

1982

# Analysis of a Monopoly

Mark Bagnoli

Follow this and additional works at: <https://ir.lib.uwo.ca/economicsresrpt>



Part of the [Economics Commons](#)

---

## Citation of this paper:

Bagnoli, Mark. "Analysis of a Monopoly." Department of Economics Research Reports, 8203. London, ON: Department of Economics, University of Western Ontario (1982).

20077

RESEARCH REPORT 8203

ANALYSIS OF A MONOPOLY\*

by

Mark Bagnoli

Department of Economics Library

JAN 23 1982

University of Western Ontario

ABSTRACT

Two issues are addressed: How robust is the standard monopoly solution to changes in the firm's strategy space, and what is a source of monopoly power? After formulating a game-theoretic model of a monopolist, I show that it is consistent with the standard monopoly model when the strategy space is restricted and the firm chooses its strategy prior to the consumers' choices. When the strategy space is enlarged, I find that the monopolist, using only information contained in the market demand curve, acts as if it were perfectly price discriminating. The last section shows that the ability to make credible threats is a source of market power. This ability is shown to be dependent upon the order of play.

Revised: January 1982

\* I would like to acknowledge my debt to Robert Willig here and to thank him for his remarkable patience and assistance. Thanks also to Glenn MacDonald, Ig Horstmann, and the Economic Theory Workshop at U.W.O. As always, I am responsible for all errors.

## INTRODUCTION

In this paper, I would like to address two issues: How robust is the standard monopoly solution to changes in the firm's strategy space, and what is the source of monopoly power.

It does not seem controversial to define a monopoly as a market which has just one seller. Somewhat more controversial is what is the source of monopoly power? Sherer [1970, p. 10] suggests the following:

"Pure monopolists, oligopolists, and monopolistic competitors share a common characteristic: each recognizes that its output decisions have a perceptible influence on price.... All three types of firms possess some degree of monopoly power over price, and so we say they possess market power or monopoly power."

Thus, a firm is said to have monopoly power if it faces a downward sloping market demand function. If one judges by the literature on barriers to entry (von Weisacker [1980]; Baumol and Willig [1981]) the arguments presented seem to suggest that Sherer's viewpoint has been adopted. The firm has monopoly power if the demand curve is negatively sloped and it is negatively sloped if there are barriers to entry.

One of the basic reasons that a firm faces a downward sloping demand curve seems to have gone unnoticed. Non-game models of the interaction of firms and consumers all have the firms choosing actions subject to the market demand function. Analyses of market power have focused on when the behavior of rival firms constrain or fail to constrain a firm from obtaining monopoly profits under this assumption. I will show that permitting the firms to choose subject to the market demand function implicitly allocates to the firms an advantage in their competition with the consumers. Furthermore, if one adopts the opposite view, that consumers choose subject to the firms'

decisions, the consumers coopt all of the gains from trade. In other words, the ability to make credible threats and thus the ability to obtain market power depends on the "order of play". One can view this as a reason why the firm faces a downward sloping market demand curve or as the reason firms can have market power.

Probably the most important modelling question is: what should be assumed about the firm's strategy space? It is traditional to assume that the firm either sets its price, or it sets the quantity that it will sell. In the case of a monopoly with no uncertainty, it is well known that the solution is not altered by assuming that the monopolist sets price instead of quantity. However, this is not true in all models. In the basic duopoly model, the outcome depends crucially upon the choice of strategy space. If it is assumed that the firm chooses quantity, one obtains the Cournot outcome, whereas, if it is assumed that the firm chooses price, one obtains the Bertrand solution.

A number of recent papers (Spence [1978], for example) have explored a model of a monopolist in which it has been assumed that the monopolist chooses a non-linear pricing schedule. The major focus of this work is to show that when resale is not possible, a non-linear pricing schedule yields larger profits than a linear pricing schedule. However, another one of the results of this work is that the standard solution (set quantity so that marginal revenue equals marginal cost and price so that demand equals supply) continues to be the solution when the monopolist's strategy space is enlarged to non-linear price schedules as long as the good can be resold.<sup>1/</sup> In other words, one obtains the same solution as was obtained under the more restrictive assumption on the monopolist's strategy space. This suggests

the possibility that the standard solution is relatively robust to changes in the monopolist's strategy space. Alternatively, one can consider this result as indicating that the strategy space is dictated by whether the good can be resold or not.

If one takes the position that placing a priori restrictions on the strategy space is a useful approach, then it is vital to understand what market conditions force the monopolist to act as if it had the particular strategy space the modeler chose. Unfortunately, this approach cannot give us an understanding of what market conditions result in important restrictions on the scope of the monopolist's actions. On the other hand, insight has resulted from the study of monopolies that are permitted to choose non-linear price schedules. This work has shown that resale makes a crucial difference to the choice of model, i.e., to the choice of an a priori strategy space.

This effort is designed to shed even more light on the question of what market factors justify which strategy space restriction. The first question I will consider is: if the monopolist sells a product that can be resold, and if the monopolist is not permitted to set personalized price schedules, will the solution correspond to the standard marginal revenue equals marginal cost solution? One of the objectives is to show that it will not be the solution when the strategy space is not artificially restricted. This will show, for instance, that transactions costs, uncertainty or something else not modelled here must be included to reproduce the standard monopoly solution.<sup>2/</sup> In addition to giving some insight about modelling a monopoly, this work may provide aid to those seeking to model situations in which it is not obvious what the agents are doing. That is to say, if one believes

that monopolists do set marginal revenue equal to marginal cost, understanding why this is, is important both in understanding a monopoly and in the construction of adequate models of other market structures.

Next, I will take up the question of what is monopoly power. The model remains the same; in particular, I continue to assume that the market demand is negatively sloped. What I show is that the "order of play" dictates who has the market power. The standard monopoly model, when modelled as a game, requires the modeler to assume that the consumers act given the monopolist's strategy choice. In other words, the game representation has the monopolist playing a "Stackelberg-type" game against the consumers.

If one reverses the order of play one obtains a number of different equilibria. If the monopolist acts given the strategy choices of all of the consumers, one equilibrium has the monopolist choosing to sell the quantity that equates demand to marginal cost. The correct way to view this is as follows: When the order of play in a model is assigned, the modeler is dictating who has credible threats in the game and who does not. The player or players who have the ability to make credible threats are able to extort some or all of the gains from trade. This then is the source of market power: the ability to make a credible threat, and it need not be linked to the slope of the market demand curve, although one can view the ability to make credible threats as assisting in the determination of the demand function the firm faces.

This paper will begin by formulating a game-theoretic model of a monopolist. The following section will show that this model is consistent with the standard monopoly model, by showing that the same solution is obtained when the strategy space is restricted, and a different solution will be presented for the monopolist when its strategy space is enlarged. The last section will show that market power comes from the "Stackelberg-type" game; it will be followed by concluding remarks.

## Section 1

In this section, I will formulate a static, complete information, game-theoretic representation of a monopoly in a market for a product which can be resold.<sup>3/</sup> This approach has the following implications. First, every player knows the payoff function. This means that every player knows his payoff for each strategy choice given the strategy choices of the other players. Second, it means that each player knows the payoff every opponent receives for every set of strategy choices (one by each player), so that each player can "put himself in every other player's shoes". Third, it means that every player knows that others can "put themselves in his shoes". Every complete information game model makes this assumption implicitly when setting out the game formulation.

In the present model, the complete information assumption means:

- (i) the monopolist knows its cost function
- (ii) each consumer knows his utility function
- (iii) the monopolist knows each consumer's utility function
- (iv) each consumer knows the monopolist's cost function
- (v) each consumer knows every other consumer's utility function.

I will assume that the monopolist has a U-shaped average cost curve,  $c(q)/q$ , and a U-shaped marginal cost curve  $c'(q)$ . Thus, I have made the traditional assumptions on the monopolist's technology. I will assume that the market demand curve is a step function, which means that I will not make the usual smoothing assumption. The product is sold in units of some size, and the product is not perfectly divisible. This generates a step, market demand function

and I will assume that it is "downward" sloping. Thus, the market demand function  $D(q)$  is defined for integer values of  $q$  such that, if  $q_1 < q_2$ ,  $D(q_1) \geq D(q_2)$  and there is a  $q_3 > q_1$ , such that  $D(q_1) > D(q_3)$ . No special assumptions will be made about individuals' demand functions nor will the "size" of the steps of the market demand function be assumed. Thus, I deal with a very general case: A consumer can purchase more than one unit at the same price and/or more than one unit at each of a number of prices. The other three assumptions are very important. I assume that the product can be resold costlessly, the monopolist cannot set personalized price schedules, and transactions costs are zero.

These assumptions correspond to those made when solving:

$$(A) \max_q D(q)q - c(q).$$

The only assumption that needs to be explained is the complete information assumption. To do this, consider what it means to write  $D(q)$  in problem A. It says that for each strategy choice by the monopolist ( $q$ ), it knows the price that equates the quantity demanded to  $q$ . In other words, it knows the aggregated outcome of each consumer's utility maximization problem. This translates directly into a game of incomplete information because the monopolist does not know the individual payoffs associated with  $q, D(q)$ . Thus, the assumption of complete information gives the monopolist more information than does the assumption that it knows the market demand curve. The extra information the monopolist has is each consumers' payoff; however, the prohibition against using personalized price schedules prevents the monopolist from directly using this extra information. In other words, even though it has enough information to construct and implement personalized price schedules, the



monopolist is not permitted to do so. Furthermore, it will be seen that its equilibrium strategy will be constructed using only aggregate information about the market demand curve. Thus, the complete information assumption is reasonable since the extra information the monopolist has gained is cancelled out by the prohibition on personalized price schedules.

Next, I will define the rules of the game. They are that the monopolist maximizes profits, the consumers maximize utility and the consumers know the monopolist's strategy choice prior to their choice of a strategy. In addition, the consumers all choose how much to buy (possibly zero) simultaneously. Finally, the resale market opens after the monopolist's market closes and the monopolist cannot participate in the resale market.

Lastly, the strategy space of each player must be defined. I will assume that the monopolist's strategy space is the set of sets of price-quantity pairs, and a set of decision rules that define the monopolist's action if the set of price-quantity pairs is not satisfied. A strategy is a set of price-quantity pairs  $\{p_i, q_i\}_{i=1}^n$  plus a decision rule which says what the monopolist does if  $q_i$  units are not contracted for at  $p_i$  for some  $i$ .<sup>4/</sup> As stated in the introduction, personalized price schedules will be excluded from the monopolist's strategy space.

Turning to the consumers, I will assume that each consumer's strategy space contains any imaginable action. This means that any threat that can be conceived of can be made and any purchasing strategy (which may or may not contain threats) are permissible, but they, too, cannot set personalized price schedules in the resale market.

I complete the definition of the game by noting that the firm's payoff is the economic profits that result from a vector of strategy choices (one by each player) and each consumer's payoff is the utility that results from the same vector of strategy choices. The equilibrium concept that will be employed is a complicated Nash equilibrium. If  $s_i$  is a strategy choice by

consumer  $i$  whose strategy space is  $S^i$ , then  $s_i$  must be a best reply to  $s_{(i)}$ .<sup>5/</sup>  
 If  $P(s)$  is the payoff function of the game, then  $s_i$  is a best reply to  $s_{(i)}$  if

$$P_i(s) \geq P_i(s_{(i)}, z) \quad \forall z \in S^i, \quad i=1, 2, \dots, n-1.$$

For the monopolist, the strategy choice  $s_n$  must maximize  $P_n(s)$  where  $s_{(n)}$  is the set of best replies by the consumers if the monopolist uses  $s_n$ . Notice that this is a Nash equilibrium adjusted to accommodate the "Stackleberg" nature of the game.

## Section 2

Logically, the first step is to show that the model represents the game formulation of the standard monopoly problem. This means that I must show that the solution to the game is the same as the solution to

$$(A) \max_q D(q)q - c(q),$$

if the firm's strategy space is restricted. I would expect to solve the game presented in Section 1 with the monopolist's strategy space restricted to a choice of quantity. However, this leaves the model incomplete because I have no recourse to an "auctioneer". In other words, a player or group of players must be choosing price in this game or no price can be determined. Looked at this way, one sees that A implies that the monopolist is choosing price too (subject to the demand function, of course).

Therefore, the monopolist will be permitted to choose a single price-quantity pair  $(p,q)$  and a rule, subject as always to the constraint prohibiting personalized price schedules. In other words, each consumer must face the same opportunities for purchase.

The Nash equilibrium has the monopolist choosing  $(p,q)$  such that  $q$  equates marginal revenue to marginal cost and  $p$  equates the quantity demanded to  $q$ ; and it has a rule which says: If  $q$  are not contracted for at  $p$ , none will be sold at all. The consumers each choose  $q_i$  as the utility maximizing amount to purchase given the monopolist's strategy. In other words, a Nash equilibrium is the standard solution to A when the monopolist's strategy space is restricted.

There are three facets of the game that cause this result. First, the monopolist must choose a single price-quantity pair. Second, the game is a game of complete information, and third, the game is "Stackelberg" against the consumers. The first implies that the price per unit facing each consumer is the same; the second implies that for each price, the monopolist can determine the utility maximizing amount for each consumer. Thus, for each price the monopolist knows how much each would purchase to maximize his or her utility and as a consequence knows how much is demanded in the aggregate. Lastly, the monopolist simply ignores the strategic role of the consumers because of the "Stackelberg" assumption.

I believe that the most enlightening way to see this, is to realize that the three facets mentioned above require that the monopolist choose a "supply point" and a take it or leave it attitude. The monopolist can do this but is constrained not to use personalized price schedules. Therefore, the monopolist seeks a price-quantity pair  $(p, q)$  that maximizes profits, but it is constrained to choosing a  $(p, q)$  which is consistent with market demand and which treats each consumer symmetrically. The monopolist must choose one point on the market demand function which maximizes profits, so it chooses the point  $(p^*, q^*)$  which may be characterized by  $q^*$ , the quantity that equates marginal revenue to marginal cost, and  $p^*$ , the price that equates  $q^*$  to the quantity demanded.

The reason no point  $(p, q)$  "below"<sup>6/</sup> the market demand function is chosen is obvious: Selling  $q$  at the price  $D(q)$  earns larger profits than selling  $q$  at a lower price. The reason a point "above" the market demand function is not chosen is the restriction that each consumer be treated symmetrically; i.e., that each face identical possibilities for purchase. In conjunction with the restricted strategy space, this means that the monopolist's price

per unit,  $p$ , must be the price that confronts each consumer. Because of this, the monopolist knows that any  $(p, q)$  "above" the market demand curve requires the consumers to choose a quantity which does not maximize utility at that price: something they will not do.

The monopolist finds the profit maximizing price-quantity pair  $(p^*, q^*)$  by ignoring the wide range of strategies available to the consumers. He does this because the game is "Stackelberg". Given his choice  $(p^*, q^*)$  the consumers seek a strategy which maximizes their utility subject to the monopolist's strategy choice. Thus, the consumers' ability to threaten, etc. is nullified by the "Stackelberg" nature of the game. Alternately, the only player in the game who can make credible threats is the monopolist. Writing the problem so that the monopolist maximizes profits subject to the market demand function requires the associated game to have this "Stackelberg" structure.

Now that I have shown that the game set out in Section 1 faithfully reproduces the standard monopoly problem A, I will turn to the task of finding an equilibrium of the game when the monopolist's strategy space is enlarged. Its new strategy space is the set of sets of price-quantity pairs plus a set of decision rules that say what the monopolist does if  $q_i$  units are not contracted for at  $p_i$  for some  $i$ . I will exhibit a set of strategies and then prove that they constitute a Nash equilibrium.

The monopolist's strategy will be defined first. It is the set of price-quantity pairs that results when the monopolist sells each unit for the largest amount that any consumer will pay for it. In other words, the monopolist determines the largest willingness-to-pay for each unit and this is the price that is associated with that unit.<sup>7/</sup>

The technical construction of the set of price-quantity pairs follows. Define the price associated with selling the first unit as  $D(1)$ , thus  $p_1 = D(1)$ . Define the function  $a(p_i)$  as the "length of the step" at  $p_i$ . In other words,  $a(p_i)$  is the number of units for which the maximum willingness-to-pay is  $p_i$ . With this, one defines the first price-quantity pair as:

$$(p_1, q_1) = (D(1), a(p_1)).$$

It should be noted that  $a(p_1)$  need not be one unit but must be at least one unit. If more than one unit could be sold at  $p_1$ , but no units sold for any higher price, then  $a(p_1)$  would be greater than one.

By making use of the step function nature of the demand function, the next price-quantity pair can be constructed. Since I wish to find the set of price-quantity pairs that is generated when the monopolist charges the most anyone is willing to pay for each unit,  $p_2$  is defined by the largest price anyone is willing to pay given  $a(p_1)$  are to be sold at  $p_1$ . This means that  $p_2$  is  $D(a(p_1) + 1)$ . The quantity associated with this price is the number of units that would be purchased at  $p_2$  if  $a(p_1)$  units are to be sold at  $p_1$ . This quantity,  $q_2$  is  $a(p_2)$  by definition of  $a(\cdot)$ . Thus,

$$(p_2, q_2) = (D(a(p_1) + 1), a(p_2)).$$

One continues to define  $(p_i, q_i)$  elements in the manner described above for all but the last pair,  $(p_n, q_n)$ . The last pair is defined as follows. Let  $\bar{q}$  be that quantity such that the monopolist's marginal cost of producing  $\bar{q}$  units is equal to the price the  $\bar{q}^{\text{th}}$  unit commands, i.e.,  $\bar{q}$  is defined as

the quantity such that  $D(\bar{q}) = c'(\bar{q})$ .<sup>8/</sup> The last price-quantity pair satisfies

$$\sum_{i=1}^{n-1} q_i \leq \bar{q} \leq \sum_{i=1}^n q_i .$$

These observations permit me to define  $(p_n, q_n)$  as:

$$p_n = \begin{cases} p_{n-1} & \text{if } \bar{q} = \sum_{i=1}^{n-1} q_i \\ D(a(p_{n-1}) + 1) & \text{otherwise,} \end{cases}$$

and

$$q_n = \begin{cases} 0 & \text{if } \bar{q} = \sum_{i=1}^{n-1} q_i \\ \bar{q} - a(p_{n-1}) & \text{otherwise.} \end{cases}$$

To complete the specification of the monopolist's strategy, I must provide a rule which defines the monopolist's response to situations in which  $q_i$  units are not contracted for at  $p_i$  for some  $i$ . The rule is straightforward: If  $q_i$  units are not contracted for at  $p_i$  for some  $i$ , the monopolist will not permit any trade to occur. One can think of this as a threat that says: The market will not open unless  $q_i$  is contracted for at  $p_i$  for each  $i$ . To preview, the "Stackelberg" nature of the game will make this a credible threat.

The consumers will each contract for every unit available for which their maximum willingness-to-pay is equal to the price set by the monopolist for those units. This is most clearly understood via an example. Suppose that the consumer is willing to pay \$5 for the first unit and \$3 for two more if the first is purchased at \$5. In this case, if units are provided at both \$5 and \$3, the consumer contracts for one unit at \$5 and two units at \$3.

To prove that these strategies do constitute an equilibrium, I must show that the strategy choices assigned above are the consumer's best reply to the presumed strategy choices of every other player, within the structure of this game.

Because of the "Stackelberg" structure of the game, consumers choose a utility maximizing strategy given the monopolist's strategy choice. This means that their strategies can be placed into two groups. For each consumer, I will refer to those units whose price equals his maximum willingness-to-pay as the "right" units. With this convention, the first group of strategies contains those strategies whose use implies that some consumer does not purchase the right units. The strategies in the other group are those that imply that each consumer does purchase the right units.

Because the monopolist's strategy is designed so that each unit is traded at its maximum willingness-to-pay, and because of the credibility of the monopolist's threat, every consumer is faced with the same choice: Buy at least the right units or else get nothing. This places each consumer in the position of having to choose whether or not the market will open. For it to open, each must choose to purchase his right units.

Consider the outcome for any consumer. If the consumer chooses not to purchase the right units the market will not open. It will not open because this consumer did not buy the right units; no one else will buy these units because resale is not profitable, and because they are already committed to purchase every unit they desire. Resale is not profitable because each unit must be purchased from the monopolist at a price which equals one of the consumer's maximum willingness-to-pay. (In fact this



consumer is the one who values this unit highest.) Thus, if someone else purchased it, he would have to pay that same price. If this person did purchase the unit in the hopes of reselling it, the largest price for which he could sell the unit would be exactly the price he paid for it because of the method the monopolist used to set the original price. Thus, the best a consumer can do in the resale market is earn zero economic profits, and he could do worse depending on the exact nature of the resale market.<sup>9/</sup>

The fact that resale earns at best zero profits, given the monopolist's strategy, implies that each consumer must contract for the right units or the market will not open. If the consumer failed to buy one of the right units, no one else would be willing to purchase it. Consequently, for some  $i$ , if  $q_i$  are not contracted for at  $p_i$  then, by the monopolist's rule, the market does not open, i.e., no trade occurs. This argument shows that each consumer is faced with a straightforward decision: Act so that the market will open or act so that it won't. The optimal decision is the one that maximizes the consumer's utility.

If the market does not open, no units are traded and the consumer receives some level of utility. On the other hand, if the market opens, the monopolist's strategy permits it to co-opt all of the gains from trade because it charges the highest maximum willingness-to-pay in the market for each unit. This means that the consumer is indifferent between choosing to purchase the right units (those for which this consumer has the highest maximum willingness-to-pay) and every other strategy available. Hence, each consumer cannot unilaterally alter his strategy and be better off. Thus,

the purchase of the right units is a best reply by each consumer to the set of strategy choices that I have asserted are a Nash equilibrium.

To complete the proof that the proposed set of strategies is a Nash equilibrium, I must show that the monopolist is choosing a profit-maximizing strategy. This means that I must show that no unit sold could be sold at a higher price and that profits cannot be increased by an alteration of the number of units sold. The first follows directly from the observation that each unit is sold to the consumer who has the highest maximum willingness-to-pay for that unit. Thus, no one will pay a larger price for any unit. Since  $D(q)$  represents the increment in revenues of selling the  $q^{\text{th}}$  unit, if the amount chosen to be sold does not equate  $D(q)$  to  $c'(q)$  then profits can be increased via the standard argument. Since  $\bar{q}$  is determined so that  $D(\bar{q}) = c'(\bar{q})$  then profits can be increased when  $q \neq \bar{q}$ . Since  $\bar{q}$  is determined so that  $D(\bar{q}) = c'(\bar{q})$ , there is no alternative decision on the amount to sell which increases profits. In other words, these arguments show that there is no strategy which yields larger profits for the monopolist. Therefore, the solution presented is an equilibrium of the game.<sup>10/</sup>

Before proceeding to an analysis of monopoly power, I wish to review what has been done. First, I have shown that the static, complete information, game-theoretic model of a monopoly described in Section 1 does reproduce the standard monopoly solution. Then, I showed that there is a strategy available, when the monopolist's strategy space is not artificially restricted, which permits the monopolist to generate a solution which is equivalent to perfect price discrimination. The monopolist is prohibited from setting personalized price schedules yet it can use this strategy which generates the same outcome. Furthermore, although the complete information game model does give the monopolist more information, its equilibrium strategy is constructed

from the market demand function. To implement this strategy the monopolist needs to know the maximum willingness to pay, unit by unit, information which is contained in the market demand curve. It does not need to know the maximum willingness to pay, person by person. This explains why the prohibition on personalized price schedules "counteracts" the extra information that is available in this complete information, game representation.

One can interpret the monopolist's strategy in two ways. One can think of the monopolist as designing an auction whose structure is such that the consumers are willing to participate but which results in the monopolist successfully extracting all of the consumers' surplus. Alternatively one can think of the monopolist's strategy as a truth revealing mechanism which, when used, gives the monopolist enough information to extract all of the consumers' surplus. Under either interpretation, the monopolist can induce the consumers to permit it to extract all of the gains from trade. Furthermore, this means that there is no deadweight social loss when the monopolist adopts this strategy.

### Section 3

As I stated in the introduction, it is my contention that focusing on the shape of the market demand function to determine monopoly power fails to identify the underlying cause of monopoly power. I assert that monopoly power is the ability of the firm (or firms) to make credible threats. In other words, the actor who has power in the market is the one who plays first, i.e., the one whose strategy choice is taken as given by the other players. To show that this is correct, I will reverse the order of play in the game described in Section 1. Instead of the consumers choosing a strategy given the monopolist's strategy choice, I will assume that the monopolist plays last. This means that the consumers simultaneously choose their strategy and the monopolist chooses his strategy given the consumers' choices. In addition, the definition of an equilibrium must be adjusted for this change in the extensive form of the game. A Nash equilibrium, now, will be a vector  $s$  which has the following properties. First,  $s_n$ , the monopolist's strategy, must be a best reply to  $s_{(n)}$ . Second,  $s_i$ , a consumer's strategy, must be a best reply to  $s_{(i,n)} = (s_1, s_2, \dots, s_{i-1}, s_{i+1}, \dots, s_{n-1})$  and the  $s_n$  which will be induced by  $(s_{(i,n)}, s_i)$ .

I will focus on one equilibrium in particular, because it provides the most dramatic contrast to the solution presented in Section 2.

From  $C(q)$ , one can calculate  $AC(q)$ , the average cost of producing  $q$  units. Define  $q'$  as the quantity that equates  $D(q)$  to  $AC(q)$ .<sup>11/</sup> Also, let  $p^*$  be  $D(q')$ . Lastly, define  $d^{-1}(p)$  as the quantity the  $i$ th consumer would demand if each unit can be purchased at  $p$ .

The set of strategies that I will show constitute a Nash equilibrium follow. Each consumer's strategy is:

$$\text{demand} \quad \begin{cases} 0 & p > p^* \\ d_i^{-1}(p) & p \leq p^* \end{cases}$$

The monopolist chooses to produce  $q'$  and sell at  $p^*$ .

Given each of the consumer's strategies, the monopolist faces a truncated market demand curve. The monopolist can sell  $q'$  at  $p^*$  or  $D(\bar{q})$  for  $p < p^*$ . Because  $q'$  is defined as the largest quantity such that  $D(q)$  equals  $AC(q)$ , for all  $q > q'$ ,  $AC(q) > D(q)$ . (Recall that  $D(q)$  is "downward" sloping.) Thus, for all choices of  $q$  such that  $q > q'$ , profits are negative, but, by the definition of  $q'$  and  $p^*$ , the profits from selling  $q'$  at  $p^*$  are 0. Therefore, the monopolist's profits are maximized by choosing to sell  $q'$  at  $p^*$ , and  $(p^*, q')$  is a best reply to the strategy choices of the consumers.

Consider any consumer. Because purchase at a lower price increases utility, and given that all of the other consumers choose

$$s_j = \begin{cases} 0 & p > p^* \\ d_j^{-1}(p) & p \leq p^* \end{cases}$$

the  $i^{\text{th}}$  consumer can choose any strategy  $s_i$  such that  $d(\sum_{j \neq i} s_j + s_i) \geq AC(\sum_{j \neq i} s_j + s_i)$ .

Clearly, this condition cannot be violated because the monopolist can always choose to shut down and coercion is prohibited. In other words, the consumer can choose any quantity between 0 and  $d_i^{-1}(p^*)$ . The consumer chooses  $d_i^{-1}(p^*)$  because any smaller quantity choice leaves the consumer with uncaptured gains from trade. Since the effect of choosing

$$s_i = \begin{cases} 0 & p > p^* \\ d_i^{-1}(p^*) & p \leq p^* \end{cases}$$

results in the consumer obtaining  $d_i^{-1}(p^*)$ , I have shown that the candidate set of strategies is a Nash equilibrium of the game under consideration in this section.

What does this equilibrium indicate? When the monopolist played first (could make credible threats), the monopolist captured all of the gains from trade. On the other hand, when all of the consumers could make credible threats (played first), they captured all of the gains from trade. The result is that the monopolist, even though it is the only seller, has no monopoly power.

The implication is that the interaction of firm(s) and consumers in a market has been modeled in the past in a way which automatically allocates to the firm(s) the power to make credible threats. In the formulation of the problem, the firm(s) have been given the market power associated with the ability to make credible threats, while there seems to be no explicit justification for this practice.

CONCLUSION

This exercise sought answers to two questions: Does the monopolist's strategy space matter? and what is monopoly power? The goal in answering the first question can be thought of as seeking an answer to how risky it is to assume a priori restrictions on the monopolist's strategy space. I believe that, in the case presented (no uncertainty, no transactions costs, no personalized price schedules and a product that can be resold) received theory would suggest that the monopolist would act as if it set  $q$  so that marginal revenue equals marginal cost and  $p$  so that the quantity demanded equals the quantity supplied,  $q$ .

In Section 1, I formulated a static, complete information game representation of the standard monopoly problem

$$(A) \max_q D(q)q - C(q).$$

After showing in section 2, that it yields the same solution as problem A when the monopolist is restricted to a choice of  $q$ , I showed that the monopolist could construct a self-selection type of mechanism which enabled it to reproduce the standard perfect price discrimination result. Therefore, we see that the choice of a strategy space does matter.

There are many ways to interpret this result. The most enlightening is: Modelling by setting a priori restrictions on the player's strategy set may not be a fruitful approach until the conditions under which the player would still choose in this set (if not restricted to it) are well understood. One can also interpret the results of Section 2 as indicating that transactions costs or uncertainty may be necessary to make the monopolist set  $q$  so that

marginal revenue equals marginal cost and  $p$  so that the quantity demanded equals  $q$ .

The other issue addressed was: What is monopoly power? I argued that it should be thought of as the ability to make credible threats. In this game, that is determined by who plays first. In other words, when it is assumed that the firm or firms play given the demand of the consumers, the modeler has assigned to the firms the opportunity to exercise monopoly power. By placing this structure on the model, one need only ask whether the firms can successfully use their advantage.

By reversing the order of play, I showed that the monopolist lost its ability to make credible threats. It is not clear why non-game-theoretic models have been constructed without serious consideration about the implicit assignment of order. Virtually every non-game, oligopoly model makes exactly this same assignment of order and then asks whether the firms are constrained by the actions of other firms into not exercising their market power. The important question in need of modelling is: When should the firms be given the ability to make credible threats to the consumers but not vice versa? There is one model in which the order of play is reversed. When a monopsonist is modelled, it is assumed that the consumer plays first or has the market power, but no argument is provided to defend this order of play either.

I am unaware of any explicit justification for this traditional assignment of the order of play, but I am convinced that it significantly affects the outcomes in most models of firm behavior and therefore should receive more critical attention.



## REFERENCES

- Baumol, W. and R. Willig (1981) "Fixed Costs, Sunk Costs, Entry Barriers, and Sustainability of Monopoly," Quarterly Journal of Economics, Vol. XCV: 405-432.
- Grossman, S. (1981) "Nash Equilibrium and the Industrial Organization of Markets with Large Fixed Costs," Econometrica, Vol. 49: 1149-1172.
- Harris, M. and A. Raviv (1981) "A Theory of Monopoly Pricing Schemes with Demand Uncertainty," American Economic Review, Vol. 71: 347-365.
- Spence, M. (1977) "Nonlinear Prices and Welfare," Journal of Public Economics, Vol. 8: 1-18.
- von Weisacker, C. C. (1980) "A Welfare Analysis of Barriers to Entry," Bell Journal, Vol. 11: 399-420.

## FOOTNOTES

<sup>1/</sup> The standard argument will be presented for a simple non-linear pricing schedule. Suppose that the price schedule is  $p_H$  per unit if 5 units or less are purchased by the consumer and  $p_L$  per unit if more than 5 units are purchased by the consumer with  $p_H > p_L$ . Consider the plight of two consumers each of whom would buy less than 5 units given the monopolist's non-linear pricing schedule. Further, let them desire to purchase 3 units each. If they coordinate they can get the 6 units at  $p_L$  per unit clearly making themselves better off. A method of implicit coordination is for one to buy all six and resell some.

More generally, one consumer will buy all of the units the monopolist seeks to sell at the lowest price per unit available and resell. This is a sketch of the argument that shows that resale can break any non-linear pricing schedule. Note that two-part tariffs are slightly more effective if the marginal consumer has positive consumer surplus.

<sup>2/</sup> Current work suggests that uncertainty will not be enough either. It is easy to show that the solution presented in Section 2 is not a solution when there is uncertainty due to the ability of consumers to free ride. My current research (joint with R. Preston McAfee) suggests that in most situations, the addition of uncertainty will not cause the standard monopoly solution to be a solution.

<sup>3/</sup> Harris and Raviv (1981) have solved a similar problem when there is uncertain demand. However, their assumption that each consumer has a reservation demand curve is very special. An example, provided in the appendix, shows that under the more general assumptions used in this paper, their central theorem is false.

4/ To make this idea clearer, consider the standard monopoly problem. The monopolist finds  $(p, q)$  such that  $q$  equates marginal revenue and marginal cost and  $p$  equates the quantity demanded to  $q$ . The question is: What does the monopolist do if  $q$  units are not purchased at price  $p$ . Any number of rules are possible. One example is: the monopolist chooses  $(p, q)$  and a rule that says trade takes place at whatever price makes the quantity demanded equal to  $q$ .

5/  $s_{(i)}$  will represent the vector  $s$  without the  $i$ th component. Thus,  $s_{(i)} = (s_1, s_2, \dots, s_n)$ .

6/ By "below" the demand curve, I mean that either  $p < D(q)$  or  $q < D^{-1}(p)$  or both.

7/ Note that this strategy does satisfy the requirement that each consumer face symmetric opportunities to purchase. Each faces the same set of price-quantity pairs and chooses a strategy given the whole schedule of price-quantity pairs. Thus, personalized price schedules are not used.

8/ Since  $c'(q)$  is U-shaped, there may be two quantities that equate  $D(q)$  to  $c'(q)$ . If there are, then  $\bar{q}$  is the larger of the two.

9/ This is possible if the resale market is not "Stackelberg" or if more than one consumer attempts to be a seller in the resale market. If either occurs, I believe that the economic profits of resale will be negative.

10/ One type of transactions costs that can be readily analyzed are those that are increasing in the number of price-quantity pairs announced by the monopolist. If transactions costs take this form then it is still likely that the monopolist will not use the marginal revenue equals marginal cost solution. The only time it will is if the per pair cost is so high that exactly 1 pair is the optimal number of pairs to announce. Otherwise, the monopolist finds the optimal number of pairs and chooses the subset of the solution presented in the body of the paper that yields largest profits.

11/ Since  $AC(q)$  is U-shaped, there may be more than one  $q$  that equates  $AC(q)$  to  $D(q)$ . If so, then  $q'$  is the largest quantity that equates  $AC(q)$  to  $D(q)$ .

Appendix

After completing this research, I was made aware of an article by Harris and Raviv since published in the June 1981 issue of the American Economic Review and titled "A Theory of Monopoly Pricing Schemes with Demand Uncertainty." I find that there is a very real distinction between their work and mine. The cause appears to be their special assumption on the individual's demand function. Harris and Raviv assumed that each consumer  $i$  buys one unit if  $p \leq R_i$ . To see the effect of this assumption, consider the following example, which will use their notation.

There is one consumer and the consumer has the demand function

$$p = R_i - q.$$

Let  $2 \leq R_i \leq 4$  and assume that the monopolist believes that

$$R_i = 2 \text{ with probability } \frac{1}{2} \text{ and } R_i = 4 \text{ with probability } \frac{1}{2}.$$

Further let the monopolist have constant marginal cost equal to 1.

This example satisfies all of the assumptions used by Harris and Raviv, except for their assumption on the consumer's demand function. Furthermore, since there is no capacity constraint, potential demand cannot exceed it. Harris and Raviv showed that the optimal marketing strategy for the monopolist is to choose a "single price strategy". In other words, the standard marginal revenue equal to marginal cost solution is the optimal marketing strategy. (See Theorem 4, p. 361.)

In my simple example, Theorem 4 fails. The firm's expected profits  $(E\pi)$  are

$$\frac{1}{2}[(2-p)p - (2-p)] + \frac{1}{2}[(4-p)p - (4-p)] = 4p - p^2 - 3.$$

The first order conditions for a maximum are

$$0 = 4 - 2p \quad \text{or} \quad p^* = 2$$

Thus, the profit maximizing price is  $p^* = 2$  and the firm's expected profits are  $E\pi(p^*) = 1$ .

To see that Theorem 4 is false under a more general assumption on individual demand, consider the effect of a two-part tariff. Define the fee,  $f$ , as

$$f = \int_0^1 (2-q) dq = \frac{3}{2},$$

where I have made use of the inverse demand function for the smallest value of  $R_i$ . If the monopolist charged the consumer  $\frac{3}{2}$  to "enter the store" and set price equal to 1 (marginal cost), the monopolist's profits are  $\frac{3}{2}$  which are larger than 1.

Thus, the two-part tariff is more profitable than the solution suggested by extending Harris and Raviv's result (Theorem 4). I make no claim that this is the most profitable strategy however and recognize that if the example had more than one consumer and costless resale, the two-part tariff would fail. Even so, an adaptation of the solution presented in the body of my paper would work, using as the demand function the analog to the  $2 - p = q$  demand function. Essentially, demand is random and could be written as  $a - bp + \epsilon = q$  where  $\epsilon \in [c, d]$  and the monopolist has priors on  $\epsilon$ . I suggest that using  $a - bp + c = q$  and the strategy proposed in my paper will yield larger profits than Harris and Raviv's suggestion for some  $c$  and  $d$  values.

My paper considered the certainty case without any special restrictions on the individual's demand functions. Each consumer's demand function is a step function that satisfies:

$$(i) \quad D_i(q_1) \cong D_i(q_2) \quad \text{for } q_1 < q_2$$

and  $(ii) \quad \text{for some } q_3 > q_1, D_i(q_1) > D_i(q_3).$

Thus, all of the results in my paper are not special cases of the work done by Harris and Raviv.