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# Maximal Growth in a Dynamic Model of International Trade

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MAXIMAL GROWTH IN A DYNAMIC MODEL

OF INTERNATIONAL TRADE

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ABSTRACT

An integration of international trade theory and the theory of growth at the maximum possible rate is presented. Results show that trade permits increased growth for two reasons. First, trade allows a country to achieve instantaneously a superior input mix. Second, and less obvious, trade may allow a permanently higher steady-state rate of growth. In a two-country model of the world economy it is shown that both countries may achieve a common faster rate of expansion through free trade. While one country may temporarily gain by imposing a tariff, both countries will suffer a lower growth rate as a consequence.

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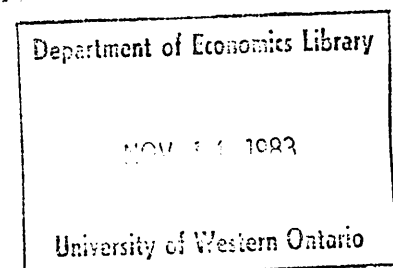
1.   Introduction

The potential for increased growth through trade is discussed in a simple, two-good, neoclassical model. A basis for trade is assumed to exist for either (or both) of two reasons: countries initially have different endowments; and/or countries have differences in technology. These differences in turn imply two sources of gains from trade (increased growth through trade). First, trade permits a country to achieve instantaneously an input bundle which increases the following period's output. Second, trade may allow a permanently higher steady-state rate of growth. While the former gain is simply the growth-maximizing equivalent of the usual static consumption-maximizing gain from trade, the latter is less obvious and carries important implications for commercial policy.

The techniques adopted by Dorfman, Samuelson, and Solow (1958) for their analysis of maximal growth in the autarkic economy are used. Both Findlay (1973) and Steedman (1979), using different techniques, have discussed the small country's potential for growth through trade. In Section 2 the conditions for the maximum sustainable rate of growth for a small open economy are developed and it is demonstrated that the country may instantaneously satisfy these. If a country is sufficiently large, its terms of trade are endogenous. This is discussed in a two-country-world model of maximal growth in Section 3. It is shown that both countries can achieve a common higher rate of growth through trade, provided their production technologies differ. In Section 4 it is shown that tariffs may be used to exploit monopoly power in trade in the short term but that they ultimately result in lower growth for both countries.

An implication of this last result for commercial policy is that a tariff to exploit monopoly power dissipates the second source of gains from trade. A

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tariff therefore cannot permanently raise income in all future periods and must result in both countries ultimately having lower incomes, although the present value of the tariff-imposing country's real income may, of course, be increased. A second implication also follows from the fact that the tariff lowers the common steady-state rate of growth: the loss per period in world real income due to a fixed tariff rate increases with time at an exponential rate. The gap between actual and potential GNP increases in absolute and in percentage terms.

2. Maximal Growth for a Small Country[1]

A country produces two goods, which may either be consumed or used as inputs in the following period's production. Both goods are produced using both inputs according to neoclassical production functions exhibiting constant returns to scale. Let the outputs of goods 1 and 2 in period (t-1) be $Q_1(t-1)$ and $Q_2(t-1)$ respectively. It is assumed that none of the output is consumed.[2] The small country may trade this output at the prevailing terms of trade in order to attain the optimal combination of inputs, $z = (S_1(t-1), S_2(t-1))$, where

$$pS_1(t-1) + S_2(t-1) = pQ_1(t-1) + Q_2(t-1) \qquad (1)$$

The technology of transforming these inputs into the next period's output may be represented as

$$Q_2(t) = F[Q_1(t); S_1(t-1), S_2(t-1)] = F(t) \qquad (2)$$

This is shown in Figure 1 as MN. The trade opportunity locus is TO which meets MN in tangency at point b, the efficient point of production.

Trade gives the country the opportunity to have different input and output mixes, each point on TO being a possible set of inputs each of which would generate a different production possibility frontier. The envelope of

these frontiers is GH, which dominates the envelope of production possibilities of an autarkic economy EF, reflecting the gains from trade. A maximal growth path is one which steps from the envelope of trade efficiency in one period to that of the next.

With initial stocks, $S_1(0)$ and $S_2(0)$, and the terminal stock of good 1, $Q_1(T)$, optimal production paths will maximize

$$Q_2(T) = F[Q_1(T); S_1(T-1), S_2(T-1)]$$

subject to efficient production in each period

$$F(t) = Q_2(t) \qquad t=1,2,\ldots,T-1$$

and trade balance

$$pS_1(t) + S_2(t) = pQ_1(t) + Q_2(t) \qquad t=1,2,\ldots,t-1$$

The Lagrangian is

$$L = F(T) + \sum_{t=1}^{T-1} \lambda(t)\{F(t) - Q_2(t)\}$$

$$+ \sum_{t=1}^{T-1} \mu(t)\{pS_1(t) + S_2(t) - pQ_1(t) - Q_2(t)\}$$

Differentiating with respect to $S_1(t)$, $S_2(t)$, $Q_1(t)$, and $Q_2(t)$, setting the derivatives to zero, and solving, the intertemporal efficiency conditions are obtained:

$$-F_1(t) = \frac{F_2(t+1)}{F_3(t+1)} = p, \qquad t=1,2,\ldots,T-1 \qquad (3)$$

The F_i are the partial derivatives of F. This may be interpreted as the marginal rate of substitution between inputs in one period is equal to the marginal rate of transformation between outputs in the previous period. This is the same condition as Dorfman, Samuelson, and Solow (1958) found for the

Figure 1
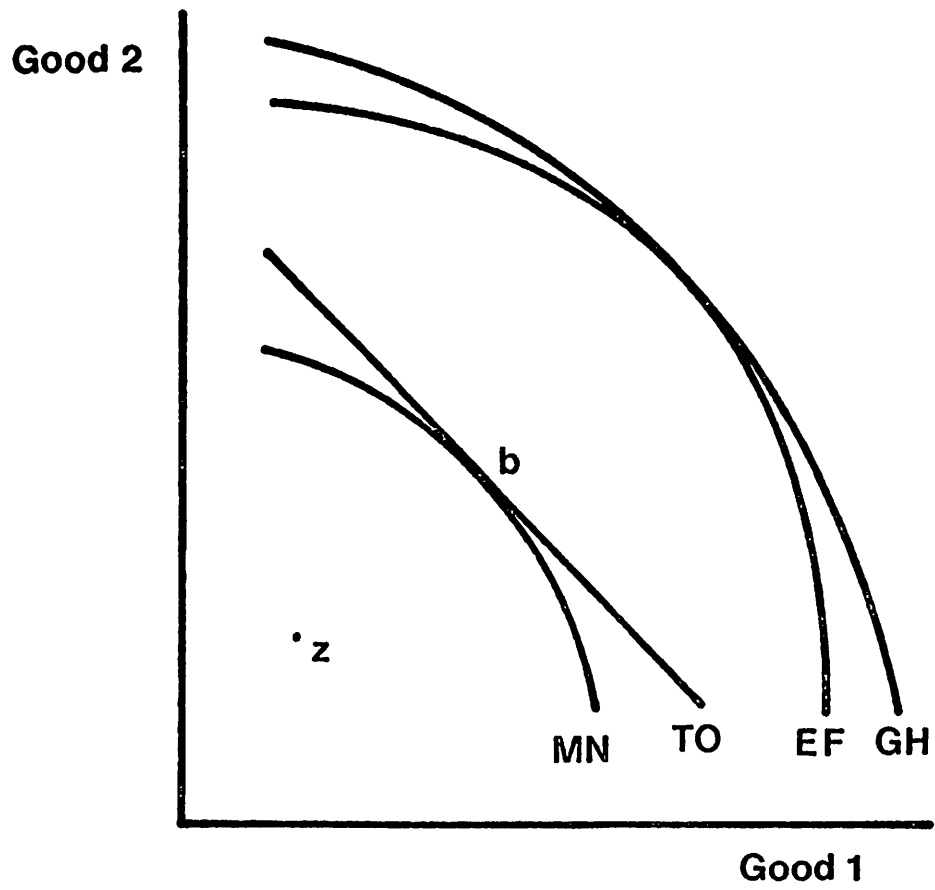
Figure 1

autarkic economy, with the additional proviso that the domestic rate of transformation equal the foreign rate of transformation, the terms of trade.

This is shown in Figure 2. The economy produces output combination b such that the marginal rate of transformation is, in absolute terms, equal to the international terms of trade. Trade occurs along TO. For intertemporal efficiency the marginal rate of substitution between inputs (the absolute slope of the isoquant) must also equal the price ratio. Thus efficient output d has an input-isoquant KL_d, which is tangential to TO at c.

It may be asked whether, of all the paths which maximize growth, there is a balanced growth path along which outputs of goods 1 and 2 grow at the same rate. Let this common growth rate be g, and let the ratio of production of good 2 to production of good 1 be b. Then

$$Q_1(t) = Q_1(0)g^t \ ,$$

$$Q_2(t) = bQ_1(t) = bQ_1(0)g^t$$

(4)

Inputs will also grow at rate g and, with unchanging terms of trade, the input ratio will remain constant, though it may differ from the output ratio. Let the ratio of input of good 2 to input of good 1 be d. Then
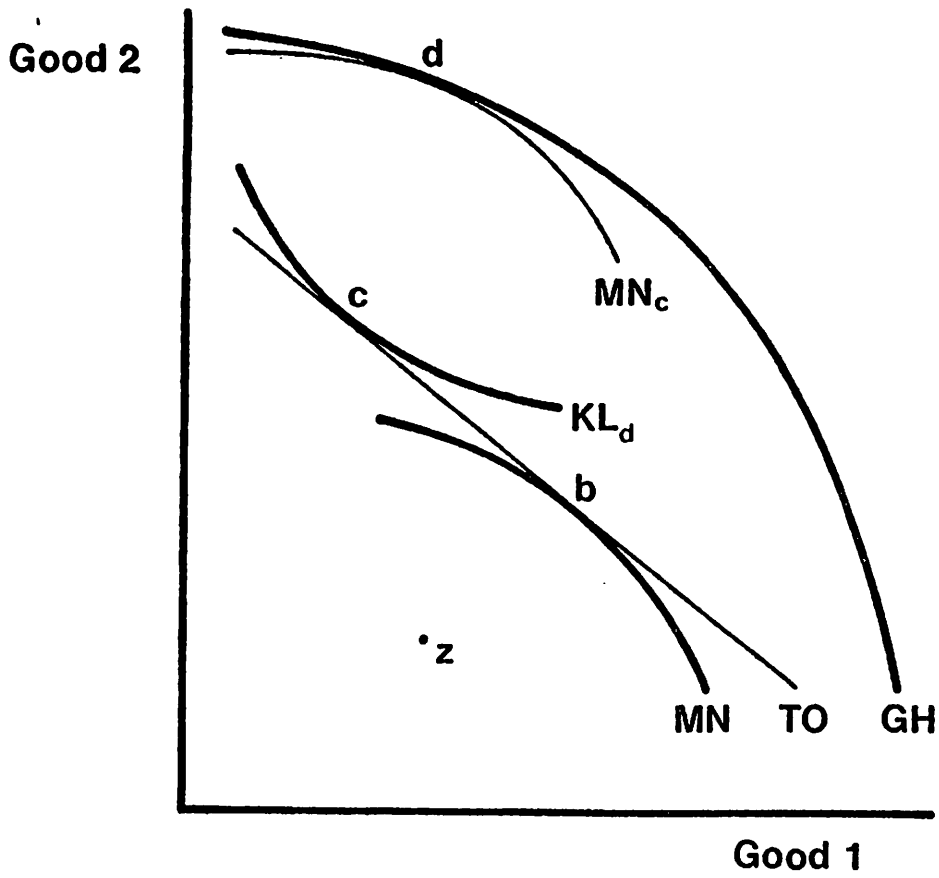
$$S_1(t) = S_1(0)g^t$$

$$S_2(t) = dS_1(t) = dS_1(0)g^t$$

(5)

Substituting these relations into the transformation function, equation (2), taking into account the homogeneity properties of F:

$$b \ a \ g = F[ag; \ 1,d]$$

(6)

where $a = (p+d)/(p+b)$. When $a = 1$, then $b = d$ and there is no trade.

The maximum rate of steady growth may be calculated by taking derivatives of (6) with respect to b and d,

Figure 2

$$\frac{\partial g}{\partial b} = \{\frac{1}{p+b} - \frac{1}{b-F_1}\}$$

$$\frac{\partial g}{\partial d} = \frac{1}{p+d}\{\frac{F_3}{b-F_1}(p+b) - g\} ,$$

and equating them to zero. The second-order conditions for a maximum are fulfilled. Thus the balanced-growth path that has the fastest attainable rate of growth has parameters that satisfy:

$$\hat{g} = F_3[\hat{a}\hat{g}; 1,\hat{d}] \tag{7}$$

$$p = -F_1[\hat{a}\hat{g}; 1,\hat{d}] \tag{8}$$

Rewriting equation (6) in terms of optimal values of parameters,

$$\hat{b}\,\hat{a}\,\hat{g} = F[\hat{a}\hat{g}; 1,\hat{d}]$$

By Euler's theorem, this may be rewritten

$$\hat{b}\,\hat{a}\,\hat{g} = \hat{a}\hat{g}F_1 + F_2 + \hat{d}F_3 \tag{9}$$

Substituting in equations (7) and (8), this reduces to

$$-F_1[\hat{a}\hat{g}; 1,\hat{d}] = \frac{F_2[\hat{a}\hat{g}; 1,d]}{F_3[\hat{a}\hat{g}; 1,d]} \tag{10}$$

which is the intertemporal efficiency condition. Therefore maximal steady-growth for a small economy is efficient.

Calling upon properties of the autarkic economy, it is proposed that, for very long planning periods, the path of balanced growth is, indeed, the only efficient growth path for the trading economy. Convergence to the required input ratio may be achieved instantaneously through trade as may the final output selection. Thus there is no additional mileage to be covered as a result of "driving down the turnpike."

Combining equations (7), (8), and (10), the relationship between the maximal rate of steady growth and the terms of trade may be determined.

$$\hat{g}(p) = \frac{F_2(p)}{p} \tag{11}$$

where $F_2'(p) > 0$, $F_2''(p) > 0$. The mapping is shown in Figure 3. $\hat{g}$ is minimized at $\bar{p}$ where $\partial \hat{g}/\partial p = 0$. Multiplying through equation (10) by $(p + \hat{b})$, taking derivatives with respect to p, and setting $\partial \hat{g}/\partial p$ to zero:

$$\frac{\partial \hat{b}}{\partial p}(p+\hat{d})\hat{g} + \hat{g}(1+\frac{\partial \hat{d}}{\partial p})(\hat{b}-F_1) = (1+\frac{\partial \hat{b}}{\partial p})(F_2+F_3\hat{d}) + \frac{\partial \hat{d}}{\partial p}F_3(p+\hat{b}).$$

For maximal steady-growth, equations (7), (8), and (9) must hold and the expression reduces to

$$\hat{b} = \hat{d}$$

Thus the potential growth of an economy is least when the ratio of inputs is the same as that of outputs--when the country does not trade.

### 3. Maximal Growth for the World Economy

Consider a world composed of two countries, each country producing the same two goods with different technologies. The transformation function for each country is assumed to be derived from neoclassical production functions where inputs are smoothly substitutable for each other and there are constant returns to scale and diminishing returns to a factor:

$$Q_2(t) = F[Q_1(t); S_1(t-1), S_2(t-1)] = F(t) \tag{12}$$

$$Q_2^*(t) = F^*[Q_1^*(t); S_1^*(t-1), S_2^*(t-1)] = F^*(t) \tag{13}$$

where asterisks denote the foreign country.

The maximum growth of the world economy, as a whole, may be found by allocating world inputs such that the sum of outputs of the two countries is maximized. Suppose that initial stocks are: for the home country, $Q_1(0)$ and $Q_2(0)$; for the foreign country, $Q_1^*(0)$ and $Q_2^*(0)$. These may be reallocated between countries as inputs for the first period's production:

Figure 3

$$\hat{g} = \frac{F_2}{p}$$
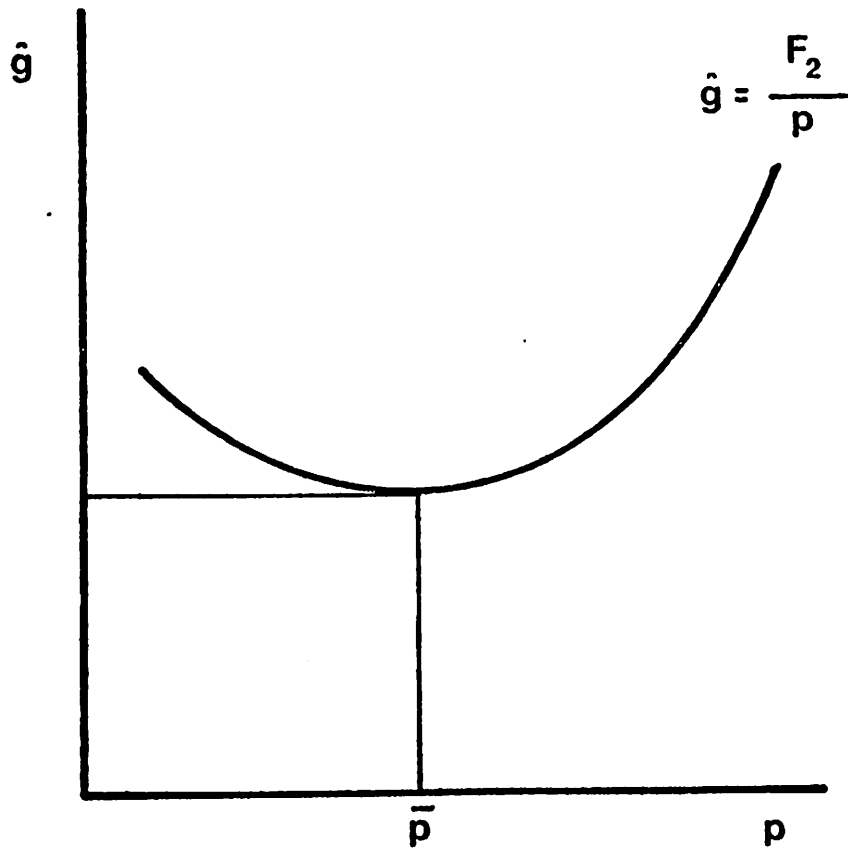
$$S_1(0) + S_1^*(0) = Q_1(0) + Q_1^*(0)$$

$$S_2(0) + S_2^*(0) = Q_1(0) + Q_1^*(0).$$

The aggregate output of good 1 in the terminal period is chosen to be $(Q_1(T) + Q_1^*(T))$. Intertemporally efficient production for the world will maximize the output of good 2,

$$Q_2(T) + Q_2^*(T) = F(T) + F^*(T),$$

subject to efficient production in each period,

$$F(t) = Q_2(t), \quad t=1,2,\ldots,T\text{-}1 ,$$
$$F^*(t) = Q_2^*(t), \quad t=1,2,\ldots,T\text{-}1,$$

and the budget constraints of each country,

$$P_t(Q_1(t) - S_1(t)) = -(Q_2(t) - S_2(t)), \quad t=1,2,\ldots,T\text{-}1,$$

$$P_t(Q_1^*(t) - S_1^*(t)) = -(Q_2^*(t) - S_2^*(t)), \quad t=1,2,\ldots,T\text{-}1,$$

and the world trade-balance condition,

$$Q_1(t) + Q_1^*(t) = S_1(t) + S_1^*(t), \quad t=1,2,\ldots,T\text{-}1.$$

First-order conditions yield:

$$F_1(t) = F_2(t) \tag{14}$$

$$-F_1(t) = F_2(t+1)/F_3(t+1), \tag{15}$$

$$-F_1^*(t) = F_2^*(t+1)/F_3^*(t+1), \tag{16}$$

The marginal rates of transformation must be equal across countries, efficient production only occurring on the world's transformation curve. That is, there must be free trade. In addition, maximum world output requires each country to follow an intertemporally efficient growth path.

Substituting relations (4) and (5) into the transformation function, yields

$$ba_t g = F[a_t g; \; 1,d] \tag{17}$$

the balanced-growth efficiency condition, where

$$a_t = \frac{p_t + d}{p_t + b} \, ,$$

the terms of trade being endogenous. The conditions for maximal balanced growth have been shown to be:

$$\hat{g} = F_3[\hat{a}_t \hat{g}; \; 1, \hat{d}] \tag{18}$$

$$p_t = -F_1[\hat{a}_t \hat{g}; \; 1, \hat{d}] \tag{19}$$

It is known that such growth is intertemporally efficient. The maximal growth rate is a function of the terms of trade

$$\hat{g}(p_t) = \frac{F_2(p_t)}{p_t} \tag{20}$$

Similarly, for the foreign country, the maximal rate of balanced growth is a function of the terms of trade,

$$\hat{g}^*(p_t) = \frac{F_2^*(p_t)}{p_t} \, . \tag{21}$$

For steady-state world equilibrium, both countries must be growing at the same rate and trading at the same terms of trade:

$$\hat{g}(\hat{p}) = \hat{g}^*(\hat{p}) \tag{22}$$

The equilibrium is determined by the intersection of the two curves in Figure 4. The relative size of one country to the other is uniquely determined by substitution into the balanced trade condition, the resulting ratio of initial stocks being:

Figure 4
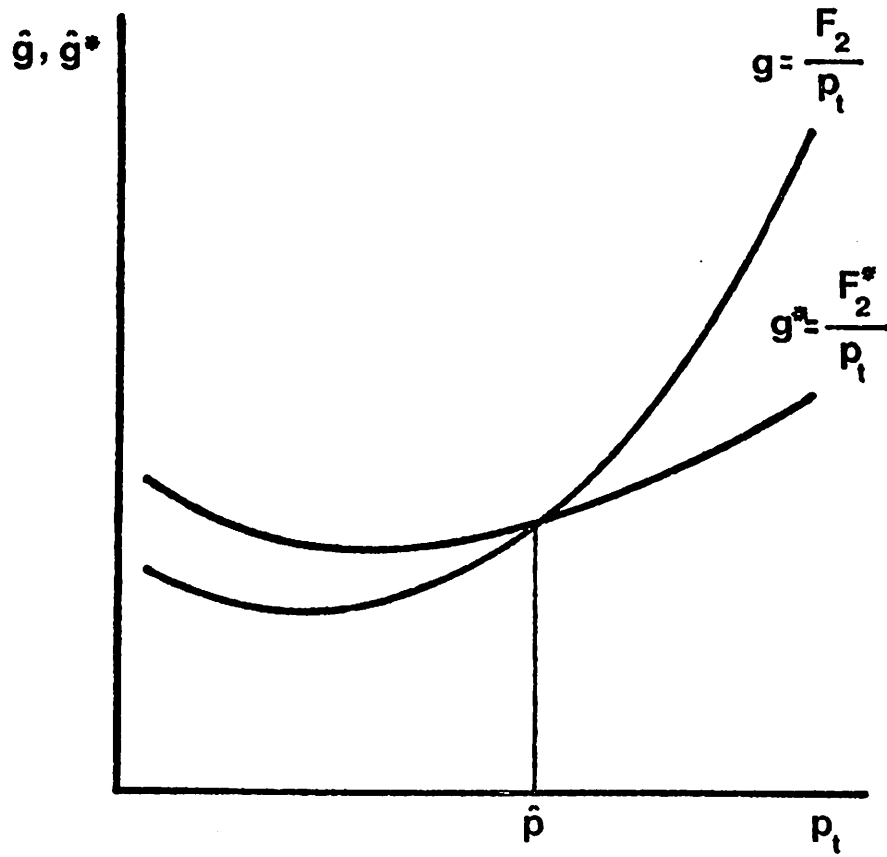
$$\frac{S_1(0)}{S_1^*(0)} = \frac{1}{\hat{p}} \frac{(\hat{b}^*\hat{a}^* - \hat{d}^*)}{(\hat{a} - 1)} \tag{23}$$

Were the production technologies of the two countries identical then there would be no long-run benefits from trade as each country could achieve the same rate of growth in an autarkic steady-state. Even were trade not advantageous in the long run, there may be short-run gains. A country's initial endowment of goods may not be in the required proportions to grow immediately at the maximal rate. Trade with another country may enable both to more closely approximate the correct ratio, moving them "closer" to the turnpike.[3]

4. Commercial Policy

It has been shown that free trade permits the maximum growth of the world economy. This does not rule out the potential for one country to attempt more rapid growth at the expense of its trading partner.

Let the home country impose an arbitrarily small tariff on imports. This distorts the domestic price ratio in each period, making the domestic price of the imported good relatively higher than the free trade price. The foreign country is passive, setting no tariffs itself, and it seeks to maximize its own growth by efficient production at international prices.

The conditions for intertemporal efficiency must still hold:

$$-F_1(t) = F_2(t+1)/F_3(t+1), \tag{15}$$

$$-F_1^*(t) = F_2^*(t+1)/F_3^*(t+1) \tag{16}$$

The marginal rates of transformation between outputs are not the same for both countries, the tariff having driven a wedge between them. Equation (14) therefore does not hold, and the world as a whole is not on a maximal growth path.

The home country has higher income as the result of the tariff, but this is achieved at the cost of lower growth. Free trade was shown to be necessary for the maximum common steady-state rate of growth, thus any steady-state with a trade distortion must necessarily entail a lower rate of growth. This is illustrated in Figure 5. Path A represents the steady-state intertemporal income path for the home country with free trade, while path B characterizes the income path under a tariff. If the objective of the country is to maximize income at some terminal point T, it is clear that for $T < \tilde{t}$ a tariff is superior to free trade but inferior for any longer time horizon. Even were income in each period discounted at some appropriate rate of importance, it is the case that, for a time horizon of sufficient length, free trade would be superior to restricted trade.

5.   Conclusions

The analysis of maximal balanced growth for a closed economy was extended to permit international trade. It was shown that, as the trade opportunity locus dominates the transformation curve, a country can achieve more rapid growth through trade. In addition, initial adjustments in factor endowments can be made in order that the economy instantly achieves the maximal growth path.

Differences in relative factor endowment and differences in technology as bases for trade were examined in a simple two-good, two-country dynamic model. It has been shown that differences in relative factor endowment can generate trade in the short run, as countries attempt to modify their stocks of inputs into production. Having achieved the optimal input ratio a country will not gain from further trade unless its technology differs from the rest of the world, in which case it may sustain a higher rate of growth through trade than it could in autarky.

Figure 5
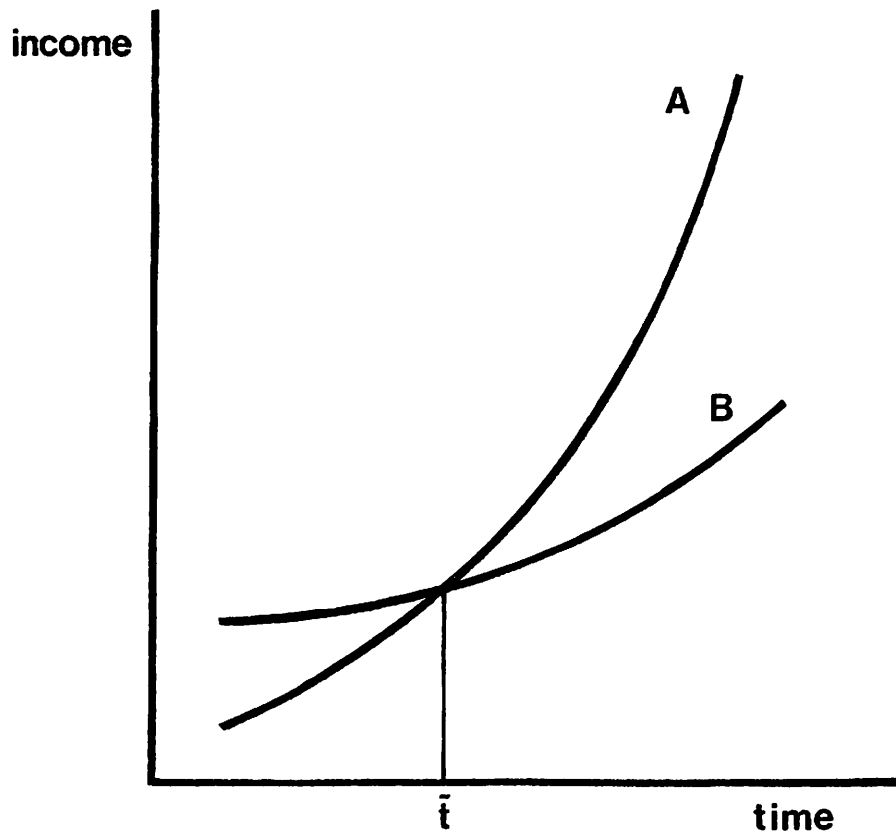
Over the short run, one country may exploit its monopoly power in trade in order to increase its own income at the expense of the rest of the world. However, such a policy will result ultimately in an income level lower than might have been achieved through free trade.

Footnotes

[*]Department of Economics, Social Science Centre, University of Western Ontario, London, Ontario N6A 5C2, Canada. Richard Brecher, Ronald Findlay, Tatsuo Hatta, Richard Manning, James Markusen, and members of the trade Workshop at the University of Western Ontario were generous in their comments and suggestions.

[1]Findlay (1973) provides an attractive analysis of this model in the context of a developing country.

[2]It may be convenient to think of the goods as forming a wage fund, used to pay labour in the following period's production.

[3]This is clearly a dynamic generalization of the bases for trade. With identical technology (which results in the same input choice) there is no incentive to trade in the long run. However, differences in endowments are sufficient to generate trade in the short run.

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