Location Estimation in Wireless Communication Systems

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Graduate Program in Electrical and Computer Engineering  
A thesis submitted in partial fulfillment of the requirements for the degree in Master of Engineering Science  
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LOCATION ESTIMATION IN WIRELESS COMMUNICATION SYSTEMS
(Thesis format: Monograph)

by

Kejun Tong

Graduate Program in Electrical and Computer Engineering

A thesis submitted in partial fulfillment
of the requirements for the degree of
Masters of Engineering in Science

The School of Graduate and Postdoctoral Studies
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Abstract

Localization has become a key enabling technology in many emerging wireless applications and services. One of the most challenging problems in wireless localization technologies is that the performance is easily affected by the signal propagation environment. When the direct path between transmitter and receiver is obstructed, the signal measurement error for the localization process will increase significantly. Considering this problem, we first propose a novel algorithm which can automatically detect and remove the obstruction and improve the localization performance in complex environment. Besides the environmental dependency, the accuracy of target location estimation is highly sensitive to the positions of reference nodes. In this thesis, we also study on the reference node placement, and derive an optimum deployment scheme which can provide the best localization accuracy. Another challenge of wireless localization is due to insufficient number of reference nodes available in the target’s communication range. In this circumstance, we finally utilize the internal sensors in today’s smartphones to provide additional information for localization purpose, and propose a novel algorithm which can combine the location dependent parameters measured from sensors and available reference nodes together. The combined localization algorithm can overcome the error accumulation from sensor with the help of only few number of reference nodes.

Keywords: Wireless localization, path loss exponent, reference node deployment, relative location estimation, accelerometer.
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<th>Description</th>
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<tbody>
<tr>
<td>AOA</td>
<td>Angle of Arrival</td>
</tr>
<tr>
<td>CRB</td>
<td>Cramer-Rao Bound</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>LBS</td>
<td>Location based Service</td>
</tr>
<tr>
<td>LSE</td>
<td>Least Square Estimation</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>IDC</td>
<td>International Data Corporation</td>
</tr>
<tr>
<td>MDS</td>
<td>Multi-dimensional Scaling</td>
</tr>
<tr>
<td>MLE</td>
<td>Maximum Likelihood Estimation</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Square Error</td>
</tr>
<tr>
<td>NLOS</td>
<td>None Line of Sight</td>
</tr>
<tr>
<td>PLE</td>
<td>Path Loss Exponent</td>
</tr>
<tr>
<td>RSS</td>
<td>Received Signal Strength</td>
</tr>
<tr>
<td>RSSI</td>
<td>Received Signal Strength Indicator</td>
</tr>
<tr>
<td>TOA</td>
<td>Time of Arrival</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time Difference of Arrival</td>
</tr>
<tr>
<td>WLSE</td>
<td>Weighted Least Square Error</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless Sensor Network</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Background

With the growing popularity of location-based services (LBSs) in recent years [1], different technologies for locating of a wireless receiver have been widely employed in various applications such as tracking [2], navigation [3], monitoring [4], and related services for emergency and safety purposes [5]. In cellular networks, the Enhanced 911 (E-911) service is mandated by the Federal Communications Commission (FCC) to locate the positions of mobile users who call the emergency number [6], [7]. In order to obtain more accurate latitude and longitude coordinates, FCC requires cellular phone manufacturers to install Global Positioning System (GPS) receivers in their products [8]. In Wireless Sensor Networks (WSNs), the measured parameters from a sensor node needs to be combined with its location information so that the data can be useful [9]. Moreover, in those network processes such as routing, topology control, coverage and boundary detection, the performances can be significantly improved when location information of the sensor nodes is exploited.

In outdoor environment, GPS is one of the most popular wireless localization schemes which can provide high accuracy. However, GPS service is not applicable in most of the indoor environments since the weak GPS signals from the satellites cannot penetrate through building materials. In urban area, the localization performance of GPS locationing is also affected by the buildings or trees, due to the signal diffractions and reflections. In addition, GPS receivers are generally expensive and have high power consumption, which can limits its application.
When GPS signal is not available, those wireless base stations, such as cell towers and WiFi access points, can be used as reference nodes, and the location dependent signal parameters can be measured from the wireless signals between the base stations and the mobile devices. With the fast evolving of today’s smartphone technologies, users can install client software in their handsets and send the measured signal parameters and identifications to remote sever to determine their current locations. WiFi-based localization is a widely applied localization scheme in indoor environment [10] since most of today’s mobile devices are equipped with WiFi modules. Many existing WiFi-based localization systems measure the received signal strength (RSS) as the location dependent parameter to estimate the target location. However, the signal measurement in indoor environment can become unreliable due to the signal attenuation caused by shadowing and multipath effect. In addition, the interference with other appliances in 2.4GHz Industrial Scientific Medical (ISM) Band is another source of error in localization using WiFi signals.

In achieving indoor localization, one of the most challenging problems is the high complexity of the signal propagation environment between the reference transmitter and the target receiver. The multipath effect in indoor environment can reduce the signal measurement accuracy and degrade the localization performance. In conventional localization algorithms using wireless signals, the location dependent signal parameters are decided based on the signal propagation model in free space. However, when there is obstruction of the direct path between the transmitter and receiver, the calculated location dependent parameter based on such signal propagation model will introduce large locationing errors. Therefore, involving those obstructed links in the localization algorithms will decrease the localization accuracy. However, it is difficult to detect the obstruction effect since the target location is unknown. In addition, the obstruction can be a human being or a movable object in indoor environment. In Chapter 3, we discuss the problem of wireless localization in obstructed environment, and propose a novel algorithm to detect and remove the obstruction in order to improve the localization performance.

Besides the impact of signal propagation environment, the placement of the reference nodes relative to the target node also plays an important role in the localization performance. For example, the reference nodes and the target should not be put on a direct line, otherwise only
one reference node will be effective in the localization process. In practice, the positions of the reference nodes are generally not adjustable after they are deployed in the wireless network. In WSNs, the sensor nodes can be deployed in those places which are not easily reachable. In large indoor environment such as a factory or a supermarket, the WiFi APs are usually installed on the ceiling. Thus, it is extremely useful to have an in-depth study on the placement of the reference nodes before they are deployed. In Chapter 4, we evaluate the localization performance based on a mathematic model, and optimize the localization accuracy with respective to the positions of the reference nodes in order to find the optimum reference node deployment scheme.

Since the conventional wireless localization schemes are based on the signal measurement between the target and reference nodes, the localization performance also depends on the number of reference nodes involved in the localization algorithm. In general, a minimum number of reference nodes are required in order to estimate the absolute position of the target node. In Chapter 5, we study the problem of localization with insufficient reference nodes. When the number of available reference nodes within the target’s communication range is less than minimum required number, we can apply distance estimation among all the nodes in the service area to construct a relative location map. The relative locations can be transferred to absolute locations when there are additional reference nodes deployed in the network. In addition, we utilize the internal sensors in today’s smartphones to provide additional location dependent parameters for localization purpose. We develop a mobile application to do experiment on real devices and propose a novel algorithm to combine the sensor data together with the parameters obtained from few available reference nodes, in order to overcome the error accumulation of the sensor output.

1.2 Wireless Localization Technologies and Challenges

The essence of any wireless localization technologies is to measure the location dependent parameters of the wireless signal between a reference transmitter and the target receiver to be located, and then to estimate the position of the target through proper processing of the measured parameters. Those location dependent parameters include time of arrival (TOA) [11],
time difference of arrival (TDOA) [12], angle of arrival (AOA) [13], received signal strength (RSS) [14] and the combination of them.

TOA-based localization measures the absolute signal propagation time between the target and the reference nodes, while TDOA-based localization measures the time difference. The main drawback of TOA and TDOA is due to their high speed signal processing requirements which mandates devices to be equipped with advanced receiver. In addition, the system has to be synchronized in time for TOA-based localization. Angle of Arrival (AOA) localization scheme measures the angle of the arrival of the received signals. Directional antenna is needed for AOA measurement method, and the antenna has to be accurately calibrated. Compared with the above discussed localization schemes, RSS-based localization is another scheme which is highly desirable in resource-constrained systems, such as WSNs, due to its low cost and easy implementation. However, RSS measurements is relatively unreliable and unpredictable due to the multipath and shadowing effect in complex signal propagation environment.

In practice, the random error existing in location dependent parameters obtained from received wireless signals is inevitable. When there is more than minimum required number of reference nodes available in the localization system, the target location can be estimated through least square estimation (LSE) by minimizing the square error of all the measurements between the target and the reference nodes. When the probability distribution of measurement error is known, the Maximum Likelihood Estimation (MLE) can be applied to maximize the joint probability of all the measurements from different reference nodes with respect to the target location. However, the variance of signal measurement error can change significantly in complex signal propagation environment. For example, the received signal strength can drop much faster in indoor environment than in open area without obstructions and obstacles. In addition, the multipath effect is another important source of error in indoor environment. It is difficult to find a statistical model of measurement error which can be generally applied in all different environments. Another drawback of LSE and MLE localization algorithm is due to the high computation complexity in solving the optimization problem when there are large number of reference nodes involved.

Besides the localization methods based on the error optimization which have been discussed above, another localization scheme - fingerprinting based localization is highly desir-
1.2. Wireless Localization Technologies and Challenges

able in complex signal propagation environment. Fingerprinting based localization methods have been already widely applied in many indoor localization applications in recent years [22]. The essence of fingerprinting method is to collect the signal features at every location in the service area, and then to determine the target location by matching the measured signal features with the previous collected ones [15]. Fingerprinting based method is considered as a low cost and low complexity localization scheme as compared to those methods based on distance estimation [16]. There are basically two phases in location fingerprinting - offline phase of signal radio map construction, and online phase of target location estimation [17]. In the offline phase, the signal fingerprinting map is constructed through site survey. The fingerprinting features of the received signals from reference nodes are recorded in the map and combined with the coordinates of the predefined spots in the measurement area. In the online phase, the signal features are measured from the corresponding reference nodes and compared with the data recorded in the fingerprinting map, in order to decide the unknown target location by choosing the most matching values. The main drawback of fingerprinting based localization is that the offline phase of fingerprinting map generation can be labour-intensive and time-consuming. Another challenge of this method is that the fingerprinting map needs to be updated every time when the indoor environment (such as the movement of furniture) and the positions of the reference nodes change in the wireless network.

In recent years, smartphone based localization has been attracting much attention. With the fast development of the smartphone technologies, more and more people are relying on mobile applications for localization and navigation [18]. Most of today’s smartphones are equipped with various modules and sensors, including GPS receiver, WiFi module, accelerometer, gyroscope, magnetometer, camera, etc. The essence of the smartphone based localization is to utilize those modules and sensors to obtain additional location dependent parameters and apply them in the localization algorithms. One of the challenging problem in smartphone based localization is the combination of the different types of parameters. Due to the limited system resource and battery capacity, the computation complexity and the energy consumption in smartphone based localization are also important issues to be considered in the localization algorithm.
1.3 Research Motivation

With the fast proliferation of wireless and mobile devices nowadays, location information has become extremely useful in wireless networks. LBSs, which refer to those wireless services depending on location information, can be supported by both short-range communication to long-range telecommunication systems [22] based on various of technologies as shown in Table 1.1.

![Table 1.1: Range of location-based services](image)

To fulfill the demands for LBSs, wireless localization has been regarded as the key enabling technology for many advanced wireless applications. In wireless health care applications [19], mobile devices such as smart phones, tablet computers, can be used to monitor the vital signs of a patient in real-time, where location information is needed for tracking the patients. In environmental monitoring applications [20], the sensor locations need to be known before the measurement activities. In smart home applications [21], location is also a key information for detecting human acclivities. In mobile advertising and marketing [23], merchants can attract customers by flashing customized coupons on mobile applications based on the location information when they are nearby. In addition, location estimation is also highly desirable in network processes. For examples, in wireless Ad-Hoc networks, location estimation is extremely useful for routing and topology control; in WSNs, the performance of coverage and boundary detection will also be enhanced when location information is available.

Wireless Localization technology is also considered as an essential feature in fifth-generation (5G) networks. Compared with the existing mobile communication systems nowadays, 5G will be characterized by wide user variety, increased mobile data volume, large number of devices connected, and high data rate [24]. A a result, 5G is facing a lot of challenges before it can be widely applied. The challenging problems include the user requirement of low latencies, scalability and reduction of signaling overhead, limited power consumption, and the mobility
management of the massive network nodes [25]. In 5G networks, different types of wireless devices need to cooperate with each other, and deal with dynamically deployed base stations in a heterogeneous manner, where location information will be extremely useful. Since most of the wireless devices in 5G networks will be equipped with localization module and combined with ground support systems and multi-band operation, 5G networks are expected to provide high localization accuracy to 1m in open sky [26].

Besides the strong demand of localization in 5G networks, another motivation of our research is on the smartphone based localization, due to the extreme fast development of today’s smartphone technologies. The worldwide smart phone market grew at an exponential rate in the past few years. According to the data from International Data Corporation (IDC) Worldwide Quarterly Mobile Phone Tracker, the market achieved 335 million units of shipments in the second quarter of 2014, and promises to reach around 1.3 billion shipments in 2014. Most of nowadays smartphones are equipped with various embedded sensors which can not only be used in interesting mobile softwares for entertainment purpose or better user interaction, but also provide us extremely useful information which for emerging applications such as wireless health care [27], social network [28], monitoring activities [29], smart homes [30], transportation and navigation [31]. Location information also plays an important role in these emerging wireless applications.

The above discussed situations and trends motivated our research in this thesis on wireless localization technologies. The technical challenges in the existing wireless localization systems have been attracting much research attention. One of the disadvantages of wireless localization technology is its difficulty to achieve high localization accuracy in harsh environment such as indoor environment, due to the large signal measurement error caused by shadowing and multipath effects. Several research works have been proposed to improve the localization performance in non-line-of-sight (NLOS) environment [33]–[35]. Another challenging problem is due to the resource constraints of the localization system. For example, in WSNs, the battery life of a sensor node is limited, so that it is not applicable to equip every sensor node with a GPS receiver with high power consumption. Moreover, the low cost sensor nodes usually don’t have the ability to do high complexity computation and high speed signal processing. Considering these constraints, the research works in [36]–[38] proposed localization schemes
to improve localization accuracy while using less system resources. Besides the problems discussed above, the number of reference nodes available in a wireless network and the placement of those reference nodes can also affect the localization performance significantly. Generally, the localization accuracy increase with the number of reference nodes involved in the localization algorithm. However, if the reference nodes are deployed improperly, such as when they are put very close to each other, or when they are put on the same line, the localization accuracy will not increase obviously even though a large number of reference nodes are involved.

Our research is motivated by the future trends and challenging problems of the wireless localization technologies in today’s emerging wireless communication systems. Several novel methods are proposed to overcome the drawbacks of the conventional localization schemes and improve the localization performance.

1.4 Contributions

In this thesis, we study on the localization technologies in today’s emerging wireless services and applications. Based on the previous discussed challenges, we propose several novel schemes and algorithms to improve the localization performance. The main contributions of this thesis are summarized as follows:

- In received signal strength (RSS)-based wireless ranging technologies, the path loss exponent (PLE) is an important parameter in RSS signal propagation model which reflects how fast the signal power decays with distance increase in a certain environment. When the direct path between transmitter and receiver is obstructed in a complex signal propagation environment, the signal power can drop significantly on the corresponding obstructed link. As a result, the PLE parameters on those obstructed links will become unpredictable. Based on our experiment, we have observed that when the obstruction of the signal is significant, it is better to discard the corresponding obstructed links rather than using them in the localization algorithm. However, it is difficult to decide which links are obstructed since the positions of the receivers and the obstructions are unknown before the localization process. In this thesis, we propose an novel algorithm based on Maximum Likelihood Estimation (MLE) in complex signal propagation environments
with unknown PLE parameter. The proposed algorithm can automatically detect the obstructed links among transmitters and receivers during the localization process, and reduce the localization error caused by obstruction effect. According to the simulation results, our proposed method shows higher localization accuracy in complex environments as compared to other existing schemes.

- Besides the signal propagation environment, the wireless localization performance is also highly sensitive to the positions of reference nodes relative to the target node. Before the deployment of reference nodes in a wireless network, a theoretical study on the optimal placement of the nodes is extremely useful for improving the localization performance while reducing the overall deployment cost. In this thesis, we propose an optimum reference node deployment scheme by minimizing the Cramer-Rao Bound (CRB). In order to find the global minimum of the CRB which is highly non-linear, a novel method is developed to solve the corresponding optimization problem. The essence of our method is to express the CRB in complex coordinates, and then to minimize the CRB with respect to the angels of reference nodes as the decision variables. The mathematical solution provides an interesting result that the highest localization accuracy is achieved when the reference nodes have uniform angular distribution around the measurement area where the target is located. In the simulations, we compare several different reference node deployment schemes, and the results show our derived optimum deployment provides the best performance.

- In achieving localization using reference nodes, the performance is generally constrained by the number of reference nodes available in the localization service area. When there is less than minimum required number of reference nodes available in the target’s communication range, relative localization algorithms can be applied to calculate a relative location map based on the distance estimations among all the nodes. In order to obtain the absolute positions with insufficient reference nodes, additional location dependent parameters are required besides the wireless signals received from available reference nodes. In this thesis, we utilize the accelerometer sensor in today’s smartphones to obtain additional location dependent parameters. The acceleration data output from the
accelerometer can be used to calculate the moving distance of the wireless device for localization purpose. However, since the distance estimation at current sampling time is calculated based on the distance estimated at previous sampling time, the existing sensor error will be accumulated with time increase. Considering this problem, we developed a mobile application to do experiment in real mobile device and show the accumulated error in distance estimation using accelerometer. In order to overcome the error accumulation, we proposed a novel algorithm which combines the location dependent parameter measured from accelerometer and available reference node together. As shown in the simulations, the performance of the combined localization algorithm can be improved significantly with help of few reference nodes involved.
Chapter 2

Localization Schemes Using Wireless Infrastructures and Signals

Location estimation has already been implemented in many emerging wireless applications nowadays. In recent years, many related technologies have been proposed in order to improve the localization performance of the conventional localization technologies. In this chapter, we introduce some existing wireless localization schemes which are well studied and widely applied.

2.1 Trilateration based Localization

Trilateration is a localization method based on distance measurements between the target and reference objects whose locations are known [39]. It is a common operation which has been widely applied in many research areas and practical applications such as kinesiology [40], aviation [41], crystallography [42], computer graphics [43], and navigation including Global Positioning Systems (GPS).

In distance-based localization schemes, the distance between a reference node and the target node is decided by the measured parameter such as TOA and RSS. Consider in a 2D plane, if the measured distance value is exactly accurate, the unknown target location will be on a circle whose center is at the reference node position, and the radius of the circle is the measured distance between the reference node and the target node, as shown in Fig. 2.1.
Figure 2.1: Target node on a circle to the center of reference node position with radius of measured distance.

When there are two reference nodes available in the network, as shown in Fig. 2.2, the two circles can intersect at two points which indicate both possible target node location.

In order to get the absolute target node location in a 2D plane, we need at least three reference nodes available in the network. As shown in Fig. 2.3, given three reference nodes, the three circles can intersect at one point which corresponds to the estimated target location. Let \((x_i, y_i)\) and \(d_i, i = 1, 2, 3\) denote the locations of the three reference nodes and the distances between the target and three reference nodes, the intersection of the three circles can be obtained by solving the system of equations

\[
\begin{align*}
(x - x_1)^2 + (y - y_1)^2 &= d_1^2, \\
(x - x_2)^2 + (y - y_2)^2 &= d_2^2, \\
(x - x_3)^2 + (y - y_3)^2 &= d_3^2.
\end{align*}
\]  

By subtracting the last equation from the first and second ones, (2.1) becomes

\[
\begin{align*}
(x - x_1)^2 - (x - x_3)^2 + (y - y_1)^2 - (y - y_3)^2 &= d_1^2 - d_3^2, \\
(x - x_2)^2 - (x - x_3)^2 + (y - y_2)^2 - (y - y_3)^2 &= d_2^2 - d_3^2.
\end{align*}
\]  

(2.2)
2.1. Trilateration based Localization

Figure 2.2: Two possible target node locations when two reference nodes available.

In order to give linear equations in \((x, y)\), (2.2) can be rearranged as

\[
\begin{align*}
2x(x_3 - x_1) + 2y(y_3 - y_1) &= (d_1^2 - d_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2), \\
2x(x_3 - x_2) + 2y(y_3 - y_2) &= (d_1^2 - d_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2).
\end{align*}
\]  \tag{2.3}

(2.3) can be expressed in matrix form as

\[
2\begin{bmatrix}
  x_3 - x_1 & y_3 - y_1 \\
  x_3 - x_2 & y_3 - y_2
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix} =
\begin{bmatrix}
  (d_1^2 - d_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\
  (d_1^2 - d_2^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2)
\end{bmatrix}.
\]  \tag{2.4}

When the three reference nodes are not located on a same line, the intersection of the three
circles which corresponds to the estimated target location can be obtained by

\[
\begin{bmatrix}
  x \\
  y 
\end{bmatrix} = \frac{1}{2} \begin{bmatrix}
  (d_1^2 - d_3^2) - (x_1^2 - x_3^2) - (y_1^2 - y_3^2) \\
  (d_2^2 - d_3^2) - (x_2^2 - x_3^2) - (y_2^2 - y_3^2) 
\end{bmatrix} \begin{bmatrix}
  x_3 - x_1 & y_3 - y_1 \\
  x_3 - x_2 & y_3 - y_2 
\end{bmatrix}^{-1}.
\]

The above derivations are based on the assumption that the distance measurements are error-free. However, measurement error always exist in realistic environment and can be caused by various factors, such as multipath channel, shadowing effect, and additive noise. As a result, the three circles in 2.3 will not intersect at one point, and there will be no solution for the system of equations in (2.1). In this circumstance, more than minimum number of reference nodes are needed to give a overdetermined system of equations. Assume \( n \) reference nodes are available
for the localization purpose, the matrix form of the equations can be expressed as

\[
2 \begin{bmatrix}
    x_n - x_1 & y_n - y_1 \\
    \vdots & \vdots \\
    x_n - x_{n-1} & y_n - y_{n-1}
\end{bmatrix}
\begin{bmatrix}
    x \\
    y
\end{bmatrix}
= \begin{bmatrix}
    (d_1^2 - d_n^2) - (x_1^2 - x_n^2) - (y_1^2 - y_n^2) \\
    \vdots \\
    (d_{n-1}^2 - d_n^2) - (x_{n-1}^2 - x_n^2) - (y_{n-1}^2 - y_n^2)
\end{bmatrix}.
\]

(2.6)

Let

\[
A = 2 \begin{bmatrix}
    x_n - x_1 & y_n - y_1 \\
    \vdots & \vdots \\
    x_n - x_{n-1} & y_n - y_{n-1}
\end{bmatrix},
\]

(2.7)

\[
p = \begin{bmatrix}
    x \\
    y
\end{bmatrix},
\]

(2.8)

and

\[
b = \begin{bmatrix}
    (d_1^2 - d_n^2) - (x_1^2 - x_n^2) - (y_1^2 - y_n^2) \\
    \vdots \\
    (d_{n-1}^2 - d_n^2) - (x_{n-1}^2 - x_n^2) - (y_{n-1}^2 - y_n^2)
\end{bmatrix},
\]

(2.9)

The system of equations in (2.6) can be expressed as

\[
Ap = b.
\]

(2.10)

The vector \( p \) which corresponds to the position of the target node can be decided by minimizing the mean square error

\[
\|Ap - b\|.
\]

(2.11)

The mean square error in (2.11) can be written in the expanded form as

\[
\]

(2.12)

Take the derivative of the mean square error in (2.12) with respect to \( p \), and set it to 0, we can get

\[
2A^T Ap - 2A^T b = 0 \leftrightarrow A^T Ap = A^T b.
\]

(2.13)
Then estimated target position can be expressed as

\[ \mathbf{p} = (\mathbf{A}^\mathsf{T} \mathbf{A})^{-1} \mathbf{A}^\mathsf{T} \mathbf{b}. \]  

(2.14)

### 2.2 Maximum Likelihood Estimation

In trilateration based localization scheme, the target location is estimated based on minimizing the mean square error of the distance estimation from available reference nodes. However, the probability distribution of the measurement error is not considered in the minimization problem. The measurement error of different signal features can have different probability distribution. In addition, the variance of the measurement error can become large in complex signal propagation environment. As a result, minimizing the mean square error of distance measurement without considering the error probability model will not give an optimum target location estimation result.

In Maximum Likelihood Estimation (MLE), the unknown parameters in a statistical model are estimated through maximizing the joint probability of having a set of independent and identically distributed observed data. Let \( \mathbf{X} \) denote the observed data samples \((x_1, x_2, \ldots, x_n)\) and \( \theta \) denote the parameter vector to be estimated in a statistical model, the joint probability density function of having \( n \) observations can be expressed as

\[ P(\mathbf{X}|\theta) = p(x_1|\theta) \cdot p(x_2|\theta) \cdots p(x_n|\theta), \]  

(2.15)

where \( p(x_n|\theta) \) is the conditional probability of having the observed data sample \( x_n \) when the parameter vector is \( \theta \). In practice, (2.15) is usually transferred to log-likelihood function as

\[ L(\mathbf{X}|\theta) = \sum_{i=1}^{n} \ln(p(x_i|\theta)). \]  

(2.16)

Then the unknown parameters in the statistical model can be estimated through minimizing the log-likelihood function in (2.16) with respect to \( \theta \).

In wireless location estimation, the observed data samples correspond to those measured location dependent parameters from the reference nodes, and the unknown parameter vector
\[ L(\mathbf{M}|x, y) = \sum_{i=1}^{n} ln(p(m_i|x, y)). \] (2.17)

Let \( \nu_i \) denote the measurement error from the \( i \)th reference node, the measured dependent parameter can be expressed as

\[ m_i = f_i(x, y) + \nu_i, \] (2.18)

where \( f(x, y) \) is the true value of the parameter between the target and the \( i \)th reference node. Therefore, (2.17) can be expressed as

\[ L(\mathbf{M}|x, y) = \sum_{i=1}^{n} g_i(x, y), \] (2.19)

where \( g_i(x, y) = ln(p(m_i - f_i(x, y))) \). Therefore, the target location can be estimated as

\[ (\hat{x}, \hat{y}) = \arg \min_{x, y} L(\mathbf{M}|x, y). \] (2.20)

Figure 2.4 shows an example of the localization process using MLE algorithm. The signal features between the reference nodes and target node are recorded and sent to the data center. The location of the target node is calculated using Maximum Likelihood Estimation (MLE) based on the statistical model of the signal measurement error.

By applying the Maximum Likelihood Estimation, the location estimation result will be more accurate than trilateration based localization where the mean square error is minimized without considering the probability distribution of the measurement errors between the target and reference nodes.

### 2.3 Fingerprinting based Localization

In trilateration based localization scheme and the MLE discussed above, the signal propagation model, which is the relationship between the distance and the measured signal feature, is as-
assumed to be known and fixed in the measurement environment. In practice, the parameters in signal propagation model between the target and reference nodes can also change significantly in complex environment. For example, in RSS-based model, there is an important parameter called path loss exponent which reflects how fast the signal strength decays with the distance increase. This parameter is highly sensitive the signal propagation environment. Moreover, in indoor environment when there is obstacles, such as tables or chairs, between the target and a reference node, the corresponding link will have a large distance estimation error, and the signal propagation model will not be applicable on that link.

Different from those wireless localization schemes based on distance estimation between target and reference nodes, fingerprinting based localization compares the received signal strength indicator (RSSI) with a radio map which is generated in offline phase, in order to decide the target position. The environment related information, such as the floor plan of a
building, is applied in the offline phase of radio map construction, so that the features of the measurement environment can be taken into account in localization process.

In the construction of signal radio map, the localization service area is divided into cells, and the signal strength is collected at a specific signal collection point inside each cell. The RSSI values measured from different reference nodes at each signal collection point can be expressed in a vector and stored in the signal radio map. Consider a localization service area with \( N \) available reference nodes, and assume the area is divided into \( M \) cells, the measured RSSI value at the \( i \)th signal collection point from the \( j \)th reference node can be expressed as \( m_{ij} \), where \( 1 \leq i \leq M, 1 \leq j \leq N \). Let \( p_i \) denote the position of the \( i \)th signal collection point, and \( \mathbf{m}_i = (m_{i1}, m_{i2}, \cdots m_{iN}) \) denote the measured RSSI vector, the signal radio map can be expressed as \( \mathbf{M} = \{\mathbf{m}_i, p_i \}_{1 \leq i \leq M} \). Before applying the constructed radio map in the online phase of target location estimation, the map can be also preprocessed for the purpose of reducing the overall cost of the localization system [44].

In the online phase of target location estimation, the measured RSSI values from reference nodes at target side are compared with the recorded values in the constructed signal radio map, in order to decide the target position. A general algorithm to estimate the target location is to first assign weights to the signal collection point in each the cell of the signal radio map, and then to calculate the target location by use of the weighted mean of all the signal collection points. Let \( w_i \) denote the weight of the \( i \)th signal collection point, the estimated target location can be expressed as

\[
\hat{\mathbf{p}} = \frac{\sum_{i=1}^{M} w_i}{W} \mathbf{p}_i, \tag{2.21}
\]

where \( W = \sum_{i=1}^{M} w_i \). The values of weights are decided based on the difference between the measured RSSI and the recorded RSSI values in the map. In [45], \( p \)-norm is applied to calculate the difference as

\[
\|\mathbf{m}_i - \mathbf{r}_i\|_p = \left( \sum_{j=1}^{N} |m_{ij} - r_{ij}|^p \right)^{\frac{1}{p}}, \tag{2.22}
\]

where \( \mathbf{m}_i \) and \( \mathbf{r}_i \) are the vectors of measured RSSI values and recorded RSSI values in the signal radio map, respectively. The Euclidean norm (when \( p = 1 \)) and Manhattan norm (when \( p = 2 \)) are widely applied in fingerprinting based localization algorithms [44], [45]. Then the
weight of the \( i \)th signal collection point can be calculated by

\[
w_i = \frac{1}{\|m_i - r_i\|}.
\]  

(2.23)

Fig. 2.5 shows the overall process of fingerprinting based localization scheme including the offline phase for radio map construction and the online phase for target location estimation.
2.3. **Fingerprinting based Localization**

Figure 2.5: Location estimation using fingerprinting based method.
Chapter 3

RSS-based Localization in Complex Environment with Unknown Path Loss Exponent

3.1 Introduction

As discussed in Chapter 1, one of the most challenging problems of wireless localization is due to the high measurement error of location dependent parameters in complex signal propagation environment. Especially in Received Signal Strength (RSS)-based localization, the signal power can drop significantly when the direct path between transmitter and receiver is obstructed. In this chapter, we study on the problem of RSS-based localization in complex environment, and propose a novel algorithm which can improve the performance of conventional localization algorithms when there are obstructions existing among transmitters and receivers.

The essence of wireless localization is to measure the location dependent parameters in the received signals, and then to estimate the location of the target by proper processing of the measured parameters. Based on different types of location dependent parameters, wireless localization schemes can be generally divided into three categories. In localization using Time of Arrival (TOA) [11] and Time Difference of Arrival (TDOA) [12], the wireless devices need to be equipped with advanced receivers with capability of high speed signal processing in
order to measure the signal propagation time. In Angle of Arrival (AOA)-based localization [13], directional antenna is needed to measure the angle of the received signal, and the antenna has to be accurately calibrated. Compared with the above two types of localization schemes, RSS-based localization technique is low cost and easily implemented. Most of today’s wireless devices have internal RF chips which can output the received signal strength indication (RSSI) directly without any additional hardware support.

The advantages of RSS based localization have been attracting great attention from researchers. In RSS-based localization, the main drawback is that the complexity of signal propagation environment can have a large impact on the localization performance. Signal attenuations can be caused by multipath and shadowing effect in complex environment. In addition, the PLE is also an environmentally dependent parameter which reflects how fast the signal power decays with distance increase. When the signal measurements are taken in an unknown environment, the PLE can be regarded as an unknown parameter. The assumption of a pre-known PLE value in previous research works is another error source of RSS-based localization [49]. RSS-based localization with unknown PLE has been recently considered in [50]-[52].

Generally, the RSS parameters are measured through a set of reference nodes whose positions are known. With more than minimum required number of reference nodes available, maximum likelihood estimation (MLE) can be applied to estimate the target location. However, when there is obstruction existing between a reference node and target, the signal power can drop significantly on the corresponding obstructed link. Based on our research, we have observed that when the obstruction of the signal is significant, it is better to discard the obstructed link rather than using it in MLE. A noticeable work which consider the obstruction effect is [53]. In this work, the authors measured a Min-Max region where the radio ranges of the reference nodes overlaps, and detected the obstruction based on whether the estimated target location is inside the Min-Max region. However, the Min-Max bound is obtained through experimental work which is labor-consuming. The performance of the proposed method can degrade when the radio ranges of the reference nodes are large. In addition, the PLE parameter is assumed to be known in [53].

In this chapter, we propose a novel algorithm which can automatically detect the obstruction with unknown PLE during the localization process. A key feature of our proposed algorithm is
that it provides a methodology to improve the accuracy of the localization results in complex environment without requiring off-line pre-processing.

### 3.2 RSS-based Localization in Obstructed Environment

#### 3.2.1 Signal Propagation Model and Problem Statement

In real wireless communication channels, the received signal power is proportional to \( d^{-\alpha} \), where \( d \) is the distance between the transmitter and receiver, and \( \alpha \) is the PLE parameter which reflects how fast the signal power decays with the distance increase in a certain environment. According to [54], the range of PLE parameters in different types of environment is shown in Table. 3.1.

<table>
<thead>
<tr>
<th>Type of environment</th>
<th>PLE range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free space</td>
<td>2</td>
</tr>
<tr>
<td>Indoor line-of-sight environment</td>
<td>1.6 - 1.8</td>
</tr>
<tr>
<td>Obstructed environment in factories</td>
<td>2 - 3</td>
</tr>
</tbody>
</table>

Table 3.1: Range of PLE parameters in different types of environment.

The relationship between the received signal power \( P_r \) and the distance \( d \) between the transmitter and receiver can be written as

\[
P_r = p_{d_0} \left( \frac{d}{d_0} \right)^{-\alpha}, \tag{3.1}
\]

where \( p_{d_0} \) is the signal power at reference distance \( d_0 \) away from the transmitter. Take the log on both side of (3.1) to express in decibel units, the equation becomes

\[
P_r = P_{d_0} + 10\alpha \log_{10} \left( \frac{d}{d_0} \right), \tag{3.2}
\]

where \( P_r = 10\log_{10} p_r \), and \( P_{d_0} = 10\log_{10} p_{d_0} \). In real environments, the received signal strength always has random variation due to the shadowing and multipath effect. Based on a large number of experiment and analytical results [55] - [58], the random signal attenuation is typically modeled as Gaussian distribution in decibels. Therefore, the relationship between the received
signal power and distance can be expressed as

$$RSS = P_{d_0} - 10\alpha \log_{10}(\frac{d}{d_0}) + \nu, \quad (3.3)$$

where $RSS$ and $P_{d_0}$ are the received signal strength in decibels at distance $d$ and reference distance $d_0$ respectively, $\alpha$ is the path loss exponent (PLE), and $\nu$ is the random signal attenuation which has Gaussian distribution with zero-mean. The reference distance $d_0$ is usually set to 1m. Therefore, (3.4) can be rewritten as

$$RSS = P_0 - 10\alpha \log_{10}d + \nu, \quad (3.4)$$

where $P_0$ is the signal power at 1m distance away from the transmitter. The rest of the discussion in this chapter will be based on the signal propagation model in (3.4).

In RSS-based localization technologies, the performance is highly sensitive to the environment complexity. In obstructed environments, the received signal can drop significantly when there is obstruction between transmitter and receiver. In the signal propagation model (3.4), the measurement error $\nu$ on the corresponding obstructed link will be much larger than on other links. In conventional localization algorithms, the variance of the random signal attenuations is usually assumed to be fixed on all the links, which can degrade the localization performance in complex environment. When the signal attenuation from one of the available reference nodes is much larger than others, it is better to discard the corresponding unreliable link between that reference node and the target in the localization algorithm. However, it is difficult to detect those unreliable links during the localization process, since the position of the target node and the obstructions are unknown.

Considering the above problem, we propose a novel algorithm which automatically detect the unreliable links. Before introducing our algorithm of localization in complex environment, we will first discuss the conventional MLE algorithm and derive the Cramer-Rao Bound (CRB) in order to show the impact of the environment complexity on the localization performance.
3.2.2 MLE algorithm for RSS-based Localization

When there are more than minimum required number of reference nodes available in the network, MLE can be applied to estimate the parameter vector \((x, y)\) which is the location of the target. Consider a wireless network with \(n\) reference nodes \((n > 3)\) whose positions are known as \((x_i, y_i)\), \(1 \leq i \leq n\). Based on the RSS-distance relationship model in (3.4), the received power at target side can be expressed as

\[
RSSI_i = P_0 - 10\alpha \log_{10} \sqrt{(x - x_i)^2 + (y - y_i)^2} + \nu_i, \quad (3.5)
\]

where \(RSSI_i\) is the observed RSSI value from the \(i\)th reference node, \(\nu_i\) is the signal attenuation caused by shadowing effect between the target and the \(i\)th reference node. Let \(g_i(x, y) = P_0 - 10\alpha \log_{10} \sqrt{(x - x_i)^2 + (y - y_i)^2}\), (3.5) can be rewritten as

\[
RSSI_i = g_i(x, y) + \nu_i, \quad (3.6)
\]

For those unobstructed links, the signal attenuation \(\nu_i\) can be modeled as random Gaussian variables with zero-mean as discussed in the signal propagation model. The standard deviation of the signal attenuations on unobstructed links can be obtained through statistical study. Let \(\sigma_i\) denote the standard deviation of signal attenuation on the \(i\)th unobstructed link, the probability density function of received signal strength on the \(i\)th unobstructed link can be expressed as

\[
f(RSSI_i|x, y) = \frac{1}{\sqrt{2\pi\sigma_i}}\exp\left\{\frac{(RSSI_i - g_i(x, y))^2}{2\sigma_i^2}\right\}, \quad (3.7)
\]

Based on the assumption that all the measurements between reference nodes and target are independent, the joint probability density function of having \(n\) observations at target side can be expressed as

\[
f(RSSI|x, y) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i}}\exp\left\{\frac{(RSSI_i - g_i(x, y))^2}{2\sigma_i^2}\right\}, \quad (3.8)
\]

where \(RSSI = (RSSI_1, RSSI_2, \ldots, RSSI_n)\). The essence of MLE method is to maximize the joint propagability density function in (3.8) with respect to the coordinates of the unknown target location \(x\) and \(y\). Maximizing the joint probability in (3.8) corresponds to minimizing
3.2. RSS-based Localization in Obstructed Environment

the log-likelihood function

\[
l(\text{RSSI}|x,y) = \min_{x,y} \sum_{i=1}^{n} \frac{(RSSI_i - g_i(x,y))^2}{2\sigma_i^2}.
\]  

(3.9)

The minimization result of (3.9) is the most likely target location which can be expressed as

\[
(\hat{x}, \hat{y}) = \arg\min_{x,y} l(\text{RSSI}|x,y).
\]  

(3.10)

In obstructed environment, the signal attenuations on certain links can be much larger than other links due to the shadowing and multipath effects. In addition, when there is obstruction between a reference node and the target, the signal strength can drop significantly in the corresponding link. Using these links in MLE of target location will decrease the localization accuracy. Therefore, we call those links with large signal attenuation as unreliable links. On those reliable links, we use a same standard deviation value for the signal attenuations. Fig. 3.1 shows the reliable links and unreliable links in an obstructed environment. Let \( \ell_u \) denote the set of unreliable links, and let \( \ell_r \) denote the set of reliable links, the MLE algorithm in obstructed

![Figure 3.1: Reliable and unreliable links in obstructed environment.](image-url)
environment can be expressed as

\[
(\hat{x}, \hat{y}) = \arg \min_{x,y} \left\{ \sum_{i \in \ell_r} \frac{(RSSI_i - g_i(x, y))^2}{2\sigma_r^2} + \sum_{i \in \ell_u} \frac{(RSSI_i - g_i(x, y))^2}{2\sigma_i^2} \right\},
\]

(3.11)

where \(l_i\) is the link between the \(i\)th reference node and target, \(\sigma_i\) is the standard deviation of the signal attenuation on \(i\)th link, and \(\sigma_r\) is the standard deviation of signal attenuations on reliable links.

### 3.2.3 Cramer-Rao Bound of RSS-based Localization

In parameter estimation problems, generally there exists random variation between the estimation result and the true value. According to Cramer-Rao Inequality \([59]\), the minimum possible variance achievable by any unbiased estimator can be lower bounded by the well-known Cramer-Rao Bound (CRB). Recently, many research works evaluate the performance of a localization algorithm through CRB analysis. In location estimation using MLE, the unknown parameters to be estimated include the coordinates of the target location. Therefore, the CRB of MLE reflects how accurate the target location estimation result can achieve, which corresponds to the performance of a localization scheme.

In real wireless communication channels, due to the random signal attenuation on each link between a reference node and the target, the location estimator will exhibit random variation between the MLE result and the true target location. The lower bound of the variance between the estimated and true target location is given by the Fisher Information

\[
F_n(\theta) = -E\left[ \frac{\partial^2 l(X|\theta)}{\partial \theta \partial \theta^T} \right],
\]

(3.12)

where \(l(\cdot)\) represents the log-likelihood function in the MLE estimation, \(X\) corresponds to the \(n\) observations, and \(\theta\) is the parameter vector to be estimated. In the RSS-based localization problems, the \(n\) observations correspond to \(n\) RSSI measurements, and the parameter vector corresponds to the coordinate of the target location \((x, y)\). Given the log-likelihood function in
3.2. RSS-based Localization in Obstructed Environment

Equation (3.9), the Fisher Information can be written as following matrix form

\[ F_n(\theta) = F_n(x, y) = \begin{bmatrix} F_{xx} & F_{xy} \\ F_{yx} & F_{yy} \end{bmatrix}, \quad (3.13) \]

where

\[ \begin{align*}
F_{xx} &= -E\left[ \frac{\partial^2 l(\text{RSSI}|x, y)}{\partial x \partial x} \right], \\
F_{xy} &= F_{yx} = -E\left[ \frac{\partial^2 l(\text{RSSI}|x, y)}{\partial x \partial y} \right], \\
F_{yy} &= -E\left[ \frac{\partial^2 l(\text{RSSI}|x, y)}{\partial y \partial y} \right].
\end{align*} \quad (3.14) \]

The covariance matrix of the location estimator is lower bounded by

\[ \text{cov}(\hat{x}, \hat{y}) - F_n^{-1}(x, y) \geq 0, \quad (3.15) \]

where

\[ \text{cov}(\hat{x}, \hat{y}) = \begin{bmatrix} E[(\hat{x} - x)^2] & E[(\hat{x} - x)^2] \\ E[(\hat{y} - y)^2] & E[(\hat{y} - y)^2] \end{bmatrix}, \quad (3.16) \]

and

\[ F_n^{-1}(x, y) = \frac{1}{|F_n(x, y)|} \begin{bmatrix} F_{yy} & -F_{xy} \\ -F_{yx} & F_{xx} \end{bmatrix}. \quad (3.17) \]

Based on (3.15), (3.16), (3.17), the CRB which corresponds to the minimum achievable localization error is given by

\[ E[(\hat{x} - x)^2 + (\hat{y} - y)^2] \geq \frac{F_{xx} + F_{yy}}{F_{xx}F_{yy} - F_{xy}^2}. \quad (3.18) \]

In [55], the elements of $F_{xx}$, $F_{xy}$, and $F_{yy}$ have been derived based on the condition that the standard deviations of signal attenuations on each link have a same value ($\sigma_i = \sigma_r, i = 1, 2 \cdots n$), and the CRB of RSS-based localization without obstruction effect is obtained as

\[ \text{CRB} = \frac{1}{c_r} \cdot \frac{\sum_{i=1}^{n} \frac{1}{d_i^2}}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(\sin \theta_i)^2}{d_i^2 d_j^2}}, \quad (3.19) \]
where
\[ c_r = \left( \frac{10\alpha}{\sigma_r \ln 10} \right)^2. \] (3.20)

As discussed in Section 3.2.2, in complex environment, the signal variation on unreliable links can be much larger than on reliable links. Therefore, the elements derived in [55] cannot be applied when there is obstruction effect. In this section, the CRB of RSS-based localization in complex environment will be derived.

Consider in a more general sense with different standard deviation \( \sigma_i \) on each link, the elements in (3.18) can be expressed as

\[
\begin{align*}
F_{xx} &= \sum_{i=1}^{N} \frac{c_i (x - x_i)^2}{d_i^4}, \\
F_{xy} &= F_{yx} = \sum_{i=1}^{N} \frac{c_i (x - x_i)(y - y_i)}{d_i^4}, \\
F_{yy} &= \sum_{i=1}^{N} \frac{c_i (y - y_i)^2}{d_i^4},
\end{align*}
\] (3.21)

where
\[ c_i = \left( \frac{10\alpha}{\sigma_i \ln 10} \right)^2. \] (3.22)

Note that \( \alpha \) is the parameter of path loss exponent which has been mentioned in Section II.

Based on (3.18) and (4.12), the CRB of RSS-based localization is derived as

\[
\text{CRB} = \frac{F_{xx} + F_{yy}}{F_{xx}F_{yy} - F_{xy}^2} = \frac{\sum_{i=1}^{n} \frac{c_i (x - x_i)^2}{d_i^4} + \sum_{i=1}^{n} \frac{c_i (y - y_i)^2}{d_i^4}}{\sum_{i=1}^{n} \frac{c_i (x - x_i)^2}{d_i^4} \sum_{i=1}^{n} \frac{c_i (y - y_i)^2}{d_i^4} - \left( \sum_{i=1}^{n} \frac{c_i (x - x_i)(y - y_i)}{d_i^4} \right)^2}
\]

\[
= \frac{\sum_{i=1}^{n} \frac{c_i}{d_i^2}}{\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{c_i c_j (x - x_i)(y - y_j)(x - x_j)(y - y_i)}{d_i^4 d_j^4}}
\]

\[
= \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{c_i c_j (x - x_i)(y - y_i)(x - x_j)(y - y_j)}{d_i^4 d_j^4}}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{c_i c_j (x - x_i)(y - y_i)(x - x_j)(y - y_j)}{d_i^4 d_j^4}}.
\] (3.23)
To simplify the result in (3.23), let \( \theta_i \) denote the angle between the x-axis and the line segment \( d_i \) which connects the target and reference node \( i \), then the angel between \( d_i \) and \( d_j \) can be expressed as \( \theta_{ij} = \theta_i - \theta_j \). According to the trigonometric identity, we can derive

\[
\sin(\theta_{ij}) = \sin(\theta_i - \theta_j) = \sin \theta_i \cos \theta_j - \cos \theta_i \sin \theta_j = \frac{y_i - y_j}{d_i} \cdot \frac{x_j - x_i}{d_j} - \frac{x_i - x_j}{d_i} \cdot \frac{y_i - y_j}{d_j}.
\]

By introducing (3.24) into (3.23), the CRB result can be simplified as

\[
\text{CRB} = \frac{\sum_{i=1}^{n} \frac{c_r^2}{d_i^2}}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{c_r \sin(\theta_{ij})^2}{d_i^2 d_j^2}}.
\]

In obstructed environment, the standard deviations of signal attenuation on unreliable links are different from reliable links. Consider in an obstructed environment, assume the first \( m \) links are reliable links and the rest of the links are unreliable links, the CRB of RSS-based localization in complex environment can be expressed as

\[
\text{CRB}_u = \frac{c_r \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} \frac{(\sin(\theta_{ij}))^2}{d_i^2 d_j^2} + c_r \sum_{i=1}^{m} \sum_{j=m+1}^{n} \frac{c_r \sin(\theta_{ij})^2}{d_i^2 d_j^2} + \sum_{i=m+1}^{n-1} \sum_{j=i+1}^{n} \frac{c_r \sin(\theta_{ij})^2}{d_i^2 d_j^2}}{c_r \sum_{i=1}^{m} \frac{1}{d_i^2} + \sum_{i=m+1}^{n} \frac{c_r}{d_i^2}}.
\]

When there are unreliable links existing in an obstructed environment, the localization performance can change significantly. In order to demonstrate the impact of obstruction effect on the localization accuracy, we compare the CRB result of RSS-based localization in unobstructed environment with the CRB in complex environment. We first place four reference nodes at the corners of a 20m by 20m square, and calculate the CRB of localization error when the target is located inside the square. Fig. 3.2 shows the CRB results with \( \alpha = 3 \) and \( \sigma_r = 5dB \).

Then we change one of the four links in Fig. 3.2 to unreliable link with higher standard deviation of signal attenuation as 10dB, and remain the other links as reliable links with standard
deviation of signal attenuation as $5\, dB$. We set the link between target and the reference node at position $(10, -10)$ as unreliable link. The CRB of the localization error is shown in Fig. 3.3.

### 3.3 RSS-based Localization with Unknown PLE

When the target nodes and the Reference Nodes (RN) are deployed in an unknown environment, the PLE $\alpha$ is an unknown parameter. Assuming that $P_0$ is a fixed parameter for all the reference nodes in the network, the localization problem in an unknown environment is to estimate the PLE and the target location. Depending on whether the PLE parameter and the target location are estimated jointly or separately, there are two kinds of estimation schemes.
3.3. RSS-based Localization with Unknown PLE

Figure 3.3: CRB of RSS-based localization in obstructed environment.

3.3.1 Joint Estimation Algorithm

Given $n$ RNs in a wireless network with pre-known positions denoted as $(x_i, y_i)$, $1 \leq i \leq n$, the received power at target side from $RN_i$ (ith reference node) can be expressed as same as in (3.5)

$$RSSI_i = P_0 - 10 \alpha \log_{10} \sqrt{(x - x_i)^2 + (y - y_i)^2} + v_i,$$

where $RSSI_i$ is the observed RSSI value from $RN_i$, $v_i$ is the signal attenuation caused by shadowing effect between the target and $RN_i$. The difference is that the PLE $\alpha$ in (3.27) is an unknown parameter.

Together with the unknown target coordinates, there are altogether three parameters ($\alpha$, $x$, and $y$) in the localization problem. For simplicity of notation, the three parameters can be collectively shown as $\theta = [\alpha, x, y]$ in joint estimation algorithm. Using the newly defined $\theta$,
equation 3.27 can be rewritten as

\[ \text{RSSI}_i = g_i(\theta) + \nu_i, \quad (3.28) \]

where \( g_i(\theta) = P_0 - 10\alpha \log_{10} \sqrt{(x - x_i)^2 + (y - y_i)^2} \). When there are more than three reference nodes available in the network, Maximum Likelihood Estimation (MLE) can be applied to estimate the parameter vector \( \theta \). Since \( \nu_i (i = 1, 2, \ldots, n) \) are independent Gaussian random variables with zero mean and standard deviation of \( \sigma_\nu \), the joint distribution of the observed RSSI values can be expressed as

\[ f(\text{RSSI}|\theta) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_\nu}} e^{\frac{-\left(\text{RSSI}_i - g_i(\theta)\right)^2}{2\sigma_\nu^2}}. \quad (3.29) \]

The MLE of PLE and target location can be obtained by jointly minimizing the following likelihood function

\[ (\hat{\alpha}, \hat{x}, \hat{y}) = \arg \min_{\theta} \sum_{i=1}^{n} \left( \text{RSSI}_i - g_i(\theta) \right)^2. \quad (3.30) \]

However, if there is obstruction between a reference node and a target node as shown in Fig. 3.4, the signal power can drop significantly on the corresponding obstructed link, which can reduce the localization accuracy of the MLE.

### 3.3.2 Separated Estimation Algorithm

Other localization schemes with unknown PLE consider to estimate the PLE parameter and the target location separately. Firstly, the PLE is estimated based on the links among reference nodes. Since the positions of reference nodes are already known, the first step of PLE estimation is a one-parameter optimization problem. The estimated PLE \( \alpha \) is then used as a known parameter in the second step for the target location estimation. In the separated estimation algorithm, the optimization computation at target side is lighter than the joint estimation algorithm, because the number of unknown parameters to be estimated reduces from three to two. This makes the separated estimation algorithm a desirable scheme for the systems with constrained computation resources.

In the first step of PLE estimation, the RSSI values at each reference node can be written...
3.3. RSS-based Localization with Unknown PLE

As

\[ RSSI_{ij} = P_0 - 10 \alpha \log_{10} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} + \nu_{ij}, \]  

(3.31)

where \( RSSI_{ij} \) is the RSSI at \( RN_i \) from \( RN_j \), \((x_i, y_i)\) and \((x_j, y_j)\) are the coordinates of \( RN_i \) and \( RN_j \), and \( \nu_{ij} \) is the signal attenuation on the link between \( RN_i \) and \( RN_j \). Since the PLE \( \alpha \) is the only unknown parameter in equation (7), we rewrite it as

\[ RSSI_{ij} = g_{ij}(\alpha) + \nu_{ij}, \]  

(3.32)

where \( g_{ij}(\alpha) = P_0 - 10 \alpha \log_{10} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \). By applying the MLE, the estimation of PLE becomes

\[ \hat{\alpha} = \min_{\alpha} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (RSSI_{ij} - g_{ij}(\alpha))^2. \]  

(3.33)

The PLE parameter obtained in equation (9) can be used as a known parameter in the next step for target location estimation, donated as \( \alpha_0 \). Therefore in target location estimation, the number of parameters to be optimized reduces from three to two compared with the joint
estimation algorithm. Equation (6) becomes

\[(\hat{x}, \hat{y}) = \arg \min_{x,y} \sum_{i=1}^{n} (RSS_{i} - g_{i}(x,y))^{2},\]  

(3.34)

where \(g_{i}(x,y) = P_{0} - 10\alpha_{0}\log_{10}\sqrt{(x-x_{i})^{2} + (y-y_{i})^{2}}\). However, if there are obstructed links among reference nodes as shown in Fig. 3.5, the PLE estimation in the first step will be inaccurate. As a result, using the inaccurate PLE in the second step reduces the localization accuracy. Moreover, the obstruction between reference nodes and target node can cause additional error in the second step of target location estimation.

![Figure 3.5: Obstruction between two reference nodes.](image)

**3.4 Proposed Algorithm**

Considering the drawbacks of the existing methods in obstructed environments described in section II, we propose a novel algorithm with unknown PLE which can detect and remove the obstructed links, and improve the localization accuracy in obstructed environments. We choose the separated estimation algorithm, so that our method can be applied to systems with
constrained computation resources. The proposed algorithm can be briefly described in two phases as shown below:

**Phase 1 PLE Estimation**

1. Use all the links among reference nodes (including obstructed links), and apply MLE to estimate a rough PLE parameter, denoted as $\alpha_0$;
2. Apply $\alpha_0$ to calculate the signal attenuation on each link, and obtain an averaged signal attenuation;
3. Compare the calculated signal attenuation on each link with the averaged one, to decide which links are obstructed links;
4. Remove the obstructed links and do MLE again to estimate a more accurate PLE parameter, denoted as $\alpha_1$.

**Phase 2 Target Location Estimation**

1. Apply $\alpha_1$ as the PLE parameter;
2. Use all the links between the target and each reference node (including obstructed links), and apply MLE to estimate a rough target location;
3. Apply the estimated rough target location to calculate the signal attenuation on each link, and obtain an averaged signal attenuation;
4. Compare the calculated signal attenuation on each link with the averaged one, to decide which links are obstructed links;
5. Remove the obstructed links and do MLE again to get a more accurate target location estimation.

The implementation details of phase 1 and phase 2 will be described in the rest of this section.

### 3.4.1 PLE Estimation

When there are obstructed links among reference nodes, the result of PLE estimation in equation (3.33) can be highly affected by the obstruction effect. However, we can utilize the inaccurate estimation result to detect those obstructed links. Denote the estimation result in equation (3.33) as $\alpha_0$, we calculate signal attenuation between $RN_i$ and $RN_j$ as

$$v_{ij} = RSS I_{ij} - g_{ij}(\alpha_0),$$

(3.35)

The calculation of signal attenuation in equation (3.35) can be inaccurate due to the inaccuracies of $\alpha_0$. However, the corresponding calculated signal attenuations of the obstructed
links are in general much larger than the rest of the links. Therefore, we compare the signal attenuation of each link with the averaged signal attenuation of all the links to decide which links are obstructed links. The averaged value of signal attenuation among reference nodes can be expressed as

\[ \bar{\nu}_r = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} |\nu_{ij}|. \]  

(3.36)

With the comparison of \( \nu_{ij} \) against \( \bar{\nu}_r \), we can determine which links are obstructed links. We consider the link between \( RN_i \) and \( RN_j \) as an obstructed link when

\[ |\nu_{ij}| > k\bar{\nu}_r, \]  

(3.37)

where \( k \) is the threshold which can be chosen according to the complexity of the environment. Based our experiments, the value of \( k \) can be set between 1.5 to 2 depending on the number of obstructed links existing in the complex environment. When the number of obstructed links is few, the performance of our proposed algorithm is not sensitive to the parameter \( k \). The averaged signal attenuation \( \bar{\nu}_r \) is used as a decision factor to detect obstructed links. Let \( L_r \) denote all the links between two reference nodes, and let \( L_{r1} \) and \( L_{r2} \) denote the links with and without obstruction, respectively. The subset \( L_{r1} \) with obstruction is chosen by (3.37), and the subset without obstruction is calculated as \( L_{r2} = L_r - L_{r1} \). Then the MLE is once again used to recalculate the PLE using only the links without obstruction \( L_{r2} \):

\[ \hat{\alpha} = \min_{\alpha} \sum_{l_{ij} \in L_{r2}} (RSS_{l_{ij}} - g_{ij}(\alpha))^2, \]  

(3.38)

where \( l_{ij} \) is the unobstructed link between \( RN_i \) and \( RN_j \). The PLE estimation in equation (3.38) is more accurate than that of equation (3.33), since the obstructed links are removed. We denote the estimation result in (3.38) as \( \alpha_1 \).
3.4.2 Target Location Estimation

Target location estimation is a similar process as in phase 1. First we estimate target location roughly by equation (10) while using the more accurate PLE $\alpha_1$. The estimated rough target location is donated as $(x_0, y_0)$. Then we detect and remove the obstructed links among the target and the reference nodes. The signal attenuation between the target node and each reference node can be calculated as

$$
\nu_i = RS S I_i - g_i(x_0, y_0),
$$

(3.39)

The averaged attenuation among the target node and reference nodes can be expressed as

$$
\bar{\nu}_t = \frac{1}{n} \sum_{i=1}^{n} |\nu_i|.
$$

(3.40)

If $|\nu_i| > k\bar{\nu}_t$, then the link between the target and $RN_i$ is considered as an obstructed link. We remove the obstructed links and estimate the target location again:

$$(\hat{x}, \hat{y}) = \arg \min_{x,y} \sum_{l \in L_2} (RS S I_l - g_l(x, y))^2,$$

(3.41)

where $L_2$ is the subset of links without obstruction between the target node and $RN_i$.

3.5 Simulation Results

In the simulations, we establish an obstructed environment to evaluate the performance of our algorithm. We use 10 reference nodes to locate the target nodes, where the reference nodes are uniformly deployed on a circle with radius of $10m$. Target nodes are randomly distributed inside the circle. We deploy 100 target nodes in the simulations, and average 100 localization results to evaluate the performance of localization algorithms. For those links (between transmitter $i$ and receiver $j$) without obstruction in the experiment area, the standard deviations of signal attenuation are set to be equal to $\sigma$. The signal transmitting power at $1m$ away from each reference node is set to be $-27dB$ ($P_0 = -27dB$). Fig. 3.6 shows the localization results of joint estimation algorithm when there is no obstructed link, and $\sigma =$
3. The red circles represent the 10 reference nodes with known location, while the black points and blue asterisks stand for the real positions and estimated location of the target nodes respectively.

![Diagram showing localization results](image)

Figure 3.6: Localization results of joint estimation algorithm without obstruction, when $\sigma = 3$.

Root Mean Square Error (RMSE) is used to calculate average localization error of 100 target nodes:

$$RMSE = \sqrt{\frac{1}{100} \sum_{i=1}^{100} ((\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2)}.$$

When there are obstructed links in the environment, localization error becomes much larger. Fig. 3.7 shows the averaged localization result of the 100 targets with obstructed links. We randomly pick 1 to 3 links among each target and the reference nodes as obstructed links, and change the signal power loss on each obstructed links from 5dB to 20dB.

To compare the performance of the proposed algorithm with the joint estimation algorithm, all the obstructed links are set among the target and reference nodes. Fig. 3.8 shows the advan-
3.5. Simulation Results

Figure 3.7: Localization error with obstructed links, when $\sigma = 3$.

tage of the proposed algorithm over joint estimation algorithm in obstructed environment with 1, 2 and 3 obstructed links. In the proposed algorithm, the threshold $k$ is set to be 2.0, 1.8, and 1.5 in the three cases, respectively. In the simulation, the signal power loss on the obstructed links is set to be 15 dB, and $\sigma$ represents the standard deviation of the signal attenuation on unobstructed links. As shown in the simulation results, when $\sigma$ is small, the performance of the proposed algorithm shows obvious advantage. With the increase of $\sigma$, the difference between the proposed algorithm and joint estimation algorithm becomes smaller. This is attributed to the fact that when signal attenuations of unobstructed links increase, the influence of obstruction effect becomes relatively less. On the other hand, the removal of an obstructed link decreases the number of reference nodes by one. When the number of obstructed links increases to three, the performance of the proposed algorithm shows fluctuation. The reason is that with more obstructed links, the rough target location calculated in (10) becomes more inaccurate. As a result, the calculated signal attenuations of unobstructed links in equation (15) will be large, so that the averaged signal attenuation gets closer to the signal attenuations of obstructed links. Therefore, the detection of obstructed links becomes harder, and it is also
the reason why we choose a smaller $k$ value. In this circumstance, the proposed algorithm can make wrong decision and discard unobstructed links, which causes the fluctuation in the simulation result.

Figure 3.8: Joint Estimation vs. Proposed Algorithm with one, two and three obstructed links.
Chapter 4

Optimum Reference Node Deployment for TOA-based Localization

4.1 Introduction

In addition to the impact of environment complexity on the wireless localization performance, the placement of reference nodes is another important factor which can influence the localization accuracy significantly. In this chapter, we study on the accuracy of Time of Arrival (TOA)-based localization algorithm based on the Cramer-Rao Bound (CRB), and derive the optimum reference node deployment scheme through minimizing the CRB with respective to the positions of reference nodes.

The location dependent parameters are generally determined by using a set of nodes that are referred to as the reference nodes. The positions of these nodes are know as priori and localization algorithms exploit this knowledge for the purpose of target localization. Most of the existing research works have mainly focused on the performance analysis of the different localization algorithms [11]-[13], [60], while little attention has been paid to the optimal deployment of the reference nodes. However, based on our numerical simulations, the location of the reference nodes plays an important role in the localization performance when the target is assumed to be located in a known region.

Motivated by the latter fact, we aim at the optimal reference node deployment in this Chapter. Particularly, we are interested in determining the optimal location of the reference nodes
Chapter 4. Optimum Reference Node Deployment for TOA-based Localization

When statistical knowledge of the target location is known. Note that such statistical knowledge can be easily obtained through the collection of the user location information from the networks such as WiFi network, cellular network, and wireless sensor network (WSN). One key challenge in the optimal deployment of the reference nodes is the geographical restrictions. Specifically, reference nodes can not be deployed in any arbitrary location, and as a result, they are confined to certain feasible regions.

A noticeable related work has been presented in [62], where the authors study the reference node deployment from an abstract point of view and without considering any practical localization schemes. In this chapter, we focus on the optimal deployment of the reference nodes for TOA-based localization due to its widespread adoption in many wireless localization applications as well as its superior performance as compared to other schemes[11],[60],[61]. Similar to many existing works [60],[61]-[66], CRB [63] is used as the performance criterion. Based on the assumption that the target is located inside a certain service area, our goal is to find the global minima of CRB which corresponds to the reference node deployment providing the highest localization accuracy. The main challenge towards the goal is that the CRB is highly non-linear. To solve the problem, we propose a novel method which expresses the CRB in complex coordinates and take the angles of reference nodes as decision variables in the minimization problem. By applying this method, the mathematical solution is shown to have a simple form indicating that the highest accuracy of the localization is achieved when the reference nodes have uniform angular distribution around the target service area. Our simulation results also show that the derived optimum deployment provides higher localization accuracy than other deployment schemes.

4.2 TOA-based Localization

When there are more than minimum required number of reference nodes available in a wireless network for localization purpose, Most Likelihood (ML) Estimation can be applied to estimate the target location. In this section, we apply TOA-based localization scheme and derive the CRB based on the ML Estimation.
4.2. TOA-based Localization

4.2.1 MLE algorithm for TOA-based Localization

Consider a network with \( N \) reference nodes. Let \((x, y)\) denote the unknown target location and \((x_i, y_i)\) denote the location of the \( i \)th reference node. The distance between the target and the \( i \)th reference node is

\[
d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}.
\]

(4.1)

Based on (4.1), the signal propagation time between the target and reference node \( i \) can be expressed as \( t_i = d_i/c \), where \( c \) is the signal propagation speed. Similar to other works, the measurement error is modeled as Gaussian random variables with zero mean \([?],[65],[66]\). Accordingly, the TOA measurement is normally distributed with mean \( d_i/c \)

\[
t_i \sim N(d_i/c, \sigma^2),
\]

(4.2)

where \( \sigma^2 \) is the variance of the measurement error. Here we assume the measurement errors among the target and the reference nodes have the same variance. The conditional probability density function (pdf) of TOA measurement between the target and reference node \( i \) can be expressed as

\[
f(t_i|x, y) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(t_i - d_i/c)^2}{2\sigma^2}}.
\]

(4.3)

Let \( t \) denote the set of \( N \) statistically independent TOA measurements between the target and the \( N \) reference nodes. Due to the independence of these measurements, the joint pdf of \( N \) measurements can be expressed as

\[
f(t|x, y) = \prod_{i=1}^{N} f(t_i|x, y),
\]

(4.4)

The ML Estimation chooses the target position for which the probability of measurements is highest. Therefore, the target location estimation can be obtained through maximizing the joint pdf in (4.4), which corresponds to minimizing the log-likelihood function

\[
l(t|x, y) = \frac{1}{2\sigma^2} \sum_{i=1}^{n} (t_i - d_i/c)^2.
\]

(4.5)
The localization result can be expressed as

\[(\hat{x}, \hat{y}) = \arg \min_{x, y} l(t|x, y), \] (4.6)

where \((\hat{x}, \hat{y})\) is the estimated location of the target.

### 4.2.2 Cramer-Rao Bound Derivation for TOA-based Localization

With an unbiased location estimator, CRB can provide the lower bound of localization error. According to Cramer-Rao Inequality [63], the Fisher information of having \(N\) observations can be written as

\[F_N(\theta) = -E\left[\frac{\partial^2 l(X|\theta)}{\partial^2 \theta}\right], \] (4.7)

where \(X\) represents the \(N\) observations, and \(\theta\) is the parameter vector to be estimated. In the TOA-based localization problems, the \(N\) observations correspond to the \(N\) TOA measurements, and the parameter vector corresponds to the coordinate of the target location \((x, y)\). Given the log-likelihood function in (4.5), the Fisher information can be written as following matrix form

\[F_N(\theta) = F_N(x, y) = \begin{bmatrix} F_{xx} & F_{xy} \\ F_{yx} & F_{yy} \end{bmatrix}, \] (4.8)

where

\[
\begin{aligned}
F_{xx} &= -E\left[\frac{\partial^2 l(t|x, y)}{\partial x \partial x}\right], \\
F_{xy} &= F_{yx} = -E\left[\frac{\partial^2 l(t|x, y)}{\partial x \partial y}\right], \\
F_{yy} &= -E\left[\frac{\partial^2 l(t|x, y)}{\partial y \partial y}\right].
\end{aligned}
\] (4.9)

Substituting \(l(t|x, y)\) by the log-likelihood function in (4.5), the element \(F_{xx}\) becomes

\[
F_{xx} = -\frac{1}{2\sigma^2} E\left[\frac{\partial^2 (\sum_{i=1}^{N} (t_i - d_i/c)^2)}{\partial x \partial x}\right] \\
= -\frac{1}{c\sigma^2} E\left[\sum_{i=1}^{N} \frac{(t_i - \frac{d_i}{c} - \frac{(x-x_i)^2}{c^2})d_i - \frac{1}{d_i}(x-x_i)^2(t_i - \frac{d_i}{c})}{d_i^2}\right]. \] (4.10)
4.2. TOA-based Localization

For an unbiased location estimator, \( E[ \sum_{i=1}^{N} (t_i - d_i/c) ] = 0 \). Therefore

\[
F_{xx} = \frac{1}{c^2 \sigma^2} \sum_{i=1}^{N} \frac{(x - x_i)^2}{d_i^2}.
\]

(4.11)

The rest of the elements can be similarly derived as

\[
\begin{align*}
F_{xx} &= \frac{1}{c^2 \sigma^2} \sum_{i=1}^{N} \frac{(x - x_i)^2}{d_i^2}, \\
F_{xy} = F_{yx} &= \frac{1}{c^2 \sigma^2} \sum_{i=1}^{N} \frac{(x - x_i)(y - y_i)}{d_i^2}, \\
F_{yy} &= \frac{1}{c^2 \sigma^2} \sum_{i=1}^{N} \frac{(y - y_i)^2}{d_i^2}.
\end{align*}
\]

(4.12)

According to Cramer-Rao Inequality, the variance between the estimated target location \((\hat{x}, \hat{y})\) and the real target location \((x, y)\) is limited by inverse of the Fisher information matrix as

\[
\text{cov}(\hat{x}, \hat{y}) - F_N^{-1}(x, y) \geq 0,
\]

(4.13)

where

\[
\text{cov}(\hat{x}, \hat{y}) = \begin{bmatrix} E[(\hat{x} - x)^2] & E[(\hat{x} - x)(\hat{y} - y)] \\ E[(\hat{y} - y)^2] & E[(\hat{y} - y)^2] \end{bmatrix},
\]

(4.14)

and

\[
F_N^{-1}(x, y) = \frac{1}{|F_N(x, y)|} \begin{bmatrix} F_{yy} & -F_{xy} \\ -F_{yx} & F_{xx} \end{bmatrix}.
\]

(4.15)

Based on (4.12), (4.13), (4.14), and (4.15), the CRB of the TOA-based location estimation is given by

\[
E[(\hat{x} - x)^2 + (\hat{y} - y)^2] \geq \frac{F_{xx} + F_{yy}}{F_{xx}F_{yy} - F_{xy}^2}
\]

\[
= \frac{c^2 \sigma^2}{\sum_{i=1}^{N} \frac{(x - x_i)^2}{d_i^2} + \sum_{i=1}^{N} \frac{(y - y_i)^2}{d_i^2} - (\sum_{i=1}^{N} \frac{(x - x_i)(y - y_i)}{d_i^2})^2}
\]

\[
= \frac{c^2 \sigma^2 N}{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{(x - x_i)(y - y_j) - (x - x_j)(y - y_i))^2}{d_i^2 d_j^2}}.
\]

(4.16)
To simplify the result in (4.16), let $\theta_i$ denote the angle between the x-axis and the line segment $d_i$ which connects the target and reference node $i$, then the angel between $d_i$ and $d_j$ can be expressed as $\theta_{ij} = \theta_i - \theta_j$. According to the trigonometric identity, we can derive

$$
\sin(\theta_{ij}) = \sin(\theta_i - \theta_j) = \sin\theta_i\cos\theta_j - \cos\theta_i\sin\theta_j
$$

By introducing (4.17) into (4.16), the CRB result can be simplified as

$$
CRB(x, y) = \frac{c^2 \sigma^2 N}{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\sin\theta_{ij})^2}.
$$

According to the derivation in (4.18), when the number of reference nodes $N$ and the variance of the TOA measurement error $\sigma^2$ are fixed, the CRB of TOA-based localization error only depends on the angles $\theta_{ij}$ which are between the line segments connecting the target and reference nodes.

### 4.3 Reference Node Deployment

In this section, we first compare two different reference node deployment schemes in order to show the impact of reference node deployment on the CRB, and then present the derivation of optimum reference node deployment.

#### 4.3.1 Impact of Reference Node Deployment

Consider a network with four reference nodes. In the first case, the four reference nodes are deployed at the corners of a $10m$ by $10m$ square. Fig. 4.1 shows the CRB of target localization error when the target is deployed at different coordinates inside the square. As shown in the figure, the target localization error is relatively lower around the center of the square and becomes higher near the corners of the square.

In the second case, we change the position of one reference node from $(10, -10)$ to $(10, 0)$. As shown in Fig. 4.2, the CRB of the localization error is much higher than in the first case.
4.3. Reference Node Deployment

Figure 4.1: CRB of target localization error with 4 reference nodes at the locations of (10, 10),
(−10, 10), (−10, −10), and (10, −10).

when the target is close to the corner (−10, −10).

4.3.2 Optimum Reference Node Deployment

Our approach of finding the optimum reference node deployment is to minimize the highly non-
linear CRB with respect to the angle variables θ_{ij}. According to the result in (4.18), minimizing
the CRB corresponds to maximizing \( \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\sin \theta_{ij})^2 \). Given \( N \) reference nodes in the
network, we can obtain

\[
\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\sin \theta_{ij})^2 = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left( \frac{1}{2} - \frac{1}{2} \cos 2\theta_{ij} \right) \\
= \frac{N(N-1)}{4} - \frac{1}{2} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \cos 2\theta_{ij}. \tag{4.19}
\]
Figure 4.2: CRB of target localization error with 4 reference nodes at the locations of (10, 10), 
(−10, 10), (−10, −10), and (10, 0).

Therefore, the problem becomes to minimize the term $\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \cos 2\theta_{ij}$. In order to find the 
solution, we transfer the problem to complex coordinates. Given $N$ complex number $z_n = e^{j\theta_n}$,
the term $\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \cos 2\theta_{ij}$ is correlated with $Z_n$ by

$$N + 2 \sum_{n=1}^{N} \sum_{m=n+1}^{N} \cos 2\theta_{nm}$$

$$= \sum_{n=1}^{N} \sum_{m=1}^{N} \cos 2\theta_{nm} + \sum_{n=1}^{N} \sum_{m=1, m \neq n}^{N} \cos 2\theta_{nm}$$

$$= \text{Re}\{\sum_{n=1}^{N} \sum_{m=1}^{N} e^{j2(\theta_n - \theta_m)}\} = \text{Re}\{\sum_{n=1}^{N} \sum_{m=1}^{N} z_n^2 \cdot (z_m^*)^2\}$$

$$= \text{Re}\{\sum_{n=1}^{N} \sum_{m=1}^{N} z_n^2 \cdot z_m^2\} = \text{Re}\{\sum_{n=1}^{N} z_n^2\} \cdot \text{Re}\{\sum_{n=1}^{N} z_n^2\}$$

$$= \sum_{n=1}^{N} z_n^2 \cdot z_n^2 \geq 0.$$  

As shown in Fig. 4.3, $|\sum_{n=1}^{N} e^{j\theta_n}|^2 = 0$ can be achieved when $\alpha_1 = \alpha_2 \cdots = \alpha_N = \frac{2\pi}{N}$. Introducing (4.20) into (4.19)

$$\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\sin \theta_{ij})^2 \leq \frac{N(N-1)}{4} + \frac{1}{4}N = \frac{N^2}{4}.$$  

Therefore, the minimum CRB of TOA-based localization is derived as

$$\text{CRB}(x, y) = \frac{\epsilon^2 \sigma^2 N}{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\sin \theta_{ij})^2} \geq \frac{4\epsilon^2 \sigma^2}{N}.$$  

The minimum CRB is achieved when $\alpha_1 = \alpha_2 \cdots = \alpha_N = \frac{2\pi}{N}$. Therefore, the corresponding reference node deployment can provide the best localization accuracy, which is the optimum reference node deployment.

### 4.4 Simulation Results

In the simulations, we deploy the reference nodes on a circle with radius of 10m. According to the derivation in Section III, the best localization accuracy can be achieved when the reference nodes are uniformly deployed on the circle. In the first simulation, we put one target at the
center of the circle and set the standard deviation of the distance measurement error $c\sigma$ to 1m. Between the target and each reference node, we do 100 times TOA measurements and calculate use Mean Square Error (MSE) to calculate the average localization error. The real localization result is estimated by ML Estimation presented in Section II. In Fig. 4.4, we change the number of reference nodes from 3 to 10 while keeping the reference nodes uniformly distributed. The derived minimum CRB and the real localization errors are shown in the figure.
4.4. Simulation Results

<table>
<thead>
<tr>
<th>Deployment</th>
<th>Reference Node 1</th>
<th>Reference Node 2</th>
<th>Reference Node 3</th>
<th>Reference Node 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Optimum)</td>
<td>$(10\cos 45^\circ, 10\sin 45^\circ)$</td>
<td>$(10\cos 45^\circ, -10\sin 45^\circ)$</td>
<td>$(-10\cos 45^\circ, 10\sin 45^\circ)$</td>
<td>$(-10\cos 45^\circ, -10\sin 45^\circ)$</td>
</tr>
<tr>
<td>2</td>
<td>$(10\cos 30^\circ, 10\sin 30^\circ)$</td>
<td>$(10\cos 30^\circ, -10\sin 30^\circ)$</td>
<td>$(-10\cos 45^\circ, 10\sin 45^\circ)$</td>
<td>$(-10\cos 45^\circ, -10\sin 45^\circ)$</td>
</tr>
<tr>
<td>3</td>
<td>$(10\cos 30^\circ, 10\sin 30^\circ)$</td>
<td>$(10\cos 30^\circ, -10\sin 30^\circ)$</td>
<td>$(-10\cos 30^\circ, 10\sin 30^\circ)$</td>
<td>$(-10\cos 30^\circ, -10\sin 30^\circ)$</td>
</tr>
<tr>
<td>4</td>
<td>$(10\cos 15^\circ, 10\sin 15^\circ)$</td>
<td>$(10\cos 15^\circ, -10\sin 15^\circ)$</td>
<td>$(-10\cos 15^\circ, 10\sin 15^\circ)$</td>
<td>$(-10\cos 15^\circ, -10\sin 15^\circ)$</td>
</tr>
</tbody>
</table>

Table 1: Locations of reference nodes in four different deployment schemes.

![Figure 4.4: Localization result with optimum reference node deployment, when $c\sigma = 1\, m$.](image)

Then we fix the number of reference nodes $N = 4$, and compare the localization performance of our derived optimum reference node deployment with other three deployment schemes. The reference node positions in the four deployment schemes are shown in TABLE 1. In Fig. 4.5, we change $c\sigma$ from $1\, m$ to $2\, m$, and show the real localization errors of the four different reference node deployment schemes. As shown in the figure, our derivation of optimum reference node deployment provides the highest localization accuracy.
In practical applications, a large number of targets can exist in a wireless network. We need to average all the target localization results over the service area in order to evaluate the localization performance of a reference node deployment scheme. For this purpose, we randomly distribute 100 targets inside the circle (service area) for the second experiment. The real localization results of the 100 targets when $c\sigma = 1m$ are shown in Fig. 4.6. After that, we average the localization results of the 100 targets in the service area and compare Deployment 1 with the other three deployment schemes. As shown in Fig. 4.7, Deployment 1 provides the best localization accuracy among the four reference node deployment schemes in TABLE 1.
4.4. Simulation Results

Figure 4.6: Localization results of 100 targets, with reference node Deployment 1, \( c_0 = 1 \text{m} \)
Figure 4.7: Averaged localization errors of 100 targets versus $c\sigma$. 
Chapter 5

Localization with Insufficient Reference Nodes

5.1 Introduction

In conventional wireless localization methods, the location dependent parameters are generally obtained from the transmitted wireless signals between the target device and reference nodes. As a result, the performance of conventional localization schemes is highly sensitive to the communication environment between the reference nodes and the target as well as the system calibration. The multipath effect and additive noise are two major sources which degrade the localization accuracy in complex signal propagation environment. In TOA and AOA based localization schemes, system calibration and synchronization also have significant impact on the localization performance. Moreover, the performance of the conventional localization schemes is further constrained by the number of reference nodes available within the communication range of the target node. As a result, insufficient number of reference nodes will prohibit the localization algorithms from estimating the absolute positions of the target nodes.

Considering the problems discussed above, we would like to develop alternative methods when there is insufficient reference nodes involved in localization algorithm. We will first discuss the Multidimensional Scaling (MDS) based relative location estimation and the smartphone based localization using accelerometer, and then propose a novel method to combine the information from available reference nodes and internal sensors in smartphone, in order to
overcome the constraint of conventional localization method as well as improving the localization performance.

Although the absolute location of the target nodes cannot be obtained when there is insufficient reference nodes, we can still construct a relative location map of the target nodes based on the estimated distances among all the nodes within the communication range. MDS can be applied to maximize the similarity of the estimated and the true distance values to calculate the relative target node positions.

Due to the exponential growth of the smartphone market in recent years, alternative localization schemes have been proposed to utilize the internal sensors embedded in smartphones to obtain additional location dependent parameters and improve localization performance [67]. Most of today’s smartphones are equipped with various built-in sensors, providing extremely useful information which is not only be used in those mobile softwares for entertainment and user interaction purpose, but also in many emerging wireless applications. Accelerometer is one of the internal sensors which can output the acceleration of the device. The moving distance of a user can be calculated through the acceleration information for location estimation. However, the accuracy of localization based on accelerometer is highly sensitive to the sensor measurement error. Moreover, the sensor error can be accumulated along with the time increase, which will degrade the localization performance significantly.

Considering the existing problems in above discussed localization schemes, we consider to combine the two different types of location dependent parameters measured from available reference nodes and internal accelerometer sensor together, in order to overcome the drawbacks and improve the localization performance. A noticeable work has been presented in [68], where the authors combine the accelerometer information with the RSS fingerprinting based on interval analysis. However, the RSS fingerprinting map generation is labor-intensive and time-consuming. In addition, the map needs to be updated when the environment in the localization system changes. In this chapter, we propose a novel algorithm to combine the information from accelerometer and available reference nodes by maximizing the joint probability of the two different measurements. As shown in our simulation results, by use of the proposed combined localization method, the error accumulation from the accelerometer can be reduced with the help of few reference nodes involved, and the localization accuracy can be
improved significantly.

5.2 Alternative Localization Schemes

In a 2D coordinate system, generally we need at least three reference nodes with predetermined positions in order to localize other nodes. In addition, the target node needs to be inside the communication range of the reference nodes. As shown in Fig. 5.1, the location of target node 1 can be estimated since it is deployed inside the communication ranges of all the three reference nodes, while the absolute positions of target node 2 and 3 cannot be determined since there are not sufficient reference nodes available in their communication ranges.

![Diagram of localization with reference nodes and target nodes]

Figure 5.1: $N$ selected reference nodes around the target
5.2.1 Relative Location Estimation

In a large wireless network with a great number of nodes, such as sensor network, it is not practical to predetermine the positions of all the nodes. When the service area is large, there is a big chance that some target nodes can only hear from less than minimum required reference nodes. Although the absolute positions of the target nodes can not be estimated, we can still construct a relative location map based on the distance estimation among all the nodes in the service area. After that, if sufficient reference nodes are provided, the relative locations can be transferred to absolute locations.

Assume the distance values among the nodes in a certain region can be obtained by signal measurements, then Multidimensional Scaling (MDS) can be applied to calculate the relative locations of the nodes in that region. Let \( d_{ij} \) denote the measured distance between the node \( i \) and node \( j \), and let \( n \) denote the number of nodes who can hear from each other in the region. The measured distance values among the nodes can be expressed as the entries of the dissimilarity matrix in MDS

\[
D = \begin{pmatrix}
  d_{1,1} & d_{1,2} & \cdots & d_{1,n} \\
  d_{2,1} & d_{2,2} & \cdots & d_{2,n} \\
  \vdots & \vdots & \ddots & \vdots \\
  d_{n,1} & d_{n,2} & \cdots & d_{n,n}
\end{pmatrix}.
\] (5.1)

The goal of MDS is to estimate the vectors \( \mathbf{p}_i = (x_i, y_i), i = 1, 2 \cdots n \), which are the positions of the nodes within the communication range. MDS can be formulated as an optimization problem, and the position vectors are obtained by

\[
\min_{\mathbf{p}_1, \mathbf{p}_2, \cdots, \mathbf{p}_n} \sum_{i=1}^{n} \sum_{\substack{j=1 \atop j \neq i}}^{n} \left( \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} - d_{ij} \right)^2.
\] (5.2)

The squared distance between node \( i \) and node \( j \) can be expressed as

\[
d_{ij} = (x_i - y_i)^2 + (x_j - y_j)^2 = \| \mathbf{p}_i - \mathbf{p}_j \|^2
= \mathbf{p}_i^T \mathbf{p}_i - 2 \mathbf{p}_i^T \mathbf{p}_j + \mathbf{p}_j^T \mathbf{p}_j.
\] (5.3)
Let $Q$ denote an $N \times 1$ vector which can be expressed as

$$Q = \begin{bmatrix} p_1^T \, p_1 \\ p_2^T \, p_2 \\ \vdots \\ p_n^T \, p_n \end{bmatrix},$$

(5.4)

and let $C$ denote $N \times 1$ vector of ones, then the matrix form of the squared distance values $D^2$ can be expressed as

$$D^2 = QC^T - 2P^T P + CQ^T,$$

(5.5)

where $P$ is the matrix form of the positions of the nodes. In order to solve the problem in (5.5), the positions of nodes are moved by multiplying with a centering matrix so that the mean value of the coordinates of all the nodes involved in MDS will become the center of the relative location map. The centering matrix of size $n$ can be expressed as

$$H_n = I_n - \frac{1}{n} C^T C$$

$$= \begin{bmatrix} \frac{n-1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & \frac{n-1}{n} & \cdots & \vdots \\ \vdots & \ddots & \ddots & -\frac{1}{n} \\ -\frac{1}{n} & \cdots & -\frac{1}{n} & \frac{n-1}{n} \end{bmatrix}.$$  

(5.6)

By multiplying with the centering matrix at both side in (5.5), we can get

$$HD^2H = H(QC^T - 2P^T P + CQ^T)H$$

$$= H(QC^T + CQ^T)H - 2(P^T H^T)(PH)$$

$$= O_n - 2\tilde{P}^T \tilde{P}$$

$$= -2\tilde{P}^T \tilde{P},$$

(5.7)

where $O_n$ is $n \times n$ zero matrix, and $\tilde{P} = PH$ is the centered coordinate matrix of the nodes in the network. Let $B = -\frac{1}{2} HD^2H$, the centered coordinate matrix which corresponds to the relative
locations of the nodes can be obtained through minimizing the square error expressed as

$$\hat{\mathbf{P}} = \min_{\mathbf{P}} ||\mathbf{B} - \hat{\mathbf{P}}^T\hat{\mathbf{P}}||^2.$$  (5.8)

The minimization problem in (5.8) can be solved by singular value decomposition (SVD).

The result of MDS is a relative location map. When there is less than minimum required reference nodes available, the map can be arbitrarily rotated or flipped. However, the relative map can be transferred to absolute map when there is additional information provided. For example, when there are two reference nodes with absolute positions available in the network, the result of MDS will be a relative map which can be flipped around the line segment connecting the two reference nodes. If a flipping will cause any of the nodes locate outside of the service area, then we can exclude it and get the absolute location map.

### 5.2.2 Smartphone based Localization using Accelerometer

Due to the fast development of today’s smartphone technologies, more and more people rely on smartphones in their daily life. In this circumstance, smartphone-based localization has been attracting much research attention. The internal sensors in today’s smartphones can be utilized for localization purpose. Accelerometer sensor is one of the most important features which can be used for location estimation. Nowadays, most of the smartphones are equipped with accelerometers, and many applications, games, smartphone activities are designed and developed based on the embedded accelerometer. In general, accelerometer is the sensor which measures the acceleration of the device, but some of the accelerometers in smartphones have built-in chips which enable them to measure not only the acceleration but also the orientation of the device.

In most of today’s smartphones, the data output from the accelerometer sensor use the coordinate system as shown in Fig. 5.2. The x axis is horizontal and points to the right of the screen, the y axis is vertical and points to the up of the screen, and the z axis points towards the outside of the front face of the screen.

In this section, we do experiment on accelerometer sensor embedded in the smartphone iPhone5S. Based on the measured acceleration, we calculate the moving distance and com-
5.2. Alternative Localization Schemes

Figure 5.2: Coordinate system of data output from accelerometer in smartphones.

Pare it with the real distance value in order to find out the existing measurement error of the accelerometer for localization purpose. The recorded data output from the accelerometer is based on the coordinate system in Fig. 5.2, and the sampling frequency is set to 30 Hz.

In the first experiment, we put the smartphone stationary on the table and the screen of the phone faces up. The data output from the acceleration sensor is recorded and shown in Fig. 5.3. Due to the earth’s gravity, the acceleration along z axis should be around \(-9.8 \, \text{m/s}^2\). Since there is no acceleration in horizontal plane, the measured data along x and y axis is close to \(0 \, \text{m/s}^2\).

Then we hold the phone on hand, keep the screen facing up, and move the phone up and down quickly for three times. The data is recorded and shown in Fig. 5.4. The acceleration changes sharply along z axis, since the movement is along the vertical direction. There is also fluctuation existing along x and y axis, because when we are moving the phone up and down quickly, our hands can also shake in the horizontal plane.

In smart-phone based localization, the measurement is usually done while the user is holding the phone and walking. The third experiment on the accelerometer is for this case. In the experiment, we hold the phone on hand, keep the screen facing up, and move forward for 5
steps. The data is recorded and shown in Fig. 5.5. From the acceleration change along y and z axis, we can easily distinguish the 5 steps movement.

Having the acceleration information, we can calculate the moving distance for localization purpose. Assume the user hold the phone and start moving from the time slot $t_0$. The acceleration at $t_0$ is 0. Let $a(t)$ denote the acceleration at time slot $t$, the moving velocity along the acceleration direction at time slot $t$ can be obtained as

$$ v(t) = \int_{t_0}^{t} a(x)dx. $$

(5.9)

Then the moving distance of the user at time slot $t$ can be calculated by

$$ d(t) = \int_{t_0}^{t} v(x)dx = \int_{t_0}^{t} \int_{t_0}^{y} a(x)dx dy $$

(5.10)

Since the data output from the accelerometer is discrete but not continuously, the above calculation needs to be transformed to discrete time. Let $\Delta t$ denote the sampling period of the
5.2. Alternative Localization Schemes

Figure 5.4: Data output from accelerometer when the device is moved up and down.

accelerometer sensor, the moving velocity along the acceleration direction at the $n$th sampling point can be expressed as

$$v(t_n) = \sum_{k=1}^{n} a(t_k) \Delta t,$$  \hspace{1cm} (5.11)

where $t_k = t_0 + k \Delta t$ is the time slot of the $k$th sampling point. Then the moving distance of the user at the $n$th sampling point can be expressed as

$$d(t_n) = \sum_{k=1}^{n} v(t_k) \Delta t = \sum_{m=1}^{n} \sum_{k=1}^{m} a(t_k) \Delta t \Delta t.$$ \hspace{1cm} (5.12)

The above derivation is transformed directly from continuous domain so that it is based on the assumption that the sampling period $\Delta t$ is small enough, and the moving velocity of the user keeps unchange during $\Delta t$. In practical, we consider the acceleration unchange during the sampling period, and calculate the moving velocity at the current sampling point based on the
previous sampling point. The moving velocity at the \( n \)th sampling point can be expressed as

\[
v(t_n) = v(t_{n-1}) + a(t_n)\Delta t,
\]

(5.13)

where \( a(t_n) \) is the acceleration during the sampling period from \( t_{n-1} \) to \( t_n \). Then the moving distance at the \( n \)th sampling point can be calculated by

\[
d(t_n) = d(t_{n-1}) + \frac{1}{2} [v(t_{n-1}) + v(t_n)]\Delta t
\]

\[
= d(t_{n-1}) + v(t_{n-1})\Delta t + \frac{1}{2} a(t_n)\Delta t^2,
\]

(5.14)

Consider a user holding the phone and moving in a 2D plane, the position of the user can be obtained based on the calculation of moving distance along \( x \) and \( y \) axis in the coordinate system of the acceleration sensor.
5.3 Combined Localization

In conventional wireless localization methods based on distance estimation between the target and reference nodes, we need at least three reference nodes to localize a target in a 2D plane (at least four reference nodes in a 3D plane). The number of reference nodes involved in the localization process plays an important role in the accuracy of target location estimation. Generally, the more reference nodes involved, the higher accuracy will be achieved. However, the computation complexity can also increase significantly when there is a large number of reference nodes involved in the localization algorithm.

As discussed in the Chapter 3, the localization performance also depends on the communication environment between the target node and the reference nodes, since the target location is estimated based on the measured signal parameters. When there is obstruction effect, the localization accuracy can decrease significantly. In Chapter 4, our experiment result showed that the localization performance is also highly sensitive to the positions of the reference nodes relative to the target node. Moreover, sometimes the target nodes in a large wireless network can only hear from less than minimum required number of reference nodes within their communication ranges.

Due to the above mentioned problems, many research works consider to utilize the internal sensors in today’s smartphones to obtain additional location dependent parameters. Accelerometer is one of the most widely used smartphone sensors for localization purpose. In achieving localization through the acceleration information output from the accelerometer sensor, the user’s location is estimated based on the calculated moving distance during the sampling period, and the location estimation at the current sampling point is estimated based on the localization result at the previous sampling point. Due to the existing error in acceleration data from the sensors, the calculated moving distance will also have errors. Moreover, the error at the previous localization result will be added to the current result, which will cause error accumulation. With the time increase, the localization accuracy will decrease significantly. Assume there is random error existing in the output data from accelerometer. Let $e_n$ denote the error of
the acceleration data \( a_n \), the velocity at the \( n \)th sampling point in (5.13) becomes

\[
v(t_n) = v(t_{n-1}) + (a(t_n) + e_n)\Delta t
\]

\[
= v(t_{n-1}) + a(t_n)\Delta t + e_n\Delta t.
\]  

(5.15)

Therefore, the moving distance at the \( n \)th sampling point in (5.14) can be calculated as

\[
d(t_n) = d(t_{n-1}) + \frac{1}{2}[v(t_{n-1}) + v(t_n)]\Delta t
\]

\[
= d(t_{n-1}) + v(t_{n-1})\Delta t + \frac{1}{2}a(t_n)\Delta t^2 + \frac{1}{2}e_n\Delta t^2.
\]  

(5.16)

As shown in (5.16), the error of distance calculation in the previous sampling time will be accumulated to the distance estimation in the current sampling time. Therefore, the measurement error will increase significantly along with the sampling time. In this section, we will propose a novel algorithm to combine the sensor data together with the information taken from the available reference nodes, so that the error accumulation from the accelerometer can be reduced.

### 5.3.1 Combined Localization Algorithm

Consider a user holding a device equipped with both accelerometer sensor and wireless communication module which enable the device to measure the signal from reference nodes, the location dependent parameters obtained from the sensor data and the received wireless signals can be combined together to improve the localization performance. Assume there are \( N \) reference nodes available within the user’s communication range, the distance measurement error between the target and the \( i \)th reference node can be expressed as

\[
e_{r,i}(x, y) = d_{m,i}(m_i) - d_{r,i}(x, y),
\]  

(5.17)

where \((x, y)\) is the true position of the user, \( m_i \) is the measured location related parameter from the \( i \)th reference node, and \( d_{m,i}(m_i) \), \( d_{r,i}(x, y) \) are the measured distance and the real distance between the target and the \( i \)th reference node, respectively. Let \((x_i, y_i)\) denote the position of
5.3. Combined Localization

the \(i\)th reference node, the real distance can be calculated as

\[
d_{r,i}(x, y) = \sqrt{(x - x_i)^2 + (y - y_i)^2}.
\] (5.18)

The measured distance \(d_{m,i}(x, y)\) is calculated through the measured location dependent parameters, such as RSS and TOA, based on corresponding signal propagation models.

In RSS-based localization, the relationship between the received signal strength from the \(i\)th reference node and the distance can be modeled as

\[
d_{m,i}(\text{rss}_i) = d_0 \cdot 10^{\frac{P_0 - \text{rss}_i}{10\alpha}}
\] (5.19)

where \(P_0\) is the signal power in decibel at the reference distance \(d_0\) away from the transmitter, \(\text{rss}_i\) is the measured signal strength from the \(i\)th reference node, and \(\alpha\) is the path loss exponent which is an environment dependent parameter. The reference distance \(d_0\) is typically set to be 1m.

In TOA-based localization, the measured time of arrival is related to the distance between the target and the \(i\)th reference node by

\[
d_{m,i}(t_i) = c \cdot t_i
\] (5.20)

where \(c\) is the parameter of signal propagation speed, and \(t_i\) is the measured signal propagation time from the \(i\)th reference node.

In combined localization algorithm, we minimize the weighted square error of the distance estimation from reference nodes and accelerometer in order to reduce the error accumulation and improve the localization accuracy. Assume \((x_{n-1}, y_{n-1})\) is the estimated target location at the \((n - 1)\)th sampling point, we first calculate the moving distances along x and y axis at the \(n\)th sampling point by

\[
\begin{align*}
x'_n &= x_{n-1} + \frac{1}{2}(2v_{x,n-1} + a_{x,n})t, \\
y'_n &= y_{n-1} + \frac{1}{2}(2v_{y,n-1} + a_{y,n})t,
\end{align*}
\] (5.21)

where \((v_{x,n-1}, v_{y,n-1})\) is the target moving velocity at the \((n - 1)\)th sampling point, and \(t\) is the
time duration between the previous and the current sampling point. Let \((v_{x,0}, v_{y,0})\) denote the initial velocity, then (5.21) can be expressed as

\[
\begin{align*}
    x'_n &= x_{n-1} + \frac{1}{2}(2v_{x,0} + 2 \sum_{k=1}^{n-1} a_{x,k} t + a_{x,n} t) t, \\
y'_n &= y_{n-1} + \frac{1}{2}(2v_{y,0} + 2 \sum_{k=1}^{n-1} a_{y,k} t + a_{y,n} t) t.
\end{align*}
\] (5.22)

Let \(\Delta a_{k,x}\) and \(\Delta a_{k,y}\) denote the accelerometer measurement error at the \(k\)th sampling point along \(x\) and \(y\) axis, respectively, the accumulated error at the \(n\)th sampling point can be calculated as

\[
\begin{align*}
e_{n,x} &= \sum_{k=1}^{n-1} \Delta a_{k,x} t^2 + \frac{1}{2} \Delta a_{n,x} t^2, \\
e_{n,y} &= \sum_{k=1}^{n-1} \Delta a_{k,y} t^2 + \frac{1}{2} \Delta a_{n,y} t^2.
\end{align*}
\] (5.23)

Then we combine the distance estimation from accelerometer and reference nodes together, based on weighted least square error algorithm. The combined weighted square error can be expressed as

\[
S(x_n, y_n) = \sum_{i=1}^{N} w_{r,i} \cdot e_{r,i}^2 + w_a \cdot (x_n - x'_n)^2 + w_a \cdot (y_n - y'_n)^2,
\] (5.24)

where \(e_{r,i}\) is the distance measurement error from the \(i\)th reference node in (5.18), \(w_{r,i}\) and \(w_a\) are the weights from the \(i\)th reference node and from the accelerometer at \(n\)th sampling point, respectively. We assign the weights to the square errors based on the corresponding variances. The variance of distance estimation error from the \(i\)th reference node can be obtained as a known parameter according to the signal propagation model, denoted as \(\sigma_{r,i}^2\), while the variance of the random error from the accelerometer can be also known as a priori, denoted as \(\sigma_a^2\). Based
on the calculated error in (5.23), we assign the weights by

\[
\begin{align*}
\frac{1}{w_{r,i}} &= \frac{1}{\sigma_{r,i}} \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_{r,i}^2} + \frac{2}{(2n-1)\sigma_d^2}}, \\
\frac{1}{w_{a}} &= \frac{1}{\sum_{i=1}^{N} \frac{1}{\sigma_{a,i}^2} + \frac{2}{(2n-1)\sigma_d^2}},
\end{align*}
\]

(5.25)

where $\sigma_d^2 = (\frac{1}{2}\sigma_a^2)^2$. Therefore, the location estimation result at the $n$th sampling point using combined localization scheme can be expressed as

\[
(x_n, y_n) = \arg \min_{x_n, y_n} S(x_n, y_n).
\]

(5.26)

By use of the combined localization scheme, we can reduce the error accumulation from accelerometer with the help of few reference nodes. Moreover, the localization performance can become more stable in harsh environment since the data output from internal accelerometer sensor is not affected by the environment change.

### 5.3.2 Simulation Results

In the simulations, we first generate a random target moving trajectory and localize the target by use of only accelerometer data. We set a random acceleration between each two sampling point from $-2m/s^2$ to $2m/s^2$, and set the sampling period to $0.05s$. The standard deviation of error from accelerometer is set to $0.1m/s^2$. Fig. 5.6 shows the generated real target trajectory and the estimated trajectory using accelerometer with 1000 sampling points.

In order to use the reference node in the combined localization algorithm, we move the target anticlockwise on a circle with radius of $10m$, starting from coordinate $(10, 0)$ and ending at the same position, and deploy the reference nodes inside the circle. The velocity of the target is set to $0.2\pi m/s$, and we output the result every 20 sampling points. The localization results of the target using only acceleration information is shown in Fig. 5.7. With the time increase, the error is accumulated and the difference between the localization result and the true target location can become very large.

Then we put one reference node at the center of the circle and apply the proposed combined
localization scheme. The variance of the distance measurement from the reference node is set to be $1 \text{ m}^2$. As shown in Fig. 5.8, the accumulated error is reduced, and the localization accuracy is increased significantly with the help of only one reference node. Fig. 5.9 and Fig. 5.10 show the localization results of combined localization scheme with two and three reference nodes, respectively, and Fig. 5.11 shows the result of using three reference nodes without accelerometer.

Figure 5.6: Real trajectory and estimated trajectory using accelerometer.

![Graph showing real and estimated trajectories using accelerometer.](image-url)
Figure 5.7: Localization results of using only acceleration information without reference nodes.
Figure 5.8: Localization results of combined localization scheme with one reference node.
Figure 5.9: Localization results of combined localization scheme with two reference nodes.
Figure 5.10: Localization results of combined localization scheme with three reference node.
Figure 5.11: Localization results of using three reference node.
Chapter 6

Conclusion and Future Work

In this thesis, our research covered various localization technologies and existing algorithms including trilateration, MLE, fingerprinting, MDS, and smartphone based localization. As discussed in the thesis, the conventional wireless localization algorithms have constraints and disadvantages when the signal propagation environment is complex and the localization system has resource limitation. In addition, the performance of target location estimation is highly sensitive to the positions of the reference nodes relative the target. Therefore, the study on the placement of reference node positions is extremely useful before deployment. When there is insufficient reference nodes available within the target’s communication range, the internal sensors equipped in the target device can be utilized to provide additional location dependent parameters for localization purpose. However, the error existing in the data output from the sensors can be accumulated along with time increase. This thesis has provided detailed study and research related to the above discussed problems. Some novel algorithms have been proposed to overcome the constraints and advantages of the conventional localization schemes. Simulations and experiments have been done to verify our proposed algorithms.

Our first proposed algorithm improved the performance of RSS-based localization in complex signal propagation environment. In RSS- based localization schemes, the distance between the target and a reference node is decided based on the received signal strength. When there are more than minimum required number of reference nodes available in the service area, MLE can be applied to estimate the target location based on the statistical error model of the received signal strength. In obstructed environment, the signal power can drop significantly
when there is obstruction between the transmitter and receiver. As a result, the distance estimation on the corresponding obstructed link can have large error. Based on our experiment, when the obstruction effect is significant, it is better to discard the corresponding reference nodes rather than involve them in the MLE algorithm. However, it is difficult to decide which link is an obstructed link, since the environment feature between the target and reference are unknown. In this thesis, we proposed a novel algorithm which can automatically detect and remove the obstruction effect in the localization process. As shown in the simulation results, our proposed localization has obvious advantages over the conventional algorithm when there are small number for obstructed links in the communication environment.

In the future work, the proposed algorithm of localization in complex signal propagation environment can be extended and applied to the localization schemes using other location dependent parameters besides RSS values. In addition, an investigation of choosing an optimum decision threshold $k$ in detecting the obstructed links can be regarded as another future work. The $k$ parameter plays an important in the performance of the proposed algorithm in complex environment.

The second contribution of this thesis is the study and derivation of optimum reference nodes deployment for TOA-based localization scheme. In achieving localization based on distance estimation, the positions of the reference nodes can affect the accuracy of target location estimation significantly. In practice, the positions of reference nodes are generally not easily adjustable after they are deployed in the wireless network. In this thesis, we have derived the optimum reference node deployment for TOA-based localization. The essence of our approach is to minimize the CRB of TOA-based localization with respect to the positions of reference nodes. A novel method has been developed to solve the highly non-linear optimization problem by transferring the problem to complex coordinates. The mathematic result of a global minima of CRB has been derived and the corresponding optimum reference node deployment has been presented. Simulation results show that our derived optimum reference node deployment scheme provide higher localization accuracy than other deployment schemes.

In this work, the global minima of the CRB is derived for one target. The derivation of minimum average CRB within a certain service area can be considered as a future work. In addition, the study of reference nodes placement for RSS-based localization is highly desir-
able, and the work can be extended to the hybrid localization schemes using multiple different location dependent parameters.

Last but not least, this thesis study on the problem of wireless localization with insufficient reference nodes. In smartphone based localization, the internal sensors equipped on the device can be utilized to provide additional location dependent parameters when there is less than minimum required reference nodes available in the services area. The accelerometer is one of the popular sensors which can be used for localization purpose. Based on the acceleration output from the accelerometer sensor, the moving distance of the device can be calculated to localize the user. However, the existing random error in the sensor data can be accumulated along with time increase, since the current location of the user is decided based on the previous location. Based on our experiment, the error accumulation can be significant in distance estimation. In order to eliminate the accumulated error, we proposed a novel algorithm to combine the sensor data together with the available reference node. As shown in the simulation results, the localization performance can be improved with help of few available reference node.

The future work of this study can be focused on the trade-off between the localization accuracy and the computation complexity. In addition, the energy consumption is another challenging problem due to the limitation of the smartphone battery capacity.

Localization has already become an essential enabling technology in today’s emerging wireless applications. It is forecasted that location based service will grow from 8.12 billion in 2014 to 39.87 billion in 2019 with annual growth rate of 37.5% from 2015 to 2019, according to a new report from a global market research and consulting company [69]. The research survey in [70] demonstrated that 74% of the adults over 18 years old rely on their smartphones to get directions and other information based on their current location, and 30% of the adults set their social media accounts to include their location information in their posts. The role of location has been changing the way of our daily life in today’s digital world. I believe the research and further development on wireless localization technologies will be highly desirable in the coming future.
Bibliography


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