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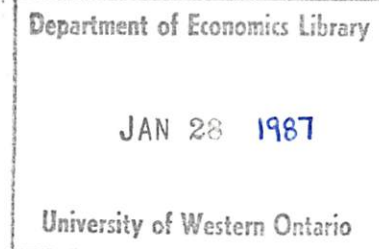
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ESTIMATION OF REDUCED FORMS  
OF RATIONAL EXPECTATION MODELS  
AND VOLCKER DEFLATION

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ESTIMATION OF REDUCED FORMS OF RATIONAL EXPECTATION MODELS AND VOLCKER  
DEFLATION

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ABSTRACT

Multivariate autoregressive moving average models are used to form the "reduced forms" of Muth's rational expectation models. An implication of the modern macroeconomic theory is that economic agents' expectations should change in the presence of major policy changes. This paper proposes a simple method for directly comparing the formulation of expectations, and illustrates it by considering the impact of a recent policy change in the US under Paul Volcker of the Federal Reserve Bank. New interpretations are based on transfer functions, "gain" calculations, Green's function matrices, solutions of difference equations as weighted sums of exponentials, one real and two complex conjugate roots of cubic polynomials, etc. The traditional distributed lag models arbitrarily assume that the gain is unity, while many time series estimators require differencing of series implicitly assuming that one of the roots is unity. Our direct estimation based on nonlinear estimation does not find either the gain or the real roots to be unity. We provide an equation for minimum mean squared error regulation, and indicate the role played by rational two step ahead speculations made by economic agents, along with changes therein emanating from the policy change.

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\* This paper borrows from Pandit and Vinod (1985), an earlier version of which was presented at the 1984 Annual meeting of the Business and Economic Statistics Section of the American Statistical Association. I thank S.M. Pandit for numerous telephone conversations, and for making his computer programs available to me. Charles Webster made many helpful comments and gave me an access to his macroeconomic data files. The remaining errors are my sole responsibility.

## 1. Introduction

The work of Lucas, Muth, McCallum, Sargent, Sims, and others has given rise to a major revision of macro economics during the past decade or so. For example, the Phillips curve used to be interpreted as showing a tradeoff between inflation ( $I_t$ ) and Unemployment ( $U_t$ ), which worsened in the late 1970's in many western countries. Lucas's (1973) model of the Phillips curve related the inflation to the rational expectation (RE) of economic agents regarding inflation, thereby suggesting the shifts in the Phillips curve by the exact amount of any increase in expected inflation, offering an explanation of the tradeoff mentioned above. An implication of the modern macroeconomic theory is that economic agents' expectations should change in the presence of major policy changes. This paper proposes a simple method for directly comparing the formulation of expectations, and illustrates it by considering the impact of a recent policy change in the US.

It is generally assumed that since the October 1979 announcement of the Federal Reserve's operating procedure, abandoning interest rate targeting in favor of money stock targeting, there has been a major policy shift in the US. Blanchard (1984) concludes that "there is no evidence of a major shift in the Phillips curve." Taylor (1984) concluded from similar data that the new policy "has become less accommodative." Haynes and Stone (1985) argue that the failures in estimating short run Phillips curves is due to a misspecification of its dynamic properties. Our methodology will be seen to be more general than in these studies, and the numerical illustration based on US quarterly data on inflation and unemployment may be of interest in its own right.

We propose a fairly general specification of the RE model structure and derive an empirically feasible "reduced form" of RE models, which replaces the subjective expectations of economic agents by conditional expectations based on appropriate econometric models. We use a data dependent systems (DDS) based autoregressive moving average vector (ARMAV) reduced form relying on certain aspects of the estimation strategy from Pandit and Wu (1983). From the known general nature (say ARMAV) of the reduced form, the method of "undetermined coefficients", Aoki and Canzoneri (1979), and suitable assumptions, we can choose the appropriate pair of RE and ARMAV models for given data. The ARMAV( $n, n-1$ ) model of Pandit and Wu (1983) generally has its autoregressive order one larger than the moving average

order, is viewed as a stochastic difference equation. The Green's function matrices  $G_j$  yield the solutions of difference equations as weighted sums of exponentials in the  $n$  roots of the autoregressive part which may appear in complex conjugate pairs, representing cyclical behavior which is often present in macro economic data. Using statistical tests we obtain a model which is more parsimonious than Pandit and Wu's ARMAV( $n, n-1$ ) by making some coefficients zero and reestimating the model.

Section 2 contains the general specification of an RE model whose reduced form is an ARMAV(4,3) model. Section 3 discusses ARMAV estimation results, where the impact of recent policy change on economic agents' REs is directly compared. Section 4 discusses the dynamics of the RE model, whereas Section 5 has our final remarks.

## 2. RE Structure and ARMAV Reduced Form

Pandit and Vinod(1985) have proposed a new methodology for studying RE models directly from ARMAV models. This section considers a slight modification of those methods. Consider a bivariate model for civilian unemployment rate  $U_t = x_{1t}$ , and inflation measured by the GNP deflator,  $I_t = x_{2t}$  based on quarterly data from the first quarter of 1949 to the fourth quarter of 1982 with 136 observations. Since Blanchard(1984) did not find any major shift till 1983, we regard 1949:1 to 1982:4 data set as representing the situation before the major policy shift will show up in economic agents' expectations. By fourth quarter of 1984 the economic agents are assumed to have perceived the policy change. Hence another data set 1949:1 to 1984:4 is also used for comparison.

Let the upper case  $X(t)$  denote an  $r \times 1$  vector containing  $x_{1t}, x_{2t}, \dots, x_{rt}$ . We have here a special case of  $r=2$  which can be generalized. Let  $X(t:t-1)$  denote the  $r \times 1$  vector of rational expectations of  $X(t)$  based on the information up to and including time  $t-1$ . In time series literature these REs are sometimes called one-step ahead least squares forecasts. In general, the formulation of REs by agents is assumed to be based on a function of three primary components:

$$X(t:t-1) = f \left[ \sum_{j=1}^p P_j X(t-j), \sum_{j=1}^p R_{j+1} X(t-j:t-j-1), \sum_{k=2}^s F_k X(t+k-1:t-1) \right], \quad (2.1)$$

The first component of (2.1) is for past data with  $P_j$  representing  $r \times r$

coefficient matrices. The  $r \times r$  matrices  $R_j$  of the second component represent the economic agents' reaction to their own experience with past rational expectations. The  $r \times r$  matrices  $F_k$  have coefficients of  $X(t+k-1:t-1)$  involving the agents' subjective  $k$  step ahead ( $k > 1$ ) optimal conditional forecasts, and represent rational speculation by agents about future values of the variables. Since the presence of rational speculation about the future makes an important difference to RE models we have distinguished between the one-step REs and  $k$  step forecasts in our notation. There is no loss of generality in using  $j=1, \dots, p$  for  $P$  and  $j=0, \dots, p$  for  $R$  matrix, because we can always choose some of the matrices to be null. Similarly there is no need to consider  $k$  step ahead forecasts made at  $t-1-q$  with  $q \neq 0$ , since the entire model is assumed to be based on information up to and including time  $t-1$ . For positive  $q$ , the terms involving  $X(t+k-1:t-1-q)$  are linear combinations of the terms included in (2.1). We assume that the agents indicate the specific components they use, i.e., indicate the appropriate choice of  $p$  and  $s$  for a given problem. Using only the past data the econometrician's task is to estimate the coefficient matrices. For example, an RE structure with  $p=3$  and  $s=2$  for the bivariate case is as follows:

$$R_1 X(t:t-1) = P_1 X(t-1) + P_2 X(t-2) + P_3 X(t-3) + R_2 X(t-1:t-2) + R_3 X(t-2:t-3) - F_2 X(t+1:t-1) \quad (2.2)$$

where we have included the additional unknown matrix  $R_1$  on the left hand side, which will be seen to be convenient. Let  $a_t$  denotes the  $2 \times 1$  vector of white noise errors at time  $t$ . A more familiar formulation of (2.2) having  $X(t)$  on the left hand side can be obtained by substituting  $X(t) - a_t$  for  $X(t:t-1)$  from the well known conditional forecasting rules, and pre-multiplying (2.2) by the inverse of  $R_1$ , assuming that  $R_1$  is nonsingular:

$$X(t) = R_1^{-1} P_1 X(t-1) + R_1^{-1} P_2 X(t-2) + R_1^{-1} P_3 X(t-3) + R_1^{-1} R_2 X(t-1:t-2) + R_1^{-1} R_3 X(t-2:t-3) - R_1^{-1} F_2 X(t+1:t-1) + a_t \quad (2.3)$$

However, it turns out that using (2.2) simplifies the comparison with the

following reduced form.

A reduced form of (2.2) or (2.3) is obtained by replacing the RE variables by their observable counterparts from the conditional forecasting rules. Compared to other empirical RE models in the literature we achieve considerable simplification by using a multivariate ARMA model as our reduced form. With some trial and error, we find that the reduced form for the bivariate model having  $p=3$  and  $s=2$  is given by the ARMAV(4,3) defined by:

$$X(t) = \phi_1 X(t-1) + \phi_2 X(t-2) + \phi_3 X(t-3) + \phi_4 X(t-4) \\ + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \theta_3 a_{t-3} \quad (2.4)$$

where the  $\theta$  and  $\phi$  matrices are all  $r \times r$  for the moving average and autoregressive terms respectively, with  $r=2$  here. We are not assuming that all matrices have nonzero terms, as will be clear by our illustration. The conditional forecasting rule  $X(t) = X(t:t-1) + a_t$  used above is generalized, Pandit and Wu(1983, Ch.5), to yield:

$$X(t+1) = X(t+1:t-1) + G_1 a_t + a_{t+1},$$

where  $G_1$  is a matrix of Green's function based on the Wold decomposition of  $X(t)$  as a sum of  $G_j a_{t-j}$  from  $j=0$  to  $j=\infty$ . Since the Green's function matrices can be recursively estimated, Pandit and Wu(1983, p.90) it can be shown that  $G_1 = \phi_1^{-1} - \theta_1$ . Now we use the assumption of rational expectations to replace  $X(t+1:t-1)$  by the observable quantity  $X(t+1) + \theta_1 a_t - \phi_1 a_t - a_{t+1}$  in (2.2) or (2.3). A novelty of this paper lies in exploiting the observability of  $X(t+1:t-1)$  based on that of  $G_1$ . Upon substituting and rearranging (2.2) we have:

$$F_2 X(t+1) = -R_1 X(t) + (P_1 + R_2) X(t-1) + (P_2 + R_3) X(t-2) + P_3 X(t-3) \\ + F_2 a_{t+1} - (R_1 + F_2 \phi_1 - F_2 \theta_1) a_t - R_2 a_{t-1} - R_3 a_{t-2} \quad (2.5)$$

where we need the matrices  $\theta_1$  and  $\phi_1$ . Hence, it is obvious that our reduced form must be of order no smaller than ARMAV(1,1), which excludes the vector

autoregressive (VAR) model used by Taylor(1984). Assuming that  $F_2$  is invertible, it can be eliminated by simply premultiplying both sides of (2.5) by its inverse. Hence, without loss of generality, we assume that the matrix  $F_2 = I$ , which provides an identifying restriction on the RE structure. If  $F_2$  is singular additional identifying information is needed. Now, by the method of undetermined coefficients we compare (2.4) and (2.5) to obtain the following relations, which give the estimates of the matrices in the RE structure (2.2) from the ARMAV reduced form (2.4).

$$F_2 = I, \quad R_1 = F_1 = -\phi_1, \quad R_2 = \theta_2, \quad R_3 = \theta_3,$$

$$P_1 = \phi_2 - R_2, \quad P_2 = \phi_3 - \theta_3, \quad \text{and } P_3 = \phi_4 \quad (2.6)$$

Using these relations and invertibility of  $R_1$  we have the following directly observable model for formulation of rational expectations:

$$X(t:t-1) = R_1^{-1} P_1 X(t-1) + R_1^{-1} P_2 X(t-2) + R_1^{-1} P_3 X(t-3) + R_1^{-1} R_2 X(t-1:t-2)$$

$$+ R_1^{-1} R_3 X(t-2:t-3) - R_1^{-1} F_2 X(t+1:t-1) \quad (2.7)$$

The important point is that we have a simple procedure for estimating fairly general RE models where agents react to certain past data points, past RE's, and two step ahead speculations made at time t-1, from ARMAV estimates; provided the order is no smaller than ARMAV(1,1), i.e., the moving average terms should not be absent. The estimation of vector models with many economic variables appears to be difficult in practice, due to multicollinearity, and conflicting indications given by statistical tests, and convergence problems associated with nonlinear maximum likelihood methods. Pandit and Wu's(1983, ch.11) computer programs give a step by step procedure which suggests a separate multivariate model for each variable, which also yields a  $\phi_0$  matrix. The vector model is obtained by premultiplying both  $\phi$  and  $\theta$  matrices by the inverse of the  $\phi_0$ , and further postmultiplying the  $\theta$  by the  $\phi_0$ . We will discuss an application to US quarterly data in the following two sections, and evaluate whether the recent macroeconomic policy shifts have indeed influenced the macro dynamics and agents' rational expectation formulation.



### 3. ARMAV estimation of US Quarterly Data

It is well known in macroeconomics literature that the aggregate demand for the economy may be specified in terms of the unemployment ( $U_t$ ) equation, and the aggregate supply (Phillips curve) may be specified in terms of an equation for the inflation variable  $I_t$ . As mentioned earlier we use US quarterly data on civilian unemployment rate and the GNP deflator (annualized percent change) for 1949:1 to 1982:4, and compare the results with 1949:1 to 1984:4 data. The recent revisions of the older data made in the 1985 Economic Report of the President have purposely not been incorporated in the first data set, since we are interested in 1982 decisions by economic agents based on the then available information. Since the bivariate system involving  $U_t$  and  $I_t$  is an important macroeconomic relation, we use it to illustrate our methodology. We are interested in understanding the formulation of rational expectations (one step ahead conditional forecasts) of these variables by economic agents, when the formation of expectations is expressed in general terms by (2.2) and (2.7). Our estimates of the  $2 \times 2$  matrices  $R_{11}^{-1} P_1$ ,  $R_{11}^{-1} P_2$  and  $R_{11}^{-1} P_3$  will indicate the influence of past values at  $t-1$ ,  $t-2$  and  $t-3$  respectively on the REs. The  $2 \times 2$  matrices  $R_{12}^{-1} R_2$  and  $R_{13}^{-1} R_3$  are attached to the one-step ahead forecasts or REs made at time  $t-2$  and  $t-3$ . Finally we estimate the role of speculation about future based on two step ahead forecasts at time  $t-1$  by  $R_{11}^{-1}$  since  $F_2 = I$ . This section is concerned with a discussion of our technique for estimation of these matrices, and comparison over the two data sets.

Time series analysis literature in Statistics mentions several techniques for estimation of multivariate ARMA models. For example, Tiao and Box(1981). or Wilson(1973). or others may be used for our purposes. We have used an approximate maximum likelihood estimation minimizing the determinant of the sum of squares and products matrix of the residuals. The software given in Pandit and Wu(1983) is used to determine the order of the dynamic system,  $n$ , in terms of solutions of difference equations as a weighted sum of  $n$  exponentials. One increases  $n$  until certain criteria are met. Bartlett has given the standard errors for autocorrelations of residuals, if we divide the observed autocorrelations by their standard errors we obtain so called "unified" autocorrelations of the residuals. We require these to be in the band (-1.96 to 1.96). We also use the F criterion to decide when to stop

increasing the value of n, Pandit and Wu(1983, pl60-164).

$$U_t = f( I_{t-1}, U_{t-1}, U_{t-2}, a_{1t-1} ) + a_{1t}, \quad (3.1)$$

$$I_t = f( U_{t-1}, \dots, U_{t-4}, I_{t-1}, \dots, I_{t-4}, a_{2t-1}, \dots, a_{2t-3} ) + a_{2t} \quad (3.2)$$

Note that our testing agrees with the fourth order specification used by Taylor(1984) in a similar bivariate model, except that it indicates a need to include moving average terms also. In any case, our methodology requires a specification of order ARMAV(1,1) or higher, if we are to have fairly general specification of the formulation of REs. One limitation may be that the computer programs in Pandit and Wu(1983) do not allow us to impose zero restrictions on subsets of coefficients. Table 1 gives our estimates of the coefficients, RSS or residual sum of squares and the F values used in determining the order ARMA(2,1) for the unemployment equation and ARMA(4,3) for the inflation equation, leading to ARMAV(4,3) from the joint tests. The unemployment equation is called the aggregate demand in Table 1, following Haynes and Stone(1985).

The presentation in Table 1 is intended to facilitate comparison of coefficients in the two time periods, the first being before the policy shift was fully believed by the economic agents, and the second after the shift. For aggregate demand and supply equations, the two data sets yield coefficients with similar signs and relative magnitudes. The decrease in the absolute value of the coefficient of  $U_{t-1}$  in the supply equation from -0.3023 to -0.0667 is balanced by an increase in the coefficient of  $U_{t-2}$ , suggesting that inflation responds to unemployment with greater sluggishness in the new policy regime. A comparison of residual variance suggests that the volatility has increased after the policy change.

Now we briefly illustrate the computations leading to the vector model in Table 1a from Table 1 for the first data set.

$$\phi_1 = \phi_0^{-1} \begin{pmatrix} 1.5162 & 0.0201 \\ -0.3023 & -0.4464 \end{pmatrix} = \begin{bmatrix} 1.517 & 0.0220 \\ -0.302 & -0.446 \end{bmatrix}$$

The matrices  $\phi_2$ ,  $\phi_3$  and  $\phi_4$  in Table 1a involve similar premultiplications. The  $\theta_1$  matrix in Table 1a is based on premultiplying the diagonal matrix (-0.1902, -1.0129) by the inverse of  $\phi_0$  and

postmultiplying by  $\phi_0$ . The matrices  $\theta_2$  and  $\theta_3$  have top diagonal term zero from Table 1.

### 3.1 Long run multipliers and the gain of transfer functions

The estimates of Table 1 may be considered in terms of polynomials in the familiar lag operator  $L$  for the dependent variable, the regressor variable, and the errors. Since the unemployment equation has zero coefficients for the lags of order 3 and 4, the polynomial is quadratic with two conjugate complex roots given at the bottom of Table 1. The inflation equation has four roots, two of which are conjugate complex and two are real. Since all roots are less than unity in absolute value, we conclude that the observed process is stationary.

In the engineering literature, the infinite sum of all weights is called the gain of the dynamic system, and when the gain is different from unity, it is interpreted as indicating a "long term" change in the equilibrium status of the dynamic system. The infinite sum can be directly computed by simply evaluating the ratio of polynomials at  $L=1$ . The gain of the inflation variable in the unemployment equation for the two data sets is respectively 0.5087 and 0.6034. This suggests that when inflation increases, (unit step input) there is a long run tendency to have excess unemployment possibly due to wage stickiness. By contrast, the gain of the unemployment variable in the inflation equation has decreased from 0.6337 for the first data set to 0.0061 for the second data set, suggesting that even if there is unemployment, it will not cause inflation in the new policy regime.

The distributed lag weights are obtained by formally dividing the error polynomial by the dependent variable polynomial. These gains for the two data sets for the unemployment equation are respectively -3.0143 and -1.9124. Similarly, for the inflation the gains associated with the error term are -4.3529 and -4.1054 for the two data sets. When econometricians use the distributed lag models based on a probability distribution which integrates to unity, they implicitly force the above mentioned gain to be unity, which is merely an unrecognized quirk of the estimation method, and may not be a realistic or justifiable constraint. The excess unemployment could not have been revealed by arbitrarily restricting the gain to be 1. The gains are not close to unity for both equations and for both data sets.

### 3.2. Efficiency of Implicit Variance Minimizing Regulation

In this subsection we consider the Phillips curve trade off between unemployment and inflation from a novel perspective. We assume that the economic decision makers wish to control the dependent variable (=output) by choosing the regressor as an instrument (=input). It is well known that the minimum mean squared error (MMSE) forecast also minimizes the variance of the forecast error, Pandit and Wu (1983, ch.5 and sec. 11.3). For the sake of discussion we consider a forecast-error-variance-minimizing (MMSE) regulation of inflation by injecting unemployment. Later we will consider the converse. All systematic error in the output is eliminated by manipulating the input control, and the only remaining error is due to the unavoidable random noise. Hence the model after regulation satisfies:

$$I_{t+1} = a_{2t+1} \quad (3.3)$$

Since the lag between the output and the input is 1, the lowest achievable variance of the output after successful regulation by the MMSE strategy is the variance of  $a_{2t}$  which from Table 1 is  $RSS/135=2.901$ . The actual variance of  $I_t$  series for the first data set is 10.02 which describes the present unregulated situation. If the goal is to minimize the variance of the output the efficiency of present unregulated situation, compared to the hypothetical MMSE regulated system is measured by the ratio of the minimum variance 2.901 to the actual variance 10.02, or 29 percent. For the second data set the efficiency has increased to 35.56% in the second data set. This suggests that the policy shift has achieved greater efficiency in controlling inflation. For the unemployment equation the efficiency has decreased from 4.24 to 3.64 percent for the second data set, as is indicated in the bottom row of Table 1.

### 4. Dynamics of Rational Expectation Models

In this Section we analyze the dynamics of the rational expectations model (2.1) and (2.3) based on the ARMAV estimates from (2.4). Using the numerical values given in Table 2 we have two equations for the two components of  $X_t$  (rounded to three digits to the right of the decimal point), which express the REs in terms of various (P, R, and F) components as in (2.7). For example, we write the first equation for the first data set as:

$$U(t:t-1) = 0.392U_{t-1} - 0.031I_{t-1} - 0.012U_{t-2} - 0.020I_{t-2} - 0.011U_{t-3} - 0.008I_{t-3} \\ + 0.027I(t-1:t-2) - 0.666U(t+1:t-1) - 0.033I(t+1:t-1) \quad (4.1)$$

Similarly, we can write all other equations from the information in Table 2.

Now, we can express the full dynamics of the RE model using the lag operator  $L$ , in terms of the transfer functions with respect to: (i) the past data, (ii) the past rational expectations and (iii) the rational speculations about the future (i.e., linear least squares forecasts). For (4.1) the polynomials are written as follows:

$$U_t = [(-0.031 - 0.020L - 0.008L^2) / D(L)] I_{t-1} + (0.027 / D(L)) I(t-1:t-2) \\ + (-0.666 / D(L)) U(t+1:t-1) + (-0.033 / D(L)) I(t+1:t-1) \quad (4.2)$$

where  $D(L) = 1 - 0.392L + 0.012L^2 + 0.011L^3 = (1 - h_1L)(1 - h_2L)(1 - h_3L)$ , with roots  $h_1$ ,  $h_2$  and  $h_3$ , and where the expressions having the polynomial  $D(L)$  in the denominator are the transfer functions.

For stable series one can judge the importance of each component on an equal footing by considering the gain of transfer functions. As we have noted above in section 3.1 the gain can be readily obtained by substituting  $L=1$  in the transfer functions. The gain represents the eventual response of a system to step input, i.e., uniformly maintained unit input. The gains from the transfer functions of (4.2) are reported in Table 3 for the unemployment equation over the two data sets. Since the absolute value of one of the roots of the inflation series is larger than unity for both data sets, it should be noted that it does not make sense to compute the gain for such unstable series by replacing the operator  $L$  by unity. The gains of all, except  $-1.0555$  for the transfer function associated with  $U(t+1:t-1)$ , are relatively small. This shows that unemployment is significantly affected by its dynamic response to speculation about its own future. The dynamic properties of the unemployment equation (4.2) are represented by the roots  $h_1 = -0.1337$ ,  $h_2 = 0.2628 + 0.1149i$ , and  $h_3 = 0.2628 - 0.1149i$ , (where  $i$  is imaginary root of  $-1$ ) of the operator polynomial  $D(L)$ . The accuracy of the roots has been verified by comparing their sum, their products taken two

and three at a time, with the real coefficients of the polynomial.

For both unemployment and inflation equation, and for both data sets our Table 3 reports: (a) the cubic equation  $D(L)$ , (b) three roots of the cubic, two of which are complex conjugate, and (c) the dynamic "period" in quarters, defined later in the following paragraph. The complex conjugate roots explain the cyclical nature of economic agents' rational expectations regarding unemployment and inflation in the US. Since the product of the complex roots in each case is fractional, it indicates that the sinusoidal components of all the series are damped, not explosive. The real root of the inflation equation is 2.5951 in the first period and 2.0818 in the second, suggesting exponential nature of the inflation in the data period covered, except that, there has been a small decrease in exponential component of inflation.

The complex conjugate roots are used to compute the "period" obtained as follows:

$$\text{period} = 2\pi / \cos^{-1} \left[ \frac{(h_1 + h_2)/2}{(h_1 h_2)^{1/2}} \right] \quad (4.3)$$

There is a slight increase in the period from 2.71 to 3.48 quarters for inflation, and from 15.25 to 20.06 quarters for unemployment.

Large gain associated with two step ahead (speculative) forecasts in Table 3 for the unemployment equation for both data sets seems to be consistent with Lucas' famous criticism of traditional macro econometric models. We find that it is rational for economic agents to give weight to their speculation about future unemployment. Of course, our illustrative model is incomplete for a formal test of Lucas' critique, because it does not also include many relevant macro economic variables including money supply.

## 5. Final Remarks

Using economic theory and intuition, we start with a fairly general specification of the mechanism of the formulation of rational expectations. This paper uses the stochastic difference equation methods to estimate an ARMAV model, then compares the coefficient matrices of the estimated model with those of the "reduced form" of the specified rational expectation model. With appropriate assumptions we choose a pair of RE model and its

ARMAV reduced form, and estimate the RE model using equations (2.6).

The dynamics of rational expectations models are explicitly studied, using transfer functions, their gains, one real and two complex conjugate roots of appropriate cubic polynomials, etc. The traditional distributed lag models arbitrarily assume that the gain is unity, while many time series estimators require differencing of series for successful estimation, which amounts to the arbitrary assumption that one of the roots is unity. Our direct estimation based on nonlinear estimation do not find either the gain or the real roots to be unity in any one of the four cases studied here.

We provide an equation for minimum mean squared error regulation, and indicate the crucial role played by rational two step ahead speculations made by economic agents. It is obvious that our methods reveal much greater insight about unemployment and inflation data than most published studies which use similar data. By studying the US data in two data sets we have considered the impact of recent change in macro economic policy on the dynamics of various series. For example, the rational expectations of market agents reveal that they have reduced the exponent associated with continuing inflation. We have explicitly studied the changes in formation of rational expectations in the two periods. It should be noted that our methods are more general than the particular application might suggest, and deserve to be studied further. We have experienced difficulties in implementing these methods for large models with economic data, possibly due to multicollinearity.

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Table 1. Estimates of Aggregate Demand (Unemployment) and Supply (Inflation)

Variable	Aggregate Demand		Aggregate Supply	
	1949.1--1982.4:1949.1--1984.4		1949.1--1982.4:1949.1--1984.4	
U(t)	Dependent Variable			
I(t)	0.0042	0.0084	Dependent Variable	
U(t-1)	1.5162	1.5163	-0.3023	-0.0667
I(t-1)	0.0201	0.0235	-0.4464	-0.4238
U(t-2)	-0.5793	-0.5859	-0.1996	-0.4683
I(t-2)	0.0078	0.0101	-0.0595	-0.0403
U(t-3)			0.3913	0.2548
I(t-3)			0.8261	0.8004
U(t-4)			0.3663	0.2827
I(t-4)			0.2762	0.2558
a(t-1)	-0.1902	-0.1331	-1.0129	-0.9449
a(t-2)	0	0	-0.9113	-0.8004
a(t-3)	0	0	0.1678	0.0707
RSS	14.8404	15.5544	391.635	484.662
F	8.4963	9.6847	8.1230	4.3452
Complex Roots	0.7581	0.7582	-0.5059	-0.4991
	+/-	+/-	+/-	+/-
	0.0680i	0.1054i	0.8203i	0.8058i
Real Roots	none	none	0.8969	0.8932
-			-0.3315	-0.3187
Efficiency	4.24	3.64	28.95	35.56

Notes: U(t-1) is  $U_{t-1}$ , and similarly for others, except that a(t-1) refers to a subscript t-1 on  $a_1$  or  $a_2$  depending on whether it is the Unemployment or Inflation equation. The statistical testing follows Pandit and Wu(1983). Section 3.2 defines the efficiency as 100 times residual variance divided by the observed variance of the series.

Table la. Estimates of Multivariate (Vector) AR and MA matrices after transformation by  $\phi_0$

Matrix 1949.1--1982.4:1949.1--1984.4				Matrix 1949.1--1982.4:1949.1--1984.4					
$\phi_1$	1.517	0.022	1.520	0.027	$\phi_2$	-0.578	0.008	-0.582	0.010
	-0.302	-0.446	-0.468	-0.424		-0.200	-0.059	-0.468	-0.040
$\phi_3$	-0.002	-0.004	-0.002	-0.007	$\phi_4$	-0.002	-0.001	-0.002	-0.002
	0.391	0.826	0.255	0.800		0.366	0.276	0.283	0.256
$\theta_1$	-0.190	0.003	-0.133	0.007	$\theta_2$	0	0.004	0	0.007
	0	-1.013	0	-0.945		0	-0.911	0	-0.800
$\theta_3$	0	-0.001	0	-0.006	$\gamma_0$	0.110	-0.012	0.109	-0.029
	0	0.168	0	0.707		-0.012	2.901	-0.029	3.389

Notes: The  $\phi_0^{-1}$  times AR and MA matrices from Table 1 are reported, except that the MA matrices are further post multiplied by  $\phi_0$ . The matrices in bottom right corner are the variance covariance matrices of residuals, Pandit and Wu(1983 p433). Double precision computations based on more accurate coefficients than in Table 1 are rounded here for presentation.

Table 2. Estimates of Rational Expectation Model of Equation (2.7)

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Matrix 1949.1--1982.4:1949.1--1984.4:Matrix1949.1--1982.4:1949.1--1984.4

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$R_1^{-1} P_1$	0.392	-0.031	0.411	-0.035	$R_1^{-1} P_2$	-0.012	-0.020	-0.009	-0.003
	-0.712	1.929	-1.559	1.832		0.885	1.488	0.612	0.225
$R_1^{-1} P_3$	-0.011	-0.008	-0.011	0.010	$R_1^{-1} R_2$	0.0	0.027	0.0	0.030
	0.828	0.624	0.679	0.614		0.0	-2.060	0.0	-1.922
$R_1$	-1.517	-0.022	-1.520	-0.027	$R_1^{-1} F_2$	-0.666	-0.033	-0.671	-0.043
	0.302	0.446	0.468	0.424		0.451	2.262	0.741	2.407

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Notes: The  $R_1^{-1}$  times matrices  $P_1$  to  $P_3$  contain coefficients associated with the past data,  $R_2$  contains those with one step ahead forecasts (REs) at time  $t-2$ , and  $F_2$  contains those with two step ahead (future speculation) forecasts made at time  $t-1$ . Double precision computations based on more accurate coefficients than in Table 2 are rounded here for presentation.

Table 3. Dynamics of Rational Expectation Model,  
Cubics similar to D(L) of (4.2), and computation of the gain

Equation	1949.1--1982.4	1949.1--1984.4
Unemployment	$1 - 0.392L + 0.012L^2 + 0.011L^3$	$1 - 0.411L + 0.009L^2 + 0.011L^3$
cubic roots	-0.1337, 0.2628 +/- 0.1149i	-0.1340, 0.2725 +/- 0.0883 i
Period	15.25 quarters	20.06 quarters
Inflation	$1 - 1.929L - 1.488L^2 - 0.624L^3$	$1 - 1.832L - 0.225L^2 - 0.614L^3$
cubic roots	2.5951, -0.3330 +/- 0.3599 i	2.0818, -0.1249 +/- 0.5285 i
Period	2.71 quarters	3.48 quarters
Unemployment	-0.094 I(t-1), 0.043 I(t-1:t-2), -0.046 I(t-1), 0.049 I(t-1:t-2),	
gain	-1.056 U(t+1:t-1), -0.052 I(t+1:t-1)	-1.102 U(t+1:t-1), -0.706 I(t+1:t-1)

Notes: Some polynomial coefficients are rounded in Table 2. The roots are complex conjugate, indicated by +/- with an i for the imaginary root of -1. Since the inflation process is unstable, with the absolute value of one root larger than unity, its gain is not reported.