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Abstract -- Currency Substitution: A Perfect Foresight Optimizing Analysis

by Russell S. Boyer and Geoffrey H. Kingston

The appropriate definition of currency substitution and the consequences of the existence of such substitution are investigated. Differential equations arise from the maximization of an integral like Brock's where real balances of two denominations yield utility. These equations are solved for a number of comparative dynamic exercises.

The model exhibits reverse transmission of inflation, but not overshooting. The higher the degree of currency substitution the lower the sustainable inflation rate differential internationally, and the higher the weight for exchange rate determination given to future (relative to the present) changes in the money supply. Exchange rate indeterminacy arises in the case of perfect currency substitution.

I. Introduction

It has long been recognized that market instruments of different denomination can be close substitutes for each other. This notion has been captured in the interest rate parity equation which states that rates of return on assets with pecuniary yield are the same. Recently, consistent with the monetary approach to the balance of payments, economists have analyzed the consequences of non-interest bearing assets being substitutes, entitling this phenomenon "currency substitution".[1]

Two types of currency substitution have been discussed in the literature: direct and effective. Bilson notes that, in reduced form, relative money demand equations are silent as to the nature of the substitution that is occurring. However, in structural form these two types of substitution have quite different consequences, so that there are two separate strands in this literature and they emphasize quite different aspects of the problem.

Calvo and Rodriguez, Mussa, and Miles discuss effective currency substitution. They emphasize that relative holdings of real balances depend on expected inflation rates, which must bear a close relationship to actual inflation rates. The exchange rate between currencies, therefore, depends upon all future values of the money supply in a typical forward-looking solution to a rational expectations problem.[2] In contrast, Boyer, Girton and Roper, Helpman and Razin, and Kareken and Wallace emphasize that the markets for foreign exchange are different from those for assets of the same denomination. A striking conclusion of this literature is that if currencies are perfect substitutes for each other then the exchange rate between them is indeterminate.

The present paper draws together these two strands of the literature by generalizing the problem so as to consider effective and direct currency substitution simultaneously. This is done in a utility-maximizing framework in which individuals have perfect foresight of deterministic profiles of future monetary policies. The utility functional is assumed to be separable in consumption and real balances, and to satisfy a condition at zero balances so as to eliminate tenuous paths as solutions to the optimal problem. In log-linearizing the resulting differential equations around the steady state equilibrium in order to obtain a solution in closed form, we take the degree of direct currency substitution as parametric.

This framework is applied to two problems which have been treated extensively in the literature: the international transmission of inflation, and exchange rate dynamics. It is found that there is a reverse transmission of inflation, and that the exchange rate rushes ahead of domestic prices, but there is no overshooting. As the degree of currency substitution increases, more weight is given to future money supply changes relative to present ones in the determination of the exchange rate. Furthermore, the sustainable inflation rate differential decreases as the degree of substitution increases. Finally, this model generates exchange rate indeterminacy in the perfect substitutability case, demonstrating that this result is a robust one.

II. Currency Substitution and Individual Optimization

Consider a representative individual who obtains utility from consumption and from holding money balances, which are available in two denominations.[3] The agent receives at every point in time a fixed endowment of the consumption good and exogenous but variable government transfers of the two currencies.

The individual faces flow and stock constraints of the form

$$c + \frac{\dot{M}_1^d}{P_1} + \frac{\dot{M}_2^d}{P_2} = y + \frac{H_1}{P_1} + \frac{H_2}{P_2} \qquad (1)$$

and

$$\frac{M_1^d}{P_1} + \frac{M_2^d}{P_2} = A \qquad (2)$$

where: c is the desired level of consumption;

M_i is holdings of money i;

y is the fixed endowment of the consumption good;

H_i is the government transfer of money i;

P_i is the price of the consumption good in terms of money i;

and A is the real value of currency holdings.[4]

A dot over a variable denotes its time derivative and a superscript d indicates its desired level.

Preferences are such that the individual's rate of time preference is a positive constant, δ, and his instantaneous utility function is of the separable form:

$$u(c) + v\left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right).$$

The function u has the standard restrictions on the signs of its derivatives

$$u' > 0, \; u'' < 0.$$

The function v is assumed to possess both a negative definite Hessian, and a dominant diagonal so as to ensure concavity, except in the polar case of perfect substitution. The two monies are assumed to be Edgeworth substitutes so that the cross partial derivatives, v_{12}, are negative.[5] Finally, it is assumed that these two currencies are essential in the sense that

$$\lim_{\frac{M_1}{P_1} \to 0} \frac{M_1}{P_1} \cdot v_1\left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right) > 0 \quad \text{for all values of } \frac{M_2}{P_2}$$

and

$$\lim_{\frac{M_2}{P_2} \to 0} \frac{M_2}{P_2} \cdot v_2\left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right) > 0 \quad \text{for all values of } \frac{M_1}{P_1} \qquad (3)$$

These are two-currency analogues of the condition found in Brock (1978).

The problem for the representative individual at any time t is to choose non-negative future time profiles of $c(\tau)$, $M_1^d(\tau)$ and $M_2^d(\tau)$ with $\tau > t$ that

$$\text{maximize} \int_t^\infty e^{-\delta(\tau-t)}\{u(c(\tau)) + v(\frac{M_1^d(\tau)}{P_1(\tau)}, \frac{M_2^d(\tau)}{P_2(\tau)})\}d\tau$$

subject to constraints (1) and (2). To solve this problem define the Hamiltonian

$$\mathcal{H} \equiv e^{-\delta(\tau-t)}\{u(c) + v(\frac{M_1^d}{P_1}, \frac{M_2^d}{P_2}) + \varphi(y + \frac{H_1}{P_1} + \frac{H_2}{P_2} - c - \dot{p}_1 \frac{M_1^d}{P_1} - \dot{p}_2 \frac{M_2^d}{P_2})$$
$$+ \psi\,(A - \frac{M_1^d}{P_1} - \frac{M_2^d}{P_2})$$

where φ and ψ have the interpretation of being shadow prices: of real hoarding, and of excess real assets respectively.

The first-order necessary conditions for an interior optimum are:

$$\frac{\partial\mathcal{H}}{\partial c} = e^{-\delta(\tau-t)}(u' - \varphi) = 0 \tag{4}$$

$$\frac{\partial\mathcal{H}}{\partial(M_1^d/P_1)} = e^{-\delta(\tau-t)}(v_1 - \varphi\,\dot{p}_1 - \psi) = 0 \tag{5}$$

$$\frac{\partial\mathcal{H}}{\partial(M_2^d/P_2)} = e^{-\delta(\tau-t)}(v_2 - \varphi\,\dot{p}_2 - \psi) = 0 \tag{6}$$

$$\frac{\partial\mathcal{H}}{\partial A} + \frac{d}{dt}[\varphi e^{-\delta(\tau-t)}] = e^{-\delta(\tau-t)}(\dot{\varphi} - \varphi\,\delta + \psi) = 0 \tag{7}$$

where p_1 and p_2 are the natural logarithms of the two price levels and perfect foresight guarantees equality between expected and actual rates of inflation. The signs imposed on the derivatives of u and v ensure that the second-order necessary conditions are satisfied.

It is noted below that at the aggregate level commodity market clearing requires that consumption is constant over time, being equal to the endowment. Equation (4), which states that income is allocated to consumption up to the point where its marginal utility equals the shadow price of real hoarding, therefore implies that that shadow price is constant. It follows from this, in conjunction with equation (7), that the shadow price of excess real assets is equal to $\varphi\,\delta$, which is the discount rate expressed in terms of utils. In turn, equations (5) and (6) have the interpretation that real money of each denomination is held up to the point where its marginal utility equals the sum, expressed in utils, of its rate of inflation (or opportunity cost) and the discount rate.

Market clearing in the commodity market for the aggregate economy ensures that consumption is equal to the constant level of the endowment. For assets, market clearing permits the substitution in these equations of actual money supplies for their desired levels. In light of this, equations (5) and (6) can be written

$$\dot{p}_1 = -\frac{\Psi}{\varphi} + \frac{v_1\left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right)}{\varphi}$$

and

$$\dot{p}_2 = -\frac{\Psi}{\varphi} + \frac{v_2\left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right)}{\varphi} \qquad (8)$$

It can be shown that the signs of v's derivative guarantee that a tenuous solution in which either price level goes to zero, so that real balances in that denomination become indefinitely large, is not optimal. In addition, when condition (3) is imposed tenuous solutions for which either price level becomes indefinitely large are not feasible, because they imply prices become negative eventually.[6]

With these tenuous solutions eliminated, it is clear that the economy must settle down to a steady state in the long run in the face of constant money supply growth after some time in the future. In fact, the local stability properties of the system of differential equations to be investigated in the next section, are such that with constant rates of growth of nominal balances, the optimal path for the economy is one of constant levels of real balances at all times.[7]

The marginal utilities in system (8) can be approximated as:

$$v_1\left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right) = \varphi[\alpha_1 - \eta_1(m_1 - p_1) - \gamma \cdot \theta^{1/2}(m_2 - p_2)]$$

$$v_2\left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right) = \varphi[\alpha_2 - \frac{\gamma}{\theta^{1/2}}(m_1 - p_1) - \eta_2(m_2 - p_2)]$$

where $\eta_1 = v_{11} \cdot \left(\frac{M_{1o}}{P_{1o}}\right)$, $\gamma = v_{12} \cdot \sqrt{\left(\frac{M_{1o}}{P_{1o}}\right) \cdot \left(\frac{M_{2o}}{P_{2o}}\right)}$, $\theta = \frac{\left(\frac{M_{2o}}{P_{2o}}\right)}{\left(\frac{M_{1o}}{P_{1o}}\right)}$, $\eta_2 = v_{22} \cdot \left(\frac{M_{2o}}{P_{2o}}\right)$

and m_1, m_2, p_1, and p_2 are the logarithms of the variables represented above by the same letters in upper case, and the subscript o denotes initial equilibrium value.

Since v has a dominant diagonal almost everywhere $v_{11}v_{22} - v_{12}^2 > 0$. In terms of the parameterization above, this implies that

$$[\eta_1\eta_2 - \gamma^2] > 0.$$

The parameter γ represents the degree of direct currency substitution in the Edgeworth sense, which is appropriate here in light of the assumed cardinality of the instantaneous utility functional. In this sense, $\gamma = 0$ is the case of zero direct substitution, for then holdings of either currency do not affect the marginal utility of the other. In the same way, the case of perfect currency substitution is that in which the marginal utilities of the two currencies are the same. This is true when

$$\alpha_1 = \alpha_2$$

and

$$\gamma = \eta_1 \theta^{1/2} = \frac{\eta_2}{\theta^{1/2}} .$$

For the intermediate case γ is less than these values, so the perfect substitution situation can be viewed as the limit as it approaches these values from below.

III. The Solution in the Neighborhood of the Steady State

The discussion to this point has noted that the optimal path of price levels in a world of currency substitution is governed by a system of first-order differential equations in real balances of the two currencies. These money holdings appear because the marginal utility of holding either currency is a function of holdings of both, in general. These marginal utilities were approximated by log-linear representations in order to represent parametrically the degree of currency substitution. This approximation also enables us to solve the system of equations in closed form near the unique steady state equilibrium.

When the approximations to the marginal utilities are substituted into system (8), the following linear differential equations arise:

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = \begin{bmatrix} \eta_1 & \gamma\theta^{1/2} \\ \frac{\gamma}{\theta^{1/2}} & \eta_2 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} - \begin{bmatrix} \eta_1 & \gamma\theta^{1/2} \\ \frac{\gamma}{\theta^{1/2}} & \eta_2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \tag{9}$$

where m_1 and m_2 are functions of time only.[9]

The solution to this system of differential equations is:

$$\begin{aligned} p_1(t) &= \frac{1}{q_1 - q_2} \int_t^\infty \{q_1(m_1 - q_2 m_2)\lambda_1 e^{\lambda_1(t-\tau)} - q_2(m_1 - q_1 m_2)\lambda_2 e^{\lambda_2(t-\tau)}\} d\tau \\ p_2(t) &= \frac{1}{q_1 - q_2} \int_t^\infty \{(m_1 - q_2 m_2)\lambda_1 e^{\lambda_1(t-\tau)} - (m_1 - q_1 m_2)\lambda_2 e^{\lambda_2(t-\tau)}\} d\tau \end{aligned} \tag{10}$$

where $$\lambda_{1,2} = \frac{(\eta_1 + \eta_2) \pm \sqrt{(\eta_1 + \eta_2)^2 - 4(\eta_1\eta_2 - \gamma^2)}}{2} \geq 0$$

so that $\lambda_1 \geq \lambda_2$ (with equality holding in the special case $\gamma = 0$ and $\eta_1 = \eta_2$),

and $$q_i = (\frac{\lambda_i - \eta_2}{\gamma})\theta^{1/2}, \ i=1,2.$$

It can be shown that $1 \leq q_1$, and $-\theta \leq q_2$ and is always non-positive, where the equalities hold when substitution is perfect. In the zero substitution case, q_1 is indefinitely large and q_2 is zero. The homogeneous portions of the complete solutions to the differential equation have been suppressed by setting to zero the arbitrary coefficients. This suppression is necessary to eliminate those solution paths which are tenuous.

It should be noted that both roots of the system are positive so that the equilibrium is not a saddle point. In other words there is no stable branch in the equilibrium space.

Because of the signs of these roots it is necessary to integrate the forcing functions in the forward direction. This is reasonable because there are no predetermined variables in this model of complete price flexibility. As a result, bygones are bygones which do not influence the nature of the economy's path in the future.

These mathematical distinctions draw attention to the fact that this model differs in its economic structure substantially from models of its genre which emphasize instead stickiness of either domestic currency prices or the stock of indebtedness relative to the rest of the world.[10] As a result, the responses of this model to standard shocks are dissimilar from those drawn from conventional models.

Of particular concern here is the movement of the exchange rate to such shocks. Since there is a single commodity in the model and there are no transactions costs, the exchange rate must be consistent with the prices of that good in terms of the two currencies. That is, the exchange rate can be viewed as determined by absolute purchasing power parity:

$$s(t) = p_1(t) - p_2(t)$$

where s is the (log of the) exchange rate quoted as the price of a single unit of currency two in terms of currency one. Specifically the exchange rate is equal to:

$$s(t) = \frac{1}{q_1 - q_2} \int_t^\infty \{ ((m_1 - q_2 m_2)(q_1 - 1)\lambda_1 e^{\lambda_1 (t-\tau)} - (m_1 - q_1 m_2)(q_2 - 1)\lambda_2 e^{\lambda_2 (t-\tau)} \} d\tau$$

showing that s depends upon the sizes of the two money supplies both at present and throughout the indefinite future.

IV. Two Problems in International Finance

This simple model is an attractive setting for analyzing two problems which have received extensive attention in the literature. They are international transmission of inflation (with the possibility of complete insulation), and the problem of exchange rate dynamics (and the possibility of overshooting). It is clear that the present model should yield different conclusions from those of conventional models because there are no predetermined variables in it.

These problems are approached most easily by solving the model for a well-known comparative dynamic exericse: a delayed step increase in the money supply of money one.

Carrying out the integration in expression (10) for this step increase occurring at time t_o in the future yields the following expressions for the price levels:

$$
\begin{aligned}
p_1(t) - p_1^i &= (m_1^f - m_1^i)\{\frac{1}{q_1 - q_2} \cdot (q_1 e^{\lambda_1 (t - t_o)} - q_2 e^{\lambda_2 (t-t_o)})\} && t < t_o \\
&= (m_1^f - m_1^i) && t_o \leq t \qquad (11) \\
p_2(t) - p_2^i &= (m_1^f - m_1^i)\{\frac{1}{q_1 - q_2} \cdot (e^{\lambda_1 (t-t_o)} - e^{\lambda_2 (t-t_o)})\} && t < t_o \\
&= 0 && t_o \leq t
\end{aligned}
$$

where p_1^i and p_2^i are the initial values for these price levels. It can be shown that the expression in brackets for p_1 is positive but less than one for $t < t_o$ and for p_2 is negative but greater than minus one. The value of the exchange rate at any time minus its initial value is the difference between these two expressions.

One difference between these expressions and those from conventional models is that these depend only on the time remaining until the money supply is increased. This solution is the same whether the increase has been fully anticipated, or only "partially anticipated".[11] This result is consistent with the model's having both roots positive so the path of the economy is determined fully by expectations of the future, and does not depend on events which occurred in the past. In contrast, solutions to models with some form of stickiness, so that they have at least one negative root, are functions also of the time at which expectations are revised.

The signs of q_1 and q_2 assure that the price level for currency one adjusts monotonically to its new value. In contrast, at times long before t_o, p_2 moves away from its long-run equilibrium; as t_o approaches p_2 moves back toward that equilibrium. The time at which its motion is reversed is given by

$$t_o + \frac{\ln \frac{\lambda_1}{\lambda_2}}{\lambda_2 - \lambda_1} .$$

What this implies about "partial anticipated" changes in the money supply is in part conventional and in part unorthodox. A revision of expectations about money supply one's behavior, such that it is, for example, higher after t_o, causes the price level in that currency to be bid up immediately bringing about a discrete jump in this variable part way towards the new long-run equilibrium. Thereafter, this price moves continuously arriving at the new long-run equilibrium at exactly t_o. In contrast, the price level in currency two has a discrete negative change when expectations are revised. Thereafter this price level moves continuously, although not necessarily monotonically, arriving at its original value at t_o.

This exercise shows that discrete, completely unanticipated changes in either money supply cause the price level in that currency to change equiproportionately. As a result there are no effects upon the price level in the other currency, such that there is complete insulation. In contrast, monetary changes that are anticipated to occur at some future date cause less than proportional changes in that currency's price level such that real balances are reduced. The inflationary path which remains to the long-run equilibrium is consistent with this reduction. It also causes individuals to substitute towards the other currency. The process by which this occurs is that the price level in the other currency falls discretely thereby generating higher real balances in that denomination. This shows that a (partially) anticipated change in either money supply has a reverse transmission upon the other currency price level.[12] This process will be investigated further in the next section.

It can be shown that as $\gamma \rightarrow 0$, q_1 gets indefinitely large, as noted above, and $\lambda_1 \rightarrow \eta_1$. The mathematical expressions (11) yield conclusions which are consistent with our intuition in this case. When direct currency substitution is low, one would expect that the movement of p_1 would depend entirely upon parameters for that market, and there would be no transmission of inflation. These are precisely the conclusions of our model, and they are consistent with, for example, Mussa. An effective currency substitution measure would take the evidence that there is a change in the ratio of real balances (i.e., relative money demands are altered) as proof that substitution is at work. In contrast, the evidence that real balances in currency two do not change supports the notion that direct currency substitution is absent.

The discussion so far has dealt entirely with the price levels and has not related it to the movement of the exchange rate. The reason is that most of the characteristics of its movement can be derived from the discussion above. In particular, a partially anticipated increase in the supply of money one causes the exchange rate to jump to a higher value immediately upon the revision of expectations. Thereafter the exchange rate stays ahead of the increase in money one prices, but this percentage dwindles so that at t_o these two prices have increased equi-proportionately.

The dependence of the exchange rate's dynamics upon the degree of currency substitution, is best analyzed by solving for the exchange rate explicitly:

$$s(t) - s^i(t) = p_1(t) - p_2(t) - (p_1^i - p_2^i) = \frac{(m_1^f - m_1^i)}{q_1 - q_2}\left((q_1 - 1)e^{\lambda_1(t-t_o)} - (q_2 - 1)e^{\lambda_2(t-t_o)}\right)$$

Given the inequalities on the characteristic vectors, it is clear that the exchange rate moves monotonically. This conclusion differs from the "overshooting" result found in Dornbusch, Flood, Gray and Turnovsky, and Wilson.

As direct currency substitution gets large, $q_1 \to 1$ and $\lambda_2 \to 0$. This has important implications for the movement of the exchange rate. Namely, for this case, $s(t) - s^i(t) = (m_1^f - m_1^i)$. That is, the exchange rate moves immediately to its long-run equilibrium value. This is true no matter how far into the future the money supply change occurs.

V. Inflation and Currency Substitution

The argument that currencies which are close substitutes for each other must have the same rate of inflation seems plausible.[13] The model above enables us to investigate this argument in order to determine the nature of a mechanism which might generate this result.

Consider the situation of the money supplies growing at rates π_1 and π_2 so that their logarithms can be written as:

$$m_1(t) = m_{1o} + \pi_1 \cdot (t - t_o)$$
$$m_2(t) = m_{2o} + \pi_2 \cdot (t - t_o).$$

The price level and exchange rate time profiles which these generate are:

$$p_1(t) = m_1(t) + \frac{\eta_2 \cdot \pi_1 - \gamma \cdot \theta^{1/2} \cdot \pi_2}{\lambda_1 \cdot \lambda_2}$$

$$p_2(t) = m_2(t) + \frac{\eta_1 \cdot \pi_2 - \frac{\gamma}{\theta^{1/2}} \cdot \pi_1}{\lambda_1 \cdot \lambda_2}$$

$$s(t) = m_1(t) - m_2(t) + \frac{(\eta_2 + \frac{\gamma}{\theta^{1/2}})\pi_1 - (\eta_1 + \gamma\theta^{1/2})\pi_2}{\lambda_1 \cdot \lambda_2}$$

In the absence of direct currency substitution, these expressions are very familiar. For that case, $\gamma = 0$ by definition, and $\lambda_1 = \eta_1$ and $\lambda_2 = \eta_2$, so that the price level in either currency depends upon only its own rate of monetary growth. The exchange rate, determined by purchasing power parity, depends upon both rates of inflation in an argument put forth notably in Bilson.[14]

Other currency rates of inflation become important for the domestic price level when direct substitution is present. The intuition from the reverse transmission example of the previous section can be applied here, to arrive at the conclusion that higher foreign rates of inflation reduce the price level at home. On the other hand, the sensitivity of domestic prices to home inflation is positively related to this substitutability. In particular the product of the roots in the denominators of the expressions for p_1 and p_2 is equal to the determinant of the matrix of the system. As γ increases, λ_2 goes to zero as does the determinant. Clearly as a consequence of these increased sensitivities, the exchange rate is more responsive to inflation rate differentials.

In the limit when λ_2 goes to zero these sensitivities get indefinitely large. Unless the numerators in the expression are equal to zero, the price levels become indefinitely large, positively or negatively in accordance with one's intuition. The perfect substitutability case is that when $\gamma = \eta_1\theta^{1/2} = \frac{\eta_2}{\theta^{1/2}}$. The numerator equals zero in the perfect substitutes case when the inflation rates are equal, $\pi_1 = \pi_2$. This shows that it is true that with perfect currency substitution rates of inflation must be equal.

Two points should be made about this case. First, π_1 and π_2 are taken to be constants over time in order to simplify the analysis. In the general case, the argument in the proceeding paragraph should be viewed as showing that the supplies of two currencies which are perfect substitutes for each other must grow at the same average rate after some time in the future. Such currencies have exchange rates that are anticipated to remain constant indefinitely. That is not possible if the supply of one is expected to have an average growth rate greater than the other's throughout the indefinite future.

Second, this is the only experiment in this paper for which the quantity of real balances is viewed as changed and sustained at a new equilibrium. This brings into question whether the logarithmic approximation procedure is a valid one. Clearly what is being argued above is that this procedure breaks down as substitution increases, because the same inflation differential causes larger changes in real balances.

In terms of the solution techniques above, what is being argued is that as $\lambda_2 \to 0$ the integrands in expressions (10) must have an average exponential rate of growth of zero after some time in the future. This will be true only if the money supplies, when growing at a positive rate, grow at the same rate.

Finally, it should be noted that there is a potential inconsistency between the definition of perfect substitutes and condition (3). The very notion of such substitution is that having one more unit of one kind enables an agent to do without one unit of another kind. Thus, it is not possible to justify condition (3) in that case because it argues that in the special case where one currency's real value goes to zero, that currency becomes essential in a particularly strong sense.

As a result of the fact that condition (3) for each currency separately cannot be justified in the perfect substitution case, does it follow that tenuous solutions with imploding real balances can be optimal? They cannot be optimal if we substitute for that condition its natural counterpart for this case. Namely

$$\lim \frac{M_i}{P_i} \cdot v_i \quad \left(\frac{M_1}{P_1}, \frac{M_2}{P_2}\right) > 0 \tag{12}$$

as both

$$\frac{M_1}{P_1} \rightarrow 0$$

and

$$\frac{M_2}{P_2} \rightarrow 0$$

for $i=1,2$. We know that perfect currency substitution means an unchanging exchange rate and, therefore, identical rates of inflation. Thus, if the solution specifies that one currency has a hyperinflation, both currencies must follow such a path. To eliminate this possibility we need to impose the condition (12) above, which is the Brock condition for the case of both currencies going to zero simultaneously.

VI. Temporary Changes in Money Supplies

The discussion above identified a curious characteristic of currency substitution. It showed that a permanent increase in a money supply caused an immediate equiproportionate rise in the exchange rate, no matter when that increase takes place. This is curious in that one's intuition would suggest that events taking place indefinitely far in the future should have no impact on the present. The major point which this exemplifies is that, for determination of the exchange rate, currency substitution reduces the weighting of the present relative to the future.

This point can be made more concretely by considering the effects of temporary changes in the two money supplies. It is assumed that the money supplies were altered at some time in the past, and at t_o in the future they both revert to their previous values. Denote the temporary level of money supply j by m_j^t and its initial level by m_j^i. Then the price levels and the exchange rate are the following functions of time for $t < t_o$:

$$
\begin{aligned}
p_1(t) - p_1^i &= (m_1^t - m_1^i) \cdot \left(1 - \frac{q_1 e^{\lambda_1(t-t_o)} - q_2 e^{\lambda_2(t-t_o)}}{q_1 - q_2}\right) \\
&\quad + (m_2^t - m_2^i) \cdot \frac{q_1 q_2}{q_1 - q_2} \cdot \left(e^{\lambda_1(t-t_o)} - e^{\lambda_2(t-t_o)}\right) \\
p_2(t) - p_2^i &= (m_2^t - m_2^i) \cdot \left(1 - \frac{q_1 e^{\lambda_2(t-t_o)} - q_2 e^{\lambda_1(t-t_o)}}{q_1 - q_2}\right) \\
&\quad + (m_1^t - m_1^i) \cdot \left(\frac{e^{\lambda_2(t-t_o)} - e^{\lambda_1(t-t_o)}}{q_1 - q_2}\right)
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
s(t) - s^i(t) &= (m_1^t - m_1^i) \cdot \left(1 - \frac{(q_1 - 1)e^{\lambda_1(t-t_o)} - (q_2 - 1)e^{\lambda_2(t-t_o)}}{q_1 - q_2}\right) \\
&\quad - (m_2^t - m_2^i)\left(1 - \frac{q_2(1 - q_1)e^{\lambda_1(t-t_o)} - q_1(1 - q_2)e^{\lambda_2(t-t_o)}}{q_1 - q_2}\right).
\end{aligned}
$$

After t_o, these prices attain their previous and long-run values of p_1^i, p_2^i and s^i.

These solutions are more complicated versions of the expressions in Section IV above. These additional complexities arise from the facts that at t_o the economy returns to its initial equilibrium, and that both money supplies are altered simultaneously. For a general level of currency substitution, these expressions do not provide any fresh insights into the nature of the economy's dynamics. However, in the case of perfect substitution, there are some insights to be garnered.

In the case of perfect substitution, the characteristic roots and characteristic vectors take on the values noted above. As a result these expressions (13) simplify to

$$p_1(t) - p_1^i = p_2(t) - p_2^i = \left(\frac{(m_1^t - m_1^i) + \theta(m_2^t - m_2^i)}{1+\theta}\right) \cdot \left(1 - e^{\lambda_1(t-t_o)}\right) \tag{14}$$

$$s(t) = s^i. \tag{15}$$

It was noted above that in this polar case the exchange rate moves to its long-run equilibrium level immediately. This is confirmed by equation (15) showing the exchange rate stays at its initial value (which is also its long-run value) so that it remains constant in the face of these temporary changes no matter how long they last and no matter which currency is changed. This shows that the movement of the exchange rate to any money supply change depends crucially upon whether the change is expected to be permanent or of finite duration. The exchange rate moves immediately to its long-run value, but that value depends only upon the permanent values of the money supplies.

The constancy of the exchange rate in the face of temporary changes in the money supply has important implications for the interpretation of the influences of such changes. Namely, in this case one can use the concept of the world money supply, and obviously, with purchasing power parity, a world price level. In logarithmic terms the world money supply is equal to

$$m_1 + \theta \cdot m_2$$

where θ clearly is the appropriate weighting factor. Equation (14) shows that changes in this world money supply, no matter what their source, have identical influences on the two individual price levels which are equal to the world price level with appropriate choice of units. It should be noted that the price level movement has the familiar exponential form derived from conventional analysis.

VII. Exchange Rate Indeterminacy

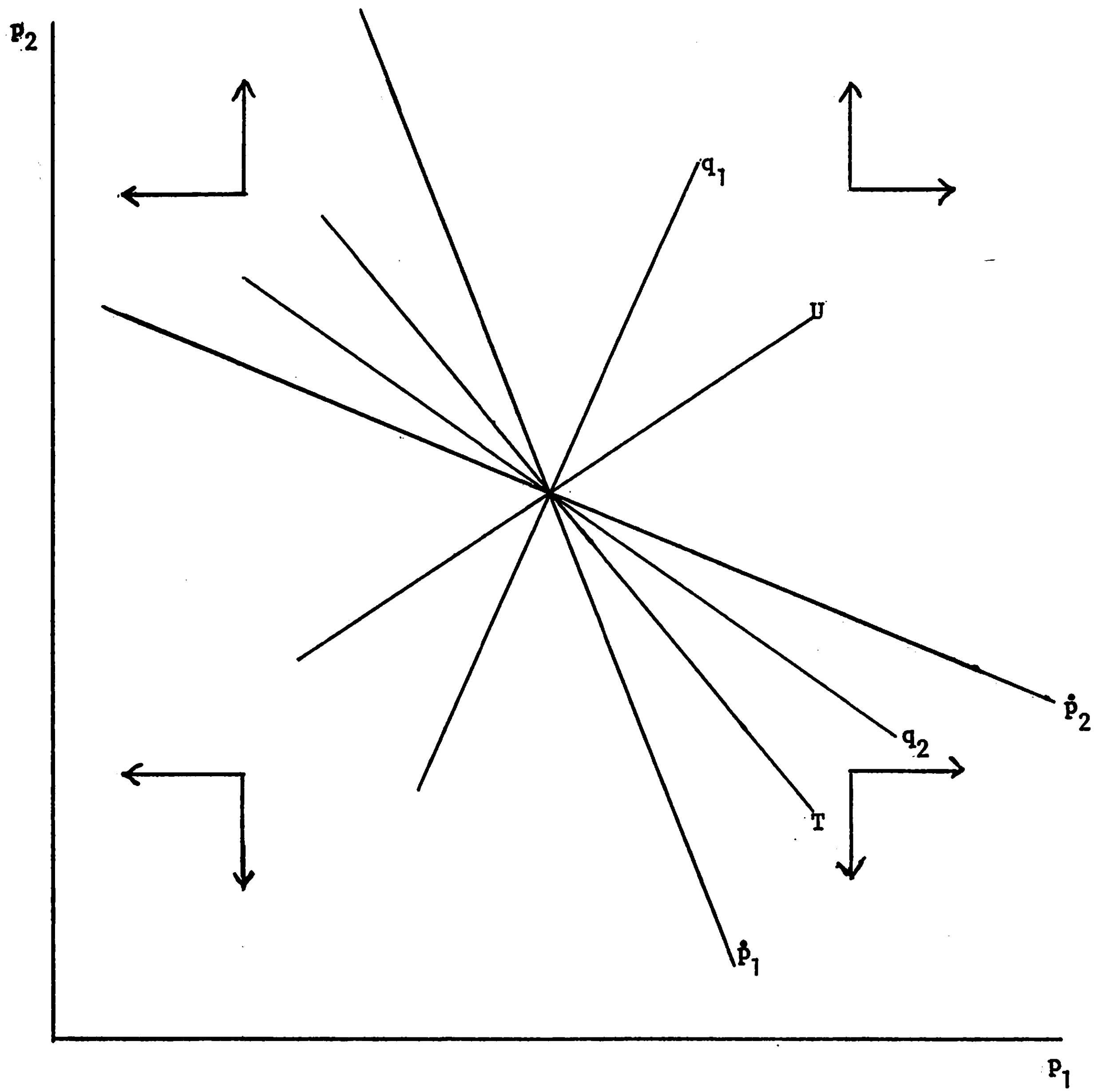
One of the central themes of the direct substitution literature is that in the perfect substitution case the equilibrium value of the exchange rate is indeterminate. It is important to see how that conclusion arises from the present model, in order to demonstrate thereby the robustness of this result.

This is done most conveniently by reiterating the argument above in a diagrammatic form. Figure 1 has the two price levels on the axes and is drawn for given current values of the money supply. The loci $\dot{p}_1$ and $\dot{p}_2$ denote the points for which the time derivatives of these variables, as represented in equation (9), are equal to zero. The characteristic vectors are identified by their slopes, q_1 and q_2. Finally, U is a unitary elastic line with slope of 1, and T has slope of $1/\theta$.

The figure is drawn for an intermediate degree of currency substitution, and all loci have slopes appropriate for that case. However, it is easy to identify the figure relevant for the polar cases of zero and perfect currency substitution. In the zero substitution case q_1 and $\dot{p}_1$ coincide, being completely vertical; and q_2 and $\dot{p}_2$ coincide, being completely horizontal.

In the perfect substitution case, the loci $\dot{p}_1$, $\dot{p}_2$, and q_2 all coincide with T, and q_1 coincides with U. The reader will remember that in this situation $\lambda_2 = 0$. As a result the only dynamics arise from exponential terms containing λ_1. That is, T is this case divides the quadrant into two, with dynamical arrows of unitary slope pointed positively northeast of that line, and similar arrows pointing negatively southwest of that line. Along T there

Figure 1



are no dynamic forces at work. Therefore every point along it is a potential equilibrium.

T can be described in more intuitive, static terms: it is the locus of points for which the world supply of real balances is constant. Moving along it the quantity of real balances of one denomination declines, but the quantity in the other denomination increases. The combination of the static interpretation and the dynamic one yields the following observation. In the case of two perfect substitutes, the division of the world's real currency supply into either denomination is indeterminate. However, the total real money supply has a well-defined equilibrium value.

This observation can be phrased algebraically as well. The solution above (10) suppressed the homogeneous portion by setting to zero the arbitrary coefficients in it. This elimination of the tenuous paths is justified so long as both roots are strictly positive. However, when λ_2 equals zero there is no need to suppress this portion of the homogeneous solution.[15] That is, if $\tilde{p}_1(t)$ and $\tilde{p}_2(t)$ are solutions of the differential equation (9) then

$$\tilde{p}_1(t) + q_2 \cdot K$$

and

$$\tilde{p}_2(t) + K$$

are as well for this case. In other words, every point along T is just as appropriate an equilibrium as every other point in the case of perfect substitutes.

VIII. Conclusion

This paper has drawn together two strands in the international financial literature which have developed along divergent paths. In the process it has shown that effective currency substitution and perfect currency substitution are polar cases of the situation of direct currency substitution.

In this general case, reverse transmission of inflation arises naturally, and there is a tendency for the exchange rate to rush ahead of domestic price movements, but there is never any overshooting. Such substitution also increases the sensitivity of domestic prices and the exchange rate to international differentials of inflation rates, so that the sustainable differential is limited by the degree of substitution.

Some new results are derived for the perfect substitution case as well. In such a situation the exchange rate moves immediately to its long-run equilibrium, no matter what temporary changes in the money supply may occur. As a result it is possible to aggregate domestic money supplies so as to define the world money supply. Such temporary changes in domestic money supplies have predictable effects on world prices, being proportional to those supplies' effects on the world money supply. In contrast with the dynamics of the exchange rate, the dynamic influences of such changes upon the price levels depend upon the time at which they are carried out in accordance with conventional analysis.

Footnotes

[1] The concern here is entirely a theoretical one: about the appropriate definition of such substitutability, and its consequences for the behavior of the economy. There is a substantial difference of opinion in the profession as to the empirical importance of currency substitution. Brittain, Evans and Laffer, and Miles, find that international financial markets evince high currency substitution. In contrast, Bordo and Choudri, and Giddy and Schadler find that in the Canada-U.S. setting the degree of such substitution is low.

[2] Such solutions are familiar since the paper by Sargent and Wallace.

[3] Such an approach is employed by Brock. As Kareken and Wallace argue, such modelling is not completely satisfactory. However, since our concern is with price dynamics rather than with microfoundations, this seems like a helpful approach to take.

[4] For some problems it may be necessary to distinguish between asset holdings an instant previous (which is the real constraint on current holdings) and current asset holdings. This does not appear to be an important distinction for this paper because all agents are taken to be identical.

[5] The fact that utility functionals are assumed to be ordinal obviates the usual objection to employing Edgeworth substitutability. Even when direct substitution is absent (so that $v_{12} = 0$), effective substitution remains. That is, a Hicks-Allen definition of substitution, based on price responsiveness, does not go to zero for this case.

[6] Both of these propositions have been proven for the single money case. The first is in Brock (1974); the second is in Brock (1979).

[7]It is well-known that the stability properties of a differential equation system can be analyzed using a local approximation around the steady state equilibrium. See, for example, Arrow and Kurz, p. 78.

[8]This parameterization of the model is useful at later stages in the analysis. In order to be consistent with the Cagan money demand function it would be necessary for η_1 and η_2 to be constant. This implies particular forms for v_{11} and v_{22}. Similarly in subsequent discussion we will consider the effects of changes in θ while holding γ constant. It is essential to realize that the approximation above is always valid, and the assumed independence among the various parameters is merely a matter of simplification.

[9]This is assumed for simplicity only. If the authorities geared monetary policy to the stabilization of prices or the exchange rate the system of equations would be altered, and the dynamics could be quite different from the flexible exchange rate case here. The portion of the solution attributable to the constants, α_i, is ignored here.

[10]Models of this sort include Brock, Calvo and Rodriguez, Dornbusch, Dornbusch and Fischer, Fischer, Flood, Gray and Turnovsky, and Wilson. Virtually all perfect foresight models of the small open economy have a saddle point equilibrium.

[11]This is Fischer's term for the possibility of revised expectations in a perfect foresight model. His discussion treats as a simple, special case a model which exhibits this sort of behavior.

[12]Burton and Mundell find that reverse transmission of monetary policy occurs under flexible exchange rates.

[13]Boyer and Girton and Roper make such an argument.

[14]That is, there still exists effective currency substitution, and in relative money demand form Bilson notes that it is not possible to distinguish this from direct currency substitution.

[15]It should be noted that the exchange rate has a determinate value in the limit as the degree of substitution becomes indefinitely large. Only when evaluated at that limit does indeterminacy arise. As Enders and Lapan emphasize, this implies that indeterminacy occurs only when currencies are perfect substitutes for all participants in the market, and there is no possibility of exchange controls ever being imposed.

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