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A Fuzzy Set Theory Based Methodology for Analysis of Uncertainties in Stage-Discharge Measurements and Rating Curve

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ABSTRACT

River stage and discharge records are essential for hydrological and hydraulic analyses. While stage is measured directly, discharge value is calculated from measurements of flow velocity, depth and channel cross-section dimensions. The measurements are affected by random and systematic measurement errors and other inaccuracies, such as approximation of velocity distribution and channel geometry with a finite number of measurements. Such errors lead to the uncertainty in both, the stage and the discharge values, which propagates into the rating curve established from the measurements. The relationship between stage and discharge is not strictly single valued, but takes a looped form due to unsteady flow in rivers.

In the first part of this research, we use a fuzzy set theory based methodology for consideration of different sources of uncertainty in the stage and discharge measurements and their aggregation into a combined uncertainty. The uncertainty in individual measurements of stage and discharge is represented using triangular fuzzy numbers and their spread is determined according to the ISO – 748 guidelines. The extension principle based fuzzy arithmetic is used for the aggregation of various uncertainties into overall stage discharge measurement uncertainty.

In the second part of the research we use fuzzy nonlinear regression for the analysis of the uncertainty in the single valued stage – discharge relationship. The methodology is based upon fuzzy extension principle. All input and output variables as well as the coefficients of the stage - discharge relationship are considered as fuzzy numbers. Two different criteria; the minimum spread and the least absolute deviation are used for the evaluation of output fuzziness. The results of the fuzzy regression analysis lead to a definition of lower and upper uncertainty bounds of the stage – discharge relationship and representation of discharge value as a fuzzy number.

The third part of this research considers uncertainties in a looped rating curve with an application of the Jones formula. The Jones formula is based on approximate form of unsteady flow equation, which leads to an additional uncertainty. In order to take into account of the uncertainties due to the use of approximate formula and measurement of discharge values, the parameters of the Jones formula are considered fuzzy numbers. This leads to a fuzzified form of Jones formula. Its spread is determined by a multi-objective genetic algorithm. We used a criterion to minimize the spread of the fuzzified Jones formula so that the measurements points are bounded by the lower and upper bound curves.

The study therefore considers individual sources of uncertainty from measurements to the single valued and looped rating curves. The study also shows that the fuzzy set theory provides an appropriate methodology for the analysis of the uncertainties in a non-probabilistic framework.

Keywords: discharge calculation, fuzzy arithmetic, fuzzy number, fuzzy nonlinear regression, hysteresis, Jones formula, measurement uncertainty, stage-discharge relationship, unsteady flow, uncertainty aggregation.

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CHAPTER 1

Introduction

1.1 General Background

River stage and discharge records are essential for hydrologic and hydraulic analyses. While stage is measured directly, discharge value is usually calculated using velocity area method from measurements of flow velocity, depth and channel cross-section. Several guidelines by the International Organization for Standardization (ISO-748 1997; ISO/TR-5168 1998), Environment Canada (Terzi 1981) and U.S. Geological Survey (Rantz et al. 1982) have outlined different sources of uncertainty in the measurement of discharge and stage. An extensive literature review of measurement uncertainty is available in Pelletier (1988). In general, the measurement uncertainty arises due to (i) random and systematic errors in measurement instrumentation; and (ii) approximation of velocity distribution and channel geometry with a finite number of measurements. Therefore, the measurements obtained from gauging stations should not always be readily accepted without the understanding and quantification of different sources of uncertainty that may affect them (Whalley et al. 2001).

The stage-discharge relationship or the rating curve is established from simultaneous measurements of stage and discharge values. Therefore, uncertainties in stage and discharge measurements propagate into the rating curve and affect the discharge values derived from it. Besides measurement uncertainty, the stage-discharge relationship is also affected by natural uncertainties due to (i) hysteresis of the rating curve (ii) changes in river cross sections due to erosion and sedimentation of river channel. If these uncertainties are not taken into account, rating curves will not be able to represent natural flows in the rivers and lead to errors in discharge values established from rating curves. These uncertainties can cause potentially large errors, influencing flood forecasting, annual maximum flood statistics and design and decisions to promote flood defence schemes (Parodi and Ferraris 2004; Samuels et al. 2002).

This report presents a comprehensive fuzzy set theory based methodology for the analysis of uncertainties in stage and discharge measurements and the rating curves. The report builds on the previous studies by Shrestha et al. (2007) and Pappenberger et al. (2006), who used fuzzy sets for the representation of uncertainty in the stage-discharge relationship and the analysis of flood inundation. Three companion papers describe the methodology developed in this research in detail: (i) Shrestha and Simonovic (2008a) deals with the analysis of uncertainties in the stage and discharge measurements, (ii) Shrestha and Simonovic (2008b) analyzes the uncertainties in stage-discharge relationship using fuzzy nonlinear regression and (iii) Shrestha and Simonovic (2008c) analyzes the hysteresis in stage-discharge relationship using fuzzified Jones formula.

1.2 Methods of discharge and stage measurements

1.2.1. Discharge measurement by current meter

The velocity area method based on the current meter measurements of velocity, is the widely accepted method for discharge determination (Herschy 1999; Whalley et al. 2001), which is the standard in Canada too (Terzi 1981; Pellitier 1988). In this method, flow velocity, water depth and cross section width are measured at a number of points distributed over a number of verticals covering the channel cross section (Figure 1.1). Point measurements are then aggregated over a cross section and total discharge in the cross section is determined using mid-section method.

The discharge Q measurement using mid section method can be expressed as:

$$Q = \sum_{i=2}^{n-1} \overline{v}_i \left(\frac{b_{i+1} - b_{i-1}}{2} \right) d_i \qquad (1.1)$$

where, b is the width measurement from a common reference point [m], d is the depth measurement [m], and $\overline{v}$ is the mean velocity [m/s].

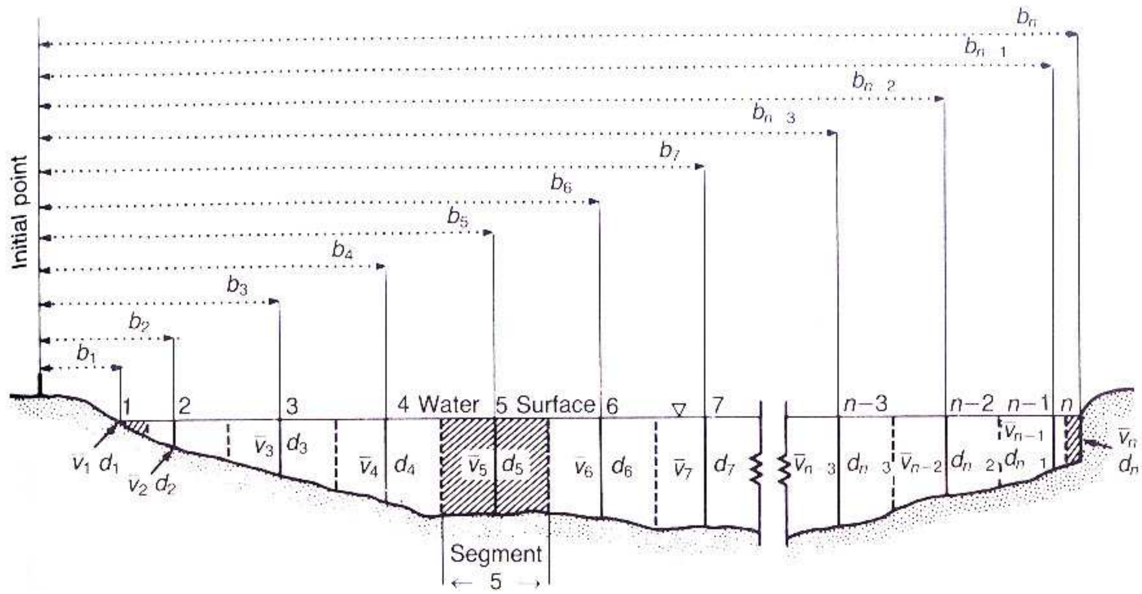


Figure 1.1. Discharge measurement using mid section method (After Herschy 1995)

According to review by Pelletier (1988) standard discharge measurement in Canada is usually performed using an individually rated Price AA current meter. A minimum of 20-25 observation verticals (for narrow streams fewer than 10) are recommended to be taken in the cross sections. The price AA current meter is calibrated on rod suspension, and on a cable suspension using a 13.6 kg Columbus type sounding weight. It is recommended that point velocity be observed for 40-80 seconds (Terzi 1981) and in practice the observation is made for 40-50 seconds (Pelletier 1988). For the estimation of mean velocity in a vertical, 0.6 depth is used for measurement where depths are less than 0.75 m and 0.2 and 0.8 depth where depths are greater than 0.75m (Terzi 1981).

1.2.2. Discharge measurement by Acoustic Doppler Current Profiler (ADCP)

Acoustic Doppler current profiler is a modern method of discharge measurement. The ADCP operates at an acoustic frequency and measures phase change caused by Doppler shift in acoustic frequency that occurs when a transmitted acoustic signal reflects off particles in the flow (Remmel 2007). The discharge measurements by ACDP can be made by either a moving boat method (Muste et al. 2004a) or the fixed boat method (Muste et al. 2004b). The first generation of the ACDP use narrow-band width, single pulse systems, while the broadband ADCP was developed in 1992 and has been increasingly used for measurements in shallower waters, such as rivers. Using broadband ADCP, velocity measurements can be obtained in waters as shallow as 1 m with relatively high spatial resolution (0.10 m) (Muste et al. 2004a).

In the broadband ADCP, the instrument transmits sound pulses at a fixed frequency in the column of water and receives returning echoes to produce successive segments, called depth cell or bin, which are processed independently. The relative velocity along acoustic beam (radial velocity) between the ADCP and particles in each depth cell is determined using frequency difference between transmitted and echoed acoustic signals using the phase difference between two superimposed echoes (Muste et al. 2004a). Velocities that are measured by the ADCP are assigned to individual depth cells constitute the center-weighted mean of velocities measured throughout the sample window (Simpson 2001).

As shown in Figure 1.2, the ADCP can only measure the central portion of total flow in the river. The areas at left, right, top and bottom areas cannot be directly measured by the instrument and is referred as the ummeasurable flows. The unmeasurable flows need to be estimated for the calculation of total discharge in the rivers.

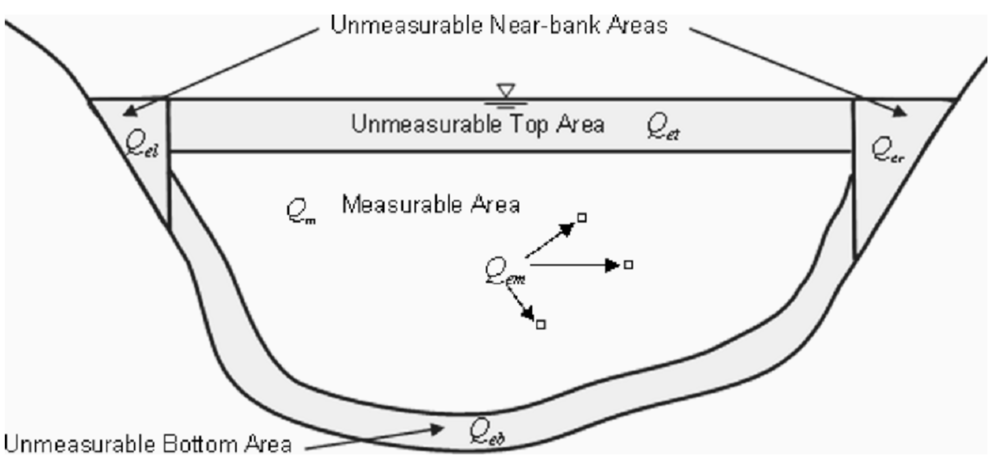


Figure 1.2. Schematic of measurable and unmeasurable areas of river cross section in ADCP discharge measurements (After González-Castro and Muste 2007)

1.3 Sources of uncertainty in discharge and stage measurements

1.3.1 Uncertainty in discharge measurement by current meter

In velocity-area method, velocities, widths and depths are measured in a finite number of verticals in a cross section. A major source of uncertainty according to the ISO-748 is in the approximation of bed profile and velocity distribution using a limited number of verticals. In general, selection of too few verticals may lead to a considerable error in discharge. ISO-748 recommends that the interval shall not be greater than 1/15 of the width in case of regular bed profiles and 1/20 of the width in case of irregular bed profiles. The ISO-748 suggested values of uncertainty for number of verticals is summarized in Table 1.1.

Table 1.1. ISO-748 suggested uncertainties for no. of verticals (at 95% confidence level)

Number of verticals	Uncertainties [%]
5	15
10	9
15	6
20	5
25	4
30	3
35	2
40	2
45	2

The velocity measurement involves three types of uncertainty: (i) number of limited points on a vertical, (ii) exposure time of velocity measurement, and (iii) current meter measurement. The first uncertainty is due to approximation of velocity distribution on a vertical using a limited number of sampling points. Common methods of determination of the mean velocity are usually based on one point, or two point methods, which involves measurement of velocity at 0.6 of the depth (0.6D), and at 0.2D and 0.8D, respectively. The ISO-748 suggested uncertainties at 95% confidence level are summarized in Table 1.2.

The second uncertainty arises due to limited exposure time of local point velocity on the vertical with an assumption of steady flow condition. An instantaneous measurement of the velocity at a point could be considerably different from mean velocity at that point. The mean flow velocity determined from measurement during finite measuring time will be therefore an approximation of true mean flow velocity at that point (Sauer and Meyer 1992). By observing the velocity for a longer time, the pulsation differences are averaged and mean velocity during exposure approaches true velocity. The ISO-748 suggested uncertainties at 95% confidence level due to exposure time are summarized in Table 1.3.

Table 1.2. ISO-748 suggested uncertainties for no. of points on a vertical (at 95% confidence level)

Method of measurement	Uncertainties [%]
Velocity distribution	1
5 points	5
2 points	7
1 point	15

Table 1.3. ISO-748 suggested uncertainties for time of exposure (at 95% confidence level)

Velocity [m/s]	Point in vertical							
	0.2D, 0.4D or 0.6D				0.8D, or 0.9D			
	Exposure time [min]							
	0.5	1	2	3	0.5	1	2	3
0.05	50	40	30	30	80	60	50	40
0.10	27	22	16	13	33	27	20	17
0.20	15	12	9	7	17	14	10	8
0.30	10	7	6	5	10	7	6	5
0.40	8	6	6	5	8	6	6	5
0.50	8	6	6	4	8	6	6	4
1.00	7	6	6	4	7	6	6	4
> 1.00	7	6	5	4	7	6	5	4

The third type of uncertainty arises in the current meter measurement of velocities, which can consist of errors due to several sources like the effect of suspension (rod or cable suspension), boundary effect (like near rough boundaries and shallow depth), effect of oblique flow, vertical motion (wave action or rocking of the boat) and effect of turbulence (Pelletier 1988). However the range of uncertainty for these sources is not available and uncertainty in the current meter measurement is usually only considered in terms of current meter rating (individual or group rating). The ISO-748 suggested uncertainties at 95% confidence level for current meter rating are summarized in Table 1.4.

Table 1.4. ISO-748 suggested uncertainties for current meter rating (at 95% confidence level)

Velocity [m/s]	Uncertainties	
	Individual rating	Group or standard rating
0.03	20.0	20
0.10	5.0	10
0.15	2.5	5
0.25	2.0	4
0.50	1.0	3
>0.50	1.0	2

Relatively small uncertainty also arises in the measurements of water depth and channel width. Under most discharge measurement conditions, measurements of the overall width and of distances between verticals can be made with reasonable precision. The ISO-748 suggested errors for width measurement is given in Table 1.5.

The instrumental error in the measurement of depth depends to a large extent on the composition of river bed. The ISO-748 suggested errors for the depth measurement is given in Table 1.6.

Table 1.5. ISO-748 suggested uncertainties for width measurements (at 95% confidence level)

Range of width [m]	Absolute errors [m]	Relative Error [%]
0 to 100	0.3	±0.3
150	0.5	±0.4
250	1.2	±0.5

Table 1.6. ISO-748 suggested uncertainties for depth measurements (at 95% confidence level)

Range of depth [m]	Absolute errors [m]	Relative error [%]	Remarks
0.4 - 6	0.04	±0.7	sounding rod
6 - 14	0.05	±0.4	log-line and air- and wet line corrections

1.3.2 Uncertainty in discharge measurement by ADCP

In most cases, ADCP discharge measurement system is dramatically faster than conventional discharge measurement systems and has comparable or better accuracy (Simpson 2001). However, the ADCP discharge measurement is also affected by a number of uncertainties, which can affect accuracy of total discharge measured by the instrument. A major source of uncertainty arises due to the unmeasurable areas at the left, right, top and bottom portions of the discharge measurement section as shown in Figure 1.2. González-Castro and Muste (2007) outlined other sources of errors in the ADCP, which include: spatial averaging of the measurement, Doppler noise, velocity ambiguity error, timing errors, side-lobe interference error, sound speed error, beam angle error, boat speed error, sample timing error, near transducer error, reference boat velocity error, depth error, cell mapping error, rotation error, edge estimation error, vertical velocity distribution error, discharge model error, finite summation error, measuring environment and operational errors. A framework for the quantification of uncertainty ranges in ADCP is still under development (WMO 2008) and suggested uncertainty values for these uncertainties are not available.

1.3.3 Uncertainty in stage measurement

The uncertainty in stage measurement depends upon the characteristics of gauging station and water surface elevation. Since the stage can be measured directly, it is reasonable to assume that errors in the measurement of stage are small compared to errors in the discharge (Clarke 1999). However, displacement of measured values from the reference point, caused by processes such as turbulent fluctuations, wind and stationary waves can lead to error in the measured stage (Schmidt 2002). Uncertainty values in different measurement instruments as suggested by Herschy (1995) are given in Table 1.7.

Table 1.7. Uncertainties for stage measurements at 95% confidence level (Herschy 1995)

Method	Uncertainty [mm]
By float operated punch tape recorder	3
By float operated autographic recorder	10
By point gauge, electrical tape gauge, tape gauge etc	1
By reference vertical or inclined gauged	3

A source of uncertainty often neglected in stage measurement is the determination of mean reference gauge height corresponding to the measured discharge. According to Rantz et al. (1982), if the change in stage is uniform or no greater than 0.05 m, the mean stage can be obtained by averaging the stage at the beginning and end of the measurement. In the case of non-uniform stage, mean stage can be obtained by weighting each stage by partial discharge. There is no suggested uncertainty range available for the determination of mean stage.

1.4 Sources of uncertainty in stage-discharge relationship

1.4.1 Propagation of measurement uncertainty

As outlined in section 1.3, each of the discharge and stage measurements consists of random and systematic uncertainties. The discharge values in particular consist of a number of uncertainties arising out of measurement uncertainties of velocity, width and depth. Therefore, the discharge values consist of aggregate of these individual uncertainties. As the rating curves are established with the measurements of discharge and stage, the measurement uncertainties propagate into the rating curve.

1.4.2 Change in river cross section

Another source of natural uncertainty in the stage-discharge relationship is due to change in river cross-section. Rivers are affected by dynamic physical processes of erosion and sedimentation. Discharge and stage measurements are made over a period of time, usually over a few years. If the change in the cross section is not taken into account, it introduces systematic error or bias in the regression data and affects the rating curve established using the regression analysis.

1.4.3 Uncertainty due to hysteresis

A major source of uncertainty in the rating curve arises due to assumption of a single-valued rating curve. In situations where a gauging station is located in sufficiently steep gradient, rate of change of discharge is low and downstream channel has sufficient capacity, the relationship between stage and discharge is sufficiently consistent with a single-valued assumption (ISO 1100-2, 1998; Rantz et al. 1982). However, assumption of the single-valued curve is not suitable if river flow is significantly affected by unsteadiness in flood wave propagation. The phenomenon may lead to a looped form of stage–discharge relationship which is commonly referred to as hysteresis. A number of factors contribute to the form of looped rating curve which include acceleration of flow in time and space, longitudinal bed slope, channel roughness and downstream boundary condition (Henderson 1966; Cunge et al. 1980; Rantz et al. 1982; Chow et al. 1988; Ponce 1989). Due to these reasons, river discharge is not just a function of stage and the assumption of single valued stage-discharge relationship becomes inconsistent.

A typical looped rating curve as shown in Figure 1.3 is characterized by peak flow always preceding peak stage and higher discharge in rising limb in comparison to falling limb of a hydrograph. The effect is due to the slope of flood wave front, which is significantly steeper on the rising limb compared to the falling limb, thus the flow is accelerating on the rise and decelerating on the fall (ISO 1100-2, 1998, USACE, 1993). The steeper slope in the rising limb allows a river channel to transit higher discharge at a particular stage compared to the falling limb. The bed slope is another important factor that affects the unsteady flow in rivers. The rating curves show more pronounced loops in rivers with flat bed slope, and greater the slope, smaller is the deviation from the single valued rating curve (Cunge et al. 1980). The channel roughness also affects shape of the loop such that higher channel roughness leads to lower peak and wider loops compared to lower channel roughness (Cunge et al. 1980).

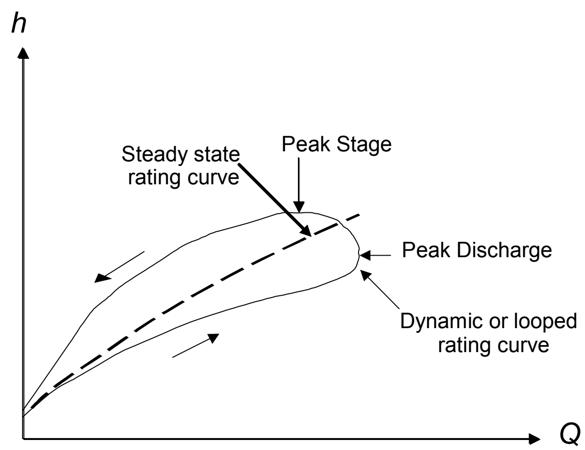


Figure 1.3. Schematic representation of steady and unsteady state rating curves

After Chow et al. (1988)

1.5 Structure of this report

Chapter 2 of this report discusses the fuzzy set theory based methodology for the uncertainty analysis. The rationale of the use of fuzzy sets for the uncertainty analysis is described. Basic concepts of fuzzy sets including fuzzy numbers, membership functions, fuzzy alpha cut, fuzzy arithmetic and fuzzy regression are introduced.

Chapter 3 of this report describes a fuzzy set theory based methodology for consideration of different sources of uncertainty in the stage and discharge measurements and their aggregation into a combined uncertainty. The uncertainty in individual measurements of stage and discharge is represented using triangular fuzzy numbers and their spread is determined according to the ISO-748 guidelines. The extension principle based fuzzy arithmetic is used for the aggregation of various uncertainties into overall stage-discharge measurement uncertainty. In addition, a fuzzified form of ISO-748 formulation is used for the calculation of combined uncertainty and comparison with the fuzzy aggregation method. This chapter also presents a methodology for the analysis of uncertainties in an Acoustic Doppler current profiler discharge measurement. The methodology is based on the representation of random uncertainties in discharge measurement at different sections in terms of fuzzy numbers and aggregation into combined fuzzy uncertainty.

Chapter 4 of this report builds on the results of chapter 3, for the analysis of uncertainty in stage-discharge relationship using fuzzy nonlinear regression. The methodology for fuzzy nonlinear regression which is based upon fuzzy extension principle is described in detail. All input and output variables as well as coefficients of the stage-discharge relationship are considered as fuzzy numbers. Two different criteria are used for the evaluation of output fuzziness: (i) the minimum spread; and (ii) the least absolute deviation criteria.

Chapter 5 of this report analyzes the uncertainties in a looped rating curve with a fuzzified form of Jones formula. A fuzzy set theory based methodology is investigated by considering the parameters of Jones formula as fuzzy numbers. The spreads of parameters of Jones formula is analyzed with a multi-objective optimization algorithm.

Chapter 6 of this report summarizes the major finding of this study and discusses dominant sources of uncertainties in the measurement and rating curve. This chapter also discusses means for the reduction of uncertainties.

CHAPTER 2

Uncertainty analysis using fuzzy set theory

2.1 Rationale for application of fuzzy sets

For the quantification of stage-discharge measurement uncertainties, the International Organization for Standardization (ISO-748) suggests the range of values at 95% confidence level for different sources of uncertainty. This recommendation is based on investigations carried out since 1968. The ISO-748 recommends independent determination of uncertainty in each measurement for the application to a particular case study. However, in most cases, independent value of confidence interval in the measurement is not available, which limits the applicability of statistical quantification of the uncertainties. It is to be noted too that randomness is not the only source of uncertainty in discharge measurements as they can be also affected by systematic uncertainty, human error and other subjective uncertainties that cannot be treated in a statistical framework. For example, the evaluation of individual current meter discharge measurement on the basis of hydrographer observation can be subjective as each measurement can receive different ratings based on the hydrographer's perception (Clemmens and Wahlin 2006). The ISO (1993) guide for expression of uncertainties has recognized these limitations by distinguishing two different categories of uncertainties according to method used to estimate their numerical values: Type A, method of evaluation of uncertainty by the statistical analysis of series of observations, and Type B, evaluation of uncertainty by means other than the statistical analysis of series of observations.

The ISO-748 also provides a statistical framework for aggregation of confidence levels of measurement uncertainties. The combined uncertainty is expressed as the ratio between the sum of percentage errors in segment discharges and the sum of segment discharges (Herschy 1995). However, such aggregation method only provides a means of combining the confidence levels and cannot provide a confidence interval of the output unless the probability distribution function that characterizes its dispersion is known (Ferrero and Salicone 2003). Therefore, there are a number of limitations in the application of the statistical methodology in the aggregation of the overall uncertainties in discharge and stage measurements.

In addition to the measurement uncertainties, the stage-discharge relationship also consists of natural uncertainties due to change in river cross sections, which can introduce bias in the regression data. The hysteresis introduces non-uniqueness in the stage-discharge relationship. The Jones formula (Jones 1916) is a popular method for reproducing hysteresis in the stage-discharge relationship. However, the modeling of hysteresis using Jones formula is affected by uncertainty due to simplifying assumptions of the formula. These uncertainties due to bias, non-uniqueness and simplification in the stage-discharge relationship cannot be directly expressed in the statistical framework using confidence intervals. Therefore, probabilistic methods of uncertainty analysis are not considered in this study.

The fuzzy set theory-based approach is explored in this study as an alternative way of analyzing various uncertainties associated with measurements and the rating curve. The fuzzy approach provides a non-probabilistic framework for representation of uncertainties using vaguely defined boundaries of fuzzy sets. El-Baroudy & Simonovic (2006) and Guyonnet et al. (2003) used fuzzy sets to treat uncertainties due to lack of knowledge and scarcity of data, respectively. In recent years, the fuzzy sets have been used for the expression of uncertainty in measurement by a number of researchers (Mauris et al. 2001; Xia et al. 2000). The study by Xia et al. (2000) considered application of fuzzy set for the estimation of uncertainty when the number of measurements is very small and the probability distribution unknown. Mauris et al. (2001) used fuzzy sets for the representation of vertical interpretation of probability distribution and nested stacks of intervals as horizontal interpretation of distribution function for representation of measurement uncertainty. The study also showed that fuzzy representation of measurement uncertainty in terms of possibility distribution is compatible with the ISO (1993) guide for expression of uncertainties, as it can characterize dispersion of observed data and provide a confidence interval that contains an important proportion of the observed values. Another approach for the consideration of measurement uncertainties uses random-fuzzy variables (Ferrero and Salicone 2003; 2004; Urbanski and Wasowski 2003) to define random properties of uncertainties in terms of probability distribution and systematic components in terms of possibility function. However, in the absence of information on the random uncertainties, purely fuzzy treatment can still be used.

The fuzzy set approach, known as fuzzy regression, can be used for addressing the non-uniqueness in the relationship between dependent and independent variables. Due to the non-unique characteristics of the stage-discharge relationship, it is more appropriate to define the upper and lower uncertainty bands around the measurement values. It is also appropriate to analyze a band of possible lower and upper values around the looped rating curve developed by Jones formula. The fuzzy regression analysis can handle such a problem by defining a band around the relationship in terms of possible upper and lower values. Following the initial work by Tanaka et al. (1982), there are numerous applications of fuzzy regression analysis in the recent years (e.g. Bárdossy et al. 1990; Lee et al. 2001; D'Urso 2003; Kao and Chyu 2003; Mousavi et al. 2007).

The study therefore considers a fuzzy set theory based methodology for the consideration of the uncertainties from the source and propagation of uncertainties in the rating curves. The rest of this chapter describes the basic principles of fuzzy sets for handling measurement and rating curve uncertainties.

2.2 Introduction to fuzzy set theory

Zadeh (1965) introduced the fuzzy set as a class of object with a continuum of grades of membership. In contrast to classical crisp sets where a set is defined by either membership or non-membership, the fuzzy approach relates to a grades of membership between [0, 1], defined in terms of the membership function of a fuzzy number. Hence, the classical notion of binary membership has been modified for the representation of uncertainty in data.

The numerical values of fuzzy numbers in a domain are assigned by membership level, which may take any value between 0 and 1, with no membership at 0 and full membership at 1. In mathematical terms, assuming X as a universe set of x values (elements), then A as a fuzzy subset of X, in ordered pairs is given by:

$$A = \{(x, \mu_A(x)); x \in X, \mu_A(x) \in [0,1]\} \tag{2.1}$$

where, $\mu_A(x)$ is the grade of membership of *x* in the fuzzy subset *A*.

A membership function can be of any shape depending on the type of a fuzzy set it belongs to. The only condition a membership function must satisfy is it should vary between 0 and 1.

2.2.1. Fuzzy Numbers

Fuzzy numbers are normal and convex fuzzy sets, whose numerical values in the domain are assigned by specific grades of membership. While Boolean operations such as union and intersection can be carried out on any fuzzy sets, the fuzzy numbers can be used to perform arithmetic operations such as addition, subtraction, multiplication and division. The commonly used fuzzy numbers are outlined below.

i. **Triangular fuzzy number.** It is based on fuzzy number $A = (a, b, c)$ with $a \leq b \leq c$. The interval (a, c) is the support of the triangular fuzzy number. This membership function is shown in Figure 2.1 and given by:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a \\ \dfrac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \dfrac{c-x}{c-b} & \text{if } b \leq x \leq c \\ 0 & \text{if } x \geq c \end{cases} \tag{2.2}$$

ii. **Trapezoidal fuzzy number.** The function is based on fuzzy number $A = (a, b, c, d)$, where $a \leq b \leq c \leq d$. The interval (a, d) is the support of the trapezoidal fuzzy number. This membership function is shown in Figure 2.2 and given by:

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a \\ \dfrac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } b \leq x \leq c \\ \dfrac{d-x}{d-c} & \text{if } c \leq x \leq d \\ 0 & \text{if } x \geq d \end{cases} \tag{2.3}$$

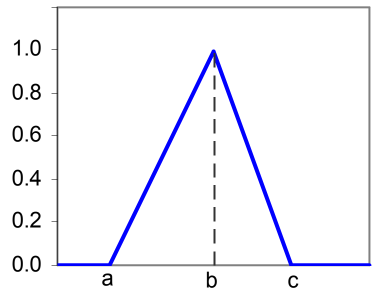


Figure 2.1. Triangular fuzzy number

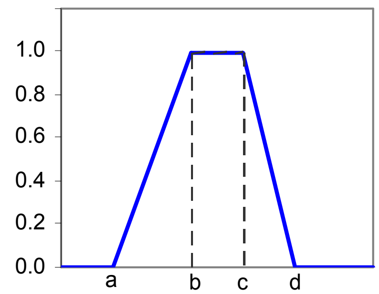


Figure 2.2. Trapezoidal fuzzy number

iii. **Left-Right (*L-R*) fuzzy number.** The linear function used in the definition of the triangular fuzzy numbers may be replaced by a monotonic function. This is called Left-Right or *L-R* representation of fuzzy numbers (Dubois and Prade 1980). For example, coefficient $\hat{A}$ is expressed as $\hat{A} = f(m, \alpha, \beta)$, where *m* is the central value and *α* and *β* are the left and right spreads respectively. The membership function $\mu_A(x)$, of the triangular *L-R* fuzzy number is given by equation 2.4 and shown in Figure 2.3 (D'Urso 2003):

$$\mu_A(x) = \begin{cases} 1 - \dfrac{m-x}{\alpha} & \text{for } m-\alpha \le x \le m,\ \alpha > 0 \\ 1 - \dfrac{x-m}{\beta} & \text{for } m \le x \le m+\beta,\ \beta > 0 \end{cases} \tag{2.4}$$

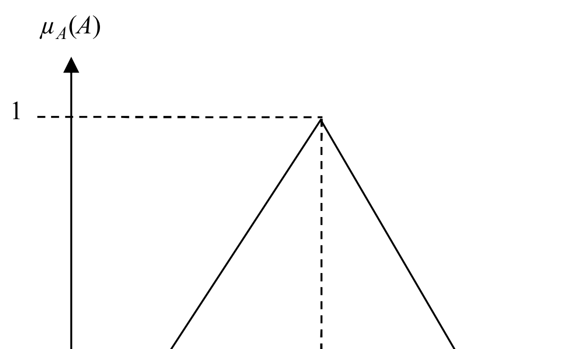


Figure 2.3. Triangular membership function of *L-R* fuzzy number

2.2.2 Alpha level cut

Fuzzy alpha-level cut (α – cut) can be used for resolving fuzzy numbers into crisp numbers, so that crisp mathematical operations such as addition, subtraction, division, square and square root can be performed (Simonovic 2008). An example of fuzzy number with α – cut and its support is shown in Figure 2.4. Let an α – cut intersect the membership function of a fuzzy number at two points a_1 and a_2 ($a_1, a_2 \in A$). Then, the subset A_α contains all possible values of the fuzzy variable A, including and between a_1 and a_2, which are referred to as the lower and upper bounds of the α – cut. The subset A_α also contains a set of elements, which have at least a membership value greater than or equal to α, as given by:

$$A_\alpha = \{a \in A, \mu_A(A) \geq \alpha\} \tag{2.5}$$

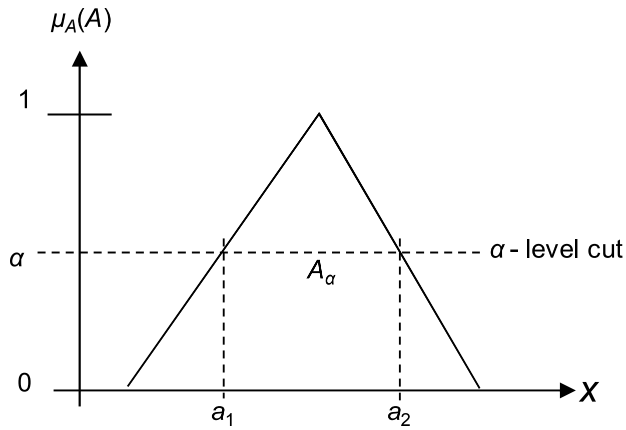


Figure 2.4. Fuzzy number and α level cut

2.2.3 Fuzzy arithmetic

The fuzzy alpha level cut based fuzzy arithmetic provides a mean to generalize crisp mathematical operations to fuzzy sets. For the fuzzy arithmetic operations, two fuzzy numbers A and B, are considered at α – level: $A_\alpha = [a_1, a_2]$, $B_\alpha = [b_1, b_2]$.

The individual arithmetic operations on the α – cut of A and B can be defined in terms of following equations (Klir 1997; Simonovic 2008):

$$[a_1, a_2] + [b_1, b_2] = [a_1 + b_1, a_2 + b_2] \tag{2.6}$$

$$[a_1, a_2] - [b_1, b_2] = [a_1 - b_2, a_2 - b_1] \tag{2.7}$$

$$[a_1, a_2] * [b_1, b_2] = [\min(a_1b_1, a_1b_2, a_2b_1, a_2b_2), \max(a_1b_1, a_1b_2, a_2b_1, a_2b_2)] \tag{2.8}$$

$$[a_1, a_2]/[b_1, b_2] = \left[\min\left(\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\right), \max\left(\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\right)\right], \text{ for } 0 \notin [b_1, b_2] \quad (2.9)$$

For the calculation of the fuzzy square root, Salicone (2007) proposed the following relation:

$$\sqrt{[a_1, a_2]} = \begin{cases} \left[\sqrt{a_1}, \sqrt{a_2}\right] & \text{when } 0 \le a_1 \le a_2 \\ \left[-\sqrt{-a_1}, \sqrt{a_2}\right] & \text{when } a_1 < 0, a_2 > 0 \\ \left[-\sqrt{-a_1}, -\sqrt{-a_2}\right] & \text{when } a_1 \le a_2 \le 0 \end{cases} \quad (2.10)$$

Similarly, for the calculation of fuzzy square, the following equation can be used:

$$[a_1, a_2]^2 = \begin{cases} \left[a_1^2, a_2^2\right] & \text{when } 0 \le a_1 \le a_2 \\ \left[-a_2^2, a_1^2\right] & \text{when } a_1 < 0, a_2 > 0 \\ \left[-a_1^2, -a_2^2\right] & \text{when } a_1 \le a_2 \le 0 \end{cases} \quad (2.11)$$

2.2.4 Fuzzy regression

The classical regression approach defines the relationship between the independent and dependent variables in terms a mathematical relationship, which can be expressed as:

$$y_i = A_0 + A_1 x_{i1} + \ldots + A_P x_{ip} \quad (2.12)$$

Where y is the dependent variable, x is the independent variable, A is the coefficient of regression, p is the number of independent variables and $i = 1, 2, \ldots n$ is the observation of each independent variable.

In real world problems, the relationships between the independent and dependent variables can rarely be expressed in terms of simple linearized equation such as (2.12). The relationships are often affected by data uncertainties and complex physical processes, which cannot be represented by simplified linear or nonlinear equations. It is more appropriate to define such relationships in terms of credible bands of lower and upper scenarios to represent the uncertainties in the data and complexities in the relationship.

Fuzzy extension principle (Zadeh, 1965) based fuzzy regression approach can handle such a problem by defining the coefficients of the relationships as fuzzy numbers, which can be expressed as:

$$y_i = \hat{A}_0 + \hat{A}_1 x_{i1} + \ldots + \hat{A}_P x_{ip} \tag{2.13}$$

The *L-R* (left–right) representation of fuzzy number provides a suitable means for representing the fuzzy coefficient $\hat{A}_j$. Due to measurement uncertainties of the independent and dependent variables, it may be necessary to define the variables and well as coefficients of the variables and fuzzy numbers, which can be expressed as:

$$\tilde{y}_i = \hat{A}_0 + \hat{A}_1 \tilde{x}_{i1} + \ldots + \hat{A}_P \tilde{x}_{ip} \tag{2.14}$$

The left and right spreads of the *L-R* fuzzy number can be extended to incorporate the uncertainty not captured in available data sets using a degree of belief, *H* (Chang and Ayyub 2001). According to this approach, each of the observed data points must be within the band around estimated regression curves at *H* level as shown in Figure 2.5. The spread of the membership function and, hence, the fuzziness of the regression variables can be controlled by specifying the *H* level between 0 and 1. Accordingly, for the degree of belief *H*, the left and right spreads of the *L-R* fuzzy number $\hat{A}$ and $\hat{B}$ can be expressed as:

$$m_j - \alpha_j(1-H) \le \hat{A}_j \le m_j + \beta_j(1-H) \tag{2.15}$$

The spread of the fuzzy regression curve also depends upon the reference point of the corresponding independent variable to which fuzzy regression analysis is performed. For example, if two fuzzy regression analyses are performed with the reference points at; (i) the minimum value of the independent variable and (ii) the maximum value of the independent variable; the spread of the regression curve will be higher around the maximum of the independent variable in case (i) compared to the case (ii). Depending upon the regression data, one or more reference points may be used, where the regression is believed to be the most accurate. The reference point should be selected where the regression is supposed to be the crispest, like around the average or the maximum value (Bárdossy et al. 1990).

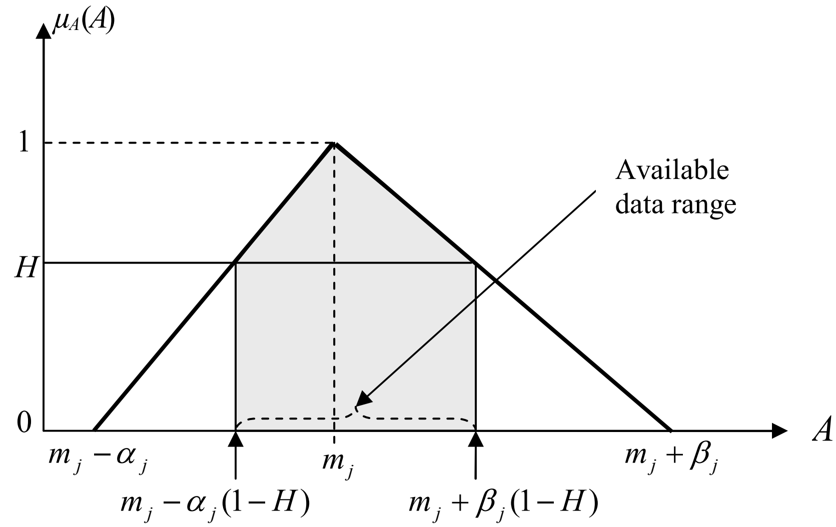


Figure 2.5. Representation of degree of belief H in L-R fuzzy number

CHAPTER 3

Analysis of uncertainties in stage-discharge measurements

The chapter presents two case studies on the analysis of stage-discharge measurement uncertainties. The first case study presents a combined methodology for uncertainty analysis of discharge and stage measurement. The methodology uses data from current meter discharge measurement and float operated stage measurement from Thompson River near Spences bridge in British Columbia, Canada. The second case study presents a methodology for uncertainty quantification in discharge measurement using Acoustic Doppler current profiler from Richelieu River in Quebec, Canada.

3.1 Methodology for uncertainty analysis

3.1.1 Analysis of current meter discharge measurement uncertainty

3.1.1.1 Aggregation of uncertainties using fuzzy arithmetic

For consideration of uncertainties in measurement of depth, width and current meter measurement of velocity, each of measurement quantities is expressed as a symmetrical triangular fuzzy number with the spread given by percentage fraction between $-x_i$ and x_i, and central value at 0 as shown in Figure 3.1.

As described in chapter 1, uncertainty in the measurement of velocity consist of three different sources: (i) uncertainty in the number of points on a vertical $\hat{X}_p$; (ii) current meter rating $\hat{X}_c$; and (iii) time of exposure $\hat{X}_e$. Each of these uncertainties is independent of each other, therefore, the total uncertainty can be considered to be less than arithmetic sum of individual uncertainties. Therefore, a method based on the ISO-748 is used, which calculates the total uncertainty as the square root of sum of squares of individual uncertainties. As the individual uncertainties are expressed in terms of percentage fraction between $-x_i$ and x_i, and central value at 0, the combined fuzzy uncertainty in the mean velocity $\hat{\bar{v}}_i$ is calculated as the sum of total uncertainties plus unity, multiplied by the crisp mean value of velocity measurement $\bar{v}_i$:

$$\hat{\bar{v}}_i = \bar{v}_i\left(1+\sqrt{\left(\hat{X}_p\right)^2+\left(\hat{X}_c\right)^2+\left(\hat{X}_e\right)^2}\right) \tag{3.1}$$

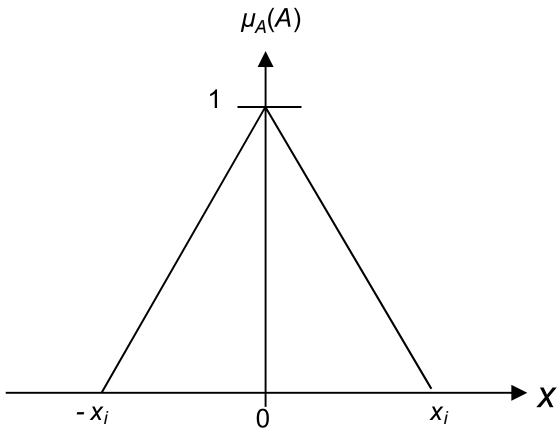


Figure 3.1. Expression of measurement uncertainty in terms of fuzzy number

Similarly, fuzzy number of the width measurement $\hat{b}_i$ and depth measurement $\hat{d}_i$ considering the measurement uncertainties is expressed as:

$$\hat{b}_i = b_i\left(1+\hat{X}_b\right) \tag{3.2}$$

$$\hat{d}_i = d_i\left(1+\hat{X}_d\right) \tag{3.3}$$

Here $\hat{X}_b$ and $\hat{X}_d$ are the fuzzy numbers of uncertainties in width b_i and depth d_i measurements, respectively.

The computation of discharge using mid section velocity area method is the standard in Canada (Pelletier 1988). For the width measurement from a common reference point, the discharge Q measurement using mid section method may be expressed as:

$$Q = \sum_{i=2}^{n-1} \bar{v}_i\left(\frac{b_{i+1}-b_{i-1}}{2}\right)d_i \tag{3.4}$$

Using the fuzzified values of velocity, depth and discharge, from equations (3.1), (3.2) and (3.3), the total discharge is calculated as a fuzzy number $\hat{Q}$:

$$\hat{Q} = \sum_{i=2}^{n-1} \hat{\bar{v}}_i \left(\frac{\hat{b}_{i+1} - \hat{b}_{i-1}}{2} \right) \hat{d}_i \tag{3.5}$$

In order to take into consideration the uncertainty due to limited number of verticals, $\hat{X}_m$, expressed as a fraction, the total uncertainty in the discharge measurement is calculated using the following relationship:

$$\hat{Q}_{tot} = \hat{Q}\left(1 + \hat{X}_m\right) \tag{3.6}$$

3.1.1.2 Aggregation of uncertainties using fuzzified ISO method

For the comparison with the above method, aggregation of uncertainties using fuzzy variables with a conventional treatment of measurement, uncertainties according to ISO-748 is used. In this case, instead of aggregation of confidence level of uncertainty, each of the uncertainties is fuzzified and aggregated using fuzzy arithmetic. According to the ISO-748 suggested formulation, a combination of confidence level of uncertainties is expressed as:

$$X_Q = \pm \left[(X_m)^2 + \left\{ \frac{\sum_{i=1}^{m} \left[(b_i d_i \bar{v}_i) \left\{ (X_b)^2 + (X_d)^2 + (X_p)^2 + (X_c)^2 + (X_e)^2 \right\}^{1/2} \right]}{\sum_{i=1}^{m} (b_i d_i \bar{v}_i)} \right\}^2 \right]^{1/2} \tag{3.7}$$

It is to be noted that the equation (3.7) is different from equation given in ISO-748. The derivation of the equations for the aggregation of uncertainties according to the ISO-748 is documented in Herschy (1995), where two different forms of equations are listed. The equation without squaring of the terms on the right hand side is used in this study as it confirms with definition of total uncertainty as a ratio of sum of percentage errors in the segment discharges to the sum of the segment discharges. Hence, the original form of equation by Herschy (1995) is used for the aggregation of uncertainties. Expressing each uncertainty in terms of fuzzy numbers leads to:

$$\hat{X}_Q = \pm\left[\left(\hat{X}_m\right)^2 + \left\{\frac{\sum_{i=1}^{m}\left[\left(b_i d_i \bar{v}_i\right)\left\{\left(\hat{X}_b\right)^2 + \left(\hat{X}_d\right)^2 + \left(\hat{X}_p\right)^2 + \left(\hat{X}_c\right)^2 + \left(\hat{X}_e\right)^2\right\}^{1/2}\right]}{\sum_{i=1}^{m}\left(b_i d_i \bar{v}_i\right)}\right\}^2\right]^{1/2} \tag{3.8}$$

The total uncertainty in discharge measurement is expressed as:

$$\hat{Q}_{tot} = Q\left(1 + \hat{X}_Q\right) \tag{3.9}$$

3.1.2 Analysis of Acoustic Doppler current profiler discharge measurement uncertainties

As described in chapter 1, the measurement of discharge using Acoustic Doppler current profiler (ADCP) consists of number of different sources of uncertainties. A major source of uncertainty arises due to unmeasurable areas at the left, right, top and bottom portions of the discharge measurement section as shown in Figure 1.2. However, suggested uncertainty values for these uncertainties are not available.

The random sources of uncertainties in the ADCP discharge measurement can be quantified from the ADCP measurements, which are usually undertaken in a number of tracks. Based on the quantified uncertainties, measurements in each section in a channel can be expressed as a fuzzy number:

$$\hat{Q}_i = Q_i\left(1 + \hat{X}_i\right) \tag{3.10}$$

where Q_i is the discharge measurement at any section of river channel, and $\hat{X}_i$ and $\hat{Q}_i$ are the fuzzy numbers of uncertainties and discharge, respectively at the measurement section.

From the uncertainty in each portion of discharge measurement defined by equation (3.10), the total uncertainty in discharge measurement by ADCP can be expressed as:

$$\hat{Q}_{total} = \hat{Q}_{left} + \hat{Q}_{top} + \hat{Q}_{middle} + \hat{Q}_{bottom} + \hat{Q}_{right} \tag{3.11}$$

where, $\hat{Q}_{left}$, $\hat{Q}_{top}$, $\hat{Q}_{middle}$, $\hat{Q}_{bottom}$ and $\hat{Q}_{right}$ are fuzzy numbers of discharge measurement at left, top, middle, bottom and right sections. $\hat{Q}_{total}$ is the fuzzy number of total discharge.

3.1.3 *Analysis of stage measurement uncertainties*

In the case of stage measurement, two different sources of uncertainty are considered: (i) error in the measuring instrument, $\hat{X}_{ins}$; and (ii) error in the determination of mean reference gauge height corresponding to the measured discharge, $\hat{X}_{ref}$. Since the uncertainty (ii) is dependent on (i), the combined uncertainty can be calculated as the sum of uncertainties (i) and (ii). Therefore the aggregated uncertainty in stage measurement can be expressed as:

$$\hat{h} = h\left\{1 + \left(\hat{X}_{ins} + \hat{X}_{ref}\right)\right\} \tag{3.12}$$

where, h is the measured stage and $\hat{h}$ is the fuzzified stage.

3.2 Case study 1: Combined uncertainty analysis of current meter discharge and stage measurements

Stage and discharge measurements from Thompson River near Spences bridge from 1970 to 2000 are used for a combined fuzzy analyses of measurement uncertainties. Thompson River is a major tributary of the Fraser River in British Columbia, Canada with a gross drainage area of 54,900 km^2 at the gauging station. The station is located in a narrow gorge with well defined banks. The analysis of the river cross sections from 1970 to 2000, showed a very little change in channel geometry indicating the cross section to be very stable.

The case study uses 58 measurements of stage and discharge values for the rating curve uncertainty analysis. The minimum, mean and maximum values of stage data are 0.44 m, 3.75 m and 8.93 m, respectively. The minimum, mean and maximum discharge values used in the analysis are 155 m^3/s, 1392 m^3/s and 4081 m^3/s, respectively. Individually rated current meter is used for the discharge measurements with 20-30 observation verticals in a cross section. Only a single point on each vertical is used for the estimation of mean velocity. The total discharge is calculated using mid-section method.

The available information from the Spences bridge gauge and general practice of discharge measurement in Canada (Terzi 1981, Pelletier 1988) are used in the analyses. The ISO-748 suggested random uncertainty values (at 95% confidence level) are used as a reference for the expression of each of the uncertainty in terms of triangular fuzzy number. As already outlined, ISO-748 recommends determination of the values independently for the application to a particular case. Therefore, in order to account for the lack of information on random uncertainties and possible systematic uncertainties, the spread of each fuzzy number, in this case, is increased by 50%. Therefore, the fuzzy number of each of the uncertainty sources is viewed as a combination of both; the random and systematic uncertainty. The left or right spread (one half of the support) of the symmetrical triangular fuzzy number taken for each of the errors is given in Table 3.1.

Table 3.1. Half of the support of fuzzy number of errors in discharge measurement due to different uncertainty sources

Uncertainty source	Half of support of fuzzy number (%)
Limited number of verticals (20-30)	7.5-4.5
Limited number of points in a vertical (single point)	22.5
Limited exposure time (one minute)	9.0
Current meter rating (individual rating for velocity > 0.5 m/s)	1.5
Depth measurement (0.4-6 m)	1.05
Width measurement (0-100 m)	0.45

In the case of uncertainty in the stage measurement, half of the support of fuzzy number of errors considered is summarised in Table 3.2.

Table 3.2. Half of the support of the fuzzy number of errors in stage measurement due to different uncertainty sources

Uncertainty source	Half of the support of fuzzy number (mm)
Measurement instrument	15
Determination of mean reference gauge height	15

3.2.1 Results and discussion

3.2.1.1 Fuzzy arithmetic aggregation method

The aggregation of uncertainties using fuzzy variables leads to nonlinear fuzzy numbers of discharge values. The membership functions of the largest measured discharge and corresponding stage are shown in Figures 3.2 and 3.3, respectively. The results show high uncertainty in discharge due to measurement uncertainties characterized by wide support of discharge fuzzy number. The central value of discharge and stage fuzzy numbers represents the values without consideration of uncertainty. The left and right spreads represent the total uncertainties in the measurement. The fuzzy numbers can be interpreted in terms of membership levels, with 0 as the highest uncertainty, i.e., the extreme possible measurement value. The closer the membership level is to 1, the lower is the uncertainty.

The independent (non-interactive) measurements of discharge and stage are combined to form a joint fuzzy number of the corresponding measurements. This leads to a tri-dimensional representation of the fuzzy number as shown in Figure 3.4. The joint membership function of the stage and discharge values provides a visualization of uncertainties in stage and discharge at any membership level. The joint membership function of the fuzzy numbers μ_{Qh} is given by:

$$\mu_{Qh} = \min(\mu_{Q,} \mu_h) \tag{3.13}$$

where, μ_Q is the discharge membership level and μ_h is the stage membership level.

Figure 3.5 shows a 'top view' of the uncertainty in observed stage and discharge values represented by spread of joint membership functions of fuzzy numbers. Each rectangle in Figure 3.5 represents four points a, b, c and d with membership level 0 as shown in Figure 3.4. It is evident from Figure 3.5 that the spread of discharge fuzzy numbers increases with the higher discharge while the spread of the stage fuzzy numbers remains constant. This is due to the fact that uncertainties in each of the elements of discharge measurement (velocity, depth and width) are expressed in terms of percentage values, while constant uncertainty is used for all stage values.

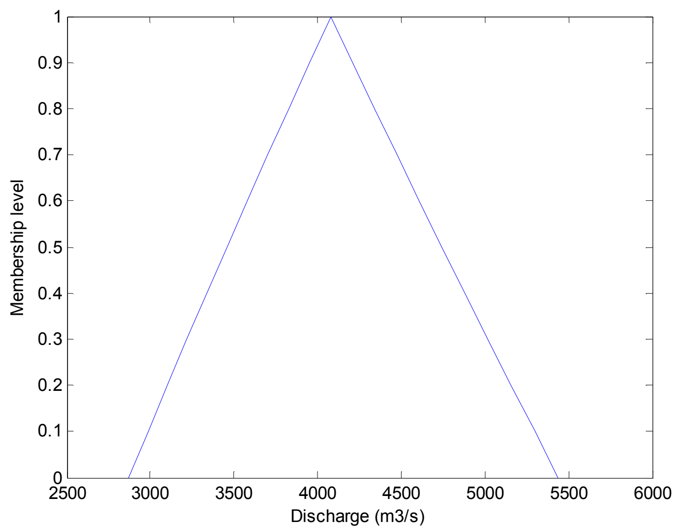


Figure 3.2. Membership function of fuzzy number of maximum observed discharge using fuzzy arithmetic aggregation method

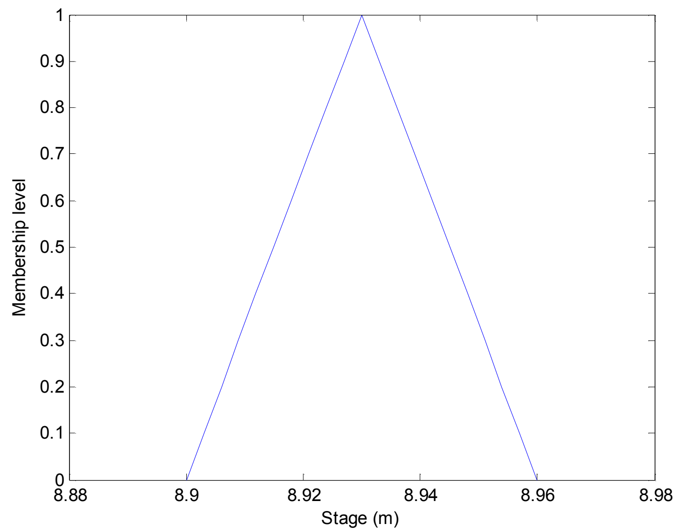


Figure 3.3. Membership function of fuzzy number of maximum observed stage using fuzzy arithmetic aggregation method

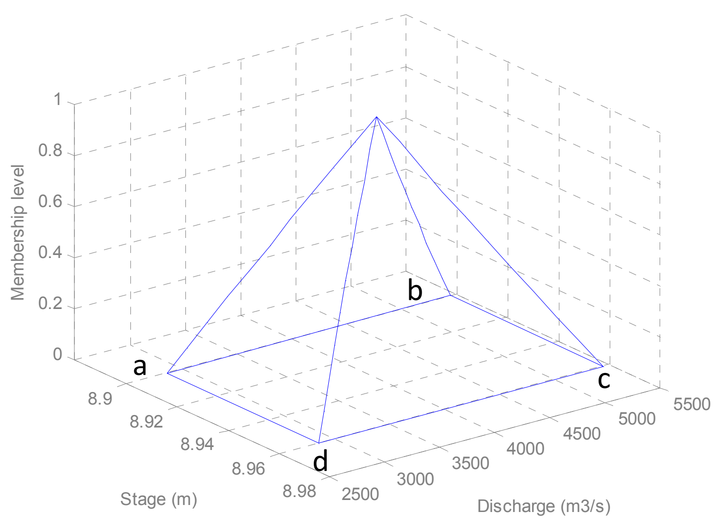


Figure 3.4. Joint membership function of maximum observed stage and discharge in terms of tri-dimensional representation of fuzzy number (fuzzy arithmetic aggregation method)

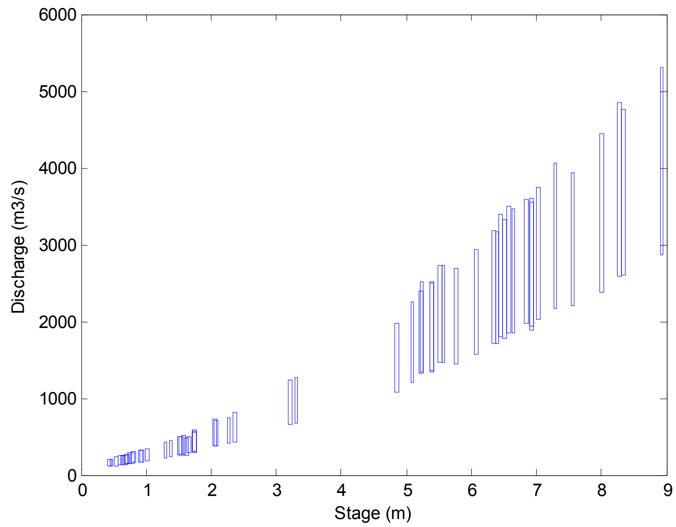


Figure 3.5. Uncertainty in observed stage and discharge represented by spread of joint membership functions using fuzzy arithmetic aggregation method

3.2.1.2 Fuzzified ISO method

The result of fuzzified ISO method also shows the large spread of the fuzzy number of largest measured discharge as shown in Figure 3.6. In this case also, there is increase in spread of discharge fuzzy numbers with the higher discharges as shown in Figure 3.7. A comparison of the left and right spread of fuzzy numbers for the minimum, mean and maximum discharge is given in Table 3.3. It can be seen from the Table that the spreads are higher in the case of fuzzy aggregation method in comparison to the ISO method. This is due to the fact that aggregation of uncertainty using fuzzy arithmetic method uses direct combination of fuzzy numbers of different uncertain quantities and there is no reduction of uncertainty. In the case of the ISO method, the fuzzified form of the ISO equation (Equation 3.10) is used, which combines the fuzzy numbers of uncertainties as a square root of the sum of squares of all the uncertainties. Therefore, there is reduction of uncertainties in the ISO method. It is to be noted too that the ISO method leads to a linear fuzzy number of discharge, and the fuzzy aggregation leads to a nonlinear fuzzy number. The right spread of the fuzzy numbers for minimum, mean and maximum are higher than the left spread in the case of fuzzy aggregation method, while the left and right spreads are equal in the case of fuzzified ISO method.

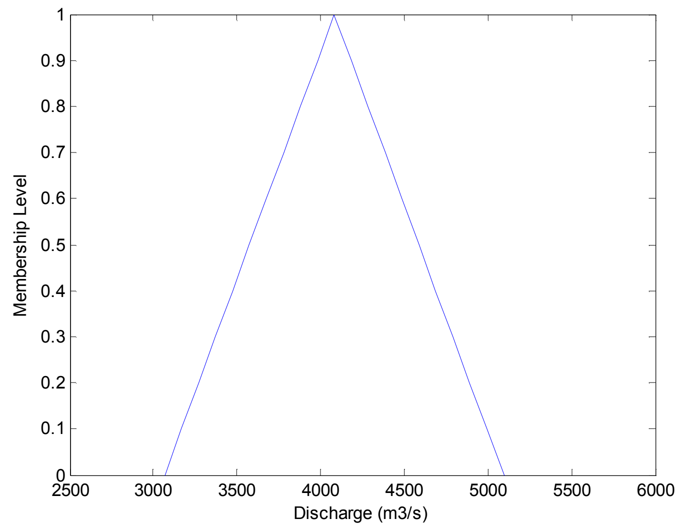


Figure 3.6. Membership function of fuzzy number of maximum observed discharge using fuzzified ISO method

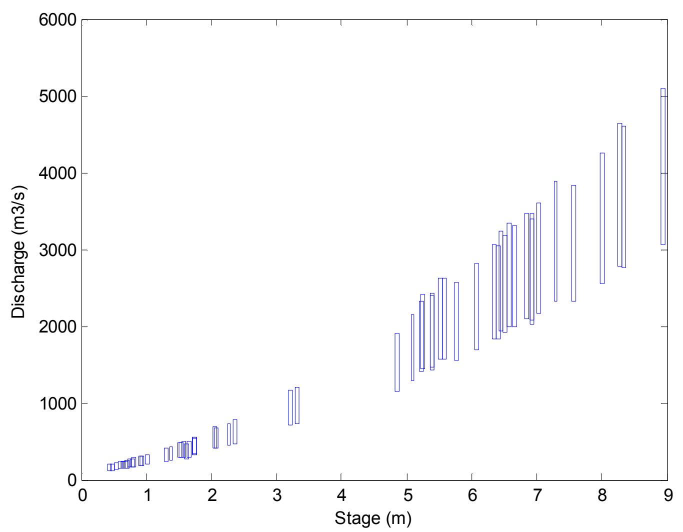


Figure 3.7. Uncertainty in the observed stage and discharge represented by spread of joint membership functions using fuzzified ISO method

Table 3.3. Comparison of the left and right spreads of the fuzzy numbers of discharge using fuzzy aggregation and fuzzified ISO method

	Left spread (m^3/s)			Right Spread (m^3/s)		
	Min	Mean	Max	Min	Mean	Max
Fuzzy arithmetic aggregation method	45	406	1211	49	459	1361
Fuzzified ISO method	38	338	1013	38	338	1031

3.2.1.3 Recommendations for reduction of uncertainties in the case study

The possibilities of reducing total uncertainty in discharge measurement is analyzed by considering different spreads of membership functions of the uncertainty sources. Three parameters with highest range of uncertainty values are chosen for the analyses, which include: approximation due to the limited number of verticals, velocity uncertainties due to the limited numbers of points on a vertical and the measurement exposure time. Different values of uncertainties used for the analyses are summarized in Table 3.4.

Table 3.4. Values of uncertainties used for the uncertainty reduction analyses

Criteria	Uncertainty values in %			
No of verticals	6.0	4.5	3.0	1.5
No of points in a vertical	22.5	15.0	10.5	2.5
Exposure time	9.0	7.5	6.0	3.0

Figures 3.8 and 3.9 show the effects of uncertainty in the number of verticals for fuzzy aggregation and fuzzified ISO methods, respectively. For the reduction of uncertainty from 6% to 1.5%, the reduction in support of fuzzy number in the fuzzy aggregation and the fuzzified ISO method are obtained to be 13.4% and 2.5%, respectively. This shows that the uncertainty due to limited number of verticals has a more significant effect with the application of fuzzy aggregation method than with the application of the fuzzified ISO method. This difference is due to the fact that ISO method combines the

uncertainties as a square root of the sum of squares of all the uncertainties and the highest value of uncertainty dominates. This leads to a lower effect of elements with the low uncertainty level.

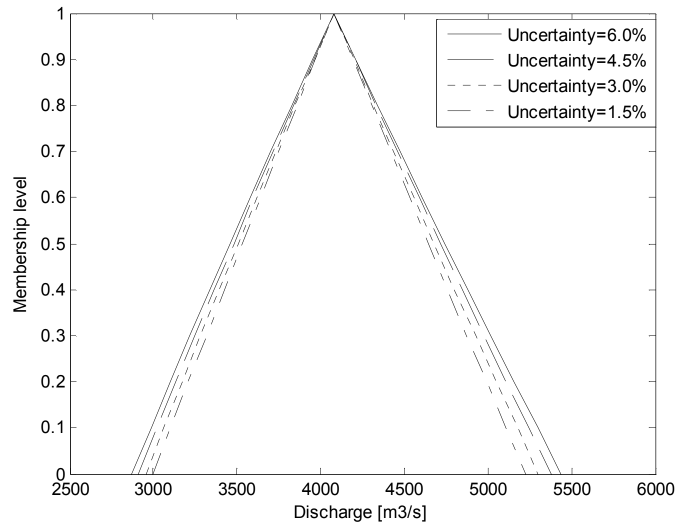


Figure 3.8. Reduction in uncertainties in the number of verticals (fuzzy arithmetic aggregation method)

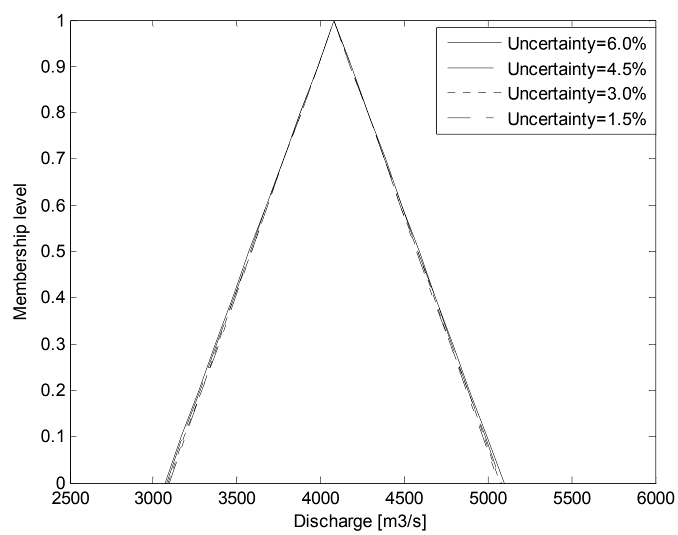


Figure 3.9. Reduction in uncertainties in the number of verticals (fuzzified ISO method)

In the case of uncertainty due to limited number of points on a vertical, higher reduction of uncertainties are observed in both methods as shown in Figures 3.10 and 3.11. The reduction in support of the fuzzy numbers of 47% and 55% are obtained in the fuzzy aggregation and the fuzzified ISO methods, respectively, when the uncertainties are reduced from 22.5% to 2.5%. The uncertainty due to limited number of points on a vertical has the highest range of values of all the uncertainties considered, which therefore provides a high possibility for the reduction of the uncertainties regardless of the method used for aggregation. However, it is interesting to note that the effect of the uncertainty is less dominating in the fuzzy aggregation method, which uses the direct combination, compared to the fuzzified ISO method, which uses square root of the sum of squares of all the uncertainties.

The effect of uncertainties due to exposure time is similar in both methods as shown in Figures 3.12 and 3.13. For the reduction of uncertainty from 9% to 3%, reduction in the support of the fuzzy number in the fuzzy aggregation and the fuzzified ISO method are obtained to be 4.8% and 5.9%, respectively. This shows that there is a limited possibility for the reduction of uncertainties for the exposure time.

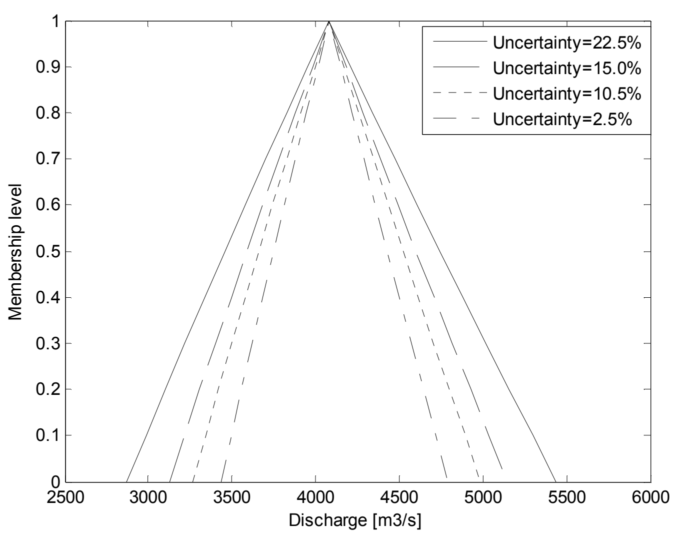


Figure 3.10. Reduction in uncertainties due to the number of points in a vertical (fuzzy arithmetic aggregation method)

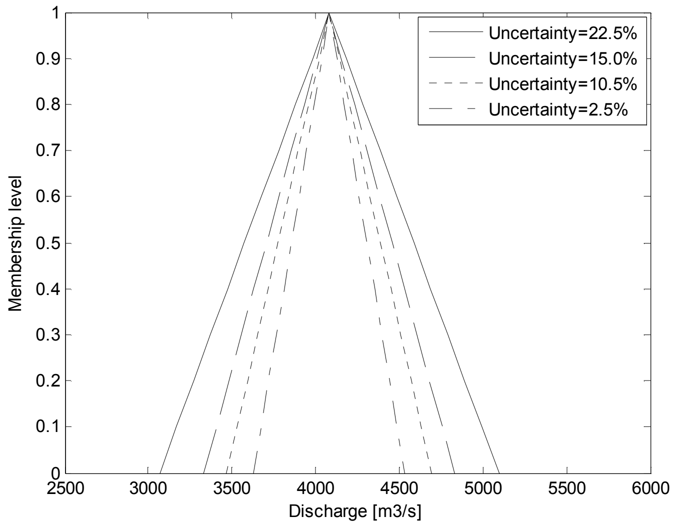


Figure 3.11. Reduction in uncertainties due to the number of points in a vertical (fuzzified ISO method)

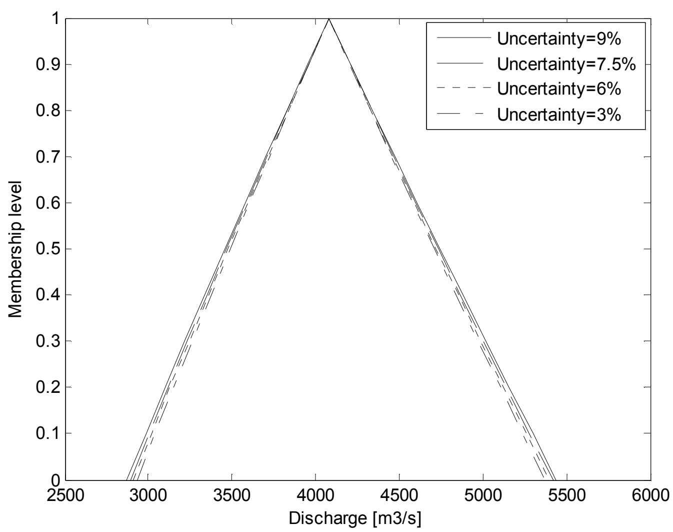


Figure 3.12. Reduction in uncertainty due to exposure time (fuzzy arithmetic aggregation method)

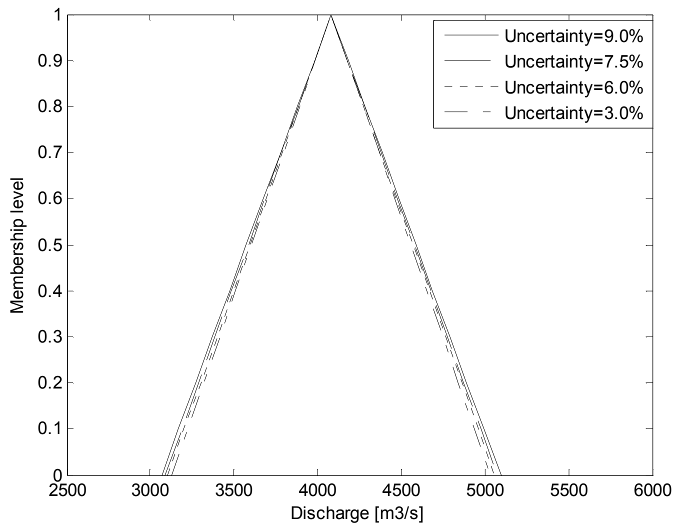


Figure 3.13. Reduction in uncertainty due to exposure time (fuzzified ISO method)

3.3 Case study 2: Uncertainty analysis of Acoustic Doppler current profiler discharge measurements

The second case study presents a methodology for uncertainty quantification in discharge measurement using Acoustic Doppler current profiler. We use seven ADCP measurement data from Richelieu River in Quebec, Canada. Each of the measurements was repeated in 4-5 tracks, based on which ensemble of discharge at each section in the river channel were calculated. We use the discharge measurement values from different tracks to calculate the uncertainties at 95% confidence intervals. The calculated 95% confidence interval uncertainty values are summarized in Table 3.5.

Table 3.5. ADCP measurements at 95% confidence interval

Measurement section	Uncertainties [%]
Left	43.20
Top	1.95
Middle	0.58
Bottom	1.22
Right	37.80

Based on calculated uncertainty values, measurements at each section in a river channel is defined as fuzzy numbers with both the left and right spreads of the fuzzy number of uncertainties increased by 50%. An example of fuzzy numbers of discharge measurement at different measurements sections are shown in Figure 3.14. Based on the defined uncertainties at different sections in a river channel, the total uncertainty is calculated using fuzzy arithmetic as shown in Figure 3.15.

It is to be noted that the uncertainties quantified in the given example only consists of random uncertainties. However, the method can be adapted to quantify other sources of random uncertainties as outlined in section 13.2 as well as systematic uncertainties. It can also be seen that the ADCP discharge measurement uncertainty (Figure 3.15) is very small compared to total uncertainty using current meter measurement (Figure 3.4). Although the uncertainties in the ADCP does not incorporate different sources of possible uncertainties, it can still be expected that the total uncertainty in the measurement will still remain small compared to the current meter discharge measurement uncertainty.

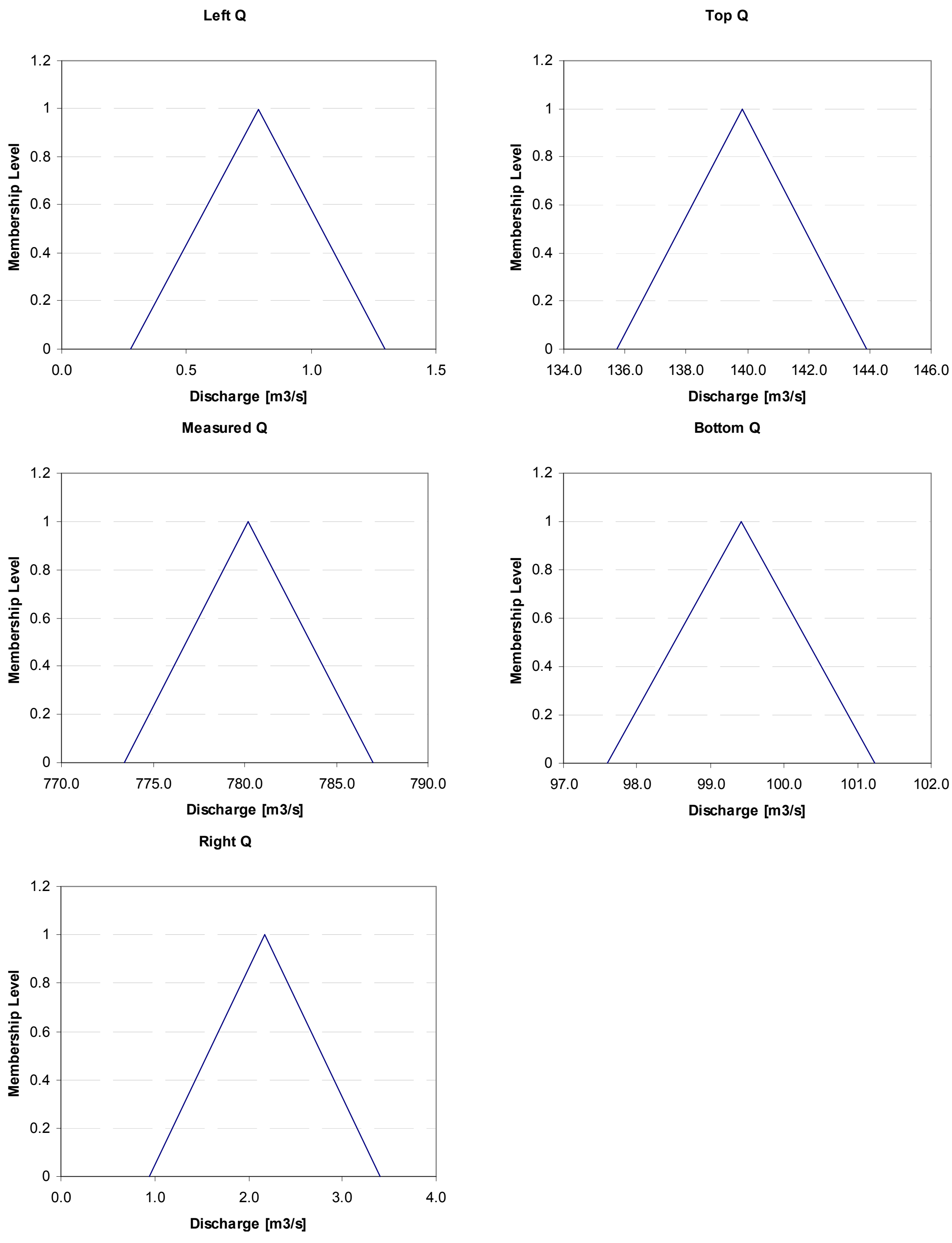


Figure 3.14. ADCP Discharge measurement uncertainties at different sections in a river channel

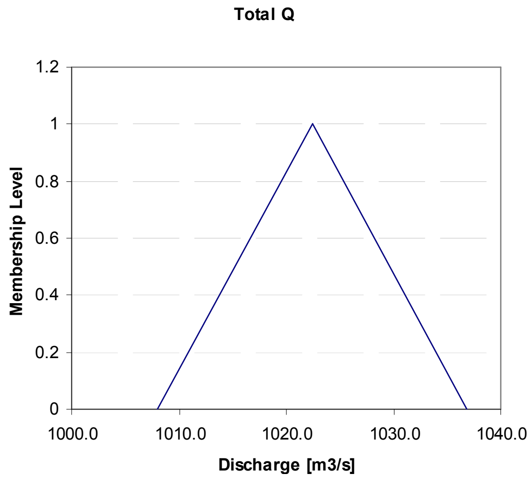


Figure 3.15. Total ADCP Discharge measurement uncertainty

CHAPTER 4

Fuzzy nonlinear regression analysis of stage-discharge relationship

This chapter presents a methodology for the analysis of uncertainty in a stage-discharge relationship. The methodology builds on the results of chapter 3, where the measurement uncertainties of stage and discharge are defined as fuzzy numbers, to define the lower and upper bounds of the stage discharge relationship. The same discharge and stage data from Thompson River near Spences bridge in British Columbia, Canada is used for the analysis.

4.1 Methodology for fuzzy regression analysis of stage-discharge relationship

4.1.1 Derivation of fuzzy regression equations for stage-discharge relationship

The relationship between the stage, h_i and the discharge, Q_i is established by statistical regression analysis using a number of simultaneous observations of stage and discharge and is expressed in the mathematical form as:

$$Q_i = Ah_i^B \tag{4.1}$$

where A and B are the coefficients of the relationship.

Expressing the stage and discharge values, as well as regression coefficients, as fuzzy numbers leads to:

$$\widetilde{Q}_i = \hat{A}\widetilde{h}_i^{\hat{B}} \tag{4.2}$$

The membership functions of fuzzy discharge and stage variables, $\widetilde{Q}_i$ and $\widetilde{h}_i$, can be derived from the measurement uncertainties. The membership functions of the fuzzy coefficients $\hat{A}$ and $\hat{B}$ can be evaluated using fuzzy regression analysis, which is based on fuzzy extension principle (Zadeh 1965). The *L-R* (left–right) representation of fuzzy number as defined in chapter 2.1.1 provides a suitable means for representing the fuzzy coefficients. As defined in section 2.2.4, it may be necessary to extend the spread of the

L-R fuzzy number to incorporate the uncertainty not captured in available data sets using a degree of belief, *H.* Accordingly, for the degree of belief *H*, the left and right spreads of the *L-R* fuzzy number $\hat{A}$ and $\hat{B}$ can be expressed as:

$$m_A - \alpha_A(1-H) \le \hat{A} \le m_A + \beta_A(1-H) \tag{4.3}$$

$$m_B - \alpha_B(1-H) \le \hat{B} \le m_B + \beta_B(1-H) \tag{4.4}$$

The spread of discharge fuzzy number $\hat{Q}_i$ and stage fuzzy number $\hat{h}_i$ obtained from measurement uncertainty analysis (chapter 3) can be expressed as follows:

$$m_{Q_i} - \alpha_{Q_i} \le \widetilde{Q}_i \le m_{Q_i} + \beta_{Q_i} \tag{4.5}$$

$$m_{h_i} - \alpha_{h_i} \le \widetilde{h}_i \le m_{h_i} + \beta_{h_i} \tag{4.6}$$

Considering the reference point for stage value as h_{ref}, as outlined in chapter 2.2.4, leads to the following modification of equation 4.6:

$$\frac{m_{h_i}}{h_{ref}} - \alpha_{h_i} \le \frac{\widetilde{h}_i}{h_{ref}} \le \frac{m_{h_i}}{h_{ref}} + \beta_{h_i} \tag{4.7}$$

For the derivation of the lower and upper bounds of the stage-discharge relationship, the results of stage and discharge measurement uncertainty analysis as described in chapter 3 can be used. Based on the uncertainty plot of combined uncertainty of stage and discharge measurement as shown in Figure 3.5, we derived conditions for fuzzy regression. From Figure 3.5, it can be seen that the lower bound of fuzzy rating curve will intersect with zero stage membership value at boundary of right spread and zero discharge membership value at boundary of left spread. Similarly, the upper bound of fuzzy rating curve will intersect with zero stage membership value at the boundary of left spread and the zero discharge membership value at the boundary of right spread. Therefore, we combined equations (4.3), (4.4), (4.5), (4.7) and (4.2), so that the lower bound of fuzzy rating curve intersects with the zero stage membership value at the boundary of right spread and the upper bound intersects with zero stage membership value at the boundary of left spread. This leads to the expressions for the lower and upper bounds of the fuzzy regression curve in the following form:

$$\{m_a - \alpha_a(1-H)\}\left(\frac{m_{h_i}}{h_{ref}} + \beta_{h_i}\right)^{\{m_b - \alpha_b(1-H)\}} \leq m_{Q_i} - \alpha_{Q_i} \tag{4.8}$$

$$\{m_a + \beta_a(1-H)\}\left(\frac{m_{h_i}}{h_{ref}} - \alpha_{h_i}\right)^{\{m_b + \beta_b(1-H)\}} \geq m_{Q_i} + \beta_{Q_i} \tag{4.9}$$

For most gauging stations it is necessary to consider two or more curves for a reasonable fit of the measured stage and discharge data. Therefore, we consider two curves, for low and high flows, meeting at a break point h_{break} in equation (4.2):

$$\widetilde{Q}_i = \hat{A}_1 \widetilde{h}_i^{\hat{B}_1} \qquad \text{for } h_i < h_{break} \tag{4.10}$$

$$\widetilde{Q}_i = \hat{A}_2 \widetilde{h}_i^{\hat{B}_2} \qquad \text{for } h_i > h_{break} \tag{4.11}$$

$$\hat{A}_1 \widetilde{h}_i^{\hat{B}_1} = \hat{A}_2 \widetilde{h}_i^{\hat{B}_2} \qquad \text{for } h_i = h_{break} \tag{4.12}$$

where the indices 1 and 2 denote the low and high range of the measurement data.

The consideration of two curves meeting at a point h_{break} leads to the expression of equation (4.8) and (4.9) as follows:

$$\left.\begin{aligned} \{m_{a1} - \alpha_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}} + \beta_{h_i}\right)^{\{m_{b1} - \alpha_{b1}(1-H)\}} &\leq m_{Q_i} - \alpha_{Q_i} \\ \{m_{a1} + \beta_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}} - \alpha_{h_i}\right)^{\{m_{b1} + \beta_{b1}(1-H)\}} &\geq m_{Q_i} + \beta_{Q_i} \end{aligned}\right\} \text{for } h_i < h_{break} \tag{4.13}$$

$$\left.\begin{aligned} \{m_{a2} - \alpha_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}} + \beta_{h_i}\right)^{\{m_{b2} - \alpha_{b2}(1-H)\}} &\leq m_{Q_i} - \alpha_{Q_i} \\ \{m_{a2} + \beta_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}} - \alpha_{h_i}\right)^{\{m_{b2} + \beta_{b2}(1-H)\}} &\geq m_{Q_i} + \beta_{Q_i} \end{aligned}\right\} \text{for } h_i > h_{break} \tag{4.14}$$

In addition, the curves for low and high flow data should also meet at lower and upper bounds as well as central value of the relationship curve. This leads to the following additional condition:

$$\left.\begin{aligned}
&\{m_{a1}-\alpha_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}+\beta_{h_i}\right)^{\{m_{b1}-\alpha_{b1}(1-H)\}} = \{m_{a2}-\alpha_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}+\beta_{h_i}\right)^{\{m_{b2}-\alpha_{b2}(1-H)\}}\\
&\{m_{a1}+\beta_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}-\alpha_{h_i}\right)^{\{m_{b1}+\beta_{b1}(1-H)\}} = \{m_{a2}+\beta_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}-\alpha_{h_i}\right)^{\{m_{b2}+\beta_{b2}(1-H)\}}\\
&m_{a1}\left(\frac{m_{h_i}}{h_{ref}}\right)^{m_{b1}} = m_{a2}\left(\frac{m_{h_i}}{h_{ref}}\right)^{m_{b2}}
\end{aligned}\right\} \text{ for } h_i = h_{break} \tag{4.15}$$

4.1.2 Fuzzy regression model fitting

The fuzzy regression model can be evaluated using a number of different criteria. Two criteria are considered for the evaluation of output fuzziness: (a) minimum spread (Wang and Tsaur 2000), and (b) least absolute deviation (Choi and Buckley 2008). The minimum spread of fuzzy numbers is obtained by minimization of output support for the total of n observations, consisting of p low flow observations as:

$$\begin{aligned}
v_1 = &\sum_{i=1}^{p}\left|\{m_{a1}+\beta_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}-\alpha_{h_i}\right)^{\{m_{b1}+\beta_{b1}(1-H)\}} - \{m_{a1}-\alpha_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}+\beta_{h_i}\right)^{\{m_{b1}-\alpha_{b1}(1-H)\}}\right|\\
&+\sum_{i=p}^{n}\left|\{m_{a2}+\beta_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}-\alpha_{h_i}\right)^{\{m_{b2}+\beta_{b2}(1-H)\}} - \{m_{a2}-\alpha_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}+\beta_{h_i}\right)^{\{m_{b2}-\alpha_{b2}(1-H)\}}\right|
\end{aligned} \tag{4.16}$$

The second criterion is implemented through the consideration of deviations between the observations and regression outputs of left and right spreads as well as central values. The deviations for the left spread l, the right spread r and the central value c considering two curves meeting at the point h_{bound} are expressed as equations (4.17), (4.18) and (4.19), respectively:

$$\begin{aligned}
l = &\sum_{i=1}^{p}\left|(m_{Q_i}-\alpha_{Q_i})-\{m_{a1}-\alpha_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}+\beta_{h_i}\right)^{\{m_{b1}-\alpha_{b1}(1-H)\}}\right|\\
&+\sum_{i=p}^{n}\left|(m_{Q_i}-\alpha_{Q_i})-\{m_{a2}-\alpha_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}+\beta_{h_i}\right)^{\{m_{b2}-\alpha_{b2}(1-H)\}}\right|
\end{aligned} \tag{4.17}$$

$$r=\sum_{i=1}^{p}\left|(m_{Q_i}+\beta_{Q_i})-\{m_{a1}+\beta_{a1}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}-\alpha_{h_i}\right)^{\{m_{b1}+\beta_{b1}(1-H)\}}\right|$$
$$\sum_{i=p}^{n}\left|(m_{Q_i}+\beta_{Q_i})-\{m_{a2}+\beta_{a2}(1-H)\}\left(\frac{m_{h_i}}{h_{ref}}-\alpha_{h_i}\right)^{\{m_{b2}+\beta_{b2}(1-H)\}}\right| \tag{4.18}$$

$$c=\sum_{i=1}^{p}\left|m_{Q_i}-m_{a1}\left(\frac{m_{h_i}}{h_{ref}}\right)^{m_{b1}}\right|=\sum_{i=p}^{n}\left|m_{Q_i}-m_{a2}\left(\frac{m_{h_i}}{h_{ref}}\right)^{m_{b2}}\right| \tag{4.19}$$

The total absolute deviation of the observations from the regression output is obtained as the sum of individual deviations of the left spread *l,* the right spread *r* and the central value *c*:

$$v_2=l+r+c \tag{4.20}$$

The set of equations (4.10) to (4.20) provides the mathematical formulation of the fuzzy regression analysis problem using fuzzy form of input and output variables. The formulation leads to an optimization problem for the evaluation of the coefficients in equations (4.10), (4.11), (4.12) in terms of the central value and the left and the right spreads. Equations (4.13), (4.14) provide nonlinear inequality constraints and equation (4.15) provides equality constraint for the optimization. There are two different objective functions for the optimisation: the least spread and the least absolute deviation given by equations (4.16) and (4.20), respectively.

4.2 Case study: Nonlinear fuzzy regression with fuzzy variables and coefficients

We use the results of stage and discharge measurement uncertainty analysis from Thompson River near Spences bridge, which is presented in chapter 4. Based on the results of uncertainty aggregation of using fuzzy arithmetic, as shown in Figure 3.5, we applied the fuzzy regression equations (4.10) to (4.20) for the analysis of uncertainty in the rating curve. For simplicity, we use symmetrical triangular *L–R* fuzzy numbers for the coefficients A_1, B_1, A_2 and B_2 with equal left and right spreads. This reduces the decision variables for fuzzy regression to eight, central values m_{a1}, m_{a2}, m_{b1}, m_{b2}, and spreads α_{a1}, α_{a2}, α_{b1}, α_{b2} for the low and high flow coefficients. We analysed the available measurements to determine the break point between low and high flows for the development of two different relationships. The break point is selected to be at the stage of $h_{break} = 3.36$ m and the same stage is selected to be a location of reference point. To capture uncertainties due to limited number of measurement points, and due to hysteresis and change in river cross section we use a degree of belief *H*, which increases the spread of the fuzzy regression curve and therefore spread of the output. Since, the uncertainty in the discharge measurements is expressed by a wide spread of their fuzzy numbers, higher values of degree of belief can be used. Three relatively high *H* values of 0.5, 0.7 and 0.9 are used to investigate the impact of subjective selection of *H* on the spread of fuzzy regression curve. The sequential quadratic programming method (Fletcher 1987), available in the MATLAB Optimization Toolbox (The MathWorks Inc. 2008) is used for the optimization of the fuzzy regression equation. The method can effectively handle nonlinear equality and inequality constraints required for the optimization of fuzzy regression equation. The method makes an approximation of the Hessian of the Lagrangian function using a quasi-Newton updating method at each iteration. The function is then used to generate an optimal solution using a line search procedure. The success of the algorithm in effective convergence to an optimal solution depends on the use of appropriate initial estimate of decision variables. The method was successfully used for the optimization of fuzzy regression equations by Shrestha et al. (2007).

4.2.1 Results and discussion

The results of fuzzy regression analysis using the criteria of (a) minimum spread and (b) least minimum deviation for the degree of belief of 0.7 are shown in Figures 4.1 and 4.2, respectively. In both cases, the analysis produces upper (*U*) and lower (*L*) curves, bounding the fuzzy stage-discharge measurement data. The uncertainty bound curves for different membership levels (between 0 and 1) represent the degree of belonging of discharge values corresponding to a particular measured stage. The closer the membership level is to 1, the higher is the degree of belonging. The spread of the fuzzy regression curves depends upon the degree of belief used during the regression analysis. Figures 4.1 and 4.2 illustrate the spread of uncertainty bound curves for degree of belief of 0.7 at two different levels of belonging, 0.0 and 0.3. The curves between 0.0*L* and

0.3L, and 0.0U and 0.3U represents (a) the total uncertainty in the stage-discharge relationship together with (b) the uncertainty that is not captured in the available data set and the uncertainty that is not directly considered in the analysis, such as rating curve hysteresis and/or change in river cross section. The use of the two different criteria for optimization of the output fuzziness generates similar results. However, some minor difference exists in the spread of fuzzy output numbers as illustrated in Figures 4.3 and 4.4. The membership functions of fuzzy discharge obtained with the minimum spread and the least absolute deviation criteria corresponding to the stage values between 2.33-2.39 m and 8.9-8.96 m are shown in Figures 4.3 and 4.4, respectively. It can be seen that the spreads of discharge membership functions obtained from fuzzy regression analyses using both criteria are higher than the spread caused by the discharge measurement uncertainty. The higher spread of membership function incorporates additional sources of uncertainty not directly considered in the analysis, such as rating curve hysteresis and change of river cross section. This also incorporates scatter of observed data around defined uncertainty band. Figures 4.3 and 4.4 also show that spreads of discharge membership functions increase with an increase in stage. Therefore, the spread caused by the discharge uncertainty corresponding to stage beyond 8.9-8.96 cm will be even higher.

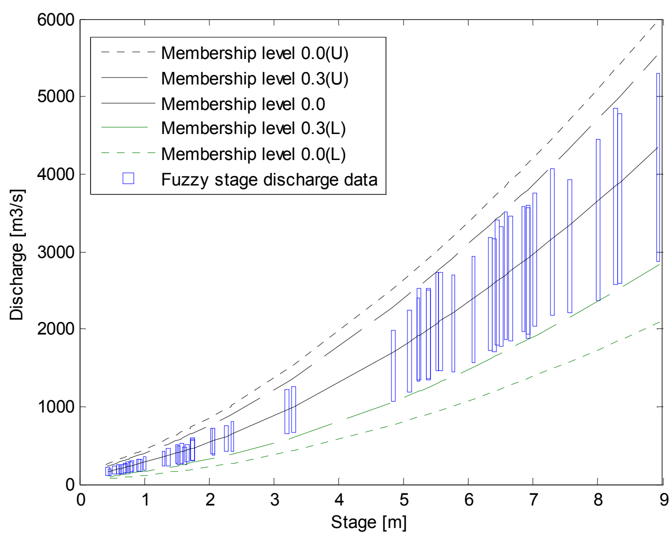


Figure 4.1. Fuzzy regression curves obtained with the minimum spread criteria and degree of belief H=0.7

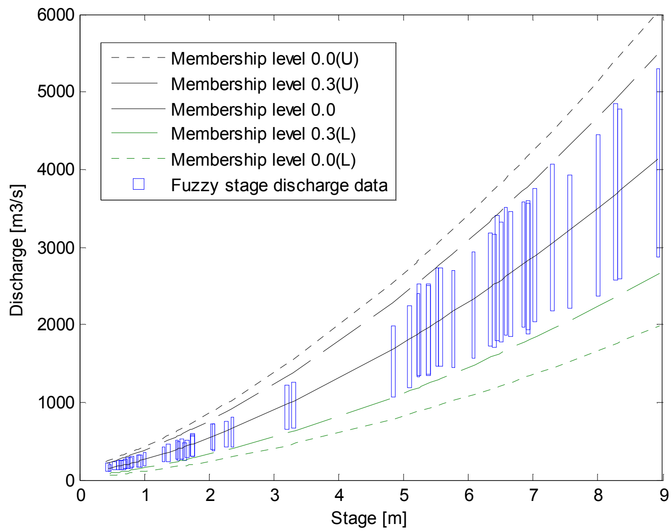


Figure 4.2. Fuzzy regression curves obtained with least minimum deviation criteria and degree of belief H=0.7

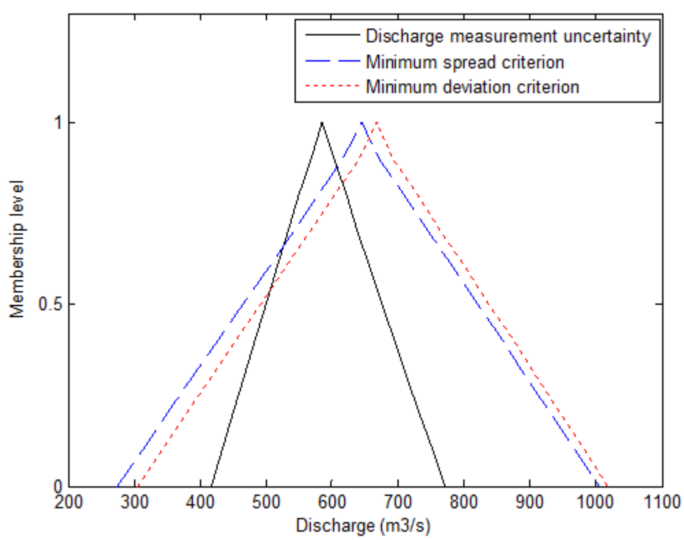


Figure 4.3. Comparison of spreads of fuzzy discharge numbers corresponding to the stage between 2.33-2.39 m and the degree of belief H=0.7

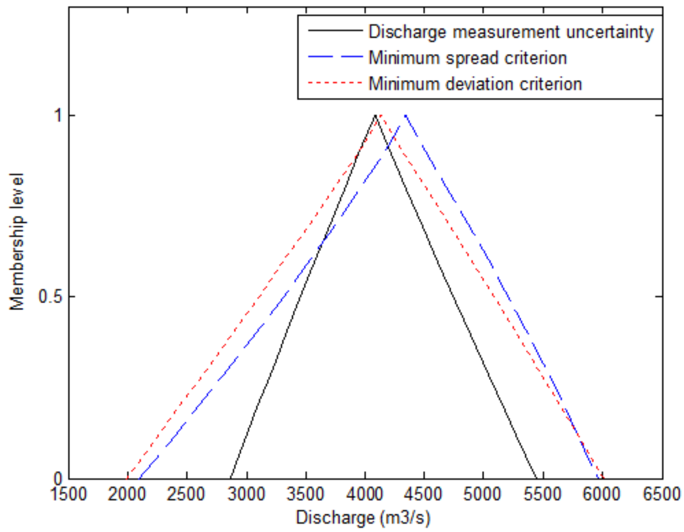


Figure 4.4. Comparison of spreads of fuzzy discharge numbers corresponding to the stage between 8.9-8.96 m and degree of belief *H*=0.7

A sensitivity of the spread of fuzzy discharge numbers obtained using two optimization criteria and the degree of belief of 0.5, 0.7 and 0.9 is summarized in Table 4.1. An increase in the degree of belief leads to a decrease in the total spread of membership function in case of both optimization criteria. However, the deviation between 1.0, and (1-*H*)*L* and (1-*H*)*U* membership level shows no change in the case of least absolute deviation criterion and a small change for the minimum spread criteria. The least absolute deviation criterion uses the difference between 1.0, and (1-*H*)*L* and (1-*H*)*U* membership levels as the objective function value for the optimization. Since the differences are calculated between the band of estimated regression curves at 1 and (1-*H*) membership levels and observations, they remain constant for each *H* level. As expected, the minimum spread criterion leads to lower spread of fuzzy regression curves compared to the least absolute deviation criterion for all values of the degree of belief. The spread of fuzzy discharge numbers is lower in the case of the minimum spread criterion compared to the least absolute deviation criterion. On the other hand, the distance criterion minimizes the deviation between 1.0, and (1-*H*)*L* and (1-*H*)*U* membership levels of fuzzy numbers of measurement uncertainty and regression curve. This therefore leads to lower deviation between the fuzzy numbers corresponding to the input discharge and the output discharge from fuzzy regression at 1.0, and (1-*H*)*L* and (1-*H*)*U* membership levels as shown in Table 4.1.

The study therefore shows two different criteria for the evaluation of output fuzziness in a fuzzy regression analysis using fuzzy input and output variables. As can be seen in Figures 4.1, 4.2, 4.3 and 4.4, and Table 4.1, the difference in results using the two criteria is small. However, there is important difference in the general applicability of the two methods. In the case when observation data is precise or the fuzziness is small, the least absolute deviation criterion is suitable for use as it minimizes the deviation between fuzzy input and output at different membership levels. In the case when the observation data is imprecise or the fuzziness is large, the differences between input and output fuzziness is not important. In such situation, it is more appropriate to minimize the total spread, which leads to a minimum uncertainty of the output. The discharge data from Thompson River used in this study is subject to considerable uncertainties characterized by large fuzziness of discharge membership functions. Therefore, the minimum spread criterion is more appropriate for use in this particular case.

The results of the study also show that the total uncertainty (natural and measurement) leads to a large uncertainty in the stage-discharge relationship characterized by a large spread of discharge fuzzy numbers. Figures 4.1 and 4.2 show higher spread of the fuzzy regression curves for high flows compared to low flows for both optimization criteria. Therefore, the extrapolation of fuzzy regression curves will lead to higher spread of the curves and hence higher discharge uncertainty. In this particular case, discharge measurement uncertainty is characterized by a large spread of it's fuzzy number. The uncertainty propagates into the development of stage-discharge relationship and leads to a wide spread of membership level curves. It can be seen from Figures 4.1, 4.2, 4.3 and 4.4 that discharge measurement uncertainty constitutes a large component of total uncertainty in the fuzzy stage-discharge relationship. The reduction in the measurement uncertainty therefore will provide the most efficient reduction in the uncertainty in the stage-discharge relationship.

Table 4.1. Sensitivity of fuzzy discharge numbers to different degree of belief

	Degree of belief	Total spread (m^3/s)			Deviation (m^3/s)		
	H	Min.	Mean	Max.	Min.	Mean	Max.
Minimum spread criteria	0.5	266	1979	5471	0	70	375
	0.7	189	1405	3864	0	71	378
	0.9	156	1090	2989	0	73	382
Minimum deviation criteria	0.5	255	1997	5647	0	67	348
	0.7	181	1424	4018	0	67	348
	0.9	140	1105	3116	0	67	348

CHAPTER 5

Hysteresis analysis using fuzzified Jones formula

This chapter presents a methodology for an analysis of a looped rating curve using fuzzified form of Jones formula. The methodology is based on consideration of parameters of Jones formula as fuzzy numbers, whose spreads are determined using a multi-objective optimization algorithm. The discharge and stage data from three stations in the Chattahoochee River in Georgia, USA is used for the analysis.

5.1 Methodology for hysteresis analysis using fuzzified Jones formula

5.1.1 Derivation of Jones formula

The Jones formula is derived from one dimensional hydrodynamic model, which is based on the conservation principles of mass and momentum, also known as the St. Venant equations. The equations are expressed in terms of the continuity (equation 5.1) and the momentum equations (equation 5.2):

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \tag{5.1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + gA\frac{\partial h}{\partial x} + gA\left(S_f - S_o\right) = 0 \tag{5.2}$$

where, h = depth of flow [m], Q = discharge [m^3/s], A = active cross sectional area of flow [m^2], g = gravitational acceleration [m/s^2], S_f = friction slope, S_0 = bed slope, x = distances along the channel [m] and t = time [s].

The momentum equation (5.2) can be rewritten as:

$$S_f = S_o - \frac{1}{gA}\frac{\partial Q}{\partial t} - \frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) - \frac{\partial h}{\partial x} \tag{5.3}$$

Neglecting the local acceleration $\frac{1}{gA}\frac{\partial Q}{\partial t}$ and convective acceleration terms $\frac{1}{gA}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)$, equation (5.3) can be rewritten as diffusion wave equation:

$$S_f = S_o - \frac{\partial h}{\partial x} \tag{5.4}$$

The friction slope S_f and the bed slope S_o in the river channel can be evaluated in terms of energy loss equations:

$$Q = K\sqrt{S_f} \tag{5.5}$$

$$Q_o = K\sqrt{S_0} \tag{5.6}$$

Where, K is the conveyance of the channel with same dimension as discharge [m^3/s] and Q_o is the reference discharge assuming steady flow conditions. The conveyance is assumed to be equal for both the unsteady discharge and steady discharge.

Combining equations (5.4), (5.5) and (5.6) leads to the equation:

$$Q = Q_o\left(1 - \frac{1}{S_o}\frac{\partial h}{\partial x}\right)^{1/2} \tag{5.7}$$

The longitudinal gradient of the water depth, $\frac{\partial h}{\partial x}$ can be replaced by an alternative term deducible from a flood record at a section as:

$$c_k\frac{\partial h}{\partial x} + \frac{\partial h}{\partial t} = 0 \tag{5.8}$$

where, c_k = kinematic wave celerity [m/s] and t = time step [s].

Combining equations (5.7) and (5.8), we obtain the following equation which is also referred to as the Jones formula:

$$Q = Q_o\left(1 + \frac{1}{S_o c_k}\frac{\partial h}{\partial t}\right)^{1/2} \tag{5.9}$$

The kinematic wave celerity can be estimated using the following equation (Ponce 1989):

$$c_k = \frac{\partial Q_o}{\partial A} \tag{5.10}$$

This leads to a modified form of the Jones formula:

$$Q = Q_o \left(1 + \frac{\frac{\partial h}{\partial t}}{S_o \frac{\partial Q_o}{\partial A}} \right)^{1/2} \tag{5.11}$$

Therefore, in the Jones formula, the term under the square root modifies the steady state discharge to looped form based on channel slope, kinematic wave celerity and rate of change of stage. The bed slope in the channel can be estimated from the hydraulic data and the water surface slope $\frac{\partial h}{\partial t}$ can be obtained from the observed stage hydrograph. For the calculation of the steady flow discharge Q_o, the single value rating curve equation can be used:

$$Q_o = ah^b \tag{5.12}$$

where, *a* and *b* are the coefficients of the relationship.

Combination of equations (5.11) and (5.12) leads to the following equation:

$$Q = ah^b \left(1 + \frac{\frac{\partial h}{\partial t}}{S_o \frac{\partial (ah^b)}{\partial A}} \right)^{1/2} \tag{5.13}$$

5.1.2 Fuzzification of Jones formula

A major source of uncertainty in the application of the Jones formula is a number of simplifying assumptions, the equation is based upon. Some of the assumptions will never be fully met in any natural river, and lead to unknown trade-off between model bias and model simplicity (Petersen-Overleir 2006). The Jones formula also needs to be calibrated with the measured discharge data. However, the discharge data consist considerable uncertainty (Shrestha and Simonovic 2008a), which will also affect the simulation using

Jones formula. These uncertainties can be taken into account through fuzzification of equation (5.13), by considering coefficients a and b as fuzzy numbers. In addition, the bed slope S_o is also unknown and uncertain parameter, which can be expressed as a fuzzy number. Since the term under the square root of the equation (5.13) modifies the steady state discharge to a looped form, only the coefficients under the square root are considered as fuzzy numbers in this work. The fuzzified form of equation (5.13) is:

$$Q = ah^b \left(1 + \frac{\dfrac{\partial h}{\partial t}}{\hat{S}_o \dfrac{\partial (\hat{a} h^{\hat{b}})}{\partial A}} \right)^{1/2} \tag{5.14}$$

5.1.3 Analysis of fuzzified Jones formula using fuzzy regression

The spreads of the fuzzy coefficients $\hat{a}$ and $\hat{b}$ can be evaluated using a fuzzy nonlinear regression. More details on fuzzy regression methodology is given in chapters 1 and 4. For a degree of belief H, the left and right spreads of the L-R fuzzy numbers $\hat{a}$, $\hat{b}$ and $\hat{S}_o$ can be expressed as:

$$m_a - \alpha_a (1-H) \le \hat{a} \le m_a + \beta_a (1-H) \tag{5.15}$$

$$m_b - \alpha_b (1-H) \le \hat{b} \le m_b + \beta_b (1-H) \tag{5.16}$$

$$m_S - \alpha_S (1-H) \le \hat{S}_o \le m_S + \beta_S (1-H) \tag{5.17}$$

The fuzzified form of the Jones formula leads to a possible bounds of inner and outer loops incorporating the observation points. Both the inner and outer loops consist of rising and falling limbs. Accordingly, the discharge values of the inner loop should be either less than or equal to the observations in the rising limb and equal to or greater than the observations in the falling limb. Similarly, the discharge values of the outer loop should be either greater than or equal to the observations in the rising limb and equal to or less than the observations in the falling limb. Therefore the equations (8), (9a), (9b) and (9c) are integrated in the following conditions:

Inner loop rising limb:

$$a\left(h/h_{ref}\right)^{b}\left(1+\frac{\dfrac{\partial\left(h/h_{ref}\right)}{\partial t}}{\left\{m_S-\alpha_S(1-H)\right\}\dfrac{\partial\left\{m_a-\alpha_a(1-H)\right\}\left(h/h_{ref}\right)^{\left\{m_b-\alpha_b(1-H)\right\}}}{\partial A}}\right)^{1/2}\leq Q_{obs} \qquad (5.18)$$

Inner loop falling limb:

$$a\left(h/h_{ref}\right)^{b}\left(1+\frac{\dfrac{\partial\left(h/h_{ref}\right)}{\partial t}}{\left\{m_S-\alpha_S(1-H)\right\}\dfrac{\partial\left\{m_a-\alpha_a(1-H)\right\}\left(h/h_{ref}\right)^{\left\{m_b-\alpha_b(1-H)\right\}}}{\partial A}}\right)^{1/2}\geq Q_{obs} \qquad (5.19)$$

Outer loop rising limb:

$$a\left(h/h_{ref}\right)^{b}\left(1+\frac{\dfrac{\partial\left(h/h_{ref}\right)}{\partial t}}{\left\{m_S+\alpha_S(1-H)\right\}\dfrac{\partial\left\{m_a+\alpha_a(1-H)\right\}\left(h/h_{ref}\right)^{\left\{m_b+\alpha_b(1-H)\right\}}}{\partial A}}\right)^{1/2}\geq Q_{obs} \qquad (5.20)$$

Outer loop falling limb:

$$a\left(h/h_{ref}\right)^{b}\left(1+\frac{\dfrac{\partial\left(h/h_{ref}\right)}{\partial t}}{\left\{m_S+\alpha_S(1-H)\right\}\dfrac{\partial\left\{m_a+\alpha_a(1-H)\right\}\left(h/h_{ref}\right)^{\left\{m_b+\alpha_b(1-H)\right\}}}{\partial A}}\right)^{1/2}\leq Q_{obs} \qquad (5.21)$$

where h_{ref} is the reference point in the stage data about which the fuzzy regression analysis is performed.

The set of equations (5.18), (5.19), (5.20) and (5.21) provides the mathematical formulation of the fuzzy regression analysis problem using the fuzzified form of Jones formula. The formulation leads to an optimization problem for evaluation of coefficients in equations (5.15), (5.16), (5.17) in terms of central value and left and the right spreads. The equations (5.18), (5.19), (5.20) and (5.21) provide nonlinear inequality constraints. During optimization, the parameters α_a, β_a, α_b, β_b, α_S, and β_S, are allowed to take positive or negative values so that they can satisfy the conditions given by equations (5.18), (5.19), (5.20) and (5.21).

For the evaluation of the spread of the loops a criteria based on minimization of the spread (Wang and Tsaur 2000) of the loop is considered, which provides an objective function for the optimization:

$$\sum_{i=1}^{n}\left| a\left(h/h_{ref}\right)^{b}\left(1+\frac{\frac{\partial\left(h/h_{ref}\right)}{\partial t}}{\{m_S+\alpha_S(1-H)\}\frac{\partial\{m_a+\alpha_a(1-H)\}\left(h/h_{ref}\right)^{\{m_b+\alpha_b(1-H)\}}}{\partial A}}\right)^{1/2} - a\left(h/h_{ref}\right)^{b}\left(1+\frac{\frac{\partial\left(h/h_{ref}\right)}{\partial t}}{\{m_S-\alpha_S(1-H)\}\frac{\partial\{m_a-\alpha_a(1-H)\}\left(h/h_{ref}\right)^{\{m_b-\alpha_b(1-H)\}}}{\partial A}}\right)^{1/2}\right| \tag{5.22}$$

where n is the number of observation data.

5.2 Case study: Application of fuzzified Jones formula for hysteresis analysis

We use the stage and discharge measurements at three stations in Chattahoochee River reach from Georgia, USA (Faye and Cherry 1980) for the analysis of hysteresis using the fuzzified Jones formula. River flow in this reach is predominantly controlled by the Buford dam located upstream. The stage and discharge data in the river reach were collected during the period of March 21-23, 1976, when the regulated discharge at Buford dam was increased to about 8000 ft^3/s (≈225 m^3/s). Continuous stage and discharge measurements at 5 to 10 minutes interval were obtained at a number of stations downstream of the dam using automatic digital recorders. For the measurement of discharge, a minimum of 17 verticals were established across the river cross section at each station. During the measurement each position was established sequentially and flow depth, mean velocity, and time were recorded. Velocity area method was used to calculate the total discharge for each measurement. A detailed description of the discharge measurement in the Chattahoochee River is available in Faye and Cherry (1980).

Petersen-Overleir (2006) used the Jones formula to reproduce rating curve hysteresis in the Chattahoochee River. He used simplifying assumptions about the hydraulic and geometric properties of the river channel and replaced physically based terms in the Jones formula with parameters. Although a good reproduction of the hysteresis curve was obtained, the use of non-physical parameters adds more uncertainty to the Jones formula. Due to this reason, a new methodology is introduced in this paper.

We use a fuzzified form of the Jones formula to reproduce the hysteresis under various sources of uncertainty. We use the data from the stations (i) Georgia Highway 141, (ii) Littles Ferry Bridge and (iii) Georgia Highway 120 at the Chattahoochee River for the analyses. Since the velocity area method is used for the aggregation of discharge, the measurement data is affected by the uncertainties that we considered in our previous work (Shrestha and Simonovic 2008a): (i) limited number of verticals; (ii) limited number of points on a vertical; (iii) limited exposure time; (iv) current meter rating, (v) depth measurement; and (vi) width measurement. However no specific information on individual measurements of velocity, width and depth are available, so we did not consider these uncertainties in this work.

The methodology used in this paper is based on the fuzzy nonlinear regression analysis with parameters expressed as fuzzy numbers. We use two different curves meeting at a point to represent the low and high flows in the steady flow equation (5.12). For simplicity, we use symmetrical triangular *L–R* fuzzy numbers with equal left and right spreads for the coefficients $\hat{a}$, $\hat{b}$ and $\hat{S}_o$ in equations (5.15), (5.16) and (5.17), respectively. This reduces the decision variables for fuzzy regression to twelve, consisting of six central values and six spreads (three each, for low and high flows). The available measurement data is analyzed to determine an appropriate break point between low and high flows and a reference point. For convenience, the brake point and reference points are selected at the same stage for each of the stations. To capture uncertainties in the measurement of discharge, we use a degree of belief *H*, which increases the spread of the fuzzy regression curve and therefore spread of the output. Due to lack of information on discharge measurement uncertainty, we use relatively low value of degree of belief of 0.5.

We initially used the sequential quadratic programming method (Fletcher 1987), available in the MATLAB Optimization Toolbox (The MathWorks Inc. 2008) for the optimization of the fuzzy regression equation. The equations (5.18), (5.19), (5.20) and (5.21) are used as nonlinear inequality constraints and the equation (5.22) as the objective function for the optimization. However, the use of equations (5.18), (5.19), (5.20) and (5.21) as constraints which could not be violated (hard constraints) did not yield any feasible solution for any of the these stations. We observed that a major problem in the use of equations (5.18), (5.19), (5.20), (5.21) and (5.22) for optimization, is the square root term in each of these equations. When the term under the square root becomes negative, it gives an imaginary number output and optimization could not be completed successfully.

To overcome this problem, we use an alternative strategy for optimization. Instead of using equations (5.18), (5.19), (5.20) and (5.21) as hard constraints, they are considered as soft constraints by allowing violations. The soft constraint is used as the objective function to minimize the number of violations. For this purpose, we specified each violation of the criteria as 1 and non-violation as 0, and expressed the sum of violation and non-violation instances as the objective function. Therefore, we have two objective functions for the evaluation of the coefficients of the fuzzified Jones formula: (i) criterion provided by equation (11); and (ii) minimization of the violation of the constraints provided by equations (5.18), (5.19), (5.20) and (5.21). In addition, the third objective function is used to evaluate the 'goodness of fit' of the solution with membership level 1 with observations. For this evaluation, the criterion provided by the Nash-Sutcliffe coefficient is used. Since the optimization problem is considered as a minimization problem, we use the Nash-Sutcliffe coefficient of efficiency subtracted from unity.

$$1 - NSCE = \frac{\sum_{j=1}^{n}\left(Q_{j,obs} - Q_{j,sim}\right)^2}{\sum_{j=1}^{n}\left(Q_{j,obs} - \overline{Q}_{obs}\right)^2} \tag{5.23}$$

where, *NSCE* is the Nash-Sutcliffe coefficient of efficiency, n is the number of observations, $Q_{j,obs}$ is the observed discharge and $Q_{j,sim}$ is the simulated discharge for membership level 1 of fuzzified Jones formula at time step j. $\overline{Q}_{obs}$ is the mean of the observed discharge.

Multi-objective optimization tool called nondominated sorting genetic algorithm-II (NSGA-II; Deb et al. 2002) is used in the analysis. It is a fast and elitist multi-objective genetic algorithm capable of finding multiple Pareto solutions in a single optimisation run. Key features of the NSGA-II are efficient sorting algorithm and maintenance of a diverse set of elite population. More details on the NSGA-II algorithm is found in original work by Deb et al. (2002), which is also summarized in Shrestha and Rode (2008). Four independent optimisation runs of NSGA-II are carried out for each of the three stations with population size 60 and 80 and number of generations between 30 and 40.

5.2.1 Results and discussion

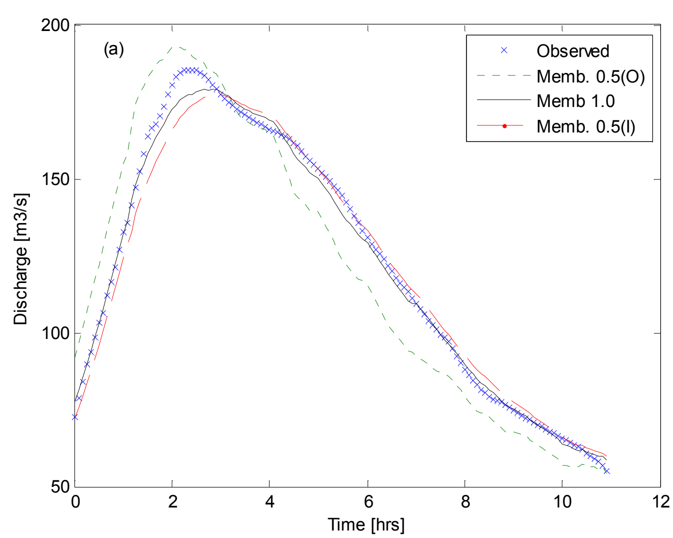
The multi-objective optimization runs at each station produce a band of Pareto solutions. The minimum values of each of the individual objective functions obtained from the optimization runs are given in Table 5.1. The results show that none of the optimization runs lead to a full satisfaction of the constraints given by the equations (5.18), (5.19), (5.20) and (5.21) at any of the stations. It can be seen from Table 5.1, that similar values of 1-*NSCE* are obtained, when two additional objective functions are also simultaneously evaluated. The sum of spreads between $0.5I$ and $0.5O$ membership levels represents the fuzziness of the output and shows large difference when three different criteria are used. Zero value of the total spread between the membership levels $0.5I$ and O.5*O* is obtained for the minimum spread criteria. It corresponds to zero spread of the coefficients of equations (5.15), (5.16) and (5.17). Since the main objective of the application of the fuzzified Jones formula is to find a solution that can incorporate the most of measurement points within a band of predefined inner and outer membership levels, we recommend the solution at the lowest number of violations as the optimal solution. The chosen solution has the least number of points outside the inner and outer bounds with 0.5 membership level. The optimal results of the multi-objective optimization of the fuzzified Jones formula with a degree of belief 0.5 for three stations (i) Georgia Highway 141, (ii) Littles Ferry Bridge and (iii) Georgia Highway 120, are shown in Figures 5.1, 5.2 and 5.3, respectively. All three cases, (a) shows the discharge hydrographs at different membership levels and (b) the fuzzified hysteresis curves for the inner (*I*) and outer (*O*) bounds at membership level 0.5. Therefore, the curves between $0.5I$ and $0.5O$ represent the combined uncertainty in measurement data and simplification of the unsteady flow equations using Jones formula.

The results of the study show that discharge values are higher at the outer membership level in comparison to the inner membership level at the rising limb. In the case of falling limb, discharge values are higher at the inner membership level in comparison to the outer membership level. The results also show that a large spreads of coefficients of fuzzified Jones formula is necessary in order to represent the dynamics of the measured data. It may be possible to obtain a smaller spread if the inner and outer loops are optimized separately. However, the optimization of the outer and inner membership levels together with fuzzy regression enables the evaluation of the not only the bounds, but also different membership levels inside or outside the bounds.

Table 5.1. Performance of the multi-objective optimization runs at three station

Station	Criteria	Sum of spreads bet. $0.5I$ & $0.5O$ memb. levels	No. of violations of constraints	Nash-Sutcliffe coefficient (1-*NSCE*)
Georgia Highway 141	Min. spread	**0**	144	0.007
	Min. no. of violations	3865	**6**	**0.009**
	Min. 1-*NSCE*	2179	32	0.004
Littles Ferry Bridge	Min. spread	**0**	151	0.006
	Min. no. of violations	2877	**3**	0.006
	Min. 1-*NSCE*	3434	20	**0.005**
Georgia Highway 120	Min. spread	**0**	155	0.006
	Min. no. of violations	2167	**14**	0.007
	Min. 1-*NSCE*	1638	66	**0.006**

The central value (membership level = 1.0) of the results in Figures 5.1, 5.2, and 5.3 represents the best fit considering the criteria of minimum violation. Similar results are also observed (not shown in the figures) for the least value of Nash-Sutcliffe coefficient subtracted from unity. The results show that the membership level 1.0 alone is not able to represent the full dynamics of the loop in all three cases considered. This indicates that the non-fuzzy (or membership level 1.0) form of the Jones formula has a limitation in the reproduction of the rating curve hysteresis. The fuzzified form of the Jones formula is therefore a useful methodology for describing the dynamics of the rating curve loops. The discharge data is characterized by measurement uncertainty and the Jones formula is affected by simplification uncertainty. Therefore, the spread of the membership levels represents an impact of combined uncertainty due to these two factors. The proposed methodology is especially appropriate as information on individual sources of uncertainty is not usually available in practice. It is to be noted too that time series of observation data representing hysteresis as shown in this study is rarely available. In such situations, the validity of the looped rating curves produced by the Jones formula cannot be fully justified. Therefore, it is more appropriate to represent the hysteresis in the fuzzified form instead of a single (crisp) loop.



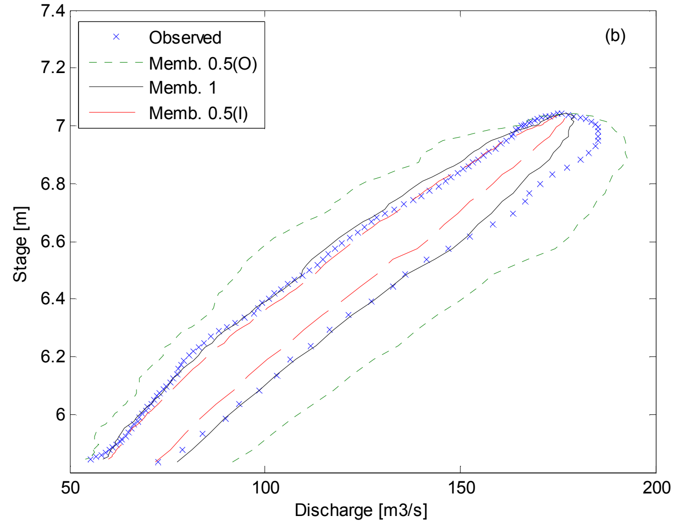
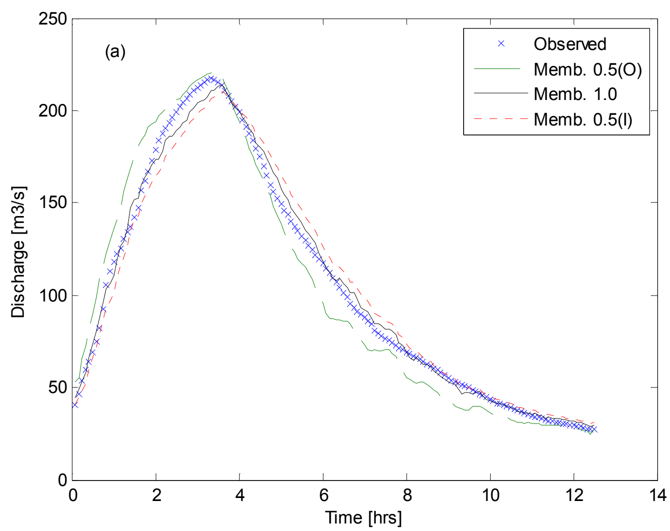


Figure 5.1. Results of the fuzzified Jones formula at Georgia Highway 141 station at different membership levels: (a) discharge hydrograph, (b) looped rating curve



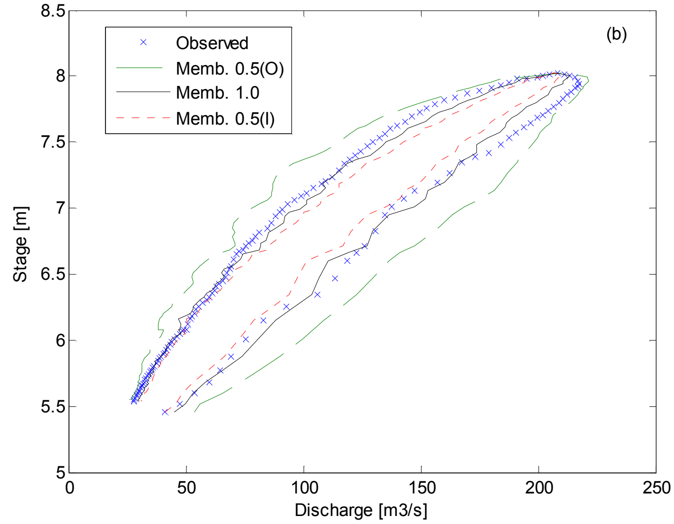
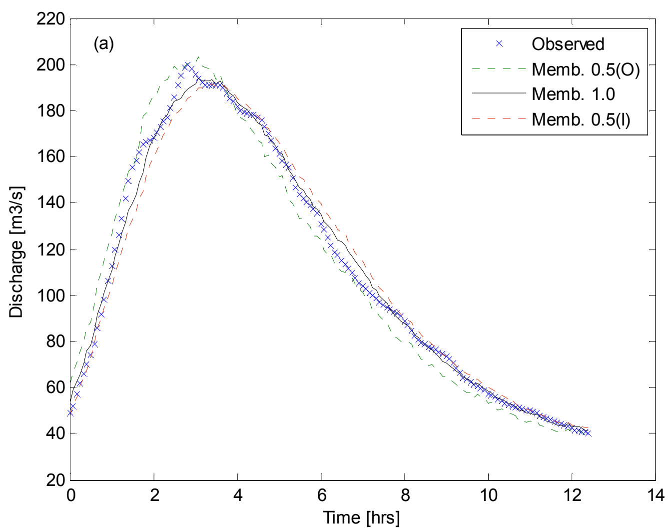


Figure 5.2. Results of the fuzzified Jones formula at Littles Ferry Bridge station at different membership levels: (a) discharge hydrograph, (b) looped rating curve



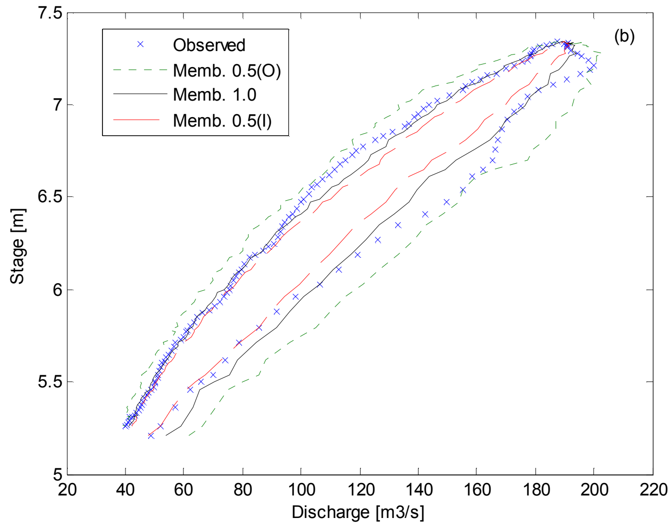


Figure 5.3. Results of the fuzzified Jones formula at Georgia Highway 120 station at different membership levels: (a) discharge hydrograph, (b) looped rating curve

CHAPTER 6

Conclusions and Recommendations

Discharge records are essential for hydrologic and hydraulic analyses of river systems. The primary purpose of a gauging station is to provide discharge records, usually by measuring a stage and converting it to discharge by means of a stage-discharge relationship. However, there are inherent uncertainties in measurement of stage and discharge values and derivation of the stage-discharge relationship. In general, the measurement uncertainty arises due to (i) random and systematic errors in measurement instrumentation; and (ii) approximation of velocity distribution and channel geometry with a finite number of measurements. On the other hand, the stage-discharge relationship is affected by natural uncertainties due to unsteady flow in the river and changes in measurement cross section. Due to unsteady flow, the stage-discharge relationship takes a loop form, referred to as hysteresis, which makes the hypothesis of single valued stage-discharge relationship incompatible. Another source of natural uncertainty is change in river cross section due to physical processes of erosion and sedimentation. Discharge and stage measurements are made over a period of time, usually over a few years. If the change in the cross section is not taken into account, it introduces systematic error or bias in the regression data and affects the rating curve established using the regression analysis.

For the quantification of stage-discharge measurement uncertainties, the International Organization for Standardization (ISO-748) suggests the range of values at 95% confidence level for different sources of uncertainty. This recommendation is based on investigations carried out since 1968. The ISO-748 recommends independent determination of uncertainty in each measurement for the application to a particular case study. However, in most cases, independent value of confidence interval in the measurement is not available, which limits the applicability of statistical quantification of the uncertainties. It is to be noted too that randomness is not the only source of uncertainty in discharge measurements as they can also be affected by systematic uncertainty, human error and other subjective uncertainties. These uncertainties due to bias, non-uniqueness and simplification in the stage-discharge relationship cannot be directly expressed in the statistical framework using confidence intervals.

The fuzzy set theory-based approach is explored in this report as an alternative way of analyzing various uncertainties associated with measurements and the rating curve. The fuzzy approach provides a non-probabilistic framework for representation of uncertainties using vaguely defined boundaries of fuzzy sets. The method is used for quantification and aggregation of individual sources of uncertainty in discharge measurement and definition of uncertainty in the stage discharge relationship.

In the first part of this report, an original fuzzy set theory based approach is used for the consideration of different sources of uncertainty in measurement of stage and discharge and their aggregation into a combined uncertainty. Each of the measurement quantities is represented as a triangular fuzzy number with the spread determined on the basis of the ISO-748 guidelines. The extension principle based fuzzy arithmetic is used for the aggregation of different uncertainties and calculation of the total measurement uncertainty. The results of the study are compared with the fuzzified form of ISO-748 formulation for the calculation of combined measurement uncertainty. The results of the Spences bridge location on the Thompson river in British Columbia, Canada show high uncertainty in the measurement of the discharge (expressed by the wide support of the discharge fuzzy number). The analysis of different uncertainty sources shows that the number of points on a vertical for the measurement of velocity is the largest source of uncertainty in the discharge measurement. Therefore, increase in the number of points on a vertical results in the largest reduction in the measurement uncertainty. Number of verticals in a cross section is another important source of uncertainty in discharge measurement. Although there is a limited reduction in uncertainty when the number of verticals is increased beyond 25, there will be a considerable increase in uncertainty when the number of verticals is reduced below 10. These results can be used as a basis for the improvement in the measurement methods and subsequent reduction in the stage discharge measurement uncertainties.

The study also presents a methodology for handling overall random uncertainties in the Acoustic Doppler current profiler discharge measurement. The results show that the ADCP discharge measurement uncertainty is very small compared to total uncertainty using current meter measurement. Although the uncertainties in the ADCP does not incorporate different sources of possible uncertainties, it can still be expected that the total uncertainty in the measurement will still remain small compared to the current meter discharge measurement uncertainty. The method can be adapted to quantify individual random uncertainties as well as systematic uncertainties.

The second part of the report builds on the results of quantification of uncertainties at Spences bridge location on the Thompson river. Based on the representation of discharge and stage measurement uncertainties using fuzzy numbers, the uncertainties in the stage-discharge relationship is analyzed using fuzzy regression. The methodology is based on the fuzzy nonlinear regression analysis with the input and output variables as well as the coefficients of the stage-discharge relationship expressed as fuzzy numbers. Therefore, the method takes into account the fuzziness in the input and output variables as well as the coefficients of the relationship. Two different criteria are used for an optimal evaluation of the output fuzziness: minimum spread and least absolute deviation criteria. The fuzzy nonlinear regression analysis leads to a definition of lower and upper uncertainty bounds on the stage-discharge relationship. The use of two optimization criteria for the evaluation of the output fuzziness leads to similar results with lower spread in the case of minimum spread criteria and the lower deviation between the fuzzy numbers of the input discharge and output discharge from fuzzy regression at different membership levels.

In this particular case study, the discharge measurement uncertainty is characterized by a large spread of its fuzzy number. The uncertainty propagates into the stage-discharge relationship and leads to a wide spread of the membership level curves. The reduction in the measurement uncertainty therefore will provide the most efficient reduction in the uncertainty in the stage-discharge relationship.

The third part of this reports deals with fuzzified form of Jones formula for the treatment of uncertainties in a looped rating curve using data from three stations in the

Chattahoochee River in Georgia, USA. The Jones formula is one of the methods used for the analysis of looped rating curves. However, it is subject to uncertainty arising from the simplification of unsteady flow equation. There are also additional uncertainties in the application of the Jones formula as discharge values used for fitting the formula are affected by the measurement uncertainties. Based on the representation of coefficients of Jones formula as fuzzy numbers, the uncertainties in the looped stage-discharge relationship are defined using an optimization scheme. A multi-objective optimization scheme NSGA-II is used for the evaluation of the output fuzziness. The measurement data from three stations in the Chattahoochee River in Georgia, USA is used for the analyses.

The multi-objective optimization scheme leads to a definition of the inner and the outer uncertainty bounds on the looped stage-discharge relationship. From the Pareto solutions obtained from the multi-objective optimization runs, a solution which has the most number of observation points inside the inner and outer loops as the optimal solutions is selected. The fuzzified form of Jones formula is able to represent the dynamics of hysteresis loop, which gives an estimation of the combined uncertainty, due to discharge measurement errors and simplification of the unsteady flow equations. It is to be noted that time series of observation data representing hysteresis is rarely available. Due to this reason, the validity of the looped rating curves produced by the non-fuzzy Jones formula cannot be fully justified and it is therefore recommended to represent the hysteresis in the fuzzified form.

The study has therefore demonstrated that the fuzzy set theory-based approach is an effective means of treating uncertainties in stage-discharge measurement and the rating curves in a non-probabilistic framework. The method considers uncertainties using vaguely defined boundaries of fuzzy sets and can incorporate different sources of uncertainty arising from random and systematic errors. The method allows quantification and aggregation of individual sources of uncertainty in discharge measurement and definition of uncertainty in the stage discharge relationship. Moreover, the method allows assessment of random and systematic uncertainties in the measurement and indirect consideration of uncertainties due to hysteresis and changes of river cross section. The method also allows the treatment of classical methods such as Jones formula in a fuzzified form, which leads to a representation of dynamics of looped rating. For most gauging stations, confidence levels of different uncertainty sources are usually unavailable for a probabilistic consideration of uncertainties. In these situations, the fuzzy set theory is recommended to be used as an alternative methodology.

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APPENDIX 1

Matlab source code

1a: Discharge uncertainty calculation using fuzzy arithmetic

fuz_arith_q.m

```
function q_res=fuz_arith_q()
% Calculation of the discharge uncertainties considering the individual
% uncertainties in the measurement of each of the quantities
% Each of the quantities: velocity, depth and width are considered as
% fuzzy numbers and total uncertainty is calculated using fuzzy
% arithmetic

% load the data with first column id second width, third depth and the
% fourth velocity
load q_data.txt;
[m1,m2]=size(q_data);


% alpha cut of fuzzy number
% alpha cut intervals
alpha=0.1;
a=[-1:alpha:1];a1=length(a);
% tot_fuz1=zeros(a1,1);
% define the id
id=q_data(1);
q_res=zeros(1,a1+1);

j=0;
while id<=q_data(m1);

    [R,C] = find(q_data==id);

% Check whether the id number exist
    if (~isempty(R));

    j=j+1;
    n=sum(C);
% select the data with the same id
    qdat0=q_data(R(1):R(n),2:4);
% sort the rows in ascending order with distance
    qdat1=sortrows(qdat0,1);

% Discharge calculation using mid-section method
% The area between the first and second, and second last and last
verticals
% are neglected
```

```
% Define the Width measurement (b(j+1)-b(j-1))/2
    b3=qdat1(3:n,1); b1=qdat1(1:n-2,1);
    b=(b3-b1)/2;
    qdat=[b qdat1(2:n-1,2:3)];

% Define uncertainties in % and increase the 95% confidence interval
value
% by 50% to cover 99.7% (within three standard deviations)
    f=1.5;
% Interpolate uncertainties according to the no of verticals
    xm=vert_uncert(n);xm1=xm*f;
% width measurment (range of width between 0 to 100 m)
    xb=0.3;xb1=xb*f;
% Depth measurement (sounding rod?)
    xd=0.7;xd1=xd*f;

% Velocity measurement depends upon the actual velocity
% one point method is used which has uncertainties as high as 15%!
    xp=15;xp1=xp*f;
% current meter rating (individual rating is used)
    xc=1;xc1=xc*f;
% time of exposure (for exposure time 1 min and velocity > 1 m.
    xe=6;xe1=xe*f;
% Total uncertainties in the velocity measurement is calculated as the
% square root of the sum of squares of all the uncertainties in
velocity
% measurement
    xv1=sqrt(xp1^2+xc1^2+xe1^2);



% fuzzify
% first column width
    width_fuz=fuzzify(qdat(:,1),xb1,alpha,'percent');
% second column depth
    depth_fuz=fuzzify(qdat(:,2),xd1,alpha,'percent');
% third column velocity
    vel_fuz=fuzzify(qdat(:,3),xv1,alpha,'percent');

% do an element wise multiplication to calculate partial fuzzy
discharge
% between the verticals
    q_fuz=width_fuz.*depth_fuz.*vel_fuz;
% calculate the total fuzzy discharge
    Q_fuz=sum(q_fuz);

% Multiply by uncertainties due to number of verticals
% first fuzzify the uncertainties
    vert_fuz=fuzzify(0,xm1,alpha,'zero');
% Calculate the total fuzzy discharge
    Q_dis=(1+vert_fuz).*Q_fuz;

% negative values for negative alpha level
%     k=1;
%     while k<=a1;
%     if a(k)<0;
```

```
%         tot_fuz1(k)=-tot_fuz(k);
%     else tot_fuz1(k)=tot_fuz(k);
%     end
%     k=k+1;
%     end

  % Calculate the total fuzzy discharge
%     Q_dis=((1+tot_fuz)*Q_fuz(11));


% write the matrix of results with id;
    res0=[id Q_dis];

    q_res(j,:)=res0;

    else
    end

id=id+1;
end
```

1b: Discharge uncertainty calculation using fuzzifed ISO method

qiso.uncert.m

```
function q_res=qiso_uncert()
% Calculation of the discharge uncertainties considering the individual
% uncertainties in the measurement of each of the quantities
% Each of the uncertainties in the measurement of velocity, depth and
width
% are considered as fuzzy numbers and aggregated using ISO 748 method

% load the data with first column id second width, third depth and the
% fourth velocity
load q_data.txt;
[m1,m2]=size(q_data);


% alpha cut of fuzzy number
% alpha cut intervals
alpha=0.1;
a=(-1:alpha:1);a1=length(a);
tot_fuz1=zeros(a1,1);
% define the id
id=q_data(1);
q_res=zeros(1,a1+1);

j=0;
while id<=q_data(m1);

    [R,C] = find(q_data==id);
```

```
% Check whether the id number exist
    if (~isempty(R));

    j=j+1;
    n=sum(C);
% select the data with the same id
    qdat0=q_data(R(1):R(n),2:4);
% sort the rows in ascending order with distance
    qdat1=sortrows(qdat0,1);

% Discharge calculation using mid-section method
% The area between the first and second, and second last and last
verticals
% are neglected

% Define the Width measurement (b(j+1)-b(j-1))/2
    b3=qdat1(3:n,1); b1=qdat1(1:n-2,1);
    b=(b3-b1)/2;
    qdat=[b qdat1(2:n-1,2:3)];

% calculate the partial discharge between the verticals
    par_dis=qdat(:,1).*qdat(:,2).*qdat(:,3);
%     sq_dis=par_dis.^2;
    tot_dis=sum(par_dis);

% Define uncertainties in % and increase the 95% confidence interval
value
% by 50% to cover 99.7% (within three standard deviations)
    f=1.50;
% Interpolate uncertainties according to the no of verticals
    xm=vert_uncert(n);xm1=xm*f;
% width measurment (range of width between 0 to 100 m)
    xb=0.3;xb1=xb*f;
% Depth measurement (sounding rod?)
    xd=0.7;xd1=xd*f;

% Velocity measurement depends upon the actual velocity
% one point method is used which has uncertainties as high as 15%!
    xp=15;xp1=xp*f;
% current meter rating (individual rating is used)
    xc=1;xc1=xc*f;
% time of exposure (for exposure time 1 min and velocity > 1 m.
    xe=6;xe1=xe*f;


% fuzzify and calculate the square of the uncertainty values
% width
    width_fuz=fuzzify(0,xb1,alpha,'zero');
% depth
    depth_fuz=fuzzify(0,xd1,alpha,'zero');
% velocity on of points in a vertical
    velp_fuz=fuzzify(0,xp1,alpha,'zero');
% current meter rating
    velc_fuz=fuzzify(0,xc1,alpha,'zero');
% time of exposure
```

```
    vele_fuz=fuzzify(0,xe1,alpha,'zero');
% Uncertainties due to limited no of verticals
    vert_fuz=fuzzify(0,xm1,alpha,'zero');


% do element wise multiplication (square) of the fuzzified
uncertainties
% (except, no of verticals)

com_fuz=width_fuz.^2+depth_fuz.^2+velp_fuz.^2+velc_fuz.^2+vele_fuz.^2;

% multiply the square area with the com_fuzzy
    i=1;
    temp=zeros(n-2,a1);
    while i<n-2;
        temp(i,:)=par_dis(i)*sqrt(com_fuz);
        i=i+1;
    end

% calculate the total uncertainties
    tot_fuz=sqrt(vert_fuz.^2+(sum(temp)/tot_dis).^2);

% negative values for negative alpha level
    k=1;
    while k<=a1;
    if a(k)<0;
        tot_fuz1(k)=-tot_fuz(k);
    else tot_fuz1(k)=tot_fuz(k);
    end
    k=k+1;
    end

% Calculate the total fuzzy discharge
    Q_fuz=((1+tot_fuz1)*tot_dis)';

% write the matrix of results with id;
    res0=[id Q_fuz];

    q_res(j,:)=res0;

    else
    end

id=id+1;
end
```

1c: Stage uncertainty calculation using fuzzy arithmetic

fuz_arith_h.m

```
function h_res=fuz_arith_h()
```

```
% Calculation of the stage measurement uncertainties considering the
%Uncertainties in gauge measurement & mean gauge height during
measurement


% load the data with first column id and second stage

load h_data.txt;
[m1,m2]=size(h_data);


% alpha cut of fuzzy number
% alpha cut intervals
alpha=0.1;
% a=[-1:alpha:1];a1=length(a);

% Define uncertainties in % and increase the 95% confidence interval
value
% by 50% to cover 99.7% (within three standard deviations)
    f=1.5;
% stage measurement (float operated autographic recorder)
% No information is available on mean gauge height determination
% uncertainties during measurement, so assuming equal to xh
    xh=0.01;xh1=2*(xh*f);

% fuzzify
% first column width
    stage_fuz=fuzzify(h_data(:,2),xh1,alpha,'value');

% write the matrix of results with id;
    h_res=[h_data(:,1) stage_fuz];
```

1d: Fuzzification the variables using user defined uncertainty

fuzzify.m

```
% fuzzify variables using user defined uncertainty level and alpha
function var_fuz=fuzzify(var,unt,alpha,operator)
% var = vector of variable to fuzzify
% unt = uncertainty value in %
% alpha = alpha level intervals to fuzzify
c=length(var);

a=(-1:alpha:1);b=length(a);
var_fuz=zeros(c,b);
j=1;
while j<=c;
        if strcmp(operator, 'percent'),
        var_fuz(j,:)=var(j,:)*(1+a*(unt/100));
        elseif strcmp(operator, 'value'),
```

```
            var_fuz(j,:)=var(j,:)+a*unt;
        elseif strcmp(operator, 'zero')
            var_fuz(j,:)=0+a*(unt/100);
        else
        end
        j=j+1;
end
```

1e: Interpolation of uncertainties due to number of verticals

Vert_uncert.m

```
function xm=vert_uncert(vert_num)
% Interpolate uncertainties due to number of verticals
xy=[5   15;
10  9;
15  6;
20  5;
25  4;
30  3;
35  2;
40  2;
45  2];
x=xy(:,1);
y=xy(:,2);

xm=interp1(x,y,vert_num);
```

1f: Fuzzy regression of fuzzy stage and discharge values with minimum distance criteria

f_regression_mindist.m

```
function [b,fval] = f_regression_mindist()

% This code requires MATLAB optimization toolbox!
% The code generates lower and upper membership bounds of the stage
% discharge relationship curves with both stage and discharge
(variables)
% treated as fuzzy numbers.
% It generates fuzzy parameters of the stage discharge relationship

% Degree of belief
dob = 0.5;
 % Initialize shared variable
% load the data
hq_fuz=load('hq_fuz.txt');
hqdat=sortrows(hq_fuz,6);

% Scale the data and devide the reference point
```

```
h11=(hqdat(1:28,5)+1)/3.36;h21=(hqdat(1:28,6)+1)/3.36;h31=(hqdat(1:28,7
)+1)/3.36;
h117=(h21-(h21-h11)*dob);h317=(h21+(h31-h21)*dob);
h12=(hqdat(28:56,5)+1)/3.36;h22=(hqdat(28:56,6)+1)/3.36;h32=(hqdat(28:5
6,7)+1)/3.36;
h127=(h22-(h22-h12)*dob);h327=(h22+(h32-h22)*dob);

% Define the coresponding discharge data
q11=hqdat(1:28,2);q21=hqdat(1:28,3);q31=hqdat(1:28,4);
q12=hqdat(28:56,2);q22=hqdat(28:56,3);q32=hqdat(28:56,4);

% Make a starting guess at 1the solution

b0 = [667   1.81 513    -0.18   667 1.68    513 -0.07];

options =
optimset('Display','iter','LevenbergMarquardt','on','MeritFunction',
'multiobj');
options.TolCon =6e-06;
OPTIONS.MaxFunEvals=200000;

[b, fval] = fminimax(@fobfun,b0,[],[],[],[],[],[],@fconst,options);

    function f = fobfun(b)
    % Objective function description
     % minimum distance criteria

    % calculate the distances
    l11=sum(abs((b(1)-b(3)*dob)*h317.^(b(2) -dob*b(4))-q11));
    l12=sum(abs((b(5)-b(7)*dob)*h327.^(b(6) -dob*b(8))-q12));

    u11=sum(abs((b(1)+b(3)*dob)*h117.^(b(2) +dob*b(4))-q31));
    u12=sum(abs((b(5)+b(7)*dob)*h127.^(b(6) +dob*b(8))-q32));

    c11=sum(abs(b(1)*h21.^b(2))-q21);
    c12=sum(abs(b(5)*h22.^b(6))-q22);

    % Nonlinear objective function
    f=l11+u11+c11+l12+u12+c12;

     end

     function [c,ceq] = fconst(b)

% Constraints descriptions

% Nonlinear inequality constraints

 c = [-q11+(b(1)-dob*b(3))*(h317.^(b(2)-dob*b(4)));-q12+(b(5)-
dob*b(7))*(h327.^(b(6)-dob*b(8)));q31-
(b(1)+dob*b(3))*(h117.^(b(2)+dob*b(4)));q32-
(b(5)+dob*b(7))*(h127.^(b(6)+dob*b(8)))];
```

```
% Nonlinear equality constraints

ceq=[b(1)*(1.^b(2))-b(5)*(1.^b(6));(b(1)-dob*b(3))*(h317(28).^(b(2)-
dob*b(4)))-(b(5)-dob*b(7))*(h317(28).^(b(6)-
dob*b(8)));(b(1)+dob*b(3))*(h117(28).^(b(2)+dob*b(4)))-
(b(5)+dob*b(7))*h117(28).^(b(6)+dob*b(8))];

    end
end
```

1g: Fuzzy regression of fuzzy stage and discharge values with minimum spread criteria

f_regression_minfuz.m

```
function [b,fval] = f_regression_minfuz()

% This code requires MATLAB optimization toolbox!
% The code generates lower and upper membership bounds of the stage
% discharge relationship curves with both stage and discharge
(varaibles)
% treated as fuzzy numbers.
% It generates fuzzy parameters of the stage discharge relationship

% Degree of belief
dob = 0.5;


 % Initialize shared variable
% load the data
hq_fuz=load('hq_fuz.txt');
hqdat=sortrows(hq_fuz,6);

% Scale the data and devide the reference point
h11=(hqdat(1:28,5)+1)/3.36;h21=(hqdat(1:28,6)+1)/3.36;h31=(hqdat(1:28,7
)+1)/3.36;
h117=(h21-(h21-h11)*0.5);h317=(h21+(h31-h21)*0.5);
h12=(hqdat(28:56,5)+1)/3.36;h22=(hqdat(28:56,6)+1)/3.36;h32=(hqdat(28:5
6,7)+1)/3.36;
h127=(h22-(h22-h12)*0.5);h327=(h22+(h32-h22)*0.5);

% Define the corresponding discharge data
q11=hqdat(1:28,2);q21=hqdat(1:28,3);q31=hqdat(1:28,4);
q12=hqdat(28:56,2);q22=hqdat(28:56,3);q32=hqdat(28:56,4);
%b0 = [2.33e-07     3.20 1.58e-07 0.040284];

% Make a starting guess at 1the solution
%b0 = [168.0527045  1.817693566 366.0680943 -0.143095172    468.0527045
1.689835877 365.4645759 -0.064609709];
b0 = [667,1.8,509.5,-0.18,667.1,1.68,508.9,-0.06;];
```

```
options =
optimset('Display','iter','LevenbergMarquardt','on','MeritFunction',
'multiobj');
options.TolCon =6e-06;
OPTIONS.MaxFunEvals=200000;

[b, fval] = fminimax(@fobfun,b0,[],[],[],[],[],[],@fconst,options);

    function f = fobfun(b)
    % Objective function description
    % minimum fuzziness criteria

     u11=sum(abs((b(1)+b(3)*dob)*h117.^(b(2) +dob*b(4))-(b(1)-
b(3)*dob)*h317.^(b(2) -dob*b(4))));
    u12=sum(abs((b(5)+b(7)*dob)*h127.^(b(6) +dob*b(8))-(b(5)-
b(7)*dob)*h327.^(b(6) -dob*b(8))));


    % Nonlinear objective function
     f=u11+u12;

     end

     function [c,ceq] = fconst(b)

% Constraints descriptions


% Nonlinear inequality constraints

 c = [-q11+(b(1)-dob*b(3))*(h317.^(b(2)-dob*b(4)));-q12+(b(5)-
dob*b(7))*(h327.^(b(6)-dob*b(8)));q31-
(b(1)+dob*b(3))*(h117.^(b(2)+dob*b(4)));q32-
(b(5)+dob*b(7))*(h127.^(b(6)+dob*b(8)))];

% Nonlinear equality constraints

ceq=[b(1)*(1.^b(2))-b(5)*(1.^b(6));(b(1)-dob*b(3))*(h317(28).^(b(2)-
dob*b(4)))-(b(5)-dob*b(7))*(h317(28).^(b(6)-
dob*b(8)));(b(1)+dob*b(3))*(h117(28).^(b(2)+dob*b(4)))-
(b(5)+dob*b(7))*h117(28).^(b(6)+dob*b(8))];

     end
end
```

1h: Fuzzified Jones formula for calculation of looped rating curve

jonesmodfuzzy.m

```
function [Q,Q1,Q2] =jonesmodfuzzy(y,h_break,xy,par)

% This code requires NSGA-II code!
```

```
% Fuzzified Jones formula for the calculation of looped rating curve
% par = parameters of the Jones formula generated by NSGA-II code
% h_break=break point between low flow and high flow in stage discharge
% relationship
% xy=cross section data
% Q=central value of Jone formula output
% Q1, Q2 =Inner and outer loops of Jone formula output
par
%Coefficients of the relationship
a1=par(:,1);a11=par(:,5);
b1=par(:,2);b11=par(:,6);
a2=a1;a21=b11;
b2=par(:,3);b21=par(:,7);
s0=par(:,4)*1e-4;s01=par(:,8)*1e-5;

% stage time series
%Smoothen the water level data using moving average function
[m1,m2]=size(y);h=zeros(m1,1);span = 5; % Size of the averaging window
window = ones(span,1)/span;
y_sm = convn(y,window,'same');
y_smooth=[y(1:2);y_sm(3:m1-2);y(m1-1:m1)];



% both y and h and stage time series, h is  y divided by reference
point
%random number is added to y in case h
% at two consecutive time steps are same.

j=2;h=y_smooth/h_break;
while j<=m1;
if y_smooth(j)==y_smooth(j-1);h(j)=y_smooth(j)/h_break+0.001;
else
end
j=j+1;
end



% Inatialise discharge variables, qhd=steady state discharge, Q is the
% discharge modified by Jones formula.
Q=zeros(m1,1);Q1=zeros(m1,1);Q2=zeros(m1,1);
qstd=zeros(m1,1);qstd1=zeros(m1,1);qstd2=zeros(m1,1);

% calculate the area at specified vertical interval using areawidth
code
[H,A,W,P] =areawidth(xy(:,1),xy(:,2),0.1);

% calculate the area at each time steps
ai = interp1(H,A,h(:,1)*h_break);

% Use fuzzified Jones formula for the power coefficients of the rating
curve
% equation
% Central value
i=1;
while i<=m1;
```

```
    if h(i)<=1;
        qstd(i)=a1*h(i)^b1;
        if i==1; Q(i)=qstd(i);
        else Q(i)=qstd(i)*sqrt(1+((h(i)-h(i-1))/300)/(s0*((qstd(i)-
qstd(i-1))/(ai(i)-ai(i-1)))));
        end
    else qstd(i)=a2*h(i)^b2;
        Q(i)=qstd(i)*sqrt(1+((h(i)-h(i-1))/300)/(s0*((qstd(i)-qstd(i-
1))/(ai(i)-ai(i-1)))));
    end
i=i+1;
end

% Lower spread
i=1;
while i<=m1;
    if h(i)<=1;
        qstd1(i)=(a1-a11*0.5)*h(i)^(b1-b11*0.5);
        if i==1; Q1(i)=qstd1(i);
        else Q1(i)=qstd(i)*sqrt(1+((h(i)-h(i-1))/300)/((s0-
s01)*((qstd1(i)-qstd1(i-1))/(ai(i)-ai(i-1)))));
        end
    else qstd1(i)=(a2-a21*0.5)*h(i)^(b2-b21*0.5);
        Q1(i)=qstd(i)*sqrt(1+((h(i)-h(i-1))/300)/((s0-s01)*((qstd1(i)-
qstd1(i-1))/(ai(i)-ai(i-1)))));
    end
i=i+1;
end

% Lower spread
i=1;
while i<=m1;
    if h(i)<=1;
        qstd2(i)=(a1+a11*0.5)*h(i)^(b1+b11*0.5);
        if i==1; Q2(i)=qstd2(i);
        else Q2(i)=qstd(i)*sqrt(1+((h(i)-h(i-
1))/300)/((s0+s01)*((qstd2(i)-qstd2(i-1))/(ai(i)-ai(i-1)))));
        end
    else qstd2(i)=(a2+a21*0.5)*h(i)^(b2+b21*0.5);
        Q2(i)=qstd(i)*sqrt(1+((h(i)-h(i-1))/300)/((s0+s01)*((qstd2(i)-
qstd2(i-1))/(ai(i)-ai(i-1)))));
    end
i=i+1;
end
end
```

1i: Cross section variables calculation for different water levels

areawidth.m

```
function [H,A,W,P] =areawidth(x,y,int)
% Cross section variables calculation for different water levels
```

```
% x= x coordinate of cross section
% y= y coordinate of cross section
% int = interval for interpolation
% H = elevation intervals
% A = Area at interval
% W = width at interval
% P = Weighted perimeter at interval

dist=x;
elev=y;
[m1,m2]=size(elev);
interv=int;

elmax=max(elev);
elmin=min(elev);

elint=elmin+interv;
elinterv=[elmin+interv:interv:elmax]';
[m3,m4]=size(elinterv);
aout=zeros(m3,1);width=zeros(m3,1);
p=1;
while elint<=elmax;
    s=1;
    [m1,m2]=size(elev);


    elev1=elev;
    dist1=dist;

    while s<=m1-1;
        if (elev1(s)>elint && elev1(s+1)<elint);
            elv11=elev1(1:s);elv12=elev1(s+1:m1);
            dis11=dist1(1:s);dis12=dist1(s+1:m1);
            disint=dist1(s)+(dist1(s+1)-dist1(s))*(elev1(s)-
elint)/(elev1(s)-elev1(s+1));
            elev1=[elv11;elint;elv12];dist1=[dis11;disint;dis12];
            m1=m1+1;
        elseif (elev1(s)<elint && elev1(s+1)>elint);
            elv11=elev1(1:s);elv12=elev1(s+1:m1);
            dis11=dist1(1:s);dis12=dist1(s+1:m1);
            disint=dist1(s)+(dist1(s+1)-dist1(s))*(elint-
elev1(s))/(elev1(s+1)-elev1(s));
            elev1=[elv11;elint;elv12];dist1=[dis11;disint;dis12];
            m1=m1+1;
        else elev1(s)=elev1(s);%dist1(r)=dist(s);
        end;
        s=s+1;
    end;

        [m1,m2]=size(elev1);

        elev2=elint-elev1;
        n=1;
        while n <= m1;
            if elev2(n)<0;elev2(n)=0;
```

```
            else elev2(n)=elev2(n);
            end;
        n=n+1;
        end

        q=1;length=zeros(m1-1,1);
        dist2=zeros(m1-1,1);
        while q <= m1-1;
        dist2(q)=dist1(q+1)-dist1(q);
        q=q+1;
        end

        r=1;
        while r <= m1-1;
            if elev2(r)==0 && elev2(r+1)==0; length(r)=0;
            else length(r)= dist2(r);
            end;
        r=r+1;
        end
        width(p)=sum(length);

        m=1;
        trap=zeros(m1-1,1);
        while m <= m1-1;
            trap(m)=abs((dist1(m+1)-dist1(m))*(elev2(m+1)+elev2(m)))/2;
            m=m+1;
        end
        aout(p)=sum(trap);

        t=1;
        hypo=zeros(m1-1,1);
        while t<=m1-1;
            if elev2(t)==0 && elev2(t+1)==0; hypo(t)=0;
            else hypo(t)=sqrt((dist1(t+1)-dist1(t))^2+(elev2(t+1)-
elev2(t))^2);
            end;
            t=t+1;
        end
        perim(p)=sum(hypo);
        p=p+1;

elint=elint+interv;
end

% Outputs
H=elinterv;
W=width;
A=aout;
P=perim';
```

APPENDIX 2

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