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Brian Coulter
Richard Ivey School of Business

Joel Shapiro
Saïd Business School

Peter Zimmerman
Saïd Business School

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A Mechanism for LIBOR*

Brian Coulter[†] Joel Shapiro[‡] Peter Zimmerman[§]

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Abstract

The investigations into LIBOR have highlighted that it is subject to manipulation. We examine a new method for constructing LIBOR that produces an unbiased estimator of the true rate. LIBOR itself is based solely on transactions. We allow for fines when a bank's transaction is different than a comparison rate, which depends on the set of transactions and non-manipulated rates elicited by a revealed preference mechanism. These non-manipulated rates will always be used in the fines, but transactions may not. We address how this approach applies to other financial benchmarks and how it works even in markets in which there are few transactions.

Keywords: LIBOR, benchmark

JEL Codes: D82, G21, G28

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[†]Richard Ivey School of Business, Western University, London, Ontario, Canada. Email: bcoulter@ivey.ca.

[‡]Saïd Business School, University of Oxford, Park End Street, Oxford OX1 1HP. Email: Joel.Shapiro@sbs.ox.ac.uk. Phone: +44 1865 288973.

[§]Saïd Business School, University of Oxford, Park End Street, Oxford OX1 1HP. Email: Peter.Zimmerman@sbs.ox.ac.uk.

1 Introduction

The London Interbank Offered Rate (LIBOR) is supposed to represent the average rate at which banks can borrow in the unsecured market. It is computed by taking the trimmed mean of the daily reported borrowing rates of the banks on the LIBOR panel.[1] Panel banks may want to manipulate LIBOR to profit off of their exposure to the benchmark. Manipulating one of the rates by even a fraction of a basis point can bring substantial gains: the market for derivative and loan products that use LIBOR rates has been estimated at greater than $300 trillion (Wheatley, 2012). The ongoing LIBOR scandal has already resulted in fines of over $9 billion for inappropriate submissions and repeated attempts to manipulate LIBOR.[2]

There is a clear need to reform the process that determines LIBOR. Regulators and ICE, the current administrator of LIBOR, have pushed to make LIBOR more transaction-based.[3] We take this as our starting point, and then refine it using two tools. First, we propose to add a revelation mechanism, which we call the 'revealed preference algorithm'. This will elicit the rates at which the banks on the LIBOR panel would lend to one another at a given point in time. Second, we create a comparison rate using the elicited rates and the set of transactions to define which banks' transactions appear to be manipulated and issue fines to those banks. This reduces manipulation and produces an unbiased estimate of the true rate. We set the fines and the comparison rate to minimize the variance of this estimate. The optimal choice may involve using only the elicited rates for fines (and will always involve those rates).

The model works as follows. First, the administrator designs the fines and the comparison rate. Banks choose their transactions, taking into account that they can potentially manipulate LIBOR but may be fined for doing so. The administrator sets LIBOR using only the banks' transactions. Then, the administrator elicits rates at which each bank would lend to one another using the 'revealed preference algorithm' (RPA). The RPA requests the rate at which every other bank would lend to a given bank. To ensure truthful reporting, a threshold rate is then chosen randomly, and if the offered rate was below this threshold rate, then the offering bank must (synthetically) lend with positive probability.[4] As the LIBOR calculation does not include RPA rates, banks cannot influence

[1] Currently, this is done by removing the top and bottom quartiles of rates submitted, and taking the arithmetic mean of the rest. See ICE, 'LIBOR: Frequently asked questions', https://www.theice.com/publicdocs/IBA_LIBOR_FAQ.pdf.

[2] To date, the scandal has hit a range of global banks, with the largest fines imposed on Deutsche Bank ($3.5 billion) and UBS ($1.5 billion). Further investigations are still underway, and the scandal has been linked to manipulation of other benchmark rates. At the time of this writing, one trader had been convicted of criminal charges in relation to LIBOR manipulation, receiving a prison sentence of 11 years. See Council on Foreign Relations, "Understanding the LIBOR Scandal," http://www.cfr.org/united-kingdom/understanding-libor-scandal/p28729.

[3] See, e.g., https://www.theice.com/publicdocs/ICE_LIBOR_Roadmap0316.pdf

[4] Lending would be to the LIBOR administrator, and the promised repayment would be made only if an equivalent loan to the bank would have been repaid. Thus, a synthetic bank loan may be created, which market participants will

LIBOR by their actions in the algorithm. Finally, the administrator may fine any of the banks for a transaction that it considers manipulated, using both the complete set of transactions and the elicited RPA rates in the comparison rate to determine if the transaction was, indeed, manipulated.

The model has several novel features. First, there are two sources of information about banks that the administrator doesn't know: banks' LIBOR exposures and banks' borrowing costs. To simplify the model, we assume that banks know each other's borrowing costs (but LIBOR exposures remain private information). Second, we note that banks may bias rates either to manipulate the LIBOR rate or to avoid any punishments that the LIBOR administrator chooses to impose.[5] Third, we show that in situations in which there are very few transactions, such as crisis times, the revealed preference algorithm can be used directly to generate the LIBOR benchmark.[6]

We examine the issue of collusion in the context of our model. Clearly, the events of the past few years have shown that a LIBOR reporting mechanism is potentially subject to inappropriate submissions both by banks in isolation[7] and by banks colluding.[8] We define collusion as an agreement by two or more banks that is not enforceable in a court of law, and we show that collusion may still exist in our mechanism although some elements of the mechanism serve as mitigants.

The LIBOR manipulation scandal triggered investigations into the manipulation of other benchmarks. We discuss how our model applies to other benchmarks that have been subject to manipulation, such as the WM/Reuters 4pm Foreign Exchange fix and ISDAFIX. In short, our model could also be used quite easily in those fixings with very little adaptation.

Our approach is complementary to the proposals made in recent regulatory reviews. Martin Wheatley (2012), in his review initiated by the British Chancellor of the Exchequer (the "Wheatley Review"), made three principal suggestions: (1) maintain LIBOR as based on a reporting mechanism; (2) tie LIBOR reports more to actual transactions; and (3) reduce the number of tenors (maturities) and currencies for which a LIBOR rate exists when there are limited transactions. This third point has already been implemented. The Financial Stability Board (2014), in a more recent and wider-reaching review (the "FSB Review"), suggested: (1) retaining the LIBOR rate but strengthening it by tying it "to the greatest extent possible [to] transactions data"; and (2) augmenting LIBOR

view as equivalent to lending directly to the bank in question. We discuss this further in the text.

[5]Another motivation for banks to manipulate the rates they submitted was to disguise their credit risk. Until recently, LIBOR rates submitted by banks were published immediately. This created the potential for stigma, particularly during the height of the financial crisis. Recent reforms now mean that LIBOR submissions are made public only after three months, so this incentive has been reduced and we do not consider it here.

[6]Duffie and Stein (2015) show this to be a relevant consideration: in 2012, even in USD, there were days on which no transactions occurred in either the one-month or the six-month tenor.

[7]In the case against UBS, for example, the Financial Services Authority (FSA) found more than 1000 examples of inappropriate submissions by UBS acting alone (Financial Services Authority, 2012).

[8]Liam Vaughan, Gavin Finch, and Lindsay Fortado, "UBS Trader Hayes exposed at core of LIBOR investigation," *Bloomberg*, 19 December 2012.

with additional, nearly risk-free reference rates.

We take the imperative to retain the LIBOR rate as a primary benchmark as given, as both reviews describe the drawbacks of moving to other measures.[9] These drawbacks include the lawsuits that could arise from a wholesale transition to a new benchmark, substantial transaction costs, and even the potential for broader financial instability.[10] Keeping in mind the two reviews' goal to increase the relevance of transactions to the benchmark, our approach uses transactions directly to calculate LIBOR. However, we show that transaction data are best augmented with the data generated through the revealed preference algorithm when assigning fines to banks for potential manipulation, to the point where transaction data is often best replaced entirely by the output of the RPA. Finally, while we do not dispute the potential benefits of augmenting LIBOR with additional benchmarks, improving the LIBOR mechanism reduces the need for a costly transition.

1.1 Related Literature

Since the LIBOR scandal came to light, some papers have examined the evidence on LIBOR manipulation. Kuo, Skeie, and Vickery (2012) find that LIBOR tracks alternative measures of interbank borrowing costs (the Federal Reserve's Term Auction Facility, Fedwire, and ICAP's New York Funding Rate) but was lower at the height of the recent crisis despite the expectation that it would be higher. LIBOR is also less diffuse than these other measures. These results suggest some degree of manipulation. Similarly, Snider and Youle (2014) demonstrate in a theoretical model that rate manipulation should lead to the bunching of reports around the cutoffs for the interquartile range, and then show that the LIBOR data have this property. Finally, using an econometric model identified by banks' rank order of LIBOR submissions, Youle (2014) finds that manipulation downwardly biased the benchmark by eight basis points in the period between 2007 and 2009.

Some papers examine possible reforms to LIBOR. Duffie and Dworczak (2014) look at a mechanism design problem in which the administrator decides on optimal weights to assign to transactions that may have been manipulated. In contrast to our paper, only one bank's transactions are considered, and the administrator is restricted to not impose fines and does not elicit information from the banks themselves. Moreover, Duffie and Dworczak allow loan sizes to vary (we do not study this aspect) and the mechanism designer assigns greater weight to larger loans. Eisl, Jankow-

[9] Duffie, Dworczak, and Zhu (2016) show that, in general, benchmarks may increase trade, improve matching, and reduce search costs.

[10] Moreover, while other benchmarks exist, they measure subtly different things than LIBOR. For example, while a benchmark based on CDS premia would accurately compile information about bank credit risk, it would fail to account for costs of funding (liquidity). Similarly, a benchmark based on repo rates would reflect bank funding costs, but fail to fully account for bank credit risk. Both reviews point out that LIBOR cannot be replicated by any single pre-existing instrument.

itsch, and Subrahmanyam (2014) show that setting LIBOR equal to the median submission (the extreme version of a trimmed mean) lowers the benefits of manipulation. Youle (2014) also argues for using the median submission and shows that it could reduce manipulation by a substantial amount. Duffie, Skeie, and Vickery (2013) show that the absence of current transactions may be mitigated by using a sample window of previous transactions. Unlike these papers, however, ours focuses on eliciting information through the revealed preference algorithm and using that information (with fines for misreporting) alongside transaction data to improve the accuracy of the mechanism.

Both the Vickrey-Clarke-Groves (VCG) and d'Aspremont and Gerard-Varet (AGV) mechanisms can incentivize agents to truthfully report their private information. These mechanisms do not work in the LIBOR setting, however, as they fail when there is more than one source of private information. In the LIBOR case, there are two important sources of private information: banks' exposures to LIBOR and banks' actual borrowing costs.[11]

The mechanism design literature also includes a number of possible approaches when agents have some information over each other's type. In this type of problem, for example, the principal may wish to maximize social welfare but does not know the individual agents' utility functions. Demski and Sappington (1984) show that, while a traditional mechanism design approach generates a mechanism that can achieve the societal first-best and maximize the principal's payoff, this mechanism is plagued by problems of multiple equilibria. Ma, Moore and Turnbull (1988), Cremer and McLean (1985), Ma (1998), and Moore and Repullo (1998) consider similar problems. The use of the revealed preference algorithm that we propose bears some similarity to Moore and Repullo (1998), as we use a mechanism to generate information from panel banks that we then use to punish a reporting bank if it is deemed to have manipulated. However, we do not require as they do that all agents to have all information about other agents; in our model, LIBOR exposures are private information.

2 Assumptions and definitions

We begin by describing the market for loans. We take a reduced form approach that is similar to the setup in Duffie and Dworczak (2014). As in their paper, manipulation of transactions cannot be eliminated, but a mechanism can be designed to minimize it. Our model differs from theirs in three important respects. First, we allow the administrator to impose a fine on banks when they are suspected of manipulation, while they rule out the use of fines. Second, we consider a panel

[11] Chen (2016) demonstrates that the AGV mechanism can solve misreporting incentives when all banks have known exposures in the same direction. We allow for exposures to be private information (and to be in either direction for any bank) and make banks' borrowing costs private, thus making the AGV and VCG mechanisms inapplicable.

of banks, whereas they consider one representative bank. This is important, as: (i) we consider the use of all banks' transactions to determine whether an individual bank is manipulating; and (ii) we must, therefore, consider the problem where there is a strategic interaction between banks' choices of their transaction rates. Third, we introduce a mechanism (the revealed preference algorithm) that elicits information that is used by the LIBOR administrator to detect and punish manipulation.

There are n banks on the LIBOR panel. LIBOR is set daily at 11am. Each bank i, where $i \in \{1,...,n\}$, carries out T transactions of size $\geq \bar{q}$ with other banks during the reporting window. A transaction consists of the bank borrowing an amount at a fixed rate from another bank in the interbank market. The lender need not be a LIBOR panel bank. We discuss the determination of the rate below.

The reporting window lasts from the previous setting of LIBOR until 10:59am. The amount $\bar{q}$ is the threshold size above which the administrator wishes to include transaction rates in LIBOR. It is exogenously determined and common knowledge.

We assume that there is a true market rate Y that measures the market risk underlying all transactions for the reporting window. The true market rate Y is unobserved by any bank or the administrator. Write:

$$Y = y + \zeta, \tag{1}$$

where y is known to be the mean, and ζ is an error term with mean zero and variance σ_Y^2.

Let $t \in \{1,...,T\}$ index the transactions. The index t does not refer to time, but simply to the $t-$th transaction for a given bank. Let X_i^t be the market rate for each transaction t — that is, the market clearing rate at which bank i can borrow when there is *no* manipulation.

Each market rate X_i^t consists of two parts:

$$X_i^t = Y + \varepsilon_i^t, \tag{2}$$

where Y is the true market rate and is unobserved, and ε_i^t is an error term that is independent and identically distributed with mean zero and variance σ_ε^2.[12]

We make the following assumption on the information structure:

Assumption 1. *All of the banks are endowed with knowledge of the market rates $\{X_i^t\}_{\forall i,t}$. The administrator does not know this information.*

We assume that banks' knowledge of market rates comes from their participation in the money

[12]Our results are robust to including a bank-specific premium term — i.e. $X_i^t = Y + \beta_i + \varepsilon_i^t$, where β_i is constant for each bank and can be observed by the administrator. There are good reasons to think that such a term, if not time-varying, may be observable by an administrator with a large archive of historical transaction data.

markets and their monitoring of counterparties. This assumption represents the fact that the banks are likely to know much more about the market than the administrator does. In reality, it is likely that banks may know only a subset of the market rates; that is, (a) bank i knows all of its own market rates (the $\{X_i^t\}_{\forall t}$), and (b) banks $j \neq i$ know those market rates X_i^t where they are either the bank who is lending to bank i or they were bidding to lend to bank i but lost out. Assumption 1 makes the model more tractable by assuming that all banks have symmetric information about market rates.

Define $\bar{X}_i$ as the average market rate for bank i:

$$\bar{X}_i \;:=\; \frac{1}{T}\sum_{t=1}^{T} X_i^t \;=\; Y + \bar{\varepsilon}_i, \qquad \text{where } \bar{\varepsilon}_i := \frac{1}{T}\sum_{t=1}^{T} \varepsilon_i^t, \tag{3}$$

and $\hat{X}$ as the average market rate for all banks:

$$\hat{X} \;:=\; \frac{1}{n}\sum_{i=1}^{n} \bar{X}_i \;=\; Y + \hat{\varepsilon}, \qquad \text{where } \hat{\varepsilon} := \frac{1}{n}\sum_{i=1}^{n} \bar{\varepsilon}_i. \tag{4}$$

Each bank i privately observes R_i, its own exposure to LIBOR. This exposure may be positive or negative.[13] We assume that this exposure is linear — i.e., the payoff from a change in LIBOR is R_i multiplied by the change in LIBOR. Each R_i is independent and identically distributed with zero mean and variance σ_R^2 and is independent of all the other random variables.[14] Let $\hat{R} = \frac{1}{n}\sum_{i=1}^{n} R_i$, the average LIBOR exposure across the banks.

Every bank i chooses a transaction rate s_i^t for each transaction t. This is the actual rate at which the bank borrows, and is observed in the market and used for the LIBOR calculation. If s_i^t is different from X_i^t, then we say that bank i is manipulating. Manipulation costs the bank an amount $d(s_i^t - X_i^t)$, where $d(\cdot)$ is a convex symmetric non-negative function. Broadly, this cost may represent borrowing at an uneconomic rate or needing to compensate a counterparty for lending at an uneconomic rate. We assume that it is quadratic, and $d(x) = \frac{\delta}{2}x^2$, where $\delta > 0$ is a fixed parameter.

We assume for simplicity that all banks choose their transaction rates simultaneously.

For a given bank i, let $\bar{s}_i$ be the simple average of its transaction rates:

$$\bar{s}_i \;:=\; \frac{1}{T}\sum_{t=1}^{T} s_i^t. \tag{5}$$

[13]The term R_i represents bank i's exposure to the LIBOR benchmark. Banks write contracts which are indexed to LIBOR, and so are exposed to changes in the benchmark. For example, a bank may issue a "fixed for floating" swap in which it enters into an agreement to pay a counterparty a stream of payments indexed to LIBOR, in exchange for a stream of payments at an agreed fixed rate. This contract moves out of the money as LIBOR rises and moves into the money as LIBOR falls. For the issuing bank R_i is negative, while for the counterparty R_i is positive.

[14]Our results are robust to assuming a non-zero mean for R_i.

After the banks carry out their transactions, the LIBOR calculation process begins. Banks' transaction rates $\{s_i^t\}_{\forall i,t}$ are automatically submitted. Let the set of all transaction rates be S. The administrator uses these to compute LIBOR according to an aggregation function $\mathbb{L}(S)$. We assume that the LIBOR aggregation function is a simple average of the banks' transaction rates.

Assumption 2 (LIBOR aggregation function)**.** *The LIBOR aggregation function is a simple average of banks' transaction rates; i.e.,*

$$\mathbb{L}(S) := \frac{1}{nT}\sum_{j=1}^{n}\sum_{t=1}^{T} s_j^t. \tag{6}$$

This assumption of a simple average is an approximation to what is done in practice, where LIBOR is a trimmed mean of the submitted rates. As long as the number of outliers discarded is small compared to the number of banks n, a simple average is a reasonable approximation for the trimmed mean.

The revealed preference algorithm (RPA) is run immediately after LIBOR is calculated. At the time that the RPA is run, a given bank j's willingness to lend to bank i is assumed to be distributed identically to the market rates X_i^t. Let the lowest rate at which bank j still profits from making a loan of size $\bar{q}$ to bank i be the willingness to lend $w_{ij} = Y + \varepsilon_i^w$, with ε_i^w distributed identically to ε_i^t (implying, most importantly, that the variance of ε_i^w is equal to σ_ε^2). The output of the RPA is the lowest *stated* rate at which bank j is willing to lend to bank i, χ_{ij}, where $j \in \{1,..i-1,i+1,..,n\}$. We will show that the unique equilibrium involves the stated rate equalling the willingness to lend, i.e., $\chi_{ij} = w_{ij}$.

The administrator then may use the actual transactions and the rates elicited from the RPA to determine whether a bank manipulated its transactions. We will later define a comparison rate, $\hat{Y}_i$, that the administrator develops for this purpose. When bank i transacts at a rate which deviates from this comparison rate by an amount $s_i^t - \hat{Y}_i$, the administrator will fine the bank an amount $p(s_i^t - \hat{Y}_i)$. We make the following assumption about the fine:

Assumption 3. *We assume that the fine is quadratic in form and that there is zero fine when a bank's transaction rate matches the comparison rate; i.e., $p(s_i^t - \hat{Y}_i) = \frac{a}{2}(s_i^t - \hat{Y}_i)^2$, where $a \in [0,A]$ is a choice function of the administrator.*

The constraint $a \leq A$ in Assumption 3 implies that the LIBOR administrator cannot levy arbitrarily high punishments. One reason for this could be industry lobbying; another could be that a financial stability regulator forbids punishments that are so large that they could threaten a bank's solvency. Similarly, $a \geq 0$ means that the administrator cannot levy a negative punishment — that is, reward a bank for transacting at rates close to the comparison rate.[15] Finally, as the setup of the

[15]One justification for this assumption is that public opinion prevents the administrator from paying banks simply for

problem is symmetric, the administrator considers only symmetric punishment functions.

We now summarize the timing of the model:

1. The administrator sets the parameter for the fine a and the structure of the comparison rate $\hat{Y}_i$.
2. A true market rate Y is realized for the reporting window.
3. For each bank $i \in \{1,\dots,n\}$, its LIBOR exposure R_i is realized and observed by bank i.
4. For each transaction $t \in \{1,\dots,T\}$, bank i's market rate X_i^t is realized and observed by all banks.
5. The transaction rates s_i^t are chosen simultaneously by all banks i for all t.
6. The administrator observes all transactions and sets LIBOR as $\mathbb{L}(S)$.
7. The administrator runs the revealed preference algorithm, which generates $n-1$ stated rates $\{\chi_{ij}\}$ for each bank i, where $j \in \{1,..i-1,i+1,..,n\}$.
8. The administrator calculates a comparison rate $\hat{Y}_i$ using the set of transactions S and the stated rates generated by the RPA $\{\chi_{ij}\}$. The administrator levies fines $p(s_i^t - \hat{Y}_i)$ based on the deviation of transaction rates from the comparison rate $s_i^t - \hat{Y}_i$.

While it is standard to proceed by solving the game backwards, we will begin by going through the revealed preference algorithm, as it is useful to understand how the algorithm elicits rates truthfully, and as it will be used as an input for the administrator to construct the comparison rate for fining misbehaving banks. We use Perfect Bayesian Equilibrium as our solution concept.

3 The Revealed Preference Algorithm

The revealed preference algorithm (RPA) is a mechanism designed to elicit the market rates from banks.[16] It is run for each bank $i \in \{1,...,n\}$. First, each bank $j \in \{1,..i-1,i+1,..,n\}$ is asked the lowest rate at which it would be willing to lend $\bar{q}$ to bank i at the time the algorithm is run. The rate stated by bank j is denoted by χ_{ij}. Then, the administrator chooses a threshold rate v_i according to a probability distribution $F(\cdot)$. The administrator commits to $F(\cdot)$ in advance and this is common knowledge. $F(\cdot)$ has full support.[17] The selection of the threshold v_i is independent of all of the stated rates χ_{ij} — i.e., it is not impacted by banks' statements of willingness to lend to bank i. If no bank j states $\chi_{ij} \le v_i$, the algorithm ends. Otherwise, one of the banks j that states $\chi_{ij} \le v_i$ is chosen with equal probability. This bank must 'synthetically' lend an amount $\bar{q}$ to bank i at a rate of v_i. At this point, the algorithm ends.

honest participation in the market.

[16] In Coulter and Shapiro (2015), we use a more elaborate mechanism, entitled the whistleblower mechanism, to elicit borrowing costs.

[17] While the particular distribution chosen will have implications for the cost of the mechanism to the administrator (we discuss this below), any distribution with full support is sufficient.

The synthetic loan is the key part of this mechanism. It occurs with a positive probability and thus incentivizes banks to put their money where their mouth is. This is where the concept of revealed preference is used. That is, if a bank is willing to lend at any amount greater than or equal to its willingness to lend w_{ij}, its optimal strategy is to state a rate that is equal to its willingness to lend, i.e. $\chi_{ij} = w_{ij}$. Bank j is required to make a synthetic loan only if $v_i \geq \chi_{ij}$, so the bank will almost surely profit from the synthetic loan. This induces bank j to reveal all rates at which it makes a positive profit, in order to secure the loan.

We state that the loan must be synthetic, because it would be difficult for the LIBOR administrator to force a panel bank to take on a loan. Instead, we envision that the administrator would itself take on the loan. The administrator demands a loan of $\bar{q}$, which is repaid with interest if and only if a similar loan to bank i would have been repaid. Therefore, this is payoff-equivalent to lending to the reporting bank itself, and could be achieved through some sort of escrow account.[18] Note that the administrator does not have to force the lending bank to make the loan, as the lending bank will find it strictly beneficial to do so.

We formally define the *revealed preference algorithm* as follows:

Revealed Preference Algorithm

The revealed preference algorithm is run for all banks $i = 1, ..., n$ within the LIBOR panel. For each bank i:

1. The administrator commits to using a distribution $F(\cdot)$ with full support to choose threshold rate v_i.
2. All non-i banks j state the lowest rate at which they are willing to lend an amount $\bar{q}$ to (synthetic) bank i. This is the 'output' variable χ_{ij}.
3. The administrator chooses a threshold rate v_i. If no bank j expressed willingness to lend at this rate or below, the algorithm ends. Otherwise, one bank, selected randomly from the set of banks that stated $\chi_{ij} \leq v_i$, must lend $\bar{q}$ to (synthetic) bank i at the rate v_i.

We now prove that truthtelling is the unique equilibrium.

Proposition 1. *The unique Perfect Bayesian equilibrium of the revealed preference algorithm is truthful reporting — that is, for every bank j to state $\chi_{ij} = w_{ij}$ with respect to every other bank i.*

Proof. See the Appendix.

Therefore, by Proposition 1, the revealed preference algorithm stated rates $\{\chi_{ij}\}$ is equivalent to the set of willingnesses to lend $\{w_{ij}\}$ at the time the algorithm is run. This output can be used by

[18] Another potential way to structure this exposure would be to have the lender sell a credit default swap on the reporting bank to the administrator, with the payment adjusted accordingly. This would mean less cash out initially.

the administrator to set the comparison rate and thus calibrate the fine for each bank.

The proof demonstrates that by revealed preference, all banks have a strict incentive to reveal the rates w_{ij} since the algorithm provides them with a positive probability of making a profitable loan by doing so.

We have not placed much importance on the distribution $F(\cdot)$ from which the threshold rate v_i is chosen. This distribution does not directly impact the mechanism — banks report truthfully no matter what the distribution is, as long as it has full support. However, when loans are probabilistically required at higher and higher rates (a thicker right-tail of the distribution), this has a cost to the administrator, who has to take the synthetic loan at a rate that is far off-market.[19]

With the RPA defined, we next turn to the full mechanism.

4 A mechanism to minimize manipulation

The revealed preference algorithm elicits rates that are reported truthfully. However, they are still noisy estimates of the underlying true rate. Gathering more of those estimates would be better, but it may be costly to run the algorithm multiple times. Instead, the administrator can exploit its access to a large source of data that has information on borrowing rates: the transactions themselves.

Regulators (see Financial Stability Board, 2014) and ICE, the current administrator of LIBOR, have determined that LIBOR should be based on transactions to the greatest extent possible. We take that as a given in our model and base LIBOR only on transaction data. Nevertheless, it is obvious that it is still possible to manipulate LIBOR since a bank may transact at rates that it chooses.[20] We counter this by allowing the administrator to set fines for transactions that appear to be manipulated. In order to determine which transactions may be manipulated, we compare each transaction to a comparison rate, consisting of a weighted average of (i) the complete set of transactions and (ii) the rates elicited from the RPA.

We will demonstrate that this produces an unbiased estimate of the true rate. Thus, the administrator sets the weights so as to minimize the variance of the benchmark. We show that the RPA rates will always be used in order to detect manipulation, while the transaction rates may not be used.

[19] Interestingly, this could make the mechanism revenue neutral in expectation, as the administrator receives the fines but pays excessive returns in the RPA.

[20] For example, a borrowing bank could refuse the lowest offers (to push up LIBOR) or solicit lower than market offers by offering direct compensation on other transactions.

4.1 The comparison rate for bank i

The administrator can impose a fine on each reporting bank i if the transaction rates deviate from a comparison rate for bank i, chosen by the administrator. We assume that, for a transaction s_i^t, the administrator uses a comparison rate $\hat{Y}_i$ consisting of a linear combination of the average transaction rate of all banks other than i, and the average of all RPA rates other than those stated by bank i:

$$\hat{Y}_i := \alpha \cdot \frac{1}{(n-1)} \sum_{j \neq i} \bar{s}_j + (1-\alpha) \cdot \frac{1}{(n-1)^2} \sum_{k=1}^{n} \sum_{j \neq k,i} \chi_{kj}, \tag{7}$$

where $\alpha \in [0,1]$ is a weighting parameter chosen by the administrator, $\bar{s}_j$ is the average transaction rate submitted by bank j, and χ_{kj} is the rate at which bank j stated that it would lend to k in the RPA.[21] The fine for each transaction is then given by the function $p(s_i^t - \hat{Y}_i)$ defined in Assumption 3. Note that $\hat{Y}_i$ is independent of t.

In Equation (7), we include all RPA rates that are not influenced by bank i. These comprise all of the RPA rates at which other banks state they would lend to bank i, plus all of the RPA rates for lending to non-i banks, other than those stated by bank i. In fact, given the setup, we could use any subset of these rates, rather than all of them. This is because when choosing a transaction rate, bank i will take expectations over $\hat{Y}_i$. The expectation of each of the χ_{kj} in Equation (7) is the same: it is the expectation of Y given bank i's information.[22] Therefore it makes no difference which subset of RPA rates are used in Equation (7).[23]

Since the comparison rate $\hat{Y}_i$ is independent of all transaction rates chosen by i and all RPA rates input by i, it cannot be directly manipulated and creates no incentive to strategically lie about lending rates in the RPA. Furthermore, there are no LIBOR-related incentives for the banks to manipulate the RPA rates as these rates are not included in the LIBOR calculation.

[21] As a robustness check, we have considered a more general comparison rate which includes bank i's own transaction rates for all $u \neq t$; i.e. it takes the following form:

$$\hat{Y}_i^t := \alpha_1 \cdot \frac{1}{(n-1)} \sum_{j \neq i} \bar{s}_j + \alpha_2 \cdot \frac{1}{(T-1)} \sum_{u \neq t} s_i^u + (1-\alpha_1-\alpha_2) \cdot \frac{1}{(n-1)^2} \sum_{k=1}^{n} \sum_{j \neq k,i} \chi_{kj},$$

where we have added a t index to the comparison rate because there is now a dependence on t. It turns out that the administrator finds it optimal to set α_2 to zero, in order to prevent the possibility of bank i being able to manipulate its own comparison rate. Therefore, we do not consider this formulation in our paper. The derivation is available upon request.

[22] We will discuss this expectation in detail in the following subsection.

[23] In fact, there are situations (not modelled) in which including as many RPA rates as possible would be useful. For example, suppose that the administrator faces an expected net revenue constraint, so that the expected amount of money raised from fines must not exceed the expected amount paid out in the RPA mechanism. As the fine is quadratic in form, its expectation is related to the variance of the comparison rate, which is strictly decreasing in the number of rates used. Thus including more RPA rates loosens the constraint.

In choosing the weight on transactions α, the administrator faces a trade-off. Increasing the value of α places more weight on the $(n-1)T$ transaction rates submitted by all other banks, but these can be distorted by those banks' manipulations. In contrast, the RPA rates are free from manipulation by any bank. However, there is an issue with the RPA rates. When the banks take expectations of the RPA rates, those expectations will use two sources of information. First, there will be information the administrator doesn't know; i.e., the market rates. It is good for the administrator for the bank to use this information when setting its transaction rate. Second, there will be information the administrator does know; i.e., the mean y of the true rate Y. It isn't useful to the administrator for the bank to use this information when setting its transaction rate, as it is common knowledge. Depending on how the banks weight these two sources of information, the administrator may benefit more from using the transactions in the comparison rate than from using the RPA rates.

4.2 The bank's problem

When banks choose their transactions rates, they do not know the value of the true market rate Y but can infer it from two pieces of information. First, they know its unconditional mean y. Second, they know all of the banks' market rates $\{X_i^t\}$, and each of these is an unbiased estimate of Y. As all banks know these two pieces of information, they have the same expected value of Y.[24]

Let each bank's expectation of the true market rate Y be:

$$e := \mathbb{E}_i[Y] = \gamma y + (1-\gamma)\hat{X}, \qquad \forall i = 1,\dots,n, \tag{8}$$

where $\mathbb{E}_i[\cdot]$ reflects expectations given bank i's information $\{\{X_j^t\}_{\forall j,t}, R_i\}$ and the term $\gamma \in [0,1]$ is a weighting parameter. The term $\hat{X}$ is the average of all the banks' market rates.

For any weight γ, the expectation e is an unbiased estimator of the true market rate Y. As each bank is risk-neutral, we will not be able to deduce an optimal value for γ, and so it remains an exogenously-determined parameter. We further assume that all banks choose the same γ, for tractability. The efficient estimator — i.e., that which minimizes $\mathbb{E}_i[(e-Y)^2]$ — corresponds to $\gamma^* = \frac{\sigma_\varepsilon^2}{nT\sigma_Y^2+\sigma_\varepsilon^2}$. If the banks had some degree of risk-aversion it would be optimal to choose $\gamma = \gamma^*$. We will discuss the effect of the parameter γ on the solution.

Given this, all banks k form the same expectations about the RPA rates $\mathbb{E}_k[\chi_{ij}] = e$ for any i and j. In other words, all banks have the same expectations about all banks' RPA rates, including their own.

[24] The only information that banks do not share are their own LIBOR exposures R_i, but there is no rationale for a bank to base its expectations of the true market rate Y on its own exposure to LIBOR.

Bank i chooses each transaction rate s_i^t, where $t \in \{1, \ldots, T\}$, to maximize its expected payoff:

$$\Pi_i := R_i \mathbb{E}_i[\mathbb{L}(S) - \mathbb{L}(S_{-i}, \{X_i^t\})] - \sum_{t=1}^{T} d(s_i^t - X_i^t) - \sum_{t=1}^{T} \mathbb{E}_i[p(s_i^t - \hat{Y}_i)], \tag{9}$$

where $(S_{-i}, \{X_i^t\})$ is the set of transaction rates with each s_i^t replaced by the corresponding X_i^t for $t \in \{1, ..., T\}$; $\mathbb{L}(\cdot)$ is the LIBOR aggregation function; $d(\cdot)$ is the cost of manipulation; and $p(\cdot)$ is the fine function. By choosing a transaction rate different from its market rate, the bank earns a payoff from manipulating LIBOR, suffers a cost of transacting at an uneconomic rate, and pays a fine to the regulator in expectation.

Given our assumptions on the quadratic cost of the manipulation and fine functions, the expected payoff becomes:

$$\Pi_i := R_i \mathbb{E}_i[\mathbb{L}(S) - \mathbb{L}(S_{-i}, X_i^t)] - \sum_{t=1}^{T} \frac{\delta}{2}(s_i^t - X_i^t)^2 - \sum_{t=1}^{T} \frac{a}{2} \mathbb{E}_i[(s_i^t - \hat{Y}_i)^2]. \tag{10}$$

In order to solve for a bank's choice of transaction, we must first find a bank's expectation of other banks' transaction rates. We do this in the following Proposition.

Proposition 2 (Expected transaction rates)**.** *For any bank j, all other banks $i \neq j$ have the same expectation of j's transaction rate $\mathbb{E}_i[s_j^t]$. This expectation is uniquely determined and is a linear combination of the parameters of the model and the banks' market rates:*

$$\mathbb{E}_i[s_j^t] = \underbrace{\frac{\delta}{\delta + a} \cdot X_j^t}_{\substack{\textit{manipulation} \\ \textit{cost effect}}} + \underbrace{\frac{\delta}{\delta + a(1-\alpha)} \left(1 - \frac{\delta + a}{\delta + a(1 + \frac{\alpha}{n-1})}\right) \cdot \left(\sum_{l \neq j} \bar{X}_l + \frac{a\alpha}{\delta + a} \bar{X}_j\right)}_{\substack{\textit{incentive to be close} \\ \textit{to average transaction rate}}} + \underbrace{\frac{a(1-\alpha)}{\delta + a(1-\alpha)} \cdot e.}_{\substack{\textit{incentive to be close} \\ \textit{to average RPA rate}}} \tag{11}$$

Proof. See the Appendix for the full proof. We briefly outline the steps of the proof here. A bank i chooses each of its transaction rates s_i^t to maximize Equation (10). Since other banks' transaction rates affect bank i's payoff, we need to solve for bank i's beliefs about each of the other banks' transaction rates. By taking each bank's expectation of each other bank's first-order condition, and considering that this must be true for all pairs of banks and all parameter values, we can uniquely find the solution.

Banks have common expectations about each other's transaction rates because they have almost entirely overlapping information sets. A bank's LIBOR exposure R_i is the only private information that it has and its peers do not. Given the assumption of zero correlation between these exposures, a bank would not expect its own exposure to LIBOR to be correlated with other banks' transaction rates.

The first term in Equation (11) represents the direct effect of the cost of manipulation δ. When $\frac{\delta}{a}$ is high, the bank's desire to reduce this cost is more important than the desire to be close to the comparison rate, and so the market rate X_j^t has a larger marginal effect on j's choice of transaction rate s_j^t. When $\frac{\delta}{a}$ is low, the bank is much more concerned about the punishment and places relatively little weight on X_j^t.

The second term in Equation (11) reflects the incentive to be close to other banks' average transaction rates to reduce the expected fine. The term is increasing in α, the weight placed on average transaction rates in the comparison rate. When $\alpha = 0$, the term is equal to zero. The term $\sum_{l \neq j} \bar{X}_l$ is the average of other banks' market rates. However, j also anticipates that other banks' transaction rates will be affected by its own market rates $\bar{X}_j$, and it needs to account for these too, albeit weighted by a term $\frac{a\alpha}{\delta + a} < 1$.

The final term in Equation (11) reflects the incentive to be close to the average RPA rate. The expected value of each RPA rate is e. The final term in Equation (11) is decreasing in α, as less weight is placed on the RPA rates in the punishment benchmark, and it equals zero when $\alpha = 1$ and no weight is placed on the RPA rates.

Given the expectations found in Proposition 2, we find each bank's optimal choice of transaction rate and plug those into LIBOR.

Proposition 3 (LIBOR solution)**.** *When each bank i chooses its optimal transaction rate s_i^t, the LIBOR rate is given by:*

$$\mathbb{L}(S) \quad = \quad \frac{\delta \hat{X} + a(1-\alpha)e}{\delta + a(1-\alpha)} \quad + \quad \frac{\hat{R}}{nT(\delta + a)}. \tag{12}$$

Proof. See the Appendix. We use Proposition 2 to solve for s_i^t and then take an average of all banks' transaction rates over $i = 1, \ldots, n$ and $t = 1, \ldots, T$.

The first term in the expression for LIBOR (Equation 12) is a weighted average of the average of the banks' market rates $\hat{X}$ and the estimated RPA rates e. If $\alpha = 1$, then each bank would care about the direct cost of manipulation and about being close to each other's transaction rates. As all other banks have the same concerns, they would all minimize the expected manipulation cost and choose an average transaction rate closer to the average of the banks' market rates $\hat{X}$. In contrast, if $\delta = 0$ and $\alpha = 0$, then banks would care only about anticipating the RPA rates and would choose transaction rates closer to e.

The second term in Equation (12) reflects the incentive for banks to manipulate their transaction rates. This is increasing in the average exposure to LIBOR $\hat{R}$, and is decreasing in nT, because the marginal effect on LIBOR of manipulating transactions is decreasing in nT. It is divided through by $\delta + a$, which reflects the importance of the cost and the fine associated with manipulation.

We now examine the properties of LIBOR and the administrator's design of the fines.

4.3 The administrator's problem

The administrator's objective is to ensure that LIBOR reflects the true market rate Y. Specifically, the administrator aims to minimize the expected squared difference between LIBOR and Y.[25] This means that the administrator will be willing to tolerate manipulations on the part of individual banks, so long as they do not have a large aggregate impact on LIBOR. Its loss function is:

$$\Lambda := \mathbb{E}_A\left[(\mathbb{L}(S) - Y)^2\right], \tag{13}$$

where $\mathbb{E}_A[\cdot]$ denotes expectation with respect to the administrator's information set. The information set contains neither the LIBOR exposure terms $\{R_i\}$, nor the banks' market rates $\{X_i^t\}$.

Examining the argument of the administrator's loss function, we find:

$$\begin{aligned} \mathbb{L}(S) - Y &= \frac{\delta(\hat{X} - Y) + a(1-\alpha)(e - Y)}{\delta + a(1-\alpha)} + \frac{\hat{R}}{nT(\delta + a)}, \\ &= \frac{\delta\hat{\varepsilon} + a(1-\alpha)(-\gamma\zeta + (1-\gamma)\hat{\varepsilon})}{\delta + a(1-\alpha)} + \frac{\hat{R}}{nT(\delta + a)}, \end{aligned} \tag{14}$$

where $\hat{\varepsilon} = \hat{X} - Y$ and $\zeta = Y - y$ represent the error terms of $\hat{X}$ and Y around their means. We can prove the following property about LIBOR.

Proposition 4. *LIBOR is an unbiased estimator of Y.*

This result is straightforward, as the LIBOR exposures $\{R_i\}$ and the error terms $\hat{\varepsilon}$ and ζ all have zero mean under the administrator's information set. Note that this property depends on the fact that LIBOR is calculated using transactions.

[25] This is in line with the objective function in Duffie and Dworczak (2014).

Using this, we can simplify the administrator's loss function:

$$
\begin{aligned}
\Lambda &= \mathbb{E}_A[(\mathbb{L}(S)-Y)^2], \\
&= Var_A[\mathbb{L}(S)-Y], \qquad \text{as } \mathbb{L}(S) \text{ is an unbiased estimator for } Y, \\
&= \underbrace{\left(\frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}\right)^2 \sigma_Y^2}_{\text{uncertainty about } Y} + \underbrace{\left(1-\frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}\right)^2 \frac{\sigma_\varepsilon^2}{nT}}_{\substack{\text{asymmetric information} \\ \text{about banks' market rates}}} + \underbrace{\frac{\sigma_R^2}{n^3T^2(\delta+a)^2}}_{\substack{\text{uncertainty about banks'} \\ \text{LIBOR exposures}}}, \qquad (15)
\end{aligned}
$$

where the final line comes from using the variance operator on Equation (14) and noting that each term is independent with zero mean.

The three terms in this expression for the administrator's loss Λ represent the three fundamental sources of uncertainty that prevent the administrator from accurately estimating the true market rate Y using our mechanism.

The first term in Equation (15) reflects the uncertainty in the value of Y. Banks make their transactions closer to the expected RPA rate when $1-\alpha$ is larger. However, when γ is also large, the expectation of RPA rates more strongly weights the mean y of Y, which is a noisier estimate when σ_Y^2, the variance of Y, is larger.

The second term in Equation (15) reflects the administrator's uncertainty about banks' market rates relative to Y. This term represents the asymmetric information between the banks and the administrator, since the banks know all the market rates with certainty. This becomes more important as banks focus on the market rates: when the cost of manipulation δ is large, banks transact close to their market rates to reduce this cost; and when α is large, banks transact close to the average market rate because the fine is based more on transactions.

The final term in Equation (15) reflects the uncertainty about banks' private exposures to LIBOR, which incentivize manipulation and are unknown to both the administrator and the other banks. This decreases as the number of panel banks n and sample of transactions T increase.

We now solve for the fine that minimizes the administrator's expected loss.

Proposition 5 (Optimal fine)**.** *To minimize the expected loss from manipulation:*

i) The administrator sets the size of fine to its maximum possible value $a^* = A$.

ii) If $\gamma \leq (1+\frac{\delta}{A})\gamma^*$, where $\gamma^* = \frac{\sigma_\varepsilon^2}{nT\sigma_Y^2+\sigma_\varepsilon^2}$, then the administrator sets $\alpha^* = 0$ and uses only the RPA rates in the comparison rate. If $\gamma > (1+\frac{\delta}{A})\gamma^*$, then the administrator sets $\alpha^* = 1-\frac{\delta\gamma^*}{A(\gamma-\gamma^*)}$.

As long as $\sigma_\varepsilon^2 > 0, \delta > 0$ and $A < \infty$, the administrator will always use the RPA rates in the mechanism for any finite panel size n and number of transactions T.

Proof. See the Appendix.

The administrator sets the fine to its maximum level by fixing $a^* = A$. It is optimal to do this when banks' LIBOR exposures are non-zero ($\sigma_R^2 > 0$), in order to disincentivize manipulation.[26] The administrator's loss is everywhere decreasing in the fine size A, so a higher permissible fine leads to more accurate LIBOR submissions. The administrator's choice of α^* (the weight on the average transaction rate in the comparison rate) is non-decreasing in A and, when $\gamma > \gamma^*$, tends to 1 as $A \to \infty$. As the punishment becomes larger, banks dare not manipulate, and so the administrator has fewer concerns about using the average transaction rates.

The administrator always uses the RPA rates in the comparison rate. This demonstrates that eliciting rates through the algorithm strictly improves upon using transactions. It is not surprising that the RPA rates would be used, as they provide additional information. The bonus feature that they are not manipulated can make them strictly more useful than transactions for fining banks. They elicit banks' private information very efficiently. We now discuss this in more detail.

The administrator's optimal weight placed on banks' transactions α^* is weakly increasing in γ, the weight that banks place on y in their estimates of the true market rate. When γ is relatively low, then e (the banks' expected value of Y) assigns more weight to the banks' private information $\hat{X}$ and less to the public information y. As this private information is relevant to the estimation of the true market rate Y, the administrator wants the banks to use it in determining their transaction rates and thus improve the accuracy of LIBOR as an estimator for Y. The administrator can do this by placing more weight on the RPA rates (i.e. by reducing α): as the banks' expectation of each RPA rate is e, they will use their knowledge of $\hat{X}$ in selecting a transaction rate. In fact, when bank places sufficiently low weight γ on y in their estimates of the true market rate Y — and thus sufficiently high weight on their knowledge of the market rates — then the RPA rates become so accurate that the administrator does not need to use transaction rates at all in the comparison rate.

As γ increases, banks place more weight on the public information y in their expectations of the RPA rates, so e becomes a less accurate estimator for Y. The accuracy of the other banks' transaction rates as an estimator for Y is not affected by the value of γ. Therefore the administrator places less weight on the RPA rates in the comparison rate and correspondingly more weight on the average transaction rate; i.e., the administrator increases α with γ.

The cut-off point for γ is related to γ^*, which is the efficient estimator for e — i.e., it is the value

[26] Note that the solution is not explicitly dependent on σ_R^2 (except insofar as the proof of Proposition 5 requires that $\sigma_R^2 > 0$) because the banks' exposures to LIBOR have mean zero. As stated earlier, this solution is robust to this assumption. Suppose that the banks' LIBOR exposures had some known non-zero mean ρ. Then, the administrator would subtract a term in ρ from the average transaction rate in order to produce a LIBOR value that remains an unbiased estimator of Y. A proof of this is available from the authors on request.

of γ that minimizes the variance of $e-Y$. Writing $\gamma^* = 1 - \frac{nT\sigma_Y^2}{nT\sigma_Y^2+\sigma_\varepsilon^2}$, we can think of γ^* as a measure of the degree of asymmetric information between banks and the administrator. The numerator in the fraction is the common uncertainty that both banks and the administrator face in anticipating the value of the true market rate Y. The denominator in the fraction is the total uncertainty faced by the administrator alone. When γ^* is high, information asymmetry is high, and the administrator fears that it will be difficult to determine whether banks are manipulating. The administrator guards against this by setting α^* low, as the RPA rates are robust against manipulation by individual banks.

As the cost of manipulation δ increases, α^* weakly decreases. Equation (12) tells us that the average transaction rate is a weighted average of the average market rate $\hat{X}$ (with weight δ) and the expected RPA rate e (with weight $a(1-\alpha)$).[27] As δ increases, the weight on the average market rate $\hat{X}$ increases, so the administrator balances this by raising $1-\alpha$. Increasing the cost of manipulation causes banks to place too much weight on their market rates, so the administrator adjusts for this by raising the weight on the RPA rates in the comparison rate.

The term γ^* is decreasing in the number of banks n and the number of transactions T, which means that α^* is weakly increasing in n and T. As the panel size or number of transactions increases, the marginal benefit of manipulation falls, and the administrator is more comfortable about using the transaction rates in the punishment function. The term γ^* approaches 0 as nT approaches infinity but never reaches it for any finite nT, so the administrator will place full weight on the transaction rates if and only if the panel size or the number of transactions (or both) becomes infinite.

Our results show that it is always optimal to use the RPA rates in the comparison rate. This is more important when the panel size (n) and the number of transactions (T) are sufficiently low. When n and T are high, punishing banks from deviating from each other's average transaction rates improves as a disciplining device, so the RPA rates become relatively less useful in the comparison rate. However, it is always optimal to place at least some weight on the RPA rates.

We have an additional result on the effectiveness of increasing panel size versus increasing the number of transactions.

Lemma 1 (Increasing the panel size delivers greater benefits than increasing the number of transactions per bank)**.** *Consider two pairs (n_1, T_1) and (n_2, T_2) such that $n_1T_1 = n_2T_2$ and $n_1 > n_2$. Then, the administrator's loss is lower under (n_1, T_1) than under (n_2, T_2). The difference between the losses is increasing in σ_R^2. The administrator's loss tends to zero as n or $T \to \infty$.*

Proof. See the Appendix.

Lemma 1 tells us that, for a given total number of transactions nT, the administrator's loss is lower when the panel size n is high than when the number of transactions per bank T is high. That

[27] There is also a term in the LIBOR exposures $\hat{R}$ which is unrelated to the administrator's choice of α.

is, it is better for the administrator to increase the panel size than the number of transactions.This is because of the law of large numbers: as n increases, the sample of R_is themselves gets closer to the actual distribution and balances out (because the mean is zero). This effect is amplified when σ_R^2 increases.

4.4 Quantifying the benefit of the mechanism

In this subsection, we compare the loss of the administrator in our proposed mechanism to an alternative: the current plan for LIBOR — that is, basing LIBOR solely on transactions with no fines. We have already shown that the solution (A, α^*) is optimal, so it is obvious that the administrator will be worse off in the current plan than in our mechanism. However, it is useful to add some facts about how the administrator's loss differential varies with the parameters of the problem.

We start by defining the administrator's loss for the transaction-based LIBOR with no fines. Setting $a = 0$ in Equation (14) gives us:

$$\Lambda^{\text{plan}} := \frac{\sigma_\varepsilon^2}{nT} + \frac{\sigma_R^2}{n^3T^2\delta^2}. \tag{16}$$

From Proposition 5, the expected loss under our mechanism is:

$$\Lambda^* = \begin{cases} (\frac{A\gamma}{\delta+A})^2\sigma_Y^2 + (1-\frac{A\gamma}{\delta+A})^2\frac{\sigma_\varepsilon^2}{nT} + \frac{\sigma_R^2}{n^3T^2(\delta+A)^2}, & \text{if } \gamma \leq (1+\frac{\delta}{A})\gamma^*, \\ \gamma^{*2}\sigma_Y^2 + (1-\gamma^*)^2\frac{\sigma_\varepsilon^2}{nT} + \frac{\sigma_R^2}{n^3T^2(\delta+A)^2}, & \text{if } \gamma > (1+\frac{\delta}{A})\gamma^*. \end{cases} \tag{17}$$

We can write this as:

$$\Lambda^* = C^2\sigma_Y^2 + (1-C)^2\frac{\sigma_\varepsilon^2}{nT} + \frac{\sigma_R^2}{n^3T^2(\delta+A)^2}, \tag{18}$$

where:

$$C := \min\left\{\gamma^*, \frac{A\gamma}{\delta+A}\right\}. \tag{19}$$

The improvement for the administrator from introducing a punishment mechanism is Δ, where:

$$\begin{aligned} \Delta \quad &:= \quad \Lambda^{\text{plan}} - \Lambda^*, \\ &= \quad 2C\frac{\sigma_\varepsilon^2}{nT} - C^2\Big(\sigma_Y^2 + \frac{\sigma_\varepsilon^2}{nT}\Big) + \frac{\sigma_R^2}{n^3T^2}\Big(\frac{1}{\delta^2} - \frac{1}{(\delta+A)^2}\Big). \end{aligned} \tag{20}$$

It is straightforward to show the following results:

Lemma 2. *The difference in losses Δ between the planned LIBOR and our mechanism is:*

i) increasing in σ_ε^2 (strictly increasing when $C > 0$);
ii) decreasing in σ_Y^2 (strictly decreasing when $C > 0$); and
iii) strictly increasing in σ_R^2.

The improvement from our mechanism, Δ, is increasing in the variance of the errors on the banks' market rates σ_ε^2. As σ_ε^2 becomes larger, the average market rate $\hat{X}$ becomes a less reliable estimator for the true market rate Y. The administrator prefers to incentivize banks to transact close to the banks' estimates of Y, rather than close to their $\{X_i^t\}$. This can be done by placing more weight on the RPA rates in the comparison rate (i.e., by reducing α). Banks estimate the RPA rates using e, which is a more reliable estimate of Y than the administrator can produce alone.

However, the improvement Δ is weakly decreasing in σ_Y^2, the variance of the true market rate. Banks' estimates of Y become less accurate as σ_Y^2 increases, and so it is not optimal to place so much weight on the RPA rates. Instead, it is better for the administrator to incentivize banks to transact close to the market rate, which is a more reliable estimator of Y than e.

The benefit of our mechanism is also increasing in σ_R^2. In the planned LIBOR, the only disincentive to the bank from manipulating comes from the unit cost δ of deviating from its market rate. With the fine mechanism, the cost per unit of manipulation increases to $\delta + A$.

The differential Δ is increasing in the overall size of fine A, as we might expect, and decreasing in the cost of manipulation δ. As manipulation becomes more expensive, the incentive to manipulate is reduced, and so the punishment mechanism is less useful. Finally, the differential decreases to zero as the number of banks n and transactions $T \to \infty$, because then the marginal effect on LIBOR of an individual bank's manipulation tends to zero.

5 Collusion

In this section, we consider the possibility of collusion between panel banks. Collusion is defined as an agreement by at least two banks that is not enforceable in a court of law. In the previous LIBOR mechanism (as administered by the BBA), collusion was problematic. While manipulation was often caused by banks operating in isolation, subpoenaed traders' communications show that banks also worked together to increase the impact of their manipulation on the LIBOR rate.[28] These traders communicated to confirm that they had similar LIBOR exposures (or, potentially, that one had an exposure and the other was hedged), and then they manipulated LIBOR to mutual benefit.

A key element of collusion is that banks must communicate their LIBOR exposures to one another to collude. Their LIBOR exposures must have the same sign for collusion to be mutually

[28] See, for example, the Financial Conduct Authority's Final Notice regarding Deutsche Bank AG at https://www.fca.org.uk/publication/final-notices/deutsche-bank-ag-2015.pdf.

beneficial. However, LIBOR exposures may change over time, making repeated collusion somewhat more difficult.

Below, we discuss possible techniques for collusion against our mechanism and some mitigants. We divide the discussion into an examination of a one-off game, in which it is more difficult to collude, and a repeated game, which facilitates collusion.

5.1 Collusion in a one-off game

Collusion is not possible in the revealed preference algorithm stage in a one-off game. In the revealed preference algorithm, the decisions that banks make about the rates at which they (synthetically) lend are the last decisions that they make in the mechanism. These decisions are made simultaneously across all banks and have no impact on the LIBOR rate — it has already been set by this point in the mechanism. Therefore, each bank maximizes its own utility.[29] Any collusive agreement will unravel during the revealed preference algorithm, as banks optimally deviate when it is their time to act.

Though unable to collude in the revealed preference algorithm stage, banks may still be able to collude in the loan-making stage. If two banks (say, i and j) discover that they have similar LIBOR exposures ($R_i \times R_j > 0$), then they may be able to make offsetting loans (wash trades) to one another either above or below their market rates. Assuming that the loans could be executed simultaneously, they could be enacted without formal contracts and would serve to bias LIBOR. In principle, such trades could be checked for (or even prohibited in the LIBOR calculations).[30] Also, if the loans had to be executed in sequence, one of the banks would refuse to either make or receive the 'last' loan, thus unraveling the collusion. Furthermore, even though this collusion is feasible, our mechanism would still punish the banks for transactions that do not correspond with other banks' transactions and revealed preference rates. Therefore, the administrator does not need to recognize collusion in order to punish it.

5.2 Collusion in a repeated game

Banks have additional scope for collusion in a repeated game. In addition to using offsetting (false) transactions to manipulate LIBOR, collusion in the revealed preference algorithm becomes possible in a repeated game by using threats of future punishment. Collusion is possible in the revealed preference algorithm in isolation, or may be paired with collusion in the loan-making stage. For example, two banks (say, i and j) that collude in making loans could also manipulate their reports

[29] Similarly, banks have no incentive to reduce other banks' punishments in a one-off game.

[30] *Nearly* offsetting loans, and/or 'circles' of loans, could be more difficult to detect but would still become clear over time.

in the revealed preference algorithm to bias the comparison rates $\hat{Y}_i$ and $\hat{Y}_j$ and reduce their punishments. These strategies rely on the future value of the collusive agreement to incentivize adherence and coordination on punishment mechanisms.

6 Application to other financial benchmarks

While we have designed our mechanism to improve the current LIBOR reporting mechanism, it has applicability to a wide range of financial benchmarks. Many of these benchmarks have been potentially subject to the same sorts of manipulation as occurred in the LIBOR benchmark since as far back as the early 1990s.[31]

LIBOR represents the rate at which a selection of global banks can borrow. Therefore, our mechanism is most easily applicable to similar borrowing and lending-based benchmarks. For example, ISDAFIX, now the ICE Swap Rate, is a benchmark of the rate at which banks are willing to enter into interest rate swap transactions. As a benchmark based primarily on self-reporting, ISDAFIX was found in 2013 to have been manipulated by inappropriate submissions, just as the LIBOR benchmark was.[32] Given the similarities between the bilateral nature of LIBOR loans and interest rate swaps, our mechanism could be applied to the ICE Swap Rate benchmark almost exactly as defined. The ICE Swap Rate could be calculated off of realized swap transactions, while comparison rates could be created using transactions and the revealed preference algorithm. The RPA could provide a second check by asking each bank the rate at which it would be willing to swap fixed-for-floating with a specified counterparty.

The broad insights of our mechanism can also be applied to other benchmarks. Gold, silver, foreign exchange, and many other commodities have benchmarks — and many have seen similar benchmark-fixing scandals.[33] Our mechanism can actually be *simplified* to apply to commodity benchmarks: whereas LIBOR is the average of the rate at which a number of banks can borrow, commodities transactions are invariant with respect to seller and buyer. An ounce of gold is the same regardless of the seller and buyer; the same is not true of a loan or a swap.

As an example, consider applying our mechanism to the foreign exchange (Forex) benchmark — likely the second largest recent manipulation scandal.[34] For a given bank i, the comparison rate $\hat{Y}_i$ could again include all Forex transactions that did not involve bank i as either buyer or seller. Then, the revealed preference algorithm could be run, but in a substantially simplified form. Rather than

[31] Douglas Keenan, "My thwarted attempt to tell of Libor shenanigans," *Financial Times*, 27 July 2012.

[32] Matthew Leising, Lindsay Fortado, and Jim Brunsden, "Meet ISDAFIX, the LIBOR Scandal's Sequel," *Bloomberg*, 18 April 2013.

[33] Matt Taibbi, "Everything is Rigged: The Biggest Price-Fixing Scandal Ever," *Rolling Stone*, 25 April 2013.

[34] Matt Levine, "Banks Manipulated Foreign Exchange in Ways You Can't Teach," *Bloomberg*, 12 November 2014.

requiring each bank to submit the level at which it would lend to every other bank, the administrator would simply request the price at which each bank would be willing to purchase or sell the foreign currency. (Whether the request is to purchase or sell could alternate daily, or banks could even be asked for their indifference point between buying and selling.) These offers would be matched up with positive probability (possibly involving the administrator as intermediary), removing the need for the 'synthetic' loan. Our mechanism would thus allow the benchmark to be based off of the volume of everyday transactions, while still generating the revealed preference algorithm results as a secondary check. The imposition of fines based on this combined information would encourage accurate transacting and discourage manipulation of the benchmarks. A larger number of transactions (in our mechanism, higher n and higher T) would help further.

Unfortunately, though, these changes would not necessarily stop collusion. Collusion was one of the main underlying causes of the recent Forex manipulation, and our mechanism can only discourage, not solve, issues of collusion. As discussed in Morrison and Shapiro (2016), changing the underlying culture — something that a single mechanism likely cannot do — may be necessary to solve manipulation in these benchmarks.[35]

Therefore, while our mechanism can be applied to a wide selection of benchmarks, collusion remains a potential problem.

7 Using the RPA when there are no transactions

An extra benefit of the revealed preference algorithm is that it allows the LIBOR mechanism to continue operating even if there are no transactions (or a sufficiently low number that the administrator chooses to ignore them). As Duffie and Stein (2015) show, this is a real concern: in 2012, even in USD, there were instances when no transactions took place at both 1-month and 6-month tenors. In less liquid currencies, and in more stressed market situations, zero-trade days are even more frequent and the problem is exacerbated.

Adapting the mechanism to a situation with few or no transactions requires two changes to the mechanism. First, rather than using realized transaction rates s_i^t to calculate LIBOR, each bank must instead report the rate at which it *could* have borrowed in the LIBOR market.[36] Banks are presumably aware of the rate at which they could borrow through activities in the market and past transactions. This is, in fact, the way that LIBOR has been calculated since its inception. Second, the comparison rates for punishment $\hat{Y}_i$ will be set entirely using rates from the revealed preference

[35] In June 2016, ICE published a code of conduct for LIBOR panel banks to encourage good governance and behavior.

[36] If there are no transactions at all, this is the rate at which it could have borrowed, but did not choose to do so. Otherwise, the borrowing cost can be set to infinite (we discuss this at length in Coulter and Shapiro (2015)).

algorithm (i.e., setting $\alpha = 0$). In this situation, each bank reports the rate at which it could have borrowed, and the LIBOR rate is set. Then, the revealed preference algorithm is run; the comparison rates $\hat{Y}_i$ are determined; and fines are levied for banks that reported transaction rates s_i^t sufficiently different from $\hat{Y}_i$. As before, by setting LIBOR early in the mechanism, LIBOR exposures are immaterial when the revealed preference algorithm is reached. Therefore, banks are incentivized to report their information truthfully.[37]

8 Conclusion

As a wealth of court evidence and subsequent fines have demonstrated, LIBOR rates have been subject to serious manipulation. In this paper, we propose a robust LIBOR mechanism that produces an unbiased estimator of the true market rate and minimizes the variance of its estimate. LIBOR itself is based solely on transactions. We augment this by allowing for fines based on a comparison rate that consists of the set of transaction rates, along with rates elicited from the panel banks using a revealed preference algorithm. The algorithm provides rates that are not manipulated, serving as an essential check on the transactions. We also discuss how this approach would be directly applicable to other financial benchmarks and how it would work even in the absence of transactions, which can occur in illiquid markets.

Since the scandals surrounding benchmarks began, panel sizes for benchmarks have been declining as banks have grown more aware of reputational risks. In addition, lower liquidity in certain markets has caused the number of transactions to drop, reducing incentives to participate on benchmark panels. There is a vicious circle here, as liquidity in certain markets depends on the availability of accurate benchmarks which, in turn, depends on panel participation. Both academics and policymakers have highlighted these issues (e.g., Powell, 2016, and Duffie and Stein, 2015). Our proposed mechanism will be more efficient and accurate than a purely transactions-based LIBOR system. A more accurate benchmark will improve liquidity and incentivize greater participation in the benchmark panel. Thus our proposed mechanism can help break the vicious circle.

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9 Appendix: proofs

9.1 Proof of Proposition 1

LIBOR is set previous to the revealed preference algorithm, and the RPA is invariant to banks' transaction choices. Therefore, LIBOR exposure is immaterial to the outcome of the RPA.

Call the strategy $\chi_{ij} = w_{ij}$ the 'truthtelling strategy'. Let the cumulative distribution function of v_i be $F(\cdot)$. Now, consider some other potential strategy, $\chi_{ij} = w_{ij} + \xi$ where $\xi \neq 0$. We shall show that any such strategy generates lower expected profitability than the truthtelling strategy $\chi_{ij} = w_{ij}$, regardless of the strategies of the other banks.

First, suppose that the bank's strategy is to choose $\xi > 0$. If $v_i \geq w_{ij} + \xi$, then both this strategy and the truthtelling strategy result in the bank lending (synthetically) at v_i with equal probability. (This is the probability that bank j is selected randomly among the set of banks that all stated willingness to lend at v_i or lower.) Similarly, if $v_i < w_{ij}$, then neither strategy results in a synthetic loan being made. A difference arises only if $w_{ij} \leq v_i < w_{ij} + \xi$. In this interval, the truthtelling strategy implies a positive probability of a synthetic loan at $v_i \geq w_{ij}$, for a non-negative profit. However, the strategy $\xi > 0$ does not result in lending when v_i lies in this interval. As v_i falls within this interval with probability $F(w_{ij} + \xi) - F(w_{ij}) > 0$, this implies that a strategy $\xi > 0$ forgoes possible profit and cannot be optimal.

Similarly, a strategy $\xi < 0$ results in the same payoff as the truthtelling strategy whenever $v_i \geq w_{ij}$ (both result in a positive probability of a synthetic loan) and whenever $v_i < w_{ij} + \xi$ (zero probability of a loan). However, it results in a positive probability of unprofitable lending whenever $w_{ij} + \xi \leq v_i < w_{ij}$, while the truthtelling strategy results in a zero payoff. This case occurs with non-zero probability $F(w_{ij}) - F(w_{ij} + \xi)$. Therefore, the strategy $\xi < 0$ has a lower expected payoff than the truthtelling strategy and cannot be optimal.

Therefore, the unique optimal strategy is to state willingness to lend truthfully: $\chi_{ij} = w_{ij}$.

9.2 Proof of Proposition 2

Differentiating Π_i with respect to s_i^t, bank i has the following first-order condition for each t:

$$\begin{aligned}\frac{R_i}{nT} &= \delta(s_i^t - X_i^t) + a\mathbb{E}_i[s_i^t - \hat{Y}_i], \\ &= (\delta + a)s_i^t - \delta X_i^t - a\mathbb{E}_i[\hat{Y}_i].\end{aligned} \tag{21}$$

Now,

$$\begin{aligned}\mathbb{E}_i[\hat{Y}_i] &= \frac{\alpha}{n-1}\sum_{j\neq i}\mathbb{E}_i[\bar{s}_j] + \frac{(1-\alpha)}{(n-1)^2}\sum_{k=1}^{n}\sum_{j\neq k,i}\mathbb{E}_i[\chi_{kj}], \\ &= \frac{\alpha}{n-1}\sum_{j\neq i}\mathbb{E}_i[\bar{s}_j] + (1-\alpha)e.\end{aligned} \tag{22}$$

We can substitute this into (21) to obtain:

$$\frac{R_i}{nT} = (\delta + a)s_i^t - \delta X_i^t - \frac{a\alpha}{(n-1)T}\sum_{j\neq i}\sum_{u=1}^{T}\mathbb{E}_i[s_j^u] - a(1-\alpha)e. \tag{23}$$

We need to solve the system of equations given by (23) for every i and t. To do this, we need to find a solution for $\mathbb{E}_i[s_j^t]$. Let us write:

$$f_{ijt}(\mathscr{P}_i) := \mathbb{E}_i\left[s_j^t \mid \mathscr{P}_i\right], \qquad \forall j \neq i. \tag{24}$$

For any i, j, t, the function f_{ijt} is a function of the known parameters of the system, which is the set $\mathscr{P}_i := \{y, \gamma, R_i, \{X_l^v\}_{\forall l,v}\}$. With a slight abuse of notation, we write $f_{ijt}(y)$ when we want to denote f_{ijt} as a univariate function of y, holding the other members of $\mathscr{P}_i$ constant. Similarly, we write $f_{ijt}(R_i)$ to denote f_{ijt} as a univariate function of R_i, and so forth for the other parameters. We shall prove the Proposition by showing that f_{ijt} is uniquely determined, independent of i, and has the solution given in the statement of the Proposition.[38]

Equation (23) becomes:

$$\frac{R_i}{nT} = (\delta + a)s_i^t - \delta X_i^t - \frac{a\alpha}{(n-1)T}\sum_{j\neq i}\sum_{u=1}^{T} f_{iju}(\mathscr{P}_i) - a(1-\alpha)e. \tag{25}$$

[38] Our proof will not require us to assume that f_{ijt} is continuous or differentiable with respect to the members of $\mathscr{P}_i$. However, we shall show that it is as part of our result.

Choose another bank $k \neq i$ and form expectations of Equation (25) with respect to k's information:

$$0 = (\delta + a) f_{kit}(\mathscr{P}_k) - \delta X_i^t - \frac{a\alpha}{(n-1)T} \sum_{j \neq i} \sum_{u=1}^{T} \mathbb{E}_k \Big[f_{iju}(\mathscr{P}_i) \Big] - a(1-\alpha)\Big(\gamma y + (1-\gamma)\hat{X}\Big). \quad (26)$$

Here, we have decomposed e using (8).

Suppose that banks have different beliefs about s_j^t. Then $f_{ijt}(\mathscr{P}_i)$ must depend on elements that are members of $\mathscr{P}_i$ but not $\mathscr{P}_k, \forall k \neq i$ — i.e., private information that i has and the other banks lack. The only such private information is R_i, but we shall show that f_{ijt} is invariant with R_i. This means that all banks must have the same beliefs.

Equation (26) is valid for all t and all $k \neq i$ and all feasible parameter values, so it is an identity. Choose any pair $R_k \neq R_k'$. Then:

$$0 = (\delta + a)\Big(f_{kit}(R_k') - f_{kit}(R_k) \Big) - \frac{a\alpha}{(n-1)T} \sum_{j \neq i} \sum_{u=1}^{T} \mathbb{E}_k \Big[f_{iju}(R_k') - f_{iju}(R_k) \Big]. \quad (27)$$

$f_{iju}(\mathscr{P}_i)$ cannot depend on R_k because $R_k \notin \mathscr{P}_i$. Therefore, the second term in Equation (27) must be equal to zero, and so f_{kit} must be constant with respect to R_k. In other words, bank k does not use its private knowledge about R_k when forming expectations about other banks. This is because the banks' exposures to LIBOR are independent of one another and so do not provide signals about each other's values. That proves the first part of the Proposition.

This means that we can drop the i index and rewrite $f_{jt}(\mathscr{P}) = \mathbb{E}_i[s_j^t | \mathscr{P}]$, for all $i \neq j$, where $\mathscr{P} = \{y, \gamma, \{X_l^v\}_{\forall l,v}\}$ (i.e., the common intersection of all the banks' information sets). Equation (26) becomes:

$$0 = (\delta + a) f_{it}(\mathscr{P}) - \delta X_i^t - \frac{a\alpha}{(n-1)T} \sum_{j \neq i} \sum_{u=1}^{T} f_{ju}(\mathscr{P}) - a(1-\alpha)\Big(\gamma y + (1-\gamma)\hat{X}\Big), \quad (28)$$

where $\mathbb{E}_k[f_{ju}(\mathscr{P})] = f_{ju}(\mathscr{P})$ because all banks have the same beliefs and use the same information set.

For notational shorthand, define:

$$\Delta_{it}(y, y') = f_{it}(y) - f_{it}(y'), \quad (29)$$

and, similarly, $\Delta_{it}(\gamma, \gamma')$ and so forth for the other parameters.

As before, Equation (26) is valid for all i and t and all possible parameter values, so it is an identity. For any pair $y \neq y'$, we have:

$$0 = (\delta + a)\Delta_{it}(y, y') - \frac{a\alpha}{(n-1)T} \sum_{j \neq i} \sum_{u=1}^{T} \Delta_{ju}(y, y') - a(1-\alpha)\gamma(y - y'). \tag{30}$$

For any given pair (y, y'), Equation (30) is true for all i and all t. This means that, for each such pair, we have a linear system with nT equations. We have nT unknown variables, which are the expressions $\Delta_{it}(y, y')$ for each i, t. The system of equations is linearly independent, so it has at most one solution. Thus, if we can find a solution, it must be unique. It is easy to check that there is indeed a solution, which occurs when each of the unknown variables has the same value:

$$\Delta_{it}(y, y') = \frac{a(1-\alpha)\gamma}{\delta + a(1-\alpha)} \cdot (y - y'). \tag{31}$$

Therefore, a solution exists, and it must be unique. As this expression holds for all (y, y'), we can see that f_{it} is a linear function of y with slope $\frac{a(1-\alpha)\gamma}{\delta + a(1-\alpha)}$.

We can employ a similar argument to show that the slope of f_{it} with respect to any of the other parameters in $\mathscr{P}$ is constant, and so f_{it} is linear with respect to each of these parameters. This occurs because the system of equations defined by Equation (28), $\forall i, t$, is linear with respect to all of these parameters and the functions f_{it}. We go through each of these in turn.

For the terms in X, the situation is a little more complicated. We can take differences of Equation (28) with respect to X_i^t:

$$0 = (\delta + a)\Delta_{it}(X_i^t, X_i^{t\prime}) - \delta(X_i^t - X_i^{t\prime}) - \frac{a\alpha}{(n-1)T} \sum_{j \neq i} \sum_{u=1}^{T} \Delta_{ju}(X_i^t, X_i^{t\prime}) - a(1-\alpha)\frac{1-\gamma}{nT}(X_i^t - X_i^{t\prime}), \tag{32}$$

and with respect to X_l^v, where at least one of $l \neq i$ or $v \neq t$ is true:

$$0 = (\delta + a)\Delta_{it}(X_l^v, X_l^{v\prime}) - \frac{a\alpha}{(n-1)T} \sum_{j \neq i} \sum_{u=1}^{T} \Delta_{ju}(X_l^v, X_l^{v\prime}) - a(1-\alpha)\frac{1-\gamma}{nT}(X_l^v - X_l^{v\prime}). \tag{33}$$

Equations (32) and (33) together define a system of n^2T^2 linear equations in n^2T^2 unknowns. Again, any solution must be unique. To solve this system, first note that, as Equation (33) is true for all i,t and all $(l,v) \neq (i,t)$, it implies that the following two equations are true for all $k \neq i$ and $v \neq t$:

$$\begin{aligned} 0 &= (\delta + a)\Delta_{kt}(X_i^t, X_i^{t\prime}) - \frac{a\alpha}{(n-1)T}\sum_{j\neq k}\sum_{u=1}^{T}\Delta_{ju}(X_i^t, X_i^{t\prime}) - a(1-\alpha)\frac{1-\gamma}{nT}(X_i^t - X_i^{t\prime}), \\ 0 &= (\delta + a)\Delta_{iv}(X_i^t, X_i^{t\prime}) - \frac{a\alpha}{(n-1)T}\sum_{j\neq i}\sum_{u=1}^{T}\Delta_{ju}(X_i^t, X_i^{t\prime}) - a(1-\alpha)\frac{1-\gamma}{nT}(X_i^t - X_i^{t\prime}). \end{aligned} \tag{34}$$

We will guess and verify a solution, which must be unique. Let us postulate the following:

$$\begin{aligned} \forall u, v \neq t: &\qquad \Delta_{iu}(X_i^t, X_i^{t\prime}) = \Delta_{iv}(X_i^t, X_i^{t\prime}), \\ \forall u, v, \forall j, k \neq i: &\qquad \Delta_{ju}(X_i^t, X_i^{t\prime}) = \Delta_{kv}(X_i^t, X_i^{t\prime}). \end{aligned} \tag{35}$$

The first line in (35) postulates that the effect of changing X_i^t on $\mathbb{E}[s_i^u]$ is the same for all $u \neq t$. The second postulates that its effect does not depend on j, for all $j \neq i$. The intuition behind the first is that a change in X_i^t affects a transaction rate s_i^u via the cost of manipulation (which applies only if $u = t$) and via the effect of X_i^t on e and, thus, the estimation of the RPA rates. In the latter channel, X_i^t matters only via its effect on $\hat{X}$ and is not dependent on u. The intuition behind the second equation is that, for $j \neq i$, a change in X_i^t affects j's transaction rate s_j^u via its effect on e and via its effect on i's average transaction rate (which affects the comparison rate for j). Thus, the marginal effect of a change in X_i^t is the same for all $j \neq i$ and all u.[39]

Using (35), equations (32) and (34) become:

$$\begin{aligned} \Big(\delta + a(1-\alpha)\frac{1-\gamma}{nT}\Big)(X_i^t - X_i^{t\prime}) &= (\delta + a)\Delta_{it}(X_i^t, X_i^{t\prime}) - a\alpha\Delta_{jt}(X_i^t, X_i^{t\prime}), \\ a(1-\alpha)\frac{1-\gamma}{nT}(X_i^t - X_i^{t\prime}) &= (\delta + a)\Delta_{jt}(X_i^t, X_i^{t\prime}) \\ &\quad - \frac{a\alpha}{(n-1)T}\Big(\Delta_{it}(X_i^t, X_i^{t\prime}) + (T-1)\Delta_{iu}(X_i^t, X_i^{t\prime}) + (n-2)T\Delta_{jt}(X_i^t, X_i^{t\prime})\Big), \\ a(1-\alpha)\frac{1-\gamma}{nT}(X_i^t - X_i^{t\prime}) &= (\delta + a)\Delta_{iu}(X_i^t, X_i^{t\prime}) - a\alpha\Delta_{jt}(X_i^t, X_i^{t\prime}) \end{aligned} \tag{36}$$

for all t and all $j \neq i$.

[39] Equations (35) do not represent an assumption. We know that any solution is unique, so if (35) were invalid, then they would not be consistent with any solution. However, we shall show that they do indeed lead to a solution, and so must be correct.

The system of equations (36) has the following unique solution, for all $j \neq i$ and all $v \neq t$:

$$\begin{aligned}
\Delta_{jt}(X_i^t, X_i^{t\prime}) &= \frac{1}{T(\delta + a(1-\alpha))}\Big(a(1-\alpha)\frac{1-\gamma}{n} + \delta\Big(1 - \frac{\delta + a}{\delta + a(1+\frac{\alpha}{n-1})}\Big)\Big) \cdot (X_i^t - X_i^{t\prime}), \\
\Delta_{iv}(X_i^t, X_i^{t\prime}) &= \frac{1}{T(\delta + a(1-\alpha))}\Big(a(1-\alpha)\frac{1-\gamma}{n} + \frac{\delta a \alpha}{\delta + a}\Big(1 - \frac{\delta + a}{\delta + a(1+\frac{\alpha}{n-1})}\Big)\Big) \cdot (X_i^t - X_i^{t\prime}), \quad (37) \\
\Delta_{it}(X_i^v, X_i^{v\prime}) &= \Delta_{it}(X_i^t, X_i^{t\prime}) + \frac{\delta}{\delta + a}(X_i^t - X_i^{t\prime}).
\end{aligned}$$

The first and second equations imply that:

$$\begin{aligned}
\Delta_{it}(X_l^v, X_l^{v\prime}) &= \frac{1}{T(\delta + a(1-\alpha))}\Big(a(1-\alpha)\frac{1-\gamma}{n} + \delta\Big(1 - \frac{\delta + a}{\delta + a(1+\frac{\alpha}{n-1})}\Big)\Big) \cdot (X_l^v - X_l^{v\prime}), \\
\Delta_{it}(X_i^v, X_i^{v\prime}) &= \frac{1}{T(\delta + a(1-\alpha))}\Big(a(1-\alpha)\frac{1-\gamma}{n} + \frac{\delta a \alpha}{\delta + a}\Big(1 - \frac{\delta + a}{\delta + a(1+\frac{\alpha}{n-1})}\Big)\Big) \cdot (X_i^v - X_i^{v\prime}).
\end{aligned} \quad (38)$$

Combining Equations (31), (37) and (38), we obtain the following solution for $f_{it}(\mathscr{P})$:

$$\begin{aligned}
f_{it}(\mathscr{P}) &= K + \frac{a(1-\alpha)\gamma}{\delta + a(1-\alpha)} \cdot y \\
&\quad + \frac{1}{T(\delta + a(1-\alpha))}\Big(a(1-\alpha)\frac{1-\gamma}{n} + \delta\Big(1 - \frac{\delta + a}{\delta + a(1+\frac{\alpha}{n-1})}\Big)\Big) \cdot \sum_{l \neq i}\sum_{v=1}^{T} X_l^v \\
&\quad + \frac{1}{T(\delta + a(1-\alpha))}\Big(a(1-\alpha)\frac{1-\gamma}{n} + \frac{\delta a \alpha}{\delta + a}\Big(1 - \frac{\delta + a}{\delta + a(1+\frac{\alpha}{n-1})}\Big)\Big) \cdot \sum_{v=1}^{T} X_i^v + \frac{\delta}{\delta + a} \cdot X_i^t,
\end{aligned} \quad (39)$$

where K is a constant unrelated to these parameters. When all of the parameters are set to zero, Equation (28) implies that $f_{it} = 0$, so we can infer that $K = 0$.

Our final step is to set $K = 0$ and rearrange Equation (39):

$$\begin{aligned}
f_{it}(\mathscr{P}) &= \frac{\delta}{\delta + a}X_i^t + \frac{\delta}{\delta + a(1-\alpha)}\Big(1 - \frac{\delta + a}{\delta + a(1+\frac{\alpha}{n-1})}\Big)\Big(\sum_{l \neq i}\frac{1}{T}\sum_{v=1}^{T} X_l^v + \frac{1}{T}\sum_{v=1}^{T} X_i^v\Big) \\
&\quad + \frac{a(1-\alpha)}{\delta + a(1-\alpha)}\Big(\gamma y + \frac{(1-\gamma)}{nT}\Big(\sum_{l=1}^{n}\sum_{v=1}^{T} X_l^v\Big)\Big).
\end{aligned} \quad (40)$$

Writing the sums of the X terms using Equations (3) and (4), we obtain the required result.

As a final check, we can take differences of Equation (28) with respect to γ (a parameter which was not used) and note that the resulting system of equations is satisfied by the solution. The set of equations produced by differencing with respect to γ is not linearly independent of those we have already studied.

For the purposes of this paper going forward, we require only the expected transaction rate, so we do not write out an exact solution for s_i^t here. This formulation can be easily derived by substituting Equation (40) into Equation (25).

9.3 Proof of Proposition 3

Take the average of Equation (23) over all $i = 1, \dots, n$ and $t = 1, \dots, T$ to obtain:

$$\frac{\hat{R}}{nT} = (\delta + a)\frac{1}{nT}\sum_{i=1}^{n}\sum_{t=1}^{T} s_i^t - \delta\hat{X} - \frac{a\alpha}{n(n-1)T^2}\sum_{i=1}^{n}\sum_{j\neq i}\sum_{t=1}^{T}\sum_{u=1}^{T}\mathbb{E}_i[s_j^u] - a(1-\alpha)e, \tag{41}$$

which becomes:

$$\frac{\hat{R}}{nT} = (\delta + a)\mathbb{L}(S) - \delta\hat{X} - \frac{a\alpha}{nT}\sum_{i=1}^{n}\sum_{t=1}^{T} f_{it}(\mathscr{P}) - a(1-\alpha)e. \tag{42}$$

Using Proposition 2, we have:

$$f_{it}(\mathscr{P}) = \frac{\delta}{\delta + a}\cdot X_i^t + \frac{\delta}{\delta + a(1-\alpha)}\Big(1 - \frac{\delta + a}{\delta + a(1 + \frac{\alpha}{n-1})}\Big)\cdot\Big(\sum_{l\neq i}\bar{X}_l + \frac{a\alpha}{\delta + a}\bar{X}_i\Big) + \frac{a(1-\alpha)}{\delta + a(1-\alpha)}\cdot e. \tag{43}$$

Now,

$$\begin{aligned}
&\frac{1}{nT}\sum_{i=1}^{n}\sum_{t=1}^{T}\Big(\frac{\delta}{\delta + a}\cdot X_i^t + \frac{\delta}{\delta + a(1-\alpha)}\Big(1 - \frac{\delta + a}{\delta + a(1 + \frac{\alpha}{n-1})}\Big)\cdot\Big(\sum_{l\neq i}\bar{X}_l + \frac{a\alpha}{\delta + a}\bar{X}_i\Big)\Big),\\
&= \frac{\delta}{\delta + a}\hat{X} + \frac{\delta}{\delta + a(1-\alpha)}\Big(1 - \frac{\delta + a}{\delta + a(1 + \frac{\alpha}{n-1})}\Big)\Big((n-1)\hat{X} + \frac{a\alpha}{\delta + a}\hat{X}\Big),\\
&= \Big(\frac{\delta}{\delta + a} + \frac{\delta}{\delta + a(1-\alpha)}\cdot\frac{\frac{a\alpha}{n-1}}{\delta + a(1 + \frac{\alpha}{n-1})}\cdot\frac{(n-1)(\delta + a) + a\alpha}{\delta + a}\Big)\hat{X},\\
&= \Big(\frac{\delta}{\delta + a} + \frac{\delta}{\delta + a(1-\alpha)}\cdot\frac{a\alpha}{(n-1)(\delta + a) + a\alpha}\cdot\frac{(n-1)(\delta + a) + a\alpha}{\delta + a}\Big)\hat{X},\\
&= \Big(\frac{\delta}{\delta + a} + \frac{\delta a\alpha}{(\delta + a(1-\alpha))(\delta + a)}\Big)\hat{X},\\
&= \frac{\delta}{(\delta + a(1-\alpha))(\delta + a)}\Big(\delta + a(1-\alpha) + a\alpha\Big)\hat{X},\\
&= \frac{\delta}{\delta + a(1-\alpha)}\hat{X}.
\end{aligned} \tag{44}$$

We use Equations (43) and (44) in Equation (42) to obtain:

$$\begin{aligned}\frac{\hat{R}}{nT} &= (\delta+a)\mathbb{L}(S) - \delta\hat{X} - a\alpha\Big(\frac{\delta}{\delta+a(1-\alpha)}\hat{X} + \frac{a(1-\alpha)}{\delta+a(1-\alpha)}e\Big) - a(1-\alpha)e, \\ &= (\delta+a)\mathbb{L}(S) - \Big(1+\frac{a\alpha}{\delta+a(1-\alpha)}\Big)\Big(\delta\hat{X}+a(1-\alpha)e\Big), \\ &= (\delta+a)\mathbb{L}(S) - \Big(\frac{\delta+a}{\delta+a(1-\alpha)}\Big)\Big(\delta\hat{X}+a(1-\alpha)e\Big),\end{aligned} \tag{45}$$

which gives the required result.

9.4 Proof of Proposition 5

The administrator wishes to solve the following problem:

$$\min_{a\in[0,A],\alpha\in[0,1]} \Lambda(a,\alpha) := \Big(\frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}\Big)^2\sigma_Y^2 \ + \ \Big(1-\frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}\Big)^2\frac{\sigma_\varepsilon^2}{nT} \ + \ \frac{\sigma_R^2}{n^3T^2(\delta+a)^2}. \tag{46}$$

Λ is continuous and differentiable everywhere in the feasible region. Optimizing with respect to α:

$$\begin{aligned}\frac{\partial\Lambda}{\partial\alpha} &= 2\Big(\frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}\sigma_Y^2 - \big(1-\frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}\big)\frac{\sigma_\varepsilon^2}{nT}\Big)\cdot\frac{-\delta a\gamma}{(\delta+a(1-\alpha))^2}, \\ &= \frac{-2\delta a\gamma}{nT(\delta+a(1-\alpha))^2}\Big(\frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}\big(nT\sigma_Y^2+\sigma_\varepsilon^2\big)-\sigma_\varepsilon^2\Big).\end{aligned} \tag{47}$$

Consider, first, the case $\gamma \le \gamma^*$. Then:

$$\begin{aligned}&\gamma\big(nT\sigma_Y^2+\sigma_\varepsilon^2\big) \le \sigma_\varepsilon^2, \\ \Rightarrow &\frac{a(1-\alpha)}{\delta+a(1-\alpha)}\gamma\big(nT\sigma_Y^2+\sigma_\varepsilon^2\big) < \sigma_\varepsilon^2, \\ &\Rightarrow \frac{\partial\Lambda}{\partial\alpha} > 0.\end{aligned} \tag{48}$$

Then the administrator sets $\alpha^* = 0$, and only RPA rates are used in the punishment function. This means that:

$$\Lambda(a,\alpha^*) = \Big(\frac{a\gamma}{\delta+a}\Big)^2\sigma_Y^2 + \Big(1-\frac{a\gamma}{\delta+a}\Big)^2\frac{\sigma_\varepsilon^2}{nT} + \frac{\sigma_R^2}{n^3T^2(\delta+a)^2}, \tag{49}$$

and:

$$
\begin{aligned}
\frac{\partial \Lambda}{\partial a}\Big|_{\alpha=0} &= \frac{2\delta\gamma}{(\delta+a)^2}\Big(\frac{a\gamma}{\delta+a}\sigma_Y^2 - \big(1-\frac{a\gamma}{\delta+a}\big)\frac{\sigma_\varepsilon^2}{nT}\Big) - \frac{2\sigma_R^2}{n^3T^2(\delta+a)^3},\\
&= \frac{2\delta\gamma}{nT(\delta+a)^2}\Big(\frac{a\gamma}{\delta+a}\big(nT\sigma_Y^2+\sigma_\varepsilon^2\big) - \sigma_\varepsilon^2\Big) - \frac{2\sigma_R^2}{n^3T^2(\delta+a)^3},\\
&\le \frac{2\delta\gamma}{nT(\delta+a)^2}\Big(\frac{a\gamma^*}{\delta+a}\big(nT\sigma_Y^2+\sigma_\varepsilon^2\big) - \sigma_\varepsilon^2\Big) - \frac{2\sigma_R^2}{n^3T^2(\delta+a)^3},\\
&< \frac{2\delta\gamma}{nT(\delta+a)^2}\Big(\gamma^*\big(nT\sigma_Y^2+\sigma_\varepsilon^2\big) - \sigma_\varepsilon^2\Big) - \frac{2\sigma_R^2}{n^3T^2(\delta+a)^3},\\
&< 0.
\end{aligned} \tag{50}
$$

Then the administrator sets a to its maximum value A.

Next, consider the case $\gamma > \gamma^*$. Using Equation (47), we can see that Λ has a local minimum with respect to α when:

$$
\begin{aligned}
\frac{\partial \Lambda}{\partial \alpha} &= 0,\\
\Rightarrow \frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)}(nT\sigma_Y^2+\sigma_\varepsilon^2) &= \sigma_\varepsilon^2,\\
\Rightarrow \frac{a(1-\alpha)\gamma}{\delta+a(1-\alpha)} &= \gamma^*,\\
\Rightarrow a(1-\alpha) &= \frac{\delta\gamma^*}{\gamma-\gamma^*}.
\end{aligned} \tag{51}
$$

This solution satisfies the second-order conditions for a minimum. It is feasible only when $\frac{\delta\gamma^*}{\gamma-\gamma^*} \in [0,A]$. As $\gamma > \gamma^*$, this condition would be violated only if $\frac{\delta\gamma^*}{\gamma-\gamma^*} > A$. In that case, the boundary condition would bind and the administrator would set $a(1-\alpha) = A$. Thus the following condition must hold at the optimum:

$$
a^*(1-\alpha^*) \;=\; \min\Big\{A, \frac{\delta\gamma^*}{\gamma-\gamma^*}\Big\} \;:=\; M. \tag{52}
$$

When $a(1-\alpha) = M$ is satisfied, then:

$$
\begin{aligned}
\Lambda(a,\alpha) &= \Big(\frac{M\gamma}{\delta+M}\Big)^2\sigma_Y^2 \;+\; \Big(1-\frac{M\gamma}{\delta+M}\Big)^2\frac{\sigma_\varepsilon^2}{nT} \;+\; \frac{\sigma_R^2}{n^3T^2(\delta+a)^2},\\
\Rightarrow \frac{\partial \Lambda}{\partial a}\Big|_{a=a^*,\alpha=\alpha^*} &= -\frac{2\sigma_R^2}{n^3T^2(\delta+a)^3},\\
&< 0,
\end{aligned} \tag{53}
$$

so the administrator sets a to its maximum value A. Thus, the solution is $a^* = A$ and $\alpha^* = 1 - \frac{M}{A}$, which implies that $\alpha^* = 0$ when $\gamma \leq (1 + \frac{\delta}{A})\gamma^*$ and is equal to $1 - \frac{\delta\gamma^*}{A(\gamma-\gamma^*)}$ otherwise.

$\alpha^* = 1$ only if $\gamma^* = 0$, which occurs if and only if $\sigma_\varepsilon^2 = 0$. The proof is complete.

9.5 Proof of Lemma 1

The solution found in Proposition 5 depends only on the value of nT. As $n_1T_1 = n_2T_2$, the administrator chooses the same values of a and α in both cases.

Let Λ_1 be the loss in case (n_1, T_1) and Λ_2 be the loss in case (n_2, T_2). Then, using Equation (46):

$$\begin{aligned}\Lambda_1 - \Lambda_2 &= \frac{\sigma_R^2}{n_1^3T_1^2(\delta+A)^2} - \frac{\sigma_R^2}{n_2^3T_2^2(\delta+A)^2}, \\ &= \frac{\sigma_R^2}{n_1^2T_1^2(\delta+A)^2}\Big(\frac{1}{n_1} - \frac{1}{n_2}\Big), \\ &< 0.\end{aligned} \tag{54}$$

We can see immediately from Equation (46) that $\Lambda \to 0$ as $nT \to \infty$.