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by

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## **Privatization, Market Liberalization and Learning in Transition Economies**

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**ABSTRACT:**

Privatization and market liberalization are widely considered to be complementary reforms in transition economies. This paper challenges this view and the closely related "big bang" approach to economic reform. Our analysis suggests that when pursued too vigorously, privatization may actually impede the transition process following market liberalization. Our result is based on an explicit model of market learning, which is a vital component of the economic transition process. Compared to a fully-functioning market in a mature market economy, a market in transition is characterized by greater uncertainty regarding market conditions, including free market equilibrium levels of prices and quantities. Market participants must learn about these conditions through their participation in the market process. When the effects of learning are incorporated into the analysis, less than full privatization is optimal when the costs of learning are sufficiently important.

Privatization and market liberalization are widely considered to be complementary reforms in transition economies. This paper challenges this view and the closely related "big bang" approach to economic reform. Our analysis suggests that when pursued too vigorously, privatization may actually impede the transition process following market liberalization and reduce social welfare. Our result is based on an explicit model of the market learning process, which is an intrinsic component of any transition from a socialist economy—in which markets and market institutions are either nonexistent or highly distorted by government interventions—to a fully-functioning market economy. The theoretical literature to date on the transition in Central and Eastern Europe has ignored the need for individuals to learn, through their participation in the market process, simultaneously about the features of a market in transition and the effects of government-instituted reforms. (See, for example, Munnell [1991] .) We will argue in this paper that because it fails to take account of the learning process, the policy advice provided by Western experts to transition economies may be seriously flawed.

An urgent task facing policymakers in a small transition economy is to identify those subsectors of the economy in which their country will have a comparative advantage. <sup>1</sup> Typically, very little information about the identity of these subsectors is provided by relative prices from the pre-transition era, since these were hugely distorted by production quotas, taxes and subsidies, and other nonmarket influences. So what economic policies will best facilitate the process of acquiring the requisite information? The standard economic advice proffered by Western economists has been to follow a "big bang" approach of simultaneous, and rapid, liberalization and privatization. Proponents of this approach place tremendous faith in the efficacy of Adam Smith's "hidden hand" as a vehicle for achieving the optimal reallocation of resources: their belief is that newly privatized producers, who will be highly responsive to the newly liberalized market signals, have the best chance of identifying the optimal path of adjustment to the new market realities.

We investigate the relationship among learning and adjustment and the degree of privatization in an extremely stylized model of the transition process. We divide the production sector into privatized and non-privatized firms or *parastatals*. The fraction of privatized firms is viewed as a policy variable. Our privatized

firms are modeled as responsive to market signals. Specifically, they base their production decisions on their private signals about market conditions and previously realized market prices. Parastatals simply select a fixed level of production. The firms produce for a world market, with deterministic world price  $w^*$ . Two inputs are required for the production process: the first is available on world markets and is perfectly elastically supplied at a price of unity; the second is a nontradable good and has a stochastic, upward-sloping residual supply curve. The source of the stochasticity is transition-related uncertainty about the demand for the input by other sectors, which are also adjusting to the transition process. The only information that our producers have about the input, in addition to their own individual signals, is its past realized prices. In particular, our producers know neither the expected intercept of the input supply curve, the number of nonresponsive producers nor the amount produced by each. Further, they do not know the structure of the market. Rather than attempt to learn the parameters of an unknown structural model, our responsive producers simply predict market prices using an adaptive expectations-style learning rule.

Since Lucas [1972], models of expectation formation such as the one we present in this paper have been widely criticized on the grounds that they postulate non-“rational” behavior by economic agents. If agents behaved in the manner we postulate, the argument runs, then arbitrage possibilities would arise and remain unexploited. This critique is certainly compelling when applied to models of long-run or steady-state behavior. Since in such contexts an abundance of econometric data would be available, agents should be able to “reverse engineer” the economic environment within which they are operating, and then base their price predictions and production decisions on an empirically validated structural model of this environment. The Lucas critique has much less force when applied to models of short-run—and, in particular, *transition*—behavior. Because they are operating in a transition environment, the agents in our model have had neither the time, the data nor the experience to “master the model” to the extent required by the rational expectations hypothesis. Given the inevitable uncertainty about market structure that characterizes all transition economies, and the inevitable transition-related noise that contaminates whatever data is available, it seems reasonable to suppose that producers might use past price observations as a forecasting tool, rather than



relying upon some structural model in which they have no basis for confidence. A related point is frequently made by econometricians in defense of their use of reduced form time-series models for short-term forecasting (see, for example, Judge [1988], p. 675). Indeed, as an empirical matter, it is well known that those very arbitrage opportunities on which the rational expectations critique is based are in fact *extremely* widespread in the early stages of transition economies. While these opportunities will no doubt be exploited eventually, if they have not already disappeared, our focus in this paper is on the period during which agents have insufficient information to exploit them.

Our approach to the gradualism versus big-bang controversy differs from the approaches that have dominated the economic transition literature. (See, for example, Gates, Milgrom and Roberts [1993] and Murphy, Shleifer and Vishny [1992].) Rather than modeling a centrally-manipulated process, in which market participants respond perfectly to incentives set by government, we focus specifically on the functioning of transition markets when information and incentives are imperfect. We ignore political-economic considerations such as those raised by Laban and Wolf [1993]. In contrast to studies such as Dewatripont and Roland [1995], we treat uncertainty as an integral component of the market transition process, and consider how individuals' responses to market signals affect production, profits, prices and social welfare.

In §1, we construct a "modified cobweb model" with time varying parameters. In §2, we distinguish three phases of the dynamic adjustment path: (i) a phase of explosive oscillations in prices and production; (ii) a phase of damped oscillations; and (iii) a phase of monotone convergence to perfect information prices. We refer to the first two phases as the *short-run* and the last phase as the *long-run*. The results in this section focus on the relationship between price and production volatility and the fraction of privatized producers. In the short-run, volatility increases with the degree of privatization while in the long-run, additional privatization reduces volatility. Moreover, the length of the short-run increases with the number of responsive producers. In §3, we specify the policymaker's performance function and examine how the optimal level of privatization depends on the various parameters of the model. For tractability, our policymaker must choose a constant level of privatization for the entire transition.<sup>2</sup> Increasing the degree of privatization has

short-run costs and long-run benefits. Price volatility results in welfare losses relative to the perfect information equilibrium: our responsive producers base their production decisions on estimated prices and hence misallocate resources when these prices differ from realized prices. A more vigorous privatization program increases volatility both in the short-run and the early long-run, and hence exacerbates this first kind of resource misallocation. On the other hand, our parastatal producers are misallocating resources by ignoring market signals, and as the number of parastatals declines, this second kind of misallocation becomes less important. Since the costs of privatization decline in the long-run, while the benefits remain constant over time, the optimal level of privatization depends on the policymaker's rate of time preference. We prove that if the short run is sufficiently important to the policymaker, there is a unique optimal level of privatization, which falls short of full privatization; on the other hand, if policymakers are sufficiently patient then full privatization is optimal. We then present three comparative statics results. First, we show that an increase in uncertainty that is equi-proportional over time reduces the optimal level of privatization. Second, we prove that an uncertainty-delaying shift in the intertemporal composition of uncertainty, holding constant the total level of uncertainty, increases the optimal degree of privatization. Third, we consider a balanced expansion in the total size of the sector and find that the effect of this on the optimal degree of privatization depends on the nature of the uncertainty that producers face. If it is primarily due to noisiness in producers' initial private signals about market conditions, then the optimal level of privatization will decline. If it is primarily due to supply shocks that occur each period, then the optimal level of privatization will increase.

#### I. A MODEL OF LEARNING IN A TRANSITION ENVIRONMENT

We consider a partial-equilibrium model in which producers learn about the market price of one of their inputs. We adopt the linear-quadratic model which is the standard for learning-theoretic papers (see Townsend [1978], Rausser and Hochman [1979], Bray and Savin [1986], etc.). We assume that market demand for *output* is perfectly elastic at the world price. The production of  $q$  units of output requires  $0.5q^2$  units of a tradable input and  $q$  units of a nontradable input. While the tradable input is elastically supplied



at a world price of unity, the supply of the other input is upward sloping with a random intercept. An interpretation of the randomness is that the *residual* supply of the input is stochastic due to stochastic demand for the input by other sectors, which are also adjusting in the course of the transition. At the start of the transition, each price-responsive producer has a point estimate of the market-clearing price for the input. As the transition progresses, producers revise their estimates of this price, based on the unfolding path of realized prices. Thus, our producers are learning about the cost of doing business in this particular sector: because of competing pressures for resources, a key component of their cost structure is unknown. With this formulation we can address in a highly stylized fashion the policy question of how a country in transition identifies those sectors in which it has a comparative advantage.

The total number of producers, denoted by  $N$ , will be held fixed for now, but varied later in the paper. All producers are risk-neutral and have identical cost functions, but a fraction  $\alpha = \frac{n}{N}$  are privatized and responsive to market signals, while the remaining fraction  $(1 - \alpha)$  are nonresponsive parastatals. Each parastatal produces the quantity  $\bar{q}$ , so that aggregate parastatal output is  $(1 - \alpha)N\bar{q}$ . Producers' common cost function is denoted by  $C(q) = pq + \frac{1}{2}q^2$ , where  $p$  is the (unknown) price of the nontradable input. Thus in period  $t$ , each privatized producer's estimated profit maximizing level of output is identically equal to the difference between the commonly known world price of output,  $w^*$ , and her (subjective) estimate of the market clearing price of the input,  $\hat{p}_{it}$ . It follows that at anticipated prices  $\{\hat{p}_{it}\}_{i=1}^n$ , aggregate demand for the input is  $N((1 - \alpha)\bar{q} + \alpha w^*) - \sum_{i=1}^n \hat{p}_{it}$ . Supply of the input in period  $t$  at price  $p$  is equal to  $(a - \delta_t + bp)$ , where  $a < 0$ ,  $b > 0$  and  $\delta_t$  is a *quantity* shock.

We consider two kinds of restrictions on supply shocks. The first are maintained throughout.

**Assumption 1.** *The  $\delta_t$ 's are independently distributed. For each  $t$ , the distribution of  $\delta_t$  is symmetric about zero and has bounded support. For every  $t$ ,  $a - \delta_t \leq 0$  for all possible realizations of  $\delta_t$ .*

The latter assumption ensures that the price of the input will be positive (since the vertical intercept of the inverse input supply curve is  $(\delta - a)/b > 0$ .) In addition, we will assume either Assumption 2 or

Assumption 2' below. Assumption 2 states that the supply shocks are essentially transitional in nature, and so eventually shrink to zero. That is, letting  $\bar{\delta}_t$  denote the upper boundary of the support of  $\delta_t$ , we assume:

**Assumption 2:**  $\lim_{t \rightarrow \infty} \sum_{\tau=1}^t \bar{\delta}_\tau$  is finite.

An implication of this assumption is that the sum of the variances of the  $\delta_t$ 's is finite also.<sup>3</sup> Our alternative assumption is:

**Assumption 2':** The  $\delta_t$ 's are identically distributed.

The main difference between the alternative assumptions is that under Assumption 2, anticipated prices asymptotically *coincide* with the perfect information price, whereas under Assumption 2', anticipated prices are asymptotically unbiased predictors of the perfect information price.

The market clearing price of the input in period  $t$  is  $p_t = b^{-1}(\delta_t + N((1 - \alpha)\bar{q} + \alpha w^*) - a - \sum_{i=1}^n \hat{p}_{it})$ . Observe that  $p_t$  depends only on the *sum* of price-responsive agents' anticipated prices. To highlight this, we define the *average anticipated price* in period  $t$ ,  $\hat{p}_t = n^{-1} \sum_{i=1}^n \hat{p}_{it}$ , and rewrite the expression as:

$$p_t = b^{-1}(M(\alpha) + \delta_t - \alpha N \hat{p}_t). \quad (1)$$

where  $M(\alpha) = N((1 - \alpha)\bar{q} + \alpha w^*) - a > 0$ .

For each  $\alpha \in [0, 1]$ , we define a benchmark input price  $p^*(\alpha)$  with the following property: if each private producer anticipates this price and produces accordingly, and *if there were no supply shocks*, then the market clearing price of the input would indeed be  $p^*(\alpha)$ . It is defined as follows:

$$p^*(\alpha) = \frac{M(\alpha)}{b + \alpha N} \quad (2)$$

Henceforth, we will refer to  $p^*(\alpha)$  as the *perfect information input price* and suppress references to  $\alpha$  except when necessary. A special case is  $p^*(1) = (Nw^* - a)/(N + b)$ , which we refer to as the *Walrasian input price*,  $p^W$ , since this is the input price that would prevail in the Walrasian equilibrium of the perfect

information version of our model with no parastatal firms. We assume that  $(w^* - p^W) \neq \bar{q}$ , i.e., that parastatals' production level differs from the level that would be Pareto optimal if all firms were responsive.

Before any production takes place, each producer has a point estimate,  $\hat{p}_{1i}$ , of the market clearing price of the input. One possible interpretation is that  $\hat{p}_{1i}$  is the view of market conditions that  $i$  acquires during her pre-transition experience. These estimates are private information. We make no assumptions at this point about the statistical distribution of producers' estimates. In particular, they may or may not be an unbiased estimate of the perfect information price  $p^*(\alpha)$ . We will, however, maintain throughout that producers have no idea whether or not they are unbiased. Indeed, producers have no other prior information about market conditions. In particular, the magnitudes  $\alpha$ ,  $N$ ,  $a$  and  $b$  are unknown, as are the parameters governing the distribution of the  $\delta_i$ 's. Furthermore, producers do not know the structure of the market. That is, they do not know that input supply is linear, or that other firms have linear supply curves. These assumptions reflect the lack of market knowledge that characterizes economies at the outset of a transition.

In period  $t > 1$ ,  $i$ 's estimate of the  $t$ 'th period input price, denoted by  $\hat{p}_{ti}$ , is a convex combination of realized market prices in previous periods and her original private signal, with higher weights placed on more recent price realizations:  $\hat{p}_{ti} = \left( \sum_{\tau=0}^{t-1} \gamma^\tau \right)^{-1} \left[ \sum_{\tau=0}^{t-2} \gamma^\tau p_{t-\tau-1} + \gamma^{t-1} \hat{p}_{1i} \right]$ . Here,  $\gamma$  is not a rate of time preference but rather reflects the rate at which producers discount past price information. We assume that  $\gamma$  is identical for all individuals. To avoid dealing with certain special cases (see p. 12 below) we impose additional bounds on the size of  $\gamma$ .

**Assumption 3.** (a)  $0 < \gamma < b^{-1}$ ; (b)  $\lim_{t \rightarrow \infty} \sum_{\tau=1}^t \gamma^\tau > b^{-1}N$ .

Note that

$$\hat{p}_{t+1,i} = \left( \sum_{\tau=0}^t \gamma^\tau \right)^{-1} \left[ p_t + \sum_{\tau=1}^t \gamma^\tau \hat{p}_{\tau i} \right] \quad (3)$$

Again aggregating anticipated prices, setting  $\Gamma_t = \sum_{\tau=0}^t \gamma^\tau$  and observing that  $\Gamma_t - 1 = \sum_{\tau=1}^t \gamma^\tau$ , we obtain the following relationship between average anticipated prices in successive periods:

$$\hat{p}_{t+1} = (\Gamma_t)^{-1} (p_t + (\Gamma_t - 1) \hat{p}_t) \quad (4)$$

Observe from equations (1) and (2), we have for all  $t \geq 1$ :

$$(p_t - p^*) = \frac{n}{b} \left( \frac{\delta_t}{n} - (\hat{p}_t - p^*) \right) \quad (5)$$

The learning rule we specify derives from the adaptive expectations literature. Muth (1960) shows that such rules are optimal prediction rules when the effect of uncertainty on a system has both a temporary and a permanent component. In the classical literature on adaptive expectations, the individuals who are predicting the system's behavior do not interact with the system. In our model, by contrast, agents' expectations influence their production decisions, which *in the aggregate* affect the behavior of the system. Nonetheless, given the pattern of behavior we assume for our agents an econometrician who does not know the structure of the model but only knows that agents utilize adaptive expectations, so that there is both a permanent and a transitory component to shocks, cannot do a better job of predicting prices than by estimating coefficients on lagged prices. Indeed, it is difficult to imagine a more sophisticated rule that producers might adopt, given their total ignorance about the parameters that determine market conditions and the structural model. Note in particular that at least in the early stages of the transition, it would be a challenging statistical problem to disentangle the effects of the per-period supply shocks from those of agents' private initial signals. For example, suppose that the first few realized prices exceed  $\hat{p}_{1i}$ . Even if she knew the underlying structure of the sector, producer  $i$  would have no way of knowing whether to attribute these unexpectedly high prices to: (a) a large negative value of  $(\hat{p}_{1i} - p^*)$ ; (b) a large negative value on average of  $(\hat{p}_{1j} - p^*)$ ,  $j \neq i$ , resulting in underproduction; or (c) a sequence of positive  $\delta_t$ 's.

## 2. THE DYNAMICS OF PRICES AND PRODUCTION

2.1. **Production and Price Paths for fixed  $\alpha$ .** In this subsection we first derive an expression for average anticipated price in period  $t$ . We then fix an arbitrary vector of private market signals and a sequence of  $s - 1$  supply shocks, and consider the dynamic path of realized input prices from period  $s$  into the future.

When  $t = 1$ , private producer  $i$ 's anticipated input price is just her initial private signal of the market price,  $\hat{p}_{1i}$ . As noted above (equation (5)), whether the difference,  $(p_1 - p^*)$ , between the market clearing price and the perfect information price is positive or negative depends jointly on whether private producers have *on average* under- or over-estimated the perfect information price—i.e., on the relationship between  $\hat{p}_1$  and  $p^*$ —and on the sign of  $\delta_1$ . In period  $t = 2$ ,  $i$ 's updated estimate of the market price,  $\hat{p}_{i2}$ , is a weighted average of her initial signal and the previous period's realized price,  $p_1$ . From (4) and (5), the expression  $(\hat{p}_2 - p^*)$ , which is the divergence from the perfect information price of the average anticipated price in period two is  $(\hat{p}_2 - p^*) = \frac{1}{1+\gamma} \left( \frac{\delta_1}{b} + (\gamma - b^{-1}n) (\hat{p}_1 - p^*) \right)$

As the transition progresses, private producers sequentially revise their estimates of the market price. While earlier price observations are increasingly discounted, each new price observation has an increasingly small role in determining producers' estimates. Combining (3) and (5), we obtain the following relationship between aggregate anticipated prices in periods  $t-1$  and  $t$ :

$$(\hat{p}_t - p^*) = \frac{\delta_{t-1}}{b \sum_{\tau=0}^{t-1} \gamma^\tau} + \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau} (\hat{p}_{t-1} - p^*) \quad (6)$$

By recursively substituting, we can express  $(\hat{p}_t - p^*)$  in terms of the realized supply shocks up to period  $t-1$  and the gap between the average initial signal and the perfect information price:

$$(\hat{p}_t - p^*) = \sum_{\tau=1}^{t-1} \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \delta_\tau + \Phi(1, t-1) (\hat{p}_1 - p^*) \quad (7)$$

where  $\Phi(\tau, \tau') = \prod_{m=\tau}^{\tau'} \frac{(\Gamma_m - 1 - b^{-1}n)}{\Gamma_m}$  if  $\tau \leq \tau'$  and 1 otherwise. For future reference, note that

$$\frac{\partial \Phi(\tau, \tau')}{\partial n} = -b^{-1} \Phi(\tau, \tau') \sum_{m=\tau}^{\tau'} (\Gamma_m - 1 - b^{-1}n)^{-1}.$$

Let  $\bar{i}(b, \gamma, n)$  denote the smallest  $t$  such that  $\sum_{\tau=1}^t \gamma^\tau \geq b^{-1}n$ . Note that  $\bar{i}(b, \gamma, n)$  increases with  $n$ . Assumption 3 guarantees that  $\bar{i}(b, \gamma, n) > 1$  for all  $n$ . To ensure that certain critical derivatives exist—specifically expression (12) below—we impose the following technical assumption:

**Assumption 4.** For all natural numbers  $n$ ,  $\sum_{\tau=1}^{\bar{i}(b, \gamma, n)} \gamma^\tau \neq b^{-1}n$ .

Observe in equation (7) that for  $m \in [\tau + 1, t - 1]$ , the  $m$ 'th element of the product  $\Phi(\tau + 1, t - 1)$  will be positive iff  $m \geq \bar{i}(b, \gamma, n)$ . An important property of our model is that the coefficients on each of the random terms in expression (7) shrink to zero as  $t$  increases:

**Lemma 1.** For all  $\tau$ ,  $\lim_{t \rightarrow \infty} \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} = 0$ .

**Proof:** Since  $\Gamma_\tau \geq 1$ , Lemma 1 can be verified by showing that  $\lim_{t \rightarrow \infty} \Phi(\tau + 1, t - 1) = \lim_{t \rightarrow \infty} \prod_{m=\tau+1}^{t-1} \frac{(\Gamma_m - 1 - b^{-1}n)}{\Gamma_m} = 0$ . Observe that for each  $m < \bar{i}(b, \gamma, n)$ ,  $\frac{(\Gamma_m - 1 - b^{-1}n)}{\Gamma_m} \in [-b^{-1}n, 0)$ . Moreover, from Assumption 3, there exists  $\zeta(n) \in (0, 1)$  such that for all  $m \geq \bar{i}(b, \gamma, n)$ ,  $\frac{(\Gamma_m - 1 - b^{-1}n)}{\Gamma_m} \in (0, \zeta(n))$ . Hence for all  $\tau$  and  $t > \tau$ ,  $|\Phi(\tau + 1, t - 1)|$  is bounded above by  $(b^{-1}n)^{\bar{i}(b, \gamma, n)}$ . Moreover, for  $t > \tau > \bar{i}(b, \gamma, n)$ ,  $|\Phi(\tau + 1, t)| < \zeta(n) |\Phi(\tau + 1, t - 1)|$ , from which Lemma 1 follows. ■

We can now construct the sequence of gaps between realized prices and the perfect information price, starting from an arbitrary vector of private market signals. First observe from (5) and (7) that for all  $t \geq 1$ , the gap between the *realized* price at  $t$  and the perfect information price is

$$(p_t - p^*) = \frac{n}{b} \left( \frac{\delta_t}{n} - \sum_{\tau=1}^{t-1} \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \delta_\tau - \Phi(1, t-1)(\hat{p}_1 - p^*) \right) \quad (8)$$

If Assumptions 2 holds, expression (8) and Assumption 3 imply that *every* sequence of *realized* prices converges to the perfect information price,  $p^*$ . This result requires no restrictions on the statistical distribution of agents' initial signals. An immediate corollary is that with certainty, average *anticipated* price will asymptotically coincide with  $p^*$ . If Assumption 2' holds rather than Assumption 2, then the best we can say is that, conditional on any vector of initial market signals and supply shocks up to time  $s$ , the *expected* paths of actual and anticipated prices, starting from period  $s + 1$ , converge to the perfect information price  $p^*$ .

**Proposition 1.** (a) If Assumption 2 holds, then for any vector of initial market signals and sequence of supply shocks,  $\lim_{t \rightarrow \infty} (p_t - p^*) = 0$  and  $\lim_{t \rightarrow \infty} (\hat{p}_t - p^*) = 0$ . (b) If Assumption 2' holds, then for any vector of initial market signals and sequence of supply shocks up to period  $s$ ,  $\lim_{t \rightarrow \infty} E_s(p_t - p^*) = 0$  and  $\lim_{t \rightarrow \infty} E_s(\hat{p}_t - p^*) = 0$ .

**Proof:** Fix  $\epsilon > 0$ . To prove part (a) of the proposition, we will show that

$$\text{there exists } T > 0 \text{ such that for all } t > T, |(\hat{p}_t - p^*)| < \epsilon \quad (9)$$

From Assumption 3, there exists  $\alpha \in (0, 1/3)$  satisfying  $(1 - 3\alpha) = \lim_{t \rightarrow \infty} \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau}$ . Thus, we can pick  $T'$  sufficiently large that for all  $t > T'$ ,  $(1 - 2\alpha) > \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau}$ . By assumption 2, we can assume additionally that  $T'$  is sufficiently large that for all  $t > T'$ ,  $\left| \frac{\delta_{t-1}}{b \sum_{\tau=0}^{t-1} \gamma^\tau} \right| < \alpha\epsilon$ . It follows from (6) that for  $t > T'$ , if  $|(\hat{p}_{t-1} - p^*)| > \epsilon$ , then

$$\begin{aligned} |(\hat{p}_t - p^*)| &= \left| \frac{\delta_{t-1}}{b \sum_{\tau=0}^{t-1} \gamma^\tau} \right| + \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau} |(\hat{p}_{t-1} - p^*)| \\ &< \alpha\epsilon + (1 - 2\alpha)|(\hat{p}_{t-1} - p^*)| < (1 - \alpha)|(\hat{p}_{t-1} - p^*)| \end{aligned}$$

It follows that there exists  $T$  such that  $|(\hat{p}_T - p^*)| = (1 - \alpha)^{T-t+1} |(\hat{p}_{t-1} - p^*)| \leq \epsilon$ , since  $\lim_{T \rightarrow \infty} (1 - \alpha)^{T-t} = 0$ . Finally, observe that if  $|(\hat{p}_{t-1} - p^*)| \leq \epsilon$ , then

$$\begin{aligned} |(\hat{p}_{t+1} - p^*)| &= \left| \frac{\delta_t}{b \sum_{\tau=0}^t \gamma^\tau} \right| + \frac{\sum_{\tau=1}^t \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^t \gamma^\tau} |(\hat{p}_t - p^*)| \\ &< \alpha\epsilon + (1 - 2\alpha)\epsilon = (1 - \alpha)\epsilon < \epsilon \end{aligned}$$

Thus we have established that statement (9) is true, which completes the proof.

To prove part (b), observe that  $E_s(\hat{p}_t - p^*) = \sum_{\tau=1}^{s-1} \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \delta_\tau + \sum_{\tau=s}^{t-1} \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} E \delta_\tau + \Phi(1, t-1)(\hat{p}_1 - p^*)$ .

Since  $E \delta_\tau = 0$ , for all  $\tau$ , part (b) follows immediately from Lemma 1. ■



**2.2. Qualitative properties of production and price paths.** Our goal in this and the following subsection is to study “the shape” of the production and price paths generated by an arbitrary vector of private market signals and supply shocks over time, and to investigate how this shape changes with  $n$ . Unless restrictions are imposed on supply shocks, however, very little can be said about any *given* path. Accordingly, we assume initially that all supply shocks are zero, which allows us to illustrate the factors influencing the effects of the initial uncertainty.

We begin by analyzing the sequence of average anticipated prices. In period one, private producer  $i$ 's anticipated input price is just her initial private signal of the market price,  $\hat{p}_{1i}$ . In period two,  $i$ 's estimate of the market price,  $\hat{p}_{i2}$ , is a weighted average of her initial signal and the previous period's realized price. Consider the expression  $(\hat{p}_2 - p^*)$ , which is the divergence from the perfect information price of the average anticipated price in period two assuming no supply shocks:  $(\hat{p}_2 - p^*) = \frac{1}{1+\gamma} (\gamma - b^{-1}n) (\hat{p}_1 - p^*)$ . Note that because  $b\gamma < 1$  (p. 7 above), the sign on  $(\hat{p}_2 - p^*)$  is different from the sign on  $(\hat{p}_1 - p^*)$ .

Now consider the behavior of the average anticipated price in period  $t$  as a function of the preceding period's average anticipated price:  $(\hat{p}_t - p^*) = \frac{\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n}{\sum_{\tau=0}^{t-1} \gamma^\tau} (\hat{p}_{t-1} - p^*)$ . Under assumption 3, we can distinguish three cases, depending on whether: (i)  $(\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n) < -\sum_{\tau=0}^{t-1} \gamma^\tau$  or equivalently  $1 + 2\sum_{\tau=1}^{t-1} \gamma^\tau < b^{-1}n$ ; (ii)  $(\sum_{\tau=1}^{t-1} \gamma^\tau - b^{-1}n) \in [-\sum_{\tau=0}^{t-1} \gamma^\tau, 0]$  or equivalently  $\sum_{\tau=1}^{t-1} \gamma^\tau < b^{-1}n < 1 + 2\sum_{\tau=1}^{t-1} \gamma^\tau$ ; (iii)  $\sum_{\tau=1}^{t-1} \gamma^\tau > b^{-1}n$ . In case (i), the coefficient on  $(\hat{p}_{t-1} - p^*)$  is less than -1; in case (ii) it belongs to  $[-1, 0]$ , while in case (iii) it belongs to  $[0, 1]$ . Let  $\underline{t}(b, \gamma, n)$  denote the largest  $t$  such that case (i) holds, and  $\bar{t}(b, \gamma, n)$  to be the smallest  $t$  such that case (ii) holds. It follows from the preceding observation that in the absence of supply shocks, the path of anticipated prices generated by any vector of initial market signals can be divided into at most three phases: phase (i) runs from period 1 to  $\underline{t}(b, \gamma, n)$ , phase (ii) from  $\underline{t}(b, \gamma, n) + 1$  to  $\bar{t}(b, \gamma, n) - 1$  and phase (iii) from  $\bar{t}(b, \gamma, n)$  on. Phase (i) is characterized by explosive oscillations, phase (ii) by damped oscillations and phase (iii) by monotone convergence. We shall refer to phases (i) and (ii) as the *short-run*, and to phase (iii) as the *long-run*. Thus, in the short-run the paths of production and anticipated prices exhibit the familiar cobweb pattern, except that the underlying parameters vary with time.

Once supply shocks are introduced, a "representative price path" is, of course, no longer meaningful. Certainly, we can no longer proceed as above and partition *any given* price sequence into three phases with qualitatively different dynamic properties.<sup>4</sup> For example, there are sequences of supply shocks whose associated price paths alternate forever between oscillatory and monotone phases. In a probabilistic sense, however, the properties of the model with supply shocks mirror the characteristics described above. For example, if  $\underline{t}(b, \gamma, n) > 1$ , then the gap between  $\hat{p}_{\underline{t}(b, \gamma, n)}$  and  $p^*$  will more likely than not be wider than the gap between  $\hat{p}_1$  and  $p^*$ . Similarly, the gap between  $\hat{p}_{\bar{t}(b, \gamma, n)}$  and  $p^*$  will more likely than not be narrower than the gap between  $\hat{p}_{\underline{t}(b, \gamma, n)}$  and  $p^*$ .<sup>5</sup>

**2.3. The effect of increasing the number of price-responsive producers.** In the absence of supply uncertainty, an increase in  $n$ , the number of private producers, has three consequences. First, there is an increase in the magnitude of oscillations during the short-run. Second, the duration of the short-run increases. More precisely, both  $\underline{t}(b, \gamma, \cdot)$  and  $\bar{t}(b, \gamma, \cdot)$  increase with  $n$  (p. 10), but  $\underline{t}(b, \gamma, \cdot)$  increases by more than  $\bar{t}(b, \gamma, \cdot)$  so that phase (i) is extended and phase (ii) is squeezed. Third, once the long-run is reached, prices and production converge to perfect information levels at a faster rate. To see this, consider the ratio  $\frac{(\Gamma_t - 1 - b^{-1}n)}{\Gamma_t}$ . In the short-run, when this ratio is negative, an increase in  $n$  makes it more negative, increasing the magnitude of oscillations. Also, an increase in  $n$  postpones the date at which the ratio turns positive. In the long-run, when it is positive, an increase in  $n$  makes it less positive, increasing the rate of convergence.

Now suppose that supply shocks are non-zero. Again, the effects of  $n$  are comparable to those above, but only in a probabilistic sense. For example, if  $n$  increases to  $n'$ , then the probability that the gap between  $\hat{p}_{\underline{t}(b, \gamma, n')}$  and  $p^*$  is wider than the gap between  $\hat{p}_1$  and  $p^*$  will exceed the probability that the gap between  $\hat{p}_{\underline{t}(b, \gamma, n)}$  and  $p^*$  is wider than the gap between  $\hat{p}_1$  and  $p^*$ . Now consider the long-run, and suppose that the gap between  $\hat{p}_{\underline{t}(b, \gamma, n')}$  and  $p^*$  is equal to the gap between  $\hat{p}_{\underline{t}(b, \gamma, n)}$  and  $p^*$ . In this case, an increase in  $n$  increases the likelihood of smooth convergence to the perfect information price, since for  $t > \bar{t}(b, \gamma, \cdot)$ , the coefficients on the  $\delta_t$ 's decline as  $n$  increases.

**2.4. The variance of market prices.** In the preceding subsections, we considered the the shape of individual dynamic price and production paths and the effect of  $n$  on these shapes in the absence of supply uncertainty. We now examine the statistical properties of these paths and the effect of  $n$  on these properties. To simplify the analysis in this section, we assume that the average initial private signal is an unbiased estimator of the perfect information price.<sup>6</sup>

**Assumption 5.**  $E(\hat{p}_1 - p^*) = 0$ .

Under Assumption 5, (7) implies that for every  $t$ ,  $\hat{p}_t$  is an unbiased estimator of the perfect information price. The variance of  $\hat{p}_t$  is obtained directly from the same expression. Letting  $\zeta^2$  denote the variance of the average initial signal and  $\sigma_\tau^2$  denote the variance of  $\delta_\tau$ , we obtain:

$$\text{Var}(\hat{p}_t) = \left\{ \sum_{\tau=1}^{t-1} \left( \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \sigma_\tau^2 + (\Phi(1, t-1))^2 \zeta^2 \right\} \quad (10)$$

To economize on notation, we set  $\Gamma_0 = b^{-1}$  and  $\sigma_0^2 = \zeta^2$ . We can now rewrite (10) as

$$\text{Var}(\hat{p}_t) = \sum_{\tau=0}^{t-1} \left( \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \sigma_\tau^2 \quad (10')$$

We calculate the variance of the  $t$ -period'th *realized* price from expressions (8) and (10)

$$\text{Var}(p_t) = b^{-2} \left\{ \sigma_t^2 + \sum_{\tau=0}^{t-1} \left( \frac{n\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \sigma_\tau^2 \right\} \quad (11)$$

Holding  $n$  fixed, the effect of time on the variances of both  $\hat{p}_t$  and  $p_t$  will be immediately apparent from expressions (10) and (11). The turning point between phases (i) and (ii) in the zero supply shock case here determines the behavior of the variances of  $\hat{p}_t$  and  $p_t$ . In the very short-run (phase (i)), each period an additional term with magnitude greater than 1 is multiplied by the  $t - 1$  pre-existing terms and another term is added to the sum, so that both variances increase with  $t$ . In phase (ii) and the beginning of phase (iii), an

additional term with magnitude less than one is multiplied by the pre-existing terms, but an additional term is added, so the effect of  $t$  is indeterminate. In the very long term, however, both variances shrink to zero.

**Proposition 2.** For  $t < \underline{t}(b, \gamma, n)$ ,  $\text{Var}(p_t) > \text{Var}(p_{t-1})$  and  $\text{Var}(\hat{p}_t) > \text{Var}(\hat{p}_{t-1})$ . For any given  $t \geq \underline{t}(b, \gamma, n)$ , the relationship between variances in successive  $t$ 's cannot be determined. However, if Assumption 2 holds, then  $\lim_{t \rightarrow \infty} \text{Var}(\hat{p}_t) = \lim_{t \rightarrow \infty} \text{Var}(p_t) = 0$ .

**Proof:** Suppose  $t < \underline{t}(b, \gamma, n)$ . Each increment in  $t$  adds another positive term to expressions (10) and (11). Moreover, the coefficients on the common terms are larger at  $t$  than at  $t-1$ . Hence  $\text{Var}(p_t) > \text{Var}(p_{t-1})$  and  $\text{Var}(\hat{p}_t) > \text{Var}(\hat{p}_{t-1})$ . For  $t \geq \underline{t}(b, \gamma, n)$ , each increment in  $t$  again adds another positive term to the expressions but the coefficients on common terms are *smaller* at  $t$  than at  $t-1$ . Hence the indeterminacy. Now assume that Assumption 2 holds, let  $t$  increase without bound and let  $S = \lim_{t \rightarrow \infty} \sum_{\tau=1}^t \sigma_\tau^2$ . (The existence of  $S$  is guaranteed by Assumption 2.) Fix  $\epsilon > 0$  and  $T' > \bar{t}(b, \gamma, n)$  such that  $\sum_{\tau=1}^{T'-1} \sigma_\tau^2 > S - 0.5\epsilon b^2$ . Now pick  $T > T'$  such that  $\sum_{\tau=1}^{T'-1} \left( \frac{\Phi(\tau+1, T-1)}{b\Gamma_\tau} \right)^2 + (\Phi(1, t-1))^2 < 0.5\epsilon/S$ .  $T$  exists because we are summing a fixed number ( $T'$ ) of terms, each of which goes to zero as  $T$  increases. Since  $T' > \bar{t}(b, \gamma, n)$ ,  $\left( \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 < b^{-2}$ , for all  $t > \tau + 1 > T'$  (since  $\Phi(\tau+1, t-1)$  is the product of terms, all of which are less than unity, and  $\Gamma_\tau$  exceeds unity.) Hence for  $t > T$ ,

$$\begin{aligned} \text{Var}(\hat{p}_t) &= \sum_{\tau=T'}^{t-1} \left( \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \sigma_\tau^2 + \sum_{\tau=0}^{T'-1} \left( \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \sigma_\tau^2 \\ &< b^{-2} \sum_{\tau=T'}^{t-1} \sigma_\tau^2 + S \left( \sum_{\tau=0}^{T'-1} \left( \frac{\Phi(\tau+1, T-1)}{b\Gamma_\tau} \right)^2 \right) < 0.5\epsilon b^2 + 0.5\epsilon b^2 = \epsilon \end{aligned}$$

This proves that  $\lim_{t \rightarrow \infty} \text{Var}(\hat{p}_t) = 0$ . It now follows from (11) that in addition  $\lim_{t \rightarrow \infty} \text{Var}(p_t) = 0$ . ■

We now consider the relationship between  $n$  and the variance of the average anticipated price in period  $t$ . We find that in the short-run, an increase in  $n$  increases volatility, while in the long-run, the effect of  $n$  is indeterminate. Recalling from p. 9 the expression for  $\frac{\partial \Phi(\tau, \tau')}{\partial n}$ , we obtain the following expressions for the

first and second derivatives of  $\text{Var}(\hat{p}_t)$  with respect to  $n$ :

$$\frac{d\text{Var}(\hat{p}_t)}{dn} = -\frac{2}{b} \left\{ \sum_{\tau=0}^{t-1} \left( \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \left( \sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1} \right) \sigma_\tau^2 \right\} \quad (12)$$

$$\begin{aligned} \frac{d^2\text{Var}(\hat{p}_t)}{dn^2} = \frac{2}{b^2} \left\{ \sum_{\tau=0}^{t-1} \left( \frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau} \right)^2 \left[ 2 \left( \sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1} \right)^2 \right. \right. \\ \left. \left. - \left( \sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-2} \right) \right] \sigma_\tau^2 \right\} \quad (13) \end{aligned}$$

Under Assumption 3,  $\Phi(\tau+1, t-1)$  is the product of terms which for sufficiently large  $t$  are eventually all less than unity. Note the sequence  $\left( (\Gamma_m - 1 - b^{-1}n)^{-1} \right)_{m=1}^\infty$  is bounded. These facts<sup>7</sup> together with Assumption 2, imply that the sequences  $\left( \frac{d\text{Var}(\hat{p}_t)}{dn} \right)_{t=1}^\infty$  and  $\left( \frac{d^2\text{Var}(\hat{p}_t)}{dn^2} \right)_{t=1}^\infty$  are bounded.

Under Assumption 2, the comparative statics of volatility with respect to  $n$  are determinate only in the short-run, when anticipated (and hence realized) prices become more volatile as  $n$  increases. Under Assumption 2', they are also determinate in the extremely long-run, when price volatility declines as  $n$  increases.

**Proposition 3.** (a) In the short-run (i.e., for  $t < \bar{i}(b, \gamma, n)$ ),  $\text{Var}(\hat{p}_t)$  is increasing and convex in  $n$ . (b) Under Assumption 2, for any given  $t \geq \bar{i}(b, \gamma, n)$ , the derivative of  $\text{Var}(\hat{p}_t)$  with respect to  $n$  cannot be signed. (c) Under Assumption 2' there exists  $T$  such that for all  $n$  and all  $t > T$ ,  $\text{Var}(\hat{p}_t)$  is decreasing in  $n$ .

**Proof:** (a) By definition of  $\bar{i}(b, \gamma, n)$ ,  $(\Gamma_m - 1 - b^{-1}n)$  is negative for every  $m < \bar{i}(b, \gamma, n)$ . Hence for every  $\tau < t < \bar{i}(b, \gamma, n)$ ,  $\sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1}$  is negative. so  $\frac{\partial \text{Var}(\hat{p}_t)}{\partial n}$  is positive. To see that  $\frac{d^2\text{Var}(\hat{p}_t)}{dn^2} > 0$ , observe that in the short-run all terms in the summation  $\left( \sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1} \right)^2$  are negative. so that the square of the sum exceeds the sum of the squares. (b) For  $t \geq \bar{i}(b, \gamma, n) > \tau$ ,  $\sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1}$  includes both positive and negative terms, so that the coefficients on the  $\sigma_\tau$ 's cannot be signed in general. (c) Let  $\sigma^2$  denote the common variance of the  $\delta_\tau$ 's and consider the expression inside the curly brackets in display (12). Note that all of the terms in the summation over  $\tau$  are positive except for when  $\tau < \bar{i}(b, \gamma, n)$ . Now consider the last term in the summation over  $\tau$ ,

$\tau = t-1$ . We can pick  $\epsilon > 0$  such that for each  $t > \bar{i}(b, \gamma, N) \geq \bar{i}(b, \gamma, n)$ , the coefficient on  $\sigma_{t-1}^2$ ,  $\left(\frac{\Phi(t, t-1)}{b\Gamma_\tau}\right)^2 (\Gamma_m - 1 - b^{-1}n)^{-1}$ , exceeds  $\epsilon$ . Moreover, from Lemma 1, there exists  $T$  such that for  $t > T$ ,  $\left|(\Phi(1, t-1))^2 \sum_{m=1}^{t-1} (\Gamma_m - 1 - \frac{n}{b})^{-1} \zeta^2 - \sum_{\tau=1}^{\bar{i}(b, \gamma, n)-1} \left(\frac{\Phi(\tau+1, t-1)}{b\Gamma_\tau}\right)^2 \left(\sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - \frac{n}{b})^{-1}\right) \sigma^2\right|$  is smaller than  $\epsilon\sigma^2$ . Thus, for  $t > T$  all of the negative terms within  $\{\cdot\}$  are exceeded in absolute value by the single positive term  $\left(\frac{\Phi(t, t-1)}{b\Gamma_\tau}\right)^2 (\Gamma_m - 1 - b^{-1}n)^{-1} \sigma^2$ . Because  $\{\cdot\}$  is preceded by a minus sign, it follows that  $\text{Var}(\hat{p}_t)$  decreases with  $n$  for  $t > T$ . ■

### 3. EXPECTED SOCIAL SURPLUS AND COMPARATIVE STATICS

**3.1. Expected social surplus in period  $t$ .** *Expected social surplus* in period  $t$ ,  $V_t$ , is defined as the sum of the expected producer surpluses accruing to private producers and parastatals and the surplus that accrues to suppliers of the nontradable input. We compare  $V_t$  to *Walrasian social surplus*,  $SS^W$ , which is the surplus that would arise if there were no uncertainty and if all production occurred at Walrasian levels. It is useful to introduce an intermediate level of social surplus, *perfect information social surplus*,  $SS_\alpha$ , which is the surplus that would arise if private producers responded optimally to parastatal production levels and there were no uncertainty (thus  $SS^W = SS_{(1)}$ ). We can now decompose  $V_t$  into three components:  $SS^W$  plus a (negative) *misallocation effect*,  $\Delta^M SS = SS_\alpha - SS^W$ , which measures the deadweight loss due to parastatals' non-Walrasian production levels, plus a (negative) *uncertainty effect*,  $\Delta^U SS = V_t - SS_\alpha$ , which measures the loss due to private producers' imperfect information about market conditions.

This formulation allows us to highlight a tradeoff that arises each period. While the tradeoff is starkest in the short-run, it also applies to the early stages of the long-run. (Under either Assumption 2 or Assumption 2', the tradeoff becomes one-sided in the extremely long-run.) For standard reasons, the misallocation effect declines as  $n$  increases: since parastatals misallocate resources, an increase of  $n$  (or, equivalently, in  $\alpha = n/N$ ) moves the perfect information equilibrium price  $p^*(\alpha)$  closer to the Walrasian price  $p^*(1)$ . On the other hand, an increase in  $n$  exacerbates the uncertainty effect. Private producers' profits are negatively related to price variance, which increases with  $n$ . Since parastatal members are assumed to be risk neutral, their expected surplus is independent of the degree of price variance. We will show that both effects are

convex in  $n$ , so that expected social surplus attains a unique maximum. These effects depend not only on the number of private producers,  $n$ , but also on the *total* number of producers in the sector,  $N$ . Accordingly, we treat the total number of producers,  $N$ , as a variable rather than a parameter.

Walrasian social surplus is the sum of aggregate profits and input producer surplus in the Walrasian equilibrium with no uncertainty. Since the Walrasian output level is  $(w^* - p^W)$ , aggregate producer profits are  $N(w^* - p^W)^2/2$ , and total input producer surplus is  $N(w^* - p^W) \times (p^W + a/b)/2$ . Thus  $SS^W = \frac{(a+bw^*)^2}{2b(N+b)}$ .

The misallocation effect,  $\Delta^M SS$ , is a function of  $\Delta\bar{q} = \bar{q} - (w^* - p^W)$ , the difference between the parastatal output level and the Walrasian level.  $\Delta^M SS$  is obtained by summing the areas of three deadweight loss triangles due to parastatal misallocation under perfect information. Aggregating the areas of the triangles yields  $\Delta^M SS = -0.5\Delta\bar{q}^2 \frac{(N-n)(N+b)}{n+b}$ . Note that  $\Delta^M SS$  is both decreasing in and convex with respect to  $n$ .

The uncertainty effect,  $\Delta^U SS$ , is the sum of two terms with opposite signs. Private producers are negatively affected by uncertainty. Whenever they over-produce, the input price exceeds  $p^*(\alpha)$  and whenever they under-produce, the input price falls short of  $p^*(\alpha)$ . In either case, profits fall short of perfect information levels. Input producers, in contrast, are positively affected by uncertainty. Their sales exceed the perfect information level whenever the input price exceeds  $p^*(\alpha)$  and fall short of this level whenever the input price is below  $p^*(\alpha)$ . Parastatal producers are unaffected by uncertainty, since the quantity they produce is independent of price. Summing the two expected differences yields  $\Delta^U SS = -\frac{n}{2} (1 + b^{-1}n) \text{Var}(\hat{p}_t)$ . Summarizing, expected social surplus in period  $t$  as a function of  $n$  is:

$$V_t(n) = \frac{(a + bw^*)^2}{2b(N + b)} - 0.5\Delta\bar{q}^2 \frac{(N - n)(N + b)}{n + b} - \frac{n}{2} (1 + b^{-1}n) \text{Var}(\hat{p}_t) \quad (14)$$

**3.2. Present discounted value of expected social surplus.** So far, we have considered the relationship between the size of the private sector and expected social surplus at each given point in time. However, the key policy issue our analysis addresses is: what fraction of firms should be privatized, assuming that this fraction will be fixed for the entire transition period? To answer this question, we consider the decision problem facing a policymaker with discount rate  $\rho$ , whose objective is to maximize the present discounted value of



expected social surplus, defined as  $V(n) = (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} V_t(n)$ , and whose only policy instrument is the level of  $n$ . Note that  $V(n)$  is a convex combination of the per-period values of expected social surplus (i.e., the weights on the per period values sum to one). Substituting from expression (14), we obtain:

$$V(n) = \frac{(a + bw^*)^2}{2b(N+b)} - 0.5\Delta\bar{q}^2 \frac{(N-n)(N+b)}{n+b} - \frac{n}{2} \left(1 + \frac{n}{b}\right) (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \text{Var}(\hat{p}_t) \quad (15)$$

We identify conditions under which a unique solution exists for the policymaker's task of maximizing  $V_t(\cdot)$  with respect to  $n$ . In general, we cannot do this because discounted expected social surplus is not in general globally concave. In the short-run, however, the per-period ESS's are concave, so that a sufficient condition for global concavity is that short-run considerations are sufficiently important to the policymaker. The following proposition makes this precise: there will be a unique optimal level of privatization provided that the policymaker is sufficiently impatient. The result holds under either Assumption 2 or Assumption 2'.

**Proposition 4.** *Given any values of  $b$ ,  $N$  and  $\gamma$ , there exists  $\bar{\rho} > 0$  such that if the policymaker's discount rate  $\rho$  is less than  $\bar{\rho}$ , then there is a unique level of privatization that maximizes discounted expected social surplus.*

**Proof:** To prove the proposition, it is sufficient to prove that for sufficiently small  $\rho$ ,  $V(\cdot) = (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} V_t(\cdot)$  is strictly concave on the interval  $[1, N]$ .<sup>8</sup>

$$\frac{dV(n)}{dn} = 0.5 \left\{ \Delta\bar{q}^2 \left( \frac{N+b}{n+b} \right)^2 - (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \left[ \left(1 + 2\frac{n}{b}\right) \text{Var}(\hat{p}_t) + n \left(1 + \frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_t)}{dn} \right] \right\} \quad (16)$$

$$\frac{d^2V(n)}{dn^2} = -\Delta\bar{q}^2 \frac{(N+b)^2}{(n+b)^3} - (1-\rho) \times \sum_{t=1}^{\infty} \rho^{t-1} \left[ \frac{\text{Var}(\hat{p}_t)}{b} + \left(1 + 2\frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_t)}{dn} + \frac{n}{2} \left(1 + \frac{n}{b}\right) \frac{d^2\text{Var}(\hat{p}_t)}{dn^2} \right] \quad (17)$$

Consider the expression for  $V_t(n)$  given by (14). Clearly for all  $n$ ,  $\frac{(N-n)}{n+b}$  is convex in  $n$ . Moreover,  $\frac{n}{2} (1 + b^{-1}n)$  is both convex and increasing in  $n$ . The product  $\frac{n}{2} (1 + b^{-1}n) \text{Var}(\hat{p}_t)$  will be convex in  $n$ ,

and hence  $V_t(\cdot)$  will be concave, provided that  $\text{Var}(\hat{p}_t)$  is convex at  $t$ . Now Proposition 3 establishes that  $\text{Var}(\hat{p}_t)$  is convex in  $n$  for  $t < \bar{t}(b, \gamma, n)$ . Part (a) of Assumption 3 guarantees that  $\bar{t}(b, \gamma, n) > 1$  for all  $n$ . Moreover,  $\bar{t}(b, \gamma, \cdot)$  increases with  $n$  (p. 10). Therefore  $V_t(\cdot)$  is concave on  $[1, N]$ , for all  $t < \bar{t}(b, \gamma, 1)$ . Indeed, there exists  $\bar{\epsilon} > 0$  such that for all  $t < \bar{t}(b, \gamma, 1)$ ,  $\left| \frac{d^2 V_t(\cdot)}{dn^2} \right| > \bar{\epsilon}$  on  $[1, N]$ . On the other hand, since both  $\left( \frac{d\text{Var}(\hat{p}_t)}{dn} \right)$  and  $\left( \frac{d^2 \text{Var}(\hat{p}_t)}{dn^2} \right)$  are bounded above (p. 16), it follows that if  $\bar{\rho}$  is sufficiently small

$$\begin{aligned} \frac{d^2 V(\cdot)}{dn^2} &= (1-\bar{\rho}) \left\{ \sum_{t < \bar{t}(b, \gamma, 1)} \bar{\rho}^{t-1} \frac{d^2 V_t(\cdot)}{dn^2} + \sum_{t > \bar{t}(b, \gamma, 1)} \bar{\rho}^{t-1} \frac{d^2 V_t(\cdot)}{dn^2} \right\} \\ &< -(1-\bar{\rho}) \left\{ \sum_{t < \bar{t}(b, \gamma, 1)} \bar{\rho}^{t-1} \bar{\epsilon} + \bar{\epsilon} \right\} \leq 0. \end{aligned}$$

■

An alternative way to guarantee uniqueness is to identify conditions under which a corner solution must obtain. Under Assumption 2', full privatization will be optimal provided that the policy-maker is sufficiently *patient*. The key to this result is that in the long-run, the variance of anticipated prices actually *decreases* with  $n$  and hence one aspect of the tradeoff between misallocation and uncertainty evaporates, while the other aspect becomes more and more one-sided. Hence if the policy-maker is sufficiently patient, long-run considerations will eventually dominate short-run concerns, and full privatization will be optimal.

**Proposition 5.** *Given any values of  $b$ ,  $N$  and  $\gamma$ , if Assumption 2 holds, there  $\bar{\rho} < 1$  such that if the policymaker's discount rate  $\rho$  exceeds  $\bar{\rho}$ , then full privatization will be the unique maximizer of discounted expected social surplus.*

**Proof:** To prove the proposition it is sufficient to show that for all  $n \in [1, N]$ ,  $\frac{dV(n; \rho)}{dn}$  is positive (see expression (16)). For each  $n \in [1, N]$ , pick  $\epsilon(n) \in \left( 0, \Delta \bar{q}^2 \left( \frac{N+b}{n+b} \right)^2 \right]$ . Choose  $T'$  so that for all  $t \geq T'$ ,  $\sum_{i=1}^{T'} \rho^{i-1} \left( 1 + 2\frac{n}{b} \right) \text{Var}(\hat{p}_i) < 2$ . Let  $\omega' = \max \left\{ \left( 1 + 2\frac{n}{b} \right) \text{Var}(\hat{p}_t) : t < T' \right\}$ . Choose  $T''$  such that for all  $t \geq T''$ ,  $n \left( 1 + \frac{n}{b} \right) \frac{d\text{Var}(\hat{p}_t)}{dn} < 0$ . Let  $\omega'' = \max \left\{ n \left( 1 + \frac{n}{b} \right) \frac{d\text{Var}(\hat{p}_t)}{dn} : t < T'' \right\}$ . Let  $T = \max\{T', T''\}$ , and let  $\omega = \max\{\omega', \omega''\}$ . Then  $(1-\rho) \sum_{i=1}^{\infty} \rho^{i-1} \left[ \left( 1 + 2\frac{n}{b} \right) \text{Var}(\hat{p}_i) + n \left( 1 + \frac{n}{b} \right) \frac{d\text{Var}(\hat{p}_i)}{dn} \right]$

$< (1 - \rho) \sum_{i=1}^{\infty} \left[ \left(1 + 2\frac{n}{b}\right) \text{Var}(\hat{p}_i) + n \left(1 + \frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_i)}{dn} \right] < (1 - \rho)(2T\omega + 2)$ . Let  $x = \min\{2, 2T\omega\}$ .

Choose  $\rho(n)$  so that  $1 - \rho(n) < \frac{\epsilon(n)}{2x}$ . Then  $(1 - \rho(n)) \sum_{i=1}^{\infty} \rho^{i-1} \left[ \left(1 + 2\frac{n}{b}\right) \text{Var}(\hat{p}_i) + n \left(1 + \frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_i)}{dn} \right] < \frac{\epsilon(n)}{2x} 2x < \epsilon(n)$ . Hence the first, positive term in expression (16) dominates the second term, establishing that  $\frac{dV(n;\rho)}{dn}$  is positive. Clearly,  $\epsilon(n)$  and  $\rho(n)$  can be chosen so that  $\rho(\cdot)$  is a continuous function on  $[1, N]$ .

Since  $[1, N]$  is compact,  $\rho(\cdot)$  attains a minimum. For this minimum value of  $\rho$ ,  $\frac{dV(\cdot;\rho)}{dn}$  is positive on  $[1, N]$ .

■

**3.3. Comparative statics properties of the model.** In this subsection we present three results. The first result establishes that if uncertainty is increased *in a uniform way* then the optimal level of privatization declines. The second result changes the *composition* of uncertainty, holding constant (in a sense to be defined) the total level of uncertainty. We show that a reduction in uncertainty in earlier periods, matched by an increase in uncertainty in later periods, increases in the optimal level of privatization. Our final result considers the effect of expanding the size of the sector in a balanced way, so that the Walrasian price of the input remains unchanged. The effect of this change turns out to depend on the relative importance of the different kinds of uncertainty: if the initial market uncertainty is small relative to the per period uncertainty, then an increase in the size of the sector increases the optimal level of privatization.

**Proposition 6.** *Assume that discounted expected social surplus attains a unique, interior maximum at  $\bar{n}$ . If for all  $\tau \geq 0$ , the  $\sigma_\tau^2$ 's are increased by the same factor of proportionality, then the optimal level of privatization declines.*

**Proof:** Let  $(\bar{\sigma}_t^2)_{t \geq 0}$  denote the initial sequence of variance and define the family of functions  $(\sigma_t^2(\beta))_{t \geq 0}$  such that for all  $t$ ,  $\sigma_t^2(\beta) = \beta \bar{\sigma}_t^2$ . Let  $\bar{n}(\beta)$  be the optimal level of privatization as a function of  $\beta$ . Since  $\bar{n}(1) < N$ , the first order condition will be satisfied with equality in a neighborhood of  $\beta = 1$ . Observing from expressions (10) and (12) that both  $\text{Var}(\hat{p}_i; \beta)$  and  $\frac{d\text{Var}(\hat{p}_i; \beta)}{dn}$  are linear in  $\beta$ , we obtain:

$$0 \equiv \frac{dV(\bar{n}(\beta); \beta)}{dn} = 0.5 \left\{ \Delta \bar{q}^2 \left( \frac{N+b}{n+b} \right)^2 - (1 - \rho) \sum_{i=1}^{\infty} \beta \rho^{i-1} \left[ \left(1 + 2\frac{n}{b}\right) \text{Var}(\hat{p}_i; 1) + n \left(1 + \frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_i; 1)}{dn} \right] \right\}$$

Using the implicit function theorem, we obtain  $\frac{d\bar{n}(\beta)}{d\beta} = -\frac{d^2V(\bar{n}(\beta); \beta)}{dn d\beta} / \frac{d^2V(\bar{n}(\beta); \beta)}{dn^2}$ . To prove the proposition, we need to show that  $\frac{d\bar{n}(\beta)}{d\beta} < 0$ . Since by assumption  $V(\cdot)$  attains a unique maximum at  $\bar{n}(\beta)$ , it follows that  $\frac{d^2V(\bar{n}(\beta); \beta)}{dn^2} < 0$ . Thus, to prove the proposition, we need to show that

$$0 > \frac{d^2V(\bar{n}(\beta); \beta)}{dn d\beta} = -(1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \left[ \left(1 + 2\frac{n}{b}\right) \text{Var}(\hat{p}_t; 1) + n \left(1 + \frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_t; 1)}{dn} \right]$$

But since  $\frac{dV(\bar{n}(\beta); \beta)}{dn} = 0$  and  $\Delta \bar{q}^2 \left(\frac{N+b}{n+b}\right)^2$  is positive, it follows from the first order condition that

$$0 < (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \left[ \left(1 + 2\frac{n}{b}\right) \text{Var}(\hat{p}_t; 1) + n \left(1 + \frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_t; 1)}{dn} \right]$$

■

Our next result establishes that the optimal level of privatization depends on the nature of the uncertainty that producers face. In particular, uncertainty about initial market signals creates more problems for price-responsive producers than per period supply shocks. To make this point precisely, we will investigate the effect of changing the *composition* of price variance facing producers, without altering the *total amount* of variance in the system. Formally, we define a *variance-preserving rightward pivot in uncertainty about  $\hat{t}$*  as a perturbation,  $(d\sigma_{\tau}^2)_{\tau=1}^{\infty}$ , of the sequence  $(\sigma_{\tau}^2)_{\tau=1}^{\infty}$  with the following properties: (a) the induced change,  $\sum_{t=1}^{\infty} \rho^{t-1} d\text{Var}(\hat{p}_t; \bar{n})$ , in the discounted sum of variance terms is zero; (b) if  $\tau \leq \hat{t}$ , then  $d\sigma_{\tau}^2$  is negative; (c) if  $\tau > \hat{t}$ , then  $d\sigma_{\tau}^2$  is positive. Proposition 7 below establishes that for an appropriately chosen point  $\hat{t}$  in the short-run, such a pivot results in an increase in the optimal level of privatization.

The explanation for this result is rather delicate. At first sight it would appear that the result holds because errors in price expectations in earlier periods have a larger effect on total variance than errors in later periods. While this observation is correct, it does not explain the result. For the correct intuition, consider the effects of adding an extra price-responsive firm. These effects differ depending on the period in which an error is realized. If an error is realized in a period  $t$  within the short-run then the additional firm's reaction to this error distorts future price expectations in every period  $\tau > t$ ; if the error is realized in period  $t' > t$ , then

expectations and prices are distorted from period  $t'$  on, but nothing changes between periods  $t + 1$  to  $t'$ . It follows that the cumulative negative impact of an additional price-responsive firm will be greater if errors are realized earlier in the transition than if they are realized later. It is for this reason that a variance-preserving rightward shift in uncertainty mitigates the negative effect of additional privatization.

**Proposition 7.** *Assume that our policymaker's problem attains a unique maximum at  $\bar{n} < N$  and that the discounted sum of variance terms, evaluated at  $\bar{n}$ , increases with  $n$ . Then there exists  $\hat{t} < \bar{t}(b, \gamma, \bar{n})$  such that a variance-preserving rightward pivot in uncertainty about  $\hat{t}$  results in an increase in the optimal level of privatization.*

**Proof:** We first rewrite expression (12) so that  $\sum_{t=1}^{\infty} \rho^{t-1} \frac{d\text{Var}(\hat{p}_t)}{dn}$  is a linear combination of the  $\sigma_{\tau}^2$ 's:

$$\begin{aligned} \sum_{t=1}^{\infty} \rho^{t-1} \frac{d\text{Var}(\hat{p}_t)}{dn} &= -\frac{2}{b} \left\{ \sum_{\tau=0}^{\infty} \frac{\sigma_{\tau}^2}{b\Gamma_{\tau}} \left[ \sum_{t=\tau+1}^{\infty} \rho^{t-1} (\Phi(\tau+1, t-1))^2 \left( \sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1} \right) \right] \right\} \quad (18) \\ &\triangleq -\frac{2}{b} \left\{ \sum_{\tau=0}^{\infty} \frac{\sigma_{\tau}^2}{b\Gamma_{\tau}} Z(\tau) \right\} \end{aligned}$$

By assumption,  $\sum_{t=1}^{\infty} \rho^{t-1} \frac{d\text{Var}(\hat{p}_t)}{dn} < 0$ . It follows from expression (18) that there exists  $\tau \geq 0$  such that  $Z(\tau) < 0$ . By definition of  $\bar{t}(b, \gamma, n)$ ,  $\tau \geq \bar{t}(b, \gamma, n)$  implies  $Z(\tau) > 0$ . Hence there exists  $\tau < \bar{t}(b, \gamma, n)$  such that  $Z(\tau) < 0$ . Next observe that  $Z(\cdot)$  satisfies the following "single-crossing property:"

$$\text{If } Z(\tau) < 0 \text{ then for all } \tau' < \tau, Z(\tau') < 0 \quad (19)$$

To verify (19), observe that

$$\begin{aligned} Z(\tau') &= \sum_{t=\tau'+1}^{\infty} \rho^{t-1} (\Phi(\tau'+1, t-1))^2 \left( \sum_{m=\tau'+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1} \right) \\ &< (\Phi(\tau'+1, \tau))^2 \sum_{t=\tau+1}^{\infty} \rho^{t-1} (\Phi(\tau+1, t-1))^2 \left( \sum_{m=\tau'+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1} \right) \end{aligned}$$

Since  $(\Gamma_m - 1 - b^{-1}n)$  is negative for all  $m \in [\tau', \tau] \subset [0, \bar{i}(b, \gamma, n))$ , so that each term on the second line of the preceding display is negative, the expression is less than  $(\Phi(\tau'+1, \tau))^2 \sum_{t=\tau+1}^{\infty} \rho^{t-1} (\Phi(\tau+1, t-1))^2 \left( \sum_{m=\tau+1}^{t-1} (\Gamma_m - 1 - b^{-1}n)^{-1} \right)$ . Since, once again,  $(\Gamma_m - 1 - b^{-1}n)$  is negative for all  $m \in [\tau', \tau] \subset [0, \bar{i}(b, \gamma, n))$ , this expression is less than  $(\Phi(\tau'+1, \tau))^2 Z(\tau') < 0$ . This establishes that statement (19) is true. It follows that there exists  $\hat{\tau} < \bar{i}(b, \gamma, n)$  such that  $Z(\tau) < 0$  if  $\tau \leq \hat{\tau}$  while  $Z(\tau) > 0$  if  $\tau > \hat{\tau}$ . We now consider a variance preserving, rightward pivot in uncertainty about  $\hat{\tau}$ . Specifically, define the family of functions  $(\sigma_{\tau}^2(\beta))$  with the property that  $\frac{d\sigma_{\tau}^2(0)}{d\beta} < 0$  if  $\tau \leq \hat{\tau}$  while  $\frac{d\sigma_{\tau}^2(0)}{d\beta} > 0$  if  $\tau > \hat{\tau}$ . To prove the proposition we need to show that  $\frac{d\bar{n}(\beta)}{d\beta} > 0$ . From the implicit function theorem,  $\frac{d\bar{n}(\beta)}{d\beta} = \frac{d^2V(\bar{n}(\beta); \beta)}{dn d\beta} / \frac{d^2V(\bar{n}(\beta); \beta)}{dn^2}$ . Since by assumption  $V(\cdot)$  attains a unique maximum at  $\bar{n}(\beta)$ ,  $\frac{d^2V(\bar{n}(\beta); \beta)}{dn^2}$  must be negative. It follows that  $\frac{d\bar{n}(\beta)}{d\beta}$  will be positive iff  $\frac{d^2V(\bar{n}(\beta); \beta)}{dn d\beta}$  is also. From (16), we obtain

$$\frac{d^2V(\bar{n}(\beta); \beta)}{dn d\beta} = - (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \left[ \left(1 + 2\frac{n}{b}\right) \frac{d\text{Var}(\hat{p}_t; \beta, \bar{n}(\beta))}{d\beta} + n \left(1 + \frac{n}{b}\right) \frac{d^2\text{Var}(\hat{p}_t; \beta, \bar{n}(\beta))}{dn d\beta} \right]$$

By construction,  $\sum_{t=1}^{\infty} \rho^{t-1} \frac{d\text{Var}(\hat{p}_t; \beta, \bar{n}(\beta))}{d\beta} = 0$ . Hence  $\frac{d^2V(\bar{n}(\beta); \beta)}{dn d\beta}$  will be positive if and only if  $(1 - \rho) \sum_{t=1}^{\infty} \rho^{t-1} \frac{d^2\text{Var}(\hat{p}_t; \beta, \bar{n}(\beta))}{dn d\beta}$  is negative. But this is clearly the case since  $\sum_{t=1}^{\infty} \rho^{t-1} \frac{d^2\text{Var}(\hat{p}_t; \beta, \bar{n}(\beta))}{dn d\beta} = -\frac{2}{b} \left\{ \sum_{\tau=0}^{\hat{\tau}} \frac{1}{b\Gamma_{\tau}} Z(\tau) \frac{d\sigma_{\tau}^2(\beta)}{d\beta} + \sum_{\tau=\hat{\tau}}^{\infty} \frac{1}{b\Gamma_{\tau}} Z(\tau) \frac{d\sigma_{\tau}^2(\beta)}{d\beta} \right\}$  and by construction,  $Z(\tau) \frac{d\sigma_{\tau}^2(\beta)}{d\beta} > 0$  for all  $\tau$ . ■

Our final comparative statics exercise is to increase the size of the sector in a balanced way. To simplify the computations we set the intercept of the input supply schedule equal to zero. We simultaneously increase the total number of producers,  $N$ , and flatten the slope of the input supply schedule so that as  $N$  changes, the Walrasian price of the input remains equal to unity. That is, we define  $b = N(w^* - 1)$ . The uncertainty of each private producer's signal is unaffected by the change in  $N$ . We impose Assumption 2'—i.i.d. per-period supply shocks—and hold constant the variance of these shocks as  $N$  increases. This is consistent with our interpretation of these shocks as originating from outside the sector. That is, we are increasing the

magnitude of our sector *relative* to the rest of the economy. Our objective in this exercise is to determine whether the optimal fraction,  $\alpha$ , of privatized firms declines or increases as the sector grows.

It turns out that the answer to this question depends on the relative magnitudes of the initial market uncertainty relative to the per-period supply shocks. This contrasts sharply with the remainder of the paper, in which we have effectively ignored the distinction between the two kinds of uncertainty. (For example, up until now we have been able to derive all of our results from the expression for  $\text{Var}(\hat{p}_t)$  in (10'), which obliterates the distinction between  $\zeta^2$  and the  $\sigma_t^2$ 's rather than (10), which highlights the distinction. In the present exercise, however, an increase in the size of the sector has effects with different signs, depending on which kind of uncertainty is involved. As  $N$  increases, holding constant the fraction  $\alpha$  of price-responsive firms, the *absolute* number of these firms  $n = \alpha N$  increases. The effects of their initial market uncertainty compound each other, and the negative effects of privatization are exacerbated. On the other hand, as the supply side of the sector grows, the input supply function flattens and this dampens the effect of the per-period quantity shocks on the market for the input good. Because of this effect, the negative effects of privatization are mitigated as  $N$  increases. Ultimately, which of these effects will dominate depends on the relative magnitudes of the two different kinds of uncertainty.

**Proposition 8.** *Given any values of  $b$ ,  $N$  and  $\gamma$ , there exists  $\bar{\rho} > 0$  such that if the policymaker's discount rate  $\rho$  is less than  $\bar{\rho}$ , then the effect of increasing  $N$  on the optimal level of privatization is determined by the relative magnitudes of  $\zeta^2$  and  $\sigma^2$ . That is, for  $\rho < \bar{\rho}$ , there exists  $\mu$  such that as  $N$  increases, the optimal level of privatization,  $\bar{\alpha}(N)$  increases if and only if  $\frac{\zeta^2}{\sigma^2} < \mu$ .*



**Proof:** We first substitute for  $b$  and replace  $n/N$  by  $\alpha$  in expression (16) for  $\frac{dV(n)}{dn}$ . We then take the derivatives required to apply the implicit function theorem.

$$\begin{aligned} \frac{d^2V_t(\alpha)}{d\alpha dN} = & \frac{\Delta\bar{q}^2}{2(1-\rho)} \left( \frac{w^*}{\alpha + w^* - 1} \right)^2 + (1-\rho) \sum_{t=1}^{\infty} \rho^{t-1} \left\{ \frac{1}{N^2} \sum_{\tau=1}^{t-1} \left( \frac{\Phi(\tau+1, t-1)}{(w^*-1)\Gamma_\tau} \right)^2 \times \right. \\ & \left[ \left( 0.5 + \frac{\alpha}{w^*-1} \right) - \frac{\alpha}{w^*-1} \left( 1 + \frac{\alpha}{w^*-1} \right) \left( \sum_{m=\tau+1}^{t-1} \left( \Gamma_m - 1 - \frac{\alpha}{w^*-1} \right)^{-1} \right) \right] \sigma^2 \\ & \left. - (\Phi(1, t-1))^2 \left[ \left( 0.5 + \frac{\alpha}{w^*-1} \right) - \frac{\alpha}{w^*-1} \left( 1 + \frac{\alpha}{w^*-1} \right) \sum_{m=1}^{t-1} \left( \Gamma_m - 1 - \frac{\alpha}{w^*-1} \right)^{-1} \right] \zeta^2 \right\} \end{aligned} \quad (20)$$

To prove the proposition we need to show that whether  $\frac{d\bar{\alpha}(N)}{dN}$  is positive or negative depends on the ratio  $\zeta^2/\sigma^2$ . From the implicit function theorem,  $\frac{d\bar{\alpha}(N)}{dN} = - \frac{d^2V(\bar{\alpha}(N); N)}{d\alpha dN} / \frac{d^2V(\bar{\alpha}(N); N)}{d\alpha^2}$ . Since by assumption  $\bar{\alpha}(N)$  maximizes  $V(\cdot; N)$ ,  $\frac{d^2V(\bar{\alpha}(N); N)}{d\alpha^2}$  is negative. Hence the sign of  $\frac{d\bar{\alpha}(N)}{dN}$  will be the same as the sign of  $\frac{d^2V(\bar{\alpha}(N); N)}{d\alpha dN}$ . The terms in expression (20) that are of the form  $(\Gamma_m - 1 - \frac{\alpha}{w^*-1})^{-1}$  are negative provided that  $m < \bar{t}(b, \gamma, \bar{\alpha})$ . Hence if  $\rho$  is sufficiently small, the coefficients of  $\zeta^2$  and  $\sigma^2$  will both be negative. However  $\sigma^2$  contributes positively to  $\frac{d^2V(\bar{\alpha}(N); N)}{d\alpha dN}$  while  $\zeta^2$  contributes negatively. Hence the expression will be negative unless  $\sigma^2$  is sufficiently large relative to  $\zeta^2$ . ■

#### 4. CONCLUSION

This paper is premised on the idea that learning is an integral part of the transition from central planning to a market economy. The learning process gives rise to a welfare tradeoff associated with privatization policy in transition economies, when market liberalization is accompanied by uncertainty over market conditions. A more vigorous privatization program increases both short-run price and production volatility as well as the time it takes for this volatility to work its way out of the system. These effects diminish welfare, so if policymakers are sufficiently concerned with the short-run a policy of less than complete privatization will be optimal. On the other hand, an increase in the number of responsive private producers reduces the misallocation effect due to parastatals' distorted production levels, which is welfare-enhancing. Thus, if policymakers are sufficiently patient a policy of full privatization will be optimal.

The magnitude and the distribution of transition-related uncertainty affect the optimal level of privatization. A uniform increase in the variance of producers' initial signals and all supply shocks reduces the optimal degree of privatization. Reducing the share of total uncertainty borne in the early stages of the transition process increases the optimal level of privatization. In the case with independent identically distributed supply shocks, an increase in the total size of the sector increases the optimal level of privatization if and only if the variance of the initial signals is less than the variance of the supply shocks. The interaction between the input price uncertainty faced by the sector and the optimal level of privatization indicates that uncertainty regarding the effects of government policies in the transition period will affect the optimal level of privatization, possibly making it more costly to privatize in a given sector.

While transition governments are more concerned with dynamic issues, such as the optimal *rate* at which parastatals should be privatized, than with static ones, such as the optimal *level* of privatization, our static analysis has some clear dynamic implications. Specifically, it suggests that the greater the degree of initial uncertainty about market conditions, the more gradually should the privatization process begin. Also, government policies that support information provision and institution-building will be particularly important in the earliest stages of transitions, when their benefits are largest. In addition, information provision will be more important in industries with more privatized producers.

Proposition 7 has implications for the hotly debated issue of reform sequencing in transition economies. In some instances, a byproduct of rapid reform may be to increase uncertainty in the short-run, with the objective of reducing long-run uncertainty. (E.g., the removal of a commodity or exchange-rate stabilization scheme may generate considerable short-run volatility, but may forestall even greater volatility in the long-run due to speculation, etc.) If privatization is one of the suite of reforms being implemented, Proposition 7 highlights an externality that should be taken into consideration when policymakers are evaluating the costs and benefits of alternative intertemporal allocations of uncertainty. The fact that short-run uncertainty is more costly than long-run uncertainty in the context of privatization may in some instances weaken the

frequently argued position that multiple, socially painful reforms should be undertaken simultaneously and at the outset of the transition process.

Rather than supporting either side of the big-bang vs. gradualism debate, our analysis adds a new dimension to the debate by emphasizing the learning process. The tradeoff we derive favors gradualism under some circumstances and big-bangs under others. Even when the learning considerations addressed in this paper would suggest a gradualist approach, gradualism may not be optimal when broader considerations, particularly political-economic ones, are taken into account. Regardless of these considerations, however, our analysis indicates that because the big-bang approach fails to acknowledge the costs of rapid privatization in a uncertain environment, its predictions will be likely to be overly optimistic except when uncertainty is minimal or policymakers are very patient and learning is correspondingly unimportant.

#### References

- Bray, M.M. and N. E. Savin. "Rational Expectations Equilibria, Learning and Model Specification." *Econometrica*, September 1986, 54(5), pp. 1129-60.
- Dewatripont, M. and G. Roland. "The Design of Reform Packages Under Uncertainty." *American Economic Review*, December, 1995, 85(5), pp. 1207-1223.
- Gates, S., P. Milgrom and J. Roberts. "Complementarities in the Transition from Socialism: A Firm-Level Analysis." Institute for Policy Reform Working Paper. August, 1993.
- Hamilton, C. and A. Winters. "Opening Up International Trade with Eastern Europe." *Economic Policy*, No. 14, (1992).
- Judge, G. G., R. C. Hill, W. E. Griffiths, H. Lutkepohl and T. Lee. *Introduction to the Theory and Practice of Econometrics*. New York: John Wiley and Sons, 1988.
- Laban, R. and H. C. Wolf. "Large-Scale Privatization in Transition Economies." *American Economic Review*, December 1993, 83(5), pp.1199-1210.
- Lucas, R. E., Jr. "Expectations and the Neutrality of Money." *Journal of Economic Theory*, April 1972, 4(2), pp. 103-24.

Michael, T. T. Revesz, P. Hare and G. Hughes. "The Competitiveness of Hungarian Industry." *Acta Oeconomica* 1993, 45(3-4), pp. 319-48.

Murphy, K. M., A. Shleifer and R. W. Vishny. "The Transition to a Market Economy: Pitfalls of Partial Reform." *Quarterly Journal of Economics*. August 1992, 107(3), pp. 889-906.

Murrell, P. "Can Neoclassical Economics Underpin the Reform of Centrally Planned Economies?" *Journal of Economic Perspectives*. Fall 1991, 5(4) p.59-76.

Muth, J. F. "Optimal Properties of Exponentially Weighted Forecasts." *Journal of the American Statistical Association*. June 1996, 55(290) p. 299-306.

Rausser, G. C. and E. Hochman. *Dynamic Agricultural Systems: Economic Prediction and Control*. New York and Oxford: North-Holland, 1979.

Townsend, R. M. "Market Anticipations, Rational Expectations, and Bayesian Analysis." *International Economic Review*. June 1978, 19(2) pp. 481-94.

## NOTES

<sup>1</sup>This concern is evidenced by the search of these governments, in particular Poland, for areas of comparative advantage. See also Hamilton and Winters [1992], and Michael, Revesz, Hare and Hughes [1993], which seek to define areas of comparative advantage based on available information.

<sup>2</sup>We do not attempt to identify the optimal *rate* at which nonprivatized firms should be converted into privatized firms. While this issue is both fascinating and an important policy issue, it is also a *much* more difficult one in the context of an explicit model of learning.

<sup>3</sup>These strong restriction imply strong and simple asymptotic properties for our model (e.g., Propositions 1 and 2 below.) We could obtain similar, but weaker results under more general conditions.

<sup>4</sup>Of course, supply shocks certainly increase path volatility, but this point is will not be addressed until later (subsection 2.4 below)

<sup>5</sup>These properties follow immediately from expression (7) and the fact that the distributions of the  $\delta_t$ 's are independent, symmetric and centered around zero.

<sup>6</sup> Our results do not critically depend on this assumption, but it simplifies our analysis and notation by allowing the mean anticipated and realized prices each period to be unbiased ex ante. If we did not make this assumption, we would have to account for the mean price each period in order to examine the variance. Since the path of mean price over time moves from its initial (biased) position toward the perfect information price, little is gained in terms of explicitness by allowing for biased initial signals, while there is a significant cost in terms of added complexity.

<sup>7</sup> FACT (roof omitted): Let  $(x_t)$  be a positive sequence such that  $\lim_{t \rightarrow \infty} x_t < 1$  and let  $(y_t)$  be a bounded sequence. Then the sequence  $\lim_{t \rightarrow \infty} \prod^t x_t \sum^t y_t$  is bounded.

<sup>8</sup>As always we ignore the fact that the  $n$ 's should really be natural numbers.