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This paper contains preliminary findings from research work still in progress and should not be quoted without prior approval of the author.

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EXTERNAL SECTOR 'CLOSING' RULES
IN APPLIED GENERAL EQUILIBRIUM MODELS[1]

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I. Introduction

In a number of recent empirically oriented general equilibrium policy models (such as Adelman and Robinson [1977]; Boadway and Treddnick [1978]; Dervis and Robinson [1978]; Deardorff and Stern [1979]; Dixon, Powell et al. [1978], Fullerton, Shoven and Whalley [1978]) a common procedure is to use a system of simple export demand and import supply functions along with a trade balance condition to represent external sector behaviour, and thus 'close' the model.[1] This paper explores some of the pitfalls in using external sector closure rules and argues that some of the 'simple' closure rules used can be easily misinterpreted.

External sector 'closure' procedures in recent applied general equilibrium models have a number of common features. Firstly, an exchange rate variable frequently appears giving the appearance of a financial magnitude affecting real variables of the system in long-run equilibrium, even though real models are used with no monetary sector specified. Secondly, equation systems are presented in some models as giving a capability of separately incorporating foreign export demand and import supply elasticities while simultaneously meeting an external sector balance condition. Thirdly, a frequent procedure is to use the Armington assumption of product heterogeneity by country to model price taking behaviour with respect to all or some of the imported goods while foreigners demand functions for domestic exports incorporate some degree of price elasticity. An external sector balance condition also applies in this case.

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[1] Usually a number of commodity classes (perhaps in the region of 10-30) are separately identified in these models with the focus of the modelling being on a price endogenous equilibrium framework allowing the analysis of detailed impacts of policy or other variations. In some cases these models are constructed for single economies and the model builder attempts to use a relatively simple external sector specification to 'close' the model; in other cases external sector policies are investigated and more attention is therefore given to these equations. Most models separately identify traded and non-traded goods and most use the so-called 'Armington' procedure in treating similar domestically produced and imported goods as qualitatively different, primarily due to the phenomenon of 'cross hauling' appearing in the international trade data to which these models are calibrated.

The main themes of this paper are that

(i) Systems of equations for behaviour of foreigners which also satisfy a trade balance condition are, by construction, complete systems of excess demand functions for foreigners and should not be viewed as a sequence of independent equations. While these systems may conveniently close the model, there is no currency exchange rate in the conventional use of the term as a financial magnitude determined from financial sector activity. This terminology can thus be misleading in the presentation of these models in that it creates the appearance of a financial variable having real long-run equilibrium effects when non-neutralities of monetary variables are not present. The exchange rate variable appearing in the specifications used can, in some cases, be removed by substitution and computation of international trade equilibria is both simplified and clarified in these models by such a procedure.

(ii) Export demand elasticities and import supply elasticities are not independent parameters in these equation systems, as is sometimes suggested, because of the trade balance condition. As is well known, in a classical two good trade model the elasticity of the foreign offer curve is simultaneously related to both foreign export demand and import supply elasticities suggesting difficulties in separately specifying both of these elasticities. Parameters that are specified in the belief that they represent export demand and import supply price elasticities in these equation systems need not be true measure of these elasticities and general equilibrium calculations for models using such specifications can therefore be misinterpreted.

(iii) Models which follow the Armington procedure and attempt to simultaneously incorporate price taking behaviour for imports, a constant price elasticity of foreign export demand, and a zero trade balance condition encounter similar difficulties. The algebra of these systems rapidly becomes intractable, but we suggest that this construction for the external sector in the two good case requires both domestic and foreign offer curves to lie one on top of the other. This procedure guarantees that the external sector is in equilibrium at any set of prices generated by the domestic portion of such models, but it also implies that in the two good case the offer curve of one country must be of opposite shape to that conventionally drawn and cannot realistically be regarded as a specification consistent with conventional trade analysis.

(iv) If models involving homogeneous goods were to be used specifying price taking behavior for all of the traded goods, a general equilibrium for the price taking economy would involve a single parameter in addition to the endogenous non-traded goods prices. This parameter can be interpreted as the relative price of a composite traded good to the non-traded goods, since the price taking economy formulation only specifies relative and not absolute prices of traded goods. The fixed relative prices of the traded goods can be used as weights in the construction of the composite traded good, and we outline a formulation of equilibrium in a small open price taking economy with both traded and non-traded goods. We sketch a modification to the conventional Gale Nikaido mapping for a pure exchange economy through which it is possible to demonstrate existence of equilibrium by Brouwer's Theorem. This equilibrium formulation indicates the need for the additional parameter mentioned above in model solution and we suggest that this parameter could be equated with the 'exchange rate' variable appearing in some of the empirically oriented general equilibrium models even though no financial variables appear in the model. A similar interpretation of the exchange rate variable appears in Srinivasan's comments [1979] on the Deardorff and Stern model [1979]. We also suggest that of the various external sector closure rules considered here this is the only one well rooted in traditional micro theory. It is also inappropriate in its simplest form for use in applied models since it will typically produce situations of complete specialization and adjustments involving a movement from complete specialization in one good to another when policy changes are considered.

(v) The external sector specification can be sufficiently important in these models that it can considerably affect the perceived impact of policy changes even when the external sector appears to account for only a relatively small fraction of total activity in the economy. We report equilibrium calculations from a simple numerical example of domestic incidence and efficiency analysis of a distorting factor tax in which we consider alternative external sector formulations and show the quantitative differences involved.

The first three points listed above are illustrated here for a simplified two good trade model in which no non-traded goods appear. We devote separate sections to formulations with and without the Armington assumption. We then consider an equilibrium formulation for a small open price taking economy where we separately identify traded and non-traded goods. We outline a proof of existence of an equilibrium for such an economy and we explore the interpretation of the additional parameter mentioned under point (iv) above. In a final section in which we present a simple numerical example and show the significance of the external sector specification, we also make some comments on the appropriateness or otherwise of the alternative specifications considered here.

II. External Sector Closure Rules in Recent Applied General Equilibrium Models

The precise structure of the external sector in the empirically oriented general equilibrium models we have in mind tend to vary somewhat from model to model although the broad characteristics are similar. While it is impossible to exactly reproduce all of these specification here, we note that the models constructed by Adelman and Robinson [1977], Boadway and Treddenick [1978], Dervis and Robinson [1978], Deardorff and Stern [1979], and Dixon, Powell, et al [1977] all have exchange rate variables appearing in the external sector of what are presented as otherwise largely conventional general equilibrium models. It is common in these models for results to be presented from policy or other experiments in which changes in exchange rates are reported as if financial variables have real effects. All of these models also separately incorporate traded and non-traded goods and use the Armington assumption of product heterogeneity across countries. The model used by Fullerton, Shoven, and Whalley [1978] has no exchange rate term in the external sector formulation and does not adopt the Armington assumption but uses an assumption that the real value of net trades is constant, implying a negative supply elasticity for imports.

In Boadway and Treddenick [1978] the focus is on analysis of the domestic impacts of variations in Canadian tariff policy and we motivate our discussion by drawing on the external sector formulations they present and discuss. They construct a model with substantial industrial and commodity detail involving C.E.S. functions for both primary and intermediate production. The data base for the model is provided by the Canadian input-output tables. Boadway and Treddnick comment on the need for a 'simple' way of closing their model by incorporating the external sector and describe their procedure as a simple 'partial' equilibrium system appended to a more elaborate general equilibrium specification of the domestic economy.

Boadway and Treddenick first describe a closure system for a model in which homogeneous products appear. They suggest a 'partial' equilibrium sub-system in which foreigners are assumed to have constant price elasticity import supply and export demand functions, and a zero trade balance condition applies. In their computations the Armington assumption of product heterogeneity is invoked and the external sector specification differs from that discussed in the earlier part of their paper. They assume foreign demand functions of constant price elasticity form and treat Canada as a taker of world prices for imported goods. This assumes a perfectly elastic foreign import supply function, although foreign and domestic goods are not perfect substitutes. The elasticity of substitution between comparable domestic and foreign goods controls the degree to which the Canadian price system is governed by world prices. An external sector balance condition involving an exchange rate variable closes the system.

Somewhat similar sub systems to that used by Boadway and Treddenick appear in the models of Dervis and Robinson [1978], Adelman and Robinson [1977], and Dixon, Powell, et al [1977]. In all cases the Armington product heterogeneity assumption is used along with constant price elasticity foreign export demand functions, domestic price taking behaviour with respect to world import prices, and an external sector balance condition. In all these models an exchange rate variable also appears. Deardorff and Stern [1979] differ a little from these formulations in linking their formulations of separate economies through an international price system although each of 18 separate country models incorporate an external sector specification approximately of the form described above.

III. A Simple Two Commodity Formulation of External Sector 'Closure'

A simple system of external sector closure is suggested by Boadway and Treddenick for a general equilibrium model in which no Armington assumption appears (products are homogeneous between countries) and where relative prices of traded goods are endogenously determined in the model. Their suggestion can be represented by the following system of "simple" foreign export demand and import supply functions.

$$(1) \qquad E = E_o\left(\frac{P_E}{e}\right)^{\eta} \qquad -\infty < \eta < 0$$

$$(2) \qquad M = M_o\left(\frac{P_M}{e}\right)^{\mu} \qquad 0 < \mu < \infty$$

where P_E and P_M are the home country prices paid for exports and received for imports. E and M are vectors of exports and imports respectively and E_o and M_o are 'base year' imports and exports. η and μ are described as export demand and import supply price elasticities respectively, explaining the restrictions on η and μ in (1) and (2). e is described as the exchange rate between domestic and foreign currency. (P_E/e) and (P_M/e) are foreign country (world) prices and thus enter the functions (1) and (2). A balance of payments condition

$$(3) \qquad P_M M = P_E E$$

closes this system.

In computing an equilibrium for a larger general equilibrium model of a domestic economy with this form of external sector closure, Boadway and Treddenick suggest the following. Domestic prices are to be determined as cost covering prices by appealing to zero profit conditions. Using the equation system (1), (2) and (3), and some form of solution for e (iterative or otherwise), M and E are determined and hence excess demands for all commodities in the model. The equilibrium vectors P_M and P_E are calculated as those guaranteeing excess demands equal zero for all commodities.

The external sector system actually used by Boadway and Treddnick [1978] in their analysis of tariff policy in the Canadian economy differs from this as will be described later but with the system they adopt they perform different calculations of the effects of policy changes in their model for alternative values of η, which they interpret as the export price elasticity. In each case impacts on the exchange rate are reported and interpreted.

We suggest that there is in fact no meaningful exchange rate term in this formulation since it can be removed by simple substitution and, moreover, the parameters of μ and η do not have the elasticity interpretation which the system above might appear to offer. The system of equations (1), (2) and (3) is a system of three equations which, in the two good case where M and E are scalars, specify a foreign offer curve of constant elasticity. This single elasticity parameter determines the two trade elasticities referred to above.

This can be seen by substituting (1) and (2) into (3) and solving for e as

$$(4) \qquad e = \left(\frac{M_o}{E_o}\right)^{\frac{1}{\mu-\eta}} \cdot \frac{P_M^{\frac{\mu+1}{\mu-\eta}}}{P_E^{\frac{\eta+1}{\mu-\eta}}}$$

Using this solution for e enables (1) and (2) to be rewritten as

$$(5) \qquad E = E_o P_E^{-1} \cdot \left(\frac{M_o}{E_o}\right)^{\frac{-\eta}{(\mu-\eta)}} P_M^{\frac{-(\mu+1)\eta}{(\mu-\eta)}} \cdot P_E^{\frac{\mu(\eta+1)}{(\mu-\eta)}}$$

$$(6) \qquad M = M_o P_M^{-1} \cdot \left(\frac{M_o}{E_o}\right)^{\frac{-\mu}{(\mu-\eta)}} \cdot P_M^{\frac{-(\mu+1)\eta}{(\mu-\eta)}} \cdot P_E^{\frac{\mu(\eta+1)}{(\mu-\eta)}} .$$

(5) and (6) describe a system of excess demand functions in which the prices P_M and P_E appear but the exchange rate e is absent. It should be noted that unlike conventional excess demand functions the sign of E and M cannot change; the direction of trade assumed by choice of E_o and M_o cannot change

irrespective of the equilibrium prices determined by the remainder of the model to which the subsystem (1)-(3) is appended.

More importantly, the system (5) and (6) does not have the same properties as suggested for the system (1), (2) and (3). The export demand elasticity (ε_E^{FD}) with respect to the endogenously determined home country price P_E is not η as might seem to be the case from (1) but

$$(7) \qquad \varepsilon_E^{FD} = \frac{\partial E}{\partial P_E} \cdot \frac{P_E}{E} = \frac{\eta(1+\mu)}{(\mu-\eta)}$$

and the import supply price elasticity (ε_M^{FS}) is not μ but

$$(8) \qquad \varepsilon_M^{FS} = \frac{\partial M}{\partial P_M} \cdot \frac{P_M}{M} = \frac{-\mu(1+\eta)}{(\mu-\eta)}$$

and for the import supply elasticity to have the appropriate sign $\eta \leq -1$ rather than $\eta \leq 0$ as in (1).

For the case where E and M are scalars, we can define the elasticity of the foreign offer curve, ε^{OC}, as

$$(9) \qquad \varepsilon^{OC} = \frac{M}{E} \cdot \frac{dE}{dM}$$

which implies that

$$(10) \qquad \varepsilon^{OC} = \frac{\varepsilon_E^{FD}}{\varepsilon_E^{FD} - 1} = \frac{\varepsilon_M^{FS}+1}{\varepsilon_M^{FS}},$$

and it follows from both (7) and (8) that

$$(11) \qquad \varepsilon^{OC} = \frac{\eta}{1+\eta} \cdot \frac{1+\mu}{\mu}$$

Thus specifying an equation subsystem (1), (2), and (3) is equivalent in the two good case to the specification of a foreign offer curve of constant elasticity. The 'true' elasticities of export demand and import supply functions are derived from the elasticity of the offer curve as in equation (10). The critical parameter in this specification is neither μ nor η taken alone, but the product $\frac{\eta}{(1+\eta)} \cdot \frac{(1+\mu)}{\mu}$. If $\eta < -1$ and $\mu > 0$ it follows that $1 < \varepsilon^{OC} < \infty$, and if η is specified close to -1 then ε^{OC} approaches ∞ independently of μ. A large positive value of μ and a large negative value of η causes ε^{OC} to approach 1. An offer curve with elasticity 1 is a straight line through the origin, the case of a small price taking economy.

In their calculations Boadway and Treddenick specify three values of η as 1, 10, and 25 which they interpret as values of export price elasticities. For the system of equations (1), (2) and (3) the values are not the true elasticities since the value of η can be the export price elasticity if μ and η are both 1. In other cases we obtain the following solutions using (7) and (8).

η	μ	ε_E^{FD}
-10	1	-1.8
-10	10	-5.5
-10	50	-10.2
-25	1	-1.9
-25	10	-7.8
-25	25	-13
-25	50	-17

Thus if a system of the form described by equations (1), (2) and (3) were to be used to represent the external sector there is some danger that the equation system can be misleading both in creating an appearance of monetary non-neutralities, and in potentially leading to misspecification of intended elasticity values.

IV. External Sector Closure Using a Mixed Price Endogenous/Fixed Price Formulation

The more common formulation in the applied general equilibrium models referenced earlier is to model the economy under investigation as facing fixed world prices for imports while simultaneously facing a foreign demand function for exports with constant own price elasticity. Domestic and foreign goods are imperfect substitutes in domestic demand functions. An external sector balance condition completes the system. Unlike the specification in Section III prices of imports are not endogenously determined in the larger model but given from outside the model as data. Following the Armington assumption common in these models we will assume that imports are not produced domestically. This specification is used by Boadway and Treddenick in the calculations they report and is common to the other models referenced.

We extend the system from Section III to such a situation to obtain an equation system of the following form.

The foreign export demand equation is as in Section III

$$(12) \qquad E = E_o(P_E/e)^{\eta} \qquad -\infty < \eta < 0$$

where P_E is the home country price of exports.

The foreign import supply function is selected to guarantee that any amount will be supplied at a fixed (exogenously determined) world price. To close the system we therefore need to consider the domestic import demand equation which, for now, we also consider to be of constant elasticity form although this is more typically part of the more complex domestic general equilibrium system with some specified substitution parameter capturing the fact domestic and foreign goods are imperfect substitutes.

This formulation gives an external sector equation subsystem as (12) plus

$$(13) \qquad M = M^S = M^D$$

$$(14) \qquad M^D = M_o(e.P_M)^{\mu} \qquad -\infty < \mu < 0$$

$$(15) \qquad P_M = \overline{P}_M$$

where M^S and M^D denote foreign import supply and domestic import demand respectively, and P_M denotes the world price of imports which is fixed at $\bar{P}_M$.

The system is completed by a balance of payments condition as in Section III

(16) $$\bar{P}_M M = \frac{P_E}{e} \cdot E$$

By substitution in this system e can be removed as in Section II

(17) $$e = \left(\frac{E_o}{M_o}\right)^{\frac{1}{\mu+\eta+1}} \cdot P_E^{\frac{n+1}{\mu+\eta+1}} \; \bar{P}_M^{\frac{-(\mu+1)}{(\mu+\eta+1)}}$$

giving the reduced forms

(18) $$M = M_o \left(\frac{E_o}{M_o}\right)^{\frac{\mu}{(\mu+\eta+1)}} \cdot P_E^{\frac{\mu(\eta+1)}{(\mu+\eta+1)}} \cdot \bar{P}_M^{\frac{\mu.\eta}{(\mu+\eta+1)}}$$

(19) $$E = E_o \left(\frac{E_o}{M_o}\right)^{\frac{-\eta}{(\mu+\eta+1)}} \cdot P_E^{\frac{(\mu.\eta)}{(\mu+\eta+1)}} \cdot \bar{P}_M^{\frac{(\mu+1)\eta}{(\mu+\eta+1)}}$$

and the identical reduced form elasticities

(20) $$\frac{\partial E}{\partial P_E} \cdot \frac{P_E}{E} = \frac{\mu \cdot \eta}{(\mu+\eta+1)}$$

and

(21) $$\frac{\partial M}{\partial P_M} \cdot \frac{P_M}{M} = \frac{\mu \cdot \eta}{(\mu+\eta+1)} \; .$$

Thus, in a similar vein to the argument in Section III specification of parameters for equations (12) and (14) could be misleading as the impression can be created of specifying separate export and import demand elasticities as η and μ when from (20) and (21) it follows that this is not the case. A single elasticity parameter is involved. Also, as in Section III, no real effects of exchange rates enter the system.

A further characteristic of this system is that in the two good case a complete foreign offer curve is not specified by the equation system. The reduced form system (18) and (19) determines equilibrium values of M and E for both domestic and foreign sectors of the model providing the export prices are consistent with zero profit conditions in the model. Equations (18) and (19) passively meet equilibrium conditions necessary for both domestic and foreign economies for any set of prices specified. Unlike the formulation in Section III, the equation system (12)-(15) does not define a foreign offer curve but a locus of external sector equilibria of constant elasticity. Instead of two intersecting offer curves as in traditional trade theory, offer curves of both domestic and foreign countries are constructed so as to simultaneously lie one on top of the other. This is, of course, counter to conventional trade theory.

Where the domestic portion of the general equilibrium system is consistent with a more conventional general equilibrium construction a similar problem nonetheless occurs with this type of formulation. As this is a common procedure in the models referred to above, it may be helpful if we amplify this point more fully for a simple case.

Suppose we consider the foreign export demand function to be as in (12), namely

$$(22) \qquad E = E_o(P_E/e)^{\eta} \qquad -\infty < \eta < 0$$

but allow the domestic import demand function to be part of a system of Cobb-Douglas demand functions for domestic and foreign goods, derived from utility maximization. For simplicity we may assume a pure trade formulation with no production where the only source of domestic income is the same of a fixed endowment $\bar{E}$ of the exportable commodity. This gives an import demand function

$$(23) \qquad M^D = \frac{\alpha(P_E\bar{E})}{\bar{P}_M \cdot e}$$

where α is the Cobb-Douglas weight on the imported good in the utility function. Use of the balance of payments condition (16) yields

(24) $$e = \left(\frac{E_o}{\alpha\bar{E}}\right)^{\frac{1}{\eta}} \cdot P_E$$

from which we obtain

(25) $$E = E_o^2/\alpha\bar{E} \quad \text{and} \quad M = \frac{(\alpha\bar{E})^{\frac{\eta-1}{\eta}} \cdot E_o^{\frac{1}{\eta}}}{\bar{P}_M} .$$

As a reduced form, this system differs sharply from the specification intended. The notable characteristic of (25) is that η from (22) does not appear in the export function, the reason being that the Cobb-Douglas demand functions must satisfy a domestic version of Walras Law and along with the balance of payments condition yields (25) independently of the value of η in (22).

V. External Sector Closure Using a Price Taking Formulation for all Tradeables

An alternative procedure to the external sector closure systems outlined in sections III and IV is to consider the domestic economy to face fixed world prices for all traded goods. This involves abandonment of the Armington assumption of product heterogeneity and the adoption of an assumption of product homogeneity among countries. Domestic prices are endogenously determined in this system with an 'exchange rate' variable determined to give external sector balance. The 'exchange rate' variable is in fact the relative price between a composite of traded and non-traded goods as suggested by Srinavasan [1979] in comments on the model constructed by Deardorff and Stern.

The role of the exchange rate in this formulation can be seen most simply by considering a pure exchange economy. Suppose we consider N commodities with the first n_o being traded goods and the remainder, $n_{o+1},\ldots,N$, non-traded goods; $n_o \leq N$. Suppose we now consider a single economy facing fixed world prices for the traded goods. We define the vector of prices $\Pi = (\Pi_1,\ldots,\Pi_N)$ to be the vector of domestic prices and we can define a set of domestic market demand functions $\xi_i(\Pi)$ $(i=1,\ldots,N)$ which are non-negative, continuous, homogeneous of degree zero in the domestic prices Π, and satisfy a domestic version of Walras Law, $\sum_{i=1}^{N} \Pi_i \xi_i(\Pi) = \sum_{i=1}^{N} \Pi_i W_i$ where the vector $w = w_1,\ldots,w_n$ define the economy wide endowments.

We suppose the world prices of traded goods $\Pi^w = (\Pi_1^w,\ldots,\Pi_{n_o}^w)$ to be given and fixed. For convenience we consider this to be a unit vector; i.e., $\sum_{i=1}^{n_o} \Pi_i^w = 1$. Equilibrium is characterized in this model by a vector of domestic prices Π^* such that demand-supply equalities hold for each non-traded good, where relative domestic prices of traded goods are the same as the fixed relative world prices and a zero trade balance condition applies.

From the homogeneity of degree zero of domestic demand functions we can arbitrarily normalize domestic prices to sum to an arbitrary constant such as unity; $\sum_{i=1}^{N} \Pi_i = 1$. Furthermore, we can impose the restrictions implied by the fixed relative world prices of traded goods by only considering domestic price vectors $(\lambda\Pi_1^W, \ldots, \lambda\Pi_{n_o}^W, \Pi_{n_{o+1}}, \ldots, \Pi_N)$ where $\Pi_1^W, \ldots, \Pi_{n_o}^W$ are the given world prices of traded goods. The normalization rule on Π is satisfied if we consider vectors $(\lambda, \Pi_{n_{o+1}}, \ldots, \Pi_N)$ whose elements sum to unity. The notation $\Pi(\lambda)$ denotes the vector $(\lambda\Pi_1^W, \ldots, \lambda\Pi_{n_o}^W, \Pi_{n_{o+1}}, \ldots, \Pi_N)$.

An equilibrium may be formally characterized by a vector $\Pi^*(\lambda^*) = (\lambda^*, \Pi^*_{n_{o+1}}, \ldots, \Pi^*_N)$ such that

(1) Demands equal supplies for non-traded goods

$$\xi_i(\Pi^*(\lambda^*)) - w_i \leq 0 \quad (= 0 \text{ if } \Pi_i^* > 0) \qquad i = n_{o+1}, \ldots, N$$

(2) Zero trade balance holds for traded goods

$$\sum_{i=1}^{n_o} \Pi_i^W(\xi_i(\Pi^*(\lambda^*)) - w_i) = 0.$$

In such a formulation λ plays the role of a relative price between traded and non-traded goods. Existence of an equilibrium in such a model follows directly from a small modification to the Gale Nikaido mapping used in conventional existence proofs which follow Brouwer's Theorem.[1] In some of the formulations adopted in the applied models mentioned earlier, a parameter such as λ appears to be equated with a financial exchange rate although it can be seen from the above formulation that this is not the case.

[1] See Mansur and Whalley [1980] in which this proof is given as part of a decomposition computational algorithm for international trade general equilibrium models.

An external sector formulation using this approach is in some ways more satisfactory than the formulations presented in Sections III and IV in the sense that this procedure directly yields an external sector closure procedure consistent with conventional micro trade theory. No specification of elasticities in the rest of the world is involved since the external sector closure involves a single parameter which sets the relative prices of traded and non-traded goods. In a two-good case the foreign offer curve is a straight line with a slope given by the world prices of traded goods. The domestic offer curve will have some elasticity to it depending on the specification of the domestic portion of the model constructed and their intersection will determine equilibrium quantities traded. The obvious disadvantage of this procedure is that complete specialization in production will typically be involved in an equilibrium; in addition policy variations may change the economy from being completely specialized in one good to complete specialization in another. A clear attraction of the Armington procedure for applied work is that cases of complete specialization are excluded.

VI. A Simple Numerical Example

To show the potential quantitative significance of the choice of the external sector specification in applied models we briefly report some calculations of alternative equilibria under different external sector closure specifications for a simple numerical example. We examine cases in which the same distorting factor tax is abolished and report large differences in the perceived impact of the tax change under different external sector specifications. While this may not be unexpected it serves to highlight the potential quantitative significance of some of the points made in earlier sections.

We consider a domestic economy with four commodities, the first two being tradeables and the second two being non-tradeables. Each commodity is produced according to a CES two-factor (capital and labour) production function with substitution elasticities all set at 0.8. We consider an equilibrium in the presence of a distorting tax characterized by equilibrium prices of all goods and factors of unity. Output 1 is exported and Output 2 is imported. The configuration of factor use in the tax distorted equilibrium is as follows:

	(Export) Sector 1	(Import Competing) Sector 2	(Non-traded Goods) Sector 3	(Non-traded Goods) Sector 4
Capital	180	20	100	100
Labour	20	180	100	100

and the trade flows involve export of 150 of good 1 and imports of 150 of good 2. Taxes on capital in sector 1 are 200 (giving a tax rate of 111%).

We construct parameters for production and a single Cobb-Douglas demand function so that all equilibrium prices in the presence of the tax are reproduced as unity. The value of output of each good is (400, 200, 200, 200) giving a national income of 1000.

Our example is deliberately constructed so that the distorting tax falls heavily on the factor intensive in the export industry to illustrate the importance of the external sector specification. Any attempt to analyze the impact of removal of the distorting tax cannot help but be highly dependent on the specification of the external sector. If the country has any influence over its terms of trade, it is possible that the distorting tax may result in a gain for the country since the terms of trade gain may outweigh the domestic distortionary cost. If, however, the country is a price taker on world markets there can be no terms of trade gain from such a tax and distorting losses are all that are involved.

We represent the difference between these cases by constructing a sequence of foreign offer curves of increasing curvature all of which are consistent with the same tax equilibrium: The foreign offer curves of increasing curvature all intersect the origin and the (150, 150) trade flow point characterizing the tax distorted equilibrium. The initial equilibrium before the tax is abolished is thus the same in all cases. We consider a sequence of circles to represent the foreign offer curves, all of which pass through the points (0,0) and (150,150). As we increase the radius of these circles the foreign offer curve becomes flatter and we move closer to a small country formulation; as we reduce the radius we move closer to a formulation in which the country has substantial influence over its terms of trade. An alternative procedure which would demonstrate the same point is to vary the elasticity of the foreign offer curve while preserving the tax distorted equilibrium. In each case we abolish the distorting tax and measure the welfare gains to the domestic economy in terms of compensating and equivalent variations.

The alternative cases we consider differ in terms of the value which determines the radius of the circle and in Table 1 we report welfare gains as a percentage of national income in the pre-change equilibrium.

Table 1

Alternative Calculations of Welfare Gains from Removal of Distorting Capital Tax in Sector 1 in Numerical Example

Radius of Foreign Offer Curve	Compensating Variation as % of National Income	Equivalent Variation as % of National Income
1,000,000	+5.58%	+7.02%
10,000	+5.08%	+6.32%
1,000	+2.35%	+2.73%
150	-1.36%	-1.40%

The calculations in Table 1 reveal the critical importance of the external sector specification. The estimates of 'loss' from the tax range all the way from +7% of national income to almost -1.5%. The 7% case is the domestic gain from the removal of a distorting tax; the -1.5% case incorporates a strong terms of trade loss which more than offsets the gain. While such sensitivity may not appear surprising in an artificial numerical example, it nonetheless highlights the critical importance of the external sector specification in models of this form.

VII. Conclusion

In this paper we have discussed the external sector closure procedures implicit in a number of recently constructed applied general equilibrium models. We argue that some of the systems currently used can be deceptive. There is no financial exchange rate variable as model descriptions sometimes suggest, nor are the trade elasticities simply characterized in the way sometimes stated. We suggest that any closure system in these models must inevitably involve a system of foreign excess demands and this could be more widely recognized in discussing these models. Even when this is done, there is substantial sensitivity of findings from such models to parametric specification and this we illustrate with a numerical example.

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