Leibniz’s analysis of change: vague states, physical continuity, and the calculus

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0. Abstract

One of the most puzzling features of Leibniz’s deep metaphysics is the apparent contradiction between his claims (1) that the law of continuity holds everywhere, so that in particular, change is continuous in every monad, and (2) that “changes are not really continuous”, since successive states contradict one another. In this paper I try to show in what sense these claims can be understood as compatible. My analysis depends crucially on Leibniz’s idea that enduring states are “vague”, and abstract away from further changes occurring within them at a higher resolution—consistently with his famous doctrine of petites perceptions. As Leibniz explains further in a recently transcribed unpublished manuscript, these changes are dense within any actual duration, which is conceived as actually divided by them into states that are syncategorematically infinite in number and unassignably small. The correspondence between these unassignably small intervals between changes and the differentials of his calculus allows processes to be conceived as continuous, despite the discontinuity of the changes that occur in actuality.
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A detailed treatment, including a mathematical formulation of Leibniz’s theory of time, is forthcoming in my Leibniz on Time, Space, and Relativity, under consideration with OUP.
1. the problem of continuous change

It is hard to see consistency in the various things Leibniz says about continuity, especially in connection with time. Bertrand Russell did not hesitate to point this out: “In spite of the law of continuity”, he complained, “Leibniz’s philosophy may be described as a complete denial of the continuous” (Russell 1900, 111). Thus in the face of Leibniz’s many proud endorsements of that law, e.g.

“Nothing takes place suddenly, and it is one of my great and best confirmed axioms that nature never makes leaps. I call this the Law of Continuity…” (from the New Essays of 1704, A VI 6, 56), Russell was able to quote, for example,

“In fact, matter is not continuous but discrete, and actually infinitely divided, though no assignable part of space is without matter. But space, like time, is something not substantial, but ideal, and consists in possibilities, that is, in an order of co-existents that is in some way possible. … And so there are no divisions in it but those that the mind makes, and the part is posterior to the whole. In real things, on the contrary, unities are prior to multiplicity, and multiplicities exist only through unities. [The same holds for changes, which are not in fact continuous.]” (To De Volder, Oct 11, 1705; LDV 327)
1. the problem of continuous change (cont.)

It is worth dwelling on this analogy between divisions in matter and changes. Matter is divided in such a way that every part of it is divided internally. But any piece of matter is not divided in all possible ways, where the points of division would correspond with all possible points in the continuum. Rather there is an actual sequence of internal divisions that has no end. Leibniz gives the idealized example of dividing every part, and part of a part, into halves, versus dividing them into thirds:

“Thus if you bisect a straight line and then any part of it, you will set up different divisions than if you trisect it.” (“Primary Truths”, [1689], A VI 4, 1648)

So, if “the same holds of changes, which are not in fact continuous”, how does it go for the temporal case? Leibniz is clear that “as a physical body is to space, so states or the series of things are to time” (To De Volder, June 30, 1704; GP II 268/LDV 303). This would then imply that, just as “in real things, that is, bodies, the parts are not indefinite … but actually assigned in a definite way” (269/305), so the same should apply to states. The duration of any real thing should be divided into states—into a certain infinite order of states of finite length, each of them further subdivided.

As we shall see, this turns out to be Leibniz’s view.
1. the problem of continuous change (cont.)

Obviously, this is difficult to reconcile with the Law of Continuity. If every duration is divided into a certain ordering of discrete states, divided from one another by discontinuous changes, then surely we have no continuity, but a series of discontinuous leaps. But Leibniz criticized the interpretations of continuous creation proposed by Pierre Bayle and Erhard Weigel in his Théodicée (1710, §384) for making precisely this error. Similarly, he wrote to De Volder in September 1699:

“Anyone who completely rejects continuity in things will have to say that motion is essentially nothing but successive leaps through intervals flowing forth not from the nature of the thing but as a result of the action of God, that is to say, reproductions in separate places, and would philosophize almost as if one were to compose matter from mere separate points.” (LDV 126/127)

“This hypothesis of leaps cannot be refuted,” Leibniz continues (126/127), except by an appeal to the “principle of order”, by which he means his Law of Continuity.

Light can be thrown on this aporia by considering Leibniz’s early dialogue, Pacidius Philalethi, written on his way to Holland in late 1676.
2. Leibniz’s analysis of change in the *Pacidius Philalethi*

In that dialogue Leibniz gives an analysis of change according to which, because successive states are mutually contradictory,

“There is no moment of change common to each of two states, and thus no state of change either, but only an aggregate of two states, old and new” (A VI 3, 566).

In support of this he gives the analogy with the infinite dividedness of body:

“(NB. Just as bodies in space form an unbroken connection, and other smaller bodies are interposed inside them in their turn, so that there is no place void of bodies; so in time, while some things last through a momentaneous leap, others meanwhile undergo more subtle changes at some intermediate time, and others between them in their turn. … At any rate, it is necessary for states to endure for some time or be void of changes. As the endpoints of bodies, or points of contact, so the changes of states. … Nor is any time or place empty. During any state whatsoever some other things are changing.)” (A VI 3, 559).

This concise note to himself in the margins of a first draft contains most of the main ingredients of Leibniz’s analysis of change.
2. the *Pacidius* analysis of change (cont.)

In this theory the only moments that are assigned are the endpoints—beginnings or ends—of finite temporal intervals, no more than two of which are ever next to one another. Thus concerning the spatial continuum, the interlocutors conclude that

“the continuum can neither be dissolved into points nor composed of them, and that there is no fixed and determinate number (either finite or infinite) of points assignable in it.” (A VI 3, 555).

and the same applies to the temporal continuum. Moreover, because the only moments are endpoints of intervals, and no two intervals have an endpoint in common, there is no moment of change, or state of change. In sum,

“at any moment that is actually assigned we will say that a moving thing is at a new point. And although the moments and points that are assigned are indeed infinite, there are never more than two of them immediately next to each other in the same line ... This does not mean, however, either that a body or space is divided into points or time into moments, because indivisibles are not parts but extrema of parts. And this is why, even though all things are subdivided, they are still not resolved all the way down into minima.” (A VI 3, 565-6).
2. the *Pacidius* analysis of change (cont.)

Leibniz is proud of this analysis of change, and continues to uphold it after 1676:

“Change is an aggregate of two opposite states in one stretch of time, with no moment of change existing, as I have demonstrated in a certain dialogue.” ([Spring-Summer 1679] A VI 4, 307)

“Change is an aggregate of two contradictory states. These states, however, are understood to be necessarily immediately next to one another, since there is no third thing between contradictories.” ([1683-85] A VI 4, 556).

And in his second last letter to De Volder of 10/11/1705, Leibniz writes:

“Endpoints of a line and unities of matter do not coincide. Three continuous points in the same straight line cannot be conceived. But two are conceived: the endpoint of one straight line [segment] and the endpoint of a second, out of which the same whole is constituted. In the same way in time there are two instants, the last instant of life and the first of death.” (GP II 278/LDV 327)

This analysis is as original as it is difficult to comprehend. Several points are in order:
(1) regarding *the discreteness of matter and change*: according to Leibniz the parts of matter are actually divided into determinate parts. This is what he means by calling the parts “discrete”—not that they are atomic or indivisible. For their being always further divided precisely precludes their being indivisible. The same goes for states: each state is determinate, having a beginning and an end, and is subdivided into determinate or discrete substates, each of which also has its own boundaries.

(2) regarding *actually infinite division*: taking Leibniz’s idealized model of infinite division, we may wonder, how does a repeated bisection of each subinterval not dissolve the continuum into an actual infinity of points or instants? The answer depends on Leibniz’s construal of the actual infinite as *syncategorematic*. This is a conception according to which, however many subdivisions one may suppose there to be, there are in fact more; but there is no number of all subdivisions. As Leibniz urged Bernoulli to acknowledge,
2. the *Pacidius* analysis of change (cont.)

“Even if I concede that there is no portion of matter that is not actually cut, one does not for this reason come to uncuttable elements or minimum portions, nor indeed to the infinitely small, but only to [portions] perpetually smaller, and yet ordinary ones; similarly to how there arise perpetually larger ones in increasing.” (19 July, 1698; GM III 524).

In the same way, each state is further divided into substates in a determinate way by the changes within it.

(3) regarding density: Changes are therefore dense within any state, since no state is so small that it is not further divided into substates by changes within. Note, however, that instants (as endpoints) are not dense. For a change is defined as the aggregate of two contradictory states, one immediately next to the other, and instants as their endpoints, so that there are no further instants between the two instants bounding such contiguous states, the end of one and the beginning of the other. Even so, such pairs of contiguous instants do not exhaust the continuum, since they are always separated by the states they bound, which have finite but arbitrarily small duration.
A duration is therefore an infinite aggregate of states, each with its own distinct endpoints. Likewise, even if every state is further divided into substates, this does not suffice for true continuity. In fact, as we shall see below, well in advance of everyone else, Leibniz distinguishes density from continuity.

Given all this, I argue, one can discern a distinct and original mereological theory of change in Leibniz, where continuous duration consists in a series of states, separated by actual changes, governed by the law of the series.

But first we must confront two outstanding difficulties.

The first is that there is ambiguity if not contradiction in Leibniz’s claims about states. Guided by the analogy with the actual division of bodies, we have depicted them as divided by changes occurring within them to infinity; yet we have seen him claim that “it is necessary for states to endure for some time or be void of changes”. In the *Pacidius* Leibniz gestures at a solution to this discrepancy, comparing the leaps to the infinitesimals of his recently completed calculus. In a passage from the first draft that he subsequently deleted, he wrote:
3. enduring states

“The spaces through which these leaps occur are smaller than can be explicated by their ratio to magnitudes known by us. And these kinds of spaces are taken in geometry to be points or null spaces, so that motion, although metaphysically interrupted by rests, will be geometrically continuous—just as a regular polygon of infinitely many sides cannot be taken metaphysically for a circle, even though it is taken for a circle in geometry, on account of the error being smaller than can be expressed by us.” (A VI 3, 569)

The idea is that even though the extended spaces through which the leaps occur are always finite, and take a finite time, such spaces and times are so small as to be “unassignable”. It is in this way that an infinite polygon can be taken for a circle. Thus even if motion is “metaphysically” interrupted by unassignable leaps (like the unassignable differences of his calculus), it will still be “geometrically continuous”. We will come back to this analogy with the infinite polygon below, as well as this justification of the vanishing difference between how things change discontinuously in reality and a true geometrical continuity (which for Leibniz is purely ideal).
3. enduring states (cont.)

But, secondly, from these premises Leibniz draws a startling conclusion: bodies do not act, and therefore do not even exist, between changes of state.

For, he argues, given that “there is no moment of change common to each of two states, and thus no state of change either”, then “if it is supposed that things do not exist unless they act, and do not act unless they change, the conclusion will follow that things exist only for a moment and do not exist at any intermediate time” (A VI 3, 557). “Hence [he concludes] it follows that proper and momentaneous actions belong to those things which by acting do not change.” (566)

Thus bodies exist at every assignable moment, but they do not exist at the unassignable times between these moments. There is therefore no temporal continuant to which the changes or actions can be ascribed.

In the dialogue, Leibniz uses this consequence to prove the necessity of “a superior cause which by acting does not change, which we call God” (567), “whose special operation is necessary for change among things” (568-9).
3. enduring states (cont.)

So here, after all his innovations, he has arrived at a version of continuous creation not unlike those he will later criticize in the *Theodicy*. As such it seems vulnerable to the same criticisms. There are no temporal continuants, just isolated instances of God’s creating bodies here, there and everywhere, but only for an instant at a time.

But Leibniz’s use of the plural in referring to “those things which by acting do not change” hints at a different solution. This is in fact the view he wants to hold, as he believes that bodies contain their own individual principles of activity. He continues to develop this view in earnest when he arrives in Hanover in 1677.

In what appears to be a draft of his intended a draft of a book on physics written in 1678, Leibniz writes:

“certainly if we consider matter alone, … no moment will be assignable at which a body will remain identical with itself, and there will never be a reason for saying that a body … is the same for longer than a moment.” (A VI 4, 1399)
3. enduring states (cont.)

But if a body cannot retain its identity through time, Leibniz urges, it must contain a substantial form as its “principle of unity and of duration” (1399). Such a form is modelled on the human Ego, which retains its form while having a succession of different perceptions. In a passage from 1683-85 discussing change, Leibniz writes:

“The only difference that occurs when everything else remains the same, and makes there be no contradiction of any kind when the same things are said to be both contiguous and separate, is the difference of time. But whether those things are really the same that we think to be so is a matter for a more profound discussion. It is enough that there are some things that remain the same while they change, such as the Ego.” (A VI 4, 562/LoC 267)

Here we see the Ego clearly identified as Leibniz’s model for “those things which by acting do not change” (A VI 3, 566/LoC 211). We shall have to say more about how this is supposed to constitute a solution to the problem of the continuum below.

But what about the other difficulty concerning the enduring states that bodies are supposed to have between changes?
3. enduring states (cont.)

In a piece dating from April-October 1686, *(Dans les corps il n’y a point de figure parfaite)*, after alluding to his analysis of change in the *Pacidius* according to which a body exists only at the assignable moment it changes state, Leibniz notes that this analysis still presupposes enduring states between the changes. He then makes the intriguing suggestion that all such enduring states must be understood to be “vague”:

“Now I believe that what exists only at a moment has no existence, since it begins and ends at the same time. I have proved elsewhere that there is no middle moment, or moment of change, but only the last moment of the preceding state and the first moment of the following state. But that supposes an enduring state. Now all enduring states are vague, and there is nothing precise about them. For example, one can say that a body will not leave some such place greater than itself during a certain time, but there is no place where the body endures that is precise and equal to it. One can thus conclude that there is no moving body of a definite shape....” *(A VI 4, 1613-14/LoC 297; my emphasis)*
Given his identification of a substance’s states with perceptions, this means that perceptions are also presupposed as enduring, rather than strictly instantaneous. This is confirmed in the continuation of the previous passage about change: “But if someone contended that not even I endure beyond a moment, he would not know whether he himself existed. For this he could know in no other way than by experiencing and perceiving himself. But every perception requires time…” (A VI 4, 562, my italics).

Now the crucial feature of the soul and its perceptions that is not possessed by a body (as conceived by the Cartesians) is memory. It is this that links the perceptions together and forges the self-identity and persistence of substances through their changes. As Leibniz wrote in another fragment from 1685:

“Certainly those things which lack forms … do not persevere the same for longer than one moment, whereas true substances persist through changes; for we experience this in ourselves, for otherwise we would not be able even to perceive ourselves, since each of our perceptions involves a memory.” (A VI 4, 627-8)
4. *petites perceptions* and the Law of Continuity

All this coheres with Leibniz’s doctrine of *petites perceptions* in the *Nouveaux Essais*:

“There is at every moment* an infinity of perceptions within us, unattended by awareness or reflection, that is to say, changes in the soul itself, of which we are unaware, because these impressions are either too small and too numerous, or too unvarying, so that they are not sufficiently distinctive on their own. … but that does not prevent them from having their effect when they are combined with others, and from making themselves felt, at least confusedly, in the aggregate.” (Preface; A VI 6, 53)

(*Here a *moment* is to be understood as an arbitrarily short duration.)

The idea is that a perception of however short a duration will still contain other perceptions in it. Even if a perception appears as one continuous state because of the limitations of sense—it is *vague*!—it is in fact infinitely divided into other smaller perceptions lasting for discrete durations. The instants that are actually assigned are the endpoints of these durations, change being the aggregate of its existence in one state at one instant and its existence in a contradictory state at the next.
4. petites perceptions (cont.)

These perceptions, Leibniz says, constitute the individual’s self-identity:

“These insensible perceptions also indicate and constitute the same individual, which is characterized by the traces or expressions they preserve of the previous states of this individual, thereby connecting these with its present state; and even when this individual itself has no sense of these traces of previous states.”

(Nouveaux Essais, A VI 6, 55)

In this connection Leibniz refers to his Law of Continuity, according to which “any change always passes from the small to the large, and vice versa, through the intermediate, in respect of degrees as well as of parts” (Nouveaux Essais, A VI 6, 56).

“All of this supports the judgement that noticeable perceptions come by degrees from those that are too small to be noticed.” (A VI 6, 56-7)

Since every perception of the same individual substance preserves traces or expressions of its previous states, it follows that the whole series of states forms a continuum, with each state or perception arising by degrees from the previous ones.
I submit that this is what Leibniz refers to on occasion as the “physical continuum”. Such a continuum is not the ideal continuum of mathematics, since it is constituted by the states or perceptions, which are divided from one another by actual changes of state, as opposed to the instants of the ideal continuum, which mark positions of possible changes. The consecutive states themselves are touching, so that they form a contiguum, rather than a true continuum.

Leibniz makes precisely this point in a fragment (from 1705?) recently discovered and transcribed by Osvaldo Ottaviani, *Locus et tempus sunt continua*. It begins:

“Place and time are continua, matter and change are contigua. A continuum does not have actual parts, except those that are assigned by an actual division. Nor does a line consist of points, or time of instants, even though there is no part of a line in which there is not an actual point, nor any part of time in which there is not an actual instant. A continuum, such as a line and time, are [sic] ideal things like numbers; and in fact they are orders of possibles in which actuals are designated.” (LH 37, 5, Bl. 134r)
5. the physical continuum (cont.)

Leibniz then gives his customary comparison with number. The parts into which a continuous line or time can be divided are potential, like the parts or fractions into which we can divide an integer, “or even the simplest integer, which is unity”. And in fact, “to every section of a line into parts there corresponds proportionally a section of unity into parts.” In contrast,

“in a spatial or ideal line infinitely many actual points can be assigned, actually distinct from one another, and endowed with different motions, which do not compose the line. And these points will be extremities of the parts into which the line is actually divided by the variety of motions in matter” (LH 37, 5, Bl. 134r)

Thus between any two such actual points there will always be another:

“In this way if an actual simultaneous division proceeds to infinity, … still, a continuous line will never therefore be resolved into points, nor into any finite division, nor will points compose a continuous line. … The same holds for time, the actual instants of which contain just as many fulgurations of the divinity, or states of the universe.” (LH 37, 5, Bl. 134r)
At this point Leibniz gives the same comparison with perpetual bi- and tri-partitions that he had used in “Primary Truths”. He concludes:

“Hence it could be understood that what is actual of place and of time is composed of points or instants or is an aggregate of them; but not place itself and time itself, which are continuous and potential things. That is, just as place could be divided in a different way, as by tripartitions, so also could time; which are no more the aggregates of points or instants, or of the lines into which they can be resolved, than number is the aggregate of the fractions into which it can be broken down.” (LH 37, 5, Bl. 134r)

Thus what is actual of place and time is constituted by a dense, infinite progression of actual points or instants. This, Leibniz says, is “equivalent to a continuum”:

“that must be said to be equivalent to a continuum [in] which there are no two actually assignable points between which another cannot be actually located.” (LH 37, 5, Bl. 134r)
5. the physical continuum (cont.)

But, Leibniz immediately clarifies,

“This is physical continuity, not geometrical; geometric points are endpoints of the continuum, not elements of it; but physical points are the elements of mass. Both are indivisibles. But mass is not something continuous, but a conjunction; for there is no separation of its parts, or no interval can be assigned between two of its parts in which there is not some part of it.” (LH 37, 5, Bl. 134v)

In his October 1705 letter to De Volder, Leibniz also writes of the “unities of matter” as not touching, but explains how their denseness is related to the “order of changes”:

“One unity is not touched by another, but there is a perpetual transcreation in motion. This is, namely, that when a thing is in such a state that it by continuing its changes through an assignable time there would have to be a penetration at the next time afterwards, each and every point would be in another place, as the avoidance of penetration and the order of changes demands”. (GP II 278)

This mention of “transcreation” (or “transproduction”) refers us back to Leibniz’s Paris notes. Leibniz introduces it in “Infinite Numbers” of c. 10 April, 1676:
5. the physical continuum (cont.)

“in transproduction, even though everything is new, still, by the very fact that this transproduction happens by a certain law, continuous motion is imitated in a way, just as polygons imitate the circle.” (LH 37, 5, Bl. 134v)

This evokes the distinction between the infinite polygon and a true geometric circle we met in the Pacidius. The curved trajectory of any body in actuality will consist in a physical continuum of arbitrarily small rectilinear, uniform motions that are divided in turn by others without limit. This can still be “taken for a circle in geometry, on account of the error being smaller than can be expressed by us” (A VI 3, 569).

As Leibniz explains in “Infinite Numbers”, it is the imagination that fills in the gaps between the points or instants, giving rise to the illusion of a perfect uniformity and true continuity:

“in the mind there is thought of uniformity, yet no image of a perfect circle: instead we apply uniformity to the image afterwards, a uniformity we forget we have applied after sensing the irregularities.” (A VI 3, 499/ LLC 91)
5. the physical continuum (cont.)

There is therefore a very strong correlation between the fictional nature of infinitesimals and the phenomenality of matter and of states. Leibniz notes this in *Locus et tempus* where he writes in the margin “That is, everything is offered to the mind as if this were so; and one monad makes up the deficit of the phenomena of the other” (LH 37, 5, Bl. 134r). In the text he explains:

“So here is a popular way of explaining the issue, suitable for those who do not grasp that material things are only phenomenal. For if the things are real, we will resolve space into a multiplicity of points, and time into a multiplicity of instants. Should we not therefore say that in fact between any two actual instants or points another must be interposed, and that we never arrive at two of them that are unassignably distant, between which nothing actual is interposed? this could be said on the subject of the continuum.” (LH 37, 5, Bl. 134v)

It should not be thought that this means that time and space are composed of instants and points. The physical points and actual instants are dense, but always separated by unassignable gaps that the imagination fills in.
Transproduction “happens by a certain law” (A VI 3, 503/ LLC 99), this being a necessary condition for actually dense states to be perceived as a continuous motion, or for a polygon of arbitrarily many sides to be depicted by the imagination as a true circle. Leibniz makes precisely this point in his *Specimen Geometriae luciferae* of around 1695:

“From these considerations the nature of continuous change can also be understood: it does not truly suffice for it that between any states that you choose an intermediate one is found, for other progressions can be thought of in which such an interpolation may be made perpetually, so that [this state of affairs] cannot be conflated with something continuous; instead, it is necessary that a continuous cause can be understood that is operating at every moment … And such changes can be understood in respect of place, species, magnitude, velocity, and indeed also of other qualities that do not belong to this consideration, like heat and light ….” (GM VII 287)
So the analogy between matter and change comes down to this. There is no assignable point in space at which there is not a unit \([unitas]\) of matter that is not actually moved with a different motion (endeavour): this is a body of arbitrary smallness, a *physical point*. Similarly, there is no assignable instant in time at which there is no actual change occurring, this being an *actual instant*.

Perceptions are always of a finite duration (even though this involves some abstraction); but because meanwhile other things are changing, and these changes must be reflected in every monadic state, these states must in fact be of vanishingly small duration, so that they are unassignable, momentaneous. The duration of any created thing must be an aggregate of such momentaneous states, produced by the changes of state at each actual instant. So we can conclude that

“the duration of things, or the multitude of momentaneous states, is the cluster of an infinity of flashes of Divinity, each of which at each instant is a creation or reproduction of all things, having no continual passage, strictly speaking, from one state to another” (to Electress Sophia, October 1705).
6. conclusion

This analysis of change, I believe, resolves the difficulties noted by Russell and other commentators about the apparent incompatibility of Leibniz’s assertions of the discreteness of the actual and the universal applicability of his Law of Continuity. Let me summarize the main theses of Leibniz’s account:

- Every duration is divided into a series of successive, contiguous states.
- These successive states are mutually contradictory.
- Change is the aggregate of two successive, contiguous states.
- Instants are the endpoints of states, so that there are never more than two of them immediately next to each other in the same time.
- All enduring states are vague. This means that no change is discernible within them, at a certain level of discrimination.
- But during any state, further changes are actually occurring, even if they are not discernible on a given level of discrimination.
6. conclusion (cont.)

- This entails that the changes are dense within any interval. But they are not continuous, since they are separated by unassignably small states.

- Any given state is actually infinitely divided by these changes within it into further states (where the infinity is understood syncategorematically).

- The states, understood as intervals between changes, are analogous to the infinitesimal differences (differentials) of Leibniz’s calculus: they are “unassignable”, momentaneous, that is, of a finite but arbitrarily short duration.

- These unassignables, however, cannot be understood as actual infinitesimals, since this would result in the paradoxes of the composition of the continuum.

- The duration of any thing is therefore divided into an actual infinity of such momentaneous states, separated by actual changes.

- Provided these states and their changes are generated by a law of progression, they constitute a physical continuum, consisting in an infinite progression of momentaneous states separated by changes.

FINIS!