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Core Formation in Partially Ionized Magnetic Molecular Clouds

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Graduate Program in Astronomy

A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy

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CORE FORMATION IN PARTIALLY IONIZED MAGNETIC
MOLECULAR CLOUDS
(Thesis format: Monograph)

by

Nicole Bailey

Graduate Program in Physics and Astronomy

A thesis submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy

The School of Graduate and Postdoctoral Studies
The University of Western Ontario
London, Ontario, Canada

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Abstract

Linear analysis of the formation of protostellar cores in planar magnetic interstellar clouds shows that molecular clouds exhibit a preferred length scale for collapse that depends on the mass-to-flux ratio and neutral-ion collision time within the cloud. This linear analysis can be used to investigate the formation of star forming clusters and the distribution of mass within star forming regions. By combining the results of the linear analysis with a realistic ionization profile for the cloud, we find that a molecular cloud may evolve through two fragmentation events in the evolution toward the formation of stars. Our model suggests that the initial fragmentation into clumps occurs for a transcritical cloud on parsec scales while the second fragmentation can occur for transcritical and supercritical cores on subparsec scales. Comparison of our results with several star forming regions (Perseus, Taurus, Pipe Nebula) shows support for a two-stage fragmentation model. Simulations of the thin-disk magnetohydrodynamic equations show that the two-stage fragmentation model is valid for a small region of parameter space assuming some form of recurrent density fluctuations within the region.

In addition, applying Monte Carlo methods to these fragmentation length scales and distributions of other environmental variables (e.g., column density and mass-to-flux ratio) allow us to produce synthetic core mass functions (CMFs) for various environmental conditions. Our analysis shows that the shape of the CMF is directly dependent on the physical conditions of the cloud. Specifically, magnetic fields act to broaden the mass function and develop a high-mass tail while ambipolar diffusion will truncate this high-mass tail. We also analyze the effect of small number statistics on the shape and high-mass slope of the synthetic CMFs. We find that observed core mass functions are severely statistically limited, which has a profound effect on the derived slope for the high-mass tail.

Keywords: diffusion – ISM: clouds – ISM: magnetic fields – magnetohydrodynamics (MHD) – stars: formation – stars: luminosity function, mass function – ISM: structure
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Finally, I would like to give my heartfelt thanks to my best friend and husband Jeff. What started out as a relationship with the adjacent desk in the office has turned into more than I could have imagined. Thank you for the encouragement, understanding, and occasional kick in the butt to get this thesis done. Thank you for being there through the highs and lows, for being my rock, and the keeper of my sanity in the last few months of madness. Three degrees down, this chapter is done, lets see what the future has in store for us... oh, and Snowy too.
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<th>Abbreviation</th>
<th>Definition</th>
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<tbody>
<tr>
<td>1D</td>
<td>One-dimensional</td>
</tr>
<tr>
<td>2D</td>
<td>Two-dimensional</td>
</tr>
<tr>
<td>AD</td>
<td>ambipolar diffusion</td>
</tr>
<tr>
<td>ALMA</td>
<td>Atacama Large Millimeter Array</td>
</tr>
<tr>
<td>amu</td>
<td>atomic mass unit: 1 amu $= 1.6605 \times 10^{-24}$ g</td>
</tr>
<tr>
<td>B##</td>
<td>Barnard object designation, e.g. B59</td>
</tr>
<tr>
<td>B1-E</td>
<td>Designation of a small clump on the east side of the B1 complex in Perseus</td>
</tr>
<tr>
<td>CMF</td>
<td>core mass function</td>
</tr>
<tr>
<td>CR</td>
<td>cosmic ray</td>
</tr>
<tr>
<td>FF</td>
<td>flux-frozen</td>
</tr>
<tr>
<td>FFT</td>
<td>fast Fourier Transform</td>
</tr>
<tr>
<td>FWHM</td>
<td>full width half maximum</td>
</tr>
<tr>
<td>GBT</td>
<td>Greenbank telescope</td>
</tr>
<tr>
<td>GMC</td>
<td>giant molecular cloud</td>
</tr>
<tr>
<td>HD</td>
<td>hydrodynamic</td>
</tr>
<tr>
<td>HI</td>
<td>neutral hydrogen</td>
</tr>
<tr>
<td>IMF</td>
<td>initial mass function</td>
</tr>
<tr>
<td>IR</td>
<td>infrared</td>
</tr>
<tr>
<td>ISM</td>
<td>interstellar medium</td>
</tr>
<tr>
<td>LTE</td>
<td>local thermodynamic equilibrium</td>
</tr>
<tr>
<td>MC</td>
<td>molecular cloud</td>
</tr>
<tr>
<td>MHD</td>
<td>magneto-hydrodynamic</td>
</tr>
<tr>
<td>MLR</td>
<td>mass-luminosity relation</td>
</tr>
<tr>
<td>Myr</td>
<td>megayear: 1 Myr $= 10^6$ years</td>
</tr>
<tr>
<td>NM</td>
<td>non-magnetic</td>
</tr>
<tr>
<td>ODE</td>
<td>ordinary differential equation</td>
</tr>
<tr>
<td>pc</td>
<td>parsec: 1 pc $= 3.086 \times 10^{18}$ cm</td>
</tr>
<tr>
<td>SFE</td>
<td>star formation efficiency</td>
</tr>
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</table>

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<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>synCMF</td>
<td>synthetic core mass function</td>
</tr>
<tr>
<td>UV</td>
<td>ultraviolet</td>
</tr>
<tr>
<td>YSO</td>
<td>young stellar object</td>
</tr>
</tbody>
</table>
Table 2: Greek Symbol Glossary

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta t_{sp}$</td>
<td>Time interval between subsequent perturbations</td>
</tr>
<tr>
<td>$\Sigma_0$</td>
<td>Initial surface density</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Mass function slope index</td>
</tr>
<tr>
<td>$\delta f$</td>
<td>Linear analysis variable that corresponds to the perturbation</td>
</tr>
<tr>
<td>$\delta f_a$</td>
<td>Linear analysis variable that corresponds to the amplitude of the perturbation</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Average rate of dissipation of turbulent kinetic energy per unit mass</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Kolmogorov length scale</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Wavelength</td>
</tr>
<tr>
<td>$\lambda_J$</td>
<td>Jeans Length</td>
</tr>
<tr>
<td>$\lambda_T$</td>
<td>Critical thermal length scale</td>
</tr>
<tr>
<td>$\lambda_{T,m}$</td>
<td>Length scale corresponding to the minimum growth time for the thermal regime</td>
</tr>
<tr>
<td>$\lambda_c$</td>
<td>Critical wavelength; $\lambda_c \equiv 2\pi/k_c$</td>
</tr>
<tr>
<td>$\lambda_{g,m}$</td>
<td>Length scale corresponding to minimum growth time ($\tau_{g,m}$); $\lambda_{g,m} = 2\lambda_{MS}$</td>
</tr>
<tr>
<td>$\lambda_{g,m,J}$</td>
<td>Length scale corresponding to minimum Jeans growth time; $\lambda_{g,m,J} = 2\lambda_J$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Statistical mean</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Initial Mass-to-Flux Ratio</td>
</tr>
<tr>
<td>$\mu_{obs}$</td>
<td>Observed mass-to-flux ratio</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Original Mass Function as defined by Salpeter (1955)</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>Initial volume density (units: g cm$^{-3}$)</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>Neutral volume density (units: g cm$^{-3}$)</td>
</tr>
<tr>
<td>$\rho_{n,0}$</td>
<td>Initial neutral volume density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Statistical variance</td>
</tr>
<tr>
<td>$\sigma_{enc}$</td>
<td>Column density enclosed within a contour</td>
</tr>
<tr>
<td>$\sigma_n$</td>
<td>Neutral column density (units: g cm$^{-2}$)</td>
</tr>
<tr>
<td>$\sigma_{n,0}$</td>
<td>Initial neutral column density (units: g cm$^{-2}$)</td>
</tr>
<tr>
<td>$\langle \sigma W \rangle_{\text{H}_2}$</td>
<td>Neutral-ion collision rate</td>
</tr>
<tr>
<td>$\tau_g$</td>
<td>Instability growth time scale</td>
</tr>
<tr>
<td>$\tau_{g,m}$</td>
<td>Minimum growth time for instability</td>
</tr>
<tr>
<td>$\tau_{g,T}$</td>
<td>Instability growth time in the thermal regime</td>
</tr>
<tr>
<td>$\tau_{ni}$</td>
<td>Neutral-ion collision time</td>
</tr>
<tr>
<td>$\tau_{ni,0}$</td>
<td>Initial neutral-ion collision time</td>
</tr>
</tbody>
</table>

Continued on next page
Table 2 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\tau}_{ni,0}$</td>
<td>Initial dimensionless neutral-ion collision time</td>
</tr>
<tr>
<td>$\chi_i$</td>
<td>Ionization fraction</td>
</tr>
<tr>
<td>$\chi_{i,0}$</td>
<td>Chapter 2: Initial ionization fraction</td>
</tr>
<tr>
<td>$\chi_{i,c}$</td>
<td>Chapter 3: Maximum ionization fraction within defined step function</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Minimum ionization fraction within defined step function</td>
</tr>
<tr>
<td></td>
<td>Complex angular frequency</td>
</tr>
</tbody>
</table>

Table 3: Roman Symbol Glossary

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Amplitude of perturbation</td>
</tr>
<tr>
<td>$A_v$</td>
<td>Visual Extinction</td>
</tr>
<tr>
<td>$A_{v,CR}$</td>
<td>Visual extinction at which transition from ultraviolet to</td>
</tr>
<tr>
<td></td>
<td>cosmic ray photoionization occurs</td>
</tr>
<tr>
<td>$A_{v,\text{crit}}$</td>
<td>Visual extinction at which decrease in step function occurs</td>
</tr>
<tr>
<td>$A_{v,d}$</td>
<td>Width of step function in units of visual extinction</td>
</tr>
<tr>
<td>$B$</td>
<td>Magnetic field strength</td>
</tr>
<tr>
<td>$B_\parallel$</td>
<td>Magnetic field parallel to the line of sight</td>
</tr>
<tr>
<td>$B_\text{tot}$</td>
<td>Total magnetic field strength in $\mu G$</td>
</tr>
<tr>
<td>$B_{z,\text{eq}}$</td>
<td>Vertical Magnetic field strength in the equatorial plane</td>
</tr>
<tr>
<td>$\vec{B}_p$</td>
<td>Planar magnetic field measured at top of the sheet;</td>
</tr>
<tr>
<td></td>
<td>$\vec{B}_p = B_x(x,y)\hat{x} + B_y(x,y)\hat{y}$</td>
</tr>
<tr>
<td>$B_{\text{ref}}$</td>
<td>Background reference magnetic field strength</td>
</tr>
<tr>
<td>$B_{x,Z}$</td>
<td>Magnetic field in x-direction measured at the top of the sheet</td>
</tr>
<tr>
<td>$B_{y,Z}$</td>
<td>Magnetic field in y-direction measured at the top of the sheet</td>
</tr>
<tr>
<td>$C_{\text{eff}}$</td>
<td>Effective sound speed</td>
</tr>
<tr>
<td>$\tilde{C}_{\text{eff}}$</td>
<td>Dimensionless effective sound speed</td>
</tr>
<tr>
<td>$C_{\text{eff,0}}$</td>
<td>Initial effective sound speed</td>
</tr>
<tr>
<td>$D_{\text{obs}}$</td>
<td>Observed diameter</td>
</tr>
<tr>
<td>$E(K)$</td>
<td>Turbulent power spectrum or Kinetic energy spectrum</td>
</tr>
<tr>
<td>$F_M$</td>
<td>Magnetic force per unit area</td>
</tr>
</tbody>
</table>

Continued on next page
Table 3 – continued from previous page

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vec{F}_T$</td>
<td>Thermal pressure force per unit area</td>
</tr>
<tr>
<td>$G$</td>
<td>Gravitational Constant: $G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$</td>
</tr>
<tr>
<td>$H$</td>
<td>Vertical Scaleheight</td>
</tr>
<tr>
<td>$I_{\text{CO}}$</td>
<td>Intensity of CO</td>
</tr>
<tr>
<td>$L_0$</td>
<td>Length scale</td>
</tr>
<tr>
<td>$L$</td>
<td>Characteristic turbulent length scale</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass</td>
</tr>
<tr>
<td>$M_{\odot}, M_{\text{sun}}$</td>
<td>Solar mass</td>
</tr>
<tr>
<td>$M_*$</td>
<td>Stellar mass</td>
</tr>
<tr>
<td>$M_J$</td>
<td>Jeans Mass</td>
</tr>
<tr>
<td>$M_{cl}$</td>
<td>Clump mass</td>
</tr>
<tr>
<td>$M_c$</td>
<td>Core mass</td>
</tr>
<tr>
<td>$N$</td>
<td>Number count</td>
</tr>
<tr>
<td>$N(\text{H}_2)$</td>
<td>Column density of molecular hydrogen (H$_2$)</td>
</tr>
<tr>
<td>$N_0(\text{H}_2)$</td>
<td>Column density of H$_2$ along a magnetic flux tube</td>
</tr>
<tr>
<td>$N_n$</td>
<td>Neutral column density (units: cm$^{-2}$)</td>
</tr>
<tr>
<td>$P_{\text{ext}}$</td>
<td>External pressure</td>
</tr>
<tr>
<td>$\tilde{P}_{\text{ext}}$</td>
<td>Dimensionless external pressure</td>
</tr>
<tr>
<td>$R$</td>
<td>Reynolds Number</td>
</tr>
<tr>
<td>$R_{\text{eff}}$</td>
<td>Effective radius</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature measured in Kelvin</td>
</tr>
<tr>
<td>$V$</td>
<td>Characteristic turbulent velocity scale</td>
</tr>
<tr>
<td>$V_{A,0}$</td>
<td>Initial Alfvèn velocity</td>
</tr>
<tr>
<td>$X$</td>
<td>Chapter 1: “X-Factor” defined as the ratio of $N(\text{H}<em>2)$ to $I</em>{\text{CO}}$</td>
</tr>
<tr>
<td></td>
<td>Chapter 3: Scaling factor relating neutral number density to ion number density</td>
</tr>
<tr>
<td>$Z, Z_0$</td>
<td>Local vertical half-thickness. Both symbols refer to the same quantity.</td>
</tr>
<tr>
<td>$c_s$</td>
<td>Sound speed</td>
</tr>
<tr>
<td>$d$</td>
<td>Cloud diameter</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Linear analysis parameter which corresponds to the unperturbed state</td>
</tr>
<tr>
<td>$\tilde{g}_p$</td>
<td>Planar gravitational potential</td>
</tr>
<tr>
<td>$k$</td>
<td>Chapter 1: Generic wave number</td>
</tr>
<tr>
<td></td>
<td>Chapter 2: Vertical wave number</td>
</tr>
<tr>
<td>$k_0$</td>
<td>Scaling constant</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
</tr>
<tr>
<td>$k_B$</td>
<td>Boltzmann Constant; $k_B = 1.3807 \times 10^{-16}$ erg/K</td>
</tr>
<tr>
<td>$k_c$</td>
<td>Critical Wave Number</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Power law exponent for the expression used to calculate the ion density as a function of neutral density</td>
</tr>
<tr>
<td>$k_x$</td>
<td>Wave number in the x direction</td>
</tr>
<tr>
<td>$k_y$</td>
<td>Wave number in the y direction</td>
</tr>
<tr>
<td>$l$</td>
<td>Viscous dissipation scale</td>
</tr>
<tr>
<td>$m_{H_2}$</td>
<td>Mass of molecular hydrogen</td>
</tr>
<tr>
<td>$m_i$</td>
<td>Ion mass</td>
</tr>
<tr>
<td>$m_k$</td>
<td>K-band Magnitude</td>
</tr>
<tr>
<td>$m_n$</td>
<td>Mean mass of a neutral particle; $m_n = 2.33$ amu in this thesis</td>
</tr>
<tr>
<td>$n_i$</td>
<td>Ion number density</td>
</tr>
<tr>
<td>$n_{n,0}$</td>
<td>Initial neutral number density</td>
</tr>
<tr>
<td>$t_0$</td>
<td>Time scale</td>
</tr>
<tr>
<td>$t_{clump}$</td>
<td>Time at which clump becomes cohesive</td>
</tr>
<tr>
<td>$t_{core}$</td>
<td>Time at which cores form; second fragmentation</td>
</tr>
<tr>
<td>$t_{diss}$</td>
<td>Dissipation time</td>
</tr>
<tr>
<td>$t_{ff}$</td>
<td>Free-fall time</td>
</tr>
<tr>
<td>$t_{frag}$</td>
<td>Time of initial fragmentation</td>
</tr>
<tr>
<td>$t_{run}$</td>
<td>Total simulation run time</td>
</tr>
<tr>
<td>$v_{los}$</td>
<td>Line of sight velocity</td>
</tr>
<tr>
<td>$\vec{v}_n$</td>
<td>Neutral particle velocity</td>
</tr>
<tr>
<td>$\vec{v}_i$</td>
<td>Ion particle velocity</td>
</tr>
<tr>
<td>$v_{n,x}$</td>
<td>Neutral particle velocity in the x-direction</td>
</tr>
<tr>
<td>$v_{n,y}$</td>
<td>Neutral particle velocity in the y-direction</td>
</tr>
<tr>
<td>$z$</td>
<td>Sheet height</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Stars and the gas that form them are part of a greater cycle of life and death within the universe. This feedback cycle continually takes the outflows from energetic stars and the remnants of old stars and forms the new generation of stars. While some of the atoms and molecules from the Big Bang may be locked in the oldest stars in the universe, others may have been recycled many times, compressed to form a star, converted to other elements through nucleosynthesis, expelled out into the interstellar medium through stellar winds or supernova explosions only to be captured again by another round of star formation.

The problem of star formation is more than just how do stars form, although that is a key part. As such, the problem of star formation can be split into two different categories: microphysics and macrophysics (McKee and Ostriker, 2007). The microphysics of star formation encompasses the questions pertaining to individual star formation. Do individual stars form from gravitational collapse of a single core? Does the medium dictate the properties of stars formed within? How do stars lose angular momentum/magnetic flux? What are the properties of disks and jets associated with young stellar objects (YSOs)?

On the flip side, the problems faced by macrophysics of star formation are on a more global scale. How are giant molecular clouds (GMCs) formed? What processes determine the distribution of physical conditions within the star forming regions of molecular clouds? What determines the mass distribution of forming stars? How do stars form within large clusters? The focus of this thesis is to use the results of the microphysics to explore the some of the current questions plaguing the research of the macrophysics. The following sections will review the underlying microphysics and the macrophysics questions that this thesis will address.
1.1 Star Formation Cycle

As alluded to above, the formation of stars is one part of an evolutionary cycle that continually occurs within galaxies. Large regions of molecular gas, known as molecular clouds, fragment into smaller denser regions known as clumps and cores which then collapse to form stars. As these stars evolve, they form elements in their cores, ranging from He to Fe. Feedback mechanisms such as stellar winds in low mass stars and a combination of stellar winds and violent supernova explosions at the end of a massive star’s life distribute these elements to the surrounding interstellar medium, enriching the medium for a future generation of stars. Swept-up material from expanding superbubbles and planetary nebulae then cools to form a new generation of molecular clouds and thus the cycle continues. Although easy to describe in the broadest sense, all of the details of this cycle are not entirely known or understood. In addition, although all of these stages are interconnected, due to the inherent complexity, the full cycle is not generally studied in extreme detail all at once. Rather, detailed research will focus on specific parts of the cycle individually (e.g., star formation, interstellar feedback, etc.), and a more detailed picture of the full cycle is pieced together from the results of these focused studies. This thesis will follow suit and focus mainly on the initial stages of the star formation cycle, specifically the fragmentation and collapse of molecular clouds into clumps and cores. Although important, the other portions of the cycle before and after this stage, i.e., the formation of the molecular cloud and the evolution of a core once it is formed, are not considered here. The following sections provide a more in depth definition of the main structures studied in this thesis.

1.1.1 Molecular Clouds

Within the star formation life cycle, the molecular cloud represents a beginning point of sorts. These structures are very large diffuse regions of molecular gas within the galaxy. They range in mass from $10^4$ - $10^6$ M$_\odot$ (McKee and Ostriker, 2007) and are typically surrounded by a layer of atomic gas which shields the molecules from the interstellar ultraviolet (UV) radiation field. Without such a shield, impinging UV radiation would easily dissociate the molecules into their constituent atoms, thus destroying the molecular cloud (McKee and Ostriker, 2007). The typical column density of a molecular cloud is on the order of $10^{21}$ cm$^{-2}$.

There are two theories regarding the formation of giant molecular clouds. The first is the Parker instability theory, whereby molecular clouds are formed via the Parker instability. On average, the interstellar magnetic field is parallel to the plane of the Galaxy and is confined by the weight of the gas that threads the field. As such, this system is subject to a universal Rayleigh-Taylor instability in which the gas plays the role of the heavy fluid and magnetic field...
plays the role of the light fluid. The result is that the interstellar gas tends to concentrate into pockets suspended within the field (Parker, 1966). This phenomenon is the Parker instability. The second is the so called “coagulation theory” (Blitz and Shu, 1980) where GMCs are grown by inelastic collision between smaller molecular clouds. Simulations show (Zhang and Song, 1999; Dobbs, 2008) that the true mechanism is likely a combination of the two.

As suggested by the name, the constituent components of molecular clouds are various different types of molecular gas, including carbon monoxide ($^{12}$CO), isotopologues of carbon monoxide ($^{13}$CO, C$^{18}$O) and molecular hydrogen (H$_2$), among other molecules (i.e. formaldehyde (H$_2$CO), ammonia (NH$_3$), etc.). Although the most important ingredient for star formation is molecular hydrogen, it is nearly impossible to detect within the infrared (IR) and radio due to its symmetric nature (i.e., it has no dipole moment). Instead, information about the density of H$_2$ within a molecular cloud is inferred from observations of carbon monoxide. The column density of molecular hydrogen is thought to be related to the intensity of the carbon monoxide line via the so called “X-factor”, ($X \equiv N(H_2)/I_{CO}$). The value of the “X-factor” is still not definitively known however researchers seem to be approaching an accepted value. For example, by subtracting the HI-associated dust emission from the total observed dust emission in the IR, Dame et al. (2001) find a value X of 1.8×10$^{20}$ cm$^{-2}$(K km s$^{-1}$)$^{-1}$ while Strong and Mattox (1996) found a similar value of 1.9×10$^{20}$ cm$^{-2}$(K km s$^{-1}$)$^{-1}$ through observation of γ rays emitted by cosmic rays interacting with the ISM. Recent observations of the X-factor within the Galaxy typically find values of 2.2 – 2.3 × 10$^{20}$ cm$^{-2}$(K km s$^{-1}$)$^{-1}$ (Shetty et al., 2011b; Galametz et al., 2013); however studies show that this value is not universal and depends on the environmental conditions within the molecular cloud, especially metallicity (Leroy et al., 2009, 2011; Shetty et al., 2011a).

Physical properties of molecular clouds and their substructures are determined through observations. The mass of a molecular cloud is inferred through measurements of the luminosity of the rotational transition from $J = 1 \rightarrow 0$ of one of the isotopologues of carbon monoxide. The formation of molecules in the first place implies a low temperature region, however the rotational transitions of the tracer molecules such as carbon monoxide and ammonia can give insight into the true temperature profile of a molecular cloud. By observing the intensity of various rotational lines in a cloud, one can gain an understanding of which lines are populated and which are not. Under the assumption of local thermodynamic equilibrium (LTE), one can then determine the temperature. Observations show that the temperature of molecular clouds ranges from 10 - 100 K with a typical temperature of ~ 30 K. As can be expected, molecular clouds are a transient phenomenon within the galaxy. The process of star formation and feedback mechanisms act to use up available gas and destroy the fragile molecular environment respectively. The typical life time of a molecular cloud is estimated to be about $3 \times 10^7$ years.
(Blitz and Shu, 1980), which is much less than a galactic rotation time.

### 1.1.2 Clumps and Cores

Observations of giant molecular clouds show the existence of varying levels of fragmentation (Frau et al., 2010; Sadavoy et al., 2012; Román-Zúñiga et al., 2009, 2010, 2012, among others). On the largest scale, the GMC itself is on the order of tens to hundreds of parsecs in diameter. Through processes that will be discussed shortly, a giant molecular cloud will fragment into smaller structures of varying length scales. Two structures of particular interest are clumps and cores. Both of these types of structures are defined as over-dense regions within the molecular cloud, however they each have distinct density thresholds.

A clump is generally defined as the largest scale fragmentation of a molecular cloud. Clumps are on the order of several parsecs across and generally exhibit a visual extinction ($A_v$) below 3 magnitudes. This corresponds to a column density of $\sim 3 \times 10^{21} \text{ cm}^{-2}$. Given the diffuse nature of material with such a small extinction value, these regions are expected to have high ionization fractions, on the order of $10^{-4}$ (Ruffle et al., 1998). Clumps may or may not be gravitationally bound, however the formation of a stellar cluster requires a gravitationally bound clump.

Cores are the intermediate-size structures between clumps and stars. These structures are on the order of a tenth of parsec in size and are defined to have a column density greater than $\sim 8 \times 10^{21} \text{ cm}^{-2}$. These regions generally exist in areas that shield the cores from incoming UV radiation. As such, they generally exhibit low ionization fractions.

### 1.2 Microphysics: The devil is in the details

The typical background density within a molecular cloud is several orders of magnitude smaller than the typical density of a protostar. Therefore there must exist some mechanism which compresses the molecular gas within a GMC into a much denser and smaller protostar. This is the basis of star formation research. The collapse of a molecular cloud into a dense core is dependent on the delicate balance between the inward force of gravity and the outward force of thermal pressure and other stabilizing forces within the environment. Such stabilizing forces include ambient magnetic fields and turbulence that can permeate molecular clouds. In the absence of such stabilizing forces, Jeans theory states that if a region obtains a minimum mass, gravitational collapse will occur. This mass is intimately related to a minimum wavelength for fragmentation known as the Jeans length. This length scale takes on different forms depending on the medium in question. The following subsections will discuss the derivation of these
length scales in a gravitationally-dominated environment as well as the effects of the stabilizing forces on the collapse of a cloud.

### 1.2.1 Gravitational Collapse and Jeans Theory

In an isothermal, uniform medium of density $\rho_0$, the dispersion relation is given by

$$\omega^2 = c_s^2 k^2 - 4\pi G \rho_0$$

(1.1)

where $c_s$ is the sound speed, $k \equiv 2\pi/\lambda$ is the wave number, and $\omega$ is the complex angular frequency. From this, the Jeans length, or minimum wavelength for gravitational fragmentation is,

$$\lambda_J = \left(\frac{\pi c_s^2}{G \rho_0}\right)^{1/2}.$$  

(1.2)

The corresponding Jeans mass is

$$M_J = \frac{4\pi \rho_0}{3} \left(\frac{\lambda_J}{2}\right)^3.$$  

(1.3)

Only clouds which have masses in excess of the Jeans mass are subject to fragmentation. The timescale for collapse due to self-gravity is expressed in terms of the free-fall time, $t_{ff}$. For an assumed pressure-free, spherical cloud, this timescale is given by

$$t_{ff} = \left(\frac{3\pi}{32G\rho_0}\right)^{1/2} = 1.37 \times 10^6 \left(\frac{10^3 \text{ cm}^{-3}}{n_H}\right)^{1/2} \text{ year}.$$  

(1.4)

The Jeans theory only applies to clouds that have a uniform density, uniform temperature, infinitesimal perturbations and infinite spatial extent (e.g. no edges). In addition, the Jeans mass is a lower limit for the required mass to initiate fragmentation. In theory, all modes with masses above this threshold will collapse; however, this is not what is observed. In fact, the fastest growing mode is found to be the one which encompasses the most mass, i.e., a contraction of the entire system. Since observations do not see infinite clouds with uniform density and temperature profiles that are monolithically collapsing, there must be something else that needs to be considered.

An alternative to an isothermal uniform system is that of an infinitely thin sheet. From the dispersion relation for surface density perturbations,

$$\omega^2 = c_s^2 k^2 - 2\pi G \Sigma_0 |k|$$  

(1.5)
where \( \Sigma_0 \) is the surface density of the sheet, one can find a critical length scale for collapse (Larson, 1985). The medium is unstable for negative values of \( \omega^2 \), which occurs when the second term on the right hand side of Equation 1.5 has greater magnitude than the first term. The critical wavenumber for collapse is then given by

\[
k_c = \frac{2\pi G \Sigma_0}{c_s^2}
\]  

(1.6)

As with the spherical case, collapse will occur for modes with wavenumbers smaller than \( k_c \) (wavelengths larger than \( \lambda_c = 2\pi/k_c \)) with one fundamental difference. Unlike the spherical case, for which the preferred mode for collapse is the largest wavelength within the system, here the maximum growth rate corresponds to \( \lambda = 2\lambda_c \), which is not on the order of the system size. This allows for clear fragmentation with a characteristic length of \( 2\lambda_c \) within the sheet (Larson, 1985).

If the sheet is not infinitely thin, under the assumptions of isothermality and self-gravity, a vertical density distribution of the form

\[
\rho(z) = \rho(0) \text{sech}^2 \left( \frac{z}{H} \right)
\]  

(1.7)

will exist in the medium. Under these circumstances, the critical wavenumber is found to be exactly half of that for the infinitely thin sheet (Ledoux, 1951, see also Larson, 1985).

### 1.2.2 Additional Environmental Effects

The fraction of gas mass converted into stars is known as the star formation efficiency (SFE). For cluster-forming regions, the SFE is between 10 and 30 percent (Lada and Lada, 2003). However, observations show that the SFE in giant molecular clouds is much lower at only a few percent (Goldsmith et al., 2008). This implies that pure gravitational fragmentation is not the primary fragmentation process within GMCs. Something within the molecular clouds must be inhibiting the collapse into dense cores, resulting in a smaller SFE. The following two sections will discuss the two mechanisms which can alter the evolutionary course of a gravitationally collapsing cloud.

**Magnetic Fields**

One of the stabilizing forces that can exist within a molecular cloud is that of an ambient magnetic field. This field is generally on the order of several tens of microgauss to several milligauss in strength and threads the entire molecular cloud. The field strength measured by
the Zeeman effect (e.g. Crutcher, 1999) tends to scale linearly with the column density. The formation of these fields is via mechanisms that form the prevailing field within a galaxy. In the presence of a magnetic field, the fragmentation and collapse of a molecular cloud becomes a delicate balance between the inward force of gravity and the stabilizing forces of the ambient magnetic fields. As will be described in more detail in the next chapter, the stability of a magnetized cloud is measured by its mass-to-magnetic flux ratio $\mu_0$. A subcritical value for the mass-to-flux ratio (i.e. $\mu_0 < 1$) indicates that the magnetic field is strong enough to support a gravitationally collapsing cloud, effectively preventing the formation of dense cores. On the other hand, supercritical values of the mass-to-magnetic flux ratio ($\mu_0 > 1$) indicate that the magnetic flux is not strong enough to support the cloud against gravitational collapse and a core has the opportunity to form.

Based on these criterion, collapse of a cloud into a dense core in the presence of a magnetic field implies that the mass-to-flux ratio is already supercritical, i.e. a magnetic field is present but is not strong enough to inhibit gravitational collapse or enough mass has been compressed together such that gravity can overcome a strong magnetic field. This leads to a fundamental problem within the field of star formation. The strength of a magnetic field will grow as the inverse of the cross-sectional area. Therefore, as a cloud collapses, the magnetic field strength should increase. Observations of the interstellar medium (ISM) and stars show that the ISM is highly magnetized while stars are generally weakly magnetized. For example, a one solar mass region of a molecular cloud would have a radius of 0.06 pc (assuming a column density of $10^{21}$ cm$^{-2}$ and a mean molecular weight of 2.33 amu), while the Sun has a radius of $2.25 \times 10^{-8}$ pc. If we assume a molecular cloud has a field strength on the order of 10 $\mu$G, the magnetic field within a region with the radius of the Sun should be on the order of $10^8$ G. However, the measured strength of the solar magnetic field is 0.5 – 4 G. This indicates that throughout the process of star formation, the mass-to-flux ratio has to increase dramatically. The question is: how? The strength of the magnetic field compared to the relative density of the cold HI phase of the ISM ensures that the cloud is very subcritical. By definition, these clouds should not collapse due to a high magnetic flux that maintains stability against the self-gravity of the cloud. However, these clouds will collapse, indicating that through some means, either mass enters the system or magnetic flux leaves the system.

Ambipolar diffusion, the drift of neutral matter with respect to the plasma and magnetic fields, occurs when there is imperfect collisional coupling between the neutral and charged particles and is the conduit for this apparent flux-leak. Due to this imperfect coupling, gravitationally-driven diffusion of matter will redistribute mass and magnetic flux in the inner flux tubes of the cloud (Ciolek and Basu, 2006). This redistribution of mass and flux allows for a situation in which the inner portions of a subcritical cloud can become supercritical, allowing for gravita-
tional collapse of this inner region. The timescale for this redistribution depends on the location within the cloud. For inner regions which are shielded from ionizing radiation of the ISM, the ambipolar diffusion timescale is on the order of 10 times the free-fall time. In the outer regions where the cloud is exposed to the radiation field of the ISM, the timescale for collapse is even longer.

**Turbulence**

The Oxford English dictionary defines turbulence as a state of “violent commotion, agitation or disturbance,” with a turbulent fluid further defined as one “in which the velocity at any point fluctuates irregularly”. Scientifically, turbulence is the break down of laminar or streamlined flow of a fluid as it flows past an obstruction. It is characterized by the Reynolds number \( R \) which is a ratio of the inertial forces to viscous forces,

\[
R = \frac{LV}{\nu}
\]  

(1.8)

where \( L \) and \( V \) are the characteristic length and velocity scales of the flow and \( \nu \) is its kinematic viscosity (Frisch, 1995). As the Reynolds number increases, the flow downstream from the object becomes more chaotic. Any flow may be subject to turbulence if the viscosity is sufficiently small or the combination of characteristic length scale and velocity are sufficiently large. Specifically, turbulence is evident in stellar atmospheres, stellar outflows and jets, and within the interstellar medium and molecular clouds. The effect of turbulence is to form eddies of various length scales within the fluid. Within turbulence, energy is generally injected on the largest scales and dissipated on the smallest scales (Frisch, 1995). The energy cascades down from the largest to smallest scales via interaction between different size eddies (Lévéque, 2009). The smallest-scale structures are small enough that they are affected by molecular diffusion and viscous dissipation. Without a driving mechanism on the largest scales, this cascade process allows turbulence to dissipate and disappear.

The length scale at which this occurs is the Kolmogorov length scale

\[
\eta = \left( \frac{\nu^3}{\epsilon} \right)^{1/4}
\]

(1.9)

where \( \epsilon \) is the average rate of dissipation of turbulent kinetic energy per unit mass. The flow of energy from the large to small scale structures is referred to as the turbulent power or kinetic energy spectrum \( E(k) \). By utilizing dimensional analysis, Kolgomorov found that the shape of
the spectrum must be (Lévêque, 2009)

\[ E(k) = k_0 e^{2/3} k^{-5/3} \quad L^{-1} \gg k \gg l^{-1} \]  

(1.10)

where \( k \) is the wave number, \( k_0 \) is a scaling constant and \( l \) is the viscous dissipation scale.

With regards to star formation, there are situations in which supercritical clouds, which by definition should collapse, will remain stable against collapse within the free-fall timescale. This stability is provided by turbulent motions within the cloud. However, this artificial stability is transient. As discussed above, turbulent motions within a medium act to transfer energy from the macroscopic to the microscopic, allowing for both energy and the turbulence itself to dissipate from the system. Simulations show that the timescale for turbulent dissipation is given by

\[ t_{\text{diss}} \approx 0.5 \frac{d}{v_{\text{los}}} \]  

(1.11)

where \( d \) is the diameter of the cloud and \( v_{\text{los}} \) is the velocity along the line of sight (McKee and Ostriker, 2007). This timescale is consistent with assumptions made by Mestel and Spitzer (1956) that turbulence in a giant molecular cloud would decay within a crossing time of the structure. Therefore, in the time it takes for a particle to traverse the diameter of a molecular cloud, the stabilizing effects of turbulence will have dissipated, allowing the supercritical cloud to finally collapse. This time must be taken into account with regards to the timescale of collapse from molecular cloud to dense core. Therefore, the timescale for collapse is increased within a turbulent medium.

1.2.3 Magnetic Fields vs. Turbulence

Within the field of star formation, there exists to this day a debate over which mechanism(s) is/are the most important for the fragmentation of molecular clouds and formation of stars. Star formation as a research area only dates back to the early 1940s. Spitzer (1941) suggested that given the short lifetimes of the most luminous stars they must have been formed recently (Larson, 2011). However, as stated by Larson (2011), this paper by Spitzer makes no actual reference to star formation. This is due to the fact that at this time, the idea that stars may now be forming out of interstellar material was considered radical. As such, the referee for the Spitzer (1941) paper said the concept of star formation was far too speculative and should be removed from the paper, which Spitzer did (Larson, 2011) However, by 1946, the concept of star formation was less radical and the study of star formation began in earnest.

At the beginning, the complexity of star forming models was entirely dependent on the technology at hand. The initial pioneering work in this field was the determination of how
a newborn star would come to exist on the main sequence. Such research was performed by
Henyey et al. (1955) and Hayashi et al. (1962). Initial calculations for the actual collapse of a
molecular cloud into a star often took on the form of analytic and self-similar solutions (e.g.
Larson, 1969; Penston, 1969a,b). Primitive computer simulations by McNally (1967) and Bod-
denheimer and Sweigart (1968) explored the pure gravitational collapse of a gaseous mass.
However, with the declassification of hydrodynamic codes originally used for nuclear bomb
simulations, star formation simulations became more sophisticated (e.g. Woodward, 1976;
Norman and Wilson, 1978). Although the importance of magnetic fields was established in
the mid 1970’s (Norman, 2011), magnetohydrodynamic (MHD) simulations lagged behind by
about a decade because even the simplest problem required 2D simulations whereas previous
hydrodynamic (HD) simulations could be done in 1D. Therefore the field split into those who
continued to add more physics to HD simulations and theories (e.g. turbulence, rotation, etc.)
(see Terebey et al., 1984; Vanajakshi and Jenkins, 1985; Henrikson, 1986, among others) and
those who forged ahead into the new territory of MHD simulations (Scott and Black, 1980,
among others).

Another reason for such a division comes from the observational side. Due to the typical
strength of the fields present in star forming regions, it is very hard to obtain reliable mea-
surements on the global scale of a molecular cloud, let alone the small local scale of a core.
In addition, the relatively weak nature of the fields themselves can be construed as evidence
for why they can be neglected. To this effect Natta (1980) stated “As long as the intensity of
magnetic fields inside molecular clouds is not directly measured, the importance of the mag-
netic field in dynamical evolution of the clouds can be a matter of opinion”. Turbulence, on
the other hand, is easily observed and can be characterized more readily. More recently, as
observations and theoretical models improve, the divide between the two branches has become
smaller (e.g. Elmegreen, 1999a; Tilley and Pudritz, 2005; Federrath and Klessen, 2012; Padoan
et al., 2012); both sides seem to acknowledge the need for both mechanisms to some extent in
all stages of star formation. In addition, it has been shown that turbulence can have an effect
on the magnetic field itself. Specifically, turbulence acts to decrease the ambipolar diffusion
timescale within a molecular cloud (Fatuzzo and Adams, 2002; Zweibel, 2002) and magnetic
reconnection timescale within core envelopes (Lazarian et al., 2012).

That being said, for this thesis, we have chosen to focus solely on the effects of ambipolar
diffusion on the formation process of clumps and cores and the possible observational infor-
mation available. As such we have chosen to ignore the effects of turbulence for the most part;
however, by doing this we are not suggesting that turbulence is not important, but rather that
we wanted to study just the magnetic effects.
1.3 Macrophysics: Fitting all the pieces together

As mentioned earlier, the macrophysics of star formation delves into the questions pertaining to the bigger picture. Here, research focuses more on how newly formed stars interact with each other and how current and future generations of stars are affected by such interaction. There are several questions which fall into this category of research, however this thesis will focus on two particular problems, specifically, the effect of ambipolar diffusion on the formation of cores within a cluster and the subsequent core mass function.

1.3.1 Cluster Formation

Although star formation can and does occur in isolation, the majority of stars form and exist within clusters. Studying cluster members provides many advantages for researchers, such as the ability to determine ages of the stars, since all stars within the cluster are assumed to have formed at approximately the same time. The study of stars which form and exist within clusters presents unique and interesting challenges that are not even thought of when considering isolated star formation. These include not only the differences in formation (i.e. competition for gas) but also the effect of the stars on their surroundings after they are formed (i.e. feedback mechanisms, changes in chemical abundances).

One of the fundamental questions pertaining to star clusters is how they are formed. The current picture of cluster star formation is one in which clusters form in filamentary clumpy clouds (Megeath et al., 2009). Observational evidence (e.g. Serpens (Davis et al., 1999; Harvey et al., 2007), Perseus including IC 348 and NGC 1333 (Enoch et al., 2006), among others) shows that these clusters form stars over regions of $\sim 1$ pc over the first few million years of their evolution. One issue with observations of cluster star formation (as well as isolated star formation for that matter) is that, given the large time frames over which stars are formed, our observations only provide a snapshot of the conditions present in the cloud at that time. In the case of cluster formation, a cluster is only generally only defined once a region shows evidence of clustered star formation. At this point in time the cloud from which the cluster formed has already evolved tremendously from its initial conditions. Due to the large timescales associated with star formation, observers cannot just go back in time to see what that specific cloud looked like before the cluster was formed. This is where theory and analytical models come in.

Using the knowledge of the known driving mechanisms behind star formation and the state of a particular region today, theory can rewind the clock to see how such structures came to be. Chapters 3 and 4 focus on the results of analytic methods in an attempt to determine the sequence of events which may cause the formation of clusters and to give current researchers an idea of the environment required to create the clusters observed today.
1.3.2 Mass Distribution

The Initial Mass Function (IMF) is the distribution of the number of stars (dN) found in different mass bins (dM). Initial studies on the “Original Mass Function” of stars were performed by Salpeter (1955) using observations of the number of stars contained in a particular luminosity bin. The monochromatic luminosity function of a cluster in, say, the $K$ band, $dN/dm_K$, is defined as the number of cluster stars per unit magnitude interval. This relation can be cast in terms of stellar masses $M_\ast$ as such:

$$dN/dm_K = dN/d\log M_\ast d\log M_\ast/dm_K.$$ (1.12)

To convert from luminosity to mass, one then needs the observed luminosity function and the derivative of the mass-luminosity relation (MLR). Using average masses which correspond to the magnitudes/luminosity of the stars observed, the first initial mass function was constructed. Figure 1.1 shows the logarithm of the “Original Mass Function” plotted as function of the logarithm of mass in solar units. Fitting the curve, it was found that between $\log(M/\text{M}_\odot) = -0.4$ and 1.0, the initial mass function $\xi \equiv dN/d\ln M$ varies as a power law of the mass:

$$\xi(M) \propto M_\ast^{-1.35}.$$ (1.13)

Subsequent to this discovery, this power of 1.35 became the fiducial reference number with regards to the slope of the initial mass function in the high mass regime, often referred to as the Salpeter IMF. Combining the apparent IMFs for different clusters, Kroupa (2001) plotted...
the power law index ($\alpha$) as a function of $\log_{10}m$. From this plot, it was shown that the initial mass function does follow the trend found by Salpeter (1955) for high mass stars, but that at $\sim 0.5 \, M_\odot$, the relation flattens and then turns over at $\sim 0.08 \, M_\odot$. From this analysis, a standard formulation for the IMF as a three-part power law has been defined (Kroupa, 2001; McKee and Ostriker, 2007):

$$\frac{dN}{d \ln m_*} \propto m_*^{-\alpha}$$

(1.14)

where

$$\begin{align*}
1.3 & \quad \text{for } 0.5 < m_*/M_\odot < 50 \\
\alpha & = 0.3 \quad \text{for } 0.3 < m_*/M_\odot < 0.5 \\
-0.7 & \quad \text{for } 0.01 < m_*/M_\odot < 0.08
\end{align*}$$

(1.15)

Within the past decade, there have been several new millimeter and sub-millimeter continuum surveys of cluster regions and large star-forming systems. These systems include $\rho$ Ophiuchi (Motte et al., 1998; Johnstone et al., 2000), the Serpens core (Testi and Sargent, 1998), Orion (Johnstone et al., 2001), Taurus (Onishi et al., 2002), and the Pipe Nebula (Alvés et al., 2007; Rathborne et al., 2009), among others. These surveys have shown that high-density cores can be identified in sufficient numbers and with sufficient resolution so as to begin to construct a statistical understanding of the number of cores that exist within a particular mass bin. This distribution of mass within the cores formed by a single cloud, referred to as the core mass function (CMF), is still not well understood. Of utmost interest is whether there exists a one-to-one mapping from the CMF to the initial mass function (IMF) of stars, i.e., does each core form one star or many? Another question is whether the value of the Salpeter slope is universal (i.e., the same everywhere) and constant (i.e., has been the same for all times). Universality and constancy of the Salpeter slope implies that all molecular clouds would follow the same evolutionary paths through all epochs. Differences in the high mass slope could indicate differences in the formation mechanisms. Currently, it is assumed that the Salpeter slope is universal and constant, however recent observations and simulations have started to reveal cracks in this assumption. Further analysis into the universality and consistency of the Salpeter slope across varying environmental conditions will be explored in Chapter 5.

1.4 Current Work

Given the broad range of topics within star formation, one can never endeavor to solve everything all at once. As such, this thesis aims to use the techniques and results of linear analysis to
delve into a couple of the macroscopic questions of star formation. As mentioned above, there are two main competing theories pertaining to the microphysics responsible for the collapse of molecular cloud material into a clump and subsequently a core. This thesis will focus on the effects of magnetic fields and ambipolar diffusion on the formation of stellar clusters as well as the core mass function of star forming regions. Chapter 2 will focus on the model used and a full discussion of the linear analysis technique employed. Chapter 3 presents a two-stage fragmentation model for the formation of clusters. Chapter 4 looks at the temporal evolution of a fragmenting molecular cloud in the context of the two-stage fragmentation model. Chapter 5 presents statistically generated core mass functions and the effects of environment on their shape and high mass slope. Finally, Chapter 6 summarizes the findings of this work and the avenues for further research.
Chapter 2

Modeling Collapse Within A Molecular Cloud

Theoretical models of astrophysical phenomena can take on several forms. The most obvious models are two- and three-dimensional (magneto/radiative) hydrodynamic computer simulations which follow the evolution of the phenomena throughout time to monitor how different physics and parameters interact with each other. Another approach is to take a more static approach by taking a system in equilibrium and seeing what happens when some external process “pokes” it, so to speak. This is the method of linear analysis. Regardless of the investigative method, both require a set of (magneto)hydrodynamic equations which describe the structure in question, in this case the molecular cloud, and how this structure evolves in time.

2.1 Cloud Model

The collapse and fragmentation of clumps and cores are assumed to occur within ionized, isothermal, magnetic interstellar molecular clouds. These clouds are modeled as a planar sheet with infinite extent in the \( x \)- and \( y \)-directions and a local vertical half thickness \( Z \). The nonaxisymmetric equations and formulations of the model, as described in Ciolek and Basu (2006) and Basu et al. (2009a,b), are as follows:

\footnote{Portions of this chapter have been published in Bailey, N.D., and Basu, S., 2012, ApJ, 761,67}
\[
\frac{\partial \sigma_n}{\partial t} = -\nabla_p \cdot (\sigma_n \vec{v}_n),
\]
\[
\frac{\partial}{\partial t} (\sigma_n \vec{v}_n) = -\nabla_p \cdot (\sigma_n \vec{v}_n) + \vec{F}_T + \vec{F}_M + \sigma_n \vec{g}_p,
\]
\[
\frac{\partial B_{z,eq}}{\partial t} = -\nabla_p \cdot (B_{z,eq} \vec{v}_n),
\]

where

\[
\vec{F}_T = -\tilde{C}_{\text{eff}} \nabla_p \sigma_n,
\]
\[
\vec{F}_M = B_{z,eq}(\vec{B}_p - Z\nabla_p B_{z,eq}) + O(\nabla_p Z),
\]
\[
O(\nabla_p Z) = \frac{1}{2} \nabla_p Z \left[ B_{x,Z}^2 + B_{y,Z}^2 + 2B_{z,eq} \left( B_{x,Z} \frac{\partial Z}{\partial x} + B_{y,Z} \frac{\partial Z}{\partial y} \right) + \left( B_{x,Z} \frac{\partial Z}{\partial x} + B_{y,Z} \frac{\partial Z}{\partial y} \right)^2 \right],
\]
\[
\vec{v}_i = \vec{v}_n + \frac{\vec{v}_{n,0}}{\sigma_n} \left( \frac{\rho_{n,0}}{\rho_n} \right)^{k_z} \vec{F}_M,
\]
\[
\tilde{C}_{\text{eff}} = \frac{\sigma_n^2 (3P_{\text{ext}} + \sigma_n^2)}{\rho_n (P_{\text{ext}} + \sigma_n^2)^2},
\]
\[
\rho_n = \frac{1}{4} (\sigma_n^2 + P_{\text{ext}} + \vec{B}_p^2),
\]
\[
Z = \frac{\sigma_n}{2\rho_n},
\]
\[
\vec{g}_p = -\nabla_p \psi,
\]
\[
\psi = \mathcal{F}^{-1} \left[ -\mathcal{F} (\sigma_n) / k \right],
\]
\[
\vec{B}_p = -\nabla_p \vec{\Psi},
\]
\[
\vec{\Psi} = \mathcal{F}^{-1} \left[ \mathcal{F} (B_{z,eq} - B_{\text{ref}}) / k \right].
\]

For the above equations, \( \sigma_n \) is the column density of neutrals, \( \vec{B}_p \) is the planar magnetic field at the top of the sheet, \( \vec{v}_n \) is the velocity of the neutrals in the plane, \( v_i \) is the corresponding velocity of the ions. The operator \( \nabla_p = \hat{x} \partial / \partial x + \hat{y} \partial / \partial y \) is the gradient in the planar directions within the sheet. The quantities \( \psi(x, y) \) and \( \Psi(x, y) \) are the scalar gravitational and magnetic potentials within the sheet, respectively. The vertical wavenumber \( k = (k_x^2 + k_y^2)^{1/2} \) is a function of wavenumbers \( k_x \) and \( k_y \) in the plane of the sheet, and the operators \( \mathcal{F} \) and \( \mathcal{F}^{-1} \) represent the forward and inverse Fourier transforms, respectively, which are calculated numerically using an FFT technique.
The volume density ($\rho_{n,0}$) is calculated from the vertical pressure balance equation

$$\rho_{n,0}c_s^2 = \frac{\pi}{2}G\sigma_{n,0}^2 + P_{\text{ext}},$$

(2.15)

where $P_{\text{ext}}$ is the external pressure on the sheet, $\sigma_{n,0}$ is the initial uniform column density of the sheet and $c_s = (k_BT/m_n)^{1/2}$ is the isothermal sound speed; $k_B$ is the Boltzmann constant, $T$ is the temperature in Kelvin and $m_n$ is the mean mass of a neutral particle ($m_n = 2.33$ amu). The evolution equations also include the effect of ambipolar diffusion, which is a measure of the coupling of the neutrals with the ions, and by extension the magnetic field. This is quantified by the timescale for collisions between neutrals and ions:

$$\tau_{ni} = 1.4\left(\frac{m_i + m_{H_2}}{m_i}\right)\frac{1}{n_i\langle\sigma w\rangle_{iH_2}}.$$

(2.16)

Here, $m_i$ is the ion mass, and $\langle\sigma w\rangle_{iH_2}$ is the neutral-ion collision rate. The typical atomic and molecular species within a molecular cloud are singly-ionized Na, Mg and HCO, which, on average, have a mass of 25 amu. Assuming collisions between $H_2$ and $HCO^+$, the neutral ion collision rate is $1.69 \times 10^{-9}$ cm$^3$ s$^{-1}$ (McDaniel and Mason, 1973). These collisions transfer information about the magnetic field to the neutral particles via the ions that are bound to the field lines. The threshold for whether a region of a molecular cloud is stable or unstable to collapse is given by the normalized mass-to-flux ratio of the background reference state,

$$\mu_0 \equiv \frac{2\pi G^{1/2}\sigma_{n,0}}{B_{\text{ref}}},$$

(2.17)

where $(2\pi G^{1/2})^{-1}$ is the critical mass-to-flux ratio for gravitational collapse in the adopted model (Nakano and Nakamura, 1978; Ciolek and Basu, 2006) and $B_{\text{ref}}$ is the magnetic field strength of the background reference state. Regions with $\mu_0 < 1$ are defined as subcritical, regions with $\mu_0 > 1$ are defined to be supercritical, and regions with $\mu_0 \simeq 1$ are transcritical. In the limit of flux freezing, the timescale for collisions ($\tau_{ni}$) is zero, implying frequent collisions between the neutral particles and ions, which therefore couples the neutrals to the magnetic field. Under these conditions, subcritical regions are supported by the magnetic field and only supercritical regions can collapse. For non-zero values of $\tau_{ni}$, a finite timescale for collisions between neutral particles and ions exists which is inversely dependent on the ion density. The variation in ion density within a cloud is linked to the type of ionizing radiation available at different depths within the cloud. This is shown in Figure 1 of Ruffle et al. (1998).

At low extinction, i.e., on the edge of the cloud, the main source of ionizing radiation is in the ultraviolet (UV) regime of the spectrum. With such energetic photons, the ionization fraction within this region is large. As the extinction increases, the amount of UV radiation that
can penetrate into the cloud decreases and thus the ionization fraction declines. This decline becomes very steep beyond a threshold column density due to shielding of the inner regions from the UV radiation that ionizes the outer layers. Ionization within these inner regions is predominantly due to cosmic rays. Ambipolar diffusion, i.e., neutral-ion slip, can lead to redistribution of mass relative to magnetic flux within a molecular cloud and cause gravitationally unstable regions to develop within subcritical regions. These regions then have the chance to collapse.

The model we use is characterized by several dimensionless free parameters. These include a dimensionless form of the initial neutral-ion collision time \( \tau_{ni,0}/t_0 \equiv 2\pi G\sigma_{n,0}\tau_{ni,0}/c_s \) and a dimensionless external pressure \( \tilde{P}_{\text{ext}} \equiv 2P_{\text{ext}}/\pi G\sigma_{n,0}^2 \), in addition to the dimensionless mass-to-flux ratio \( \mu_0 \) that was introduced earlier. We normalize column densities by \( \sigma_{n,0} \), length scales by \( L_0 = c_s^2/2\pi G\sigma_{n,0} \) and time scales by \( t_0 = c_s/2\pi G\sigma_{n,0} \). Based on these parameters, typical values of the units used and other derived quantities are

\[
\sigma_{n,0} = \frac{3.63 \times 10^{-3}}{(1 + \tilde{P}_{\text{ext}})^{1/2}} \left( \frac{n_{n,0}}{10^3 \text{ cm}^{-3}} \right)^{1/2} \left( \frac{T}{10 \text{ K}} \right)^{1/2} \text{ g cm}^{-2},
\]

(2.18)

\[
c_s = 0.188 \left( \frac{T}{10 \text{ K}} \right)^{1/2} \text{ km s}^{-1},
\]

(2.19)

\[
t_0 = 3.98 \times 10^5 \left( \frac{10^3 \text{ cm}^{-3}}{n_{n,0}} \right)^{1/2} (1 + \tilde{P}_{\text{ext}})^{1/2} \text{ yr},
\]

(2.20)

\[
L_0 = 7.48 \times 10^{-2} \left( \frac{T}{10 \text{ K}} \right)^{1/2} \times \left( \frac{10^3 \text{ cm}^{-3}}{n_{n,0}} \right)^{1/2} (1 + \tilde{P}_{\text{ext}})^{1/2} \text{ pc},
\]

(2.21)

\[
\tau_{ni,0} = 3.74 \times 10^4 \left( \frac{T}{10 \text{ K}} \right) \left( 0.01 \text{ g cm}^{-2} \right)^{2} \times \left( \frac{10^{-7}}{\chi_{i,0}} \right) (1 + \tilde{P}_{\text{ext}})^{-1} \text{ yr},
\]

(2.22)

where \( n_{n,0} \) is the initial neutral number density and \( \chi_{i,0} \) the ionization fraction. Note that \( \tau_{ni,0} \) is inversely proportional to \( \chi_{i,0} \) for a fixed \( \sigma_{n,0} \). For our analysis, we assume a dimensionless external pressure \( P_{\text{ext}} = 0.1 \) and a temperature of 10 K. Observations of star forming regions usually quote visual extinction \( (A_v) \) in place of column density. Following the prescription of Pineda et al. (2010), the conversion from visual extinction to column density can be achieved by combining the ratio of H\(_2\) column density to colour excess (Bohlin et al., 1978) with the ratio of total-to-selective extinction (Whittet, 2003) to yield a conversion factor of \( N(H_2) = 9.35 \times 10^{20}(A_v/1 \text{ mag}) \text{ cm}^{-2} \). Although this conversion is specifically for H\(_2\), the abundance ratios of other molecules present in the cloud are so much smaller (i.e., the abundance ratio of CO to H\(_2\) is on the order of \( 10^{-4} \)) that they do not contribute significantly to the overall number density. Therefore we assume that the H\(_2\) number density is representative of all species.
2.2. Linear Analysis

2.2.1 Derivation of Dispersion Relation for a Magnetic Cloud

Assuming the unperturbed zero-order or background state of the model is static and uniform, Equations 2.1 - 2.3 are linearized to first order for any physical quantity via

\[ f(x, y, t) = f_0 + \delta f \]

where \( f_0 \) is the unperturbed background state, \( \delta f \) is the perturbation, \( \delta f_a \) is the amplitude of the perturbation, \( k_x, k_y \) and \( k \) are the \( x, y \), and \( z \) wavenumbers respectively such that \( k^2 \equiv k_x^2 + k_y^2 \) and \( \omega \) is the complex angular frequency. The perturbations are necessarily small such that \( |\delta f_a| \ll f_0 \). With this assumed perturbation, \( \partial/\partial t \rightarrow -i\omega \), \( \partial/\partial x \rightarrow ik_x \), and \( \partial/\partial y \rightarrow ik_y \).

Applying the perturbation to the dimensional equations for a model cloud, and retaining only the first order terms, one obtains the following system of equations,

\[ \omega \delta \sigma_{n,0} = k_x c_s \delta v_{n,x} + k_y c_s \delta v_{n,y}, \]

\[ \omega c_s \delta v_{n,x} = \frac{k_x}{k}(C_{\text{eff},0} k - 2\pi G \sigma_{n,0}) \delta \sigma_{n,0} 
+ \frac{k_x}{k}(2\pi G \sigma_{n,0} \mu_0^{-1} + k V_{A,0}^2 \mu_0) \delta B_{z,\text{eq}}, \]

\[ \omega c_s \delta v_{n,y} = \frac{k_y}{k}(C_{\text{eff},0} k - 2\pi G \sigma_{n,0}) \delta \sigma_{n,0} 
+ \frac{k_y}{k}(2\pi G \sigma_{n,0} \mu_0^{-1} + k V_{A,0}^2 \mu_0) \delta B_{z,\text{eq}}, \]

\[ \omega \delta B_{z,\text{eq}} = \frac{1}{\mu_0} k_x c_s \delta v_{n,x} + \frac{1}{\mu_0} k_y c_s \delta v_{n,y} 
- i\tau_{ni,0}(2\pi G \sigma_{n,0} \mu_0^{-2} k + k^2 V_{A,0}^2) \delta B_{z,\text{eq}}, \]

where \( V_{A,0} \) is the Alfvén speed. This speed is related to the mass to flux ratio \( (\mu_0) \) as follows

\[ V_{A,0}^2 \equiv \frac{B_{\text{ref}}}{4\pi \rho_{n,0}} = 2\pi G \sigma_{n,0} \mu_0^{-2} Z_0 \]
where $Z_0$ is the half thickness of the sheet and $\rho_{n,0}$ is the mass volume density.

A mode is unstable if the imaginary part of the complex frequency $\omega_{IM} > 0$. The growth timescale of such an instability is $\tau_g = 1/\omega_{IM}$. The dispersion relation, under the assumption of ambipolar diffusion, is found to be

\[
(\omega + i\theta)(\omega^2 - C_{\text{eff},0}^2 k^2 + 2\pi G \sigma_{n,0} k) = \omega(2\pi G \sigma_{n,0} k \mu_0^2 + k^2 V_{A,0}^2)
\]  

(2.31)

where

\[
\theta = \tau_{ni,0} (2\pi G \sigma_{n,0} k \mu_0^2 + k^2 V_{A,0}^2)
\]  

(2.32)

In the limit of flux freezing, $\tau_{ni,0} \to 0$, which gives the reduced dispersion relation

\[
\omega^2 + 2\pi G \sigma_{n,0} k (1 - \mu_0^{-2}) - k^2 (C_{\text{eff},0}^2 + V_{A,0}^2) = 0.
\]  

(2.33)

The gravitationally unstable mode corresponds to one of the roots of $\omega^2 < 0$ and occurs for $\mu_0 > 1$. The growth time for this mode can be written as

\[
\tau_g = \frac{\lambda}{2\pi [G\sigma_{n,0} (1 - \mu_0^{-2}) (\lambda - \lambda_{MS})]^{1/2}}
\]  

(2.34)

for $\lambda \geq \lambda_{MS}$, where

\[
\lambda_{MS} = \frac{C_{\text{eff},0}^2 + V_{A,0}^2}{G \sigma_{n,0} (1 - \mu_0^{-2})}.
\]  

(2.35)

The length scale corresponding to the minimum growth time is $\lambda_{g,m} = 2\lambda_{MS}$. For the case with no magnetic field, Equation 2.35 reduces to the thin disk equivalent of the Jeans length,

\[
\lambda_J = \frac{C_{\text{eff}}}{G \sigma_{n,0}}.
\]  

(2.36)

Again, the length scale corresponding to the minimum growth time is $\lambda_{g,m,J} = 2\lambda_J$.

The addition of ambipolar diffusion complicates the process somewhat. In these cases, the gravitationally unstable mode corresponds to one of the roots of the full dispersion relation (Equation 2.31). However since it is a cubic function, there is no simple expression to describe these roots. Therefore, each length scale is computed numerically.
2.2. Linear Analysis

2.2.2 Derived Relations: Previous and Current

There are several relations that can be derived from the dispersion relations and their roots. The first is the relation between the growth timescale and the wavelength for specific values of the mass-to-flux ratio \( \mu_0 \) and neutral-ion collision time \( \tilde{\tau}_{ni,0} \).

Figure 2.1 shows the growth time \( \tau_g/t_0 \) as a function of the wavelength for four cases of the neutral-ion collision time \( \tau_{ni,0}/t_0 \). Each panel shows the dependence for several labeled values of \( \mu_0 \). Note that these figures are presented in normalized form. These different cases were chosen to represent various areas within a molecular cloud: diffuse regions with high ionization fractions (\( \tau_{ni,0}/t_0 = 0.001 \)), dense core forming regions with low ionization fractions (\( \tau_{ni,0}/t_0 = 0.2 \)) and an intermediate region that is somewhat between the two extremes (\( \tau_{ni,0}/t_0 = 0.04 \)). The flux-frozen case serves as a reference point.

For the flux-frozen case (\( \tau_{ni,0}/t_0 = 0 \)), note the dependence on the value of \( \mu_0 \); there are no unstable, gravitationally collapsing modes for \( \mu_0 < 1 \). This is consistent with the discussion earlier that for a flux-frozen cloud, only initially supercritical clouds can collapse. For the other three cases, however, the addition of ambipolar diffusion allows for unstable, gravitationally collapsing modes to exist for regions with \( \mu_0 < 1 \). For all four plots and all cases of \( \mu_0 \) shown, note that each curve has a distinct minimum. This minimum represents the shortest growth time (fastest growth rate) and corresponding preferred length scale for instability. We find the location of these minima for a series of values of \( \mu_0 \) for fixed values of \( \tau_{ni,0}/t_0 \). The left panel of Figure 2.2 shows the minimum growth time of the gravitationally unstable mode \( \tau_{g,m}/t_0 \) as a function of the critical mass-to-flux ratio \( \mu_0 \). In the limit of flux-freezing, the curve shows that for the supercritical regime \( \mu_0 > 1 \), the growth time for instability is short, essentially the dynamical time \( 2t_o \approx Z_0/c_s \). As the mass-to-flux ratio approaches the critical value \( \mu_0 = 1 \) the growth timescale for instability becomes infinitely long. Consistent with the discussion above, this implies that in the absence of ambipolar diffusion, it would take an infinite amount of time for a region with a critical or subcritical mass-to-flux ratio to collapse into a core. As the mass-to-flux ratio approaches infinity, this implies negligible magnetic support. In this regime, the growth time \( \tau_{g,T} \) is dependent on the critical thermal length scale \( \lambda_T \equiv C_{eff,0}^2/G\sigma_{n,0} \) as follows,

\[
\tau_{g,T} = \frac{\lambda}{2\pi[G\sigma_{n,0}(\lambda - \lambda_T)]^{1/2}}. \quad (2.37)
\]

The minimum growth time for the unstable mode occurs at \( \lambda_{T,m} = 2\lambda_T \). With the addition of ambipolar diffusion, the curves show that the subcritical regime has a finite growth timescale for instabilities. The magnitude of this timescale depends on the degree of ambipolar diffusion, i.e., the timescale for neutral-ion collisions. For short neutral-ion collision times \( \tau_{ni,0}/t_0 < 0.2 \), the timescale for collapse of a subcritical region is \( 10 - 50 \) times longer...
Figure 2.1: Growth timescale of gravitationally unstable mode ($\tau_g/t_0$) as a function of wavelength ($\lambda/L_0$) for models (a) $\tau_{ni,0}/t_0 = 0$, (b) $\tau_{ni,0}/t_0 = 0.001$, (c) $\tau_{ni,0}/t_0 = 0.04$, and (d) $\tau_{ni,0}/t_0 = 0.2$. Each panel shows timescale curves for models with mass-to-flux ratios $\mu_0 = 0.5$, 1.0, 1.1, 2.0, and 6.0 (labeled).
than that of a supercritical region; frequent collisions between the neutrals and ions will slow the redistribution of matter across the field lines. This is the origin of the often quoted result that the ambipolar diffusion time is $\approx 10$ times the free-fall time. As the neutral-ion collision time increases (i.e., $\tau_{ni,0}/t_0 > 0.2$), the time between collisions increases to a point that neutrals rarely/never collide with an ion, and therefore have no knowledge of the magnetic field present in the molecular cloud. In this case, the timescale for collapse of the subcritical regions approaches that of thermal collapse ($\tau_{g,T}$).

The right panel of Figure 2.2 shows the wavelength with minimum growth time, $\lambda_{g,m}/L_0$, as a function of $\mu_0$ (see also Morton, 1991; Ciolek and Basu, 2006). There are several features of this graph that are worth noting here. In the flux-frozen case, as the mass-to-flux ratio approaches the critical value ($\mu_0 = 1$) the wavelength with the minimum growth time approaches infinity. For a non-zero neutral-ion collision time, the wavelength with the minimum growth time becomes finite for mass-to-flux ratios less than or equal to the critical mass-to-flux ratio. This results in a peak in the region of $\mu_0 \sim 1$. Note that the location of this peak changes, becoming slightly more supercritical as the neutral-ion collision time increases. For highly subcritical regions, the length scale again asymptotes to the thermal limit. In this case, this is
due to the fact that the magnetic field is so strong that material flowing past cannot distort the field. As such, there are no magnetic tension forces to inhibit the flow and for all intent and purposes, it is as if the magnetic field is not there at all. The effect of the magnetic field is however evident in the increased fragmentation timescale for subcritical regions.

An alternate way to look at the data is to plot the minimum growth time and length scales as a function of the neutral-ion collision time for a constant mass-to-flux ratio. This is shown in Figure 2.3. The panels show the minimum timescale for collapse ($\tau_{g,m}/t_0$, left) and corresponding wavelength ($\lambda_{g,m}/L_0$, right) as a function of the dimensionless neutral-ion collision time ($\tau_{ni,0}/t_0$) for the five cases of the initial mass-to-flux ratio ($\mu_0$) displayed in Figure 2.1. Focusing on the left panel first, for $\mu_0 \leq 1.0$, the growth times decrease linearly until the neutral-ion collision time reaches unity, after which they asymptote to the thermal collapse time. Conversely, for $\mu_0 = 2.0$, there is a plateau for highly ionized regions where the collapse time is longer, while for low ionization fractions the collapse time is again the thermal collapse time. For large enough values of $\mu_0$, the gravitational effects outweigh the magnetic effects and the cloud will just collapse on the thermal timescale. Moving to the right panel, the general trends shown by all curves indicate that the wavelength with the minimum growth
time is a maximum for short neutral-ion collision times and a minimum for long neutral-ion collision times. For regions with high ionization fractions ($\tau_{ni,0}/t_0 \sim 0.1$), as $\mu_0$ increases, the wavelength increases from the thermal wavelength ($\lambda_T$) to a maximum at $\mu_0 = 1.1$, and then decreases back to the thermal wavelength as $\mu_0$ rises further (see Figure 2.2). On the other extreme, for low ionization fractions, the variation between the different mass-to-flux ratios is almost indistinguishable since the neutrals are poorly coupled to the ions and the magnetic field. Note however, that for intermediate values of neutral-ion collision times, $\tau_{ni,0}/t_0 \gtrsim 1.0$, the preferred wavelength for $\mu_0 = 2.0$ becomes larger than those for $\mu_0 = 1.0$ and $\mu_0 = 1.1$. This is a result of the complex interplay between ambipolar diffusion and field-line dragging in this hybrid regime of moderate coupling.

2.3 Summary and Future Directions

As shown in the previous sections, the application of a linear analysis to the magnetohydrodynamic equations has shown that the value of the length scale and timescale for fragmentation depend strongly on the mass-to-flux ratio and ionization fraction. A flux-frozen (i.e., highly ionized) region yields infinite length scales and timescales for fragmentation in the limit of a critical mass-to-flux ratio, and yields thermal length scales and timescales for highly supercritical regions. Conversely, the addition of ambipolar diffusion allows subcritical regions to fragment. This results in large but finite length scales for fragmentation in the transcritical regime ($\sim 0.8 < \mu_0 < 1.2$) and long fragmentation timescales for subcritical regions. By increasing the degree of ambipolar diffusion (i.e., decreasing the ionization fraction), the length and timescales in the trans- and subcritical cases tend back toward the thermal limit.

Due to the link between the environment and the length and timescales for fragmentation, the linear analysis technique has been able to help observers understand the probable conditions within a particular cloud, e.g., the magnetic field strength and ionization fraction. However, this analysis has not covered all of the environmental conditions within a cloud. For example, to date, there has been little to no research performed on the relationship between the fragmentation length and timescales and the main observable, i.e., column density/visual extinction. Consequently, the effects of a magnetic field and the ionization fraction on such a relationship are not known. The following chapters will address this deficiency in the context of clustered star formation.
Chapter 3

Two-Stage Fragmentation for Cluster Star Formation

3.1 Background

Molecular clouds are observed to have complex morphology existing on a wide range of scales (e.g., McKee and Ostriker, 2007; Goldsmith et al., 2008), and exhibit numerous clumpy and filamentary structures. They are the birthplaces of stars, and the observed distribution of young stellar objects (YSOs) also yields a wide range of object densities. The YSOs are measured to be anywhere from loosely clustered to highly clustered (Megeath et al., 2009). Many molecular cloud maps do however reveal an interesting pattern of dark clouds within which parsec-scale clusters are being formed. The dark clouds are often themselves separated by several parsecs. Even the Taurus molecular cloud, often associated with so-called “isolated” star formation, demonstrates several dark clouds (Onishi et al., 1998) of pc-scale and masses \( \sim 100 \, M_\odot \) that are separated by several pc, each containing embedded weak clusters of YSOs. It was suggested long ago that the dark clouds within a molecular cloud complex may be the result of some early fragmentation process (Dutrey et al., 1991; Gaida et al., 1984; Schneider and Elmegreen, 1979), and subsequently the suggestion was made that the fragmentation within the dark clouds may be controlled by the local Jeans length (Hartmann, 2002). Based on the observed morphology of clustering within parsec-scale clouds, there is indeed a case to be made for a two-stage fragmentation process. The fragmentation within the dark clouds also shows a possible scaling with Jeans length, e.g., the typical separation between cores in the Taurus dark clouds is 0.25 pc, consistent with the Jeans length corresponding to a column density associated with the visual extinction \( A_V \approx 5 \) magnitudes. The question then arises as to

\footnote{Portions of this chapter have been published in Bailey, N.D., and Basu, S., 2012, ApJ, 761,67}
what controls the fragmentation process at the parsec scale, leading to the formation of individual dark clouds within a molecular cloud complex. We look for an explanation for large initial fragmentation scales that does not depend on the scale of the modeled system.

3.2 Physical Model

Observational studies of star forming regions have shown that there are several different extinction thresholds which define the state of a molecular cloud. Ruffle et al. (1998) identify a threshold of $A_v \sim 3$ mag whereby above this value clumps within the Rosette Molecular cloud are believed to have embedded stars. Studies of the Ophiuchus and Perseus clouds by Johnstone et al. (2004) and Kirk et al. (2006), respectively, suggest that there is a core formation threshold of $A_v > 5$ mag and a star formation threshold of $A_v \sim 7 - 8$ magnitudes (see also Onishi et al., 1998; Froebrich and Rowles, 2010). Here we aim to investigate these thresholds to determine at what visual extinction values different structures within a molecular cloud will from.

We consider clump/core collapse within partially ionized, isothermal, magnetic interstellar molecular clouds. These clouds are modeled as a planar sheet with infinite extent in the $x$- and $y$-directions and a local vertical half-thickness $Z$. The nonaxisymmetric equations and formulations of this model are described in Chapter 2. As shown in the previous chapter, the natural result of the linear analysis is the correlation between fragmentation length and time scales with respect to the mass-to-flux ratio or neutral-ion collision time. However, these are not the typical quantities that are observed in a molecular cloud, the typical observed quantity being the column density/visual extinction of the region. The missing link required for this analysis is the ionization profile of the cloud. The following sections will describe the method for linking the fragmentation scales, $\tau_{g,m}$ and $\lambda_{g,m}$, to the column density or visual extinction within the cloud.

3.2.1 Ionization Profile

The ionization state of a molecular cloud depends on its density and consequently the visual extinction. As demonstrated by Ruffle et al. (1998), the ionization profile of a molecular cloud is one in which the outer layers are highly ionized due to photoionization by background ultraviolet (UV) starlight, while the inner layers are shielded from the UV photons and ionized by cosmic rays. This picture assumes a quiescent molecular cloud which has no evidence of star formation in its past; radiation from previous generations of stars can greatly complicate this picture.
In their investigation, Ruffe et al. (1998), modeled the chemical evolution of a collapsing clump using a simplified model. Here, we have constructed a model ionization profile for the cloud similar to those shown in Figure 1 of Ruffe et al. (1998). Unlike their ionization profiles, however, ours takes into account both the UV and cosmic ray (CR) ionization regions. The UV region is described by a step function of the form

\[ \log \chi_i = \log \chi_{i,0} + 0.5(\log \chi_{i,c} - \log \chi_{i,0}) \times \left(1 + \tanh \frac{A_v - A_{v,\text{crit}}}{A_{v,d}}\right), \]

where \( \chi_i \) is the ionization fraction, and \( \chi_{i,0}, \chi_{i,c}, A_{v,\text{crit}} \) and \( A_{v,d} \) are the step function parameters, namely the maximum ionization, minimum ionization, location and width of the step respectively. The cosmic ray regime is derived from the well known power-law expression for the ion density \( n_i \) as a function of neutral density \( n_n \), namely

\[ n_i = X n_n^{1/2}, \]

where \( X = 10^{-5}\text{cm}^{-3/2} \) (Elmegreen, 1979; Umebayashi and Nakano, 1980; Tielens, 2005). As shown by Caselli et al. (1998, Figure 11), clear correlations between \( n_i \) and \( n_n \) are difficult to establish observationally. Detailed calculations using grain chemistry (Ciolek and Mouschovias, 1994) show that the exponent in the above relation is not a fixed value. However, a value of 1/2 is a reasonable theoretical average for the relatively low density regime considered here. We combine Equation (3.1) with vertical hydrostatic equilibrium (Equation 2.15). A smooth transition between these two regimes is chosen to occur at a visual extinction of \( A_{v,\text{CR}} = 6.365 \text{mag} \). The full profile is then described by

\[ \log \chi_i = \begin{cases} \log \chi_{i,0} + 0.5(\log \chi_{i,c} - \log \chi_{i,0}) \times \\ \left(1 + \tanh \frac{A_v - A_{v,\text{crit}}}{A_{v,d}}\right) \end{cases} \]

\[ \begin{array}{ll} A_v \leq A_{v,\text{CR}} & \text{log}[1.148 \times 10^{-7}(1 + \tilde{P}_{\text{ext}})^{1/2} \times \\ \left(\frac{T}{10^4 \text{K}}\right)^{1/2} \left(\frac{2.75 \text{ mag}}{A_v}\right)] & A_v > A_{v,\text{CR}}, \end{array} \]

where the values of the step function parameters are \( \log \chi_{i,0} = -4, \log \chi_{i,c} = -7.3203, A_{v,\text{crit}} = 4.0 \text{ mag} \) and \( A_{v,d} = 1.05 \text{ mag} \). Four decimal place precision is used for some of the profile parameters to ensure a smooth transition, rather than for empirical reasons. The midpoint of the drop in ionization \( (A_{v,\text{crit}}) \) has been adopted to be closer to \( A_v \sim 5 \text{ mag} \) for comparison to observations in the Perseus region (see Section 3.4). All numbers above are to be taken as typical values, and our intention in this paper is to outline a general scenario rather than performing a detailed parameter search. The above profile can also be expressed in terms of the column density instead of the visual extinction via Equation 2.23.
Figure 3.1: Model ionization curve for a molecular cloud. Ionization fraction ($\chi_i$) is plotted against neutral column density ($\sigma_n$) and corresponding visual extinction ($A_v$) based on Equation (2.23). The curve is a composite which captures both the photoionized region ($A_v \leq 6.365$ mag; $\sigma_n < 0.0232$ g cm$^{-2}$) and the cosmic-ray-ionized regions ($A_v > 6.365$ mag; $\sigma_n > 0.0232$ g cm$^{-2}$).

Figure 3.1 shows the resulting profile for $\chi_i$ as a function of $\sigma_n$. As shown, the adopted model is one which has a steep step function as the ionization fraction drops off at $A_v \sim 3$ mag. However instead of leveling off at the bottom as the step function would normally, it continues to decrease as dictated by the cosmic ray relation. With this ionization profile in hand, we now have all the necessary pieces for constructing the relationship between the fragmentation time and length scales and column density or visual extinction. Taking the ionization profile, for each visual extinction value we can find the corresponding neutral-ion collision time via Equation 2.22. Assuming particular values of the mass-to-flux ratio, we can then use linear analysis to produce $\tau_g$ versus $\lambda$ plots akin to Figure 2.1 for each neutral-ion collision time. Finally, the location of the minimum for each $\tau_g$ - $\lambda$ curve yields the corresponding fragmentation time and length scale for each visual extinction value. The relation of these fragmentation scales with respect to column density can be determined using Equation 2.23.

Figure 3.2 shows the minimum timescale (left) and corresponding length scale (right) as
a function of $\sigma_n$ for assumed mass-to-flux ratios of 0.5 (solid line), 1.1 (dotted line) and 6.0 (dash-dotted line). Also plotted for reference is the ionization curve from Figure 3.1 (dashed line). Note that in the lower panel, the derived values for the $\mu_0 = 0.5$ and $\mu_0 = 6.0$ curves are almost equal and lie nearly on top of one another. As depicted in the right hand panel of Figure 2.2, this correspondence of values is evident in the locations of the curves at these values of the mass-to-flux ratio.

### 3.3 Two-Stage Fragmentation Model

For a diffuse molecular cloud, (i.e., $A_v < 3$ mag), the mass-to-flux ratio can be expected to be either sub- or transcritical. Figure 3.2 shows that while the length scale for fragmentation of a subcritical region is small, the timescale is essentially infinite, therefore direct fragmentation of a diffuse subcritical cloud into cores is unlikely. Instead, the more likely scenario is the fragmentation of a transcritical cloud. Under this assumption, the length scale for fragmentation is on the order of several parsecs and the timescale for collapse is no longer effectively infinite (see Figure 3.2, dotted line); therefore, clumps can form. As the newly formed clump starts to contract, the density, and consequently the visual extinction, increases. This contraction will eventually push the clump past a critical $A_v$ threshold ($A_{v,\text{crit}}$) where the ionization fraction and the length and time scales for collapse decrease dramatically. When this occurs, high density regions within the clump may subfragment and start collapsing independently from the clump. The timescale for the collapse of these subfragments is dictated by the local mass-to-flux ratio.
3.3. Two-Stage Fragmentation Model

Table 3.1: Derived time and corresponding length scales for visual extinction thresholds

<table>
<thead>
<tr>
<th>$\mu_0$</th>
<th>$\lambda_{g,m}$ (pc)</th>
<th>$\tau_g$ (Myr)</th>
<th>$\lambda_{g,m}$ (pc)</th>
<th>$\tau_g$ (Myr)</th>
<th>$\lambda_{g,m}$ (pc)</th>
<th>$\tau_g$ (Myr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.363</td>
<td>931</td>
<td>0.215</td>
<td>3.07</td>
<td>0.134</td>
<td>0.997</td>
</tr>
<tr>
<td>1.1</td>
<td>4.717</td>
<td>2.44</td>
<td>1.520</td>
<td>1.09</td>
<td>0.883</td>
<td>0.528</td>
</tr>
<tr>
<td>2.0</td>
<td>0.650</td>
<td>0.44</td>
<td>0.372</td>
<td>0.26</td>
<td>0.224</td>
<td>0.157</td>
</tr>
<tr>
<td>6.0</td>
<td>0.368</td>
<td>0.29</td>
<td>0.218</td>
<td>0.17</td>
<td>0.136</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Trans- and supercritical subfragments will collapse faster than the parent clump while subcritical subfragments will not.

From these plots, relevant time and length scales corresponding to the collapse of clumps and cores can be determined. Table 3.1 shows these scales for the three extinction thresholds and four values of $\mu_0$. For each extinction threshold, the variation of $\lambda_{g,m}$ and $\tau_g$ across the different values of $\mu_0$ further highlights the trends shown in Figure 2.2. Looking at a specific $\mu_0$ value, the length and time scales for collapse become progressively smaller for increasing extinction. Of particular interest are the comparisons between the length and time scales for the formation of clumps versus cores. Assuming a clump has $\mu_0 = 1.1$ and $A_v \sim 3$ mag and a subfragment has an $A_v \sim 5$ mag, $\lambda_{\text{clump}}/\lambda_{\text{core}} = 3 - 22$ and $\tau_{\text{clump}}/\tau_{\text{core}} = 2.2 - 14$ for $\mu_0 \geq 1.1$ in the core. For $\mu_0 < 1$, $\tau_{\text{clump}}/\tau_{\text{core}} \leq 1$ indicating that a subcritical subfragment will not collapse before the parent clump does.

Note that these time scales represent one $e$-folding of the density. Depending on the initial density, it may take several $e$-folding times for a structure to reach the threshold density that defines it as a core. Taking all of this into account, this analysis suggests that a smaller, more dense region can form within a larger, more diffuse region and can collapse within the timescale of collapse for the larger region. The disparity of time and length scales for collapse between the clump and core is far greater for a model with magnetic fields and ambipolar diffusion than it is for a simple hydrodynamic model.

The underlying model and subsequent results for the two-stage fragmentation scenario, as outlined above, propose various constraints on the conditions required for such a model to work in nature. Namely, the clump must form in an initially transcritical diffuse environment while core formation requires a denser supercritical region within the parent clump. Our analysis also shows that the fragments within these clumps exhibit length scales that are indicative of regions that are either very subcritical or very supercritical. In order to truly delineate between...
these two extremes, high resolution magnetic field data is required, however without such data, some conjectures about the region can still be drawn. As shown by Ciolek and Mouschovias (1994), the mass-to-flux ratio increases as the density of the region increases. The denser subfragments within the clump would be expected to have a greater mass-to-flux ratio than the global clump, allowing it to collapse faster than the clump itself (see Figure 3.2). It is possible for a minority of subfragments to locally have a subcritical mass-to-flux ratio, however these subfragments would likely not be able to collapse faster than their surroundings due the long timescale required. Although the lack of magnetic field data for most of the regions makes definitively choosing one mass-to-flux regime over the other impossible, it can be assumed that regions with active star formation must be within locally supercritical subfragments, while regions with little to no star formation are either too young for the subfragments to have collapsed or are largely subcritical.

Given the variability of the conditions within cloud regions, Figure 3.3 shows an enhanced picture of the two-stage fragmentation model. The right-hand panel shows the fragmentation length scale as a function of column density. In the diffuse regions of the clouds, as indicated earlier, we expect the mass-to-flux ratio be in the transcritical regime. The parameter space where we expect the fragmentation of the gas into parsec size clumps is indicated by the hatched region to the left of $A_v,\text{crit}$. As shown in the left-hand panel, this hatched region corresponds to fragmentation time scales on the order of 2-10 Myr. As the density of the region increases, it will cross over to the right hand side of $A_v,\text{crit}$. The hatched region on this side of $A_v,\text{crit}$ represents the parameter space for subfragmentation within the clump. As shown, a wide range of mass-to-flux ratios are considered able to fragment based on the decrease of the fragmentation length scale. However, the equivalent region in the upper panel of Figure 3.3 shows
that although regions within this range of mass-to-flux ratios can fragment, only the ones with trans- or supercritical mass-to-flux ratios (dotted and dash-dot lines respectively) are able to collapse at a rate that is faster than that of the clump itself.

3.4 Comparison to Observations

The above analysis reveals several physical conditions that are important for the collapse of clumps and subclumps into cores. First, for regions with transcritical mass-to-flux ratios, the preferred length scale decreases as the initial column density/visual extinction of the region increases. Based on our ionization model (see Figure 3.1), less dense regions \( (A_v < 3 - 5 \text{ mag}) \) have larger ionization fractions (smaller neutral-ion collision times) and thus yield a larger length scale for collapse while the denser regions \( (A_v > 5 \text{ mag}) \) have smaller ionization fraction (larger neutral-ion collision times) and thus yield much smaller collapse length scales. From a timescale point of view, it seems favorable for regions within a clump to fragment and start to collapse before the overall clump structure collapses or fragments.

Observations of nearby molecular clouds, including (but not limited to) the Taurus molecular cloud (Rebull et al., 2010; Hartmann, 2002; Schmalzl et al., 2010), Rosette Molecular cloud (Williams et al., 1995), Pipe Nebula (Frau et al., 2010; Román-Zúñiga et al., 2010) and Perseus molecular cloud (Kirk et al., 2006; Sadavoy et al., 2012) show clumpy filamentary structure both with and without active star formation. In the following discussion we choose a small sample of clumps which show evidence of substructure, and perform in-depth case studies comparing our two-stage fragmentation model to the observed clump data. The three regions that we have chosen come from the Perseus Molecular cloud, the Taurus Molecular cloud, and the Pipe nebula. These case studies are by no means a thorough compilation of all possible regions, but rather a representation of some interesting cases. This analysis can shed some insight into the possible magnetic field strengths needed within these structures. Note that the existence of subfragments within these chosen areas indicates that the clump has evolved to a point in the parameter space of Figure 3.2 that is to the right of \( A_v, \text{crit} \), although the initial fragmentation event that created the clumps would have occurred at lower densities on the left hand side of \( A_v, \text{crit} \).

3.4.1 Case Study A: Perseus B1-E

The Perseus molecular cloud is located at a distance of about 250 pc in the constellation Perseus. It shows clustered star formation primarily in two regions (NGC 1333 and IC 348) while elsewhere in the cloud it is relatively quiescent. A particularly interesting region is a
Chapter 3. Two-Stage Fragmentation for Cluster Star Formation

Table 3.2: Derived length scales for B1-E

<table>
<thead>
<tr>
<th>$A_v$ (mag)</th>
<th>$N_n$ ($10^{21}$ cm$^{-2}$)</th>
<th>$\mu_0 = 0.5$</th>
<th>$\mu_0 = 1.1$</th>
<th>$\mu_0 = 2.0$</th>
<th>$\mu_0 = 6.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9</td>
<td>2.7</td>
<td>0.373</td>
<td>4.85</td>
<td>0.668</td>
<td>0.378</td>
</tr>
<tr>
<td>6.8</td>
<td>6.3</td>
<td>0.160</td>
<td>1.05</td>
<td>0.268</td>
<td>0.162</td>
</tr>
<tr>
<td>10.6</td>
<td>9.9</td>
<td>0.102</td>
<td>0.66</td>
<td>0.170</td>
<td>0.103</td>
</tr>
</tbody>
</table>

$\sim 0.1$ deg$^2$ clump roughly 0.7 degrees east of the B1 clump, coined B1-E (Sadavoy et al., 2012). This region is sandwiched between the two star forming regions (NGC 1333 to the west, and IC 348 to the east), however previous submillimeter observations of this region showed that despite its high extinction ($A_v > 5$ mag), it contained neither dense cores nor young stellar objects (Kirk et al., 2006; Jørgensen et al., 2007, among others). However, recent data from the Herschel Gould Belt survey and subsequent observations from the Green Bank Telescope (GBT) have revealed that there is substructure within this region (Sadavoy et al., 2012). In their paper, Sadavoy et al. (2012) suggest that the delay in the formation of cores within this region could be due to a strong magnetic field. In this case study, we use our linear analysis methods described above to see if theoretical models exhibiting the same density as the observations result in similar observed length scales. We also comment on the mass-to-flux ratio that is likely required to generate these observed length scales.

First, the B1-E clump as a whole was found to have a column density $N_n = (6.3 \pm 3.6) \times 10^{21}$ cm$^{-2}$ and a radius of 0.46 pc (Sadavoy et al., 2012). Taking into account the error range on the density, this corresponds to a visual extinction range of 2.89 to 10.59 magnitudes, with the mid-point value corresponding to a visual extinction of $\sim 6.75$. Table 3.2 shows the derived length scales for collapse to be compared with the diameter of the structure, for the low, mid and high values of $A_v$ for $\mu_0 = 0.5$, 1.1, 2.0 and 6.0. From this analysis, we see that one scenario stands out as a good fit. The diameter of the B1-E clump is $\sim 1$ pc which matches the model with $A_v = 6.8$ and $\mu_0 = 1.1$ ($\lambda_{g,m} = 1.05$ pc). All the other models can be ruled out because they result in length scales that are much smaller than the diameter of the region. The model with $A_v = 6.8$ and $\mu_0 = 1.1$ corresponds to a growth timescale of about 0.5 Myr.

We now turn to the substructures identified within this clump. Sadavoy et al. (2012) list a range of densities for each fragment. Table 3.3 shows the length scales, $\lambda_{g,m}$, for each fragment. We have calculated $\lambda_{g,m}$ for both the upper and lower limits of these densities. For clarity, we present only the mean value of $\lambda_{g,m}$ for each of the four mass-to-flux ratios. Looking at the derived values, we see that those found for both $\mu_0 = 0.5$ and $\mu_0 = 6.0$ are consistent with the observed diameter $D_{\text{obs}}$ for each fragment. This suggests that the cores are in either a strongly or weakly magnetized region depending on whether the mass-to-flux ratio is sub- or
3.4. Comparison to Observations

Table 3.3: Mean derived length scales for fragments within B1-E

<table>
<thead>
<tr>
<th>Object</th>
<th>( N_n ) (10(^{21}) cm(^{-2}))</th>
<th>( D_{\text{obs}} ) (pc)</th>
<th>( \lambda_{p,m} ) (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mu_0 = 0.5 )</td>
<td>( \mu_0 = 1.1 )</td>
<td>( \mu_0 = 2.0 )</td>
</tr>
<tr>
<td>B1-E1</td>
<td>12.1 - 35.0</td>
<td>0.066</td>
<td>0.056</td>
</tr>
<tr>
<td>B1-E2</td>
<td>13.4 - 30.0</td>
<td>0.064</td>
<td>0.054</td>
</tr>
<tr>
<td>B1-E3</td>
<td>8.9 - 26.8</td>
<td>0.072</td>
<td>0.075</td>
</tr>
<tr>
<td>B1-E4</td>
<td>7.5 - 31.3</td>
<td>0.056</td>
<td>0.083</td>
</tr>
<tr>
<td>B1-E5</td>
<td>6.5 - 24.5</td>
<td>0.087</td>
<td>0.098</td>
</tr>
<tr>
<td>B1-E6</td>
<td>9.8 - 29.1</td>
<td>0.055</td>
<td>0.069</td>
</tr>
<tr>
<td>B1-E7</td>
<td>9.6 - 29.1</td>
<td>0.055</td>
<td>0.070</td>
</tr>
<tr>
<td>B1-E8</td>
<td>6.5 - 25.1</td>
<td>0.068</td>
<td>0.098</td>
</tr>
<tr>
<td>B1-E9</td>
<td>9.1 - 28.2</td>
<td>0.051</td>
<td>0.073</td>
</tr>
</tbody>
</table>

The lack of star formation in this region is evidence for either a slower fragmentation process or a later fragmentation event. The former possibility requires the presence of a strong magnetic field in the region. Formation of subcritical cores within a transcritical clump would require fragmentation along the magnetic field lines, which is not currently included in our model. Sadavoy et al. (2012) note that this region is likely isolated from the surrounding regions given that it does not fall into the age gradient provided by the two closest regions (IC 348 and NCG 1333). This lends credence to the possibility that this region is younger than the surroundings. However, if the clump/core/star formation threshold values hold, \( A_v \) values for the cores of ~ 7-13 mag suggest that they should have formed stars. The lack of observed stars within this region therefore suggests that collapse is being slowed. Sadavoy et al. (2012) concluded that this delay is due to a strong magnetic field.

Currently there are no magnetic field measurements specifically for the B1-E region, however measurements of the nearby B1 region have shown the existence of a strong magnetic field \( (B = 19 - 27 \mu G) \) (Goodman et al., 1989; Crutcher et al., 1993). The mass-to-flux ratio of a cloud can be written in terms of the density and field strength (Chapman et al., 2011) as

\[
\mu_{\text{obs}} = 7.6N_{||}(H_2)/B_{\text{tot}} \tag{3.3}
\]

where \( N_{||}(H_2) \) is the column density in units of \( 10^{21} \) cm\(^{-2}\) along a magnetic flux tube and \( B_{\text{tot}} \) is the total magnetic field strength in \( \mu G \). Observable quantities are the column density of molecular hydrogen, \( N(H_2) \), and the magnetic field component parallel to the line of sight \( (B_{||}) \). Projection effects between \( N_{||}(H_2)/B_{\text{tot}} \) and the observed quantity \( N(H_2)/B_{||} \) will overestimate \( \mu_{\text{obs}} \) by an average factor of 3 assuming a random orientation of the magnetic field with respect
to the line of sight (Heiles and Crutcher, 2005). By combining this with the conversion factor for $H_2$ number density to visual extinction, namely $N(H_2) = 9.4 \times 10^{20} A_v$, the mass-to-flux ratio of a region can be calculated in terms of the visual extinction and line-of-sight component of the magnetic field,

$$
\mu_{\text{obs}} = 2.4 A_v / B_{\parallel},
$$

where $A_v$ is in mag and $B_{\parallel}$ is in $\mu$G.

Assuming that the magnetic field in B1-E is similar to its neighboring region B1 and applying this equation to the values for the cores, we find that the majority of the mass-to-flux ratios fall into the transcritical to supercritical regimes (0.7-4.42). Some of the lower end densities for the cores give significantly subcritical values ($\mu_{\text{obs}} \sim 0.32$), however given the range in the observed densities for each core, these lower values are likely not indicative of the true density. Less extreme mass-to-flux ratios (i.e., $\mu_{\text{obs}} \sim 4$ vs $\mu_{\text{obs}} \sim 6$) suggest that the cores may be larger than defined in Sadavoy et al. (2012). Following this scenario, a strong magnetic field combined with higher densities results in regions that are transcritical which collapse on longer time scales than their supercritical counterparts.

Based on the above analysis, we conclude that this region is evolving at a slower pace than the neighboring regions and will need to attain greater column densities than the typical threshold value ($A_v \sim 8$) to push the cores into the supercritical regime, due to the strong magnetic field present. Observations of the entire Perseus region by Goodman et al. (1990) qualitatively indicate that the magnetic field strengths in NGC 1333 and IC 348 are weaker than in the B1 and B1-E regions. Assuming these observations are fairly complete, the presence of a weaker field in the regions which exhibit star formation and a greater field in those regions which do not show signs of star formation are generally consistent.

### 3.4.2 Case Study B: Taurus B218

The Taurus molecular cloud is one of the closest star forming regions, at a distance of only $137\pm10$ pc (Torres et al., 2007). It is known to be host to at least 250 young stellar objects (Rebull et al., 2010) that are distributed along the filamentary structures within the cloud (Hartmann, 2002). Schmalzl et al. (2010) present a high resolution column density map of the L1495 filament. Extinction maps of this region show that the lowest visual extinction is on the order of $A_v = 5$ mag and numerous dense fragments exist with peak extinction values of $A_v \geq 15$ mag. Specifically, there are 4 fragments that coincide with fairly large dense core populations. These fragments are four known Barnard Objects (B213, B216, B217 and B218) containing 14, 10, 8 and 6 dense cores respectively. The size of the Barnard Objects are $\sim 1$ pc across while the radii of the cores are $\sim 0.1$ pc. For our analysis, we choose to focus on B218.
B218 is an elongated object with an area of 0.13 pc\(^2\). The major axis is 0.82 pc while the minor axis is 0.35 pc. Schmalzl et al. (2010) quote a surface density \(N_n = 9.39 \times 10^{21}\) cm\(^{-2}\) for this region, corresponding to a visual extinction of about 10 mag. However, this only includes regions that have \(A_v \geq 5\) mag. Other observations of the region suggest an overall visual extinction of the B218 object to be about 3 mag (Gaida et al., 1984). For our analysis, we will look at three extinction values, namely \(A_v = 3, 5, 10\) mag. Table 3.4 shows the derived length scales \(\lambda_{g,m}\) for \(\mu_0 = 0.5, 1.1, 2.0, 6.0\). From this analysis, assuming that the length of the major axis corresponds to the collapse length, we see there is no exact match between the derived and observed length scale for the extinction values used. However, based upon the size scales derived for the various mass-to-flux ratios, we can conclude that the clump is transcritical; the length scales for the other mass-to-flux ratios are too small under all \(A_v\) considerations. Specifically, the observed length scale falls within the parameter space defined by 5 mag \(\leq A_v < 10\) mag and 0.9 \(\leq \mu_0 \leq 1.1\). A better match could be found by either increasing the visual extinction or decreasing value of the mass-to-flux ratio. Given that the mass-to-flux ratio will increase as the density increases, we suggest the former possibility; the visual extinction of the region is likely between 5 and 10 mag. Looking back at Table 3.1, we see that a region with an \(A_v = 8\) mag and \(\mu_0 = 1.1\) gives a length scale more on par with the major axis of B218.

From this analysis we can see that by identifying the major axis as the collapse length, the mass-to-flux ratio of the clump is estimated to be 0.9 \(\leq \mu_0 \leq 1.1\). Recent polarization observations of the Taurus region that estimate the magnetic field strength based on the dispersion of polarization directions (Chapman et al., 2011) show that the entire cloud complex which contains B213, B216, B217 and B218 (defined collectively in their paper as B213) is significantly subcritical. These two results are in direct conflict with each other. The main reason for this discrepancy is the lumping together of the four Barnard objects into one larger region. Specifically there is a discrepancy in the visual extinction of the regions. Chapman et al. (2011) cite a visual extinction for their B213 region of \(\sim 2\) mag while all four of the Barnard objects that make up this region are quoted to have visual extinction values on the order of 10 mag (Schmalzl et al., 2010). However, based on the analysis above, we see that the actual visual

![Table 3.4: Derived length scales for B218](image-url)

<table>
<thead>
<tr>
<th>(A_v) (mag)</th>
<th>(N_n) ((10^{21}) cm(^{-2}))</th>
<th>(\lambda_{g,m}) (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.80</td>
<td>0.363</td>
</tr>
<tr>
<td>5</td>
<td>4.67</td>
<td>0.215</td>
</tr>
<tr>
<td>10</td>
<td>9.39</td>
<td>0.1074</td>
</tr>
</tbody>
</table>

\(\lambda_{g,m}\) for \(\mu_0 = 0.5, 1.1, 2.0, 6.0\)
extinction of the B218 region is implied to be about 8 mag. Using this value of $A_v$ and the magnetic field strengths of B213 measured by Chapman et al. (2011) in Equation (3.4) results in only somewhat subcritical values ($\mu \sim 0.7$) as compared to those computed using $A_v = 2$ mag. Also, given that the region defined as B213 in Chapman et al. (2011) covers a much larger area than the B218 region, it is plausible that the local magnetic field strength in B218 could be greater than the global field of their B213 region. Taking errors into account, it is possible for these regions to have mass-to-flux ratios that are closer to the critical value, as suggested by our analysis.

Turning to the substructures, Schmalzl et al. (2010) list six high density regions within B218. Table 3.5 shows the derived length scales $\lambda_{g,m}$ for each fragment. In this case, there is no clear cut mass-to-flux ratio regime that yield a match between $\lambda_{g,m}$ and $D_{obs}$ for all of the cores. Fragments B218-2-4 have diameters that agree well with $\lambda_{g,m}$ in the $\mu_0 = 2.0$ regime while fragments 5 and 6 agree well with $\lambda_{g,m}$ in the $\mu_0 = 0.5$ and $\mu_0 = 6.0$ regimes. Finally, fragment 1 is consistent with a mildly supercritical mass-to-flux ratio ($1.1 < \mu_0 < 2.0$). As with the B1-E region, the lack of observed star formation within B218 is likely due to the strong magnetic field. Overall, the analysis of this region yields results that are consistent with the results of Section 3.3.

### 3.4.3 Case Study C: Pipe Nebula

Finally, the Pipe Nebula is slightly further away than the Taurus molecular cloud, at a distance of 145 pc. Unlike the previous two regions, the Pipe Nebula, although filamentary, shows little evidence of star formation; the only evidence is in B59. Recent studies (Frau et al., 2010; Román-Zúñiga et al., 2010) show that there exists evolutionary variation across the nebula. Frau et al. (2010) propose that the fragmentation in the “bowl” corresponds to early stages of evolution, the collapsing material in the “stem” corresponds to an intermediate phase, and the star formation within B59 corresponds to the latest stage of evolution. This in itself makes
the Pipe Nebula an interesting observational case on its own, as it seems to be providing us with several of the snapshots needed to decipher cloud to star evolution in one location. Dust emission maps show that there is a difference in the amount of structure within each region (Román-Zúñiga et al., 2010) that follows the evolutionary gradient proposed by Frau et al. (2010). The maps of the shank/stem regions show that they are much more diffuse than the B59 region, suggesting that they are indeed less evolved. Román-Zúñiga et al. (2010) find 220 extinction peaks within the stem, shank, bowl, and smoke regions of the nebula. All of the extinction peaks are within regions that have visual extinctions greater than the threshold value quoted by Ruffle et al. (1998). In addition, they all have radii that are on the order of a tenth of a parsec or less, which is in line with the sizes found in our analysis. Analysis of the location of peaks within the various regions of the cloud show that there is a high density of peaks within the bowl and B59 (Román-Zúñiga et al., 2010).

The previous two case studies have looked at the relationship between the length scales for clumps and cores in regions with little to no star formation. For this region, rather than repeating this type of analysis, we use our model to look at two different phenomena. First, we will apply our model to B59 as a whole. Second, we examine the four cores studied by Frau et al. (2010). The four cores in question are core 14, 40, 48 and 109, which come from three distinct regions of the Pipe Nebula; 14 resides in B59, 40 and 48 reside in the stem and core 109 is in the bowl.

B59 is the only region within the Pipe Nebula that exhibits active star formation. It is itself split up into several clumps as shown by Román-Zúñiga et al. (2009, Figure 6). Although all clumps show evidence of cores within them, only B59-09 harbours visible YSOs. As such, B59-09 provides the unique opportunity to look at all three levels of evolution within star formation: clump, cores and YSOs. Figure 2 of Román-Zúñiga et al. (2012) shows the centrally condensed \( A_v \) profile of the B59-09ab cores, centered on the star forming region. Assuming a minimum visual extinction threshold for the region of 10 mag, we find that the B59-09 clump has a radius of \( \sim 30000 \) AU, which corresponds to a diameter \( D_{\text{obs}} = 0.3 \) pc. Recent studies of B59 by Román-Zúñiga et al. (2012) show that the star forming core(s) within B59, specifically B59-09a and 09b, have a molecular hydrogen surface density of \( N_{H_2} = 2.96 \times 10^{22} \) cm\(^{-2} \) and a diameter \( D_{\text{obs}} = 0.11 \) pc. This surface density corresponds to a visual extinction of 31.6.

Table 3.6 shows the results of our analysis for both the extended and local clump regions of B59-09, with \( A_v = 10 \) mag and 31.6 mag respectively. Focussing on the extended region first and comparing the derived length scales to the observed, we conclude that the mass-to-flux ratio of the region is somewhere between 1.1 and 2.0. This means that the star forming clump B59-09 as a whole is moving out of the transcritical regime and is becoming supercritical. Diving a little deeper into this region, the second entry in Table 3.6 shows that the high density
region which contains the YSOs, if considered separately, is also globally between $\mu_0 = 1.1$ and $\mu_0 = 2.0$.

Going one step deeper, if the spacing between the YSOs ($\sim 0.06$ pc) is assumed to have a near one-to-one relationship with the scale of the earlier fragmentation, our analysis can shed some light on the probable conditions in the region at that time. Table 3.6 shows that for both $A_v$ values, in order to match the YSO spacing, the region is either significantly subcritical or supercritical. Evidence of YSOs within the region suggests the latter as the former have longer collapse times.

With that in mind, our analysis implies that in order for the YSOs to form with such spacing, the local mass-to-flux ratio in the locations of current YSOs must have been larger than the surrounding region to promote collapse. Observations show that the magnetic field strength in B59 is $17 \mu G$ (Alves et al., 2008). We can therefore narrow the visual extinction range in which the YSO fragmentation occurred. In order for a subregion to collapse, the mass-to-flux ratio must be sufficiently different from the background value. If we assume a minimum mass-to-flux ratio $\mu_0 = 2.0$, for $B = 17 \mu G$, Equation (3.4) yields a minimum visual extinction of 14 mag. Therefore, for the extinction range, $14 \text{ mag} < A_v < 31.6 \text{ mag}$, the observed magnetic field results in a corresponding mass-to-flux ratio range for the cores to be $2.0 < \mu_{\text{obs}} < 4.5$.

We now consider the four cores from the various regions in the Pipe Nebula. Table 3.7 shows the results of this analysis. Comparing the derived values of $\lambda_{g,m}$ with $D_{\text{obs}}$, cores 14, 40 and 48 seem to agree well with those in either the subcritical regime or highly supercritical regime ($\mu_0 = 6.0$). Core 109 does not agree well with any of the three regimes, however it can be concluded that it is not extremely sub- or supercritical but rather somewhere between $\mu_0 = 1.1$ and 2.0. With regards to the evolutionary gradient proposed by Frau et al. (2010), our length scales do show a trend to smaller length scales for cores 48, 40 and 14. Core 109 does not seem to fit in with this trend at all and given its size compared to core 40 could be mistaken as a star forming core. Comparison between the length scales for B59-09 and core 14 is not applicable since the latter is not contained in B59-09.

The evolutionary gradient between B59 and the bowl of the Pipe Nebula is a direct consequence of the different magnetic field strengths present in the different regions; $B = 17, 30,$

<table>
<thead>
<tr>
<th>$A_v$ (mag)</th>
<th>$N_n$ ($10^{21}$ cm$^{-2}$)</th>
<th>$D_{\text{obs}}$ (pc)</th>
<th>$\lambda_{g,m}$ (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0 = 0.5$</td>
<td>$\mu_0 = 1.1$</td>
<td>$\mu_0 = 2.0$</td>
<td>$\mu_0 = 6.0$</td>
</tr>
<tr>
<td>10</td>
<td>9.35</td>
<td>0.3</td>
<td>0.1078</td>
</tr>
<tr>
<td>31.6</td>
<td>29.6</td>
<td>0.11</td>
<td>0.0341</td>
</tr>
</tbody>
</table>
3.4. Comparison to Observations

<table>
<thead>
<tr>
<th>Object</th>
<th>(N_n (10^{21} \text{ cm}^{-2}))</th>
<th>(D_{\text{obs}} ) (pc)</th>
<th>(\lambda_{p,m} ) (pc)</th>
<th>(\mu_0 = 0.5)</th>
<th>(\mu_0 = 1.1)</th>
<th>(\mu_0 = 2.0)</th>
<th>(\mu_0 = 6.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>13.3</td>
<td>0.071</td>
<td>0.076</td>
<td>0.50</td>
<td>0.127</td>
<td>0.077</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>11.1</td>
<td>0.104</td>
<td>0.091</td>
<td>0.60</td>
<td>0.152</td>
<td>0.092</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>6.1</td>
<td>0.127</td>
<td>0.164</td>
<td>1.08</td>
<td>0.274</td>
<td>0.166</td>
<td></td>
</tr>
<tr>
<td>109</td>
<td>47.6</td>
<td>0.063</td>
<td>0.021</td>
<td>0.14</td>
<td>0.035</td>
<td>0.021</td>
<td></td>
</tr>
</tbody>
</table>

and 65 \(\mu G\) in B59, the stem, and the bowl respectively (Alves et al., 2008). The region with the lowest magnetic field has formed stars while the other two regions have only formed cores. This is consistent with the results of our other two case studies (see Sections 3.4.1 and 3.4.2), whereby the delay in star formation is due to a strong magnetic field which forces the region to attain a higher column density before it becomes supercritical and can collapse.

The current state of B59 (i.e., shows evidence of several cores/YSOs yet has not collapsed itself) is supported by its apparent transcritical nature. The age of the stellar cluster within B59-09 indicates that the clump has survived for longer than 10\(_{ff}\) times (Román-Zúñiga et al., 2012). This again suggests that the magnetic field within the region is strong enough to delay the overall collapse of the B59-09 clump (Román-Zúñiga et al., 2012) while the inner regions have attained sufficient conditions to form stars. Based on the current data, Román-Zúñiga et al. (2012) propose that B59-09 is currently contracting toward a denser configuration. Our model of a transcritical clump is consistent with this assertion.

### 3.4.4 Summary of Comparison to Observation

From the three case studies presented above, we have found that the observed length scales and corresponding mass-to-flux ratios determined from our analysis tend to support the two-stage fragmentation scenario. In general it seems that parent clumps exhibit mass-to-flux ratios that are transcritical (0.9 < \(\mu_0\) < 1.1). The column densities of these regions place them to the right hand side of \(A_v,\text{crit}\) (see Figure 3.2). This is consistent with our model since formation of subfragments requires that the clump has already crossed over the critical \(A_v\) threshold. The column density of all of the cores tend to suggest that they are either highly sub- or supercritical. This discrepancy is due to the coincidence between fragmentation length scales in these two regimes. A lack of comprehensive magnetic field observations in these regions is a major contributor to this ambiguity, however the more likely scenario points toward supercritical cores given that the timescale for collapse of subcritical core is very long even at high densities. For regions with magnetic fields measurements, we found that the existence of a gradient in
the field strength results in a gradient of star formation. For example, as noted above, in the Pipe Nebula, regions with the weakest magnetic field strength exhibit the formation of young stellar objects, while regions with the strongest magnetic field strengths currently only exhibit starless cores.

### 3.5 Summary and Limitations

We have studied the effect of ambipolar diffusion on the formation of stellar clusters using the results of linear analysis. By combining the linear analysis with realistic ionization profiles for a molecular cloud, our analysis has yielded a two-stage fragmentation model for clustered star formation that includes the formation of clumps and their subsequent subfragmentation. We present several interesting concepts that are worth noting.

- **Linear analysis** shows that there are varying length and time scales for collapse depending on both the mass-to-flux ratio and the neutral-ion collision time of the region (Morton, 1991; Ciolek and Basu, 2006, this work). Transcritical mass-to-flux ratios can have significantly larger fragmentation scales compared to super- and subcritical regions (Figure 2.2; right). Subcritical cores have significantly longer collapse time scales compared to their trans- and supercritical counterparts (Figure 2.2, left). Increased neutral-ion collision times serve to decrease the length and time scales for collapse, particularly for the transcritical and subcritical regimes.

- A combination of the linear analysis and ionization profile shows that molecular cloud conditions are favorable for a two-stage fragmentation process for the formation of stars. This model suggests that an initially diffuse, transcritical (i.e., \( \mu_0 \) is approximately unity) cloud can undergo an initial fragmentation on parsec scales and then undergo a second fragmentation event. This second event occurs once the density of the clump increases past the threshold where the ionization fraction drops dramatically and the length scale for fragmentation decreases steeply (Figure 3.2).

- Comparison with several observed core and star forming regions show that the clump sizes are consistent with their mass-to-flux ratio being transcritical, while the core length scales imply they could be sub- or supercritical. Lack of star formation to date within the observed regions (excluding B59) could suggest subcritical cores. However, analysis of the measured magnetic field for Perseus and Taurus showed that these regions are at least mildly supercritical.
3.5. **Summary and Limitations**

- The threshold values for clump/core/star formation as outlined in Section 3.4 are dependent on the region in question and should not be used as strict values. As shown in our case studies, regions with higher intrinsic magnetic fields will require a greater column density (higher visual extinction) to push the region to a transcritical and supercritical mass-to-flux ratio. The Pipe Nebula is a perfect example of this whereby cores in all three regions (B59, stem and bowl) have densities that exceed the star formation threshold \( A_v \sim 8 \) in other clouds, but only the region with the lowest magnetic field strength (B59) currently shows evidence of young stellar objects.

Although we have shown through this analysis that molecular cloud conditions are favourable for a two-stage fragmentation process, there are some limitations to this analysis.

- This analysis lacks any sort of time evolution. It is well accepted that structures within a molecular cloud generally collapse over some timescale, and thus that at some point a structure that has formed within the diffuse gas will eventually end up passing the critical visual extinction threshold where the ionization fraction drops. However, without implicit time evolution we do not know exactly how long this collapse takes. As mentioned above, the fragmentation time scales quoted via linear analysis are \( e \)-folding times. However depending on the starting column density of a particular structure, it may take several \( e \)-folding times to reach the necessary density required to undergo another fragmentation event. Keeping track of the time that has passed is required to get a handle on the formation and evolution of the structures within the cloud.

- Fragmentation of a molecular cloud is a delicate balance between gravitational forces and external forces such as those from magnetic fields that act to prevent collapse. All of the physical processes which go into fragmentation are dynamic and inter-related. For example, in the above analysis, we assume a particular value of the mass-to-flux ratio (e.g. \( \mu_0 = 1.1 \)) to describe the two stage-fragmentation process, however as the cloud evolves, it is unrealistic to assume that this value will remain constant throughout all regions of the cloud, especially within a collapsing region. As shown by Figure 2.2 (left), an increase or decrease in the mass-to-flux ratio (and neutral-ion collision time) can have significant effects on the collapse time. Therefore the evolution of all of the constituents must be tracked at all times.

The following chapter will explore the time evolution of cloud fragmentation to determine the veracity of the two-stage fragmentation scenario.
Chapter 4

Two-Stage Fragmentation Model: Numerical Simulations

4.1 Background

As outlined in Section 3.3, the two stage fragmentation model proposes that as the clump evolves/collapses, regions within will increase in column density to the point where the length scale for collapse is much smaller than the size of the clump itself allowing, theoretically, for the formation of subclumps/cores within the clump. In addition, the analysis showed that the timescales for collapse are such that at the time when they are formed, a clump will take longer to collapse than a subclump. However, as mentioned at the end of the previous chapter, these timescales are not static over the evolution of the objects. As the column density increases, the timescale for collapse decreases (see Figure 3.2) in a non-linear fashion such that the new timescale for collapse is not necessarily equivalent to the original timescale less the time elapsed. This change in the timescale cannot be captured by the linear analysis. In addition, this scenario cannot account for the time in between the formation of the clump and the subsequent formation of the subclumps. Therefore, to explore these dynamical features, simulations involving time evolution must be performed.

4.2 Numerical Model

To model the fragmentation of a molecular cloud and the subsequent evolution of the substructures formed, we utilize the IDL code developed by Basu and Ciolek (2004) (see also Ciolek and Basu, 2006; Basu et al., 2009b). This multi-fluid non-ideal MHD code solves Equations 2.1 - 2.14 numerically in \((x, y)\) coordinates. This code employs the numerical method of
4.3. Model Parameters

We ran several simulations to test various realizations within the parameter space. Unlike previous studies using this code (Basu and Ciolek, 2004; Ciolek and Basu, 2006; Basu et al., 2009b), the addition of the step-like ionization profile requires that the background column density within the simulations be initially defined. Since the aim of this investigation is to test the two-stage fragmentation model, we require an initially diffuse cloud and as such set the background column density to correspond to a visual extinction $A_{V,0} = 1$ mag. This corresponds to a column density $\sigma_{n,0} = 3.638 \times 10^{-3}$ g cm$^{-2}$ (see Equation 2.23) or number density $N_n = 9.3 \times 10^{20}$ cm$^{-2}$ (assuming $m_n = 2.33$ amu). All simulations begin with an initial linear column density perturbation. The amplitude of the perturbation in each pixel is a random number between zero and one. To maintain the requirement that the perturbation be necessarily small (i.e., $|\delta f_a| \ll f_0$), this random number is then scaled by a fixed value $A$. From here on, the
value $A$ will be referred to as the amplitude even though for all intents and purposes it is not the true amplitude present in any of the pixels. The scaled random value for each pixel is then added to the current column density value within that pixel. For some simulations, subsequent perturbations are applied at specific intervals ($\Delta t_{sp}/t_0$).

All simulations have an external pressure $\bar{P}_{\text{ext}} \equiv 2P_{\text{ext}}/\pi G \sigma_{n,0}^2 = 0.1$, temperature $T = 10$ K and are performed on a 512 x 512 periodic grid. Each dimension has a dimensionless length of $64\pi$. This translates to a physical box size which is 15.16 pc on a side, or a physical pixel size of 0.0296 pc. Finally, in order to compare the differences in the outcomes between the different models, we have fixed the random number generator to give the same realizations of the perturbations for each event, i.e., the initial perturbation in all simulations is generated by the same seed. Therefore, any differences detected between the models are due to actual changes in the parameters and not based upon random stochastic events. However, maintaining the same seed for all further perturbations in the same model can artificially favour structure in certain regions of the simulation. To avoid this, all subsequent perturbations in a single model are produced using the next sequential seed realization. All simulations are set to stop when $\sigma_n/\sigma_{n,0} \geq 10$ within any pixel. Runaway collapse is always underway at this stage, and a protostar would be expected to form in a short time. The initial parameters for the specific models can be found in Table 4.1. The values quoted for the initial neutral-ion collision time indicate whether the assumed ionization profile as a function of column density is step-like ($\tau_{ni,0}/t_0 = 0.001$) or cosmic ray only ($\tau_{ni,0}/t_0 = 0.2$).

<table>
<thead>
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<th>Model</th>
<th>$\mu_0$</th>
<th>$\tau_{ni,0}/t_0$</th>
<th>$\Delta t_{sp}/t_0$</th>
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<td>0.03</td>
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<td>0.001</td>
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</tr>
<tr>
<td>G</td>
<td>1.1</td>
<td>0.2</td>
<td>10</td>
</tr>
</tbody>
</table>
4.4 Simulations and Results

4.4.1 Effect of Linear Perturbations

The following sections will describe the qualitative and some quantitative features of each model. The first four models presented will investigate the effect of varying the length of time between subsequent perturbations while the models discussed in Section 4.4.2 will explore the effects of changing other parameters. In depth comparisons between the models, as well as comparisons to previous studies (e.g., Basu et al., 2009b) will be presented in Section 4.4.3. Finally, an extensive analysis of the clump and core features within each model will be discussed in Section 4.4.4.

Model A: Initial Perturbation Only

This model represents both a fiducial model with which our subsequent models can be compared as well as a natural progression from the similar simulations presented in Basu et al. (2009b), i.e., inclusion of the step-function ionization profile. Analysis of this simulation shows that large scale structure starts to emerge at $t/t_0 \approx 40$ while cohesive clump-like structures emerge at $t/t_0 \approx 135$. As the simulation continues, the clump starts to contract. At $t/t_0 \approx 130$, the ionization fraction drops dramatically in regions where the column density has passed the critical value, however there is no evidence of a second fragmentation event. Figure 4.1 shows the column density enhancement ($\sigma_n/\sigma_{n,0}$) map at the final time ($t/t_0 \approx 143$). The left panel shows the entire region while the right panel shows a zoom in of the clump region centered at $(x, y) = (348, 260)$. As shown in the left panel of this figure, the region fragments into two distinct clump regions (due to the periodic boundary conditions, the dense regions in the top left and bottom left regions of the simulation represent one continuous clump). The spacing between the centroids of these clumps is about 11 pc. At the time of initial fragmentation ($t/t_0 \approx 40$), the initial size of these clumps are $\sim 4.5$ pc x $\sim 10$ pc. At the end of the simulation, the clump has contracted to $\sim 1.9$ pc x $\sim 7.6$ pc. These sizes are consistent with those predicted by the linear analysis as well as those seen in observations. As shown in the right panel of Figure 4.1, there is no evidence of a second fragmentation event. Rather, the clump exhibits an onion-like structure with a dense central region and layers of more subsequently more diffuse material at larger radial distances from the center.

Figure 4.2 shows the magnetic field strength (left) and mass-to-flux ratio (right) of the clump centered at $(x, y) = (348, 260)$. Comparing the left panel of Figure 4.2 to the right panel of Figure 4.1 we can see that the magnetic field traces the column density enhancement structure. This implies that the magnetic field has been dragged in as the cloud has fragmented.
Figure 4.1: Column density enhancement ($\sigma_n/\sigma_{n,0}$) map for Model A at $t/t_0 = 143.6$. Panels show the full simulation region (left) and a zoom in of the clump region centered at $(x, y) = (348, 260)$ (right). Axes show the $x$ and $y$ pixel locations for the 512 x 512 grid. Each pixel is 0.0296 pc across. Colourtables show the range of ($\sigma_n/\sigma_{n,0}$) values on a linear scale.

Figure 4.2: Magnetic field strength ($B_z$) map for clump region centered at $(x, y) = (348, 260)$ (left) and $\mu_0$ map for full simulation region (right) at $t/t_0 = 143.6$. Axes show the $x$ and $y$ pixel locations for the 512 x 512 grid. Each pixel is 0.0296 pc across. Colourtables show ranges on a linear scale for both panels.
Figure 4.3: Ionization fraction $\chi_i$ map of clump region centered at $(x, y) = (348, 260)$ for Model A at $t/t_0 = 143.6$. Axes show the $x$ and $y$ pixel locations for the 512 x 512 pixel grid. Each pixel is 0.0296 pc across. Colourtable shows range of ionization fractions on logarithmic scale. Note reverse colourtable as compared to previous figures.

and collapsed. Switching our attention to the mass-to-flux ratio, at early times this value is random. This is due to its dependence on the column density within each pixel. As the simulation evolves, the structure of the mass-to-flux ratio begins to follow the column density and magnetic field strength structures. Over the course of the simulation, the value of the mass-to-flux ratio within any pixel does not deviate far from the initial value ($\mu_0 = 1.1$); the minimum and maximum values at the final time are $\sim 1.05$ and $\sim 1.17$ respectively. The right panel of Figure 4.2 depicts the final structure of the mass-to-flux ratio within the clump. As shown, the extreme inner region of the clump has a very high mass-to-flux ratio (relatively speaking) while just outside are two regions of very low mass-to-flux ratio. Finally, Figure 4.3 shows the ionization profile of the same clump at the final time. As with the magnetic field, the ionization structure follows that of the column density with the highest ionization existing on the outer edges of the clump and while the inner regions are shielded and exhibit the typical lower ionization fraction.

Model B: $\Delta t_{sp} = 5t_0$

This model is the first simulation which includes additional random column density perturbations throughout the simulation. This model has the shortest time between perturbations, $\Delta t_{sp} = 5t_0$. This corresponds to a physical time of once every $\sim 2$ Myr. For this model, the first hint of fragmentation occurs at $t/t_0 \approx 10$, with large scale structure forming between
Figure 4.4: Column density enhancement ($\sigma_n/\sigma_{n,0}$) map for Model B shortly after large scale structure forms within the region at $t/t_0 = 60.1$. Boxes outline the visually defined clump regions in this simulation. Colour bars show the range of column density enhancement values on a linear scale.

$t/t_0 = 20 \& 30$. The column density within these structures starts to increase shortly after formation. Five very distinct regions emerge at $t/t_0 \approx 50$. Figure 4.4 shows the large scale density structure within the simulation at the time when distinct clump regions emerge in this model ($t/t_0 \approx 60$). Boxes outline the general areas where clumps are visually identified. Numbers indicate the region identification. As shown, most clumps have a somewhat elliptical shape, however the perturbations have destroyed the symmetry that was evident in Model A. The clump in Region 3 deviates from the elliptical shape in favour of a long filamentary shape. The ionization drops between $t/t_0 = 60 \& 65$ at which point cores emerge in all of the 5 structures. The spacing between the clumps is between 5 to 9 parsec with the clump within Region 4 being ~3.7 pc x ~2.4 pc. Figure 4.5 shows the column density structure of the cloud at the final time ($t/t_0 = 70.0$). Looking at the lower panel, we see that the clump itself has two larger regions with one smaller one in between and 7 distinct cores as indicated by the white circles. These cores are on the order of 0.1-0.2 pc across and have an average spacing of 0.25 - 0.75 pc between. More in depth analysis of the cores and clumps will be presented in Section 4.4.4.

As shown in the left panel of Figure 4.6, the magnetic field strength again follows the column density structure. The right panel of the figure shows the mass-to-flux ratio structure. White contours show the location of the clump in Region 4 for reference. As shown, unlike Model A, this model has a much more chaotic mass-to-flux ratio structure. This is due to the fact that the column density is perturbed multiple times throughout the course of the simulation, which results in regions where the column density (and therefore mass) is enhanced in a
4.4. Simulations and Results

Figure 4.5: Column density enhancement maps for Model B at $t/t_0 = 70.0$. Panels show the full simulation region (left) and a zoom in of the clump in Region 4 (right). White circles indicate visually identified cores. Axes show the $x$ and $y$ pixel locations for the 512 x 512 pixel grid. Each pixel is 0.0296 pc across. Colourtable shows the $(\sigma_n/\sigma_{n,0})$ range on a linear scale (upper panel) and square root scale (lower panel).

Figure 4.6: Magnetic field strength, $B_z$ map for clump in Region 4 (left) and $\mu_0$ map for full simulation region (right) for Model B at $t/t_0 = 70.0$. Axes show the $x$ and $y$ pixel locations for the 512 x 512 pixel grid. Each pixel is 0.0296 pc across. Colour bars show the ranges on a linear scale for both panels. White contours (lower panel) depict $\sigma_n/\sigma_{n,0}$ levels from 2.0 - 8.0 in 2.0 increments.
pixel but the magnetic field is not. Therefore the mass-to-flux ratio does not get the chance to relax back to a uniform value as was observed in Model A. Looking at the full region, some coherent structure forms around the high column density regions, however it is hard to pick out a particular clump within the mass-to-flux ratio map unless the the location is already known. At the final time, the mass-to-flux ratio ranges from minimum of 0.7 to a maximum of 1.53. The subcritical regions generally correspond to low $\sigma_n/\sigma_{n,0}$ regions. All cores exhibit enhanced $\mu_0$ values on the order of 1.3, however, the highest $\mu_0$ value does not occur in a region that coincides with one of the five clumps. Finally, Figure 4.7 shows that the ionization profile again mimics the column density structure.

**Model C: $\Delta t_{sp} = 10t_0$**

For this model, the time between subsequent perturbations has been doubled as compared to the previous model. This results in an increase in the total simulation time to $t_{run} \sim 80t_0$. Overall, as shown by Figures 4.8 - 4.10, the macro structures formed in this simulation are very similar to those seen in Model B, resulting in the same sizes and spacing for the clumps formed. However, the evolution of the clumps and cores occurs over a longer time frame. The large scale structure still appears by $t/t_0 \approx 20$, but a coherent clump does not emerge in this model until $t/t_0 \approx 70$ as compared to at $t/t_0 \approx 60$ for Model B. Likewise, cores develop between $t/t_0 \approx 70 - 75$. 

Figure 4.7: Ionization fraction $\chi_i$ map of clump within Region 4 for Model B at $t/t_0 = 70.0$. Axes show the $x$ and $y$ pixel locations for the 512 x 512 pixel grid. Each pixel is 0.0296 pc across. Colourtable shows range of ionization fractions on logarithmic scale. Contours are the same as Figure 4.6. Note reverse colourtable as compared to previous figures.
4.4. Simulations and Results

Figure 4.8: Column density enhancement maps for Model C at $t/t_0 = 80.7$. Panels, white circles, and colour bars are the same as Figure 4.5.

Figure 4.9: Magnetic field strength (left) and mass-to-flux ratio (right) maps for Model C at $t/t_0 = 80.7$. Panels, contours, and colour bars are the same as Figure 4.6.

Figure 4.10: Ionization fraction map for Model C at $t/t_0 = 80.7$. Contours and colour bar are the same as Figure 4.7.
On smaller scales, the effect of the less frequent perturbations becomes more evident. The left panel of Figure 4.8 shows the column density enhancement at $t/t_0 = 80.7$, while the lower panel shows the close up of clump in Region 4. As with the previous model, the white circles indicate the regions of visually identified cores. Comparing the final structure of this clump to the same region in Model B (see right panel of Figure 4.8) we can see that the clump is more cohesive as opposed to looking more like two to three smaller clumps. In addition, there are fewer cores within this clump at the end of the simulation than were observed for Model B. This is likely due to the fact that with less frequent perturbations being introduced to the system, the medium has more time in between to readjust, resulting in less structure. Figure 4.9 shows the magnetic field strength within clump in Region 4 (left panel) and the mass-to-flux ratio map of the full simulation box (right panel). The magnetic field strength again follows the column density structure while the mass-to-flux ratio shows the same chaotic nature as Model B. Finally, Figure 4.10 shows the ionization structure of the clump in Region 4. Again, we see that the densest regions exhibit a very low ionization fraction while the more diffuse regions have a higher ionization fraction.

**Model D: $\Delta t_{sp} = 15t_0$**

For this model, the time between subsequent perturbations has been increased to $\Delta t_{sp} = 15t_0$. This increases the total simulation time to $t/t_0 \approx 88$. As with the previous models the same overall structures are formed, however the time frame for formation is again delayed. The first hint of large scale structure emerges again at about $t/t_0 = 20$, however the pattern of clumps observed in the previous two simulations does not emerge until $t/t_0 \approx 50$ with coherent clumps finally emerging at $t/t_0 \approx 75$. The ionization fraction drops between $t/t_0 = 75 & 80$ and cores form shortly thereafter. At $t/t_0 \approx 83$ there are 5 distinct column density peaks in the clump within Region 4 that are or could become cores. The lack of cores in the other four regions is due to the fact that the column density enhancement in these regions is on average about 1.5, which is hardly enough to define the region as a clump.

As with the other models, Figures 4.11 - 4.13 show the other physical model parameters at the final time. From the right panel of Figure 4.11, we see that at the end of the simulations, the number of column density peaks has decreased to 3 as compared to other models. Finally, as with the previous two models, the magnetic field strength and ionization fraction structure follows the column density structure (Figure 4.12 (left) and Figure 4.13 respectively) while the mass-to-flux ratio map again shows a very chaotic structure (Figure 4.12, right).
4.4. Simulations and Results

Figure 4.11: Column density enhancement maps for Model D at $t/t_0 = 87.7$. Panels, white circles and colour bars are the same as Figure 4.5.

Figure 4.12: Magnetic field strength (left) and mass-to-flux ratio (right) maps for Model D at $t/t_0 = 87.7$. Panels, contours and colour bars are the same as Figure 4.6.

Figure 4.13: Ionization fraction map for Model D at $t/t_0 = 87.7$. Contours and colour bar are the same as Figure 4.7.
4.4.2 Extra Models

In addition to the previous four models which explored the effects of varying the frequency of perturbations, we ran three other models which explored the effects of changing other parameters. For all three models, the time between perturbations is set to that of Model C (i.e., $\Delta t_{sp} = 10t_0$). For Model E, we explore the effect of changing the amplitude of the perturbations while in Model F, we explore the effect of setting the initial mass-to-flux ratio within the simulation region to a more supercritical value. Finally, for Model G, we explore the effect of assuming a cosmic ray only ionization profile with an initial neutral-ion collision time, $\tau_{ni,0}/t_0 = 0.2$. We discuss the effect of these individual changes below. A full comparison between our models and those presented in Basu et al. (2009a) will follow in Section 4.4.3.

Model E: $\Delta t_{sp} = 10t_0$, $A = 0.015$

This model explores the effect of changing the amplitude of the column density perturbations. Namely, we have decreased the amplitude by a factor of 2 to $A = 0.015$. The main effect of this change is an increase to the time it takes for structures to form and collapse. In this case, the first sign of fragmentation is at $t/t_0 = 10 - 20$ with clumps forming between $t/t_0 = 80 - 85$. At $t/t_0 \approx 80$ the ionization fraction drops and cores form at $t/t_0 \approx 85$. As with the previous models, Figures 4.14 - 4.16 show the maps of the physical quantities. Again, as shown by the magnetic field strength maps, mass-to-flux ratio maps and the ionization fraction map, this simulation shows the same typical features as the previous models.
Figure 4.15: Magnetic field strength (left) and mass-to-flux ratio (right) maps for Model E at $t/t_0 = 95.2$. Panels, contours and colour bars are the same as Figure 4.6.

Figure 4.16: Ionization fraction map for Model E at $t/t_0 = 95.2$. Contours and colour bar are the same as Figure 4.7.
For this model, we wished to test the effect of a supercritical initial mass-to-flux ratio ($\mu_0 = 2.0$), on the evolution of the cloud. The most obvious difference is the total run time of the simulation. Model C had a total simulation time of $t/t_0 = 80.7$ while this model has a total simulation time of only $t/t_0 = 16.8$. Due to such a shortened time frame, structure formation occurs much faster. The first hint of fragmentation occurs at $t/t_0 < 10$, with coherent clump/cores visible at $t/t_0 = 15$. Figure 4.17 shows the column density enhancement maps for the entire region (left panel) and a close up of the three prominent peaks (right panel). At first glance, the column density enhancement map is very different from those presented in the previous models. Instead of five clump regions, there are about a dozen or so regions with column density peaks with $\sigma_n/\sigma_{n,0} > 2$. The distance between these column density peaks is about 3 pc while the size of the column density peaks themselves is on the order of 0.2 pc. On closer inspection of the full simulation map, subregions of the map exhibit the same macro structure as Models B, C, D, and E except on a smaller scale.

Focusing on the right panel of Figure 4.17, we see that the clump/core regions do not exhibit distinct cores, but rather the same onion type structure as those in Model A. This is a direct consequence of the initial mass-to-flux ratio. For Models B, C, D, and E, the initial mass-to-flux ratio is set to a value of 1.1, which undergoes two distinct drops in the fragmentation length scale as a function of column density, (see Figure 3.2, right), and thus undergoes two distinct fragmentation episodes. Comparatively, this model has an initial mass-to-flux ratio value of 2.0. As such, it only undergoes one decrease in fragmentation length scale directly to values on the order of $l/l_0 < 1$ at very low column density values.

Figure 4.18 shows the magnetic field strength within the region containing the three most prominent cores (left panel) and the mass-to-flux ratio map of the full region (right panel).
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Figure 4.18: Magnetic field strength (left) and mass-to-flux ratio (right) maps for Model F at $t/t_0 = 16.8$. Panels, contours and colour bars are the same as Figure 4.6.

Figure 4.19: Ionization fraction map for Model F at $t/t_0 = 16.8$. Contours and colour bar are the same as Figure 4.7.
Figure 4.20: Column density enhancement maps for Model G at $t/t_0 = 40.9$. Panels, white circles and colour bars are the same as Figure 4.5.

while Figure 4.19 shows the ionization profile of the three most prominent cores. The white contours are the column density contours to help show the locations of the cores. As with the previous four models, the magnetic field strength and ionization fraction again follow the column density enhancements while the mass-to-flux ratio exhibits a very chaotic nature.

Model G: $\Delta t_{sp} = 10t_0, \tau_{ni,0} = 0.2t_0$

Finally, this model investigates the effects of periodic perturbations on a region with a cosmic ray only ionization profile. The fragmentation scale for this model is smaller than Model C. Fragmentation occurs at an earlier time, $t/t_0 < 10$ compared to $t/t_0 = 10 - 30$ in Model C. Clump/core formation occurs at $t/t_0 \approx 20$. The clumps themselves are smaller than those observed in Model C, but larger than those in Model F with a size on the order of 1.5 pc. This time the decrease in fragmentation length and timescale is due to the linear ionization profile. As shown in Figures 2.2 & 2.3, for a mass-to-flux ratio of 1.1, the length and time scales for fragmentation decrease dramatically as the neutral-ion collision time increases.

Figure 4.20 shows the column density enhancement map at the final time. Although the clumps are smaller than in Model C, we do see that there is some evidence of substructure in some of them. Although several of the regions show column density enhancements, only one region reaches a column density enhancement of 10, while the other ones end the simulation with column density enhancements on the order of 2. The right panel shows the single core that collapses within the simulation region. As shown, this core is much smaller than those in previous models. The peak is only $\sim 0.03$ pc and the entire envelope is only $\sim 0.15$ pc whereas in the other models, the peak itself has a visual extent on the order of 0.15 pc.

Figure 4.21 shows the magnetic field strength and mass-to-flux ratio maps. As with the other models, the magnetic field strength again follows the column density maps and the mass-
4.4. Simulations and Results

Figure 4.21: Magnetic field strength (left) and mass-to-flux ratio (right) maps for Model G at $t/t_0 = 40.9$. Panels, contours and colour bars are the same as Figure 4.6.

to-flux ratio exhibits a very patchy nature. The maximum mass-to-flux ratio does match with the one core, however as in the other models, there are some regions of high mass-to-flux ratio that do not correspond to a column density peak.

4.4.3 Model Comparisons

The previous subsections presented the overall results and features of each of the models. Within these results we found several features that changed between the various models. These include time scales for various events such as large scale fragmentation, clump formation, core formation and total run times, trends within the formation of clumps and cores themselves, and trends regarding the maximum $\sigma_n/\sigma_{n,0}$ and maximum and minimum $\mu_0$ values attained. In addition, we can compare the effects of changing individual parameters on the evolution of the cloud and compare to the previous investigations by Basu et al. (2009b). The following will discuss these different features and comparisons.

Time Scales

The most apparent difference between the seven models are the times at which various structures form. Table 4.2 shows the total run time ($t_{\text{run}}$), large scale fragmentation time ($t_{\text{frag}}$), clump formation time ($t_{\text{clump}}$), and core formation time ($t_{\text{core}}$) for each model. Ranges of ages indicate that the event occurred at some point within that time range, however the exact time is not known due to the frequency at which data was output from the simulations. Comparing Models A, B, C, and D, we see that the total run time of the simulation increases as the the frequency of perturbations decreases. With the additional perturbations, the overall simulation time is shortened by approximately a factor 2 for Model B, to a factor 1.6 for Model D. Models
Table 4.2: Model timescales

<table>
<thead>
<tr>
<th>Model</th>
<th>$t_{\text{run}}/t_0$ Myr</th>
<th>$t_{\text{frag}}/t_0$ Myr</th>
<th>$t_{\text{clump}}/t_0$ Myr</th>
<th>$t_{\text{core}}/t_0$ Myr</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>143.6</td>
<td>56.3</td>
<td>40 - 50</td>
<td>135</td>
</tr>
<tr>
<td>B</td>
<td>70.0</td>
<td>27.5</td>
<td>~10 ~3.9</td>
<td>~60 ~23.5</td>
</tr>
<tr>
<td>C</td>
<td>80.7</td>
<td>31.7</td>
<td>10 - 20</td>
<td>70</td>
</tr>
<tr>
<td>D</td>
<td>87.7</td>
<td>34.4</td>
<td>10 - 20</td>
<td>75 - 80</td>
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<tr>
<td>E</td>
<td>95.2</td>
<td>37.3</td>
<td>10 - 20</td>
<td>80 - 85</td>
</tr>
<tr>
<td>F</td>
<td>16.8</td>
<td>6.6</td>
<td>&lt;10 &lt;3.9</td>
<td>~15 ~5.9</td>
</tr>
<tr>
<td>G</td>
<td>40.9</td>
<td>16.0</td>
<td>&lt;10 &lt;3.9</td>
<td>~20 ~7.8</td>
</tr>
</tbody>
</table>

E, F and G have varying effects on the full run time. The decrease in perturbation amplitude within Model E results in an increase in total run time. Compared to Model C, this run takes about 19% longer to run. Conversely, the increase to $\mu_0$ in Model F and linear ionization profile within Model G have the opposite effect, reducing the run time by a factor of 4 and 1.75 respectively.

There are three other notable times within the evolution of the cloud. These are the initial large scale fragmentation time of the cloud, the clump formation time defined as the time when the material defining the outer boundaries of the clump region reaches $\sigma_n/\sigma_{n,0} > 2$, and the small scale fragmentation or core formation time. As shown in Table 4.2 the initial fragmentation within the region typically occurs very early in the simulation for each model. This process takes up about 10 – 20% of the simulation time. The majority of the simulation time (≈ 65%) goes toward forming the clump while the final ≈ 5% of the simulation is the formation and collapse of the core region. Further comparison to the two-stage fragmentation model will be presented in Section 4.5.

**Maximum $\sigma_n/\sigma_{n,0}$**

Figure 4.22 shows the evolution of the value of the maximum column density enhancement as a function of time. The left panel shows the evolution over the full simulation time. For each model, the maximum column density increases gradually until it reaches a column density enhancement between 2 and 3, at which point the column density increases rapidly over a very short time. From this figure we can again see the trend in simulation run times, i.e., the run time increases as the time between perturbations increases, to the limiting value of the model with no perturbations (Model A).

The right panel of Figure 4.22 shows a zoom in of the maximum column density enhancement at initial times for all models. As shown, all models except Model E start at the same
initial maximum column density while Model E starts at a lower value. This difference is a consequence of the smaller perturbation amplitude within Model E. Comparing the initial evolution of the maximum column density enhancement value for each model, we see that there are four different evolutionary paths, depending on the initial parameters. The first of these paths is defined by Models A, B, C, and D. For all four models, the instantaneous value of the maximum column density enhancement has fluctuations while the overall trend is increasing. Following the evolution, we see all four models have the same initial evolution until Models B, C, and D reach the time of their next perturbations, at which point the column density enhancement jumps abruptly. After each perturbation, the fluctuations in the column density enhancement increases. This continues until the column density enhancement reaches between 2 and 3 and the maximum value increases dramatically as discussed above. Model E depicts the second path. This model shows the same trend as Models A, B, C, and D, however the decrease in perturbation amplitude results in a curve that resembles that of Models A and C. The third path is defined by Model F. Here, the increased mass-to-flux ratio within the simulation region results in larger fluctuations in the maximum column density enhancement at early times. This allows the simulation to reach a point of runaway collapse faster than the other simulations, specifically Model C. The final path is that of Model G. Here we see that after the initial perturbation, the maximum column density enhancement drops dramatically compared to the other models. There is also less variability overall, resulting in a smoother curve. After each perturbation event, the column density enhancement continues to exhibit an initial drop, however the extent of the decrease is smaller each time and the overall trend is increasing.
Table 4.3: Maximum and Minimum $\mu_0$ values

<table>
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<th>Model</th>
<th>Minimum $\mu_0$</th>
<th>Maximum $\mu_0$</th>
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<tr>
<td>A</td>
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<td>0.76</td>
<td>1.54</td>
</tr>
<tr>
<td>C</td>
<td>0.76</td>
<td>1.43</td>
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<td>D</td>
<td>0.90</td>
<td>1.29</td>
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<td>E</td>
<td>0.93</td>
<td>1.24</td>
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<tr>
<td>F</td>
<td>1.73</td>
<td>2.25</td>
</tr>
<tr>
<td>G</td>
<td>0.85</td>
<td>1.93</td>
</tr>
</tbody>
</table>

**Maximum and Minimum $\mu_0$ values**

Another feature of these simulations is the evolution of the mass-to-flux ratio throughout the simulations. As shown by the mass-to-flux ratio maps above (see right hand panels of Figures 4.2, 4.6, 4.9, 4.12, 4.15, 4.18, and 4.21), the addition of multiple perturbations during the evolution of the cloud results in a randomized field of values. Table 4.3 shows the minimum and maximum mass-to-flux ratio values within the full simulation region for each model. For all models, we see that over the course of the simulation, the value of the mass-to-flux ratio increases in some regions while subsequently decreasing in others. This results in a maximum and minimum mass-to-flux ratio within each simulation. For Model A, this divergence is minimal while for the other models, the maximum and minimum values increase/decrease by up to a factor of 1.75 and 1.45 respectively. Due to these changes, some regions within Models B, C, D, E, and G become subcritical.

Figures 4.23 - 4.25 show the evolution of the mass-to-flux ratio as a function of time. The solid curves show the value of the maximum mass-to-flux ratio within the simulation volume while the dashed curves show the value of the mass-to-flux ratio in the pixel associated with the maximum column density enhancement value (see Figure 4.22). Note that the location of the maximum value within the simulation may not be at the same pixel at all times. From these figures, there are several features worth noting. First, looking at Figure 4.23 for Model A, we see that the evolution of the maximum mass-to-flux ratio within the simulation is the same as the evolution of the mass-to-flux ratio for the pixel with the maximum column density enhancement. In addition, both curves exhibit the same trend as the column density enhancement curves in Figure 4.22. From this, we can conclude that without the additional perturbations, the maximum mass-to-flux ratio coincides with the location of the column density peak.

Looking at the other models however, we see that with the addition of extra perturbations, the mass-to-flux ratio within the pixel with the maximum column density enhancement value does not necessarily correspond to the maximum value within the simulation. The lower left
Figure 4.23: Mass-to-flux ratio values as a function of time for Models A-E and G. Solid lines depict maximum $\mu$ value within the simulation. Dashed lines show the $\mu$ value for the pixel with the maximum column density enhancement ($\sigma_n/\sigma_{n,0}$). Top: Comparison between solid and dashed curves. Bottom: Individual plots for maximum $\mu$ value within simulation (left) and $\mu$ for pixel with maximum column density enhancement.
Figure 4.24: Mass-to-flux ratio values as a function of time for Models F. Solid and dashed curves depict the same information as in Figure 4.23.

Figure 4.25: Evolution of mass-to-flux ratio at early times. Solid and dashed curves depict the same information as in Figure 4.23.
4.4. Simulations and Results

Panel of Figure 4.23 shows the evolution of the maximum mass-to-flux ratio value within the simulation. Looking at Models B, C, D, and E, we see that the evolution follows a saw-tooth like pattern. The origin of this pattern is a direct consequence of the perturbations. The maximum value at the end of the simulation depends on the frequency and amplitude of the perturbations. The lower right hand panel of Figure 4.23 shows the evolution of the mass-to-flux ratio within the pixel with the maximum column density. Looking at Models B, C, D, and E, this evolution is extremely variable. At early times (see Figure 4.25), this variability is due to the fact that the pixel with the maximum column density value is not necessarily the same at each time step. Near the end of each simulation for Models B, C, D, and E, the amount of variability decreases suggesting that the pixel with the maximum column density enhancement is now fixed. For all models, the mass-to-flux ratio within this pixel increases rapidly toward the end of the simulation, following the trend of the column density enhancement.

Finally, Model G exhibits a different evolution of the global and local maximum mass-to-flux ratios. Recall that this simulation assumes a linear ionization profile with an initial neutral-ion collision time $\tau_{ni}/t_0 = 0.2$. For this simulation, the global and local maxima coincide throughout the evolution except just after a perturbation event. At these times, the local mass-to-flux ratio drops faster than the global. This model exhibits the largest mass-to-flux ratio increase both locally and globally out of all the models presented. This is a direct consequence of the value of the neutral-ion collision time within this model. For this model, even for low column densities, the time frame between subsequent collisions of neutral particles and ions is long enough that the neutral particles are able to slip past the magnetic field lines to form high column density regions. As such, the column density increases while the magnetic field strength stays relatively constant resulting in a significant increase to the mass-to-flux ratio. In the other models, by using a step-like ionization profile, at lower column densities, the ionization fraction is much higher resulting in an almost flux-frozen medium. Initial redistribution of mass within these simulations would also drag the magnetic field with it, thus maintaining a mass-to-flux ratio near the initial value.

Comparison of Models A, C, F & G

Finally, Models A, C, F, and G represent a set from which we can study the effect that changing various initial parameters has on the evolution of the region. These simulations can be directly compared to those of Basu et al. (2009b). First as mentioned previously, Models A and G represent two distinct extensions of the Basu et al. (2009b) models: Model A allows us to investigate the effect of adding a step-like ionization profile while Model G allows us to investigate the effect of just adding the perturbations to the simulation.

Starting with Model A, the clump structures presented in Figure 4.1 are very similar to those
presented by Basu et al. (2009b), in that both show the onion-like column density structure. However our clump regions are much larger. This implies the overall shape of the structures formed does not depend on the ionization profile, but that the size of the regions does. This is evident in Figure 3.2. The added ionization profile does however affect the simulation time, increasing it by almost a factor of $2/3$ from $t_{\text{run}}/t_0 = 88$ to $t_{\text{run}}/t_0 = 143.6$. This is directly due to the fact that with the chosen ionization curve (see Figure 3.1), at early times the low column density gas is almost flux frozen. Looking at Model G, the structures formed are almost identical to those depicted in Basu et al. (2009b). The main difference between the two is again the total simulation run time. In this case, our model runs for $t_{\text{run}}/t_0 \approx 40$ compared to the $t_{\text{run}}/t_0 = 88$ for the equivalent Basu et al. (2009b) model. The shorter run time is a direct consequence of the added perturbations.

Models C and F differ in the initial value for the mass-to-flux ratio within the simulation region. As shown by Figures 4.9 & 4.18, this change has a significant effect on the evolution of the cloud. Due to the greater mass-to-flux ratio in Model F, the fragmentation scale is much smaller. In addition, the corresponding timescale is also much shorter, resulting in a significantly shorter evolution time. Model F can also be compared to simulations presented by Basu et al. (2009b). Overall, this model produces structures similar to those found in their corresponding simulation, however the run time has been reduced to $t_{\text{run}}/t_0 \approx 17$ from $t_{\text{run}}/t_0 \approx 23$.

### 4.4.4 Clump and Core Analysis

Our simulations show many similar features to various observed star forming regions, therefore we can treat our simulations as synthetic observations. One of the main analyses performed on a collapsing molecular cloud is the identification of clump and core regions for the purpose of determining their physical properties (e.g., radius, column density, mass, etc.). Accurate accounting for all of the clump/core regions within star forming regions is required for determining several evolutionary problems within star formation. These include determining how the environment affects the collapse of clumps and cores into stars (i.e., why do some regions have evidence of young stellar objects while others do not) as well as trying to ascertain the possible one-to-one correlation between the Initial Mass Function (IMF) and the Core Mass Function (CMF). Our simulations are well suited for an analysis of the number of clumps and cores.

A preliminary visual inspection of the clump in Region 4 at the end of the simulations shows that for Models B, C, and D, the number of visible cores decreases as the frequency of perturbations decreases. This is likely due to the fact that with fewer perturbations, the
medium has more time in between to readjust, resulting in less structure (see right hand panels of Figures 4.5, 4.8, and 4.11). Another trend evident in Model B, C, and D is the decreasing average density of the clumps in regions other than Region 4 as the frequency of perturbations decreases. This is likely another consequence of the medium having more time to readjust after a perturbation event. Comparing Model C ($A = 0.03$) to Model E ($A = 0.015$), we see that both have the same number of cores, however the spacing between the cores is smaller in Model E than in Model C. Looking at the structure of the clump in Region 4 itself, we see that the frequency and amplitude of the perturbations has an effect on the structure of the clump itself. Model B ($\Delta t_{sp} = 5t_0$) exhibits three distinct regions, however as the time between subsequent perturbations increases, we see that the number of distinct regions decreases: Model C ($\Delta t_{sp} = 10t_0$) has two regions while Model D ($\Delta t_{sp} = 15t_0$) has only one. Comparing Models C and E, we see that the effect of decreasing the amplitude of the perturbation that the two clumps evident in Model C are replaced by only one in Model E (see right hand panels of Figures 4.8 and 4.14). Finally looking at the velocity maps, we see that the magnitude of infall speed within the core is typically subsonic to transonic, in agreement with other numerical models (e.g. Basu and Ciolek, 2004; Basu et al., 2009b) and analytic models (Adams and Shu, 2007) of core formation driven by ambipolar diffusion.

In addition to these general trends, we can also quantitatively define clump and core regions and determine the physical properties (e.g., radius and mass). Observationally, clump and core regions are probed via different molecular tracers that map the appropriate column densities associated with these structures. As described in Section 3.2, the definition of a clump/core depends on the region in which they are observed. Clump regions are generally defined as coherent regions where the visual extinction is $A_v > 1 - 3$ magnitudes. These regions are typically traced by various isotopologues of carbon monoxide (CO). Studies of the Ophiuchus and Perseus clouds by Johnstone et al. (2004) and Kirk et al. (2006), respectively, suggest that the core formation threshold is $A_v > 5$ magnitudes and a star formation threshold is $A_v \approx 7 - 8$ magnitudes (see also Onishi et al., 1998; Froebrich and Rowles, 2010). These denser regions are typically traced by line emission from NH$_3$, CCS, and HC$_5$N (Sadavoy et al., 2012; Román-Zúñiga et al., 2012). With regards to our simulations, although the specific chemistry of the regions has not been added explicitly (i.e., the code only follows the evolution of neutral particles and ions in general), the column density enhancement panels can be thought of as a composite plot of the different tracers used in observations.

In general, the threshold column density for the definition of a core is on the order of $N_{H_2} = 10^{22}$ cm$^{-2}$. Within their analysis of cores produced by the thin-disk code used here, Basu et al. (2009b) defined the background column density to be equal to this threshold, i.e., $N_{n,0} = 1.0 \times 10^{22}$ cm$^{-2}$. With this definition, they assumed any region with a column density
enhancement of 2 or above to be a core. For the simulations presented here, in order to be able to test the two-stage fragmentation model, the initial column density needed to be low and therefore the background column density was set to $\sigma_{n,0} = 3.638 \times 10^{-3} \text{ g cm}^{-2}$ ($N_{n,0} = 9.3 \times 10^{20} \text{ cm}^{-2}$) which corresponds to a visual extinction $A_v = 1.0 \text{ mag}$. Therefore, at the end of the simulations, the column density enhancement value of 10 corresponds to a column density that is just below the typical threshold definition of a core. For our simulations, given the limitations of the data, we will look at the mass enclosed within the regions defined by the typical visual extinction thresholds for clumps and cores as discussed above. Specifically, we will define a clump region to be defined by the contour where the column density is on the order of a factor 2 above the background and a core region to be those defined by a contour where the column density is $\sim 8$ times the background.

**Method**

In order to determine the outer boundaries of a structure of interest within a set of observations, a systematic method for determining the thresholds has to be defined. One such tool which helps with this determination is `clumpfind2d` (Williams et al., 1994). This is a set of IDL routines which first determines the location and extent of structures within observations or simulated data and then uses the found clump/core regions to determine the size and intensity of these structures. The routine assumes linearly spaced contours based upon a user defined minimum and interval and traces structures by connecting pixels that are within one resolution element of each other (Williams et al., 1994). The `clumpfind2d` suite of programs contains two main procedures, `clfind` and `clstat`. The `clfind` procedure is the main program which finds the coherent regions that define the particular structures of interest based upon user defined values for the minimum contour level and spacing between contours. The `clstat` procedure then performs analysis on these regions to determine the radius and sum of the data enclosed within the regions defined by `clfind`. Other values determined by `clstat` include the $x$ and $y$ full width half maximums (FWHMs) and the velocity data of the region.

Observationally, there are a couple of techniques that are used to determine the properties of cores found in different regions. Typically, identified core regions within a data set are fit with a Gaussian profile from which the core radius is defined to be half of the full width half maximum (FWHM) (i.e., $R = \text{FWHM}/2$). Using `clumpfind2d`, one would independently find the FWHM of the region of interest and then sequentially run `clfind` and `clstat`, varying the value of the contour levels until the FWHM values from `clstat` match the predetermined values. Once they match, the extent of the structure and value of the observed quantity within the structure can be determined (e.g., the column density or mass within the core). For our simulations, we will use `clfind` and `clstat` to find the regions with column density enhancements above our defined
clump and core thresholds (\(A_v = 2\) and 8 magnitudes respectively) and determine the radii and enclosed mass within these regions.

**Results**

We have performed clump/core analysis on the column density enhancement data for all seven models. Specifically, we have used clfind to determine the locations of the large and small scale structures within the region to compare with the visual determinations listed in Section 4.4.1. We also performed in-depth core finding analysis on the clump in Region 4 for Models B, C, D, and E and the prominent cores in Models F and G to determine how the change in simulation parameters affected the number of cores formed as well as the characteristics of these cores. For consistency, we have chosen to perform this analysis on the final times for each of the simulations.

Figure 4.26 shows the locations of clump regions within each of the models. The radii of the circles correspond to the derived radii of the contour defining the clump regions for Models B, C, D, and E and the major axis of the clump region in Model A. Table 4.4 shows the derived values for these clumps. The mass is calculated by multiplying the average density by the total area of the region. Comparing the derived masses, we see that for models B, C, and D, as the frequency of perturbations decreases, the mass contained within the clumps increases. In addition, as shown by Model E, the mass contained within the clumps increases as the amplitude of the perturbations decreases. As with the visual analysis above, we also note that the number of distinct clumps decreases with the frequency of perturbations.

Figure 4.27 shows the locations and designations of the cores analyzed. The radii of the circles correspond to the derived radii of the contour defining the core. Table 4.5 shows the derived values for these cores. The mass was calculated via the same method as the clump masses. For each core, we calculated the mass of the region defined by the contour indicated. For the densest cores in each model, we also calculated the mass within a smaller region corresponding to a larger contour value, indicated by the “a”, “b”, etc. subcategorization (e.g., B1a, C1a, etc.). Although some of the core regions contained pixels with column density enhancements values above the threshold value of 8, in some cases the number of pixels above this threshold were not enough to be identified as a core by clumpfind2d. In these cases, we lowered the value of the contour until these regions contained enough pixels to be found by clumpfind2d.

Comparing all cores analyzed, we see that the size of the cores are all on the order of 0.1 pc across (\(R_{\text{eff}} \approx 0.05\) pc) and the masses range from \(\sim 0.7 - 3.6\) M\(_{\odot}\). The exact values of the radii and mass are directly linked to the contour level used to define the outer boundaries of the regions. As discussed above, for the regions containing the maximum column density
Figure 4.26: Maps of clump locations within each model. Top Left: Model A, Top right: Model B, Upper Middle left: Model C, Upper Middle right: Model D, Lower Middle left: Model E, Lower Middle right: Model F, Bottom center: Model G. Circles identify the core location, labels indicate their designation from Table 4.4. Radius of the circles correspond to the derived radii of the contour defining the clump. Colourtable indicates the column density enhancement on a linear scale. Axes show $x$ and $y$ pixel positions within the simulation (1 pixel = 0.0296 pc across).
### Table 4.4: Clump parameters

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<td>5.19</td>
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Figure 4.27: Maps of core locations within Region 4 for the various models. Top left: Model B, Top right: Model C, Middle left: Model D, Middle right: Model E, Bottom left: Model F, Bottom right: Model G. Circles identify the core location, labels indicate their designation from Table 4.5. Radius of the circles correspond to the derived radii of the contour defining the core. Colour table indicates the column density enhancement on a linear scale. Axes show $x$ and $y$ pixel positions within the simulation (1 pixel = 0.0296 pc across).
Table 4.5: Core parameters

<table>
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<tr>
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<th>Core Mass (M☉)</th>
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<td>0.041</td>
<td>0.76</td>
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</tbody>
</table>
enhancement within the simulation, we determined the core properties for multiple contour levels. As expected, higher contour values resulted in smaller derived parameters due to the smaller region defined by them. Comparing our values to typical observational values (see Tables 3.3, 3.5 & 3.7) we see that our simulations are producing clumps and cores with similar sizes and masses.

4.5 Simulations in Context of the Two-Stage Fragmentation Model

The aim of these simulations was to determine whether a collapsing cloud would follow the two-stage fragmentation scenario described in the previous chapter, and what environmental effects are necessary for the development of clumps that exhibit several cores within. As a brief reminder, the two-stage fragmentation scenario is as follows. Assuming an initially transcritical cloud, it will undergo an initial fragmentation event at low densities and a subsequent fragmentation event at higher densities when the length and timescale for fragmentation decrease significantly. In order to verify this scenario, our simulations must undergo two separate fragmentation events and exhibit the predicted length and time scales. Of the seven models presented, four exhibit the features of the two-stage fragmentation scenario, those being Models B, C, D, and E. This not only indicates that perturbations are necessary for the development of substructures within a collapsing clump, but also that the initial environment has a significant impact on the evolutionary path the region will take. The following will discuss the evolution of all models within the context of the two-stage fragmentation model.

For all models, at early times, the initial column density within the region is very low ($A_{v,0} = 1$ magnitude). At this stage, the fragmentation length and time scales depend on the ionization fraction and mass-to-flux ratio within the cloud. For Models A, B, C, D, and E, at $A_{v,0} \approx 1$ mag, the preferred fragmentation length scale is on the order of 20 pc, which is larger than the entire region. The growth time for this unstable mode is on the order of 10-20 Myr. However, as the column density and mass-to-flux ratio change within the region, so too will the fragmentation length scale and growth time. An increase in either column density or mass-to-flux ratio within a region will decrease the fragmentation length scale and timescale. Therefore, as the cloud evolves, there will be a point in time where regions within the simulation will accumulate enough mass such that the column density and/or mass-to-flux ratio values increase and the length scales decrease enough to allow fragmentation to occur. This dependence on environmental parameters is clearly exhibited by the fragmentation times shown in Table 4.2. For Model A, as shown by Figures 4.22 & 4.23, the value of the column
Figure 4.28: Minimum growth time (left panel) and fragmentation length scale (right panel) as a function of column density. Curves shown are the same as in Figure 3.2. Arrows depict the evolutionary paths for an increase in column density (1) and an increase in mass-to-flux ratio (2) as the cloud evolves. Due to the linked nature of the mass-to-flux ratio and column density, these changes will occur simultaneously. The red arrow depicts the likely path regions within the cloud will take under the effect of an increase in both column density and mass-to-flux ratio. Arrows are not to scale.

density enhancement and mass-to-flux ratio are more or less constant. As such, it takes between 15 and 20 Myr for fragmentation to occur. When fragmentation does occur, the length scale is on the order of 10 pc, which corresponds to the predicted length scale from the model. In the other models, the timescale for fragmentation is much shorter. This is due to various reasons specific to the models. In Models B, C, D and E, the addition of perturbations causes both the column density and mass-to-flux ratio within some regions to increase. This causes the length and time scales to decrease for two reasons as depicted in Figure 4.28. The effect of the column density increase is shown by arrow 1 while the effect of a mass-to-flux ratio increase is shown by arrow 2. Given that these two values evolve in tandem, the actual evolution of the cloud within the parameter space would likely follow the red arrow. Note that the sizes of the arrow are not to scale. As such, the clouds in these models take less time to fragment initially as compared to Model A. Although not definitive given the data at hand, it is likely that as the time between column density perturbations increases from Model B to D, the actual time at which fragmentation occurs likely also increases. Continuing on in the evolution, the other fragmentation event occurs after the ionization fraction has dropped down to the cosmic ray induced value. The minimum growth time at this point is on the order of 1-2 Myr. In Table 4.2, this time corresponds to the time between $t_{\text{clump}}$ and $t_{\text{core}}$. Looking at Models B, C, D and E, we see that all exhibit roughly the correct time frame for the growth of these core regions.

Compared to the other models, Models F and G follow a much different path. Focusing on Model F first, analysis shows that it only experienced one fragmentation event early on in the
evolution. As shown by Figure 4.29, this is due to the fact that for \( \mu_0 = 2.0 \) (solid line) there is only one sharp decrease in length and time scales which occurs at very low column densities as opposed to the two sharp decreases exhibited for \( \mu_0 = 1.1 \) (dotted line). Similarly, the length and timescale trends for Model G also only undergo one sharp decrease at low densities, resulting in only one fragmentation event. In this case, this is a consequence of the cosmic ray only ionization profile.

### 4.6 Summary

Based on the above analysis and discussion, we can conclude that our simulations do follow the trends shown by the two-stage fragmentation model. However, through testing, we have found that the occurrence of two fragmentation events within the evolution of a cloud is highly dependent on the environment within which the region is evolving. Based on the simulations, we can say that the cloud must meet three criteria in order for it to experience two fragmentation events. First, the cloud must start out diffuse with an initially transcritical mass-to-flux ratio (\( \mu_0 \approx 1.1 \)). Second, the cloud must pass through a rapid drop in \( \chi_i \). Finally, we also found that there must be some form of perturbations occurring within the region in order for structures formed from the second fragmentation event to remain distinct from each other. The clump and core regions formed by these simulations exhibited sizes and masses on the same order as those seen in observations.

In addition to supporting the two-stage fragmentation model, our simulations have also shed some light on other properties within a collapsing cloud. First, from the mass-to-flux ratio maps of the various models, we found that the addition of perturbations destroys the smoothness.
of the mass-to-flux ratio that is observed in Model A. This can have significant implications for observations. As shown in the simulation maps, for a region undergoing several sudden column density perturbations, the value of the mass-to-flux ratio can vary on sub-parsec scales. However, current magnetic field measurements cannot achieve fine enough resolution to detect such variation. Therefore, an observed region with such small scale variation to the mass-to-flux ratio, regardless of how it may arise, would be smoothed out to some average value.
Chapter 5

The Effect of Magnetic Fields and Ambipolar Diffusion on Analytic CMFs

Observations of the stellar initial mass function (IMF) and the core mass function (CMF) show similarities in the shape and high-mass slope of these two functions (Motte et al., 1998; Testi and Sargent, 1998; Johnstone et al., 2000; Alvés et al., 2007; Nutter and Ward-Thompson, 2007; Simpson et al., 2008; Enoch et al., 2008; Sadavoy et al., 2010, among others). As such, much theoretical effort has been invested in order to explain these similarities. Various different approaches to this problem have been explored, including analytic and numerical studies which invoke gravitational fragmentation or accretion (Silk, 1995; Inutsuka, 2001; Basu and Jones, 2004), turbulence (Padoan et al., 1997; Padoan and Nordlund, 2002; Ballesteros-Paredes et al., 2006; Hennebelle and Chabrier, 2008, 2009), independent stochastic processes (Larson, 1973; Elmegreen, 1997) and magnetic fields (Dib et al., 2008), among others. Results of these studies vary from those which seem to agree with the fiducial Salpeter form, \( \frac{dN}{d \log M} \propto M^{-\alpha} \) where \( \alpha = 1.35 \) is the value of the Salpeter slope, to those that do not.

As described in Chapter 1, the high mass slope of the IMF was initially derived by Salpeter (1955) and later improved upon by Kroupa (2002) and Chabrier (2003a,b, 2005). Despite variations in observed and theoretically derived IMF slope values, it is often assumed that the shape and high mass slope of the IMF and CMF are identical and universal. From a theoretical view, such a one-to-one correspondence between these two functions implies that high-mass cores beget high-mass stars and likewise for low-mass cores. The need for extensive simulations of how a complex of cores turns into a cluster of stars is simplified tremendously if it is assumed that each core will collapse into a single star with some mass loss to account for the mass shift between the CMF and IMF.

The underlying tenet of universality is that all star-forming regions are the same and un-

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1Portions of this chapter have been published in Bailey, N.D., & Basu, S., ApJ, 2013, 766, 27
dergo the same process to form stars, however recent observations and simulations have started to reveal cracks in this assumption. In a study of the effect of turbulence on the formation of the CMF, Hennebelle and Chabrier (2008, 2009) find that comparisons between their IMF and observations for different cloud conditions suggest that star formation should predominantly occur in clouds five times denser than characterized by Larson (1981). This led them to question the universality of the IMF since, as they say, choosing different cloud parameters would lead to a different CMF/IMF. Several studies of the IMF also tend to disagree with the assumed universality. Observations of different star clusters in both the Milky Way and the Large Magellanic Cloud (LMC) show a wide scatter of slopes: $\alpha = 0.5 - 2.0$ (Elmegreen, 1999b). A survey of high mass slope values for different stars (i.e., cluster stars versus association stars versus field stars) yields a wide range of values; $\alpha = 2.0 - 4.0$ for extreme field stars to $\alpha = 1.5 - 2.0$ for cluster stars (Elmegreen, 1997). Further to this, Elmegreen (1999b) shows that through stochastic fractal sampling of a cloud, the derived IMF slopes can vary from $\alpha$ as low as 1.0 to as high as 1.7. Clark et al. (2007) note that if the lifetime of a more massive core is longer than a less massive one, the slope of the CMF should be shallower in order to obtain the IMF. Finally, Zaritsky et al. (2012) show that there may be evidence for two distinct stellar IMFs that depend on the age and metallicity of the cluster in question. Based on the above evidence and arguments, it is not clear why one should insist on using $\alpha = 1.35$ as the universal slope for both the CMF and IMF.

The majority of the work in this area has focused on the effects of turbulence within the molecular clouds on the formation and shape of the CMF. Research which considers the effect of magnetic fields and ambipolar diffusion on the CMF is sparse. Kunz and Mouschovias (2009) used the results of a non-ideal MHD linear analysis of a partially ionized sheet (Morton, 1991; Ciolek and Basu, 2006) to generate a broad CMF, assuming ambipolar-di ffusion initiated core formation. Their model assumed subcritical to critical initial conditions with a uniform distribution of mass-to-flux ratios between 0.1 and 1.0 times the critical value for gravitational instability (see Section 5.4 for more discussion of their model).

In this study, we use the results of the linear analysis of a partially ionized sheet along with a lognormal distribution of initial column density and various distributions of mass-to-flux ratio. We explore both subcritical and supercritical initial conditions. Mildly supercritical initial conditions are the most likely to lead to massive core formation, as seen in e.g., Figure 2 of Ciolek and Basu (2006). Furthermore, we use a lognormal distribution of column densities, as expected in molecular clouds on both theoretical grounds for a turbulent medium (Padoan et al., 1997) and from observations (Kainulainen et al., 2009). The aim of this research is two-fold. In the first part we show the effects of a magnetic field on the shape of the CMF. Starting from an assumption of lognormal column density probability we show the broadening effect
of neutral-ion drift via ambipolar diffusion and differing mass-to-flux ratio distributions. In the second part, we address the inherent limitations of observed core mass functions, i.e. sample size and bin size. Specifically, we aim to compare small-sample synthetic CMFs to large sample synthetic CMFs to show the effect of small number statistics on the observed features of the CMF. Section 5.1 outlines the model and methods for constructing our synthetic CMFs. Section 5.2 shows the results for the different distribution models considered. Section 5.3 shows the effect of small number statistics and the variance in derived analytic slopes. Finally Sections 5.4 and 5.5 give our discussion and summary.

5.1 Synthetic Core Mass Functions

To better understand the effects of the environment on the shape and peak of the core mass function, we produce synthetic CMFs (synCMFs) based upon varying physics and properties of molecular clouds. These include the column density \( \sigma_n \), ionization fraction \( \chi_i = \log \left[ \frac{n_e}{n_H} \right] \), mass-to-flux ratio \( \mu_0 \), and neutral ion-collision time \( \tau_{ni} \). The synCMFs are produced by randomly sampling predefined column density and mass-to-flux ratio distributions (where applicable) and using a preferred fragmentation length scale to calculate the core mass. We choose to use such methods due to the random nature of molecular cloud properties. This allows us to statistically determine the shape of the CMF for a wide range of randomly chosen \( \sigma_n - \mu_0 \) pairs.

5.1.1 Physical Model

We again consider the formation of cores and the resulting CMF within ionized, isothermal, interstellar molecular clouds. These clouds are modelled as planar sheets with infinite extent in the \( x \) and \( y \) directions and a local vertical half thickness \( Z \). The nonaxisymmetric equations and formulations of our assumed model have been described in Chapter 2. For this work we consider three models: nonmagnetic, flux-frozen magnetic field and a magnetic field with ambipolar diffusion.

The key ingredient to this analysis is the assumed length scale for the core. As shown in Chapter 2, linear analysis of the magnetohydrodynamic equations result in a cubic dispersion relation (Equations 2.31 & 2.32). Given the cubic nature of this dispersion relation, the fragmentation timescale and corresponding length scale are determined through numerical methods. The value of this length scale is related to the degree of ambipolar diffusion, i.e., the degree of ionization within the cloud, and the mass-to-flux ratio of the region. Previous...
5.1. Synthetic Core Mass Functions

Figure 5.1: Wavelength with minimum growth time as a function of initial mass-to-flux ratio. Displayed curves are for $\tau_{ni,0}/t_0 = 0$ (solid curve, flux freezing) and $\tau_{ni,0}/t_0 = 0.2$ (dotted curve).

studies show that the ionization fraction within a molecular cloud resembles a step function (Ruffle et al., 1998; Bailey and Basu, 2012) such that the outer layers are highly ionized due to UV photoionization while ionization of denser inner regions is primarily due to cosmic rays. For this study, we choose to fix the neutral-ion collision time to the dimensionless value $\tau_{ni,0}/t_0 = 2\pi G\sigma_{n,0}\tau_{ni,0}/c_s = 0.2$, a value typical of the denser inner regions where most cores are likely to form (Basu et al., 2009a, and references within). This corresponds to an ionization fraction $\chi_i = 5.2 \times 10^{-8}$ at a neutral column density $\sigma_{n,0} = 0.023$ g cm$^{-2}$. Figure 5.1 (dotted line) shows the relation between the collapse length scale and the mass-to-flux-ratio for this neutral-ion collision time. By fixing the neutral-ion collision time, our ambipolar diffusion models have only two free parameters, the column density and mass-to-flux ratio distributions. Our choices for these two parameters are discussed in the following sections.

In the case of flux freezing, recall that the dispersion relation reduces to that of Equation 2.33 which has a fragmentation length scale given by Equation 2.35:

$$\lambda_{MS} = \frac{C_{eff,0}^2 + V_{A,0}^2}{G\sigma_{n,0}(1 - \mu_0^{-2})}$$

The length scale corresponding to the minimum growth time is $\lambda_{g,m} = 2\lambda_{MS}$. This is the length scale used to produce our synCMFs for models with flux freezing. The variation of this length scale as a function of $\mu_0$ is shown by the solid line in Figure 5.1. Finally, for the case with no
magnetic field, Equation 2.35 reduces to the thin disk equivalent of the Jeans length given by Equation 2.36

\[ \lambda_J = \frac{C^2_{\text{eff}}}{G \sigma_{n,0}}. \]

Again, the length scale corresponding to the minimum growth time is \( \lambda_{g,m,J} = 2 \lambda_J \), which is the scale used in our nonmagnetic model.

### 5.1.2 Column Density Distribution

A survey of column density (\( \sigma_n \)) distributions within various molecular clouds shows that they generally exhibit log-normal distributions either with or without a high density tail (Kainulainen et al., 2009). Correlation of these different shapes with the conditions within the clouds suggest that regions with a pure lognormal distribution tend to be quiescent while those with high density tails show signs of active star formation.

Since the aim of this analysis is to investigate the shape of the core mass function as an initial condition for star formation, we choose a simple log-normal distribution as shown in Figure 5.2. This plot shows the distribution as a function of both the column density (\( \sigma_n \), lower axis) and the visual extinction (\( A_v \), upper axis). The conversion from visual extinction to column density is again calculated via Equation 2.23.
The variance and mean ($\sigma^2$ and $\mu$) of this distribution were chosen based upon observational information. Previous studies of molecular clouds show visual extinction thresholds for core and star formation to be on the order of $A_v = 5$ mag (Johnstone et al., 2004; Kirk et al., 2006) and $A_v = 8$ mag (see Johnstone et al., 2004; Froebrich and Rowles, 2010, among others) respectively. As such, we adopted a mean visual extinction value of 8 magnitudes for our lognormal column density distribution. The variance reflects the typical width of the lognormal fits to cloud density functions presented by Kainulainen et al. (2009).

### 5.1.3 Mass-to-Flux Ratio Distributions

Although column density/visual extinction maps are fairly commonplace, measurements of magnetic field strengths within molecular clouds are difficult to obtain. Due to limitations in techniques and resolution, studies of magnetic fields within clouds are generally on a more global scale (see Crutcher, 1999; Heiles and Troland, 2004; Troland and Crutcher, 2008; Falgarone et al., 2008; Crutcher et al., 2010; Chapman et al., 2011, among others) which does not give much insight into the exact nature of $\mu_0$ within denser small scale regions. Therefore, the mass-to-flux ratio of specific regions are not generally known, let alone a distribution over an entire cloud. Recent simulations of cloud formation with magnetic fields (Vázquez-Semadeni et al., 2011) show that the mass-to-flux ratio distribution seems to exhibit a lognormal shape. On the other hand, analysis of the likelihood of different magnetic field distributions (Crutcher et al., 2010) show that the magnetic field strengths for various regions (HI diffuse clouds, OH dark clouds, etc) exhibit a uniform distribution ranging from very small values up to a maximum value. This seems to disagree with the simulations of Vázquez-Semadeni et al. (2011). With these results in mind, we choose to explore both options (i.e., uniform and lognormal distributions).

As shown by the linear analysis results presented in Chapter 2 (see also Bailey and Basu, 2012) and Figure 5.1, the length scale for collapse is dependent on the value of the mass-to-flux ratio. The value of $\mu_0$ is selected from a predefined distribution that is independent of the distribution of $\sigma_n$. This implies that the magnetic field strength is not constant and varies according to the choices of $\sigma_n$ and $\mu_0$. The independent sampling of values of $\sigma_n$ and $\mu_0$ does not then allow for any systematic dependence of one quantity on the other. We believe this is an acceptable first approximation since the initial conditions of the mass-to-flux ratio distribution in a molecular cloud are poorly constrained. We test several possible $\mu_0$ distributions in an attempt to determine if the shape of an observed CMF could reveal information about the underlying mass-to-flux ratio distribution. We consider both uniform and lognormal distributions. Figures 5.3 & 5.4 show the adopted lognormal mass-to-flux ratio distributions for the
Chapter 5. The Effect of Magnetic Fields and Ambipolar Diffusion on Analytic CMFs

Figure 5.3: Model mass-to-flux distributions for flux freezing models. Left: Broad Lognormal Distribution (FF2). Right: Narrow Lognormal distribution (FF3).

Figure 5.4: Model mass-to-flux distributions for ambipolar diffusion models. Left: Broad Lognormal Distribution (AD4). Right: Narrow Lognormal (AD5).

flux freezing and ambipolar diffusion models respectively. Specifically, all distributions sample the transcritinal peak in fragmentation scale, $\lambda_{g,m}$ (see Figure 5.1). The properties of all $\mu_0$ distributions considered are given in Table 5.1.

5.1.4 Producing Synthetic Core Mass Functions

To produce a synthetic CMF, we randomly sample the column density distribution for the nonmagnetic case and both the column density and mass-to-flux ratio distributions for the
Table 5.1: Model Parameters

<table>
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<tr>
<th>Model Name</th>
<th>$\mu_0$ Distribution</th>
<th>Mean (log $\mu$)</th>
<th>Variance ($\sigma^2$)</th>
<th>$\mu_0$ Range</th>
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<tr>
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<td>1.0 - 1.5</td>
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<td>0.1 - 1.0</td>
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<td>-</td>
<td>1.0 - 3.0</td>
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<td>Narrow lognormal</td>
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<td>0.01</td>
<td>0.6 - 1.5</td>
</tr>
</tbody>
</table>

magnetic cases. These values are then used to find the preferred length scale for collapse from the linear analysis. Finally, the mass is determined by multiplying the column density by the square of the corresponding length scale. By randomly sampling each model distribution $10^6$ times, a synthetic CMF is produced.

### 5.2 Models and Results

Our analysis covers several different mass-to-flux ratio distributions and assumptions about the neutral-ion collision time and column density distribution. As stated earlier, the column density distribution is the same for all models (see Figure 5.2) and the neutral-ion collision time for the ambipolar diffusion models is set to a normalized value, $\tau_{ni,0}/t_0 = 0.2$. In addition to the models listed in Table 5.1, we also present a nonmagnetic (NM) fiducial case. The following subsections present the results for each model individually. An in-depth comparison between all the models and implications regarding observed CMFs will be discussed in Sections 5.2.4 & 5.3 respectively.

#### 5.2.1 Non-Magnetic Model

The nonmagnetic model serves as a baseline for our investigation. The left panel of Figure 5.5 shows the resulting core mass function from this technique. As discussed in Section 5.1.2, we choose the peak of our density distribution to correspond to the apparent visual extinction threshold for the creation of star forming cores: $A_v \sim 8$ magnitudes. The right hand panel of Figure 5.5 shows the contributions from high density gas ($A_v > 8$ mag, dotted line) and low density gas ($A_v < 8$ mag, dashed line). As expected from the Jeans theory, the core mass
distribution mimics the column density distribution, with high mass cores formed from low density gas and low mass cores formed from high density gas. The distribution of masses for this model peaks at a value of $\log(M/M_\odot)=0.4$, or $M \approx 2.5M_\odot$, which is consistent with observations (Nutter and Ward-Thompson, 2007).

### 5.2.2 Flux Frozen Magnetic Model

A main aim of this paper is to show the effect of a magnetic field on the CMF. A flux frozen field represents the simplest case. Such a scenario arises in highly ionized regions where frequent collisions between ions and neutral particles would ensure perfect coupling to the magnetic field. Figures 5.6-5.8 show the resulting synthetic core mass function for the three models FF1, FF2, and FF3 respectively. Under the assumption of a uniform mass-to-flux ratio distribution (FF1), the resultant CMF (Figure 5.6, left) exhibits a narrow peak with a distinct high mass tail. The right hand panel of Figure 5.6 again shows the contributions to the CMF from the two column density regimes ($A_v < 8$ mag (dashed line) and $A_v > 8$ mag (dotted line)). This composite plot shows that like the NM case, and in line with the Jeans theory, the low density gas forms high mass cores, while high density gas forms low mass cores. However, unlike the
5.2. Models and Results

Figure 5.6: Synthetic core mass functions for a flux frozen magnetic cloud assuming a uniformly distributed mass-to-flux ratio (FF1). Left: Total core mass function. Right: Contributions to the core mass function from cores with $A_v < 8$ mag (dashed line) and cores with $A_v > 8$ mag (dotted line).

Jeans theory and NM case, we see that with the addition of a magnetic field, the high density gas also contributes to the formation of high mass cores, albeit to a lesser extent. Compared to the NM case, the peak of this core mass function is shifted to $M \approx 10^{0.7} \, M_\odot \approx 5.0 \, M_\odot$. On the right hand side of this peak, the trend can be described by two distinct slopes. For $0.7 < \log(M/M_\odot) < 1.2$, $\alpha = 0.8$ while for $\log(M/M_\odot) > 1.2$ the slope becomes shallower: $\alpha \approx 0.6$. Neither of these values corresponds to the typical Salpeter and observational values. This discrepancy will be discussed further in Section 5.3.

The formation of the high mass tail is due to the relationship between $\mu_0$ and $\lambda$ as defined by Equation 2.35. For $\mu_0 - \sigma_n$ pairs which have mass-to-flux ratios closer to the critical value ($\mu_0 = 1$), the corresponding length is up to 23 times larger than the thermal Jeans length for the same column density (see Figure 5.1). This increase in length scale has a direct effect on the mass of the core that is formed. Conversely, the low mass distribution is formed by $\mu_0 - \sigma_n$ pairs that have high column density and mass-to-flux values that are closer to the other limit ($\mu_0 = 3$), where $\lambda$ is only about 1.5 times larger than the thermal length scale.
Figure 5.7 shows the resulting synCMF for a broad lognormal \( \mu_0 \) distribution (FF2). The two panels again show the total and composite CMFs as described above. This distribution results in a CMF that is similar to that of model FF1 (Figure 5.6), with a few minor differences. First, the high mass tail exhibits a steeper slope that results in a more pronounced peak region. Second, the peak of the mass function has shifted to a slightly smaller value of \( M \approx 10^{0.5} M_\odot = 3.16 M_\odot \). As before, the high-mass trend can be described by two distinct slopes. For \( 0.5 < \log(M/M_\odot) < 1.0 \), \( \alpha = 1.31 \) while for \( \log(M/M_\odot) > 1.0 \) the slope becomes shallower: \( \alpha = 0.63 \).

Figure 5.8 shows the resulting synCMF for a narrow log-normal \( \mu_0 \) distribution (FF3). Unlike the previous two models, this one does not exhibit a narrow log-normal type peak, but rather shows a broad peak that leads directly into a high mass tail. As a result, the post peak trend for this model can be described by a single slope, \( \alpha = 0.44 \). Also, note that the function itself has been shifted toward higher masses as compared to the other two flux frozen models. As such, this CMF peaks at \( M \approx 10^{1.3} M_\odot \sim 20 M_\odot \). This shift in the mass range is due entirely to the narrow peak distribution of the mass-to-flux ratio; all of the chosen mass-to-flux ratios result in length scales that are \( \sim 6 - 23 \) times larger than the thermal length scale (see Figure 5.1) and therefore, the low mass cores that are formed in the other two models are absent in this model. Overall, as shown by all three models, the effect of adding a flux-frozen field is the appearance of a broad shallow tail at the high mass end of the core mass function.
5.2. Models and Results

5.2.3 Ambipolar Diffusion Magnetic Model

In the previous section we looked at the effect of a simple flux-frozen field on the shapes of the resulting CMF(s). Here we look at how the addition of neutral-ion slip via ambipolar diffusion affects the shape of the CMF. As discussed above, we have fixed the normalized neutral-ion collision time to $\tau_{ni,0}/t_0 = 0.2$. This implies a high degree of ambipolar diffusion and therefore less frequent collisions between the neutrals and ions. Such a situation would occur in the inner regions of a molecular cloud where the main ionization mechanism is cosmic rays.

Figures 5.9-5.13 show the resulting synCMFs for all five mass-to-flux ratio distributions respectively. To establish how the sub- and supercritical regions of the mass-to-flux ratio affect the shape of the CMF, we start our analysis by presenting two cases that isolate each regime. Figures 5.9 & 5.10 show the resulting synCMFs for the subcritical and supercritical uniform mass-to-flux ratio distributions (AD1 and AD2) respectively. The two panels show the total and constituent core mass functions as described in the previous section.
Figure 5.9: Synthetic core mass functions for a magnetic cloud including the effects of ambipolar diffusion assuming a uniform subcritical distributed mass-to-flux ratio (AD1). Panels depict the same curves as Figure 5.6.

Figure 5.10: Synthetic core mass functions for a magnetic cloud including the effects of ambipolar diffusion assuming a uniform supercritical distributed mass-to-flux ratio (AD2). Panels depict the same curves as Figure 5.6.
Focusing on model AD1, Figure 5.9, the left panel shows that the core mass function is very similar to the nonmagnetic model (see Figure 5.5, left). This is due to the fact that the curve on the subcritical side of Figure 5.1 converges to the nonmagnetic limit faster than in the trans- and supercritical regions. Upon closer comparison, AD1 peaks at the approximately the same value as NM, however the density composite CMF (Figure 5.9, right) reveals differences between these two models. Unlike the nonmagnetic model, AD1 shows evidence that a portion of the high column density gas goes toward forming high mass cores (Figure 5.9, right).

Figure 5.10 shows the resulting synCMF under the assumption of a uniform supercritical distribution (AD2). The left panel shows that the total CMF is a hybrid between the nonmagnetic and flux-frozen models presented above. Specifically, this CMF shows the same peaked nature with high mass tail as the flux frozen model, however this tail abruptly declines at about 100 $M_\odot$. This truncation makes the overall shape of the CMF resemble the nonmagnetic case, albeit broader, with the beginnings of a “shoulder” feature between 10 and 100 $M_\odot$. Looking at the composite column density CMF (Figure 5.10, right), we see that the lowest and highest mass cores are formed by the highest and lowest density gas respectively, while the middle has contributions from both density regimes. The peak of the mass function for this model occurs at about $\log(M/M_\odot) = 0.7$.

Model AD3 assumes a uniform mass-to-flux ratio distribution that samples the peak of the $\lambda$ versus $\mu_0$ graph (see Figure 5.1). The resulting CMF (Figure 5.11, left) is very similar to the one produced by AD2. Looking at the contributions from the low and high column density gas (Figure 5.11, middle), we again see that the lowest and highest mass cores are formed by the highest and lowest density gas respectively while the middle range has contributions from both density regimes.

The right panel of Figure 5.11 shows the contributions from the subcritical ($\mu_0 \leq 1$, dashed line) and supercritical ($\mu_0 > 1$, dotted line) gas. We see that the total synCMF for AD3 (Figure 5.11, left) is a combination of models AD1 and AD2. Specifically, we see that the majority of the cores are formed from supercritical gas, while the subcritical gas yields a minor contribution to the population of low mass cores. By mentally combining the middle and right hand plots in Figure 5.11, one can determine that the highest mass cores are formed by supercritical gas and fall into the non-star-forming regime while low-mass cores are formed by both supercritical and subcritical gas, and fall into both the star-forming and non-star-forming regimes. The peak of the mass function for this model occurs at about $\log(M/M_\odot) = 0.7$ and the average slope of the high mass ‘tail’ is $\alpha = 1.42$. 
Figure 5.11: Synthetic core mass functions for a magnetic cloud including the effects of ambipolar diffusion assuming a uniformly distributed mass-to-flux ratio (AD3). Left: Total core mass function. Middle: Contributions to the core mass function from cores with $A_v < 8$ mag (dashed line) and cores with $A_v > 8$ mag (dotted line). Right: Contributions to the core mass function from cores with $\mu_0 < 1$ (dashed line) and cores with $\mu_0 > 1$ (dotted line).

Figure 5.12: Synthetic core mass functions for a magnetic cloud including the effects of ambipolar diffusion assuming a broad, lognormal mass-to-flux ratio distribution (AD4). Panels depict the same curves as Figure 5.11.
5.2. Models and Results

Finally, Figures 5.12 & 5.13 show the resulting synCMFs for the two lognormal $\mu_0$ distributions, AD4 and AD5, respectively. The broad lognormal distribution (AD4) is similar to models AD2 and AD3, however this model shows a more distinct ‘peak’ and ‘shoulder’ region as compared to the other two. Looking at the composite mass-to-flux ratio plot (Figure 5.12,right) we see that the peak region is mainly formed by subcritical gas while the shoulder region is formed mainly by contributions from supercritical gas. This model peaks at $M = 10^{0.7} \, M_\odot \approx 5.0 \, M_\odot$, and the average slope of the high mass tail is $\alpha = 1.18$.

Switching to the narrow lognormal distribution (Figure 5.13), we see that this model results in a double peaked function. Examination of the composite plots show that the low mass peak is formed by the subcritical material while the second peak is formed by supercritical material. These peaks occur at $\log(M/M_\odot) \approx 0.7$ and $\log(M/M_\odot) \approx 1.5$ respectively. The formation of the high mass peak is due to the extremely narrow mass-to-flux ratio distribution that is centered on the peak of the $\lambda - \mu_0$ curve (with $\tau_{ni,0}/t_0 = 0.2$) in Figure 5.1.

5.2.4 Assessment of Synthetic Core Mass Functions

The previous subsections presented the overall results and features of each of the models. Within these results we found three main features that changed between the different models. These are the overall shape of the core mass function, the location of the peak(s) and the slope of the high mass tail (if it exists). Here we discuss these three features across all models.
Shape

Within the nine models presented, there were three distinct recurring shapes; pure lognormal as represented by the NM and AD1 models, lognormal peak with a shoulder as represented by AD2, AD3, AD4 and AD5, and the lognormal peak with high mass tail as represented by FF1, FF2, and FF3. The appearance of these shapes are directly connected to the state of the magnetic field in the region. In the absence of a magnetic field, the CMF is a pure lognormal function. This shape is also observed in model AD1. As mentioned earlier, the reason that this AD model shows such a shape while the other ones do not is due to the shape of the $\lambda - \mu_0$ curve on the subcritical side of Figure 5.1; the curve asymptotes to the nonmagnetic limit faster on that side than on the supercritical side. Therefore one would expect a model with only subcritical mass-to-flux values to look similar to the nonmagnetic model, but with a slight broadening due to a narrow region of mass-to-flux ratios with $\lambda$ larger than the non-magnetic limit.

For models with an increasing supercritical regime, the broadening becomes more pronounced as a shoulder develops. This shoulder is due to an increase in higher mass cores that are the product of the larger length scales picked out by the supercritical mass-to-flux ratios. The extent of the shoulder depends on the mass-to-flux ratio distributions. For uniform distributions, the CMF is narrower with a less defined shoulder region, while for a broad lognormal distribution, the shoulder region is much broader and distinct. Finally, the appearance of the double peaked CMF in AD5 is an example of an extreme shoulder. This second peak is due solely to the extremely narrow mass-to-flux ratio range used in this model. This preferentially picks out only mass-to-flux ratios with length scales much larger than the nonmagnetic model.

The appearance of the pure high mass tail is entirely a product of flux-freezing. This is due to the asymptotic nature of the flux-frozen curve as it nears the critical mass-to-flux ratio (see Figure 5.1). This allows for transcritical mass-to-flux ratio values to produce much larger masses for the same column density.

Peak Location

The location of the CMF peak depends on the distribution of the mass-to-flux ratio. The location of the peak in the nonmagnetic case, which occurs at $\log(M/M_\odot) = 0.4$ ($M \approx 2.5 M_\odot$) serves as the comparison point. For magnetic models, the location of the peak was generally larger than this value as long as the mass-to-flux ratio distribution was uniform with some contribution from the supercritical regime (see models FF1, AD2, and AD3). Model AD1, although also assuming a uniform mass-to-flux ratio distribution, exhibits a similar peak value to NM due to the exclusion of supercritical mass-to-flux ratio values. When considering the
lognormal mass-to-flux ratio distributions, we find that the peak location is dependent on the width of the distribution. Specifically, broader distributions exhibit values closer to the NM peak value, while narrower distributions exhibit peak values that are higher than the nonmagnetic case. Model AD5 is an anomaly and does not fit within these trends given that it exhibits two peaks.

**High Mass Slope**

As alluded to earlier, the shape and extent of the high mass slope was found to be variable and connected to the influence of the magnetic field. Specifically, the appearance of the ‘shoulder’ feature is directly connected to the presence of ambipolar diffusion. The degree of the shoulder in the ambipolar diffusion models was found to be dependent on the range of allowed mass-to-flux ratio values. Overall, these differences in shapes result in a wide range of slopes. For the flux-frozen models, the slopes were as steep as \( \alpha = 1.31 \) for \( 0.5 < \log(M/M_\odot) < 1.0 \) in the case of FF2, and as shallow as \( \alpha = 0.44 \) in the case of FF3. For the ambipolar diffusion models, the average high mass slope ranges between \( \alpha = 1.18 \) and \( \alpha = 1.42 \). Although some of these slopes are consistent with the Salpeter value, \( \alpha = 1.35 \) (Salpeter, 1955), others are significantly different. Further analysis of this discrepancy is given in the following section.

### 5.3 Scaling to Observations

Unlike our synCMFs, typical observational CMFs usually contain on the order of 200 cores, not \( 10^6 \). Therefore, to make our analysis relevant for typical observed CMFs, we must scale our sample sizes to those typically observed. The following two sections explore the effect of two observational constraints, sample size and bin size, on the shape and slope of observed CMFs.

#### 5.3.1 Effect of Sample Size

To test the effect of the sample size on the resultant CMF, we scaled three synCMFs (NM, FF1, and AD3) down to plausible observational sample sizes (100, 200, 300, 400, and 500 cores). Figure 5.14 shows the resulting synCMFs for each of the fifteen cases. In addition to scaling the sample size, we have also truncated the mass range considered to one more typically found in observed CMFs \((-1.0 < \log(M/M_\odot) < 1.3)\). Under these scaled conditions, we see that the nonmagnetic CMFs still maintain the overall shape exhibited by the full sample curve (Figure 5.5), however the two magnetic cases are fairly different. The high mass tail and truncated shoulder features present in the full sample curves for FF1 and AD3 respectively
are no longer quite as distinct at these sample sizes. For a definitive difference between the ambipolar diffusion and flux-frozen cases, observations would have to extend up to objects with masses between $10^2$ and $10^3$ solar masses. Therefore, on typical observational scales, conclusions about the nature of the magnetic field from the shape of the CMF are possible, but highly uncertain.

### 5.3.2 Effect of Bin Size

Constructing histograms for the purposes of determining a CMF requires binning data into predetermined mass bins. For the above synCMFs, we used $\Delta \log(M/M_\odot) = 0.1$ size bins. Variations in the bin size acts to change the resolution of the resulting curve; smaller bins yield more detail while larger bins show only the broad strokes. To determine the effect of the bin size on the resulting CMF, we re-binned the histograms for AD3 in Figure 5.14 (bottom row) using $\Delta \log(M/M_\odot) = 0.25$ bins. Figure 5.15 shows the comparison of the original bin size ($\Delta \log(M/M_\odot) = 0.1$, bottom row) to the new bin size (top row). As expected, with the larger
Figure 5.15: Bin size comparison for small sample core mass functions. Panels show the effect of the bin size on the resulting curve for $\Delta \log(M/M_\odot) = 0.25$ bins (top row) and $\Delta \log(M/M_\odot) = 0.1$ bins (bottom row). Model used in all panels is AD3.

bin size, the detail becomes smeared out, resulting in an average curve.

5.3.3 Effect of sample size and bin size on CMF slopes

The main piece of data generally extracted from a CMF is the slope of the high mass tail. This information is then used to compare different regions to each other, and to the initial mass function (IMF) in an attempt to determine the true nature of star formation and the possible relation between the CMF and IMF. However, as discussed above, the sample size and bin size have a profound effect on the shape of the curve. This effect translates over to the derived slopes. To determine the extent of this effect, we generate 2000+ CMFs for each sample size and compute the average slope. Figure 5.16 shows the results of this analysis for models FF1 and AD3 for both mass bin sizes. The filled symbols show the average slope for each of the smaller sample sizes while the open symbols depict the slope of the full sample ($10^6$). Tests with larger numbers of samples for each sample size showed differences in the average slope of up to 0.01, which is encompassed in the size of the symbols.

As shown in Figure 5.16, the size of the bin clearly affects the average slope. The larger bin size yields slopes that are steeper than the Salpeter slope, while the smaller bin size shows
Figure 5.16: Average slope as a function of sample size. Symbols represent the derived slopes for the two models and two bin sizes: AD3, $\Delta \log(M/M_\odot) = 0.1$ (squares), AD3, $\Delta \log(M/M_\odot) = 0.25$ (circles), FF1, $\Delta \log(M/M_\odot) = 0.1$ (diamonds), and FF1, $\Delta \log(M/M_\odot) = 0.25$ (triangles). Average slopes computed over a minimum of 2000 samples. Open symbols indicate the slopes of the full sample size.
an overall shallower average slope. The size of the sample also affects the slope. Smaller samples generally result in steeper slopes than those derived using the full sample.

Furthering this analysis we look at both the minimum and maximum slopes calculated for each filled point in Figure 5.16, as well as the distribution of slopes. Figure 5.17 shows the distribution of slopes for four of the points on Figure 5.16 as indicated (Top row: AD3, Bottom Row, FF1. Left column: 100 cores, Right column: 500 cores) assuming a CMF constructed with $\Delta \log(M/M_\odot) = 0.1$ bins. All four cases show that the preferred slope value is close to the average slope value. The maximum and minimum computed slopes exhibit a very wide range for the small sample sizes (i.e., $\alpha = -0.1$ to $\alpha = 11$ for AD3, 0.1, 100 cores) while the larger sample sizes exhibit a smaller maximum-minimum range (i.e., $\alpha = 0.34$ to $\alpha = 1.79$ for AD3, 0.1, 500 cores). This decrease in the slope variance is evident when comparing the left column
to those in the right column in Figure 5.17. From these plots, we conclude that although there can be a wide variance in possible slope values, the preferred slope value is in general smaller than the typical Salpeter value, $\alpha = 1.35$, and the range of slopes decreases as the number of samples increases. Figure 5.18 shows the distribution of slopes for two CMFs (left: AD3, right: FF1) assuming a 100 core sample size and $\Delta \log(M/M_\odot) = 0.25$ bins. Comparing to the left hand plots in Figure 5.17, we see that the larger bin size results in a bimodal distribution with peaks occurring at $\alpha \approx 2.0$ and $\alpha \approx 1.5$ for AD3 and $\alpha \approx 1.9$ and $\alpha \approx 1.45$ for FF1. The result of this bimodal distribution is to shift the average slope values to smaller values than that of the dominant peak. This is particularly evident in Figure 5.16 in the trend of slopes for the smallest sample sizes of the triangles (FF, $\Delta \log(M/M_\odot) = 0.25$). Further analysis of the effect of the original column density distribution on the variance and mean of the resulting slope histogram showed that a larger variance in the column density distribution shifts the mean in the slopes to smaller values ($< 1$) while a smaller variance results in a greater mean value, $\alpha \approx 1.35$.

5.4 Discussion

Our analysis shows that the shape of the CMF is highly dependent on the magnetic field strength and neutral-ion coupling within the cloud. Specifically, a flux-frozen magnetic field broadens the nonmagnetic lognormal distribution to have a significant power-law high mass tail, though it is much shallower than the Salpeter value. When ambipolar diffusion is taken into account, there is an intermediate mass tail and a high mass cutoff. The extent of all these features depend on the range of mass-to-flux ratio values in the initial cloud.

Kunz and Mouschovias (2009) (KM09) carried out a more focused study of the effect of magnetic fields and ambipolar diffusion in creating a broad CMF. Their model explored only
the subcritical portion of the fragmentation scales seen in Figure 5.1. Furthermore, they assumed a uniform distribution of subcritical mass-to-flux ratios and a fixed Jeans mass in order to generate their mass distribution.

The low-mass tail in their distribution originates in the assumption that the subcritical clouds ultimately form dense cores with masses that are scaled by $\mu_0$ for subcritical values of $\mu_0$. This is because numerical simulations of Basu and Mouschovias (1995) show that only an inner region where the mass-to-flux ratio exceeds the critical value undergoes rapid collapse. We do not make that assumption in this study, since cores that form by ambipolar drift have an appearance that is similar to those that are forming by a more rapid gravitationally-dominated process (see Basu et al., 2009b). Since the resultant CMF in our model is generated from an underlying lognormal function, it has an intrinsic peak even when binned in linear mass bins. An advantage of the KM09 model is that they do not need to assume an underlying lognormal distribution to obtain a lognormal-like CMF, however their CMF is peaked only when binned in log mass.

Upon scaling our models to observational sample sizes and ranges, we found that the distinction between the different models is lost within typical observational mass ranges and therefore no information regarding the magnetic field can be reliably gleaned from the shape of the observed CMFs. Further to this, analysis of the slopes for each of the sample sizes showed that the smaller sample sizes result in slopes that are 1.1 – 1.4 times greater than the slope derived from the full sample, while the derived slopes for the larger bin size are $\sim 1.3 – 2.0$ times greater than the corresponding smaller bin size slope measurements. Although we have taken care to scale our analysis down to those typically used in observations, the question still remains as to how well our results and conclusions correspond to actual observations. A recent study of the CMF for five separate star forming regions (Ophiuchus, Taurus, Perseus, Serpens and Orion) performed by Sadavoy et al. (2010) provides the perfect platform for comparison. Looking at the core mass distributions for these regions, as expected, it is hard to definitively discern any characteristic features that are indicative of a particular magnetic field model. With limited data, it is plausible that the CMFs for Ophiuchus, Taurus and Perseus could exhibit the indicative shoulder of the ambipolar diffusion models, while the full Orion CMF could show evidence of a flux frozen field. Looking at the slopes of the CMFs for these regions, Sadavoy et al. (2010) showed each region gave slope values that are close to the $\alpha = 1.35$ Salpeter slope, within their adopted errors. Comparing their slope values to those in Figure 5.16, most of them would fall somewhere in the lower half of the graph in amongst the diamonds and squares while the Orion with OMC slope would fall in amongst the triangles and circles. However, looking at the bin size of the observations, all of the slopes should be within the triangle/circle regime of the graph. Comparing these values to the corresponding slope histograms (see Figure 5.18)
we see that these values all fall within the regime of possible slopes. On the surface, this seems to be a huge discrepancy between our results and observations; however, each of these five observational slopes represents a single slope within our 2000+ values used to derive an average slope. Looking at the range of slopes derived from our analysis, these observed slopes fall within this range. As shown in Figure 5.17, the only way to produce a narrower range of slope values is to increase the sample size, which is not always possible observationally since the number of objects detected depends entirely upon the number of objects actually present and the sensitivity of the instrument.

Based on our analysis and the above comparison to the work by Sadavoy et al. (2010), we argue that the observed CMFs are extremely statistically limited, both in the size of the sample and the number of samples over which the slope of the CMF is averaged. Through our analysis, we have shown that with larger number statistics, not only is the measured slope of the CMF much different than the typical Salpeter value $\alpha = 1.35$, but it is also highly dependent on the size of the mass bin. In addition, the range of individual slope values decreases as the number of cores in the sample size increases. This is analogous to the results found by Elmegreen (1999b), where although it was determined that the most probable value for the IMF slope is the Salpeter value, $\alpha = 1.35$, it is a highly reduced average of all possible outcomes. Subsequently, we argue that based on our analysis and the results of Elmegreen (1999b), there seems to be no clear cut correlation between the slope of the CMF and the IMF, and that the shape and slope of the CMF are entirely controlled by the conditions within the cloud itself. Since it is unfeasible to claim that all clouds exhibit identical conditions, it is therefore unrealistic to expect a universal shape and slope value for the CMF in all star forming regions.

5.5 Summary

We have studied the effect of magnetic fields on the formation and properties of the core mass function using a combination of the results from linear analysis and Monte Carlo methods. In addition, we have studied the effects of low number statistics on the slope of the high mass tail. Here we summarize the main results of our analysis.

- The synthetic CMFs show that the presence of a magnetic field has several effects on the shape of the CMF. In general, a magnetic field acts to broaden the core mass function compared to the nonmagnetic CMF. In addition, the magnetic CMFs exhibit a high mass tail. The form of this tail depends on whether the field is flux frozen or allows for neutral-ion drift across the field lines. In the former case, the tail exhibits a continuous power law while in the latter case, the high mass tail truncates to form a “shoulder”.
• The nonmagnetic model shows that the high mass cores are formed from low density gas and vice versa. Analysis of the contributions of low and high density gas to the low and high mass regions of the CMF shows that the addition of magnetic fields results in additional contributions of high mass cores formed from high density gas.

• Scaling of the synthetic CMFs down to typical observational sample sizes and bin sizes show that the ability to distinguish between the different models is no longer possible for the smallest sample sizes (100 cores) and typical bin sizes ($\Delta \log(M/M_\odot) = 0.25$). This shows that the current observations of core mass functions are statistically limited.

• Statistical analysis of the derived slope from a large sample of synthetic CMFs show that the slope of the high mass tail is systematically steeper for smaller core sample sizes than for larger sample sizes. In addition, the average slope is also systematically steeper for larger bin sizes.

• Analysis of the minimum, maximum and distribution of calculated slopes shows that the most probable slope does not necessarily correspond to the canonical Salpeter value. In addition, the most probable slope value becomes shallower as the sample size increases.
Chapter 6

Conclusions

New high-resolution observations with the *Herschel* telescope have revealed the existence of previously unknown structure within star forming regions (e.g., Sadavoy et al., 2012) and upcoming observations with ALMA promise to reveal even smaller structures. With such data on the horizon, theoretical models and computational simulations are required to help understand the underlying physical mechanisms and trends found in the observations. This thesis has helped to advance the theory of clump and core formation both in the context of clustered star formation and the core mass function.

Previous studies of core collapse via linear analysis of the magnetohydrodynamic equations for a molecular cloud within the thin-disk approximation revealed that there are various length and time scales for fragmentation depending on the conditions within the medium. Changes in the mass-to-flux ratio and ionization fraction of the region were shown to have significant effects on the size of the structures formed and how long it takes for them to fragment and collapse. This work has extended this analysis to the formation of clumps, the effect of the environment on the formation of hierarchical clump-core complexes within molecular clouds, and the effect of magnetic fields and ambipolar diffusion on the core mass function.

By combining the results of linear analysis with a realistic ionization profile for a cloud, we found how the length and timescales vary with respect to column density for constant mass-to-flux ratios. This lead to the discovery of a two-stage fragmentation process within molecular clouds that can easily explain the formation of star forming clusters. This two-stage fragmentation scenario shows that an initially diffuse transcritical cloud could undergo two separate fragmentation events, allowing for the formation of several cores within a larger clump region. Temporal evolution of the MHD equations showed that the two-stage fragmentation scenario requires very particular conditions within the cloud, including an initially transcritical mass-to-flux ratio, a step-like ionization profile, and the addition of random column density perturbations throughout the region. In addition to the two-stage fragmentation scenario, the
numerical simulations gave some insight into the possible small scale magnetic field and mass-to-flux ratio values within clumps and cores for varying environmental conditions. Specifically, the chaotic nature of the mass-to-flux ratio map for simulations with the step-like ionization profile show that highly ionized regions do not have the chance to relax back to a more uniform state. Future advances in the resolution and measurement techniques of magnetic fields within molecular clouds will hopefully help to verify and constrain the parameter space for fragmentation within the two-stage fragmentation model by providing accurate magnetic field strengths for clumps and cores.

Combining the linear analysis results with Monte Carlo methods showed that magnetic fields, and more specifically, the degree of ambipolar diffusion has an effect on the shape, width, and high mass slope of the core mass function. The magnetic field was found to broaden the mass distribution while the addition of ambipolar diffusion produced a “shoulder” feature within the high mass tail. Scaling the synthetic core mass functions down to typical observational sample and bin sizes showed that the observations are severely statistically limited and that the derived high mass slope is not necessarily equal to the fiducial Salpeter (1955) value. This analysis has shown that the core mass function likely does not have a one-to-one correlation with the IMF. Variations in slope values indicate the CMF is not universal for all regions, but rather depends on the specific environmental conditions within the region being studied. Future high resolution observations will hopefully reveal regions with sufficient numbers of cores to overcome the statistical limitations of current CMF observations and help verify the results presented here.

Although these investigations have been successful in shedding some light on the process of star formation, they do not constitute an exhaustive search of the parameter space. Given that the fragmentation length and timescales are highly dependent on the state of the environment, to fully understand the process of star formation and the observations of star forming regions, we must explore the effect all possible environmental variations. Our studies have covered the full spectrum of mass-to-flux ratio values, however we have only explored two representative ionization profiles. With regard to the two-stage fragmentation model, future avenues include investigating the effects of different ionization profiles and finding physical mechanisms for the perturbations in the thin disk simulations (e.g., small scale turbulence). We can also temporally extend our simulations past the point where the first core collapses in order to investigate the evolution as subsequent cores collapse. With regards to the core mass function studies, we can extend this analysis to the clump regime by adding the step-like ionization profile to the Monte Carlo analysis. In addition, by running many realizations of the numerical simulations, we can construct simulated core mass functions to compare with the synthetic core mass functions.
Bibliography


BIBLIOGRAPHY


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