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INFORMATION IN PRODUCTION*

by

Glenn M. MacDonald

May, 1980

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ABSTRACT

A model of job-worker matching with imperfect information is presented. Earlier work has focused on the information acquisition process. The nature of the manner in which such information would be used was given a rather neo-classical treatment; output could be ascribed to a particular worker. Herein, the analysis takes place in a familiar microeconomic setting.

The impact of changes in information quality is considered at both the firm and industry levels. The relationship between information on worker skills and output is made precise.

INFORMATION IN PRODUCTION

I. Introduction

Implicit in virtually all discussions of information in the labor market is the vague notion that more information increases output. The intuitive argument is that output depends on the way workers and jobs are paired off, and that more information somehow facilitates the matching process. In this essay I model the job-worker matching process and examine the manner in which improved information affects the production process.

The inquiry is carried out at both the firm and market levels. Improving information quality alters the productive opportunities faced by the firm, as well as firm's cost function. Viewed in isolation, the (representative) firm may be adversely affected by better information. When the constraints imposed by market clearing are imposed, however, the net effect on output must be beneficial, though there are some surprises at the market level as well.

Earlier models of job-worker matching with imperfect information (e.g., Johnson (1978), Jovanovic (1979), Mortensen and Burdett (1979) and MacDonald (1980)) have dealt with situations in which the job-worker match depends only on the firm and the particular worker; the implicit production function being the sum of the outputs of the individual workers. Herein, the analysis takes place in a more familiar microeconomic setting wherein output may not be ascribed to any particular individual.¹ Workers arrive with some information on their productive capacity. This information is utilized to assign them to "jobs". The content of a job, the set of tasks which the worker attempts to perform and the time spent at each, is endogenous. In full market equilibrium, job content is determined by information quality, technology and supply of factors.

The nonstochastic theory of how workers are assigned to jobs has existed in intuitive form at least since the writings of Adam Smith. Recently Rosen (1979), building on the work of Koopmans and Beckmann (1957), formalized the nonstochastic theory at the firm level. My work builds upon this formal structure.

II Individual Skills and the Nature of Uncertainty

Individuals are assumed to be of two varieties, A and B.² Groups A and B differ only with respect to their skills, which are taken to be endowed.

Individual skills take the following form. There is a continuum of tasks, indexed by $s \in [0,1]$. The quantity of task s that a member of group A (B) can perform, if pursuing the task full-time, is $t_A(s) > 0$ ($t_B(s)$). It is assumed that there are no costs of changing tasks, and that there are no supply side complementarities between tasks. It follows that the amount of task s performed is a function solely of group, and fraction of time spent on task s (and not the fraction of time spent on task s' ; $s \neq s'$).

There is a continuum of individuals, indexed by i , arranged on $[0,1]$ as depicted in Figure 1.³ Individuals for whom $i \in [0,\delta]$ comprise group A; group B is made up of $i \in (\delta,1]$. It is assumed that δ , summarizing the "macro" distribution of skills, is known.

We are interested in analyzing the impact of information on the structure of production. Accordingly the process generating information is very simple. Each and every individual receives a test, the results of which are summarized by a label "a" or "b". Let $P(a|A)$ be the probability of receiving an a given that one is truly a type A, and define $P(\cdot|\cdot)$ similarly. It is assumed that

- (i) the test is symmetric: $P(a|A) = P(b|B) \equiv P$;
- (ii) the test is informative but imperfect: $P \in (\frac{1}{2}, 1)$;
- (iii) the information structure is symmetric: a and b are freely

observable by all, and no one has any prior information except δ .

Figure 1

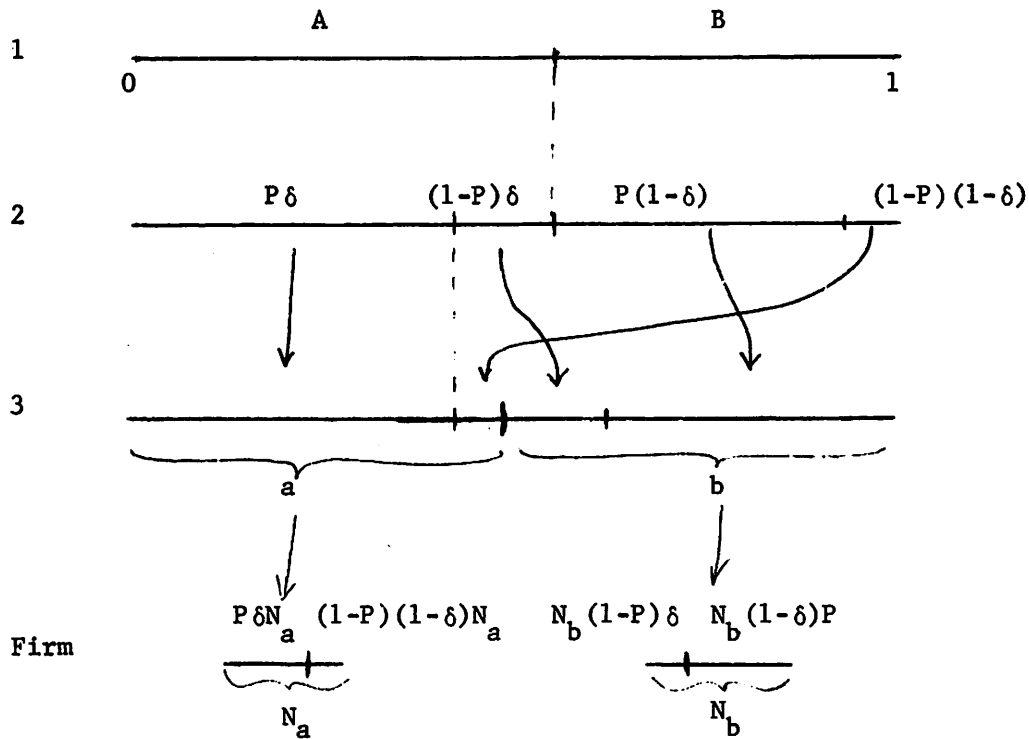
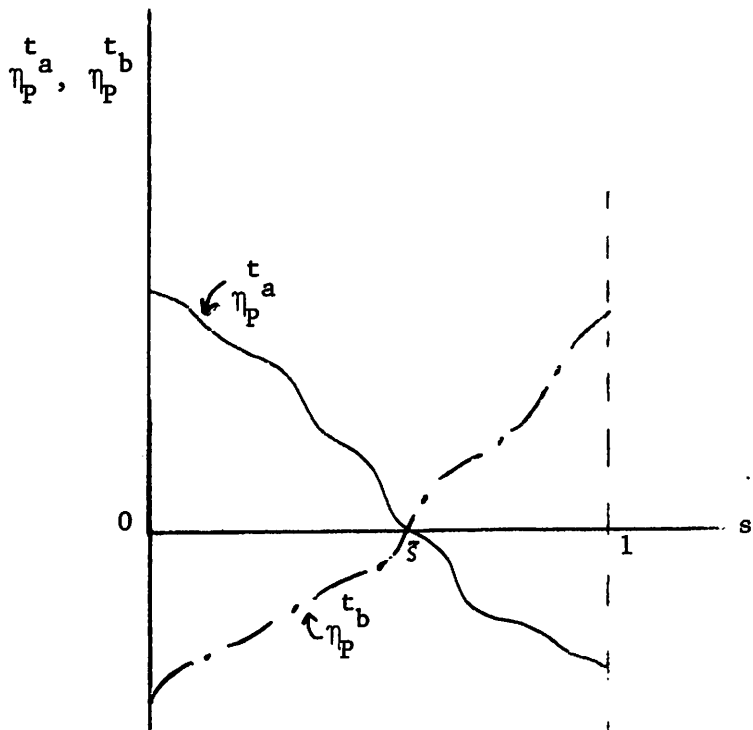


Figure 2



A second "macro" assumption is that the realized number of individuals in the population that are correctly identified by the test is P . This may not be justified with reference to the law of large numbers. The law of large numbers refers to the proportion correctly identified, whereas this assumption deals with the total number correctly identified. The two are numerically the same here (though conceptually different) because the labor force is normalized to unity (i.e., $\int_0^1 di = 1$). However, if the economy-wide testing procedure were repeated many times, the number correctly identified would be P on average; so in that sense I am assuming a "representative economy".

The second line of Figure 1 depicts the labeling process. Without loss of generality, it is assumed that of the δ individuals in group A, the correctly labeled individuals ($P\delta$ of them) are those for whom $i \in [0, P\delta]$. Table 1 summarizes the process.

Table 1[†]

Test Result

		a	b
Group	A	$P\delta$ $(0, P\delta]$	$(1-P)\delta$ $(P\delta, \delta]$
	B	$(1-P)(1-\delta)$ $(\delta + P(1-\delta), 1]$	$P(1-\delta)$ $(\delta, \delta + P(1-\delta)]$

[†]The upper figure in each cell is the number of individuals, while the interval is their location in $[0, 1]$

It follows immediately that the probability of being labeled a or b (which under our assumptions equals the number labeled a or b) is given by the column sum in Table 1.

$$P(a) = P\delta + (1-P)(1-\delta)$$

(1)

and
$$P(b) = (1-P)\delta + P(1-\delta)$$

Further, the probability that one is actually an A given the label a (which equals the number of A's labeled a, divided by the number labeled a) is

$$P(A|a) = \frac{P\delta}{P(a)} .$$

Similarly,

$$P(B|a) = \frac{(1-P)(1-\delta)}{P(a)}$$

$$P(A|b) = \frac{(1-P)\delta}{P(b)} \quad (2)$$

and
$$P(B|b) = \frac{P(1-\delta)}{P(b)}$$

In line three of Figure 1, individuals are re-ordered so that people labeled a come first. The figure is drawn for $\delta > \frac{1}{2}$ in which case $P(a) < \delta$ and $P(b) > 1-\delta$. This occurs because for $\delta > \frac{1}{2}$, group A is larger than group B. The number of people labeled a is the number of A's labeled a, $P\delta$, plus the number of B's labeled a, $(1-P)(1-\delta)$. The latter is smaller than the number of A's labeled b, $(1-P)\delta$, because the same fraction of each group is mis-labeled, but the A group is larger. Accordingly the number of A's, δ , exceeds the number of a's, $P\delta + (1-P)(1-\delta)$.

The macro process is now complete. A population consisting of δ A's and $(1-\delta)$ B's has been broken into two groups, a and b, of size $P(a)$ and $P(b)$ respectively.

The firm will hire some number of a's and b's. Denote these choices by N_a and N_b . It is assumed that the sample chosen by the firm reflects the population fractions perfectly. That is, for example, of the N_a chosen, exactly $P(A|a) \cdot N_a$ are really A's. Again this may not be justified with reference to the law of large numbers, but makes sense when interpreted as the experience of a representative firm. While this assumption may be somewhat

unappealing, it allows a relatively straightforward analysis, and some preliminary work suggests that relaxing this assumption has little impact. The firm therefore knows how many A's and B's it has, but does not know who they are. This is depicted as line four of Figure 1.

Thus, the uncertainty in the model is person-specific. Firms have good information regarding the total productive capacity at their command, but imperfect information on its specific location.

III Comparative Advantage

It is assumed that tasks are ordered so that

$$r(s) \equiv \frac{t_A(s)}{t_B(s)} \quad (3)$$

has the property $\partial r / \partial s < 0$. That is, A's comparative advantage declines as we move from task s to task $s + ds$.⁴ Uniform absolute advantage, $r > 1$ (or < 1) for all s , is not ruled out a priori.

Equation (3) summarizes the relationship between productivities across groups A and B. The firm, however, only has access to the label and the probabilities (2). Let $t_a(s)$ ($t_b(s)$) be the amount of task s individuals labeled a (b) are expected to perform:

$$t_a(s) = P(A|a)t_A + P(B|a)t_B \quad (4)$$

and
$$t_b(s) = P(A|b)t_A + P(B|b)t_B$$

t_a and t_b are convex combinations of t_A and t_B . It follows immediately from (2) that

$$t_A(s) \geq t_a(s) \geq t_b(s) \geq t_B(s) \Leftrightarrow r \geq 1, P \in \left(\frac{1}{2}, 1\right) \quad (5)$$

That is to say, t_a and t_b follow the same ranking as t_A and t_B , but the differences are attenuated by the fact that the labeling process is imperfect.

Differentiation of (4) also yields⁵

$$\eta_p^t_a = \eta_p^{P(A|a)} \left(1 - \frac{t_B}{t_a}\right) = \frac{1-\delta}{P(a)} \left(1 - \frac{t_B}{t_a}\right)$$

$$\eta_p^t_b = \eta_p^{P(B|b)} \left(1 - \frac{t_A}{t_b}\right) = \frac{\delta}{P(b)} \left(1 - \frac{t_A}{t_b}\right)$$
(6)

As neither $\eta_p^{P(A|a)}$ nor $\eta_p^{P(B|b)}$ depend on s , it follows that the pattern of $\eta_p^t_a$ and $\eta_p^t_b$ in s is determined by the bracketed terms in (6). From (5) it is immediate that

$$\text{sign}\{\eta_p^t_a\} = - \text{sign}\{\eta_p^t_b\} = \text{sign}\{r-1\}$$
(7)

That is, for s such that $r > 1$, A's are more productive than B's. Increasing P implies that the fraction of a's that are in fact A's rises, raising the productivity of the a's.

Further since

$$\text{sign}\left\{\frac{\partial}{\partial s} \frac{t_B}{t_a}\right\} = - \text{sign}\left\{\frac{\partial}{\partial s} \frac{t_A}{t_b}\right\} > 0,$$

it follows that

$$\text{sign}\left\{\frac{\partial}{\partial s} \eta_p^t_a\right\} = - \text{sign}\left\{\frac{\partial}{\partial s} \eta_p^t_b\right\} < 0.$$
(8)

Equation (8) reveals the obvious but important fact that changes in the accuracy of the labeling process have the greatest impact on productivity when the productivities of the groups are most different. Under the assumption that for some \bar{s} , $r(\bar{s}) = 1$, Figure 2 (page 3) illustrates (7) and (8)

It is also useful to have an analogue to r , referring to the relative productivities of a's and b's. Consequently, let

$$R(s) \equiv \frac{t_a(s)}{t_b(s)}.$$

Using (5) and (7), it is easily shown that

$$\frac{\partial R}{\partial s} < 0,$$

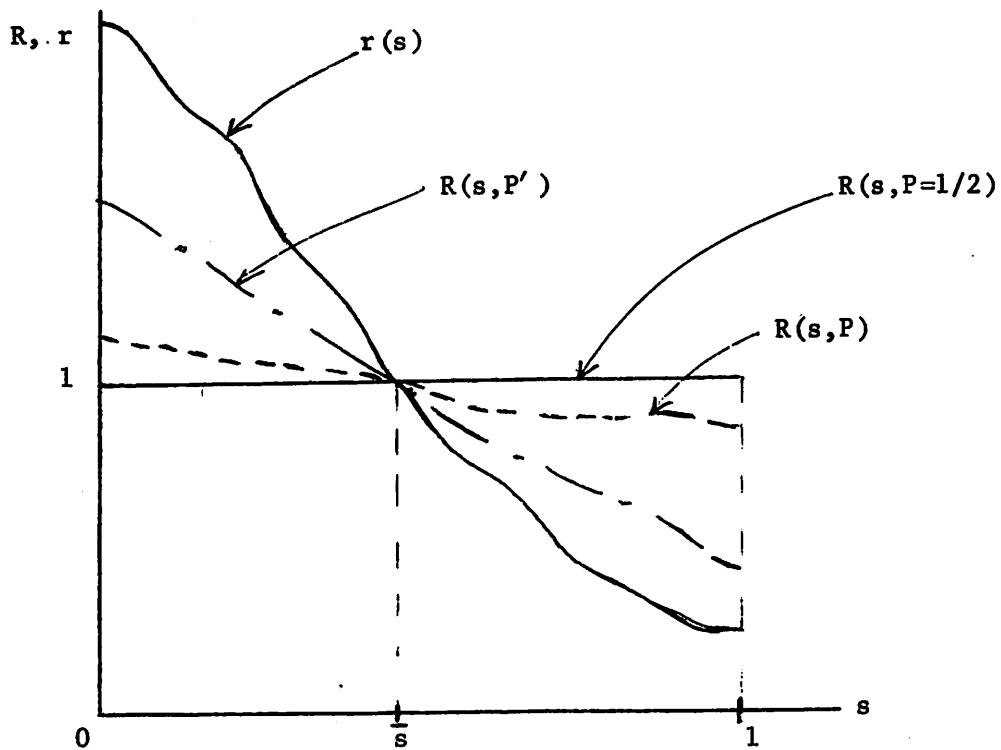
$$\text{sign}\left\{\frac{\partial R}{\partial P}\right\} = \text{sign}\{r-1\}$$
(9)

and that

$$1 \lesssim R \lesssim r \text{ as } r \lesssim 1, P \in (\frac{1}{2}, 1). \quad (10)$$

Equations (9) and (10) indicate that R has the same essential properties as r , and that R more closely approximates r the higher is P . Figure 3 illustrates the point.

Figure 3
($P' > P$)



IV The Allocation of Workers to Tasks--Endogenous Job Content

In this section I consider the optimal assignment of a given stock of workers, N_a and N_b , to tasks. To avoid repetition, a and b will be summarized by x (i.e., N_a and N_b are N_x , $x=a,b$). The solution to the problem may be summarized by (i) a derived production function expressing output as a function of the N_x ; and (ii) two "jobs", one for each type of worker (x). A job has two characteristics: (i) a set of tasks to which the worker is assigned, S_x , where $\bigcup_x S_x = [0,1]$; (ii) an assignment function satisfying

$$\int_{S_x} \phi_x(s) ds = 1$$

$$\phi_x \geq 0 \quad \forall s \in [0,1] \quad (10)$$

$$\phi_x > 0 \Leftrightarrow s \in S_x.$$

$\phi_x(s)ds$ represents the fraction of time an individual with label x spends working at job s . The total amount of task s done by all of group x , T_x , is then given by

$$T_x \equiv N_x \int_{S_x} \phi_x(s) ds. \quad (11)$$

A job of type x , J_x is defined by the pair $\langle S_x, \phi_x(s) \rangle$.

For simplicity, the firms' technology (production functional) is assumed to be of the form

$$q = \min_{s \in [0,1]} \{T(s)/\alpha(s)\}, \quad (12)$$

where q denotes output, $T(s)$ is the total amount of task s used, and $\alpha(s) > 0$ is the (scale independent) task-output coefficient for task s .

Recalling that tasks were re-ordered so that $r' < 0$, there is no reason to suppose that $\alpha(s)$ is even continuous. The theory to follow remains valid

provided $\alpha(s)$ has only finitely many discontinuities. For simplicity I shall suppose $\alpha(s)$ to be continuous.

The important characteristics of (12) are that all tasks are necessary and that $\min\{\cdot\}$ is linear homogeneous in $T(s)$. The former is conventional, and the latter makes industry analysis reasonably straightforward, as one can ignore the number and size distribution of firms.

The fixed coefficient assumption is relatively innocuous, and allows explicit solutions for the optimal T_x . If factor proportions (actually "task proportions" herein) are optimally chosen, then task-output ratios are constant (to the first order) in any case. Accordingly, for those questions whose solutions depend only on the first order differentials of q , the solutions do not depend on fixed-coefficients. The essential issue of how information quality (P) affects output falls into this category.

The problem faced by the firm is how to obtain the maximum quantity of output from a given stock of workers, N_x . Formally, the problem may be written

$$\begin{aligned} \max_{T_a(s), T_b(s)} \quad & \min_s \left\{ \frac{T_a(s) + T_b(s)}{\alpha(s)} \right\} \\ \text{S.T.} \quad & \int_0^1 \frac{T_a(s)}{t_a(s)} ds = N_a \\ & \int_0^1 \frac{T_b(s)}{t_b(s)} ds = N_b \\ & T_a(s), T_b(s) \geq 0 . \end{aligned} \tag{13}$$

$T_x(s)/t_x(s) \equiv N_x(s)$ (not to be confused with N_x) is the number of type x workers, measured in efficiency units, assigned to task s .⁶ The problem may be posed in terms of optimal choice of jobs, J_x . That is,

$$S_x = \{s \in [0,1] \mid T_x > 0\}$$

and

$$\varphi_x(s) = \frac{T_x(s)}{N_x t_x(s)}.$$

It follows that (13) may be restated as

$$\begin{aligned} \max_{J_x} \min_s \left\{ \frac{N_a t_a(s) \varphi_a(s) + N_b t_b(s) \varphi_b(s)}{\alpha(s)} \right\} \\ \text{S.T. } \int_{S_a} \varphi_a(s) ds = 1 \\ \int_{S_b} \varphi_b(s) ds = 1 \\ \varphi_a(s), \varphi_b(s) \geq 0 \end{aligned} \quad (14)$$

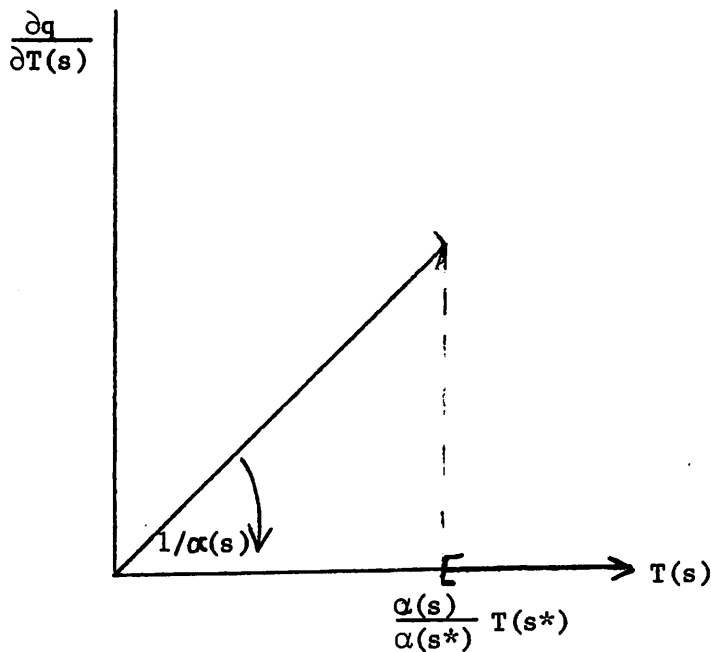
For the present discussion it is more convenient to stick to the problem as posed in (13). (13) is a relatively straightforward control problem. However, although more space is required, it is revealing to solve the problem in a series of steps.

First of all, it is obvious that $T(s)/\alpha(s) = (T_a(s) + T_b(s))/\alpha(s)$ is constant for all s . If $T(s)/\alpha(s) > T(s')/\alpha(s')$ for some $s \neq s'$, q could be increased by augmenting $T(s')$ and reducing $T(s)$. It is useful to think of this as equalizing the marginal product of the $T(s)$. That is, if $\min_{s' \neq s} \{T(s')/\alpha(s')\}$ is attained at s^* , the marginal product of $T(s)$ is the discontinuous function

$$\frac{\partial q}{\partial T(s)} = \begin{cases} 1/\alpha(s) & \text{for } T(s) < \alpha(s)T(s^*)/\alpha(s^*) \\ 0 & \text{for } T(s) \geq \alpha(s)T(s^*)/\alpha(s^*) \end{cases}$$

This is depicted in Figure 4.

Figure 4



Next, the sets of tasks performed by each group are nonintersecting, except for possibly at one point. Suppose this were not true. Then there would exist tasks s , s' and s'' , where $s'' > s' > s$, such that $T_a(s) > 0$, $T_b(s') > 0$ and $T_a(s'') > 0$. Now consider the process of transferring dN_b workers to task s'' and dN_a workers to task s' , holding $T(s'')$ fixed. It is easily shown that the resulting change in $T(s')$ is given by

$$dT(s') = \frac{dN_b}{t_a(s')} \left\{ \frac{1}{R(s'')} - \frac{1}{R(s')} \right\}$$

which is strictly positive as $\partial R / \partial s < 0$. On the other hand, transferring dN_b workers to task s and dN_a workers to task s' , so as to hold task s constant, yields a strictly negative change in $T(s')$. Accordingly, it follows that optimal assignment must involve nonintersecting jobs, except possibly at the

boundary. Since all tasks are necessary, it follows that there is a number, $\rho \in (0,1)$, such that⁷

$$\begin{aligned} T_a(s) &> 0 & s \in [0, \rho] \\ &= 0 & \text{otherwise} \end{aligned}$$

and

$$\begin{aligned} T_b(s) &> 0 & s \in (\rho, 1] \\ &= 0 & \text{otherwise,} \end{aligned}$$

which is to say $S_a = [0, \rho]$ and $S_b = (\rho, 1]$. Also, $T(s) = T_x(s)$ for $s \in S_x$.

Given (14), the full solution is readily obtained. Consider the choice of $T_a(s)$ and $T_b(s)$ conditional on fixed ρ . For $s \in [0, \rho]$, $T(s)/\alpha(s) = T_a(s)/\alpha(s)$ is a constant, say $q_a(\rho)$:

$$q_a(\rho) = \frac{T_a(s)}{\alpha(s)}, \quad s \in [0, \rho]$$

Then

$$\int_0^\rho \frac{T_a(s)}{\alpha(s)} ds = N_a$$

translates into

$$q_a(\rho) = N_a \left[\int_0^\rho \frac{\alpha(s)}{t_a(s)} ds \right]^{-1} \quad (15)$$

whence

$$T_a(s) = N_a \alpha(s) \left[\int_0^\rho \frac{\alpha(s)}{t_a(s)} ds \right]^{-1} . \quad (16)$$

Similarly

$$q_b(\rho) = N_b \left[\int_\rho^1 \frac{\alpha(s)}{t_b(s)} ds \right]^{-1} \quad (17)$$

and

$$T_b(s) = N_b \alpha(s) \left[\int_\rho^1 \frac{\alpha(s)}{t_b(s)} ds \right]^{-1} . \quad (18)$$

(16) and (18) are the optimal choices of T_a and T_b given ρ . It follows that

$$q(\rho) = \min\{q_a(\rho), q_b(\rho)\}.$$

Noting that

$$\lim_{\rho \rightarrow 0} q_a(\rho) = \lim_{\rho \rightarrow 1} q_b(\rho) = \infty,$$

and that

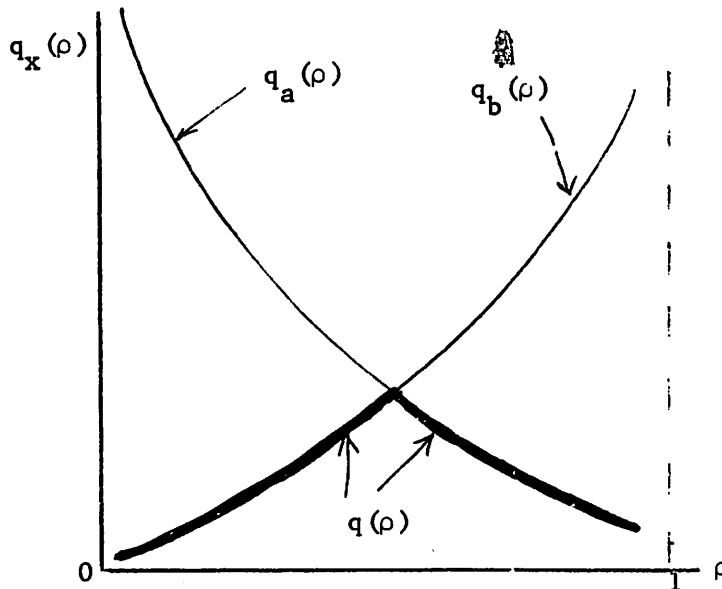
$$\text{sign}\left\{\frac{dq_a}{d\rho}\right\} = - \text{sign}\left\{\frac{dq_b}{d\rho}\right\} < 0$$

(i.e., the same number of workers assigned to a larger set of tasks must produce less) it follows that $\rho \in (0,1)$ and that ρ is the solution to

$$q_a - q_b = 0, \tag{19}$$

The optimal choice of ρ is depicted in Figure 5

Figure 5



To reiterate, the firm obtains the maximal output attainable from a given stock of workers by assigning type a workers to tasks $s \in [0, \rho]$, and type b workers to tasks $s \in (\rho, 1]$. Given ρ , the fraction of time workers spend at each task is

$$\phi_x(s) = \frac{T_x(s)}{N_x t_x(s)} = \frac{\alpha(s)}{t_x(s)} \left[\int_{S_x} \frac{\alpha(s)}{t_x(s)} ds \right]^{-1}. \quad (20)$$

The maximal output attainable from this division of tasks and intra-group time allocation is a function $q(N_a, N_b; P, \delta)$ which corresponds to the usual notion of a production function with inputs N_a and N_b . The next section examines the properties of this production function.

V. The Production Function

This section examines the properties of the production function resulting from the optimal assignment of workers derived in the previous section. Before doing so it is useful to state the precise nature of the conceptual experiment being performed. The production function is the solution to the following question. What is the maximum output attainable from any given N_a and N_b ? This maximum output depends parametrically on p , δ , $r(s)$ and $\alpha(s)$.

In particular, examining $\partial q / \partial p$ corresponds to asking the question: how would output change if the firm hired the same number (not necessarily the same people) of a's and b's in two different situations defined by distinct levels of P ?

The production function, $q(N_a, N_b; P, \delta)$, is defined implicitly by the equations

$$q - N_a \left[\int_0^{\rho} \frac{\alpha(s)}{t_a(s)} ds \right]^{-1} = 0 \quad (21)$$

$$\text{and } N_a \left[\int_0^{\rho} \frac{\alpha(s)}{t_a(s)} ds \right]^{-1} - N_b \left[\int_{\rho}^1 \frac{\alpha(s)}{t_b(s)} ds \right]^{-1} = 0. \quad (22)$$

Note that (21) and (22) are not a simultaneous system. Rather, they are recursive. (22) (just (19) expanded) defines the value of ρ that maximizes output given N_a and N_b . (21) (just $q - q_a(\rho) = 0$) then determines the output that follows from optimal choice of ρ .⁸

The recursive nature of the definition of $q(\cdot; \cdot)$ implies that for any v

$$\frac{dq}{dv} = \frac{\partial q}{\partial v} + \frac{\partial q}{\partial \rho} \frac{d\rho}{dv} \quad (23)$$

where $d\rho/dv$ is derived from differentiation of (22) alone. Below I shall focus on $v = N_x$ and P . The following are easily obtained⁹

$$\frac{d\rho}{dN_a} = \frac{N_b}{N_b + R(\rho)N_a} \cdot \frac{t_a(\rho)}{q\alpha(\rho)} > 0, \quad (24)$$

$$\frac{d\rho}{dN_b} = \frac{-R(\rho)N_a}{N_b + R(\rho)N_a} \cdot \frac{t_b(\rho)}{q\alpha(\rho)} \quad (25)$$

$$= -\frac{N_a}{N_b} \frac{d\rho}{dN_a} < 0 \quad (25')$$

$$\text{and } \frac{d\rho}{dP} = \frac{\frac{1}{P} \left\{ \int_0^{\rho} \varphi_a(s) \eta_p^a ds - \int_{\rho}^1 \varphi_b(s) \eta_p^b ds \right\}}{\frac{q\alpha(\rho)}{N_a t_a(\rho)} \left\{ \frac{N_b + R(\rho)N_a}{N_b} \right\}} \quad (26)$$

$$= \frac{N_a}{P} \frac{d\rho}{dN_a} \{ \cdot \} \quad \text{using (24)}$$

$$= \frac{-N_b}{P} \frac{d\rho}{dN_b} \{ \cdot \} \quad \text{using (25')}$$

(24) and (25) simply state that an increase in one factor makes it efficient to extend the use of that factor to tasks at which it is relatively less efficient. (26) is generally ambiguous, the sign being determined by the expression in braces.

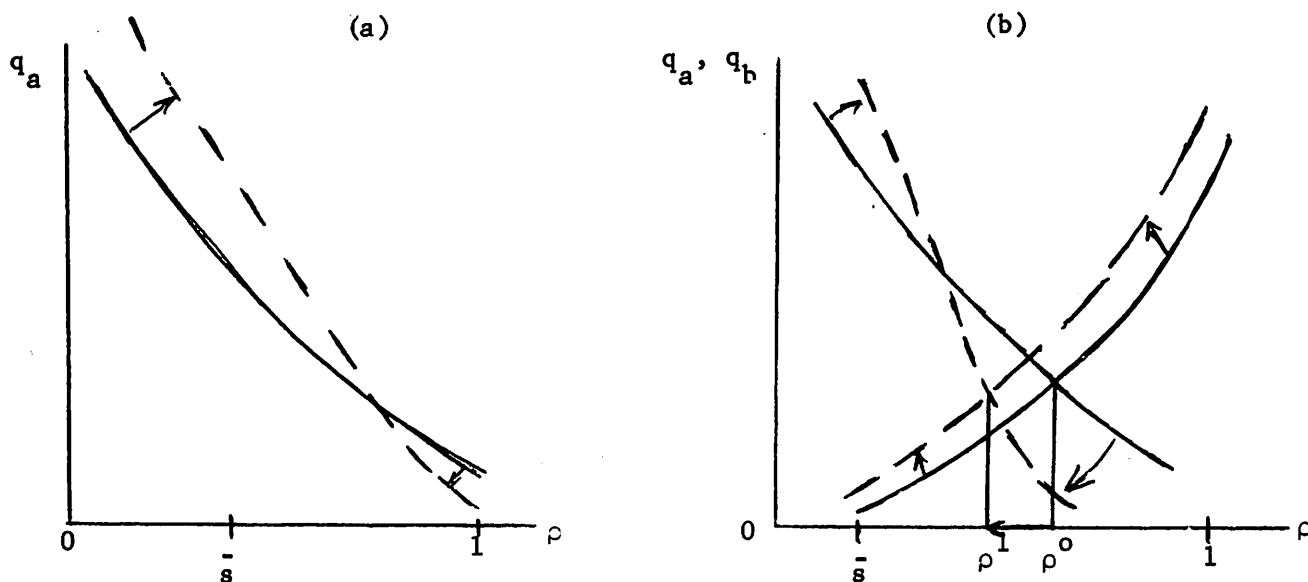
The intuition is relatively straightforward. ρ is determined by the intersection of q_a and q_b , both of which are shifted by P (see Figure 6). Consider q_a . For any given N_a an increase in P implies an increase in the number of a's that are correctly labeled, and a corresponding decline in incorrect labelings. If $R(\rho) > 1$, a's are assigned only to tasks at which A's are relatively more efficient. As a consequence, $t_a(s)$ rises for every task s to which a's are assigned, and

$$q_a = N_a \int_0^{\rho} \frac{\alpha(s)}{t_a(s)} ds$$

rises. Suppose however that ρ is such that $R(\rho) < 1$. In that case, A's are not more productive at each and every task to which a's are assigned, and increasing the number of A's reduces, leaves unchanged, or increases $t_a(s)$

as $R(s) \gtrless 1$. For given ρ , a's must spend more of their time at tasks for which $R(s) < 1$, and less at those for which $R(s) > 1$. If ρ is only slightly in excess of \bar{s} (recall $R(\bar{s}) = r(\bar{s}) = 1$), then greater time intensity in the less efficient region involves a small absolute amount of time, and q_a will still rise. Otherwise, $q_a(\rho)$ must fall. This is depicted in Figure 6(a).

Figure 6



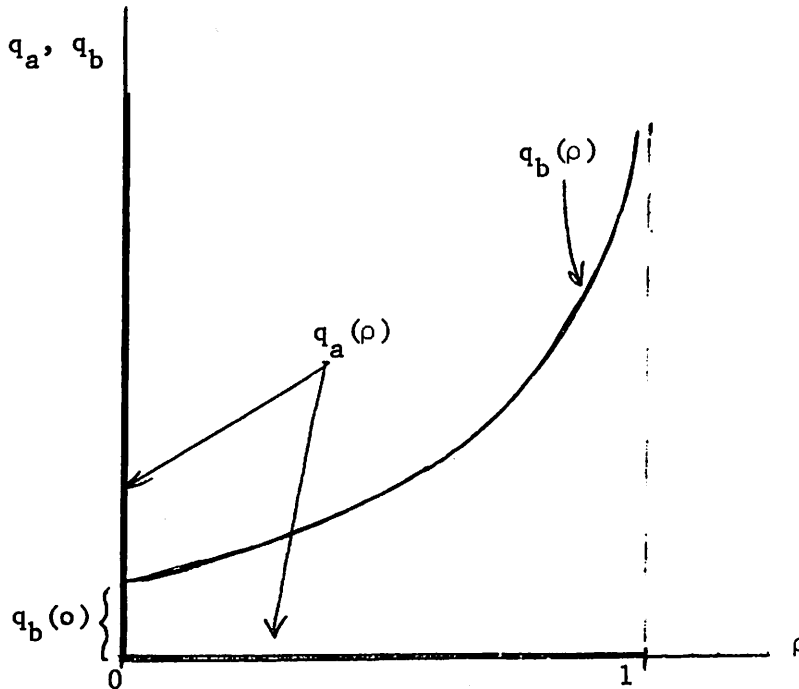
The opposite holds for $q_b(\rho)$. That is, it may fall if ρ is sufficiently below \bar{s} . In the end, either both, or one of, q_a and q_b (both evaluated at ρ) must rise with P . Referring again to (26), the first (second) term in braces captures the change in q_a (q_b). Part (b) of Figure 6 illustrates the case for which $\rho > \bar{s}$ and $d\rho/dP < 0$. The point to remember is that changes in P yield changes in t_x that induce optimal alterations in q_x over the entire range of tasks to which N_x is assigned.

Given (24) and (25), some basic properties of $q(\cdot; \cdot)$ are readily obtained.

First of all, provided $t_A(s)$ and $t_B(s) > 0$ for all s , there are no necessary inputs. For example, as $N_a \rightarrow 0$, $q_a(\rho)$ becomes the right angled function depicted in Figure 7. $N_a = 0$ implies $q = q_b(0)$. Similarly $N_b = 0$ implies $q = q_a(1)$.

Figure 7

$$N_a = 0$$



Next, q is linear homogeneous in N_a and N_b . From (22), ρ depends only on the ratio of N_a to N_b . Thus, for N_a/N_b fixed, q is linear in N_a . Linear homogeneity implies that q , being a maximum function, is weakly concave, and hence concave contoured, in N_a and N_b . More generally, if (12) is not linear homogeneous q is concave contoured with the same scale characteristics as (12).

Third, the marginal products of q are given by

$$\frac{\partial q}{\partial N_a} = \frac{q}{N_a} \cdot \frac{R(\rho)N_a}{N_b + R(\rho)N_a} \quad (27)$$

and

$$\frac{\partial q}{\partial N_b} = \frac{q}{N_b} \cdot \frac{N_b}{N_b + R(\rho)N_a} \quad (28)$$

It follows that the output elasticities are given by

$$\eta_{N_a}^q = \frac{R(\rho)N_a}{N_b + R(\rho)N_a} \quad (29)$$

and

$$\eta_{N_b}^q = \frac{N_b}{N_b + R(\rho)N_a} \quad (30)$$

Further, the marginal rate of substitution of N_a for N_b is

$$\frac{\partial q / \partial N_a}{\partial q / \partial N_b} = R(\rho), \quad (31)$$

and the elasticity of substitution is given by

$$\sigma = \frac{\frac{q\alpha(\rho)}{N_a t_a(\rho)} \left[\frac{N_b + R(\rho)N_a}{N_b} \right]}{- \left. \frac{\partial \log R(s)}{\partial s} \right|_{s=\rho}} \quad (32)$$

σ is smaller, the more quickly R declines with s : essentially the more different are A's and B's. In particular, σ falls as P rises.

We are now in a position to ask how P affects the function q .

The intuition is straightforward. For given numbers N_x , a change in P has two effects. One is that a larger fraction of N_x is correctly labeled. This augments output. Second, to hold N_x fixed as P changes, the number of individuals of types A and B will generally change. This may diminish output. For example, consider how much output 100 a's and 0 b's can produce. Assume $r < 1$ so that B's are more productive at every task. Suppose the labeling process has $P = 1/2$. Then when the firm hires 100 a's it gets 50 A's and 50 B's. Now suppose we start all over and label with $P = 1$. This time the firm gets 100 A's when it hires 100 a's. $r < 1$ implies that less output is produced under the second regime.

At this point one may object that the firm would not hire 100 a's if $P = 1$. While this may or may not be true, it is entirely irrelevant at this point (but see Section VI) because the question at hand is how does P affect

the derived technology available to the firm, when that technology is expressed in terms of the observable factors of production N_x .

Straightforward differentiation of (21), coupled with (26), yields

$$\eta_P^q = \frac{R(\rho)N_a \int_0^\rho \varphi_a(s) \eta_P^a ds + N_b \int_\rho^1 \varphi_b(s) \eta_P^b ds}{N_b + R(\rho)N_a} \quad (33)$$

which, using (29) and (30) becomes ($\eta_{N_x}^q$ evaluated at ρ):

$$\eta_P^q = \eta_{N_a}^q \int_0^\rho \varphi_a(s) \eta_P^a ds + \eta_{N_b}^q \int_\rho^1 \varphi_b(s) \eta_P^b ds. \quad (34)$$

Making use of (6) yields

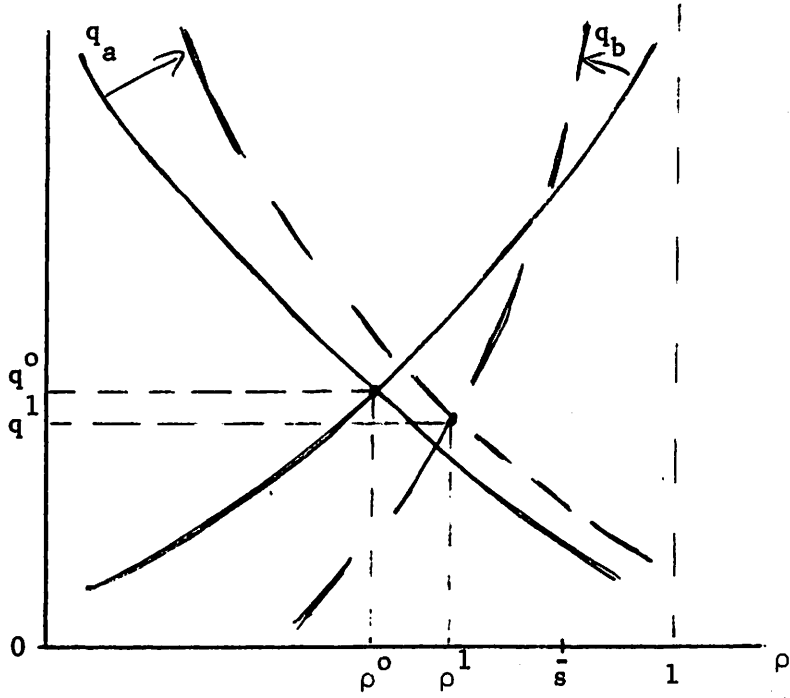
$$\eta_P^q = \frac{1-\delta}{P(a)} \eta_{N_a}^q \left[1 - \int_0^\rho \varphi_a(s) \frac{t_B(s)}{t_A(s)} ds \right] + \frac{\delta}{P(b)} \eta_{N_b}^q \left[1 - \int_\rho^1 \varphi_b(s) \frac{t_A(s)}{t_B(s)} ds \right] \quad (35)$$

(35) > 0 is necessary and sufficient for increased information quality to raise output for given N_a and N_b . As explained above, it is possible that as P rises, one of q_a or q_b may fall. ρ adjusts so as to equalize q_a and q_b . Whether q rises or falls depends (supposing for example that $dq_a/dP > 0$ and $dq_b/dP < 0$) on how fast q_a declines as the set of tasks performed by N_a expands. One case where $dP > 0$ implies $dq > 0$ is illustrated in part (a) of Figure 8. A set of isoquants consistent with this is provided in part (b). Figure 8 corresponds to a case where N_b/N_a is relatively large. Raising P generally pulls the unit isoquant in at the centre and may push out at the "edges". That is, for any P there is some N_b/N_a ray for which $R = 1$.¹⁰ As P rises, this ray generally rotates (it declines in Figure 8(b)). To the northwest of the new $R = 1$ ray ($\rho < \bar{s}$), the marginal rate of substitution rises, falling to the southeast of the ray. Accordingly, increases in P generate a lower substitution elasticity along the new unit isoquant.

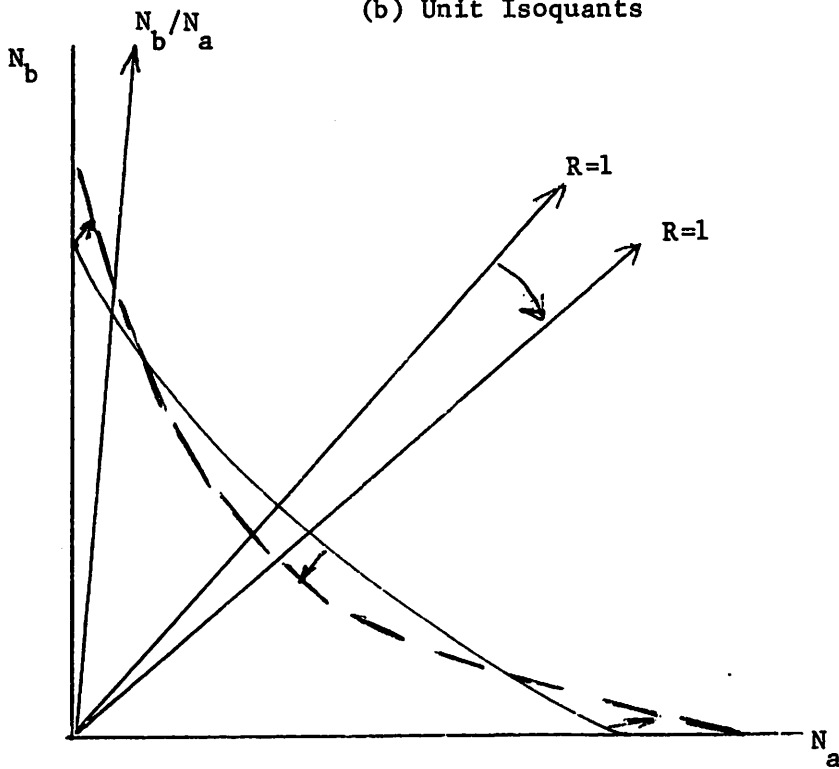
Thus, the impact of increased P on the labeling process increases output for some factor combinations and reduces it for others.

Figure 8

(a)



(b) Unit Isoquants



It is easy to find conditions that are sufficient for $\eta_P^q > 0$ at a particular (N_a, N_b) . For example, define ρ_a and ρ_b by $\partial q_a(\rho_a)/\partial P = \partial q_b(\rho_b)/\partial P = 0$. Then $\rho_b \leq \rho \leq \rho_a$ is sufficient for $\eta_P^q > 0$.

A natural question is: what kind of restrictions yield $\eta_P^q > 0$ for all pairs (N_a, N_b) . That is, when can increases in P never reduce q ?

Returning to (35), it is immediately evident that:

$$\eta_P^q > 0 \forall (N_a, N_b) \Leftrightarrow \text{both (i) } \int_0^\rho \varphi_a(s) \frac{t_B(s)}{t_a(s)} ds < 1 \quad (36a)$$

$$\text{and (ii) } \int_\rho^1 \varphi_b(s) \frac{t_A(s)}{t_b(s)} ds < 1 \quad (36b)$$

hold $\forall (N_a, N_b)$.

Take (36a) for example. If N_a/N_b is very large, ρ approaches 1. (36a) will be satisfied essentially if the fraction of time that workers with label a must spend performing tasks at which they are relatively inefficient is not too large.

Thus, for any $\alpha(s)$, $r(s)$, P and δ , (36a) and (36b) provide a complete characterization of whether increased information quality yields greater output for all possible factor choices.

When thinking about what kinds of restrictions imply that (36a) and (36b) are satisfied, one frequently thinks in terms of restrictions on $\alpha(s)$ (operating through $\varphi_x(s)$) and/or $r(s)$. An interesting question is whether $\alpha(s)$ and $r(s)$ can be separated. That is, are there restrictions one can place on $r(s)$ that yield $\eta_P^q > 0$ for all (N_a, N_b) regardless of $\alpha(s)$? Are there restrictions on $\alpha(s)$ that eliminate dependence on $r(s)$? The answer to both questions is no. Define

$$\Psi_a(s) = \frac{\varphi_a(s)}{k_0 + r(s)} \left[\int_0^1 \frac{\varphi_a(s)}{k_0 + r(s)} ds \right]^{-1} \geq 0 \quad (37a)$$

and

$$\Psi_b(s) = \frac{\varphi_b(s)}{1 + k_1 r} \left[\int_0^1 \frac{\varphi_b(s)}{1 + k_1 r} ds \right]^{-1} \geq 0 \quad (37b)$$

where

$$k_0 = \frac{(1-P)(1-\delta)}{P\delta} \quad \text{and} \quad k_1 = \frac{1-P}{P} \cdot \frac{\delta}{1-\delta}.$$

Clearly

$$\int_0^1 \Psi_a(s) ds = \int_0^1 \Psi_b(s) ds = 1 \quad (38)$$

Using (4), it is easy to show that the necessary and sufficient conditions, (36a) and (36b) (which must hold $\forall N_x$), are equivalent to

$$\int_0^1 \Psi_a(s) \cdot r(s) ds > 1 \quad (37a)$$

and

$$\int_0^1 \Psi_b(s) r(s) ds < 1. \quad (37b)$$

Suppose $\alpha(s)$ is arbitrary. Then $\Psi_a(s)$ and $\Psi_b(s)$ are arbitrary (except that they satisfy (38)). It follows that both (37a) and (37b) hold regardless of $\Psi_x(s)$ only if $r(s)$ is both less than and greater than 1 for all s , which cannot hold.

Suppose $r(s)$ is arbitrary. Is there an $\alpha(s)$, and subsequent $\Psi_a(s)$ and $\Psi_b(s)$ such that (37a) and (37b) hold regardless of $r(s)$? Obviously not. The choice $r > 1$ always violates (37b) irrespective of $\alpha(s)$, while $r < 1$ always violates (37a).

In sum, improvements in information quality change the maximal output obtainable from given N_a and N_b through altering the number of workers in each group that are correctly labeled, and changing the true skills available to the firm. This may or may not raise output, depending upon factor proportions, task-output requirements $\alpha(s)$, and the structure of skills $r(s)$.

For given $\alpha(s)$ and $r(s)$, conditions are provided which determine whether increased information quality results in increased output for every factor combination. These conditions are shown to necessarily depend on both $\alpha(s)$ and $r(s)$. That is, whether information raises output depends on both technology and relative skill endowments.

VI. Alternative Characterizations

Above it was shown that raising P may alter the firm's productive opportunities in a manner detrimental to the firm. It is readily demonstrated that the same result extends to the cost function. That is, for given factor prices, increasing P may raise the cost of producing a given output. Consequently, allowing for optimal factor choice does not remove the possibility that increases in information quality may prove harmful to the firm.

To that end, consider the problem of minimizing the cost of producing one unit of output, subject to fixed factor prices w_a and w_b .¹¹ The firm's problem is

$$\min_{N_a, N_b} w_a N_a + w_b N_b$$

$$\text{S.T. } q = 1 .$$

The appropriate Lagrangian is

$$\mathcal{L} = w_a N_a + w_b N_b + \lambda [1 - q(N_a, N_b)]$$

yielding first order (here both necessary and sufficient conditions)

$$w_a = \lambda \frac{\partial q}{\partial N_a}$$

(39)

$$w_b = \lambda \frac{\partial q}{\partial N_b}$$

$$\text{and } 1 - q = 0 .$$

Let the optimal choices be N_a^* , N_b^* and λ^* . λ^* is of course marginal cost.

The minimum cost is

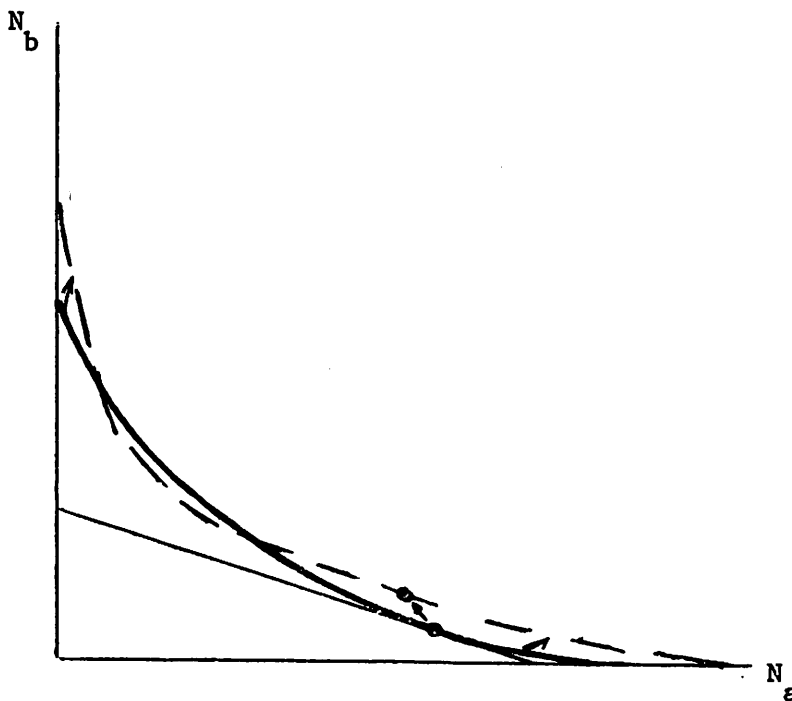
$$C^* = w_a N_a^* + w_b N_b^* + \lambda^*[1 - q(N_a^*, N_b^*)].$$

It follows that

$$\begin{aligned} \frac{dC^*}{dP} &= w_a \frac{dN_a^*}{dP} + w_b \frac{dN_b^*}{dP} + \frac{d\lambda^*}{dP} [1 - q(N_a^*, N_b^*)] - \lambda^* \left[\frac{\partial q}{\partial N_a} \frac{dN_a^*}{dP} + \frac{\partial q}{\partial N_b} \frac{dN_b^*}{dP} + \frac{\partial q}{\partial P} \right] \\ &= - \lambda^* \frac{\partial q}{\partial P} \text{ using (39),} \\ &\neq 0 \text{ as } \frac{\partial q}{\partial P} > 0. \end{aligned}$$

This states that while changes in P may induce adjustments in factor proportions, the alterations cannot be so as to reduce costs if $\partial q / \partial P < 0$ at (N_a^*, N_b^*) . This is illustrated in Figure 9.

Figure 9



We may view all of this from another angle. Let N_A and N_B be the true numbers of A's and B's corresponding to N_x . Clearly

$$N_A = P(A|a)N_a + P(A|b)N_b$$

and
$$N_B = P(B|a)N_a + P(B|b)N_b.$$

Alternatively,

$$N_a = \frac{P(B|b)N_A - P(A|b)N_B}{P(A|a)P(B|b) - P(B|a)P(A|b)}$$

and

$$N_b = \frac{P(A|a)N_B - P(B|a)N_A}{P(A|a)P(B|b) - P(B|a)P(A|b)}.$$

Define the maximum output attainable from given numbers N_A and N_B as $\hat{q}(N_A, N_B)$.

Clearly

$$\hat{q}(N_A, N_B) = q \left[\frac{P(B|b)N_A - P(A|b)N_B}{P(A|a)P(B|b) - P(B|a)P(A|b)}, \frac{P(A|a)N_B - P(B|a)N_A}{P(A|a)P(B|b) - P(B|a)P(A|b)} \right]$$

where $q(.,.)$ is as defined above. For given N_A and N_B , if we let $\delta = N_A / (N_A + N_B)$ and choose units so that $N_A + N_B = 1$, the proof in the next section can be modified to show that $\partial \hat{q} / \partial P > 0$. That is, given the underlying skills, better information has one effect, better allocation to tasks.

However, consider the cost minimization problem cast in terms of N_A and N_B . Let

$$W_A = \frac{W_a P(B|b) - W_b P(B|a)}{P(A|a)P(B|b) - P(B|a)P(A|b)}$$

and
$$W_B = \frac{W_b P(A|a) - W_a P(A|b)}{P(A|a)P(B|b) - P(B|a)P(A|b)}.$$

Both W_A and W_B depend on P .

Then the problem is

$$\begin{aligned} \min \quad & W_A N_A + W_B N_B \\ & N_A, N_B \\ \text{S.T.} \quad & q = 1. \end{aligned}$$

Proceeding as above, it is immediate that

$$\frac{dc^*}{dP} = N_A^* \frac{\partial W_A}{\partial P} + N_B^* \frac{\partial W_B}{\partial P} - \lambda^* \frac{\partial \hat{q}}{\partial P}.$$

This analysis gives the same answer as before. Here $\partial c^*/\partial P > 0$ may occur because of the effects of P on the cost of purchasing N_A (N_B) when one must acquire it through purchasing the imperfectly labelled N_x (which, after all, are all one can observe).

VII. The Competitive Market

The previous sections considered the behavior of an individual firm, first obtaining as much output as possible from fixed N_a and N_b , then choosing N_a and N_b optimally. It was shown that improved information quality may have detrimental effects on the firm. This occurred because the firm could consider hiring any N_a and N_b it wished. As P changed, the number of A's and B's in any (N_a, N_b) choice was not constant.

The market does not have this freedom. There are δ A's and $(1-\delta)$ B's. Thus, for example, it is not generally possible to vary P holding N_a and N_b fixed.

Thus we are led to ask how information affects industry output in equilibrium.

For simplicity, assume that the price of output is fixed at unity. As all firms are identical and have linear homogeneous technologies, they may be treated as a single large competitive firm. The industry is completely characterized by

$$N_a = P(a) \quad (40)$$

$$N_b = P(b) \quad (41)$$

$$\frac{N_b}{N_a} = \frac{\int_0^1 \frac{\alpha(s)}{t_b(s)} ds}{\int_0^\rho \frac{\alpha(s)}{t_a(s)} ds} \quad (42)$$

$$q = N_a \left[\int_0^\rho \frac{\alpha(s)}{t_a(s)} ds \right]^{-1} \quad (43)$$

$$\frac{w_a}{w_b} = R(\rho) \quad (44)$$

$$q = w_a N_a + w_b N_b \quad (45)$$

(40) and (41) are the "factor supply" equations. Even assuming that the number of A's and B's is fixed at total of 1, N_x is not fixed, although $N_a + N_b = 1$ must hold. Note that (40) and (41) completely determine N_x (independently of w_x) because the labor force is inelastically supplied.

Given N_a and N_b , (42) determines the optimal value of ρ in the usual way. Given ρ and N_x , output is given by (43). (44) is the cost minimization condition from the previous section. This determines relative wages. The zero profit condition, (45), determines the level of wages. Obviously (44) and (45) could have been replaced by equality of wages and value of marginal product.

The system (40) to (45) is completely recursive. Indeed, for purposes of examining dq/dP , (44) and (45) may be ignored. It follows that

$$\frac{dq}{dP} = \frac{\partial q}{\partial P} + \frac{\partial q}{\partial N_a} \frac{dN_a}{dP} + \frac{\partial q}{\partial \rho} \frac{d\rho}{dP} \quad (46)$$

where

$$\frac{d\rho}{dP} = \frac{\partial \rho}{\partial P} + \frac{\partial \rho}{\partial N_a} \frac{dN_a}{dP} + \frac{\partial \rho}{\partial N_t} \frac{dN_b}{dP} \quad (47)$$

and

$$\frac{dN_a}{dP} = (2\delta - 1) = - \frac{dN_b}{dP} \quad (48)$$

Differentiating (43) and simplifying yields

$$\frac{dq}{dP} = \frac{q^2}{N_a} \int_0^\rho \frac{\alpha(s)}{t_a^2(s)} \frac{\partial t_a(s)}{\partial \rho} ds + \frac{(2\delta-1)q}{N_a} - \frac{q^2 \alpha(\rho)}{N_a t_a(\rho)} \frac{d\rho}{dP}. \quad (49)$$

Using the results of Section V,

$$\frac{d\rho}{dP} = \frac{q/P}{\frac{q^2 \alpha(\rho)}{N_a t_a(\rho)} (1 + R(\rho) \frac{N_a}{N_b})} \left\{ \int_0^\rho \varphi_a(s) \eta_P^a ds - \int_\rho^1 \varphi_b(s) \eta_P^b ds + (2\delta-1) \left[\frac{1}{N_a} + \frac{1}{N_b} \right] \right\}. \quad (50)$$

Substituting (50) into (49), collecting terms, and using (29) and (30) yields

$$\eta_P^q = \eta_{N_a}^q \left[\int_0^\rho \varphi_a(s) \eta_P^a ds + \frac{P(2\delta-1)}{N_a} \right] + \eta_{N_b}^q \left[\int_\rho^1 \varphi_b(s) \eta_P^b ds + \frac{P(1-2\delta)}{N_b} \right].$$

Using (40) and (41), and the definition of $\eta_P^{t_x}$ yields

$$\eta_P^q = \eta_{N_a}^q \left[1 - \frac{1-\delta}{P(a)} \int_0^\rho \varphi_a(s) \frac{t_B(s)}{t_a(s)} ds \right] + \eta_{N_b}^q \left[1 - \frac{\delta}{P(b)} \int_\rho^1 \varphi_b(s) \frac{t_A(s)}{t_b(s)} ds \right]$$

which is similar to (35). Noting that $t_B(s)/t_a(s)$ is rising in s , and that $t_A(s)/t_b(s)$ is falling, it follows that

$$\int_0^\rho \varphi_a(s) \frac{t_B(s)}{t_a(s)} ds < \frac{t_B(\rho)}{t_a(\rho)}$$

and that

$$\int_\rho^1 \varphi_b(s) \frac{t_A(s)}{t_b(s)} ds < \frac{t_A(\rho)}{t_b(\rho)}.$$

Then

$$\eta_P^q > \eta_{N_a}^q \left[1 - \frac{1-\delta}{P(a)} \frac{t_B(\rho)}{t_a(\rho)} \right] + \eta_{N_b}^q \left[1 - \frac{\delta}{P(b)} \frac{t_A(\rho)}{t_b(\rho)} \right]. \quad (51)$$

Noting that $\eta_{N_a}^q / \eta_{N_b}^q = R(\rho)P(a)/P(b)$, it is easily shown (just expanding and collecting terms) that the right-hand side of (51) is zero. Therefore $\eta_P^q > 0$ in industry equilibrium.

Thus, while it is possible for increases in P to reduce the output attainable from given N_a and N_b , the facts that N_a and N_b change in response to P , and that the labor market must clear, jointly imply that equilibrium output must always rise with P .

A second question of interest is how information quality affects wages. Using (44), (45) and (29)

$$w_a = q \cdot \frac{R(\rho)}{N_b + R(\rho)N_a}.$$

As q , ρ , N_a and N_b are all endogenous, dw_a/dP is a messy and apparently uninformative expression which I shall not burden the reader with. The result is that increases in P can raise both, one, or neither wage rate. To see why this is true, consider

$$q = w_a N_a + w_b N_b.$$

Then

$$\frac{dq}{dP} = P(a) \frac{dw_a}{dP} + P(b) \frac{dw_b}{dP} + (2\delta - 1)[w_a - w_b].$$

All that may be shown in general is that $dq/dP > 0$. Suppose, for example, that $\delta > \frac{1}{2}$ and $R(\rho) > 1$. It follows that a's are paid more than b's, and that $dP > 0$ increases the number of a's and reduces the number of b's.

Accordingly it is possible for increased output to be exhausted by greater aggregate payments to a's and smaller aggregate payments to b's. The former may be achieved via smaller per person payments to a larger number of individuals. In elasticities (k_x is the share of q received by x's) ¹²

$$\eta_P^q = k_a \left[\eta_P^{N_a} + \eta_P^{w_a} \right] + k_b \left[\eta_P^{N_b} + \eta_P^{w_b} \right].$$

In the example just described $k_a \eta_P^{N_a} + k_b \eta_P^{N_b} > 0$ and $k_b \eta_P^{w_b} < 0$. For $\delta < \frac{1}{2}$ and $R(\rho) < 1$, only the latter inequality is reversed. Thus whenever the more abundant factor is also relatively more productive at task ρ , it is possible for $dP > 0$ to reduce both, or one, or neither wage.

If the more abundant factor is less productive at task ρ , at least one wage must rise. Nonetheless it is always true that expected (prior to labelling) wages must rise since $q = w_a N_a + w_b N_b = w_a P(a) + w_b P(b) = \text{expected wages}$.

Another interesting question is the effect of P on specialization. Given that a 's are assigned to tasks on $[0, \rho]$ and b 's assigned to $(\rho, 1]$, a sensible definition of specialization is in terms of $\varphi_a(s)$ and $\varphi_b(s)$. That is, if a change in P results in less time being allocated to jobs near ρ , then the worker may be said to be more specialized. More precisely (analogous to the standard definition of risk): a is said to be more specialized under $\varphi_a^1(s)$ than $\varphi_a^0(s)$ if

$$\int_0^\tau \varphi_a^1(s) ds \geq \int_0^\tau \varphi_a^0(s) ds \quad \forall \tau \in [0, 1].$$

b is more specialized under $\varphi_b^1(s)$ than $\varphi_b^0(s)$ if

$$\int_0^\tau \varphi_b^1(s) ds \leq \int_0^\tau \varphi_b^0(s) ds \quad \forall \tau \in [0, 1].$$

I shall focus on φ_a . φ_b is entirely analogous. Assume $d\rho/dP = 0$ for the moment. From (11)

$$\begin{aligned} \varphi_a(s) &= \frac{T_a(s)}{N_a t_a(s)} \\ &= \frac{q}{N_a} \frac{\alpha(s)}{t_a(s)} \text{ since } T_a(s) = \alpha(s)q \quad \forall s. \end{aligned}$$

Then, for any s , a little rearrangement yields

$$\frac{d\varphi_a(s)}{dP} = \frac{\varphi_a(s)}{P} \left\{ P \frac{N_a}{q} \frac{d}{dP} (q/N_a) - \eta_p^a \right\} \quad (52)$$

The first term in braces does not depend on s , and $\varphi_a(s)/P \geq 0$. Accordingly, the sign pattern of $d\varphi_a(s)/dP$ in s is determined solely by η_p^a :

$$\text{sign} \left\{ \frac{d\varphi_a(s)}{dP} \right\} = \text{sign} \{ \text{constant} - \eta_p^a \}.$$

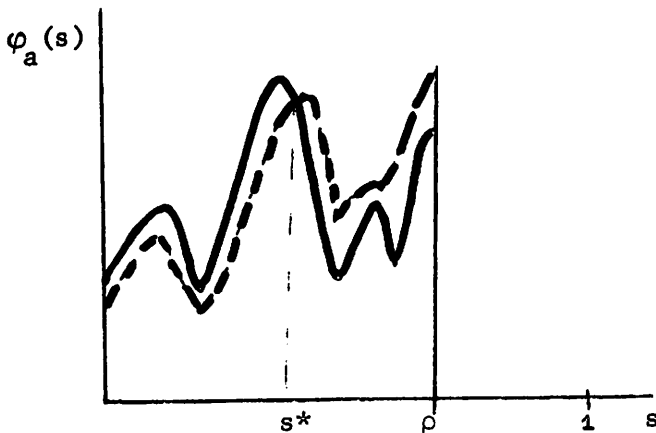
Now $\eta_p^a = \eta_p^{P(A/a)} (1 - t_B/t_a)$ is strictly declining in s . Therefore $d\varphi_a(s)/dP$ is strictly rising in s . Since $\varphi_a(s)$ must integrate to 1 in any case, it follows that there is some s^* for which

$$\text{sign}\left\{\frac{d\varphi_a(s)}{dP}\right\} = \text{sign}\{s-s^*\}.$$

Equivalently, increases in information quality reduce specialization.

See Figure 10 (drawn for $d\rho/dP = 0$).

Figure 10[†]



[†] corresponds to higher P

More generally ($d\rho/dP \neq 0$), the fact that $\int_0^\rho \varphi_a ds = 1$ yields

$$\varphi_a(\rho) \frac{d\rho}{dP} + \int_0^\rho \frac{d\varphi_a}{dP} ds = 0$$

If $d\rho/dP < 0$, $\partial\varphi_a/\partial P$ must go from negative to positive as s rises, and the conclusion follows unaltered. For $d\rho/dP > 0$, it is possible for $d\varphi_a/dP$ to be negative on $[0, \rho]$, but the fact that $\varphi_a(\rho + d\rho) > 0$, where it was zero before, yields the conclusion trivially.

The intuitive explanation is quite simple. Again assume that ρ is fixed. $T_a(s)/\alpha(s)$ must always be equalized across s . Now $T_a(s)/\alpha(s) = [N_a \varphi_a(s)] \cdot t_a(s)/\alpha(s)$; that is, the number of a 's assigned to task s , times output per man. The number of a 's assigned to task s depends on P only through $\varphi_a(s)$, while output per man depends on P only

through $t_a(s)$. Thus if $q = T_a(s)/\alpha(s)$ were to remain constant as P varies, $\varphi_a(s)$ and $t_a(s)$ would have to vary in opposite directions. As $t_a(s)$ responds more positively to P the smaller is s , $\varphi_a(s)$ must follow the opposite pattern. q is not constant as P changes, but this does not affect the impact of P on the pattern of specialization because $\int \varphi_a(s) ds = 1$ must hold.

VIII Welfare Considerations

So far I have paid little attention to the choice of information quality P . Let us suppose that individuals may purchase a test, which yields the label x , at a cost $C(P)$. If individuals are risk neutral, they will choose P to maximize expected wages net of testing costs. $P > \frac{1}{2}$ will be chosen so long as $C(\frac{1}{2})$ is not too large. Firms' expectations (in particular that P is the same for all) will be realized. The resulting equilibrium will be ex ante Pareto efficient.

If workers are risk averse, the same efficient choice of P will be chosen. Firms may, and will find it to their advantage to, offer contracts that guarantee workers a fixed income of $E(q) = P(a)w_a + P(b)w_b$. That is, any equilibrium contract offer must yield zero expected profit and hence must have expectation equal to $E(q)$. Among the class of contracts with given expectation, risk averse individuals will always prefer the one with no variance. Given this contract, workers again solve the problem

$$\max_P E(q) - C(P)$$

If such contracts are assumed not to be feasible (for example, it may pay firms to renege, ex post), then it is clear that a tax on those who receive the larger income ex post, that is used to finance a subsidy to those who receive the lower income, will (i) induce the Pareto efficient choice; and (ii) yield zero net revenue, so long as the tax is of the amount $\max(w_a, w_b) - q$, and the subsidy is of the amount $q - \min(w_a, w_b)$. Here is a case where a progressive tax on labor earnings induces efficient choice.

IX Observables?

The analysis so far has been relatively abstract. However, it suggests some interesting observable hypotheses, mainly with regard to job content. Although not considered in the theoretical section, it is useful to think of firms as heterogeneous. That is, there may be different industries. They must, however, all use the same set of tasks.¹³

Regardless of $\alpha(s)$, the marginal rate of substitution turns out to be $R(\rho)$. Accordingly, if wage rates are equalized across industries, ρ must be the same across industries. Differences in $\varphi_x(s)$ are determined by $\alpha(s)$. This suggests why jobs tend to be named according to tasks rather than time-intensity. For example, "economist" means someone who does some mix of economic theorizing and empirical work. Inter-firm differences in jobs for economists tend to be in terms of the intensity with which tasks are pursued, $\varphi_x(s)$.

Along the same lines, recall that $\varphi_x(s)$ is proportional to $\alpha(s)/t_x(s)$. This implies that one spends more time working at tasks which are important for production (high $\alpha(s)$), but less time (relative to $\alpha(s)$) at those tasks for which one feels more qualified (higher $t_x(s)$). This of course only applies within a homogeneous group, and only to those tasks to which one is optimally assigned. As an example, even given the importance we attach to research, we always feel that we don't get enough time at it. The reason is that teaching is necessary and that it is necessary to make up for relative inefficiency at teaching by applying more time to it.

I have argued elsewhere that schooling is a source of person-specific information. One of the results above is that increased information quality may have some unusual implications for wage rates, although average wages must always rise. This suggests that one may find considerable differences

in ex post returns to schooling by occupation (occupation defined by "job"). Also, if one does not account for the effects of schooling on the number of individuals in each occupation, measured average returns to schooling may have little content. For example, consider a simple standardized comparison of the wages of high school graduates to those of college graduates. Let γ_i^{HS} (γ_i^C) denote the fraction of high-school (college) graduates in job i , and let w_i^{HS} (w_i^C) be the corresponding wage. The theory tells us only that

$$\sum_i \gamma_i^C w_i^C / \sum_i \gamma_i^{HS} w_i^{HS} > 1. \quad (53)$$

Both "standardized" indices

$$\sum_i \gamma_i^C w_i^C / \sum_i \gamma_i^C w_i^{HS}$$

and $\sum_i \gamma_i^{HS} w_i^C / \sum_i \gamma_i^{HS} w_i^{HS}$

have little content, essentially because the γ 's are functions of the level of schooling. Indeed, we know that it is possible for $w_i^C < w_i^{HS}$ for all i (although (53) must hold), in which case the standardized indices, by capturing only the wage effects, give an unusual picture of the returns to schooling.¹⁴ Realistically, one feels that w_i^C is less than w_i^{HS} for some i , and greater for others (electricians and accountants?). Many students appear to choose college as a bet (which the progressive tax system encourages) that college will help them get a "good job". That is, a larger fraction of college graduates than high school graduates end up accountants.

A final point is that as schooling (hence information quality) rises, measured substitution elasticities ought to fall. Better information makes factors more different. This is consistent with the stylized fact that elasticities of substitution decline with skill (usually measured by duration of training).

X. Extensions

Extending the model to allow for different qualities of information across people is straightforward, provided the "representative firm" type assumptions of Section II are maintained. Suppose, for example, that there are two tests, corresponding to P' and P ($P' > P$), and that given the fraction of the population taking each quality of test is known. Using a "' to denote values corresponding to P' , it is immediate that

$$t_A \lesseqgtr t'_a \lesseqgtr t_a \lesseqgtr t_b \lesseqgtr t'_b \lesseqgtr t_B \quad \text{as } r \lesseqgtr 1.$$

It follows that there are values ρ_0 , ρ_1 and ρ_2 such that

$$s'_a = [0, \rho_0]$$

$$s_a = [\rho_0, \rho_1]$$

$$s_b = [\rho_1, \rho_2]$$

and $s'_b = [\rho_2, 1]$.

The rest of the analysis follows similarly.

The essential weakness of the analysis is that several relevant sources of uncertainty have been assumed away. While interesting, taking these into account greatly complicates the problem. Present indications are that the conclusions of the analysis herein are unlikely to be altered in any significant way.

Footnotes

¹The problem here is not unlike that addressed by the early marginal productivity theorists. That is, the existing literature treated output as if it could be thought of as "ultimately" produced by a single factor. The more general analysis recognizes the essential interdependence between factors.

²The basic non-stochastic setup of skills and technology is due to Rosen.

³In Figure 1 it is assumed that individuals are re-ordered at each stage.

⁴ $r(s)$ is only assumed differentiable for convenience. What is required is that $s < s' \Rightarrow r(s) > r(s')$.

⁵Throughout, the notation η_Y^X denotes the elasticity of X with respect to Y: $\eta_Y^X = \frac{Y}{X} \frac{dX}{dY}$.

⁶Actually, all workers with a given label do all the tasks to which their group is assigned. $N_x(s)$ essentially measures man-hours on task s.

⁷It is assumed that task ρ is not shared. This solution is arbitrarily close to one for which ρ is shared if the latter is optimal.

⁸Occasionally it is useful to use (21) in the form $q - q_b(\rho) = 0$, which is obviously equivalent to (21).

⁹Note that (25') implies that

$$\eta_{N_a}^{\rho} = -\eta_{N_b}^{\rho} > 0 .$$

¹⁰This N_b/N_a is defined by

$$\frac{N_b}{N_a} = \frac{\int_0^{\bar{s}} \frac{\alpha(s)}{t_a(s)} ds}{\int_{\bar{s}}^1 \frac{\alpha(s)}{t_b(s)} ds} .$$

¹¹Here it is assumed that w_a and w_b do not depend on P . Clearly, one can make the cost of production vary with P in any fashion one wants by letting w_x change appropriately with P . The point about optimal factor choice is therefore best made for fixed w_x . Equilibrium response of w_x to P is examined below.

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$$\eta_P^{P(a)} = \frac{P(2\delta-1)}{P(a)}, \quad \eta_P^{P(b)} = \frac{P(1-2\delta)}{P(b)} .$$

¹³Of course we can let $\alpha(s)$ be as small as we like for various tasks in different industries. Accordingly, while all tasks must be used, virtually no time need be allocated to some tasks, which is a reasonable approximation to not having to do the task at all.

¹⁴ $w_i^C \neq w_i^{HS}$ is possible so long as the information structure is symmetric-- in particular, one cannot claim to have not have gone to college when one has. This symmetry is crucial for the optimality properties of informational equilibrium. See Mortensen and Burdett (1979).

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