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by

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ABSTRACT

The Hopkinson rate is the most common of all rate structures used for pricing electricity for industrial use. It consists of an "energy charge" for total kilowatt hour consumption plus an additional "demand charge" based on the maximum usage by the plant during any quarter-hour period during the month. Despite its apparent drawbacks, it is shown that it can have optimal properties in the pricing of demand variance. The analysis is greatly simplified by the use of the extreme value probability distribution for peak demand. This device is also used to comment on optimal generation capacity calculations and as the basis for some econometric analysis of the effect of the Hopkinson rate on the peak demands of a sample of eight Ontario pulp and paper mills between 1970 and 1977.

I. Introduction

Electricity for industrial use comprises roughly 40 per cent of total North American electricity consumption.[1] As industrial electricity rates may therefore substantially affect total electricity demand, an understanding of the impact of various rate structures is very important in utility planning. The nature of the rate structure will influence the desired amount of generation capacity, the probability and effects of a shortage and the consumption patterns of users as well as other aspects of the economic efficiency of the electricity system.

There has been considerable economic research in the electricity area. Much of the theoretical work has focused on optimal pricing when generation capacity is sometimes (but not always) a binding constraint. Early contributions towards solving this "peak load pricing" problem were made by Boiteux (1949) and Steiner (1957). Subsequent research by Brown and Johnson (1969), Meyer (1975), Sherman and Visscher (1976), Crew and Kleindorfer (1978), Dansby (1979) and Koenker (1979) (to cite just a small fraction of a very large literature) has extended the original Boiteux-Steiner analysis, including taking it from a deterministic to a stochastic context.

A second branch of the research has concentrated on the empirical analysis of experiments with residential time-of-use (TOU) rates. These experiments have been conducted in several U.S. states with subsequent econometric studies (e.g., Aigner and Hausman (1980), Atkinson (1979), Hausman, Kinnucan and McFadden (1979), Hendricks, Koenker and Poirier (1979) and Lawrence and Braithwait (1979)). These studies have been notable not only because of the potential importance of their results but also for their econometric innovation in dealing with such problems as the self-selection bias present in voluntary experiments or the non-linear budget constraints of declining block rates.

However as noted by Taylor (1975), there has been much less work specific to the industrial demand for electricity. Moreover the research that has been done in this area is mostly on TOU rates (see Denton (1979), Panzar and Willig (1979) and Chung and Aigner (1980)). While these rates represent an important pricing innovation, they are not nearly so common as the Hopkinson rate which has received little attention in modern research. This latter rate structure will be the focus of this paper.

The Hopkinson rate consists of an "energy charge" for every kilowatt hour a customer uses plus an additional "demand charge", a peak demand charge on the maximum usage by the plant for any 15 minute period during the month.[2] Kahn (1970, p. 95) writes that the Hopkinson rate is "almost universally used by electric and gas utilities for large-volume sales at wholesale and to industrial users". More recently, Mitchell, Manning and Acton (1978, p. 13) indicate that the Hopkinson rate is employed by nearly all North American utilities and note (Chapter 4) that it has much in common with pricing methods used in several European nations.

Why is this rate system used so widely? The most likely answer is that it seemed to be a simple method of dividing the capacity costs according to one view of each customer's capacity requirement.[3] The problem is that the maximum demand charge is based on individual peak demand which may not be related to system peak. For example, consider a user whose peak demand is normally at 7:00 a.m. when system peak is at 7:00 p.m. This user faces an incorrect incentive to use less electricity when there is idle generation capacity but has no special incentive to reduce usage during the peak period when capacity may be strained. (The rate may even cause the user to shift consumption into the system peak period.)[4] Because of this, most economists advocate TOU rates of some form.[5]

However, instead of viewing a firm's peak demand and its "coincident demand" (consumption at the time of system peak) as substitutes, suppose they are both outcomes of the same stochastic process and that the firm's choice variables are in effect the parameters of that process. Then as the optimal level of capacity is a function of each user's demand variance (as in Boiteux (1951) and Drèze (1964)) and each individual peak is also a function of that user's demand variance, the Hopkinson rate's individual maximum demand charge can be used to price each user's contribution to system variance and hence its marginal effect on the costs of capacity provision. While this argument does not provide much support for the Hopkinson rate as commonly applied, it may justify its use in combination with TOU rates when the demand charge is for maximum quarter-hour usage during the "on-peak" period. This kind of Hopkinson rate was part of the TOU rate structure introduced in 1978 for large users by two large California utilities, Pacific Gas and Electric and Southern California Edison.

The formal analysis of the above argument is contained in Section II. Each peak demand is modelled as the maximal order statistic drawn from a large number of "peak-eligible" 15 minute demands, suggesting the use of the extreme value probability distribution for peak demand. This approach is employed to study the Hopkinson rate as well as to comment on some general problems of forecasting peak demand. It should be emphasized that the intention here is not to provide exact solutions to real-world peak load pricing problems, which will generally require detailed utility-specific empirical analysis. Instead this work explains one reason why individual peak demand charges may be efficient and suggests ways in which their application may be improved.

Section III estimates the effect of the Hopkinson rate on the peak demands of several Ontario pulp and paper mills. The estimation procedure maximizes a likelihood function which is based on the extreme value distribution discussed above. The results indicate that the Hopkinson rate has a small but significant dampening effect on individual peak demand.

Section IV presents the summary and conclusions of this research.

II. The Hopkinson Rate

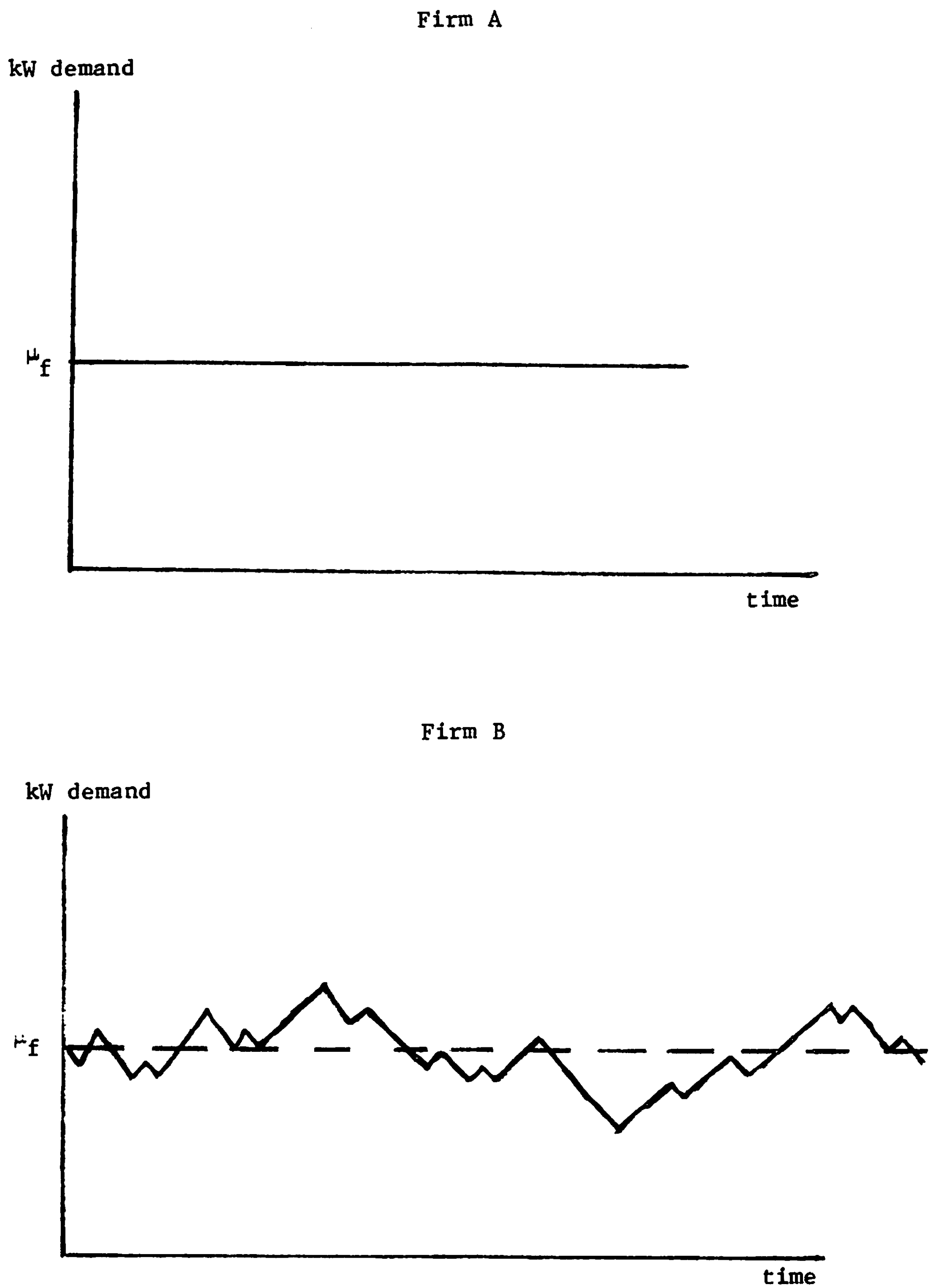
Outline

The purpose of this Section is to illustrate a case in which an individual peak demand charge may be desirable, even though individual maximum usage is not coincident with system peak. The basic argument is outlined by Figure 1. Firms A and B have the same overall consumption over the peak period but firm B has a demand which varies stochastically (but not systematically with time of use). If all J firms in the economy were identical to A then their demands could all be met with capacity equal to $J\mu_f$. But if some of the plants were like B, a higher capacity would be required to meet the peak of the fluctuating demand. Note that if the only charge were the straight kW•h tariff, both A and B would have the same electricity bill. But with an additional peak demand charge, B would pay a higher bill as would seem to be appropriate.

The reasoning behind this is that the expected system peak over some period is a function of the variance of system demand and hence of each user's demand variance. As individual peak is also a function of each user's demand variance, the individual maximum demand charge can be used

Figure 1

Hypothetical Demand Patterns During Peak Period



to price each user's contribution to system variance and hence its marginal effect on the capacity cost of meeting expected system peak.

A charge based on coincident demand (the usage by the individual firm at the time system peak occurs) could also be used to price correctly the impact of the firm's actions on expected system peak, as system peak is just the sum of all the coincident demands. However an individual peak demand charge can be more efficient. This is because the utility will generally provide some reserve over expected peak and the optimal size of this reserve will clearly vary with the variance of system demand. The individual maximum demand charge can therefore be used to price the effect of the user's variance on the costs of providing optimal reserve capacity as well as on the costs of the capacity to meet expected system peak.

Clearly if there are many users with small consumption variance and uncorrelated demands, the effects of an individual user's variation on total system variation will be small. (This is analogous to "risk-pooling" in insurance markets.) However if users are large or their demands are correlated, variance charges may be important.

A simplifying assumption used in this paper is that the firm's only choice variables are the parameters of <u>one</u> stochastic process which generates <u>both</u> individual peak demand and coincident demand. In this sense individual peak and coincident demand will be "perfect complements" and the Hopkinson rate can be a first best measure. If peak demand and coincident demand were generated by related but not identical stochastic processes then the Hopkinson rate could still be more efficient than a flat kilowatt hour charge but could generally only be a second best solution.

The idea of variance charges has a long history, originating with the French marginalists of the 1950's. In his summary of this work, Drèze (1964) builds on results he attributes to Boiteux and calculates optimal rates for user variance in a simple one-period electricity demand model. Marchand (1974) extends Drèze's work to a situation where users buy "interruptible" electricity and can choose among several types of supply, each differentiated by the probability with which delivery is guaranteed. Marchand calculates the optimal prices associated with these different kinds of contracts.

The main innovation here is that these concepts are related to the most common existing rate structure, the Hopkinson rate. There are no real-world examples of Drèze's pure variance charges and while the contracts modelled by Marchand are provided by many utilities, they generally cover a much smaller fraction of electricity sales than the Hopkinson rate. Also, unlike this study, Drèze and Marchand assume that each user's demand is uncorrelated with every other user's and that all demand occurs during one period, which therefore must provide peak demand. Marchand assumes that each user's demand is uniformly distributed; in this paper the normal distribution will usually be assumed, although in some cases this can be relaxed to allow any distribution with finite mean and variance.

The Model

The formal model will aim at providing an example of the calculation of an optimal capacity charge. It is assumed that the utility has three choice variables: total generation capacity C, the price of electricity per kilowatt hour $P^{kW.h}$ and the price per kilowatt of the maximum 15 minute demand P^{kW}. There is no treatment of the problems of transmission or the optimal mix of peaking, baseload and intermediate capacity.[6] Variable costs (e.g. fuel costs) are assumed to be billed as separate kilowatt hour charges so that $P^{kW \cdot h}$ and P^{kW} are therefore pure capacity charges.

There are T quarter-hour periods in a month which are "peak-eligible" (i.e. periods when a system peak can occur) and the stochastic system demands x_i during these periods have identical probability distributions $f(x_i)$ with mean μ and variance σ^2. The uncertainty associated with capacity unreliability could be accommodated by incorporating stochastic generation failure as a component of system demand. However for simplicity it is assumed that the marginal capacity is completely reliable so that incremental changes in C do not affect the parameters of f.

It is possible that demand will exceed available capacity. It can be shown that under the sufficient assumption that the costs of a shortage are proportional to the size of the shortage, the optimal rule for the utility is to build capacity to ensure some probability q that demand will not exceed capacity for the entire month. (Probability q will in general be a function of the costs of shortage and of generation capacity and in what follows it is assumed that q is close to one.) The analysis then continues using the extreme value distribution.

Suppose a researcher recorded the heights of all the boys in a number of similar sixth grade classes. It might be expected that the distributions for each of the classes would be approximately the same and would likely be normal. However, if the researcher now examined only the distribution of the heights of the tallest boy in each class, the normal distribution would no longer be appropriate. In fact, if each class were large enough the appropriate distribution would almost always have one particular form: that of the extreme value distribution.

Similarly, consider the peak monthly demand x^{max} as the maximal order statistic from the quarter-hour demands $x_1, \ldots, x_T$. Gnedenko (1943) has shown that if x^{max} has a limiting distribution as T increases, it must take one of three forms. If for example the x_i's have the normal distribution, the probability distribution function of x^{max} is:

$$G(x^{max}) = \exp(-\exp - (x^{max} - \ell_T)/s_T) \qquad (1)$$

where ℓ_T is the mode (the location parameter) and s_T is a scale parameter called the intensity. The distribution is called the extreme value, double exponential or Gumbel distribution[7] and has had many engineering applications to such natural peak problems as flood probabilities (see Gumbel (1958)).

Its parameters ℓ_T and s_T can be characterized in terms of μ and σ of the parent normal[8] as in Berman (1964):

$$\ell_T = \mu + b_T\sigma$$
$$s_T = \sigma_T\sigma$$

where

$$a_T = (2\log T)^{-(1/2)}$$
$$b_T = (2\log T)^{(1/2)} - 1/2(2\log T)^{-(1/2)}(\log\log T + \log 4\pi) \qquad (2)$$

It might be thought that the existence of the asymptotic distribution (1) requires that the x_i's be independent, which would be unlikely in this case. However (1) will hold without such independence, provided the dependency tends to vanish as the separation between underlying events increases. More formally, Loynes (1965) shows that an extreme value distribution above is applicable provided the following "strong mixing" condition holds for the parents:[9]

$$|F(x_{i+j}|x_i) - F(x_{i+j})| \le g(j) \qquad (3)$$

where $g(j) \to 0$ as $j \to \infty$. Such a "decay of dependence" condition is satisfied for example by all ARMA processes. However it should be noted that such dependence can slow convergence to the asymptotic distribution, particularly in the tails.[10]

The calculation of C* as a function of T and q is straightforward. As G(C*) = q, (1) can be inverted to yield:

$$(C^* - \ell_T)/sT = -\log(-\log q) \qquad (4)$$

or using (2):

$$C^* = \mu + k\sigma$$

$$\text{where } k = b_T - a_T \log(-\log q) \qquad (4')$$

Note that the expectation of a random variable with the extreme value distribution (Hastings and Peacock (1978, p. 60) is $\ell_T + \gamma s_T$ where γ is Euler's constant (approximately .57721) so that using (2):

$$E(x^{max}) = \mu + k^e\sigma$$

$$\text{where } k^e = b_T + \gamma a_T \qquad (5)$$

The difference between C* and $E(x^{max})$ (equal to $a_T(-\log(-\log q) - \gamma)\sigma$) might be called the "optimal reserve margin". As an example if T = 160 and q = .99, $a_T = .31$ and $b_T = 2.53$ so C* is $\mu + 3.99\sigma$ and $E(x^{max})$ is $\mu + 2.71\sigma$.

Now consider an individual industrial electricity user firm f. Suppose that the charges considered here are only applied to individual demands during the T system peak eligible periods and that all these individual demands $x_{f,1},\ldots,x_{f,T}$ are identically distributed with mean μ_f and variance σ_f^2, both choice variables for the firm.[11] In addition assume that the probability of shortage will be small enough that the firm ignores it in its cost minimization.

Suppose that the firms pay a charge for both the mean μ_f and the standard deviation σ_f of its electricity usage and it takes these prices as given. Changes in μ_f and σ_f will affect μ and σ and hence C*. Equating the marginal costs of μ_f and σ_f to the utility to the marginal effects on user f's electricity bill yields optimal prices (see Appendix 1) for user f of:

$$P^{\mu_f} = C'$$
$$P^{\sigma_f} = C'k\left(\frac{\sigma_f + \rho\sigma_s}{\sigma}\right) \qquad (6)$$

where C' is the marginal cost of one unit of capacity, σ_s^2 is the variance of demand for the rest of the system and ρ is the correlation between the demand of user f and demand of the rest of the system during system peak eligible periods.

As a one unit increase in mean demand will increase C* by one unit, P^{μ_f} is as expected. P^{σ_f} is more complicated and depends on the contribution of user f's variance to system variance.

Prices (6) are analogous to those in Drèze (1964) for the case of independent demands. To find corresponding Hopkinson rates, assume the $x_{f,i}$'s are strong mixing and normally distributed, so that individual peak demand therefore has the extreme value distribution. Following (5), $E(x_f^{max})$ is $\mu_f + k^e\sigma_f$ and the expected value of the firm's electricity bill[12] is therefore

$$P^{kW.h}\, T\mu_f + P^{kW}(\mu_f + k^e\sigma_f) \qquad (7)$$

Assuming the firm minimizes expected costs, P^{μ_f} and P^{σ_f} of (6) may therefore be replaced by

$$P^{kW} = C' \frac{k}{k^e} \left(\frac{\sigma_f + \rho\sigma_s}{\sigma}\right)$$
$$P^{kW \cdot h} = (C' - P^{kW})/T \qquad (8)$$

where $P^{kW \cdot h}$ could be negative. It is shown in Appendix 1 that price sets (6) and (8) will each be "break-even" in the sense that expected returns cover costs if electricity generation capacity has constant marginal cost.

Consider first the case when user and system demand are perfectly correlated. In this case system peak and individual peak are exactly coincident. For this example, a one unit increase in individual peak increases utilized capacity by one unit so it might be thought that P^{kW} should be exactly C'. However, optimal P^{kW} is $C' \cdot \frac{k}{k^e}$ which will generally be greater than C'.[13] This is because an increase in individual peak may reflect an increase in user variance which will increase system variance. With a loss of load probability rule such an increase in system variance will raise the optimal reserve margin.

Now consider (8) when user and system demands are perfectly uncorrelated so that $\rho = 0$. In this case if a user's variance is very small relative to system variance then P^{kW} will be very small. This reflects the "risk pooling" associated with uncorrelated events. However if ρ is non-zero, even if σ_f is small P^{kW} of (8) becomes $C' \cdot \frac{k}{k^e} \rho$ which could be substantial.[14]

This sort of analysis might be applicable to a situation where a common set of electricity prices is applied to a large group of similar customers. However if prices like (6) or (8) were applied to individual customers who realized how the prices were determined, the result would be inefficient, as Drèze (1964) points out in a similar context. This is because the user will perceive that its electricity prices are functions of σ_f, a parameter under its control. In this situation, these prices violate the MRS = MRT condition (see Appendix 1).

This complicates matters, but only slightly. If one assumes (as Drèze does and as is implicit above) that user f does not consider the second order effect of its actions on system variance or on the probability of shortage, then it is shown in Appendix 1 that the optimal P^{kW} and $P^{kW \cdot h}$ are:

$$P^{kW} = C' \frac{k}{k^e} \left(\frac{\sigma_f + 2\rho\sigma_s}{2\sigma}\right) \tag{9}$$

$$P^{kW \cdot h} = (C' - P^{kW})/T$$

where $P^{kW \cdot h}$ could be negative. Note that P^{kW} will generally be smaller than in (8).

The prices in (8) covered expected costs if electricity generation capacity has constant marginal cost. In contrast, price set (9) will generate an expected loss. This is because even with constant per unit capacity costs, electricity generation with uncertainty will still have decreasing costs because of the risk pooling effect of adding random demands together (see Drèze (1964), p. 22). This fundamental non-convexity could make efficient pricing difficult for a utility with a "no-deficit" constraint although in reality, this effect may be offset as unit capacity costs may be increasing.

Two further points about prices (9) are shown in the Appendix. First these prices are also efficient if ρ is one of the firm's choice parameters as well as μ_f and σ_f. Second, in the reasonable case where σ_f is small relative to σ then (9) is equivalent to

$$P^{kW} = C' \frac{k}{k^e} \left(\frac{X_f^c - \bar{X}_f}{X_f^{max} - \bar{X}_f}\right) \tag{10}$$

$$P^{kW \cdot h} = (C' - P^{kW})/T$$

where X_f^c is coincident demand and $\bar{X}_f$ is the firm's average demand. Presumably (10) could be the basis for a Hopkinson rate schedule where the prices would be based on X_f^c and X_f^{max} and would be efficient. Turvey and Anderson (1977, Chapter 16) indicate that some utilities do use coincident demands to calculate P^{kW}.

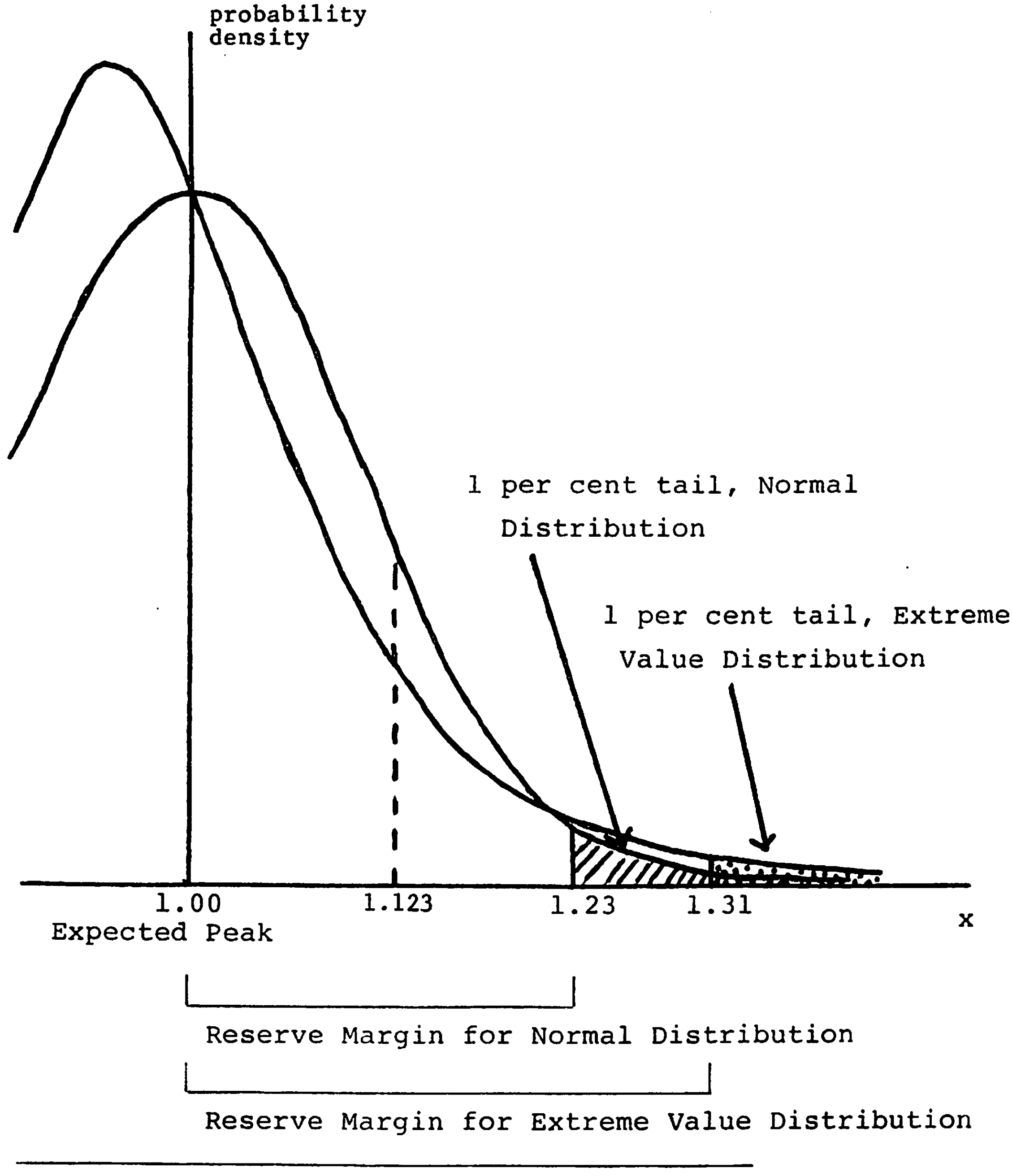
This section has shown that individual peak demand charges can be efficient in some circumstances.[15] The following uses the same methods to discuss optimal capacity.

Optimal Capacity

Again assume that the utility sets capacity so that the loss of load probability is q . Assume also that the utility knows the exact mean and variance of peak demand but uses the normal distribution when the extreme value distribution would be more appropriate. Under these conditions the utility will construct too little capacity because it has not allowed for skewness in the distribution of peak demand (see Figure 2). With a loss of load probability of .01 and a mean-variance ratio of 10:1, the optimal capacity under the extreme value distribution assumption exceeds the capacity that will be built by 7%. In terms of the reserve margin over expected peak, this means that the optimal reserve is 35% larger than that actually built. If the loss of load probability is only .001 with the same mean-variance ratio, optimal capacity will be 14% greater than the actual capacity or the optimal reserve will be 60% greater than the actual reserve.[16] However, it can also

Figure 2

Optimal Capacity when Peak
has Extreme Value Distribution



For this diagram, the mean variance ratio is 10:1 and the loss of load probability is .01. 1.123 is the boundary of the 11 per cent tail for each distribution.

be seen in Figure 2 that for higher loss of load probabilities, it is possible that the optimal reserve will be smaller than the actual reserve.

There are also implications for forecasting future peak load capacities. Standard utility practice is now to spend considerable effort in forecasting average demand for some time in the future and then obtain a forecast of peak demand by multiplying by some constant factor. This technique ignores that there may be a change in demand variance which will lead to a divergence in the trend of average demand and peak demand. Such a change could be due, for example, to the increase in air conditioning usage which will make demand fluctuate more with the weather.

As a simple numerical example, suppose that consumption consists of two normally distributed components: one with mean 60 and a standard deviation of 12 and the other with mean 40 and a standard deviation of 16. Assuming the demands are independent, their sum has mean 100 and standard deviation 20. Assuming the remaining conditions to be as in the previous numerical example ($T = 160$ and $q = .99$), expected peak demand is $100 + (2.716)(20) = 154.32$ and the ratio of expected peak demand to expected demand will be 1.5432. Now suppose the first component's mean grows to 80 and the second's mean grows more rapidly and becomes 75. The same mean/standard deviation ratios are kept so that the standard deviations become 16 and 30 respectively. Total demand now has mean 155 and standard deviation 34. Expected peak demand is now 247.34 and the ratio of expected peak demand to expected demand has grown to 247.34/155 = 1.5958. The more rapid growth in the high-variance usage has led to an increase in the ratio of expected peak to expected demand of about 3.4 per cent. (The expected value of the ratio of peak to total demand, which is not exactly the same, will have grown slightly faster.)

Do the data suggest that this kind of effect has any importance? By summing up the peak demands of all utilities in the United States and dividing by mean U.S. hourly demand, a weighted average of the utilities' peak demand/mean hourly demand ratios can be calculated (where the weights are in proportion to each utility's mean hourly demand). Using data from the Edison Institute (1980), this figure averaged 1.556 between 1966 and 1970. For 1971 to 1975 it was 1.611 and for 1976 to 1980 it was 1.619.[17] While these changes do not seem large, the 4.0 per cent difference between 1.619 and 1.556 given 1980 total consumption would imply a difference in expected peak of over 16,000 megawatts.[18] Data from Baughman, Joskow and Kamat (1979, p. 242) indicate that the construction costs of 16,000 megawatts of relatively cheap natural gas turbine peaking capacity would have been close to 3 billion dollars in 1980.[19]

III. Estimation of the Effect of Peak Demand Charges

The application of the extreme value distribution was very useful in the preceding analysis. This section uses it to compute maximum likelihood (ML) estimates of the response of individual peak demand to maximum demand charges. This will help determine whether the peak dampening effects studied in the previous section are important and may also be of use in other aspects of system planning, such as the response of revenues to changes in rates. The work reported here will be from a sample of individual data on eight Ontario pulp and paper mills between 1970 and 1977.

The "standard" method of estimating peak charge effects over a time series would be linear regression, with peak demand as the dependent variable and the peak charge and other variables on the right-hand side. Least squares estimates have the least variance of all linear and unbiased estimators and if the dependent variable is normally distributed, OLS will also be ML and hence asymptotically the most efficient of all consistent estimators. However if the dependent variable has the extreme value distribution, OLS will not be ML, but ML estimates can be calculated which will have the property of asymptotic efficiency.

In the theoretical exposition of the previous section, some strong assumptions were made, for example that the parent distributions were normal. In order to relax these assumptions, assume that there are T "peak-eligible" quarter-hour periods in a month.[20] Assume also that the natural logarithm of a plant's electricity demand for quarter-hour period i of month m is $x_{i,m}$, and that all the $x_{i,m}$'s have an identical probability distribution F:

$$x_{i,m} \sim F(x_{i,m} | Z_{m,1}, \ldots, Z_{m,J}) \qquad (11)$$

where $Z_{m,1}, \ldots, Z_{m,J}$ are monthly observations on the underlying explanatory variables. As mentioned before, if the distribution of x_m^{max} has a limiting distribution as T increases, Gnedenko (1943) has shown that it must take one of three forms: If x is not bounded above and

$$\lim_{t \to \infty} \frac{1-F(tx)}{1-F(t)} \neq x^{-\gamma} \qquad \text{for all } x > 0 \text{ and some } \gamma > 0 \qquad (12)$$

then x_m^{max} must have extreme value distribution[21] of form (1). Assumption (12) is appropriate for many parent distributions, including the Normal, Gamma, Poisson, Logistic, Exponential and Lognormal. As also mentioned, the parents need not be independent provided strong-mixing condition (3) holds.[22] Note also that the successful applications discussed in Gumbel (1958) suggest that (1) may be reasonably robust to small violations of these basic assumptions.

To determine whether the distribution of x_m^{max} responds to events such as changes in the peak price, the location parameter ℓ_m is in turn reparameterized in terms of the explanatory variables. The log likelihood function associated with (1) then becomes:

$$L(x_m^{max}) = -\log(s_m) - e_m - \exp(-e_m) \tag{13}$$

where $e_m = (x_m^{max} - \ell_m)/s_m$

$\ell_m = a_0 + a_1 \log(Z_{m,1}) + \ldots + a_J \log(Z_{m,J})$

and $s_m = s_o$

As x_m^{max} is already a logarithm, $a_1, \ldots, a_J$ will be elasticities. Maximum likelihood estimation consists of maximizing L in (13) over $\hat{a}_0, \ldots, \hat{a}_J$ and $\hat{s}_o$.

The remaining formulation of the model essentially consists of choosing the variables $Z_1, \ldots, Z_J$. The key constraint on model building is that the only available customer-specific data are the "standard billing data" on their electricity purchases and in particular there are no data on the output of individual plants. The very simple model used here is similar to that used by Corio and Trimmell (1978) which employed some of these same data (although their empirical methods were very different).

While the model is described in more detail in Veall (1981), the basic idea is that it would be very difficult with the available data to model the entire electricity purchase decision, as it depends on all input and output prices, market conditions, inventories etc., most of which are not observed. Instead it is assumed that the firm considers all this information in determining monthly output and that there are no short-run substitutes for electricity[23] so that output determines the number of kilowatt hours (kW.h) that will be purchased during the month. The work here focusses on the determination of peak demand conditional on the kW.h decision, and therefore kW.h is one right-hand side variable.[24]

Given kW·h, the plant's electricity bill is minimized by keeping its consumption over the month as smooth as possible. In particular, if the peak demand price rises, plants will attempt to reduce peak demand and make up lost output by increasing the level of production at other times. This will result in extra wage costs, for example in the form of overtime or shift differentials.[25] Therefore peak demand will be a decreasing function of the peak demand price relative to the wage[26] as well as an increasing function of total monthly electricity usage.

The original data set consisted of 768 monthly observations on eight Ontario pulp and paper mills between 1970 and 1977.[27] Some observations were excluded because of strikes or because normal charges and procedures were suspended under the "force majeure" provision of the standard contract. This left 706 observations for estimation. A description of the data is provided in Appendix 2.

The first step in the estimation procedure[28] was to pool the data for all the mills (in a time series cross-section panel) and maximize the likelihood function (13) with the mode re-parameterized in terms of kW.h, the ratio of the peak demand price to the wage and intercept dummies for mill and month of the year. The intensity was re-parameterized in terms of the mill dummies only. The resulting estimates were reasonable (for example the coefficient of the peak demand price/wage ratio was estimated as -.06 with a standard error of .02). However when compared to individual mill-by-mill estimation over the time series, the null hypothesis that the coefficients of kW·h, the peak demand price/wage ratio and the monthly dummy coefficients do not vary across firms is rejected decisively at the 99 per cent level using a likelihood ratio test. It was therefore decided to shift to a mill-by-mill investigation.

As a first pass with this new approach, likelihood function (13) was maximized for each mill with the monthly dummies retained and the assumption of a constant scale parameter, s_o. As a preliminary test, the residuals were then pooled and graphed as a histogram along with the theoretical density (see Figure 3).

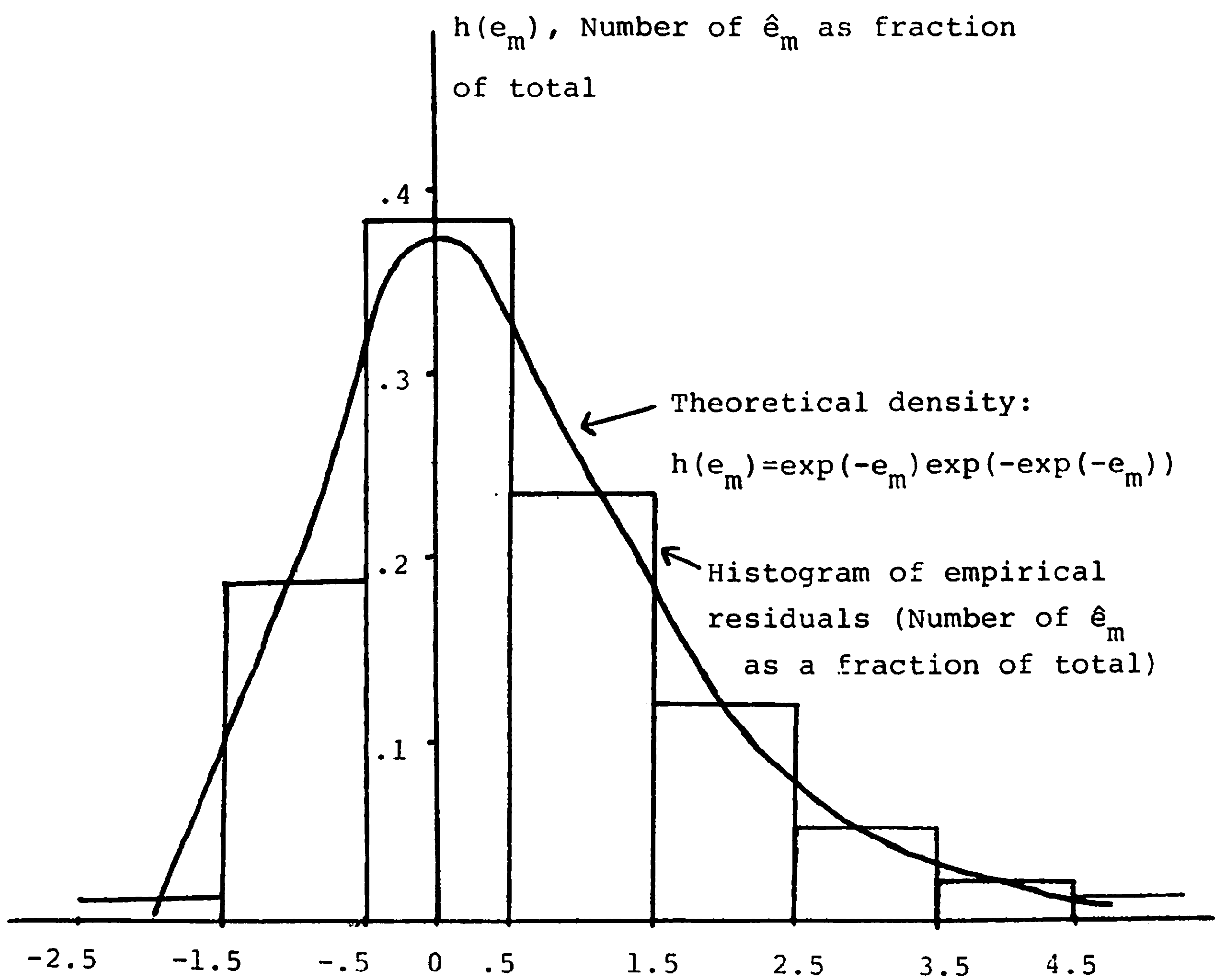
While this provides striking visual evidence in support of the extreme value distribution, the ordinary chi-square critical level for the goodness-of-fit test is not strictly applicable to residuals from ML parameter estimation, as discussed in Kendall and Stuart (1961, pp. 425-430). The correct test, as described by Chernoff and Lehman (1954), is extremely complicated. However, it is known that the "true" critical level is between the chi-square value with G-1 degrees of freedom (where G is the number of groups for the test) and the chi-square level with G-K-1 degrees of freedom (where K is the total number of parameters estimated). However in this research K will generally exceed (or be very close to) reasonable values for G. (For example, in the pooled residual case, K is 120.) Therefore the goodness-of-fit statistics will be generally compared to a chi-square with G-1 degrees of freedom (although that test is admittedly less powerful).

The results for the individual mill-by-mill estimation are in Table 1. (The monthly dummy coefficients are not reported to reduce clutter.) The key results are the goodness-of-fit tests on the residuals. For one of the eight mills, the null hypothesis of the extreme value distribution is rejected. For the OLS residuals, the null hypothesis of normality can be rejected three times (and for seven of the eight cases this goodness-of-fit statistic is greater than for the ML case).

Figure 3

Theoretical and Empirical Densities of Residuals:

Pooled Sample for All Eight Firms



Goodness-of-fit $\chi^2 = 4.3$ $\hat{e}_m = (x_m^{max} - \hat{I}_m)/\hat{s}_m$

(95 per cent critical level = 14.1)

Number of residuals = 706

TABLE 1

ML and OLS Estimates under the Assumption of Constant Scale Parameters with Monthly Dummies Estimated

Mill	Method	Constant	P^{kW}/W	kW.h	DW	Goodness of fit	Number of Obs.
1	ML	4.5043 (.3957)	-.1134 (.0537)	.5743 (.0381)	1.17	24.3	90
	OLS	7.3777 (.6309)	-.1504 (.0403)	.2965 (.0602)	.96	29.7	90
2	ML	5.8124 (.2280)	-.1655 (.0916)	.3352 (.0251)	1.08	8.9	88
	OLS	5.8541 (.2281)	-.2229 (.0855)	.3332 (.0265)	1.30	20.5	88
3	ML	6.0745 (.2870)	-.0933 (.0670)	.4078 (.0288)	.64	23.3	88
	OLS	6.6611 (.2377)	-.0332 (.0412)	.3559 (.0235)	.62	31.6	88
4	ML	6.4383 (.2619)	-.0887 (.0413)	.3523 (.0268)	.66	22.1	88

TABLE 1 (cont'd.)

Mill	Method	Constant	P^{kW}/W	kW.h	DW	Goodness of fit	Number of Obs.
4	OLS	7.2130 (.2786)	-.0945 (.0358)	.2722 (.0291)	.89	24.4	88
5	ML	9.1378 (.2728)	-.0030 (.0296)	.0992 (.0267)	.77	14.3	88
	OLS	9.3397 (.2034)	-.0080 (.0239)	.0797 (.0202)	.87	6.3	88
6	ML	4.1003 (.7143)	.1930 (.0533)	.6535 (.0620)	1.09	12.8	87
	OLS	5.0883 (.5786)	.2081 (.0493)	.5663 (.0505)	1.17	18.0	87
7	ML	1.1918 (.4568)	-.1387 (.0502)	.8874 (.0439)	1.50	12.3	85
	OLS	2.0107 (.4172)	-.1807 (.0581)	.8081 (.0404)	1.51	21.4	85

TABLE 1 (cont'd.)

Mill	Method	Constant	P^{kW}/W	kW.h	DW	Goodness of fit	Number of Obs.
8	ML	4.1515 (.3612)	-.1898 (.0487)	.5645 (.0386)	1.50	10.4	92
	OLS	4.7030 (.3673)	-.2436 (.0642)	.5059 (.0391)	1.71	13.4	92

The monthly dummy coefficients are not reported. For the Durbin-Watson test the critical level using the asymptotic normal approximation to Durbin-Watson is just under 1.6. For the goodness-of-fit test, the null hypothesis of the extreme value distribution for the ML residuals and of the normal distribution for the OLS estimates can be rejected if the statistic exceeds $\chi_{14}^{2\ .95} = 23.7$ (statistic is based on 15 groups).

Looking at the ML coefficient estimates, the estimated peak demand price/wage mode elasticity is negative for seven of the eight mills, ranging in those cases from -.0030 to -.1898. The negative sign was expected as an increase in the peak demand price relative to the wage should tend to dampen peak. For five of these seven cases the coefficient is significantly different from zero (using a one-tailed test at the 95 per cent level). For mill 6, the estimated coefficient is positive and statistically significant. (It may be recalled from footnote 23 that mill 6 is the only mill capable of electricity generation by steam.) It would be expected that the kW·h coefficient would be between zero and one and the ML estimates confirm this. However these coefficients do range considerably, from .0992 to .8874.

The OLS coefficients are not very different from the ML estimates. The ML coefficients of the peak demand price/wage ratio coefficient tend to be slightly closer to zero (except for the mill 3 estimates). The kW.h coefficients are reasonably close except for mill 1. As the OLS estimates should be consistent in this case,[29] this similarity of estimates is reassuring.

Another issue is whether the peak demand price/wage ratio should be a single variable or whether the peak demand price and the wage should be included as separate variables. The t-tests on this restriction indicated that the null hypothesis of using only one variable could not be rejected for seven of the eight cases. In the case that it could reject, the peak demand price coefficient had the expected negative sign while the wage coefficient had the expected positive sign.

However there is one disturbing aspect to these results. The Durbin-Watson statistics for both ML and OLS residuals tend to indicate serial dependence. In attempts to correct this problem, such additional explanatory variables as a time trend, fossil fuel prices, industry shipments and lagged independent variables were tried, all without success. The usual procedure in this case is an ad hoc first-order autocorrelation transform (i.e., "quasi first-differencing"). However this approach is inappropriate in this case as the set of variates with the extreme value distribution is not closed under addition, unlike the set of normally distributed variables. This makes the problem very difficult.[30]

Veall (1981) attempts a complicated approach to deal with the autocorrelation problem in the extreme value distribution context and is partially successful. It is a problem which requires further research both for this particular case and for cases with other kinds of non-normal variates. Appendix 3 presents the results of a simple first-order autocorrelation transform (i.e., Cochrane-Orcutt technique). While the standard errors are slightly larger, the inferences from above remain intact (namely that the peak demand charge appears to have a peak dampening effect for all the mills except number 6). However the important point here is that the extreme value distribution can potentially be used when the dependent variable is peak demand and some support has been provided for this approach in the data. It may be a feasible approach for other kinds of peak demand estimation problems (e.g. system electricity demand, public transit) or any time when the data are reported as maxima or minima.

IV. Summary and Conclusions

The most prevalent method of charging for electricity sold to industrial customers is the Hopkinson rate. This consists of an energy charge per kilowatt hour plus a demand change for the maximum usage during any quarter-hour of the month. This paper studies the effects of this charge, both in a theoretical and empirical context.

The primary motivation for the former was the observation that recent time-of-use rates in California still had a maximum demand charge for maximum quarter-hourly usage during the on-peak period. The theoretical example studied provides a potential justification. The basic argument is that a user's demand variance is positively related to both its own-peak demand and to system variance. System demand variance is in turn positively related to both the expected system peak and the size of the optimal reserve margin over expected peak. The individual maximum demand charge can therefore be used to price the effect of a firm's actions on the costs of providing the optimal level of capacity.

The analysis was greatly simplified by the use of the extreme value distribution. This device was also used to comment on the calculation of optimal capacity. If the utility is building capacity to maintain some low loss of load probability, the use of the extreme value distribution will suggest a larger optimal capacity than if peak demand is treated as a normal variate. In addition, it is illustrated that the practice of predicting peak demand by multiplying the average demand by an exogenous factor may be inaccurate if demand variance changes. An example of a possible reason for an increase in system demand variance is the increased use of air conditioners which make electricity consumption fluctuate more with the weather.

Section III attempted to estimate the effect on individual peak demand of the Hopkinson rate's maximum demand charge. The sample was the monthly standard billing data (total and maximum demand) of eight Ontario pulp and paper mills from 1970 to 1977. The estimation technique treated individual peak demand as the maximal order statistic from many draws and hence used the extreme value distribution. The parameters of the extreme value distribution were again re-parameterized in terms of "explanatory" variables to yield a likelihood function. The results from maximizing this likelihood function tended to support the use of the extreme value distribution and also suggested that for most of the mills, a one per cent increase in the peak demand price per kilowatt relative to the wage would lead to a reduction in peak demand of up to two-tenths of a percentage point.

Appendix 1

Optimal Prices

Note the relationship of the mean and variance of the usage of firm f to the system mean and variance:

$$\mu = \mu_f + \mu_s$$
$$\sigma = (\sigma_f^2 + 2\rho\sigma_f\sigma_s + \sigma_s^2)^{(1/2)}$$

where μ_s and σ_s^2 are the mean and variance of the rest of the system's usage and ρ is the correlation of firm f's usage with the rest of the system's usage.

Then the optimal prices P^{μ_f} and P^{σ_f} will reflect the marginal cost of μ_f and σ_f. Letting P^C be the average unit cost of capacity and recalling $C^* = \mu + k\sigma$

$$\begin{aligned} P^{\mu_f} &= \frac{\partial P^C C^*}{\partial \mu_f} \\ &= \frac{\partial P^C C^*}{\partial C^*} \cdot \frac{\partial C^*}{\partial \mu} \cdot \frac{\partial \mu}{\partial \mu_f} \\ &= C' \end{aligned}$$

$$\begin{aligned} P^{\sigma_f} &= \frac{\partial P^C C^*}{\partial \sigma_f} \\ &= C' \cdot \frac{\partial C^*}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial \sigma_f} \\ &= C' \cdot k \cdot \left(\frac{\sigma_f + \rho\sigma_s}{\sigma}\right) \end{aligned} \tag{A1:1}$$

defining C' as the marginal cost of capacity at $C = C^*$.

Expected proceeds of this pricing plan if all users face these prices would be

$$\sum_f \left(C' \cdot \mu_f + C' \cdot k \cdot \left(\frac{\sigma_f + \rho\sigma_s}{\sigma}\right)\sigma_f\right) = C' \cdot (\mu + k\sigma)$$

Expected costs under the assumption of constant returns to scale are also

$$C' \cdot (\mu + k\sigma)$$

so the plan breaks even.

Now examining the MRS/MRT conditions, using (A1:1),

$$\text{MRT} = \frac{\dfrac{\partial P^C{}_{C^*}}{\partial \sigma_f}}{\dfrac{\partial P^C{}_{C^*}}{\partial \mu_f}} = k\left(\frac{\sigma_f + \rho\sigma_s}{\sigma}\right)$$

The user's electricity bill is

$$B = C'\mu_f + C'k\left(\frac{\sigma_f + \rho\sigma_s}{\sigma}\right)\sigma_f$$

$$\text{MRS} = \frac{\dfrac{\partial B}{\partial \sigma_f}}{\dfrac{\partial B}{\partial \mu_f}} = \frac{k}{\sigma}(2\sigma_f + \rho\sigma_s) - \frac{k(\sigma_f + \rho\sigma_s)^2\sigma_f}{\sigma^3}$$

which does not equal the MRT even if it is assumed that the second term can be neglected.

If prices are instead

$$p^{\mu_f} = C'$$

$$p^{\sigma_f} = C' \frac{k}{2\sigma} (\sigma_f + 2\rho\sigma_s) \qquad \text{(A1:2)}$$

then

$$MRS = \frac{k}{2\sigma} (2\sigma_f + 2\rho\sigma_s) - C' \frac{k(\sigma_f + 2\rho\sigma_s)\sigma_f(\sigma_f + \rho\sigma_s)}{2\sigma^3}$$

$$\doteq k\left(\frac{\sigma_f + \rho\sigma_s}{\sigma}\right)$$

If the second term is neglected under the assumption that the firm does not consider the effect of σ_f on σ (which will be small unless user f's variance is a substantial portion of the system variance), then MRS = MRT.

If ρ is a choice parameter

$$MRS_{\rho,\mu_f} = \frac{k\sigma_s\sigma_f}{\sigma}$$

$$MRS_{\rho,\sigma_f} = \frac{\sigma_s\sigma_f}{\sigma_f + \rho\sigma_s}$$

again ignoring the effect the user has on system variance. The corresponding MRT's are identical.

Prices (Al:2) yield an expected bill of

$$C'\mu_f + C'\frac{k}{2\sigma}(\sigma_f + 2\rho\sigma_s)\sigma_f \qquad \text{(A1:3)}$$

Hopkinson rate (9) also yields this expected bill and therefore is equivalent for expected cost minimizers. Moreover if P^{kW} and $P^{kW\cdot h}$ are as in (10), the expected bill will be

$$E((C' - P^{kW})\bar{x}_f + P^{kW}x_f^{max})$$

$$= E(C'\bar{x}_f + C'\frac{k}{k^e}(\frac{x_f^c - \bar{x}_f}{x_f^{max} - \bar{x}_f})(x_f^{max} - \bar{x}_f))$$

$$= C'\mu_f + C'\frac{k}{k^e}E(E(x_f^c | x = x^{max}) - \mu_f)$$

$$= C'\mu_f + C'\frac{k}{k^e}(\mu_f + \rho\frac{\sigma_f}{\sigma}(E(x^{max}) - \mu) - \mu_f)$$

$$= C'\mu_f + C'\frac{k}{k^e}(\rho\sigma_f k^e) \qquad \text{as } E(x^{max}) = \mu + k^e\sigma$$

$$= C'\mu_f + C'k\rho\sigma_f$$

If σ_f is small relative to σ, (Al:3) also becomes $C'\mu_f + C'k\rho\sigma_f$. Therefore price sets (Al:2), (9) and (10) are equivalent.

Appendix 2

The Data

Because of Ontario Hydro confidentiality requirements, the mills were referred to only by number and the only information on individual mills was their standard billing data. There were no mill data on output or on capital acquisitions, although there were no sustained jumps or falls in consumption which a major change in plant utilization might cause. The data are described as:

(i) x_m^{max}, the log of the maximum kilowatt consumption in any 15 minute period during month m

(ii) the peak charge, or price per kilowatt at peak as set by Ontario Hydro

(iii) KWH_m, or the log of the total consumption of kilowatt hours during month m

(iv) the kilowatt hour charge, which is the Ontario Hydro price per kilowatt hour and which is the same for all mills studied

(v) the price of natural gas, defined as the upper limit of the band which gas companies negotiate with their largest customers. The price is that of the relevant regional supplier

(vi) the price of oil, which is the wholesale price of No. 6, 3% fuel. The price is that of the relevant regional supplier

(vii) the wage, which is the average hourly wage for the 3-digit S.I.C. division Pulp and Paper mills in Ontario. This is the only non-mill-specific variable used in the basic model

(viii) shipments, based on Statistics Canada data for Ontario shipments of the 2-digit S.I.C. classification Paper and Allied Industries.

The term "real" applied to a variable implies it has been divided by the Statistics Canada Industry Selling Price Index for Ontario for the 2-digit S.I.C. classification Paper and Allied Industries. Variables (iv), (v), (vi) and (viii) are not used in reported results but only in attempts to deal with the autocorrelation discussed on pp. 26-27.

Appendix 3

Coefficient Estimates from GLS Estimation

Mill	Constant	pkW/W	kW.h
1	7.8518 (.4932)	-.1516 (.0536)	.2526 (.0473)
2	6.2358 (.2494)	-.2165 (.1268)	.2887 (.0299)
3	9.1559 (.2052)	-.0160 (.0571)	.1147 (.0202)
4	7.4733 (.3173)	-.0040 (.0600)	.2510 (.0336)
5	9.6444 (.1737)	-.1054 (.0441)	.0420 (.0177)
6	7.2941 (.7078)	.2347 (.0891)	.3690 (.0627)
7	3.4294 (.5664)	-.1787 (.0962)	.6786 (.0544)
8	4.7038 (.4026)	-.2107 (.0809)	.5073 (.0425)

First order serial correlation adjustment using Cochrane-Orcutt method. Standard errors are in parentheses.

Footnotes

[1]Morgan and Talukdat (1979), p. 293.

[2]Alternatively these periods can be 20 minutes or 30 minutes (and are even six hours in Sweden). The billing period of one month does not seem to vary.

[3]Bauer and Gold (1939) suggest the original justification was that "each additional customer could be regarded roughly as adding to plant requirements in proportion to maximum demand... .The special charge was thus conceived as paying return on plant investment, while the kW.h rate conveyed costs due to production and delivery of energy used" (p. 88).

[4]A less important defect is that the intervals are usually measured consecutively (12:00 to 12:15, 12:15 to 12:30, etc.) which could induce unusually low usage near the end of each period. One solution is overlapping periods (12:00 to 12:15, 12:01 to 12:16, etc.).

[5]One new suggestion is "ex post" peak pricing with an "after-the-fact" charge on user's demand at time system peak occurred. Another is what Vickrey (1971) called "responsive" pricing and is sometimes known as "homeostatic" or "spot" pricing. Under this system, the price of electricity would be adjusted very frequently (perhaps every five minutes) so as to keep demand approximately equal to production capacity with the restriction that price would always exceed marginal operating cost.

[6]See Ellis (1980) for analysis of this latter issue.

[7]The original derivation is due to Fisher and Tippett (1928).

[8]As the system demands x_i are the sum of many individual demands, the Central Limit Theorem suggests the normal to be most appropriate. However the normal parent assumption is not necessary to obtain (1) as will be discussed in Section III. A different assumption for the parents would lead to different formulae (2).

[9]A somewhat more general condition has been derived by Leadbetter (1974). Berman (1964) discusses the case with normal parent distributions.

[10]Even with independence, convergence is not rapid in the tails with normal parent distributions. See Gumbel (1958), pp. 221-223.

[11]Billing is actually done on a plant rather than a firm basis. Note also that the firm demand is the amount of electricity which would be purchased <u>if</u> it were available, although this distinction will not be important as the probability of shortage has been assumed small.

[12]Assuming the probability of shortage is ignored.

[13]Provided q is greater than .57. For example, if as before $q = .99$ and $T = 160$, $k^e = 2.71$ and $k = 3.98$, $\frac{k}{k^e} = 1.47$.

[14]Using a conservative estimate of the marginal cost of peaking generation capacity of \$2.50 per month and the example's $\frac{k}{k^e}$ of 1.47 and ρ of only .1, the peak demand charge would be at least \$.37 per kilowatt per month, or over \$1800 per month for a moderately large 5000 kW user.

[15]Many of the simplifying assumptions made above are not required for individual peak demand charges to be efficient. For example, the identical probability distribution assumption is not necessary for either the x_i's or the $x_{f,i}$'s. However in Veall (1981), it is suggested that a reasonably accurate and tractable approach would be to retain the identical distribution assumption for the demands of each individual firm, as most firms will be

in continuous operation for at least the few hours likely to contain system peak. Due to the residential component of system demand, the identical distribution assumption may not be realistic for the system as a whole. In that case, it is probably feasible to allow each x_i to have its own mean and variance and calculate C* as a function of $\mu_1, \ldots, \mu_T$ and $\sigma_1, \ldots, \sigma_T$. This will affect the formulae above for $P^{kW.h}$ by replacing C' with $\sum_{i=1}^{T} \frac{\partial C}{\partial \mu_i}$. For P^{kW}, $C'k$ would be replaced by $\sum_{i=1}^{T} \frac{\partial C}{\partial \sigma_i}$.

[16]These figures may be somewhat overstated as in the right-hand tail, the true density of the maximum draw approaches the extreme value density from the left. The accuracy of the approximation depends on the number of draws, the true parent density and the degree of dependence between parent densities.

[17]As an example for an individual utility, the peak demand/mean hourly demand ratio of Southern California Edison averaged 1.518 between 1966 and 1970. This figure rose to 1.596 for 1971 to 1975 and was 1.697 for 1976 to 1980. In addition, the difficulty cannot be solved by merely applying the fixed factor approach to data for peak months. Southern California Edison's peak comes most frequently during August. The August ratio of peak demand to mean hourly demand averaged 1.365 for 1966 to 1970, 1.419 for 1971 to 1975 and 1.505 for 1976 to 1980. The figures for July and September display a similar pattern.

[18]Furthermore, the confidence intervals for predictions of peak demand may be much larger than those for mean hourly demand (see Veall (1981), p. 88).

[19]These effects may be offset by the point of Telson (1975) that most utilities already provide too high a level of reliability.

[20]It is allowable if there are some other periods with a probability of providing the peak, as long as that probability approaches zero for large T.

[21]This condition essentially rules out parent distributions which are Cauchy. Note that it is also being assumed that electricity utilization capacity is not an important constraint. If it were, the appropriate limiting distribution of x_m^{max} (if non-degenerate) would be the three parameter Weibull. This more complicated distribution would be used instead of (1).

[22]Suppose a firm which experiences an unusually high peak early in the month realizes that for the remainder of the month there is no longer an incentive to smooth demand as long as the initial peak is not exceeded. If the firm reacts by shifting the parent distribution for the rest of the month, the resulting dependence is not strong-mixing.

[23]Ontario Hydro research indicates 97 percent of electricity is used for machine operation and it is probable (although not known) that these machines are not steam-compatible. Customers 1, 2 and 7 have hydroelectric generation but as that is likely "run-of-the-river" (i.e., no dam) this gives no particular advantage in terms of reducing peak, as it is likely the generators work more or less constantly. Customer 6 can generate electricity with steam (possibly by burning wastes) and as will be seen, its results do not match theoretical expectations.

[24]No special methods are used for treating kW.h as an endogenous variable. The reader can either regard the estimation as conditional on kW.h or be willing to assume that randomness in peak demand is of sufficiently small importance in plant operation that it does not feed back to the output/kW.h decision. Electricity charges constitute less than 5 percent of total value added by pulp and paper mills, according to Statistics Canada industry data.

[25]The marginal cost of all these alternatives is assumed proportional to the wage, which is what the model requires. Another method of reducing peak would be appropriate scheduling of maintenance activities. Note also that while these mills are 24 hour-a-day operations, they generally shut down for a few shifts every month, often around a holiday.

[26]The argument is essentially that the wage is the appropriate deflator for the nominal peak demand price.

[27]Thanks are due to D. Fretz and F. Trimmell of Ontario Hydro for providing and interpreting the data set.

[28]Maximization of likelihood functions used the algorithm proposed by Berndt, Hall, Hall and Hausman (1974). Because the likelihood function is not globally concave in the parameters, different sets of starting values were used as a check against finding maxima which were local but not global.

[29]With a constant intensity, both ML and OLS are estimating shifts in the location of the density. However the constants need not be similar as the OLS estimates correspond to the expected value (which exceeds the mode for the extreme value distribution). It would be therefore expected that the OLS constants would exceed the ML constants, which is true in all eight cases.

[30]This problem is present in other cases where the dependent variable is non-normal, such as in the limited dependent variable literature. For this latter case, labour supply studies based on panel data seldom present test results for serial correlation along the time series.

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