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Behaviour of Reinforced Concrete and Composite Conical Tanks Under Hydrostatic and Seismic Loadings

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Graduate Program in Civil and Environmental Engineering

A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy

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ABSTRACT

Conical liquid storage tanks are widely used to store different liquids and to provide water supply at cities and municipalities. However, no comprehensive guidelines currently exist in the codes of practice for the structural analysis and design of such tanks. The walls of a conical tank can be made of steel, reinforced concrete, or a combination of the two materials in a composite type of construction in which steel and concrete walls are connected using steel studs. The research conducted in this thesis provides a comprehensive understanding of the structural behaviour of reinforced concrete and composite conical tanks under hydrostatic and seismic loadings. Finite element models for both reinforced concrete and composite tanks are developed and validated. In these models, a 3-D consistent shell element that accounts for the material nonlinear effect is used. The composite model also includes a 3-D contact element simulating the steel studs. The numerical models are utilized to study different behavioural aspects of reinforced concrete and composite conical tanks. An Equivalent Cylinder Method (ECM) is introduced and assessed for the analysis and design of reinforced concrete conical tanks. A set of charts that can be used to determine the adequate thickness and the straining actions for reinforced concrete conical tanks under hydrostatic pressure is developed. An Equivalent Section Method (ESM) for the analysis of composite tanks, which is based on using an equivalent single wall, is introduced and assessed. Both the ECM and ESM are found to be inadequate for the analysis of reinforced concrete and composite conical tanks, respectively. The composite finite element model is extended to include an optimization routine for minimization of the cost of composite conical tanks. The optimization of the design of a real composite conical tank using the developed
scheme resulted in a reduction of 32% in the material cost. The study is proceeded by examining the seismic behaviour of composite conical tanks. This is done by extending a previously developed numerical model that takes into account the fluid-structure interaction that occurs during the seismic vibration of a conical tank. A simplified procedure for the analysis of composite conical tanks under seismic loadings is introduced. The procedure is found to be adequate for preliminary design as the differences in the prediction of the natural frequencies and seismic forces are shown to be less than 17% compared to those predicted by the sophisticated numerical model.

**Keywords**

Reinforced concrete, Conical, Composite, Tanks, Studs, Finite element, Hydrostatic, Seismic, Analysis, Design.
CO-AUTHORSHIP

This thesis has been prepared in accordance with the regulations for an Integrated Article format thesis stipulated by the School of Graduate and Postdoctoral Studies at Western University. Statements of the co-authorship of individual chapters are as follows

Chapter 2: NONLINEAR BEHAVIOUR OF REINFORCED CONCRETE CONICAL TANKS UNDER HYDROSTATIC PRESSURE

All the numerical work was conducted by A. A. Elansary under close supervision of Dr. A. A. El Damatty and Dr. A. M. El Ansary. Drafts of Chapter 2 were written by A. A. Elansary and modifications were done under supervision of Dr. A. A. El Damatty and Dr. A. M. El Ansary. A paper co-authored by A. A. Elansary, A. A. El Damatty, and A. M. El Ansary has been published in the Canadian Journal of Civil Engineering.

Chapter 3: ASSESSMENT OF EQUIVALENT CYLINDER METHOD AND DEVELOPMENT OF CHARTS FOR ANALYSIS OF CONCRETE CONICAL TANKS

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Chapter 4: BEHAVIOR OF COMPOSITE CONICAL TANKS UNDER HYDROSTATIC PRESSURE

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Chapter 5: OPTIMUM DESIGN OF COMPOSITE CONICAL TANKS UNDER HYDROSTATIC PRESSURE

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Chapter 6: SEISMIC ANALYSIS OF LIQUID STORAGE COMPOSITE CONICAL TANKS

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To my beloved parents *Alaa Elansary* and *Salwa Abdelaakher*

To my lovely and wonderful wife *Lobna Mady*

To my sister *Marwa* and my brother *Mahmoud*

For patience, support, encouragement, and sharing these years of hard work

To my beloved country, *Egypt*

To my colleagues at *Cairo University*

To my supervisor, *Dr. Ashraf A. El Damatty*
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<td>$a_1, a_2, a_3$</td>
<td>Failure surface constants</td>
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<td>$a_V^t$</td>
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<tr>
<td>$A_{sf}$</td>
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<td>$A_s$</td>
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<tr>
<td>$A_c$</td>
<td>Area of concrete</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Area of contact element</td>
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<tr>
<td>$A_g$</td>
<td>Gross area of concrete</td>
</tr>
<tr>
<td>$A_{s, hoop}$</td>
<td>Area of steel in the hoop direction</td>
</tr>
<tr>
<td>$A_{st}$</td>
<td>Area of steel in meridional direction</td>
</tr>
<tr>
<td>$A_{Sr}$</td>
<td>Surface area of tank</td>
</tr>
<tr>
<td>$A_T$</td>
<td>Total surface area of tank’s vessel</td>
</tr>
<tr>
<td>$B$</td>
<td>Peel force in the stud</td>
</tr>
<tr>
<td>$B_{u,1}$</td>
<td>Ultimate peel force in stud</td>
</tr>
<tr>
<td>$B_{u,2}$</td>
<td>Ultimate peel force in stud to avoid bond slippage</td>
</tr>
<tr>
<td>$C$</td>
<td>Shrinkage coefficient</td>
</tr>
<tr>
<td>$[C]$</td>
<td>Damping matrix</td>
</tr>
<tr>
<td>$C_{St,One}$</td>
<td>Cost of one stud</td>
</tr>
</tbody>
</table>
$C_M$  Total material cost of tank’s vessel
$C_S$  Cost of steel shell
$C_{SB}$  Cost of steel bars
$C_{PC}$  Cost of plain concrete
$C_{St}$  Cost of studs
$C_{PC/m3}$  Cost of 1 m$^3$ of plain concrete
$C_{SB/m3}$  Cost of reinforcing steel bars per 1 m$^3$ of concrete
$C_{S/tonne}$  Cost of the steel plate per 1 tonne
$D_{ax}$ or $d_b$  Stud’s diameter
$[D_{ep}]$  Elasto-plastic material matrix of concrete
$[D^e]$  Elastic material matrix of concrete
$[D^s]$  Material matrix of steel
$[DM]_V$  Fluid added masses of vertical free vibration
$[DM]_H$  Fluid added masses of horizontal free vibration
$E_x$  Modulus of elasticity of steel bars in X-direction
$E_y$  Modulus of elasticity of steel bars in Y-direction
$E_s$  Modulus of elasticity of steel
$E_c$  Modulus of elasticity of concrete
$E_{eq}$  Modulus of elasticity of equivalent section
$f$  Yield surface of concrete
$f'_c$  Concrete strength
$f_y$  Yield stress of steel or yield stress of studs
$f_u$  Ultimate stress of steel
\( f_{\text{Shrinkage}} \)  
Tensile stresses due to shrinkage

\( f(X_{i,j,k,l}) \)  
Unpenalized objective function

\( f_p(X_{i,j,k,l}) \)  
Penalized objective function

\( \{f\}^t \)  
Total load vector corresponding to stresses at increment \( t \)

\( \{f_c\}^t \)  
Load vector corresponding to stresses in concrete wall at increment \( t \)

\( \{f_s\}^t \)  
Load vector corresponding to stresses in steel shell at increment \( t \)

\( \{f_{st}\}^t \)  
Load vector corresponding to stresses in studs at increment \( t \)

\( F \)  
Failure surface

\( F_Q \)  
Factor for the base shear force

\( F_N \)  
Factor for the base normal force

\( g(\theta) \)  
Deviatoric plane shape function

\( H \)  
Vessel’s height

\( H_e \)  
Elastic hardening modulus of concrete

\( H_p \)  
Plastic hardening modulus of concrete

\( H_{eq} \)  
Height of equivalent cylinder

\( \{H\} \)  
Vector of unity corresponding to active horizontal degrees of freedom

\( I \)  
Confining pressure

\( I_{eq} \)  
Moment of inertia of equivalent section

\( I_{cr} \)  
Moment of inertia of cracked section
$I_{eq}$  
Moment of inertia of uncracked section

$K_p$  
Stiffness of the peel spring per unit area

$K_s$  
Stiffness of the shear spring per unit area  
or stiffness of sloshing

$K_v$  
Stiffness of flexible component in vertical direction  
or stiffness of shaft in vertical direction

$k_H$  
Stiffness of shaft in horizontal direction

$[K_0]$  
Global initial stiffness matrix

$[K_{ss}]$  
Initial stiffness matrix of steel shell

$[K_{cw}]$  
Initial stiffness matrix of concrete wall

$[K_{ce}]$  
Initial stiffness matrix of contact element

$[K]^{t(k-1)}$  
Global stiffness matrix at load increment $t$ and  
itration $(k-1)$

$[K_c]^{t(k-1)}$  
Stiffness matrix of concrete wall at load increment $t$ and  
itration $(k-1)$

$[K_s]^{t(k-1)}$  
Stiffness matrix of steel shell at load increment $t$ and  
itration $(k-1)$

$[K_{st}]^{t(k-1)}$  
Stiffness matrix of studs at load increment $t$ and  
itration $(k-1)$

$l$  
Stud’s length

$L$  
Height of shaft

$m_s$  
Equivalent sloshing mass

$m_f$  
Equivalent flexible mass
Equivalent rigid mass

Direction cosines for angles between local and global directions

Meridional moment

Mass of contained fluid

Ultimate meridional moment

Nominal meridional moment

Effective mass matrix

Mass matrix of steel shell

Mass matrix of concrete wall

Modular ratio between the steel and concrete

Base normal force

Number of studs connected to a certain element

Total number of studs for the tank

Shape function at node i

Cubic shape function at node n

Total number of studs

Meridional axial force

Ultimate meridional compression force

Peel force in one stud

Total peel forces carried by contact element

Base shear force

Vessel’s radius at the bottom
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_F$</td>
<td>Ring tension force</td>
</tr>
<tr>
<td>$R_{\text{max}}$ or $R_u$</td>
<td>Ultimate ring tension force</td>
</tr>
<tr>
<td>$R_{\text{eq}}$</td>
<td>Radius of equivalent cylinder</td>
</tr>
<tr>
<td>${R}^t$</td>
<td>Global load vector at increment $t$</td>
</tr>
<tr>
<td>${R_c}^t$</td>
<td>Load vector due to own weight of concrete wall</td>
</tr>
<tr>
<td>${R_S}^t$</td>
<td>Load vector due to own weight of steel shell</td>
</tr>
<tr>
<td>${R_f}^t$</td>
<td>Load vector due to liquid pressure</td>
</tr>
<tr>
<td>$S_{st}$</td>
<td>Spacing between studs</td>
</tr>
<tr>
<td>$S_u$</td>
<td>Ultimate shear force in stud</td>
</tr>
<tr>
<td>$SF$</td>
<td>Scale factor for earthquake</td>
</tr>
<tr>
<td>$S_{M}(T)$</td>
<td>Acceleration from Manual of Civil Structures in Mexico</td>
</tr>
<tr>
<td>$S_{E}(T)$</td>
<td>Acceleration from earthquakes’ spectra</td>
</tr>
<tr>
<td>$t$</td>
<td>Vessel’s wall thickness or time</td>
</tr>
<tr>
<td>$t_{eq}$</td>
<td>Thickness of equivalent cylinder or equivalent section</td>
</tr>
<tr>
<td>$t_f$</td>
<td>Thickness of the flange of a steel beam</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Concrete wall’s thickness</td>
</tr>
<tr>
<td>$t_s$</td>
<td>Steel shell’s thickness</td>
</tr>
<tr>
<td>$T$</td>
<td>Steel shell’s minimum thickness</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Maximum hoop tension force in concrete wall</td>
</tr>
<tr>
<td>$T_F$ or $T$</td>
<td>Fundamental period</td>
</tr>
<tr>
<td>$T_{\text{min}}$</td>
<td>Minimum thickness of steel shell</td>
</tr>
<tr>
<td>$u$</td>
<td>Ultimate bond stresses</td>
</tr>
<tr>
<td>${\ddot{U}^t}$</td>
<td>Total nodal accelerations vector at time $t$</td>
</tr>
</tbody>
</table>
\{\dot{u}^t\} \quad \text{Total nodal velocities vector at time } t

V_{elm.} \quad \text{Total shear forces carried by contact element}

V_{sd} \quad \text{Shear force in one stud}

V_S \quad \text{Ultimate shear force in a stud}

V_P \quad \text{Ultimate peel force in a stud}

V_{Pb} \quad \text{Ultimate peel force on the stud to avoid bond slippage}

\{V\} \quad \text{Vector of unity corresponding to active vertical degrees of freedom}

X_{i,j,k,l} \quad \text{Solution instance i.e: combination of variables } i, j, k, l

Y_{act.} \quad \text{Actual values for the function } Y

Y_{all.} \quad \text{Allowable values for the function } Y

\beta \quad \text{Stress ratio}

\gamma_{xy}, \gamma_{xz}, \text{ and } \gamma_{yz} \quad \text{Shear strains}

\gamma \quad \text{Liquid specific weight}

\gamma_s \quad \text{Specific weight of steel}

\delta W \quad \text{Virtual work done by the forces in the springs}

\delta \Delta \bar{u}^s, \delta \Delta \bar{v}^s, \delta \Delta \bar{w}^s \quad \text{Virtual incremental local displacements at steel shell}

\delta \Delta \bar{u}^c, \delta \Delta \bar{v}^c, \delta \Delta \bar{w}^c \quad \text{Virtual incremental local displacements at concrete wall}

\Delta u^s, \Delta v^s, \Delta w^s \quad \text{Incremental global displacements at steel shell}

\Delta u^c, \Delta v^c, \Delta w^c \quad \text{Incremental global displacements at concrete wall}

\Delta \bar{u}^s, \Delta \bar{v}^s, \Delta \bar{w}^s \quad \text{Incremental local displacements at steel shell}

\Delta \bar{u}^c, \Delta \bar{v}^c, \Delta \bar{w}^c \quad \text{Incremental local displacements at concrete wall}

\Delta U_n^c, \Delta V_n^c, \Delta W_n^c \quad \text{Nodal degrees of freedom at concrete wall}

xxx
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta U_n^s, \Delta V_n^s, \Delta W_n^s)</td>
<td>Nodal degrees of freedom at steel shell</td>
</tr>
<tr>
<td>(\bar{\varepsilon})</td>
<td>Effective strain</td>
</tr>
<tr>
<td>(\varepsilon_1, \varepsilon_2, \varepsilon_3)</td>
<td>Principle strains</td>
</tr>
<tr>
<td>(\varepsilon_x)</td>
<td>Normal strain in X-direction</td>
</tr>
<tr>
<td>(\varepsilon_y)</td>
<td>Normal strain in Y-direction or yield strain of steel</td>
</tr>
<tr>
<td>(\varepsilon_u)</td>
<td>Ultimate strain of steel</td>
</tr>
<tr>
<td>(\varepsilon_{ESM})</td>
<td>Strain of equivalent section</td>
</tr>
<tr>
<td>({\Delta U})</td>
<td>Global displacement vector</td>
</tr>
<tr>
<td>(\zeta_1)</td>
<td>Damping ratio for first mode</td>
</tr>
<tr>
<td>(\zeta_2)</td>
<td>Damping ratio for second mode</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>Vessel’s angle of inclination</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Coefficient of friction</td>
</tr>
<tr>
<td>(\nu_c)</td>
<td>Poisson’s ratio of concrete</td>
</tr>
<tr>
<td>(\nu_s)</td>
<td>Poisson’s ratio of steel</td>
</tr>
<tr>
<td>(\rho_N)</td>
<td>Violation factor for the constraint N</td>
</tr>
<tr>
<td>(\rho_x)</td>
<td>Reinforcement ratio in X-direction</td>
</tr>
<tr>
<td>(\rho_y)</td>
<td>Reinforcement ratio in Y-direction</td>
</tr>
<tr>
<td>(\rho_{s,min})</td>
<td>Minimum reinforcement ratio</td>
</tr>
<tr>
<td>(\sigma_x)</td>
<td>Normal stress in the X-direction</td>
</tr>
<tr>
<td>(\sigma_y)</td>
<td>Normal stress in the Y-direction</td>
</tr>
<tr>
<td>(\sigma_c)</td>
<td>Maximum effective stress</td>
</tr>
<tr>
<td>(\bar{\sigma})</td>
<td>Applied effective stress</td>
</tr>
<tr>
<td>(\sigma_y)</td>
<td>Yield stress of steel</td>
</tr>
</tbody>
</table>
\( \sigma_u \)  
Ultimate stress of steel

\( \sigma_c \)  
Stress in concrete

\( \sigma_s \)  
Stress in steel

\( \sigma_1, \sigma_2, \text{ and } \sigma_3 \)  
Principle stresses

\( \sigma_{x'}, \sigma_{y'} \text{ and } \sigma_{z'} \)  
Stresses in the local directions \( x', y' \text{ and } z' \), respectively

\( \tau_{xy}, \tau_{xz}, \text{ and } \tau_{yz} \)  
Shear stresses

\( \phi \)  
Strength reduction factor

\( \Psi \)  
Plastic potential surface of concrete

\( \Psi_N(X_{i,j,k,l}) \)  
Violation function for constraint \( N \)

\( \omega \)  
Angular frequency
CHAPTER 1

INTRODUCTION

1.1. General

Elevated storage tanks are used all over the world to store different types of liquids and to provide supply of water to cities and municipalities. These tanks provide the pressure head necessary to deliver water to distant locations. Typically, elevated tanks consist of a containment vessel mounted on a supporting system in form of shaft or space frame. The tank’s vessel can be made of steel, reinforced concrete, or composite concrete-steel. Different geometrical shapes can be used for water storage tanks, such as a rectangle, cylinder, sphere, or cone. Rectangular tanks occupy large land area and they are not aesthetically appealing as conical tanks. Spherical tanks are not as easy to be constructed as rectangular and conical tanks especially if made of reinforced concrete, where the formwork is difficult to construct. In general, conical vessels have two common shapes: pure and combined. The pure conical vessel consists solely of a truncated cone, while the combined conical vessel consists of a pure cone with a top superimposed cylindrical cap. Conical vessels have a larger liquid retaining capacity for the same base radius of cylindrical vessels. In addition, conical vessels can have a smaller height for the same storage volume compared to cylindrical vessels. Moreover, elevated conical tanks can provide large storage capacity without the need of large thick cantilevered base that exists in elevated cylindrical tanks.

Large liquid storage capacities can be provided by steel vessels using a relatively small wall thickness. However, they require regular maintenance frequently to prevent corrosion. Moreover, they are highly susceptible to buckling as well as significant strength reduction.
due to geometric imperfections. On the contrary, reinforced concrete tanks require less maintenance and are less susceptible to strength reduction due to geometric imperfections. Conical tanks are subjected to tensile and compressive forces in the hoop and meridional directions, respectively. Reinforced concrete tanks are efficient in resisting the compressive meridional forces, while they are weak in resisting the tensile hoop forces. Composite concrete-steel tanks, which will be referred to as “composite tanks”, consist of an internal concrete wall connected through studs to an external steel shell, as shown in Fig. 1-1. These tanks combine the advantages of reinforced concrete and steel tanks because they provide adequate resistance to both the compressive meridional and tensile hoop forces.

![Figure 1-1](image.png)

**Fig. 1-1.** Cross section plan in composite concrete-steel tank.

According to the ACI-371 (2008) design code, the capacity of elevated water tanks ranges between 1,900 and 11,000 m³. **Fig. 1-2** (a) shows an example of a reinforced concrete conical tank in China, while **Fig. 1-2** (b) shows an example of a composite conical tank in Mexico. Both tanks are mounted on a shaft with a circular cross section.
In the following sections, the motivation of conducting this thesis is presented, afterwards a review of previous research studies on the analysis and design of liquid storage tanks under hydrostatic loading is reported. Then, a review on the analysis and design of liquid storage tanks under seismic loading is presented. Finally, the author reports the objectives and scope of the thesis as well as a summarized description of each chapter.

1.2. **Motivation**

A number of serious failures of elevated water tanks occurred in the past and triggered researchers to study the behaviour, analysis, and design of these structures under both hydrostatic and seismic loadings. Vandepitte (1977) reported one of the elevated steel tanks failures, which occurred in Belgium, in 1972. Korol (1991) reported the failure of another elevated steel tank, which occurred in Fredericton, Canada, in 1990. Both investigations revealed that these tank collapsed due to buckling when they were subjected to hydrostatic pressure. Godden (1985) reported a failure of an elevated reinforced concrete tank under an earthquake, which occurred in El-Asnam, Algeria in 1980, as shown in Fig. 1-3(a). This
investigation showed that the failure of this tank was due to the poor reinforcement detailings at the beam-column connections.

Memari and Ahmadi (1992) reported the failure of one reinforced concrete water tank and the severe cracking of another tank in Iran due to the earthquake that occurred in 1990. They concluded that the single degree of freedom model was not adequate for the design of elevated water tanks. Moreover, they reported that the water sloshing and the P-delta effects had minor effects on the calculated seismic force on these structures. Fig. 1-3(b) shows another failure for an elevated reinforced concrete tank collapsed during an earthquake in Kocaeli, Turkey in 1999. Saatcioglu et al. (2001) reported that the main factor for the collapse of this tank under the earthquake was the inadequate and sometimes nonexistent guidelines of both design and construction. Other failures of reinforced concrete tanks were reported by Rai (2002) who showed a failure of elevated water tanks in Jabalpur earthquake in 1997 and Bhuj earthquake in 2001. He found that these failures occurred due to the flexural-tension cracks that developed near the tank’s base. Sezen et al. (2008) reported a collapse of two reinforced concrete tanks in Turkey in 1999 due to the failure of the supporting columns.

To the best of the author’s knowledge, no attempt has been previously made to study the behaviour of reinforced concrete conical tanks under hydrostatic loading. Also, no research is reported in the literature to study the behaviour of composite tanks under hydrostatic and seismic loadings. The optimization of these tanks was not studied in order to evaluate their minimum cost. Furthermore, no guidelines are found in the current codes of practice regarding the analysis and design of these structures under such loadings. Therefore, it seemed imperative that research studies are needed to understand the behaviour of
reinforced concrete and composite conical tanks under hydrostatic pressure and seismic loading and to develop tools to optimize the design of such structures.

![Image](http://nisee.berkeley.edu/elibrary/Image/GoddenJ35)  
(a)  

![Image](http://vis.eng.uci.edu/~curee/e3/)  
(b)  

**Fig. 1-3.** Failure of elevated reinforced concrete tanks (a) in Algeria, 1980 (b) in Turkey, 1999.

1.3. **Literature Review**

Different nonlinear models for concrete are presented and the adequacy of adopting the model by Pietruszczak et al. (1988) for modelling of concrete conical tanks is first discussed. Then, a review on the analysis and design of steel and concrete tanks under hydrostatic and seismic loadings is presented. Afterwards, a review on the composite concrete-steel slabs, to provide an insight on the behaviour of composite tanks, is presented. Finally, the author provides a literature review on the optimization of concrete and steel tanks.
1.3.1. Concrete models

Different concrete models were developed in the literature to capture the nonlinear behaviour of concrete. A plasticity model that accounts for the ductile behaviour of concrete under high confining pressure was developed by Pietruszczak et al. (1988) and Jaing (1988). Their model was based on a number of theories including the theory of elasto-plasticity and the progressive evolution of the yield surface suggested by Poorooshasb and Pietruszczak (1985). This model had the advantage of accounting for the enhancement in the concrete properties due to confinement pressure. Later afterwards, different constitutive models were developed but none of them was tested to be compatible with the 13-node subparametric shell element that was developed by Koziey and Mirza (1997). For example, a concrete constitutive model was proposed by Selby and Vecchio (1997) that was based on the modified compression field theory. They tested the adequacy of their concrete model by modelling a set of concrete walls and comparing the results with their counterparts obtained from experiments. Wang et al. (2004) proposed another constitutive model and implemented it into a numerical model to simulate different reinforced concrete slabs. This model showed good performance when it was used to model a set of slabs with normal strength concrete. However, the same model predicted inaccurate deflections for a set of slabs with high strength concrete. Etse and Folino (2010) developed an elasto-plastic constitutive model that is not only valid for normal concrete but also high strength concrete. The material model by Pietruszczak et al. (1988) has different advantages. First, Koziey (1993) showed that this material model is compatible with a shell element, which can be used to model the tanks’ walls, without showing numerical instabilities. Second, this model captures typical features in concrete behaviour, such as the ductile behaviour, enhancement
in the strength due to confining pressure, as well as the nonlinear behaviour of the stress-strain relation.

1.3.2. Tanks under hydrostatic loading

Many studies are found in the literature covering the analysis and design of concrete cylindrical tanks under hydrostatic pressure, such as Chau and Lee (1991), Ramanjaneyulu et al. (1993), and Ghali (2014). Few studies were conducted on concrete conical tanks under hydrostatic pressure, such as El Mezaini (2006), Bruder (2011), Elansary and El Damatty (2013), and Azabi (2014). Steel conical tanks under hydrostatic pressure were studied by various researchers, such as Vandepitte et al. (1982), El Damatty et al. (1997a,b), El Damatty et al. (1999), Sweedan and El Damatty (2009), and Niloufari et al. (2014). No studies are reported in the literature on the analysis and design of composite tanks under hydrostatic pressure. Some pioneering works on concrete and steel tanks are presented below.

Chau and Lee (1991) analyzed cylindrical as well as rectangular tanks using a self-developed computer program, RCTANK. Validation of the program was conducted by comparing the results for four different reinforced concrete tanks with those obtained from manual methods. The developed program was limited to analysis of tanks with a maximum height of 6 m. Shortly afterwards, Ramanjaneyulu et al. (1993) developed another computer program, TANK, to evaluate the load carrying capacity of reinforced concrete cylindrical water tanks. In their study, they obtained the collapse loads for a set of tanks with variable reinforcement along the height by applying the limit analysis approach. Both RCTANK and TANK programs were developed using analytical solutions. A recent study on cylindrical tanks was conducted by Ghali (2014) who developed a Finite Element Model.
(FEM) based on a conical shell element. The numerical model was validated by comparing the straining actions for a set of tanks with their counterparts obtained from analytical solutions. The FEM was used to develop a set of tables that can be utilized to determine the straining actions developed in cylindrical tanks under hydrostatic pressure. The aforementioned studies and programs on reinforced concrete tanks focused on rectangular or cylindrical tanks and did not cover conical tanks. El Mezaini (2006) and Bruder (2011) analyzed a set of reinforced concrete cylindrical tanks with a conical base using SAP 2000 software. Azabi (2014) analyzed a set of reinforced concrete pure conical tanks using a FEM that was based on the 13-node subparametric shell element developed by Koziey and Mirza (1997). Azabi (2014) assessed the accuracy of a simplified approach for the analysis and design of reinforced concrete conical tanks. El Mezaini (2006), Bruder (2011), and Azabi (2014) compared between the internal forces obtained from their numerical models with the results obtained from the Portland Cement Association, PCA design aids (1993). They found that significant discrepancies exist between the internal forces from their finite element models and the PCA design aids (1993). The above mentioned studies on reinforced concrete tanks did not account for the nonlinear behaviour of reinforced concrete. The FEM by Azabi (2014) was extended by Elansary and El Damatty (2013) to account for the nonlinear behaviour of concrete. This nonlinearity was considered by including a concrete constitutive model previously developed by Pietruszczak et al. (1988) and Jaing (1988). Elansary and El Damatty (2013) used the developed FEM to study the behaviour of twelve reinforced concrete conical tanks with a wide range of practical dimensions. They reported that the maximum deflection of the tank’s wall occurs at the
middle one-third of the tank’s height and the maximum hoop stress occurs at 1/5 to 1/6 of the tank’s height.

Regarding steel conical tanks, Vandepitte et al. (1982) studied the behaviour of these tanks under hydrostatic pressure by conducting an extensive experimental investigation on mylar, brass, aluminum, and steel conical tanks. They experimentally evaluated the buckling capacities of 610 conical tanks with different dimensions. Afterwards, El Damatty et al. (1997b) modelled the steel conical tanks using a 13-node subparametric shell element. This element was developed by Koziey and Mirza (1997) and then extended by El Damatty et al. (1997a) to include the nonlinear behaviour of steel. Shortly afterwards, the extended model was used by El Damatty et al. (1999) to develop a simplified design procedure for steel pure conical tanks. This procedure took into account the main parameters controlling the design of steel tanks, such as instability, yielding, large deformations, geometric imperfections, and residual stresses. This simplified design approach was extended by Sweedan and El Damatty (2009) to cover the design of steel combined conical tanks. Sabir and Mousa (1995) conducted a study on the analysis of combined conical tanks with girder stiffeners using a linear elastic FEM. The analysis is done in this model using cylindrical and conical elements. Results of the analysis showed that large stresses exist at the connection between the cylindrical and conical parts of these tanks. The effect of geometric imperfections on the buckling capacities of combined conical tanks was studied by Niloufari el al. (2014). They performed an experimental and numerical investigations on the buckling and post-buckling behaviour of steel tanks under hydrostatic pressure. Results of their analysis showed that the geometric imperfections may have a decreasing or an increasing effect on the buckling resistance of steel tanks. The above mentioned studies on
steel tanks revealed that inelastic bucking at the vessel’s base was usually the main cause of failure.

1.3.1. Tanks under seismic loading

Large number of studies are found in the literature regarding the seismic analysis of cylindrical and conical tanks. However, no studies are found in the literature on composite conical tanks under earthquake excitations. A review on the seismic analysis of elevated water tanks was conducted by Madhuri and Madhukar (2013). They reported that the analysis of elevated water tanks should be done for three cases: empty, partially filled, and fully filled conditions. When they were subjected to earthquakes, partially filled tanks suffer less than half of the force to which the fully filled tanks experience. Early work on studying the behaviour of cylindrical tanks under seismic horizontal excitation was conducted by Haroun and Housner (1981). They showed that the flexibility of the tank’s wall has a significant effect on the earthquakes induced-forces. In their investigation, they developed a simple and efficient procedure to estimate the seismic forces on cylindrical tanks under horizontal excitations. Shortly afterwards, Haroun and Housner (1982) extended their study by developing a FEM where the liquid was modelled analytically, meanwhile the tank’s walls were modelled using ring elements. Haroun and Ellaithy (1985) extended this FEM by accounting for rocking to estimate the seismic forces on the supporting towers of elevated tanks. The seismic analysis of conical steel tanks under horizontal excitations was extensively studied by El Damatty (1995), El Damatty et al. (1997c,d), and El Damatty and Sweedan (2006). In the first three studies, the authors developed a numerical model for studying the stability of liquid-filled conical tanks subjected to seismic loading. Their model involved a previously formulated consistent shell
element and includes both the geometric and material nonlinearities. In the later study, the authors developed an equivalent mechanical analogue which considered the fluid–structure interaction to estimate the hydrodynamic pressure forces on the tank’s wall. In the same study, a simple procedure was proposed to calculate the forces on the tank’s walls due to the horizontal ground excitations.

Regarding the effect of vertical excitations, early theoretical model concerning the evaluation of the vertical natural frequencies and mode shapes for cylindrical tanks was conducted by Haroun and Tayel (1985a). Shortly afterwards, the response of cylindrical steel tanks to vertical earthquake excitations was studied by Haroun and Tayel (1985b). They modelled the tank’s walls using ring elements that was previously utilized by Haroun and Housner (1982) to study the behaviour of cylindrical tanks under vertical excitation. Their investigation showed that the hydrodynamic pressure on tank’s wall due to the vertical excitation had two components: short period impulsive and long period sloshing components. They conducted an experimental program and concluded that the sloshing component due to the vertical excitation had a negligible value if it was compared with the impulsive component. They found that the vertical ground excitations can significantly increase the hoop stresses on cylindrical tanks. Therefore, the authors concluded that the vertical ground excitation was important for reinforced concrete tanks more than steel tanks due to the sensitivity of reinforced concrete to the hoop stresses. It is worth mentioning that Haroun and Tayel (1985a) modelled the tank’s walls using the finite elements and the liquid region was treated mathematically. The same concept was adopted by Sweedan and El Damatty (2005) who modelled steel conical tanks under vertical ground excitation using the 13-node shell element. In their study, they developed a mechanical model to estimate
the fundamental natural frequency and the seismic forces on steel conical tanks subjected to vertical seismic excitations. They presented the mechanical model parameters in the form of charts depending on the layout dimensions of the tank’s vessels. Results of their analysis for a set of tanks showed that the fundamental axisymmetric frequency decreased with the increase in the vessel’s inclination angle. They also concluded that an insignificant effect on the axisymmetric fundamental frequency occurred when the shell mass was included in the analysis.

Nonlinear dynamic analysis of steel conical tanks under the effect of both horizontal and vertical ground excitations was performed by El Damatty et al. (1997c,d). They concluded that steel conical tanks were very sensitive to seismic loading and must be designed for large static load factors in order to survive strong ground motions. Seismic analysis of concrete conical tanks under vertical and horizontal excitations was performed by Moslemi (2011). In his study, he modelled the tank’s walls and the fluid using shell and fluid elements, respectively. He found that the effect of including the roof in the FEM on the dynamic response of conical tanks was insignificant. He also found that the variations of the cone angle had a significant effect on the impulsive component and insignificant effect on the sloshing component.

1.3.2. Composite concrete-steel structures

Composite concrete-steel structures consist of a steel shell attached to a reinforced concrete slab by studs. These connecting studs are welded onto the steel plate and embedded into the concrete slab. An early experimental program on the behaviour and strength of one-way composite slabs was conducted by Daniels and Crisinel (1993a,b). Their investigation revealed that the behaviour and strength of the connection between the steel plate and
concrete slab may be estimated using the pull-out and push-out tests. Good agreement was noted between the experimental results and those obtained from analytical solutions. Other two studies by Eldib et al. (2009) and Shanmugam et al. (2002) focused on the finite element modelling of two way composite slabs using the commercial software, COSMOS and ABAQUS, respectively. Their models were validated by carrying out experiments on a set of two way composite slabs. From their studies, they found that the composite slabs exhibit good flexure characteristics and highly ductile behaviour. The above mention studies showed that the connecting studs were subjected to significant shear and peel forces when the composite slabs were subjected to external loads. The behaviour of these studs under static shear loading was studied by Choi et al. (1999), Shim et al. (2004), Nguyen and Kim (2009), and Xu and Sugiura (2013a,b). The studs’ behaviour was captured by plotting the load-slip curves from push-out tests for different concrete properties and studs’ configurations. A linear behaviour was observed for the load-slip curves from the start of loading up to 50% of the peak load and a nonlinear behaviour was noted beyond this point. These tests showed that the main factors affecting the behaviour of studs were the stud’s diameter and strength of the concrete slab. Regarding the behaviour of studs under tension peel forces, Choi et al. (1999), Ožbolt et al. (1999), and Siwei et al. (2008) carried out a set of pull-out tests on studs with different diameters. Their tests showed that the failure of studs embedded in concrete under tension peel forces can be brittle if the concrete cone around the studs fails. However, the failure can be ductile if it occurs due to the yielding or bond slippage of studs.
1.3.3. Optimization of tanks

The optimum wall’s thickness of reinforced concrete cylindrical tanks were obtained by Thevendran and Thambiratnam (1987) who used a direct search optimization method. One year later, the same authors optimized concrete conical tanks with a piecewise linearly tapered wall thickness. In their optimization technique, they considered the bending and hoop stresses in the tank’s wall as design constraints. Reinforced concrete tanks were also optimized by Chau and Lee (1991) where they included more design parameters, such as the reinforcement bar size and spacing. However, their work was confined to concrete cylindrical and rectangular tanks. In their optimization technique, they relied on design variables enumeration which was only applicable to a limited number of design variables. Cylindrical concrete reinforced tanks were also optimized by Tan et al. (1993) who used a direct search method. In their work, a larger number of constraints were accounted for in the design including constraints on the wall’s thickness, ultimate moment and shear, cracking, and concrete cover. Moreover, they considered constraints on the ultimate tension force in reinforcing steel, spacing, and minimum reinforcement ratio. In the above mentioned studies on optimization of reinforced concrete tanks, the objective function sought by researchers was the materials minimization without considering the optimization of the layout dimensions, such as the wall’s height and radius. A study by Barakat and Altoubat (2009) was conducted on the optimization of concrete cylindrical and conical tanks using different global optimization techniques. These global techniques had the advantage of avoiding being trapped in a local optima, especially at the constraint boundaries. The objective function considered six design variables which covered the whole geometry of the tank including: top and bottom wall thicknesses, base thickness,
vessel height, and wall inclination angle. In addition to the volume minimization, they constrained the search to the limiting values of stresses used in previous research. Optimization of steel conical tanks was studied by El Ansary et al. (2010, 2011a) using a coupled finite element genetic algorithm technique. El Ansary et al. (2010) optimized the thickness of a steel conical tank subjected to hydrostatic loading. Shortly afterwards, the same authors extended their study on the stiffened steel conical tanks in El Ansary et al. (2011a). In this investigation the design variables were the shell thickness, geometry of the vessel as well as dimensions and number of stiffeners, which covered the whole parameters defining the tank geometry. Their optimization algorithm hybridized a genetic algorithm with a quasi-Newton search to eliminate the random effect on the final solution inherent to the genetic algorithm. Such work was extended to other shell structures, such as cooling towers which were studied by El Ansary et al. (2011b).

1.4. Objectives of Thesis

The main objectives of this thesis are outlined in the following points:

1. Develop a FEM, which accounts for the nonlinear behaviour of concrete, to study the behaviour of reinforced concrete pure conical tanks under hydrostatic pressure.

2. Check the adequacy of analyzing and designing of reinforced concrete conical tanks using a simplified method based on utilizing an equivalent cylinder. In this method, referred to as “ECM”, an equivalent cylinder is used in combination with the PCA design aids (1993) to analyze and design conical tanks.

3. Develop a set of charts to obtain the adequate wall thicknesses and straining actions for a set of concrete conical tanks under hydrostatic water pressure.
4. Develop a Finite Element Model for Composite tanks (CFEM) to study their behaviour under hydrostatic pressure. The concrete wall, steel shell, and studs are included in the CFEM.

5. Assess the adequacy of a simplified approach, which is referred to as equivalent section method (ESM), for the analysis of composite tanks. This approach is based on transforming the composite section to a single material section having an equivalent thickness and Young’s modulus.

6. Develop an optimization tool using a genetic algorithm technique in conjunction with the CFEM to obtain the optimum concrete wall and steel shell thicknesses and the studs’ configuration for composite conical tanks.

7. Extend the CFEM to perform free vibration and time history analyses for composite conical tanks and compare the results with those obtained from the ESM.

1.5. **Scope of Thesis**

The thesis has been prepared in an “Integrated-Article” format. In the present chapter, a review of the studies related to the analysis and design of cylindrical and conical tanks is presented. Then, the objectives of the thesis are outlined. The following five chapters presents the thesis objectives. Chapter 7 presents the conclusions from this study along with the suggestions for further research work.

1.5.1. **Nonlinear Behaviour of Reinforced Concrete Conical Tanks under Hydrostatic Pressure**

In Chapter Two, a Finite Element Model (FEM), which accounts for material nonlinearity experienced in reinforced concrete, is developed. This nonlinearity is considered by implementing a concrete plasticity constitutive model in the developed FEM. Analysis of a set of twelve tanks with different practical dimensions is performed under hydrostatic
water pressure. The analysis is done in two stages: the first stage is performed under working loads and the second stage is done under ultimate loads. The thicknesses required to prevent concrete cracking, when the tanks are filled with water, are obtained for the studied tanks. The variations of meridional and hoop stresses through the thickness of the tanks are determined by plotting the stresses at the outer faces of the tank’s wall. The effect of including material nonlinearity in the FEM on the deformed shape is assessed. The developed FEM is used to find the location of maximum deflection and stresses. The variations in the maximum deflection and stresses with the dimensional parameters of the conical vessel are reported.

1.5.2. Assessment of Equivalent Cylinder Method and Development of Charts for Analysis of Concrete Conical Tanks

In Chapter Three, a common simplified approach used in the design of conical tanks, which involves replacing the conical vessels with equivalent cylinders, is presented. The adequacy of this simplified method is assessed in this chapter through comparison with the detailed finite element results, which are obtained from the nonlinear FEM developed in Chapter Two. The FEM is then used to develop a set of charts which can be used to determine the adequate thickness as well as straining actions that develop in a liquid filled reinforced concrete conical tank. The use of this set of charts in designing reinforced concrete conical tanks is illustrated through worked examples.

1.5.3. Behavior of Composite Conical Tanks under Hydrostatic Pressure

In Chapter Four, a Finite Element Model for Composite tanks (CFEM), which accounts for both the geometric and material nonlinearities, is developed. The material nonlinearity is considered by including nonlinear models for both steel and concrete. The developed
CFEM also considers nonlinear behaviour of studs by including the nonlinear load-slip and load-peel curves obtained from test results reported in the literature. In the CFEM, both the concrete and steel walls are modelled using 13-node subparametric shell elements, while the connecting studs between the two walls are modelled using 26-node contact elements using a smearing approach. Validation of the CFEM is conducted by modelling two composite slabs from the literature and comparing the results with their counterparts obtained from the conducted experiments. The CFEM is used to evaluate the deflections, stresses, and internal forces in the concrete and steel walls as well as steel studs. An Equivalent Section Method (ESM) for the analysis of composite tanks, which is based on using an equivalent single wall, is introduced. Deflections, stresses, and internal forces in the steel and concrete walls predicted using this simplified approach are compared to those predicted by the detailed finite element model.

1.5.4. Optimum Design of Composite Conical Tanks under Hydrostatic Pressure

In Chapter Five, a comparison is conducted between the material costs of steel, reinforced concrete, and composite conical tanks having the same layout dimensions. This comparison showed that composite tanks provide the most economical solution. An optimization tool is developed to obtain the optimum design of composite conical tanks under hydrostatic pressure. The developed numerical tool incorporates the CFEM, which is developed in Chapter Four, and a genetic algorithm optimization technique. The developed numerical tool is used to obtain the optimum design of a case study composite conical tank that was recently constructed. The optimum design provides the thicknesses of the concrete and steel walls and studs’ configuration corresponding to the minimum material cost. A
comparison between the optimized and unoptimized case study composite tank showed that a reduction of 32% in the material cost can be achieved when the tank is optimized.

1.5.5. Seismic Analysis of Liquid Storage Composite Conical Tanks

In Chapter Six, the analysis of composite conical tanks under seismic forces is performed. A previously developed numerical model, which takes into account the fluid-structure interaction during the seismic vibration, is extended to study the seismic behaviour of composite conical tanks. A simplified method based on using an Equivalent Section (ESM) is provided to calculate the fundamental frequencies for composite tanks. This method is also used in combination with the response spectrum in order to obtain the forces at the vessel’s base under earthquakes excitations. The adequacy of this method is assessed by comparing the results obtained from the ESM with those resulting from the extended numerical model. Time histories of stresses at the concrete and steel walls, forces at the base, and forces in the studs for composite tanks subjected to different earthquake excitations are reported. The increase in stresses in the walls, forces at the base, and forces in the studs than their counterparts from hydrostatic loading is evaluated.

1.6. References


CHAPTER 2

NONLINEAR BEHAVIOUR OF REINFORCED CONCRETE CONICAL TANKS UNDER HYDROSTATIC PRESSURE

2.1. Introduction

Elevated conical tanks typically consist of a shaft and a containment conical vessel that is usually made of either steel or reinforced concrete. Conical vessels have two common shapes: pure that has a pure truncated cone shape and combined that consists of a pure cone with a top superimposed cylindrical cap. This study focuses on the analysis of reinforced concrete elevated conical tanks with a pure conical vessel similar to the structure shown in Fig. 2-1.

Fig. 2-1. Reinforced concrete elevated conical tank located in China.

The current study is motivated by the lack of provisions for the analysis and design of such structures in the existing relevant design codes such as API (2005), AWWA (2005), and ACI 371 (2008). In the literature, the authors found that most of the work on the behaviour
of conical tanks was done for steel tanks and the available studies on the behaviour of reinforced concrete tanks are very limited. Therefore, it was decided to cover part of the literature for steel conical tanks because they have the same state of stresses as do the concrete conical tanks. Previous researchers, such as Vandepitte et al. (1982), El Damatty et al. (1997b), El Damatty et al. (1998), and Niloufari et al. (2014), showed that steel conical tanks are significantly affected by buckling and instability. An early work on studying the buckling of conical shells was done by Vandepitte et al. (1982) who conducted an extensive experimental investigation on steel, aluminum, brass, and mylar conical shells under hydrostatic pressure. They provided an expression for the wavelength of the axisymmetric buckling mode and found that welding stresses had an insignificant effect on the buckling strength. Later, El Damatty et al. (1998) used a self-developed FEM to study the inelastic stability of a number of small-scale steel conical models that were tested by Vandepitte et al. (1982). El Damatty et al. (1998) considered the nonlinear behaviour of steel by including a nonlinear material model in the developed FEM to accurately capture the behaviour of steel conical vessels. Results of their investigation showed that the failure of steel conical tanks occurs usually due to inelastic buckling.

Regarding the design of conical tanks, El Damatty et al. (1999) proposed a simplified design procedure for liquid-filled steel conical tanks that took into account instability, yielding, large deformations, geometric imperfections, and residual stresses. In that design approach, the hydrostatic pressure, roofing, and snow loads were also considered. This design approach was extended by Sweedan and El Damatty (2009) to cover steel combined conical tanks where they introduced a magnification function that relates the maximum overall stresses to the theoretical membrane stresses. The two aforementioned studies were
conducted using the numerical model developed by El Damatty et al. (1997b) for steel conical tanks. The base of this numerical model is a special finite shell element, called the “consistent shell element” that was developed by Koziey and Mirza (1997) and then extended by El Damatty et al. (1997a) to account for the geometric nonlinear effect.

Limited number of studies on the analysis and design of reinforced concrete tanks are found in the literature, such as Chau and Lee (1991) and Ramanjaneyulu et al. (1993). However, all these studies were limited to the linear elastic range without including nonlinear material modelling. Chau and Lee (1991) developed a computer program, RCTANK, for rectangular and circular liquid retaining tanks. In this program the maximum dimension for rectangular tanks and the maximum diameter for circular tanks are limited to 6 m. The analysis was performed for the tank’s roof, walls, and base separately without considering the structural connection such that hinged or fixed boundary conditions were assumed for each element separately. Ramanjaneyulu et al. (1993) developed another computer program, TANK, to evaluate the load-carrying capacity for reinforced concrete water tanks. They obtained the collapse loads for a set of tanks with variable reinforcement along the tanks’ height by applying the limit analysis approach. The two computer programs above are based on analytical solutions that did not account for the material nonlinearity and they were not validated for conical-shaped tanks. To the best of the author’s knowledge, no research has been reported in the literature on nonlinear analysis of reinforced concrete conical tanks under hydrostatic loading.

In the current study, the numerical model developed by El Damatty et al. (1997b) for the analysis of steel conical tanks is extended to model reinforced concrete conical tanks. This
work required the incorporation of a material model for reinforced concrete into this numerical tool. Pietruszczak et al. (1988) and Jaing (1988) developed a plasticity concrete model that captures the ductile behaviour of concrete as well as the enhancement of concrete properties due to confinement pressure. This model is based on a number of theories including the theory of elasto-plasticity and the progressive evolution of the yield surface suggested by Poorooshasb and Pietruszczak (1985).

Many other concrete models were developed in the literature that account for the nonlinear behaviour of concrete and the effect of the confinement pressure on the properties of concrete, as presented in the Selby and Vecchio (1997) and Wang et al. (2004) models. In the current study, the material model that was developed by Pietruszczak et al. (1988) is included in a FEM to simulate the nonlinear behaviour of reinforced concrete conical tanks under hydrostatic loading. This material model is selected for a couple of reasons. First, Koziey (1993) showed that this material model is compatible with the 13-node shell element without showing numerical instabilities. Second, this model is able to capture the ductile behaviour of concrete, enhancement in strength due to confining pressure, and the nonlinear behaviour of the stress-strain relation.

A validation of the developed FEM is conducted by modelling two reinforced concrete slabs and comparing the results predicted by the model to their corresponding experimental results. After being validated, this FEM is used to study the behaviour of reinforced concrete pure conical tanks under hydrostatic pressure. Different behavioral aspects are reported including the load-deflection curves, deformed shapes, distributions of hoop and meridional stresses, and variations in the stresses along the tanks’ wall thicknesses for a set
of tanks. Also, a comparison between the displacements obtained from the linear and nonlinear analyses for the studied tanks is presented. Furthermore, the locations of the maximum deformation and stresses along each tank’s height are determined and the variations of the maximum deflection and stresses with the tanks dimensions are also reported.

2.2. Identification of the problem

Fig. 2-2 shows the geometry of a typical pure conical tank. As shown in this figure, the tank’s vessel is specified by four geometric variables: the vessel’s height, H; the angle of inclination with the vertical, θ; the radius at the base, R; and the wall’s thickness, t. Fig. 2-2 also shows the normal and shear stress components that are resulted in the vessel’s wall when the tank is filled with a fluid with a specific weight, γ. The horizontal component of the fluid hydrostatic pressure results in tensile stresses in the hoop direction, σₓ, while the weight of the fluid and the vessel’s own weight result in compressive stresses in the meridional direction, σᵧ. The shear stress component, τᵧz results in the x-z plane with nonzero values, while the shear stress components τₓy and τₓz are zeros due to the symmetry of the geometry and loading of the problem. It is worth mentioning that the strain components εₓ, εᵧ, γₓy, γₓz, and γᵧz, not shown in Fig. 2-2, correspond to the stress components σₓ, σᵧ, τₓy, τₓz and τᵧz, respectively.
Fig. 2-2. Geometry and stress components of pure conical tanks.

2.3. **Finite Element Model (FEM)**

The tank’s vessel is modelled using a 3-D FEM based on a triangular subparametric shell element that was developed by Koziey and Mirza (1997). This element has the advantage of avoiding spurious shear stress variation. This stress variation is found in isoparametric shell elements that were introduced by Ahmad et al. (1970). The formulation of these elements are based on the Mindlin (1951) plate bending theory. When these elements are used to model shell structures, they result in overly stiff solutions, as reported by Koziey (1993). This behaviour is due to the presence of spurious shear modes in the elements formulation resulting mainly from using same order for in-plane interpolation of displacements and through thickness rotations in isoparametric shell elements. To overcome these problems, a consistent 13-node shell element that was developed by Koziey and Mirza (1997) is utilized in the current study.
As shown in Fig. 2-3, the consistent shell element has 13 nodes: three corner nodes, three mid-side nodes, six one third-side nodes, and one node at the centroid of the element. The corner nodes have both displacement (u, v, w) and rotational (α, β, φ, ψ) degrees of freedom, the mid-side nodes have only rotational degrees of freedom, and the one-third nodes have only translational degrees of freedom. One of the advantages of this element is being free from spurious shear modes, which was achieved by approximating the displacements using cubic interpolation functions, and the through thickness rotations using quadratic shape functions. Another advantage of this element is that it includes special rotational degrees of freedom which lead to a cubical variation of the displacement through the thickness of the shell. The rotations α and β provide linear variation of displacements through the thickness simulating bending deformations, while the rotations φ and Ψ vary cubically simulating transverse shear deformations. Therefore, a quadratic distribution of the transverse shear stress can be predicted by the element. These special rotational degrees of freedom are important for modelling thick shells where shear deformations are significant. In the proposed FEM, the walls of the reinforced concrete tanks, which are considered thick shells, are modelled using the 13-node element.

Fig. 2-3. The 13-node subparametric shell element.
The 13-node element was extended by El Damatty et al. (1997a) to include geometric nonlinearity as well as a material nonlinear model for steel structures. This element is further extended in this research to consider the material nonlinearity in concrete by including a nonlinear elasto-plastic concrete material model that was developed by Pietruszczak et al. (1988) and Jaing (1988). The failure surface of this model is defined by three constants: $a_1$, $a_2$, and $a_3$; therefore, this model is considered to be a three parameter model. Fig. 2-4 shows the failure surface in the principal stress space $(\sigma_1, \sigma_2, \sigma_3)$ which is defined according to the following equation:

$$F = a_1 \left( \frac{\bar{\sigma}}{g(\theta) f_c^l} \right) + a_2 \left( \frac{\bar{\sigma}}{g(\theta) f_c^l} \right)^2 - \left( a_3 + \frac{1}{f_c^l} \right) = 0 \quad (2-1)$$

where $I$ is the confining pressure, $g(\theta)$ is a function specifying the shape of the deviatoric or $\pi$ plane and $\bar{\sigma}$ is the effective stresses due to applied loads, which is calculated from the principle stresses $\sigma_1$, $\sigma_2$, and $\sigma_3$, using the following equation:

$$\bar{\sigma} = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (2-2)$$

Jaing (1988) showed that the concrete properties are significantly affected by the confinement pressure. This can be observed from the typical curves for stress ratio ($\beta = \bar{\sigma}/g(\theta) f_c^l$) versus effective strain ($\bar{\varepsilon}$) in the compression domain, as shown in Fig. 2-5 (a). The effective strain can be calculated from the principle strains $\varepsilon_1$, $\varepsilon_2$, and $\varepsilon_3$, using the following equation:

$$\bar{\varepsilon} = \sqrt{\frac{1}{2} [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]} \quad (2-3)$$
Also, the typical stress ratio versus the effective strain in the tension domain are found to be nonlinear, as shown in Fig. 2-5 (b). It is worth mentioning that the provided stress-strain curves in Fig. 2-5, obtained from the concrete model by Jaing (1988), were compared with experimental results from the literature by Kotsovos and Newman (1979) and showed good agreement.

![Failure surface in principal stress space (Jaing 1988).](image)

**Fig. 2-4.** Failure surface in principal stress space (Jaing 1988).

![Stress ratio-effective strain curves (Jaing 1988).](image)

**Fig. 2-5.** Stress ratio-effective strain curves (Jaing 1988).
In the developed FEM, stresses at each load increment are calculated based on the strains and confinement stresses from the previous load increment. The material matrix at each stress integration point is updated at each load increment using the elasto-plastic material model. The updated material matrix is calculated using the stress and strain determined from the previous load increment using the following equation:

$$[D^{ep}] = [D^e] - \frac{[D^e] \left( \frac{\partial \Psi}{\partial \sigma} \right) \left( \frac{\partial f}{\partial \sigma} \right) [D^e]}{H_e + H_p}$$

(2-4)

where $[D^{ep}]$, $[D^e]$, $\Psi$, $f$, $H_e$ and $H_p$ are the elasto-plastic and elastic material matrices, the plastic potential surface and yield surface, and the elastic and plastic hardening moduli, respectively.

Cracks in concrete are assumed to be uniformly distributed within the element based on the smeared cracking approach, which does not require specifying a predetermined location or orientation for the cracks before implementing the analysis. Since the crack pattern in concrete vessels under hydrostatic loading is unknown, the smeared approach is suitable for this application. Jaing (1988) reported that a degradation in stiffness occurs due to the propagation of micro-cracks, sliding along the aggregate faces as well as the nonhomogeneous and localization of deformation into the shear bands. These shear bands are formed due to the microscopic fracturing inside the concrete when it is subjected to external loads.

To implement the material model for concrete, the 13-node shell element is divided into five layers through the thickness, i.e. five integration points are utilized in the thickness direction, as shown in Fig. 2-6. The authors tried using more than five integration points in the thickness direction and they found insignificant differences in the results. At each
layer, the stresses are calculated due to the existing strains at seven Gauss integration points using the following equation:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix}
= [D^{ep}]
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix}
\]

(2-5)

**Fig. 2-6.** Seven Gauss integration points in five layers for the 13-node element.

Cracking is assumed to occur when the effective stresses at any point exceed the maximum allowable effective strength of concrete, according to the following criteria provided by Jaing (1988):

\[
F = \bar{\sigma} - g(\theta)\bar{\sigma}_c = 0
\]

(2-6)

\[
\bar{\sigma}_c = \frac{-a_1 + \sqrt{a_1^2 + 4a_2(a_3 + \frac{1}{f'_c})}}{2a_2} f'_c
\]

(2-7)

where \(\bar{\sigma}_c\) is the allowable effective concrete stresses that depend on the material constants and the confining pressure.

After cracking occurs in concrete at any integration point, the reinforcement matrix replaces the concrete matrix. According to Vecchio (1989), the reinforcement material
matrix for the orthogonally reinforced panels can be calculated using the following equation:

\[
[D^s] = \begin{bmatrix}
\rho_x E_x & 0 & 0 & 0 & 0 \\
0 & \rho_y E_y & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(2-8)

where \(\rho_x\) and \(\rho_y\) are the reinforcement ratios in the X and Y directions, respectively, and \(E_x\) and \(E_y\) are the modulus of elasticity for the steel bars in the X and Y directions, respectively. The bilinear stress-strain curve is adopted in order to model the reinforcing steel bars, as shown in Fig. 2-7. The main stress component in the steel bars is the axial normal stress, while the other stress components are neglected.

![Bilinear stress-strain curve for the reinforcing steel.](image)

**Fig. 2-7.** Bilinear stress-strain curve for the reinforcing steel.

Shrinkage is included in the FEM by adding additional tensile stresses as they are considered by the PCA (1993) design aids in the tank’s thickness calculation. The PCA (1993) design aids suggest using a value of 0.0003 for the shrinkage coefficient and provides the following equation for calculating the tensile stresses due to shrinkage:
\[ f_{\text{Shrinkage}} = \frac{C E_s A_s}{A_c + n A_s} \]  

(2-9)

where \( C, A_s, A_c, \) and \( n \) are the shrinkage coefficient, area of steel, area of concrete and modular ratio between the steel and concrete, respectively.

The failure criteria from the ACI 318 (2005) design code are included in the developed FEM. The tank is assumed to fail when the axial force in the hoop direction exceeds the maximum tension force in the hoop reinforcement. Additionally, the tank is assumed to fail when the meridional axial force exceeds the maximum axial force or the meridional moment exceeds the allowable meridional moment, according to the following equations:

\[ R_{\text{max}} = A_{s, \text{hoop}}(\phi f'_y) \]  

(2-10)

\[ P_{\text{max}} = 0.80\phi[0.85E'(A_g - A_{st}) + f_y A_{st}] \]  

(2-11)

\[ M_{\text{max}} = \phi M_n \]  

(2-12)

where \( R_{\text{max}}, P_{\text{max}}, M_{\text{max}} \) and \( M_n \) are the maximum ring tension force, maximum meridional compression force, maximum meridional moment, and nominal moment, respectively. And \( \phi \) is a strength reduction factor, which is equal to 0.9 and 0.65 in Equations (2-10) and (2-11), respectively, as well as \( \phi \) is equal to 0.9 for tension-controlled sections and 0.65 for compression-controlled sections in Equation (2-12).

**2.4. Finite element model validation**

To validate the developed FEM, two simply supported reinforced concrete slabs are analyzed. The first slab has a square shape and was tested by Taylor et al. (1966) under uniform lateral load. The second slab has a rectangular shape and was tested twice by Ghoneim and MacGregor (1994). In the first test, Ghoneim and MacGregor (1994) applied
a uniform lateral load on the rectangular slab. The second test was carried out on the same slab under a combined effect of lateral uniform load and an axial compressive line load of 584.5 kN/m applied at the short edges of the rectangular slab. The dimensions of the square and rectangular slabs as well as the applied loads are shown in Fig. 2-8. A mesh of uniformly distributed bars is placed at the bottom of the two slabs such that the bars are high grade steel with reinforcement ratios of 0.004 in the X and Y directions, respectively. The square slab was modelled by Jaing (1988) in order to validate his FEM using rectangular plate bending elements along with beam column elements for steel bars. Later, the same slab was modelled by Koziey (1993) in order to validate his FEM at which he used 13-node triangular shell elements for concrete and one-dimensional isoparametric bar elements for steel bars. He also used the constitutive model developed by Pietruszczak et al. (1988) since this model adequately reflects the important features in concrete behaviour such as a progressive transition from compaction to dilatancy and the sensitivity of a material to confining pressure.

![Slab edge](image)  
Support line - - -  
Lateral load  
Axial load  

(a) Square slab

(b) Rectangular slab

Fig. 2-8. Reinforced concrete slabs under different loadings.
In the FEM developed by Koziey (1993) and the FEM developed by Jaing (1988) for the square slab, the reinforcement bars are modelled based on the discrete reinforcement approach. However, the smeared reinforcement approach is implemented in the proposed FEM where the steel bars are assumed to be uniformly distributed along the shell element. This approach is used in the current study because the arrangement and locations of the steel bars are not well predetermined before performing the analysis and design of the studied conical tanks.

Due to double symmetry of geometry and loading, only one quarter of each slab is modelled in the current study using the proposed FEM. As shown in Fig. 2-9, the square and rectangular slabs are modelled using 8 and 16 triangular shell elements, respectively. The analyses are done in a load-controlled manner and the load is applied incrementally. The lateral load-deflection curves for the slabs obtained from the proposed FEM are compared in Fig. 2-10 with the corresponding curves obtained from the experiments. The figure shows a good agreement between the experimental and numerical results. Referring to Figs. 10 (b) and (c), one can observe the significant effect of the axial line load on the maximum lateral load carried by the slab. A similar state of biaxial stresses occurs in the conical tanks where the hydrostatic pressure develops both hoop and meridional stresses.
Fig. 2-9. Finite element meshes for two simply supported slabs modelled using the 13-node triangular elements.

Fig. 2-10. Load-deflection curves for reinforced concrete slabs.
2.5. Basic assumptions

A number of assumptions are made in the analysis of the reinforced concrete tanks considered in this study. First, the vessel’s top edge is assumed to be free. This assumption was justified by El Damatty et al. (1997b) as they have shown that the radial displacements at this location due to hydrostatic pressure are negligible even when free boundary conditions were assumed at the top edge. Second, the base of the vessel is assumed to be hinged due to the nature of the connection between the wall and the base. At this connection, the thickness of the vessel’s base is not significantly larger than the thickness of the vessel’s wall. Therefore, the vessel’s wall cannot be assumed to be completely fixed in the vessel’s base. Third, only one quarter of the tank’s vessel is modelled due to the symmetrical behaviour experienced in the conical tanks subjected to a hydrostatic liquid pressure, as shown in Fig. 2-11. Finally, at the lines of symmetry at the two meridians of the vessel’s quarter, the boundary conditions are set such that the displacements in the hoop direction are prevented while the vertical displacements are permitted.

![Fig. 2-11. A sketch of the conical tank's vessel.](image)
A mesh consisting of 128 elements is used to model the pure conical vessel, as shown in Fig. 2-12. A coarse mesh is used near the top edge of the vessel, while a fine mesh is used near the bottom edge to capture the expected concentration of stresses in this area. The mesh has 16 elements in the ring direction and 16 elements in the meridional direction. A mesh sensitivity analysis is performed to assess the adequacy of the selected mesh by modelling a tank using four different meshes. The modelled tank has a radius at the bottom, an inclination angle, a height and a wall’s thickness of 4, 45°, 7 m, and 200 mm, respectively. The number of elements in the ring and meridional directions for the studied meshes are varied between 4 and 16. Fig. 2-13 shows the maximum deflection along the tank’s height for each mesh and the solution is observed to converge to 5.9 mm for an 8x16 mesh or finer with a reasonable tolerance.

![Figure 2-12](image.png)

**Fig. 2-12.** A 16x16 finite element mesh for the conical tank’s vessel.
The load-deflection curve for each mesh at a point of maximum displacement along the vessel’s height is shown in Fig. 2-14 where it is observed that the specific weight and displacement at failure are overestimated using the 4x4 mesh. Therefore, by using the 8x16 mesh or finer, the specific weight at failure converges to 27 kN/m³ and the displacement at failure converges to 5.9 mm. Fig. 2-15 shows the meridional stress distribution in concrete along the tank’s height for each mesh where it is shown that the meridional stresses converge for an 8x16 mesh or finer. Although an 8x16 mesh is sufficient to obtain an accurate solution for the displacement and the specific weight at failure, a mesh with a size of 16x16 (128 elements) is chosen for the analysis of the studied tanks. This is done in order to obtain an accurate displacement and stress distributions along the tank’s height as well as the locations of their maximum values.
Twelve pure conical reinforced concrete tanks having different dimensions are modelled using the developed FEM. For practical dimensions, the angle of inclination varies between $30^\circ$ and $60^\circ$, while the vessel’s height varies between 6 and 9 m as shown in Table 2-1.

The radius at the base is chosen to be 4 m to simulate a constant footprint for all of the studied tanks while the thicknesses for the studied tanks are calculated based on the concrete cracking limit, as will be described in detail in the section on Analysis.
methodology. The volumes of the 12 tanks are chosen to be within the range specified by the ACI-371 (2008) which is from 1,900 to 11,000 m$^3$. Detailed recommendations for the tank sizes can be found in WEF- ASCE (2010) and Tchobanoglous et al. (1991).

<table>
<thead>
<tr>
<th>Tank No.</th>
<th>H (m)</th>
<th>$\theta$ (degree)</th>
<th>t (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>6</td>
<td>30</td>
<td>120</td>
</tr>
<tr>
<td>T2</td>
<td>6</td>
<td>45</td>
<td>160</td>
</tr>
<tr>
<td>T3</td>
<td>6</td>
<td>60</td>
<td>320</td>
</tr>
<tr>
<td>T4</td>
<td>7</td>
<td>30</td>
<td>140</td>
</tr>
<tr>
<td>T5</td>
<td>7</td>
<td>45</td>
<td>200</td>
</tr>
<tr>
<td>T6</td>
<td>7</td>
<td>60</td>
<td>420</td>
</tr>
<tr>
<td>T7</td>
<td>8</td>
<td>30</td>
<td>160</td>
</tr>
<tr>
<td>T8</td>
<td>8</td>
<td>45</td>
<td>280</td>
</tr>
<tr>
<td>T9</td>
<td>8</td>
<td>60</td>
<td>500</td>
</tr>
<tr>
<td>T10</td>
<td>9</td>
<td>30</td>
<td>180</td>
</tr>
<tr>
<td>T11</td>
<td>9</td>
<td>45</td>
<td>320</td>
</tr>
<tr>
<td>T12</td>
<td>9</td>
<td>60</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 2-1. Dimensions of the studied tanks.

Ductility is assumed to be the ratio between the displacement at the peak load to the displacement at the first yielding of the reinforcement, as proposed by Park (1989). The maximum displacement is defined as the displacement corresponding to the peak of the load-carrying capacity, while the displacement at yielding is defined as the displacement when yielding first occurs in a system.

The material properties of concrete are assumed to be as follows: characteristic strength $f'_c = 30$ MPa; Poisson’s ratio $\nu_c = 0.2$; and modulus of elasticity $E_c = 25,743$ MPa.

The concrete plasticity model parameters are used as reported in Jaing (1988). The material properties of steel are assumed to be as follows: yield stress $f_y = 400$ MPa; ultimate stress $f_u = 520$ MPa; Poisson’s ratio $\nu_s = 0.3$; and modulus of elasticity $E_s = 200,000$ MPa.
2.6. **Analysis methodology**

The (ACI 350-06 2006) recommends the use of both the Allowable Stress Design method and the Strength Design method to analyze liquid retaining structures. Therefore, the analysis process can be classified into two stages: Stage 1 is the analysis under working loads and Stage 2 is the analysis under ultimate loads.

Stage 1, according to the PCA (1993) design aids, the working loads are applied on the tank to obtain the distribution of hoop stresses along the vessel’s height. The wall’s thickness should be chosen such that the maximum hoop tensile stresses do not exceed the concrete strength. The tensile stresses due to shrinkage should be included according to Equation (2-9). By applying the above criterion, the proposed FEM is used to determine the appropriate walls’ thickness for the studied tanks. The analysis is repeated several times for each tank by assuming a specific thickness each time, and then the liquid specific weight is applied incrementally for each analysis until the first crack occurs. When the assumed thickness is very large, the first crack occurs at a liquid specific weight larger than 10 kN/m$^3$ and vice versa. The relation between the liquid specific weight at cracking and the tanks’ assumed thicknesses is plotted in Fig. 2-16. Knowing that the water specific weight is 10 kN/m$^3$, the wall’s thickness for each tank corresponding to the start of cracking is obtained.
Stage 2 includes plotting the hoop and meridional stress distributions along the vessel’s height under ultimate loads. These loads can be obtained by multiplying the working loads by specific load factors. The ACI-350 (2006) recommends using an ultimate factor of 1.4 and a durability factor of 1.93 for environmental structures with steel bars that have a yield stress of 400 MPa. Therefore, a factor of 2.7 is adopted in this study when the analysis is performed using the Strength Design method. Reinforcement ratios in both the hoop and meridional directions are obtained in this stage. The hoop reinforcement must resist all of the tensile stresses in the hoop direction while the meridional reinforcement must resist the combined action of meridional axial force and moment. It should be noted that in Stage 2 the tensile strength of concrete is neglected in both the hoop and meridional directions, according to ACI-350 (2006). The reinforcement ratios in the ring direction for the studied tanks are assumed such that the stresses in the ring reinforcement do not exceed $\phi f_y$. A minimum reinforcement ratio in the meridional direction for all of the studied tanks is assumed according to the following equation from ACI-318M (2005):

![Graph showing variation of tank thicknesses with liquid specific weight.](image)
\[ \rho_{s,\text{min}} = \frac{0.25 \sqrt{f_c}}{f_y} \geq \frac{1.4}{f_y} \] (2-13)

Additionally, the twelve tanks are analyzed using the linear elastic model to test the significance of including the nonlinear concrete model in the FEM. In the linear analysis, the tanks’ thicknesses that result from the nonlinear FEM are used. Both the linear and nonlinear analyses are carried out under both working and ultimate loads.

2.7. Results

The results obtained from the analyses of the 12 tanks analyzed under both working and ultimate loads are discussed in this section. The following behavioral aspects of reinforced concrete conical tanks are investigated: load-deflection relation, deformed shape, normal and shear strains, hoop and meridional stresses, variation of stresses along the wall’s thickness, and ductility of the tanks.

2.7.1. Load-deflection

Fig. 2-17 shows the linear and nonlinear load-deflection curves for tank T5 using both the linear and nonlinear analyses at the location of maximum displacement along the vessel’s height. It is clear that the material nonlinearity is reflected on the displacement. The maximum displacement from the nonlinear analysis is 10% larger than the maximum displacement from the linear analysis. Fig. 2-17 also shows that the tank experiences a gradual stiffness degradation as the load increases when a nonlinear analysis is carried out.
The maximum transverse displacements from both the linear and nonlinear analyses under both working and ultimate loads for the studied tanks are shown in Table 2. Under working loads, the ratio between the maximum transverse displacements from the linear analyses to the nonlinear analyses is approximately 0.9 for all of the studied tanks. However, under ultimate loads, this ratio is 3.1, 2.4, and 1.8 for the tanks with an inclination angle of 30°, 45°, and 60°, respectively. Therefore, these results show the importance of using nonlinear analysis to accurately predict the displacements of reinforced concrete conical tanks.

**Table 2-2. Maximum transverse displacements for the 12 studied tanks (mm).**

<table>
<thead>
<tr>
<th>Tanks No.</th>
<th>Working (mm)</th>
<th>Ultimate (mm)</th>
<th>Working (mm)</th>
<th>Ultimate (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.6</td>
<td>1.7</td>
<td>0.7</td>
<td>5.4</td>
</tr>
<tr>
<td>T2</td>
<td>1.2</td>
<td>3.2</td>
<td>1.3</td>
<td>8.7</td>
</tr>
<tr>
<td>T3</td>
<td>2.8</td>
<td>7.5</td>
<td>3.1</td>
<td>14.9</td>
</tr>
<tr>
<td>T4</td>
<td>0.7</td>
<td>1.9</td>
<td>0.8</td>
<td>6.0</td>
</tr>
<tr>
<td>T5</td>
<td>1.4</td>
<td>3.8</td>
<td>1.6</td>
<td>9.4</td>
</tr>
<tr>
<td>T6</td>
<td>3.3</td>
<td>8.9</td>
<td>3.7</td>
<td>16.0</td>
</tr>
<tr>
<td>T7</td>
<td>0.8</td>
<td>2.1</td>
<td>0.9</td>
<td>6.7</td>
</tr>
<tr>
<td>T8</td>
<td>1.3</td>
<td>3.6</td>
<td>1.5</td>
<td>8.0</td>
</tr>
<tr>
<td>T9</td>
<td>4.1</td>
<td>11.0</td>
<td>4.6</td>
<td>19.8</td>
</tr>
<tr>
<td>T10</td>
<td>0.9</td>
<td>2.4</td>
<td>1.0</td>
<td>6.8</td>
</tr>
<tr>
<td>T11</td>
<td>1.6</td>
<td>4.3</td>
<td>1.8</td>
<td>8.9</td>
</tr>
<tr>
<td>T12</td>
<td>4.8</td>
<td>13.0</td>
<td>5.6</td>
<td>22.6</td>
</tr>
</tbody>
</table>
Fig. 2-18 shows the load-deflection curves for tanks T2, T5, and T8, all of which have an inclination angle of 45°. The last point at each curve represents the maximum displacement at cracking. Also, these points provide the liquid specific weight at cracking, which is 10 kN/m^3 for all of the tanks. Cracking occurs exactly at a liquid specific weight of 10 kN/m^3 because the thicknesses of all the tanks are optimized to achieve that. A sudden increase in the deflection occurs at the beginning of the loading because the liquid specific weight is applied by increments of 1 kN/m^3 while the total value for the wall’s own weight is applied at the first load increment.

Fig. 2-19 shows the maximum transverse displacements at cracking for the studied tanks. It can be noted that the maximum deflection significantly increases as the inclination angle increases. For tanks T1, T2, and T3, the ratios between the maximum deflections are 1: 1.8: 4.2, while the ratios between their thicknesses is 1: 1.3: 2.6. Fig. 2-19 also shows that the maximum transverse displacement increases as the tank’s height increases. For tanks T1, T4, T7, and T10, the ratios between the maximum deflections are 1: 1.1: 1.2: 1.4, while the ratios between their thicknesses are 1: 1.1: 1.3: 1.5. Therefore, the effect of increasing
the angle of inclination is more significant than the effect of increasing the tank’s height on the maximum transverse displacement.

**Fig. 2-19.** Maximum transverse displacements for the studied tanks (under working loads).

**Fig. 2-20** shows the load-deflection curves under ultimate loads for both linear and nonlinear analyses for tanks T2, T5, and T8, which have an inclination angle of 45°. In this figure, the letters N and L refer to the load-deflection curves obtained from nonlinear and linear analyses, respectively. Both analyses provide the same deflection before cracking. However, after cracking, the nonlinear analysis provides deflections significantly larger than their counterparts obtained from linear analysis. The adopted material model in the FEM is nonlinear but the response of the studied conical tanks from the nonlinear analysis tends to be bilinear. This can be observed from the nonlinear load-deflection curves in **Fig. 2-20** where a significant reduction in the stiffness occurs once concrete cracking is initiated in the hoop direction. After cracking, the stiffness in the hoop direction is mainly provided by the hoop reinforcing steel which is significantly less than the stiffness of the uncracked concrete. This leads to excessive deformations in the radial direction. The nonlinearity is not clear after concrete cracking because the global behaviour of the tanks is controlled by the vessel’s expansion in the hoop direction. For the load-deflection curves from the
nonlinear analysis, a smooth transition is observed for all of the tanks, which indicates the gradual propagation of cracking in different elements where the concrete tensile strength is exceeded. Fig. 2-20 shows that all of the tanks fail at a liquid specific weight of 27 kN/m$^3$ which is the factored specific weight adopted by ACI-350 (2006) as an ultimate load factor.

Fig. 2-20. Linear and nonlinear load-deflection curves for tanks T2, T5, and T8 (under ultimate loads).

Fig. 2-21 shows the maximum transverse displacements for the studied tanks at failure. From this figure, it can be observed that the maximum transverse displacement at failure increases as the tank’s height increases for those tanks with an inclination angle of 30˚ and 60˚. The ratios between the maximum transverse displacement for tanks T3, T6, T9, and T12 are 1: 1.1: 1.3: 1.5. However, no clear trend is observed for the tanks with an inclination angle of 45˚. This figure also shows that the maximum transverse displacement increases with the increase in the tank’s inclination angle. The ratios between the maximum transverse displacements for tanks T4, T5, and T6 are 1: 1.7: 3. Therefore, the effect of increasing the angle of inclination is more significant than the effect of increasing the tank’s height on the maximum transverse displacement.
2.7.2. Deformed shape

Fig. 2-22 shows the deformed shapes for tanks T7, T8, and T9 under both working and ultimate loads. It is clear from this figure that transverse displacements at the top of each tank are not zero due to the significant vertical displacements at this location.

The maximum displacement due to working loads occurs at 0.3, 0.5, and 0.6 of the tank’s height for the tanks with an inclination angle of 30°, 45°, and 60°, respectively. The maximum displacement due to ultimate loads occurs at 0.3, 0.4, and 0.5 of the tank’s height for the tanks with an inclination angle of 30°, 45°, and 60°, respectively. It can be observed that the ratio between the maximum transverse displacements due to working loads to the maximum transverse displacements due to ultimate loads is 7, 5.4, and 4.3 for tanks T7, T8, and T9, respectively. It is clear that the ratios from the nonlinear analysis are significantly larger than the ones from the linear analysis. For this reason, the maximum transverse displacements due to the ultimate loads cannot be obtained by multiplying the ones from the analysis under working loads by the ultimate load factor.
2.7.3. Strain distribution

The maximum normal and shear strains along the tank’s height under ultimate loads are allocated for tanks T2, T5, and T8. The distributions of these strains along the wall’s thicknesses are plotted in Fig. 2-23. In this figure, \( z \) is the coordinate measured from mid-surface of the wall perpendicular to the meridional direction and it varies from (-1) to (+1) at the outer and inner faces, respectively. Because the problem is axisymmetric, the shear strains \( \gamma_{xy} \) and \( \gamma_{xz} \) have zero values. One can observe from Fig. 2-23 (a) that the distribution of normal strain along the wall’s thickness in the circumferential direction, \( \varepsilon_x \) is approximately constant because no bending exists in this direction. Fig. 2-23 (b) shows that the distribution of normal strain along the wall’s thickness in the meridional direction, \( \varepsilon_y \) is approximately linear due to the significant meridional axial and moment in this direction. The distribution of the shear strain, \( \gamma_{yz} \) reflects clearly the capability of the 13-node element to capture the parabolic distribution of the shear strains along the wall’s
thickness. Similar observations are noted for the strain distributions of the remaining studied tanks.

![Graphs of strain distributions](image)

**Fig. 2-23.** Normal and shear strain distributions along the wall’s thicknesses of tanks T2, T5, and T8 (under ultimate loads).

### 2.7.4. Hoop stresses

**Fig. 2-24** shows the hoop stress distributions at the outer faces for tanks T7, T8, and T9 under working loads. Evidently, the maximum hoop stresses do not exceed the concrete tensile strength which is approximately $0.1 f_c' = 3$ MPa, according to Jaing (1988) and the PCA (1993) design aids. **Fig. 2-24** also shows that the maximum hoop stresses occur at $(0.15 - 0.3)$ of each tank’s height. The same trend for the hoop stress distribution is observed for the remaining studied tanks.
The hoop stress distributions in the outer and inner steel bars for tanks T7, T8, and T9 due to the ultimate loads are plotted in Fig. 2-25 from which it is clear that the maximum hoop stresses occur at (0.2 - 0.35) of each tank’s height. This figure also shows that these tanks fail when the actual hoop stresses reach the allowable tensile stresses in steel, which is 0.9 \( f_y = 360 \) MPa. The same notes are found to be valid for the remaining studied tanks.

According to ACI-350 (2006), under ultimate loads, the concrete is allowed to crack and the reinforcing steel must resist all tension stresses in the hoop direction.

**Fig. 2-24.** Hoop stress distributions in concrete for tanks T7, T8, and T9 (under working loads).
Fig. 2-25. Hoop stresses distributions at steel bars for tanks T7, T8, and T9 (under ultimate loads).

The tanks with an inclination angle of 60° showed compression stresses at the bottom edge of the vessel due to the high confining pressure experienced at this location. This occurs because the stresses near the vessel’s base are affected by two factors: the first, is the liquid pressure acting outward perpendicularly to the tank’s wall, which tends to produce tensile hoop stresses, while the second is the confining pressure exerted by the meridional stresses, which tends to cause compression hoop stresses.

The variation of the hoop stresses throughout the tank’s thickness is plotted for the tanks T7, T8, and T9 in Fig. 2-25 where it is found that the variation in the hoop stresses is not significant for tanks T7 and T8. However, this variation is found to be significant for tank T9 due to the significant bending effect experienced in the tanks with an inclination angle of 60°.
2.7.5. **Meridional stresses**

Meridional stress distributions due to the ultimate loads at both the outer steel and concrete faces for tanks T7, T8, and T9 are plotted in Figs. 2-26 and 2-27, respectively. These figures show that the meridional stresses at the top of the tank are equal to zero and they increase near the vessel’s base. The maximum values for the meridional stresses are observed to occur at approximately 0.1 of the tank’s height. A similar distribution is observed for the remaining studied tanks. Fig. 2-26 shows that the maximum values for the meridional stresses are approximately equal among all of the studied tanks. Fig. 2-27 shows that the maximum meridional stresses in concrete increase with the increase in the inclination angle. Figs. 2-26 and 2-27 show that a significant variation exists in the meridional stresses along the tank’s thickness due to the bending effect.

![Meridional stress distributions](image)

**Fig. 2-26.** Meridional stress distributions at steel bars for tanks T7, T8, and T9 (under ultimate loads).
2.7.6. **Ductility**

Ductility is calculated for tanks T2, T5, and T8 to measure their ability to dissipate a significant amount of energy during severe loading. The maximum displacements for tanks T2, T5, and T8 can be obtained from the nonlinear load-deflection curves that are shown in Fig. 2-20. From these curves, the displacements at the reinforcement first yielding are 0.48, 0.5, and 0.41 mm for tanks T2, T5, and T8, respectively. Therefore, the ductility for tanks T2, T5, and T8 are 11.9, 12.2, and 12.1, respectively. It can be observed that no significant difference is noted between the ductility of these tanks.

2.7.7. **Characteristics reflected from the nonlinear material model**

This section outlines some characteristics of the adopted constitutive model that are reflected on the behaviour of the studied tanks. Firstly, a smooth transition after concrete cracking without a sharp or sudden change is shown clearly in the load-deflection curves. Secondly, nonlinear behaviour is observed in the load-deflection curves under both working and ultimate loads. Thirdly, the initiation of cracking occurs when the hoop tensile
stresses in concrete are less than the tensile strength of concrete. This occurs because the cracking in the elasto-plastic model not only depends on the stress component in the ring direction but also on the stress component in the meridional direction. Therefore, the cracking occurs when the effective stresses exceed the concrete failure surface.

2.8. Summary and conclusions

The behaviour of 12 reinforced concrete pure-conical tanks is studied under both working and ultimate loads. The analysis is done using a finite element program that considers the nonlinear behaviour of reinforced concrete structures. The load-deflection curves, deformed shapes, strains, hoop, and meridional stresses are plotted. From this study, it can be concluded that the material nonlinearity and concrete cracking clearly affect the response of reinforced concrete conical tanks under both working and ultimate loads. Based on the performed analysis, the following conclusions can be drawn:

1. Under working loads, the ratio between the maximum transverse displacements from the linear analyses to the nonlinear analyses is approximately 0.9 for all of the studied tanks. However, under ultimate loads, this ratio is 3.1, 2.4, and 1.8 for the tanks with an inclination angle of 30°, 45°, and 60°, respectively.

2. The transverse displacement of a tank increases with the increase in the inclination angle due to both working and ultimate loads.

3. The maximum displacement due to working loads occurs at 0.3, 0.5, and 0.6 of a tank’s height for the tanks with an inclination angle of 30°, 45°, and 60°, respectively, while the maximum displacement due to ultimate loads occurs at 0.3, 0.4, and 0.5 of a tank’s height for the tanks with an inclination angle of 30°, 45°, and 60°, respectively.
4. The normal strain in the meridional direction and the shear strain distributions through the wall’s thickness are parabolic. However, insignificant change occurs in the normal strain distributions through the wall’s thickness in the hoop direction.

5. Maximum hoop stresses in concrete occur at (0.15 - 0.3) of the tank’s height due to both working and ultimate loads.

6. Hoop stresses do not change significantly through the tank’s wall for those tanks with an inclination angle of 30° and 45°; however, this variation is significant for those tanks with an inclination angle of 60°.

7. Maximum meridional stresses occur within 0.1 of the tank’s height at the bottom edge of the tank’s vessel for both the steel and concrete due to both the working and ultimate loads.

8. A significant variation is observed in the meridional stresses along the tank’s thickness due to bending effects.

9. All of the studied tanks have approximately the same ductility.

2.9. **Acknowledgements**

The authors would like to take this opportunity to thank the Ontario government for their generous financial support through the Ontario Trillium Scholarship (OTS). This work was made possible by the facilities of the Shared Hierarchical Academic Research Computing Network, SHARCNET, and Compute Canada.

2.10. **References**


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CHAPTER 3
ASSESSMENT OF EQUIVALENT CYLINDER METHOD AND DEVELOPMENT OF CHARTS FOR ANALYSIS OF CONCRETE CONICAL TANKS

3.1. Introduction

Elevated conical tanks usually consist of a conical vessel supported on a shaft, as shown in Fig. 3-1. Often the vessel has a superimposed cylindrical part and in this case the tank is referred to as “combined conical tank”, whereas a conical vessel without a top cylindrical part is referred to as “pure conical tank”. The shaft is usually made of reinforced concrete, while the vessel is made of either steel or reinforced concrete.

Fig. 3-1. Reinforced concrete elevated conical tank in Saudi Arabia (Alriyadh News Paper, 2009).

Conical steel tanks were modelled and studied by El Damatty et al. (1997b) using a 3-D consistent sub-parametric shell element that was developed by Koziey and Mirza (1997). This element has the advantage of avoiding the spurious shear stress variations which were found in isoparametric shell elements. El Damatty et al. (1997a) extended this element to
account for the geometric nonlinear effect and included as well a nonlinear strain-hardening material plasticity model for steel. Their numerical model accounted for the effects of geometric imperfections and residual stresses, which are quite important for thin-walled shell structures. This numerical model was then used by El Damatty et al. (1999) to develop a simplified design procedure for liquid-filled steel conical tanks. This simplified approach was extended by Sweedan and El Damatty (2009) to cover the design of combined conical tanks.

Sabir and Mousa (1995) analyzed combined conical steel tanks with girder stiffeners under hydrostatic pressure using a linear elastic Finite Element Model (FEM). Cylindrical and conical elements were implemented in the formulation of this model. Results from their investigation showed that large stresses develop at the connection between the upper cylindrical and lower conical parts of the tanks. These stresses were found to be significantly reduced by incorporating a circular girder near this connection. Zielnica (2002) performed an analytical study on the buckling capacity of conical tanks under twisting shear loads using inelastic analysis. It was concluded that the critical moment of twist that triggers buckling for conical tanks is significantly affected by the vessel’s thickness and layout dimensions. El Damatty et al. (2001) found that a significant enhancement of the buckling capacity of steel pure conical tanks can be provided by welding longitudinal stiffeners to the bottom part of the tanks. Their analysis was performed on both existing and newly designed conical tanks. The buckling behaviour of combined conical steel tanks was studied by Niloufari et al. (2014). They conducted an experimental investigation on a set of steel tanks under hydrostatic pressure. Their investigation revealed that buckling capacities for the studied tanks with \( t/R = 0.003 \)
reduce significantly due to geometric imperfections, where t and R are the wall’s thickness and bottom radius, respectively.

Regarding reinforced concrete tanks, Chau and Lee (1991) developed a computer-aided package, RCTANK, to analyze and design rectangular and circular reinforced concrete tanks. In this computer code, the base, wall, and roof of a tank were analyzed using analytical equations under hydrostatic water pressure. Based on the analysis results, this computer package can be utilized to predict rectangular and circular tanks’ wall thickness and required reinforcement. However, this numerical tool is limited to certain tank dimensions of 6×6×6 m for rectangular tanks and 6 m diameter × 6 m depth for circular tanks. Ramanjaneyulu et al. (1993) developed another computer package, based on analytical equations, to evaluate the load carrying capacities of reinforced concrete cylindrical tanks. This package can estimate the collapse loads for short, medium, and long cylindrical tanks without any limitations on tank dimensions. In their program, they considered the effect of variation in reinforcement along the tank’s height. El Mezaini (2006) analyzed reinforced concrete cylindrical tanks with a conical base using SAP 2000 software that was developed by Wilson and Habibullah (2003). The tanks were modelled using a 3-D shell element available in SAP 2000 software. El Mezaini (2006) compared the internal forces obtained from the numerical model to their counterparts from the Portland Cement Association, PCA design aids (1993). Bruder (2011) extended the work of El Mezaini (2006) by considering a larger number of concrete tanks with practical dimensions. He concluded that the PCA design aids (1993) provides inadequate design for cylindrical tanks with conical base. El Mezaini (2006) and Bruder (2011) reached the same conclusion that significant discrepancies were noticed between the internal forces from the
numerical model and the PCA design aids (1993). Therefore, they recommended that designers should utilize finite element analysis to properly design concrete conical tanks. Recently, Ghali (2014) analyzed circular storage tanks by developing a numerical model based on a conical shell element. The developed numerical model was validated by comparing the results obtained from the analysis of a set of concrete tanks to those based on closed form analytical solutions. This validated numerical model was then utilized to develop a set of tables to determine the straining actions of circular tanks. The aforementioned numerical and analytical investigations were conducted on rectangular or circular tanks and were not validated for conical tanks under hydrostatic loads. Azabi (2014) analyzed a set of reinforced concrete conical tanks using a FEM that is based on the 3-D consistent element developed by Koziy and Mirza (1997). He assessed the accuracy of a simplified approach for the analysis and design of reinforced concrete conical tanks. This approach is based on the PCA design aids (1993) combined with the equivalent cylindrical approach proposed by the American Water Works Association, AWWA (2005). His study concluded that the simplified design approach, which is based on the PCA design aids (1993) combined with the AWWA (2005), provides an inadequate design if applied to conical tanks. It is worth mentioning that Azabi (2014) did not take into account the nonlinear behaviour and cracking of concrete.

The investigation conducted by Azabi (2014) was extended by Elansary et al. (2015) by accounting for shrinkage and the nonlinear behaviour of concrete. Shrinkage is considered by adding initial tensile stresses in the finite element model based on the PCA design aids (1993). The nonlinearity of concrete was considered by including a concrete constitutive model previously developed by Pietruszczak et al. (1988) and Jaing (1988). A set of twelve
reinforced concrete conical tanks covering a wide range of practical dimensions were analyzed by Elansary and El Damatty (2013) and Elansary et al. (2015). The analysis was conducted under both working and ultimate loads using the extended finite element program that considers shrinkage and nonlinear behaviour of concrete. Results of the analysis showed that the maximum deflection for the tank’s wall occurs at the middle one-third of the tank’s height and the maximum hoop stresses occur at 1/5 to 1/6 of the tank’s height. Elansary et al. (2015) found that the ratio between the maximum displacements from the linear to nonlinear analyses is 0.9. Furthermore, results from their analysis revealed that the maximum meridional stresses in the concrete wall and reinforcing bars occur within the bottom 10% region of the tank’s vessel. This was noted for the studied tanks under either working or ultimate loads.

Elansary and El Damatty (2013) selected the material concrete model by Pietruszczak et al. (1988) and Jaing (1988) for two reasons. First, Koziey (1993) showed that this material model was compatible with the 3-D consistent element, which was adopted by Elansary and El Damatty (2013), without showing numerical instabilities. Second, this model was able to capture the ductile behaviour of concrete, enhancement in strength due to confining pressure, and the nonlinear behaviour of the stress-strain relation. It is worth mentioning that the nonlinear behaviour of concrete was studied in many other investigations, same as Selby and Vecchio (1997), Wang et al., (2004), and Chen (2007). However, the compatibility of these models with the 3-D consistent element was not validated in their investigations. These investigations revealed that the nonlinear behaviour of concrete should be considered in the analysis of concrete structures in order to model them accurately. They reported that the nonlinearity in concrete occurs due to the formation of
cracks, hardening/softening, aggregate interlock, reinforcement slippage, and dowel action.

The current paper has three main objectives. The first objective is to assess the effect of shrinkage and effect of change in concrete strength on the design of reinforced concrete conical tanks. The effect of shrinkage is assessed by evaluating the required concrete wall thickness while considering/ignoring shrinkage. Meanwhile, the effect of change in concrete strength is assessed by evaluating the required concrete wall thickness using two different practical concrete strengths. The second objective is to check the adequacy of analyzing and designing reinforced concrete conical tanks using a simplified method based on the AWWA design code (2005) and the PCA design aids (1993). In this method, the AWWA design code (2005) is used to obtain the dimensions of an equivalent cylinder and then the PCA design aids (1993) are used to analyze and design the equivalent cylindrical tanks. The adequacy of this method is assessed by using a nonlinear FEM, developed by Elansary et al. (2015), to determine failure loads of a set of reinforced concrete conical tanks. The load factors at the determined failure loads are then compared with the ultimate load factors obtained from the ACI-350M-06 (2006) design code. The third objective is to develop a set of charts that can be used to determine wall thicknesses and straining actions for reinforced concrete conical tanks under hydrostatic water pressure. These charts cover a wide range of practical dimensions for reinforced concrete conical tanks and they are developed for two different practical concrete strengths. Finally, an example is presented to illustrate the prediction of thicknesses and straining actions for six reinforced concrete conical tank using the developed charts.
3.2. **State of stresses in conical tanks**

Fig. 3-2 (a) shows the dimensions of a typical shape of a pure conical vessel where \( H \), \( \theta \), and \( R \) are the vessel’s height, inclination angle, and bottom radius, respectively. The water applies hydrostatic pressure with a zero value at the vessel’s top edge and a maximum value of \( P_r \) at the vessel’s base, as shown in Fig. 3-2 (b). Fig. 3-2 (a) shows that the water inside the vessel can be divided into two volumes: volume \( V_1 \) that is bounded by an imaginary cylinder with a radius “\( R \)” and volume \( V_2 \) that is bounded by the imaginary cylinder and the vessel’s wall. The weight of \( V_1 \), “\( W_1 \)”, is resisted by the vessel’s base, while the weight of \( V_2 \), “\( W_2 \)”, is resisted by the vessel’s wall. The weight “\( W_2 \)” can be resolved into a meridional component, \( F_m \), and a hoop component, \( F_h \), as shown in Fig. 3-2 (a). Those lead to two components of stresses resulting from the hydrostatic pressure: compressive stresses in the meridional direction, \( \sigma_{\text{Meridional}} \), and tensile stresses in the hoop direction, \( \sigma_{\text{Hoop}} \).

![Fig. 3-2](image)

**Fig. 3-2.** (a) Dimensions of a pure conical vessel (b) Applied hydrostatic pressure and stress components developed in the tank’s vessel.
As the radius of the vessel reduces when approaching the base, high stress concentration will occur at the bottom part of the vessel. The maximum tensile hoop stress occurs at distance of 1/5 to 1/6 of the vessel’s height relative to the base, as reported by Elansary and El Damatty (2013). According to the PCA design aids (1993), this maximum tensile hoop stress is used to determine the wall’s thickness and the reinforcement in the ring direction.

3.3. Finite Element Model (FEM)

A set of pure conical reinforced concrete tanks are analyzed using a FEM that is based on a 13-node shell element, which was originally developed by Koziey and Mirza (1997). This element was extended by El Damatty et al. (1997a) to include the nonlinear behaviour of steel. Elansary et al. (2015) extended this model to include the nonlinear material model for reinforced concrete. In this material model, the smearing approach is adopted to model both concrete cracking and reinforcing bars, same as done by Vecchio (1989). In the smeared cracking approach, the cracks are assumed to occur due to the average deformation spread over the area of the finite element. Pietruszczak et al. (1988) considered the strain softening in the smeared cracking approach by using a path-independent criterion which assumed the existence of a strain softening surface. The smearing of reinforcing bars approach assumes that steel bars are uniformly distributed over the area of the finite element. The smearing approach is adopted because previous studies available in the literature did not provide any guidance for predicting the cracks’ patterns and locations for reinforcing bars in reinforced concrete conical tanks.

Cracks in concrete are assumed to occur when the actual tensile stresses due to working loads and shrinkage exceed the concrete tensile strength. The tensile stresses due to shrinkage are included in the FEM according to the PCA design aids (1993), which suggest
using a value of 0.0003 for the shrinkage coefficient. The PCA design aids (1993) provides the following equation to calculate tensile stresses due to shrinkage:

\[ f_{\text{Shrinkage}} = \frac{CE_s A_s}{A_c + nA_s} \]  

(3-1)

where \( C \), \( A_s \), \( A_c \), and \( n \) are the shrinkage coefficient, area of steel, area of concrete, and modular ratio between steel and concrete, respectively.

The FEM used in the current study was developed and validated by Elansary et al. (2015). In their work, the FEM was validated by analyzing different reinforced concrete slabs from the literature under lateral and edge loads. The numerical results obtained from the FEM were found to be in an excellent agreement with the experimental results from the literature.

The authors adopted the FEM mesh and boundary conditions that were used by Elansary et al. (2015). In their investigation, a sensitivity analysis revealed that using 256 shell elements is adequate for predicting accurate values for the displacements and stresses. Elansary et al. (2015) included the ACI 318-05 (2005) and PCA design aids (1993) failure criteria in their FEM. They assumed that failure occurs when the force in the ring tension reinforcement exceeds that of the ultimate tension force. They also assumed that failure occurs when the meridional axial force or moment exceed the corresponding ultimate values, according to the following equations:

\[ R_{\text{max}} = A_{s,\text{hoop}} (\phi f_y) \]  

(3-2)

\[ P_{\text{max}} = 0.80 \phi [0.85f'_c (A_g - A_{st}) + f_y A_{st}] \]  

(3-3)

\[ M_{\text{max}} = \phi M_n \]  

(3-4)

where \( R_{\text{max}} \), \( P_{\text{max}} \), \( M_{\text{max}} \), and \( M_n \) are the maximum ring tension force, maximum meridional compression force, maximum meridional moment, and nominal moment, respectively. \( A_{s,\text{hoop}} \), \( A_{st} \), and \( A_g \) are the area of steel in the hoop and meridional directions and gross
area of the concrete section, respectively. $\phi$ is a strength reduction factor, which is equal to 0.9, 0.65 in Equations (3-2) and (3-3), respectively. $\phi$ is equal to 0.9 for tension controlled sections and 0.65 for compression controlled sections in Equation (3-4).

3.4. **Method of analysis**

All studied tanks are subjected to a hydrostatic liquid pressure such that the liquid level is kept constant throughout the loading process. As shown in Fig. 3-2 (b), the liquid pressure is applied in a triangular pattern with a zero value at the top of the vessel and a maximum value, $P_r = p\gamma_lH$, at bottom, where $\gamma_l$ is the specific weight of water and $p$ is the load factor. The analysis is conducted incrementally by gradually increasing the value of $p$. This analysis is performed twice; first under working loads and then under ultimate loads. The thicknesses of the studied tanks are obtained by trial and error procedure under working loads. In this procedure, the thickness of each tank is assumed and the liquid pressure is applied incrementally until first cracking occurs. If cracking occurs at a load factor $p > 1$, a smaller thickness is assumed and the analysis is repeated until concrete cracking occurs exactly at a load factor $p = 1$. After choosing the wall’s thickness for each tank, another set of analyses is carried out using factored (ultimate) loads to determine the straining actions in the tank’s wall. The charts for straining actions are obtained by applying the water pressure incrementally up to a load factor of $p = 2.7$. This value results from multiplying the environmental durability factor by the ultimate load factor specified in the ACI 350-08 (2008).

3.5. **Tank dimensions and material properties**

The capacity of commonly built elevated water tanks ranges from 1,900 to 11,000 m³ according to the ACI 371R-08 (2008) design code. Recommendations for the tanks’
dimensions are provided in the Water Environment Federation-ASCE (2010). Tchobanoglous et al. (1991) also provided recommendations for tank sizes and dimensions. A set of 160 tanks with different dimensions is chosen to be studied in this research. As shown in Table 3-1, the first 40 tanks have cylindrical vessels, i.e: have an inclination angle of 0°, while the remaining 120 tanks have pure conical vessels with an inclination angle of 30°, 45°, and 60°. The 40 cylindrical tanks are studied in order to compare the thicknesses and straining actions predicted by the FEM with those obtained from the PCA design aids (1993). In the conducted parametric study, the radius at the base is varied from 4 to 8 m with an increment of 1 m whereas the tanks’ height is varied from 5 to 12 m with an increment of 1 m. The tanks’ wall thicknesses are calculated according to the ACI 350M-06 (2006) and PCA design aids (1993) such that the first crack occurs when the tank is full of water.

Two different normal strength concrete, which are used for conventional structures, are considered for the developed charts; 30 and 40 MPa. These strengths are adopted in the current work to avoid using high strength concrete, which requires special design precautions and it is more expensive than normal strength concrete. According to the ACI-350M-06 (2006) design code, the Young’s modulus for the normal strength concrete can be calculated as:

$$E_c = 4700 \sqrt{f_c'}$$

The concrete plasticity model parameters are used same as those reported by Jaing (1988). Steel properties are assumed as follows: yield strength $f_y = 400$ MPa, ultimate strength $f_u = 520$ MPa, Poisson’s ratio $\nu_s = 0.3$, and modulus of elasticity $E_s = 200,000$ MPa.
Table 3-1. Tank dimensions.

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<th>Tank #</th>
<th>$\theta$</th>
<th>R (m)</th>
<th>H (m)</th>
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3.6. Equivalent Cylinder Method (ECM)

The ECM is an approximate approach for the analysis and design of conical tanks. This approach is based on calculating the height and radius for an equivalent cylinder from the conical vessel dimensions, as shown in Fig. 3-3. The AWWA design code (2005) recommends using an equivalent radius for the conical vessel’s by calculating the average
of its top and bottom radii. The following three equations can be used to obtain the dimensions of the equivalent cylinder:

\[ H_{eq} = \frac{H}{\cos \theta} \]  (3-6)

\[ R_{eq} = \frac{2R + H \tan \theta}{2 \cos \theta} \]  (3-7)

\[ t_{eq} = t \]  (3-8)

where \( H, R, t \), and \( \Theta \) are the vessel’s height, radius, thickness, and angle of inclination, respectively. \( R_{eq}, t_{eq}, \) and \( H_{eq} \) are the equivalent cylinder’s radius, thickness, and height, respectively.

Fig. 3-3. Equivalent cylinder dimensions.

The PCA design aids (1993) provide a set of charts that can be used to obtain the straining actions for the equivalent cylinder. These charts are used to calculate the ring tension force and meridional moment corresponding to different boundary conditions at the top and base of the tank’s wall. The boundary conditions are either free top and fixed base or free top and hinged base. The straining actions from the charts vary depending on the applied load distribution i.e. triangular, trapezoidal, top shear, or top moment. Although the ECM is relatively easy to apply, it has the following drawbacks:
a. Replacing the conical vessel with an equivalent cylinder is an empirical method, which means it has no scientific rationale.

b. The state of stresses in conical tanks is not accurately represented due to the fact that these tanks are subjected to significant meridional axial forces, which is not considered by the PCA design aids (1993).

c. The PCA design aids (1993) provide charts for cylindrical tanks which do not take into account the own weight of the tank’s wall. This is acceptable for cylindrical tanks because the wall’s own weight causes an axial meridional compression force only on the wall. If this force is neglected, the wall will be designed under moment only in the meridional direction, which is usually more conservative. However, the own weight of the walls of conical tanks causes meridional bending moment in addition to the meridional compression force due to the inclination of the wall. The tank own weight in this case will cause significant meridional moments that cannot be neglected.

d. The PCA design aids (1993) are based on linear analysis, so they do not take into account the nonlinear behaviour of reinforced concrete.

The ACI 350M-06 (2006) suggests using a durability factor of 1.93 for hoop tension members with reinforcing steel having a yield strength of 400 MPa under normal environmental exposures. However, the durability factor for flexure members ranges between 1.07 and 1.93 corresponding to a reinforcing steel spacing of 50 and 300 mm, respectively. This study focuses on water tanks subjected to normal environmental exposures; therefore, an ultimate load factor of 2.7 is adopted. It is worth mentioning that
the PCA design aids (1993) and ACI 350M-06 (2006) require a minimum thickness of 300 mm to be used for liquid filled reinforced concrete tanks with a height of 3 m or higher.

3.7. **Effect of shrinkage**

The effect of ignoring shrinkage on the calculated thicknesses for reinforced concrete conical tanks is presented in this section. The thicknesses for a set of tanks with a bottom radius of 6 m, a height of 8 m, and an inclination angle of 0°, 30°, 45°, and 60° are determined twice, considering/neglecting shrinkage, using the nonlinear FEM. For this set of tanks, it is found that the difference between the thicknesses calculated considering/neglecting shrinkage increases with the increase of the tank’s inclination angle, as shown in Fig. 3-4. For the four tanks, one can notice that the thicknesses considering shrinkage are larger than those calculated neglecting shrinkage by a factor of 1.3. The same factor is observed for the studied 160 tanks. Fig. 3-4 shows that shrinkage has a significant effect on increasing the calculated thickness and therefore it must be considered in the thickness calculations.

![Fig. 3-4. Thicknesses of tanks using nonlinear FEM considering/neglecting shrinkage (R = 6 m, H = 8 m) (fc’=30 MPa).](image-url)
3.8. **Effect of changing concrete strength**

The effect of change in concrete strength on the designed thickness of the conical tank’s wall is provided in this section. The nonlinear FEM is used to evaluate the thicknesses of the set of 160 tanks. **Fig. 3-5** shows the thickness for four tanks with a bottom radius of 6 m, a height of 8 m, and an inclination angle of 0°, 30°, 45°, and 60° using $f_{c'} = 30$ and 40 MPa. It is clear that the compressive strength of concrete has a significant effect on the calculated thicknesses.

**Fig. 3-5.** Thicknesses for tanks using nonlinear FEM for $f_{c'}=30$ MPa and $f_{c'}=40$ MPa ($R = 6$ m, $H = 8$ m).

A significant reduction in thicknesses occurs due to the increase in concrete strength from 30 MPa to 40 MPa. The ratio between the thicknesses calculated using $f_{c'} = 30$ MPa and $f_{c'} = 40$ MPa is 1.29, 1.31, 1.43, and 1.5 for tanks with an inclination angle of 0°, 30°, 45°, and 60°, respectively. Therefore, if the design yields significantly large wall’s thickness, it is recommended to use a concrete with a higher strength especially for wide tanks with large inclination angles.
3.9. **Comparison between FEM and ECM**

The wall’s thicknesses, maximum ring tension forces, and meridional moments obtained from the nonlinear FEM are compared with their counterparts obtained from the ECM for a set of tanks with a bottom radius of 6 m, a height of 8 m, and an inclination angle of 0°, 30°, 45°, and 60°. 

**Fig. 3-6** shows the thicknesses obtained from the nonlinear FEM and ECM for tanks with an inclination angle of 0°, 30°, 45°, and 60°. A difference less than 10% in walls’ thicknesses is noted for tanks with an inclination angle of 0°, i.e. for cylindrical tanks. This difference exists because the FEM accounts for the wall’s own weight, which is neglected in the PCA design aids (1993). **Fig. 3-6** shows that the ECM estimates larger thicknesses for tanks with an inclination angle of 30°, 45°, and 60°. The ratio between the thicknesses from the ECM to FEM is 1.2, 1.3, and 1.4 for tanks with an inclination angle of 30°, 45°, and 60°, respectively. Therefore, the ratio between the thicknesses from the two approaches increases with the increase of inclination angle. It can be concluded that the thicknesses obtained from the ECM are typically over-conservative compared to those obtained from the FEM.

![Thicknese for tanks using ECM and the nonlinear FEM (R = 6 m, H = 8 m).](image)
For the same tanks, the maximum ring tension forces due to ultimate loads obtained from the FEM and ECM are plotted in Fig. 3-7. It is clear that the difference in ring tension forces does not exceed 10% for tanks with an inclination angle of 0°. Fig. 3-7 shows that the ECM overestimates the ring tension for the tanks with an inclination angle of 30°, 45°, and 60°. The ratio between the ring tension forces from the ECM to FEM is 1.4, 1.7, and 2.2 for tanks with an inclination angle of 30°, 45°, and 60°, respectively. Consequently, one can conclude that the ratio between the ring tension forces obtained from the two approaches increases with the increase of the inclination angle.

![Fig. 3-7. Ring tension force using ECM and the nonlinear FEM (R = 6 m, H = 8 m).](image)

The maximum meridional moment due to the ultimate loads for the same tanks from the FEM and ECM are plotted in Fig. 3-8. A good agreement between the ECM and FEM is observed in predicting the meridional moments for tanks with an inclination angle of 0° and 30°. Fig. 3-8 shows that the ECM overestimates the meridional moment for tanks with an inclination angle of 45° and 60°. The ratio of discrepancy in meridional moment obtained from the ECM to FEM is 1.6 and 2.3 for tanks with an inclination angle of 45°
and 60°, respectively. Therefore, the ratio of discrepancy in meridional moment obtained from the two approaches increases with the increase of inclination angle.

![Chart](https://via.placeholder.com/150)

**Fig. 3-8.** Meridional moment using ECM and the nonlinear FEM (R = 6 m, H = 8 m).

From the previously shown results, it can be concluded that the ECM overestimates the values of the wall thickness, ring tension force, and meridional moment that lead to an uneconomical solution. Therefore, a set of charts based on the nonlinear FEM is proposed in this paper for the design of reinforced concrete conical tanks to provide an economical solution.

### 3.10. Adequacy of the ECM

Section 3.9 shows that significant differences exist between the thicknesses and straining actions resulting from the ECM and FEM. However, this section did not answer the question if the ECM is conservative when it is used to design reinforced concrete conical tanks or not. The answer of this question is provided in the current section where the adequacy of using the ECM is checked by modelling the 160 tanks using the nonlinear FEM. The thicknesses and the reinforcement ratios in both directions are calculated using the ECM. Afterwards, these thicknesses and reinforcement ratios are used as input to the nonlinear FEM, which includes the ACI 318R-05 (2005) failure criteria. The load factors
at failure are recorded for the 160 studied tanks and plotted in Fig. 3-9. It can be observed that the load factors at failure for the studied tanks are 2.8~3.2, 3.1~4.2, 3.9~4.7, and 3.8~6.3 for tanks with an inclination angle of 0°, 30°, 45°, and 60°, respectively. Therefore, the nonlinear FEM shows that the ECM yields a safe design for all of the studied tanks in terms of thicknesses and reinforcement ratios. As shown in Fig. 3-9, the load factors at failure decreases as the bottom radius increases for tanks with an inclination angle of 0°, 30°, and 45°. For tanks with an inclination angle of 60°, it is found that the load factors at failure increases as the bottom radius increases. Considering 2.7 as a safe ultimate load factor, one can conclude that the ECM yields an uneconomical design for all of studied tanks.

Fig. 3-9. Load factors at failure using thicknesses and reinforcement from ECM.
3.11. **Charts for thickness**

The thicknesses for the 160 studied tanks are obtained using the nonlinear FEM for two different concrete strengths: 30 and 40 MPa, as shown in Figs. 3-10 and 3-11, respectively. A minimum thickness of 300 mm is adopted, as required by PCA design aids (1993) and ACI 350M-06 (2006) design code. For all tanks, these figures show that the thicknesses increase with the increase of the inclination angle. For example at $f_c' = 30$ MPa, tanks with a bottom radius of 7 m and a height of 9 m, the ratio between the thicknesses is 1.0: 1.1: 1.6: 3.3 for tanks with an inclination angle of 0°, 30°, 45°, and 60°, respectively. However, at $f_c' = 40$ MPa, this ratio becomes 1.0: 1.0: 1.1: 2.2 for the same tanks with an inclination angle of 0°, 30°, 45°, and 60°, respectively.

Figs. 3-10 and 3-11 show that the thicknesses of tanks increase with the increase of the bottom radius. For $f_c' = 30$ MPa, tanks with an inclination angle of 45° and a height of 9 m, the ratio between the thicknesses is 1.0: 1.2: 1.3: 1.5: 1.6 for tanks with bottom radius of 4, 5, 6, 7, and 8 m, respectively. For $f_c' = 40$ MPa, this ratio becomes 1.0: 1.0: 1.0: 1.1: 1.2 for the same tanks. It can be noted that the thicknesses of walls increase with the increase of the tanks’ height. For $f_c' = 30$ MPa, tanks with an inclination angle of 45° and a bottom radius of 6 m, the ratio between the thicknesses is 1.0: 1.2: 1.6: 2.0 for tanks with height of 6, 8, 10, and 12 m, respectively. For $f_c' = 40$ MPa, this ratio becomes 1.0: 1.0: 1.1: 1.4 for the same tanks.
Fig. 3-10. Thickness for the studied tanks using the nonlinear FEM ($f_c' = 30$ MPa).

Fig. 3-11. Thickness for the studied tanks using the nonlinear FEM ($f_c' = 40$ MPa).
3.12. Straining actions charts

This section shows the results obtained from the analysis of the studied tanks under hydrostatic pressure. A set of charts are plotted to show the axial forces and bending moments in both ring and meridional directions. The moment in ring direction is negligible due to the symmetrical nature of the geometry and hydrostatic pressure about the vertical axis of the conical vessel. However, the axial force in the ring direction is significant. Hence, the tanks’ wall should be designed under pure tension force in the ring direction. In the meridional direction, the tank’s wall is subjected to significant meridional axial force and moment. Therefore, the wall should be designed under the combined effect of both axial force and moment. A set of charts for calculating ring tension forces, meridional moments, and meridional axial forces is presented for the studied tanks in Sections 3.12.2, 3.12.3, and 3.12.4, respectively. Those straining actions are obtained due to ultimate loads considering a load factor of 2.7.

3.12.1. Straining actions distribution

The typical distributions of ring axial force, $R_F$, meridional moment, $M$, and meridional axial force, $N$, along the tank’s height are shown in Fig. 3-12. It is clear that the maximum ring axial force occurs at the middle one third of the tanks’ height, while the maximum meridional moment occurs at the bottom one third of the tanks’ height. Fig. 3-12 (c) shows that the maximum meridional axial force occurs near the vessel’s base. The ring axial force is tension along the whole height of the vessel except for the region near the base where it changes to compression, as shown in Fig. 3-12 (a). This compression force exists due to the constraint provided by the boundary conditions in this region. Fig. 3-12 (c) shows that the meridional axial force does not reverse its direction along the whole height of the vessel.
This means that the whole vessel is subjected to meridional compression axial force. However, the meridional bending moment reverses its direction at the upper half of the vessel, as shown in Fig. 3-12 (b). This indicates that tensile stresses exist at the inner and outer faces of the tank’s vessel in the meridional direction. Moreover, Elansary et al. (2015) reported that the inner and outer faces of the tank’s wall are usually subjected to a tensile stresses in the ring direction. Therefore, two meshes of reinforcement are required to be used in the inner and outer faces of the vessel’s wall. The reinforcement bars in the two meshes should be extended along the meridional and ring directions to resist the tensile stresses.

![Fig. 3-12. Typical distribution for the ring tension force, \( R_F \), meridional moment, \( M \), and meridional axial force, \( N \), along the tank’s height.](image)

In this study, the reinforcement is assumed to be uniformly distributed in both the ring and meridional directions. Fig. 3-12 shows that no significant reduction is observed in the ring tension force at the upper half of the vessel, while significant reductions in both the meridional moment and axial force are noted. Therefore, the meridional reinforcement can
be curtailed at the upper half of the vessel for more economical solution. The maximum values for the ring axial force, meridional moment and meridional axial force for the studied 160 tanks are presented in the following three sub-sections.

3.12.2. Ring tension force (R_F)

The maximum ring tension forces due to ultimate loads for the studied tanks for concrete strengths of 30 and 40 MPa are shown in Figs. 3-13 and 3-14, respectively. These figures show that the ring tension forces increase with the increase of the tank’s height. For the tanks with an inclination angle of 0°, 30°, 45°, and 60°, when the height increases from 5 m to 12 m, the ring tension force increases by 180%, 190%, 230%, and 280%, respectively. It is observed that when the bottom radius increases, the ring tension force increases. As an example, for tanks with an inclination angle of 0°, 30°, 45°, and 60°, when the bottom radius increases from 4 m to 8 m, the ring tension force increases by 72%, 55%, 40%, and 35%, respectively. Figs. 3-13 and 3-14 demonstrate that the change of the tank’s inclination angle has a significant effect on the resulting ring tension forces. For example, when the bottom radius and the tank height are 6 m and 8 m, respectively, the ratios between the maximum ring tension force for the tanks with inclination angle of 30°, 45°, and 60° to that of the cylindrical tank are 1.3, 1.8, and 3.2, respectively. It is worth mentioning that the ratio mentioned above is found to be applicable for all studied tanks with concrete strengths of 30 and 40 MPa.
Fig. 3-13. Ring tension force ($f_c' = 30$ MPa).

Fig. 3-14. Ring tension force ($f_c' = 40$ MPa).
3.12.3. Meridional moment (M)

Figs. 3-15 and 3-16 show the maximum meridional moments for the studied tanks for concrete strengths of 30 and 40 MPa. It is obvious that the change in the tank’s height has a significant effect on increasing the meridional moment for all of the studied tanks. For tanks with bottom radius of 6 m and an inclination angle of 0°, 30°, 45°, and 60°, when the height increases from 5 m to 12 m, the meridional moment increases by 220%, 370%, 670%, and 1300%, respectively.

Figs. 3-15 and 3-16 show that the increase of the bottom radius has a significant effect on increasing the meridional moment for all of the studied tanks. For instance, for tanks with a height of 8 m and an inclination angle of 0°, 30°, 45°, and 60°, when the bottom radius increases from 4 m to 8 m, the meridional moment increases by 150%, 180%, 240%, and 200%, respectively.

Fig. 3-15. Meridional moment ($f'_c = 30$ MPa).
Figs. 3-15 and 3-16 show that the change in the inclination angle greatly affects the resulting meridional moments. The ratio between the meridional moments is 1: 1.7: 2.9: 12.3 for tanks with bottom radius of 6 m, height of 8 m and an inclination angle of 0°, 30°, 45°, and 60°, respectively. It should be noted that the ratio mentioned above is for tanks with concrete strength of 30 MPa. However, this ratio is found to be 1: 1.2: 2.2: 8.1 for tanks with concrete strength of 40 MPa.

3.12.4. Meridional axial force (N)

The maximum meridional axial forces for the studied tanks for concrete strengths of 30 and 40 MPa are plotted in Figs. 3-17 and 3-18, respectively. From these figures, it can be observed that meridional axial forces increase significantly with the increase of tank’s height. For tanks with a bottom radius of 6 m and an inclination angle of 0°, 30°, 45°, and 60°, when the height increases from 5 m to 12 m, the meridional axial force increases by
140%, 610%, 710%, and 830%, respectively. It is found that the meridional axial force decreases when the bottom radius increases for tanks with an inclination angle of 30°, 45°, and 60°. This occurs because the water volume $V_2$, shown in Fig. 3-2 (a), increases when the bottom radius decreases for tanks with the same vessel’s height. The increase of this water volume causes an increase of the meridional axial force. As an example, for tanks with height of 8 m, when the bottom radius increases from 4 m to 8 m, the meridional axial force decreases by 12%, 24%, and 28%, respectively.

![Graphs showing meridional axial force for different inclination angles and bottom radii.](image)

**Fig. 3-17.** Meridional axial force ($f_c' = 30$ MPa).
Figs. 3-17 and 3-18 show that the meridional axial forces increase when the inclination angle increases. For a tank with bottom radius of 6 m and height of 8 m, the ratios between the meridional axial forces for tanks with an inclination angle of 45° and 60° to that of the tank with an inclination angle of 30° are 2.5 and 7.5, respectively. It is noteworthy to mention that same trends are observed for tanks with concrete strengths of 30 MPa and 40 MPa.

3.13. **Illustrative numerical example**

In this section, a numerical example is presented to demonstrate the use of the charts proposed in the current study to predict the design thicknesses and straining actions of a new set of six reinforced concrete conical tanks. This set of tanks is selected within the range of dimensions specified in the developed charts but differs from the geometry of the studied 160 tanks. In addition, for further validation, the resulting design thicknesses and
straining actions obtained from the developed charts are compared to their counterparts obtained from the nonlinear FEM. The dimensions, storage capacities, and concrete strengths of the set of conical tanks considered in this example are presented in Table 3-2. The tanks are assumed to be filled with water such that they are subjected to hydrostatic water pressure considering the water specific weight is 10 kN/m³.

Table 3-2. Dimensions and concrete strength of the analyzed tanks.

<table>
<thead>
<tr>
<th>Tank #</th>
<th>H (m)</th>
<th>R (m)</th>
<th>θ°</th>
<th>f’c (MPa)</th>
<th>Capacity (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tk1</td>
<td>10.5</td>
<td>5.5</td>
<td>35</td>
<td>30</td>
<td>2926</td>
</tr>
<tr>
<td>Tk2</td>
<td>10.5</td>
<td>5.5</td>
<td>35</td>
<td>40</td>
<td>2926</td>
</tr>
<tr>
<td>Tk3</td>
<td>7.5</td>
<td>6.5</td>
<td>40</td>
<td>30</td>
<td>2270</td>
</tr>
<tr>
<td>Tk4</td>
<td>7.5</td>
<td>6.5</td>
<td>40</td>
<td>40</td>
<td>2270</td>
</tr>
<tr>
<td>Tk5</td>
<td>9.5</td>
<td>7.5</td>
<td>55</td>
<td>30</td>
<td>6547</td>
</tr>
<tr>
<td>Tk6</td>
<td>9.5</td>
<td>7.5</td>
<td>55</td>
<td>40</td>
<td>6547</td>
</tr>
</tbody>
</table>

The charts in Sections 3.11 and 3.12 are developed for cylindrical and conical tanks with an inclination angle of 30°, 45°, and 60°. Therefore, a linear interpolation is used to determine the thicknesses and straining actions of the set of six tanks, as listed in Table 3-3. These straining actions extracted from the charts can then be used to design these tanks under factored hydrostatic pressure. In order to validate the charts, the same six conical tanks are then analyzed using the nonlinear FEM. Analysis is performed twice; first under working loads and then under ultimate loads. Concrete cracking started in the six analyzed tanks at a load factor $p \geq 1$, as shown in Table 3-4. For tanks Tk1, Tk2, Tk3, and Tk4, the FEM yields load factors at cracking $p > 1$. This occurs because the charts for thickness are developed, as previously mentioned in Section 3.11, such that the tanks’ thicknesses must not be less than 0.3 m. In other words, when the resulted thickness of a certain tank obtained from the nonlinear FEM, at a load factor $p = 1$, is less than the minimum thickness, a value
of 0.3 m is selected and plotted in the charts. Therefore, the analysis of tanks Tk1, Tk2, Tk3, and Tk4 using the selected thickness of 0.3 m yields load factors at cracking $p > 1$. An excellent agreement is found between the straining actions obtained from the nonlinear FEM and the charts with difference less than 5%.

### Table 3-3. Thicknesses and straining actions of a new set of six tanks from the charts.

<table>
<thead>
<tr>
<th>Tank #</th>
<th>$t$ (m)</th>
<th>$R_F$ (kN/m)</th>
<th>$M$ (kN.m/m)</th>
<th>$N$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tk1</td>
<td>0.37</td>
<td>2000</td>
<td>117</td>
<td>-1967</td>
</tr>
<tr>
<td>Tk2</td>
<td>0.31</td>
<td>2150</td>
<td>90</td>
<td>-1833</td>
</tr>
<tr>
<td>Tk3</td>
<td>0.35</td>
<td>1417</td>
<td>87</td>
<td>-1067</td>
</tr>
<tr>
<td>Tk4</td>
<td>0.30</td>
<td>1500</td>
<td>77</td>
<td>-1100</td>
</tr>
<tr>
<td>Tk5</td>
<td>0.83</td>
<td>3567</td>
<td>750</td>
<td>-5067</td>
</tr>
<tr>
<td>Tk6</td>
<td>0.55</td>
<td>3733</td>
<td>467</td>
<td>-5000</td>
</tr>
</tbody>
</table>

### Table 3-4. Load factors at cracking and ultimate straining actions of a new set of six tanks from the nonlinear FEM.

<table>
<thead>
<tr>
<th>Tank #</th>
<th>Load factor</th>
<th>$R_F$ (kN/m)</th>
<th>$M$ (kN.m/m)</th>
<th>$N$ (kN/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tk1</td>
<td>1.1</td>
<td>1910</td>
<td>112</td>
<td>-1895</td>
</tr>
<tr>
<td>Tk2</td>
<td>1.3</td>
<td>2252</td>
<td>93</td>
<td>-1813</td>
</tr>
<tr>
<td>Tk3</td>
<td>1.2</td>
<td>1477</td>
<td>85</td>
<td>-1014</td>
</tr>
<tr>
<td>Tk4</td>
<td>1.4</td>
<td>1518</td>
<td>78</td>
<td>-1088</td>
</tr>
<tr>
<td>Tk5</td>
<td>1</td>
<td>3544</td>
<td>734</td>
<td>-5053</td>
</tr>
<tr>
<td>Tk6</td>
<td>1</td>
<td>3700</td>
<td>446</td>
<td>-4926</td>
</tr>
</tbody>
</table>

### 3.14. Summary and conclusions

A nonlinear Finite Element Model (FEM) is used to analyze a set of 40 cylindrical tanks and 120 pure conical tanks under hydrostatic water pressure. The effect of considering shrinkage and effect of changing in concrete strength on the designed wall’s thickness are determined. For all studied tanks, the ratio between the thicknesses considering shrinkage to those neglecting shrinkage is found to be 1.3. It is also concluded that increasing the
concrete strength significantly reduces the calculated tank’s thickness. The ratio between thicknesses of the tanks with $f_{c'} = 30$ MPa to those with $f_{c'} = 40$ MPa can reach up to 1.5.

A simplified design method for analysis and design of conical tanks is presented and referred to as Equivalent Cylinder Method (ECM). This method is based on transforming the geometry of conical tanks to an equivalent cylinders based on the equations provided in the AWWA (2005) code. The PCA (1993) design aids are then utilized to obtain the thicknesses and straining actions of the equivalent cylinders. The resulted thicknesses and straining actions are then compared to those obtained from the nonlinear FEM. It is found that the ratio between the thicknesses obtained from the ECM to those resulting from the nonlinear FEM is 1.2, 1.3, and 1.4 for tanks with an inclination angle of 30°, 45°, and 60°, respectively. For ring tension forces, this ratio is observed to be 1.4, 1.7, and 2.2 for tanks with an inclination angle of 30°, 45°, and 60°, respectively. This indicates that overestimated thickness and straining actions are obtained when the ECM is used to analyze conical tanks.

The adequacy of utilizing the ECM as an approximation to apply the PCA (1993) provisions on an equivalent cylinder of conical shape tanks is assessed. The studied tanks are designed using the ECM to obtain the wall’s thicknesses and reinforcement ratios in the hoop and meridional directions. These thicknesses and reinforcement ratios are then used as input to the nonlinear FEM to test the adequacy of using the ECM in the design of conical tanks. The load factor at failure for each of the studied tanks is recorded. It is concluded that the ECM provides uneconomical solutions when it is used for the design of conical tanks.
The distributions of straining actions along the vessel’s height are studied for the 160 tanks. It is concluded that the maximum ring tension force and meridional moment occur at the middle one third and the bottom one third of the tanks’ height, respectively. It is also found that the maximum meridional axial force occurs near the vessel’s base. Two inner and outer uniformly distributed reinforcement meshes are used because the inner and outer faces of the tank’s wall are subjected to tensile stresses in the hoop and meridional directions. These stresses cannot be resisted by the concrete tensile resistance. A set of charts are developed to determine the adequate wall thicknesses, ring tension forces, meridional axial forces, and meridional moments for tanks with concrete strengths of 30 and 40 MPa. Straining actions resulting from ultimate loads are plotted for the studied 40 cylindrical and 120 conical tanks. Based on these charts, both the wall’s thickness and straining actions increase with the increase of the inclination angle, bottom radius, or tank’s height. An illustrative example is provided where the thicknesses and straining actions for a new set of six reinforced concrete conical tanks are obtained using the proposed charts. The dimensions of the new set of tanks are selected to be different from those for the studied 160 tanks. A linear interpolation is used to obtain the thicknesses and straining actions of the new set of tanks. The resulting straining actions are then validated by analyzing the same set of tanks using the nonlinear FEM. A good agreement is noticed between the results predicted by the nonlinear FEM and their counterparts obtained from the charts.

3.15. **Acknowledgements**

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3.16. References


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CHAPTER 4

BEHAVIOUR OF COMPOSITE CONICAL TANKS UNDER HYDROSTATIC PRESSURE

4.1. Introduction

Vessels with truncated conical shapes are commonly used as liquid containments in elevated conical tanks. The main structural components of conical tanks are the supporting system and the vessel, as shown in Fig. 4-1. The vessels can be made of steel or reinforced concrete, with steel being more common especially in North America. Recently, composite steel-reinforced concrete construction has been used for conical vessels combining the benefits of both materials as explained later. In this type of construction, the vessels consist of an external steel shell made of curved steel panels and an internal reinforced concrete shell that is cast-in-situ. The steel and reinforced concrete shells are connected together using steel studs that are welded to the steel shell and embedded into the reinforced concrete shell, which will be referred to here as “concrete wall”. The state of stress in liquid-filled conical vessels was described in detail by El Damatty et al. (1997b). The hydrostatic pressure associated with the contained liquid leads to tensile stresses in the hoop (circumferential) direction and compressive in the meridional (axial) direction. While steel conical vessels are efficient in resisting the tensile hoop stresses, they are susceptible to buckling under the meridional compressive stresses. In fact, a number of steel conical tanks collapsed in the past because of buckling, such as in Belgium, in 1972 and in Fredericton,
Canada, in 1990, which were reported by Vandepitte (1977) and Korol (1991), respectively. In contrast, reinforced concrete conical vessels have strong resistance to buckling under compressive meridional stresses, but they are weak in resisting the tensile hoop stresses. Composite conical tanks overcome the disadvantages of reinforced concrete and steel conical tanks by making full use of the capacity of the two materials. As a result, the construction of composite conical tanks has been recently spreading in different locations around the globe. However, no guidelines exist in the current codes of practice regarding the analysis and design of this type of composite structures. The literature review on studies related to structural behaviour of hydrostatically loaded conical tanks indicates that a number of studies were done for steel tanks, a few exists for reinforced concrete tanks, and none is available for composite tanks.

Following the collapse of a steel conical tank that happened in Belgium, 1972, which was reported by Vandepitte (1977), an extensive experimental program was conducted by Vandepitte et al. (1982). In this experimental program, a large number of small-scale truncated conical shell models, made of mylar, brass, aluminum, and steel, were tested. The models were filled gradually with water and the height of water at which buckling occurred was recorded. The study considered geometric imperfections, which are known to affect significantly the buckling capacity of thin-shell structures. Based on the experimental results, expressions were developed to determine the adequate thickness of conical vessels required to prevent buckling for different magnitudes of geometric imperfections. Following the failure of the steel conical tank that occurred in Fredericton, Canada in 1990, an extensive research program was conducted on the stability of hydrostatically loaded steel conical tanks by El Damatty et al. (1997a, b). In this research
program, an in-house numerical model for the analysis of steel conical tanks was developed by El Damatty et al. (1997b). This element was developed by Koziy and Mirza (1997) and then extended by El Damatty et al. (1997a) to include the nonlinear behaviour of steel. Results of the aforementioned studies showed that inelastic bucking at the vessel’s base is usually the main cause of failure of conical steel vessels subjected to hydrostatic pressure. A study by El Damatty et al. (1998) showed that inelastic buckling of steel conical tanks was most sensitive to axisymmetric imperfections. El Damatty et al. (2001) found that welding stiffeners to the bottom part of steel conical tanks significantly enhances the buckling capacity of conical tanks.

Many investigations are found in the literature covering the analysis of concrete rectangular or circular tanks under hydrostatic pressure, such as the studies conducted by Green and Perkins (1980), Chau and Lee (1991), and Ghali (2014). None of these investigations accounted for the nonlinear behaviour of concrete. Green and Perkins (1980), and later Anchor (1992) presented a simplified procedure for the analysis and design of reinforced concrete rectangular tanks. In their studies, they used the classical beam theory to obtain the internal forces in rectangular tanks subjected to hydrostatic water pressure. Chau and Lee (1991) analyzed circular in addition to rectangular tanks using a self-developed computer program, RCTANK. They validated this program by comparing the results for four different reinforced concrete tanks with their counterparts obtained from manual methods. Two years later, Ramanjaneyulu et al. (1993) developed another computer program, TANK, to evaluate the load capacity of reinforced concrete tanks by applying the limit analysis approach. The four aforementioned studies are based on analytical solutions that are valid for rectangular or circular tanks and cannot be used for conical tanks.
Recently, Ghali (2014) analyzed circular storage tanks using a Finite Element Model (FEM) based on a conical shell element. The FEM was validated by comparing the analysis results for a set of tanks with those obtained from closed form analytical solutions. Ghali (2014) developed a set of tables that show straining actions of circular tanks with a wide range of practical dimensions. However, the adequacy of using the developed tables to determine the straining actions of conical tanks was not tested by Ghali (2014). El Mezaini (2006) and Bruder (2011) analyzed a set of reinforced concrete cylindrical tanks with a conical base using SAP 2000 software. Azabi (2014) analyzed a set of reinforced concrete pure conical tanks using a FEM that is based on the 3-D consistent element developed by Koziey and Mirza (1997). Azabi (2014) assessed the accuracy of a simplified approach for the analysis and design of reinforced concrete conical tanks. This approach is based on the Portland Cement Association, PCA design aids (1993) combined with the equivalent cylindrical approach by the American Water Works Association, AWWA (2005). El Mezaini (2006), Bruder (2011), and Azabi (2014) compared between the internal forces obtained from their numerical models with those resulting from the PCA design aids (1993). Significant discrepancies were obtained between the internal forces resulting from the numerical models and the PCA design aids (1993). The FEM by Azabi (2014) was extended by Elansary et al. (2015), as presented in Chapter 2, to account for shrinkage and the nonlinear behaviour of concrete. The nonlinearity of concrete was considered by including a concrete constitutive model previously developed by Pietruszczak et al. (1988) and Jaing (1988). Elansary et al. (2015) used the developed FEM to study the behaviour of twelve reinforced concrete conical tanks with a wide range of practical dimensions. They reported that the maximum deflection of the tank’s wall occurs at the middle one-third of
the tank’s height and the maximum hoop stress occurs at 1/5 to 1/6 of the tank’s height. Their study showed that the maximum meridional stress in the concrete wall and reinforcing bars occur within the bottom 10% region of the tank’s vessel.

The aforementioned studies on reinforced concrete tanks revealed the necessity of using significantly large thickness to prevent cracking for large capacity tanks. Meanwhile, the investigations on steel tanks showed that they can suffer from buckling if inadequate thickness is used. The facts that concrete is superior in resisting buckling and steel is superior in resisting tensile stresses led practitioners to combine the two materials in composite structures. As mentioned earlier, no studies are found in the literature regarding the behaviour of composite conical tanks. The behaviour of composite slabs can provide an insight on that of composite tanks. An early study on the behaviour and strength of one-way composite slabs was performed by Daniels and Crisinel (1993a, b). They conducted experiments on a set of one way composite slabs and found that the behaviour and strength of the connection between the steel plate and concrete slab may be estimated using the pull-out and push-out tests. Good agreement was noted between the experimental results and those obtained from analytical solutions. Another two studies by Eldib et al. (2009) and Shanmugam et al. (2002) focused on the finite element modelling of two-way composite slabs using the commercial software, COSMOS and ABAQUS, respectively. They validated their models by carrying out experiments on a set of composite slabs, which consisted of steel plates connected through studs to concrete slabs. Their studies revealed that the composite slabs exhibit good flexure characteristics and highly ductile behaviour. When composite slabs are subjected to external loads, shear and peel forces develop in the connecting studs. The behaviour of studs under shear forces was studied by Choi et al.
The main differences between the aforementioned studies are the properties of studs and concrete slab. In these investigations, a set of push-out tests with different studs’ configurations was carried out and the resulting load-slip curves were obtained. The results of these tests showed that the load-slip curves of studs are linear up to 50% of the peak load and they are nonlinear beyond this value. The same tests showed that the behaviour of studs under shear forces is affected by the diameter, ultimate and yielding strengths of studs, as well as the strength of the concrete slab. Regarding the behaviour of studs under tension peel forces, Choi et al. (1999), Ožbolt et al. (1999), and Siwei et al. (2008) carried out a set of pull-out tests on studs with different diameters. Their tests showed that the failure of studs embedded in concrete under tension peel forces can be brittle if the failure mode is in the form of concrete cone pull-out. The failure can be ductile if it is governed by yielding or bond slippage of studs. Based on the aforementioned discussion on studs, the authors decided to include the nonlinear behaviour of studs under shear and peel forces in the numerical model developed in this study for assessment of the behaviour of composite conical tanks.

Finite shell element modelling is used in the current study to simulate composite conical tanks. Ahmad et al. (1970) introduced isoparametric shell elements which are based on the Mindlin plate bending theory (1951). However, the transverse shear stresses predicted by those isoparametric shell elements were found to be very large with arbitrary magnitudes. This behaviour is due to the presence of spurious shear modes in the elements’ formulation resulting mainly from using same order for in-plane interpolation of displacements and through thickness rotations. A consistent sub-parametric 13-node shell element that
overcomes these drawbacks was developed by Koziey and Mirza (1997). Shortly afterwards, El Damatty et al. (1997a) extended this element by considering the geometric nonlinearity and the nonlinear behaviour of steel. Recently, Siddique and El Damatty (2012) developed a 26-node contact element to model the contact surface between steel and Glass Fibre Reinforced Polymer (GFRP) plates. This element can be used in conjunction with the 13-node shell element to model multilayer shells, same as composite slabs and walls.

The first objective of the current paper is to develop and validate a comprehensive Finite Element Model for Composite conical tanks (CFEM) to study their nonlinear behaviour under hydrostatic pressure. The geometric nonlinear effect, as well as the nonlinear behaviour of steel, concrete, and studs are included in the CFEM formulation. The second objective is to assess the adequacy of a simplified approach for the analysis of composite conical tanks, which is referred to as the Equivalent Section Method (ESM). This approach is based on transforming the composite section to a single material section having an equivalent wall thickness and an equivalent Young’s modulus. The validity of this approach is assessed by comparing the load factor, displacements, stresses, and forces at failure obtained from this approach to their counterparts resulting from the developed CFEM.

The paper starts by discussing the behaviour of studs under both shear and peel forces. Then, the details of the CFEM are presented including the modelling of the studs, concrete wall, and the steel shell. This is followed by a validation of the developed CFEM through modelling two composite slabs that were reported in the literature. Then, details of the ESM, including its advantages and drawbacks, are discussed. Afterwards, the dimensions,
material properties, and studs’ configuration of a case study composite conical tank are reported. The displacements, stresses, and forces obtained from the CFEM are compared to those resulting from the ESM.

4.2. **Behaviour of studs under shear forces**

The nonlinear behaviour of studs under shear forces can be studied by conducting push-out experiments or simulated push-out tests using numerical models. Many push-out experiments and simulated tests were carried out by previous researchers, same as Choi et al. (1999), Shim et al. (2004), Nguyen and Kim (2009), as well as Xu and Sugiura (2012, 2013). The configuration of this test and the typical load-slip curve according to the Eurocode (1994) are shown in **Fig. 4-2**. In the push-out test, a group of studs are subjected to shear forces, then the load-slip curve is plotted by recording the applied shear force and the resulting slip.

![Fig. 4-2. Push-out test configuration and the typical load-slip curve (Eurocode 1994).](image)

From the push-out tests, conducted by the aforementioned researchers, it was observed that both the concrete strength and studs’ diameter have significant effects on the shear capacity per stud. Xu et al. (2012) reported that the ratio between the shear capacity per stud was
1:1:1.2 obtained from three identical push-out tests with a concrete strength of 30, 40, and 50 MPa, respectively. The same investigation revealed that the ratio between the shear capacity per stud was 1: 1.2: 1.4: 2 obtained from four identical push-out tests performed with studs having a diameter of 13, 16, 19, and 22 mm, respectively. The investigation by Xu et al. (2012) showed that the number of studs and spacing have insignificant influence on the shear capacity per stud. In other words, insignificant group effect was observed from the push-out tests. The difference in the shear capacity per stud did not exceed 10% obtained from two identical push-out tests with different number of studs and spacing. The first test was carried out using two studs with a spacing of 80 mm, while the second test was carried out using nine studs with a spacing of 50 mm. One can conclude that if the studs’ spacing is larger than 50 mm, it will have insignificant effect on the shear capacity per stud. Shim et al. (2004) reported that the failure of studs embedded in concrete may occur due to failure of the stud’s shank or the concrete slab. As shown in Fig. 4-3, the applied shear force may trigger four different modes of failure: shear failure in the stud’s shank, concrete cone failure, slab cracking failure, or slab shear failure. Their experimental investigation is performed using studs with diameters of 25, 27, and 30 mm. In the push-out tests with 25 mm-diameter studs, a shank failure was observed; however, in the push-out tests with 27 or 30 mm-diameter studs, shank failure, concrete cone, slab cracking, or slab shear failures occurred. Therefore, they concluded that the concrete cone, slab cracking, or slab shear failures are most likely to occur for large-diameter studs. While, a shank shear failure is most likely to occur for studs having a diameter smaller than 25 mm. The load-slip curves obtained from a database of studs with different diameters and concrete strengths are included in the CFEM to consider the nonlinear behaviour of studs.
This database is developed from the experimental and numerical work performed by Choi et al. (1999), Shim et al. (2004), Nguyen and Kim (2009), as well as Xu and Sugiura (2012, 2013). In this database, the studs’ diameter is varied from 13 mm to 30 mm, while the concrete strength is varied from 25 MPa to 60 MPa.

![Fig. 4-3. Failure modes for a stud under shear force by Shim et al. (2004)](image)

(a) Shank failure (b) Cone failure (c) Slab cracking failure (d) Slab shear failure.

### 4.3. Behaviour of studs under peel forces

The nonlinear behaviour of studs under tension peel forces has been extensively studied by Choi et al. (1999), Ožbolt et al. (1999), and Siwei et al. (2008). In their work, several pull-out experiments and simulated tests were carried out to study the behaviour of the studs under a pulling force and they plotted resulting load-peel curves. In Fig. 4-4, Choi et al. (1999) showed the typical load-peel curves in the cases of brittle cone failure or bond slippage failure. From this figure, it can be noted that the behaviour of the stud under tension peel is almost linear up to half of the pull-out load and then it starts to be nonlinear beyond this value. The load-peel curves obtained from a database of studs with different diameters and concrete strengths are included in the CFEM to consider the nonlinear behaviour of studs under peel forces. This database is developed from the experimental
and numerical work performed by Choi et al. (1999) and Tastani and Pantazopoulou (2009). In this database, the studs’ diameter is varied from 10 mm to 12 mm, while the concrete strength is varied from 28 MPa to 51 MPa. On the other hand, the behaviour of studs under compression axial force is included in the CFEM assuming full contact between the concrete wall and steel shell.

Fig. 4-4. Pull-out test configuration and the typical load-displacement curve by Choi et al. (1999).

### 4.4. Composite Finite Element Model (CFEM)

The behaviour of composite conical tanks under hydrostatic water pressure is studied by modelling the tank’s vessel using an in-house numerical model. The CFEM incorporates the Newton-Raphson method that was reported by Bathe (1982) to obtain the incremental displacements by solving a system of nonlinear equations:

\[
[K]^{(k-1)}[\Delta U] = \{R\}^t - \{f\}^t \tag{4-1}
\]

\[
[K]^{(k-1)} = [K_c]^{(k-1)} + [K_s]^{(k-1)} + [K_{st}]^{(k-1)} \tag{4-2}
\]

\[
\{f\}^t = \{f_c\}^t + \{f_s\}^t + \{f_{st}\}^t \tag{4-3}
\]

\[
\{R\}^t = \{R_c\}^t + \{R_s\}^t + \{R_f\}^t \tag{4-4}
\]
where \( [K]_t^{(k-1)} \), \([K_c]_t^{(k-1)}\), \([K_s]_t^{(k-1)}\), and \([K_{st}]_t^{(k-1)}\) are the stiffness matrix for the whole structure, concrete wall, steel shell, and studs, respectively, at a load increment \( t \) and iteration \((k-1)\). \( \{R\}_t - \{f\}_t \) is the unbalanced load vector. \( \{f_c\}_t \), \( \{f_s\}_t \), and \( \{f_{st}\}_t \) are the incremental nodal forces, corresponding to the stresses at a load increment \( t \), in the concrete wall, steel shell, and studs, respectively. \( \{\Delta U\} \) and \( \{R\}_t \) are the incremental global displacement vector and total applied load vector, respectively. \( \{R_f\}_t \), \( \{R_c\}_t \), and \( \{R_s\}_t \) are the load vector due to the applied external liquid pressure, own weight of the concrete wall and steel shell, respectively. The following subsections show a description of the elements that are used to model the concrete wall, steel shell, and studs. Afterwards, modelling of the nonlinear behaviour of studs and the equations used for calculating the studs’ forces are presented.

### 4.4.1. Modelling of concrete wall and steel shell

Both the inner concrete wall and outer steel shell of composite tanks are modelled using the 13-node consistent sub-parametric shell element. This element was first developed by Koziey and Mirza (1997) and then extended by El Damatty et al. (1997a) to consider the nonlinear behaviour of steel and the geometric nonlinearity. The element does not exhibit the locking phenomenon that is found in the isoparametric shell elements when they are used to model thin shells. This is achieved by using a consistent formulation that includes a cubic interpolation function for in-plane displacements and a quadratic interpolation function for through-thickness rotations. The consistent shell element consists of 13 nodes and has a triangular shape, as shown in Fig. 4-5. The element has one node at each corner of the triangle and three nodes at each side. The corner nodes have displacement and rotational degrees of freedom; the mid-side nodes only have rotational degrees of freedom,
whereas the one-third side nodes only have displacement degrees of freedom. The details of calculating both the stiffness matrix and the unbalanced load vector of the 13-node element are provided in El Damatty et al. (1997a). The same element was further extended by Elansary et al. (2015) to consider the nonlinear behaviour of concrete by including a nonlinear concrete model that was developed by Pietruszczak et al. (1988) and Jaing (1988).

The 13-node shell element by Koziey and Mirza (1997).

Fig. 4-5. The 13-node shell element by Koziey and Mirza (1997).

The concrete wall in the composite tank is assumed to fail when the effective stresses in concrete, $\bar{\sigma}$, reach the failure surface. The equation of the failure surface, $F$, is proposed by Pietruszczak et al. (1988) and Jaing (1988) as:

$$F = \bar{\sigma} - g(\theta) \bar{\sigma}_c = 0$$  \hspace{1cm} (4-5)

$$\bar{\sigma}_c = \frac{-a_1 + \sqrt{a_1^2 + 4a_2(a_3 + \frac{\psi}{\bar{\sigma}_c})}}{2a_2} f'_c$$  \hspace{1cm} (4-6)

where $g(\theta)$ is a function specifying the shape of the deviatoric plane. The constants $a_1$, $a_2$, and $a_3$ define the failure surface of the concrete model. $\bar{\sigma}_c$ is the ultimate effective stresses in concrete that depend on the material constants and confining pressure, $I$. The
steel shell in the composite tank is assumed to fail when the effective stresses in the steel shell reach the ultimate steel effective stresses, according to Von Mises failure criterion.

4.4.2. Modelling of studs

A 26-node contact element is used to model the studs between the concrete wall and steel shell such that the studs are smeared along the surface of the contact elements. Using the smearing approach for the studs allows to change easily the number of studs in the finite element mesh without the need to model each stud separately. The smearing approach also allows to use a mesh with different size elements, which is required to capture the stress concentration locations. The 26-node contact element was first developed by Siddique and El Damatty (2012) to model the interface between steel and GFRP plates connected with an adhesive. The element consists of two 13-node triangular elements connected by springs. Each node in the first element is connected to a corresponding node in the second element by three springs in the three local directions x’, y’, and z’, as shown in Fig. 4-6. Two springs are parallel to the plane of the element and transfer shear forces. These springs are oriented in the direction of the local axes x’ and y’. Meanwhile, the third spring, which is oriented in the direction of the local axis z’, is perpendicular to the plane of the element and transfers peel forces. In the present study, the same element is extended to model the studs between the curved concrete wall and steel shell, as shown in Fig. 4-6. The nodes of the 13-node element simulating the concrete wall are connected to the nodes of one face of the contact element. Meanwhile, the nodes of the 13-node element simulating the steel shell are connected to the nodes of the other face of the contact element.

The incremental global displacements of the concrete wall and steel shell \( (\Delta u^c, \Delta v^c, \Delta w^c, \Delta u^s, \Delta v^s, \Delta w^s) \) can be written in terms of the nodal degrees of freedom as:
\[
\begin{align*}
\begin{bmatrix}
\Delta u^c \\
\Delta v^c \\
\Delta w^c
\end{bmatrix} &= \sum_{n=1}^{10} \bar{N}_n \begin{bmatrix}
\Delta U^c \\
\Delta V^c \\
\Delta W^c
\end{bmatrix} \\
&= \sum_{n=1}^{10} \bar{N}_n \begin{bmatrix}
\Delta U^s \\
\Delta V^s \\
\Delta W^s
\end{bmatrix}
\end{align*}
\] (4-7)

Fig. 4-6. The 26-node contact element connecting the concrete wall and steel shell.

\[
\begin{align*}
\begin{bmatrix}
\Delta u^s \\
\Delta v^s \\
\Delta w^s
\end{bmatrix} &= \sum_{n=1}^{10} \bar{N}_n \begin{bmatrix}
\Delta U^s \\
\Delta V^s \\
\Delta W^s
\end{bmatrix} \\
&= \sum_{n=1}^{10} \bar{N}_n \begin{bmatrix}
\Delta U^s \\
\Delta V^s \\
\Delta W^s
\end{bmatrix}
\end{align*}
\] (4-8)

where \( \bar{N}_n \) is the cubic shape function at node \( n \). The superscripts \( c \) and \( s \) refer to the nodes connected to the concrete wall and steel shell, respectively. It is worth mentioning that the mid-side nodes do not have displacement degrees of freedom, therefore the summation is done for the ten nodes having active displacement degrees of freedom. The transformation matrix \( [\theta] \) of the relative direction cosines between the local and global axes is given by:

\[
[\theta] = \begin{bmatrix}
l_1 & m_1 & n_1 \\
l_2 & m_2 & n_2 \\
l_3 & m_3 & n_3
\end{bmatrix}
\] (4-9)
Therefore, the incremental local displacements can be written in terms of the nodal incremental degrees of freedom as:

\[
\begin{align*}
&\left\{ \Delta \bar{u}^c \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \Delta U_n^c \right\} \\
&\left\{ \Delta \bar{v}^c \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \Delta V_n^c \right\} \\
&\left\{ \Delta \bar{w}^c \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \Delta W_n^c \right\}
\end{align*}
\] (4-10)

\[
\begin{align*}
&\left\{ \Delta \bar{u}^s \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \Delta U_n^s \right\} \\
&\left\{ \Delta \bar{v}^s \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \Delta V_n^s \right\} \\
&\left\{ \Delta \bar{w}^s \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \Delta W_n^s \right\}
\end{align*}
\] (4-11)

The initial stiffness matrix of the contact element, derived by Siddique and El Damatty (2012), is adopted in the CFEM to analyze composite conical tanks. The principle of virtual work is used to obtain the incremental nodal forces in studs, \( \{f_{st} \}^t \), corresponding to the stresses at a load increment \( t \). The virtual work exerted by the forces transferred by springs of the contact element, \( \delta W \), can be obtained using the following equations:

\[
\delta W = \int_{A_e} \left[ \sigma_{x'}(\delta \Delta \bar{u}^c - \delta \Delta \bar{u}^s) + \sigma_{y'}(\delta \Delta \bar{v}^c - \delta \Delta \bar{v}^s) + \sigma_{z'}(\delta \Delta \bar{w}^c - \delta \Delta \bar{w}^s) \right] dA_e \] (4-12)

\[
\begin{align*}
&\left\{ \delta \Delta \bar{u}^c \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \delta \Delta U_n^c \right\} \\
&\left\{ \delta \Delta \bar{v}^c \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \delta \Delta V_n^c \right\} \\
&\left\{ \delta \Delta \bar{w}^c \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \delta \Delta W_n^c \right\}
\end{align*}
\] (4-13)

\[
\begin{align*}
&\left\{ \delta \Delta \bar{u}^s \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \delta \Delta U_n^s \right\} \\
&\left\{ \delta \Delta \bar{v}^s \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \delta \Delta V_n^s \right\} \\
&\left\{ \delta \Delta \bar{w}^s \right\} = \sum_{n=1}^{10} \overline{N}_n [\theta] \left\{ \delta \Delta W_n^s \right\}
\end{align*}
\] (4-14)

The components of the incremental nodal forces in studs in the global directions can be obtained by differentiating the virtual work with respect to the global degrees of freedom, as shown in the following equations:

\[
f_{x, n} = \int_{A_e} \left[ \overline{N}_n \sigma_{x'} l_1 + \overline{N}_n \sigma_{y'} l_2 + \overline{N}_n \sigma_{z'} l_3 \right] dA_e \] (4-15)

\[
f_{y, n} = \int_{A_e} \left[ \overline{N}_n \sigma_{x'} m_1 + \overline{N}_n \sigma_{y'} m_2 + \overline{N}_n \sigma_{z'} m_3 \right] dA_e \] (4-16)
\[
f_{z,n} = \int_{A_e} \left[ \bar{N}_n \sigma_{x'} n_1 + \bar{N}_n \sigma_{y'} n_2 + \bar{N}_n \sigma_z n_3 \right] dA_e \tag{4-17}
\]

The shear stresses \(\sigma_{x'}\) and \(\sigma_{y'}\) are developed in the contact element due to the relative displacements occurring between the two surfaces of the contact element. These stresses are obtained from the nonlinear load-slip curves by smearing the stud’s stiffness over the area served by each stud. The adequate load-slip curve is obtained from the push-out test that has studs’ configuration and concrete strength similar to those used for the studied composite tank. The peel stresses, \(\sigma_z\), are obtained from the nonlinear load-peel curve from the pull-out test if the contact element is subjected to tension forces. However, if the contact element is subjected to compression forces a full contact is assumed between the concrete wall and steel shell.

The calculation procedure of the incremental nodal forces in studs from the nodal displacements is shown in the flow chart in Fig. 4-7. The same chart shows the incorporation of the nodal forces in studs into the main program, where the global stiffness, load vectors, and nodal displacements are evaluated. As presented in the flow chart, the incremental local displacements at the concrete wall \(\Delta \bar{U}_c\), \(\Delta \bar{V}_c\), and \(\Delta \bar{W}_c\) and those at the steel shell \(\Delta \bar{U}_s\), \(\Delta \bar{V}_s\), and \(\Delta \bar{W}_s\) are first obtained from the main program. Second, the relative displacements parallel to the contact element, \((\Delta \bar{U}_c - \Delta \bar{U}_s)\) and \((\Delta \bar{V}_c - \Delta \bar{V}_s)\) are calculated in the \(x'\) and \(y'\) directions, respectively. The relative displacements perpendicular to the contact element, \((\Delta \bar{W}_c - \Delta \bar{W}_s)\) are calculated in the \(z'\) direction. Third, the relative displacements \((\Delta \bar{U}_s - \Delta \bar{U}_c)\) and \((\Delta \bar{V}_s - \Delta \bar{V}_c)\) are used to obtain the shear stresses \(\sigma_{x'}\) and \(\sigma_{y'}\) in the contact elements using the adequate nonlinear load-slip curve. The shear stresses are obtained by dividing the load corresponding to the relative displacement by the
served area by one stud. If the relative displacement \((\Delta W^c - \Delta W^s)\) leads to tension force, the adequate nonlinear load-peel curve is used to obtain the peel stresses, \(\sigma_{z'}\). If the relative displacement \((\Delta W^c - \Delta W^s)\) leads to compression force, a full contact between the concrete wall and steel shell is assumed. Fourth, the stresses \(\sigma_{x'}, \sigma_{y'}, \text{ and } \sigma_{z'}\) are integrated over the surface of the contact elements to yield the incremental nodal forces in studs using Equations (4-15), (4-16), and (4-17). Finally, the system of equations for the whole structure is solved using Equation (4-1) to find the new incremental displacements.

The total shear forces at each contact element, in the meridional and hoop directions, \(V_{elm,x}\) and \(V_{elm,y}\), can be obtained by performing numerical integration of the stresses along the surface of the contact elements:

\[
V_{elm,x} = \int_{A_e} \sigma_{x'} \, dA_e 
\]  
(4-18)

\[
V_{elm,y} = \int_{A_e} \sigma_{y'} \, dA_e 
\]  
(4-19)

where \(\sigma_{x'}\) and \(\sigma_{y'}\) are the shear stresses in the hoop and meridional directions, respectively. Similarly, the total peel force transferred by a contact element, \(P_{elm}\), is calculated using the following equation:

\[
P_{elm} = \int_{A_e} \sigma_{z'} \, dA_e 
\]  
(4-20)

where \(\sigma_{z'}\) is the peel stresses in the \(z'\) local direction. The smearing approach is adopted, assuming no variation of the shear and peel forces along the contact element, to obtain the shear and peel forces per stud using the following equations:
Fig. 4-7. Flow chart for calculation of incremental nodal forces in studs.

\[ V_{sd, x} = \frac{V_{elm, x}}{N_s} \]  \hspace{1cm} (4-21)

\[ V_{sd, y} = \frac{V_{elm, y}}{N_s} \]  \hspace{1cm} (4-22)
\[ P_{sd} = \frac{P_{elm.}}{N_s} \]  
\[ N_s = N_{s,T} \frac{A_e}{A_T} \]

where \( V_{sd, x}, V_{sd, y}, \) and \( P_{sd} \) are the shear in the hoop and meridional directions and peel forces in one stud. \( N_s \) is the number of studs connected to a certain contact element, \( N_{s,T} \) is the total number of studs for the tank, and \( A_T \) is the total surface area of the tank’s vessel.

It is worth mentioning that the type of failure of each stud’s configuration is implicitly included in the load-slip curve. Therefore, the failure criterion of studs is implicitly considered because the load-slip curves are included in the CFEM.

### 4.5. CFEM validation

The CFEM is validated by modelling two simply supported composite slabs under a concentrated load that were tested and modelled by Shanmugam et al. (2002). The plan dimensions of the slabs are 1500 mm × 1500 mm with effective spans of 1400 mm × 1400 mm. As shown in Fig. 4-8, each slab consists of two steel plates connected to the upper and lower faces of a concrete slab, respectively. The steel plates are attached to the concrete slab using shear studs having a diameter of 13 mm. Shanmugam et al. (2002) modelled the tested slabs using the commercial finite element software, ABAQUS. They used four-node shell elements to model the steel plates and eight-node solid elements to model the concrete slab. Shanmugam et al. (2002) reported that the shear studs between the concrete slab and steel plates were modelled using a set of parallel springs.

The properties of the concrete slabs and steel plates for the double skin composite slabs, DSCS4 and DSCS5, are shown in Table 4-1. In the current study, only one quarter of each slab is modelled in the CFEM due to the double symmetry in geometry and loading. A
mesh consisting of 32 triangular elements for each steel plate and 32 triangular shell elements for the concrete slab is used, as shown in Fig. 4-9. In addition, 64 contact elements are used to model the connecting studs between the concrete slab and steel plates. As depicted in Fig. 4-9, simply supported boundary conditions, SSBCs, are placed at the two adjacent edges of the concrete slab and steel plates. In the other two edges, boundary conditions accounting for symmetry, SBCs, are assigned to the concrete slab and steel plates.

![Diagram showing typical cross section of the double skin composite slab by Shanmugam et al. (2002).](Image)

**Fig. 4-8.** Typical cross section of the double skin composite slab by Shanmugam et al. (2002).

<table>
<thead>
<tr>
<th>Property</th>
<th>DSCS4</th>
<th>DSCS5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concrete slab</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Young’s modulus, $E_c$ (MPa)</td>
<td>22,000</td>
<td>26,000</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_c$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Concrete strength, $f_{c'}$, (MPa)</td>
<td>26</td>
<td>34</td>
</tr>
<tr>
<td><strong>Steel plate</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>Upper 4.6</td>
<td>Upper 4.6</td>
</tr>
<tr>
<td></td>
<td>Lower 4.6</td>
<td>Lower 4.6</td>
</tr>
<tr>
<td>Young’s modulus, $E_s$ (MPa)</td>
<td>200,000</td>
<td>200,000</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_s$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Yielding stress, $f_y$ (MPa)</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>Ultimate stress, $f_u$ (MPa)</td>
<td>600</td>
<td>600</td>
</tr>
</tbody>
</table>

**Table 4-1.** Properties of concrete and steel for composite slabs DSCS4 and DSCS5.

**Fig. 4-10** shows the load-deflection curves obtained from the numerical model using ABAQUS software and those resulting from the experiment by Shanmugam et al. (2002).
The same figure also shows the load-deflection curves obtained from the CFEM. A good agreement is observed between the load-deflection curve obtained from the CFEM and the experiment for the two composite slabs. For slab DSCS5, the CFEM predicts the load-deflection curve more accurately than that predicted by ABAQUS. Fig. 4-10 shows that the CFEM accurately predicts the initial structure stiffness in the linear range as well as the ultimate load carried by each slab. The failure predicted by the CFEM was due to buckling at the top steel plate, same as that predicted by Shanmugam et al. (2002).

Fig. 4-9. Finite element mesh for the double skin composite slabs DSCS4 and DSCS5.

Fig. 4-10. Load-deflection curves for DSCS4 and DSCS5 slabs from an experiment and ABAQUS software by (Shanmugam et al. 2002), as well as from the CFEM.
4.6. **Equivalent Section Method (ESM)**

This section shows an approximate approach for the design of composite tanks where both the concrete wall and steel shell are replaced by an equivalent single wall. The Young’s modulus and the thickness of the equivalent wall are calculated, according to the American Standards for the composite steel floor deck (2006), using the following equations:

\[
E_{eq}t_{eq} = E_s t_s + E_c t_c \tag{4-25}
\]

\[
I_{eq} = \frac{I_u + I_{cr}}{2} \tag{4-26}
\]

where \(E_{eq}\), \(E_s\), and \(E_c\) are Young’s modulus of the equivalent wall, steel shell, and concrete wall, respectively. \(t_{eq}\), \(t_s\), and \(t_c\) are thickness of the equivalent wall, steel shell, and concrete wall, respectively. \(I_{eq}\), \(I_u\), and \(I_{cr}\) are moment of inertia of the equivalent wall, the section with uncracked concrete, and the section with cracked concrete, respectively.

Equations (4-25) and (4-26) yield an equivalent section having axial and bending stiffness equal to those of the original composite section. Calculation details of \(I_u\) and \(I_{cr}\) can be found in the American Standards for the composite steel floor deck (2006).

In the ESM, the properties of the equivalent wall are utilized to analyze the tank’s vessel using a Linear Finite Element Model (LFEM). This model can predict the strain at the outer faces of the equivalent wall assuming compatibility between the concrete wall and steel shell. The stresses in the concrete wall and steel shell can be obtained as:

\[
\sigma_c = \varepsilon_{ESM} \times E_c \tag{4-27}
\]

\[
\sigma_s = \varepsilon_{ESM} \times E_s \tag{4-28}
\]

where \(\sigma_c\) and \(\sigma_s\) are the stresses in the concrete wall and steel shell, respectively, \(E_c\) and \(E_s\) are the Young’s modulus of the concrete and steel, respectively, \(\varepsilon_{ESM}\) the strain obtained from the LFEM for the equivalent wall. In ESM, the failure criteria is adopted from the
ACI 318-08 (2008) design code. The composite tank is assumed to fail when the strain in the concrete wall reaches a value of 0.003 or when the stress in the steel shell reaches a value of 0.85 $f_y$.

The ESM has various advantages: First, it is simple because only one mesh is used to model both the concrete wall and steel shell in the LFEM. Second, the complications of modelling of studs are excluded. Third, significant running time can be saved due to limiting the number of degrees of freedom to one mesh instead of two meshes. However, this method has a number of drawbacks:

- a. No researchers reported using this method in the analysis of composite tanks.
- b. Real dimensions and material properties of the concrete wall and steel shell are not directly utilized.
- c. Forces in the studs cannot be obtained because the studs are not modelled.
- d. The nonlinear behaviour of studs is not considered.
- e. Calculating the forces and stresses in the concrete wall and steel shell separately based on the relative stiffness may not be as accurate as the CFEM, which models the concrete wall and steel shell using two separate meshes.

### 4.7. Boundary conditions

A set of boundary conditions for displacements and rotations are applied at the edges of the two meshes for the concrete wall and steel shell. First, the vessel’s top edge is assumed to be free. This assumption was justified by El Damatty et al. (1997b) where they showed that the radial displacements at this location are negligible under hydrostatic pressure. A pin connection is assumed at the intersection between the vessel’s bottom edge and the concrete shaft allowing local rotations of the walls to occur at this location. The real
boundary conditions will be between pin and fix conditions. The assumption of a pin connection is on the conservative side. As shown in Fig. 4-11, two meridional lines of symmetry at each mesh are applied due to the symmetry in geometry and loading. It is worth mentioning that the concrete shaft is not included in the CFEM and LFEM because it has an insignificant effect on the behaviour of the vessel under hydrostatic loading, as reported by Sweedan and El Damatty (2009). Therefore, the vertical displacements are prevented at the vessel’s base assuming infinite axial stiffness of the concrete shaft.

4.8. Loading

Two loads are considered to act on the tank’s vessel; first, the vessel’s own weight that is applied at the first increment, second, the hydrostatic liquid pressure load that is applied incrementally. The water level is kept constant during the whole loading process. The load from hydrostatic pressure is multiplied incrementally by a load factor $p$ which is increased gradually until the structure fails.

4.9. Finite element mesh

The composite conical tanks under hydrostatic pressure are axisymmetric in geometry and loading, therefore only one quarter of the tank’s vessel is modelled. The vessel is modelled using one mesh for the mid-surface of the concrete wall and a similar mesh for that of the steel shell, as shown in Fig. 4-11. Each mesh consists of 256 triangular shell elements that are connected together using 256 contact elements. The elements in the meshes are chosen to be fine near the tank’s base so that the concentration of stresses at this location can be captured.

The aforementioned mesh is chosen after conducting a sensitivity analysis by modelling a composite conical tank using three different meshes. The tank’s bottom radius, height, and
inclination angle are 4 m, 9 m, and 51.6°, respectively. The three meshes are generated such that the number of divisions is varied from 4 to 16 in each of the ring and meridional directions. Fig. 4-12 (a) shows the load-deflection curves for a point at an elevation of 5 m, where the maximum deflection occurs along the tank’s height, obtained from the three meshes. The 4 x 8 mesh overestimates the load factor at failure, while both the 8 x 8 and 8 x 16 converge to the load factor at failure of p = 2.5.

Fig. 4-11. Mesh of the concrete wall or steel shell and the meridional lines of symmetry.

Fig. 4-12 (b) shows the hoop stress distribution in the steel shell along the vessel’s height for each mesh. One can observe that the differences between the hoop stresses from the 8 x 8 and 8 x 16 meshes do not exceed 5%. Although the 8 x 8 mesh is enough to obtain an accurate solution for displacements and stresses, the 8 x 16 mesh is selected for the analysis of the studied tank. The 8 x 16 mesh is chosen in order to evaluate the displacements and
stresses at large number of points along the tank’s height. This is done in order to accurately obtain the locations of maximum displacement and stress along the vessel’s height.

![Load-deflection curves](image1.png) ![Hoop stress distributions in the steel shell](image2.png)

(a) Load-deflection curves  
(b) Hoop stress distributions in the steel shell

**Fig. 4-12.** Results from the mesh sensitivity analysis for a composite conical tank.

### 4.10. Case study

One practical example of composite conical tanks, which has been recently constructed, is considered in this paper as a case study. The tank has a 15 m-high reinforced concrete shaft and a truncated pure conical vessel, as shown in **Fig. 4-13**. The tank has a bottom radius, height, and inclination angle of 4 m, 9 m, and 51.6˚, respectively. The tank’s vessel consists of an external steel shell and internal reinforced concrete wall, as shown in the cross section plan view in **Fig. 4-14**. The concrete wall has a thickness that varies along the vessel’s height ranging from 125 mm at the bottom to 62 mm at the top, while the steel shell has a constant thickness of 8 mm constant along the vessel’s height. One mesh of reinforcement exists at the concrete wall such that the bars’ diameter is 10 mm with spacing of 200 mm and 300 mm in the meridional and hoop directions, respectively. The concrete strength and
Young’s modulus are 24.5 MPa and 23264 MPa, respectively. The steel Yielding and Ultimate stresses, and Young’s modulus are 248 MPa, 400 MPa, and 200000 MPa, respectively. The steel shell and concrete wall are connected using studs with a diameter and spacing of 13 mm and 400 mm, respectively.

Fig. 4-13. Layout dimensions of the studied elevated composite conical tank.

Fig. 4-14. Cross section plan view in the vessel of the composite conical tank.
The nonlinear load-slip and load-peel curves, corresponding to the studs’ configuration in the case study tank, are included in the database for studs in the CFEM. The load-slip curve is obtained from the push-out test by Xu and Sugiura (2013), as shown in Fig. 4-15 (a). The failure mode in this test was a stud shear fracture with bending deformations and nearby concrete crush at a push-out load of 66 kN. The load-peel curve is obtained from the pull-out test by Choi et al. (1999), as shown in Fig. 4-15 (b). The failure mode in this test was due to the pull-out of the stud.

![Load-slip curve and Pull-out load-displacement curve](image)

**Fig. 4-15.** Nonlinear load-displacement curves for studs under shear and peel forces.

### 4.11. Results

This section shows the results obtained from the analysis of the considered composite conical tank under hydrostatic loading using both the developed Composite Finite Element Model (CFEM) and the Equivalent Section Method (ESM). The load-deflection curves and displacement distributions along the vessel’s height are plotted. Afterwards, the stress and force distributions in the concrete wall and steel shell are presented. Finally, the meridional shear force distribution in studs along the vessel’s height is presented.
4.11.1. Load-deflection curve and deformed shape

The load-radial displacement curve for a point at an elevation of 5 m, where the maximum displacement occurs along the vessel’s height, obtained from both the CFEM and ESM are shown in Fig. 4-16. In this figure, the vertical axis shows the load factor, p, while the horizontal axis shows the radial displacement at the considered point. One can note that the stiffness predicted by the ESM is constant from the start of loading to the failure, which is predicted by the ESM to occur at a load factor of p = 1.9. The same stiffness value is predicted by the CFEM from the start of loading until a load factor of p = 0.6. This is followed by a degradation in the stiffness up to the tank’s failure, which occurs at p = 2.5. This significant degradation in stiffness occurs due to including the nonlinear behaviours of concrete, steel, and studs in the CFEM. The load-deflection curve obtained from the ESM is linear because the material nonlinearity and the nonlinear behaviour of the studs are not included in this method. A failure in the steel shell is predicted by both the ESM and the CFEM analyses. In the CFEM, the failure occurs when the effective stress in the steel shell reaches the failure surface. However, the failure in the ESM occurs when the axial stress in the steel shell reaches 0.85\( f_y \).

Fig. 4-16 shows that the CFEM predicts a significantly larger displacement and moderately larger load capacity compared to the ESM. The ratios between the displacements and load factors at failure for the CFEM to ESM are 2.3 and 1.3, respectively. It can be concluded that the ESM underestimates both the displacement and load carrying capacity.

Fig. 4-17 shows the transverse displacement distributions at failure along the vessel’s height for both the concrete wall and steel shell obtained from the CFEM and ESM.
The CFEM has the ability to predict the displacements for the concrete wall and steel shell separately but the ESM predicts displacements representing the two walls together. This occurs because the CFEM utilizes one mesh for each wall, whereas the ESM uses only one mesh for the two walls.

![Load-deflection curve for a node at an elevation of 5 m.](image)

**Fig. 4-16.** Load-deflection curve for a node at an elevation of 5 m.

The transverse displacements can be obtained as a result of the resolution of both the radial and vertical displacements in the direction perpendicular to the shell. The transverse displacements are found to be nonzero at the vessel’s top edge due to the assumed free boundary conditions. The radial displacements vanish at this location but the downward vertical displacements are significant, which results in the nonzero transverse displacement values. **Fig. 4-17** shows that the maximum displacements obtained from both the CFEM and ESM occur at an elevation of 5 m. The ratio between the maximum transverse displacements from the CFEM to ESM at failure is 3.1. **Fig. 4-17** (a) depicts that the relative radial displacements between the concrete wall and steel shell are not significant which indicates a full contact between the two walls as a result of the strong peel stiffness of the studs.
4.11.2. Stress distributions

The stresses in the meridional direction at the two faces of the concrete wall at failure obtained from the CFEM and ESM are plotted in Fig. 4-18. One can observe that the ESM significantly underestimates the meridional stresses at the inner and outer faces of the concrete wall. The meridional stresses at the inner and outer faces obtained from the ESM are less than those resulting from the CFEM by 25% and 28%, respectively. Fig. 4-18 shows that both approaches show a clear effect of local bending at the bottom of the concrete wall. From the same figure, it can be observed that both the inner and outer faces of the concrete wall are subjected to compressive stresses along the whole height of the vessel. Regarding the stresses in the steel shell, Figs. 4-19 and 4-20 show the meridional and hoop stresses at failure obtained from the CFEM and ESM. The hoop and meridional stress distributions through the thickness of the steel shell are constant, as shown in Figs. 4-19 (a) and 4-20 (a).
The stresses at the inner and outer faces of the steel shell are identical because its thickness is significantly smaller than the concrete wall’s thickness. This also the main cause of not having a significant local bending effects in the steel shell in the meridional direction. The ratio between the meridional stresses in the steel shell obtained from the CFEM and ESM is 1.3. Figs. 4-18 (a) and 4-19 (a) show that the ratio between the maximum meridional stresses in the outer face of the steel shell to those at the concrete wall is 9. It is obvious from Fig. 4-20 that the tensile hoop stresses in the steel shell do not exceed the tensile strength of steel. The ESM significantly underestimates the hoop stresses in the steel shell such that the ratio between the maximum hoop stress obtained from the ESM to that obtained from the CFEM is 0.5. The maximum hoop stress resulting from both the CFEM and ESM occurs at an elevation of 1 m.
Fig. 4-19. Meridional stress distributions in the steel shell at failure along the vessel’s height.

Fig. 4-20. Hoop stress distributions in the steel shell at failure along the vessel’s height.

4.11.3. Force distributions

The distributions of hoop and meridional axial forces in the concrete wall and steel shell at failure obtained from the CFEM and ESM are shown in Figs. 4-21 and 4-22, respectively.
Both the CFEM and ESM predict that the hoop and meridional axial forces along the whole height of the vessel are tension and compression, respectively. The CFEM has the ability to directly evaluate the axial forces in the concrete wall and steel shell separately because each of these walls is modelled using one separate mesh. However, the ESM predicts approximate values for the axial forces in the concrete wall and steel shell separately based on the axial stiffness for each wall. From Fig. 4-21 (a), one can find that the CFEM predicts hoop axial forces in the concrete wall smaller than their counterparts in the steel shell. The ratio between the maximum hoop tensile force in the concrete wall and steel shell is 0.4. However, the ESM predicts hoop axial forces in the concrete wall larger than their counterparts in the steel shell, as shown in Fig. 4-21 (b). This occurs because the CFEM accounts for the concrete cracking which is not considered in the ESM. Both the CFEM and ESM predict that the maximum hoop axial forces in the concrete wall and steel shell occur between elevations of 2 and 4 m.

From Fig. 4-22, it can be observed that both the CFEM and ESM predict meridional axial forces in the concrete wall larger than their counterparts in the steel shell. The ratios between the maximum meridional axial forces in the concrete wall to the steel shell are 1.5 and 1.7 obtained from the CFEM and ESM, respectively. The behaviour of both the concrete wall and steel shell in the meridional direction predicted by both the CFEM and ESM is approximately similar because no cracking exists in the concrete wall in this direction. The maximum meridional axial forces in the concrete wall and steel shell from both the CFEM and ESM occur at the vessel’s base, as shown in Fig. 4-22. The same figure shows that the CFEM predicts meridional axial forces in the concrete wall and steel shell larger than those predicted by the ESM.
Fig. 4-21. Hoop axial force distributions along the vessel’s height at failure.

Fig. 4-22. Meridional axial force distributions along the vessel’s height at failure.

The distributions of the meridional bending moment in the concrete wall along the vessel’s height at failure from the CFEM and ESM are shown in Fig. 4-23. The CFEM can directly evaluate the meridional moment in the concrete wall and steel shell separately because each of them is modelled using one separate mesh. However, the ESM predicts
approximate values for the meridional moment in the concrete wall and steel shell separately based on the bending stiffness of each wall. Both the CFEM and ESM predict meridional bending moment in the steel shell significantly smaller than that in the concrete wall. The ratio between the maximum bending moment in the steel shell to that in concrete wall does not exceed 0.008. This occurs due to the large difference between the thicknesses of the two walls. The meridional bending moment in the steel shell is negligible when compared with the meridional moment in the concrete wall. **Fig. 4-23** shows that the maximum local bending moments obtained from both the CFEM and ESM occur at an elevation of 0.5 m. The meridional bending moments in both the concrete wall and steel shell decrease significantly above 0.2 of the vessel’s height.

![Graph of meridional bending moment distribution](image1.png)

**Fig. 4-23.** Meridional bending moment distributions in concrete along the vessel’s height at failure.

### 4.11.4. Meridional shear force distribution in studs

**Fig. 4-24** shows the discrete distributions of the Studs’ Meridional Shear Force (SMSF) along the vessel’s height obtained from the CFEM. The distribution of the SMSF for the
case study tank, where the studs’ spacing is 0.4 m, is plotted in Fig. 4-24 (a). Meanwhile, Figs. 4-24 (b) and (c) show the SMSF distributions if the studs’ spacing, for the same case study tank, is increased by a factor of 2 and 3, respectively. The SMSF cannot be evaluated using the ESM, where both the concrete wall and steel shell are modelled using one mesh without modelling the studs. The plotted forces in Fig. 4-24 represent the average SMSF at each contact element, which are calculated using Equation (4-22). It is observed that the maximum SMSF at failure occurs at the lower half of the vessel. One can also observe that the SMSF are approximately constant at the lower half of the vessel’s height, whereas their values decrease beyond this region. Fig. 4-24 (a) shows that the SMSF does not reverse their direction, which means that the slip does not change its direction along the vessel’s height. However, the SMSF are found to change their direction, for the same case study tank, when the studs’ spacing is increased by a factor of 2 and 3, as shown in Figs. 4-24 (b) and (c), respectively. The maximum SMSF is 4.7 kN, 23.5 kN, and 62.3 kN corresponding to studs’ spacing of 0.4 m, 0.8 m, and 1.2 m, respectively. In view of Fig. 4-15 (a), one can observe that the SMSF are within the linear range when the studs’ spacing is 0.4 m or 0.8 m, but they are within the nonlinear range when the studs’ spacing is 1.2 m. When the studs’ spacing is 0.4 m or 0.8 m, no failure occurs in the studs because the SMSF in all of the studs do not exceed the maximum shear force, as shown in Figs. 4-24 (a) and (b). The failure in these cases occurs a load factor of $p = 2.5$, when the effective stresses in the steel shell reach the failure surface. However the maximum SMSF exceeds the maximum shear force, when the studs’ spacing is increased to 1.2 m, and the tank fails at a load factor of $p = 2.3$. Therefore, one can concluded that the studs’ spacing can be
significantly increased, or the number of studs can be decreased, without significant reduction in the load carrying capacity of the case study tank.

(a) Stud’s spacing =0.4 m  (b) Stud’s spacing =0.8 m  (c) Stud’s spacing =1.2 m

Fig. 4-24. Meridional shear force distribution in a set of studs along the vessel’s height at failure.

4.12. Summary and conclusions

A Composite Finite Element Model (CFEM) is developed in the present study to analyze liquid filled tanks constructed of steel/concrete conical vessels under hydrostatic water pressure. Two separate meshes of 13-node shell elements are used to model the concrete wall and steel shell, whereas one mesh of 26-node contact elements is utilized to model the connecting studs using a smearing approach. The nonlinear behaviours of concrete, steel, and studs are included in the CFEM. The model is validated by modelling two composite slabs that were reported in the literature. An Equivalent Section Method (ESM), as a simplified approach for analysis of composite tanks, is introduced. In this method, a virtual cross section with an equivalent thickness and equivalent Young’s modulus replaces the concrete and steel walls. One case study composite conical tank, which has been recently
constructed, is analyzed using both the developed CFEM and ESM. The following conclusions can be withdrawn from the current study:

1. Both the CFEM and ESM can predict the stresses and forces at the concrete wall and steel shell, separately.

2. The displacements obtained from the CFEM are significantly larger than those resulting from the ESM because the CFEM accounts for the concrete cracking.

3. The predicted failure by both the CFEM and ESM is due to the failure in the steel shell.

4. Insignificant relative transverse displacements are observed between the concrete wall and steel shell due to the full contact existed between two walls.

5. A ratio of 3.1 is observed between the maximum transverse displacements obtained from the CFEM to those resulting from the ESM.

6. The locations of maximum displacements, stresses, and forces along the vessel’s height obtained from both the CFEM and ESM are identical.

7. The meridional stresses at the inner and outer faces of the concrete wall obtained from the ESM are smaller than their counterparts resulting from the CFEM by 25% and 28%, respectively.

8. The CFEM predicts a constant distributions of both the hoop and meridional stresses through the thicknesses of the steel shell.

9. In the steel shell, the ratios between the maximum meridional and hoop stresses resulting from the CFEM to those obtained from the ESM are 1.3 and 0.5, respectively.
10. In both the concrete wall and steel shell, the maximum meridional axial force and bending moment occur at the vessel’s base and at an elevation of 0.5 m, respectively.

11. The meridional bending moment and meridional stresses in the concrete wall are significantly greater than their counterparts in the steel shell.

12. Both the CFEM and ESM show that the meridional bending moments in both the concrete wall and steel shell significantly decrease above 0.2 of the vessel’s height.

13. The meridional shear forces in studs are approximately constant at the lower half of the vessel’s height where the maximum values exist.

14. The meridional shear forces in studs are within the linear range in the load-slip curve for studs with a spacing of 0.4 m or 0.8 m, while the forces are within the nonlinear range when the spacing is increased to 1.2 m.

15. For the case study tank, the studs’ spacing can be increased by a factor of 3 without a significant reduction in the load carrying capacity.

16. Based on the comparison with the CFEM, the ESM is not adequate for the analysis of composite conical tanks because it yields small values for forces and stresses in both the concrete wall and steel shell.

4.13. **Acknowledgements**

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4.14. **References**


CHAPTER 5

OPTIMUM DESIGN OF COMPOSITE CONICAL TANKS UNDER HYDROSTATIC PRESSURE

5.1. Introduction

Under hydrostatic water pressure, large capacity conical tanks are subjected to tensile hoop as well as compression meridional stresses. Steel conical tanks are strong in resisting the tensile hoop stresses but they are weak in resisting buckling resulting from the compressive meridional stresses. On the other hand, reinforced concrete conical tanks are strong in resisting meridional buckling, but they are weak in resisting the hoop tensile stresses. Composite tanks consist of an inner reinforced concrete wall connected through studs to an outer steel shell. These tanks combine the advantages of both steel and reinforced concrete tanks, where the steel walls resist the hoop stresses and the concrete walls resist the buckling. Given the advantage of composite tanks over the mere use of either steel or concrete, this research focuses on the optimal design of composite tanks.

Much research was done in the analysis and design of steel tanks, such as Vandepitte et al. (1982), El Damatty et al. (1997b), and Niloufari et al. (2014). Reinforced concrete tanks were extensively studied in the literature such as Ramanjaneyulu et al. (1993), El Mezaini (2006), and Bruder (2011). Although, the optimization of these tanks received less attention than their design and analysis, there were some pioneering works that are reported below, highlighting the limitations and gaps in their studies.

Regarding the optimization of reinforced concrete tanks, Thevendran and Thambiratnam (1987) used a direct search method to find the optimum shapes of concrete cylindrical tanks, and later (1988) to concrete conical tanks, with a piecewise linearly tapered wall
thickness. Their work was confined to finding the optimal concrete thickness and did not take into account the other parameters that define the geometry of the tank. Moreover, they only considered bending and hoop stresses as design constraints.

More design parameters were optimized by Chau and Lee (1991), who optimized the concrete thickness in addition to the reinforcement bar size and spacing. However, their work was confined to concrete circular and rectangular tanks. Additionally, in their optimization technique, they relied on design variables enumeration, which is only applicable to a limited number of design variables.

Tan et al. (1993) also used a direct search method to optimize concrete cylindrical tanks with piecewise linearly tapered thickness. They considered larger number of constraints for the design including constraints for concrete on the thickness, ultimate moment and shear, cracking and concrete cover. They also considered constraints for reinforcing steel on ultimate tension force, spacing, and minimum reinforcement ratio. Prior to their work, the objective function sought by researchers was the materials minimization, which inherently means cost minimization, while constraining the stresses to their allowable values.

Recently, Barakat and Altoubat (2009) optimized concrete cylindrical and conical tanks using different global optimization techniques, hence they avoided being trapped in a local optima, especially at the constraint boundaries. The objective function included six design variables, which covered the whole geometry of the tank including: top and bottom wall thicknesses, base thickness, vessel height, and wall inclination angle. In addition to the volume minimization, they constrained the search to the limiting values of stresses used in previous research.
Some investigations were conducted on the optimization of steel conical tanks, such as El Ansary et al. (2010, 2011a). In these studies, a coupled finite element genetic algorithm technique for the optimum design of steel conical tanks under hydrostatic loading was proposed in El Ansary et al. (2010). Shortly afterwards, the same authors extended their study on the stiffened steel conical tanks in El Ansary et al. (2011a). In this investigation the design variables were the shell thickness, geometry of the vessel as well as dimensions and number of stiffeners, which covered the whole parameters defining the tank geometry. Their optimization algorithm hybridized a genetic algorithm with a quasi-Newton search to eliminate the random effect on the final solution inherent to the genetic algorithm. Such work was extended to other shell structures, such as cooling towers, (El Ansary et al., 2011b).

The current work investigates the use of composite concrete-steel conical tanks, which is inspired by previous research on the analysis and design of composite slabs. These slabs consisted of a concrete slab connected through studs to steel plates. An early study on the analysis and design of one way composite slabs was performed by Daniels and Crisinel (1993a, b). They tested a set of composite slabs under concentrated loads and compared the experimental results with those obtained from analytical solutions. Other analytical studies by Shanmugam et al. (2002) and Eldib et al. (2009) focused on the analysis of two way composite slabs under a concentrated load. These studies revealed that composite slabs exhibit good flexure characteristics and highly ductile behaviour.

The analysis, design, and optimization of composite tanks were lacking in the literature. A primitive study on the analysis of composite conical tanks was conducted by Elansary and El Damatty (2015). Both the concrete and steel walls were modelled using 13-node shell
elements, while the studs were modelled using contact elements. The current study is conducted to cover the area of the optimization of these structures. Due to the advantages of the optimization technique that is used by El Ansary et al. (2010, 2011a), the same technique is adopted in the current work.

This paper has three main objectives. The first objective is to compare between the material cost of steel, reinforced concrete, and composite conical tanks, before conducting optimization of the composite tank. The second objective is to develop an optimization tool capable to estimate the optimum design parameters for composite conical tanks. This tool will be used to determine the reduction in the material cost that can be achieved from the optimization. The third objective in this paper is to examine the sensitivity of the optimum variables to the material prices. This is done by changing the material prices by (+/-) 50% of the current prices of concrete, steel, and studs in Canada. The paper starts by defining the geometry and components of the composite tanks. Then, a comparison between the material costs for steel, reinforced concrete, and composite conical tanks with the same layout dimensions is reported. Afterwards, the structural analysis and design of composite conical tanks, which is included in the optimization tool, are discussed. The parameters of the optimization tool are then presented including design variable, constraints, and the objective function. Finally, a sensitivity analysis is performed to test the effect of changing the material prices on the optimized design variables.

5.2. **Geometry and material properties**

**Fig. 5-1** shows the typical cross section plan for the vessel of composite conical tanks. As shown this figure, the vessel consists of an external steel shell connected through studs to
an internal concrete wall. The contact between the concrete wall and steel shell, including the studs, transfers the shear and peel forces.

Fig. 5-1. Cross section plan in composite concrete-steel tank.

A composite conical tank, which has been recently constructed with a capacity of 2800 m³, is considered as a reference tank for this study. First, a reinforced concrete and steel conical tanks with the same layout dimensions of the reference tank are designed to compare between their material costs with the composite reference tank. Second, the reference composite tank is redesigned using a coupled finite element-optimization tool, which is developed in the current study, in order to obtain the minimal possible material cost. The reference tank is elevated above the ground by a supporting shaft that has a height of 15 m. The tank’s vessel has a truncated cone shape with a height of 9 m, a bottom radius of 4 m, and an inclination angle with the vertical of 51.6°, as shown in Fig. 5-2. The concrete wall and steel shell are connected using shear studs that have a diameter and spacing of 13 mm and 400 mm, respectively. The thicknesses of the concrete wall and steel shell are 113 mm and 8 mm, respectively. The inside wall is made of conventional concrete with a strength, Poisson’s ratio, and Young’s modulus of 24.5 MPa, 0.2, and 23,264 MPa,
respectively. The outside steel shell has a yielding stress, Poisson’s ratio, and Young’s modulus of 248 MPa, 0.3, and 200,000 MPa, respectively.

Fig. 5-2. Elevated composite tank (reference tank) and the vessel’s approximated shape.

5.3. **Material prices**

In this study, the cost of materials is based on the prices in Canada in 2015. For the sake of comparison between the reinforced concrete, steel, and composite tanks, it is assumed that the labour and maintenance costs are approximately equal, and therefore they are excluded from the cost of the tanks. The total material cost of any tank is calculated as:

\[
C_M = C_S + C_{SB} + C_{PC} + C_{St}
\]

(5-1)

where \(C_M\) is the total material cost of the tank, \(C_S\), \(C_{SB}\), \(C_{PC}\), and \(C_{St}\) are the cost of the steel shell, steel bars, plain concrete, and studs, respectively. In is worth mentioning that \(C_{SB} = C_{PC} = C_{St} = 0\) for steel tanks, and \(C_S = C_{St} = 0\) for reinforced concrete tanks.

5.3.1. **Plain concrete cost**

The cost of plain concrete is based on the volume unless a special aggregate and additives are used. A cost of $180 is adopted for 1 m³ of plain concrete ready mix with a conventional
strength of 25 MPa, according to the Canada Building Materials (CBM) general contractors’ price list in Canada (2015). The actual cost of plain concrete for the reinforced concrete wall is calculated as:

\[ C_{PC} = A_{Sr} \times t_c \times C_{PC/m^3} \]  

(5-2)

where \( A_{Sr} \), \( t_c \), and \( C_{PC/m^3} \) are surface area of the tank, thickness of the concrete wall, and the cost of 1 m³ of plain concrete, respectively.

5.3.2. **Steel bars cost**

The cost of reinforcing steel deformed bars with regular sizes is mainly governed by the weight. In this research, a cost of $600 is adopted for a tonne of reinforcing steel bars. The actual cost of the steel rebars is calculated as:

\[ C_{SB} = A_{Sr} \times t_c \times C_{SB/m^3} \]  

(5-3)

where \( C_{SB/m^3} \) is the cost of reinforcing steel bars per 1 m³ of concrete.

5.3.3. **Steel shell cost**

The steel shells are used for the construction of the steel tanks or they are used as an external wall in composite tanks. A cost of $800 is adopted for a tonne of a steel plate. The actual cost of the steel shells is calculated as:

\[ C_{SP} = A_{Sr} \times t_S \times \gamma_S \times C_{SP/tonne} \]  

(5-4)

where \( C_{SP/tonne} \), \( t_S \), and \( \gamma_S \) are the cost of the steel plate per 1 tonne, thickness of the steel shell, and the specific weight of steel, respectively.

5.3.4. **Studs’ cost**

A stud is specified by its length and diameter. For different stud’s length, no significant change is observed between the prices of studs that have the same diameter. The prices for studs with different diameters, supplied by Fastenal Company (2015), are shown in Table
The total cost of the studs is calculated based on the number of studs used in the composite tank, according to the following equation:

\[ C_{St} = N_{St} \times C_{St,One} \]  

(5-5)

where \( N_{St} \) and \( C_{St,One} \) are the total number of the studs and cost of one stud.

Knowing the surface area of a composite tank, \( A_{Sr} \), and the area served by one stud, \( S_{St} \), the number of studs can be calculated as:

\[ N_{St} = \frac{A_{Sr}}{S_{St}} \]  

(5-6)

Table 5-1. Prices of studs with different diameters.

<table>
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<th>Diameter (mm)</th>
<th>Price ($)</th>
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<tr>
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<tr>
<td>22</td>
<td>3.90</td>
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<tr>
<td>26</td>
<td>5.12</td>
</tr>
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</table>

5.4. Cost of steel, concrete, and composite tanks

A steel and a reinforced concrete tanks that have the same layout dimensions of the reference composite tank are designed under hydrostatic water pressure to compare between their material costs. In this paper, the steel and reinforced concrete tanks are referred to as “equivalent steel” and “equivalent concrete” tanks, respectively. The equivalent steel tank is designed under hydrostatic water pressure using “the simplified procedure” by Sweedan and El Damatty (2009). Based on this method, the thickness of the steel conical vessel is calculated by magnifying the theoretical membrane stresses by a magnification function to obtain the overall shell stresses. This function depends on the imperfections in steel whether the tank is good or poor. The equivalent steel tank is initially
designed as a good tank, assuming moderate imperfections amplitudes, and then as a poor tank, assuming it has large imperfections amplitudes. The design of the equivalent steel conical tanks resulted in a steel shell with a thickness of 25 mm and 35 mm for the good and poor tanks, respectively. Regarding the equivalent concrete tank, it is analyzed using a Finite Element Model (FEM) that was developed by Elansary et al. (2015). Using this FEM, both the minimum thickness and reinforcement required for a reinforced concrete tank under hydrostatic water pressure can be estimated. The design of the equivalent concrete tank is based on the ACI 318-08 (2008) and ACI 350-06 (2006) design requirements as well as the PCA design aids (1993). The design of this tank resulted in a concrete wall of a thickness of 500 mm and a reinforcement mesh with reinforcement ratios of 0.015 and 0.007 in the hoop and meridional directions, respectively. A comparison between the cost of the equivalent (good/poor) steel tanks, equivalent concrete tank, and reference composite tank is shown in Fig. 5-3. It is observed that the cost of the equivalent poor steel tank is the highest, while the cost of the composite tank is the lowest. Fig. 5-3 also shows that the cost of the equivalent concrete tank is insignificantly less than the cost of the equivalent good steel tank by 8%. It can be observed that the ratio between the cost of the equivalent good steel, poor steel, and equivalent concrete tanks to the cost of the composite tank is 2.65, 1.89, and 1.75, respectively.

From this section, it can be noted that the material cost of the composite tanks is significantly less than the cost of the steel and reinforced concrete tanks. Consequently, the composite tank is the most economical solution for conical tanks when they are designed under hydrostatic water pressure.
5.5. **Structural analysis and design**

The optimization of the composite tanks is performed using a tool that combines both the finite element theory and a global search optimization technique. This optimization technique assumes a large number of solution instances during the optimization process. For each solution instance, the composite tank is analyzed using a Finite Element Model for Composite tanks (CFEM) that was developed by Elansary and El Damatty (2015). The concrete wall and steel shell in the composite tanks are modelled in the CFEM using two separate meshes of a 13-node triangular shell element that was developed by Koziey and Mirza (1997). This element has three corner nodes, three side nodes, and one node at its centroid, as shown in Fig. 5-4(a). The uniqueness of this element is that a cubic and a quadratic interpolation functions are employed for the in-plane displacements and through-
thickness rotations, respectively. Using these functions leads to a consistent formulation for the displacement field. This results in avoiding the shear locking phenomenon, which was found in isoparametric elements. The 13-node shell element includes special rotational degrees of freedom which lead to a cubical variation of the displacement through the thickness of the element. The rotations $\alpha$ and $\beta$ provide linear variation of displacements through the thickness simulating bending deformations, while the rotations $\phi$ and $\Psi$ vary cubically simulating transverse shear deformations. Therefore, a quadratic distribution of the transverse shear stress can be predicted by the element. The connecting shear studs are modelled using a 26-node contact element that was developed by Siddique and El Damatty (2012). As shown in Fig. 5-4(b), the element consists of two 13-node elements connected by 13 springs that are located at the 13-nodes. The smearing approach is adopted such that the properties of studs are assumed to be distributed along the surface area of the contact element. The forces in the studs are obtained based on the resulted slip between the two ends of the studs using the nonlinear load-slip curve. The suitable load-slip curves are obtained from the push-out tests or analytical models that were carried out by previous researchers.

Only one quarter of the tank’s vessel under hydrostatic pressure is modelled due to the double symmetry in geometry and loading, then the suitable boundary conditions are chosen. Two separate meshes consisting of 128 triangular shell elements are used to model each of the concrete wall and steel shell as well as a mesh of 128 contact elements is used to model the contact surface between the two walls. A mesh sensitivity analysis is performed for this mesh and showed that the selected mesh is suitable to predict the straining actions, forces in studs, and the tank’s failure load.
Fig. 5-4. (a) The 13-node shell element (b) The 26-node contact element.

The analysis of the large number of solution instances in the optimization needs a significant computational time, which might grow to prohibitive levels. Conducting a nonlinear analysis for any solution instance requires using incremental loading which leads to a significant running time for each solution instance. The reference tank is analyzed twice (linear/nonlinear) to test the effect of considering the geometric and material nonlinearities and the nonlinear behaviour of studs on the composite tanks’ capacity. The CFEM included the nonlinear concrete model by Pietruszczak et al. (1988) to account for the nonlinear behaviour of concrete. The nonlinear load-slip curves obtained from a database of studs with different diameters and concrete strengths are included in the CFEM
to consider the nonlinear behaviour of studs. This database are developed from the experiments and analytical parametric studies performed by Choi et al. (1999), Shim et al. (2004), Nguyen and Kim (2009), as well as Xu and Sugiura (2012, 2013). In this database, the studs’ diameter is varied from 13 mm to 30 mm, while the concrete strength is varied from 25 MPa to 60 MPa. An example of the nonlinear load-slip curves, corresponding to a concrete strength of 30 MPa and studs’ diameter of 13 mm, is shown in Fig. 5-5. The load deflection curves obtained from the linear and nonlinear analyses for a point at height of 5 m from the tank’s base, where the maximum deflection along the tank’s height occurs, is shown in Fig. 5-6. One can observe that excluding the geometric and material nonlinearities has insignificant effect on the tank’s capacity while it has significant effect on the displacements.

The analysis of the reference tank using the CFEM also showed that the forces in the studs are within the linear range. This eliminated the need to consider the whole nonlinear load-slip curve of the studs in the CFEM. Therefore, the initial stiffness of studs from the push-
out tests is adopted in the linear CFEM. Values of the initial shear stiffness of studs are obtained from the push-out tests or analytical models by Choi et al. (1999), Shim et al. (2004), Nguyen and Kim (2009), as well as Xu and Sugiura (2012, 2013).

![Figure 5-6](image)

**Fig. 5-6.** Linear and nonlinear load-deflection curves for the reference tank.

Based on the aforementioned discussion, it is decided not to include the geometric nonlinearity and the nonlinear behaviour of studs, concrete, and steel. This eliminated the need to apply incremental loading in the CFEM and lead to reducing the running time significantly.

The composite conical tanks in the developed tool are designed to resist both the own weight of the concrete and steel walls as well as the hydrostatic water pressure. The ultimate load factor for composite tanks was not reported in the literature. However, El Damatty et al. (1999) and Elansary et al. (2015) used an ultimate load factor of 1.4 and 2.7 for steel and reinforced concrete tanks, respectively. A significant difference exists between the two factors because the ultimate load factor for concrete includes an environmental durability factor of 1.93, according to the ACI 350-06 (2006) design code. This durability factor is used for reinforced concrete tanks to avoid concrete cracking and subsequently protect the reinforcing steel from being exposed to moisture. Frequent
exposure to moisture can lead to rusting of the reinforcing steel. This reinforcing steel is the major element that resists the tensile hoop stresses developed in reinforced concrete tanks. For composite tanks, the major element that resists the tensile hoop stresses is the steel shell, which is located at the outer face of the tank, i.e. away from the water inside the tank. Therefore, in the current research, an ultimate load factor of 1.4, same as that of steel tanks, is adopted for composite tanks.

The CFEM is used to calculate the maximum straining actions on the concrete and steel walls including the ring tension and meridional axial forces as well as bending moment. Moreover, the same CFEM is used to obtain the maximum shear and peel forces in studs. The resulted straining actions and forces are compared with the ultimate values from the ACI 318-08 (2008) and ACI 350-06 (2006) design codes. The failure criterion for the steel wall is set such that when the maximum ring tension force exceeds the ultimate tension force. Also, the failure is assumed to occur in the concrete or steel walls when the maximum meridional moment exceeds the ultimate meridional moment under a certain meridional axial force, according to the following equations:

\[
R_u = A_{n,\text{hoop}}(\phi f_y) \tag{5-7}
\]

\[
P_u = 0.80\phi \left[ 0.85f'_c(A_g-A_{st}) + f_y A_{st} \right] \tag{5-8}
\]

\[
M_u = \phi M_n \tag{5-9}
\]

where \( R_u \) and \( P_u \) are the ultimate ring tension and meridional compression forces, \( M_u \) and \( M_n \) are the ultimate meridional and nominal moments. \( \phi \) is the strength reduction factor which is equal to 0.9, 0.65, and (0.9 for tension controlled sections and 0.65 for compression controlled sections) in Equations (5-7), (5-8), and (5-9), respectively.
The CFEM also includes three failure criteria for studs, namely peel, shear, and bond slippage such that any stud fails when the shear or peel forces exceed the ultimate values. According to the ACI 408R-03 (2003), the ultimate shear and peel forces are calculated using the following equations:

\[ S_u = \phi \mu A_s f_y \]  \hspace{1cm} (5-10)
\[ B_{u,1} = \phi A_s f_y \]  \hspace{1cm} (5-11)

where \( S_u \) is the ultimate shear force in a stud, \( B_{u,1} \) is the ultimate peel force in a stud, \( \phi \) is the strength reduction factor, which is equal to 0.75, \( A_s \) is the cross sectional area of a stud, \( f_y \) is the yielding stress of a stud, \( \mu \) is the coefficient of friction, \( \mu = 0.7\lambda \) for concrete anchored to steel plate by studs, and \( \lambda = 1 \) for normal weight concrete.

According to the ACI 408R-03 (2003) design code, a bond slippage failure occurs for a stud when the bond stresses exceed the ultimate bond stresses that are calculated using the following equations:

\[ u = 20 \sqrt{\frac{f_c}{d_b}} \leq 5.52 \text{ MPa} \] \hspace{1cm} (5-12)
\[ A_{sf} = \pi d_b l \]  \hspace{1cm} (5-13)
\[ B_{u,2} = u A_{sf} \]  \hspace{1cm} (5-14)

where \( u \) is the ultimate bond stresses, \( d_b \) is the stud’s diameter, \( B_{u,2} \) is the ultimate peel force on the stud corresponding to the bond slippage, \( A_{sf} \) is the surface area of the stud embedded in concrete, and \( l \) is the stud length.

5.6. Optimization problem modelling

Modern optimization techniques can efficiently solve the optimization problem in the current work because they can be applied without having a closed form for the cost and
constraints, as well as without having an initial estimation for the design variables. The Genetic Algorithm (GA) technique is adopted in this study to obtain the minimum cost of composite tanks. This technique can be binary coded or real coded, as reported by Gaffney et al. (2010). In Binary Coded Genetic Algorithms (BCGA), the design variables are assumed to be binary numbers. While, for the Real Coded Genetic Algorithms (RCGA), the design numbers are assumed to be real numbers, which is suitable for any practical problem, such as the analysis of composite tanks. GA has some parameters that are defined in the beginning of the solution. These parameters are the design variables, objective function, constraints, upper and lower bounds for the variables, number of generations, number of population, and genetic operators. The details of these parameters are presented in the following subsections.

5.6.1. Design variables

The composite tanks have a number of design variables, which can be classified into three main categories: material, layout, and structural design variables. The material variables are the strength and Young’s modulus of concrete, and the yielding stress and ultimate strength of steel. The layout variables are the vessel’s height, bottom radius, and vessel’s inclination angle. The structural design variables are the thickness of the concrete wall and steel shell, the studs’ diameters and spacing.

In this research, the adopted concrete and steel variables are chosen same as the material variables of the reference composite conical tank. The concrete and steel that are used in this tank have conventional properties and they are available at regular prices in the location where the reference tank is constructed. This choice is done in order to avoid the higher cost that will be achieved when nonconventional steel or concrete are used for the
construction of the tank. Additionally, the layout variables in the current research are limited to the reference composite conical tank. This is done so as to adhere to the architectural requirements that necessitates adopting the current shape. The structural design variables are optimized to obtain the minimum cost of a composite tank that has the same layout and material design variables as the reference composite conical tank. In light of the above, the adopted design variables in the current work are: (1) thickness of the concrete wall, (2) thickness of the steel shell, (3) diameter of the studs, and (4) spacing of the studs.

5.6.2. Objective function

The material cost is chosen in the current research to be the objective function in the optimization tool. The calculation details of the total material cost of the composite tank are previously presented in Section 5.3. Based on the material costs in Canada, the following objective function is used:

\[ f(X_{i,j,k,l}) = A_{sr} \times (6240 \, t_s + 205 \, t_c + \frac{C_{St,One}}{S_{St}}) \]  \hspace{1cm} (5-15)

where \( f(X_{i,j,k,l}) \) is the unpenalized objective function; \( X_{i,j,k,l} \) is the solution instance i.e: the combination of the design variables \( i, j, k, l \).

5.6.3. Design constraints

Constraints are included in the optimization technique to guarantee that the solution instances do not violate the design criteria. The ACI 318-08 (2008), ACI 350-06 (2006), and ACI 408R-03 (2003) failure criteria are included in the developed optimization technique as constraints, as previously shown in Section 5.5.
A penalty should be applied to the solution instances that violate the design criteria. These criteria are violated when one of the actual axial force or moment exceed the corresponding ultimate value. The penalty function is developed based on the difference between the actual and ultimate forces in the ring and meridional directions. Also, a penalty is applied to the tank if the thickness of the steel shell is less than the minimum thickness that is reported in PCI (2010). Moreover, a penalty is applied on the tank if the forces in studs exceed the ultimate values.

The following penalty function is included in the developed optimization tool:

\[ f_p(X_{i,j,k,l}) = f(X_{i,j,k,l}) + \sum_{N=1}^{6} \Psi_N (X_{i,j,k,l}) \delta_N \]

(5-16)

where \( f_p(X_{i,j,k,l}) \) is the penalized objective function; \( f(X_{i,j,k,l}) \) is the unpenalized objective function; \( \delta_N = 1 \) if the constraint \( N \) is violated, while \( \delta_N = 0 \) if the constraint \( N \) is not violated; and \( \Psi_N(X_{i,j,k,l}) \) is the violation function that should be applied due to the violation of the constraint \( N \). The violation functions are defined according to the following equations:

\[ \Psi_R(X_{i,j,k,l}) = \rho_R (R_{\text{act.}} - R_u) \]

(5-17)

\[ \Psi_P(X_{i,j,k,l}) = \rho_P (P_{\text{act.}} - P_u) \]

(5-18)

\[ \Psi_M(X_{i,j,k,l}) = \rho_M (M_{\text{act.}} - M_u) \]

(5-19)

\[ \Psi_B(X_{i,j,k,l}) = \rho_B (B_{\text{act.}} - B_u) \]

(5-20)

\[ \Psi_S(X_{i,j,k,l}) = \rho_S (S_{\text{act.}} - S_u) \]

(5-21)

\[ \Psi_T(X_{i,j,k,l}) = \rho_T (T_{\text{act.}} - T_{\min}) \]

(5-22)

where \( R \), \( P \), and \( M \) refer to the ring tension force, meridional axial force, and meridional moment, respectively; \( B \) and \( S \) refer to the peel and shear forces in the studs, respectively;
$T_{\text{min}}$ refers to the steel shell’s minimum thickness; and $Y_{\text{act}}$ and $Y_{\text{u}}$ refer to the actual and ultimate values for the function $Y$, respectively.

$\rho_N$ is the violation factor for the constraint $N$ that is used to avoid the dominance of the search by one of the constraints. These factors adjust the terms in the penalty function to have the same order of magnitude of the expected material cost. The Monte Carlo simulation, which was reported by Kroese et al. (2011), is used to calculate the violation factors by assuming a set tanks with different dimensions randomly. The maximum forces in the concrete wall and steel shell as well as in studs are obtained using the CFEM. Then, the differences between the actual forces and the corresponding ultimate values are recorded. The violation factors are found to be 0.05, 0.1, and 5 for the axial ring tension force, meridional axial force and moment, and they are found to be 5, 5, and 13000000 for the peel and shear forces in studs, and the steel shell’s minimum thickness.

5.6.4. Upper and lower bounds

According to the Eurocode 4 (1994) and American National Standard Institute (ANSI)/Steel Deck Institute (SDI) (2011) specifications, the minimum thickness of the steel plate is 0.75 mm. PCI (2010) handbook requires the minimum thickness of the steel shell to be one-half of the stud’s diameter. Both of the aforementioned requirements are considered in both the penalty function and the lower bound of the steel shell’s thickness. Regarding the concrete wall, the minimum thickness above the top of the steel deck for composite slabs must be 50 mm, according to the (ANSI/ SDI) (2011). Based on the requirements mentioned above and on some practicality requirements, the upper and lower bounds for the design variables are determined and listed in Table 5-2. The available studs in the market have diameters ranging between 10 mm to 25 mm, which are adopted in this study.
Table 5-2. Upper and lower bounds for the design variables.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete wall thickness (mm), $t_c$</td>
<td>50</td>
<td>500</td>
</tr>
<tr>
<td>Steel shell thickness (mm), $t_s$</td>
<td>0.75</td>
<td>10</td>
</tr>
<tr>
<td>Spacing between studs (m), $S_{st}$</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>Stud diameter (mm), $D_{st}$</td>
<td>10</td>
<td>25</td>
</tr>
</tbody>
</table>

5.7. Optimization method

In the current work, a global search method is used in the optimization technique in order to avoid trapping the solution into a local minimum. This is done based on the recommendations of Barakat and Altoubat (2009), as previously mentioned in Section 5.1. All global search methods (simulated annealing, genetic algorithms, particle swarm optimization, etc.) have a random element in their search techniques, as reported by Rao (2009). This randomness may result in inconsistency in finding the final solution unless a suitable set of operators are selected carefully. Usually, they are able to find a near to global optimum solution, but there exist an extra mile to zero on the global optimality. To tackle this problem, El Ansary et al. (2010) applied Sequential Quadratic Programming (SQP) after applying the global search method, where the solution found by the global search is fed as an initial search point to the SQP.

In the current research, the objective function is linear while the constraints are highly nonlinear. The SQP cannot be used to conduct the analysis after the global search analysis, since the SQP approximates the search space with quadratic functions, and it is only suitable for nonlinear objective functions. Consequently, a direct search method (Nelder-
Mead (1965), which do not use any derivative of the objective function, is adopted in the current work. The solution obtained from the global search is fed as an initial search point to the Nelder-Mead direct search. A Real Coded Genetic Algorithm (RCGA) is used to conduct the global search using the following operators and parameters:

- Four pairs of tanks undergo uniform mutation
- Four pairs of tanks undergo boundary mutation
- Four pairs of tanks undergo non-uniform mutation
- Two pairs of tanks undergo simple arithmetic cross over
- Two pairs of tanks undergo whole arithmetic cross over
- Two pairs of tanks undergo heuristic cross over
- No. of generations = 100, No. of populations = 100,

The structural analysis and design are coupled with the RCGA and Nelder-Mead direct search to perform the optimization of the composite conical tanks. A preprocessor is developed to generate the input file for each combination of variables suggested by the optimization algorithm. The applied procedure of the GA is shown in the flow chart in Fig. 5-7 and it is summarized in the following steps:

1. The parameters of the RCGA are determined at the beginning of the solution which include: the Upper Bound (UB) and Lower Bound (LB) number of generation, \( G_{\text{max}} \), number of population, \( P \), and operator’s parameters.

2. The objective function is determined in terms of the design variables based on the current prices in Canada.

3. An initial population is created by randomly assuming a set of composite tanks with different design variables between the upper and lower bounds. The design
variables are the concrete wall thickness, $t_c$, steel shell thickness, $t_s$, spacing between the studs, $SS$, and cross sectional area of one stud, $A_s$.

4. For each tank, a three dimensional model is created in the form of a data file that includes the geometry of the tank, the chosen thicknesses of the concrete wall and steel shell, and the studs’ configuration.

5. For each tank, the CFEM is used to analyze the tank and calculates the straining actions in the concrete wall and steel shell, and the maximum shear and peel forces in studs.

6. If any of the actual straining actions exceed the ultimate corresponding values, the penalty function is executed based on the differences between them. These differences are provided by the CFEM and they are used by the optimization code for the penalty functions.

7. The objective function, the cost in this case, is calculated for each tank. The tanks (or candidates) of the initial population are sorted in an ascending order based on the cost such that the first ranked candidate has the minimum cost.

8. A new generation of tanks is produced by applying the mutation and cross over operators, which are specified at the beginning of the solution. The operators are applied to the highly ranked candidates that are obtained from the initial generation.

9. The initial population is replaced by the newly developed tanks from the operators. These new tanks have better fitness i.e. lower costs.

10. Steps 2 to 9 are repeated for a certain number of generations until the global minimum of the cost is reached. The best fitness-Generation i.e. the Cost-Generation curve is plotted.
Fig. 5-7. Flow chart for optimization of composite conical tanks using genetic algorithm.
11. The resulted solution instance from the RCGA is utilized as a starting search point for the direct search analysis.

5.8. Results

This section shows the results obtained from the optimization of the case study tank using the developed numerical tool. The cost-generation curve that is obtained from the analysis using the GA is plotted to determine the number of generations required for the analysis. Afterwards, the optimized tank is analyzed using the nonlinear CFEM to examine the validity of excluding the nonlinear behaviour in the developed optimization tool. This section also shows a comparison between cost of the unoptimized and optimized composite tanks as well as the achieved reduction in the material cost when optimization is applied. At the end of this section, the results for a sensitivity study of the optimized design variables to the material prices are presented.

5.8.1. GA cost-generation curve

The optimization analysis is implemented to obtain the structural design variables corresponding to the minimum cost using the developed numerical tool. The minimum material cost in CAD is obtained by plotting the cost-generation curve, as shown in Fig. 5-8. It is clear that cost of the tank decreases when the generation increases. The curve has the shape of an exponential function with a number of sudden drops in the cost. These drops occur when the new generation produces composite tanks with significant reduction in the material cost. It can be observed that the number of generations after which the cost becomes constant is approximately 35. Therefore, the analysis is performed for the subsequent runs using a number of generation of 35 instead of 100, which inherently decreased the running time by 65%. Fig. 5-8 also shows that the optimum solution is
obtained with a material cost of $50,000. This solution is corresponding to a concrete wall and steel shell’s thickness of 70 mm and 6.5 mm as well as a stud’s spacing and diameter of 890 mm and 13 mm, respectively. It is decided to perform the analysis multiple times until the differences between design variables obtained from the optimization runs become less than a tolerance of 5%. This is done to guarantee that the minimum cost is reached and make sure that the obtained solution is not a local minimum.

![Cost-Generation curve from the genetic algorithm stage](image)

**Fig. 5-8.** Cost-Generation curve from the genetic algorithm stage.

### 5.8.2. Nonlinear analysis of the optimized tank

The optimized composite conical tank is analyzed using the linear and nonlinear CFEM in order to test the validity of using the linear CFEM in the developed optimization tool. The load-deflection curve for the optimized tank from the linear and nonlinear analyses are shown in **Fig. 5-9**. From this figure, it can be observed that insignificant difference is obtained between the failure load resulting from both the linear and nonlinear analyses. The failure load predicted by both the linear and nonlinear analyses is due to the crushing of concrete in the meridional direction. The maximum meridional shear force in studs
resulted from analysis of the optimized composite tank is 24 kN. Referring to the load-slip curve for studs with a diameter of 13 mm in Fig. 5-5, one can note that the forces in studs are within the linear range.

![Linear and nonlinear load-deflection curves for the optimized tank.](image)

**Fig. 5-9.** Linear and nonlinear load-deflection curves for the optimized tank.

Based on the above mentioned discussion, it can be concluded that excluding the geometric and material nonlinearities as well as the nonlinear behaviour of studs has insignificant effect on the optimization results of composite tanks.

### 5.8.3. Comparison between the reference and optimized tank

The dimensions and studs’ configurations of the reference and optimized tanks are shown in Table 5-3. It is observed that a cost of $50,000 is achieved from the optimization which is significantly cheaper than that of the reference tank. The ratio between the cost of the optimized tank to the reference tank is 0.68. The details of the cost for both the optimized and reference tanks, including the cost of concrete wall, steel shell, and studs, are plotted in Fig. 5-10. It is found that the cost of the steel shell is significantly higher than the cost of the concrete wall and studs for both the optimized and reference tanks. The ratio between the costs of the steel shell of the optimized to the reference tanks is 0.82. The cost of the
concrete wall for the optimized tank is 0.65 of the cost of the concrete wall for the reference tank. The cost of the studs for the optimized tank is 0.25 of the cost of the studs for the reference tank. From Fig. 5-10 and Table 5-3, it is notable that a significant reduction in the total material cost of the composite tank can be achieved by reducing the thicknesses of both the concrete wall and steel shell, as well as increasing the stud’s spacing.

Table 5-3. Dimensions and studs’ configuration for the reference and optimized composite tanks.

<table>
<thead>
<tr>
<th></th>
<th>t_c (mm)</th>
<th>t_s (mm)</th>
<th>S_st (mm)</th>
<th>D_st (mm)</th>
<th>Cost ($1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference tank</td>
<td>113</td>
<td>8</td>
<td>400</td>
<td>13</td>
<td>73</td>
</tr>
<tr>
<td>Optimized tank</td>
<td>70</td>
<td>6.5</td>
<td>890</td>
<td>13</td>
<td>50</td>
</tr>
</tbody>
</table>

Fig. 5-10. Cost of the concrete wall, steel shell, and studs for the reference and optimized composite tanks.

5.8.4. Sensitivity to material prices

Sensitivity of the optimum variables to material prices is examined by increasing/reducing price of the concrete, steel plate, and studs by 50%, as shown in Table 5-4. The same table also shows the material prices in Canada in 2015 which are considered as “datum” prices.
for the sensitivity study. The optimum variables are obtained for six cases where the objective function is calculated at each case based on the 50% increase/reduction in the price of concrete, steel plate and studs, as shown in Table 5-5.

Table 5-4. Datum and changed material prices ($).

<table>
<thead>
<tr>
<th></th>
<th>Concrete</th>
<th>Steel plate</th>
<th>Shear stud, Dst (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Datum</strong></td>
<td>180</td>
<td>800</td>
<td>1.07 1.5 2.1 2.73 3.9 5.12</td>
</tr>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in Concrete</td>
<td>+50%</td>
<td>270</td>
<td>800 1.07 1.5 2.1 2.73 3.9 5.12</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-50%</td>
<td>90</td>
<td>800 1.07 1.5 2.1 2.73 3.9 5.12</td>
</tr>
<tr>
<td><strong>Case 3</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in Steel Plate</td>
<td>+50%</td>
<td>180</td>
<td>1200 1.07 1.5 2.1 2.73 3.9 5.12</td>
</tr>
<tr>
<td><strong>Case 4</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-50%</td>
<td>180</td>
<td>400 1.07 1.5 2.1 2.73 3.9 5.12</td>
</tr>
<tr>
<td><strong>Case 5</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in Studs</td>
<td>+50%</td>
<td>180</td>
<td>800 1.6 2.3 3.2 4.1 5.9 7.7</td>
</tr>
<tr>
<td><strong>Case 6</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-50%</td>
<td>180</td>
<td>800 0.5 0.8 1.1 1.4 2.0 2.6</td>
</tr>
</tbody>
</table>

Table 5-5. Objective functions for the datum and six changed-price cases.

<table>
<thead>
<tr>
<th></th>
<th>Objective function, ( f(X_{i,j,k,l}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Datum</strong></td>
<td>( A_{sr} \times (6240 , t_c + 205 , t_c + C_{St,One} \div S_{St}^2) )</td>
</tr>
<tr>
<td><strong>Case 1</strong></td>
<td>( A_{sr} \times (6240 , t_c + 295 , t_c + C_{St,One} \div S_{St}^2) )</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td>( A_{sr} \times (6240 , t_c + 115 , t_c + C_{St,One} \div S_{St}^2) )</td>
</tr>
<tr>
<td><strong>Case 3</strong></td>
<td>( A_{sr} \times (9360 , t_c + 205 , t_c + C_{St,One} \div S_{St}^2) )</td>
</tr>
<tr>
<td><strong>Case 4</strong></td>
<td>( A_{sr} \times (3120 , t_c + 205 , t_c + C_{St,One} \div S_{St}^2) )</td>
</tr>
<tr>
<td><strong>Case 5</strong></td>
<td>( A_{sr} \times (6240 , t_c + 205 , t_c + C_{St,One} + 50% \div S_{St}^2) )</td>
</tr>
<tr>
<td><strong>Case 6</strong></td>
<td>( A_{sr} \times (6240 , t_c + 205 , t_c + C_{St,One} - 50% \div S_{St}^2) )</td>
</tr>
</tbody>
</table>

The optimization runs for the case study composite conical tank are repeated for the six cases using 35 instead of 100 generations. This number of generations is found to be enough in order to obtain the optimum variables, as previously presented in Section 5.8.1. Table 5-6 lists the design variables obtained from the six cases and the percentages of change of the variables from the datum tank’s variables. The change in a variable \( X \) from the variable of the optimized datum tank is calculated as:
\[
\% \text{ Change in } X = \frac{X - X_{\text{Datum}}}{X_{\text{Datum}}} \times 100\%
\] (5-23)

For Cases 1 and 4, where price of the concrete increases and price of the steel plate decreases by 50%, respectively, the optimum solution results in a reduction in the concrete thickness by 29% and an increase in the steel thickness by 31%. On the contrary, the optimum solution results in an increase in the concrete thickness by 14% and a reduction in the steel thickness by 15% for Cases 2 and 3, where price of the concrete decreases and price of the steel plate increases by 50%. One can note that no change in both the stud’s spacing and diameter occurs when the concrete price changes by (+/-) 50%, as presented in Cases 1 and 2. However, a reduction in the stud’s diameter is observed in Cases 3 and 4 where the steel plate price change by (+/-) 50%. For Case 5, where the studs’ prices are increased by 50%, a reduction in the studs’ diameters occur and no change is observed in the studs’ spacing. For Case 6, where the studs’ prices are decreased by 50%, a reduction in the studs’ spacing occurs and no changes are observed in the studs’ diameters. It can be concluded that the optimum thickness of the concrete wall and steel shell as well as studs’ configuration are sensitive to the change in the material prices. This conclusion is drawn when the price of the concrete, steel plate, and studs are changed by (+/-) 50%.

The costs of the concrete wall, steel shell, and studs for the six cases of the sensitivity analysis are listed in Table 5-7. One can observe that the costs of the studs are significantly smaller than the cost of both the concrete wall and steel shell. The ratio between studs’ cost to total cost of the composite tank does not exceed 0.04 for the six cases. Table 5-7 also shows that the steel shell’s cost is usually significantly larger than the concrete wall’s cost. The ratio between steel shell’s cost to concrete wall’s cost ranges between 2.6~4.1 for the six cases. This reflects the significant effect of the steel shell’s cost on the optimization of
the composite tanks. The total costs of the equivalent concrete and equivalent steel tanks, which are estimated for six cases of the sensitivity analysis, are provided in Table 5-7. The cost of the equivalent concrete tank corresponds to the minimum attainable concrete wall thickness and reinforcement, while the cost of the equivalent steel tank corresponds to the minimum attainable steel wall thickness. One can observe that the cost of composite tank is always significantly smaller than the costs of both equivalent concrete and equivalent steel tanks.

Table 5-6. Percentages of change in optimized design variables due to change in material prices.

<table>
<thead>
<tr>
<th></th>
<th>t&lt;sub&gt;c&lt;/sub&gt; (mm)</th>
<th>% Change</th>
<th>t&lt;sub&gt;s&lt;/sub&gt; (mm)</th>
<th>% Change</th>
<th>S&lt;sub&gt;st&lt;/sub&gt; (mm)</th>
<th>% Change</th>
<th>D&lt;sub&gt;st&lt;/sub&gt; (mm)</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Datum</td>
<td>70</td>
<td>--</td>
<td>6.5</td>
<td>--</td>
<td>890</td>
<td>--</td>
<td>13</td>
<td>--</td>
</tr>
<tr>
<td>Case 1</td>
<td>50</td>
<td>-29%</td>
<td>8.5</td>
<td>31%</td>
<td>890</td>
<td>0%</td>
<td>13</td>
<td>0%</td>
</tr>
<tr>
<td>Case 2</td>
<td>80</td>
<td>14%</td>
<td>5.5</td>
<td>-15%</td>
<td>890</td>
<td>0%</td>
<td>13</td>
<td>0%</td>
</tr>
<tr>
<td>Case 3</td>
<td>80</td>
<td>14%</td>
<td>5.5</td>
<td>-15%</td>
<td>870</td>
<td>-2%</td>
<td>10</td>
<td>-23%</td>
</tr>
<tr>
<td>Case 4</td>
<td>50</td>
<td>-29%</td>
<td>8.5</td>
<td>31%</td>
<td>890</td>
<td>0%</td>
<td>10</td>
<td>-23%</td>
</tr>
<tr>
<td>Case 5</td>
<td>60</td>
<td>-14%</td>
<td>8</td>
<td>23%</td>
<td>890</td>
<td>0%</td>
<td>10</td>
<td>-23%</td>
</tr>
<tr>
<td>Case 6</td>
<td>60</td>
<td>-14%</td>
<td>8</td>
<td>23%</td>
<td>800</td>
<td>-10%</td>
<td>13</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 5-7. Material cost of the composite conical tank and its equivalent all steel or all concrete tanks for the six sensitivity analysis cases in ($1000).

<table>
<thead>
<tr>
<th></th>
<th>Composite Tank</th>
<th>Equivalent Concrete Tank</th>
<th>Equivalent Steel Tank (Good)</th>
<th>Equivalent Steel Tank (Poor)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Concrete wall</td>
<td>Steel shell</td>
<td>Studs</td>
<td>Total cost</td>
</tr>
<tr>
<td>Datum</td>
<td>13</td>
<td>36</td>
<td>1.7</td>
<td>50</td>
</tr>
<tr>
<td>Case 1</td>
<td>13</td>
<td>47</td>
<td>1.7</td>
<td>61</td>
</tr>
<tr>
<td>Case 2</td>
<td>8</td>
<td>30</td>
<td>1.7</td>
<td>40</td>
</tr>
<tr>
<td>Case 3</td>
<td>14</td>
<td>45</td>
<td>1.2</td>
<td>61</td>
</tr>
<tr>
<td>Case 4</td>
<td>9</td>
<td>23</td>
<td>1.2</td>
<td>34</td>
</tr>
<tr>
<td>Case 5</td>
<td>11</td>
<td>44</td>
<td>1.8</td>
<td>57</td>
</tr>
<tr>
<td>Case 6</td>
<td>11</td>
<td>44</td>
<td>1.0</td>
<td>56</td>
</tr>
</tbody>
</table>
5.9. **Summary and conclusions**

A robust numerical tool is developed in this study to find the minimum cost of a composite conical tank using both optimization and the finite element theories. In the optimization part, the Genetic Algorithm (GA) is used in combination with Nelder-Mead direct search to find the optimum structural parameters for the composite tanks. In the Finite Element Model for Composite tanks (CFEM), the concrete wall and steel shell of the composite tanks are modelled using 13-node shell elements, while the interface between the two walls, including the studs, is modelled using a set of 26-node contact elements.

A comparison is conducted between the cost of a reference composite, steel, and reinforced concrete tanks having the same layout dimensions as the reference composite conical tank. It is found that cost of the composite conical tank is less than the cost of reinforced concrete, good steel, and poor steel tanks by 62%, 47%, and 43%. Consequently, the composite tank is found to be the most economical solution for conical tanks when they are designed under hydrostatic loading.

For composite tanks, the cost of studs is always less than the cost of concrete wall and steel shell. The ratios between the costs of the steel shell, concrete wall, and studs of the optimized to the reference tank are 0.82, 0.65, and 0.25, respectively. The cost of the steel shell is significantly higher than the cost of concrete wall and studs for both the optimized and reference tanks.

Using the developed optimization tool results in a reduction in the material cost of the reference composite conical tank by 32%. The optimum material cost of the case study composite conical tank is $50,000. This solution requires using a concrete wall and steel shell with a thickness of 70 mm and 6.5 mm, respectively, as well as studs’ spacing and
diameter of 890 mm and 13 mm, respectively. Therefore, it can be concluded that a significant reduction in the total cost can be achieved by decreasing the number of studs and reducing the thickness of both the concrete wall and steel shell.

A sensitivity study is conducted by changing the price of concrete, steel plate, and studs by (+/-) 50% and obtaining the corresponding optimum design variables. This sensitivity study revealed that the optimum thicknesses of the concrete wall and steel shell as well as studs’ configuration are sensitive to the change in the material prices. Therefore, the accurate material prices must be obtained at the time of designing of the composite conical tanks in order to obtain their optimum design. It is concluded that the costs of steel and concrete walls are always significantly larger than the cost of studs. The ratio between the costs of studs to the total cost of optimized tanks do not exceed 0.04 for the six cases in the sensitivity analysis. This analysis revealed that the steel shell’s cost is always significantly larger than the concrete wall’s cost such that the ratio between the two costs is always larger than 2.6.

5.10. Acknowledgements

The authors are grateful to the Government of Ontario, Canada for their generous support through the Ontario Trillium Scholarship, OTS.

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CHAPTER 6

SEISMIC ANALYSIS OF LIQUID STORAGE COMPOSITE CONICAL TANKS

6.1. Introduction

Earthquakes are often followed by fire events that might cause devastating property damage and human losses. These fires are usually initiated from the rapture of gas lines and power lines, as reported by Scawthorn et al. (2006). As such, the availability of a large water supply after earthquakes is crucial to extinguish fires. Therefore, large capacity water reservoirs must be safe and need to remain functional after earthquakes. Among various tank types, conical tanks are quite common in many locations around the globe. These tanks consist of vessels that have truncated conical shapes. Traditionally, the vessels were made of either steel or reinforced concrete. From stress analysis point of view, each material has its own advantages and disadvantages. Under hydrostatic loading, conical vessels are subjected to meridional compressive stresses and tensile hoop stresses. Concrete tanks have good resistance to the meridional compressive stresses, while their resistance to the tensile hoop stresses is weak. On the other hand, steel conical tanks, as thin shell structures, are controlled by their buckling capacity in resisting the meridional compressive stresses, while they are strong in resisting the tensile hoop stresses. Recently, attempts have been made to construct composite conical tanks that benefit from the advantages of both materials. Fig. 6-1 shows a composite conical tank that has been recently constructed in a location which is considered as a seismically active zone. The vessel consists of an inner reinforced concrete wall and an outer steel shell connected through shear studs. This type of composite construction represents an efficient system to provide large water storage capacity with reasonable thicknesses for the concrete wall and
steel shell. To the best of the author’s knowledge, the seismic behaviour of composite conical tanks was not studied previously in the literature.

Madhuri and Madhukar (2013) conducted a review on the seismic analysis of elevated water tanks. They stated that generally three cases should be considered while analyzing elevated water tanks: empty, partially filled, and fully filled conditions. When they are subjected to earthquakes, partially filled tanks suffer less than half of the force to which the fully filled tanks experience. Early work on the analysis of cylindrical tanks under seismic horizontal excitations was conducted analytically by Haroun and Housner (1981). They divided the seismic forces resulting from hydrodynamic pressure acting on a tank into three components; the first component is associated with the rigid motion of the tank, the second component is due to the flexibility of the tank’s walls, and the third component is associated with the top surface liquid sloshing. Shortly afterwards, Haroun and Housner (1982) extended their study by developing a finite element model where the tank’s wall was modelled using ring elements, while the liquid was modelled using the boundary
element theory. They developed a mechanical model to simulate the hydrodynamic pressure developed inside flexible tanks without considering the rocking at the base. Haroun and Ellaithy (1985a) extended this mechanical model to account for the rocking. The extended model was used by Haroun and Ellaithy (1985b) to study the response of cylindrical tanks under horizontal excitations.

Regarding the analysis of conical tanks, El Damatty et al. (1997b) developed a numerical model to study the stability of steel conical tanks under seismic loading. They evaluated the horizontal and vertical forces resulting from the hydrodynamic pressure using the boundary element theory. These forces were considered by incorporating a fluid added-mass matrix in the nonlinear dynamic analysis. The developed numerical model was used by El Damatty et al. (1997c) to study the behaviour of steel conical tanks under seismic loading. Free vibration and nonlinear time history analyses were carried out on a set of tall and broad tanks where the material and geometric nonlinearities were considered. In their investigation, they found that steel conical tanks, which are designed under hydrostatic loading, are sensitive to seismic loading and they have high tendency to develop localized buckling. El Damatty et al. (2005) validated the previously developed numerical model by conducting shake table testing on a set of small-scale steel conical tanks. A good agreement was achieved between the experimentally predicted and numerically evaluated results with differences not exceeding 10%. A parametric study on a set of full scale steel combined conical tanks was carried out and charts for their natural frequencies were developed. Shortly afterwards, El Damatty and Sweedan (2006) developed an equivalent mechanical analogue to calculate the forces at the base of steel and concrete pure conical tank’s under horizontal excitations. The parameters of the developed analogue were provided in the
form of charts in terms of the tanks’ geometric layout dimensions. The mechanical model parameters were validated by extrapolating the curves to an inclination angle of 0° and comparing them with those of cylindrical tanks, which were reported by Haroun and Housner (1981). A recent study was conducted by Jolie et al. (2013), where they assessed the design procedure of conical tanks under horizontal excitations in the current codes. This procedure was based on replacing the conical tank with an equivalent cylindrical tank. They implemented this procedure to determine the response of a number of steel conical tanks under horizontal excitations. The results were compared with those predicted by the equivalent mechanical model that was previously developed by El Damatty and Sweedan (2006). Jolie et al. (2013) found that the procedure in the current codes is not adequate for designing of conical tanks under horizontal ground motions.

Regarding the effect of vertical excitations, Haroun and Tayel (1985a) developed a theoretical model to evaluate the vertical natural frequencies and mode shapes of cylindrical tanks. About the same time, Haroun and Tayel (1985b) suggested an analysis method to predict the response of cylindrical tanks under vertical excitations. They concluded that the vertical component of earthquake excitations develops axial stresses much smaller than those resulting from the horizontal component of earthquake excitations. The sloshing component due to the vertical excitation has been experimentally observed to have a negligible value when it was compared with the impulsive component. They also found that the response due to the higher vibration modes was minimal when it was compared with the response of the fundamental mode. Veletsos and Tang (1986) suggested a simple practical procedure for evaluating the response of cylindrical tanks under the vertical component of ground excitations. In their procedure, not only they
accounted for the interaction between the tank’s wall and contained liquid but also they considered the interaction between the supporting system and the soil medium. A good agreement was observed between the results obtained from the suggested procedure and their counterparts resulting from exact analytical solutions.

Sweedan and El Damatty (2005) proposed an equivalent model for conical tanks under vertical ground excitations. In their study, they developed a simple procedure for estimating the fundamental natural frequency and the seismic forces on conical tanks subjected to vertical seismic excitations. They presented the mechanical model parameters in the form of charts that depend on the vessel’s dimensions. The mechanical model parameters were validated by extrapolating the curves to an inclination angle of 0° and comparing the results with those corresponding to the cylindrical tanks reported by Veletsos and Tang (1986). They showed that including the shell mass in the model has an insignificant effect on the axisymmetric fundamental frequency. Recently, Jolie et al. (2014) assessed the importance of considering the vertical component of the ground acceleration when analyzing steel conical tanks. In their study, the normal forces due to the vertical seismic excitations were evaluated using the mechanical analogue that was previously developed by Sweedan and El Damatty (2005). They found that the vertical ground accelerations have a significant effect on the meridional stresses compared with those resulting from hydrostatic pressure.

In the current study, a Finite Element Model for Composite tanks (CFEM) is developed to perform free vibration and time history analyses. As previously mentioned, these tanks consist of an inner reinforced concrete wall connected through studs to an outer steel shell. Since the CFEM is not available for general users, and it is also relatively complicated for
practicing engineers, a simpler approach is proposed for the seismic analysis of such tanks. This approach, denoted as Equivalent Section Method (ESM), involves transforming the composite section to a single material section having an equivalent wall thickness. This equivalent section is used along with the mechanical analogue developed by Sweedan and El Damatty (2005) and El Damatty and Sweedan (2006) to analyze composite tanks under seismic excitations. The ESM is used to predict the response of these tanks resulting from the hydrodynamic pressure. The validity of this approach is assessed by comparing the results obtained from the CFEM with those resulting from ESM. The second objective of the current investigation is to assess, through a case study, the increase in stresses in the concrete wall, steel shell, and shear studs as a result of applying seismic loads.

The paper starts by presenting the details of the CFEM and ESM that are used to perform the seismic analysis of composite conical tanks. The description and material properties of a set of four composite conical tanks with different dimensions are then reported. Afterwards, a comparison between the fundamental frequencies obtained from the CFEM and ESM for the four tanks is carried out. A time history analysis is performed to the four tanks using the CFEM. The forces at the tanks’ bases are then evaluated and compared with those obtained from the ESM. Finally, as a case study, time history analysis is conducted for a real composite conical tank using the CFEM to assess the increase in stresses in the concrete wall, steel shell, and studs due to horizontal and vertical excitations.

6.2. Finite Element Model for Composite tanks (CFEM)

The concrete wall and steel shell are modelled using a 13-node triangular shell subparametric element that was first introduced by Koziey and Mirza (1997). This element was extended by El Damatty et al. (1997a) to include the geometric nonlinearity and the
nonlinear behaviour of steel. Using a smearing approach, the studs between the concrete wall and steel shell are modelled using a 26-node contact element that was developed by Siddique and El Damatty (2012). This element was extended by Elansary and El Damatty (2015), as presented in Chapter 4, to model the studs in composite tanks by smearing the studs’ properties along the surface of contact elements. In the formulation of the 13-node triangular shell element, cubic and quadratic polynomials are assumed for the approximation of the in-plane displacements and through thickness rotations, respectively. This formulation ensures that the spurious transverse shear modes, which were found in the isoparametric shell elements, are avoided. The 26-node contact element consists of two triangular 13-node elements. Each node on the first triangular element, either the concrete wall or steel shell, is connected to a node on the second element using three-dimensional springs.

The numerical model for free vibration of liquid-filled steel conical tanks, which was developed by El Damatty et al. (1997b), is extended in the current study to perform free vibration analyses of liquid-filled composite conical tanks. The natural frequencies for the vertical and horizontal free vibrations are calculated by solving the following Eigen value problems:

\[
\begin{align*}
\left| [K_0] - \omega^2 \{ [M_s] + [DM]_V \} \right| &= 0 \\
\left| [K_0] - \omega^2 \{ [M_s] + [DM]_H \} \right| &= 0
\end{align*}
\]  

(6-1)  

(6-2)

where \([DM]_V\) and \([DM]_H\) are the fluid added masses for the vertical and horizontal free vibrations, respectively. \([K_0]\) and \([M_s]\) are the structure stiffness matrix and the mass matrix of the tank’s wall, which are calculated using the following equations:

\[
[K_0] = [K_{ss}] + [K_{sw}] + [K_{ce}]
\]  

(6-3)
\[ [M_s] = [M_{ss}] + [M_{cw}] \]  \hspace{1cm} (6-4)

where \([K_{ss}], [K_{cw}],\) and \([K_{ce}]\) are the stiffness matrix of the steel shell, concrete wall, and contact element simulating the studs, respectively. \([M_{ss}]\) and \([M_{cw}]\) are the mass matrix of the steel shell and concrete wall, respectively.

The numerical model for the time history analysis of liquid-filled steel conical tanks, which was developed by El Damatty et al. (1997b), is also extended in the current study to perform time history analysis of composite conical tanks. This analysis is performed by solving the following dynamic equation of motion:

\[ [M]\{\ddot{U}\} + [C]\{\dot{U}\} + [K^{t(k-1)}]\{\Delta U\} = \{R^t\} - \{F^{t(k-1)}\} - [M]\{H\}a^t_H - [M]\{V\}a^t_V \]  \hspace{1cm} (6-5)

where \(\{\ddot{U}\}\), \(\{\dot{U}\}\), and \(\{U\}\) are the total nodal accelerations, total nodal velocities and incremental nodal displacements, respectively. \(\{R^t\}\) is the load vector due to the hydrostatic water pressure acting on the tank’s walls. \(\{F^{t(k-1)}\}\) is the load vector, corresponding to the stresses at load increment \(t\). \(a^t_H\) and \(a^t_V\) are the ground acceleration components in the horizontal and vertical directions, respectively. \(\{H\}\) and \(\{V\}\) are vectors of unit value corresponding to the active horizontal and vertical degrees of freedom, respectively. \([K^{t(k-1)}]\) is the tangential stiffness matrix, which is evaluated using Equation (6-3). The effective mass matrix \([M]\) can be calculated as:

\[ [M] = [M_{ss}] + [M_{cw}] + [DM]_H + [DM]_V \]  \hspace{1cm} (6-6)

\([C]\) is the damping matrix that is obtained using Rayleigh method as a linear combination of the mass matrix, \([M]\), and the tangential stiffness matrix, \([K^{t(k-1)}]\).

Previous studies on cylindrical and conical tanks under seismic loads revealed that the fundamental sloshing frequencies are much lower than those of the vibrating walls. Therefore, the coupling between sloshing and the vibrating walls was usually neglected.

In
the current study, the time histories of the base shear force and overturning moment due to sloshing are calculated using Newmark method. Equivalent sloshing mass and height are obtained from the charts developed by El Damatty and Sweedan (2006) with a damping ratio of 0.5%. These time histories are added to their counterparts resulting from the CFEM in order to obtain the total base shear force and moment. Sloshing is not considered in the vertical direction because it has insignificant effect on the normal force at the tank’s base, as reported by Haroun and Tayel (1985b).

Different failure criteria for the tanks’ walls are included in the CFEM based on the ACI 318-08 (2008) and the ACI 350-06 (2006) design codes. The tank’s wall is assumed to fail when the ring tension or meridional forces exceed the corresponding ultimate forces. The CFEM also includes three failure criteria for studs, namely peel, shear, and bond slippage, based on the ACI 408R-03 (2003) design code. The stud is assumed to fail when the shear force exceeds the ultimate shear resistance or when the peel force exceeds either the ultimate peel or bond slippage resistance.

The assumed boundary conditions for the free vibration and time history analyses are as follows: first, the vessel’s top edge is assumed to be free in order to allow for lateral and vertical displacements. Second, constraints are applied to the horizontal and vertical displacements of all nodes at the vessel’s base to account for the large axial rigidity of the base slab. Third, one half of the vessel is modelled due to the symmetry in geometry and loading, as shown in Fig. 6-2, and the symmetry boundary conditions are applied along the lines of symmetry.

Each of the concrete wall and steel shell are simulated in the CFEM using an 8×8 mesh, where 8 divisions are used in each of the hoop and meridional directions.
Therefore, 128 shell elements per mesh are used for the concrete wall and steel shell while 128 contact elements are used to simulate the studs in the CFEM. This mesh is selected based on the results from a mesh sensitivity analysis that is performed using 4×4, 4×8, and 8×8 meshes. The difference between the radial displacements and meridional shear forces in the studs obtained from the 4×8 and 8×8 meshes is shown to be less than 5% and 4%, respectively.

The analysis under earthquake histories is performed for composite conical tanks by applying an incremental analysis at each time step, as presented by El Damatty et al. (1997c). In the free vibration and time history analyses, the tank’s vessel is assumed to be filled with water and the water level is kept constant throughout the analysis process.

Knowing the vessel’s layout dimensions and the fluid’s mass, $M_F$, the maximum base shear force, $Q_b$, and maximum base normal force, $N_b$, obtained from the time history analysis can be expressed as:

$$ Q_b = F_Q M_F a_h $$

(6-7)
\[ N_b = F_N M_f a_V \]  \hspace{1cm} (6-8)

where \( F_Q \) and \( F_N \) are factors for the base shear and normal forces that depend on the applied earthquake excitation. \( a_H \) and \( a_V \) are the peak ground accelerations of the horizontal and vertical components, respectively, for the selected earthquakes.

6.3. **Equivalent Section Method (ESM)**

In addition to using the sophisticated CFEM, the seismic response of composite conical tanks is determined in this study using a simplified approach, which is denoted as Equivalent Section Method (ESM). In this approach, an equivalent steel wall thickness of the composite tank is evaluated as:

\[ t_{eq} = t_s + t_c \left( \frac{E_c}{E_s} \right) \]  \hspace{1cm} (6-9)

where \( t_{eq}, t_s, \) and \( t_c \) are the thickness of the equivalent section, steel shell, and concrete wall, respectively. \( E_c \) and \( E_s \) are the Young’s modulus of concrete and steel, respectively. The base shear force and overturning moment due to the horizontal earthquake excitations are obtained using the mechanical model developed by El Damatty and Sweedan (2006), as shown in **Fig. 6-3** (a). This model is based on replacing the tank’s vessel and contained water by equivalent masses \( m_0, m_f, \) and \( m_s \) corresponding to the rigid, flexible, and sloshing components at heights \( H_0, H_f, \) and \( H_s, \) respectively. The base normal force due to the vertical earthquake excitations is calculated using the mechanical model developed by Sweedan and El Damatty (2005), as shown in **Fig. 6-3** (b). In this model, the tank’s vessel and contained water are replaced by equivalent masses \( m_0 \) and \( m_f \) corresponding to the flexible and rigid components, respectively. In **Fig. 6-3**, the stiffness of the flexible and sloshing components in the horizontal direction are \( K_f \) and \( K_s, \) respectively, while the stiffness of the flexible component in the vertical direction is \( K_v. \)
Fig. 6-3. Schematic presentation of mechanical models (a) under horizontal excitation (b) under vertical excitation.

The stiffness $K_s$ and $K_f$ are calculated from the fundamental frequencies, $f_s$ and $f_f$ that are obtained from a set of charts developed by El Damatty and Sweedan (2006) for horizontal free vibration. A sample of these charts for frequency of the flexible component, $f_f$, at $\Theta = 45^\circ$ is provided in Fig. 6-4 (a). The value of $f_f$ can be obtained using the $H/R$ and $t/R$ ratios, where $H$, $R$, and $t$ are the vessel’s height, bottom radius, and wall’s thickness, respectively. In Fig. 6-4 (a), $E$ and $\rho_F$ represent the Young’s modulus of the tank’s wall and fluid density, respectively. Another set of charts, developed by Sweedan and El Damatty (2005), are used to obtain frequency of the flexible component in the vertical direction, $f_v$. A sample of these charts for $f_v$ at $\Theta = 45^\circ$ is provided in Fig. 6-4 (b).

Spectral accelerations are obtained from the acceleration response spectrum corresponding to the earthquakes excitation. Square Root of Sum of Squares (SRSS) is used to calculate the normal and shear forces as well as bending moment at the tank’s base. More details on the equations used in the mechanical analogy can be found in Sweedan and El Damatty (2005) and El Damatty and Sweedan (2006).
Fig. 6-4. Variation of fundamental frequencies $f_t$ and $f_v$ with conical tank’s dimensions.

6.4. Description of selected tanks

Four composite conical tanks with different dimensions and wall’s thicknesses are considered in this study. The layout dimensions of the tanks’ vessels and wall thicknesses are listed in Table 6-1. The tanks T1, T2, and T3 are considered to compare between the fundamental frequencies obtained from free vibration using both the CFEM and ESM. These tanks have the same height and bottom radius but different inclination angles. The concrete wall and steel shell thicknesses are chosen such that the equivalent wall thicknesses are within the practical range that was reported by Sweedan and El Damatty (2005). Another composite conical tank, denoted by T4, is considered as a case study and is analyzed to compare between the base forces obtained from the CFEM and ESM. The layout dimensions and walls’ thickness of tank T4 are chosen same as those of a composite conical tank that has been recently constructed at a high seismic region in Mexico. The vessel of this tank is supported on a 15 m high shaft which transfers the tank’s loads to the foundation. The shaft consists of internal and external steel plates with a diameter of 8 m and 7.5 m, respectively, while concrete is placed between the two plates. The horizontal
stiffness, $k_H$, and vertical stiffness, $k_V$, of the shaft are found to be 1.44E9 N/m and 14.6E9 N/m, respectively, and they are calculated as:

$$k_H = \frac{3(E_c I_c + E_s I_s)}{L^3} \quad (6-10)$$

$$k_V = \frac{(E_c A_c + E_s A_s)}{L} \quad (6-11)$$

where $E_c$ and $E_s$ are the modulus of elasticity of concrete and steel, respectively. $I_c$ and $I_s$ are the moment of inertia of the infill concrete and steel plates, respectively, $A_c$ and $A_s$ are the area of the infill concrete and steel plates, respectively. $L$ is the shaft height.

The concrete wall and steel shell for the studied tanks’ vessels are connected together with studs having a diameter and a spacing of 13 mm and 400 mm, respectively. The concrete properties are as follows: strength $f_c = 24.5$ MPa; Poisson’s ratio $\nu_c = 0.2$; and modulus of elasticity $E_c = 22,285$ MPa. Material properties of steel are as follows: yield stress $\sigma_y = 248$ MPa; ultimate stress $\sigma_u = 400$ MPa; Poisson’s ratio $\nu_s = 0.3$; and modulus of elasticity $E_s = 200,000$ MPa.

<table>
<thead>
<tr>
<th>H(m)</th>
<th>R(m)</th>
<th>$\Theta^\prime$</th>
<th>$t_c$ (mm)</th>
<th>$t_s$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>8</td>
<td>4</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>T2</td>
<td>8</td>
<td>4</td>
<td>45</td>
<td>80</td>
</tr>
<tr>
<td>T3</td>
<td>8</td>
<td>4</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>T4</td>
<td>9</td>
<td>4</td>
<td>51.6</td>
<td>113</td>
</tr>
</tbody>
</table>

For all of the studied tanks, the studs’ configuration and material properties are chosen same as those of tank T4. The typical section plan and elevation views for the composite conical tank’s vessel are shown in Fig. 6-5. Also, the same figure shows the sections where the stress histories are reported. Sections 1 and 4 are located at the vessel’s top and bottom.
edges, respectively, while Sections 2 and 3 are located at an elevation of 4.5 m and 2.25 m measured from the vessel’s base, respectively.

![Elevation and plan views for the tank’s vessel.](Image)

**Fig. 6-5.** Elevation and plan views for the tank’s vessel.

### 6.5. Natural frequency estimated by the CFEM and ESM

A comparison between the fundamental frequencies obtained from the CFEM and ESM is carried out for tanks T1, T2, and T3. For the purpose of comparison between those two approaches, the masses of the concrete wall and steel shell are set to zero. This is done because the shell masses are excluded in the charts for the horizontal and vertical free vibrations presented by El Damatty and Sweedan (2006) and by Sweedan and El Damatty (2005), respectively. **Fig. 6-6** shows the fundamental frequencies of the horizontal free vibration obtained from the two approaches for tanks T1, T2, and T3. From this figure, it can be seen that the fundamental frequency of the horizontal vibration decreases with the
increase in the tank’s inclination angle. **Fig. 6-6** also shows that percentages of discrepancy between the frequencies obtained from the two approaches for tanks T1, T2, and T3 are 8%, 17%, and 10%, respectively.

Regarding the free vertical vibration, a comparison between frequencies obtained from the CFEM and ESM is carried out for tanks T1, T2, and T3. **Fig. 6-7** shows the fundamental frequencies of the vertical free vibration resulting from the two approaches. From this figure, it can be observed that no significant difference is obtained from the two approaches as the differences are 12%, 8%, and 14% for tanks T1, T2, and T3, respectively. It is clear that the fundamental frequency of the vertical vibration decreases with the increase in the tank’s inclination angle, as shown in **Fig. 6-7**.

![Fig. 6-6. Fundamental frequencies for horizontal free vibration from the CFEM and ESM.](image)

The differences between the frequencies obtained from the CFEM and ESM results from a couple of reasons. First, the approximation implemented in the ESM which results from using an equivalent steel section instead of modelling the two walls using their actual thicknesses and material properties. The CFEM is more accurate because it considers both
the steel shell and the concrete wall. Second, the inaccuracy in the ESM when frequencies are obtained from the charts developed by El Damatty and Sweedan (2006) and by Sweedan and El Damatty (2005). One can conclude that the ESM can predict the fundamental frequencies for composite tanks with an error not exceeding 17%.

![Graph](image)

**Fig. 6-7.** Fundamental frequencies for vertical free vibration from the CFEM and ESM.

### 6.6. Input ground motion selection

Four earthquake histories are selected to perform the time history analysis of the composite conical tank case study: Big-Bear, Chi-Chi, North-Ridge, and San-Fernando occurred in 1992, 1999, 1994, and 1971, respectively. These earthquake excitations are selected because their dominant frequencies contain the fundamental modes of vibration of the studied tank. The acceleration time histories of the selected earthquakes as well as the response spectra are given by the Pacific Earthquake Engineering Research Center, PEER (2013). For each earthquake history, a scale factor is obtained from the spectral acceleration and the response spectrum from the Manual of Civil Structures in Mexico, MOC (2008). The spectrum from MOC is selected because the case study composite tank is constructed in Mexico. This factor is calculated at a damping ratio of 5%, according to the following equation:
where SF, \( S_M(T) \), and \( S_E(T) \) are the scale factor for earthquake, acceleration from the MOC (2008), and acceleration from the earthquakes’ spectra. The acceleration response spectrum at the location of the tank from the MOC (2008) is plotted in Fig. 6-8.

\[
SF = \frac{S_m(T)}{S_h(T)}
\]  

(6-12)

**Fig. 6-8.** Elastic response spectrum for the horizontal component a damping ratio of 5% (MOC, 2008).

**Table 6-2** shows the period of the first four vibration modes calculated using the CFEM. The same table reports the spectral accelerations obtained from the MOC (2008), \( S_M(T) \), and the spectral accelerations for the selected earthquakes, \( S_E(T) \) corresponding to the first four vibration periods. For each earthquake record, the scale factor is evaluated at the periods of the first four vibration modes, as shown in **Table 6-2**. The scale factor is also evaluated at the range of periods between the maximum and minimum periods, i.e 0.514 sec and 0.149 sec. The scale factor is chosen for each earthquake history such that the scaled spectral accelerations are always equal to or larger than the spectral accelerations prescribed at the MOC (2008) at the considered range of periods. The chosen scale factors
are 2.88, 1.33, 0.82, and 1.81 corresponding to the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquake excitations, respectively.

**Table 6-2.** Scale factors for the selected earthquakes.

<table>
<thead>
<tr>
<th>Item</th>
<th>Earthquake</th>
<th>Horizontal free vibration</th>
<th>1\textsuperscript{st} mode</th>
<th>2\textsuperscript{nd} mode</th>
<th>3\textsuperscript{rd} mode</th>
<th>4\textsuperscript{th} mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>TF (sec)</td>
<td></td>
<td></td>
<td>T\textsubscript{1}</td>
<td>T\textsubscript{2}</td>
<td>T\textsubscript{3}</td>
<td>T\textsubscript{4}</td>
</tr>
<tr>
<td></td>
<td>0.514</td>
<td>0.449</td>
<td>0.321</td>
<td>0.149</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SM(T) (g)</td>
<td></td>
<td></td>
<td>0.21</td>
<td>0.21</td>
<td>0.21</td>
<td>0.14</td>
</tr>
<tr>
<td>SE(T) (g)</td>
<td>Big-Bear</td>
<td>0.075</td>
<td>0.099</td>
<td>0.213</td>
<td>0.381</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chi-Chi</td>
<td>0.195</td>
<td>0.167</td>
<td>0.185</td>
<td>0.105</td>
<td></td>
</tr>
<tr>
<td></td>
<td>North-Ridge</td>
<td>0.263</td>
<td>0.322</td>
<td>0.272</td>
<td>0.171</td>
<td></td>
</tr>
<tr>
<td></td>
<td>San-Fernando</td>
<td>0.116</td>
<td>0.136</td>
<td>0.18</td>
<td>0.204</td>
<td></td>
</tr>
<tr>
<td>Scale factor</td>
<td>Big-Bear</td>
<td>2.80</td>
<td>2.12</td>
<td>0.99</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chi-Chi</td>
<td>1.08</td>
<td>1.26</td>
<td>1.14</td>
<td>1.33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>North-Ridge</td>
<td>0.80</td>
<td>0.65</td>
<td>0.77</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td></td>
<td>San-Fernando</td>
<td>1.81</td>
<td>1.54</td>
<td>1.17</td>
<td>0.69</td>
<td></td>
</tr>
</tbody>
</table>

The time history of the strongest ten seconds of each earthquake is chosen in order to limit the running time for the time history analysis. A time step of 0.02 sec is chosen because it is significantly smaller than that of the fundamental period of the horizontal and vertical vibrations. The ratio between the chosen time step and the lowest period obtained from the horizontal free vibration for the case study tank is 0.13.

**6.7. Base forces from CFEM and ESM**

The base shear forces and overturning moments are evaluated using the CFEM and ESM for tank T4 under the horizontal excitation of the selected earthquakes without scale factors. The earthquakes’ spectrum are utilized in the ESM to obtain the spectral accelerations using a damping ratio of 5% and 0.5% for the impulsive and sloshing components, respectively. The mechanical model parameters of tank T4 as well as the peak
ground accelerations and spectral accelerations under the four earthquakes are presented in **Table 6-3 and Table 6-4**, respectively. In those tables, \(m_{0\text{-w}}\) and \(m_{e\text{-w}}\) are the effective and rigid wall masses, respectively. One can notice that the fundamental period of the sloshing component is significantly larger than that of flexible component. This difference between the two periods is clearly reflected on the spectral accelerations obtained from the four earthquakes spectra. From **Table 6-4**, it can be observed that the spectral accelerations of the sloshing component, \(S_a(T_s)\), are significantly smaller than that of the flexible component, \(S_a(T_f)\). **Table 6-5** shows the base shear forces due to the rigid, flexible, and sloshing components \(Q_0\), \(Q_f\), and \(Q_s\), respectively. This table also shows the total base shear forces, which are calculated ignoring sloshing, \(Q_{\text{without sloshing}}\), and those obtained considering sloshing, \(Q_{\text{with sloshing}}\). One can notice that the contribution of the sloshing component in the base shear force is not significant such that the increase of the base shear forces due to including sloshing does not exceed 1.6%. The overturning moments due to the rigid, flexible, and sloshing components \(M_0\), \(M_f\), and \(M_s\), respectively, are presented in **Table 6-6**. The same table shows the total overturning moments, which are calculated ignoring sloshing, \(M_{\text{without sloshing}}\), and those obtained considering sloshing, \(M_{\text{with sloshing}}\). The contribution of the sloshing component in the overturning moments is noted to be insignificant such that the increase of the overturning moments due to the inclusion of the sloshing does not exceed 2.7%.

The base shear forces and overturning moments obtained from the CFEM and ESM under horizontal excitation of the selected earthquakes for tank T4 are shown in **Figs. 6-9 and 6-10**. It can be noted from **Fig. 6-9** that the base shear forces resulting from the CFEM are
different than their counterparts from the ESM by 9%, 12%, 8%, and 8% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively.

Table 6-3. Mechanical model parameters for tank T4.

<table>
<thead>
<tr>
<th>Periods (sec)</th>
<th>Masses (kg×1000)</th>
<th>Heights (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_f$</td>
<td>$T_s$</td>
<td>$m_{0-w}$</td>
</tr>
<tr>
<td>0.4</td>
<td>7.1</td>
<td>206.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>86.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>21.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2933.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>432.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2423.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23.2</td>
</tr>
</tbody>
</table>

Table 6-4. Peak ground and spectral accelerations of the selected earthquakes.

<table>
<thead>
<tr>
<th>Units</th>
<th>$G_{max}$</th>
<th>$S_s (T_f)$</th>
<th>$S_s (T_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Big Bear</td>
<td>1.22</td>
<td>0.88</td>
<td>0.04</td>
</tr>
<tr>
<td>Chi-Chi</td>
<td>0.71</td>
<td>1.64</td>
<td>0.06</td>
</tr>
<tr>
<td>North Ridge</td>
<td>1.02</td>
<td>3.1</td>
<td>0.02</td>
</tr>
<tr>
<td>San Fernando</td>
<td>0.91</td>
<td>1.3</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 6-5. Base shear force for tank T4 under the selected earthquakes.

<table>
<thead>
<tr>
<th>Units</th>
<th>(kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake</td>
<td>$Q_0$</td>
</tr>
<tr>
<td>Big Bear</td>
<td>277</td>
</tr>
<tr>
<td>Chi-Chi</td>
<td>162</td>
</tr>
<tr>
<td>North Ridge</td>
<td>233</td>
</tr>
<tr>
<td>San Fernando</td>
<td>208</td>
</tr>
</tbody>
</table>

Table 6-6. Overturning moment of tank T4 under the selected earthquakes.

<table>
<thead>
<tr>
<th>Units</th>
<th>(kN.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earthquake</td>
<td>$M_0$</td>
</tr>
<tr>
<td>Big Bear</td>
<td>2425</td>
</tr>
<tr>
<td>Chi-Chi</td>
<td>1416</td>
</tr>
<tr>
<td>North Ridge</td>
<td>2034</td>
</tr>
<tr>
<td>San Fernando</td>
<td>1819</td>
</tr>
</tbody>
</table>

Fig. 6-10 shows that the overturning moments obtained from the time history analysis are different than their counterparts from the mechanical models by 12%, 7%, 3%, and 1% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. No
A trend can be noted for the differences in the base shear forces and overturning moments obtained from the two approaches. For the Big-Bear earthquake, the shear forces and overturning moments obtained from the CFEM are found to be smaller than their counterparts from the ESM. However, for Chi-Chi, North-Ridge, and San-Fernando earthquakes, the opposite trend is observed. For the four earthquakes, the differences between the shear forces and overturning moments from the two approaches do not exceed 12%.

**Fig. 6-9.** Base shear force of tank T4 subjected to selected earthquakes.

**Fig. 6-10.** Overturning moment of tank T4 subjected to selected earthquakes.
The vertical normal forces at the base obtained from the CFEM and ESM under the vertical excitation of the selected earthquakes of tank T4 are shown in Fig. 6-11. It is observed that the differences between normal forces resulting from the two approaches are 11%, 9%, 9%, and 10% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. No trend is observed for the differences between the vertical normal forces obtained from the two approaches. In some cases, the forces obtained from the CFEM are larger than those obtained from ESM, in other cases, the opposite trend is observed such that the differences in either of the two trends do not exceed 11%.

![Fig. 6-11. Normal force at the vessel's base of tank T4 subjected to selected earthquakes.](image)

The differences between the forces at the base obtained from the CFEM and ESM are due to various reasons. First, the fundamental periods obtained from the CEFM are different than those from the ESM due to the inaccuracy in using the charts by Sweidan and El Damatty (2005) and El Damatty and Sweidan (2006). Second, the inaccuracy of using the charts of equivalent masses and heights in the ESM. Third, the sensitivity of the acceleration obtained from acceleration spectra of earthquakes to the period because these spectra are highly fluctuated. Fourth, the approximation in the ESM due to using the SRSS
approach in evaluating the total forces at the base. Fifth, the approximation in the ESM due to using an equivalent wall instead of modelling the concrete and steel wall, separately. In the ESM, the whole system is replaced by a limited number of degrees of freedom instead of accounting from the whole degrees of freedom, which are considered in the CFEM.

6.8. Time history from CFEM

The time histories of stresses in the concrete wall and steel shell of tank T4, under the scaled horizontal and vertical earthquake excitations, are presented in this section. Also, the time history of shear force in the connecting studs under the same earthquake excitations is reported. The increase in stresses in the concrete wall and steel shell and the increase in the studs’ shear forces are presented. The forces at the base for the case study tank under the selected earthquake excitations are reported. It is worth mentioning that sloshing is neglected because it has insignificant effect on the forces at the base, as shown in Section 6.7.

6.8.1. Time history response of concrete wall

The time history of the meridional stresses at Section 4 in the outer face of the concrete wall for tank T4 under the Big-Bear earthquake is plotted in Fig. 6-12. From this figure, it is clear that the meridional stress at \( t = 0 \) sec due to the hydrostatic loading is -10.9 MPa. This figure also shows that the minimum meridional stress in concrete due to the earthquake loading is -16.3 MPa, which occurs at \( t = 5.82 \) sec. Therefore, the meridional stresses are amplified by 50% when tank T4 is subjected to the horizontal and vertical components of the Big-Bear earthquake excitation. The minimum meridional stresses at the outer and inner faces of the concrete wall under the selected earthquakes at Section 4
are shown in Figs. 6-12 and 6-13. The dotted lines in these figures refer to the meridional stresses due to the hydrostatic water pressure.

Fig. 6-12. Meridional stresses at the outer face of the concrete wall of tank T4 under the Big-Bear earthquake (at Section 4).

The meridional stresses at the outer face of the concrete wall due to the selected earthquakes are observed to exceed those resulting from the hydrostatic water pressure by 50%, 45%, 42%, and 44% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. The average increase in the meridional stresses at the concrete outer face due to the selected earthquakes is 45%. At the inner face of the concrete wall, these percentages become 20%, 14%, 12%, and 15% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. The average increase in the meridional stresses at the inner face of the concrete wall due to the selected earthquakes is 15%. It can be noted that the percentages of increase in the meridional stresses at the inner face of the concrete wall are smaller than those at the outer face. Also, one can observe that the stresses at the inner concrete face are smaller than those at the outer face. This occurs due to the local bending effect at the bottom part of the concrete wall.
6.8.2. Time history response of steel shell

The time history of hoop stresses at the outer face of the steel shell at Section 2 under the horizontal and vertical Big-Bear earthquake components is plotted in Fig. 6-15. From this figure, it can be noted that the hoop stress at $t = 0$ sec due to the hydrostatic loading is 90.1 MPa. This figure also shows that the maximum hoop stress in the steel shell under the Big-Bear earthquake excitation is 123 MPa, which occurs at $t = 5.78$ sec. Therefore, the Big-
Bear earthquake amplifies the hoop stresses in the steel shell by 37%. The time history of the meridional stresses at the outer face of the steel shell at Section 4 under the Big-Bear earthquake is plotted in Fig. 6-16. From this figure, it can be observed that the meridional stress at t = 0 sec due to the hydrostatic water pressure is -80.8 MPa. This figure also shows that the minimum meridional stress in steel under the Big-Bear earthquake excitation is -137.9 MPa, which occurs at t = 5.8 sec. Therefore, the Big-Bear earthquake magnifies the meridional stresses in the steel shell of tank T4 by 71%.

![Graph showing hoop stresses at outer face of steel shell of tank T4 under Big-Bear earthquake excitation.](image)

**Fig. 6-15.** Hoop stresses at outer face of the steel shell of tank T4 under the Big-Bear earthquake excitation (at Section 2).

It is found that insignificant differences exist between the hoop stresses at the inner and outer faces of the steel shell, such that the maximum difference does not exceed 1%. This indicates that the steel shell is subjected to uniform tensile stresses with no evidence of bending in the hoop direction. The hoop stresses at the inner and outer faces of the steel shell at Section 2 due to the selected earthquakes are shown in Fig. 6-17. The dotted line in this figure refers to the hoop stresses due to the hydrostatic water pressure. It can be observed that the hoop stresses due to the selected earthquakes exceed those resulting from
the hydrostatic loading by 37\%, 23\%, 18\%, and 21\% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. Therefore, the average increase in the hoop stresses in the steel shell under the applied earthquakes is 25\%.

**Fig. 6-16.** Meridional stresses at outer face of the steel shell of tank T4 under the Big-Bear earthquake excitation (at Section 4).

**Fig. 6-17.** Hoop stresses at the inner and outer faces of the steel shell of T4 (at Section 2).

The meridional stresses at the inner and outer faces of the steel shell due to the selected earthquakes are shown in **Figs. 6-17** and **6-18**. The dotted lines in these figures refer to the meridional stresses due to the hydrostatic water pressure. It can be observed that meridional
stresses due to the selected earthquakes exceed the meridional stresses due to the hydrostatic loading for the inner face of the steel shell by 71%, 70%, 66%, and 69% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. At the outer face of the steel shell, these percentages become 76%, 73%, 69%, and 72% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively.

**Fig. 6-18.** Meridional stresses at the inner face of the steel shell of tank T4 (at Section 4).

**Fig. 6-19.** Meridional stresses at the outer face of the steel shell of tank T4 (at Section 4).
Therefore, the average increase in the meridional stresses in steel under the applied earthquakes is 71%. It can be noted that the percentages of increase in the meridional stresses at the inner face of the steel shell are insignificantly smaller than those at the outer face of the steel shell for the four earthquakes. This occurs because no local bending exists in the steel shell because the thickness of the steel shell is significantly smaller than that of the concrete wall.

6.8.3. Time history response of studs

The time histories of the meridional shear forces in studs located at Sections 3 and 4 under the horizontal and vertical components of the Big-Bear earthquake are plotted in Fig. 6-20. The meridional shear forces in studs are obtained by smearing the studs’ properties over the area of the contact element. The total meridional shear force on each contact element is obtained using numerical integration of stresses over the surface of the element. Then, the average meridional shear force in studs is obtained by dividing the total meridional shear force by the number of studs at each contact element. The variation in the meridional shear forces in the studs located at Section 3 is negligible, while this variation is significant for the studs located at Section 4. This indicates that the relative displacements between the concrete and steel walls near the tank’s base are larger than the relative displacements beyond this location. Fig. 6-20 shows that the meridional shear forces in the studs located at Section 4 at $t = 0$ sec due to the hydrostatic loading are 7.2 kN. The same figure also shows that the maximum meridional shear forces in the studs located at Section 4 due to the earthquake loading are 9.1 kN, which occur at $t = 5.24$ sec. Therefore, the Big-Bear earthquake amplifies the meridional shear forces in the studs near the base by 26%. 
The meridional shear forces in the studs located at Section 4 due to the selected earthquakes are shown in Fig. 6-21. The dotted line in this figure refers to the meridional shear forces in studs due to the hydrostatic water pressure. It can be observed that meridional shear forces in the studs under the selected earthquakes exceed their counterparts due the hydrostatic loading by 26%, 26%, 24%, and 25% for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively.

**Fig. 6-20.** Meridional shear in studs for tank T4 under the Big-Bear earthquake excitation (at Sections 3 and 4).

**Fig. 6-21.** Meridional shear force in studs for tank T4 (at Section 4).
It is worth mentioning that the maximum shear force that can be carried by one stud is 28 kN, according to the ACI 408R-03 (2003) for a stud with a diameter and yield strength of 13 mm and 400 MPa, respectively. Fig. 6-21 shows that the maximum shear forces on the studs due to the applied selected earthquakes do not exceed the ultimate shear force.

**6.8.4. Time history of forces at tank’s base**

The time histories of the base shear, overturning moment, and normal force at the base of tank T4 under the scaled horizontal and vertical components of the Big-Bear earthquake are shown in Figs. 6-22, 6-23, and 6-24, respectively. The maximum base shear force at the base is 922 kN and it occurs at \( t = 5.98 \) sec, as shown in Fig. 6-22. The maximum overturning moment at the base is 13000 kN.m and it occurs at \( t = 5.98 \) sec, as shown in Fig. 6-23. The maximum normal force at the base is 3070 kN and it occurs at \( t = 6.88 \) sec, as shown in Fig. 6-24. It can be noted that both the shear force and overturning moment occur at the same time, while the base normal force occurs at different time.

![Graph](image)

**Fig. 6-22.** Base shear force of tank T4 due to the horizontal and vertical components of the Big Bear earthquake.

The maximum values of the shear force, overturning moment, and normal force at the base of tank T4 under the horizontal and vertical excitations of the selected earthquakes are
shown in Figs. 6-25, 6-26, and 6-27, respectively. It is found that the studied tank does not fail when it is subjected to any of the selected earthquake records. Fig. 6-27 shows that the maximum and minimum normal forces occur when tank T4 is subjected to the Big-Bear and San-Fernando earthquakes, respectively.

One can notice from Fig. 6-27 that the difference between the maximum normal force at the base from the Big-Bear and San-Fernando earthquakes is 61%. This significant difference is due to the fact the scale factors for the earthquakes are calculated based on
the fundamental periods of the horizontal free vibrations regardless the fundamental periods of the vertical free vibrations.

![Graph showing base shear force](image1)

**Fig. 6-25.** Base shear force of tank T4 under the selected earthquakes obtained from CFEM.

![Graph showing overturning moment](image2)

**Fig. 6-26.** Overturning moment at the base of tank T4 under the selected earthquakes obtained from CFEM.

Referring to Equations (6-7) and (6-8), the factors $F_Q$ and $F_N$ can be evaluated for the selected earthquakes. Knowing the layout dimensions of the vessel and assuming a full tank, the mass of the contained water is found to be $M_F = 2.9 \times 10^6$ kg. The peak ground accelerations of the horizontal components, $a_H$, are 0.3g, 0.1g, 0.1g, and 0.2g, while the
peak ground accelerations for the vertical components, \( a_v \), are 0.24g, 0.07g, 0.07g, and 0.09g for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. \( F_Q \) is found to be 0.2, 0.6, 0.6, and 0.3, while \( F_N \) is found to be 0.9, 1.8, 1.5, and 0.9 for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. Therefore, one can conclude that the factors \( F_Q \) and \( F_N \) depend on the earthquake for both the horizontal and vertical excitations, respectively.

![Graph showing forces for different earthquakes](image)

**Fig. 6-27.** Normal force at the base of tank T4 under the selected earthquakes obtained from CFEM.

### 6.9. Summary and conclusions

A Finite Element Model for Composite tanks (CFEM) is developed to perform free vibration and time history analyses for a set of tanks under both horizontal and vertical excitations. The CFEM accounts for the fluid-structure interaction resulting from the seismic excitation of the tank. Both the concrete wall and steel shell are modelled using 13-node subparametric shell elements, while the connecting studs are modelled using 26-node contact elements using the smearing approach. A simplified approach, denoted as Equivalent Section Method (ESM), is proposed to obtain the fundamental frequencies and
base forces for composite tanks. This approach is based on replacing the concrete wall and steel shell by a wall with an equivalent thickness. This equivalent wall is used in combination with a mechanical analogue from the literature to find the fundamental frequencies and base forces under earthquake loadings.

A free vibration analysis is carried out of a set of composite conical tanks with different dimensions using the CFEM and ESM. The fundamental frequencies obtained from the CFEM are compared with their counterparts from the ESM and they are observed not to be identical. The discrepancy percentages are found to be 8%, 17%, and 10% for the horizontal free vibration and 12%, 8%, and 14% for the vertical free vibration for the tanks with an inclination angle of 30°, 45°, and 60°, respectively. It is concluded that the ESM can predict the fundamental frequencies for composite conical tanks with an error not exceeding 17%.

A time history analysis, using the CFEM, is performed for a case study composite conical tank in Mexico under four selected earthquake histories. The time history analysis is carried out under the unscaled history of the selected earthquakes, then the maximum forces at the base are obtained. These forces are compared with their counterparts from the ESM. The base forces obtained from the CFEM are noted to be smaller or larger than their counterparts from the ESM without showing a specific trend. It is concluded that the ESM can predict the forces at the base for composite conical tanks with an error not exceeding 12%. Sloshing is found to have insignificant effect on the calculated base forces under horizontal earthquake excitations. The increases in the base shear forces and overturning moments due to including sloshing do not exceed 1.6% and 2.7%, respectively.
The CFEM is used to perform another time history analysis for the case study tank to evaluate the increase in forces and stresses under the selected earthquakes. A scale factor for each earthquake history is calculated by dividing the acceleration from the Manual of civil structures (MOC) in Mexico by the acceleration from each earthquake spectrum. Neither the concrete wall, steel shell, nor studs experience failure when the tank is subjected to any of the selected earthquake records. The average increases in the meridional stresses in the outer and inner faces of the concrete wall due to the applied earthquakes are 45% and 15%, respectively. The average increases in the hoop and meridional stresses in steel due to the applied earthquakes are 25% and 71%, respectively. The average increase in the meridional shear force in studs due to the selected earthquakes is 25%.

Significant differences in the base forces are observed from the time history analyses for the case study tank under the selected earthquakes. The percentage of amplification of the total fluid mass that contributes to the base shear force is found to be 0.2, 0.6, 0.6, and 0.3 for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively. However, the percentage of amplification of the total fluid mass that contributes to the normal force is found to be 0.9, 1.8, 1.5, and 0.9 for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively.

6.10. **Acknowledgements**

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6.11. References


Pacific Earthquake Engineering Research Center (PEER), 2013, University of California Berkeley.


CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1. General

The research conducted in this thesis consists of four main parts. In the first part, a Finite Element Model (FEM) is developed to study the behaviour of reinforced concrete conical tanks under hydrostatic pressure. This model accounts for the nonlinear behaviour of concrete by including a concrete plasticity constitutive model. This model is used to study different behavioural aspects including the deformed shape, hoop, and meridional stresses for a set of reinforced concrete conical tanks. The same FEM is used to develop charts for a set of tanks, with a wide range of practical dimensions, which can be used to determine the adequate thickness and straining actions. A simplified method for the analysis of conical tanks, based on using an Equivalent Cylinder (ECM), is introduced. In this method, the dimensions of an equivalent cylinder, thickness, and straining actions are obtained using the provisions available for cylindrical tanks. In the second part of the thesis, a Finite Element Model for Composite tanks (CFEM) is developed to study the behaviour of composite conical tanks under hydrostatic pressure. In this model, the concrete wall and steel shell are modelled using shell element, while the connecting studs are modelled using contact elements using a smearing approach. The CFEM accounts for the nonlinear behaviour of concrete, steel, and studs. The nonlinear behaviour of studs is considered by including the nonlinear load-slip and load-peel curves in the CFEM. A simplified approach for analyzing composite conical tanks under hydrostatic pressure is introduced. This approach, referred to as “Equivalent Section Method (ESM)”, is based on analyzing composite tanks using a virtual section. This section utilizes an equivalent
wall thickness and equivalent Young’s modulus instead of using the actual thickness and material properties of the concrete wall, steel shell, and connecting studs. The adequacy of using this method in analyzing composite tanks is tested by comparing the results from this method with those obtained from the CFEM. In the third part of the thesis, the CFEM is incorporated into an optimization tool, which is based on the genetic algorithm, to obtain the optimum design parameters for composite conical tanks. This tool is used to calculate the minimum thicknesses of the concrete wall and steel shell as well as the optimum configuration of studs. The objective function in the optimization technique is calculated based on the current prices of the steel plates, reinforcing bars, concrete, and studs. In the fourth part of the thesis, a FEM, which was developed in the literature to perform free vibration and time history analyses for steel tanks, is combined with the CFEM to study the behaviour of composite conical tanks under seismic loading. A simplified approach for calculating the fundamental frequencies and the base forces is introduced. This approach is based on using the section properties obtained from the ESM and conducting the seismic analysis using a mechanical analogue that was reported in the literature.

### 7.2. Conclusions

The following are the main conclusions from the study of reinforced concrete conical tanks under hydrostatic pressure:

1. The ratio between the maximum transverse displacements from the linear analyses to the nonlinear analyses is 0.9 under working loads. However, this ratio becomes 3.1, 2.4, and 1.8 for the tanks with an inclination angle of 30°, 45°, and 60°, respectively under ultimate loads.
2. Transverse displacement of conical tanks increases with the increase in the inclination angle or tank’s height under either the working or ultimate loads.

3. Under either the working or ultimate loads, the maximum hoop stress in concrete occurs at 0.15 -0.3 of the tank’s height. However, the maximum meridional stress occurs within 0.1 of the tank’s height at the bottom edge of the tank’s vessel for both the steel and concrete.

4. For the tanks with an inclination angle of 60˚, the ratio between the tensile hoop stresses at the two faces of the tank’s wall reaches 1:2. However, this ratio does not exceed 1% for the tanks with an inclination angle of 30˚ and 45˚.

5. A significant variation in the meridional stresses exists along the tank’s thickness due to the bending effect. The ratio between the meridional stresses in the two faces of the tank’s wall reaches 1:10.

6. Maximum ring tension force and meridional moment occur at the middle one third and the bottom one third of the tanks’ height, respectively. However, the maximum meridional axial force occurs near the vessel’s base.

7. Considering shrinkage in the design of reinforced concrete conical tanks increases the tanks’ thicknesses by a factor of 1.3.

8. Concrete strength has a significant effect on the calculated thickness of reinforced concrete conical tanks. The ratio between the thickness calculated using $f' \text{c}= 30$ MPa and $f' \text{c}= 40$ MPa reaches 1.5.

9. Uneconomical solutions are obtained when reinforced concrete conical tanks are designed using the ECM.
10. When the inclination angle, bottom radius, or the tank’s height increase, the tank’s thickness, which prevents the concrete cracking under working loads, increases.

11. Ring tension forces, meridional axial forces, and meridional moments increase with the increase of the inclination angle, bottom radius, or the tank’s height.

The main conclusions from the study of composite conical tanks are listed below:

1. The displacements obtained from the CFEM, which account for the concrete cracking, are significantly larger than those obtained from the ESM. The ratio between the maximum transverse displacements obtained from the CFEM to ESM reaches 3.1.

2. The full contact between the concrete wall and steel shell results in insignificant relative transverse displacements such that they do not exceed 0.1%.

3. Insignificant change in the meridional stresses is experienced through the thickness of the steel shell, however the opposite is noted for the concrete wall.

4. The ratio between the meridional stresses in the steel shell obtained from the CFEM and ESM is 1.3.

5. Maximum meridional axial forces in the concrete wall and steel shell occur exactly at the vessel’s base, meanwhile the maximum bending moments in both the concrete wall and steel shell occur at an elevation of 0.06 of the vessel’s height.

6. Significant reduction in the meridional bending moments in both the concrete wall and steel shell above 0.2 of the vessel’s height.

7. The CFEM predicts same locations of maximum displacements, stresses, and forces along the vessel’s height as the ESM.
8. Meridional shear forces in studs are approximately constant at the lower half of the vessel’s height.

9. The forces in studs and the adequate studs’ configuration can be evaluated using the CFEM but cannot be predicted by the ESM.

10. The ESM is not adequate for the analysis of composite conical tanks because it yields nonconservative values for the forces and stresses in the tank’s wall.

11. The cost of a composite tank is less than the cost of a reinforced concrete, good steel, and poor steel tanks having the same layout dimensions by 62%, 47%, and 43%.

12. The cost of the steel shell is significantly higher than the costs of the concrete wall and studs such that it reaches 70% of the total material cost.

13. An economical solution for composite conical tanks can be obtained by using the genetic algorithm in combination with the CFEM. A reduction of 32% in the total material cost can be achieved by optimizing the thickness of the concrete wall and steel shell as well as the studs’ configuration.

14. The developed optimization tool provides an economical solution for a reference composite tank by reducing the thicknesses of the concrete wall and steel shell’s thicknesses as well as decreasing the number of studs.

15. A sensitivity analysis showed that the optimized thickness of the steel and concrete walls as well as the studs’ configuration are significantly affected by the material prices.

16. Differences between the frequencies obtained from the CFEM and ESM for the tanks with an inclination angle of 30°, 45°, and 60° are 8%, 17%, and 10% for horizontal free vibration and 12%, 8%, and 14% for vertical free vibration, respectively.
17. Normal forces, base shear forces, and overturning moments resulting from the CFEM are different than their counterparts from the ESM with a difference not exceeding 12%.

18. The average increases in the meridional stresses due to seismic loading in the outer and inner faces in the concrete wall are 45% and 15%, respectively.

19. The average increases in the hoop and meridional stresses due to seismic loading in the steel shell are 25% and 71%, respectively, while the average increases in the meridional shear in studs is 25%.

20. Significant difference in the forces at the base that are obtained from the CFEM for a case study tank under a set of selected earthquakes although these earthquakes are scaled to the same response spectrum.

21. The total fluid mass contributes to the base shear force with a factor of 0.2, 0.6, 0.6, and 0.3 for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively.

22. The total fluid mass contributes to the base normal force with a factor of 0.9, 1.8, 1.5, and 0.9 for the Big-Bear, Chi-Chi, North-Ridge, and San-Fernando earthquakes, respectively.

7.3. **Recommendation for future work**

The work of this thesis can be extended by conducting the following investigations:

- Study the effect of using variable wall thickness and variable reinforcement along the height of reinforced concrete tanks on reducing the overall material quantities.
• Study the effectiveness of using one or multiple ring beams at different elevations for reinforced concrete conical tank and evaluate the reduction in thickness and reinforcement than those obtained for the conventional tanks.

• Study the behaviour of reinforced concrete tanks under conventional and high intensity wind loading and develop design charts that can be used by practitioners.

• Conduct a parametric study on composite conical tanks with a wide range of practical dimensions subjected to hydrostatic pressure to assess the validity of the results obtained from this thesis.

• Derive simplified design equations for composite conical tanks under hydrostatic and seismic loadings that can be used by professional engineers.

• Use the developed optimization tool to obtain the optimum design for composite conical tanks with a wide range of practical dimensions.

• Conduct a small-scale experiment on a composite conical tank under cyclic loading to extend the understanding of its dynamics characteristics.

• Conduct a parametric study on composite conical tanks with a wide range of practical dimensions subjected to different earthquakes to assess the validity of the results obtained from this thesis.

• Study the behaviour of composite conical tanks under conventional and high intensity wind loadings and validate the results by conducting an experimental program.
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