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INDUSTRIAL STRATEGY WITH COMMITTED FIRMS

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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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Abstract

A country can gain by increasing its share of imperfectly competitive international industries that earn rents. Recent work suggests that some markets might involve commitment: where firms use investments in capital to pursue strategic advantages. This paper develops an international commitment model in which no firm has "first-mover" advantages. A domestic government will have an incentive to use either a capital subsidy or export subsidy to increase domestic rents. If both tools are available, the optimum policy involves subsidizing exports and taxing capital to undo the capital bias caused by commitment.

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Industrial Strategy With Committed Firms

1. Introduction

It is well-known that imperfectly competitive international markets provide incentives for governments to carry out policies designed to extract rent from foreign trade. The obvious policies include export taxes and encouragement of export cartels to exploit the monopoly power of domestic firms[1], (see, for example, Basevi (1970), Frenkel (1971), Pursell and Snape (1973) and Auquier and Caves (1979)), or the use of tariffs to extract rent from foreign firms (see Brander and Spencer (1981)). A rather different set of policy motives arises from the idea that it is to the advantage of a country to capture a large share of the production of rent-earning industries. This paper is concerned with this latter aspect of industrial strategy, which is likely to be particularly important in industries which are exploiting new technologies and which could therefore experience major changes in trade patterns.

The capture idea is similar to the classic infant industry argument in trade theory. (References include Kemp (1960), Johnson (1965) and Chacholiades (1978) Ch. 21). The difference here is that the benefits do not arise from positive externalities or because capital markets are imperfect, but simply from obtaining a larger share of industries which earn rent from foreign sales. The capture idea is likely to be most important in industries which are involved in capital formation which will influence future market shares. It therefore seems appropriate to develop the analysis using insights from the recent "commitment" models in oligopoly theory, (see Spence (1977, 1979), Dixit (1980), and Eaton and Lipsey (1980, 1981)) which emphasize the role of irreversible investments in establishing market power.

To introduce the idea of commitment we observe that oligopoly theory is concerned with the behaviour of firms in strategic environments, in which each firm must consider the likely behaviour of its rivals. Schelling (1960) argued that in such strategic situations it is important to distinguish between "threats" and "commitments". A firm makes a threat or commitment in order to persuade its rivals that it will respond to some possible action by them in such a way as to make them regret the action. If one firm can so persuade its rivals, then it can manipulate their actions so as to gain an advantage. The difference between a threat and a commitment is that a threat involves a response that is not in the threatening firm's best interests once its rivals have made the action in question, while a commitment involves a response that is acceptable or even optimal to the firm in the face of its rivals' actions.

An example of the distinction between threats and commitments arises in the theory of entry deterrence. In the Bain-Sylos limit output model the established firm produces an output large enough so that if it continued to produce this output in the face of entry, an entrant would make losses. In essence, the established firm threatens to continue to produce the limit output even if entry should occur. However, if entry does occur, the original firm would not find production of the limit output a very attractive strategy and would prefer to cut back output. Thus the limit output model does not involve a commitment. (References on the limit output model are Bain (1956), Modigliani (1958), and Sherer (1970, ch. 8).)

Because the limit output is only a threat and not a commitment, potential entrants might not take it seriously. However, Spence (1977) recognized that a firm might be able to commit itself to producing an entry-deterring level of output by making irreversible investments. Furthermore even if established firms do not (or cannot) actually prevent entry they can use commitments to manipulate the post-entry equilibrium to their advantage. Several recent papers have taken up this idea. (See Spence (1979) Dixit (1980) and Eaton and Lipsey (1980, 1981).)

In each of these papers there is an important asymmetry among firms. Some firms are in the industry first and are able to realize "first-mover" advantages by making commitments.[2] This asymmetry of opportunity is natural when entry deterrence is being considered. In this paper, however, we develop a model in which commitment is important despite symmetry of opportunity for the firms. Such a model seems appropriate in an international setting in which firms in two or more countries may simultaneously be jockeying for position by undertaking investments in similar products.

The particular industrial strategies considered here are investment subsidies (or taxes) and export subsidies. We show that a single country can increase its monopoly rent by using these policies to increase the domestic share of imperfectly competitive international industries. However, if two or more national governments play the industrial strategy game using capital subsidies, the countries involved will end up worse off with considerable excess capital.

Section 2 sets out a simple symmetric commitment model based on a two-stage Nash equilibrium concept. It is a natural counterpart, for an industry involving commitment, of the Cournot model. It is shown that commitment has a significant effect even in this symmetric model and, in particular, that firms use "too much" capital in the sense that cost is not minimized for the output chosen. Section 3 examines industrial strategy policies in an international market where commitment is important. We show that incentives to use export subsidies or capital subsidies do exist, despite the fact that the use of the latter worsens the bias toward capital. However, if both capital subsidies and export subsidies are available, the optimum policy is to tax capital so as to offset the tendency to overuse capital, and to use the export subsidy to increase the rent of domestic firms.

2. A Symmetric Commitment Model

This section sets out a simple duopoly model. (In the next section it is assumed that the two firms are located in different countries.) The commitment aspect of the model arises because firms go through a two stage decision process involving capital and output.

In the first stage of the decision process firms decide on the amount of capital to install. In the second stage the output decision is made. This structure could reflect the development of a new market in which there is a growth phase, during which capital is put in place, followed by a mature phase. Alternatively, the two stage decision could be part of either a repeated process or a continuing process. Every few years capital wears out or is rendered obsolete and a new capital decision must be made. The essential point is that capital, although not completely fixed, varies slowly compared to the rate at which other factors and output can be varied so that current output decisions are always made taking the capital stock of both firms as given.

Additions to capital lower the variable cost of extra output and therefore influence the position of a firm's reaction function in quantity space. Capital could be interpreted quite broadly including "goodwill" generated by advertising and marketing expenditures which lower the further sales effort required to sell a unit of output. Similarly, instead of "capital" we could equally well regard commitment as arising from cost-reducing research and development. For reasons of economy our discussion is confined to the capital interpretation.

As in Dixit (1980) it is assumed that the second stage, in which output is determined, is resolved as a Nash game on quantities, taking capital levels as fixed. The equilibrium is defined by the property that each firm is maximizing profit with respect to own output, given its capital stock and the output of the other firm. Although the Nash assumption is extreme, it is only one example of the class of models sometimes referred to as conjectural variation models. If the conjectural variation, λ, is defined as the expected response of the rival's output to a change in own output, the Nash case arises if $\lambda = 0$. We use the simple Nash structure so as to keep alegebra to a minimum, but the results obtained generalize directly to other conjectural variation models.

Both firms are aware of how the second stage is to be resolved and take this into account in choosing their capital levels in the first stage. The question arises concerning why the firms should acquiesce in a Nash quantity game if they know the rules of the game in advance. More directly, why should they act as if their rival's output is fixed, when they know that it responds to their own. One explanation is that firms may simply accept the Nash outcome as preferable to more cutthroat forms of competition as, for example, the Stackelberg leader-leader outcome that would result if each firm tried to act on its belief about the other. In any case, firms must have some perception about how quantities depend on capital stock. The Nash case provides a concrete example than can be analyzed. Alternatively, we could just specify that output as a function of capital satisfies certain reasonable properties and proceed.

Profits are represented by the variable π, revenue by R, variable cost by C, output by x, price by p and cost of capital by v. Each variable is superscripted to indicate the associated firm. Profit functions of the two firms are

$$\pi^i(x^1,x^2;k^i) = R^i(x^1,x^2) - C^i(x^i;k^i) - v^i k^i \tag{1}$$

where $R^i = x^i p^i(x^1,x^2)$ for $i = 1, 2$. We assume x^1 and x^2 are slightly different products but they are substitutes so that $R^i_j < 0$ where $i \neq j$. (We often use i and j to refer to the firms and it is to be understood that if i represents 1 in a particular expression, then j represents 2, and vice versa. Subscripts are generally used to denote derivatives.) We also assume that x^1 and x^2 are what we refer to as strong substitutes: each firm's perceived marginal revenue is decreasing in the other firms output.

$$R^i_{ij} < 0 \tag{2}$$

In addition, extra capital is assumed to reduce variable cost, C^i for a given x^i, but at a decreasing rate. Thus $C^i_k < 0$; $C^i_{kk} > 0$. Marginal cost, $\partial C^i/\partial x^i$, is denoted c^i, and is assumed to fall as capital increases: $c^i_k < 0$. (These conditions are consistent with neoclassical production.)

With subscripts indicating derivatives (except for $\partial C^i/\partial x^i \equiv c^i$) the first order conditions for the Nash quantity equilibrium are

$$\partial \pi^i/\partial x^i \equiv \pi^i_i \equiv R^i_i(x^1,x^2) - c^i(x^i;k^i) = 0 \tag{3}$$

with second order condition

$$\pi^i_{ii} < 0 \tag{4}$$

We also assume that the Routh stability condition[3] is satisfied

$$A \equiv \pi^1_{11}\pi^2_{22} - \pi^1_{12}\pi^2_{21} > 0 \tag{5}$$

The Routh condition is sufficient for local stability and for many systems is also necessary. This condition is met under the reasonable requirement that own effects of output on marginal profit dominate cross effects.

First order conditions (3) are output reaction functions for firms 1 and 2 in implicit form, given particular values of k^1 and k^2. Their solution yields the Nash equilibrium quantities, x^1 and x^2, as functions of k^1 and k^2.

$$x^1 = x^1(k^1, k^2); \; x^2 = x^2(k^1, k^2) \tag{6}$$

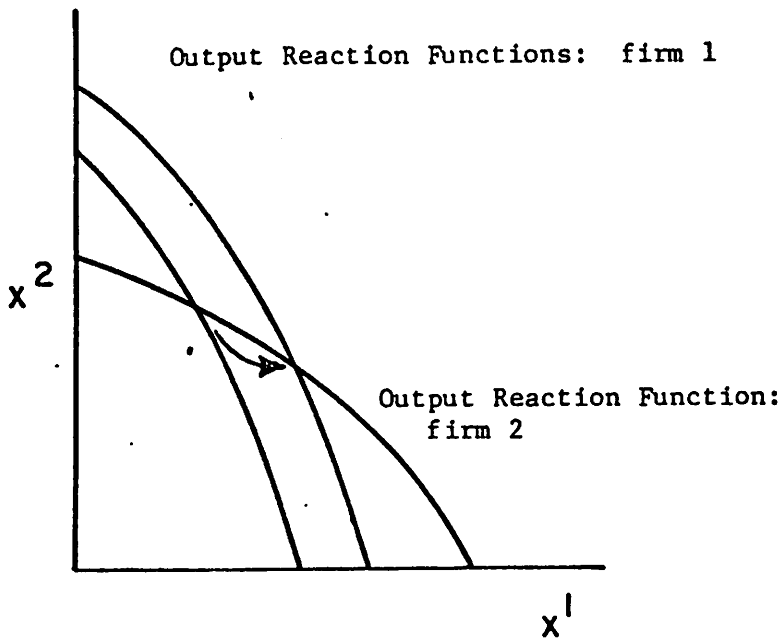
A comparative static exercise shows that x^i is increasing in k^i and that, provided x^1 and x^2 are strong substitutes, x^j is decreasing in k^i. Totally differentiating (3) with respect to k^i holding k^j constant yields the comparative static matrix equation

$$\begin{bmatrix} \pi^i_{ii} & \pi^i_{ij} \\ \pi^j_{ji} & \pi^j_{jj} \end{bmatrix} \begin{bmatrix} dx^i \\ dx^j \end{bmatrix} = \begin{bmatrix} c^i_k dk^i \\ 0 \end{bmatrix} \tag{7}$$

where $c^i_k = \partial c^i/\partial k^i$. Thus, using Cramer's rule $x^i_i \equiv \partial x^i/\partial k^i = c^i_k\pi^j_{jj}/A$ where A is the determinant of the left most matrix, which is positive by stability condition (5). Since $\pi^j_{jj} < 0$ by second order condition (4) and $c^i_k < 0$, x^i_i is positive. Similarly, $x^j_i \equiv \partial x^j/\partial k^i = -c^i_k\pi^j_{ji}/A < 0$ provided (2) holds since $\pi^j_{ji} \equiv R^j_{ji}$.

As illustrated in figure 1, additions to own capital shift a firm's reaction function outward, increasing own output and decreasing the output of the other firm. The reaction functions in the diagram slope downward. This follows from total differentiation of (3) with respect to x^i and x^j, holding k^i and k^j constant, which yields $dx^j/dx^i = -R^j_{ji}/\pi^j_{jj}$, which is negative from (2) and (4).

Figure 1



Having seen how output is determined once capital stocks are in place, it remains to characterize the capital decision, which is made in the first stage. We use the Nash equilibrium concept for the capital game as well as for output. The equilibrium occurs when each firm is maximizing profit with respect to own capital, given the level of capital chosen by the other firm. We let the function g (for gain) represent profit as a function of k^1 and k^2. Since each firm knows the functions $x^1(k^1,k^2)$ and $x^2(k^1,k^2)$,

$$g^i(k^1,k^2) \equiv \pi^i(x^1(k^1,k^2),\ x^2(k^1,k^2);\ k^i) \qquad (8)$$

where π^i is given by (1). We assume

$$g^1_{12} < 0 \text{ and } g^2_{21} < 0 \tag{9}$$

where $g^i_{ij} \equiv \partial^2 g^i/\partial k^i \partial k^j$.

Condition (9) means that each firm's perceived marginal profit with respect to its own capital stock is declining in its rival's capital stock. Each firm i, i = 1, 2, chooses k^i to maximize profit g^i. The Nash equilibrium for the capital game occurs when

$$g^1_1 = 0, \; g^2_2 = 0 \tag{10}$$

where $g^i_i \equiv \partial g^i/\partial k^i$. Second order conditions must also hold

$$g^1_{11} < 0; \; g^2_{22} < 0 \tag{11}$$

and we assume the Routh stability condition for the capital game:

$$D \equiv g^1_{11}g^2_{22} - g^1_{12}g^2_{21} > 0 \tag{12}$$

which follows under the condition that own effects of capital on marginal profit dominate cross effects. Thus without considering the question of stability, expression (12) could quite reasonably be imposed simply as a regularity condition.

Imposing stability, which is an explicitly dynamic concept, raises questions about the dynamic nature of the model. As already mentioned, the basic idea is that investment in capital takes place before output is produced. Capital can normally be added during the second stage; however, since capital is chosen optimally in the first stage, firms will never choose to add capital in the second stage unless external factors change. Over time capital does wear out, either gradually, or at once in "one hoss shay" fashion. Thus the capital decision may be repeated many times (or continuously in a continuous model), so that there is an underlying dynamic element to the model. The commitment aspect arises not because capital is absolutely immutable but because its speed of adjustment is very slow compared with the speed of adjustment of output.

For many industries, however, one might argue that adjustment in capital is so slow as to make questions of stability irrelevant since external factors change sufficiently rapidly that the industry would never have time to approach a capital stock equilibrium anyway. In such industries, the initial decision on any major investment is so important and takes so long to implement that it makes sense to think of the Nash game as being played with investment plans. Each firm has time to learn about its rivals' plans and investments in progress and can adjust its own plans or investments in progress accordingly.

In a standard Cournot model firms choose capital and output simultaneously and do not take into account the effect that changes in own capital have on the other firms' output. In symbols, in a pure Cournot model firms assume $\partial x^i/\partial k^j = 0$, whereas here, in our Nash commitment model, firms realize that $\partial x^i/\partial k^j < 0$.

The first important implication to be drawn from the model is that firms overuse capital and therefore do not minimize cost. Total cost for output x^i is $C^i(x^i;\ k^i) + v^i k^i$, which, when minimized with respect to k^i holding x^i constant, yields first order condition

$$C_k^i + v^i = 0 \tag{13}$$

where $C_k^i = \partial C^i/\partial k^i$. However, from (8) and (1) the first order condition (10) for profit maximization implies

$$g_i^i \equiv \pi_i^i x_i^i + \pi_j^i x_i^j - C_k^i - v^i = 0 \quad \text{where } i \neq j$$

Since $\pi_i^i = 0$ and $\pi_j^i \equiv R_j^i$ this yields

$$C_k^i + v^i = R_j^i x_i^j \quad \text{where } i \neq j \tag{14}$$

R_j^i and x_j^i are both negative, therefore $C_k^i + v$, which is marginal cost with respect to own capital, is positive, which indicates that cost is not minimized. Furthermore, (14) implies that C_k^i is too small in absolute value, which means that capital is being overused since $C_{kk}^i > 0$. Note that a tax on capital of $R_j^i x_i^j$ will induce cost-minimization. This overcapitalization of the industry is purely a result of commitment and does not rely on any asymmetry of opportunity for the firms, and can be regarded as an extension of the results in Dixit (1980).

3. Industrial Strategy

Our objective is to examine whether industrial strategy, in the form of capital subsidies and export subsidies, can enable a domestic firm to capture a larger share of the world market so as to increase profits and rent net of the subsidy to the domestic country. Our approach is to characterize the subsidies that would maximize rent. In practise, of course, governments have neither the information nor the singlemindedness necessary to implement such finely tuned policies. Nevertheless, if the optimum subsidies are positive, we shall at least know that incentives for such subsidies can arise from pure rent-seeking motives.

The setting is as follows. There are two firms in the industry, one located in the "domestic" country and one located in a foreign country. In order to focus on the purely rent-seeking rationale for industrial strategy we assume that all output is for export to other countries. The possibility of domestic consumption would generally strengthen any incentives for subsidization of the domestic firm since it tends to increase quantities and decrease prices, but we do not analyze that issue here. The domestic government wishes to maximize net rent accruing to the domestic country, which in this simple context is just the profit of the domestic firm minus the cost of any policies carried out.

The first type of policy to consider is a subsidy (or tax) on capital by itself. Since the international community discourages direct export subsidies the pure capital subsidy is perhaps the most relevant policy. The government offers a subsidy, s, per unit of capital so that from (1) and (8) the profit of the domestic firm is

$$g^1(k^1, k^2; s) = R^1(x^1, x^2) - C^1(x^1; k^1) - (v^1 - s)k^1 \tag{15}$$

where $x^1 = x^1(k^1, k^2)$, $x^2 = x^2(k^1, k^2)$.

Before characterising the optimum subsidy some comparative static properties of the commitment model should be discussed. As one might expect, increases in the subsidy, s, tend to increase the capital stock chosen by the domestic firm and reduce the capital stock chosen by the foreign firm: $k^1_s > 0$; $k^2_s < 0$.

These effects are obtained by total differentiation of the first order conditions $g^1_1 = 0$ (from (15)), and $g^2_2 = 0$ to yield

$$\begin{bmatrix} g^1_{11} & g^1_{12} \\ g^2_{21} & g^2_{22} \end{bmatrix} \begin{bmatrix} k^1_s \\ k^2_s \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \tag{16}$$

which implies that $k^1_s = -g^2_{22}/D$ and $k^2_s = g^2_{21}/D$. Then $k^1_s > 0$ by (11) and (12) and $k^2_s < 0$ by (9) and (12). Note that the slope, dk^2/dk^1, of the foreign firm's capital reaction function is $-g^2_{21}/g^2_{22}$ so that

$$k^2_s = k^1_s dk^2/dk^1 \tag{17}$$

The optimum subsidy is found by maximizing net rent (or benefit), B:

$$B(k^1, k^2; s) = g^1(k^1, k^2; s) - sk^1 \tag{18}$$

Using the expression dB/ds to denote the total derivative, the first order condition is

$$dB/ds = g_1^1 k_s^1 + g_2^1 k_s^2 + g_s^1 - k^1 - sk_s^1 = 0 \tag{19}$$

We have $g_s^1 = k^1$ from (15), $g_1^1 = 0$ (the first order condition for profit maximization), and $k_s^2 = k_s^1\, dk^2/dk^1$ from (17) so (19) reduces to

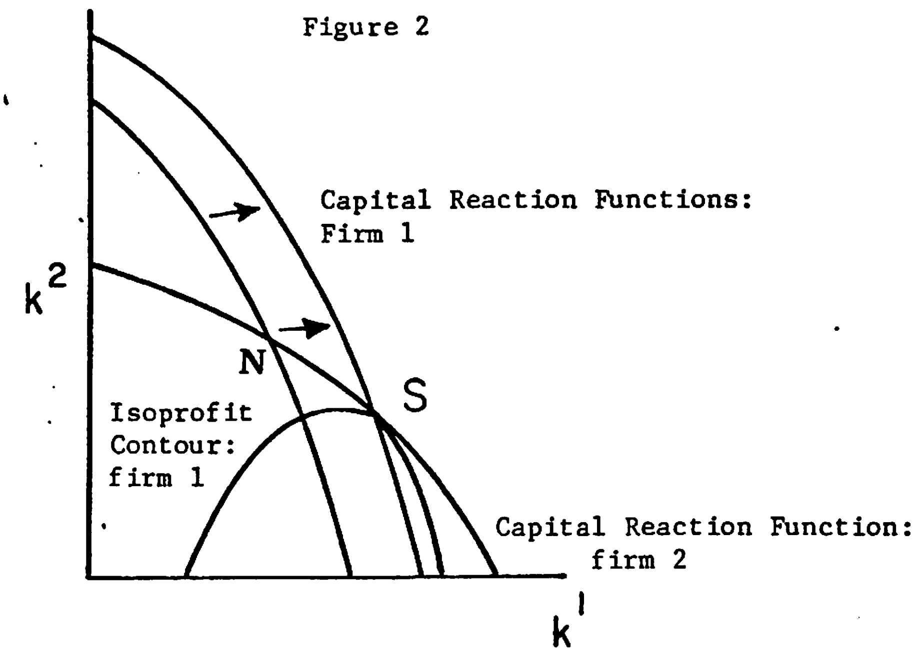
$$s = g_2^1 dk^2/dk^1 \tag{20}$$

We have established that dk^2/dk^1 is negative under our regularity condition that $g_{21}^2 < 0$, and it is easily shown that g_2^1 is also negative: $g_2^1 = \pi_1^1 x_2^1 + \pi_2^1 x_2^2$. $\pi_1^1 (\equiv \partial\pi^1/\partial x^1)$ is zero, $\pi_2^1 = R_2^1$ which is negative since x^1 and x^2 are substitutes, and $x_2^2 (\equiv \partial x^2/\partial k^2)$ has been shown to be positive from (7). Therefore $g_2^1 = R_2^1 x_2^2 < 0$ and $s > 0$.

The optimum subsidy is positive. Correspondingly, dB/ds is positive at $s = 0$ indicating that there is a rent-seeking incentive for the government to introduce a subsidy on capital, despite the fact that a subsidy increases the bias toward excessive use of capital.[4]

In effect the optimum capital subsidy moves the domestic firm from the Nash point in capital space to the Stackelberg point in capital space. As illustrated in Figure 2, each firm has a Nash reaction-function in capital space. Both reaction functions are satisfied, without the subsidy, at point N. Isoprofit contours show different combinations of k^1 and k^2 that would yield equal profit to firm 1. Therefore, given the Nash reaction function of firm 2, point S, the Stackelberg leader-follower point, is the most profitable position for firm 1. The subsidy shifts the reaction function of firm 1 so that it intersects the reaction function of firm 2 at the Stackelberg point.

Figure 2



That the optimal subsidy achieves the Stackelberg point can be seen by noting that if firm 1 were a Stackelberg leader with s = 0, it would choose k^1 to satisfy $g_1^1 + g_2^1 dk^2/dk^1 = 0$. This coincides with first order condition $g_1^1(k^1,k^2;s) = 0$, obtained from differentiation of (15), when s is at its optimum value, $g_2^1 dk^2/dk^1$, for Nash firms.

One might object that it is unrealistic to assume that the government knows the true nature of the industry, and can charge the optimum subsidy, while firms are "naive". However, the important point remains that a government will perceive the kind of incentive described here whenever its perception of the industry differs from the perception held by the domestic firm even if that perception is wrong. Secondly, the firms may not be naive but may simply accept the Nash rules as an alternative to less profitable forms of competition. Also, of course, the domestic firm can do better when it is moved to the Stackelberg point by subsidies than when it goes there by itself. Persuading the government that it has a movable reaction function allows it to enjoy the best of both worlds, unless the government recovers the full cost of the subsidy by an export license fee or some equivalent lump sum tax which offsets the redistribution of income from taxpayers to the shareholders of the firm.

One country, if left to pursue this policy of increasing its share of a profitable industry through capital subsidies, can certainly benefit. However, if both governments try to follow this policy, both countries end up worse off at the Stackelberg leader-leader position and with considerable excess capital in the industry. Thus, the governments involved have an incentive to undertake direct negotiation to prevent the use of industrial strategy policies. They also have an incentive to cheat on any resulting agreements.

The second tool of industrial strategy to consider is the export subsidy. There are two cases to examine: the export subsidy by itself and the combined policy of export subsidy and capital subsidy. The export subsidy, denoted σ, is more complicated in its effect than the capital subsidy because it affects both the capital decision and the output decision given capital.

Before proceeding, we recall that one possible technological structure for the industry is that capital, once in place, is completely fixed, and cannot be augmented or reduced. If an export subsidy is offered after capital is in place, without advance warning, then the export subsidy cannot affect capital and alters only the output decision. However, we are generally concerned with a situation in which either capital is not yet in place or in which capital is in place but slowly variable.

Net rent or benefit now depends on σ:

$$B(k^1, k^2; \sigma) = g^1(k^1, k^2; \sigma) - \sigma x^1 \tag{21}$$

where $g^1(k^1, k^2; \sigma) \equiv R^1(x^1, x^2) - C^1(x^1; k^1) - v^1k^1 + \sigma x^1$ (22)

Note that σ influences the choice of x^1 and x^2 through $x^1 = x^1(k^1, k^2; \sigma)$ and $x^2 = x^2(k^1, k^2; \sigma)$ in addition to its affect on the values of k^1 and k^2. The first order condition for a maximum of B is obtained by setting total derivative $dB/d\sigma$ to zero.

$$dB/d\sigma = g_1^1k_\sigma^1 + g_2^1k_\sigma^2 + g_\sigma^1 - x^1 - \sigma(dx^1/d\sigma) = 0 \tag{23}$$

where total derivative $dx^1/d\sigma = x_1^1k_\sigma^1 + x_2^1k_\sigma^2 + x_\sigma^1$ where $x_\sigma^1 = \partial x^1/\partial\sigma$. Manipulating (23) and using $g_\sigma^1 = \pi_1^1x_\sigma^1 + \pi_2^1x_\sigma^2 + x^1$, $\pi_1^1 = 0$, $\pi_2^1 = R_2^1$ and $g_1^1 = 0$ yields

$$\sigma = (g_2^1k_\sigma^2 + R_2^1x_\sigma^2)/(dx^1/d\sigma) \tag{24}$$

In order to determine the sign of σ it is necessary to sign the comparative static effects $dx^1/d\sigma$, k_σ^2, and $x_\sigma^2 (\equiv \partial x^2/\partial\sigma)$. In the appendix it is shown that $dx^1/d\sigma > 0$, $k_\sigma^2 < 0$, and $x_\sigma^2 < 0$. These results require some effort to obtain since the sign of k_σ^1, on which $dx^1/d\sigma$ partly depends, is indeterminate. An export subsidy does increase total exports as we expect, but it also tends to reduce the capital bias by increasing output for any level of capital, and it may even reduce the equilibrium capital stock of the domestic firm. Despite this possibility the equilibrium capital stock of the foreign firm does fall, and also the direct effect of σ on x^2, holding capital fixed, is negative. These results imply that, from (24), the optimum subsidy on exports is definitely positive.

In the case in which capital is already completely fixed and inflexible, which implies $k_\sigma^1 = k_\sigma^2 = 0$, then $dx^1/d\sigma = x_\sigma^1$, and recognizing that $x_\sigma^2 = (dx^2/dx^1)x_\sigma^1$, the optimum subsidy is $\sigma = R_2^1 dx^2/dx^1 > 0$ (substituting in (24)). This is also the optimum subsidy for a pure Cournot model without commitment: for any given capital stock it induces the domestic firm to move to the (rent-maximizing) Stackelberg point in the output space. If the capital stock is not yet determined, the capital subsidy is increased by a term which represents the effect of a larger σ on the domestic firm's profitability via its negative effect on the foreign firm's capital stock.

The most difficult case is the combined policy of a capital subsidy and an output subsidy. The profit function of the domestic firm is

$$g^1(k^1, k^2; s, \sigma) = R^1(x^1, x^2) - C^1(x^1; k^1) + \sigma x^1 - (v^1 - s)k^1 \quad (25)$$

where $x^1 = x^1(k^1, k^2;\sigma)$; $x^2 = x^2(k^1, k^2;\sigma)$, and k^1 and k^2 themselves depend on σ and s. The benefit function is

$$B(k^1, k^2; s, \sigma) = g^1(k^1, k^2; s, \sigma) - \sigma x^1 - sk^1 \qquad (26)$$

The first order conditions for an optimum policy are

$$dB/ds = (g_1^1 - s)k_s^1 + g_2^1 k_s^2 + g_s^1 - k^1 - \sigma dx^1/ds = 0 \qquad (27)$$

where $dx^1/ds = x_1^1 k_s^1 + x_2^1 k_s^2$

$$dB/d\sigma = (g_1^1 - s)k_\sigma^1 + g_2^1 k_\sigma^2 + g_\sigma^1 - x^1 - \sigma dx^1/d\sigma = 0 \qquad (28)$$

where $dx^1/d\sigma = x_1^1 k_\sigma^1 + x_2^1 k_\sigma^2 + x_\sigma^1$

Using $k_s^2 = (dk^2/dk^1)k_s^1$ we have $dx^1/ds = (dx^1/dk^1)k_s^1$ where $dx^1/dk^1 = x_1^1 + x_2^1(dk^2/dk^1)$. Then since $g_s^1 = k^1$ and $g_1^1 = 0$, expression (27) implies

$$s = g_2^1 dk^2/dk^1 - \sigma dx^1/dk^1 \qquad (29)$$

Comparing (29) with (20), which gives the optimum capital subsidy in isolation, we see that if $\sigma = 0$, the two expressions coincide. However, if $\sigma > 0$, the optimum subsidy on capital is reduced by the term $\sigma dx^1/dk^1$. This term reflects the effect of a unit increase in capital on the cost of the output subsidy to the government.

Turning to the export subsidy, since $\pi_1^1 = 0$, we have $g_\sigma^1 = R_2^1 x_\sigma^2 + x^1$ and substituting this and $g_1^1 = 0$ in (28) we obtain

$$dB/d\sigma = -sk_\sigma^1 + g_2^1 k_\sigma^2 + R_2^1 x_\sigma^2 - \sigma dx^1/d\sigma = 0 \qquad (30)$$

Therefore the output subsidy is

$$\sigma = (g_2^1 k_\sigma^2 + R_2^1 x_\sigma^2)/(dx^1/d\sigma) - sk_\sigma^1/(dx^1/d\sigma) \qquad (31)$$

Comparison with (24) shows that the first term of (31) coincides with the optimum export subsidy when there is no capital subsidy.

It is no longer clear that s and σ are both positive. Expressions (29) and (34) are two equations in the two policy variables s and σ. They are solved in the appendix to yield

$$s = -R_2^1 x_1^2 \tag{32}$$

$$\sigma = (g_2^1(dk^2/dk^1) + R_2^1 x_1^2)/(dx^1/dk^1) \tag{33}$$

The optimum export subsidy, σ, is certainly positive under our regularity conditions. However, the optimum capital subsidy is negative: capital should be taxed. In the commitment model, firms overuse capital causing them to incur higher costs than technologically necessary for the output they produce. With an export subsidy available it becomes rent-maximizing for the country to undo the capital bias. Indeed, referring back to equation (14) in Section 2 shows that the optimum tax (or negative subsidy) on capital is precisely what is required to induce the domestic firm to minimize costs.

The export subsidy now has an extra term to compensate for the export-reducing effects of the tax on capital. It is as if the capital tax (subsidy) is assigned to correct the departure from cost-minimization, and the export subsidy is assigned to maximize rent contingent on the capital tax. In fact, however, the optimum values for the two tools are arrived at by joint maximization.

Concluding Remarks

The idea that firms might use investments in capital to commit themselves to certain courses of action for strategic reasons seems an important development in oligopoly theory. Recent work has examined the role of commitment in asymmetric environments: where one firm has the opportunity to act before another. We have examined a model in which firms are on an equal footing with respect to timing, although they may differ in other respects. The tendency towards overcapitalization that arises in asymmetric models persists in our symmetric model, showing that symmetry is not essential to the overcapitalization result.

The model we examine is perhaps the simplest symmetric commitment model one could construct: a two stage Nash model with only two firms. However the assumptions that there are only two firms, that firms take the capital decisions of other firms as given, and that the second stage is resolved as a Nash quantity game are not essential to the results. Generalizations to n firms and to conjectural variations other than zero are straightforward in principle, but are considerably more difficult algebraically and obscure the essential nature of the model.

One important aspect of high capital industries is that national governments often intervene with investment subsidies and thinly disguised export subsidies. The main contribution of this paper is to analyse the incentives for such policies in international industries where commitment is important. The essential idea is that export subsidies or capital subsidies induce a domestic firm to gain a larger market share of rent-earning industries. A domestic government, if left to pursue such policies in isolation, can increase domestic net rent accruing from such industries by using either a capital subsidy or an export subsidy. However, if both (or many) countries try to follow the same policy all will lose. If, moreover, the only tool used is the capital subsidy, the industry will become heavily overcapitalized.

A fairly striking result is obtained for the case in which both export and capital subsidies can be used. The optimum policy is to have a tax (or negative subsidy) on capital that is just sufficient to exactly offset the tendency for overcapitalization by the domestic firm, so that it minimizes costs. The corresponding subsidy on exports is larger than in the case where a capital tax is not used, and enables the domestic firm to capture a larger share of the industry than it would unaided.

Appendix

Comparative Static Effects

The following comparative static effects of the output subsidy σ were needed in examining industrial strategy incentives and effects: i) $x^1_\sigma > 0$, $x^2_\sigma < 0$, ii) $k^2_\sigma < 0$, and iii) $dx^1/d\sigma > 0$. In deriving these results we include the possibility that the capital subsidy s is not zero. This generalization is analytically trivial and makes no difference to the results, but strictly speaking is required for the simultaneous derivation of the optimum s and σ.

First we derive x^1_σ and x^2_σ. These are the effects on the domestic and foreign firms' outputs (respectively), arising from a change in the output subsidy to domestic firms, holding capital constant. From (25), $g^1(k^1, k^2; \sigma) \equiv \pi^1(x^1, x^2, k^1; s, \sigma)$ where $x^1 = x^1(k^1, k^2; \sigma)$ and $x^2 = x^2(k^1, k^2; \sigma)$. Therefore the first order conditions for the Nash quantity equilibrium given k^1 and k^2 are $\pi^1_1 = R^1_1 - c^1 + \sigma = 0$, $\pi^2_2 = R^2_2 - c^2 = 0$. Totally differentiating with respect to x^1, x^2 and σ, we obtain:

$$\begin{bmatrix} \pi^1_{11} & \pi^1_{12} \\ \pi^2_{21} & \pi^2_{22} \end{bmatrix} \begin{bmatrix} x^1_\sigma \\ x^2_\sigma \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Solving for x^1_σ and x^2_σ yields

$$x^1_\sigma = -\pi^2_{22}/A \tag{1a}$$

$$x^2_\sigma = \pi^2_{21}/A \tag{2a}$$

where A is the determinant of the left hand matrix. A is positive by the Routh stability condition, π^2_{22} is negative by the second order condition for profit maximization in the Nash quantity game and

$\pi^2_{21} \equiv R^2_{21} < 0$ by our assumption of strong substitutability. Therefore $x^1_\sigma > 0$ and $x^2_\sigma < 0$, as was to be shown.

The effects k^1_σ and k^2_σ are the effects on the equilibrium capital stocks for the domestic and foreign firms (respectively) of a change in σ. These effects are obtained by totally differentiating $g^1_1 = 0$ and $g^2_2 = 0$ with respect to k^1, k^2 and σ to yield

$$\begin{bmatrix} g^1_{11} & g^1_{12} \\ g^2_{21} & g^2_{22} \end{bmatrix} \begin{bmatrix} k^1_\sigma \\ k^2_\sigma \end{bmatrix} = \begin{bmatrix} -g^1_{1\sigma} \\ -g^2_{2\sigma} \end{bmatrix}$$

where $g^1_{1\sigma} = \partial^2 g^1/\partial k^1 \partial\sigma$ and $g^2_{2\sigma} = \partial^2 g^2/\partial k^2 \partial\sigma$.

The solutions are

$$k^1_\sigma = [g^2_{2\sigma} g^1_{12} - g^1_{1\sigma} g^2_{22}]/D \tag{3a}$$

$$k^2_\sigma = [-g^2_{2\sigma} g^1_{11} + g^1_{1\sigma} g^2_{21}]/D \tag{4a}$$

where D is the determinant of the left hand matrix, which is positive by the Routh stability condition. $g^1_{1\sigma}$ and $g^2_{2\sigma}$ require some effort to sign. Since $g^1 \equiv \pi^1(x^1(k^1, k^2, \sigma), x^2(k^1, k^2, \sigma), k^1; s, \sigma)$ we have

$$\begin{aligned} g^1_1 &= \pi^1_1 x^1_1 + \pi^1_2 x^2_1 - C^1_k - (v^1 - s) \\ &= R^1_2 x^2_1 - C^1_k - (v^1 - s) \end{aligned} \tag{5a}$$

$$g^1_{1\sigma} = R^1_2 x^2_{1\sigma} + x^2_1(dR^1_2/d\sigma) - c^1_k x^1_\sigma \tag{6a}$$

We proceed by relating $g^1_{1\sigma}$ to g^1_{11} which is negative from the second order conditions. From differentiation of (5a) with respect to k^1

$$g^1_{11} = R^1_2 x^2_{11} + x^2_1(dR^1_2/dk^1) - c^1_k x^1_1 \tag{7a}$$

We have from (7) of the text

$$x^2_1 = -c^1_k\pi^2_{21}/A \text{ and } x^1_1 = c^1_k\pi^1_{11}/A \tag{8a}$$

Together with (2a), (8a) implies

$$x^2_\sigma = -x^2_1/c^1_k \tag{9a}$$

so that differentiating (9a) with respect to k^1,

$$x^2_{1\sigma} \equiv x^2_{\sigma 1} = -(c^1_k x^2_{11} - x^2_1 c^1_{kk})/(c^1_k)^2 \tag{10a}$$

Similarly from (1a) and (8a)

$$x^1_\sigma = -x^1_1/c^1_k. \tag{11a}$$

Using (9a) and (11a) together with $dR^1_2/d\sigma = R^1_{21}x^1_\sigma + R^1_{22}x^2_\sigma$ yields

$$\begin{aligned} dR^1_2/d\sigma &= (R^1_{21}x^1_1 + R^1_{22}x^2_1)/c^1_k \\ &= -(dR^1_2/dk^1)/c^1_k \end{aligned} \tag{12a}$$

Substituting (10a), (11a), and (12a) in (6a) gives

$$\begin{aligned} g^1_{1\sigma} &= -R^1_2(c^1_k x^2_{11} - x^2_1 c^1_{kk})/(c^1_k)^2 - x^2_1(dR^1_2/dk^1)/c^1_k + x^1_1 \\ &= [R^1_2(x^2_{11} + x^2_1 c^1_{kk}/c^1_k) + x^2_1 dR^1_2/dk^1 - c^1_k x^1_1]/c^1_k \\ &= -g^1_{11}/c^1_k + R^1_2 x^2_1 c^1_{kk}/(c^1_k)^2 \quad \text{from (7a)} \end{aligned} \tag{13a}$$

Also $g^2_{2\sigma} = R^2_1 x^1_{2\sigma} + x^1_2(dR^2_1/d\sigma) - c^2_k x^2_\sigma$ (14a)

Since $x^1_\sigma = -x^1_1/c^1_k$ (11a), $x^1_{\sigma 2} = -x^1_{12}/c^1_k$. Similarly to (12a) $dR^2_1/d\sigma = -(dR^2_1/dk^1)/c^1_k$.

Substituting into (14a), we obtain,

$$g^2_{2\sigma} = -g^2_{21}/c^1_k \text{ where } g^2_{21} = R^2_1 x^1_{21} + x^1_2(dR^2_1/dk^1) - c^2_k x^2_1 \tag{15a}$$

Substituting (13a) and (15a) in (3a) and (4a) gives

$$k^1_\sigma = 1/c^1_k - (g^2_{22}R^1_2x^2_1c^1_{kk})/(c^1_k)^2D \tag{16a}$$

$$k^2_\sigma = g^2_{21}((g^1_{11}/c^1_k) + g^1_{1\sigma})/D$$

$$= g^2_{21}R^1_2x^2_1c^1_{kk}/(c^1_k)^2D < 0 \text{ using (13a)} \tag{17a}$$

k^2_σ is definitely negative: increases in the output subsidy for firm 1 (the domestic firm) lower the equilibrium capital stock for firm 2. However k^1_σ is indeterminate; the term $1/c^1_k$ prevents k^1_σ from being definitely positive. The interpretation is that increases in σ reduce the profitability of lowering marginal cost through additional capital.

The third effect to consider is the total effect of σ on x^1, taking into account the changes in k^1 and k^2.

$$dx^1/d\sigma = x^1_1k^1_\sigma + x^1_2k^2_\sigma + x^1_\sigma \tag{18a}$$

Note that from (16a) and (17a)

$$k^2_\sigma = (k^1_\sigma - 1/c^1_k)(dk^2/dk^1) \text{ where } dk^2/dk^1 = -g^2_{21}/g^2_{22} \tag{19a}$$

Substituting (11a) and (19a) into (18a) and using $dx^1/dk^1 = x^1_1 + x^1_2dk^2/dk^1$,

$$dx^1/d\sigma = (dx^1/dk^1)(k^1_\sigma - 1/c^1_k) \tag{20a}$$

Since from (16a), $k^1_\sigma - 1/c^1_k > 0$, we have $dx^1/d\sigma > 0$. For completeness we also note that $dx^2/d\sigma < 0$. Subsidizing output (exports) of firm 1 increases equilibrium output of firm 1 and decreases equilibrium output of firm 2.

Simultaneous Solution for the Optimum s and σ

The problem is to solve the first order conditions (27) and (30) for s and σ. Rather than solving directly using Cramer's rule, it is more efficient to begin by substituting (19a) and (20a) into (30) and gathering terms in k_σ^1.

$$dB/d\sigma = (-s + g_2^1 dk^2/dk^1 - \sigma dx^1/dk^1)k_\sigma^1$$
$$+ R_2^1 x_\sigma^2 - g_2^1(dk^2/dk^1)/c_k^1 + \sigma(dx^1/dk^1)/c_k^1 = 0 \quad (21a)$$

By expression (27) for s the first term is zero. Substituting for x_σ^2 (9a) in (21a),

$$dB/d\sigma = [-R_2^1 x_1^2 - g_2^1(dk^2/dk^1) + (dx^1/dk^1)\sigma]/c_k^1 = 0 \quad (22a)$$

which rearranged yields the expression (31) for σ. The expression (32), $s = -R_2^1 x_1^2$ is then obtained by substituting (31) into (29) of the text.

Footnotes

1. By domestic firms we mean firms that are owned domestically.

2. In the papers by Dixit (1980) and Eaton and Lipsey (1980, 1981) there is only one established firm. In Spence (1979) firms enter a new market in sequence.

3. The Routh stability condition (sometimes called the Routh-Hurwicz condition) is for the adjustment mechanism $\dot{x}_1 = \alpha^1(x_1^* - x_1)$; $\dot{x}_2 = \alpha^2(x_2^* - x_2)$. A good reference is Takayama (1974).

4. With a capital subsidy, the first order conditions for profit maximization given by $g_1^1 = 0$ from (15) reduce to $C_k^1 + v^1 = R_2^1 x_1^2 + s$. Since with no capital subsidy (see (14)), $C_k^1 + v^1 = R_2^1 x_1^2$, the effect of a positive s is to increase the positive value of $C_k^1 + v^1$ which (for the same output) implies a greater deviation from the cost-minimizing solution: $C_k^1 + v^1 = 0$.

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Referee's Notes

These notes derive

a) stability condition (5) (pg. 8 of text)
b) sign of x^i_i and x^i_j (pg. 8 of text)
c) comparative static effects k^1_s and k^2_s (pg. 14 of text)
d) expression (17) (pg. 14 of text)

Derivation of stability condition (5) in text (pg. 8). Assume the standard local adjustment mechanism:

$$\dot{x}^1 = \alpha^1(x^{1*} - x^1); \; \dot{x}^2 = \alpha^2(x^{2*} - x^2) \qquad \text{(i)}$$

where $\dot{x}^i = dx^i/dt$ (the time derivative), α^i is the (positive) speed of adjustment and x^{i*} is the optimum value of x^i given the current value of $x^j (j \neq i)$. From first order conditions (3) of the text x^{i*} depends on x^j so we can write

$$x^{1*} = r^1(x^2); \; x^{2*} = r^2(x^1) \qquad \text{(ii)}$$

where r stands for the reaction function. For local stability analysis r^i can be replaced by the first order Taylor series expansion

$$r^i(x^j) \simeq \bar{x}^i + r^i_j(\bar{x}^j)(x^j - \bar{x}^j) \qquad \text{(iii)}$$

where upper bars denote final equilibrium values. Letting y^i denote $x^i - \bar{x}^i$, and substituting (iii) in (i) allows the adjustment system to be written

$$\dot{y} = My \qquad \text{(iv)}$$

where $\dot{y} = \begin{bmatrix} \dot{y}^1 \\ \dot{y}^2 \end{bmatrix}$, $M = \begin{bmatrix} -\alpha^1 & \alpha^1 r^1_2 \\ \alpha^2 r^2_1 & -\alpha^2 \end{bmatrix}$, $y = \begin{bmatrix} y^1 \\ y^2 \end{bmatrix}$

Local stability of system (i) is equivalent to local stability of system (iv). System (iv) is locally stable if the real part of every eigenvalue of M is negative, that is, if the solutions, λ, to the determinant equation

$$\det(\lambda I - M) = 0$$

have negative real parts. (I is the 2 x 2 identity matrix.) Written out, this equation, the characteristic equation, is

$$(\lambda)^2 + (\alpha^1 + \alpha^2)\lambda - \alpha^1\alpha^2(r^1_2 r^2_1 - 1) \qquad \text{(v)}$$

Next, totally differentiate first order conditions (3) to obtain expressions for r^1_2 and r^2_1:

$$\pi^1_{11} r^1_2 + \pi^1_{12} = 0; \quad \pi^2_{21} + \pi^2_{22} r^2_1 = 0$$

$$\text{so} \qquad r^1_2 = -\pi^1_{12}/\pi^1_{11}; \qquad r^2_1 = -\pi^2_{21}/\pi^2_{22} \qquad \text{(vi)}$$

and condition (v) can be written

$$(\lambda)^2 + (\alpha^1 + \alpha^2)\lambda + \alpha^1\alpha^2(1 - \pi^1_{12}\pi^2_{21}/\pi^1_{11}\pi^2_{22}) \qquad \text{(vii)}$$

The Routh conditions (see, for example, Takayama, p. 310) are necessary and sufficient for the real parts of the eigenvalues to be negative and are therefore sufficient for local stability. These conditions are (in this case)

$$\alpha^1 + \alpha^2 > 0 \qquad \text{(viii)}$$

$$\alpha^1\alpha^2(1 - \pi^1_{12}\pi^2_{21}/\pi^1_{11}\pi^2_{22}) > 0 \qquad \text{(ix)}$$

Condition (viii) is automatic from the assumption that α^1 and α^2 are both positive. Since π^1_{11} and π^2_{22} are negative by second order conditions (4), condition (ix) holds if and only if

$$A \equiv \pi^1_{11}\pi^2_{22} - \pi^1_{12}\pi^2_{21} > 0$$

as was to be shown.

*   *   *

Derivation of x^i_i and x^i_j (pg. 8)

x^i_i is $\partial x^i/\partial k^i$, the rate of change in output of firm i if k^i changes while k^j remains constant. First order condition (3) is $\pi^i_i = 0$ so totally differentiating with respect to x^i, x^j, and k^i yields

$$\pi^i_{ii}dx^i + \pi^i_{ij}dx^j + (\partial\pi^i_i/\partial k^i)dk^i = 0$$

and totally differentiating $\pi^j_j = 0$ with respect to x^i, x^j, and k^i yields

$$\pi^j_{ji}dx^i + \pi^j_{jj}dx^j + (\partial\pi^j_j/\partial k^i)dk^i = 0$$

Since $\partial\pi^i_i/\partial k^i = -c^i_k$ and $\partial\pi^j_j/\partial k^i = 0$ these two equations are equivalent to (7) of the text.

*   *   *

Stability condition (12) (pg. 10) is derived in the same manner as stability condition (5).

*   *   *

Comparative Static Effects k^1_s and k^2_s (pg. 14)

Firm 1 maximizes (15) with respect to k^1 so that

$$g^1_1 \equiv R^1_1x^1_1 + R^1_2x^2_1 - c^1x^1_1 - C^1_k - (v^1 - s) = 0$$

Firm 2 maximizes $g^2 \equiv R^2(x^1, x^2) - C^2(x^2; k^2) - v^2k^2$ so that

$$g_2^1 \equiv R_1^2 x_2^1 + R_2^2 x_2^2 - c^2 x_1^2 - C_k^2 - v^2 = 0$$

Totally differentiating $g_1^1 = 0$ and $g_2^2 = 0$ with respect to k^1, k^2 and s yields

$$g_{11}^1 dk^1 + g_{12}^1 dk^2 + (\partial g_1^1/\partial s)ds = 0$$

$$g_{21}^2 dk^1 + g_{22}^2 dk^2 + (\partial g_2^2/\partial s)ds = 0$$

Since $\partial g_1^1/\partial s = 1$ and $\partial g_2^2/\partial s = 0$ these two equations can be written in matrix form as (16) of the text. Then, using Cramer's rule,

$$k_s^1 = -g_{22}^2/D \text{ and } k_s^2 = -g_{21}^2/D \qquad \text{(x)}$$

*   *   *

derivation of (17) (p. 14)

For the capital stock chosen by one firm the other firm has on optimum reaction. This defines a Nash reaction function in capital space. The slope of the foreign firm's reaction function is found by totally differentiating $g_2^2 = 0$ with respect to k^1 and k^2:

$$g_{21}^2 dk^1 + g_{22}^2 dk^2 = 0$$

to obtain $dk^2/dk^1 = -g_{21}^2/g_{22}^2$

Then, using (x), $k_s^2 = k_s^1 dk^2/dk^1$

*   *   *

CUMULATIVE INDEX

1981

1982