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B. G. Sidharth · S. R. Valluri

Abstract It is shown that a collection of photons with nearly the same frequency exhibits a "condensation" type of phenomenon corresponding to a peak intensity. The observed cosmic background radiation can be explained from this standpoint. We have obtained analogous results by extremization of the occupation number for photons with the use of the Lambert W function. Some of the interesting applications of this function are briefly discussed in the context of graphene which exhibits an interesting two dimensional structure with several characteristic properties and diverse practical applications.

In a previous communication [1] it was suggested that the origin of the Cosmic Background Radiation is the random motion of interstellar Hydrogen. We will now deduce the same result from a completely different point of view.

We start with the formula for the average occupation number for photons of momentum $\vec{k}$ for all polarizations [2]:

$$\langle n_{\vec{k}} \rangle = \frac{2}{e^{\beta \hbar \omega} - 1} \tag{1}$$

Let us specialize to a scenario in which all the photons have nearly the same energy so that we can write,

$$\langle n_{\vec{k}} \rangle = \langle n_{\vec{k}'} \rangle \, \delta \left(k - k' \right), \tag{2}$$

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where $\langle n'_k \rangle$ is given by (1), and $k \equiv |\vec{k}|$. The total number of photons N, in the volume V being considered, can be obtained in the usual way,

$$N = \frac{V[k]}{(2\pi)^3} \int_0^\infty dk 4\pi k^2 \langle n_k \rangle \tag{3}$$

where V is large. Inserting (2) in (3) we get,

$$N = \frac{2V}{(2\pi)^3} 4\pi k'^2 \left[\epsilon^\Theta - 1\right]^{-1} [k], \; \Theta \equiv \beta\hbar\omega \tag{4}$$

In the above, $[k] \equiv [L^{-1}]$ is a dimensionality constant, introduced to compensate the loss of a factor k in the integral (3), owing to the δ-function in (2): That is, a volume integral in $\vec{k}$ space is reduced to a surface integral on the sphere $[\vec{k}] = k'$, due to our constraint that all photons have nearly the same energy.

We observe that, $\Theta = \hbar\omega/k_B T \approx 1$ (k_B is the Boltzmann constant) , since by (2), the photons have nearly the same energy $\hbar\omega$. We also introduce,

$$v = \frac{V}{N}, \lambda = \frac{2\pi c}{\omega} = \frac{2\pi}{k} \text{ and } z = \frac{\lambda^3}{v} \tag{5}$$

λ being the wavelength of the radiation. We now have from (4), using (5),

$$(e-1) = \frac{vk'^2}{\pi^2}[k] = \frac{8\pi}{k'z}[k]$$

Using (5) we get:

$$z = \frac{8\pi}{k'(e-1)} = \frac{4\pi}{(e-1)}[k] \tag{6}$$

From (6) we conclude that, when

$$\lambda = \frac{e-1}{4}[L] \approx 0.43[L] \tag{7}$$

then,

$$z \approx 1 \tag{8}$$

or conversely. We are working in the cgs system, so that the units in (7) are *cm*. A further confirmation of this is as follows: We get an additional input, in the order of magnitude sense, from the system under consideration to fix the units.

There may not be a real need to say that $\Theta = \frac{\hbar\omega}{k_B T} \approx 1$ at this point. The extremization of the expression in (4) may lead to a solution in terms of the Lambert W or the polylogarithm functions with essentially the same results. These functions have created a renaissance in the solutions of problems in many diverse fields of human knowledge with particular emphasis in science and engineering [3–5].

Extremization of N given in (4) results in

$$\frac{dN}{dk} = \frac{V}{\pi^2}\left(\frac{2k}{e^{\beta\hbar kc} - 1} - \frac{k^2}{\left(e^{\beta\hbar kc} - 1\right)^2}e^{\beta\hbar kc} \cdot \beta\hbar c\right) = 0$$

therefore

$$2\left(e^{\beta\hbar kc} - 1\right) = \beta\hbar kc \, e^{\beta\hbar kc}$$

that is

$$2\left(1 - e^{-\beta\hbar kc}\right) = \beta\hbar kc$$

A better rearrangement gives;

$$(2 - \beta\hbar kc)e^{\beta\hbar kc} = 2$$

that is

$$(-2 + \beta\hbar kc)e^{\beta\hbar kc} = -2$$

or

$$(-2 + \beta\hbar kc)e^{\beta\hbar kc-2} = -2e^{-2} \tag{9}$$

Equation (9) is in the form typical of the Lambert W function [3, 4]

$$\beta\hbar kc - 2 = W\left(-2e^{-2}\right) \rightarrow \beta\hbar kc = 2 + W\left(-2/e^2\right)$$

$$\frac{\hbar kc}{k_B T} = 2 + W_j\left(-2/e^2\right)$$

where j denotes the branch of the multivalued inverse Lambert W function (4). The real values for the different branches $j = 0$ and -1 are $W_0\left(-2/e^2\right) = -0.4$ and $W_{-1}\left(-2/e^2\right) = -2$.

For these two cases physically relevant values are:

$$\frac{\hbar kc}{k_B T} = 1.6$$

and

$$\frac{\hbar kc}{k_B T} = 0$$

In Table 1 the values of the temperature for different wavelengths in the microwave region are given. A detailed summary of temperature values for different frequencies (units in cm^{-1}, rather than cm) are given in [6].

The last value in the above table corresponds to the values obtained from the COBE satellite observations [7].

The values of the temperature obtained for λ=0.3cm and 0.33cm are in fairly good agreement with those given [8, 9]. The above approach using the Lambert W function is generic and was initially applied to obtain Wien's displacement law in the context of the Planck spectral distribution of the blackbody radiation [3]. It is worth noting that the blackbody spectrum is preserved as the universe expands, with the effective temperature decreasing as S^{-1} where the scale factor S(t) is the expansion factor [6].

We would like to reiterate that the Cosmic Background Radiation (CBR) is commonly considered a relic of the Big Bang. It is thermal radiation peaking at the microwave wavelength (T=2.735 ± 0.06 [7]) filling the observable universe almost uniformly, and follows a black-body form [10–13]. Precise measurements of the CBR are fundamental for cosmology, because any viable proposed cosmological model of the universe must explain this radiation. The Planck satellite, launched on May 14, 2009, observes the entire sky twice a

Table 1 Temperature value extrema of N at different wavelengths in the microwave region

λ (cm)	T (K)
0.3	3.00272
0.33	2.72974
0.4	2.25204
0.5	1.80163
0.63	1.42987
0.05-1	2.735 ± 0.06 [7]

year using 74 detectors sensitive over nine frequency bands between 30 and 857 GHz [10–13]. The CBR radiation displays hot regions (red) and cold regions (blue) and this is called the Dipole Anisotropy. This is a purely kinematic effect caused by the Dopler shift originating from the Earths motion, or more accurately the motion of the group of galaxies to which our own galaxy belongs. The average velocity of this group is about 600 kilometers per second with respect to the background radiation itself. So if we are approaching a region of the CBR, we have the reddish colour and if we are receding away then we have a bluish colour, as far as the Dipole Anisotropy is concerned. The NASA COBE Satellite has revealed variations in the CBR which it turns out now, is not completely uniform. Again there are reddish and bluish regions which represent the fluctuations around the average value of about hundred thousandth of the average temperature. The main objective of Planck is to measure the spiral anisotropies in the temperature of the cosmic microwave background within the astrophysical limits. It was also designed to measure and map the CMB polarization anisotropies which encode cosmological information and provide a unique probe of the early history of the universe. It also produces information on the properties of extragalactic sources and on dust and gas in the Milky Way, using component separation to produce a clean CMB map and power spectra on large angular scales, where diffuse galactic emission is the main contaminant [10, 11].

The primary conclusion of the Planck mission is that the universe fits a six-parameter, vacuum-dominated, cold dark matter model. According to the data, subtle fluctuations in the CBR temperature were imprinted on the deep sky during the recombination era, i.e., when the universe was about 370 000 years old in the standard big-bang model. In such a standard framework, that imprint reflects ripples that arose from the early era, at about 10^{-30}s after the initial singularity in the big-bang model. It is commonly held that such ripples should give rise to the current cosmic structure of galactic clusters and dark matter. However another of the possible consequences of the initial Big Bang namely gravitational waves, are still tantalizingly elusive. Recently it was claimed that such ripples and gravitational waves as also inflation were observed in the BICEP 2 Experiment [14, 15]. But subsequently there has been a partial retraction because these observed features could well have been due to cosmic dust [16–18].

Alternative cosmologies such as the Quasi Steady State Cosmology (QSSC) [19] considers the CMB to come from a large number of discrete sources. The QSSC model also explains the CMB well and gives rise to a near black-body curve that fits all of the COBE data. According to the Quasi Steady State Cosmology, the thermalization of the CMB is an ongoing process. Although the Planck data are in accord with the flat ΛCDM model, there are hints of model-dependent tension. It will be interesting to see if and how the Planck data would allow interpretation of the different theories of cosmology.

The solution obtained in terms of the Lambert W function on extremization of the occupation number of bosons confirms the solution obtained by considering the condensation of photons. The solution obtained is a typical application of the remarkable Lambert W function not only in quantum statistics but also in several other diverse fields [3]. Other useful applications of the Lambert W function are in problems relevant to Fermi Dirac statistics. A problem of special interest is that of graphene which has interesting parallels to high energy fermions [21]. For nearly mono-energetic fermions or bosons, there would be a loss of dimensionality and the collection would behave as it were in two dimensions. The extremization of the occupation number of a Fermion gas would also lead to a Lambert W solution for the energy or equivalently of the momentum in terms of the chemical potential μ and temperature T, albeit a different one and will be further pursued elsewhere.

There are many other interesting application for graphene that include the study of surface waves (edge states) existing near the edge of a graphene nanoribbon [20] that give informative solutions in terms of the Lambert W function [22] that are exactly analogous to those of the Schroedinger equation for the one dimensional square well potential in Quantum Mechanics.

The bosonization of fermions property leads to interesting applications. The work of Vega Monroy [23] on the Bose-Einstein Condensation (BEC) of paired photons and dressed electrons in Graphene shows that the condensate fraction N and the BCS like critical temperature T_c can be usefully connected in terms of the multi branched Lambert W function of the temperature emphasizing the logarithmic structure dependance of N and T. Such a BEC is experimentally feasible [23] and BEC and superfluidity of 2D quasiparticles in graphene in a high magnetic field has been predicted [24]. The above solutions can also be expressed in terms of the polylogarithm functions which also have a remarkable range of applications [5, 25, 26].

In conclusion, we have obtained values for the microwave background temperature that are in good accord with cosmic microwave background observations. Furthermore, we have provided a brief overview of cosmic background radiation with the hope that it will shed further light on proposed models of the universe. We have also briefly shown the potential applicability of the Lambert W and polylogarithm functions in problems relevant to quantum statistics.

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