$$E(Dr) = \left[\frac{(0.013)(0.6)}{0.31(1)\sqrt{\left(\frac{0.5}{100}\right)}}\right]^{\frac{3}{8}} = 0.6787\ m \tag{3.48}$$

The result, as in the deterministic case calls for a pipe with the diameter of 700 mm. It is the next largest size of standard diameter pipe available able to receive the expected flow rate.

ii) The 90% percentile value of pipe size

Step 1. The distribution from the problem definition is assumed to be normal with the properties as shown in Table 3.7.

For the time of concentration of 15 minutes that is assumed to be equal to the critical storm duration, the peak flow is determined using the modified rational method in equation (3.38).

The probabilistic value for intensity i, correspond to the 90 percentile of the normal distribution function. Using equation (3.11), the integral becomes difficult to solve due to the error functions. Alternatively using the normal deviate z corresponding to the 90th percentile (z is 1.28) and the transformation equation (Eq. 3.12) we can solve for "x" or i, given the assumed statistical parameters in Table 3.7.

$$i = x = z\sigma + \mu = 1.28(2) + 3 = 5.56\ mm/min \tag{3.49}$$

The intensity value and other given variables from the problem definition are substituted in the modified rational method equation (3.38) in order to solve for the peak flow.

$$\widetilde{Q_p} = 0.6 * 20000 * \frac{5.56}{(60 * 1000)} = 1.112\ m^3/s \tag{3.50}$$

Step 2. Size of pipe is obtained using equation (3.40).

$$\widetilde{Dr} = \left[\frac{(0.013)(1.112)}{0.31(1)\sqrt{(\frac{0.5}{100})}}\right]^{\frac{3}{8}} = 0.855\ m \tag{3.51}$$

The pipe diameter is taken as the next largest available standard diameter increment which is 900 mm.

3.2.3.4 Solution using implicit probabilistic approach

i) Expected value

Step 1. The same example from the deterministic case is addressed using probabilistic simulation for the expected value. For the time of concentration of 15 minutes and the same critical storm duration, the modified rational method, equation (3.39) is used to determine the expected peak flow. The log-normal cumulative distribution equation, transformation and other properties are presented by equations (3.18) to (3.23).

Monte Carlo Simulation (MCS) technique is used to solve this problem. MCS consists of artificially recreating a chance process by adding a probability density function around each intensity mean parameter in Table 3.7 in order to describe the uncertainty properties of the statistics.

For example, the point corresponding to the critical 15 minute storm duration has a built in log-normal shaped probability density function. The log-normal probability density function with the mean of 3 mm/min and standard deviation of 2 mm/min is defined for the input.

Then 2000 input combinations are selected and evaluated through a use of a random number generator in an automated process. In this example the MATLAB software is used to evaluate the expected value for the probabilistic approach. The program source code is available in Appendix B. Each run of the program results in a slightly different expected value of intensity due to the random nature of the process (example output Figure 6.3 Appendix B). Using MCS the 15 minute storm expected random intensity is found to be:

$$E(\hat{\imath}) = \; 3.0474 \; mm/min \tag{3.52}$$

Substituting the value in Eq. (3.52) into Eq. (3.42), the modified rational method, the implicitly determined expected peak flow value becomes:

$$E(\widetilde{Q_p}) = 0.6 * 20000 * \frac{3.0474}{(60 * 1000)} \tag{3.53}$$

$$E(\widetilde{Q_p}) = 0.61 m^3/s$$

Step 2. The sewer pipe is determined using equation (3.43). The expected diameter of pipe is a random variable given that the discharge input from Eq. (3.53) is a random variable as well.

$$E(\widetilde{Dr}) = \left[\frac{nE(\widetilde{Q_p})}{0.31 k_n \sqrt{S_0}}\right]^{\frac{3}{8}}$$

$$E(\widehat{Dr}) = \left[\frac{(0.013)(0.61)}{0.31(1)\sqrt{(\frac{0.5}{100})}}\right]^{\frac{3}{8}} \tag{3.54}$$

$$E(\widehat{Dr}) = 0.6829\ m$$

The result, as in the deterministic case, is a pipe with the diameter of 700 mm. This is the next largest size of standard diameter pipe available in order to receive the expected flow rate.

ii) The sizing of pipe which will account for the intensity value at or below 90 percent.

Step 1. For the time of concentration of 15 minutes that is assumed to be equal to the critical storm duration, the peak flow is determined using the rational method previously formulated in equation (3.36)

The probabilistic value for intensity corresponds to the 90th percentile of the log-normal distribution function.

$$0.9 = F(x) = \Pr[X \leq x] = \int_0^x \frac{1}{x\sigma_1\sqrt{2\pi}} e^{-(lny-\mu_1)^2/2\sigma_1^2}\ dy \tag{3.55}$$

Using the Monte Carlo Simulation technique as described in the previous section the intensity that will occur at or below 90 percent of the time can be determined. The same MATLAB program used to evaluate the expected random value of intensity is used to evaluate 90th percentile value of intensity; the source code is available in the Appendix B.

The 90th percentile intensity as a result of MCS random number generation is found to be:

$$\hat{\imath} = 5.43\ mm/min \tag{3.56}$$

The intensity value and other given variables from the problem definition are substituted in Eq. (3.42) in order to solve for the peak flow corresponding to the 90th percentile intensity.

$$\widehat{Q_p} = 0.6 * 20000 * \frac{5.43}{(60 * 1000)} \tag{3.57}$$

$$\widehat{Q_p} = 1.086\ m^3/s$$

Step 2. Finally the pipe is sized using equation (3.43).

$$\widehat{Dr} = \left[\frac{n\widehat{Q_p}}{0.31 k_n \sqrt{S_0}}\right]^{\frac{3}{8}} \tag{3.58}$$

$$\widehat{Dr} = \left[\frac{(0.013)(1.086)}{0.31(1)\sqrt{(\frac{0.5}{100})}}\right]^{\frac{3}{8}}$$

$$\widehat{Dr} = 0.847\ m$$

The pipe diameter size is taken as the next highest standard diameter increment which is 900 mm.

The deterministic problem of pipe sizing is transformed into a fuzzy domain and solved using the fuzzy rule-based Mamdani inference method presented in section 3.2.2.3 of the report.

First, the input space "flow" is partitioned into five simple partitions in the interval [0 m^3/s, 1 m^3/s], and the output space "pipe size" is partitioned in the interval [-0.4888 m, 1 m] into five membership functions as shown in Fig.3.10 and 3.11 respectively.

The input variable flow corresponds to a fuzzy set which has five linguistic partitions describing a discharge flow; with the partitions labeled in Fig. 3.10. The output variable pipe size corresponds to the pipe diameter and the fuzzy set partitions are labeled in Fig. 3.11. The triangular fuzzy set membership functions shape has been assumed for illustrative simplicity. The flow input space and the pipe size output space parameters are subjectively chosen. The height of the triangle is defined by the value which is subjectively assumed to hold the full membership in the given membership function and the base of the triangle is the range of ambiguous values holding some degree of membership in the fuzzy set.

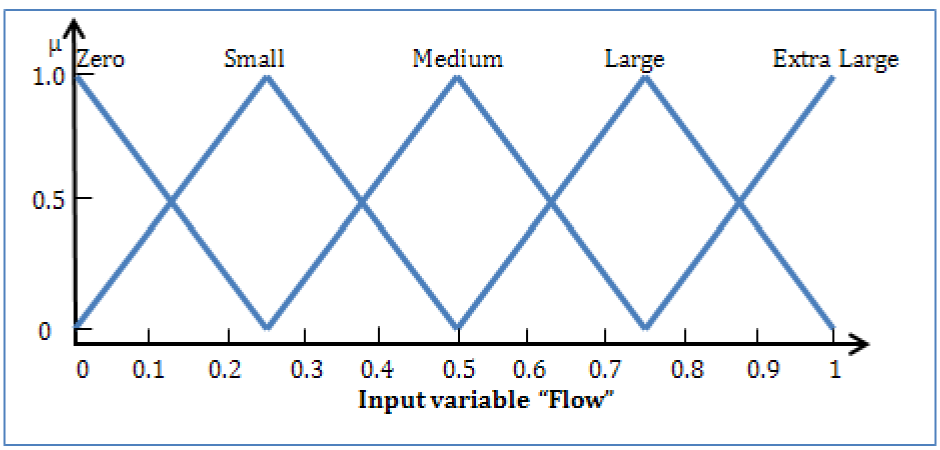


FIGURE 3.10- FIVE PARTITIONS FOR THE INPUT VARIABLE, FLOW (M^3/S).

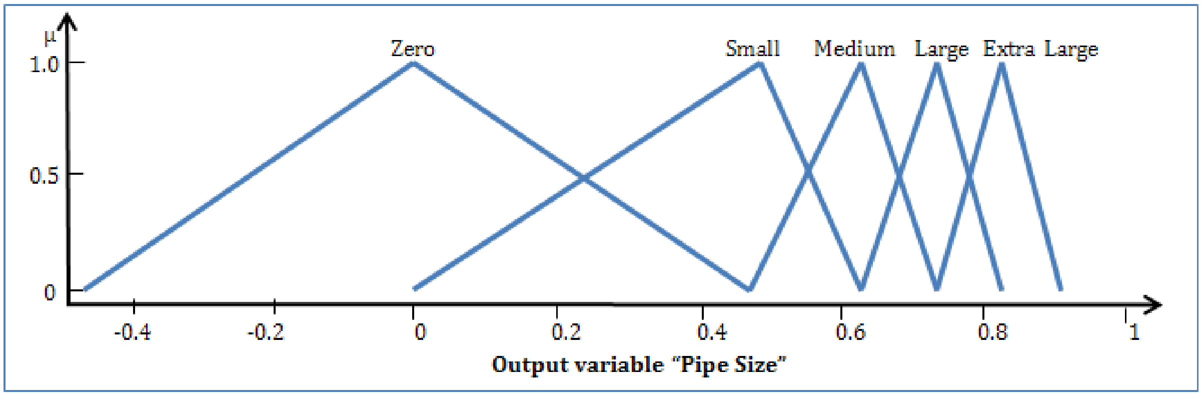


FIGURE 3.11- FIVE PARTITIONS FOR THE OUTPUT VARIABLE, PIPE SIZE [M].

In order to find the approximate solution for the pipe size output a few input points are selected and the Mamdani graphical inference method is employed. The centroid method is used for defuzzification.

Let us choose eleven crisp singletons for inputs:

$$flow = \{0, 0.1, 0.2, 0.3, 0.4, 0.5\ ,0.6\ ,0.7, 0.8, 0.9, 1.0\} m^3/s \tag{3.59}$$

To illustrate the procedure, for flow input of $0.1m^3/s$, rules 1 and 2 are fired as shown in Fig. 3.12. The resulting aggregated output after applying the union operator (disjunctive rules) is found and the fuzzy set is defuzzified using the centroid method yielding a result of *0.0847 m* for pipe size as shown in Figure 3.12.

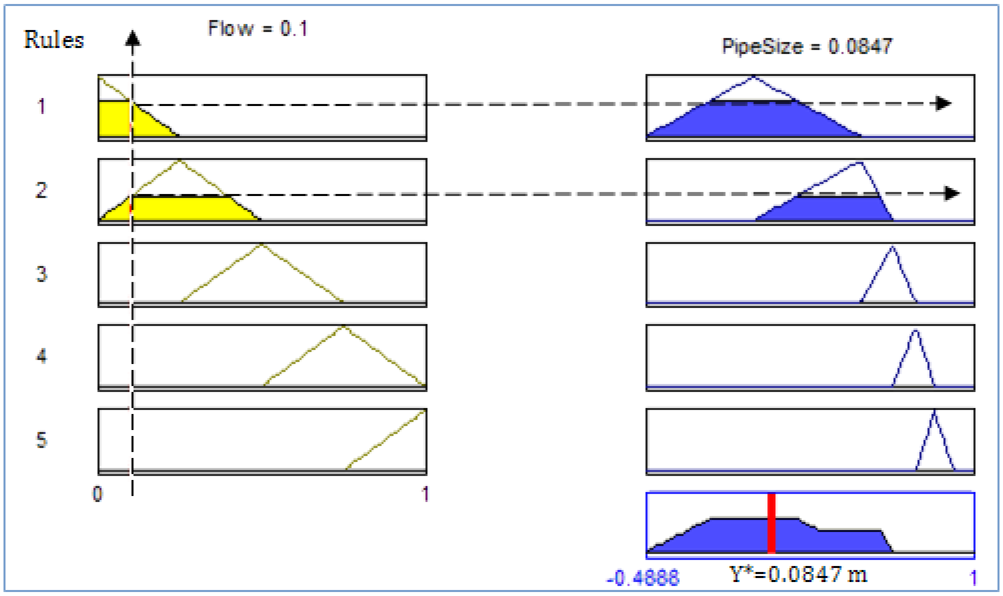


FIGURE 3.12- GRAPHICAL INFERENCE METHOD - MEMBERSHIP PROPAGATION AND DEFUZZIFICATION.

The results for each input, once aggregated and defuzzified are summarized in Table 3.8 and compared to those values determined by using the deterministic model. The graphical comparison is available as well, in the plot shown in Figure 3.13. As we can see, the results using the fuzzy approach are very similar to the true solution. The precision may be increased by increasing the number of additional rules.

TABLE 3.8- COMPARISON OF PIPE DIAMETER(FUZZY AND DETERMINISTIC MODELS)

Discharge (m^3/s)	Pipe size(m) Deterministic	Pipe size(m) Fuzzy
0.0	0.000	0.000
0.1	0.347	0.085
0.2	0.450	0.233
0.3	0.523	0.391
0.4	0.583	0.425
0.5	0.634	0.620
0.6	0.679	0.654
0.7	0.719	0.693
0.8	0.756	0.751
0.9	0.790	0.781
1.0	0.822	0.823

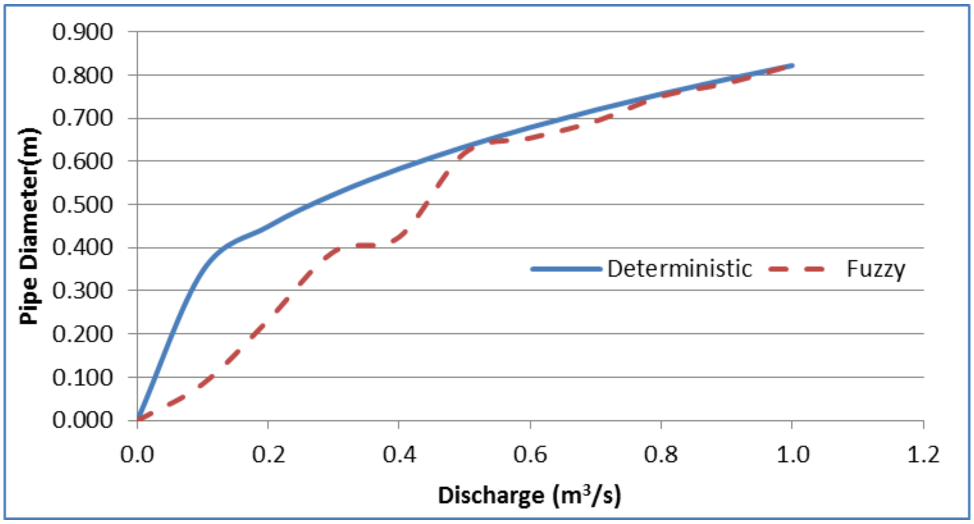


FIGURE 3.13- COMPARISON OF PIPE DESIGN RESULTS: FUZZY APPROACH AND DETERMINISTIC APPROACH

3.3 Single reservoir operation

Reservoirs are used to store water; they take different structural forms depending on their design functions (recreation, flood protection, water supply, etc.). The reservoirs may be created in a river valley by the use of a dam, by excavation in the ground, or by conventional construction techniques such as concrete.

Different design, planning, operation and management requirements lead to different formulations of models for optimization. Planning, design, operation and management of reservoirs require knowledge of various stream flow characteristics.

Consider a reservoir operation problem concerned with finding the operation release schedule for stored water with the goal to minimize the damage as a result of reservoir water inundation to surrounding property. The problem is an operation one as the reservoir is already constructed and cannot be modified in order to avoid potential water damages to nearby property. The problem presented illustrates the importance of optimization towards finding the appropriate operator controlled releases.

3.3.1 Problem identification

Reservoir operation is challenging, in that the reservoir operator must make long term release schedules to accommodate incoming periods of floods and droughts so that the overall reservoir design goals are met. Generally, for reservoir operation optimization, inflow data must be given and discharge or release is the decision variable. The inflow data is from historic records, it is assumed to be an adequate representation for future inflows. This assumption may hold critical error and uncertainty (in the form of natural hydrologic variability) in making decisions concerning reservoir operation. It is of importance then to deal with the reservoir operation problem under uncertainty. In addition the reservoir operation problem includes the inability of operators to formulate sharp (crisp) boundaries or constraints, due to uncertainty in knowledge. Crisp constraints are required for the implementation of traditional deterministic optimization models. Therefore, the goal of this optimization exercise is to take into account the hydrologic variability and allow formulation of constraints with some range of uncertainty.

3.3.2 MATHEMATICAL FORMULATION

Reservoir problems by nature deal with random parameters due to the hydrological inflow input, thus it is at no surprise that the reservoir optimization problems are solved in the literature using both, deterministic and stochastic methods. Stochastic reservoir optimization may take two forms: (a) implicit (deterministic models with the generated sequences of random variables); and (b) explicit (uncertainty incorporated directly in the objective function and/or constraints). The latter explicit method will be looked in detail through the chance constrained probabilistic method, an approach that has been extensively used in water resources (Simonović, 2002). In addition, the problem will be addressed using a fuzzy optimization approach when data is not available and constraints are not crisply formulated.

3.3.2.1 DETERMINISTIC APPROACH

A deterministic optimization model is formulated to optimize the reservoir operation by determining the optimal release from the reservoir in various time intervals, under the objective of minimizing flood damage due to excess storage of water in the reservoir. The flood damage is a function of storage and therefore to minimize active storage in the reservoir is to minimize potential flood damage. In mathematical form, the objective of optimization can be stated as:

$$Minimize\ Z = S \tag{3.60}$$

where S, is the active volume of water stored in the reservoir.

The model is governed by the continuity equation:

$$S_{t-1} + i_t - R_t = S_t \qquad t = 1,2,\ldots\ldots,n \tag{3.61}$$

where: S_t is the volume of water in the reservoir at time t, i_t is the inflow into reservoir in the time interval (t-1, t), and R_t is the amount of water discharged/released downstream in time interval (t-1, t). The known values in the continuity equation are inflows; other known values are physical features of the reservoir, that is; maximum and minimum storage capacity, initial volume of stored water, and maximum release through the outlet structure.

There are three constraints:

1) A deterministic constraint on the reservoir release

$$0 \leq R_t \leq R_{max} \qquad\qquad t = 1,2, \ldots, n \tag{3.62}$$

2) A deterministic constraint that prohibits the storage of water below a certain operational level S_{min} and in excess of the reservoir capacity, C.

$$S_{min} \leq S_t \leq C \qquad\qquad t = 1,2, \ldots, n \tag{3.63}$$

3) The last deterministic constraint states that the storage at the end of the critical period must be at least as great as the unknown starting storage. This last constraint prevents "borrowing water" to artificially inflate the amount of water that can be delivered steadily throughout the course of the critical period (ReVelle, 1999).

$$S_0 \leq S_n \tag{3.64}$$

The optimization problem described above can be solved, using classical linear programming algorithm based on the simplex method.

$$\max c^t u \ \ under\ the\ conditions \tag{3.65}$$

$$Au \leq b$$

$$u \geq 0$$

where *c* represents column vector of the objective function coefficients, *u* is the column vector of the decision variables, *A* is the matrix of the coefficients in the constraints and *b* is the column vector of the right-hand sides of the constraints.

Finally putting all terms together, the reservoir optimization linear programming problem is formulated:

$$Minimize\ Z = S \tag{3.66}$$

$$Subject\ to, mathematical\ model$$

$$S_{t-1} + i_t - R_t = S_t \qquad t = 1,2, \ldots\ldots, n$$

$$Subject\ to\ constraints,$$

$$0 \le R_t \le R_{max} \qquad t = 1,2, \ldots\ldots, n$$

$$S_0 \le S_n$$

$$S_{min} \le S_t \le C \qquad t = 1,2, \ldots\ldots, n$$

The constraints are linear, the state equation is linear and the objective function is chosen in linear form. The optimal solution can be obtained using various software tools readily available.

3.3.2.2 Probabilistic approach

The same reservoir deterministic model already developed will be transformed here in the probabilistic form to deal with some uncertain inputs. The transformation to stochastic optimization as discussed in section 2.1.2 is done through the introduction of an additional probabilistic constraint, shown below.

$$P\{\tilde{s}_t \le S_{goal}\} \ge \propto \qquad t = 1,2, \ldots\ldots, n \tag{3.67}$$

where $\tilde{s}_t$ is the random equivalent of s_t, the storage at the end of period t, S_{goal} is the known decision maker specified target storage level of the reservoir and $\propto$ is the decision maker specified reliability of not violating constraint (3.67). It takes values between (0 and 1).

The stochastic problem is reduced to a deterministic equivalent by the method of chance constraints. It is assumed that the random components are additive from one period to the next. Then the probability density function of their sum can be obtained by convolution regardless of whether or not they are independent (Simonović, 1979).

The probabilistic constraint is transformed into a deterministic chance constraint by the following procedure.

Step 1. Continuity equation (3.61) is substituted into probabilistic constraint equation (3.67) allowing constraint to be rewritten as:

$$P\{S_{t-1} + \tilde{\imath}_t - R_t \leq S_{goal}\} \geq \propto \qquad t = 1,2,\ldots\ldots,n \tag{3.68}$$

where, $\tilde{\imath}_t$ is the random equivalent of i_t, the inflow during t.

Step 2. A deterministic equivalent of the equation (3.68) is found by inversion and rearrangement leading to:

$$S_{goal} - S_{t-1} + R_t \leq F_{x_t}^{-1}(1-\propto) \qquad t = 1,2,\ldots\ldots,n \tag{3.69}$$

where, $F_{x_t}^{-1}(1-\propto)$ is the inverse value of the cumulative distribution function of the convoluted $\tilde{\imath}_t$, evaluated at (1-∝) . Hence forth it will be replaced by $x_t^{1-\alpha}$.

Step 3. The expression for two deterministic chance constraint time steps are given below,

for t=1

$$P\{S_1 \leq S_{goal}\} \geq \propto \tag{3.70}$$

$$P\{S_0 + \tilde{\imath}_1 - R_1 \leq S_{goal}\} \geq \propto$$

$$P\{\tilde{i_1} \leq R_1 + S_{goal} - S_0\} \geq \propto$$

$$x_1^{1-\alpha} \geq R_1 + S_{goal} - S_0$$

for t=2,

$$P\{S_2 \leq S_{goal}\} \geq \propto \tag{3.71}$$

$$P\{S_1 + \tilde{i_2} - R_2 \leq S_{goal}\} \geq \propto$$

$$P\{(S_0 + \tilde{i_1} - R_1) + \tilde{i_2} - R_2 \leq S_{goal}\} \geq \propto$$

$$P\{\tilde{i_1} + \tilde{i_2} \leq R_1 + R_2 + S_{goal} - S_0\} \geq \propto$$

$$x_2^{1-\alpha} \geq R_1 + R_2 + S_{goal} - S_0$$

Equation (3.69) can thus be expressed in final simplified chance constraint deterministic form as:

$$S_{goal} - S_0 + \sum_{t=1}^{n} R_t \leq x_t^{1-\alpha} \qquad t = 1,2, \ldots\ldots, n \tag{3.72}$$

Note that the summation of random variable inflows takes place here. For time interval, t=1 we have $\tilde{i_1}$, t=2 we have $\tilde{i_1} + \tilde{i_2}$, …, t=n we have $\tilde{i_1} + \tilde{i_2} + \cdots + \tilde{i_n}$.

The random variable inflow has a known marginal PDF, $f(i_t)$, as a result of fitting a distribution to available historical data. However the distributions of the sums have to be found.

This is accomplished though a step by step iterative convolution method from t=2 to t=n, expressed in general (recursive equation for convolution) form as (Simonović, 1979):

$$p_r(t) = \sum_{i-j=r} P\{i_t = i\} p_j(t-1) \tag{3.73}$$

$$\min r \leq r \leq \max r$$

The magnitudes of min r and max r are found from *min r= min I - max j = a-d, max r= max I - min j=b-c* under the constraints $a \leq i \leq b$ *and* $c \leq j \leq d$.

The mathematical formulation presented in this report will be using the log-normal distribution for marginal inflow. Some distributions of the independently assumed random variable inflows can be easily summed based on the distribution regenerative properties. They, for example include the normal distribution and gamma distribution. For these cases the summation of two identical regenerative functions results in the same function with parameters solvable in closed form. Distributions not falling in this category, such as the log-normal distribution can be approximated based on equation (3.73).

The log-normal distribution is not a regenerative function and as such cannot be solved in closed form and it is very difficult to solve numerically (Beaulieu, 2004). The convolution method must be employed. However, based on evidence, the sum of two independent log-normal random variables can be approximated by another log-normal random variable (Beaulieu, 2004). Knowing the additive property of a log-normal distribution, Monte Carlo Simulation(MCS) technique may be used instead of the generalized convolution method.

MCS technique uses the known log-normal marginal continuous probability distribution function and randomly samples them to produce hundred or even thousands of scenarios or iterations (Vose, 1996). Consider adding two marginal log-normal PDF for t=1 and t=2 using MCS. The simulated values from each marginal log-normal PDF are determined first using MCS, these values are added together (i_1+i_2=X_2), and the expected value and standard deviation is found of the new summed value, these parameters are than fitted back into the log-normal distribution function.

The problem formulation becomes similar to linear formulation in the deterministic approach, Eq. (3.66) with the addition of the deterministic chance constraint, Eq. (3.72), and as such can be solved with the same linear programming approach as the deterministic model formulation.

3.3.2.3 Fuzzy approach

The reservoir operation optimization model formulation will be expanded to utilize the fuzzy linear optimization approach in doing so it will depart from the classical assumptions that all coefficients of the constraints need to be crisp numbers and that the objective function must be minimized or maximized (Zimmermann, 1996).

Using the fuzzy optimization approach for linear programming discussed in section 3.4.1, and using the deterministic model given by Eq. (3.66) with modification for considering linear membership function for “greater than” constraints, the fuzzy formulation becomes:

$$maximize \qquad \lambda \tag{3.74}$$

$$subject\ to$$

$$\frac{(Bx)_i}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \quad , i = 0, \ldots, 36$$

$$\frac{(Bx)_i}{p_i} - \lambda \geq \frac{d_i}{p_i} \quad , i = 37, \ldots, 60$$

$$x \geq 0$$

Expanding by substituting for $(Bx)_I$

$$maximize \qquad \lambda \tag{3.75}$$

$$subject\ to\ objective\ function\ constraint$$

$$\frac{\sum_{t=1}^{12} s_t}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \quad , i = 0$$

$$all\ other\ constraints$$

$$\frac{(S_t + R_t - S_{t-1})}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \quad , i = 1, \dots, 12, t = 1, \dots, 12,$$

$$\frac{s_t}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \quad , i = 13, \dots, 24\ , t = 1, \dots, 12,$$

$$\frac{R_t}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \quad , i = 25, \dots, 36\ , t = 1, \dots, 12,$$

$$\frac{s_t}{p_i} - \lambda \geq \frac{d_i}{p_i} \quad , i = 37, \dots, 48, t = 1, \dots, 12,$$

$$\frac{(S_t + R_t - S_{t-1})}{p_i} - \lambda \geq \frac{d_i}{p_i} \quad , i = 49, \dots, 60, t = 1, \dots, 12,$$

$$R_t \geq 0 \qquad t = 1, 2, \dots., 12$$

3.3.3 Numerical example

The following demonstrates the deterministic procedure for a single reservoir operation optimization and its modification for the implementation in the probabilistic and fuzzy domains.

3.3.3.1 Problem

The reservoir optimization case study is the Fanshawe reservoir on the North Thames River located in Ontario, Canada (just outside the City of London). An optimization problem is formulated for 12 month time period (t=12) as discussed in preceding section 3.3.2 and solved using data provided from the Upper Thames Conservation Authority (UTRCA). The pertaining data consists of physical constraints for the reservoir such as the maximum and the minimum storage capacity. Monthly inflow historical data was also provided covering a time period between 1953 and 2009.

The goal here is to present an example with realistic numerical data pertaining to the current available optimization knowledge. The reservoir operation problem is to be solved using:

a. Deterministic optimization approach
b. Probabilistic optimization approach based on the chance constraint method and
c. Fuzzy optimization approach

The preceding section of the report includes the mathematical models (objectives and constraints). The result includes a series of release rules for the 12 month operating period that reservoir operators can follow in order to meet the defined objective.

The data is given below:

Maximum reservoir capacity, $C=0.22503 \times 10^8$ m^3

Dead or Minimum reservoir storage, $S_{min}=0.055 \times 10^8$ m^3

Sill of dam elevation operator goal storage, $S_{GOAL}=0.1235 \times 10^8$ m^3

Initial storage, $S_0=0.1482 \times 10^8$ m^3

Maximum possible release for non-flooding condition, $R_{max}=370$ m^3/s

The release is transformed to consistent units with the rest of the variables by finding the maximum release allowable in each month, given in Table 3.9.

TABLE 3.9- MAXIMUM MONTHLY RELEASE FLOWS [10^8 M^3]

Month , T=	1	2	3	4	5	6	7	8	9	10	11	12
R_{max}	9.91008	8.95104	9.91008	9.5904	9.91008	9.5904	9.91008	9.91008	9.5904	9.91008	9.5904	9.91008

The inflow parameters based on the available UTRCA provided data is given below.

For illustrating the deterministic approach 2009 historical inflow data is used as input for optimization given in Table 3.10 below.

TABLE 3.10- FANSHAWE RESERVOIR INFLOWS [10^8 M^3]

Month , T=	1	2	3	4	5	6	7	8	9	10	11	12
Inflow 2009	0.34284	1.80472	1.21867	0.72058	0.54104	0.20062	0.12133	0.09508	0.07206	0.12294	0.10446	0.38033

For illustrating the probabilistic approach statistical parameters are given in Table 3.11 below.

TABLE 3.11- MONTHLY INFLOW STATISTICS [10^8 M^3]

Month , T=	1	2	3	4	5	6	7	8	9	10	11	12
Mean, μ	0.5036	0.5685	1.2499	0.9164	0.3722	0.1708	0.1329	0.1120	0.1797	0.2615	0.4689	0.6218
Std,σ	0.3968	0.5231	0.5572	0.5551	0.2954	0.1395	0.1706	0.1157	0.2636	0.3161	0.4033	0.4592

In the fuzzy approach, consider that the decision makers wanted some leeway in the constraint to account for the knowledge uncertainty, which is unavailable with the crisp constraint requirements of the deterministic model. Furthermore, the decision makers assessed that the combined annual maximum acceptable storage to avoid costly damage due to inundation should not exceed $1.6 \times 10^8\ m^3$. Since the decision makers felt that they were forced into specifying the precise constraints in spite of the fact that they would rather have given some intervals due to the imprecision in the hydrologic data and other uncertainties, the fuzzy linear programming model was selected as satisfactory in order to account for these perceptions. The lower bounds and the upper bounds of the tolerance interval, d_i and spread of tolerance, p_i, were estimated as shown in Table 3.12 and 3.13 respectively.

TABLE 3.12- ESTIMATED RESERVOIR LOWER BOUND PARAMETERS [10^8 M^3]

i	d_i	p_i	Comments
0	1.1000	0.5000	Corresponding to objective function
1	0.3428	0.0686	Corresponding to inflow (equation of state), based on 2009 data of potential inaccuracy. Note: first entry represents first month and last the twelfth month.
2	1.8047	0.3609	
3	1.2187	0.2437	
4	0.7206	0.1441	
5	0.5410	0.1082	
6	0.2006	0.0401	
7	0.1213	0.0243	
8	0.0951	0.0190	
9	0.0721	0.0144	
10	0.1229	0.0246	
11	0.1045	0.0209	
12	0.3803	0.0761	
13	0.2250	0.0001	Maximum reservoir capacity, based on physical constraint. Note: first entry represents first month and last the twelfth month.
14	0.2250	0.0001	
15	0.2250	0.0001	
16	0.2250	0.0001	
17	0.2250	0.0001	
18	0.2250	0.0001	
19	0.2250	0.0001	
20	0.2250	0.0001	
21	0.2250	0.0001	
22	0.2250	0.0001	
23	0.2250	0.0001	
24	0.2250	0.0001	
25	9.9101	0.0001	Corresponding to maximum possible release for non-flooding condition, based on physical constraint. Note: first entry represents first month and last
26	8.9510	0.0001	
27	9.9101	0.0001	
28	9.5904	0.0001	

29	9.9101	0.0001	the twelfth month.
30	9.5904	0.0001	
31	9.9101	0.0001	
32	9.9101	0.0001	
33	9.5904	0.0001	
34	9.9101	0.0001	
35	9.5904	0.0001	
36	9.9101	0.0001	

TABLE 3.13-ESTIMATED RESERVOIR UPPER BOUND PARAMETERS [10^8 M^3]

i	d_i	p_i	Comments
37	0.1235	0.0900	Corresponding to Dead or Minimum reservoir storage, physical constraint. Note: first entry represents first month and last the twelfth month. The value of 0.1482 corresponds to the last month storage requirement of being less than initial month
38	0.1235	0.0900	
39	0.1235	0.0900	
40	0.1235	0.0900	
41	0.1235	0.0900	
42	0.1235	0.0900	
43	0.1235	0.0900	
44	0.1235	0.0900	
45	0.1235	0.0900	
46	0.1235	0.0900	
47	0.1235	0.0900	
48	0.1482	0.0000	
49	0.2743	0.0686	Corresponding to inflow (equation of state), based on 2009 data of potential inaccuracy. Note: first entry represents first month and last the
50	1.4438	0.3609	
51	0.9749	0.2437	
52	0.5765	0.1441	
53	0.4328	0.1082	
54	0.1605	0.0401	
55	0.0971	0.0243	
56	0.0761	0.0190	
57	0.0576	0.0144	

58	0.0984	0.0246	twelfth month.
59	0.0836	0.0209	
60	0.3043	0.0761	

The above parameters make up the linear membership functions to be used for fuzzy linear programming. For the case of inflow (state equation) a triangular membership function is used.

3.3.3.2 Solution using deterministic optimization approach

The deterministic formulation for the Fanshawe reservoir operation optimization has been presented in section 3.3.2.1. It is repeated here for convenience:

$$Minimize\ Z = S$$

$$Subject\ to, mathematical\ model$$

$$S_{t-1} + i_t - R_t = S_t \qquad t = 1,2, \dots ..,12$$

$$Subject\ to\ constraints,$$

$$0 \leq R_t \leq R_{max} \qquad t = 1,2, \dots .,12$$

$$S_0 \leq S_n$$

$$S_{min} \leq S_t \leq C \qquad t = 1,2, \dots .,12$$

Substituting the given data from Table 3.10, the above problem with 12 balance equations and 25 constraints becomes readily solvable.

The linear programming optimization solution can be found using Microsoft Excel Solver, MATLAB or other software packages. The optimal solution is shown in Table 3.14.

TABLE 3.14- THE FANSHAWE RESERVOIR OPTIMIZATION RESULTS- DETERMINISTIC APPRAOCH

Optimization Summary(10^8 m^3)		
Objective Function Z= 0.753		
Month, T	Storage	Release
1. Jan	0.055	0.436035
2. Feb	0.055	1.804723
3. Mar	0.055	1.218672
4. Apr	0.055	0.720576
5. May	0.055	0.541037
6. Jun	0.055	0.200621
7. Jul	0.055	0.121332
8. Aug	0.055	0.095083
9. Sep	0.055	0.072058
10. Oct	0.055	0.122939
11. Nov	0.055	0.104458
12. Dec	0.1482	0.287133

3.3.3.3 Solution using probabilistic optimization approach

The probabilistic form of the Fanshawe reservoir operation optimization has already been presented in section 3.3.2.2. It is repeated here for convenience.

$$Minimimze\ Z = S$$

$$Subject\ to, mathematical\ model$$

$$S_{t-1} + i_t - R_t = S_t \qquad t = 1,2, \dots\dots, 12$$

$$Subject\ to\ constraints,$$

$$0 \le R_t \le R_{max} \qquad t = 1,2, \dots., 12$$

$$S_0 \le S_n$$

$$S_{min} \leq S_t \leq C \qquad t = 1,2, \ldots .,12$$

deterministic chance constraint,

$$S_{goal} - S_0 + \sum_{t=1}^{n} R_t \leq x_t^{1-\alpha} \qquad\qquad t = 1,2, \ldots \ldots,12$$

Historical Inflow data was fitted with log-normal distribution, as the flows are always positive, and generally have standard deviations that increase as the mean increases. These characteristics are common to the log-normal distribution.

The value for the reliability tolerance (α) is taken as 0.9. The corresponding cumulative distribution values for x_t, the result of summation of random inflow variable are found. It is assumed that the random variables are additive from one period to the next. The probability density function of their sum can be obtained by convolution regardless if they are independent or not (Curry et al, 1973).

Parameters given in Table 3.11 are used for fitting marginal log-normal inflow distribution. Summing the known marginal log-normal distributions approximately yield a log-normal distribution. That is to determine cumulative distribution x_t, January, t=1 to December, t=12 a convolution process must be performed first following equation (3.73) such that distributions are convoluted through iterative process. The designation of the function has been simplified, and has the following interpretation:

January = January

February = January + February

March = (January + February) + March

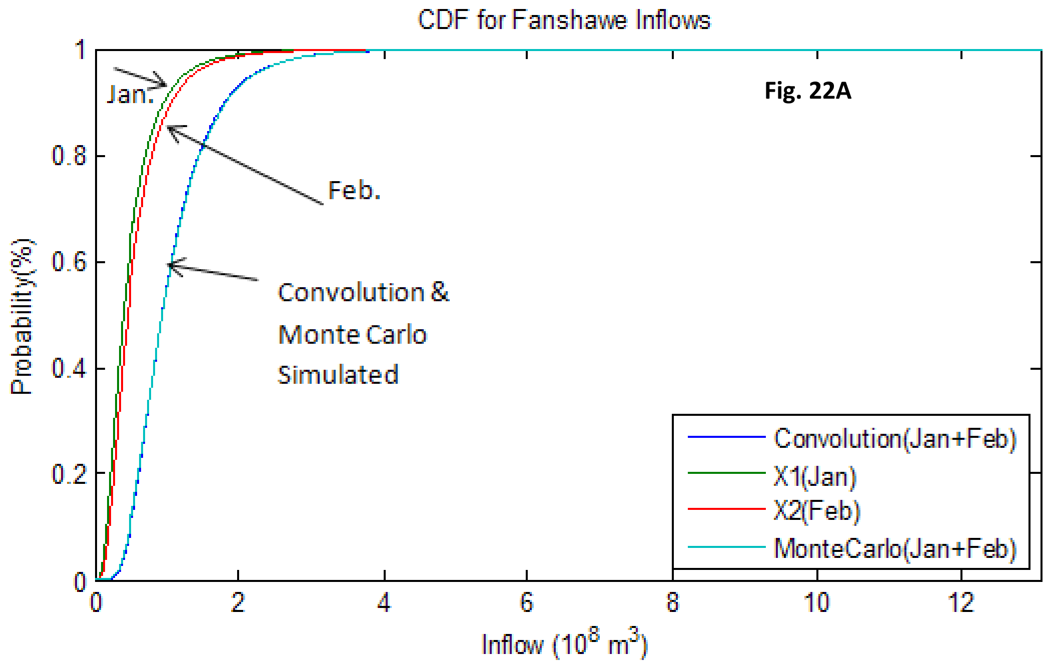
April = (January + February + March) + April

Etc.

The iterative convolution procedure is done through a discrete numerical approximation using equation (3.73) in a program developed in MATLAB (computer code is available in Appendix C). The solution for each time step using convolution is found and converted to an empirical distribution. The empirical distribution is confirmed to be approximately equal to a log-normal distribution. This is achieved by MCS of random variables generated from two distributions summing the random values and fitting them to a log-normal distribution. When compared graphically in Fig. 3.14 the Monte Carlo simulated log-normal distribution overlaps with the empirical distribution that had been convoluted. Therefore we may conclude that the distribution is indeed a Log-normal one with parameters as used in the simulated Monte Carlo distribution. Figure 3.15 shows the result of convoluted random variables for the Fanshawe reservoir inflows. The corresponding cumulative distribution values for x_t, obtained by the summation of log-normal inflow distributions with reliability tolerance of 0.9 are summarized in Table 3.15. The summation procedure of marginal inflow log-normal distributions to find x_t is included in the MATLAB program developed. The source code for the program is available in Appendix C.

TABLE 3.15- APPROXIMATE LOG-NORMAL CUMULATIVE DISTRIBUTION DETERMINISTIC EQUIVALENT INFLOW FOR A =0.9 [10^8 M^3]

Jan.	Feb.	Mar.	Apr.	May.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
0.1623	0.4331	1.3695	2.0714	2.3786	2.5305	2.6579	2.7855	2.9234	3.1131	3.4732	4.0035



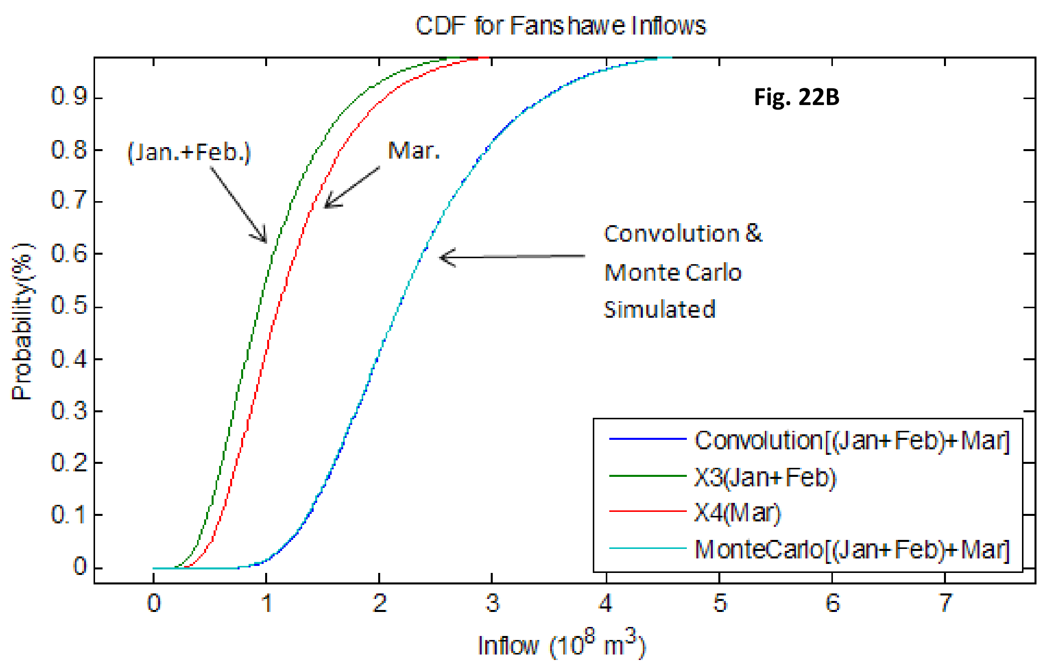


FIGURE 3.14(A-B)- THE FANSHAWE RESERVOIR, LONDON ONTARIO, CANADA, (A) CONVOLUTION OF INFLOWS IN JANUARY AND FEBRUARY, (B) CONVOLUTION OF INFLOWS (JANUARY+FEBRUARY) AND MARCH.

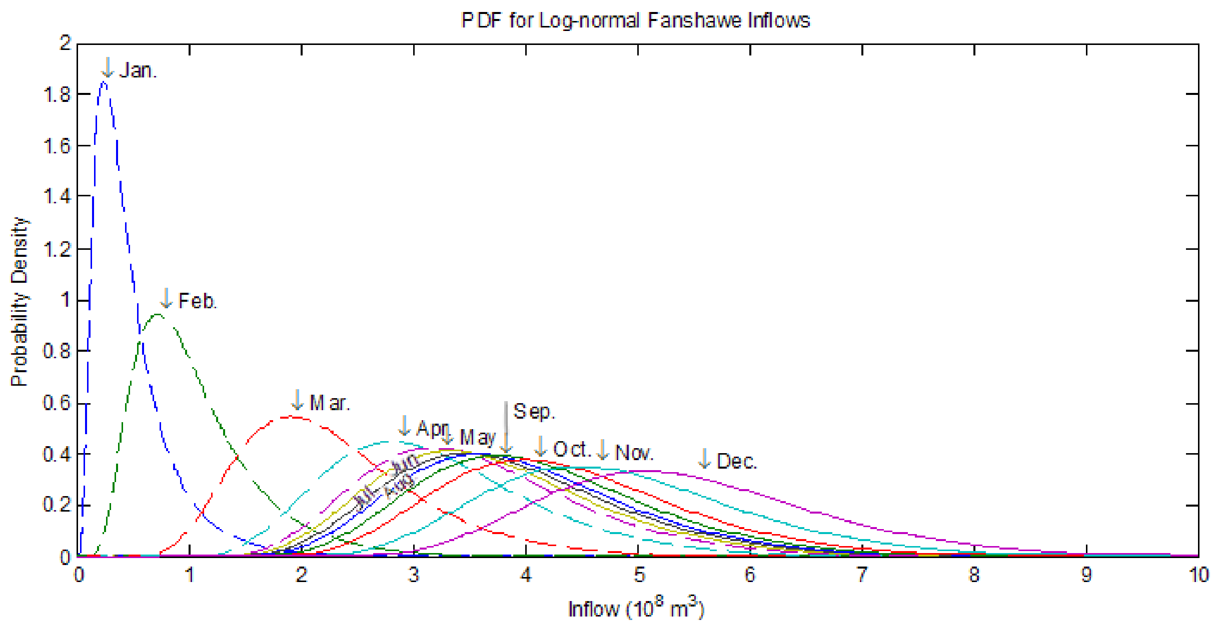


FIGURE 3.15- THE FANSHAWE RESERVOIR, LONDON, ONTARIO, CANADA – PROBABILITY DISTRIBUTION OF THE INFLOW

Once the convolution process is complete and inflow convoluted values corresponding to the reliability index selected are found, as shown in Table 3.15, the problem may be solved using linear optimization as in the case of the deterministic formulation. The linear programming optimization for convenience has been conducted within the same MATLAB program developed for convolution and is available in Appendix C. The optimization toolbox is required for the program to successfully run. The optimal solution is shown in Table 3.16.

TABLE 3.16- RESULT SUMMARY OF RELIABILITY OPTIMIZATION FOR A=0.9 (LOG-NORMAL CUMULATIVE DISTRIBUTION)

Optimization Summary(10^8 m^3)		
Objective Function Z= 1.5067		
Month, T	Storage	Release
1. Jan	0.1235	0.187
2. Feb	0.1235	0.2708
3. Mar	0.1235	0.9363
4. Apr	0.1235	0.7019
5. May	0.1235	0.3072
6. Jun	0.1235	0.152
7. Jul	0.1235	0.1274

8. Aug	0.1235	0.1276
9. Sep	0.1235	0.1379
10. Oct	0.1235	0.1897
11. Nov	0.1235	0.3602
12. Dec	0.1482	0.5055

3.3.3.4 Solution using fuzzy optimization approach

The fuzzy optimization model for this problem has been formulated in section 3.3.2.3 but for convenience is repeated here.

$$maximize \qquad \lambda$$

$$subject\ to\ objective\ function\ constraint$$

$$\frac{\sum_{t=1}^{12} s_t}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \qquad ,i = 0$$

$$all\ other\ constraints$$

$$\frac{(S_t + R_t - S_{t-1})}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \qquad ,i = 1, \ldots ,12, t = 1, \ldots ,12,$$

$$\frac{s_t}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \qquad ,i = 13, \ldots ,24\ ,t = 1, \ldots ,12,$$

$$\frac{R_t}{p_i} + \lambda \leq 1 + \frac{d_i}{p_i} \qquad ,i = 25, \ldots ,36\ ,t = 1, \ldots ,12,$$

$$\frac{s_t}{p_i} - \lambda \geq \frac{d_i}{p_i} \qquad ,i = 37, \ldots ,48, t = 1, \ldots ,12,$$

$$\frac{(S_t + R_t - S_{t-1})}{p_i} - \lambda \geq \frac{d_i}{p_i} \qquad ,i = 49, \ldots ,60, t = 1, \ldots ,12,$$

$$R_t \geq 0 \qquad\qquad t = 1,2, \ldots ,12$$

Substituting the values given in Tables 3.12 and 3.13 into the 61 constraints listed above and solving using a linear programming solver software package readily available yields λ equal to *0.0626* with corresponding storage and release as summarized in Table 3.17.

TABLE 3.17- FUZZY LINEAR PROGRAMMING RESULTS

Optimization Summary(10^8 m^3)		
Objective Function Z= 1.5687		
Month, T	Storage	Release
1. Jan	0.129136	0.297626
2. Feb	0.129136	1.46638
3. Mar	0.129136	0.9902
4. Apr	0.129136	0.585485
5. May	0.129136	0.439605
6. Jun	0.129136	0.163009
7. Jul	0.129136	0.098585
8. Aug	0.129136	0.077257
9. Sep	0.129136	0.058548
10. Oct	0.129136	0.09989
11. Nov	0.129136	0.084874
12. Dec	0.148201	0.289964

3.4 Multipurpose reservoir planning

In the area of water resources management much larger weight is being placed on replacing single-objective optimization with multi-objective analysis. Consider the planning of a multipurpose reservoir that may call for a number of different conflicting and non-commensurable objectives. An example of conflicting objectives could be minimization of reservoir storage for flood protection and maximization of storage for irrigation water supply. Unlike dealing with single optimization problems, it is no longer clear on what the optimum solution is that will satisfy different objectives. A decision must be made by selecting a solution from a set of alternatives, as the single optimum solution does not exist in the case of multi-objective analysis. The set of solutions being selected from are known as non-dominated solutions.

Determining the non-dominated solutions involves asking decision makers about their preferences regarding different objectives. In addition, as the number of decision makers or stakeholders increases it becomes more challenging to arrive at preferences to be used for the selection of the best solution.

3.4.1 Problem identification

The physical parameters used for modeling the multipurpose reservoir may be subject to various uncertainties. The uncertainty caused by variability in parameters must be quantified so that the levels of uncertainty can be communicated and decision makers can voice their preference through trade-off of uncertainty. Some of the model parameters as inputs may only be known to a group of decision makers and stakeholders. The formulated model should allow for uncertainty to be quantified based on the subjective judgment of those decision makers familiar with the parameters desired. The decision makers involved with the multipurpose reservoir may also have preconceived loose aspirations for the objective functions.

3.4.2 Mathematical formulation

A mathematical model for planning multipurpose reservoir under uncertainty using probabilistic and the fuzzy multi-objective optimization methodology is presented below as discussed in sections 2.1.3 and 2.2.4, respectively.

The following model has been adopted from Simonovic (2009) and modified to illustrate stochastic PROTRADE method and the fuzzy multi-objective optimization.

A regional water agency is responsible for the operation of a multipurpose reservoir used for (a) municipal water supply, (b) groundwater recharge, and (c) the control of water quality in the river downstream from the dam. Allocating the water to the first two purposes is, unfortunately, in conflict with the third purpose. The agency would like to minimize the negative effect of the water quality in the river, and at the same time maximize the benefits from the municipal water supply and groundwater recharge.

The problem formulation requires two decision variables: x_1 - the number of units of water delivered for water supply; and x_2 - the number of units of water delivered for groundwater recharge.

3.4.2.1 Probabilistic approach

For the stochastic multi-objective optimization, the objective functions and constraints are formulated as follows.

Objective Functions - From the problem description we note that there are two objectives: minimization of the increase in river pollution, and maximization of benefits. Trade-offs between these two objectives are sought to assist the water agency in the decision-making process.

The objective function for maximization of benefits can be written as:

$$Maximize\ Z_1(\boldsymbol{x}) = \tilde{c}_{11}\ x_1 + \tilde{c}_{12} x_2 \qquad (3.76)$$

and the objective function for minimization of water pollution as:

$$Minimize\ Z_2(\boldsymbol{x}) = \tilde{c}_{21}\ x_1 + \tilde{c}_{22} x_2 \tag{3.77}$$

where the objective function for pollution can be rewritten as:

$$Maximize\ \ -Z_2(\boldsymbol{x}) = -\tilde{c}_{21}\ x_1 - \tilde{c}_{22} x_2 \tag{3.78}$$

to provide for maximization of both objectives, where $\tilde{c}_{ij}$ is the *i*th objectives probabilistic coefficient for each decision variable *j*.

Constraints - Feasible region constraints are defined by following five constraints.

Technical constraints due to pump capacity:

$$A_{11} x_1 + A_{12} x_2 \leq B_1 \tag{3.79}$$

labour capacity:

$$A_{21} x_1 + A_{22} x_2 \leq B_2 \tag{3.80}$$

and water availability:

$$A_{31} x_1 + A_{32} x_2 \leq B_3 \tag{3.81}$$

with all decision variables being non-negative:

$$x_1, x_2 \geq 0 \tag{3.82}$$

where A_{ij} is the *i*th constraints coefficient for each decision variable *j* and B_i is the right hand side deterministic value for constraint *i*.

The mathematical structure of the above problem shows that both objective functions and all constraints are linear functions of decision variables. Therefore, this mathematical model of the multi-purpose reservoir can be classified as linear multi-objective analysis problem. The problem is stochastic as the objective function has parameters that are not known with certainty but are random instead.

3.4.2.2 Fuzzy set approach

Assume that now the decision makers wish to model the same multipurpose reservoir problem above but with a certain aspiration for the objectives. In order to satisfy the new requirements the fuzzy multi-objective optimization approach is used. From the model given by Eq. (2.44) the fuzzy multi-objective problem is converted to a conventional linear programming problem.

For the fuzzy multi-objective optimization, the objective function and constraints are:

$$maximize \qquad \lambda \tag{3.83}$$

$$subject\ to\ objective\ function\ constraints$$

$$\frac{c_{11}}{T_1}x_1 + \frac{c_{12}}{T_1}x_2 - \lambda \geq \frac{z_1^0}{T_1}$$

$$\frac{c_{21}}{T_2}x_1 + \frac{c_{22}}{T_2}x_2 + \lambda \geq \frac{z_2^0}{T_2}$$

$$all\ other\ constraints$$

$$\frac{A_{11}}{p_1}x_1 + \frac{A_{12}}{p_1}x_2 + \lambda \leq 1 + \frac{d_1}{p_1}$$

$$\frac{A_{21}}{p_2}x_1 + \frac{A_{22}}{p_2}x_2 + \lambda \leq 1 + \frac{d_2}{p_2}$$

$$\frac{A_{31}}{p_3}x_1 + \frac{A_{32}}{p_3}x_2 + \lambda \leq 1 + \frac{d_3}{p_3}$$

$$x_1, x_2 \geq 0$$

3.4.3 Numerical example

The following demonstrates the application of stochastic (PROTRADE method) and fuzzy multi-objective analysis for the formulations presented in section 3.4.2.

3.4.3.1 Problem

The reservoir planners (the regional water agency) wish to find the optimal solution that minimizes negative effects of water quality in the river while maximizing benefits from municipal water supply and groundwater recharge. The problem has already been mathematically formulated in section 3.4.2. The numerical example presented here illustrates how a multipurpose reservoir problem under uncertainty may be solved using:

a. Probabilistic (stochastic) multi-objective analysis (PROTRADE method) and
b. Fuzzy multi-objective analysis

The available data for solving the stochastic multipurpose reservoir problem are in Table 3.18. In addition, the following assumptions are made:

- One time period is involved; t= 0, 1.
- Allocation is limited to two restrictions: (a) pump capacity is 8 hours per period and (b) labor capacity is 4 person-hours per period.
- The total amount of water in the reservoir available for allocation is 72 units.

- The pollution in the river increases following a normal distribution. Water coming from water supply increases pollution with mean and variance of 3 and 2 units per unit of water used for water supply, respectively. Likewise pollution as a result of groundwater recharge increases with a mean and variance of 2 and 1 units per unit of water used for groundwater recharge, respectively.
- The contribution margin (selling price/unit less variable cost/unit) for municipal water supply and groundwater recharge is assumed to have a normal distribution. For municipal water supply the contribution margin for population mean and variance are given as 3 and 2 respectively. For groundwater recharge the contribution margins are having a population mean of 2 and variance of 1.

TABLE 3.18- AVAILABLE DATA FOR AN ILLUSTRATIVE EXAMPLE

	Water supply	Groundwater recharge
Number of units of water delivered	x_1	x_2
Number of units of water required	1	5
Pump time required (hours)	0.50	0.25
Labour time required (person-hour)	0.2	0.2

3.4.3.2 Solution using probabilistic multi-objective analysis (PROTRADE method)

The objective functions Eq. (3.76) and Eq. (3.78) are rewritten for convenience:

The objective function for benefits:

$$Maximize\ Z_1(\boldsymbol{x}) = \tilde{c}_{11}\ x_1 + \tilde{c}_{12}x_2$$

and the objective function for pollution is:

$$Maximize - Z_2(\boldsymbol{x}) = -\tilde{c}_{21}\ x_1 - \tilde{c}_{22}x_2$$

where:

$$\tilde{c}ij \sim Normal\ [E(\tilde{c}_{ij}), Var(\tilde{c}_{ij})] \tag{3.84}$$

$$E(\tilde{c}_{11}) = 1 \quad Var(\tilde{c}_{11}) = 1$$

$$E(\tilde{c}_{12}) = 3 \quad Var(\tilde{c}_{12}) = 2$$

$$E(\tilde{c}_{21}) = 3 \quad Var(\tilde{c}_{21}) = 2$$

$$E(\tilde{c}_{22}) = 2 \quad Var(\tilde{c}_{22}) = 1$$

Substituting the available data in Table 3.18 into the constraints formulated by Eq. (3.79-3.82):

Technical constraint due to pump capacity becomes:

$$0.5x_1 + 0.25x_2 \leq 8 \tag{3.85}$$

labor capacity becomes:

$$0.2x_1 + 0.2x_2 \leq 4 \tag{3.86}$$

water availability becomes:

$$x_1 + 5x_2 \leq 72 \tag{3.87}$$

and non-negativity of decision variables:

$$x_1, x_2 \geq 0 \tag{3.88}$$

In the following section the best solution to the above problem is presented using the PROTRADE method (introduced in section 2.2.4) in collaboration with the regional water agency decision maker in order to determine the compromised solution of the non-dominated pareto solutions set.

Step 1. Definition of objective functions using the expected value

$$maximize\ Z_1(\boldsymbol{x}) = x_1 + 3x_2$$

$$maximize\ -Z_2(\boldsymbol{x}) = -3x_1 - 2x_2 \tag{3.89}$$

subject to constraints

$$0.5x_1 + 0.25x_2 \leq 8$$

$$0.2x_1 + 0.2x_2 \leq 4$$

$$x_1 + 5x_2 \leq 72$$

$$x_1, x_2 \geq 0$$

The feasible region in the decision space for the problem is given in Figure 3.16.

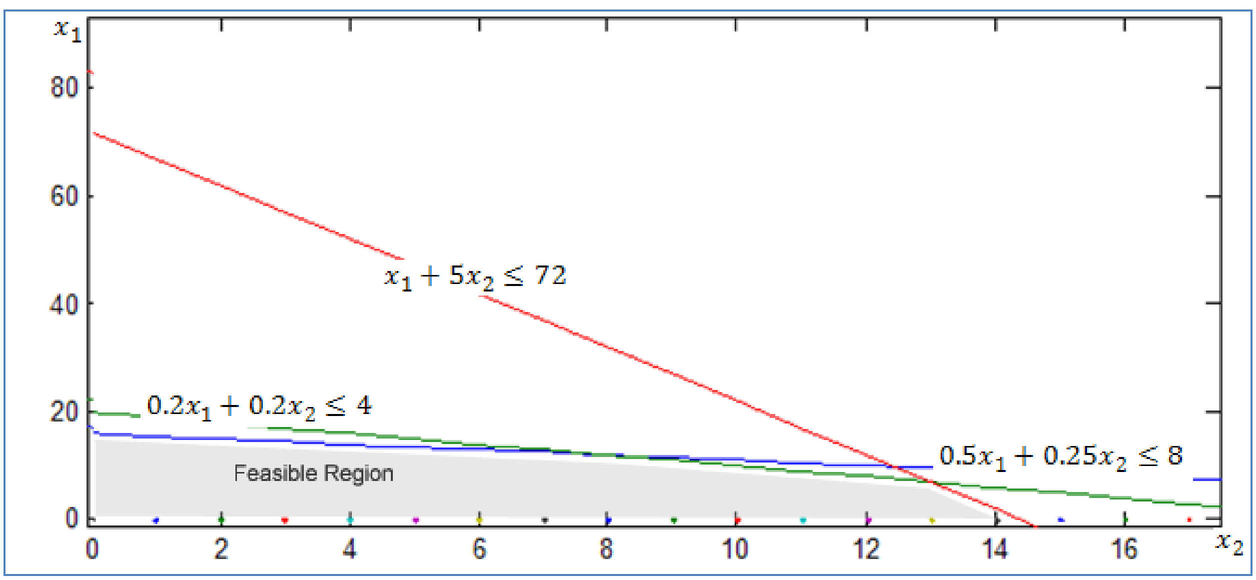


FIGURE 3.16- THE FEASIBIE REGION

Step 2. Range for the objective function

$$Z(\boldsymbol{x}) = [Z_1(\boldsymbol{x}), Z_2(\boldsymbol{x}), \ldots\ldots, Z_p(\boldsymbol{x})]$$

$$Z_i(\boldsymbol{x}_i^*) = max \quad Z_i(\boldsymbol{x}), \quad i\epsilon I[1,p]$$

$$g_q(\boldsymbol{x}) \leq 0 \; where \; q \in I[1,Q]$$

$$\boldsymbol{U_1} = \begin{bmatrix} Z_1(\boldsymbol{x}_1^*) \\ Z_2(\boldsymbol{x}_2^*) \end{bmatrix} = \begin{bmatrix} 46 \\ 0 \end{bmatrix} \tag{3.90}$$

(3.91)

$$\boldsymbol{B} = \{\boldsymbol{x}_i^*, \quad i\epsilon I[1,p]\} = \{(7,13), (0,0)\}$$

$$\boldsymbol{M} = \begin{bmatrix} Z_{1min} \\ Z_{2min} \end{bmatrix} = \begin{bmatrix} 0 \; (0,0) \\ -52 \; (12,8) \end{bmatrix} \tag{3.92}$$

Step 3. Formulation of an initial surrogate function:

$$F(\boldsymbol{x}) = \sum\nolimits_{i=1}^{p} G_i(\boldsymbol{x})$$

(3.93)

$$G_1 = \frac{Z_1(\boldsymbol{x}) - 0}{46 - 0} = \frac{1}{46}x_1 + \frac{3}{46}x_2$$

(3.94)

$$G_2 = \frac{Z_2(\boldsymbol{x}) - 52}{0 - 52} = \frac{-3}{52}x_1 - \frac{2}{52}x_2 + 1$$

(3.95)

$$F(\boldsymbol{x}) = -0.036x_1 + 0.027x_2 + 1$$

Step 4. An initial solution $\mathbf{x_1}$ is obtained maximizing F($\mathbf{x}$), subject to constraints $g_q(\boldsymbol{x}) \leq 0$. $\mathbf{x_1}$ solution is (0, 14.4). This solution is used to generate a goal vector $\mathbf{G_1}$:

$$\boldsymbol{G}_1 = \begin{bmatrix} 0.939 \\ 0.446 \end{bmatrix} \tag{3.96}$$

Step 5. A multidimensional utility function is defined in a multiplicative form following recommendation by Giocoechea et al. (1979):

$$1 + ku(\boldsymbol{G}) = \prod_{i=1}^{p} [1 + kk_i c_i (1 - e^{b_i G_i})] \tag{3.97}$$

This function is used to reflect the DM's goal utility, where k and k_i are constants determined by questions posed to the DM.

The following parameters are assumed in this example:

$$K = -0.555 \tag{3.98}$$

$$K_1 = 0.9$$

$$K_2 = 0.2$$

$$C_1 = 1.156$$

$$C_2 = 1.018$$

$$b_1 = -2.00$$

$$b_2 = -4.00$$

Step 6. A new surrogate objective function is defined:

a) Compute u($\mathbf{G_1}$)

$$1 + kk_1 u_1(G_1) = 0.510 \tag{3.99}$$

$$1 + kk_2 u_2(G_2) = 0.906$$

$$u(\boldsymbol{G_1}) = \frac{0.510 * 0.906 - 1}{-0.555} = 0.968$$

b) Decide on the utility increment 0 ≤ Δu(**G**) ≤ 1.

Let Δu(**G**) be equal to 0.1

$$\Delta \mathrm{u}(\mathbf{G}) = \mathrm{u}[G_1 + r\nabla u(\boldsymbol{G_1})] - u(\boldsymbol{G_1}) \tag{3.100}$$

c) Solve for the step size r,

$$G_1 + r\nabla u(\boldsymbol{G_1}) = \begin{bmatrix} 0.939 \\ 0.446 \end{bmatrix} + r \begin{bmatrix} 0.288 \\ 0.070 \end{bmatrix} \tag{3.101}$$

Solving for r in the equation,

$$u[G_1 + r\nabla u(\boldsymbol{G_1})] - 1.068 \tag{3.102}$$

yields:

$$r = 1.82 \tag{3.103}$$

Hence,

$$w_1 = 1 + \frac{1.82}{0.939} * 0.288 = 1.557 \tag{3.104}$$

$$w_2 = 1 + \frac{1.82}{0.446} * 0.070 = 1.285 \tag{3.105}$$

and the new surrogate objective function becomes,

$$S_1(\boldsymbol{x}) = \sum_{i=1}^{2} w_i G_i(x) = -0.04x_1 + 0.052x_2 + 1.285 \tag{3.106}$$

Step 7. An alternative solution is generated maximizing the surrogate solution S_1 finding a solution $\mathbf{x_2}$ = (0, 14.4) used to generate $\mathbf{G_2}$ and $\mathbf{U_2}$:

$$\boldsymbol{U_2} = \begin{bmatrix} 43.2 \\ -28.8 \end{bmatrix} \quad \boldsymbol{G_2} = \begin{bmatrix} 0.939 \\ 0.446 \end{bmatrix} \tag{3.107}$$

Step 8. A vector $\mathbf{V_1}$ that expresses the tradeoff between the goal value and its probability of achievement is generated:

$$\boldsymbol{V_1} = \begin{bmatrix} 0.939, & 0.500 \\ 0.446, & 0.500 \end{bmatrix} \tag{3.108}$$

The probability of achieving level $\mathbf{G_1}$ is 0.500 or better.

Step 9. Assume that after speaking with DM $\mathbf{U_2}$ is not to their satisfaction.

Step 10. The $Z_k(\mathbf{x})$ with the least satisfactory pair of ($G_k(\mathbf{x_2})$, $1-\alpha_k$) is selected and the DM specifies a new probability for that pair. Assume that the DM is not satisfied with what is obtained in $\mathbf{G_2}$ for example (0.446, 0.500) and would like to specify that,

$$prob[\tilde{Z}_2(\boldsymbol{x}) \geq -28.8] \geq 0.70 \tag{3.109}$$

Step 11. The solution space is redefined creating a new x-space (decision space).

$$X \, \varepsilon \, D_2 \tag{3.110}$$

$$0.5x_1 + 0.25x_2 \leq 8$$

$$0.2x_1 + 0.2x_2 \leq 4$$

$$x_1 + 5x_2 \leq 72$$

and

$$-3x_1 - 2x_2 - 0.53[2x_1^2 + x_2^2]^{\frac{1}{2}} \geq -28.8$$

where

$$\Phi\ (-0.53) = 1 - 0.700 = 0.300 \tag{3.111}$$

Step 12. A new surrogate objective function is generated and a sequential search for a satisfactory solution is performed going back to step 7 as many times as necessary.

$$S_2(\boldsymbol{x}) = \sum_{i=1}^{1} w_i G_i(x) = 0.034x_1 + 0.102x_2 \tag{3.112}$$

Maximize S_2 subject to constraints D_2

yields,

$$\boldsymbol{x}^3 = (0, 11.383) \tag{3.113}$$

and

$$\boldsymbol{G_3} = \begin{bmatrix} G_1(\boldsymbol{x_3}) \\ G_2(\boldsymbol{x_3}) \end{bmatrix} = \begin{bmatrix} 0.742 \\ 0.562 \end{bmatrix} \tag{3.114}$$

Now determine 1-α_k for i = 1 (for U_2)

$$1(0) + 3(11.383) + k_{\alpha i}[0^2 \ + 2(11.383)^2\ ]^{\frac{1}{2}} \geq e_i \tag{3.115}$$

$$34.149 + k_{\alpha 1}(16.098) > 43.2$$

$$k_{\alpha 1} \geq 0.562$$

where

$$\alpha_1 = \Phi\ (k_{\alpha 1}) = \Phi(0.562)\ = 0.713 \tag{3.116}$$

that is

$$1 - \alpha_1 = 0.287 \tag{3.117}$$

also determine 1-α_k for i = 1 (for 'best' G_1 can do from U_1)

$$1(0) + 3(11.383) + k_{\alpha i}\left[0^2 \ + 2(11.383)^2 \ \right]^{\frac{1}{2}} \geq e_i \tag{3.118}$$

$$34.149 + k_{\alpha 1}(16.098) > 46$$

$$k_{\alpha 1} \geq 0.736$$

where

$$\alpha_1 = \Phi\ (k_{\alpha 1}) = \Phi(0.736)\ = 0.769 \tag{3.119}$$

that is

$$1 - \alpha_1 = 0.231 \tag{3.120}$$

Therefore, the DM can achieve Z_1 of 46 at probability 0.231 or better, or Z_1 of 43.2 at probability of 0.287 or better and still maintain Z_2 of -28.8 at probability 0.700 or better.

Similarly for i=2

$$1(0) + 3(11.383) + k_{\alpha i}\left[0^2 \ + 2(11.383)^2 \ \right]^{\frac{1}{2}} \geq e_i \tag{3.121}$$

For z_2 (e_2) equal to -28.8 we already know the probability is 0.700 or better.

For z_2=e_2=0 (from $\mathbf{U_1}$)

$$-3(0) - 2(11.383) + k_{\alpha 2}\left[2(0)^2 \ + 11.383^2 \ \right]^{\frac{1}{2}} \geq 0 \tag{3.122}$$

$$k_{\alpha 2} \geq 2.00$$

where

$$\alpha_2 = \Phi\ (k_{\alpha 2}) = \Phi(2.00)\ = 0.977 \tag{3.123}$$

that is

$$1 - \alpha_2 = 0.023 \tag{3.124}$$

Thus DM can choose $\mathbf{V_2}$ to be,

$$\boldsymbol{V}_2 = \begin{bmatrix} 43.2, & 0.287 \\ -28.8, & 0.700 \end{bmatrix} \tag{3.125}$$

In summary the DM's preferences lead to the solution of 43.2 units of profit at probability of 0.287 and 28.8 units of pollution at a probability of 0.700 or better.

3.4.3.3 Solution using fuzzy multi-objective analysis

The multipurpose reservoir planning problem is solved here using the fuzzy mathematical multi-objective optimization formulation given by (3.83). The decision makers wish more flexibility in the constraints and estimate the lower bound aspiration level for constraints as:

$$d_1 = 6\ , \qquad d_2 = 3\ , \qquad d_3 = 65 \tag{3.126}$$

with the spread of tolerance of

$$p_1 = 3, \qquad p_2 = 2, \qquad p_3 = 10 \tag{3.127}$$

Furthermore, the decision makers have a certain aspiration that they wish to achieve for the objectives based on the results of the independent deterministic maximization and minimization of each objective function given here.

$$maximize\ Z_1(\boldsymbol{x}) = x_1 + 3x_2 \tag{3.128}$$

$$minimization\ Z_2(\boldsymbol{x}) = 3x_1 + 2x_2$$

For the benefit objective the valid range is found to be:

$$z_1^{max} = 46\ and\ z_1^{min} = 0 \tag{3.129}$$

For the pollution objective the valid range is found to be:

$$z_2^{max} = 52\ and\ z_2^{min} = 0 \tag{3.130}$$

From the above valid ranges the decision makers agree on the objective goals and tolerance

$$\begin{aligned} z_1^0 &= 40, \quad T_1 = 5 \\ z_2^0 &= 35, \quad T_2 = 10 \end{aligned} \tag{3.131}$$

In order to satisfy the new requirements the fuzzy multi-objective analysis approach is used, from the model given by (3.83) the fuzzy multi-objective problem is converted into a conventional linear programming problem.

$$maximize \qquad \lambda \tag{3.132}$$

$$subject\ to\ objective\ function\ constraints$$

$$0.2x_1 + 0.6x_2 - \lambda \geq 8$$

$$0.3x_1 + 0.2x_2 + \lambda \geq 3.5$$

$$all\ other\ constraints$$

$$0.167x_1 + 0.083x_2 + \lambda \leq 3$$

$$0.1x_1 + 0.1x_2 + \lambda \leq 2.5$$

$$0.1x_1 + 0.5x_2 + \lambda \leq 7.5$$

$$x_1, x_2 \geq 0$$

Solving the above formulation using a linear programming solver yields λ equal to *0.484* with corresponding municipal water supply x_1 equal to *0.80645* units, and groundwater recharge x_2 equal to *13.871* units. The objective function value for benefits z_1 is equal to 42.42 units and for pollution, z_2 is equal to *30.16* units.

4. Final remarks

Transforming deterministic problems into the fuzzy and probabilistic domains as seen in the selected examples, allow for decision makers to be more involved in the decision making process and in return more aware of the uncertainty and its consequences. As demonstrated, the probabilistic approach may deal with quantifying objective uncertainties while the fuzzy approach proves to be beneficial in dealing with subjective uncertainties. Therefore, utility of these two approaches is dependent on the available information in addition to the quality of the mathematical formulation.

The fuzzy set and probabilistic approach can increase the quality of information beyond traditional approaches, as evident from the reservoir operation and multipurpose reservoir planning case. Problems with extreme uncertainties may be solved with some precision as demonstrated by the stormwater sewer pipe sizing and dike height design cases. The pipe size estimated by the fuzzy simulation was comparable to the "realistic" case - the state that we are only able to assess based on the retrospective knowledge of the already available deterministic models. The fuzzy approach is robust in its ability to deal with different sources of uncertainties as demonstrated by the cases considered here. However, its robustness to handle different sources of uncertainties is not sufficient to justify its use under all circumstances. Caution must be taken, pending on the level of precision desired the stochastic approach may be the better alternative. But the probabilistic approach can be implemented only if uncertainties are quantifiable (objective) and sufficient historical data is available.

It should be emphasized that the methodologies presented for simulation, optimization, and multi-objective analysis in this report are adoptable to many other decision making problems. The selected cases are proof of the wide range of possibilities in water resource decision making applications. In conclusion, water resource decision making is subject to various sources of uncertainty. Uncertainty may compromise our ability to make appropriate decisions. This further emphasizes the importance of methods presented in this report.

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6. APPENDICES

6.1 APPENDIX A: DIKE DESIGN

MATLAB Monte Carlo simulation code for discharge

```
%MATLAB Lognormal distribution Monte Carlo Intensity Simulation

%number(n) of random number generated iterations
n=2000;

%format of Lognormal distribution is Lognormal(m,v) where
%'mu' is equal to the discharge population mean and 'sigma' is equal to the
discharge population
%standard deviation
mu = 538.5;
sigma = 100;
%Transformation of 'mu' and 'sigma' to lognormal location, 'm' and shape,
%'s'parameters
m = log((mu^2)/sqrt(sigma^2+mu^2));
s = sqrt(log(sigma^2/(mu^2)+1));


%X is the random variables generated using lognormal distribution
X = lognrnd(m,s,n,1);

%MX and STD are the expected value and standard deviation respectively
MX = mean(X);
STD = std(X);
%Percentiles
percentile=quantile(X,[.90]);


%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%%%%%%%%Summary of Results: comparing Population & Sample Distribution
%%%%%%%%%%%%%%%%%%%%%%%%
z = (0:0.02:1000);

%lognormal pdf of population distribution
y = lognpdf(z,m,s);

subplot(2,1,1),plot(z,y),  title  ({'LogNormal  Population  pdf',;['mean:',
num2str(mu),', Std:', num2str(sigma)]}),xlabel('x'); ylabel('p');

%lognormal sample distribution from Monte Carlo simulation
subplot(2,1,2),hist(X,100),title ({'LogNormal Random Simulation',;['Expected
value:',  num2str(MX),'  m^3/s,  90th  Percentile:',  num2str(percentile),'
m^3/s']});

%%%%%%%%%%%%%%%%%%%%%%%%%



%END of Program
```

MATLAB CODE FOR MONTE CARLO SIMULATION OF STAGE

```
%MATLAB Lognormal distribution Monte Carlo Intensity Simulation

%number(n) of random number generated iterations
n=2000;

%format of Lognormal distribution is Lognormal(m,v) where
%'mu' is equal to the stage population mean and 'sigma' is equal to the stage
population
%standard deviation
mu = 6.763;
sigma = 0.3

%Transformation of 'mu' and 'sigma' to lognormal location, 'm' and shape,
%'s'parameters
m = log((mu^2)/sqrt(sigma^2+mu^2));
```

```
s = sqrt(log(sigma^2/(mu^2)+1));


%X is the random variables generated using lognormal distribution
X = lognrnd(m,s,n,1);

%MX and STD are the expected value and standard deviation respectively
MX = mean(X);
STD = std(X);
%Percentiles
percentile=quantile(X,[.90]);


%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%Summary of Results: comparing Population & Sample Distribution
%%%%%%%%%%%%%%%%%%%%%%%%
z = (0:0.02:20);

%lognormal pdf of population distribution
y = lognpdf(z,m,s);

subplot(2,1,1),plot(z,y),   title   ({'LogNormal   Population   pdf',;['mean:',
num2str(mu),', Std:', num2str(sigma)]}),xlabel('x'); ylabel('p');

%lognormal sample distribution from Monte Carlo simulation
subplot(2,1,2),hist(X,100),title  ({'LogNormal  Random  Simulation',;['Expected
value:', num2str(MX),' m, 90th Percentile:', num2str(percentile),' m']});

%%%%%%%%%%%%%%%%%%%%%%%%%


%END of Program
```

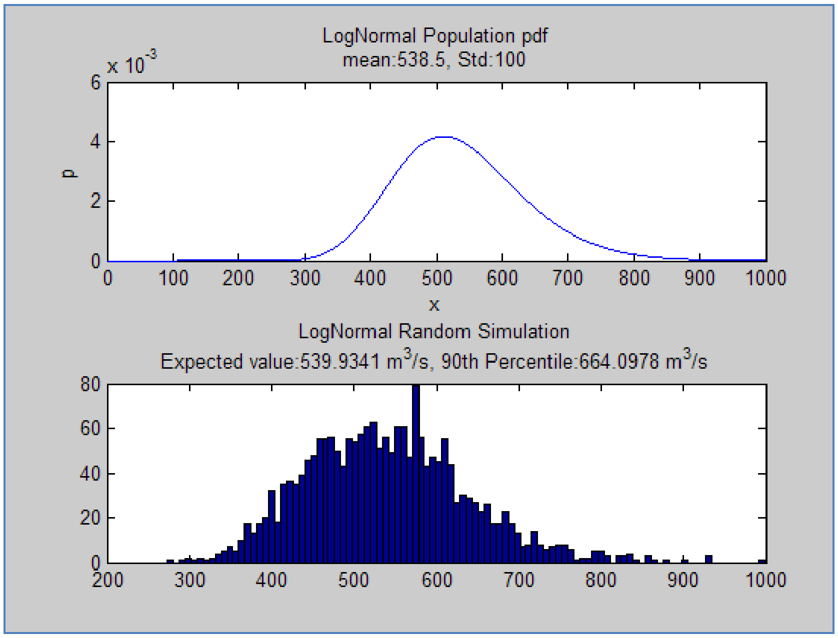

FIGURE 6.1- OUTPUT GRAPH OF MONTE CARLO DISCHARGE SIMULATION

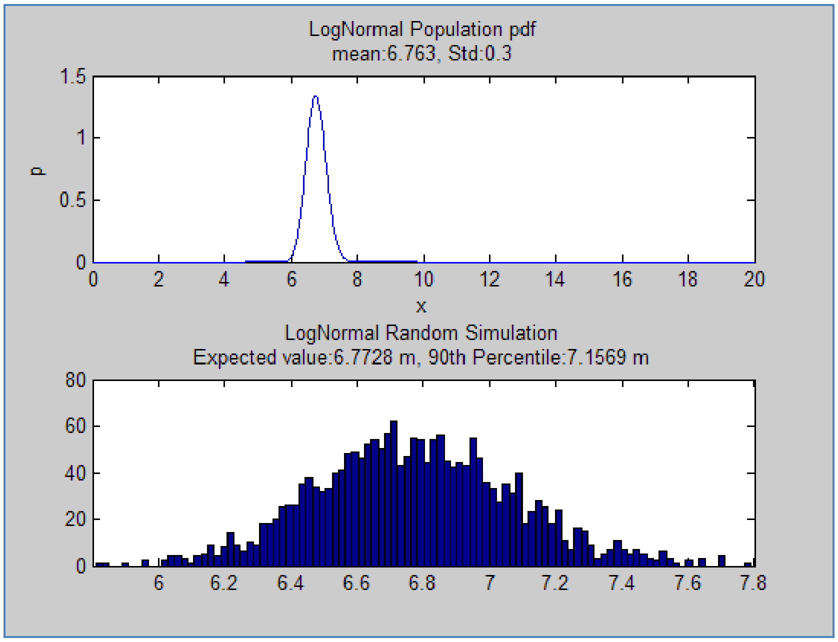

FIGURE 6.2- OUTPUT GRAPH OF MONTE CARLO STAGE SIMULATION

6.2 Appendix B: Stormwater sewer pipe design

MATLAB program code for Monte Carlo simulation.

```
%Matlab Lognormal distribution Monte Carlo Intensity Simulation

%number(n) of random number generated iterations
n=2000;

%format of Lognormal distribution is Lognormal(m,v) where
%'mu' is equal to the intensity population mean and 'sigma' is equal to the
intensity population
%standard deviation
mu = 3;
sigma = 2;
%Transformation of 'mu' and 'sigma' to lognormal location, 'm' and shape,
%'s'parameters
m = log((mu^2)/sqrt(sigma^2+mu^2));
s = sqrt(log(sigma^2/(mu^2)+1));



%X is the random variables generated using lognormal distribution
X = lognrnd(m,s,n,1);

%MX and STD are the expected value and standard deviation respectively
MX = mean(X);
STD = std(X);
%Percentiles
percentile=quantile(X,[.90]);


%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%Summary of Results: comparing Population & Sample Distribution
```

```
%%%%%%%%%%%%%%%%%%%%%%%%
z = (0:0.02:20);

%lognormal pdf of population distribution
y = lognpdf(z,m,s);

subplot(2,1,1),plot(z,y),   title   ({'LogNormal   Population   pdf',;['mean:', num2str(mu),', Std:', num2str(sigma)]}),xlabel('x'); ylabel('p');

%lognormal sample distribution from Monte Carlo simulation
subplot(2,1,2),hist(X,100),title  ({'LogNormal  Random  Simulation',;['Expected value:',   num2str(MX),'  mm/min,   90th   Percentile:',   num2str(percentile),' mm/min']});

%%%%%%%%%%%%%%%%%%%%%%%%%


%END of Program
```

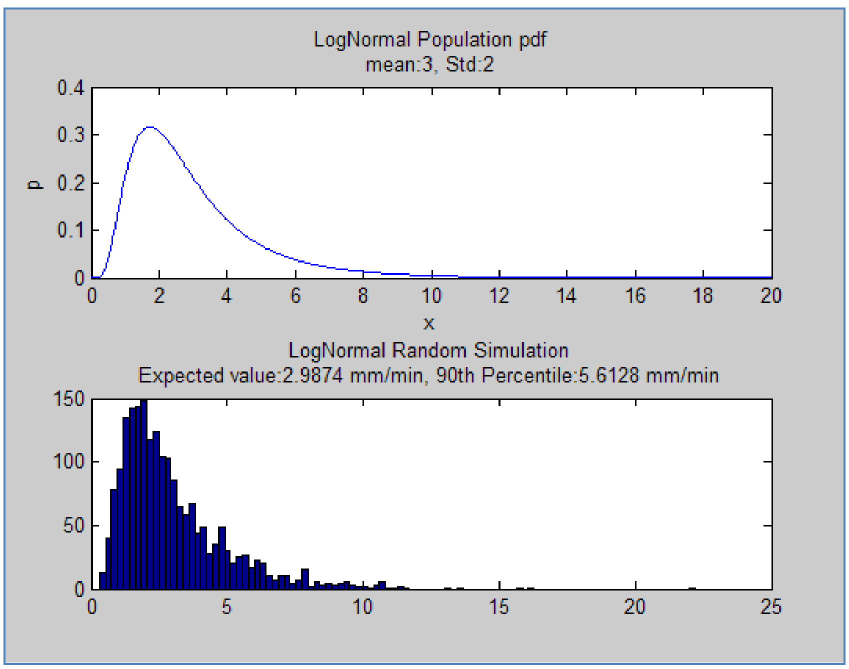

FIGURE 6.3- OUTPUT GRAPH OF MONTE CARLO INTENSITY SIMULATION

6.3 Appendix C: Reservoir optimization

MATLAB program code for Fanshawe Reservoir optimization using chance constrained method.

```
%Linear Programming Optimization of Fanshawe Reservoir using chance
%constrained method
%Inflow Chance constraint lognormal inverse
alpha=0.9; %Input
P=1-alpha;
n=10000;%for Random number generator
%Historical Statistical Parameters Inputs for lognormal distribution---------
----
%%%%%%%%%%INPUTS%%%%%%%
zm1=0.5036;
zs1=0.3968;

zm2=0.5685;
zs2=0.5231;

zm3=1.2499;
zs3=0.5572;

zm4=0.9164;
zs4=0.5551;

zm5=0.3722;
zs5=0.2954;

zm6=0.1708;
zs6=0.1395;

zm7=0.1329;
zs7=0.1706;

zm8=0.1120;
```

```
zs8=0.1157;

zm9=0.1797;
zs9=0.2636;

zm10=0.2615;
zs10=0.3161;

zm11=0.4689;
zs11=0.4033;

zm12=0.6218;
zs12=0.4592;
%-------------
mu1 = log((zm1^2)/sqrt((zs1^2)+zm1^2))
sigma1 = sqrt(log((zs1^2)/(zm1^2)+1))

mu2 = log((zm2^2)/sqrt((zs2^2)+zm2^2))
sigma2 = sqrt(log((zs2^2)/(zm2^2)+1))

mu3 = log((zm3^2)/sqrt((zs3^2)+zm3^2))
sigma3 = sqrt(log((zs3^2)/(zm3^2)+1))

mu4 = log((zm4^2)/sqrt((zs4^2)+zm4^2))
sigma4 = sqrt(log((zs4^2)/(zm4^2)+1))

mu5 = log((zm5^2)/sqrt((zs5^2)+zm5^2))
sigma5 = sqrt(log((zs5^2)/(zm5^2)+1))

mu6 = log((zm6^2)/sqrt((zs6^2)+zm6^2))
sigma6 = sqrt(log((zs6^2)/(zm6^2)+1))

mu7 = log((zm7^2)/sqrt((zs7^2)+zm7^2))
sigma7 = sqrt(log((zs7^2)/(zm7^2)+1))

mu8 = log((zm8^2)/sqrt((zs8^2)+zm8^2))
```

```
sigma8 = sqrt(log((zs8^2)/(zm8^2)+1))

mu9 = log((zm9^2)/sqrt((zs9^2)+zm9^2))
sigma9 = sqrt(log((zs9^2)/(zm9^2)+1))

mu10 = log((zm10^2)/sqrt((zs10^2)+zm10^2))
sigma10 = sqrt(log((zs10^2)/(zm10^2)+1))

mu11 = log((zm11^2)/sqrt((zs11^2)+zm11^2))
sigma11 = sqrt(log((zs11^2)/(zm11^2)+1))

mu12 = log((zm12^2)/sqrt((zs12^2)+zm12^2))
sigma12 = sqrt(log((zs12^2)/(zm12^2)+1))
%%%%Convolution Process
%---------------1-------------------------

x = linspace(0,50,10000);
dx = (x(end) - x(1))/(length(x) - 1);
%Summation
X1 = lognpdf(x,mu1,sigma1);   %Jan
X2 = lognpdf(x,mu2,sigma2);%Feb
fc1 = conv(X1,X2)* dx;
x3 = (2*x(1)):dx:(2*x(end));
%convert to CDF
y1=cumtrapz(fc1)/200;
%Cdf for comparison
C1 = logncdf(x,mu1,sigma1);   %logninv....
C2 = logncdf(x,mu2,sigma2);
%%%%Random summation


R1 = lognrnd(mu1,sigma1,n,1);
R2 = lognrnd(mu2,sigma2,n,1);
m1=mean(R1+R2);
s1=std(R1+R2);
v1=(s1)^2;
mr1 = log((m1^2)/sqrt(v1+m1^2));
```

```
sr1 = sqrt(log(v1/(m1^2)+1));
D1 = logncdf(x,mr1,sr1);
%----------------------2-----------------------

%Summation
X3 = lognpdf(x,mr1,sr1);   %Jan+Feb
X4 = lognpdf(x,mu3,sigma3);%March
fc2 = conv(X3,X4)* dx;

%Convert to CDF
y2=cumtrapz(fc2)/200;
%Cdf for comparison
C3 = logncdf(x,mr1,sr1);
C4 = logncdf(x,mu3,sigma3);
%%%%Random summation



R3 = lognrnd(mr1,sr1,n,1);
R4 = lognrnd(mu3,sigma3,n,1);
m2=mean(R3+R4);
s2=std(R3+R4);
v2=(s2)^2;
mr2 = log((m2^2)/sqrt(v2+m2^2));
sr2 = sqrt(log(v2/(m2^2)+1));
D2 = logncdf(x,mr2,sr2);

%-------------------3-----------------

%Summation
X5 = lognpdf(x,mr2,sr2);   %Jan+Feb+Mar
X6 = lognpdf(x,mu4,sigma4);%Apr
fc3 = conv(X5,X6)* dx;

%Convert to CDF
y3=cumtrapz(fc3)/200;
%Cdf for comparison
C5 = logncdf(x,mr2,sr2);
```

```
C6 = logncdf(x,mu4,sigma4);
%%%%Random summation



R5 = lognrnd(mr2,sr2,n,1);
R6 = lognrnd(mu4,sigma4,n,1);
m3=mean(R5+R6);
s3=std(R5+R6);
v3=(s3)^2;
mr3 = log((m3^2)/sqrt(v3+m3^2));
sr3 = sqrt(log(v3/(m3^2)+1));
D3 = logncdf(x,mr3,sr3);
%-------------------4------------------

%Summation
X7 = lognpdf(x,mr3,sr3);   %Jan+Feb+Mar+Apr
X8 = lognpdf(x,mu5,sigma5);%May
fc4 = conv(X7,X8)* dx;

%Convert to CDF
y4=cumtrapz(fc4)/200;
%Cdf for comparison
C7 = logncdf(x,mr3,sr3);
C8 = logncdf(x,mu5,sigma5);
%%%%Random summation



R7 = lognrnd(mr3,sr3,n,1);
R8 = lognrnd(mu5,sigma5,n,1);
m4=mean(R7+R8);
s4=std(R7+R8);
v4=(s4)^2;
mr4 = log((m4^2)/sqrt(v4+m4^2));
sr4 = sqrt(log(v4/(m4^2)+1));
D4 = logncdf(x,mr4,sr4);

%-------------------5------------------
```

```
%Summation
X9 = lognpdf(x,mr4,sr4);   %Jan+Feb+Mar+Apr+May
X10 = lognpdf(x,mu6,sigma6);%Jun
fc5 = conv(X9,X10)* dx;

%Convert to CDF
y5=cumtrapz(fc5)/200;
%Cdf for comparison
C9 = logncdf(x,mr4,sr4);
C10 = logncdf(x,mu6,sigma6);
%%%%Random summation



R9 = lognrnd(mr4,sr4,n,1);
R10 = lognrnd(mu6,sigma6,n,1);
m5=mean(R9+R10);
s5=std(R9+R10);
v5=(s5)^2;
mr5 = log((m5^2)/sqrt(v5+m5^2));
sr5 = sqrt(log(v5/(m5^2)+1));
D5 = logncdf(x,mr5,sr5);
%-------------------6------------------

%Summation
X11 = lognpdf(x,mr5,sr5);   %Jan+Feb+Mar+Apr+May+Jun
X12 = lognpdf(x,mu7,sigma7);%Jul
fc6 = conv(X11,X12)* dx;

%Convert to CDF
y6=cumtrapz(fc6)/200;
%Cdf for comparison
C11 = logncdf(x,mr5,sr5);
C12 = logncdf(x,mu7,sigma7);
%%%%Random summation
```

```
R11 = lognrnd(mr5,sr5,n,1);
R12 = lognrnd(mu7,sigma7,n,1);
m6=mean(R11+R12);
s6=std(R11+R12);
v6=(s6)^2;
mr6 = log((m6^2)/sqrt(v6+m6^2));
sr6 = sqrt(log(v6/(m6^2)+1));
D6 = logncdf(x,mr6,sr6);

%-------------------7------------------

%Summation
X13 = lognpdf(x,mr6,sr6);   %Jan+Feb+Mar+Apr+May+Jun+Jul
X14 = lognpdf(x,mu8,sigma8);%Aug
fc7 = conv(X13,X14)* dx;

%Convert to CDF
y7=cumtrapz(fc7)/200;
%Cdf for comparison
C13 = logncdf(x,mr6,sr6);
C14 = logncdf(x,mu8,sigma8);
%%%%Random summation



R13 = lognrnd(mr6,sr6,n,1);
R14 = lognrnd(mu8,sigma8,n,1);
m7=mean(R13+R14);
s7=std(R13+R14);
v7=(s7)^2;
mr7 = log((m7^2)/sqrt(v7+m7^2));
sr7 = sqrt(log(v7/(m7^2)+1));
D7 = logncdf(x,mr7,sr7);
%-------------------8------------------

%Summation
X15 = lognpdf(x,mr7,sr7);   %Jan+Feb+Mar+Apr+May+Jun+Jul+Aug
```

```
X16 = lognpdf(x,mu9,sigma9);%Sep
fc8 = conv(X15,X16)* dx;

%Convert to CDF
y8=cumtrapz(fc8)/200;
%Cdf for comparison
C15 = logncdf(x,mr7,sr7);
C16 = logncdf(x,mu9,sigma9);
%%%%Random summation



R15 = lognrnd(mr7,sr7,n,1);
R16 = lognrnd(mu9,sigma9,n,1);
m8=mean(R15+R16);
s8=std(R15+R16);
v8=(s8)^2;
mr8 = log((m8^2)/sqrt(v8+m8^2));
sr8 = sqrt(log(v8/(m8^2)+1));
D8 = logncdf(x,mr8,sr8);

%-------------------9------------------

%Summation
X17 = lognpdf(x,mr8,sr8);   %Jan+Feb+Mar+Apr+May+Jun+Jul+Aug+Sep
X18 = lognpdf(x,mu10,sigma10);%Oct
fc9 = conv(X17,X18)* dx;

%Convert to CDF
y9=cumtrapz(fc9)/200;
%Cdf for comparison
C17 = logncdf(x,mr8,sr8);
C18 = logncdf(x,mu10,sigma10);
%%%%Random summation



R17 = lognrnd(mr8,sr8,n,1);
```

```
R18 = lognrnd(mu10,sigma10,n,1);
m9=mean(R17+R18);
s9=std(R17+R18);
v9=(s9)^2;
mr9 = log((m9^2)/sqrt(v9+m9^2));
sr9 = sqrt(log(v9/(m9^2)+1));
D9 = logncdf(x,mr9,sr9);

%-------------------10------------------

%Summation
X19 = lognpdf(x,mr9,sr9);   %Jan+Feb+Mar+Apr+May+Jun+Jul+Aug+Sep+Oct
X20 = lognpdf(x,mu11,sigma11);%Nov
fc10 = conv(X19,X20)* dx;

%Convert to CDF
y10=cumtrapz(fc10)/200;
%Cdf for comparison
C19 = logncdf(x,mr9,sr9);
C20 = logncdf(x,mu11,sigma11);
%%%%Random summation



R19 = lognrnd(mr9,sr9,n,1);
R20 = lognrnd(mu11,sigma11,n,1);
m10=mean(R19+R20);
s10=std(R19+R20);
v10=(s10)^2;
mr10 = log((m10^2)/sqrt(v10+m10^2));
sr10 = sqrt(log(v10/(m10^2)+1));
D10 = logncdf(x,mr10,sr10);

%-------------------11------------------

%Summation
X21 = lognpdf(x,mr10,sr10);   %Jan+Feb+Mar+Apr+May+Jun+Jul+Aug+Sep+Oct+Nov
X22 = lognpdf(x,mu12,sigma12);%Dec
```

```
fc11 = conv(X21,X22)* dx;

%Convert to CDF
y11=cumtrapz(fc11)/200;
%Cdf for comparison
C21 = logncdf(x,mr10,sr10);
C22 = logncdf(x,mu12,sigma12);
%%%%Random summation

R21 = lognrnd(mr10,sr10,n,1);
R22 = lognrnd(mu12,sigma12,n,1);
m11=mean(R21+R22);
s11=std(R21+R22);
v11=(s11)^2;
mr11 = log((m11^2)/sqrt(v11+m11^2));
sr11 = sqrt(log(v11/(m11^2)+1));
D11 = logncdf(x,mr11,sr11);

%%%%%---INVERSE LOGNORMAL CDF-------------------------------------------------
----------------
in1 = logninv(P,mu1,sigma1)
in2 = logninv(P,mr1,sr1)
in3 = logninv(P,mr2,sr2)
in4 = logninv(P,mr3,sr3)
in5 = logninv(P,mr4,sr4)
in6 = logninv(P,mr5,sr5)
in7 = logninv(P,mr6,sr6)
in8 = logninv(P,mr7,sr7)
in9 = logninv(P,mr8,sr8)
in10 = logninv(P,mr9,sr9)
in11 = logninv(P,mr10,sr10)
in12 = logninv(P,mr11,sr11)
%___________________________________________________________________
%Linear Optimization
%First, enter the coefficients

%Objective Function
```

```
f = [1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];
 %   1  2  3  4  5  6  7  8  9  0  1  2  3  4  5  6  7  8  9  0  1  2  3  4
%Inflow Chance Constraint deterministic
A =  [0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  0  0  0  0  0  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  0  0  0  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  0  0  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  0  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  0  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  1  0  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  1  1  0
      0  0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  1  1  1
];
b  =  [-(0.1235-in1-0.1482);  -(0.1235-in2-0.1482);  -(0.1235-in3-0.1482);  -
(0.1235-in4-0.1482);  -(0.1235-in5-0.1482);  -(0.1235-in6-0.1482);  -(0.1235-
in7-0.1482);   -(0.1235-in8-0.1482);   -(0.1235-in9-0.1482);   -(0.1235-in10-
0.1482); -(0.1235-in11-0.1482); -(0.1235-in12-0.1482)];
Aeq= [1  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
      0  1  0  0  0  0  0  0  0  0  0  0  1  1  0  0  0  0  0  0  0  0  0  0
      0  0  1  0  0  0  0  0  0  0  0  0  1  1  1  0  0  0  0  0  0  0  0  0
      0  0  0  1  0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0
      0  0  0  0  1  0  0  0  0  0  0  0  1  1  1  1  1  0  0  0  0  0  0  0
      0  0  0  0  0  1  0  0  0  0  0  0  1  1  1  1  1  1  0  0  0  0  0  0
      0  0  0  0  0  0  1  0  0  0  0  0  1  1  1  1  1  1  1  0  0  0  0  0
      0  0  0  0  0  0  0  1  0  0  0  0  1  1  1  1  1  1  1  1  0  0  0  0
      0  0  0  0  0  0  0  0  1  0  0  0  1  1  1  1  1  1  1  1  1  0  0  0
      0  0  0  0  0  0  0  0  0  1  0  0  1  1  1  1  1  1  1  1  1  1  0  0
      0  0  0  0  0  0  0  0  0  0  1  0  1  1  1  1  1  1  1  1  1  1  1  0
      0  0  0  0  0  0  0  0  0  0  0  1  1  1  1  1  1  1  1  1  1  1  1  1
];
beq=[(0.1482+in1);
(0.1482+in2);(0.1482+in3);(0.1482+in4);(0.1482+in5);(0.1482+in6);(0.1482+in7)
;(0.1482+in8);(0.1482+in9);(0.1482+in10);(0.1482+in11);(0.1482+in12)];
```

```
%Lower Bound Constraints
%lb = zeros(24,1);
lb=[0.055;  0.055;  0.055;  0.055;  0.055;  0.055;  0.055;  0.055;  0.055;  0.055;
0.055; 0.1482; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0; 0];

%Upper Bound Constraints
ub=[0.22503;     0.22503;     0.22503;     0.22503;     0.22503;     0.22503;
0.22503;0.22503;0.22503;                                        0.22503;0.22503;
0.22503;9.91008;8.95104;9.91008;9.5904;9.91008;9.5904;9.91008;9.91008;9.5904;
9.91008;9.5904;9.91008];
%ub=inf(24,1);

%Next, call a linear programming routine.

[x,fval,exitflag,output,lambda]= linprog(f,A,b,Aeq,beq,lb,ub)

%%%---OUTPUT RESULTS SUMMARY-----
RR=inf(24,1);

fprintf('Objective Function Z= %8.3f  m^3\n', fval)
fprintf('\n\n')
fprintf('Optimization Summary (10^8 m^3)\n Storage\n\n')
for i= 1:1:12
    fprintf(num2str(i))
    fprintf('. Month Storage:%8.3f\n', x(i))
end

fprintf('\n\n')
fprintf('Optimization Summary (10^8 m^3)\n Release\n\n')
for i= 13:1:24
    if ((2*x(i)-RR(i-12))<0)
    fprintf(num2str(i-12))
    fprintf('. Month Release:%8.3f  \n', (x(i)))
    else
     fprintf(num2str(i-12))
    fprintf('. Month Release:%8.3f  \n', RR(i-12))
    end
```

```
end
```

6.4 APPENDIX D: LIST OF PREVIOUS REPORTS IN THE SERIES

ISSN: (print) 1913-3200; (online) 1913-3219

(1) Slobodan P. Simonovic (2001). Assessment of the Impact of Climate Variability and Change on the Reliability, Resiliency and Vulnerability of Complex Flood Protection Systems. Water Resources Research Report no. 038, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 91 pages. ISBN: (print) 978-0-7714-2606-3; (online) 978-0-7714-2607-0.

(2) Predrag Prodanovic (2001). Fuzzy Set Ranking Methods and Multiple Expert Decision Making. Water Resources Research Report no. 039, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 68 pages. ISBN:
(print) 978-0-7714-2608-7; (online) 978-0-7714-2609-4.

(3) Nirupama and Slobodan P. Simonovic (2002). Role of Remote Sensing in Disaster Management. Water Resources Research Report no. 040, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 107 pages. ISBN: (print) 978-0-7714-2610-0; (online) 978-0-7714- 2611-7.

(4) Taslima Akter and Slobodan P. Simonovic (2002). A General Overview of Multiobjective Multiple-Participant Decision Making for Flood Management. Water Resources Research Report
no. 041, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 65 pages. ISBN: (print) 978-0-7714-2612-4; (online) 978-0-7714-2613-1.

(5) Nirupama and Slobodan P. Simonovic (2002). A Spatial Fuzzy Compromise Approach for Flood Disaster Management. Water Resources Research Report no. 042, Facility for Intelligent
Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 138 pages. ISBN: (print) 978-0-7714-2614-8; (online) 978-0-7714-2615-5.

(6) K. D. W. Nandalal and Slobodan P. Simonovic (2002). State-of-the-Art Report on Systems Analysis Methods for Resolution of Conflicts in Water Resources Management. Water Resources Research Report no. 043, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 216 pages. ISBN: (print) 978-0-7714-2616-2; (online) 978-0-7714-2617-9.

(7) K. D. W. Nandalal and Slobodan P. Simonovic (2003). Conflict Resolution Support System – A Software for the Resolution of Conflicts in Water Resource Management. Water Resources Research Report no. 044, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 144 pages. ISBN: (print) 978-0-7714- 2618-6; (online) 978-0-7714-2619-3.

(8) Ibrahim El-Baroudy and Slobodan P. Simonovic (2003). New Fuzzy Performance Indices for Reliability Analysis of Water Supply Systems. Water Resources Research Report no. 045, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 90 pages. ISBN: (print) 978-0-7714-2620-9; (online) 978-0-7714- 2621-6.

(9) Juraj Cunderlik (2003). Hydrologic Model Selection for the CFCAS Project: Assessment of Water Resources Risk and Vulnerability to Changing Climatic Conditions. Water Resources Research Report no. 046, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 40 pages. ISBN: (print) 978-0-7714- 2622-3; (online) 978-0-7714- 2623-0.

(10) Juraj Cunderlik and Slobodan P. Simonovic (2004). Selection of Calibration and Verification Data for the HEC-HMS Hydrologic Model. Water Resources Research Report no. 047, Facility for Intelligent Decision Support,

Department of Civil and Environmental Engineering, London, Ontario, Canada, 29 pages. ISBN: (print) 978-0-7714-2624-7; (online) 978-0-7714-2625-4.

(11) Juraj Cunderlik and Slobodan P. Simonovic (2004). Calibration, Verification and Sensitivity Analysis of the HEC-HMS Hydrologic Model. Water Resources Research Report no. 048, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 113 pages. ISBN: (print) 978- 0-7714-2626-1; (online) 978-0-7714- 2627-8.

(12) Predrag Prodanovic and Slobodan P. Simonovic (2004). Generation of Synthetic Design Storms for the Upper Thames River basin. Water Resources Research Report no. 049, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London,
Ontario, Canada, 20 pages. ISBN: (print) 978- 0-7714-2628-5; (online) 978-0-7714-2629-2.

(13) Ibrahim El-Baroudy and Slobodan P. Simonovic (2005). Application of the Fuzzy Performance Indices to the City of London Water Supply System. Water Resources Research Report no. 050, Facility for Intelligent Decision Support, Department of Civil and Environmental
Engineering, London, Ontario, Canada, 137 pages. ISBN: (print) 978-0-7714-2630-8; (online) 978-0-7714-2631-5.

(14) Ibrahim El-Baroudy and Slobodan P. Simonovic (2006). A Decision Support System for Integrated Risk Management. Water Resources Research Report no. 051, Facility for Intelligent
Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 146 pages. ISBN: (print) 978-0-7714-2632-2; (online) 978-0-7714-2633-9.

(15) Predrag Prodanovic and Slobodan P. Simonovic (2006). Inverse Flood Risk Modelling of The Upper Thames River Basin. Water Resources Research Report no. 052, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 163 pages. ISBN: (print) 978-0-7714-2634-6; (online) 978-0-7714-2635-3.

(16) Predrag Prodanovic and Slobodan P. Simonovic (2006). Inverse Drought Risk Modelling of
The Upper Thames River Basin. Water Resources Research Report no. 053, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 252 pages. ISBN: (print) 978-0-7714-2636-0; (online) 978-0-7714-2637-7.

(17) Predrag Prodanovic and Slobodan P. Simonovic (2007). Dynamic Feedback Coupling of Continuous Hydrologic and Socio-Economic Model Components of the Upper Thames River Basin. Water Resources Research Report no. 054, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 437 pages. ISBN: (print) 978-0-7714-2638-4; (online) 978-0-7714-2639-1.

(18) Subhankar Karmakar and Slobodan P. Simonovic (2007). Flood Frequency Analysis Using Copula with Mixed Marginal Distributions. Water Resources Research Report no. 055, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London,
Ontario, Canada, 144 pages. ISBN: (print) 978-0-7714-2658-2; (online) 978-0-7714-2659-9.

(19) Jordan Black, Subhankar Karmakar and Slobodan P. Simonovic (2007). A Web-Based Flood Information System. Water Resources Research Report no. 056, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 133 pages. ISBN: (print) 978-0-7714-2660-5; (online) 978-0-7714-2661-2.

(20) Angela Peck, Subhankar Karmakar and Slobodan P. Simonovic (2007). Physical, Economical, Infrastructural and Social Flood Risk – Vulnerability Analyses in GIS. Water Resources Research Report no. 057, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 80 pages. ISBN: (print) 978-0- 7714-2662-9; (online) 978-0-7714-2663-6.

(21) Predrag Prodanovic and Slobodan P. Simonovic (2007). Development of Rainfall Intensity Duration Frequency Curves for the City of London Under the Changing Climate. Water Resources Research Report no. 058, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 51 pages. ISBN: (print) 978-0- 7714-2667-4; (online) 978-0-7714-2668-1.

(22) Evan G. R. Davies and Slobodan P. Simonovic (2008). An integrated system dynamics model for analyzing behaviour of the social-economic-climatic system: Model description and model use guide. Water Resources Research Report no. 059, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 233 pages. ISBN: (print) 978-0-7714-2679-7; (online) 978-0-7714-2680-3.

(23) Vasan Arunachalam (2008). Optimization Using Differential Evolution. Water Resources Research Report no. 060, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 42 pages. ISBN: (print) 978-0-7714- 2689-6; (online) 978-0-7714-2690-2.

(24) Rajesh Shrestha and Slobodan P. Simonovic (2009). A Fuzzy Set Theory Based Methodology for Analysis of Uncertainties in Stage-Discharge Measurements and Rating Curve. Water Resources Research Report no. 061, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 104 pages. ISBN: (print) 978-0-7714-2707-7; (online) 978-0-7714-2708-4.

(25) Hyung-Il Eum, Vasan Arunachalam and Slobodan P. Simonovic (2009). Integrated Reservoir Management System for Adaptation to Climate Change Impacts in the Upper Thames River Basin. Water Resources Research Report no. 062, Facility for Intelligent Decision Support, Department of Civil and Environmental Engineering, London, Ontario, Canada, 81 pages. ISBN:
(print) 978-0-7714-2710-7; (online) 978-0-7714-2711-4.

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