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Getting what you Ordered: Symbolic and Non-Symbolic Ordinality as Predictors of Exact and Approximate Calculation in Adults

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Abstract

Performance on symbolic and non-symbolic numeric order determination tasks was examined as predictors of Woodcock Johnson calculation (exact) and computation estimation (approximate) scores among university adults (Median age = 21, range = 18-49 years, 61 female). For Woodcock Johnson scores, only the symbolic task variant was found to be a significant predictor of performance outcomes after entering both task variants into a multiple regression. For the computational estimation task, only the non-symbolic task variant was found to be a significant predictor of performance outcomes after entering both task variants into a multiple regression. These findings suggest that symbolic and non-symbolic number system acuity uniquely relate to different mathematical outcome measures.

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Estimation ability is an important aspect of day-to-day life. For example, estimation plays an important role in day planning (e.g. how long will it take to get to a destination?), situational judgements (e.g. how long will I have to wait in that line?) and financial decision-making (e.g. about how much will these groceries cost after tax?). Put simply, estimation is often suitable for day-to-day activities, especially when exact calculation is time consuming (e.g. determining an exact grocery bill) or indeterminable due to unknown factors (e.g. traffic) (Booth & Sielger, 2006). Given this widespread use and practical importance of estimation ability in daily life, it is important to understand the cognitive underpinnings of such skill, especially when imprecise estimation can have negative impacts for both one-self and others (e.g. over-estimating how far you can drive on a near-empty gas tank). However, little work has been done to examine the cognitive underpinnings of such skill, with most numerical research focusing on processes involved in exact calculation (Booth & Sielger, 2006). This paper looks to address this issue by investigating how number representation systems relate to approximate calculation ability.

As humans, we represent number concepts through two different cognitive systems; one non-symbolic, and the other symbolic (Ansari, 2008). The non-symbolic system is an innate cognitive system (Ansari, 2008), approximate in nature, and does not involve the use of learned number symbols for numeric representation. The symbolic system, on the other hand, is a developed cognitive system that is exact in nature, due to its use of specific learned number symbols for numeric representation (Ansari, 2008). Regarding how these two systems relate to each other, many have suggested that the non-symbolic number system, which provides a fundamental sense of numeric quantity, forms the mapping base on which the symbolic number representation system develops (Dehaene, 1997; Verguts & Fias, 2004; Nieder & Dehaene, 2009). This claim is supported by evidence suggesting a similar neural substrate for symbolic and non-symbolic processing (Piazza et al., 2007; Eger et al., 2009), along with findings that tasks indexing non-symbolic number system performance are related to mathematical performance (Halberda et al., 2008; Mundy and Gilmore, 2009; Gilmore et al., 2010; Piazza et al., 2010; Lyons and Beilock, 2011; Wagner and Johnson, 2011).

Recent findings, however, suggest that these claims of a link between the symbolic and non-symbolic number representation systems have been made without consideration of how these systems differ when task requirements demand the determination of numeric ordinality (Lyons & Beilock, 2013). Here, it is suggested that there are two aspects of number representation; one being a sense of quantity, and the other being a sense of relative order (Lyons & Beilock, 2009), with previous research focusing almost exclusively on the former. In these numeric “ordinality” tasks (i.e., Are these in order?), differences in performance on symbolic and non-symbolic versions of the tasks have been consistently demonstrated with regards to the distance effect (Lyons & Beilock, 2009, 2013, 2014; Newton, Waring, & Penner-Wilger, 2014). In tasks of numeric magnitude comparison (i.e. “Which is more: 2 or 3?”), the distance effect is the robust finding that participants are slower to correctly respond when the distance between the stimuli is small (e.g. 5-6) than when the distance is large (e.g. 3-8). For numeric ordinality tasks, this standard distance effect holds true for non-symbolic trials, but for symbolic trials, this distance effect is reversed (i.e., determining 1,2,3 is in order is easier than determining 3, 5, 7 is in order). These findings have led some to suggest that the symbolic and non-symbolic number systems are not as closely tied to each other as previously (Lyons & Beilock, 2013; Newton et al., 2014). Indeed, recent neural research has suggested that there is a qualitatively different coding of symbolic and non-symbolic numbers in the brain (Lyons, Ansari & Beilock, 2015). Essentially, it is suggested that although both systems employ the parietal cortex to code for number (the similar neural substrate argument mentioned previously), symbolic numbers are represented in a discrete manner (little to no overlap between number concepts), whereas non-symbolic numbers are represented in an analogue manner (with numeric overlap increasing with number size) (Lyons et al., 2015). Overall, these findings suggest that the symbolic number system is in fact different from the non-symbolic system and does not develop by mapping onto the non-symbolic system.

Existing research on these number representation systems has also looked at how each system (symbolic and non-symbolic) relates to performance on measures of exact calculation, which has yielded some mixed results. For example, Bugden et al. (2012) found that stronger performance on a symbolic magnitude comparison task was related to stronger performance on a measure of math fluency. This finding was echoed by that of Vogel, Remark, and Ansari (2014), who also found that performance on a symbolic magnitude comparison task was related to performance on two standardized arithmetic achievement tests. Halberda, Mazzocco and Feigenson (2008), on the other hand, examined how the non-symbolic number system acuity of 14-year old children related to their past math performance. In this non-symbolic measure, participants saw a screen with both blue and yellow dots, and had to indicate which colour (blue or yellow) was more numerous (Halberda et. al., 2008). They found that non-symbolic number system acuity was significantly correlated with previous measures of math achievement, even when controlling for other cognitive measures like IQ, visual-spatial reasoning and working memory (Halberda et. al., 2008). Halberda et. al. (2008) took these findings to suggest that non-symbolic number system acuity is related to exact calculation ability. Another study also examined the relationship between non-symbolic number system acuity and mathematic performance, only this time with younger children (Libertus, Feigneson & Halberda, 2011). Libertus et. al. (2011) tested 3- to 5-year old children using the same non-symbolic magnitude comparison task as Halberda et. al. (2008). They found that even in preschool aged children, non-symbolic number system acuity correlated with math ability, despite the relative lack of formal mathematics education of this population, suggesting a link between non-symbolic representation acuity and math ability that starts in early life.

One issue with the above studies, however, is that none examined the relation between both symbolic and non-symbolic number system acuity and exact calculation ability. Of note, when the impact of both number representation systems on exact calculation is examined, it has been found that only the symbolic system is predictive of exact calculation ability (Holloway & Ansari, 2009; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Kolkman, Kroesbergen, & Leseman, 2013; Newton, Waring & Penner-Wilger, 2014) . Holloway and Ansari (2009), for example, looked at whether or not performance on symbolic and non-symbolic magnitude comparison tasks was related to individual differences on standardized math scores in children aged 6-8. It was found that only performance on the symbolic magnitude comparison task was related to arithmetic scores. Similar results were also observed by Newton, Waring and Penner-Wilger (2014) in university aged adults. For that study, Newton et. al. (2014), used measures of symbolic and non-symbolic magnitude comparison and order determination (i.e. are these numbers in numeric order) tasks to create factors indexing symbolic and non-symbolic number system performance. It was found that of these two factors, only the symbolic factor was predictive of a measure of arithmetic fluency. Both of these studies demonstrate that when both number representation systems are examined, only the symbolic system remains predictive of exact calculation ability.

It is important to note, however, that although the symbolic number representation system is the primary predictor of tasks involving exact calculation, it may not be a predictor of approximate calculation tasks. Notably, past research has suggested that approximate and exact calculation processes differ on both behavioral and neural levels (Dehaene & Cohen, 1991, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene, Spelke, Stanescua, Pinel, & Tsivkin, 1999; Lui, 2013). For example, it has been found behaviourally that approximate calculation performance does not show a problem size effect – the phenomenon that as the size of the operands increase, it takes longer to solve the problem and is more likely that the answer will be wrong – a robust phenomenon in exact calculation (Lui, 2013, Zbrodoff & Logan, 2005). Despite increases in problem difficulty, RT and errors rates did not increase during approximate calculation trials, suggesting that approximation is a cognitively different process than exact calculation.

Furthermore, it has been found that the horizontal segment of the intraparietal sulcus -- believed to be involved in quantity determination -- is more active during approximate (in comparison to exact) calculation tasks (Dehaene et al., 1999). Comparably, the left angular gyrus – believed to be involved in number processing that requires additional verbal processing – has been found to be more active during exact (in comparison to approximate) calculation tasks (Dehaene et al., 1999). As such, it seems fair to posit a difference in the way number representation systems relate to tasks of exact versus approximate calculation. In particular, the approximate nature of the non-symbolic system may play a role in approximate calculation that is not accounted for by symbolic number understanding.

The present study examined the relation of both symbolic and non-symbolic number representation systems to measures of exact and approximate calculation skill. The hypotheses for this study are as follows: 1) Only symbolic system performance – measured using a symbolic order determination task -- will predict performance in the exact calculation measure (consistent with previous findings), and 2) Only non-symbolic system performance – measured using a non-symbolic order determination task -- will predict performance in the approximate calculation measure. Examining the relative predictive power of symbolic and non-symbolic ordinality for approximate calculation represents a novel contribution to the field of numerical cognition.

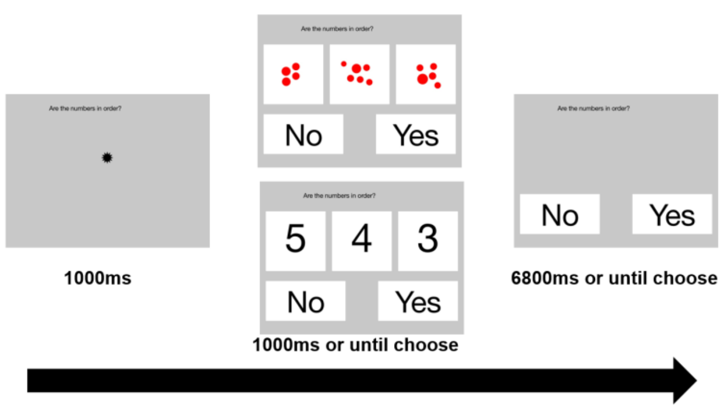
Method

Participants

The participants for the study consisted of 85 undergraduate students (Median age = 21 years, range = 18-49 years, 61 female) from King's University College and Western University. All participants completed their elementary and secondary education in Canada. Participation for this study was either on a voluntary basis or for course credit.

Materials

Figure 1. Example of a symbolic and a non-symbolic trial of the numeric order determination task



Numeric order determination task (see Figure 1). Participants were presented with three single digit numbers (ranging from 1 to 9) on an iPad screen, and were asked to choose if the sequence of numbers were in order (ascending or descending) or were not in order. Problems appeared in both symbolic (Arabic digits) and non-symbolic (dots) formats. For each format, participants completed 72 randomized trials. The same triplets, shown in Appendix A, were used for both the symbolic and non-symbolic task (i.e. If the sequence 1 3 5 appeared in the symbolic format, that same sequence would also appear in the non-symbolic format. Within each format, each triplet occurred in ascending, descending or mixed (i.e. not in order) orders. For each trial, stimuli remained on the screen for 1000ms or until the participant made a choice. After 1000ms, if a choice had not been made, the stimuli would disappear from the screen, and the participants were given up to an additional 6800ms to make a choice. The inter-stimulus interval for each trial was 1000ms. Before each block, participants were given 10 practice trials to familiarize themselves with the task. The dependent measure for both order tasks was a composite variable that incorporated RT and error rate data. Composite variables were created using the formula: $P = RT(1 + 2ER)$, previously used in Lyons et al. (2014), where RT refers to response time on correct trials and ER refers to error rate. Higher *P* scores indicate poorer performance.

Approximate calculation measure. Participants completed a computational estimation task (Appendix B: previously used by Hanson & Hogan, 2000) on an iPad. In this task, participants were presented with a series of 20 mathematical questions (including whole number and decimal addition, subtraction, multiplication and division questions, percentage questions, and fraction addition, subtraction, and multiplication questions), and were explicitly told that they would not have enough time to solve for the correct answer and would have to estimate. During trials, stimuli remained on the screen for 12 seconds before both the question and the answer box disappeared. If participants did not answer within 12 seconds, they did not get to enter an answer for that question. Participants were given five practice trials to familiarize themselves with the task. The dependent measure for this task was the number of points scored. Points were awarded in the same way as in Hanson and Hogan (2000), with estimates within 10% of the correct answer receiving three points, answers within 20% receiving two points, answers within 30% receiving one point, and answers either not given or not within 30% receiving zero points.

Exact calculation measure. Participants completed the Math Calculation subtest of the Woodcock-Johnson III Battery (Woodcock, McGrew & Mather, 2007). The Woodcock Calculation subtest is a paper-and-pencil task that consists of mathematical problems of increasing difficulty. Participants were asked to correctly answer as many questions as they could, and were given no time limit for completion. The dependent measure was the total number of correct answers provided.

Procedure

Participants were seated in a quiet room in front of an iPad. Once comfortable, participants completed the symbolic and non-symbolic variants of the order determination measure. Presentation order (symbolic then non-symbolic or the reverse) was counterbalanced across participants. Following the order determination tasks, participants completed the computational estimation task. Following the iPad tasks, the iPad was removed and participants completed the Math Calculation subtest of the Woodcock-Johnson III Battery. These tasks were completed in one session as part of a larger study which lasted approximately one hour and 45 minutes.

Results

Descriptive statistics and correlations

Descriptive statistics for all measures are reported in Table 1. Correlations among measures are reported in Table 2. Pearson's bivariate correlation was used to analyze the relation among the exact calculation measure, the approximate calculation measure, the symbolic composite measure, and the non-symbolic composite measure. There was a significant correlation between the exact calculation measure and the approximate calculation measure, $r = -.520$, $p < .001$. This correlation indicates that higher exact calculation scores were associated with higher approximate calculation scores. There was a significant correlation between the symbolic composite measure and the exact calculation measure, $r = -.403$, $p < .001$. This correlation indicates that lower symbolic composite scores (indicative of better performance) were associated with higher levels of exact calculation skill. There was a significant correlation between the non-symbolic composite measure and exact calculation measure, $r = -.217$, $p = .046$. This correlation indicates that lower non-symbolic composite scores (indicative of better performance) were associated with higher levels of exact calculation skill. There was a significant correlation between the symbolic composite measure and the approximate calculation measure, $r = -.385$, $p < .001$. This correlation indicates that lower symbolic composite scores (indicative of better performance) were associated with higher levels of approximate calculation skill. There was a significant correlation between the non-symbolic composite measure and exact calculation measure, $r = -.403$, $p < .001$. This correlation indicates that lower non-symbolic composite scores (indicative of better performance) were associated with higher levels of exact calculation skill.

Table 1
Descriptive Statistics (N = 85)

	Mean	SD
Approximate calculation	15.53	7.91
Exact calculation	30.08	7.97
Symbolic composite	1425	469.02
Symbolic RT (ms)	1209	358.46
Symbolic error (% error)	8.8%	6.5%
Non-symbolic composite	1895	559.24
Non-symbolic RT (ms)	1449	306.37
Non-symbolic error (% error)	15.2%	11.7%

Table 2
Correlations among measures (N = 85)

	1	2	3	4
1. Approximate Calculation				
2, Exact Calculation	.520**			
3. Symbolic Composite	-.385**	-.403**		
4. Non-symbolic Composite	-.403**	-.217*	.577**	

*p < .05, **p<.01

Predicting Exact Calculation

Data were analyzed using multiple regression to determine whether performance on symbolic and non-symbolic order determination trials predicted exact calculation skill, and both the symbolic and non-symbolic composite scores were entered into a single step. The results of the multiple regression indicated that the two predictors explained 16% of the variance in the exact calculation measure, $R^2 = .16$, $F(2, 82) = 8.00$, p =.001.It was found that symbolic composite scores were a significant predictor of exact calculation scores, $\beta = -.42$, $t = -3.38$, p = .001 (see Figure 2). The non-symbolic composite was not found to be a significant predictor of calculation fluency, $\beta = .02$, $t = .19$, $p = .846$. These results support the hypothesis that only symbolic representation system acuity predicts exact calculation skill.

Predicting Approximate Calculation

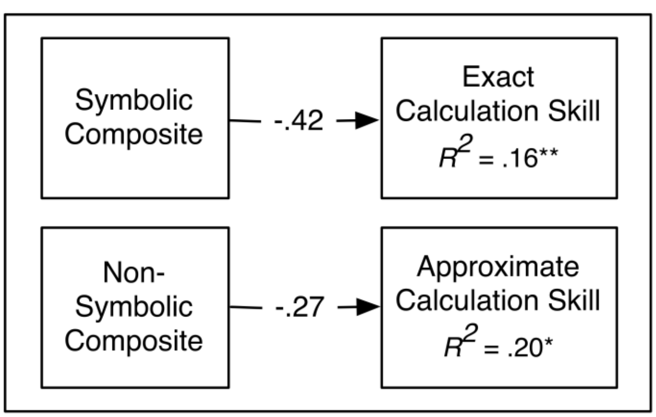
Data were analyzed using multiple regression to determine whether performance on symbolic and non-symbolic order determination trials predicted approximate calculation skill, and both the symbolic and non-symbolic composite scores were entered into a single step. The results of the multiple regression indicated that the two predictors explained 20% of the variance in the exact calculation measure, $R^2 = .20$, $F(2, 82) = 10.05$, $p < .001$.It was found that non-symbolic composite scores were a significant predictor of exact calculation scores, $\beta = -.27$, $t = -2.24$, $p = .028$ (see Figure 2). The symbolic composite was not found to be a significant predictor of calculation fluency, $\beta = -.23$, $t = -1.88$, $p = .063$. These results support the hypothesis that only non-symbolic representation system acuity predicts approximate calculation skill.

Table 3
Regression Analyses Predicting Exact and Approximate Calculation Skill From Measures of Symbolic and Non-Symbolic Number Representation System Acuity

	Calculation skill measure	
	Exact Skill	Approximate Skill
Predictor	β	β
Symbolic composite	-.42**	-.23
Non-symbolic composite	.02	-.27*
Total R^2	.16	.20
n	85	85

*$p < .05$, **$p<.01$

Figure 2. Separate regression models predicting exact and approximate calculation skill. Standardized regression coefficients shown. * indicates $p < .05$, ** indicates $p < .01$



Discussion

The present study looked to examine how both symbolic and non-symbolic number system acuity relate to measures of exact and approximate calculation. Past research has yielded a variety of findings with regards to how symbolic and non-symbolic number representation systems relate to exact calculation ability. It has been shown that both non-symbolic system acuity (Halberda et. al., 2008; Libertus, Feigneson & Halberda, 2011) and symbolic system acuity (Bugden et al., 2012; Vogel, Remark, & Ansari, 2014) individually relate to exact calculation. Furthermore, it has recently been shown that when the impact of both representation systems are examined in relation to exact calculation skill, only symbolic system acuity remains a predictor (Holloway & Ansari, 2009; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Kolkman, Kroesbergen, & Leseman, 2013; Newton, Waring & Penner-Wilger, 2014). Consistent with previous findings, the present study found that both symbolic and non-symbolic measures were correlated with a measure of exact calculation. Also consistent with previous findings, the present study found that when both symbolic and non-symbolic measures were entered into a single step regression predicting exact calculation, only the symbolic measure remained a significant predictor. Overall, these findings, along with past research, provide convincing evidence that only the symbolic (and not the non-symbolic) number representation system is predictive of performance on measures of exact calculation.

The novel contribution of this study is its examination of how both symbolic and non-symbolic number representation systems relate to a measure of approximate calculation. Similar to exact calculation findings, both symbolic and non-symbolic number system acuity were correlated with a measure of approximate calculation. However, after entering both symbolic and non-symbolic measures into a single step regression predicting approximate calculation, only the non-symbolic measure remained a significant predictor. This finding provides the first evidence that only non-symbolic (and not symbolic) number system acuity is predictive of performance on a measure of approximate calculation.

Each of the above findings fit well within the current understanding of how both (1) symbolic and non-symbolic systems differ and (2) exact and approximate calculation differ. For example, it has been suggested that at the neural level, symbolic and non-symbolic number representation systems represent numerosity in two different ways; one in a more digital manner (symbolic) and the other in a more analogue manner (non-symbolic) (Lyons, Ansari & Beilock, 2015). This suggests that the symbolic system may be better suited for determining exact numeric answers, whereas the non-symbolic system may be better suited for giving approximations. Furthermore, given that exact and approximate calculation has been found to differ on both behavioral and neural levels (Dehaene & Cohen, 1991, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Dehaene, Spelke, Stanescua, Pinel, & Tsivkin, 1999; Lui, 2013), it is not so hard to conceive that one representation system underlies one type of calculation process, while the other underlies another type of calculation process (with neither representation system underlying both calculation processes). As such, it is suggested here that (1) there is a fundamental aspect of exact calculation that is accounted for by an exact (symbolic) understanding of numerosity, but not by by approximate (non-symbolic) number sense and (2) there is a fundamental aspect of the approximate calculation process that is accounted for by an approximate number sense, but not by a symbolic (exact) understanding of numerosity.

One limitation of the present study is that while the approximate calculation measure was designed to limit efforts to exactly calculate correct answers, it is possible that some participants could have exactly calculated some answers in the time allotted. Another limitation of this measure was that by requiring participants to manually enter in their estimates, they may have felt required to calculate a more exact answer (despite clear instructions to provide an estimate). In future studies, we look to reduce such confounds by adding another estimation measure in which participants are forced to choose from a set of possible answers, as such a task would allow us to more easily limit the answer time frame (preventing exact calculation) while possibly being less demanding on participants.

In summary, the present study adds to current research suggesting that only symbolic number representation system acuity is predictive of exact calculation performance, while also presenting the novel finding that only non-symbolic number system acuity is predictive of approximate calculation performance. The next logical step for this research is training studies (which we are currently designing) to assess whether or not training individual’s non-symbolic number representation system can improve their approximate calculation ability in absence of general math training.

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Appendix A

List of numeric order determination task stimuli triplets (36 total, each repeated twice for a total of 72 trials

(1, 2, 3), (3, 2, 1), (1, 3, 2), (3, 1, 2), (2, 3, 4), (4, 3, 2), (2, 4, 3), (4, 2, 3), (1, 3, 5), (5, 3, 1), (1, 5, 3), (5, 1, 3), (3, 5, 7), (7, 5, 3), (3, 7, 5), (7, 3, 5), (3, 4, 5), (5, 4, 3), (3, 5, 4), (5, 3, 4), (2, 4, 6), (6, 4, 2), (2, 6, 4), (6, 2, 4), (4, 6, 8), (8, 6, 4), (4, 8, 6), (8, 4, 6), (4, 5, 6), (6, 5, 4), (4, 6, 5), (6, 4, 5), (5, 7, 9), (9, 7, 5), (5, 9, 7), (9, 5, 7)

Appendix B

Computational Estimation Skill (Hanson & Hogan, 2000) question list

1. 97 ´ 26
2. 5869 + 642 + 8190 + 16 =
3. 58.4 ´ .13
4. 58,795 - 2,989
5. 6123 ¸ 54
6. 16 is what percentage of 38?
7. $\frac{12}{19}+\frac{2}{17}$
8. 123 ´ 48
9. 443.7 ¸ 19.1
10. 7 is what percentage of 65?
11. $\frac{9}{32}+\frac{4}{15}$
12. 369 ´ .89
13. $\frac{9}{11}-\frac{12}{26}$
14. 382 + 294 + 316 + 270
15. $\frac{20}{21}-\frac{5}{19}$
16. 8402 ¸ 41
17. 104 is 21% of __
18. 64 ´ $\frac{14}{17}$
19. .69 ¸ .91
20. 48% of 41 = __