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MACROECONOMIC EFFECTS OF FISCAL POLICY*

by

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I. Introduction

Traditional macroeconomic models have been constructed on the presumption of private market failure. Labor markets, capital markets, or goods markets are seen to be incapable of allocating resources in a Pareto efficient manner, at least without significant time lags. The primary role for fiscal policy in such an environment is to stimulate aggregate demand in response to an adverse supply or, more frequently, demand shock to the economy. The persistent belief in widespread market inefficiency then leads quite naturally to the conclusion that such policy actions will be capable of altering real economic outcomes in a welfare enhancing manner.

Recently, certain macroeconomists have begun to question the validity of the basic premise of pervasive market failure. Approaching fiscal policy questions from the opposite perspective of market efficiency, these economists have reconsidered the positive and normative aspects of government tax and spending changes. On the side of positive analysis, they have been concerned with the effects of fiscal policy actions on real variables such as employment, output, investment [Bailey (1971), Grossman and Lucas (1974), Hall (1980), Barro (1981, 1984), Aschauer (1982) and Bryant (1983)], and the current account [Greenwood (1983), Sachs (1983) and Kimbrough (1985)]. On the normative side, they have been interested in determining whether fiscal stabilization policy is welfare improving [Kydland and Prescott (1980a)] and in specifying the optimal tax structure [Barro (1979), Kydland and Prescott (1980b), Lucas and Stokey (1983) and Razin and Svensson (1983)].

This paper is intended to bring together and elaborate upon such issues in the common framework of a small choice-theoretic intertemporal general equilibrium model. While the model employed is simple, it still allows for government services to yield consumption benefits for individuals and production benefits for firms. It also permits government investment in public capital which has the potential of enlarging society's future production possibilities and of augmenting the rate of return on private capital. The incorporation of distortional taxes on the returns to labor service and investment makes possible a discussion of the positive and normative effects of tax changes. Finally, a slight extension of the model allows for a consideration of open economy issues arising from domestic fiscal policy actions.

An important characteristic of the modeling strategy adopted here is that economic agents make their consumption, investment, labor effort, and production decisions in a rational manner based upon forward-looking behavior about both government spending and taxation policies. One benefit of this approach is that it highlights the importance of distinguishing clearly between anticipated versus unanticipated as well as temporary versus permanent fiscal policy actions in tracing out the effects such policy is likely to have on the economy.

In Sections II and III the maximization problem of the representative agent and the economy's general equilibrium are presented. A positive analysis of the effects of changes in tax rates is pursued in Section IV. The question of the desirability of tax policy to stabilize macroeconomic variables then is considered in Section V. The discussion in Section VI focuses on the positive effects of public expenditure, after which the question of optimal fiscal policy is taken up in Section VII. In Section VIII some simulation results are presented to bring together the issues raised earlier in terms of government spending and taxation. Finally, some of the effects of fiscal policy in an open economy setting are outlined in Section IX and concluding comments offered in Section X.

II. The Representative Agent's Maximization Problem

Consider the following model of a "closed" economy. The world is inhabited by a representative agent who lives for two periods. The agent's goal is to maximize the value of the following lifetime utility function $\underline{U}(\cdot)$ as given by

$$\underline{U} = U(\tilde{c}_1) + V(\ell_1) + \beta[U(\tilde{c}_2) + V(\ell_2)] \quad \beta \in (0,1) \tag{1}$$

$$(\text{with } U' > 0, \text{ and } V', V'', U'' < 0)$$

where β is the individual's (constant) subjective discount factor, $\tilde{c}_1$ and $\tilde{c}_2$ represent his "effective" consumption in the first and second periods; and ℓ_1 and ℓ_2 denote his labor supply in these periods. Effective consumption in a period, say t, is taken to be a function of private consumption expenditure, c_t, and government expenditure on consumption goods, g_t^c. Specifically, it is assumed that $\tilde{c}_t = c_t + \alpha(g_t^c)$ where $\alpha(\cdot)$ is an increasing concave function.

As can be seen, government purchases are allowed to influence utility directly by providing a current substitute for private consumption goods with no interaction with leisure. The marginal rate of substitution between private and public consumption goods, α', is assumed to lie between 0 and 1 so that an incremental unit of publicly provided goods yields only a fraction of the utility to be derived from an extra unit of privately purchased goods. This assumption is crucial for this modeling strategy since it implies that increases in government spending will impose negative wealth effects on the representative agent. The recent empirical work of Kormendi (1983) and Aschauer (1985) report values for $\alpha'(\cdot)$ in the range of .20 to .40, however, so that it does not appear that this assumption is unrealistic.

The individual derives his income in each period through the owner-operation of a firm. The firm produces one good by use of two factors of production, labor, ℓ, and capital, i. Also, in each period the government provides services, g^{ℓ}, which aid private production in that period, and undertakes public investment, g^i, which will augment future private production. In particular, period-t output, y_t, of the firm is characterized by the following production function:

$$y_t = \delta_t + f(\ell_t, g_t^{\ell}) + h(i_t, g_t^i) + i_t + g_t^i \qquad t=1,2 \qquad (2)$$

where δ_t represents a time-varying constant.

It is assumed that the marginal product of current public services, $f_2(\cdot)$, is less than unity. This is analogous to the negative wealth effect discussed above for the public consumption goods case although here no hard empirical evidence is available to lay a foundation for the claim of public sector "inefficiency". It also will be assumed that public investment is inefficient in the sense that the marginal product of public capital, $h_2(\cdot)$, is less than that of private capital, $h_1(\cdot)$. Note that the production technology is specified such that there is no direct interplay between g^{ℓ} and the marginal productivity of private capital or between g^i and the marginal product of labor. This may seem restrictive but it still allows for analysis of how changes in the level of government spending may affect the marginal product of labor, and consequently the demand for labor, as well as how such changes might impact on the rate of return to private capital, and therefore private investment demand.

In addition to earning income each period through the owner-operation of a firm, it will be assumed that the individual receives a transfer payment, τ, from the government. The agent can use the after-tax income from his firm and this transfer payment in three ways--taxes will be discussed momentarily. These earnings can be used to finance consumption, purchase capital goods for use next period, or to buy real denominated bonds. The real denominated bonds have a return of r so that a bond purchased in the first period for one unit of consumption pays 1 + r units of consumption in the second period.

Recall that in each period the government engages in four types of spending. It provides consumption and production services, public investment goods, and transfer payments. This spending can be financed by bond issuance or by use of the following tax instruments. In each period t the government levies a proportional tax in the amount λ_t on that portion of output that is attributable to labor effort. Essentially, λ_t is the labor-income tax rate in period t. Also, the value added from the firm's production derived from capital investment is taxed at the rate θ_t. One can view θ_t as being the period-t corporate income tax rate.

The maximization problem facing the representative agent is shown below with the agent's choice variables being c_1, c_2, ℓ_1, ℓ_2 and i.[1,2,3]

$$W(\cdot) \equiv \max\{U(c_1 + \alpha(g_1^c)) + V(\ell_1) + \beta[U(c_2 + \alpha(g_2^c)) + V(\ell_2)]\} \quad (3)$$

subject to

$$c_1 + \frac{c_2}{(1+r)} = \delta_1 + (1-\lambda_1)f(\ell_1, g_1^\ell) + \frac{(1-\lambda_2)f(\ell_2, g_2^\ell) + (1-\theta)h(i, g^i) - ri}{(1+r)} + \tau_1 + \frac{\tau_2}{(1+r)}$$

[Note that for simplicity it has been assumed that $\delta_2 = 0$.]

The first-order conditions associated with this maximization problem--in addition to the above budget constraint--are shown below. They are:

$$U'(c_1 + \alpha(g_1^c)) = \beta(1+r)U'(c_2 + \alpha(g_2^c)) \quad (4)$$

$$-V'(\ell_1) = (1-\lambda_1)f_1(\ell_1, g_1^\ell)U'(c_1 + \alpha(g_1^c)) \quad (5)$$

$$-V'(\ell_2) = (1-\lambda_2)f_1(\ell_2,g_2^\ell)U'(c_2+\alpha(g_2^c)) \tag{6}$$

$$(1-\theta)h_1(i,g^i) = r. \tag{7}$$

These conditions have the usual interpretation, (4) being an intertemporal efficiency condition in effective consumption, (5) and (6) being intratemporal efficiency conditions in effective consumption and work effort, and (7) an intertemporal efficiency condition in production undertaken through the use of physical capital. Note that r is equal to the after-tax real rate of return on capital investment.

III. The Model's General Equilibrium

In the model the goods market must clear each period, implying that the two market-clearing conditions shown below must hold:

$$c_1 + i + g_1^c + g_1^\ell + g^i = \delta_1 + f(\ell_1,g_1^\ell) \tag{8}$$

$$c_2 + g_2^c + g_2^\ell = f(\ell_2,g_2^\ell) + h(i,g^i) + i + g^i. \tag{9}$$

By utilizing the above two conditions in conjunction with the first-order conditions (4) to (7), it can be seen that solutions for ℓ_1, ℓ_2, and i in the model's general equilibrium are implicitly characterized by the three equations (10), (11) and (12):[4]

$$-V'(\ell_1) = U'(\delta_1 + f(\ell_1,g_1^\ell) - i - g_1^c + \alpha(g_1^c) - g_1^\ell - g^i)(1-\lambda_1)f_1(\ell_1,g_1^\ell) \tag{10}$$

$$-V'(\ell_2) = U'(f(\ell_2,g_2^{\ell}) + h(i,g^i) + i + g^i - g_2^c + \alpha(g_2^c) - g_2^{\ell}))(1-\lambda_2)f_{\ell}(\ell_2,g_2^{\ell}) \tag{11}$$

$$U'(\delta_1 + f(\ell_1,g_1^{\ell}) - i - g_1^c + \alpha(g_1^c) - g_1^{\ell} - g^i) = \beta[1 + (1-\theta)h_1(i,g^i)]U'(f(\ell_2,g_2^{\ell}) + h(i,g^i) + i + g^i - g_2^c + \alpha(g_2^c) - g_2^{\ell}). \tag{12}$$

This system of equations can be subjected to various comparative static exercises to determine how changes in tax parameters, λ_1, λ_2, and θ, or government spending variables, g_1^c, g_2^c, g_1^{ℓ}, g_2^{ℓ} and g^i, affect the economy's general equilibrium. These questions will be addressed in subsequent sections of the paper.

Finally, before proceeding further it should be noted that the government, like any other actor in the economy, must satisfy a budget constraint. Its budget constraint is

$$g_1 + \tau_1 + \frac{g_2 + \tau_2}{(1+r)} = \lambda_1 f(\ell_1,g_1^{\ell}) + \frac{\lambda_2 f(\ell_2,g_2^{\ell}) + \theta h(i,g^i)}{(1+r)} \tag{13}$$

where $g_1 \equiv g_1^c + g_1^{\ell} + g^i$ and $g_2 \equiv g_2^c + g_2^{\ell} - g^i$ represent the government's absorption of resources in periods one and two, respectively.

IV. Changes in Income Tax Rates

In traditional macroeconomic models, tax changes have been imagined to be important principally for their ability to affect the current flow of disposable income and hence the aggregate demand for goods and services. In contrast, the model developed in this paper implicitly adopts conditions sufficient to produce a "Ricardian" equivalence between lump-sum tax and debt financing of a particular stream of government spending.[5] Consequently, the focus shifts to an analysis of the role changes in tax rates might play in the determination of employment, output, consumption, and investment by altering the incentives to engage in market activity (consumption, employment, and production) in one or the other period (temporary tax changes) or in both periods (permanent tax changes). In the subsequent analysis, changes in labor-income tax rates as well as corporate-income tax rates are considered.

a) Changes in Labor-Income Tax Rates

Imagine that the government announces that it intends to increase the future level of income taxes, i.e., $d\lambda_1 = 0$, $d\lambda_2 > 0$. The increase in revenue arising from this anticipated tax hike will be used to finance lump-sum transfer payments to the representative agent. Since the timing of these transfer payments is inconsequential just their present value, τ, will be focused on here, where $\tau = \tau_1 + (1/(1+r))\tau_2$. By subjecting (10), (11), and (12) to the required comparative statics exercise, it can be seen that (see Appendix A for details)

$$\frac{d\ell_1}{d\lambda_2} > 0, \; \frac{d\ell_2}{d\lambda_2} < 0, \text{ and } \frac{di}{d\lambda_2} > 0. \tag{14}$$

With the help of the above solutions, the effect of an increase in future taxes on first-period consumption can be determined readily from (8). One obtains (again, see Appendix A for details)

$$\frac{dc_1}{d\lambda_2} = f_1(1)\,\frac{d\ell_1}{d\lambda_2} - \frac{di}{d\lambda_2} < 0. \tag{15}$$

The above results can be interpreted intuitively. First, as can be seen, an increase in future income taxation raises current and reduces future work effort. This reflects an intertemporal substitution effect as agents substitute away from working in the future, where the after-tax rate of return is now smaller, toward working in the present, where the rate of return is now relatively higher. Second, note that current investment is increased as a result of the rise in future income taxes. This follows because future output can be obtained either by working in the future or through investing in capital during the current period. In general, agents would like to obtain a relatively smooth profile of consumption over time, so by investing more in the current period they can partially compensate for the loss in future output due to the reduction in future labor effort. Third, as can be seen from (15), the increase in future taxes leads to a reduction in current consumption. This arises because the increase in first-period investment, while being partly financed by an increase in current labor supply, also is financed partly by a reduction in current consumption.

The effects of an anticipated labor-income tax rate increase on current real activity outlined above depend crucially on the inclusion of physical investment in the model. Given the time-separable specification of preferences, without physical investment, there would be no link between real activity in adjacent periods, a fact Barro and King (1985) have emphasized. Thus, an increase in future labor taxation would have no effect whatsoever on current real activity. This is easily confirmed in the current setting by noting that without investment (10) alone would implicitly describe the determination of current labor effort, ℓ_1. Furthermore, note that another implication of the time-separable preference structure is that anticipated future shocks must affect current consumption and labor supply in opposite directions as is easily discerned from (5).

The welfare effect of a change in future labor income taxes is not difficult to uncover. To determine the impact on welfare of a change in the period-t labor income tax rate, differentiate both sides of equation (3) with respect to λ_t while applying the standard envelope theorem. One obtains

$$\frac{dW}{d\lambda_t} = \frac{\partial W}{\partial \lambda_t} + \frac{\partial W}{\partial \tau}\frac{d\tau}{d\lambda_t} + \frac{\partial W}{\partial r}\frac{dr}{d\lambda_t}$$

$$= U'(\tilde{c}_1)\{-(\frac{1}{1+r})^{t-1} f(t) + \frac{d\tau}{d\lambda_t} - (\frac{1}{1+r})^2 [(1-\lambda_2)f(2) + (1-\theta)h(\cdot) + i + \tau_2 - c_2]\frac{dr}{d\lambda_t}\} \quad t=1,2.$$

This expression can be simplified further by using the government's budget constraint (13) and the goods market-clearing condition (9) to find that

$$\frac{dW}{d\lambda_t} = U'(\tilde{c}_1)\{\lambda_1 f_1(1)\frac{d\ell_1}{d\lambda_t} + \frac{\lambda_2 f_1(2)}{(1+r)}\frac{d\ell_2}{d\lambda_t} + \frac{\theta h_1(\cdot)}{(1+r)}\frac{di}{d\lambda_t}\} \gtrless 0. \tag{16}$$

In general, the effect on welfare of an increase in the period-t labor-income tax rate is ambiguous since the sign of (16) is uncertain. It is not difficult to see why. Take the case under consideration of an increase in the future labor-income tax rate. Now, suppose that the tax on capital's income is zero, or that

$\theta = 0$, and that initially $\lambda_1 = 0$ and $\lambda_2 > 0$. Here an increase in future income taxes unambiguously lowers economic welfare. When there are no other taxes in place, the anticipation of an increase in future income taxes reduces welfare. Now contrast this with the case where initially $\lambda_1 > 0$ and $\lambda_2 = 0$. Here an increase in future income taxes raises economic welfare. This may seem paradoxical until one realizes that this is a second-best situation. Note that the effect of initially having an income tax solely in the first period is to create a distortion whereby agents tend to favor second-period labor effort vis-à-vis first-period labor effort. This distortion reduces welfare, ceteris paribus. The institution of a small income tax in the second period improves economic welfare since it works against this intertemporal substitution effect caused by the original distortion. That is, it tends to increase labor effort in the first period and reduce it in the second which helps to ameliorate the situation.

b) Changes in the Corporate Income Tax

Suppose that the government increases the corporate income tax rate, θ.[6] Again, the system of equations (10), (11), and (12) describing the economy's general equilibrium, can be used to find

$$\frac{d\ell_1}{d\theta} < 0, \frac{d\ell_2}{d\theta} > 0, \frac{di}{d\theta} < 0 \text{ and } \frac{dc_1}{d\theta} > 0. \tag{17}$$

To begin with, as undoubtedly expected, current investment, i, falls as a result of the increase in the corporate income tax rate. This occurs because the after-tax rate of return, r, on investment is now reduced. Since current investment falls, more first-period output is available for alternative uses. In particular, the agent uses these extra resources to increase his current consumption and to reduce his current labor effort, both of these decisions being partly motivated by the drop in the (after-tax) real interest rate, r. Finally, note that the future supply of labor, ℓ_2, increases. This is because the reduction in current investment causes future output, y_2, and hence consumption, c_2, to fall. This fall in future output due to a lower capital stock is partially offset by the agent increasing his labor supply in that period.

To conclude this section of the paper, Table 1 is presented which summarizes the model's main conclusions about changes in tax rates.[7] As can be seen, when analyzing the impact of shifts in the labor-income tax rate, it is important to distinguish whether the tax rate movement is transitory or permanent in character, and whether it reflects a current unanticipated event or unexpected future one.

V. Tax Policy and Business Cycle Stabilization

It has often been suggested that tax instruments should be used to dampen business cycle fluctuations. In particular, economists often advocate the use of procyclical tax policies in response to an adverse shock to the system. In this section, through the use of a simple example, the "feasibility and desirability" of such policies is contemplated.

Table 1

TAX CHANGE	ℓ_1 (and y_1)	ℓ_2	i	c_1
(i) Anticipated increase in future income tax rate, i.e., $\Delta\lambda_1 = 0$, $\Delta\lambda_2 > 0$.	(+)	(-)	(+)	(-)
(ii) Unanticipated temporary increase in current income tax rate, i.e., $\Delta\lambda_1 > 0$, $\Delta\lambda_2 = 0$.	(-)	(+)	(-)	(-)
(iii) Unanticipated permanent increase in the current income tax rate, i.e., $\Delta\lambda_1 = \Delta\lambda_2 > 0$.	(-)	(-)	(0)[1]	(-)
(iv) An increase in the corporate income tax rate, θ.	(-)	(+)	(-)	(+)

[1] Some initial conditions have been assumed in deriving this result. First, it has been assumed that $g_1^c = g_2^c$, $g_1^\ell = g_2^\ell$, and $\theta = 0$. Second, note from (7) that investment, i, can be written as a function of the real interest rate, r, and government spending on public investment, g^i, so that $i = i(r, g^i)$. Now also assume that $\delta_1 = h(i(\frac{1-\beta}{\beta}, g^i), g^i) + 2i(\frac{1-\beta}{\beta}, g^i) + 2g^i$. These initial conditions make the first and second periods identical from the agent's perspective and start the model off from a steady-state situation.

To begin with, abstract from the revenue-raising motives for taxation by assuming that there is no government spending on goods in the artificial economy modeled here. Also, assume that all taxes and lump-sum transfer payments are initially set at zero. Now let the second-period production function be subject to an additive shock, δ_2, so

$$y_2 = \delta_2 + f(\ell_2) + h(i) + i$$

where δ_2, a mean zero random variable, is governed by the probability density function $p(\delta_2)$.

The economy is supervised by a central planner who desires to maximize the representative agent's welfare. The planner's first-period maximization problem is shown below where he is choosing i, ℓ_1, and a state-contingent value for second-period labor supply $\ell_2 = \ell_2(i,\delta_2)$. It is assumed that the policymaker has no informational advantage over the representative agent in regard to the particular realization for δ_2. Thus, one has

$$W \equiv \max U(\delta_1 + f(\ell_1) - i) + V(\ell_1) + \beta\int[U(\delta_2 + f(\ell_2) + h(i) + i) + V(\ell_2)]p(\delta_2)d\delta_2. \quad (18)$$

The first-order necessary and sufficient conditions arising from this problem are

$$U'(\delta_1 + f(\ell_1) - i) = \beta[1+h_1(i)]\int U'(\delta_2 + f(\ell_2) + h(i) + i)p(\delta_2)d\delta_2 \quad (19)$$

$$-V'(\ell_1) = f_1(\ell_1)U'(\delta_1 + f(\ell_1) - i) \quad (20)$$

$$-V'(\ell_2) = f_1(\ell_2)U'(\delta_2 + f(\ell_2) + h(i) + i) \quad (21)$$

where (21) implicitly describes the state-contingent value for ℓ_2 as a function of i and δ_2.

A natural question to ask is: how does increased variability in the random variable δ_2 affect the representative agent's expected welfare, W? To answer this question, let δ_2 be a linear function of another variable $\hat{\delta}_2$ so $\delta_2 = \sigma\hat{\delta}_2$, where σ is a constant and $\hat{\delta}_2$ is a zero mean random variable with density function $\hat{p}(\hat{\delta}_2)$, which implies $p(\delta_2) = \frac{1}{\sigma} \cdot \hat{p}(\hat{\delta}_2/\sigma)$. Now, to obtain the effect of such an increase in the dispersion of δ_2 on the agent's welfare differentiate (18) with respect to σ while utilizing the first-order conditions (19), (20), and (21). One obtains

$$dW/d\sigma = \beta \int U'(c_2)\hat{\delta}_2 p(\hat{\delta}_2)d\hat{\delta}_2 = \beta\,\mathrm{cov}(U'(c_2),\hat{\delta}). \tag{22}$$

The above expression is unambiguously negative as long as the agent is risk averse since c_2 is an increasing function of δ_2 (see Appendix A) and the covariance between a variable and a decreasing (increasing) function of itself is negative (positive). Thus, a mean preserving increase in the dispersion of the random variable δ_2 lowers the representative agent's expected welfare, W. It may not be surprising, therefore, to find pressure being placed on the fiscal authorities to attempt to reduce the variability of second-period income.

Suppose that the government accedes to this pressure and decides to stabilize the fluctuating component of second-period output. The only component of output which actually varies in the second period is $\delta_2 + f(\ell_2)$. Let the government choose to peg this stochastic component of output at some constant level $\bar{y}$ so that $\delta_2 + f(\ell_2) = \bar{y}$. There are three interesting questions associated with this policy: (i) Will the stabilization policy enhance societal welfare? (ii) In what way can output be stabilized? (iii) What will be the effects of the policy on macroeconomic variables such as current employment, output and investment?

Given this stabilization policy, the central planner's problem is now

$$W \equiv \max U(\delta_1 + f(\hat{\ell}_1) - \hat{i}) + V(\hat{\ell}_1) + \beta \int [U(\delta_2 + f(\hat{\ell}_2) + h(\hat{i}) + \hat{i}) + V(\hat{\ell}_2) + \phi(\bar{y} - \delta_2 - f(\hat{\ell}_2))] p(\delta_2) d\delta_2 \qquad (23)$$

with $\hat{i}$, $\hat{\ell}_1$, $\hat{\ell}_2 = \hat{\ell}_2(\hat{i}, \delta_2)$ again being the choice variables. Here, the "^" is meant to denote that the choice variables are being determined optimally in the presence of the stabilization constraint. The first-order necessary and sufficient conditions for a constrained maximum are

$$U'(\delta_1 + f(\hat{\ell}_1) - \hat{i}) = \beta[1 + h_1(\hat{i})] \int [U'(\delta_2 + f(\hat{\ell}_2) + h(\hat{i}) + \hat{i})] p(\delta_2) d\delta_2 \qquad (24)$$

$$-V'(\hat{\ell}_1) = f_1(\hat{\ell}_1) U'(\delta_1 + f(\hat{\ell}_1) - \hat{i}) \qquad (25)$$

$$-V'(\hat{\ell}_2) = [1 - \phi / U'(\delta_2 + f(\hat{\ell}_2) + h(\hat{i}) + \hat{i})] f_1(\hat{\ell}_2) U'(\delta_2 + f(\hat{\ell}_2) + h(\hat{i}) + \hat{i}) \qquad (26)$$

$$\delta_2 + f(\hat{\ell}_2) = \bar{y}. \qquad (27)$$

Note that the implied solutions for $\hat{i}$, $\hat{\ell}_1$, $\hat{\ell}_2(\hat{i},\delta_2)$, and ϕ will be unique given the concavity of the objective function together with the convexity of the constraint.

The answer to the first of the questions posed above is immediate. Income stabilization cannot improve the welfare of this artificial economy since the addition of the stabilization constraint to problem (18) can only reduce the value of the maximand by restricting the economy's opportunity set. This conclusion would be robust to any other source of aggregate uncertainty, such as second-period multiplicative shocks to the functions $f(\cdot)$ and $h(\cdot)$.

The answer to the second question is almost as immediate. Inspection of equation (26) reveals that the government can stabilize second-period output fluctuations by imposing a state-contingent labor-income tax, λ_2, in the amount $\lambda_2 = \phi/U'(\bar{y}+h(\hat{i})+\hat{i})$. Given the constraint, the tax rate must move in a procyclical fashion with respect to the productivity shock, with the movement being governed by

$$\frac{d\lambda_2}{d\delta_2} = -\frac{V''(f^{-1}(\bar{y}-\delta_2)) + f_{11}(f^{-1}(\bar{y}-\delta_2))U'(\bar{y}+h(\hat{i})+\hat{i})(1-\lambda_2)}{f_1(f^{-1}(\bar{y}-\delta_2))^2 U'(\bar{y}+h(\hat{i})+\hat{i})} > 0 \qquad (28)$$

which shows clearly that the movement in the tax rate will depend intimately on the elasticities of the marginal disutility of labor and the marginal product of labor or, roughly speaking, the supply of and demand for labor. Consequently, a completely successful state contingent policy will require a detailed knowledge of the characteristics of preferences and technology.

Finally, how is the stabilization scheme likely to affect first-period production, labor supply and consumption? To facilitate answering this question, suppose that the government decides to stabilize the random component of output at the mean level it takes in the absence of intervention. Thus

$$\bar{y} = \int [\delta_2 + f(\ell_2(i,\delta_2))] p(\delta_2) d\delta_2 .$$

Also, assume that the momentary utility function in consumption $U(\cdot)$ is characterized by decreasing absolute risk aversion which requires that $U'''(\cdot) > 0$.

Now, to see how stabilization policy of the sort above will affect investment, first note that as a consequence of (20) and (25) first-period labor supply--with or without government intervention--can be written solely as a single, increasing function of investment. In other words, it is possible to write $\ell_1 = \ell_1(i)$ and $\hat{\ell}_1 = \ell_1(\hat{i})$. Second, by taking a second-order Taylor expansion of the marginal utility of consumption around $\bar{y}$ in the right-hand side of (19) it can be seen that

$$\begin{aligned} U'(\delta_1 + f(\ell_1(i)) - i) &\geq \beta[1 + h_1(i)]U'(\bar{y} + h(i) + i) \\ &\quad + \{\tfrac{1}{2}\beta[1+h_1(i)]\int[\delta_2 + f(\ell_2(\delta_2, i)) - \bar{y}]^2 p(\delta_2) d\delta_2\} \\ &\quad \cdot \inf_{\delta_2} U'''(\delta_2 + f(\ell_2(i,\delta_2)) + h(i) + i) \\ &\geq \beta[1 + h_1(i)]U'(\bar{y} + h(i) + i) \end{aligned} \tag{29}$$

Next, from (24) and (27) it follows that in the presence of stabilization policy equation (29) must hold with equality if i is replaced by $\hat{i}$. Since the right-hand side of (29) is decreasing in i, while the left-hand side is increasing in this variable, it follows immediately that $\hat{i} < i$. Consequently, it then occurs that $\hat{\ell}_1 < \ell_1$ and $\hat{c}_1 > c_1$ (see Appendix A).

Thus, in the presence of the stabilization policy it is seen that agents respond to the reduction in uncertainty about future income by increasing current consumption as well as decreasing current work effort, output and investment. In this sense, the oft-stated macroeconomic goals of "economic growth" and "stability" may be contradictory; the pursuit of the latter has been shown to reduce the former in this simple example. Further, although individuals are better off from the standpoint of their current period utility calculation (consumption rising and work effort falling) they experience a loss in their future expected utility (due to the elimination of the ability to respond to random shocks reflecting changes in the future opportunity set facing society) which dominates the former effect and, on net, their expected welfare declines.

VI. Changes in Government Spending

This section directs attention toward the macroeconomic impact of public purchases. In the model developed here, government spending of various sorts may affect employment, output, consumption, and investment by altering the wealth of the representative agent or by directly affecting the marginal productivity of labor and private capital. So as to isolate the effects of government spending, per se, it will be assumed that all revenue is raised through lump-sum taxation (i.e., let $\lambda_1 = \lambda_2 = \theta = 0$).[8] As has been mentioned, due to the Ricardian equivalence theorem the timing of lump-sum taxation is irrelevant for the determination of the real variables in the system. Changes in public expenditure on services and on public capital are considered in turn.

a) Changes in Government Spending on Services

To begin with, consider an unanticipated temporary increase in government spending on services. To perform this experiment, define g_1^s as first-period total government spending on services so that $g_1^s = g_1^c + g_1^\ell$. Now let ρ be the fraction of total government expenditure on services devoted to the provision of government consumption services so that $(1-\rho)$ represents the fraction assigned to the provision of production services. Consequently, it follows that a temporary increase in government expenditure on services implies that $dg_1^c = \rho dg_1^s$, $dg_1^\ell = (1-\rho)dg_1^s$, and $dg_2^c = dg_2^\ell = 0$.

The impact on the agent's welfare resulting from the temporary increase in government services can be seen by differentiating (3) to be

$$\frac{dW}{dg_1^s} = \rho\,\frac{\partial W}{\partial g_1^c} + (1-\rho)\frac{\partial W}{dg_1^\ell} + \frac{\partial W}{\partial \tau}\,\frac{d\tau}{dg_1^s} + \frac{\partial W}{\partial r}\,\frac{dr}{dg_1^s}$$

(where again, $\tau \equiv \tau_1 + (1/(1+r))\tau_2$)

$$= -U'(\tilde{c}_1)\{1-\rho\alpha'(1) - (1-\rho)f_2(\ell_1, g_1^s)\} < 0 \qquad (30)$$

[Using the standard envelope theorem result, and (9) and (13)].

As can be seen, when government expenditure is increased temporarily, the agent suffers a welfare loss since by assumption both $\alpha'(\cdot)$ and $f_2(\cdot)$ lie between zero and one.

The effect of a temporary change in g_1^s on ℓ_1, ℓ_2, and i can be deduced from the system of equations (10), (11), and (12).[9] Under the assumption that the private production process is separable in labor and government services (to be relaxed momentarily), the following results obtain

$$\frac{d\ell_1}{dg_1^s} > 0,\ \frac{d\ell_2}{dg_1^s} > 0,\ \text{and}\ \frac{di}{dg_1^s} < 0. \tag{31}$$

Consequently, the effects on output in the first and second periods, respectively, are given by

$$\frac{dy_1}{dg_1^s} = f_1(1)\underset{(+)}{\frac{d\ell_1}{dg_1^s}} + (1-\rho)f_2(1) > 0 \tag{32}$$

and

$$\frac{dy_2}{dg_1^s} = f_1(2)\underset{(+)}{\frac{d\ell_2}{dg_1^s}} + (1+r)\underset{(-)}{\frac{di}{dg_1^s}} < 0. \tag{33}$$

Note that the negative wealth effect associated with the temporary rise in government purchases induces the agent to decrease consumption and increase labor supply in both periods. Further, the temporal incidence of the rise in government purchases lies in the current period. That is, the impact effect of the fiscal shock is to reduce the amount of first-period resources available for private consumption in that period. In an attempt to smooth effective consumption and leisure over time, therefore, the agent decreases capital accumulation which, in turn, raises the real rate of return and promotes an intertemporal substitution of work effort to the present, and of consumption to the future. On net, output rises in the current period and falls in the future. In the latter case, the increased output due to increased labor effort is dominated by the fall in output due to decreased capital accumulation. This insures that consumption in both periods is reduced in the final equilibrium.

The direct impact which higher government spending may have on the marginal product of labor is now considered. If government services are technical complements with labor, then the positive effect on current work effort is reinforced as labor is substituted across periods in response to the rise in the relative wage $(1+r)f_1(1)/f_2(2)$. Ambiguities arise given a sufficiently large value of the complementarity term $f_{12}(1)$ since it becomes possible for the rise in the relative wage to induce a reduction in second-period work effort and an increase in capital accumulation. Ambiguities also become evident in the opposite case of technical substitutability since the decrease in the relative wage in the first period tends to reduce current work effort, acting against the rise in labor prompted by the negative wealth effect of higher government expenditure. Note that if $\rho\alpha'(1) + (1-\rho)f_2(1)$ equals unity--so that there would be no wealth effect associated with a marginal increase in government spending--this channel would still

still allow for real effects of government purchases. For the case of technical complementarity and zero wealth effects it is possible to state unambiguously that current work effort would increase at the expense of future work effort and capital accumulation would rise to carry forward part of the production of the relatively favorable first period.

Next consider a rise in government expenditure in the second period which is foreseen by the agent. Again assuming separability in production between labor and government services, one finds

$$\frac{d\ell_1}{dg_2^s} > 0, \frac{d\ell_2}{dg_2^s} > 0, \text{ and } \frac{di}{dg_2^s} > 0. \tag{34}$$

Further, the effect on output is (clearly) positive in both periods. The anticipated government expenditure imposes a negative wealth effect, as before, and the agent responds by reducing consumption and increasing work effort in both periods. In his attempt to prepare for the extraordinary call for resources in the second period, the agent increases saving which, in turn, lowers the rate of return and causes a secondary shift in work effort from the present to the future.

Notice that the main qualitative difference in the effects of unanticipated versus anticipated changes in government expenditure lies in the behavior of private investment and the capital stock. In a more general model with multiple periods, anticipated increases in government spending would tend to lead to increased capital accumulation prior to the fiscal policy action, an effect which would be absent from the case where the fiscal policy change is unexpected. That is, the ability to accumulate (or decumulate) capital allows the agent partially to buffer fiscal shocks. Consequently, it would appear that the effect on work effort at the time of the fiscal change would be smaller in the anticipated case since the agent has had time to prepare for the expected excess demand for resources at that time.

Finally, a permanent increase in government spending of an equal amount in both periods will be considered (i.e., $dg_1^s = dg_2^s = d\bar{g}^s$). Assuming, once again, separability in production, we get in the lump-sum tax environment

$$\frac{d\ell_1}{d\bar{g}^s} > 0, \quad \frac{d\ell_2}{d\bar{g}^s} > 0, \quad \text{and} \quad \frac{di}{d\bar{g}^s} \overset{?}{\gtrless} 0. \tag{35}$$

Furthermore, output rises and consumption falls in both periods. As before, the rise in government spending is a drain on wealth and labor effort and consumption react accordingly, the first rising and the latter falling in both periods. Note the ambiguity in the response of investment to the permanent shock in government spending.

In the benchmark case where the real rate of return and time preference are equal--in a steady-state situation of optimizing models along the lines of Sidrauski (1967)--the effect on capital accumulation is nil. In this situation the agent desires to distribute the burden of the government spending shock equally across both periods. Concrete predictions outside of the benchmark case seem hard to obtain.[10] To the extent that the borderline condition holds, however, there arises an important empirical distinction between (unanticipated) temporary and permanent changes in government expenditures, with investment falling in the former case and remaining unchanged in the latter case.

b) Public Investment

As a final exercise, consider a rise in the level of public investment, $dg^i > 0$. Recall that it is assumed that the public capital is less productive at the margin than private capital (i.e., $h_2(i,g^i) < h_1(i,g^i)$). By following the line of argument employed in the previous section, it can be seen that the welfare loss associated with an increase in public investment is given by

$$\frac{dW}{dg^i} = -U'(\tilde{c}_1)[r - h_2(i,g^i)]/(1+r) < 0. \tag{36}$$

The net effects on work effort in both periods and private capital accumulation under the assumption that there is no complementarity between the two types of capital [or $h_{12}(i,g^i) = 0$] are

$$\frac{d\ell_1}{dg^i} > 0, \quad \frac{d\ell_2}{dg^i} \overset{?}{\gtrless} 0, \text{ and } -1 < \frac{di}{dg^i} < 0.$$

There are two factors playing a role in these results. First, as usual, the negative wealth effect arising as a result of excessive public capital accumulation tends to raise work effort and lower consumption in each period. Second, the impact effect of the increased public investment is to reduce the amount of first-period resources available for consumption and increase their second-period availability. In his desire to smooth his time profiles for consumption and leisure, the agent partially reacts to this scarcity by a less than one-to-one reduction in private investment. In other words, the individual borrows from the future to ease the burden of the shock in the current period. Note that total investment still has increased, as is evidenced by the fact that current labor supply has risen while current consumption has dropped. The fall in private investment, however, is associated with an increase in the private rate of return which promotes a reduction in second-period work effort relative to the first period, and thus an ambiguity in the response of second-period labor supply arises. Note that if public and private capital were equally as efficient at the margin (i.e., $h_1 = h_2$), so that there was no wealth effect associated with an increase in public investment, then second-period labor effort would unambiguously decline.

It is also useful to investigate the effects of a rise in public investment which is complementary with private investment, e.g., infrastructure investment. The impact effect of such an increase in public investment would be to raise the marginal product of private capital and hence its real return since $\partial r/\partial g^i = h_{12}(i,g^i) > 0$. This would tend to promote an intertemporal reallocation of labor to the current period and an increase in private investment to take advantage of private capital's higher marginal productivity.

To conclude this section, the effects of various changes in government spending are provided in Table 2. As before, it is particularly important to distinguish between changes which are

Table 2[1]

SPENDING CHANGE	ℓ_1	ℓ_2	i	c_1
(i) Anticipated increase in future spending, i.e., $\Delta g_2^s > 0$, $\Delta g_1^s = 0$.[2]	(+)	(+)	(+)	(-)
(ii) Unanticipated temporary increase in current spending, i.e., $\Delta g_1^s > 0$, $\Delta g_2^s = 0$.[2]	(+)	(+)	(-)	(-)
(iii) Unanticipated permanent change in spending, i.e., $\Delta g_1^s = \Delta g_2^s = \Delta \bar{g}^s$.	(+)	(+)	(0)[3]	(-)
(iv) Increase in public investment, i.e., $\Delta g^i > 0$.[4]	(+)	(?), (-)[5]	(-)	(-)

[1]The results obtained in this table are based on the assumptions that $0 < \alpha'(\cdot)$, $f_2(\cdot) < 1$, and $h_2(\cdot) < h_1(\cdot)$.

[2]Assuming that $f_{12}(\cdot) = 0$.

[3]The initial conditions mentioned in Table 1, footnote 1 have been assumed in deriving this result.

[4]Assuming that $h_{12}(\cdot) = 0$.

[5]This result obtains if $h_2(\cdot) \cong h_1(\cdot)$.

regarded as temporary versus those which are permanent and also between those which are anticipated versus unanticipated. Further, the composition of the change in government spending is crucial to the various results.

VII. Optimal Taxation

One way to analyze the effects of various fiscal policy programs entailing both spending and distortional tax changes would be to simultaneously reference Tables 1 and 2 above so as to determine the impact of the particular spending cum tax shift in mind. For example, a temporary increase in current government expenditure financed totally by future labor-income taxes can be seen to cause current employment and output to rise and so on. However, the model remains indeterminate in the sense that there is no theory of government behavior tying various ad hoc spending and tax plans together.

The approach taken by Barro (1979), and Lucas and Stokey (1983) is to assume an exogenous stream of government spending and to derive the tax structure which minimizes the deadweight loss associated with income taxation.[11] This approach is taken here except, as in Kydland and Prescott (1980b) and Kimbrough (1984), government spending is also allowed to be optimally chosen. Note that the model utilized in the present paper is less general than the models contained in Barro (1979) and Lucas and Stokey (1983) in that it extends only over two periods. Nevertheless, it is more general in that it is genuinely dynamic, involving capital accumulation. This latter aspect is important since it allows the private economy to smooth consumption and leisure over time in response to current or anticipated fiscal shocks and it expands the tax base to include capital income taxation.

The determination of the government's optimum spending cum tax program is just a variation on the Ramsey (1927) tax problem. The government should pick the various tax rates and components of government expenditure so as to maximize the agent's welfare, as given by the outcome of the optimization problem posed in (3), subject to its own budget constraint (13). Formally, the government's problem is

$$\max_{G} W(\cdot) + \phi[\lambda_1 f(1) + \frac{\lambda_2 f(2) + \theta h(\cdot)}{(1+r)} - g_1 - \frac{g_2}{(1+r)}] \tag{38}$$

with its choice variables being given by the fiscal policy vector $G \equiv (\lambda_1, \lambda_2, \theta, g_1^c, g_1^\ell, g_2^c, g_2^\ell, g^i)$ and where ϕ is defined to be the Lagrange multiplier associated with the government's budget constraint. The first-order conditions--in addition to the budget constraint (13)--arising from this maximization problem are:

$$\frac{\partial W}{\partial G_j} + \frac{\partial W}{\partial \tau}\frac{d\tau}{dG_j} + \frac{\partial W}{\partial r}\frac{dr}{dG_j} = -\phi\{k_j + \lambda_1 f(1)\frac{d\ell_1}{dG_j} + \frac{\lambda_2 f_1(2)}{(1+r)}\frac{d\ell_2}{dG_j} + \frac{\theta h_1(\cdot)}{(1+r)}\frac{di}{dG_j} - \frac{[\lambda_2 f(2) + \theta h(\cdot) - g_2]}{(1+r)^2}\frac{dr}{dG_j}\} \quad (38+j) \quad \forall j=1,2,\ldots 8.$$

where G_j is the j^{th} component of the vector G and similarly k_j is the j^{th} element of the vector $k \equiv (f(1), \frac{f(2)}{(1+r)}, \frac{h(\cdot)}{(1+r)}, -1, \lambda_1 f_2(1) - 1, \frac{-1}{(1+r)}, \frac{\lambda_2 f_2(2) - 1}{(1+r)}, \frac{\theta h_2(\cdot) - r}{(1+r)})$.[12]

The above set of first-order conditions are readily interpretable. To begin with, consider the set of first-order conditions (39) to (41) describing the government's optimal tax policy. The left-hand sides of these equations can be simplified, through the use of the envelope theorem and equations (9) and (13), to obtain

$$U'(1)\left[\lambda_1 f_1(1)\frac{d\ell_1}{dG_j}+\frac{\lambda_2 f_1(2)}{(1+r)}\frac{d\ell_2}{dG_j}+\frac{\theta h_1(\cdot)}{(1+r)}\frac{di}{dG_j}\right]=-\phi\{\cdot\} \quad (38+j) \quad \forall j=1,2,3.$$

To see intuitively the implications of these equations, divide both sides of (39)--which is the first-order conditions governing the choice of λ_1--by minus the term in brackets on the right-hand side of this equation. The term on the left-hand side of the resulting equation illustrates the marginal welfare loss per extra dollar raised via an increase in the first-period tax rate, λ_1. The right-hand side of this new equation, or ϕ, represents the marginal cost of an extra dollar raised in revenue through distortional taxation. Note that one could also perform an analogous operation on both sides of equations (40) and (41). Then the right-hand sides of these new versions of (39), (40), and (41) are identical, each being equal to ϕ. Consequently, an optimal tax policy requires that the marginal welfare loss per extra (present-value) dollar raised through each tax instrument be equivalent.

Next, focus on the first-order conditions (42) to (46) which determine the efficient choice of government spending. The left-hand sides of these expressions can be once again simplified by using the envelope theorem in conjunction with equations (9) and

(13) to get

$$U'(1)\, m_{j-3} = -\phi\{\cdot\} \qquad (38+j) \quad \forall j = 4,5,\ldots,8$$

where $m_{s=j-3}$ is the s^{th} component of the vector $m \equiv (\alpha'(1) - 1, f_2(1) - 1, \frac{\alpha'(2)-1}{1+r}, \frac{f_2(2)-1}{1+r}, \frac{h_2(2)-r}{1+r})$. Again, this set of first-order conditions can be interpreted intuitively. So as to better understand these conditions, divide both sides of (42)--the efficiency condition determining the optimal choice of g_1^c--by minus the term in brackets on the right-hand side of the expression. The resulting term on the left of the new equation represents the marginal net benefit of first-period government spending on consumption services (per net tax dollar spent). Again, the right-hand side of this equation, or ϕ, shows the marginal welfare cost of an extra dollar raised in revenue through distortional taxation. An long as government revenue cannot be raised costlessly, the net marginal benefit of this government spending, or the marginal value of g_1^c over and above its resource cost, should be set greater than zero. Finally, note that by performing analogous operations on equations (43) to (46), it can be seen that the net marginal benefit per (net) tax dollar spent should be equalized across the various components of government expenditure.

A complete characterization of the government's optimal income tax program is implicitly given by equations (39) to (46) which are the efficiency conditions governing the tax policy, (13) representing the government's budget constraint, and (10), (11), and (12) describing the economy's general equilibrium. This is a system of twelve equations in twelve unknowns, λ_1, λ_2, θ, g_1^c, g_1^ℓ, g_2^c, g_2^ℓ, g^i, ϕ, ℓ_1, ℓ_2, and i. As can be seen, even a basic understanding of the optimal tax policy in this simple model requires a detailed knowledge of the interaction between tastes and technology.[13] An elementary question one could ask is whether or not labor income tax rates are likely to be constant through time. That is, will there be uniform labor-income taxation across time here? A glance at the system of eleven equations describing the economy's general equilibrium would seem to indicate that in general the answer is no.

In order to further focus on this question, suppose that only labor-income taxation is available to the government and that the pattern of government expenditure is exogenously imposed on the economy [i.e., drop equations (42) to (46)] with $g_1^c = g_2^c$ and $g_1^\ell = g_2^\ell$. Next, note that from the first-order condition (7), private investment, i, can be written as a function of the real interest rate, r, and government investment in public goods, g^i. Thus, one could write $i = i(r,g^i)$. Evaluate this function at $r = (1-\beta)/\beta$ and set $\delta_1 = h(i(\frac{1-\beta}{\beta},g^i),g^i) + 2i(\frac{1-\beta}{\beta},g^i) + 2g^i$. Now suppose that labor-income tax rates were the same across time and test whether this provides a solution to the model. If $\lambda_1 = \lambda_2$, it can be seen that equations (10), (11), and (12) describing the model's general equilibrium would imply that $\ell_1 = \ell_2$ and $i = i(\frac{1-\beta}{\beta},g^i)$. Consequently, it follows that $c_1 = c_2$. Also, note that (13) implies that the government must have a balanced budget in each period here, so that $g_1 = \lambda_1 f(\ell_1, g_1^{\ell})$ and $g_2 = \lambda_2 f(\ell_2, g_2^{\ell})$. Finally, this solution also satisfies equations (39) and (40). This follows because in this circumstance $\frac{d\ell_1}{d\lambda_1} + (\frac{1}{1+r}) \frac{d\ell_2}{d\lambda_1} = \frac{d\ell_2}{d\lambda_2} + (1+r) \frac{d\ell_1}{d\lambda_2}$, while the budget deficit terms vanish. Thus, a sufficient condition to have uniform labor-income taxation across time in the model is that the real equilibria in the first and second periods are identical. How departures from this benchmark case will influence the structure of income taxes is a question which will be explored in the subsequent section.

Before proceeding further, however, it will be noted that certain restrictions can be placed on the forms of taste and technology which will guarantee uniform labor-income taxation across time in the absence of investment-income taxation. As is discussed by Razin and Svennson (1983) and Kimbrough (1984), if preferences are implicitly separable between consumption and leisure and technology is linear, with government spending being excluded from the functions U(•), f(•) and h(•), then labor-income tax rates will be constant across time for arbitrary values of government spending in each period.[14] (See Appendix B for further discussion.)

VIII. Simulations

In this section the analysis of government spending shocks and optimal taxation is brought together through the use of numerical simulations. For simplicity, the pattern of government spending is exogenously imposed on the economy implying that equations (42) to (46) will be excluded from the government's optimal fiscal program. The constants, β and δ_1, and the functions $U(\cdot)$, $V(\cdot)$, $f(\cdot)$, and $h(\cdot)$ are parameterized as follows: $\beta = .95$, $\delta_1 \cong 20.13$, $U \cong -40.2\exp(-.025c)$, $V = -\exp(\ell)$, $f \cong 20.09\ell$, and $h = .5\ \ln(i)$. The first simulation is conducted under the constraint that the set of tax instruments available to the government consists of labor-income tax rates alone while in the second simulation the rate of return on investment is allowed to be taxed as well. This setup makes it possible to direct attention to three questions: 1) What are the effects of temporary and permanent changes in the level of government spending on employment, output, investment and the rate of return? 2) To what extent are labor-income taxes smoothed across time? 3) What is the implication for the structure of labor-income taxes of the addition of an investment tax?

The analysis begins by studying the case of labor-income taxation solely. Table 3 reports the results of various experiments involving pure government spending shocks. Permanent increases in government spending result in a negative wealth effect which tends to increase work effort in each period. However, associated with the higher government spending are permanently higher distortionary labor-income taxes, which induce a substitution away from market (employment, output) to non-market activity (leisure). On net, the substitution effect dominates and employment and output fall in each period.[15] Further, as the temporal incidence of the spending/tax burden is even across time there is no incentive for

Table 3

Case	g_1	g_2	λ_1	λ_2	Deficit $g_1 - \lambda_1 f(1)$	ℓ_1	ℓ_2	i	c_1	c_2	r
I. No Government Spending	0	0	0	0	0	1.8238	1.8238	9.50	47.3	47.3	.053
II. Temporary Government Spending	5	0	.0709	.0705	2.40	1.8239	1.8108	7.46	44.3	44.8	.067
	10	0	.1446	.1427	4.71	1.8213	1.7897	5.63	41.1	42.4	.089
	15	0	.2232	.2184	6.87	1.8144	1.7563	4.08	37.5	40.7	.122
III. Anticipated Government Spending	0	5	.0669	.0672	-2.44	1.8137	1.8227	11.67	44.9	44.5	.043
	0	10	.1348	.1357	-4.88	1.8008	1.8158	13.93	42.4	41.7	.036
	0	15	.2043	.2059	-7.32	1.7835	1.8024	16.25	39.7	38.9	.031
IV. Permanent Government Spending	5	5	.1377	.1377	0.0	1.8080	1.8080	9.5	41.9	41.9	.053
	10	10	.2814	.2814	0.0	1.7694	1.7694	9.5	36.2	36.2	.053
	15	15	.4443	.4443	0.0	1.6810	1.6810	9.5	29.3	29.3	.053

this burden to be shifted from one period to the other. Thus, output is reduced by equal amounts in each period and investment and the rate of return are left unaffected. Finally, in accordance with the example in the previous section, the government finds it optimal to equalize labor-income tax rates across both periods of the model.

Next, consider a temporary rise in current government spending which involves, in Case II, a strong intertemporal substitution effect on work effort since the temporal incidence of the government expenditure in the first period creates an excess demand for goods and a rise in the rate of return. In order to isolate, roughly, the impact of the rise in this relative return to work effort, compare the second line of Case II with the first line of Case IV which have approximately the same values for the permanent level of government spending, i.e., $\bar{g}^{II} = (1+r)/(2+r) \cdot 10 = (1.089)/(2.089) \cdot 10 = 5.21 \cong 5 = \bar{g}^{IV}$. In response to the rise in the relative return to work effort, employment and output rise in the first period relative to the constant employment and output path which would have been forthcoming had the temporal incidence of the government spending been equal across periods.

Further, notice the remarkable tendency for the government to redistribute the burden of financing the first-period public expenditure over the two periods by running a deficit nearly equal to one-half the size of the expenditure. Tax rates are only slightly higher in the first than in the second period, with the greatest difference between tax rates across time being two percent. The fact that labor is taxed at a (slightly) higher rate in the first period relative to the second appears to arise because the increase in government spending creates a rise in the interest rate and promotes an intertemporal substitution of work effort from the second to the first period. Consequently, to minimize the aggregate area of the sum of welfare loss triangles, the government taxes relatively more the good with the larger tax base, which is first-period labor supply.

In sum, the conclusions to be drawn from Table 3 are, first, that permanent increases in government spending reduce output while temporary increases in government spending--relative to the permanent level--increase output. These simulation results are in basic agreement with the empirical results contained in Barro (1981), although in the latter study a tendency for permanent increases in public expenditure to raise output was also found. Second, the assumption of constant income tax rates appears to be a reasonable approximation to the optimal tax structure. Of course, the latter conclusion depends crucially on the assumption that government spending does not interact directly with the marginal product of labor and thereby cause an asymmetry in labor market conditions in the two periods. Finally, there exists a positive correlation between government budget deficits and high interest rates. However, there is no causal relationship between these two variables. Rather, it is the extraordinary demand for real resources in the period in which government spending actually occurs which is the source of the movements in interest rates (to eliminate the excess demand) and the deficit (as the government spreads the tax burden across time).

Consider now Table 4, where the set of tax instruments has been expanded to include the investment tax. The first thing to notice is that there is less of a tendency for labor-income tax rates to be constant across time. For example, in the case of permanent government spending the tendency is for the first-period tax rate to be lowered and the second-period tax rate to be raised from a position of equality. The reason would appear to be to lessen the negative impact on investment of the introduction of the investment tax by reducing the current labor-income tax and raising the future labor-income tax rate, which shifts the relative burden of labor-income taxation to the future and promotes capital accumulation. Nevertheless, on net the after-tax real rate of return falls and the consumption profile has a negative incline.

This general pattern for tax rates carries through to the cases of temporary current and anticipated government spending as well. In both cases the introduction of the investment tax tends to switch the <u>relative</u> burden of labor-income taxation away from the first period and toward the second period. This, it should be noted, leads to current temporary government spending having larger stimulative (or at least less detrimental) effects on first-period labor effort and output than in the case where the investment tax was absent.

To conclude, the addition of the investment tax seems to bias the time profile of labor-income taxation so as to encourage first relative to second-period labor effort. Nevertheless, it

Table 4

Case	g_1	g_2	λ_1	λ_2	θ	Deficit $g_1 - \lambda_1 f(1)$	ℓ_1	ℓ_2	i	c_1	c_2	r	$r/(1-\theta)$
I. No Government Spending	0	0	0	0	0	0	1.8238	1.8238	9.50	47.3	47.3	.053	.053
II. Temporary Government Spending	5	0	.0696	.0708	.032	2.5	1.8243	1.8112	7.43	44.3	44.8	.065	.067
	10	0	.1426	.1433	.047	4.8	1.8220	1.7903	5.57	41.2	42.4	.086	.090
	15	0	.2208	.2191	.054	7.0	1.8153	1.7572	4.01	37.6	40.0	.118	.125
III. Anticipated Government Spending	0	5	.0647	.0674	.065	-2.4	1.8144	1.8234	11.62	45.0	44.5	.040	.043
	0	10	.1288	.1354	.195	-4.7	1.8033	1.8182	13.80	42.5	41.6	.029	.036
	0	15	.1918	.2035	.423	-6.9	1.7898	1.8086	16.01	40.1	38.7	.018	.031
IV. Permanent Government Spending	5	5	.1342	.1381	.099	.12	1.8092	1.8092	9.41	42.1	41.9	.048	.053
	10	10	.2732	.2813	.236	.27	1.7735	1.7733	9.29	36.5	36.0	.041	.054
	15	15	.4273	.4401	.457	.46	1.6941	1.6933	9.08	30.1	29.2	.030	.055

still appears to be a reasonable conclusion that, to a first approximation, optimal taxation requires a fairly smooth time-path for labor-income tax rates.

IX. Open Economy Extensions

The above model can be modified to analyze the effects of taxation and government spending in a "small" open economy. In a "small" open economy version of the model domestic residents would be free to borrow and lend on international capital markets. Suppose that the world real interest rate is r^* and assume that the government taxes (subsidizes) the interest rate on foreign lending (borrowing) at the rate of θ. The domestic after-tax real interest rate, r, would thus be given by $r = (1-\theta)r^*$. The agent's maximization problem would again be described by (3).

Now define b_1 to be the first-period trade balance. Thus

$$b_1 \equiv f(\ell_1, g_1^{\ell}) - c_1 - i - g_1.$$

Note that b_1 represents the amount of net foreign lending that the domestic economy performs in the first period. By substituting an open economy version of the government's budget constraint (13), which incorporates a modification to reflect the fact that the government now taxes the earnings on foreign lending, into the representative agent's budget constraint, shown in (3), a relationship stating that trade must balance intertemporally is obtained:

$$c_1 + g_1 + i + \left(\frac{1}{1+r^*}\right)[c_2 + g_2] = \delta_1 + f_1(\ell_1, g_1^{\ell}) + \left(\frac{1}{1+r^*}\right)[f(\ell_2, g_2^{\ell}) + h(i, g^i) + i] \quad (47)$$

The small open economy's general equilibrium can be described by the first-order conditions (4) to (7) in addition to the economy's intertemporal budget constraint (47). Specifically, these conditions yield the following three equations which implicitly define solutions for ℓ_1, ℓ_2, and b_1:

$$-V'(\ell_1) = U'(f(\ell_1, g_1^{\ell}) + \alpha(g_1^c) - \bar{i} - b_1 - g_1)(1-\lambda_1) f_1(\ell_1, g_1^{\ell}) \tag{48}$$

$$-V'(\ell_2) = U'(f(\ell_2, g_2^{\ell}) + \alpha(g_2^c) + h(\bar{i}, g^i) + i + (1+r^*) b_1 - g_2) f_1(\ell_2, g_2^{\ell}) \tag{49}$$

$$U'(f(\ell_1, g_1^{\ell}) + \alpha(g_1^c) - \bar{i} - b_1 - g_1) = [(1+(1-\theta) r^*] \beta U'(f(\ell_2, g_2^{\ell}) + \alpha(g_2^c) + h(\bar{i}, g^i) + \bar{i} + (1+r^*) b_1 - g_2) \tag{50}$$

[with $\bar{i} \equiv \bar{i}(r^*, g^i)$, cf. (7)].

Note that equation (7) implies that private investment, i, is solely a function of the world real interest rate, r^*, and the size of the public capital stock, g^i. Thus, one could write $\bar{i} = \bar{i}(r^*, g^i)$, where $\bar{i}$ is the level of private investment which is undertaken in the open economy. It is easy to see that the "small" open economy version of the model closely parallels the closed economy one. Basically, net foreign lending, b_1, in the open economy reacts the same way in response to many shocks as investment, i, does in the closed economy.

To see this, consider the effect of a temporary increase in current labor-income taxes on labor effort in each period and on net external savings. By performing the required comparative statics exercise on (48), (49), and (50), it is easy to show that current labor effort, ℓ_1, falls, future labor effort, ℓ_2, rises, and net foreign lending, b_1, decreases. The intuition is clear. Since a temporary increase in current labor-income taxes creates a disincentive to work effort in this period, agents will substitute intertemporally toward working more next period where the after-tax marginal product of labor is now relatively higher. The reduction in current work effort will cause a loss in current income. Current consumption will not drop by the full loss in current income since agents will smooth out the effects from this loss in income over both periods. Consequently, individuals will reduce consumption in the first period by less than the reduction in current income. This can be achieved by lending less (or borrowing more) on international capital markets. Thus b_1 will fall. Due to the reduction in net foreign lending that part of second-period income derived from first-period net foreign savings will be smaller. This shortfall in income from net foreign savings will be met by a reduction in second-period consumption as well as by an increase in second-period labor effort.

The important point to note here is that a temporary increase in the current labor-income tax rate causes first-period savings to decrease in both the closed and open economy versions of the model. In the closed economy, this causes the interest rate to rise and investment to fall, while in the open economy the trade balance tends to swing into a deficit. It happens that in many situations the trade balance deficit of a small open economy responds in the same fashion to shocks as the real interest rate does in a closed economy. Since the trade balance is more readily observable than the real interest rate, it may be more useful to test the open economy version rather than the closed economy version of the above model.

There is one important difference, however, between the closed and open economy versions of the model. In the closed economy domestic fiscal shocks cause movements in the after-tax real interest rate which in turn generate intertemporal substitution effects which affect agents' consumption-leisure decision-making. In the small open economy this channel of effect is no longer operational since the domestic after-tax real interest rate is now exogenous, given by $r = (1-\theta)r^*$. Fiscal policy shocks impact on agents' consumption-leisure decision-making only to the extent that they are either associated with wealth effects or with changes in incentives to work or to invest induced by changes in proportional taxation.[16]

To see this more clearly, consider the case where the government increases public investment and assume that there is no complementarity between private and public capital. As analyzed previously, such a change in fiscal policy exerts two effects on the closed economy's general equilibrium. First, to the extent that public capital is less efficient than private capital, a negative wealth effect is created. This tends to stimulate labor effort and reduce consumption in both periods. Second, this increased public investment tends to reduce the economy's resources available for first-period vis-à-vis second-period consumption and leisure. This drives up the real interest rate which works to reduce current consumption, investment, and future labor supply effort and stimulate current labor supply effort and future consumption. In the small open economy this second channel of impact is not operational. Consequently, consumption falls and labor supply effort rises in both periods with no effect on private investment. Note that the economy finances this increased current public investment by reducing current consumption, increasing current labor effort, and by borrowing from abroad against its increased future output--derived from both a higher level of work effort in the future and an increased public capital stock. Finally, to the extent that public and private capital are complements in production, a greater level of public investment will induce an upward movement in private investment which is required in order to equilibrate the return on private investment with the world interest rate. The agent will finance this new higher level of private investment by borrowing on world markets and this will tend to further exacerbate the deterioration in the trade balance.

To conclude this section, the effects in the small open economy of various shocks in fiscal policy are presented in Table 5.

X. Conclusion

A small neoclassical general equilibrium is constructed in this paper to investigate the macroeconomic effects of fiscal policy. The two-period model presented probably represents the simplest choice-theoretic paradigm that can be utilized to address fiscal policy adequately. Despite its simplicity, the framework employed

Table 5

	ℓ_1 (and y_1)	ℓ_2	I	c_1	b_1
TAX CHANGE					
(i) Anticipated increase in future income tax rate, i.e., $\Delta\lambda_1 = 0$, $\Delta\lambda_2 > 0$.	(+)	(-)	(0)	(-)	(+)
(ii) Unanticipated temporary increase in current income tax rate, i.e., $\Delta\lambda_1 > 0$, $\Delta\lambda_2 = 0$.	(-)	(+)	(0)	(-)	(-)
(iii) Unanticipated permanent increase in the current income tax rate, i.e., $\Delta\lambda_1 = \Delta\lambda_2 > 0$.	(-)	(-)	(0)	(-)	(0)[1]
(iv) An increase in the tax rate on investment income, θ.	(-)	(+)	(0)	(+)	(-)
SPENDING CHANGE[2]					
(i) Anticipated increase in future spending, i.e., $\Delta g_2^s > 0$, $\Delta g_1^s = 0$.[3]	(+)	(+)	(0)	(-)	(+)
(ii) Unanticipated temporary increase in current spending, i.e., $\Delta g_1^s > 0$, $\Delta g_2^s = 0$.[3]	(+)	(+)	(0)	(-)	(-)
(iii) Unanticipated permanent change in spending, i.e., $\Delta g_1^s = \Delta g_2^s = \Delta\bar{g}^s$.	(+)	(+)	(0)	(-)	(0)[4]
(iv) An increase in public investment, $\Delta g^i > 0$.[5]	(+)	(+)	(0), (+)[6]	(-)	(-)

[1] Some initial conditions have been assumed in deriving this result. They are: $g_1^\ell = g_2^\ell$, $\lambda_1 = \lambda_2$, $\theta = 0$, and $1/\beta = (1+r^*)$.

[2] It has been assumed that: $0 < \alpha'(\cdot)$, $f_2(\cdot) < 1$, and $h_2(\cdot) < h_1(\cdot)$.

[3] Assuming that $f_{12}(\cdot) = 0$.

[4] In deriving this result it has been assumed that $g_1^\ell = g_2^\ell$, and $1/\beta = (1+r^*)$.

[5] Assuming that $h_{12}(\cdot) = 0$.

[6] This holds when $h_{12}(\cdot) > 0$.

allows economic actors to make a consumption and labor supply choice in each period and decisions about how much real and financial capital to carry over between the two periods. It can also be used to address issues on both the expenditure and taxation sides of fiscal policy. On the expenditure side of fiscal policy, government services were modeled as yielding consumption and production benefits for the private sector while government investment in public capital augmented society's future production possibilities. On the taxation side, government revenue could be raised through either labor-income taxation, corporate income taxation, or bond financing. A salient feature of the analysis is that when investigating the impact of fiscal policy changes, it is important to distinguish whether they are transitory or permanent in character, and whether they reflect current but unanticipated events or expected future ones. The framework was also flexible enough to model both the closed and "small" open economies.

While the simplistic framework used can generate a qualitative picture about fiscal policy issues, it provides no insight about the likely quantitative impact of various fiscal programs. Obtaining quantitative estimates of the effects of alternative fiscal policies is likely to be an important avenue for future research. One way to proceed toward this end would be to construct a numerical dynamic general equilibrium model and then simulate the impact of alternative fiscal programs. By judiciously picking functional forms and parameter values in the model, a quantitative estimate of the welfare gains and losses associated with various government policies could perhaps be obtained. Such a modelling strategy would seem to be in the spirit of Kydland and Prescott's (1980a, 1982) work. The model presented in this paper, hopefully, is a stepping stone toward this goal.

Appendix A

This appendix is presented to provide the interested reader with a taste for some of the technical aspects of the comparative statics results discussed in the text. The results of those comparative static exercises not discussed here can be easily deduced by mimicking the line of argument utilized below. To begin with, the impact of a change in λ_2 on ℓ_1, ℓ_2 and i can be discussed by taking the total differential of equations (10), (11), and (12). The resulting system of three equations is:

$$
\begin{bmatrix}
-[V''(1) + U''(1) f_1(1)^2 \tilde{\lambda}_1 + U'(1) \tilde{\lambda}_1 f_{11}(1)] & U''(1) f_1(1) \tilde{\lambda}_1 & 0 \\
0 & -U''(2)(1 + h_1) \tilde{\lambda}_2 f_1(2) & -[V''(2) + U''(2) \tilde{\lambda}_2 f_1(2)^2 + U'(2) \tilde{\lambda}_2 f_{11}(2)] \\
U''(1) f_1(1) & -[U''(1) + \beta \tilde{\theta} h_{11} U'(2) + \beta(1 + \tilde{\theta} h_1)(1 + h_1) U''(2)] & -\beta(1 + \tilde{\theta} h_1) U''(2) f_1(2)
\end{bmatrix}
\begin{bmatrix} d\ell_1 \\ di \\ d\ell_2 \end{bmatrix}
= \begin{bmatrix} 0 \\ -U'(2) f_1(2) \\ 0 \end{bmatrix} d\lambda_2 \qquad \text{(A.1)}
$$

where $\tilde{\lambda}_1 \equiv (1-\lambda_1)$, $\tilde{\lambda}_2 \equiv (1-\lambda_2)$, $\tilde{\theta} \equiv (1-\theta)$ and the notation x(t) means that the arguments in the function $x(\cdot)$ are being evaluated at their date t values. Define $-\Omega$ as the determinant of the 3 x 3 matrix on the left-hand side of the above equation system. The expression for Ω is

$$\Omega = -[V''(1) + U''(1)\tilde{\lambda}_1 f_1(1)^2 + U'(1)\tilde{\lambda}_1 f_{11}(1)]\{\beta\tilde{\theta}h_{11}U'(2)[V''(2) + U''(2)\tilde{\lambda}_2 f_1(2)^2$$
$$+ U'(2)\tilde{\lambda}_2 f_{11}(2)]$$
$$+ \beta(1+\tilde{\theta}h_1)(1+h_1)U''(2)[V''(2) + U''(2)\tilde{\lambda}_2 f_{11}(2)]\}$$
$$- U''(1)[V''(1) + U'(1)\tilde{\lambda}_1 f_{11}(1)][V''(2) + U''(2)\tilde{\lambda}_2 f_1(2)^2 + U'(2)\tilde{\lambda}_2 f_{11}(2)]$$
$$> 0.$$

Solving the system of equations (A.1) yields

$$\frac{d\ell_1}{d\lambda_2} = U'(2)f_1(2)U''(1)f_1(1)\tilde{\lambda}_1\beta(1+\tilde{\theta}h_1)U''(2)f_1(2)/\Omega > 0 \tag{A.2}$$

$$\frac{di}{d\lambda_2} = U'(2)f_1(2)[V''(1) + U''(1)f_1(1)^2\tilde{\lambda}_1 + U'(1)f_{11}(1)\tilde{\lambda}_1]\beta(1+\tilde{\theta}h_1)U''(2)f_1(2)/\Omega > 0 \tag{A.3}$$

$$\frac{d\ell_2}{d\lambda_2} = -U'(2)f_1(2)\{[V''(1) + U''(1)f_1(1)^2\tilde{\lambda}_1 + U'(1)f_{11}(1)\lambda_1][\beta\tilde{\theta}h_{11}U'(2)$$
$$+ \beta(1+\tilde{\theta}h_1)(1+h_1)U''(2)] + [V''(1) + U''(1)\tilde{\lambda}_1 f_{11}(1)]U''(1)\}/\Omega < 0 \tag{A.4}$$

Consequently, it follows from (A.2), (A.3), and (A.4) that the derivatives presented in the text in (14) have the signs shown.

Also, through the use of (A.2) and (A.3) it can be seen that

$$\frac{di}{d\lambda_2} = f_1(1)\,\frac{d\ell_1}{d\lambda_2} + \{U'(2)f_1(2)[V''(1) + U'(1)\tilde{\lambda}_1 f_{11}(1)]\beta(1+\tilde{\theta}h_{11})U''(2)f_1(2)\}/\Omega \tag{A.5}$$

where the last term in this expression is unambiguously positive. Using (A.5) together with (15) in the text it immediately follows that $dc_1/d\lambda_2 < 0$, as was stated.

Next, some of the results in Section V will be derived. To begin with, how is c_2 related to δ_2 in the economy without government intervention? From (21) it can be seen that

$$\frac{d\ell_2}{d\delta_2} = \frac{-f_1(2)U''(2)}{[V''(2) + f_{11}(2)U'(2) + f_1(2)^2U''(2)]} < 0 \qquad \text{(A.6)}$$

By using the above result (A.6) in conjunction with (9) it follows immediately that

$$\frac{dc_2}{d\delta_2} = \frac{V''(2) + f_{11}(2)U'(2)}{[V''(2) + f_{11}(2)U'(2) + f_1(2)^2U''(2)]} > 0$$

In an entirely similar fashion the response of first-period labor supply and consumption to an increase in investment for either the economy with or without intervention can be deduced from (20) or (26), and (8) to be

$$\frac{d\ell_1}{di} = \frac{f_1(1)U''(1)}{[V''(1) + f_{11}(1)U'(1) + f_1(1)^2U''(1)]} > 0$$

and

$$\frac{dc_1}{di} = \frac{-[V''(1) + f_{11}(1)U'(1)]}{[V''(1) + f_{11}(1)U'(1) + f_1(1)^2U''(1)]^2} < 0 .$$

Finally by taking the total differential of equations (10), (11) and (12) the impact that a temporary increase in government spending on services has on ℓ_2, ℓ_2 and i can easily be uncovered. It is easy to see that when doing this exercise the 3 x 3 matrix on the left-hand side of (A.1) remains the same and all that changes is the 3 x 1 displacement vector on the right-hand side of this equation. The results obtained are:

$$\frac{d\ell_1}{dg_1^s} = -[1-\alpha'(1)\rho - (1-\rho)f_2(1)]U''(1)\{f_1(1)\beta h_{11}U'(2)[V''(2)+U''(2)f_1(2)^2 + U'(2)f_{11}(2)] + f_1(1)\beta(1+h_1)^2U''(2)[V''(2)+U'(2)f_{11}(2)]\}/\Omega > 0, \quad (A.7)$$

$$\frac{d\ell_2}{dg_1^s} = -[1-\alpha'(1)\rho - (1-\rho)f_2(1)]U''(1)\{[V''(1)+U'(1)f_{11}(1)]U''(2)(1+h_1)f_1(2)\}/\Omega > 0 \quad (A.8)$$

and

$$\frac{di}{dg_1^s} = [1-\alpha'(1)\rho - (1-\rho)f_2(1)]U''(1)\{[V''(1)+U'(1)f_{11}(1)][V''(2)+U''(2)f_1(2)^2 + U'(2)f_{11}(2)]\}/\Omega < 0. \quad (A.9)$$

Appendix B

In this appendix the implications for uniform labor income taxation of a preference structure which is implicitly separable between consumption and leisure and linear technology are examined. To begin with, assume that labor income taxation is the only tax instrument available to the government. Then, as mentioned in the text, if preferences are characterized by implicit separability between consumption and leisure, and production is linear, with the functions $U(\cdot)$, $f(\cdot)$, and $h(\cdot)$ being independent of government spending, then uniform labor income taxation will obtain. This is easy to show. Let $f(\ell_t) = w\ell_t$ (actually for the argument being employed the marginal product of labor, w, can be different across time) and $h(i) = hi$. From the agent's optimization problem (3) it can be seen that his period-t labor supply, ℓ_t, is given by the compensated labor supply function $\ell_t = \ell_t^s(1, \tilde{w}_1, D\tilde{w}_2, D, \underline{U})$ where $\tilde{w}_t \equiv (1-\lambda_t)w_t$ and $D \equiv 1/(1+h)$. In this situation, the first-order conditions (39) and (40) governing the optimal determination of λ_1 and λ_2 can be rewritten as

$$\lambda_1 w_1^2 \ell_{1,2}^s + \lambda_2 D w_1 w_2 \ell_{2,2}^s = (\phi/U'(c_1))[w_1\ell_1 - \lambda_1 w_1^2 \ell_{1,2}^s - \lambda_2 D w_1 w_2 \ell_{2,2}^s] \qquad (B.1)$$

$$\lambda_1 w_1 D w_2 \ell_{1,3}^s + \lambda_2 D^2 w_2^2 \ell_{2,3}^s = (\phi/U'(c_1))[Dw_2\ell_2 - D\lambda_1 w_1 w_2 \ell_{1,3}^s - D^2\lambda_2 w_2^2 \ell_{2,3}^s] \ . \qquad (B.2)$$

with $\ell_{t,j}^s$ being defined as the derivative of ℓ_t^s with respect to its j^{th} argument. The two first-order conditions (B.1) and (B.2) can be manipulated to obtain the following formula

$$\frac{\lambda_1/(1-\lambda_1)}{\lambda_2/(1-\lambda_2)} = \frac{[D\tilde{w}_2\ell^s_{1,3}/\ell_1 - D\tilde{w}_2\ell^s_{2,3}/\ell_2]}{[\tilde{w}_1\ell^s_{2,2}/\ell_2 - \tilde{w}_1\ell^s_{1,2}/\ell_1]}$$

[note that $\ell^s_{1,3} = \ell^s_{2,2}$ by symmetry of the Slutsky matrix]

Now, implicit separability between consumption and leisure implies that $\ell^s_{1,1}/\ell_1 = \ell^s_{2,1}/\ell_2$ and $\ell^s_{1,4}/\ell_1 = \ell^s_{2,4}/\ell_2$ which together with the standard "adding up" condition from consumer theory, or that $\ell^s_{t,1} + \tilde{w}_1\ell^s_{t,2} + D\tilde{w}_2\ell^s_{t,3} + D\ell^s_{t,4} = 0$, it follows that the numerator and denominator of the right-hand side of the above expression are equal.[17] Consequently, labor income taxation is uniform across time. Note implicit separability between consumption and leisure implies that the sum of the proportional effects of a change in first-period after-tax real wage rate on first- and second-period labor supply exactly equals the sum of the proportional effects of the second-period after-tax real wage rates on these labor supplies.

In the intersection between strongly separable preferences assumed in the text and the implicitly separable preferences assumed here lies the logarithmic utility function. The first simulation undertaken in the text was rerun with logarithmic preferences: $U(c\ ) = \frac{1}{3}\ln c$ and $V(\ell) = \frac{2}{3}\ln(\bar{\ell}-\ell)$ with $\bar{\ell} = 6.53$. It was hoped that this simulation would highlight the implications for uniform labor income taxation across time of variability in the real interest rate, since when $h(\cdot) = h\ln i$ the economy can only transform resources across time with diminishing returns. The results of this (perhaps more controlled) simulation were quite similar to those reported for the first-one with there again being a remarkable tendency to smooth tax rates across time. (It should be noted that no serious attempt was made in any simulation to choose parameter values, or a functional form for $h(\cdot)$, etc., which would maximize the variability in the tax rates between the two periods.)

Footnotes

*Helpful comments from Robert Barro, V. V. Chari, Peter Howitt and Michael Parkin are gratefully acknowledged.

[1]Note that it is being assumed that the world "starts up" at the beginning of period one. Consequently, in the first period the agent does not have either any physical capital or bonds which he has brought over from the past. Since there is only physical capital in the second period there is no need to index i (or g^i) with a subscript. Also, it trivially follows that first-period private investment equals the second-period capital stock. Alternatively if one liked, δ_1 could be viewed as capturing the effects of capital investment undertaken prior to period one. Value added from this period-zero capital investment is not taxed.

[2]The agent's intertemporal budget constraint can be derived by eliminating his holdings of bonds, b, from his first- and second-period budget constraints: $c_1 + i + b = \delta_1 + (1-\lambda_1)f(\ell_1, g_1^\ell) + \tau_1$ and $c_2 = (1-\lambda_2)f(\ell_2, g_2^\ell) + i + (1-\theta)h(i, g^i) + (1+r)b + \tau_2$.

[3]The arguments of the function $W(\cdot)$ are: λ_1, λ_2, θ, τ_1, τ_2, g_1^c, g_2^c, g_1^ℓ, g_2^ℓ, g^i and r.

[4]An alternative, and perhaps more intuitive, representation of the model's general equilibrium describing the system of demand and supply functions implicit in (8) to (12) is given by the following four equations

$\ell_t^d(w_t, g_t^\ell) = \ell_t^s(1, \tilde{w}_1, D\tilde{w}_2, D, \underline{U})$ (with $\tilde{w}_t \equiv (1-\lambda_t)w_t$ and $D \equiv 1/(1+r)$) $\forall t=1,2$

$$c_t^d(1,\tilde{w}_1,D\tilde{w}_2,D,g_1^c,g_2^c,\underline{U}) + i^d(D,\theta,g^i) + g_1 = \delta_1 + f(\ell_1^d(\cdot),g_1^\ell)$$

$$\text{(with } g_t \equiv g_t^c + g_t^\ell + g_t^i - g_{t-1}^i)$$

and

$$E(1,\tilde{w}_1,D\tilde{w}_2,D,g_1^c,g_2^c,\underline{U}) + i^d(\cdot) + g_1 + Dg_2 = \delta_1 + f(\ell_1^d(\cdot),g_1^\ell) + D[f(\ell_2^d(\cdot),g_2^\ell) + h(i^d(\cdot),g^i) + i^d(\cdot) + g^i] - \tilde{w}_1\ell_1^s(\cdot) - D\tilde{w}_2\ell_2^s(\cdot)$$

where w_t and $\tilde{w}_t$ are the before- and after-tax period-t real wages, D is the after-tax market discount factor, and $E(\cdot)$ is the expenditure function associated with the problem $\min\{c^1 + Dc^2 - \tilde{w}^1\ell^1 - D\tilde{w}^2\ell^2 | g_1^c, g_2^c, \underline{U}\}$. [Note that the endogenous variables here are w_1, w_2, D and $\underline{U}$.] Greenwood and Kimbrough (1984) use a framework similar to this to analyze the international transmission of fiscal policy shocks in a two-country world--with and without capital controls.

[5]This can easily be seen by noting that the system of equations (10), (11) and (12) describing the model's general equilibrium does not involve any transfer payment terms. For a full discussion of the theorem see Barro (1974) and Chan (1983).

[6]One could also view θ as a tax on private savings. To see this, suppose that the government taxes <u>both</u> the real return on bonds, r, and the value-added from capital $h(i,g^i)$ at the rate θ. Now denote $\tilde{r} = (1-\theta)r$. Solving the agent's optimization problem in this circumstance leads to almost the identical set of first-order conditions as those shown above; equations (5) and (6) remain the same, whereas now $\tilde{r}$ replaces r in (4), (7), and the agent's budget constraint. Note that the representative agent's choices are implicitly described by his first-order conditions (4), (5), (6),

and his budget constraint, with (7) being eliminated by substituting it into both (4) and the budget constraint. However, this system of equations is identical in both circumstances.

[7] All the results reported in Table 1 can be readily obtained by following the standard comparative static procedure outlined in Appendix A.

[8] As has already been demonstrated, the timing of distortional taxes has important implications for the macro economy. To combine the effects of a government spending scheme with a distortional tax financing policy would be to run the risk of confounding the effects of government spending with income taxation. Also, there would be many distortional tax schemes capable of financing a given change in the present-value of government spending and it would be hard to know how to choose among them.

[9] Note that the definition for a temporary change in government spending employed here is different from that of Barro (1981). Barro's definition holds, at the original interest rate, constant at the present value of government spending. That is, in the two-period setting adopted here he would fix the value of $g_1^s + (1/(1+r))g_2^s$. This would imply, at the initial interest rate, that an increase in current government spending, g_1^s, must be offset by a reduction in future government spending, g_2^s. The analogous exercise in the current model would be to reduce second-period government expenditure, g_2^s, by an amount which would keep the representative agent's level of utility, $\underline{U}$, constant. Barro deletes the wealth effects from a temporary increase in current government spending so as to emphasize

the scarcity of private disposable resources in the current period vis-à-vis the future that results. This tends to drive up the real interest rate and consequently increase current labor supply and output. The definition employed here incorporates the negative effect that a temporary increase in government spending will have on agents' wealth. Presumably, temporary government spending, such as for wars, could have significant adverse effects on agents' wealth positions. This negative wealth effect would tend to increase labor supply effort and output in the first and second periods.

[10]It may seem reasonable to conjecture that the effect on capital accumulation will depend upon whether the time profiles of consumption and leisure are positively or negatively inclined through time. For instance, one may speculate that if $(1+r) > \beta$ so that the time profiles of consumption and leisure are upward sloping, ceteris paribus, then the bulk on the burden of the shock will be absorbed in the future. The original conjecture, however, turns out to be false. It seems that how the burden of the shock is distributed through time depends upon the time profiles of the marginal propensities to consume goods and leisure (see Greenwood and Kimbrough (1984)). These marginal propensities to consume in general may be either increasing or decreasing functions of the real interest rate.

[11]Persson and Svensson (1984) also discuss optimal taxation policy and provide an intuitive explanation of the time inconsistency problem associated with it.

[12]Note from Section III describing the economy's general equilibrium that ℓ_1 and ℓ_2 in (38) are implicit functions of λ_1, λ_2, θ, g_1^c, g_2^c, g_2^ℓ and g^i, i.e., $\ell_t = \ell_t(\lambda_1, \lambda_2, \theta, g_1^c, g_2^c, g_1^\ell, g_2^\ell, g^i)$ $\forall t=1,2,$.

[13]Note that the derivatives contained in (39) to (46) are themselves dependent upon taste and technology, as can be verified by glancing at the solutions for (14) contained in Appendix A.

[14]Implicit separability is a form of separability imposed on the consumer's expenditure function. In the case under study it would imply that the agent's expenditure function, E(·),--c.f. footnote 4--can be written in the following form: $E(1,\tilde{w}_1,D\tilde{w}_2,D,\underline{U}) = E(c(1,D,\underline{U}), \ell(\tilde{w}_1,D\tilde{w}_2,\underline{U}),\underline{U})$ where $c(1,D,\underline{U})$ and $\ell(\tilde{w}_1,D\tilde{w}_2,\underline{U})$ are group price indices for consumption and leisure, respectively, over which the macro expenditure function E(·) is defined.

[15]The maximum level of permanent government expenditure that the model economy could sustain was 18.8. That there is such a maximum follows from the Laffer curve effect. Note that as the level of permanent government expenditure is increased so does the labor income tax rate, and this induces a drop in labor supply. At high enough tax rates the gain in revenue resulting from higher tax rates is outweighed by the fall in revenue caused by the cut in labor effort.

[16]Fiscal shocks emanating from within a large open economy can obviously affect the world real interest rate. For an analysis of the international transmission of fiscal policy in a two-country world, where such an effect is operated, see Frenkel and Razin (1984) and Greenwood and Kimbrough (1984).

[17]The first of these facts is easily deduced from the form of the implicitly separable expenditure function given in footnote 14.

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