1984

A Rational Expectations Model of Equilibrium Entry and Market Contestability

Elie Appelbaum

Chin Lim

ISSN: 0318-725X
ISBN: 0-7714-0602-9

RESEARCH REPORT 8420

A RATIONAL EXPECTATIONS MODEL OF EQUILIBRIUM ENTRY AND MARKET CONTESTABILITY

by

Elie Appelbaum
and
Chin Lim*

November, 1984

Elie Appelbaum
Department of Economics
York University
Downsview, Ontario, Canada

and

Chin Lim*
Department of Economics
University of Western Ontario
London, Ontario, Canada

*Research support for Chin Lim by the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

1. INTRODUCTION

The question of entry is of great importance to both positive and normative economics. In fact, the central motivation behind the market contestability theory (see Baumol, Panzar and Willig (1982)), is the issue of entry. This theory argues forcefully that the mere existence of potential frictionless entry would discipline the behaviour of incumbents, and depending on the industry cost structures, produce a particular form of market structure which is at the same time efficient. Entry conditions therefore become a crucial factor in determining the mode and efficiency of market structures.

In most of the literature[1] dealing with entry, the existence of incumbent(s) is taken as a historical fact, so that the potential, or actual entrants they refer to, are basically late entrants into a market which is already served by incumbents. The static and non-stochastic nature of the existing models preclude the possibility of explaining both early and late entries.

In any market one cares to observe, it is a fact that firms do not arrive simultaneously; some enter early and others late. Early entrants may have the advantage of reaping temporary economic profits which are competed away only after late entrants arrive. In a model of complete certainty, this suggests of course that all agents should wish to enter early. The 'endowment' of the advantage of early entry to some agents and do not to others is, therefore, restrictive and arbitrary.

It is clear that the entry decision itself should be examined within a choice model of rational behaviour. But at the same time, the choice between early and late entry into a market is meaningful only if there is some trade-off between the advantages and disadvantages of early and late entrance. Such a trade-off does not exist in a full information static model, where as we have noted above, early entrance is always preferred. In a world of imperfect information, however, it is not so clear that early entry will always be preferred. There will now exist a trade-off between the possible higher profit associated with early arrival and the informational (or 'flexibility) advantage of postponing entry till at least some uncertainty is resolved. Thus, incumbency has its advantages and disadvantages.

In this paper, we present a model that exploits this trade-off. Demand is uncertain, and in such an environment, producing agents may choose between entering early (before the state of demand is revealed) or late (after the state of demand is known). The market is assumed to be fully contestable in the sense that agents may contest in the early and late production stages. This is in contrast to existing contestable market models which assume but do not explain the nature and modes of contestability.

In our model, firms making entry decisions have rational expectations regarding the stochastic price distribution, in the sense that agents' expectations are mutually consistent with the equilibrium price distribution. The model will explain the equilibrium modes of entry (number of early and late entrants), their levels of production and the price distribution. In addition, although both early and late production processes are fully contestable, the model will also determine the "degree of contestability" (or probability of entry) by late entrants. Such a market will also be shown to be stable and efficient.

Furthermore, we will show how the various equilibruim variables are affected by the parameters of the cost and stochastic demand distribution functions.

II. CHOICE OF ENTRY

(i) Demand Condition

We consider a market in which demand conditions are uncertain so that the industry demand function is characterized by

$$Q = Q(p, \theta) \qquad \text{with } Q_p < 0,\ Q_\theta > 0 \tag{1}$$

where Q is aggregate demand, p is price, and $\theta \in [\underline{\theta}, \bar{\theta}]$ is continuous random variable with cumulative density $\Phi(\theta;k)$ where k is a distribution shifting parameter.[2]

The firms who choose to participate in this market are assumed to be small relative to the size of the market. The equilibrium price facing them is a random variable $p(\theta)$, whose probability distribution depends on $\Phi(\theta;k)$ and the various exogenous variable determining the industry equilibrium. Facing the equilibrium price distribution and cost conditions, firms choose the mode of their participation in the market, i.e., whether to enter early (before prices are revealed) or late (after prices are revealed) and how much to produce given each entry mode.

(ii) Cost Conditions

Let the technology of producing output level q be given by the (Austrian) production possibility[3] set $\{A: \ (x(t)\Big|_{t=o}^{T}, q(T))\}$ where x(t) is the input vector used at time t necessary to produce output level q(T) at time T. Suppose the firm faces input price vector w(t), then its cost function is given by

$$\underset{\{x(t),T\}}{\text{Min}} \; \{ \sum_{t=o}^{T} w(t) \cdot x(t): \; \{x(t) \Big|_{t=o}^{T}, q\} \in A\} \equiv g^{o}(q)$$

where in deriving the cost function $g^{o}(q)$, both the distribution of inputs over time, and the production period T itself are chosen optimally to minimize the cost of producing q. Consider next a program where the firm faces a time constraint $T \leq T^1$. Then its cost function is given by

$$\underset{\{x(t),T\}}{\text{Min}} \; \{ \sum_{t=o}^{T} w(t) \cdot x(t): \; \{x(t) \Big|_{t=o}^{T}, q\} \in A, \; T \leq T^1\} \equiv g^{1}(q)$$

and clearly we must have

$$g^{o}(q) \leq g^{1}(q)$$

because of the tighter time constraint faced by the latter program.

Alternatively, it is often assumed that the input prices themselves depend on the time constraint. In particular, it seems reasonable that input prices will be higher, for a tighter time constraint, due to, for example, overtime rates, faster input delivery costs etc. In cases like this the inequality above again holds, since for early and late input price vectors w^o and w^1 respectively, which satisfy $w^o \leq w^1$, we must have $g(q;w^o) \leq g(q;w^1)$.

The choice of production techniques becomes even more important when, in addition, the firm faces some form of uncertainty. In the face of uncertainty there is an informational advantage in postponing commitments. This informational advantage will, however, have to be balanced against cost-efficiency considerations since postponement may require a tighter time constraint and hence higher costs later on.

To simplify matters, suppose the firm can start its production process early (before prices are known) or late (after prices are revealed). According to the above discussion, early production processes will be more efficient but will involve greater uncertainty. Late production, on the other hand, will be less efficient, but will be carried out <u>after</u> uncertainty is resolved.

Let the cost functions corresponding to the two processes be given by

$$g^o(q) \equiv s^o + c^o(q) \qquad \text{and } g^1(q) = s^1 + c^1(q) \tag{2}$$

where s^o, s^1 are the fixed costs of the early and later processes, and $c^o(\cdot)$, $c^1(\cdot)$ are the respective variable costs. We assume that $c^i_q > 0$, $c^i_{qq} > 0$, i.e., variable costs are increasing and convex and $c^i(0) = 0$, $i=0,1$. In line with the above discussion we assume that the late process has either higher fixed costs (due to speedier organization and coordination), or higher variable costs (or both). The higher variable costs may occur if either $w^1 \geq w^o$, i.e., when late input prices are higher, or if, for the same input prices, the late production, because of a tighter time constraint, is less efficient. Thus, in (2) we assume that either $s^1 \geq s^o$ or $c^1 \geq c^o$ (or both). We parameterize the two variable cost functions by the parameters α^o and α^1 where, for the sake of simplicity (but without great loss of generality) we write

$$c^o(q) = \alpha^o c(q) \quad \text{and} \quad c^1(q) = \alpha^1 c(q) \tag{2a}$$

with $\alpha^1 \geq \alpha^o > 0$ capturing the cost advantage of early production (due to possible lower input prices on a more efficient production process).

We can interpret the two processes as two types of plants and the cost functions are, thus, plant cost functions.

(iii) <u>Entry Decisions</u>

In the following discussion we will identify "firms" with production processes, i.e., with plant types. Each "firm" may choose whether to use the early or late production processes. Of course, there is no reason why "real" firms, i.e., the entrepreneurial unit, should be restricted to one process only. In our formulation, however, since firms are identified with production processes, an organizational unit using both processes will be considered as two "firms". A firm who chooses to employ the early process is said to enter the market early, whereas a firm who chooses the late process is said to enter late. In view of our terminology it is of course possible for a "real firm" to enter both early and late, i.e., choose both processes.

We assume that firms are risk neutral so that when considering the two processes they are concerned with the expected profits corresponding to these processes.

Consider first the option of late entry, i.e., the choice of the late process. Given an observed price $p(\theta)$ a late entrant will (if he in fact chooses to enter) choose an output level z which solves,

$$\max_{z \geq 0} \; p(\theta)z - \alpha^1 c(z) - s(z) \tag{3}$$

where

$$s(z) = \begin{cases} s^1 & \text{if } z > 0 \\ 0 & \text{if } z = 0 \end{cases} \tag{4}$$

Let the maximum in (3) be written as $J(p(\theta),\alpha^1) - s(z)$, where $J(\cdot)$ is the usual[4] variable profit function and is, thus, increasing and convex in $p(\theta)$ and decreasing in α^1. If $J \geq s^1$, the firm will, in fact, choose, ex-post, to use the late production process and produces a positive z, otherwise it will choose not to use this process. Thus, defining $p^1(\alpha^1, s^1)$ by

$$J(p^1,\alpha^1) = s^1 \quad \text{or} \quad p^1 \equiv \min \frac{\alpha^1 c(z) + s^1}{z} \tag{5}$$

we get the optimal z by Hotelling's Lemma as

$$z^1 = \begin{cases} \partial J(p(\theta),\alpha^1)/\partial p & \text{if } p(\theta) \geq p^1 \\ 0 & \text{if } p(\theta) \leq p^1 \end{cases} \tag{6}$$

For the sake of notational convenience we suppress the parameters in J and write $J(p(\theta),\alpha^1) \equiv J^1[p(\theta)]$. Denoting <u>actual</u> profits by π^1 we then get

$$\pi^1 = \max[J^1(P(\theta)) - s^1, 0] \tag{7}$$

and the corresponding expected profit is

$$E[\pi^1] = E[J^1(P(\theta)) - s^1 | p(\theta) \geq p^1] = \int_{\theta^1}^{\bar{\theta}} [J^1(p(\theta)) - s^1] d\Phi(\theta) \tag{8}$$

where θ^1 (which will be defined more precisely later) is the lowest θ state where a late firm will begin producing; formally, θ^1 staisfies $p(\theta^1) = p^1(\alpha^1, s^1)$. Clearly, for late entry to be desirable <u>ex-ante</u>, it must yield non-negative expected profits.

Consider now the option of early entry, i.e., of choosing the early process. An early entrant has to choose whether or not to enter and how much output y to produce, <u>before</u> he knows the state of demand. The expected profits from this option are obtained from the solution to the problem

$$\max_{y \geq 0} \{E[p(\theta)]y - \alpha^o c(y) - s(y)\} \tag{9}$$

where

$$s(y) = \begin{cases} s^o & \text{if } y > 0 \\ 0 & \text{if } y = 0 \end{cases} \tag{10}$$

The maximum in (9) is written as $J(\bar{p},\alpha^o) - s^o$ where $\bar{p} \equiv E[p(\theta)]$ is the expected price, and $J(\bar{p},\alpha^o)$ is the variable profit evaluated at the mean price $\bar{p}$ for the early process parameterized by α^o.

If the expected profit in (9), $E[\pi^o] \equiv J(\bar{p},\alpha^o) - s^o \equiv J^o(\bar{p}) - s^o < 0$, early entry will not be chosen. For a firm to enter early, we require that

$$E[\pi^o] \equiv J^o(\bar{p}) - s^o \geq 0 \tag{11}$$

If (11) is satisfied, the optimal output of an early entrant, y^o, is given by (Hotelling's Lemma)

$$y^o = \partial J^o(\bar{p})/\partial\bar{p} \quad \text{or} \quad \bar{p} = \alpha^o \partial c(y^o)/\partial y \tag{12}$$

Both conditions (11) and (12) require that

$$\bar{p} \geq \text{Min} \frac{\alpha^o c(y^o) + s^o}{y^o} \equiv p^o(\alpha^o, s^o) \tag{13}$$

Whether or not the early or the late process or both are chosen depends on the profitability of the alternative options. Clearly, both processes would be chosen as long as $E[\pi^o] > 0$ and $E[\pi^1] > 0$. But the preference between the early and late processes depends on the various parameters and is in general not unambiguous:

$$E[\pi^o] - E[\pi^1] = [J^o(\bar{p}) - \int_{\theta^1}^{\bar{\theta}} J^1(p(\theta))d\Phi(\theta)] + [(1 - \Phi(\theta^1))s^1 - s^o] \tag{13}$$

Equation (13) highlights the advantages and disadvantages of early and late entry. On the one hand, late entry has an informational advantage on two counts: (i) the fixed costs, s^1, are only incurred if late entry <u>actually</u> occurs and that occurs only with probability $(1 - \Phi(\theta^1))$; in contrast the fixed costs of an early entrant, s^o, is incurred with certainty or probability 1. (ii) Holding the variable costs the same $(\alpha^o = \alpha^1)$, we have, from the convexity of the profit function, that $J^o(\bar{p}) \equiv J(\bar{p}, \alpha^o) \leq E[J(p(\theta), \alpha^o] = E[J(p(\theta), \alpha^1)] \equiv E[J^1(p(\theta))]$ where the inequality indicates the flexibility advantage that late production has over early production. Against these two informational advantages, late entry, compared to early entry, is relatively cost inefficient since $s^1 \geq s^o$ and $\alpha^1 \geq \alpha^o$. There is therefore a trade-off between <u>informational</u> and <u>cost</u> advantages between the two processes, and whichever is preferred depends on the cost and stochastic demand parameters.

In allowing the firm the options of choosing the early process, the late process, or both, our model provides a generalization of all previous models on the subject of the competitive firm under price uncertainty. For instance, the well-known models of Sandmo (1971), Baron (1970) and Tisdell (1963) limit the firm to the early process only. On the other extreme, the models of Oi (1961), Dreze and Gabszewicz (1967), Sheshinski and Dreze (1976), Lippman and McCall (1981), limit the firm to the late process only. Turnovsky's (1973) model does allow the firm the option of engaging in both processes but the analysis was limited to examine decisions at the firm level and was never directed at studying the properties of equilibrium. Our ensuing equilibrium analysis can be seen, among other things, as filling a void in this literature.

Our task is to analyze how the modes of entry, the price distribution and the degree of contestability of the early and late processes are determined in equilibrium and how these equilibrium variables are affected by the various cost and stochastic demand parameters.

Before we begin the formal analysis in the remaining sections, two important features of the model should be noted. First, the mode of entry and production are governed not only by cost conditions but also by the firms' subjective evaluation of the stochastic price distribution. Firms are rational in the model, including their expectations so that their expectations of the price distribution are mutually consistent with the actual equilibrium price distribution. In other words, all agents are assumed to have rational expectations.

Finally, the market that we consider is perfectly contestable in the sense that subject to the production cost structures, there is no artificial barriers to entry in both the early and late processes; they are both contestable. In a typical equilibrium, some will choose to enter and produce early while others would wait and enter only if the "good states" of demand occur; and all early and late entrants earn only zero expected profit. Since late entries will depend on the states of demand, the model therefore will distinguish between (actual) early entrants, potential (or expected number of) late entrants and the state-dependent number of late entrants.

III. EQUILIBRIUM ENTRY

(i) Equilibrium Conditions

For the sake of exposition, we shall reduce the complexity of the problem by linearizing the random demand condition (1) as:

$$Q(p,\theta) = \theta - \beta p \ , \quad \beta > 0 \tag{1a}$$

Now, let m and $n(\theta)$ be the number of early and late entrants. Then equilibrium is defined by the vector $(m, y^o, n(\theta), z^1(\theta), p(\theta))$ where y^o and z^1 are the optimal production values previously defined, and where the following conditions are satisfied:

$$\begin{array}{ll} \text{(i)} & Q(p,\theta) = my^o + n(\theta)z^1(\theta) \qquad \text{all } \theta \\ \text{(ii)} & E[\pi^o] \le 0 \\ \text{(iii)} & E[\pi^1] \le 0 \\ \text{(iv)} & E[\pi^o] + E[\pi^1] = 0 \qquad ?? \end{array} \tag{14}$$

Condition (i) requires that the market clears in every state of the world, conditions (ii) and (iii) state that (because of free entry) neither early nor late processes will yield positive expected profits and the last condition (iv) is simply to ensure that the market does not close in <u>every</u> state of the world.

There are therefore three possible equilibrium configurations depending on demand and cost conditions: either $\{m>0,\ n(\theta)=0$ for all $\theta\}$ or $\{m=0,\ n(\theta)>0$ for some $\theta\}$ or $\{m>0,\ n(\theta)>0$ for some $\theta\}$. In other words, either all enter early with absolutely no late entrants; or none enter early so that the market, depending on state θ, is served by late entrants; or early entry may coexist with late entry. Formally, the last case is the more general one (or the interior solution), with the two preceding cases being the corner solutions depicting two extremes.

We shall focus our analysis on the general case (the interior solution). Our task will be to describe the factors that determine such an equilibrium and then to examine, how the equilibrium is affected by changing cost, and the stochastic demand parameters.

Consider first the zero expected profit condition for late entry. From the solution for z in (6) to (8) we know that every late entry state requires non-negative profit. But then the market equilibrium condition (14) (iii) requires non-positive profit for every state. Hence, it follows that every state must yield zero profit; in other words we must have $\pi^1=0$ for all θ. This, in turn implies that the equilibrium price distribution must satisfy

$$p(\theta) = p^1(\alpha^1,s^1) \quad \text{for } \theta \in [\theta^1,\bar{\theta}] \tag{15}$$

In other words, the <u>equilibrium</u> price is bounded from above by p^1; the minimum average cost of late entrants.

Using the demand function (1) we can define θ^1 more precisely by $Q(p^1,\theta^1)=my^o$ which after using the linearization (1a) gives

$$\theta^1 = \beta p^1(\alpha^1,s^1)+my^o \equiv \theta^1(y^o,m;\alpha^1,s^1,\beta) \tag{16}$$

Thus, we can now write the equilibrium price distribution as

$$p(\theta) = \begin{cases} (\theta - my^o)/\beta & \theta \le \theta^1 \\ p^1(\alpha^1, s^1) & \theta \ge \theta^1 \end{cases} \tag{17}$$

Taking its expectations we get the expected price as

$$\bar{p} = \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^1} (\theta - my^o)d\Phi(\theta) + \int_{\theta^1}^{\bar{\theta}} p^1 d\Phi(\theta)$$

which after integration by parts yields[5]

$$\bar{p} = p^1(\alpha, s^1) - \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^1} \Phi(\theta)d\theta \equiv \bar{p}(y^o, m; \alpha^1, s^1, \beta) \tag{18}$$

The full set of equilibrium conditions can now be summarized by

$$\frac{\partial J^o(\bar{p})}{\partial \bar{p}} - y^o = 0 \equiv G(y^o, m) \tag{19}$$

$$J^o(\bar{p}) - s^o = 0 \equiv H(y^o, m) \tag{20}$$

$$z^1(\theta) = \frac{\partial J[p^1, \alpha^1]}{\partial p} \qquad \theta \ge \theta^1 \tag{21}$$

$$n(\theta) = [\theta - my^o - \beta p^1(\alpha^1, s^1)]/z^1(\theta) \qquad \theta \ge \theta^1 \tag{22}$$

where $p(\theta)$ is defined in (17), $\bar{p}$ in (18) and θ^1 in (16). The system of equations (16) - (22) is a complete characterization of the equilibrium in the industry; it determines all of the endogenous variables $\{m, y^o, n(\theta), z^1(\theta), p(\theta)$ for all θ which satisfy the equilibrium conditions in (14). Note also that f expectations are based on the equilibrium $p(\theta)$ defined (17) so that firms ind have rational expectations.

(ii) Number of Late Entrants and Late Plant Size

If we examine the system of market equilibrium equations we notice from (21) that the optimal "size of late plant" (the amount z^1) is immediately determined for all $\theta \ge \theta^1$ as a function of p^1, α^1 and s^1, i.e., we get

$$z^1(\theta) = \begin{cases} \dfrac{\partial J(p^1,\alpha^1)}{\partial p} \equiv z^1(\alpha^1,s^1) > 0 & \text{if } \theta \geq \theta^1 \\ 0 & \text{if } \theta < \theta^1 \end{cases} \tag{23}$$

It should be noticed, however, that $z^1(\theta)$ is a random variable depending on θ and is, therefore, also dependent on the distribution of θ. Consequently, parameter changes which shift the distribution of θ, or θ^1, will also shift the probability distribution of $z^1(\theta)$. To emphasize this we can write the distribution of $z^1(\theta)$ as

$$z^1(\theta) = \begin{cases} z^1(\alpha^1,s^1) & p(\theta \geq \theta^1) \\ 0 & p(\theta < \theta^1) \end{cases} \tag{24}$$

Furthermore, from the convexity of J in p and the fact that $\partial p^1/\partial s^1 > 0$, $\partial p^1/\partial \alpha^1 > 0$, $\partial J/\partial \alpha^1 < 0$ we get[6]

$$\frac{dz^1}{d\alpha^1} < 0, \quad \frac{\partial z^1}{\partial s^1} > 0 \qquad \text{for } \theta \geq \theta^1 \tag{25}$$

In other words, an increase in variable costs will decrease the optimal plant size whereas an increase in fixed costs will increase it. This simply corresponds to the fact that while both increases in α^1 and s^1 will increase the minimum average costs, the first will shift the minimum to the left whereas the latter will shift it to the right.

From (24) we can write the expected size of the late plant as:

$$E[z^1(\theta)] = z^1(\alpha^1,s^1)[1 - \Phi(\theta^1)] \tag{26}$$

which depends on both the distribution of θ and θ^1.

Another variable of interest is the total amount produced by late entrants. Defining this amount to be $Z = n(\theta)z^1(\theta)$, we have from (22)

$$Z(\theta) = \begin{cases} [\theta - my^o - \beta p^1] = \theta - \theta^1 & \theta \geq \theta^1 \\ 0 & \theta < \theta^1 \end{cases} \tag{27}$$

and the expected total amount of late-plant output is given by

$$E[Z(\theta)] \equiv \int_{\theta^1}^{\bar{\theta}} (\theta - \theta^1) d\Phi(\theta) = (\bar{\theta} - \theta^1) - \int_{\theta^1}^{\bar{\theta}} \Phi(\theta) d\theta \tag{28}$$

after integration by parts.

In addition we can use (27) to get the number of late entrants as

$$n(\theta) = \begin{cases} [\theta - \theta^1]/z^1(\alpha^1, s^1) & \theta \geq \theta^1 \\ 0 & \theta < \theta^1 \end{cases} \tag{29}$$

and expected number of late entrants,

$$E[n(\theta)] = [(\bar{\theta} - \theta^1) - \int_{\theta^1}^{\bar{\theta}} \Phi(\theta) d\theta]/z^1(\alpha^1, s^1) \tag{30}$$

The number of late entrants, is a random variable whose probability distribution (and obviously also its expected value) is determined, among other things, by the distribution of θ and by θ^1. Finally, the probability of late entry (or the "degree of contestability" by late entrants) is given by $(1 - \Phi(\theta^1))$.

Before we conclude this section, notice that the late plant operates at capacity, i.e., at the minimum average cost level and furthermore, from (20) it follows that early plants also operate at capacity. This efficiency result which arises from our more general model contradicts the excess capacity result of Dreze and Gabcszewicz (1967) and Sheshinski and Dreze (1976). The reason for their excess capacity result is their specialized model which does not permit early production.

IV. COMPARATIVE STATICS

It should be noted that in what follows we assume firms are small relative to the market size so that they can be treated as a continuum of agents. This removes the awkward integer problems and discontinuities that would otherwise arise. Secondly, firms are price takers so that in the calculus that follows, a firm's output choice, whether y or z, does not affect prices but changes in equilibrium prices do affect each firm's output choices.

From (19) and (20) we can (using (18) for $\bar{p}$ and (16) for θ^1) determine y^o and m. Then, $z^1(\theta)$, $n(\theta)$ and $p(\theta)$ are given by (21), (22) and (17) respectively. Further, given y^o and m, we can also calculate expected total late production, $E[Z(\theta)]$ (using (28)), expected number of late entrants, $E(n)$ (using (30)) and the probability of late entry, or the degree of contestability by late entrants $(1 - \Phi(\theta^1)$ (given θ^1 and the probability distribution of θ).

Totally differentiating equations (19) and (20) we get (the superscripts for y^o and z^1 will now be dropped for notational simplicity):

$$\begin{bmatrix} G_y & G_m \\ H_y & H_m \end{bmatrix} \begin{bmatrix} dy \\ dm \end{bmatrix} = \begin{bmatrix} -G_{s^o}ds^o - G_{s^1}ds^1 - G_{\alpha^o}d\alpha^o - G_{\alpha^1}d\alpha^1 - G_k dk \\ -H_{s^o}ds^o - H_{s^1}ds^1 - H_{\alpha^o}d\alpha^o - H_{\alpha^1}d\alpha^1 - H_k dk \end{bmatrix} \tag{31}$$

The effects on the other variables of interest such as the total early output $Y = my$, the state-dependent number of late entrants $n(\theta)$, a late entrant's output $z(\theta)$, late entrants' total output $Z(\theta) = n(\theta) \cdot z(\theta)$, the expected number of late entrants $E[n(\theta)]$, the expected total amount of late production $E[Z(\theta)] = E[n(\theta) \cdot z(\theta)]$, the probability of late entry

$[1-\Phi(\theta^1)]$, the critical state θ^1 where late entry appears, and the equilibrium expected price $\bar{p} \equiv E[p(\theta)]$ will be derived from the other auxiliary equilibrium equations.

For the equilibrium to be stable it is required that the Routh-Hurwicz conditions are satisfied, namely

$$G_y + H_m < 0 \qquad (32)$$
$$-(G_y + H_m)\Delta > 0$$

where $\Delta = G_y H_m - H_y G_m$ is the determinant of the LHS matrix in (31).

<u>Theorem 1</u>: The equilibrium defined above satisfies the Routh-Hurwicz stability conditions (32).

<u>Proof</u>: It can be easily verified that

$$G_y = -1 < 0 \qquad (33)$$

$$G_m = J^o_{pp} \frac{\partial \bar{p}}{\partial m} = -J^o_{pp} \; \Phi(\theta^1)y/\beta < 0 \qquad (34)$$

(since $J_{pp} > 0$ from the convexity of J)

$$H_y = 0 \qquad (35)$$

$$H_m = J^o_p \frac{\partial \bar{p}}{\partial m} = J^o_p \frac{G_m}{J_{pp}} < 0 \qquad (36)$$

Thus we have $G_y + H_m < 0$ and

$$\Delta = G_y H_m = J^o_p \; \Phi(\theta^1)y/\beta > 0 \; . \qquad (37)$$

The signs in (33)-(37) have a simple intuitive interpretation. The negativity of G_y implies that the firms' stability conditions (second-order condition) are satisfied. The fact that $H_y = 0$ follows from the assumption that individual firms cannot effect the price distribution (and the mean price). The negativity of both G_m and H_m implies that an increase in m will reduce the expected price and thus will decrease both expected profits and output of each firm. $H_m < 0$ will, therefore, guarantee that when expected profits are positive, entry of new firms will push down the expected price and hence reduce expected profits back to zero.[7]

To get the comparative statics results we require, in addition to (33)-(37), the signs of all the other partial derivatives on the RHS of (31). These partial derivatives are as follows:

$$G_{\alpha^o} = J^o_{pp}\frac{\partial\bar{p}}{\partial\alpha^o} + J^o_{p\alpha^o}(\cdot) = J^o_{p\alpha^o}(\cdot) < 0 \qquad (38)(i)$$

$$G_{\alpha^1} = J^o_{pp}(\cdot)\frac{\partial\bar{p}}{\partial\alpha^1} = J^o_{pp}(\cdot)[1-\Phi(\theta^1)]\frac{\partial p^1}{\partial\alpha^1} > 0 \qquad (ii)$$

since $\Phi(\theta^1) \leq 1$, $\partial p^1/\partial\alpha^1 > 0$

$$G_{s^o} = J^o_{pp}(\cdot)\frac{\partial\bar{p}}{\partial s^o} = 0 \qquad (iii)$$

$$G_{s^1} = J^o_{pp}(\cdot)\frac{\partial\bar{p}}{\partial s^1} = J^o_{pp}(\cdot)[1-\Phi(\theta^1)]\frac{\partial p^1}{\partial s^1} > 0 \qquad (iv)$$

$$G_k = J^o_{pp}(\cdot)\frac{\partial\bar{p}}{\partial k} = -\frac{J^o_{pp}}{\beta}\int_{\underline{\theta}}^{\theta^1}\Phi_k(\theta)d\theta \qquad (v)$$

$$H_{\alpha^o} = J^o_p\frac{\partial\bar{p}}{\partial\alpha^o} + J^o_{\alpha^o} = J^o_{\alpha^o} < 0 \qquad (vi)$$

$$H_{\alpha^1} = J^o_p(\cdot)\frac{\partial\bar{p}}{\partial\alpha^1} = G_{\alpha^1}J^o_p/J^o_{pp} > 0 \qquad (vii)$$

$$H_{s^o} = J^o_p\frac{\partial\bar{p}}{\partial s^o} - 1 = -1 < 0 \qquad (viii)$$

$$H_{s^1} = J^o_p(\cdot)\frac{\partial\bar{p}}{\partial s^1} = G_{s^1}J^o_p/J^o_{pp} > 0 \qquad (ix)$$

$$H_k = J^o_p(\cdot)\frac{\partial\bar{p}}{\partial k} = G_kJ^o_p/J^o_{pp} \qquad (x)$$

It is useful to notice that, with the exception of G_{α^o}, H_{α^o} and H_{s^o}, all the partial effects of parameter changes are completely determined by the partial effect on the expected price, $\bar{p}$.

To determine the signs of G_k and H_k we have to specify the type of shift in distribution that is being considered. We consider two types of shifts in distribution: (i) A shift in the mean of θ, (ii) a mean-preserving increase in risk as characterized by Rothschild and Stiglitz (1970). Let us denote the first type of shift parameter by k^1 and the second by k^2. Consider a change in k^1. If the mean of θ increases (when k^1 increases) we must have for any θ

$$\Phi_{k^1}(\theta) = \partial[\int_{\underline{\theta}}^{\theta} d\Phi(\theta)]/\partial k^1 \leq 0 \qquad (39)$$

Thus, in (38)(v) and (x) we get that the partial effect of an increase in the mean of θ is to increase the expected price and hence $G_{k^1} > 0$, $H_{k^1} > 0$.

Consider now a mean-preserving increase in risk. Let an increase in k^2 represent a mean-preserving increase in risk. Then we have, as shown by Rothschild and Stiglitz [1970] and Diamond and Rothschild [1974], that

$$\int_{\underline{\theta}}^{\theta*} \Phi_{k^2}(\theta)d\theta \geq 0 \quad \text{all } \underline{\theta} \leq \theta* \leq \bar{\theta} \qquad (40)$$

with $\int_{\underline{\theta}}^{\bar{\theta}} \Phi_{k^2}(\theta)d\theta = 0$ to preserve the mean. Thus, in (38)(v) and (x) we get that the partial effect of a mean-preserving (preserving the mean of θ) increase in risk is to <u>reduce</u> the <u>expected price</u>. Hence, $G_{k^2} < 0$, $H_{k^2} < 0$.

Using (33)-(40) we obtain the following results (where the proofs are given in the appendix).

<u>Theorem 2</u>: An increase in late production variable costs (α^1) will have the following effects: $dy/d\alpha^1 = 0$; $dm/d\alpha^1 > 0$; $dY/d\alpha^1 > 0$; $d\theta^1/d\alpha^1 > 0$; $dz(\theta)/d\alpha^1 < 0$; $dZ(\theta)/d\alpha^1 < 0$, $dn(\theta)/d\alpha^1 = ?$ for $\theta \geq \theta^1$; $dE(Z)/d\alpha^1 < 0$; $d[1-\Phi(\theta^1)]/d\alpha^1 < 0$; $dE(n)/d\alpha^1 = ?$; $d\bar{p}/d\alpha^1 = 0$; $dp(\theta)/d\alpha^1 < 0$ for $\theta < \theta^1$, $dp(\theta)/d\alpha^1 > 0$ for $\theta \geq \theta^1$.

To interpret these results, notice that as α^1 increases the upper bound of the price distribution increases and tends to increase the mean (for given y and m). In other words, the partial effect of α^1 on $\bar{p}$ is positive. This however cannot be an equilibrium situation since the profitability of early entry increases which will attract a greater number of early entrants. This in turn will push the expected price down until it reaches its original level, with a larger number (m) of early entrants but with the same plant size (y) at minimum average costs. Consequently the total amount ($Y = my$) produced by early entrant's increases.

As for late entrants, since α^1 is higher, their optimal plant size, $z(\theta)$, is reduced (to the new minimum average cost level) for all actual late entry states $\theta \geq \theta^1$. Furthermore, since both total output supplied by early entrants, Y, and minimum average costs of late entrants, p^1, have increased, the "cutoff state", θ^1, must increase, so that the probability of late entry (or the degree of contestability by late entrants), $1 - \Phi(\theta^1)$, is lower. For the same reason, the total amount produced by all late firms, $Z(\theta)$, must be lower for any late entry state $\theta \geq \theta^1$ and hence also expected total late production, $E(Z)$, decreases ($Z(\theta)$ is lower and θ^1 higher). Since $z(\theta)$ and $Z(\theta)$ are both lower for $\theta \geq \theta^1$, we cannot tell whether the number of late entrants, $n(\theta)$, given an entry state, will be higher or lower and hence it is also impossible to determine the effect on the expected number of late entrants, $E[n(\theta)]$.

Finally looking at the effect on the price distribution itself we note that the cutoff price p^1 increases as does the cutoff state θ^1. Thus, for all $\theta \geq \theta^1$ (which is now a smaller set of states) $p(\theta) = p^1$ is higher; but for all $\theta < \theta^1$ (a larger set of states) $p(\theta)$ is lower. The mean of $p(\theta)$, however, remains the same.

Theorem 3: An increase in early production variable costs (α^o) will have the following effects: $dy/d\alpha^o < 0$; $dm/d\alpha^o < 0$; $dY/d\alpha^o < 0$; $d\theta^1/d\alpha^o < 0$; $dz(\theta)/d\alpha^o = 0$, $dZ(\theta)/d\alpha^o > 0$, $dn(\theta)/d\alpha^o > 0$ all $\theta \geq \theta^1$; $dE[Z]/d\alpha^o > 0$; $dE(n)/d\alpha^o > 0$; $d[1 - \Phi(\theta^1)]/d\alpha^o > 0$; $d\bar{p}/d\alpha^o > 0$; $dp(\theta)/d\alpha^o \begin{cases} = 0, & \theta \geq \theta^1 \\ > \theta, & \theta < \theta^1 \end{cases}$.

The effects of an increase in α^o are "almost" the opposite of the effects of an increase in α^1. As α^o increases the optimal size of the early plant is reduced and there is a tendency for expected profits to decrease (become negative) leading to exit of some early firms. But as the number of early firms decreases the expected price increases, increasing expected profit back to zero. Thus, we have a smaller number of early firms (m), each producing a smaller amount (y) at the new and higher minimum average cost.

The optimal size of the late plant, $z(\theta)$, does not change since late production conditions have not changed. However, since the share of early production is now lower in every state, the probability of late entry, $1 - \Phi(\theta^1)$, increases and so does the number $(n(\theta))$ and total production of late entrants, $Z(\theta)$, in every entry state $\theta \geq \theta^1$. Obviously, the corresponding expected values $E[n(\theta)]$ and $E[Z(\theta)]$, must also increase.

Finally, because the new price distribution has the same upper bound p^1, we have the same price as before for late entry states, $\theta \geq \theta^1$; but the set of entry states is larger (since θ^1 decreases). In contrast, for every non-late-entry state $\theta < \theta^1$ (which is a smaller set of states) $p(\theta)$ is now higher since early production, $Y = m \cdot y$, is lower in every state. Thus, the mean of the price distribution, $\bar{p}$, is higher.

<u>Theorem 4</u>: An increase in late-plant fixed costs (s^1) will have the following effects: $dy/ds^1 = 0$; $dm/ds^1 > 0$; $dY/ds^1 > 0$; $d\theta^1/ds^1 > 0$; $dz(\theta)/ds^1 > 0$, $dZ(\theta)/ds^1 < 0$, $dn(\theta)/ds^1 < 0$ for $\theta \geq \theta^1$; $dE(Z)/ds^1 < 0$; $d[1 - \Phi(\theta^1)]/ds^1 < 0$; $dE(n)/ds^1 < 0$; $d\bar{p}/ds^1 = 0$; $dp(\theta)/ds^1 \begin{cases} > 0, & \theta \geq \theta^1 \\ < 0, & \theta < \theta^1 \end{cases}$.

As s^1 increases, the upper bound of the distribution of p increases which will have a tendency to increase the expected price (for a given y and m). Therefore, the number of early entrants, m, will increase, reducing their expected profits back to zero, at the previous expected price. The optimal size of the early plant, y, is thus, unchanged (at minimum average cost level) but the number of early entrants is higher. Hence, the total amount produced by early entrants, $Y = m \cdot y$, increases.

As for late entrants, the increase in s^1 will increase the optimal size of their plant, $z(\theta)$, for every late entry state $\theta \geq \theta^1$, since by doing so fixed costs are further spread to reduce (the higher) average costs. The increase in both p^1 and $Y = m \cdot y$ will increase the "cutoff state" θ^1 and hence reduce the probab of late entry, $1 - \Phi(\theta^1)$. Furthermore, since the cut-off state increases, the total amount produced by late entrants, $Z(\theta)$, given any late-entry state, will be lower and since $z(\theta)$ is higher, so also the number of late entrants, $n(\theta)$, will be lower. This, of course, implies that the expected values, $E\{Z\}$ and $E\{n\}$, must be lower.

Finally, for any entry state, $\theta \geq \theta^1$ (which is now a smaller set of states) the price p^1 (the upper-bound) is now higher, but for every non-late-entry state $\theta < \theta^1$ (which is now a larger set of states), $p(\theta)$ is lower since total early production, Y, is higher in every state. The mean price $\bar{p}$, however, remains unchanged.

<u>Theorem 5</u>: An increase in early-plant fixed costs (s^o) will have the following effects:

$dy/ds^o > 0$; $dm/ds^o < 0$; $dY/ds^o < 0$; $d\theta^1/ds^o < 0$;

$dz(\theta)/ds^o = 0$, $dZ(\theta)/ds^o > 0$, $dn(\theta)/ds^o > 0$ all $\theta \geq \theta^1$;

$dE(Z)/ds^o > 0$; $dE(n)/ds^o > 0$; $d[1 - \Phi(\theta^1)]/ds^o > 0$;

$d\bar{p}/ds^o > 0$; $dp(\theta)/ds^o \begin{Bmatrix} = 0, & \theta \geq \theta^1 \\ > 0, & \theta < \theta^1 \end{Bmatrix}$.

As s^o increases, the optimal size of early plants, y, will increase, but since an increase in s^o reduces expected profits, the number of early entrants, m, will decrease. Since y increases and m decreases, it may seem that it is not clear what happens to total output, $Y = m \cdot y$, produced by early entrants. But note that an increase in s^o must be accompanied by a corresponding increase in expected price, $\bar{p}$, to maintain the zero expected profit condition. And also note that an increase in s^o does not affect the upper bound, p^1, of the price distribution. Consequently, $Y = my$ must be smaller in order to support a higher expected price, $\bar{p}$. Thus, the increase in s^o increases early plant size, but decreases the number of early plant firms and their total output.

As a consequence, the probability of late entry, $1 - \Phi(\theta^1)$, increases (since a lower Y reduces the cut-off state, θ^1) and hence increases the set of late entry states. Furthermore, for any late entry state, $\theta \geq \theta^1$, the number of late entrants, $n(\theta)$ increases and so does total late production $Z(\theta)$. Since both $Z(\theta)$ and $n(\theta)$ and the probability of late entry, $1 - \Phi(\theta^1)$, increase, we also have that the expected values $E[Z(\theta)]$ and $E[n(\theta)]$ increase. Furthermore, since the cost conditions of late production have not changed neither does the optimal size, $z(\theta)$, of the late plant.

Finally, because Y is now smaller, price $p(\theta)$ for $\theta < \theta^1$ must rise. In addition to this, we have noted previously that θ^1 is smaller so that the set of non-late entry states, $\theta < \theta^1$ is now smaller. Hence the expected equilibrium price, $\bar{p}$, increases.

<u>Theorem 6</u>: An increase in the mean of the distribution of θ: (increasing k^1), will have the following effects:

$dy/dk^1 = 0$; $dm/dk^1 > 0$; $dY/dk^1 > 0$; $d\theta^1/dk^1 > 0$;

$dz(\theta)/dk^1 = 0$, $dZ(\theta)/dk^1 < 0$, $dn(\theta)/dk^1 < 0$ for $\theta \geq \theta^1$;

$dE[Z]/dk^1 = ?$; $dE(n)/dk^1 = ?$; $d[1 - \Phi(\theta^1)]/dk^1 = ?$;

$d\bar{p}/dk^1 = 0$; $dp(\theta)/dk^1 \begin{cases} = 0, & \theta \geq \theta^1 \\ < 0, & \theta < \theta^1 \end{cases}$.

An increase in the mean of θ tends to increase the expected price, thus increasing expected profits and consequently attracting a larger number, m, of early entrants. This, in turn, will reduce the expected price, $\bar{p}$, back to its original level, reducing expected profits to zero, and the optimal size of the early plant, y, to its previous level. Total early production, Y, is thus higher.

Since late production conditions are unchanged, so is the optimal size, $z(\theta)$, of the late plant. But because total early production is higher, we have lower total late production, $Z(\theta)$, for any late entry state $\theta \geq \theta^1$ and hence also a lower number, $n(\theta)$, of late entrants. The increase in total early production increases the cut-off state θ^1, but, since the distribution of θ has also "shifted to the right" we do not know what happens to the probability of late entry, $1 - \Phi(\theta^1)$. Consequently, we also do not know what happens to the expected number, $E[n]$, of late entrants and expected total late production $E[Z]$.

Finally, for any late entry state $\theta \geq \theta^1$ we have the same price, p^1, as before but for any non-late entry state $\theta < \theta^1$, we have a lower price, $p(\theta)$, since total early output is higher. The expected price, $\bar{p}$, as described before, remains unchanged.

<u>Theorem 7</u>: A mean-preserving increase in risk (increasing k^2) will have the following effects

$$dy/dk^2 = 0;\ dm/dk^2 < 0;\ dY/dk^2 < 0;\ d\theta^1/dk^2 < 0;$$

$$dz(\theta)/dk^2 = 0,\ dZ(\theta)dk^2 > 0,\ dn(\theta)/dk^2 > 0 \quad \text{for all } \theta > \theta^1;$$

$$dE(Z)/dk^2 > 0;\ dE(n)/dk^2 > 0;\ d[1 - \Phi(\theta^1)]/dk^2 > 0;$$

$$d\bar{p}/dk^2 = 0;\ dp(\theta)/dk^2 \begin{cases} = 0, & \theta \geq \theta^1 \\ > 0, & \theta < \theta^1 \end{cases}.$$

Since all firms are risk neutral, an increase in risk will clearly not affect the output level, y, of an early entrant if the expected price remains unchanged. A mean-preserving increase in the risk of θ will initially, however, <u>not</u> necessarily preserve the mean of <u>$p(\theta)$</u>. In fact, since $p(\theta)$ in (17) has an upper horizontal bound for high θ, a mean-preserving spread in θ which shifts probability weights from, say, the center to the upper and lower ranges of $\theta \in [\underline{\theta},\bar{\theta}]$ will result in an initial <u>decrease</u> in the mean of $p(\theta)$. Thus, an increase in risk will <u>tend</u> to decrease the mean of p and hence reduce expected profits of early entry. This will lead to a reduction of the number, m, of early entrants, which in turn restores the expected price, $\bar{p}$, and the expected profit to their original equilibrium levels. Thus, although a mean-preserving spread of $\Phi(\theta)$ will initially reduce $\bar{p}$, the final restoration of $\bar{p}$ to its original level is a result of equilibrium adjustments. With equilibrium $\bar{p}$ unaltered so would the early plant size y since the minimum average cost is not affected. But with y unchanged and m being reduced, we have total early output Y becoming smaller.

Turning to late entrants, it is clear that since $z(\theta)$ is chosen after uncertainty is resolved, an increase in risk will not affect $z(\theta)$. It will, however, affect the probability of late entry. Since weight has been shifted away from the center of the distribution $\Phi(\theta)$ and, in addition, total early production is lower, the probability of late entry, $1 - \Phi(\theta^1)$, is higher; and at every late entry state $\theta \geq \theta^1$, actual total late production, $Z(\theta)$, is also higher. Thus, at every late entry state the number of late entrants, $n(\theta)$, will be higher, since late plant size remains at the same level as before. This, of course, implies that the <u>expected</u> total late output, $E(Z)$ and <u>expected</u> number of entrants, $E(n)$, will also be higher.

Hence, even though both early and late firms are risk neutral, their numbers are affected in an opposite manner by an increase in risk; whereas the number of early entrants, m, decreases the expected number, $E[n]$ (and actual number, $n(\theta)$, for $\theta \geq \theta^1$) of late entrants increases.

The new price distribution will have the same upper bound, p^1, which now occurs in a larger set of late entry states. But the price $p(\theta)$ for all non-late entry states $\theta < \theta^1$ (which is now a smaller set of states) is higher, since early production share is lower for all states. The mean price, $\bar{p}$, however, remains unchanged.

A summary of all the above results are given in Table I below.

	y	m	Y	θ^1	$z(\theta)$ ($\theta \geq \theta^1$)	$Z(\theta)$ ($\theta \geq \theta^1$)	$n(\theta)$ ($\theta \geq \theta^1$)	$E[Z]$	$E(n)$	$1 - \Phi(\theta^1)$	$E(p)$	$p(\theta) \vert \theta < \theta^1$	$p(\theta) \vert \theta \geq$
α^0	-	-	-	-	0	+	+	+	+	+	+	+	0
s^0	+	-	-	-	0	+	+	+	+	+	+	+	0
α^1	0	+	+	+	-	-	?	-	?	-	0	-	+
s^1	0	+	+	+	+	-	-	-	-	-	0	-	+
k^1	0	+	+	+	0	-	-	?	?	?	0	-	0
k^2	0	-	-	-	0	+	+	+	+	+	0	+	0

<u>Table I</u>

V. SUMMARY AND CONCLUSION

Table I demonstrates the cost efficiency and informational trade-offs facing the firm in this market. Greater relative efficiency in the early production process (lower levels of (s^o, α^o) and/or higher values of (s^1, α^1)) will tend to result in a larger number (m) of early entries and a larger total market share $(Y = my)$ of early entrants corresponding to a lower expected number $(E[n])$, lower expected market share $(E[Z])$, and a smaller degree of contestability $(1 - \Phi(\theta^1))$ by late entrants. On the other hand, greater uncertainty (increasing k^2) will tend to give the opposite result.

Other results of interest is that more favorable expected demand conditions (increasing k^1 or $E[\theta]$) will tend to cause more early entries (m) and total early output (Y), although the predictions on the late entry variables are not unambiguous.

It is also useful to notice how this model generalizes earlier literature on the theory of the firm under uncertainty. On one end of the spectrum, the model of Baron (1970), Sandmo (1971), and Tisdell (1963) which assume only early production is a special case of this model when all late entrants' variables (such as $n(\theta)$ and $z(\theta)$) vanish. In this model, this tends to occur the greater the relative efficiency of early production over late production (i.e., smaller (s^o, α^o) and/or larger (s^1, α^1)); and the smaller the demand uncertainty (i.e., smaller k^2) since a smaller uncertainty decreases the informational advantage of entering late. On the other end of the spectrum, the models of Dreze and Gabcszewicz (1967), Sheshinski and Dreze (1976) and Lippman and McCall (1981), which assume only late entry, are also special cases of this model whenever the early entrants' variables (such as m and y) vanish. In this model, this tends to occur whenever early production is relatively less cost efficient (i.e., higher (s^0,α^0) and/or smaller (s^1,α^1)); and whenever there is greater uncertainty (i.e., large k^2).

Another implication is that although we have not explicitly discussed the question of forward markets, this issue is implicit and is easily pursued within the model. The early entrants in this model produce y which are sold in the spot market after demand is revealed. Arbitrage by forward market agents implies that in equilibrium, the forward market price p^f must be equal to the equilibrium expected price $\bar{p}$ so that arbitraging agents earn only zero expected profits. Consequently, the equilibrium expected price $\bar{p}$ in this model can be interpreted as the equilibrium forward price. In Table I, it is interesting to note that the equilibrium expected price $\bar{p}$ is invariant to k^1 (the mean of demand distribution), k^2 (the fluctuations in the demand distribution) and also to s^1 and α^1 (the late production cost parameters). The equilibrium value of $\bar{p}$, is only affected by, and varies positively with, the early production cost parameters, s^o and α^o.

Finally, the model could be extended in several directions. One interesting extension is to model equilibrium entry decisions in a multi-product economy; this would provide a natural extension of the work of Baumol, Panzar and Willig (1982). Another direction which could be pursued is to introduce risk aversion.

FOOTNOTES

[1]An exception is a recent paper by Harris and Lewis (1983) which provides a model in which incumbency is related to informational advantages.

[2]For the sake of notational brevity, we will in most cases suppress k and write $\Phi(\theta,k)$ as $\Phi(\theta)$ whenever k is not used in discussions.

[3]See for example Hicks (1973) and Appelbaum and Harris (1977).

[4]See for example Diewert (1974).

[5]
$$\bar{p} = \frac{1}{\beta}\int_{\theta^1}^{\bar{\theta}} (\theta - my^o)\,d\Phi(\theta) + [1-\Phi(\theta^1)]p^1$$

$$= \frac{1}{\beta}\int_{\theta^1}^{\bar{\theta}} (\theta - my^o - \beta p^1)\,d\Phi(\theta) + p^1$$

$$= p^1 - \frac{1}{\beta}\int_{\underline{\theta}}^{\theta^1} \Phi(\theta)\,d\theta$$ after integration by parts and using the fact that $\theta^1 \equiv \beta p^1 + my^o$.

[6]$\frac{dz}{d\alpha^1} = Jpp\,\frac{\partial p^1}{\partial \alpha^1} + Jp\alpha^1$, but since $J(p^1, \alpha) = S^1$

$\partial P^1/\partial \alpha^1 = P^1\alpha^1 = -J\alpha^1/Jp$. Thus,

$dz/d\alpha^1 = \{Jp\,Jp\alpha^1 - J\alpha^1\,Jpp\}/Jp = -JppJ/Jp$,

using the homogeneity of J in α^1,p.

[7]See Derez and Sheshinshi (1983)

<u>Appendix</u>

<u>Theorem 2</u>:

$$\frac{dy}{d\alpha^1} = \{-G_{\alpha^1}H_m + H_{\alpha^1}G_m\}/\Delta = \{-G_{\alpha^1}G_m y + yG_{\alpha^1}G_m\}/\Delta = 0$$

$$\frac{dm}{d\alpha^1} = -G_y G_{\alpha^1} y > 0$$

Thus, $dY/d\alpha^1 > 0$ and since $dP^1/d\alpha^1 > 0$

we also have $d\theta^1/d\alpha^1 > 0$ and

$d\{1-\phi(\theta^1)\}/d\alpha^1 < 0.$

It was shown above that $dz/d\alpha^1 < 0.$

Now,

$$\frac{dZ}{d\alpha^1} = \frac{d(\theta-\theta^1)}{d\alpha^1} < 0$$

$$\frac{dE(Z)}{d\alpha^1} = -\beta \frac{\partial\theta^1}{\partial\alpha^1}\{1-\phi(\theta^1)\} < 0$$

$$\frac{dn}{d\alpha^1} = d\left(\frac{\theta-\theta^1}{z}\right)/d\alpha^1 ? \quad \text{since } \frac{\partial\theta^1}{\partial\alpha^1} > 0 \text{ but } \frac{\partial z}{\partial\alpha^1} < 0$$

$$\frac{dE(n)}{d\alpha^1} \quad ?$$

$$\frac{dp}{d\alpha^1} = d[\{\theta-my\}/\beta]/\, d\alpha^1 \quad < 0 \qquad \theta \leq \theta^1$$

$$\frac{dp^1}{d\alpha^1} \quad > 0 \qquad \theta > \theta^1$$

<u>Theorem 3</u>:

$$\frac{dy}{d\alpha^o} = \{G_m H_{\alpha^o} - G_{\alpha^o} H_m\}/\Delta = \frac{\{yc^1-c\}}{\alpha^o c^{11}}\frac{\partial \bar{p}}{\partial m}\frac{1}{\Delta} < 0$$

$$\frac{dm}{d\alpha^o} = J^o_{\alpha^o}/\Delta < 0$$

$$\frac{dY}{d\alpha^o} = \frac{dm.y}{d\alpha^o} < 0$$

$$\frac{d\theta^1}{d\alpha^o} = \frac{dY}{d\alpha^o} < 0$$

thus $d\{1-\phi(\theta^1)\}/d\alpha^o > 0$

Theorem 3 cont'd

$$\frac{dz}{d\alpha^o} = 0 \qquad \frac{dZ}{d\alpha^o} = \frac{-d\theta^1}{d\alpha^o} > 0$$

$$\frac{dn}{d\alpha^o} = \frac{d(\theta-\theta^1)/d\alpha^o}{z} > 0 \qquad \theta \geq \theta^1$$

$$\frac{dE(Z)}{d\alpha^o} = \frac{-d\theta^1}{d\alpha^o}\{1-\phi(\theta^1)\} > 0$$

Thus also $\frac{dE(n)}{d\alpha^o} > 0$

Finally,

$$\frac{dp}{d\alpha^o} = \begin{cases} -d\left(\frac{my}{\beta}\right)/d\alpha^o > 0 & \theta \leq \theta^1 \\ dP^1/d\alpha^o = 0 & \theta \geq \theta^1 \end{cases}$$

Theorem 4:

$$\frac{dy}{ds^1} = \{-G_{s^1}H_m + G_mH_{s^1}\}/\Delta = \frac{1}{\Delta}\{-G_{s^1}G_my + G_mG_{s^1}y\} = 0$$

$$\frac{dm}{ds^1} = -G_yG_{s^1}y/\Delta > 0$$

Thus $dY/ds^1 > 0$ and $\frac{d\theta^1}{ds^1} = \frac{dY}{ds^1} + \beta\frac{\partial P^1}{\partial s^1} > 0$

and hence $\frac{d\left\{\frac{1}{}\phi(\theta^1)\right\}}{ds^1} > 0$

$dz/ds^1 > 0$

$$\left.\begin{aligned} \frac{dZ}{ds^1} &= \frac{-d\theta^1}{ds^1} < 0 \\ \frac{dn}{ds^1} &= \left\{\frac{-d\theta^1}{ds^1}z - (\theta-\theta^1)\frac{\partial z}{\partial s^1}\right\}/z^2 < 0 \\ \frac{dE(Z)}{ds^1} &= -\beta\frac{d\theta^1}{ds^1}\{1-\phi(\theta^1)\} < 0 \end{aligned}\right\} \quad \theta \geq \theta^1$$

$$\frac{dE(n)}{ds^1} = \frac{1}{z}\frac{dE(Z)}{ds^1} - \frac{E(Z)}{z^2}\frac{dz}{ds^1} < 0$$

Theorem 5:

$$\frac{dy}{ds^o} = -G_m/\Delta > 0$$

$$\frac{dm}{ds^o} = G_y/\Delta < 0$$

$$\frac{dY}{ds^o} = -\frac{\phi(\theta^1)}{\beta}\frac{d\bar{p}}{ds^o} = -\frac{\phi(\theta^1)}{\beta}\frac{1}{J^o_p} < 0$$

$$\frac{d\theta^1}{ds^o} = \frac{dY}{ds^o} < 0 \qquad \frac{d(1-\phi(\theta^1))}{ds^o} > 0$$

$$\frac{dz}{ds^o} = 0$$

$$\left.\begin{aligned}\frac{dZ}{ds^o} &= \frac{-d\theta^1}{ds^o} > 0\\ \frac{dn}{ds^o} &= \frac{d(Z/z)}{ds^o} > 0\end{aligned}\right\} \quad \theta \geq \theta^1$$

$$\frac{dE(Z)}{ds^o} = -\{1-\phi(\theta^1)\}\frac{dY}{ds^o} > 0$$

and thus also

$$\frac{dE(n)}{ds^o} > 0$$

Theorem 6:

$$\frac{dy}{dk^1} = \{yG_mG_{k^1} - yG_mG_{k^1}\}/\Delta = 0$$

$$\frac{dm}{dk^1} = -yG_yG_{k^1}/\Delta > 0$$

$$\frac{dY}{dk^1} = \frac{d(my)}{dk^1} > 0$$

$$\frac{d\theta^1}{dk^1} = \frac{dY}{dk^1} > 0$$

$$d\{1-\phi(\theta^1)\}/dk^1 = -\{\phi_{k^1}(\theta^1) + \phi'(\theta^1)\frac{d\theta^1}{dk^1}\} \ ?$$

$$\frac{dz}{dk^1} = 0$$

Theorem 6 cont'd:

$$\left.\begin{aligned} \frac{dZ}{dk^1} &= \frac{-d\theta^1}{dk^1} < 0 \\ \frac{dn}{dk^1} &= \frac{dZ/z}{dk^1} < 0 \end{aligned}\right\} \quad \theta \geq \theta^1$$

$$\frac{dE(Z)}{dk^1} = \{\phi(\theta^1)-1\}\frac{d\theta^1}{dk^1} - \int_{\theta^1}^{\bar{\theta}} \phi_{k^1}(\theta)d\theta \quad ?$$

and therefore also

$$\frac{dE(n)}{dk^1} \quad ?$$

Theorem 7:

$$\frac{dy}{dk^2} \{-yG_{k^2}G_m + G_mG_{k^2}y\}/\Delta = 0$$

$$\frac{dm}{dk^2} = \frac{-yG_yG_k}{\Delta} < 0$$

$$\frac{dz}{dk^2} = 0$$

$$\frac{dY}{dk^2} = \frac{dmy}{dk^2} < 0$$

$$\frac{d\theta^1}{dk^2} = \frac{dmy}{dk^2} < 0$$

$$\frac{d(1-\phi(\theta^1))}{dk^2} > 0$$

$$\left.\begin{aligned} \frac{dZ}{dk^2} &= \frac{-d\theta^1}{dk^2} > 0 \\ \frac{dn}{dk^2} &= \frac{d(Z/z)}{dk^2} > 0 \end{aligned}\right\} \quad \theta \geq \theta^1$$

$$\frac{dE(Z)}{dk^2} = -\{1-\phi(\theta^1)\}\frac{d\theta^1}{dk^2} + \int_{\underline{\theta}}^{\theta^1} \phi_k(\theta)d\theta > 0$$

$$\frac{dE(n)}{dk^2} = \frac{dE(Z)}{dk^2}\frac{1}{z} > 0$$

REFERENCES

Appelbaum, E. and Harris, R.J. (1977), "Estimating Technology in an Intertemporal Framework; A Neo-Austrian Approach," Review of Economics and Statistics.

Baron, D. (1970), "Price Uncertainty, Utility and Industry Equilibrium in Pure Competition," International Economic Review, II, 463-480.

Baumol, W.J., Panzar, J.C. and Willig, R.D. (1982), Contestable Markets and the Theory of Industry Structure, New York: Harcourt, Brace, Jovanovich, Inc.

Diamond, P. and Stiglitz, J. (1974), "Increases in Risk and Risk Aversion," Journal of Economic Theory, 8, 337-360.

Dreze, J. and Gabszewicz, J.J. (1967), "Demand Fluctuations, Capacity Utilization and Prices," Operations in Research Verfabren, 3, 119-141.

Dreze, J. and Sheshinski, E. (1983) "On Industry Equilibrium Under Uncertainty", mimeo.

Harris, R. and Lewis, T. (1982), "Strategic Commitment Under Uncertainty with Private Information" University of British Columbia, Discussion Paper 82-24.

Hicks, J.R. (1973), Capital and Time, London: Oxford University Press.

Lippman, S.A. and McCall, J. (1981), "Competitive Production and Increases in Risk," American Economic Review, 71, 207-211.

Oi, W. (1961), "The Desirability of Price Instability Under Price Uncertainty," Econometrica, 29, 58-64.

Rothschild, M. and Stiglitz, J.E. (1970), "Increasing Risk: I. A Definition," Journal of Economic Theory, 2, 225-243.

Sandmo, A. (1971), "On the Theory of the Competitive Firm Under Price Uncertainty,' American Economic Review, 61, 65-73.

Sheshinski, E. and Dreze, J. (1976), "Demand Fluctuations, Capacity Utilization, and Costs," American Economic Review, 66, 731-742.

Tisdell, C. (1963), "Uncertainty, Instability, Expected Profit," Econometrica, 31 243-247.

Turnovsky, S.J. (1973), "Production Flexibility, Price Uncertainty and the Behaviour of the Competitive Firm," International Economic Review, 14 395-412.