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INFLATION AND STOCK PRICES IN A CASH-IN-ADVANCE ECONOMY*

by

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ABSTRACT

The relation between inflation and stock prices is analysed in this paper. It is shown, within the framework of a choice problem, that if transactions are subject to a cash-in-advance constraint then a rise of the steady-state inflation rate will have a depressing effect on the stock market. This result arises for two reasons. Firstly, the consumption value attached to a given dividend stream is reduced when inflation rises because the latter acts as a specific tax on dividends. Secondly, the firm's profits are lowered because inflation rate changes distort the labour-leisure choice. However, the real rate of interest stays invariant accross different inflationary regimes.

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INTRODUCTION

In the theory of finance stock prices represent the discounted value of of future dividend. For instance, Lucas (1978) derives this from a well specified choice problem. Within this framework, equities are claims on the income generated by real assets, so that in principle, they should be a hedge against long-run movements in the inflation rate.

Empirically this hypothesis is not well supported, more specifically, Fama (1981) amongs others (cited in his introduction) documents a negative relation between stock prices and inflation for the post-1953 period. If causality runs from inflation to stock prices, then what must be explained is how a change of the inflation rate reduces future real dividends, or why it produces a rise of the discount factor used to capitalize equity earnings. Feldstein (1980) singles out money illusion built into the taxation structure as the relevant explanation, while Modigliani and Cohn (1979) attribute this effect to irrational behavior.

Alternatively, Fama hypothesizes a complete dichotomy between the real and the monetary sectors of the economy. The observed negative correlation is assumed to be the result of two independent forces: future income growth is positively correlated with stock price on the real side of the economy while it is negatively correlated with inflation on the monetary side. Under those circumstances, if the private sector formed expectations rationally then inflation and stock prices move in opposite directions.

Although Fama's hypothesis is instructive, it assumes that no costs are associated with anticipated inflation. However, if money exists because it economizes on transactions costs, then variation in the inflation rate will produce a reallocation of resources that will in general affect relative prices.

This paper is an attempt to formalize this concept along the line of Stockman (1981), Helpman and Razin (1981) and Aschauer and Greenwood (1983), where money is introduced via a cash-in-advance constraint instead of directly into the utility function. A competitive stock market is added to this framework, which permits the derivation of an explicit solution for stock prices. It will be shown that the negative correlation between stock prices and inflation is consistent with rational behavior and does not require money illusion built into the taxation structure.

The paper is organized as follow; section I states and derives the solutions of the problems faced by individuals, while section II considers the market equilibrium conditions.

I THE MODEL

Consider an economy inhabited by two representative agents, a firm that maximizes its present value, and a household that maximizes lifetime utility as given by :

$$\underline{U} = \sum_{t=1}^{\infty} \beta^{t-1} U(c_t, L_t) \quad ; \qquad \beta = 1/(1+\rho) \ , \quad \rho > 0$$

where U(.) is the momentary utility function and is assumed to be strictly quasi-concave; c_t and L_t are current consumption and current work effort; ρ is the constant rate of time preference. Consumption and labour are assumed to be normal and inferior goods respectively.

The household receives an income each period from three different sources. It sells labour services to the firm at the competitively determined wage rate W_t, it receives a nominal transfer from the government, and a portion of the firm's profit determined by how many shares of the firm's equity it holds,-the firm has one equity outstanding which is perfectly divisible.

At each period, the consumer optimally allocates his resources between consumption, the share of equity, labour supply and end-of-period real balances. His transaction must however satisfy a cash-in-advance constraint, so that before he can transform his wage income or his dividends into consumption goods and equity, money has to be held for one period. The timing of transactions plays an important role and a better understanding of our results will be gained by considering the sequence of transactions occuring during any given period. At the beginning of period t, the agent receives a government transfer of value TR_t, so that his nominal balances are equal to $M_t = M^d_{t-1} + TR_t$. These balances are then used to purchase consumption goods c_t and/or shares of the firm, z_t. Once this allocation is made, the goods and stock markets close. At the end of the period the agent gets his wage income $W_t L^s_t$ if he has supplied any labour services and his dividend, $P_t z_t D_t$, where P_t is the price level prevailing during t and D_t is the real profit earned by the firm during the period. The end-of-period money holding M^d_t is formed by the wage income, the dividend and the fraction of M_t that has not been spent on c_t or z_t. The important point about all this is that a share bought at the beginning of period t, or a unit of labour supplied during t receives compensation only at the end of period t when all markets are closed. From the individual point of view these money balances are useful only to increase his consumption in period t + 1 .

The firm's problem is simpler. It chooses the level of employment L^d_t which maximizes its present value. Labour inputs used in one period do not affect the firm's output in any other period, so that the firm's present value will be maximized so long as L^d_t is chosen each period to maximize current profit.

Equilibrium is found by imposing market clearing conditions on the solutions of the firm and household problems. These are:

$$c_t = f(L^d_t) \qquad (1)$$

$$L^s_t = L^d_t = L_t \qquad (2)$$

$$M^d_t = M_t \qquad (3)$$

$$z_t = 1 \qquad (4)$$

Condition (1) implies that output cannot be stored and must be consumed each period - f(.) describes the technology available to the firm which is assumed to be strictly concave. Condition (2) imposes equilibrium in the labour market. Condition (3) expresses that all money must be willingly held, and finally, condition (4) states that the representative household must own all the outstanding equity of the firm[1].

Before considering the general equilibrium properties of this economy, let us characterize the solution of the problem faced by the firm and by the household. The representative household is assumed to solve the following dynamic programming problem, with the choice variables being ($c_t, L^s_t, z_t, M^d_t/P_t$), and where all the prices are taken as given:

$$V(a_t, s_t) = \underset{(c_t, L^s_t, z_t, M^d_t/P_t)}{\text{MAX}} \{U(c_t, L^s_t) + \beta V(a_{t+1}, s_{t+1})\} \qquad (5)$$

subject to

$$M_t/P_t + W_t/P_t\, L^s_t + z_t D_t - q_t(z_t - z_{t-1}) - c_t - M^d_t/P_t = 0 \qquad (6)$$

$$M_t/P_t - c_t - q_t(z_t - z_{t-1}) \geqslant 0 \qquad (7)$$

where

q_t : price of z_t in terms of current consumption goods

$a_t = (z_{t-1}, D_t, M_t/P_t)$

$s_t = (W_t/P_t, q_t)$

$M_t = M^d_{t-1} + TR_t$

(7) is the cash-in-advance constraint which formalizes the need to finance expenditure in the goods and equity markets with money balances, while (6) is the standard budget constraint that total expenditure must equal total income each period.

Beside the budget constraint (6) and the liquidity constraint (7), the following set of Euler equations describes the choice of our representative household through time:

$$U_c(\cdot_t) = \alpha_t + \gamma_t \qquad t = 1,2,\ldots,\infty \qquad (8)$$

$$-U_L(\cdot_t) = \beta U_c(\cdot_{t+1})\, P_t/P_{t+1}\, W_t/P_t \qquad t = 1,2,\ldots,\infty \qquad (9)$$

$$q_t U_c(\cdot_t) = \beta U_c(\cdot_{t+1}) \{ P_t/P_{t+1}\, D_t + q_{t+1} \} \qquad t = 1,2,\ldots,\infty \qquad (10)$$

$$\alpha_t = \beta U_c(\cdot_{t+1})\, P_t/P_{t+1} \qquad t = 1,2,\ldots,\infty \qquad (11)$$

α_t and γ_t are the Kuhn-Tucker multipliers associated with constraints (6) and (7) respectively[2].

Some insights will be gained by considering the intuition behind these Euler equation. Condition (8) states that the individual will consume up to the point where the marginal utility of current consumption equals its marginal cost, which is the marginal indirect utility of holding one more real dollar, $\partial V(\cdot_t)/\partial M_t/P_t$. Condition (9) makes sure that the marginal disutility of work equals to the marginal benefit of work. To be more explicit, suppose that our individual is contemplating the possibility of supplying one more unit of labour during period t. For this extra effort he will receive W_t at the end of the period; this nominal amount can be transformed into consumption goods only at t+1 when it will have a value equal to W_t/P_{t+1}. To express this extra consumption in terms of utility,

W_t/P_{t+1} has to be multiplied by the discounted value of period t+1 marginal utility, $\beta U_c(\cdot_{t+1})$. So that the right-hand side of (9) is the benefit of working one more unit at time t; at the margin, this gain must equal the cost of working this extra unit, $-U_L(\cdot_t)$. Condition (10) dictates individual's choice in the equity market. If the individual acquires one more equity at the beginning of t, he can increase his t+1 consumption by $P_t/P_{t+1}\, D_t + q_{t+1}$ - i.e. $P_t/P_{t+1}\, D_t$ is the value in period t+1 of the dividend received at the end of t, while q_{t+1} is the price in terms of t+1 consumption goods for which this extra equity can be sold. This sum, when multiplied by the discounted value of t+1 marginal utility, forms the marginal benefit received from holding this extra equity. In equilibrium this gain must be just outweighed by the cost of buying this equity, $q_t U_c(\cdot_t)$. Condition (11) determines the end-of-period real balances. An extra real dollar at the end of t will be worth P_t/P_{t+1} at the beginning of t+1, and multiplying this amount by $\beta U_c(\cdot_{t+1})$ gives the marginal benefit from acquiring this dollar. Once again, in equilibrium, this gain must equal the marginal cost of raising this dollar α_t.

Three points should be made before we go on to consider the problem faced by the firm. First, when the individual makes his labour supply decision, he does not take into account the effect of his choice on the firm's dividend. This follows because our consumer is representative of many identical individuals, and it is assumed that he perceives his <u>individual</u> action as having no effect on the firm's profit. From the individual point of view dividends are exogeneous. Second, the individual's choices depend on the inflation rate π_t which enters as $P_t/P_{t+1} = 1/(1+\pi_t)$ in the equilibrium condition (9) - (11). Third, condition (10) defines the implicit real rate of interest r_t prevailing throughout t as:

$$U_c(\cdot_t)/\beta U_c(\cdot_{t+1}) = (1 + r_t) = 1/q_t(P_t/P_{t+1}\ D_t + q_{t+1}) \qquad (10')$$

by definition, q_{t+1} equals $q_t(1 + \Delta q_t/q_t)$, where Δq_t is the change in q between t+1 and t, so that:

$$r_t = (\ P_t/P_{t+1}\ D_t\ )/q_t + \Delta q_t/q_t \qquad (12)$$

In each period the dividend price ratio plus the proportional rate of change of q must equal the real rate of interest, r.

As mentioned earlier, the problem faced by the firm is simpler. It maximizes the present value of its dividend stream available for consumption:

$$PV = \sum_{t=1}^{\infty} \beta^t\ P_t/P_{t+1}\ D_t \qquad (13)$$

This present value is computed using the same discount factor as for the consumer since the firm is managed so as to maximize the household's welfare. D_t, the dividend paid to the stock holder is defined as:

$$D_t = f(L_t^d) - W_t/P_t\ L_t^d \qquad t = 1,2, \ldots, \infty$$

The firm perceives itself as operating in perfectively competitive markets, both for its input and for its output. This means that (13) will be maximized if at each period, the firm chooses L_t^d such that:

$$f'(L_t^d) = W_t/P_t \qquad t = 1,2, \ldots, \infty \qquad (14)$$

So the marginal product of labour equals the real wage paid by the firm at the end of each period.

II EQUILIBRIUM AND PREDICTIONS

We are now in a position to consider the general equilibrium properties of our economy. In such a state, the decisions made by the representative household must be consistent with those of the firm. This will be the case when (1) - (4) and (14) are satisfied. To keep the analysis simple, only the steady-state solution when all real variables $(L,c,M/P,W/P,q,\alpha,\gamma)$ are constant through time will be consider[3]. In this situation, the conditions characterizing the individual behavior in market equilibrium are:

$$U_c(f(L),L) = \alpha + \gamma \qquad (15)$$

$$-U_L(f(L),L) = \beta\, U_c(f(L),L)(1+\pi)^{-1} f'(L) \qquad (16)$$

$$q_t U_c(f(L),L) = \beta\, U_c(f(L),L)(\, D/(1+\pi) + q_{t+1}\,) \qquad (17)$$

$$c = W/P\ L + D \qquad (18)$$

(18) is the steady-state budget constraint and asserts that consumption must equal the real value of labour income plus the firm's profit paid as a dividend. This budget constraint can, however, be expressed in a different way. Using (14) and the definition of D, (18) is equivalent to:

$$c = f(L) \qquad (19)$$

In steady-state equilibrium, the representative agent's cash-in-advance constraint holds with equality, so that we also have[4]:

$$M/P = c$$

Using the definition of M/P we find that :

$$c_t = P_{t-1}/P_t\,(\,M_{t-1}/P_{t-1} - c_{t-1} + W_{t-1}/P_{t-1}\,L_{t-1} + D_{t-1}\,) + TR_t/P_t$$

As the liquidity constraint is binding at each period, the last expression can be rearranged to yield:

$$c_t = (1 - \pi/(1+\pi))(W/P\ L + D) + TR/P \qquad (20)$$

(20) is an alternative way of writing the individual's steady-state budget constraint. It highlights the tax that inflation levies on the representative household. Dividend and wage income must be held in the form of currency before they can be spent on consumption goods. With a positive inflation rate, the real value of those balances available for consumption is reduced, so that each real dollar received either as wage income or dividend is taxed by a factor of $\pi/(1+\pi)$. However, the real value of the transfer received at the beginning of each period equals the sum of those taxes. This can be seen upon substitution of (18) into (20), so that :

$$\pi/(1+\pi)\ (W/P\ L + D) = TR/P$$

The taxes levied on the household as worker and as share-holder are used to subsidize it as consumer. In this set up, inflation plays the role of a specific tax on work effort and on dividend. As is usually the case, this distorts resource allocation and relative prices.

The effect of inflation on the labour-leisure choice has previously been investigated by Aschauer and Greenwood (1983). This model is similar to theirs, so that their results carry over to the present case; the steady-state values of c and L are negatively related to inflation[5]. Market activities use fiat money intensively compared to the non-market activity of leisure. So, when inflation goes up, it becomes advantageous to substitute into this last activity.

Once the market equilibrium for c and L are determined, condition (17) can be used to price the firm's outstanding equity. In the steady-state, the marginal utility of consumption is constant, so that (17) can be rewritten as:

$$q_t = \beta((1 - \pi/(1+\pi))D + q_{t+1}) \qquad (21)$$

In each period, the market price of the firm's equity depends on its value in the future and on the real dividend once inflation has been taken into account. In long-run equilibrium, π and D are constant, so that (21) can be solved for the steady-state value of q.

$$q = 1/(1+\pi)\ D/\rho \qquad (22)$$

In steady-state equilibrium, the price of the firm's equity is equal to the discounted present value of its real dividend stream available for consumption.

We now have all the necessary ingredients to derive the effect of inflation on the steady-state value of q. Differentiating (22) with respect to π yields:

$$dq/d\pi = - 1/\rho\ (1/(1+\pi))^2\ D + 1/\rho\ (1/(1+\pi))\ dD/d\pi \qquad (23)$$

The sign of this expression clearly depends on the effect of inflation on the steady-state value of D. This effect is negative under the assumption that f(.) is strictly concave.

$$dD/d\pi = -f''(L)L\ dL/d\pi < 0$$

Given this last result, equation (23) can unambiguously be signed negative, so that a rise in the steady-state value of π has a depressing effect on the firm's equity price, q, i.e.

$$dq/d\pi = -1/\rho \ (\ 1/(1+\pi) \ )(\ (\ 1/(1+\pi) \ ) \ D + f''(L)L \ dL/d\pi \ ) < 0$$

Inflation affects q in two different ways. Firstly, it lowers the consumption value attached to a given dividend stream. Secondly, it distorts the labour-leisure choice so that an increase in π reduces the steady-state value of L and consequently the steady-state value of D. These effects can be seen more easily if we express the last derivative in terms of elasticities:

$$\pi/q \ dq/d\pi = -\pi/(1+\pi) - (\ f''(L)L^2/D \ ) \ \pi/L \ dL/d\pi \qquad (24)$$

The first term on the right-hand side of (24) is the elasticity of q with respect to π keeping constant the stream of dividends. The second term is the product of the elasticity of D with respect to L and of L with respect to π. The total response of q_t following a rise in π will depend on the different magnitudes of these elasticities.

This result arises because the firm's equity represents a claim on the nominal profit of the firm. This dividend can be spent on consumption goods only after money has been held for one period. When inflation rises, the real value of those dividends fall for the two reasons mentioned above, and so does the attractiveness of this claim on future profit. The stock market must however clear, so that the price of the equity has to fall.

In steady-state equilibrium, the nominal value of the firm's equity, P_tq_t, rises at the same rate as the price level. If the inflation rate unexpectedly goes up, q_t fall discontinuously to a new lower level while its price in terms of money starts rising at the speed of the new higher inflation rate.

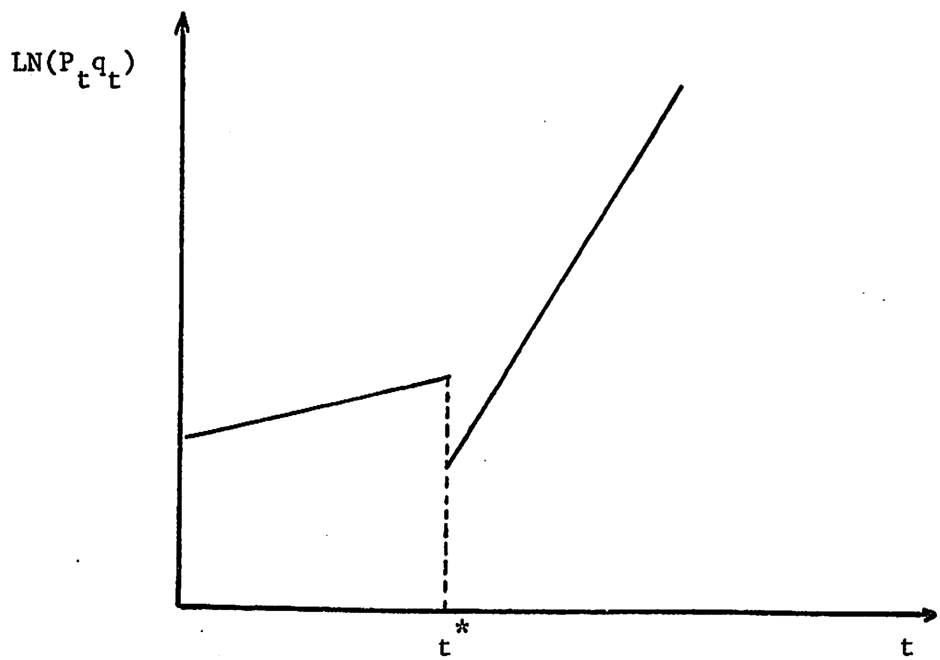
Figure I below describes the adjustment to an increase in the steady-state inflation rate occuring at t^*.

Finally, using (22) we can deduce that the steady-state value of the implicit real rate of interest, r, always equals ρ, the rate of time preference. The price of the firm's equity will adjust to insure that this equality holds. In this set-up, the real rate of interest is invariant across different inflationary regimes .

CONCLUSION

Contrary to conventional belief, the data shows that inflation and stock prices tend to move in opposite directions. This paper has presented a simple theoretical model that can account for this fact. The assumption driving this result is that transactions in the goods and equity markets are subjected to a cash-in-advance constraint. A change in the inflation rate reduces the consumption value attached to all future dividends and requires a fall of the equity's price.

FIGURE I



FOOTNOTES

1 - Only three of these conditions are independent. The fourth condition is derived from the others by using (6) as Walras' law.

2 - The reader should consult Stockman (1981) to see how the Euler equations are derived from the first order conditios of the household choices problem.

3 - Even though this model may have problems with speculative bubbles, only the stable solutions will be considered.

4 - In steady-state, $U_c(.)$ must be constant through time; from (8) the sum $\alpha_t + \gamma_t$ has to be constant through time; suppose γ_t equals zero - i.e. the liquidity constraint is not binding - then we have a contradiction since (11) implies that α_t varies across time. In steady-state equilibrium the cash-in-advance constraint must be binding. This proof is basically taken from Stockman (1981) and it will do for any π greater than $-\rho$.

5 - This result is found in differentiating (16) ,

$$dL/d\pi = - U_L/\beta\{f'(L)^2(U_{cc}-U_c/U_L\ U_{cL})-f'(L)(U_c/U_L\ U_{LL}-U_{cL})+f''(L)U_c\} < 0$$

under the assumption that labour is inferior and consumption normal, then $dL/d\pi$ is negative.

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