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INFLATION, WELFARE AND THE CHOICE OF EXCHANGE RATE REGIMES

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Abstract

What effect does the choice of exchange rate regime have on a "small" open economy's real allocations and level of welfare? This question is discussed within the construct of a cash-in-advance economy. It is shown that the economy's steady state levels of consumption, labor force participation, output and welfare are inversely related to its steady-state inflation rate. Consequently, as is shown, a flexible exchange rate regime is preferable to a fixed one because it allows a nation to optimally choose its rate of inflation.

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In a recent paper in the Journal of Political Economy, Elhanan Helpman develops a simple two country model to compare the welfare levels that are achieved under different exchange rate regimes. In a world characterized by perfect foresight, his basic finding is that equilibrium consumption allocations, and hence welfare levels, are identical under a floating exchange rate system and a (one sided peg) fixed exchange rate regime. This result is due to Helpman's assumption that output is determined exogenously which allows a separation of the real and monetary sides of the economy. The purpose of this paper is to extend Helpman's model by making two alterations in his setup: (1) to allow production to be endogenously determined as a function of labor force participation and (2) to allow welfare levels to be dependent upon labor service as well as upon consumption. The main conclusion then obtained is that a country may achieve a higher welfare level under a flexible exchange rate regime than under a fixed exchange rate regime since it is able to choose an optimal rate of domestic inflation (or currency depreciation).

I The Model

Consider the following model of a "small" open economy inhabited by an immortal representative agent who is blessed with perfect foresight. The agent's goal is to maximize his lifetime utility, $\underline{U}(\cdot)$, as given by

$$\underline{U} = \Sigma_{t=0}^{\infty}\rho^{t}U(c_{Ht}+c_{Ft},l_{t}) \qquad \rho\varepsilon(0,1)$$

where ρ is the individual's (constant) subjective discount factor, $U(\cdot)$ his momentary utility function, c_{Ht} and c_{Ft} are his period t consumption of the home country's output and the foreign country's output respectively, and l_t is the quantity of labor services he supplies in period t. The momentary utility function, $U(\cdot)$, is assumed to be strictly quasi-concave and twice differentiable with consumption being a normal good and labor service being an inferior one.

The representative agent has two sources of income. His primary source of income is through the owner operation of a firm. The firm produces the consumption good using labor, l, supplied by the individual. The firm's output of this good, y, is described by the following production process

$$y_t = f(l_t)$$

where the function $f(\cdot)$ is taken to be twice differentiable and to exhibit diminishing returns to scale. Also, in each period t the individual receives a nominal transfer payment, X_t, from the domestic government (this transfer payment may be negative).

There is an international bond market in which the individual can freely participate. In this market he can issue (or buy) one period real bonds denominated in terms of the home produced good, b_H, or the foreign produced good, b_F. These two kinds of bonds pay the internationally determined real rates of return r_H and r_F

respectively. If, for instance, the representative agent were to issue domestically denominated bonds worth b_H units of the home produced good he would have to repay the equivalent of $(1+r_H)b_H$ units of the home produced good next period to the buyer of the bond.

In the model the individual must use currency to purchase goods. For example, if in period t the individual purchases c_{Ht} units of the home produced good this must be bought using currency from the individual's current holdings of domestic money, M_{Ht}. Likewise, in this period his purchases of the foreign produced good must be bought using currency from his holdings of foreign money, M_{Ft}.

A time profile of the individual's life in a typical period, t+1, will now be given so as to highlight the circulation of money in the model. The representative agent enters this period with a certain amount of domestic and foreign money to spend left over from the previous period,t. Now at the beginning of period t+1 the individual receives in domestic currency the income his firm accrued from sales during the previous period. This amounts to $P_{Ht}f(l_t)$ where P_{Ht} represents the domestic nominal price of the home produced good in period t. At this time the agent also receives a nominal transfer payment from the government in the amount X_{t+1}.

The individual then takes his cash to the international bond-cum-foreign exchange market and liquidates the debts that he incurred during the previous period. These debts now amount in nominal terms to $(1+r_{Ht})P_{Ht+1}b_{Ht} + (1+r_{Ft})e_{t+1}P_{Ft+1}b_{Ft}$ where P_{Ft+1} is the foreign nominal price of their output in period t+1 and e_{t+1} is the price in domestic currency of a unit of foreign currency in period t+1. After doing this the individual issues new domestic and foreign denominated real bonds worth in nominal terms $P_{Ht+1}b_{Ht+1}$ and

$e_{t+1}P_{Ft+1}b_{Ft+1}$ respectively. His resulting new holdings of cash are then allocated between holding domestic and foreign currency in the magnitudes M_{Ht+1} and M_{Ft+1}.

During the remainder of the period the individual uses his holdings of domestic and foreign currency to purchase his consumption quantities of the home produced good c_{Ht+1} and the foreign produced good, c_{Ft+1}. He also supplies labor input to his firm. The agent then enters period t+2 with $M_{Ht+1} - P_{Ht+1}c_{Ht+1}$ units of domestic currency and $M_{Ft+1} - P_{Ft+1}c_{Ft+1}$ units of foreign currency and the process begins again. It has of course been assumed in the above discussion that in period t+1 the individual made his decisions about how much money to hold, bonds to issue, goods to consume, and labor to supply in an optimal fashion. This is the matter to which we now turn.

Formally the recursive equation of the representative agent's dynamic programming problem is[1]

$$V(s_t;\, P_t) = \max_{\substack{c_{Ht},c_{Ft}\\ l_t,s_{t+1}}} \{U(c_{Ht} + c_{Ft},\, l_t) + \rho V(s_{t+1};\, P_{t+1})\}$$

subject to

$$\text{(1)} \quad \frac{M_{Ht+1}}{P_{Ht+1}} + \frac{e_{t+1}P_{Ft+1}}{P_{Ht+1}}\left[\frac{M_{Ft+1}}{P_{Ft+1}}\right] \leq \frac{P_{Ht}f(l_t)}{P_{Ht+1}} + \frac{P_{Ht}}{P_{Ht+1}}\left[\frac{M_{Ht}}{P_{Ht}} - c_{Ht}\right]$$

$$+ \frac{e_{t+1}P_{Ft}}{P_{Ht+1}}\left[\frac{M_{Ft}}{P_{Ft}} - c_{Ft}\right] - (1+r_{Ht})b_{Ht} - \frac{(1+r_{Ft})e_{t+1}P_{Ft+1}b_{Ft}}{P_{Ht+1}}$$

$$+ b_{Ht+1} + \frac{e_{t+1}P_{Ft+1}}{P_{Ht+1}}\, b_{Ft+1} + \frac{X_{Ht+1}}{P_{Ht+1}}$$

$$\text{(2)} \quad c_{Ht} \leq M_{Ht}/P_{Ht}$$

$$\text{(3)} \quad c_{Ft} \leq M_{Ft}/P_{Ft}$$

where $s_t \equiv (b_{Ht}, b_{Ft}, M_{Ht}/P_{Ht}, M_{Ft}/P_{Ft})$ and $P_t \equiv (P_{Ht}, P_{Ft}, e_t)$. The first constraint, or (1), is the individual's budget constraint while the two constraints (2) and (3) represent the agent's cash-in-advance constraints which reflect the fact that the individual must have currency to purchase goods.

The following set of seven Euler equations can be derived from the above dynamic programming problem in the usual manner where α_t, β_t and γ_t are the Kuhn-Tucker multipliers associated with the constraints (1), (2) and (3) respectively and the notation $\cdot_t$ means that the functional arguments are being evaluated at their date t values.[2]

$$\partial U(\cdot_t)/\partial c_{Ht} = \alpha_t P_{Ht}/P_{Ht+1} + \beta_t \tag{4}$$

$$\partial U(\cdot_t)/\partial c_{Ft} = \alpha_t P_{Ft} e_{t+1}/P_{Ht+1} + \gamma_t \tag{5}$$

$$\partial U(\cdot_t)/\partial l_t = -\alpha_t P_{Ht}/P_{Ht+1} f'(l_t) \tag{6}$$

$$\rho(\alpha_{t+1} P_{Ht+1}/P_{Ht+2} + \beta_{t+1}) = \alpha_t \tag{7}$$

$$\rho(\alpha_{t+1} P_{Ft+1} e_{t+2}/P_{Ht+2} + \gamma_{t+1}) = \alpha_t e_{t+1} P_{Ft+1}/P_{Ht+1} \tag{8}$$

$$\rho\alpha_{t+1}(1+r_{Ht+1}) = \alpha_t \tag{9}$$

$$\rho\alpha_{t+1}(1+r_{Ft+1})\frac{e_{t+2}P_{Ft+2}}{P_{Ht+2}} = \frac{\alpha_t e_{t+1} P_{Ft+1}}{P_{Ht+1}}. \tag{10}$$

The first two Euler equations, (4) and (5), set the marginal utilities of consuming home and foreign produced goods equal to their marginal costs. To be more explicit by way of an example, the marginal cost of consuming an extra unit of the home produced good would be the marginal (indirect) utility obtained from holding an additional unit of domestic real balances. In other words, the right-hand side of (4) turns out to be equal to $\partial V(\cdot t)/\partial(M_{Ht}/P_{Ht})$.

The fourth and fifth equations, (7) and (8), set the marginal benefits of holding extra units of domestic and foreign balances at the beginning of period t+1 equal to their marginal costs. For example, by holding an extra unit of domestic real balances at the beginning of t+1 the individual can increase his consumption of the home good by one unit during the remainder of the period. Thus the marginal benefit of holding an extra unit of domestic real balances at the beginning of t+1 is the gain in utility derived from consuming an extra unit of the home good in t+1. That is, the left-hand side of (7) is equivalent to $\rho\partial U(\cdot t+1)/\partial c_{Ht+1}$. The marginal cost of holding this unit of real balances, α_t, is the marginal (indirect) utility a bond would have yielded instead. In other words, $\alpha_t = -\rho\partial V(\cdot t+1)/\partial b_{Ht+1}$.

The next two Euler equations, (9) and (10), state that the marginal benefits from and costs of issuing domestic and foreign bonds must be equalized.[3] By issuing a domestic real bond in t+1 the individual can increase his consumption of the home good by one unit this period. Therefore the marginal cost, $-\alpha_t$, of issuing a domestic real bond at the beginning of t+1 is the gain in t+1 utility that this would allow, or $-\rho\partial U(\cdot t+1)/\partial c_{Ht+1}$. The benefit of issuing such a bond, as shown by minus the left-hand side of (9), is equal to the loss in t+2 utility that the individual will suffer when this bond must be repaid. That is, the left-hand side of this equation equals $-[(1+r_{Ht+1})\rho^2]\partial U(\cdot t+2)/\partial c_{Ht+2}$.

Lastly (6) sets the marginal disutility of work equal to the marginal benefit of such work. Using (4) and (7) this equation can be rewritten as:

$$\partial U(\cdot t)/\partial l_t = -[f'(l_t)P_{Ht}/P_{Ht+1}]\rho\partial U(\cdot t+1)/\partial c_{Ht+1}$$

An extra unit of work in period t leads to an increase in the nominal value of the firm's output in this period of $P_{Ht}f'(l_t)$. But the individual does not receive the firm's earnings until t+1. At this time the real value of these extra earnings will be $P_{Ht}f'(l_t)/P_{Ht+1}$. To see what this extra unit of labor is worth in utility terms the increment in the real value of the firm's earnings must be multiplied by the (discounted) marginal utility of home consumption in period t+1. Thus the expression on the right-hand side of the above equation is obtained. By defining π_{Ht} as the domestic rate of inflation in period t, so that $\pi_{Ht} \equiv P_{Ht+1}/P_{Ht} - 1$, the above equation can be rewritten as:

$$\text{(11)} \qquad \partial U(\ t)/\partial l_t = -[f'(l_t)/(1+\pi_{Ht})]\rho \partial U(\cdot\ t+1)/\partial c_{Ht+1}$$

II Purchasing Power and Interest Rate Parities

The Euler equations (4) to (10) may be used to show that the economy's equilibrium implies Purchasing Power Parity and Interest Rate Parity hold in each period. Equations (4) and (5) yield as the marginal rate of substitution between consuming home and foreign goods

$$\text{(12)} \qquad \frac{\partial U(\cdot_t)/\delta c_{Ht}}{\partial U(\cdot_t)/\delta c_{Ft}} = \frac{\alpha_t P_{Ht}/P_{Ht+1} + \beta_t}{\alpha_t P_{Ft} e_t/P_{Ht+1} + \gamma_t}$$

while equations (7) and (8) yield the condition

$$\text{(13)} \qquad \frac{\alpha_t P_{Ht}/P_{Ht+1} + \beta_t}{\alpha_t P_{Ft} e_t/P_{Ht+1} + \gamma_t} = \frac{P_{Ht}}{P_{Ft} e_t}.$$

Further, the assumption that c_H and c_F enter additively in the momentary utility function implies a constant, unitary marginal rate of substitution between consuming home and foreign goods so that it can be infered from (12) and (13) that Purchasing Power Parity holds or[4]

$$P_{Ht}/e_t P_{Ft} = 1.$$

Next consider equations (9) and (10) which yield

$$(1+r_{Ht}) = (1+r_{Ft})(e_{t+1}P_{Ft+1}/P_{Ht+1})(P_{Ht}/e_tP_{Ft})$$

Given Purchasing Power Parity, we then obtain Interest Rate Parity in terms of the real rates of return in each country[5]

$$(1+r_{Ht}) = (1+r_{Ft}).$$

This result obtains because the bonds are denominated in terms of home or foreign produced goods which are perfect substitutes in consumption. A positive (or negative) differential between the home and foreign real rates of return would allow our representative agent to make an unlimited profit by buying (selling) domestic denominated bonds while issuing foreign denominated real bonds worth the same amount in terms of the consumption good. The individual could then pocket the positive difference between the real interest payments he would earn and owe as a result of the above transactions. The real rates of return on domestic and foreign denominated bonds must equalize to prevent such obvious arbitrage between the two types of bonds and the unbounded profit opportunity that this allows.

III The Steady State Equilibrium

For equilibrium in the model to hold the goods market must clear each period or

(14) $$c_{Ht} + c^{*}_{Ht} = f(l_t) \qquad \forall\ t=1,\ldots,\infty$$

where c^*_{Ht} is the period t foreign demand for the home produced good. Likewise the money market must always clear so that

$$(15) \qquad M_{Ht} + M^*_{Ht} = M^s_{Ht} \qquad \forall\ t=1,\ldots,\infty$$

where M^*_{Ht} is the foreign demand for domestic currency in period t and M^s_{Ht} is the supply of domestic currency in this period. Lastly,[6] the government's budget constraint for period t is

$$(16) \qquad X_t = M^s_{Ht} - M^s_{Ht-1} \qquad \forall\ t=1,\ldots,\infty.$$

The model's steady state equilibrium is characterized by constant (over time) values for all real variables, the domestic and foreign inflation rates, and for the Kuhn-Tucker multipliers.[7] Thus dropping time subscripts on variables so as to denote their steady state values one can rewrite equation (11) as

$$(17) \qquad (\partial U/\partial c_H)/(\partial U/\partial l) = -(1+\pi_H)/\rho f'(1).$$

In the steady state equilibrium the representative agent's cash-in-advance constraints hold as strict equalities[8] so that his budget constraint (1) can be written as

$$c = f(1) - f(1)[\pi_H/(1+\pi_H)] + (X/P_H) - rb$$

where $c \equiv c_H + c_F$ and $b \equiv b_H + b_F$. The term $f(1)[\pi_H/(1+\pi_H)]$ represents the tax that inflation levies on the individual. This inflation tax term arises in the model because the firm is constrained to accumulate and hold cash over each period. However, in the model's general equilibrium the negative impact this tax has on real income is exactly offset by the real value of the transfer payments the individual receives from the government. The real value of these transfer payments, (X/P_H), is equal to the amount of real revenue, $[\pi_H/(1+\pi_H)](M^S/P_H)$, that the goverment earns through money creation--a fact easily deduced from the goverment's budget constraint (16). Thus,

$$(X/P_H) = [\pi_H/(1+\pi_H)](M^S/P_H).$$

Since in the steady state both domestic and foreign individuals only hold domestic currency so as to purchase home goods (i.e. $P_{Ht}c_{Ht} = M_{Ht}$ and $P_{Ht}c^*_{Ht} = M^*_{Ht}$ for all t) it immediately follows from (14) and (15) that

$$(X/P_H) = f(1)[\pi_H/(1+\pi_H)].$$

Thus in the model's steady state equilibrium the representative agent's budget constraint will look like

$$c = f(1) - rb. \tag{18}$$

Equations (17) and (18) implicitly define solutions for the steady state values of consumption, c, and labor services, 1. It can therefore be seen that the direct effect of inflation on the consumer's consumption-labor supply decision must occur solely through the manner in which it affects the margin of substitution between consuming and working as given by the right-hand side of (17).

Formally, an increase in the domestic rate of inflation,[9] π_H, leads to a decline in the representative agent's steady state participation in the labor force and consumption:[10,11]

$$(19) \qquad dl/d\pi_H < 0$$

$$(20) \qquad dc/d\pi_H = f'(l)(dl/d\pi_H) < 0.$$

The economic intuition behind these results is straightforward. Recall that the firm holds the individual's nominal earnings for one period before distributing them to him. A high rate of inflation will cause the purchasing power of these earnings to be reduced when the individual actually receives them. Thus when inflation rises the value of an extra unit of work to the individual is diminished. The individual will therefore reduce his labor supply and consequently there will be a fall in output and consumption.[12] Market activity--here, the operation of a firm--requires the use of money and is taxed by inflation while non-market activity--leisure--does not require the use of money and hence escapes the inflation levy. As a result when the rate of inflation rises the individual moves out of the market activities of production and consumption and into the non-market activity of leisure.

IV Fixed versus Flexible Exchange Rates

In the steady state the representative agent's equilibrium amounts of consumption, c, and labor supplied, l, were shown to be decreasing functions of the domestic inflation rate, π_H, or

$$c = c(\underset{-}{\pi_H})$$

$$l = l(\underset{-}{\pi_H}).$$

Thus the agent's steady state equilibrium level of welfare, W, may be written as

$$W = U[c(\underset{-}{\pi_H}), l(\underset{+}{\pi_H})]. \tag{21}$$

Now purchasing power parity states that the following must hold

$$P_{Ht} = e_t P_{Ft} \qquad \forall\ t=1,\ldots,\infty$$

which implies that

$$\pi_{Ht} = \pi_{Ft} + \delta_t + \pi_{Ft}\delta_t \qquad \forall\ t=1,\ldots,\infty \tag{22}$$

where $\pi_{Ft} \equiv (P_{Ft+1}-P_{Ft})/P_{Ft}$ is the foreign inflation rate in period t and $\delta_t \equiv (e_{t+1}-e_t)/e_t$ is the rate of depreciation of the exchange rate in period t. Hence with fixed exchange rates (i.e. $\delta = 0$) it must be the case that the domestic and foreign countries have the same inflation rate, or that $\pi_H = \pi_F$. Consequently, with fixed exchange rates the domestic economy has its welfare level, W, exogenously determined by the foreign inflation rate, π_F:

$$W = U[c(\pi_F), l(\pi_F)].$$

Under a flexible exchange rate regime, however, the "small" domestic economy may choose the rate of domestic inflation, π_H (or equivalently the rate of currency depreciation, δ), which maximizes domestic welfare, W. Maximizing the domestic welfare function (21) with respect to the domestic rate of inflation yields

$$dW/d\pi_H = (\partial U/\partial c)(dc/d\pi_H) + (\partial U/\partial l)(dl/d\pi_H) = 0.$$

Using the result in (20) that $dc/d\pi_H = f'(l)(dl/d\pi_H)$ we have the condition that

$$(\partial U/\partial c)/(\partial U/\partial l) = -1/f'(l).$$

Now comparing this result to equation (17) we see that to maximize welfare we must set

$$(1+r)(1+\pi_H) = 1$$

or

$$\pi_H = -r/(1+r). \tag{23}$$

This is the familiar optimum quantity of money result as discussed in Friedman(1969). Let us define the variable i_H as

$$i_H = \pi_H + r + \pi_H r.$$

It can then be seen that the above rule for the optimum quantity of money would set the opportunity cost of holding money, or $-i_H/(1+i_H)$, equal to zero.

As has been mentioned when the domestic economy picks its inflation rate, π_H, it is also simultaneously picking its rate of exchange rate depreciation, δ--a fact evident from (22). In particular (22) and (23) tell us that when the domestic country follows the rule for obtaining the optimum quantity of money its exchange rate will appreciate at the rate

$$-\delta = i_F/(1+i_F) \qquad i_F \equiv \pi_F + r + \pi_F r$$

where the right-hand side of the above equation is the opportunity cost of holding foreign currency.

Thus unless the foreign country is adhering to the rule for the optimum quantity of money the domestic country should adopt a flexible exchange rate system and follow the rule itself. This allows the domestic economy to maximize its own welfare as opposed to the adoption of a fixed exchange rate system which constrains it to accept the foreign suboptimal rate of inflation and the associated inferior level of welfare.

V Conclusions

This paper has extended Helpman's model by allowing production to be endogenous and welfare levels to be dependent upon labor service as well as consumption. Higher inflation reduces both consumption and labor service and, if the inflation rate exceeds the optimal rate, reduces the economy's welfare level as well. A case for flexible exchange rates may be made unless the foreign country maximizes its welfare level by choosing the optimal rate of inflation. Then the choice between adopting a system of fixed or flexible exchange rates is a matter of indifference.

Appendix

As was mentioned in the text equations (17) and (18) implicitly define solutions for the steady state values of c and l. By totally differentiating these equations and then solving the effects of a change in π_H on l can be uncovered. The result of this exercise is

$$\frac{\partial l}{\partial \pi_H} = \frac{\partial l/\partial \pi \big|_{u=\tilde{u}}}{1 - [i_H/(1+i_H)]f'(l)\partial l/\partial y - f''(l)\partial l/\partial f'(l)\big|_{u=\tilde{u}}} < 0$$

where $\partial l/\partial \pi_H\big|_{u=\tilde{u}}$ and $\partial l/\partial f'(l)\big|_{u=\tilde{u}}$ show the substitution effects of a change in the domestic inflation rate and the marginal product of labor on labor supply (around the equilibrium level of utility, $\tilde{u}$), respectively, and $\partial l/\partial y$ shows the effect of an increase in real income on labor supply.

The above expression for $\partial l/\partial \pi_H$ is unambiguously negative. The numerator of the above solution is negative showing that a rise in inflation causes a substitution effect which leads to a drop in labor supply. That is, when the inflation rate rises the agent substitutes out of market activity--production and consumption--and into non-market activity--leisure. The denominator of the expression is unambiguously positive since all of the terms composing it are positive. The last term in the denominator will be discussed first. As the rate of inflation rises the individual cuts down on the amount of labor he supplies to his firm. But this causes the marginal product of labor, $f'(l)$, to rise. Specifically, a unitary fall in labor service causes the marginal product of labor to rise by $f''(l)$. But this rise in the marginal product of labor causes a substitution effect which makes the individual work more. That is, now working is more profitable and this increases the individual's labor force participation mitigating the adverse effect that a rise in the inflation rate had on the agent's labor supply decision.

The second term in the denominator can be explained intuitively as follows. In the model the individual's opportunity cost of holding real balances, $(i_H/1+i_H)$, is not equal to the social cost of holding money, which is zero. This causes the individual's marginal rate of substitution between consuming and working, $(\partial U/\partial l)/(\partial U/\partial c_H)$, to differ from the rate at which the economy can transform labor into consumption as given by the marginal product of labor, $f'(l)$. Consequently for each unit of labor that the individual cuts back on due to the increase in the rate of inflation there is a loss in the agent's welfare (when measured in terms of the consumption good) of the amount $(i_H/1+i_H)f'(l)$. In effect the term $(i_H/1+i_H)$ represents the implicit tax which is levied on production due to the requirement that money be used to buy goods. The loss in the agent's real welfare, mentioned above, causes the agent to work more and consequently has an offsetting effect on the drop in labor service caused by the rise in the inflation rate. Note that if the economy had been following the optimum quantity of money rule (i.e., $i_H= 0$) the term in question would have disappeared from the denominator of the above expression.

Footnotes

(1) It is being assumed that the rest of the world is facing an analogous maximization problem.

(2) Stockman(1981) illustrates, in a similar context, how the Euler equations for a representative agent's dynamic programming problem can be obtained.

(3) The Euler equations (4), (7) and (9) tell one that when the agent's consumption-saving decisions are being made optimally the following condition must hold

$$\partial U(\cdot t)/\partial c_{Ht} = [\rho(1+r_{Ht})]\partial U(\cdot t+1)/\partial c_{Ht+1}. \qquad \forall\ t=1,\ldots,\infty.$$

This equation implies that the marginal rate of substitution between current and future (home) consumption is equal to the market rate of transformation between current and future (home) consumption. This sort of condition is derived and more fully discussed in Lucas(1978).

(4) Equation (12) and the Purchasing Power Parity condition would then imply that $\beta_t = \gamma_t$ for all t.

(5) For notational convenience in the remainder of this paper

the following definition will be employed: $r_t \equiv r_{Ht} = r_{Ft}$.

(6) Obviously, an analogous set of equilibrium conditions will hold for the rest of the world.

(7) Constancy of the Kuhn-Tucker multipliers may be derived in the following manner. Constancy of c_{Ht}, c_{Ft} and l_t implies the constancy of the marginal utility of consumption of the domestic good and thereby of the right-hand side of equation (4). Using this result, equation (7) then ensures that $\alpha_t = \alpha$ (a constant). Employing equation (4) once again and recognizing that the domestic rate of inflation is constant allows the result that $\beta_t = \beta$. Finally, since $\beta_t = \gamma_t$ (see footnote 3), $\gamma_t = \gamma$.

Next, note that equation (7) ensures the $\beta \neq 0$ unless $\rho = (1+\pi_H)$. Suppose $\beta = 0$. Then equation (7) may be rewritten as $\alpha_{t+1} = \alpha_t(1+\pi_H)/\rho$. Unless $\rho = (1+\pi_H)$, $\alpha_{t+1} \neq \alpha_t$, contradicting the result in the previous paragraph. For constancy of the domestic and foreign inflation rates see footnote (8).

Note also equation (9) implies that $\rho = (1+r)^{-1}$.

(8) Note that the general condition $\beta \neq 0$ (see footnote 5) implies $P_{Ht}c_{Ht} = M_{Ht}$ while the analogous general condition for the foreign individual, $\beta^* \neq 0$, implies $P_{Ht}c^*_{Ht} = M^*_{Ht}$. Since $c_{Ht} + c^*_{Ht} = c$ is constant in the steady state, and also $M_{Ht} + M^*_{Ht} = M^S_t$, the domestic inflation rate equals the steady state growth rate of the domestic money stock. The same result can easily be shown to hold for the foreign country as well.

(9) Note that the foreign inflation rate does not enter into the system of equations (17) and (18). This is because domestic residents--consumers and firms--do not have to hold foreign currency over adjacent time periods. Consequently, the rate of foreign inflation inflicts no burden on domestic residents.

(10) A more detailed discussion of these results is contained in the Appendix.

(11) It can be shown that the domestic economy jumps immediately to the new steady state and that individuals do not change their holdings of real bonds.

(12) Note the implication of a positively sloped long run Phillips curve. For a more general discussion see Stockman(1981) or Aschauer(1981).

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