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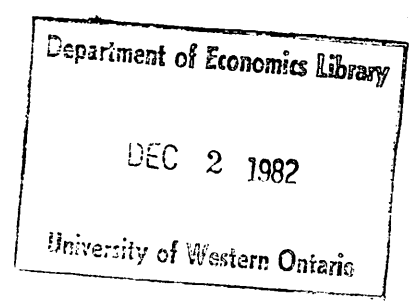
ACTIVIST MONETARY POLICY, ANTICIPATED IN-  
FLATION, AND THE VALUE OF INFORMATION

by

Peter Howitt

March, 1981

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ACTIVIST MONETARY POLICY, ANTICIPATED INFLATION, AND  
THE VALUE OF INFORMATION

This paper argues that a case for activist monetary policy can be made even under the assumptions of rational-expectations equilibrium theory, if account is taken of the real effects of perfectly anticipated inflation. The case is more clearly related to traditional arguments for stabilizing the price-level than it is to Keynesian demand-management. The paper presents an example in which agents choose whether or not to learn the monetary authority's information. The model allows for explicit calculations of each agent's expected utility. Conditions are derived under which the optimal monetary policy is necessarily an activist one. The paper also shows how this case for activist policy arises from a divergence between the private and social values of information.

1. Introduction

Activist monetary policy may be defined in the following terms. Suppose that the change in the money supply from time  $t$  to  $t+1$ ,  $x_t$ , is determined by the monetary authority in an economy according to the rule, or policy:  $x_t = x(n_t; M_t, M_{t-1}, \dots)$  where  $M_t$  is the money supply and  $n_t$  a random variable representing any information that the monetary authority possesses. An activist policy is one that depends non-trivially on  $n_t$ ; i.e., one for which  $x(n; M_t, M_{t-1}, \dots) \neq x(n'; M_t, M_{t-1}, \dots)$  for some  $n, n'$ . The recent rational-expectations critique of activist policy is based on an "ineffectiveness proposition", valid in a certain class of models (Barro, 1976; Lucas, 1972; Sargent and Wallace, 1975), according to which if people's expectations are formed rationally, if markets clear, if people know the form of the policy

$x(\cdot)$ , and if people have access to the monetary authority's information, then the behavior of all real variables in the economy is independent of the form of the rule. Thus, in particular, there can be no welfare gain from the adoption of an activist policy.

The "ineffectiveness proposition" has been premised on the idea that real output can be affected by errors in forecasting the price-level, but not by the forecasts themselves, or by the actual time-path of the price level. The argument is that under the foregoing assumptions monetary policy cannot generate forecast errors and hence cannot affect real output.

There is, however, a large body of literature describing the effects that actual and forecast time-paths of the price-level can have on real magnitudes, even if they are the same paths and no forecast errors occur. (For a sample, see Friedman, 1977, Leijonhufvud, 1977.) Real resources are used in changing prices and altering the nominal values in contracts; the quantity of such resources used will vary with the rate of inflation, even if fully expected. The timing of transactions, and hence the use of resources for storing and trading commodities will be affected by expected inflation because it affects the real cost of holding money. The stock of capital and its rate of accumulation will be affected by the expected rate of inflation as firms and households adjust their portfolios to changes in cost of holding money. Political pressures and social tensions can be influenced by perfectly foreseen inflation that induces more frequent bargaining over the terms of contracts, and hence more frequent acts of conflict and confrontation. Expected inflation can also affect the real wages of anyone whose receipts will be stored, even temporarily, in the form of money or any other liquid asset whose real pecuniary yield is reduced by expected inflation. Finally, real resources can be used in forecasting the price-level; thus an expectation

that the price-level will fluctuate in ways that can be predicted on the basis of currently available information will generally cause more resources to be devoted to gathering and processing such information than if people expect the monetary authority to act in such a way as to make the future price level independent of any current information.

The "ineffectiveness proposition", by abstracting from all these considerations, can actually be used in defense of activist policies. For it has as a direct corollary a "harmlessness proposition" to the effect that under the same assumptions activist policies, because they affect no real variables, can do no real harm, no matter how vigorously pursued. In particular, the counter-argument that a too vigorous activist policy would cause too much variability in the price-level carries no force if anticipated changes in the price level have no real effects.

On the other hand if it is admitted that, for any of the above reasons, anticipated changes in the price-level can have real effects, then there is an obvious case to be made for activist policies in strict preference to non-activist policies. For example the literature on the optimum quantity of money (e.g., Friedman 1969, Grandmont-Younès 1973) makes it clear that under some circumstances the expected rate of inflation should be just such as to make the holding of money privately costless. To do this when future fluctuations can occur in population or technology that can be foreseen and that would alter the rate of inflation the absence of an activist policy would require an activist policy to offset the effects of these fluctuations on the expected rate of inflation. Formal examples of such justifications for activist policy are now beginning to appear in the literature (see, for example, Bulow and Polemarchakis, 1978; Weiss, 1980a, 1980b).

The argument over activist vs. non-activist policy is ultimately an argument over who should be entrusted with the production and use of a certain kind of information; a centralized authority or decentralized private agents. The rational expectations critique has argued that there is nothing to be gained from centralizing these decisions. Any argument for activist policies must establish that private agents will not be motivated to make the socially optimal decisions.

Either argument should take into account the costs of gathering and using information both to private agents and the monetary authority. The models in which the "ineffectiveness proposition" has been demonstrated do this only trivially by supposing that some kinds of information are available at no cost and the rest not available at any cost.<sup>1</sup> In a previous paper (Howitt, 1981) I argued that, when agents face a non-trivial choice of how much information to use, a Keynesian role for activist monetary policy can be rationalized as Pareto-optimal because the private and social values of information may differ. In that model selling prices were set by each agent in anticipation of future sales, and were costly to adjust. The present paper argues that the role for activist policy that arises in a model where anticipated inflation can have real effects can be justified by a similar argument, even when prices are always adjusted costlessly by a Walrasian auctioneer to their market-clearing levels.

More specifically, the paper sets out a model in which agents may or may not invest resources to obtain a forecast of next period's population, which enables them to predict next period's price. The forecast will have effects on the amount of labour supplied in the current period because the workers store their wages in non-interest-bearing money. The private value of the information contained in these forecasts is almost always positive.

But the social value is not, because the variability in the forecasts just causes socially wasteful variability in the current level of real output. An optimal monetary policy in some circumstances is one that alters the relationship between the future population and price-level, in just such a way as to render privately useless the population forecasts. This saves society the resources that would otherwise go into obtaining the forecasts, as well as eliminating this source of variability in today's output. Such a policy turns out almost always to be an activist one.

This role for activist monetary policy differs in important respects from the Keynesian role. It bears a clear resemblance to the more traditional role stressed by several generations of quantity theorists; that of stabilizing the price-level or, more generally, that of providing "a stable background for the economy" (Friedman, 1968, p. 13). The paper also discusses briefly the relationship between these various roles.

## 2. The Model

I use a simple model of overlapping generations, with random population growth. Time is indexed by  $t = 0, 1, \dots$ . At any date  $t$  there are  $n_t$  agents born, who are referred to collectively as "generation  $t$ ". Each agent lives two periods. There is only one good (other than money). Money is storeable but the good is not. Each member of generation  $t$  when young produces  $q_t$  units of the good in excess of his own consumption that period. He also consumes when old the amount  $c_t$  in excess of his current production. His utility is given by the utility function  $w_t = -v(q_t) + u(c_t)$ . The supply of money in period  $t$  is  $M_t$ . At the beginning of each period this money is held entirely by the old generation ( $t-1$ ), who spend it all on buying the good from the young. The monetary

authority gives each young agent a lump-sum grant of  $x_t$  units of money (possibly negative) after he has produced and sold  $q_t$ . The auctioneer chooses the price  $p_t$  so as to clear the market in period  $t$ .

Each period the old take  $p_t$  as given by the auctioneer, and their per capita holding of money  $M_t/n_{t-1}$  as given by history; they each demand the quantity  $M_t/p_t n_{t-1}$  of the good. The young also take  $p_t$  as given but they must decide how much to produce before learning its value.<sup>2</sup> They choose  $q_t$  so as to maximize their expectation of  $-v(q_t) + u((p_t q_t + x_t)/p_{t+1})$ . The auctioneer sets  $p_t = M_t/q_t n_t$ .

The old have no interest in forecasting  $x_t, p_t$ , or  $p_{t+1}$ , but the young do. Before choosing  $q_t$  a young agent can choose, at a cost of  $Z_m (\geq 0)$  utils, to learn the exact value of  $n_{t+1}$  (i.e., to "monitor"  $n_{t+1}$ ), which will help in his forecasts.<sup>3</sup> Assume that:

(A.1) The  $n_t$ 's are independently, identically distributed, according to the continuous distribution function  $f(\cdot)$ , whose support is an interval  $[\underline{n}, \bar{n}]$ , with  $0 < \underline{n} < \bar{n}$ , and:

(A.2)  $u(\cdot)$  and  $v(\cdot)$  are thrice continuously differentiable on  $\Omega$  (the non-negative real line) with  $u' > 0$ ,  $v' \geq 0$ ,  $u'' \leq 0$ ,  $v'' > 0$  on  $\Omega$ , such that  $v'(\cdot)$  defines a one-to-one correspondence from  $\Omega$  onto  $\Omega$ .

### 3. Optimal Allocations

I shall take a short-run perspective in evaluating monetary policies. That is, suppose that there is a new monetary authority each period. Then the authority at any date must base his policy upon his expectations of the behavior of future authorities. Suppose that in periods  $t=2,3,\dots$ , the authorities are expected to act so as to bring about the level of output per young given by the function  $\bar{q}_t(n_t, n_{t+1})$ , where:



(A.3)  $\bar{q}_t(\cdot)$  is continuous and strictly positive, on  $[\underline{n}, \bar{n}] \times [\underline{nx}\bar{n}] \forall t = 2, 3, \dots$

[It will become clear below just how the future authorities could accomplish this for any such set of  $\bar{q}_t(\cdot)$ 's.] Then from the point of view of the first authority, which will also be our point of view, the only generations whose welfare can be affected, and hence whose welfare need be considered, are 0 and 1. Let social welfare be measured by the function  $\phi = E(w_0 + \lambda w_1)$ , where E denotes the expectation taken before the authority learns the value of  $n_2$ ; that is, the value conditional upon  $q_0$ ,  $n_0$  and  $n_1$ ; and where  $\lambda > 0$ .

An "allocation" is defined as any continuous function  $q_1(\cdot): [\underline{n}, \bar{n}] \rightarrow \Omega$ . If generation 1 produces  $q_1$  each then generation 0 consumes  $(q_1 n_1 / n_0)$  each. Thus associated with any allocation is a pair of expected utilities:  $Ew_0 = -v(q_0) + \int_{\underline{n}}^{\bar{n}} u(q_1(n_2)n_1/n_0)f(n_2)dn_2$ ,  $Ew_1 = -\int_{\underline{n}}^{\bar{n}} v(q_1(n_2))f(n_2)dn_2 + \int_{\underline{n}}^{\bar{n}} \int_{\underline{n}}^{\bar{n}} u(\bar{q}_2(n_2, n_3)n_2/n_1)f(n_2)f(n_3)dn_2dn_3$ .

An "efficient allocation" is an allocation whose associated expected utilities are not dominated in the Pareto-sense by those of any other allocation. An allocation is efficient if and only if it is independent of  $n_2$ , for any dependence on  $n_2$  just adds useless, and costly, uncertainty to each agent's prospects. To prove this suppose first that  $q_1(n') \neq q_1(n'')$  for some  $n', n'' \in [\underline{n}, \bar{n}]$ . Define  $\tilde{q} \equiv E q_1(n)$ . By (A.1) and the continuity of  $q_1(\cdot)$ ,  $q_1(n) \neq \tilde{q}$  with positive probability. Thus by (A.2) and Taylor's Theorem,  $Eu(q_1(n)n_1/n_0) \equiv u(\tilde{q}n_1/n_0)$  and  $Ev(q_1(n)) > v(\tilde{q})$ . Thus  $q_1(\cdot)$  is dominated by  $\tilde{q}$  and cannot be efficient. Suppose next that  $q_1(n) \equiv \tilde{q}$ . Take any allocation  $q'(n) \neq \tilde{q}$ . Using Taylor's Theorem as above we see that  $u(\tilde{q}n_1/n_0) > Eu(q'(n)n_1/n_0)$  if  $\tilde{q} > E\tilde{q}(n)$  and  $v(\tilde{q}) < Ev(q'(n))$  otherwise. Thus  $\tilde{q}$  is not Pareto-dominated by  $q'(\cdot)$ . Since  $q'(\cdot)$  is arbitrary  $\tilde{q}$  is efficient.

An "optimal allocation" is an allocation whose expected utilities yield a higher value of  $\phi$  than those of any other allocation. Since  $\lambda > 0$  an optimal allocation must also be efficient. Thus it can equivalently be defined as a scalar  $q$  that maximizes  $\Psi(q) \equiv u(qn_1/n_0) - \lambda v(q)$  subject to  $q \geq 0$ . By (A.2),  $\Psi'( )$  is continuous on  $\Omega$ ,  $\Psi'(0) = u'(0) n_1/n_0 > 0$  and  $\lim_{q \rightarrow \infty} \Psi'(q) = -\infty$ . Thus, there exists a  $\hat{q} > 0$  such that  $\Psi'(\hat{q}) = 0$ . By (A.2),  $\Psi( )$  is strictly concave on  $\Omega$ . Thus  $\hat{q}$  is the unique maximizer of  $\Psi$  on  $\Omega$  and is therefore the unique optimal allocation.

#### 4. Monetary Policy

Let the money supply in period 1 be given as  $M_1 > 0$ . The "policy" of the first authority can be defined as a continuous function  $x_1( ) : [\underline{n}, \bar{n}] \rightarrow \mathbb{R}$  such that:

$$(1) \quad M_2 \equiv M_1 + n_1 x_1(n_2) > 0 \quad \forall n_2 \in [\underline{n}, \bar{n}].$$

An "activist policy" is defined as a policy such that  $x_1(n') \neq x_1(n'')$  for some  $n', n'' \in [\underline{n}, \bar{n}]$ . We shall be concerned with showing that under some circumstances activist policies can perform better than nonactivist policies. Assume that there is a fixed cost,  $Z_a \geq 0$ , in units of social welfare, for administering an activist policy rather than a monetarist policy, but that this is the only administrative cost.

As a simplifying assumption suppose that each  $\bar{q}_t(n_t, n_{t+1})$  is independent of  $n_{t+1}$ ; so we may write:

$$(2) \quad \bar{q}_t(n_t, n_{t+1}) = \bar{q}_t(n_t); \quad t \geq 2.$$

Thus the price in period 2 will be:

$$(3) \quad \bar{p}_2(n_2) = \frac{M_1 + n_1 x_1(n_2)}{n_2 \bar{q}_2(n_2)}.$$

5. Equilibrium Without Monitoring

Suppose no agent monitors  $n_2$ . Suppose that before choosing  $q_1$  each young agent can predict with certainty the value of  $p_1$  that the auctioneer will set, and that he knows the exact form of the functions  $f(\cdot)$ ,  $\bar{p}_2(\cdot)$  and  $x_1(\cdot)$ . Then his decision problem is to choose  $q$  so as to :

$$(4) \quad \text{Max} \quad -v(q) + \int_{\underline{n}}^{\bar{n}} u((p_1 q + x_1(n_2))/\bar{p}_2(n_2))f(n_2)dn_2, \text{ subject to } q \geq 0, \text{ and}$$

$$p_1 q + x_1(n_2) \geq 0 \quad \forall n_2 \in [\underline{n}, \bar{n}].$$

Define a "nonmonitoring equilibrium" with respect to the policy  $x_1(\cdot)$  as a number  $q^* > 0$  such that:

$$(5) \quad q^*v'(q^*) = \int_{\underline{n}}^{\bar{n}} \frac{M_1 n_2 \bar{q}_2(n_2)}{(M_1 + n_1 x_1(n_2))n_1} u'(n_2 \bar{q}_2(n_2)/n_1) f(n_2) dn_2.$$

According to (A.2),  $qv'(q)$  varies strictly monotonically from 0 to  $\infty$  as  $q$  varies from 0 to  $\infty$ . According to (A.1), (A.2), (A.3), and (1), the RHS of (5) is strictly positive. Thus there exists a unique nonmonitoring equilibrium with respect to any given policy. Suppose the auctioneer sets the price  $p > 0$ . Then, by (A.2) and (3) the quantity demanded,  $q \equiv M_1/n_1 p$  is the quantity supplied; i.e., the quantity that solves the problem (4), if and only if it satisfies (5). Thus  $q^*$  constitutes an equilibrium in the usual sense, and we shall assume that when no agent monitors  $n_2$  the market generates the allocation  $q_1(n_2) \equiv q^*$ .

A nonmonitoring equilibrium is efficient; it must be independent of  $n_2$  because the young choose their output before learning the value of  $n_2$ . Also, for every efficient allocation  $q > 0$  there is a nonactivist policy yielding  $q$  as the nonmonitoring equilibrium. The policy is computed from (5) as:

$$(6) \quad x_1(n) \equiv x = \frac{M_1}{n_1} \left( \int_{\underline{n}}^{\bar{n}} \frac{n_2 \bar{q}_2(n_2)}{q v'(q) n_1} u'(n_2 \bar{q}_2(n_2)/n_1) f(n_2) dn_2 - 1 \right)$$

In particular, there is a nonactivist policy that yields the optimal allocation  $\hat{q}$ . Thus as long as no monitoring occurs there is no need for an activist policy.

#### 6. Equilibrium With Monitoring

Next suppose that all agents monitor  $n_2$ . Assume that once the young agent learns the value of  $n_2$  he can predict perfectly the auctioneer's price,  $p_1(n_2)$  before deciding how much to produce,  $q_1(n_2)$ . His problem is to choose  $q(n_2)$  so as to:

$$(7) \quad \text{Max } -v(q(n_2)) + u((p_1(n_2)q(n_2) + x_1(n_2))/(\bar{p}_2(n_2))) \text{ subject to:}$$

$$q(n_2) \geq 0, \text{ and } p_1(n_2) q(n_2) + x_1(n_2) \geq 0.$$

Define a "monitoring equilibrium" with respect to  $x_1(\cdot)$  as an allocation  $q^*(\cdot)$  such that  $q^*(n_2) > 0$  on  $[\underline{n}, \bar{n}]$ , and:

$$(8) \quad q^*(n_2) v'(q^*(n_2)) = \frac{M_1 n_2 \bar{q}_2(n_2)}{(M_1 + n_1 x_1(n_2)) n_1} u'(n_2 \bar{q}_2(n_2)/n_1) \quad \forall n_2 \in [\underline{n}, \bar{n}]$$

The same argument that established the existence of  $q^*$  establishes the existence of a unique function  $q^*(n_2) > 0$  satisfying (8). By (A.2) and the inverse function theorem the function:

$$(9) \quad \varphi(q) \equiv q v'(q)$$

has a unique, continuous inverse function  $\xi: \Omega \rightarrow \Omega$ , where

$$(10) \quad \xi(\varphi(q)) \equiv q, \quad \xi'(\varphi(q)) \equiv [\varphi'(q)]^{-1} > 0, \text{ on } \Omega.$$

By the continuity of  $\xi(\cdot)$  and the continuity of the RHS of (8) with respect to  $n_2$ ,  $q^*(\cdot)$  is continuous. Therefore  $q^*(\cdot)$  is a unique monitoring equilibrium. As before this is a unique equilibrium in the usual sense and we shall assume that when all agents monitor the market generates the allocation  $q^*(\cdot)$ .

When agents monitor there is a need for activist policy. Under a nonactivist policy the monitoring equilibrium will not be efficient except by accident, because it will not generally be independent of  $n_2$ . When  $n_2$  varies, the young will generally anticipate a change in next period's price by shifting their supply schedules this period, thereby causing this period's equilibrium output to vary. But an activist policy can always be found to eliminate this costly variability in output.

More precisely, take any efficient allocation with positive output; that is, any  $q_1(n_2) \equiv \bar{q} > 0$ . Then, from (8),  $q_1(n_2)$  is the monitoring equilibrium if and only if the authority pursues the policy:

$$(11) \quad x_1^*(n_2) \equiv \frac{M_1}{n_1} \left( \frac{n_2 \bar{q}_2(n_2) u'(n_2 \bar{q}_2(n_2)/n_1)}{n_1 \bar{q} v'(\bar{q})} - 1 \right).$$

But it follows directly from (11) that  $x_1^*(n_2)$  is a nonactivist policy if and only if the restriction of  $u(\cdot)$  to the set  $C \equiv \{c = n_2 \bar{q}_2(n_2)/n_1 \text{ for some } n_2 \in [\underline{n}, \bar{n}]\}$  has the form:  $a + b \ln c$ , for some constants  $a, b$ , with  $b > 0$ . In what follows I assume that this special case does not hold.

Thus, in particular, the optimal allocation  $\hat{q}$  can be generated as a monitoring equilibrium by the appropriately chosen policy, but this policy must be an activist policy. Furthermore, if the authority pursues a nonactivist policy, not only will it fail to achieve  $\hat{q}$ , it will fail to achieve any efficient allocation. Hence the need for activist policy when agents monitor.

## 7. The Source of Inefficiency

What is the source of the inefficiency in a monitoring equilibrium that gives rise to the need for activist policy? In one sense it is the way in which we have defined efficiency. Specifically, we have defined it in terms of expected utility, where the expectation is taken ex ante; i.e., before the value of  $n_2$  is known. Suppose instead we look at utilities ex post:  $w_0(q_1(n_2), n_2) \equiv -v(q_0) + u(n_1 q_1(n_2)/n_0)$ , and  $w_1(q_1(n_2), n_2) \equiv -v(q_1(n_2)) + u(n_2 \bar{q}_2(n_2)/n_1)$ . An allocation  $q_1(\cdot)$  is said to be "ex post efficient" if there is no other allocation  $q_1'(\cdot)$  such that:

$$(12) \quad \begin{cases} w_i(q_1(n_2), n_2) \leq w_i(q_1'(n_2), n_2) \text{ for all } n_2 \in [\underline{n}, \bar{n}] \text{ and } i=0,1, \text{ with} \\ \text{a strict inequality for at least one such } n_2, i. \end{cases}$$

This corresponds to the efficiency concept used by Lucas (1972),<sup>4</sup> who showed, in terms of his somewhat different model, that every "k-percent policy" generates an efficient allocation. Likewise in the present model, as we shall see, every nonactivist policy generates an efficient allocation in the ex post sense. Thus the inefficiency described in the previous section can be attributed to our choice of an ex ante rather than ex post concept of efficiency.

The ex post concept is, however, particularly uninteresting in the present model. As Muench (1977) and Polemarchakis and Weiss (1977) have noted, it is an extremely broad concept. Indeed all allocations are ex post efficient in the present model. For according to (A.2),  $w_1(q_1(n_2), n_2) < (>) w_1(q_1'(n_2), n_2)$  if and only if  $w_0(q_1(n_2), n_2) > (<) w_0(q_1'(n_2), n_2)$ , so that (12) can never hold.

We may also regard the inefficiency of monitoring equilibria as a case of missing markets. In particular, suppose that the young could sell, and the

old buy, claims to output contingent upon the value of  $n_2$ . Each old agent would choose to buy a set of claims; that is, a function  $c_o(\cdot): [\underline{n}, \bar{n}] \rightarrow \Omega$ , so as to:

$$(13) \quad \max \int_{\underline{n}}^{\bar{n}} u(c_o(n_2))f(n_2)dn_2 \quad \text{subject to} \quad \int_{\underline{n}}^{\bar{n}} \pi(n_2)c_o(n_2)dn_2 = M_1/n_o$$

where  $\pi(n_2)$  is the money price of claim to a unit of output in the event of  $n_2$ . Each young agent would choose to sell a set of claims  $q_1(\cdot)$  and acquire thereby a sum of money  $m$ , so as to:

$$(14) \quad \max \int_{\underline{n}}^{\bar{n}} \{-v(q_1(n_2)) + u((m + x_1(n_2))/\bar{p}_2(n_2))\}f(n_2)dn_2$$

$$\text{subject to} \quad \int_{\underline{n}}^{\bar{n}} \pi(n_2)q_1(n_2)dn_2 = m.$$

An equilibrium would be a continuous function  $\pi(\cdot): [\underline{n}, \bar{n}] \rightarrow \Omega$  such that the function  $c_o(\cdot) \equiv n_1 q_1(\cdot)/n_o$  solves (13), where  $q_1(\cdot)$  solves (14). It follows from classical theory of the calculus of variations and the concavity assumptions of (A.2) that  $\pi(\cdot)$  is an equilibrium and  $q_1(\cdot)$  the associated allocation if and only if there exist constants  $\lambda_o$  and  $\lambda_1$  such that:

$$(15) \quad \frac{u'(n_1 q_1(n_2)/n_o)}{\lambda_o} \equiv \frac{\pi(n_2)}{f(n_2)} \equiv \frac{v'(q_1(n_2))}{\lambda_1}$$

$$(16) \quad \int_{\underline{n}}^{\bar{n}} n_1 q_1(n_2) \pi(n_2) dn_2 = M_1$$

$$(17) \quad \lambda_1 = \int_{\underline{n}}^{\bar{n}} (\bar{p}_2(n_2))^{-1} u'(n_2 \bar{q}_2(n_2)/n_1) f(n_2) dn_2$$

It follows directly from (15) that the equilibrium allocation  $q_1(\cdot)$  is efficient.<sup>5</sup> For, from (A.2), the equation  $u'(n_1 q_1(n_2)/n_o) = (\lambda_1/\lambda_o)v'(q_1(n_2))$  has a unique solution  $q_1(n_2) = \tilde{q} \quad \forall n_2$ . Thus the equilibrium allocation  $q_1(n_2)$  equals the constant  $\tilde{q}$  and hence is efficient. The inefficiency of the monitoring

equilibrium can therefore be attributed to the absence of this market in contingent claims.

We must be careful in interpreting this result. The essential feature of this contingent claims market is not that it permits agents to have their claims vary with  $n_2$ . For as we have just seen they do not end up using this option. In fact, this equilibrium is essentially identical to the nonmonitoring equilibrium in which contingent trading was ruled out by assumption.

More precisely, take any policy  $x_1(\cdot)$ . Then  $\tilde{q}$  is the equilibrium allocation in the contingent claim market if and only if that  $\tilde{q}$  is the nonmonitoring equilibrium. Define  $\lambda_1$  by (17) and  $\lambda_0, \pi(\cdot)$  by (15), with  $q_1(n_2) \equiv \tilde{q}$ . The continuity of  $\pi(\cdot)$  follows from (A.1). Thus to show that  $(\pi(\cdot), \tilde{q})$  is the equilibrium in the contingent claim market I need only show that (16) is satisfied. By the second equality of (15)  $\int_{\underline{n}}^{\bar{n}} n_1 \tilde{q} \pi(n_2) dn_2 = \int_{\underline{n}}^{\bar{n}} (n_1 \tilde{q} v'(\tilde{q}) / \lambda_1) f(n_2) dn_2 = n_1 \tilde{q} v'(\tilde{q}) / \lambda_1$ . But, by (3), (5), and (17) this last expression equals  $M_1$ , which establishes (16). Next, suppose that  $(\pi(\cdot), \tilde{q})$  is the equilibrium in the contingent claim market. To show that  $\tilde{q}$  is the nonmonitoring equilibrium I need only show that it solves (5). From the second equality of (15),  $\tilde{q} v'(\tilde{q}) = \int_{\underline{n}}^{\bar{n}} \tilde{q} v'(\tilde{q}) f(n_2) dn_2 = \int_{\underline{n}}^{\bar{n}} \tilde{q} \lambda_1 \pi(n_2) dn_2$ . By (16), this expression equals  $\lambda_1 M_1 / n_1$ . By (3) and (17) this equals

$$\int_{\underline{n}}^{\bar{n}} \frac{M_1 n_2 \bar{q}_2(n_2)}{(M_1 + n_1 x_1(n_2)) n_1} u'(n_2 \bar{q}_2(n_2) / n_1) f(n_2) dn_2, \text{ which establishes (5).}$$

Instead, the essential feature of this contingent claim market is that it convenes before agents know the value of  $n_2$ . If they knew  $n_2$  before it convened then no one would buy a claim contingent on any but the actual event, and the resulting market situation would be exactly as described in the monitoring equilibrium.



Indeed the basic cause of the inefficiency of monitoring equilibria is that  $n_2$  is known before trading occurs. Whether or not contingent trading is permitted is irrelevant. If  $n_2$  is known beforehand you get the same inefficient result in either case. Otherwise you get the same efficient result in either case. This is a case of missing markets. What is missing is not a market that permits contingent trading but a market that convenes before  $n_2$  is known.

#### 8. The Social Benefit of Monitoring

Alternatively we have a case where, as in the examples discussed by Hirshleifer (1971), the social benefit from spending resources to acquire information (i.e. to monitor  $n_2$ ) is negative. As in Hirshleifer's examples this information has no social value. Any efficient allocation can be achieved without the agents or the authority being informed. But start with a nonactivist policy yielding as the nonmonitoring equilibrium the optimal allocation  $\hat{q}$ . Now suppose that agents start monitoring. The authority may restore  $\hat{q}$  with the appropriate activist policy, but this will cost  $Z_a$ . Or, it may continue with a nonactivist policy, in which case the monitoring equilibrium will yield a lower level of social welfare than  $\hat{q}$ . In either case the agent's decision to monitor causes a reduction in social welfare. Perhaps a new market would open which convened before the now advanced date at which  $n_2$  becomes known. But even in this case society must pay the cost of opening the previously missing market, or at least of advancing the date of the previously existing one.<sup>6</sup>

As a result of their decision to monitor even the young themselves may be made worse off, with no change in policy. For if  $x_1(\cdot)$  is anything but  $x_1^*(\cdot)$  of (11), then the resulting change in expected utility,

$\Delta \equiv Ew_1(q^*(n_2), n_2) - Ew_1(q^*, n_2) \stackrel{\geq}{\leq} 0$  respectively if  $R'(q) \stackrel{\geq}{\leq} 0 \forall q \in \bar{\Omega}$ , where  $R(q) \equiv v''(q) \cdot q / v'(q)$ , the elasticity of marginal cost. To prove this, define  $\varphi^* \equiv \varphi(q_1^*)$  and  $\varphi^*(n_2) \equiv \varphi(q_1^*(n_2))$ . Then  $\Delta = \zeta(\varphi^*) - E\zeta(\varphi^*(n_2))$ , where:

$$(18) \quad \zeta(\cdot) \equiv v(\xi(\cdot)) \text{ on } \bar{\Omega}$$

Since  $x_1(\cdot) \neq x_1^*(\cdot)$ , therefore  $q_1^*(n_2)$  is not independent of  $n_2$ , and hence, by (A.2),  $\varphi^*(n_2)$  is not independent of  $n_2$ . But, from (5) and (8),  $\varphi^* = E\varphi^*(n_2)$ .

Therefore, by the usual argument,  $\Delta \stackrel{\geq}{\leq} 0$  if  $\zeta''(\cdot) \stackrel{\geq}{\leq} 0$  on  $\bar{\Omega}$ . But, from

$$(9), (10), \text{ and } (18), \quad \zeta'' \equiv v''(\xi')^2 + v' \xi'' \equiv \left(\frac{1}{\varphi'}\right)^3 (v'' \varphi' - v' \varphi'') \equiv \left(\frac{1}{\varphi'}\right)^3$$

$$(q(v'')^2 - v' v'' - qv' v''') \equiv - \frac{v'(q)}{\varphi'(q)^3} R'(q), \text{ which gives the desired result.}$$

9. The Private Benefit to Monitoring

There is a need for activist policy if and only if monitoring would occur. Thus before discussing optimal monetary policy we must know whether or not agents will monitor. It might be thought that since the information contained in  $n_2$  has no social value, and since the social benefit to monitoring  $n_2$  is negative, therefore no one will be interested in monitoring it. But this turns out to be false. This section shows that the private benefit to monitoring will be positive, unless the authority pursues an "ideal" policy of the form (11), in which case it will be zero.

The size of this gain to a given agent depends upon whether or not the others are monitoring.<sup>7</sup> Consider first the case where no one else is monitoring. For any  $n_2 \in [\underline{n}, \bar{n}]$ , define  $\tilde{q}(n_2)$  as the solution to the problem:  $\max[-v(q) + u((p_1^*q + x_1(n_2))/\bar{p}_2(n_2))]$ , subject to  $q \geq \max(0, -x_1(n_2)/p_1^*)$ , where  $p_1^* \equiv M_1/n_1q^*$ . Then  $\tilde{q}(n_2)$  is uniquely defined by the conditions:

$$(19) \quad \begin{cases} -v'(\tilde{q}(n_2)) + \frac{p_1^*}{\bar{p}_2(n_2)} u'((p_1^*\tilde{q}(n_2) + x_1(n_2))/\bar{p}_2(n_2)) \leq 0 & \text{with equality} \\ \text{if } q > \max(0, -x_1(n_2)/p_1^*). \end{cases}$$

From (19), (A.2), and the appropriate second-order Taylor series expansion:

$$(20) \quad \begin{aligned} & -v(\tilde{q}(n_2)) + u((p_1^*\tilde{q}(n_2) + x_1(n_2))/\bar{p}_2(n_2)) \geq \\ & -v(q^*) + u((p_1^*q^* + x_1(n_2))/\bar{p}_2(n_2)) \text{ with equality} \\ & \text{if and only if } q^* = \tilde{q}(n_2). \end{aligned}$$

The isolated agent who monitors gets the expected utility

$$\tilde{w} \equiv \int_{\underline{n}}^{\bar{n}} \{-v(\tilde{q}(n_2)) + u((p_1^*\tilde{q}(n_2) + x_1(n_2))/\bar{p}_2(n_2))\} f(n_2) dn_2. \quad \text{The private gain to}$$

monitoring in this case is  $B^n \equiv \tilde{w} - Ew_1(q^*, n_2)$ . Note that

$(p_1^*q^* + x_1(n_2))/\bar{p}_2(n_2) \equiv n_2\bar{q}_2(n_2)/n_1$ . Therefore, from (A.2) and (20)  $B^n \geq 0$  with equality if and only if  $\bar{q}(\cdot) \equiv q^*$ . But, inspection of (11) and (19) reveals that  $\bar{q}(\cdot) \equiv q^*$  if and only if  $x_1(\cdot)$  has the form (11). Thus  $B^n \geq 0$  with equality if and only if  $x_1(\cdot)$  the form (11).

Next consider the case where everyone else is monitoring. Define  $q'$  as the solution to the problem:  $\text{Max}[-v(q) + \int_{\underline{n}}^{\bar{n}} u((p_1^*(n_2)q + x_1(n_2))/\bar{p}_2(n_2))f(n_2)dn_2]$ , subject to  $q \geq \text{Max}(0, -x_1(n_2)/p_1^*(n_2))$ , where  $p_1^*(\cdot) \equiv M_1/n_1q^*(\cdot)$ . The agent who decides not to monitor when others are gets the expected utility  $w' = -v(q') + \int_{\underline{n}}^{\bar{n}} u((p_1^*(n_2)q' + x_1(n_2))/\bar{p}_2(n_2))f(n_2)dn_2$ , and the gain to monitoring in this case is  $B^m \equiv Ew_1(q_1^*(n_2), n_2) - w'$ . In the problem (7) the agent could have chosen  $q(\cdot) \equiv q'$ . Thus, by the same argument as in the previous paragraph,  $B^m \geq 0$  with equality if and only if  $q^*(\cdot) \equiv q'$ . So, by the result leading up to (11),  $B^m \geq 0$  with equality if and only if  $x_1(\cdot)$  has the form (11).

For future reference note that, from (A.2) and the maximum theorem (Debreu, 1959, p. 19), when a nonactivist policy  $x_1(\cdot) = x$  is followed, both  $B^n$  and  $B^m$  can be regarded as continuous functions,  $B^n(\cdot), B^m(\cdot)$  on  $[-M_1/n_1, \infty)$ .

#### 10. Optimal Monetary Policy

Define an "informational equilibrium" with respect to any policy  $x_1(\cdot)$  as the nonmonitoring equilibrium w.r.t.  $x_1(\cdot)$  if  $B^n \geq Z_m$  or the monitoring equilibrium w.r.t.  $x_1(\cdot)$  if  $B^m \geq Z_m$ . From the results of the previous section it follows that if  $Z_m > 0$  and policy has the form (11) then the unique informational equilibrium will be the nonmonitoring equilibrium.

The objective of monetary policy is the social welfare  $\phi$  minus the costs, if any, of monitoring and of administering policy. An optimal monetary policy

is one yielding an informational equilibrium with a value of this objective no less than that of any informational equilibrium with respect to any other policy. A complete analysis of optimal policy is complicated by the fact that an informational equilibrium with respect to some policies may not exist or be unique. But consider the following two special cases. First, consider the nonactivist policy  $x_1(\cdot) \equiv \hat{x}$  that yields  $\hat{q}$  as the nonmonitoring equilibrium. Let  $\hat{B}^n, \hat{B}^m$  be the respective private benefits with respect to this policy. Suppose that  $Z_m > \max(\hat{B}^n, \hat{B}^m)$  and  $Z_a > 0$ . Then the optimal policy is obviously to set  $x_1(\cdot) \equiv \hat{x}$ , in which case the informational equilibrium will be the nonmonitoring equilibrium with the optimal allocation  $\hat{q}$ , and no monitoring or administration costs. Any activist policy would yield no higher value of  $\phi$  but would cost  $Z_a > 0$ . Any other nonactivist policy would yield a lower  $\phi$  and might also incur monitoring costs.

Next, consider the case where:

$$(21) \quad Z_m < \min(\hat{B}^n, \hat{B}^m).$$

Such cases exist because, from the results of the previous section, and the result that the policy (11) is activist,  $\min(\hat{B}^n, \hat{B}^m) > 0$ . Define the functions  $\phi^n(x), \phi^m(x)$  as the value of  $\phi$  resulting from the nonactivist policy  $x$  in the non-monitoring and monitoring equilibria. It is easily verified that  $\phi^n(\cdot)$  and  $\phi^m(\cdot)$  are continuous on  $[-M_1/n_1, \infty)$ . Since  $\hat{q}$  is uniquely optimal and  $\hat{x}$  is the only nonactivist policy yielding  $\hat{q}$  through (6),

$$(22) \quad \phi^n(x) < \hat{\phi} \text{ for all } x \in [-M_1/n_1, \infty), \text{ except } x = \hat{x}.$$

Since monitoring equilibria with monetarist policies are never optimal allocations, we have:

$$(23) \quad \phi^m(x) < \hat{\phi} \text{ for all } x \in [-M_1/n_1, \infty).$$

Define  $\bar{\phi}^m \equiv \sup\{\phi^m(x) \mid x \in [-M_1/n_1, \infty)\}$ , and  $\bar{\phi}^n \equiv \sup\{\phi^n(x) \mid B^n(x) \leq Z_m, x \in [-M_1/n_1, \infty)\}$  if any such  $x$  exists, or else any number less than  $\hat{\phi}$ . It can

be shown<sup>8</sup> that:

$$(24) \quad \bar{\phi}^m < \hat{\phi}, \quad \bar{\phi}^n < \hat{\phi}.$$

Suppose that:

$$(25) \quad Z_a < \min(\hat{\phi} - \bar{\phi}^m + Z_m, \hat{\phi} - \bar{\phi}^n).$$

Such cases exist, by (24). Then the following argument shows that (i) the activist policy (11) yielding  $\tilde{q} = \hat{q}$  is optimal, and (ii) no nonactivist policy is optimal. If this activist policy is pursued, then by the results of the previous section  $B^m = B^n = 0$ . Therefore monitoring will occur only if  $Z_m = 0$ . In any case no monitoring costs will be incurred. Also, by comparing (11) with (5) we see that the nonmonitoring equilibrium with respect to this policy is also  $\hat{q}$ . Thus in any case the value of the policy is  $\hat{\phi} - Z_a$ . The argument proceeds by showing that no other activist policy has a higher value, and no nonactivist policy does as well.

Any other activist policy will yield a value of  $\phi \leq \hat{\phi}$ , by the definition of  $\hat{q}$ . Thus whether or not it induces monitoring it will have a value no more than  $\phi - Z_a \leq \hat{\phi} - Z_a$ .

Consider any nonactivist policy  $x$ , and any equilibrium with respect to this policy. (a) If it is a monitoring equilibrium, it has a value  $\phi^m(x) - Z_m \leq \bar{\phi}^m - Z_m$ . (b) If it is nonmonitoring equilibrium, it has a value  $\phi^n(x) \leq \bar{\phi}^n$ . By (25) both these values are strictly less than  $\hat{\phi} - Z_a$ .

Thus we have a class of examples in which an activist monetary policy is optimal, because of the inefficiency caused by people (mis)using socially useless information. The activist policy can be thought of as working in two ways. It is calculated to yield an efficient allocation even if people continue to use the information. At the same time it removes the incentive for such use and hence saves the cost of any private resources devoted to that purpose.<sup>9</sup>

11. The Stationary Case

This section looks briefly at the present model from the stationary perspective of Lucas (1972). A stationary allocation is a continuous function:  $q(\cdot): [\underline{n}, \bar{n}]^3 \rightarrow \Omega$ . Thus  $q(n, n', n'')$  is the output of each young agent when there are  $n$  old and  $n'$  young this period and when there will be  $n''$  young next period. An optimal stationary allocation maximizes the expected utility of the unborn:

$$(26) \quad \int_{\underline{n}}^{\bar{n}} \int_{\underline{n}}^{\bar{n}} \int_{\underline{n}}^{\bar{n}} \{-v(q(n, n', n''))n' + u(n'q(n, n', n'')/n)n\} f(n)f(n')f(n'') dndn'dn''$$

(Note that  $-v$  and  $u$  must be weighted by  $n'$  and  $n$  respectively to capture the probability of being in a generation of a given size.) A stationary policy is a function  $x(\cdot): [\underline{n}, \bar{n}]^3 \rightarrow (-1, \infty)$ . The money supply grows according to:

$M_{t+1} = M_t (1 + x(n_{t-1}, n_t, n_{t+1}))$ . A monitoring equilibrium is the solution  $q^*(\dots)$  to:

$$(27) \quad q^*(n, n', n'')v'(q^*(n, n', n'')) = \int_{\underline{n}}^{\bar{n}} \frac{n''q^*(n', n'', n''')/n'}{1 + x(n, n', n''')} u'(n''q^*(n', n'', n''')/n') f(n''') dn'''$$

A nonmonitoring equilibrium is the solution  $q^*(\dots)$  to:

$$(28) \quad q^*(n, n')v'(q^*(n, n')) = \int_{\underline{n}}^{\bar{n}} \frac{n''q^*(n', n'')/n'}{1 + x(n, n', n'')} u'(n''q^*(n', n'')/n') f(n'') dn''.$$

Once again there is a case for activist policy because in a monitoring equilibrium people will (mis)use the socially useless information concerning the size of the next generation. Specifically, it follows from (26) that the optimal allocation is the solution  $\hat{q}(\dots)$  to:

$$(29) \quad -v'(\hat{q}(n, n')) + u'(n'\hat{q}(n, n')/n) = 0$$

This allocation will be yielded as both the monitoring and nonmonitoring equilibria if:

$$(30) \quad x(n, n', n'') = \frac{(n''\hat{q}(n', n'')/n') u'(n''\hat{q}(n', n'')/n')}{\hat{q}(n, n') v'(\hat{q}(n, n'))} - 1$$

It can also be yielded as the nonmonitoring equilibrium if:

$$(31) \quad x(n, n', n'') = x(n, n') = \frac{\int_{\underline{n}}^{\bar{n}} (n'' \hat{q}(n', n'')/n') u' (n'' \hat{q}(n', n'')/n')}{\int_{\underline{n}}^{\bar{n}} \hat{q}(n, n') v' (\hat{q}(n, n'))} f(n'') dn'' - 1$$

But it cannot be yielded as the monitoring equilibrium by any nonactivist policy. For the optimal allocation is independent of  $n''$ , whereas, according to (27) the monitoring equilibrium will not be independent of  $n''$  if  $x(\dots) \equiv x$ .

In this stationary case there is a further reason for activist policy. According to (A.2) and (29), the optimal allocation depends upon  $n$  unless  $u'' \equiv 0$ . But according to (27) and (28) under a nonactivist policy both equilibria are independent of  $n$ . Thus even a nonmonitoring equilibrium can usually be improved upon by an activist policy in the stationary case. The optimal social contract would have the young produce more the more old mouths there were to feed. But with no government intervention each short-run market allocation will be independent of the number of old, for the old influence the market only by the total amount of money they bring to market, in relation to the amount that will be brought next period. The optimal activist policy increases that relative amount when the number of old increases, so as to increase the young's output by creating the (rational) expectation of less inflation, and hence of a higher marginal return to producing now to acquire money to spend later. Thus the additional rationale for activist policy thrown up by the stationary case is that it might be used as a substitute for social security.



12. The Full Intertemporal Case

The short-run case had no future, and the stationary case no history. Consider now the full intertemporal case where the monetary authority knows the values of  $q_0$ ,  $n_0$ , and  $n_1$  given by history, can predict  $n_2$ , and can choose policy not only for period 1 but also for all  $t \geq 1$ .<sup>10</sup>

To deal with this case and avoid chain letter paradoxes suppose that the utility function  $u(\cdot)$  is bounded:

$$(32) \quad u(q) \leq \bar{u} \quad \text{for some } \bar{u} > 0, \text{ all } q \geq 0.$$

(Alternatively we could suppose that the amount of  $q$  producible by the young is bounded.)

Allocations are sequences of functions:  $\{q_t(n_0, \dots, n_{t+1})\}_{t=0}^{t=\infty}$  where  $q_0(n_0, n_1) = q_0$ . Efficiency is defined in terms of the expected utilities  $Ew_t = E[n_t(-v(q_t) + u(q_{t+1}n_{t+1}/n_t))]$ ;  $t=0,1,\dots$ ; where  $E$  is the expectation with respect to  $n_2, n_3, \dots$ . Define also:  $z_t = -n_t v(q_t) + n_{t-1} u(q_t n_t / n_{t-1})$ ;  $t=0,1,\dots$ . For any  $T \geq 0$  define the accumulated utility as:

$$(33) \quad V_T \equiv \sum_{t=0}^T Ew_t = -n_0 v(q_0) + \sum_{t=0}^{T-1} Ez_t + En_T u(q_{T+1} n_{T+1} / n_T)$$

Consider the allocation  $\{q_t(\cdot)\} \equiv \{\hat{q}(\cdot)\}$ , where  $\hat{q}(\cdot)$  is the stationary optimum defined by (29). This is clearly sustainable as a monitoring equilibrium if and only if the activist policy (30) is pursued. To show that the case for activism can be made in this context we need only show that this allocation is efficient.

To show this we need only compare it with other allocations of the form  $\{q'_t(n_{t-1}, n_t)\}$ , because any other allocation  $\{q_t(n_0, \dots, n_{t+1})\}$  is Pareto-dominated by the sequence  $\{Eq_t(n_0, \dots, n_{t+1}) | n_{t-1}, n_t\}$ . Suppose that  $\hat{q}(\cdot)$  is not efficient. Then there is an allocation  $\{q'_t(\cdot)\}$  such that:

$$(34) \quad \begin{cases} E(\hat{w}_t - w'_t) \leq 0 & \forall t \geq 0 \\ E(\hat{w}_{t_0} - w'_{t_0}) = -\bar{w} < 0 & \text{for some } t_0 \geq 0. \end{cases}$$

Note that, by the definition of  $\hat{q}(\cdot)$ ,

$$(35) \quad E(\hat{z}_t - z'_t) \geq 0 \quad \forall t \geq 0.$$

From (33) ~ (35):

$$(36) \quad E n_t [u(\hat{q}(n_t, n_{t+1}) n_{t+1} / n_t) - u(q'_{t+1}(n_t, n_{t+1}) n_{t+1} / n_t)] \leq \hat{V}_t - V'_t \leq -\bar{w} \quad \forall t \geq t_0$$

Define the norm:  $\|\hat{q} - q'_t\| = E(\hat{q} - q'_t)^2$ . Note that, by (A.2),  $u(\cdot)$  is uniformly continuous on  $\Omega$ . Thus, from (36):

$$(37) \quad \|\hat{q} - q'_t\| \geq \epsilon > 0 \quad \forall t \geq t_0 + 1.$$

Therefore, by (A.2) and the definition of  $\hat{q}(\cdot)$ :

$$(38) \quad E(\hat{z}_t - z'_t) \geq \delta > 0 \quad \forall t \geq t_0 + 1$$

Therefore, by (A.1), (32), (33), (35) and (38):

$$(39) \quad \hat{V}_t - V'_t \geq (t - 1 - t_0)\delta + \bar{n}[u(0) - \bar{u}] \quad \forall t \geq t_0 + 1$$

But, (39) contradicts (36).

### 13. Conclusion

We have shown how, depending upon the costs and benefits of using information and the cost of administering activist policy, such a policy may be justified by a divergence between the private and social values of information. This happens despite rational expectations. It also may happen even if there is no cost-advantage enjoyed by the monetary authority over private agents in using information, because (25) does not exclude the case of  $Z_a > Z_m$ .

Indeed the monetary authority can successfully pursue its activist policy even if it cannot at any cost obtain the information available to private agents. For the policy works by persuading people that next period, once the information becomes freely available to all, the money supply will

be adjusted to it in a predetermined fashion. There is no need for the monetary authority to have that information before today's price is determined in order to persuade people that this is how they will behave. (A similar point has been made by Weiss, 1980a, and by Bulow and Polemarchakis, 1978.)

Conversely, if the monetary authority is better informed than private agents it certainly does not follow that, as Barro suggests (1976, p. 23) the authority should disseminate that information. On the contrary, under condition (25) above the authority would be better to conceal the information if possible.

This case for activist policy is quite different from the more usual Keynesian case (for a modern version see Fischer, 1977, or Phelps and Taylor, 1977) which argues that monetary policy should be used to dampen fluctuations in aggregate demand which, because of stickiness in nominal prices, would otherwise lead to fluctuations in real output. In the present model monetary policy affects current fluctuations in aggregate supply, not aggregate demand. It has its effects on current output not by changing the current money supply but by affecting expectations of future changes in the price-level. Finally, it works even with perfectly flexible nominal prices. One similarity between the two cases is that, as I have tried to show in this paper and my earlier one (1981), they can both be rationalized by a divergence between the private and social values of information.

The case actually bears a close resemblance to the traditional argument, used by quantity theorists, that monetary policy ought to be used to stabilize the price-level. This argument was made by Fisher (1922), and Friedman (1968). Both of these authors, as well as Simons (1936) recognized explicitly that a stable price-level would require a variable money supply

(in our terminology an activist policy).<sup>11</sup> In all these cases monetary policy was assumed to work by affecting the way in which the price-level would be affected by exogenous disturbances. What the present argument makes clear is that this case can be made even if these effects on the price-level can be perfectly anticipated.<sup>12</sup>

In our model optimal policy does stabilize the current price-level, as a by-product of stabilizing current output, since  $p = M/q$ , and  $M$  is given historically. But there is also a special case in which optimal policy keeps the price-level constant from one period to the next. As can be seen from (3) and (11) this is the case where the marginal utility of consumption is constant. More generally, optimal policy reduces the variability of next period's price below what it would be next period under a non-activist policy, whenever the elasticity of marginal utility is less than unity.<sup>13</sup> (This is also the condition required in the Lucas model to make the Phillips curve slope "the right way".)

Finally, it should be noted that Friedman's (1968) argument goes on to recommend that in our less than ideal world policy should be aimed at stabilizing the money supply rather than the price level. His further argument is not refuted by the present example. The example suggests that in a much more general class of models the optimal monetary policy is often an activist one. But to implement it we would first have to know which model was the "true" one, and how to devise a system that would ensure the right decisions were made by the monetary authority. Friedman rightly points out that both of these requirements are far from being met, and argues that stabilizing the money supply is the best practical way of achieving approximate price level stability. His argument does not depend in any way upon the "ineffectiveness" argument addressed by the present paper.

Footnotes

<sup>1</sup>The limitations of such extreme informational assumptions have also been pointed out by such authors as Poole (1976), Laidler (1978) and Friedman (1979).

<sup>2</sup>Requiring the young to decide on  $q_t$  before learning the value of  $p_t$  allows us to avoid the complications of the young taking a "free ride" by inferring from  $p_t$  the information of his better-informed contemporaries. Under our assumptions he may be able to make such an inference but only after it is too late for him to make any use of the information.

<sup>3</sup>The model is the same as that of Bulow and Polemarchakis (1978) except that (a) the random variable is population instead of productivity, (b) the utility function is not restricted to the class  $-v(q) + u(c) = -q + \frac{1}{\alpha} c^\alpha$  ( $\alpha \neq 0$ ,  $\alpha < 1$ ), being restricted only by (A.2) below, (c) the agents may choose not to be informed. The subsequent analysis also differs in that we consider more than the stationary properties of the model

<sup>4</sup>Except that Lucas takes the stationary perspective of section 11 below. The Lucas model differs from the present one also in that (a) it assumes monetary transfers take the form of interest-payments on money rather than lump-sum transfers, (b) the random allocation of young people to separate markets takes the place of our random size of population, (c) the production decision in the Lucas model is made after observing the current price instead of before, which is why in his model there is an inference problem not present here, and (d) in the Lucas model no one chooses whether or not to be informed.

<sup>5</sup>The existence of this equilibrium allocation follows as an obvious corollary of the result proved two paragraphs below.

<sup>6</sup>Tom Courchene has pointed out to me that from this point of view the inefficiency arises from too many markets rather than too few. One solution, if feasible, would be to destroy the market for information.

<sup>7</sup>We are not considering cases where some proper fraction of agents monitor.

<sup>8</sup>To prove the first inequality note that there exists, by definition, a sequence  $\{x^r\}$  such that  $\phi^m(x^r) \rightarrow \bar{\phi}^m$ . Suppose (a) that  $\{x^r\}$  is bounded. Then it has a subsequence converging to  $\bar{x} \in [-M_1/n_1, \infty)$ . By the continuity of  $\phi^m(\cdot)$ ,  $\bar{\phi}^m = \phi^m(\bar{x})$ . Thus, by (23),  $\bar{\phi}^m < \hat{\phi}$ . Suppose (b) that  $\{x^r\}$  is unbounded. Then by (8) and (A.2), the sequence of monitoring equilibria with respect to  $x^r$  uniformly approaches the degenerate allocation  $q(\cdot) \equiv 0$ , which, by the definition of  $\hat{q}$  and the fact that  $\hat{q} > 0$ , yields a social welfare  $\phi^0 < \hat{\phi}$ . In this case  $\phi^m(x^r) \rightarrow \phi^0$ . Thus  $\bar{\phi}^m = \phi^0 < \hat{\phi}$ . The second inequality can be proved analogously.

<sup>9</sup>Activist policy in this case also ensures that an informational equilibrium will exist. This might be regarded as another argument in its favor.

<sup>10</sup>It is not obvious that stationary optima are also intertemporally optimal for in many models of overlapping generations the stationary optima may require a sacrifice of some initial generation. See, for example, Fried (1980).

<sup>11</sup>It is interesting that Friedman advocated the policy of stabilizing the money supply as the best practical alternative to the ideal policy of stabilizing the price level, for reasons that we discuss below, whereas Simons advocated just the reverse. Simons thought that ideally the whole financial system should be reformed so as to make practical the first best policy of stabilizing the money supply, but that in the absence of such radical

reform the best practical alternative was a policy of stabilizing the price level. The connection between these two positions is discussed by Friedman (1967).

<sup>12</sup>It is not clear that Fisher would have agreed with this argument. For after pointing out that the major problems revealed by the data on the price level are its secular and cyclical variability, he says (p. 321): "One method of mitigating both of these evils is the increase in knowledge as to prospective price levels. As we have seen the real evils of changing price levels do not lie in these changes per se but in the fact that they usually take us unawares. It has been shown that to be forewarned is to be forearmed, and that a foreknown change in price-levels might be so taken into account in the rate of interest as to neutralize its evils." This clearly foreshadows the recent, "ineffectiveness" argument by implying that his proposals for monetary policies to stabilize the price level depend for their usefulness upon the absence of what we would call rational expectations.

<sup>13</sup>Proof: With a nonactivist policy,  $p_2(n_2) = \bar{p}(n_2) \equiv \frac{M_1 + n_1 x_1^*}{n_2 \bar{q}_2(n_2)}$ .  
 With the activist policy (11),  $p_2(n_2) = p_2^*(n_2) \equiv \frac{M_1 u'(n_2 \bar{q}_2(n_2)/n_1)}{n_1 \tilde{q}v'(\tilde{q})}$ . Define  $Q_2 \equiv n_2 \bar{q}_2(n_2)$ . Then we can write  $p_2^*$  and  $\bar{p}_2$  as the functions:  $\bar{p}_2(Q_2) = \frac{M_1 + n_1 x_1^*}{Q_2}$  and

$p_2^*(Q_2) = \frac{M_1 u'(Q_2/n_1)}{n_1 \tilde{q}v'(\tilde{q})}$ . Note that if  $u'$  is less than unit elastic then

$$(i) \quad \frac{d \log \bar{p}_2(Q_2)}{dQ_2} < \frac{d \log p_2^*(Q_2)}{dQ_2} < 0$$

Suppose we consider the particular non-activist policy such that:

$$(ii) \quad E p_2^* = E \bar{p}_2 \equiv E p.$$

Then there is some  $\tilde{Q}$  such that  $\bar{p}_2(\tilde{Q}) = p_2^*(\tilde{Q}) \equiv \tilde{p}$ . From (i) and this.

$$(iii) \quad \begin{cases} \tilde{p} < p_2^* < \bar{p}_2 & \text{when } Q_2 < \tilde{Q} \\ \bar{p}_2 < p_2^* < \tilde{p} & \text{when } \tilde{Q} < Q_2 \end{cases}$$

From (iii):

$$(iv) \quad (p_2^* - \bar{p})^2 < (\bar{p}_2 - \tilde{p})^2 \vee Q_2 \neq \tilde{Q}$$

From (ii) and (iv):

$$\text{var } p_2^* = E(p_2^* - \tilde{p})^2 - (\tilde{p} - E p)^2 \cong E(\bar{p}_2 - \tilde{p})^2 - (\tilde{p} - E p)^2 = \text{var } \bar{p}_2,$$

with a strict inequality unless  $Q_2 \equiv \tilde{Q}$ . If we consider non-activist policies such that  $E(\bar{p}_2(n_2)) \neq E(p_2^*(n_2))$  the analogous argument can be used to show that  $\bar{p}_2(n_2)$  has a larger coefficient of variation than  $p_2^*(n_2)$ .



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