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DUAL EXCHANGE RATE SYSTEMS AND  
CAPITAL CONTROLS:  
AN INVESTIGATION

by

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and  
Jeremy Greenwood

ABSTRACT

The welfare aspects of dual exchange rate systems are analyzed in this paper. This exchange rate system is shown to be equivalent to a tariff being levied on domestic financial transactions with the rest of the world. The adoption of a dual exchange rate system is found to distort both agents' consumption-saving and money balance decision-making. Also, dual exchange rate systems and systems of capital controls are shown to be essentially equivalent to one another. Finally, it is argued that a flexible exchange rate system where the optimum quantity of money rule is being followed is superior to a dual exchange rate system.

(Preliminary)

June, 1983

We would like to thank Peter Garber for helpful comments. Any remaining errors are ours.

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## I. Introduction and Conclusions

Macroeconomic policymakers often view fluctuations in the capital account as an undesirable phenomenon. In some sense, they believe capital inflows or outflows to be associated with economic instability. Tranquility in the capital account is seen as a desirable trait. The dual exchange rate system is one exchange rate system which is intended to smooth out perturbations in the capital account. Under this exchange rate system, by controlling a separate price of foreign exchange at which financial transactions with the rest of the world must be undertaken, the policymaker can regulate movements in the capital account.

Recent work in international finance, such as Obstfeld (1981), Sachs (1982), Svensson and Razin (1983), and Greenwood (1983a), has stressed that movements in the capital account are reflections of optimizing agents' consumption-saving and investment decisions. In a distortion-free competitive equilibrium the outcomes of private sector decision-making are Pareto-efficient and cannot be improved upon by government stabilization policies. The observed fluctuations in the capital account mirror the welfare maximizing responses of private agents to changes in their environment caused by shifts in such things as technology, the international terms of trade, world real interest rates, inflation rates, and government spending. Furthermore, taken by itself the capital account may be a meaningless economic statistic. This is because, theoretically, the correlation between movements in the capital account and economic welfare is ambiguous.

In this paper, the theoretical underpinnings and the welfare properties of the dual exchange rate system are analyzed.<sup>1</sup> It is shown that dual exchange rate systems and systems of capital controls are isomorphic to one another in the same sense that tariffs and quotas are equivalent in the standard trade

literature. Finally, as compared to a flexible exchange rate where the optimum quantity of money rule is being followed, the adoption of a dual exchange rate system can only immiserize the welfare of a competitive economy.

## II. The Individual's Optimization Problem

Consider the case of the following "small" open economy that has adopted a dual exchange rate system. For simplicity, it will be assumed that this economy has a life span of two periods.<sup>2</sup> It is inhabited by a representative agent whose goal is to maximize his lifetime utility,  $\underline{U}(\cdot)$ , as given by:

$$\underline{U}(\cdot) = \sum_{t=1}^2 \beta^{t-1} U(c_t)$$

where  $\beta$  is this individual's subjective rate of time preference and  $c_t$  his period  $t$  consumption of an imported good.

In each period  $t$  the agent is endowed with a certain quantity of an export good denoted by  $X_t$ . This export good can be exchanged for the imported good on world commodity markets. Under a dual exchange rate system, all transactions with foreigners involving the purchase or sale of goods are done at the commercial exchange rate. Let  $e_t$  be the commercial exchange rate in period  $t$ , or the worth of a unit of foreign currency in terms of domestic currency when it is being used to facilitate the transactions of goods with the rest of the world. Arbitrage in world commodity markets would ensure that the law of one price (2.1) holds in each period.

$$p_t = e_t p_t^* \quad \forall t=1,2 \quad (2.1)$$

$$p_{xt} = e_t p_{xt}^*$$

where  $P_t$  and  $P_{xt}$  are the domestic period  $t$  nominal prices of the imported and exported goods, respectively, and  $P_t^*$  and  $P_{xt}^*$  are the foreign nominal prices of these goods in this period. Under a dual exchange rate system, the commercial exchange rate is often pegged or fixed at some level,  $\bar{e}$ , implying that  $e_t = \bar{e}$  for all  $t$ . Defining  $\rho_t$  to be the world terms of trade in period  $t$  so that  $\rho_t \equiv P_{xt}^*/P_t^* = P_{xt}/P_t$ --that is,  $\rho$  is the relative price of exports in terms of imports--it can be seen that the individual's endowment of exports in this period is worth  $y_t$  units of imports where  $y_t = \rho_t X_t$ . Also, in period  $t$  the representative agent receives a real transfer payment,  $t_t$ , from the government.

The individual can hold two assets in this economy: domestic money and foreign nominal bonds. The individual holds money in order to economize on the transactions costs of exchange. In particular, in each period a certain fraction  $v$  of the agent's real income,  $y$ , is absorbed in transactions costs. It will be assumed that  $v$  is a convex function of the ratio of the individual's nominal money balances,  $M$ , to his nominal income,  $P_t y_t$ . In other words, there are diminishing returns to holding money. By increasing his holdings of money, the agent can economize on the proportion of his real income which is being absorbed in transactions costs. As the ratio of money,  $M$ , to nominal income,  $P_t y_t$ , rises, however, the reduction in  $v$  brought about by holding an extra unit of money is reduced. Formally,

$$v_t = v(M_t/P_t y_t) \quad v_t = 1, 2 \quad \text{with } v' < 0, v'' > 0$$

$$\text{and } 0 < v < 1$$

The representative agent, in the first period, can also buy (or sell) a foreign denominated nominal bond which earns (or pays) the foreign nominal

interest rate  $i^*$ . For instance, if in period one the individual was to buy a bond then worth one unit of foreign currency, he would earn  $(1+i^*)$  units of foreign currency in period two. Now, under a dual exchange rate system, all financial transactions with the rest of the world must be made at the financial exchange rate. Specifically, if  $s_t$  is the financial exchange rate in period  $t$ , then a unit of foreign currency used for financial transactions would be worth  $s_t$  units of domestic currency. Thus, if in period one the representative agent invested a unit of domestic currency in foreign bonds, he would receive  $1/s_1$  units of these bonds. In the second period, these bonds would pay off  $(1+i^*)/s_1$  units of foreign currency which at that time would be worth  $(1+i^*)s_2/s_1$  units of domestic currency. Under a dual exchange rate system, the government manages the financial exchange rate in such a manner so as to reduce capital account transactions to some target level.

The representative agent in this dual exchange rate economy would face the following constrained maximization problem with his choice variables being  $c_1, m_1 \equiv M_1/P_1$ , and  $m_2 \equiv M_2/P_2$ :

$$\begin{aligned} \text{Max } & U(c_1) + \beta U(c_2) \\ \text{s. t. } & \\ c_2 = & (1 - v(\frac{m_2}{y_2}))y_2 + t_2 + (1+i^*)\left(\frac{s_2}{s_1}\right)\left(\frac{P_1}{P_2}\right)\left[(1 - v(\frac{m_1}{y_1}))y_1 + t_1\right. \\ & \left. - c_1 - m_1\right] - m_2 + (P_1/P_2)m_1 \end{aligned} \quad (2.2)$$

The first-order conditions associated with this maximization problem--in addition to (2.2)--are:

$$\begin{aligned} U'(c_2) &= \beta \left[ \frac{(1+i^*)}{(1+\pi)(1+f)} \right] U'(c_2) \\ &= \beta(1+r)U'(c_2) \quad \text{with } 1+r \equiv \left[ \frac{(1+i^*)}{(1+\pi)(1+f)} \right] \end{aligned} \quad (2.3)$$

$$\begin{aligned}
 -v' \left( \frac{m_1}{y_1} \right) &= \frac{i^* - f}{(1+i^*)} & (2.4a) \\
 &= \frac{i}{(1+i)} & \text{with } 1+i \equiv (1+\pi)(1+r)
 \end{aligned}$$

and

$$-v' \left( \frac{m_2}{y_2} \right) = 1 \quad (\text{Note that this implies } m_2/y_2 = k \equiv v'^{-1}(-1).) \quad (2.4b)$$

where  $\pi$  is the domestic inflation rate, or  $\pi \equiv (P_2 - P_1)/P_1$ , and  $f$  is the rate of appreciation in the financial exchange rate here defined as  $f = -(s_2 - s_1)/s_2$ . Only the first two first-order conditions will be focussed on now. Equation (2.3) states that the individual should save until the loss in first-period utility resulting from the drop in consumption due to shifting a small amount of resources into bonds exactly offsets the discounted gain in second-period utility following from increased future consumption. Note that by reducing current consumption by a real unit the individual could purchase  $P_1/s_1$  units of foreign bonds that would pay off the equivalent of  $(1+i^*)(P_1/P_2)(s_2/s_1) = (1+i^*)/[(1+\pi)(1+f)]$  consumption units next period. Thus, if  $r$  is denoted as the real interest rate facing domestic residents, then  $1+r = (1+i^*)/[(1+\pi)(1+f)]$ .

Equation (2.4a) sets the marginal product of money in period one, or  $-v'(m_1/y_1)$ , equal to the opportunity cost of holding money in this period, or  $(i^* - f)/(1+i^*)$ . It is easy to see that  $(i^* - f)/(1+i^*)$  is the opportunity cost of holding a unit of real balances in period one. To finance the holding of an additional unit of real balances in period one, the individual could issue a foreign nominal bond in the amount  $P_1/s_1$ . The individual would then have to pay the purchaser of this bond  $(1+i^*)P_1/s_1$  units of foreign currency next period. This would be worth  $(1+i^*)(P_1/P_2)(s_2/s_1) = (1+i^*)/[(1+f)(1+\pi)]$

units of domestic consumption at that time. But this would not be the real cost that the individual would incur in the second period in order to increase his first period holdings of real balances by a unit. In the second period, the individual would still have  $P_1$  units of nominal balances that he has carried over from the first period and that would now be worth  $P_1/P_2 = 1/(1+\pi)$  units of consumption goods. Thus, the second period cost the agent would incur would be  $(1+i^*)/[(1+f)(1+\pi)] - 1/(1+\pi) = (i^* - f)/[(1+f)(1+\pi)]$ . To obtain the cost in terms of first-period consumption, one should discount the above term by one plus the real interest rate, or  $(1+i)/[(1+\pi)(1+f)]$ . By doing this, it can be seen that the opportunity cost of holding money in our dual exchange rate environment is  $(i^* - f)/(1+i^*)$ .

Now, define  $i$  as the domestic nominal interest rate so that  $1+i = (1+\pi)(1+r)$ . It is easy to see that the opportunity cost of holding money may be written as  $i/(1+i)$ . That is,  $i/(1+i) = (i^* - f)/(1+i^*)$ . The rate of appreciation in the financial exchange rate, or  $f$ , affects the opportunity cost of holding money because it influences the domestic nominal interest rate,  $i$ , via its impact on the domestic real interest rate,  $r$ .<sup>3,4</sup>

The above optimization problem of the consumer suggests that the agent's compensated demand function for first-period consumption should take the following form:

$$c_1 = c(f, \omega) \quad (2.5)$$

(+)

where  $\omega$  is an index of the individual's lifetime utility. It also suggests that his first-period demand for money function should appear as

$$m_1 = m(f)y_1 \quad (2.6)$$

(+)

The sign under an argument in one of the demand functions shows the sign implied



by the consumer's problem of the partial derivative of that demand function with respect to the argument in question.

Finally, note that since the commercial exchange rate is fixed under a dual exchange rate, the law of one price (2.1) implies that domestic and foreign inflation rates must be the same. That is,  $\pi = \pi^*$  where  $\pi^*$  is defined as the foreign rate of inflation, so that  $\pi^* \equiv (P_2^* - P_1^*)/P_1^*$ . Thus, one could replace  $\pi$  by  $\pi^*$  in equations (2.3) and (2.4a) and elsewhere in the model. The world real interest rate,  $r^*$ , is given by the equation  $1+r^* = (1+i^*)/(1+\pi^*)$ . Consequently, domestic and foreign real interest rates,  $r$  and  $r^*$ , have the following relationship with one another:

$$\begin{aligned} 1+r &= (1+r^*)/(1+f) \\ &= (1+r^*)\left[1 - \frac{f}{1+f}\right] \end{aligned} \tag{2.7}$$

As can be seen, a dual exchange rate system can be viewed as effectively imposing a tax--or tariff--on domestic financial transactions with the rest of the world. A tax at the rate  $\frac{f}{1+f}$  is being levied on the principle and interest derived from the savings that the domestic economy does with the rest of the world.

### III. The Model's General Equilibrium

Like any other actor in the economy, the government must satisfy a budget constraint. To facilitate the analysis it is helpful to artificially split up the government into two sectors: viz a central bank and an authority which manages the financial exchange rate. This dichotomization does not affect the model's generality. The central bank's purpose is to provide the economy's

money stock and to maintain the commercial rate fixed at the level  $\bar{e}$ . The central bank's budget constraint for the first-period is

$$b + \mu_1 = m_1 \quad (3.1)$$

where  $b$  is defined to be the real value--measured in terms of imports--of the central bank's acquisitions of foreign nominal bonds in the first period and  $\mu_1$  is the value of transfer payments that it makes to domestic residents in this period. Changes in  $b$  can be thought of as reflecting movements in the current balance of payments while fluctuations in  $\mu_1$  can be regarded as shifts in domestic credit. The central bank's second-period budget constraint appears as

$$\mu_2 = m_2 - \left(\frac{1}{1+\pi}\right)m_1 + (1+r^*)b \quad (3.2)$$

(Recall that  $1+r^* \equiv (1+i^*)/(1+\pi^*)$ .)

where  $\mu_2$  is the real value of transfer payments made by the central bank in period two.

As was just mentioned, it is convenient to split up the government into two agencies. The second agency is responsible for managing the financial exchange rate and will be dubbed the Financial Exchange Rate Authority (FERA). The FERA's first-period budget constraint is

$$b^f + \tau_1 = \left(1 - \frac{\bar{e}}{s_1}\right)a \quad (3.3)$$

with  $a \equiv (1 - v(1))y_1 + t_1 - c_1 - m_1$  [Note  $v(1) \equiv v\left(\frac{m_1}{y_1}\right)$ .]

where  $b^f$  is defined to be the real value--measured in terms of imports--of the FERA's purchases of foreign nominal bonds,  $\tau_1$  is real value of transfer

payments that it makes to domestic residents, and  $a$  is the real amount of savings undertaken by the private sector. (Note that consistency in the model requires that aggregate value of first-period transfer payments made by the government is equal to the sum of transfer payments made by the central bank,  $\mu_1$ , and the FERA,  $\tau_1$ , i.e.,  $t_1 = \mu_1 + \tau_1$ .) The left-hand side of the above equation represents the FERA's first-period expenditure while the right-hand side shows its revenues in this period. To gain some insight into the nature of this revenue, note that domestic residents in order to purchase foreign bonds sell resources to the FERA worth  $a$  units of imports. In exchange for these resources, the FERA gives them  $(\frac{P_1}{s_1})a$  nominal units of foreign currency. At world prices this would be worth  $(\frac{P_1}{s_1 P_1^*})a$  units of imports. Using the law of one price (2.1) it follows that the foreign exchange authority is giving the public foreign exchange worth  $(\frac{\bar{e}}{s_1})a$  units of imports in resources for domestic resources worth  $a$  units. Thus the revenue earned by the FERA in the first period is the difference between these two sums or  $(1 - \frac{\bar{e}}{s_1})a$ .

The FERA's second-period budget constraint is

$$\tau_2 = (1+r^*)b^f - [(\frac{s_2 - \bar{e}}{s_1})a] \quad (3.4)$$

where  $\tau_2$  is the real value of its second-period transfer payments. (Again, note that  $t_2 = \mu_2 + \tau_2$ .) The term in brackets on the far right-hand side of the above expression illustrates (minus) the earning the FERA makes from its operations in the second period. This term is easy to explain. In the second period domestic residents will have foreign currency worth  $(\frac{\bar{e}}{s_1})(1+r^*)a$  units of imports--again the foreign exchange is being evaluated at international prices. The foreign exchange authority, however, gives them domestic currency

worth only  $(\frac{s_2}{s_1})(1+r^*)$  units of imports. The FERA's earnings in the second period is given by the difference between these two sums.<sup>5</sup>

How the FERA allocates its first-period revenue,  $(1 - \frac{\bar{e}}{s_1})a$ , between transfer payments,  $\tau_1$ , and foreign bonds,  $b^f$ , is important. That is, it turns out that Ricardian equivalence does not hold in this model, a fact which will be elaborated on later. There are, of course, many schemes by which the FERA could allocate its revenues accruing from its management of the financial exchange rate, between transfer payments and bond holdings. It seems that the most natural and innocuous assumption to make is that in the first period the FERA makes transfer payments to domestic residents amounting to the present value of its revenues--calculated using the world real interest rate,  $r^*$ . In other words, let

$$\tau_1 = [(1 - \frac{\bar{e}}{s_1}) - (\frac{s_2 - \bar{e}}{s_1})]a = (\frac{f}{1+f})a \quad (3.5)$$

Intuitively, one would then expect the FERA's second-period transfer payment to be zero, so that  $\tau_2 = 0$ , since the agency has already distributed all of its profits (or losses) in the first period. This is correct. This pattern of transfer payments turns out to be a useful neutral benchmark case from which to evaluate the macroeconomic implications of other transfer payment schemes.

Under this scheme it follows from the FERA's first-period budget constraint (3.3) and equation (3.5), that the FERA must be acquiring bonds,  $b^f$ , in the magnitude  $(\frac{s_2 - \bar{e}}{s_1})a$ . This, in some ways is made out to be a sensible thing for the FERA to do. Recall that in the first period domestic residents are saving a units of imports. At the world real interest rate,  $r^*$ , this should

return  $(1+r^*)a$  units of imports in principle and interest next period. The FERA authority effectively taxes this principle and interest at the rate  $f/(1+f)$ , however, so that domestic residents actually only receive the amount  $\frac{(1+r^*)}{(1+f)} a$ . Thus in order to obtain this additional amount of imports in the second period the economy as a whole--that is, domestic residents cum the FERA but excluding the central bank--need only really purchase foreign bonds in the first period worth  $\frac{a}{(1+f)}$  units of imports--when evaluated at world prices.

Now note that domestic residents in fact purchase foreign nominal bonds in the first period worth only  $(\frac{\bar{e}}{s_1})a$  units of imports--again when evaluated at world prices. This shortfall in the total required purchases of foreign nominal bonds can be corrected by the FERA purchasing bonds,  $b^f$ , in the first period in the amount  $[(\frac{1}{1+f}) - \frac{\bar{e}}{s_1}]a = (\frac{s_2 - \bar{e}}{s_1})a$ . In fact, as was discussed, this is the amount they do purchase when they distribute the present-value of their earnings in the first period.

Lastly, it can be immediately seen from (3.4) that if the FERA does purchase this amount of bonds in the first period then second-period transfer payments will be zero. This occurs because the second-period earnings on the FERA's bonds will just offset the losses it occurs from its financial exchange rate operation in that period. By adopting the above transfer payment scheme the FERA ensures that in the second period the additional amount of imports that private citizens have been saving for in the first period will be exactly provided for from the first-period savings by the economy as a whole--again, domestic residents cum the FERA.

As has been mentioned, the objective of a dual exchange rate system is to reduce capital account transactions to some target level. The FERA sets the rate of appreciation in the financial exchange rate,  $f$ , so as to achieve

this target. In other words, as can be seen from (2.7), the FERA taxes--or subsidies when  $f$  is negative--the proceeds from domestic saving so as to manipulate--one plus--the domestic real interest rate in order to bring domestic financial transactions with the rest of the world into line with this target. Assume that the government wants the magnitude of the capital account in the first period to be  $\bar{c}_a$ . This implies that it must set net external savings by the economy--domestic residents cum the FERA--to  $-\bar{c}_a$  implying that<sup>6</sup>

$$\frac{-a}{(1+f)} = \bar{c}_a \quad (3.6)$$

By using the above condition (3.6) in conjunction with the representative agent's budget constraint (2.2), the government's budget constraints (3.1), (3.2), (3.3) and (3.4), and the transfer scheme (3.5), it is apparent that an implication of a dual exchange rate system is that

$$c_1 = (1 - v(1))y_1 - b + \bar{c}_a \quad (3.7)$$

$$c_2 = (1 - v(k))y_2 + \frac{(1+i^*)}{(1+\pi^*)} b - \frac{(1+i^*)}{(1+\pi^*)} \bar{c}_a \quad (3.8)$$

$$m_1 = \mu_1 + b \quad (3.9)$$

$$m_2 = \mu_2 + \left(\frac{1}{1+\pi^*}\right)m_1 - \frac{(1+i^*)}{(1+\pi^*)} b \quad (3.10)$$

The interpretation of the above equations is obvious. For instance, (3.7) shows that a familiar identity holds for the first period: the sum of the current account,  $(1 - v(1))y_1 - c_1$ , plus the capital account,  $\bar{c}_a$ , must equal the balance of payments,  $b$ .

#### IV. Two Comparative Statics Exercises

In order to gain some further insight into the nature of dual exchange rate systems, two comparative statics exercises will be undertaken. First, suppose future income,  $y_2$ , rises a fact of which the public is fully cognizant. How will this affect the rate of appreciation in the financial exchange rate,  $f$ , and the balance of payments,  $b$ ? To answer this question one should focus on equations (2.5), (2.6), (3.7) and (3.9) which imply in equilibrium that the following holds:

$$c(f, \omega) = (1 - v(\frac{m_1}{y_1}))y_1 - b + \bar{ca} \quad (4.1)$$

$$m(f)y_1 = \mu_1 + b \quad (4.2)$$

Now, an increase in future real income,  $y_2$ , leads to a change in the agent's real welfare--see Appendix for further details--of the amount

$$\begin{aligned} d\omega &= (1+f) \frac{(1+\pi^*)}{(1+i^*)} (1-v(k))dy_2 + f db - v'(1)db \\ &= (1+f) \frac{(1+\pi^*)}{(1+i^*)} (1-v(k))dy_2 + \frac{i^*(1+f)}{(1+i^*)} db \quad (\text{using (2.4a)}) \end{aligned} \quad (4.3)$$

where  $d\omega$  is the individual's welfare gain when measured in terms of current consumption. As can be seen from (4.3), this change in the agent's real welfare can be broken down into three components. The first component represents the present discounted value of the change in agent's future real income net of transactions costs. (Recall that the appropriate discount factor for the individual is  $(1+f)(1+\pi^*)/(1+i^*) \equiv 1/(1+r)$ ).

The second component represents the welfare gain realized by the individual when the government increases its holdings of foreign reserves in the first period by  $db$ . Note that when the government increases its foreign

reserves in the first period by a unit, equation (3.7) implies that private consumption falls in this period by a unit. Now, as can be seen from (3.8), the agent's future consumption will increase by  $(1+i^*)/(1+\pi^*)$  units when the government's current reserves increase by a unit. But the agent evaluates a unit of future consumption as being worth  $(1+f)(1+\pi^*)/(1+i^*)$  units of current consumption, a fact which (2.3) illustrates. Thus, to the individual an increase in future consumption by the amount  $(1+i^*)/(1+\pi^*)$  units is valued at  $(1+f)$  units of present consumption. Consequently, on net, the representative agent incurs a welfare gain in the amount of  $f$  units of present consumption when the government increases its current reserves by a unit. In a distortion-free competitive equilibrium changes in the central bank's holding of interest bearing reserves should not affect agents' welfare levels, a fact both Obstfeld (1982) and Stockman (1983) have shown. Ricardian equivalence fails to hold in the current model because the real interest rate at which the government borrows and lends,  $r^*$ , is not the same as that used by private agents in their consumption-savings decision,  $r$ . [See (2.7) where  $r = (r^* - f)/(1+f)$ .]

The third component illustrates the welfare gain the individual enjoys due to the fact that current transactions costs fall when today's real supply of money,  $m_1$ , rises because the government's holdings of foreign reserves increases by the amount  $db$ . When real balances increase by a unit, transactions costs decrease by the amount  $(i^* - f)/(1+i^*)$ --note that  $-v'(1) = (i^* - f)/(1+i^*)$  from (2.4a). The net effect of the second and third terms on the agent's real welfare is positive as is shown.



The impact of an increase in  $y_2$  on  $f$  and  $b$  can be uncovered by subjecting equations (4.1) and (4.2) to the usual sort of comparative statistics exercise while making use of (4.3). The results of this exercise are

$$\frac{df}{dy_2} = \frac{-\{[(1+\pi^*)(1+f)/(1+i^*)](1-v(k))\} \frac{\partial c}{\partial \omega}}{\Omega} < 0, \quad (4.4)$$

where  $\Omega \equiv \left\{ \frac{\partial c}{\partial f} \right\}_{\omega=\bar{\omega}} + [(1+f)(1+i^*) \frac{\partial c}{\partial \omega} / (1+i^*)] \left( \frac{\partial m}{\partial f} \right) y_1 \} > 0$ , and

$$\frac{db}{dy_2} = \left( \frac{\partial m}{\partial f} \right) y_1 \left( \frac{df}{dy_2} \right) < 0 \quad (\text{since } \frac{df}{dy_2} < 0) \quad (4.5)$$

An increase in future real income,  $y_2$ , causes the current financial exchange rate to depreciate relative to its future value, i.e.,  $f$  falls. This is what (4.4) shows. This expression can be explained intuitively as follows. When the agent's future real income,  $y_2$ , rises he feels wealthier today. At the initial value of  $f$ , rather than consume this increase in wealth solely in the second period, the agent instead wants to smooth out this gain in his wealth over both periods, and hence desires to increase his current consumption. When  $f$  remains fixed, the agent's current disposable income remains unaltered so that the only way the individual would increase his current consumption would be by borrowing on the international bond market. This attempt to borrow causes  $f$  to fall, so that one plus the domestic real interest rate on loans,  $1+r = (1+i^*)/(1+\pi^*)(1+f)$ , increases and the capital account,  $\bar{ca}$ , remains fixed.

This decrease in  $f$  is positively associated with  $\partial c / \partial \omega$  which shows how an increase in wealth--at a constant value of  $f$ --affects current consumption,  $c_1$ . Also, note that this movement in  $f$  is inversely related to the size of  $\partial c / \partial f \big|_{\omega=\bar{\omega}}$  which represents the substitution effect of a change in

$f$  on current consumption. The bigger this substitution effect is, or the more willing the agent is to substitute away from current consumption toward future consumption as  $f$  falls, the smaller will be the decline in  $f$ .

The magnitude of this downward movement in  $f$  is negatively related to the size of  $(\partial m / \partial f) y_1$  which shows the impact of a change in  $f$  on the current demand for real balances. As  $f$  falls, the opportunity cost,  $i / (1 + i) = (i^* - f) / (1 + i^*)$ , of holding real balances increases since the domestic nominal interest rate has risen and this results in people holding less real balances. An outflow of foreign reserves will then occur. Recall from the above discussion that a unit loss in the government's foreign reserves will lead to a reduction in individuals' welfare by the amount  $(1 + f) i^* / (1 + i^*)$ . On this account, agents will desire to reduce their current consumption by  $[(1 + f) i^* / (1 + i^*)] \partial c / \partial \omega$ . Also, a unit reduction in the government's foreign reserves will lead to an increase in the supply of current consumption goods available to private citizens by the amount  $1 + v'(1) = (1 + f) / (1 + i^*)$ , a fact which is readily apparent from the right-hand side of (4.1). Thus, the net effect of a unit reduction in the government's foreign reserves, brought about by a unit reduction in the demand for money, would be to create an excess supply of current consumption goods of  $(1 + f) [1 + i^* (\partial c / \partial \omega)] / (1 + i^*)$  units. Therefore, there would be less need for current borrowing and this would have a dampening effect on the fall in  $f$ .

Finally, as was just mentioned, when  $f$  falls this causes the demand for real balances to drop, and hence the government's holdings of foreign reserves to fall as (4.5) portrays. First, this drop in the government's foreign reserves will be related to the sensitivity of the demand for real

balances with respect to changes in  $f$  which is governed by the size of  $(\partial m / \partial f) y_1$ . Second, it will be proportional to the magnitude of the change in  $f$  itself as given by  $df/dy_2$ .

Note from (3.7) that as a consequence of the increase in future income,  $y_2$ , the trade balance will tend to swing into deficit. That is,  $(1 - v(1))y_1 - c_1$  will tend to fall since  $b$  drops while  $\bar{c}_a$  remains fixed. In a distortion-free competitive equilibrium one would expect a trade balance deficit to occur in response to an increase in future income as domestic residents dipped into their savings or borrowed in order to finance greater current consumption. This result is discussed by Sachs (1982), Svensson and Razin (1983), and Greenwood (1983a). In the present setting the increase in private consumption,  $c_1$ , is being financed by the central bank running down its reserves,  $b$ . The central bank is effectively acting as a financial intermediary here.

For the second comparative statics exercise suppose that the government decides to increase its target level for the capital amount,  $\bar{c}_a$ . What would the impact of this decision be on the rate of appreciation in the financial exchange rate,  $f$ , and the balance of payments,  $b$ ? Following the procedure outlined in the discussion of the previous comparative statics exercise the change in the agent's real welfare as a consequence of this policy would be

$$d\omega = \frac{i^*(1+f)}{(1+i^*)} db - f d\bar{c}_a \quad (4.6)$$

The change in the agent's real welfare is made up of two components, as the right-hand side of (4.6) illustrates. These two components were explained in the previous comparative statics exercise and hence will not be discussed again here.

By undertaking the desired comparative statics exercise on equations (4.1) and (4.2), while utilizing (4.6), one obtains the following results

$$\frac{df}{d\bar{c}a} = \frac{[1 + f \frac{\partial c}{\partial \omega}]}{\Omega} > 0 \quad (4.7)$$

(since  $1 + f \frac{\partial c}{\partial \omega} > 0$  as  $\frac{\partial c}{\partial \omega} < 1$  and  $f > -1$ )

and

$$\frac{db}{d\bar{c}a} = \left(\frac{\partial m}{\partial f}\right) y_1 \frac{df}{d\bar{c}a} > 0 \quad (4.8)$$

Equation (4.7) illustrates that an increase in the government's target level for the capital account,  $\bar{c}a$ , will lead to a rise in the rate of appreciation in the financial exchange rate,  $f$ , or equivalently will cause a downward movement in the domestic real interest rate,  $r$ . This is intuitively obvious: a lower equilibrium domestic real interest rate is required in order to entice domestic residents to save the reduced requisite amount--recall that  $\bar{c}a = -a/(1+f)$ .

The analogy between dual exchange rate systems and capital controls is readily apparent. Under a dual exchange rate system the government picks a target for the capital account,  $\bar{c}a$ , and sets the rate of appreciation in the financial exchange rate,  $f$ , so as to force domestic savings into line with this target. As (2.7) illustrates, effectively the government is levying a tax on private financial transactions undertaken with the rest of the world. This is parallel to a government levying a tariff on imports so as to achieve some import goal. Now, capital controls are analogous to quotas on imports. It is well-known in the international trade literature that tariffs and quotas are identical under certain conditions, as is discussed by Bhagwati (1965). In the current situation one could imagine the government setting quotas on private financial transactions with the rest of the world in the amount  $\bar{c}a$ . If the

government allocated the rights to these quotas so as to maintain perfect competition among quota holders then the domestic real interest would be identical under either a system of dual exchange rates or capital controls.<sup>7</sup> That is, in this situation the two systems are isomorphic to each other.<sup>8</sup>

Finally, also note from (4.8) that when the government increases its target level for the capital account a balance of payments surplus is likely to occur. This is because as the domestic real interest,  $r$ , falls so does the opportunity cost of holding money,  $i/(1+i)$ . The resulting rise in the demand for money will create a propensity toward a balance of payments surplus.

#### V. The Optimum Choice of Exchange Rate Regime

How should an economy choose its exchange rate regime? This question has been examined by Helpman (1981), and subsequently by Aschauer and Greenwood (1983), Greenwood (1983a), and Persson (1983). To address this question in the current context, imagine that this economy is controlled by a central planner whose aim is to maximize individuals' lifetime utility. The optimization problem the central planner should solve is shown below where  $c_1$  and  $m_1$  are his choice variables.

$$\begin{aligned} \text{Max } & U(c_1) + \beta U(c_2) \\ \text{s.t. } & c_2 = (1 - v(k))y_2 + \frac{(1+i^*)}{(1+\pi^*)}[(1 - v(1))y_1 - c_1] \end{aligned} \quad (5.1)$$

The first-order conditions arising from this maximization problem--in addition to the general equilibrium budget constraint (5.1)--are

$$U'(c_1) = \beta \frac{(1+i^*)}{(1+\pi^*)} U'(c_2) \quad (5.2)$$

$$-v' \left( \frac{m_1}{y_1} \right) = 0 \quad (5.3)$$

To help understand the inefficiencies that a dual exchange rate system introduces into an economy, it is useful to contrast the above two first-order conditions with those that arise in the dual exchange rate economy.

A comparison of (5.2) with its analogue (2.3) is the dual exchange rate economy illustrates how agents' consumption-saving decisions are distorted in a dual exchange rate environment. It can easily be seen from (5.2) that economic efficiency requires that the marginal rate of substitution between future and current consumption,  $U'(c_1)/\beta U'(c_2)$ , be equated to one plus the world real interest rate,  $(1+i^*)/(1+\pi^*)$ . In the dual exchange rate economy, a wedge of the amount  $(1+\pi^*)/[(1+\pi)(1+f)]$  is driven between those two quantities. Now, let  $\delta$  represent the rate of appreciation in the commercial exchange rate where  $\delta$  is formally defined as  $\delta = -(e_2 - e_1)/e_2$ . The law of one price (2.1) implies in general that  $(1+\delta) = (1+\pi^*)/(1+\pi)$ . Consequently, the wedge in question could be rewritten as  $(1+\delta)/(1+f) = [1 + (\delta - f)/(1+f)]$ . Thus, it follows that the distortion in the agent's consumption-saving decisions induced by the dual exchange rate system could be removed by letting the commercial and financial exchange rates appreciate at the same rate. In other words, economic efficiency dictates that  $f = \delta$ . This, of course, implies that the dual exchange rate system should be abandoned.

By comparing equation (5.3) with its dual exchange rate system analogue, (2.4a), it can be seen how agents' money demand decisions are distorted in a dual exchange rate system environment. Equation (5.3) states that economic efficiency necessitates that money's marginal product be set equal to its marginal (social) cost which is zero when money is costless to produce. Individuals under a dual exchange rate system, however, perceive the opportunity

cost of holding money to be  $(i^* - f)/(1 + i^*)$  which is not equal to the social opportunity cost--zero--of holding money. This results in money being underutilized, and the real transaction costs of exchange being too high, in the dual exchange rate economy. The private opportunity cost of holding money can be reduced to the social opportunity cost of holding money by letting the financial exchange rate appreciate at the rate  $f = i^*$ . That is, the financial exchange rate should appreciate at the foreign nominal interest rate.

In conclusion, economic efficiency requires that  $\delta = f = i^*$ . In other words, a "small" open economy should adopt a flexible exchange rate system and follow the optimum quantity of money rule in order to maximize its welfare. This is easy to see since the law of one price (2.1) implies that when  $\delta = i^*$ , the domestic economy is deflating according to the optimum quantity of money rule. That is,

$$\pi = -r^*/(1 + r^*) \quad \text{where again } (1 + r^*) \equiv (1 + i^*)/(1 + \pi^*)$$

where  $r^*$  is the world real interest rate. This result is also shown in Aschauer and Greenwood (1983) and Greenwood (1983a). The fact that a "small" open economy should abandon a dual exchange rate system should be appealing to the international economist.<sup>9</sup> Free trade in goods for a "small" open economy is taken for granted as being desirable for such an economy, and it consequently seems a small step to argue that free trade in financial assets is also inherently desirable.

APPENDIX

In this appendix a demonstration of the welfare gain the agent realizes today when his future real income,  $y_2$ , is anticipated to improve will be undertaken. Recall that the individual's lifetime utility is

$$\underline{U} = U(c_1) + \beta U(c_2)$$

so that

$$d\underline{U} = U'(c_1)dc_1 + \beta U'(c_2)dc_2$$

Now, divide both sides of the above equation by  $U'(c_1)$  and define  $d\omega \equiv d\underline{U}/U'(c_1)$  as the change in the agent's real welfare evaluated in terms of current imports.

$$d\omega = dc_1 + \beta \frac{U'(c_2)}{U'(c_1)} dc_2$$

From the agent's first-order condition (2.3), it can be seen that

$\beta U'(c_2)/U'(c_1) = (1+f)(1+\pi^*)/(1+i^*)$ , implying that

$$d\omega = dc_1 + \frac{(1+f)(1+\pi^*)}{(1+i^*)} dc_2$$

But from equations (3.7) and (3.8), as well as from equations (3.9) and (2.4b), it is known that in the dual exchange rate economy's general equilibrium the following must be true:

$$dc_1 = -db - v'(1)db$$

$$dc_2 = (1 - v(k))dy_2 + \frac{(1+i^*)}{(1+\pi^*)} db$$

Thus,

$$d\omega = \frac{(1+f)(1+\pi^*)}{(1+i^*)} (1 - v(k))dy_2 + f db - v'(1)db$$

which is equation (4.3) in the text.



FOOTNOTES

<sup>1</sup>Flood and Marion (1982) also examine dual exchange rate systems. Their primary concern is with how various exchange rate systems insulate an economy from different sorts of disturbances. Exchange rate regimes are evaluated on their ability to minimize output variance. The model they use is of the reduced-form Phillips curve variety and does not appear to be well suited to address the welfare properties of the dual exchange rate system that are being analyzed here. A discussion of the welfare aspects of any exchange rate regime is probably best done within a choice-theoretic framework, such as that outlined by Helpman (1981). This is the approach taken here.

<sup>2</sup>Similar two-period setups have been used by Persson (1983), Sachs (1982), Stockman (1983), Svensson and Razin (1983), and Greenwood (1983a,b) to address various problems in international finance.

<sup>3</sup>Note that it is the rate of appreciation in, and not the level of, the financial exchange rate that is relevant for the representative agent's decision-making. This is because the domestic real interest rate, which is central to both the individual's consumption-saving and money balance decisions, is determined by the rate of appreciation in the financial exchange rate.

<sup>4</sup>An alternative form of the dual exchange system is one which lets domestic residents repatriate interest income at the commercial exchange rate--see Flood and Marion (1982). The reader can check for himself that in the current model this would imply that the domestic real interest rate,  $r$ , is now given by the expression  $1+r = [1 + (\frac{\bar{e}}{s_2})i^*] / [(1+\pi)(1+f)]$ . Also, the opportunity cost of holding money,  $i/(1+i)$ , would now be described by the formula

$i/(1+i) = [(\frac{\bar{e}}{s_2})i^* - f]/[1 + (\frac{\bar{e}}{s_2})i^*]$ . This alternative setting involves no new substantive considerations in the analysis but it is algebraically more onerous.

<sup>5</sup>Note that the government's revenues in each period depend upon the spread between the values of the financial and commercial exchange rates. (Specifically in period one they depend upon  $s_1 - \bar{e}$  while in period two they depend on  $s_2 - \bar{e}$ .) If the financial and commercial exchange rates deviate too far apart from each other incentives will develop for private individuals to try to illegally arbitrage between these two rates. This may be one reason why the life expectancy of dual exchange rate systems is short relative to the more traditional exchange rate systems.

<sup>6</sup>Note that bond acquisitions or sales by the FERA should be included in the capital account, and not the balance of payments. Changes in the FERA's holdings of foreign bonds are not directly related to either changes in the money supply, or domestic credit as is the case with movements in the central bank's holdings of foreign reserves. The FERA is essentially a fiscal institution which levies imports duties--which may be negative--on the proceeds from domestic savings with the rest of the world. In the model, the FERA only purchases or sells foreign bonds to ensure that the additional second-period imports that private citizens have been saving for in the first period will be exactly provided for. That is, the FERA purchases or sales of foreign bonds are only meant to round-off any shortfalls or excesses in the purchases of foreign bonds by private agents. In other words, the FERA's transactions on the international bond market are only meant to backup the planning of private citizens. Thus, the FERA's acquisitions or sales of foreign bonds should be counted as capital account transactions.

<sup>7</sup>Also, it is being assumed that the rest of the world is characterized by perfect competition in all markets.

<sup>8</sup>In the present setup, it is easy to show formally that the relationship between  $f$  and  $\bar{c}_a$  is univalent, *ceteris paribus*.

<sup>9</sup>It might be the case that a large country could benefit from the imposition of a dual exchange rate system if it could affect the world real interest rate, or the terms of trade at which it borrows or lends. This is, of course, directly related to the optimum tariff question in the standard trade literature. Alternatively, if there were some distortions in a "small" open economy then a dual exchange rate system might be preferable to free trade in assets. However, it is hard to believe that there wouldn't be alternative commercial policies, aimed at ameliorating these distortions, that would be preferable to a dual exchange rate system or capital controls. The general rule of thumb seems to be that one should use the commercial policy that most directly affects the source of the distortion. This type of issue has been extensively addressed by trade theorists--for example see Bhagwati (1981).

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