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INFLATION AND UNEMPLOYMENT

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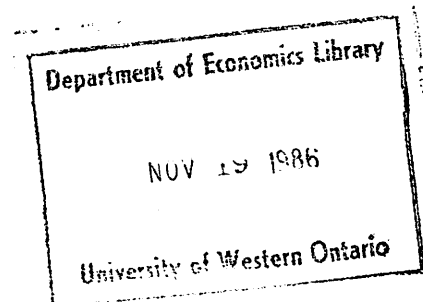
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I. INTRODUCTION

There has been a long-standing debate within the economics profession concerning the effects of inflation and its impact on unemployment. Following the statistical work of Phillips (1958), a growing belief developed that unemployment and inflation had a negative influence on each other. At the time it seemed a short step to conclude, as did many in the profession, that the new-found Phillips curve was a structural relationship, and consequently offered a 'menu of choice' between the levels of unemployment and inflation, which could be exploited by the policymaker. After some years of experimentation with this relationship, disillusionment began to set in as both unemployment and inflation rose. The theoretical underpinnings of the Phillips curve trade-off began to be questioned by Friedman (1968) and Phelps (1970), and received their ultimate blow with the work of Lucas (1972). Friedman (1977) eyeing emerging data at the time, in fact, suggested that the operational Phillips curve was upward sloping. He conjectured that inflation has disruptive effects on economic activity with high rates of inflation going hand-in-hand with high unemployment. To date, the theoretical links between inflation and unemployment seem to have been difficult to ascertain.

Perhaps as a consequence of the perceived failure of existing models, which emphasized the dominant role of monetary factors as the driving force behind cyclical fluctuations, recently a literature has developed which suggests that business cycles are primarily a real phenomenon (see, for example, Kydland and Prescott (1982) and Long and Plosser (1983)). This line of research has tended to de-emphasize the role that monetary factors play in causing economic fluctuations. As a result, it has not been oriented toward seeking an explanation of the observed anomalous correlations between inflation, and unemployment or output.

The present paper attempts to capture part of the spirit of the real business cycle literature in a model in which money is introduced. A stochastic general equilibrium model is constructed which is capable of examining the covariance properties between unemployment and inflation, both conditioned and unconditioned upon exogenous factors such as the current growth rate of the money supply, the level of productivity, etc. The model exhibits some interesting features. First, it is shown that natural forces in the economy may operate to generate an endogenous negative relationship between unemployment and inflation. This is interesting since it has been argued that real business cycle models are incapable of explaining the procyclical nature of prices (see Lucas (1977)). Second, despite this apparent negative association between unemployment and inflation, the actual trade-off which the policymaker confronts is one in which these two variables are positively related. Also, in contrast to the work of Lucas (1972), the model employed does not rely on agents' misperceptions about the current rate of monetary expansion.

The current work borrows ingredients from several sources. First Aschauer (1980), Aschauer and Greenwood (1983), and Carmichael (1985 a,b) have analyzed the deleterious impact that inflation can have on equilibrium employment. Their papers build on Stockman (1981) which investigates the adverse effect that inflation can have on an economy's steady-state capital stock. The current study models the disruptive effects of inflation in a similar fashion. Second, a role for real shocks as a driving force behind aggregate fluctuations is introduced in a manner similar to that of Kydland and Prescott (1982), and Long and Plosser (1983). Third, King and Plosser (1984) suggested that some components of the money supply may react

endogenously to real disturbances. This observation is incorporated in the model of this paper. Fourth, drawing the link in equilibrium models between shifts in unemployment (rather than employment) on the one hand, and monetary and nonmonetary disturbances on the other, is tenuous at best. The present study attempts to overcome this obstacle by introducing a nonconvexity into agents' labor supply decisions along the lines proposed by Rogerson (1985). Such nonconvexities are capable of generating unemployment in equilibrium models, and Rogerson (1985) shows how they can often be easily handled by a simple extension to the standard competitive equilibrium construct.

The remainder of the paper is organized as follows: In Section II the underlying physical environment of the economy is described. The representative agent/firm's optimization problem and decision rules are presented in Section III. Next, Section IV characterizes the model's general equilibrium. The joint behavior of unemployment and inflation in response to monetary and real shocks is investigated in Section V. A discussion of the results and their implications is contained in Section VI.

II. THE PHYSICAL ENVIRONMENT

Consider the following model of a closed economy inhabited by a continuum of identical agents distributed uniformly over the interval $[0,1]$. An agent's goal is to maximize the expected value of his lifetime utility as given by

$$E\left\{\sum_{t=0}^{\infty} \beta^t [U(c_t) + V(l_t)]\right\} \quad \beta \in (0,1)$$

where β is the agent's (constant) subjective discount factor, and c_t and l_t denote his period- t consumption and work effort, respectively.¹ $U(\cdot)$ is increasing, $V(\cdot)$ is decreasing, and both $U(\cdot)$ and $V(\cdot)$ are strictly concave

functions. Each agent is additionally endowed with K units of capital in every period, which he chooses to supply inelastically.

The aggregate output of a given market in period t is given by the constant-returns-to-scale production process

$$y_t = f(\phi_t l, K, \lambda_t)$$

with $\phi_t l$ and K representing the aggregate amount of labor and capital employed respectively, and λ_t representing a technology shock which is known at the beginning of period t . The value of λ_t , which is realized at the beginning of period t , is generated by a stochastic process whose distribution function is denoted by $G(\lambda_t)$ and defined on the domain $Z = [\underline{\lambda}, \bar{\lambda}]$. It will be assumed that the technology factor has a direct and significant influence on the marginal product of labor. Given the behavior of agents with respect to supplying capital, note that $K = \int_0^1 K ds$. Capital has been added to the model solely to produce a constant-returns-to-scale technology together with a diminishing marginal product of labor. The constant-returns-to-scale assumption combined with the assumed diversified ownership of capital work to guarantee that a competitive equilibrium will exist.

Each agent has two sources of income which he receives at the beginning of the period. First, he earns a wage-cum-dividend payment associated with the operation of a firm running the production process $f(\cdot)$ in the previous period. Second, he receives a transfer payment in the amount of τ_t from the government. The individual can hold this period- t income in either of two assets: bonds, b_t , or real cash balances, m_t . A real bond purchased for one unit of consumption in period t pays back the cash equivalent of $(1+r_t)$ units of the consumption good in period $t+1$, so that r_t is the period- t (possibly

state-contingent) real interest rate. All transactions in the model must be effected using currency.² Thus, for instance, if in period t the individual buys c_t units of the consumption good this must be purchased using currency from his period- t holdings of real cash balances, m_t .

Each agent is assumed to face a nonconvexity in his labor supply decision. Specifically, he either works the fixed amount $l > 0$ or does not work at all.³ It has long been recognized that the Pareto optimality property of a competitive equilibrium is not necessarily destroyed by the presence of nonconvexities in either agents' tastes or firms' production technologies--see Rothenberg (1960). As is discussed in Rogerson (1985), when there are nonconvexities in agents' choice sets, however, economic welfare in a competitive equilibrium can be potentially improved by extending individuals' choice sets to include the possibility of a lottery over their consumption, labor and asset holding allocations. The lottery convexifies their choice sets, so to speak. Specifically, imagine the production process as being owned by a firm who offers the agent an income-employment contract of the following form: In each period t the firm and the agent agree on a probability ϕ_t^W that the agent will be called to work l units. From an agent's point of view ϕ_t^W is then a choice variable. An individual who is chosen to be employed receives a wage-cum-dividend payment from the firm designed to allow the agent to undertake c_t^W units of consumption, buy b_t^W units of bonds, and acquire m_t^W units of real currency. The probability of being unemployed is $\phi_t^U = (1 - \phi_t^W)$, and in this state the agent receives a wage-cum-dividend payment to provide for c_t^U units of consumption, b_t^U units of real bonds, and m_t^U units of real currency.

To digress for a moment in order to focus on the issues surrounding the lumpiness in each agent's labor supply decision, consider the static, deterministic, nonmonetary version of the economy currently under study

portrayed in Figure 1. To begin with, in a world where agents are not constrained in their labor supply decision, the outcome of decentralized competitive decision-making would be represented in standard fashion by the point F. As can be seen, each agent consumes the amount c^* , works l^* , and enjoys the level of welfare U^* . Now suppose that an agent must either work the amount l^N or not at all. One possible competitive equilibrium for such an environment is illustrated by the point N where all agents work the amount l^N , consume c^N , and realize the welfare level U^N . Now let the following lottery be offered to agents: An agent will work the amount l^N with probability ϕ or not work with probability $(1-\phi)$, but in either case he will be provided with the constant level of consumption c^L [equal to per capita production, $f(\phi l^N, K)$]. As is easy to deduce, if an agent is employed he obtains a welfare level of U^W , while if he is unemployed he realizes a welfare level of U^U . The expected utility level associated with these two events is shown by $U^L [= \phi U^W + (1-\phi) U^U]$. Note that this ex ante level of welfare, U^L , is superior to that obtained in the economy without the lottery, U^N , but inferior to that arising in the economy without the nonconvexity, U^* . A numerical example corroborating the existence of equilibria of the type illustrated in Figure 1 is provided in Appendix B.

There is a government in this economy whose purpose is to provide transfer payments, τ_t , in each period t to its citizens via money creation. Its period- t budget constraint is

$$P_t \tau_t = M_{t-1}^S (\mu_t - 1), \quad (\text{with } \mu_t \equiv M_t^S / M_{t-1}^S)$$

where P_t is the period- t nominal price level, M_t^S is the nominal supply of money in this period, and μ_t is the gross growth rate of the nominal money supply between periods t and $t-1$. It is assumed that the value of μ_t is generated by a stochastic process whose transition distribution function is

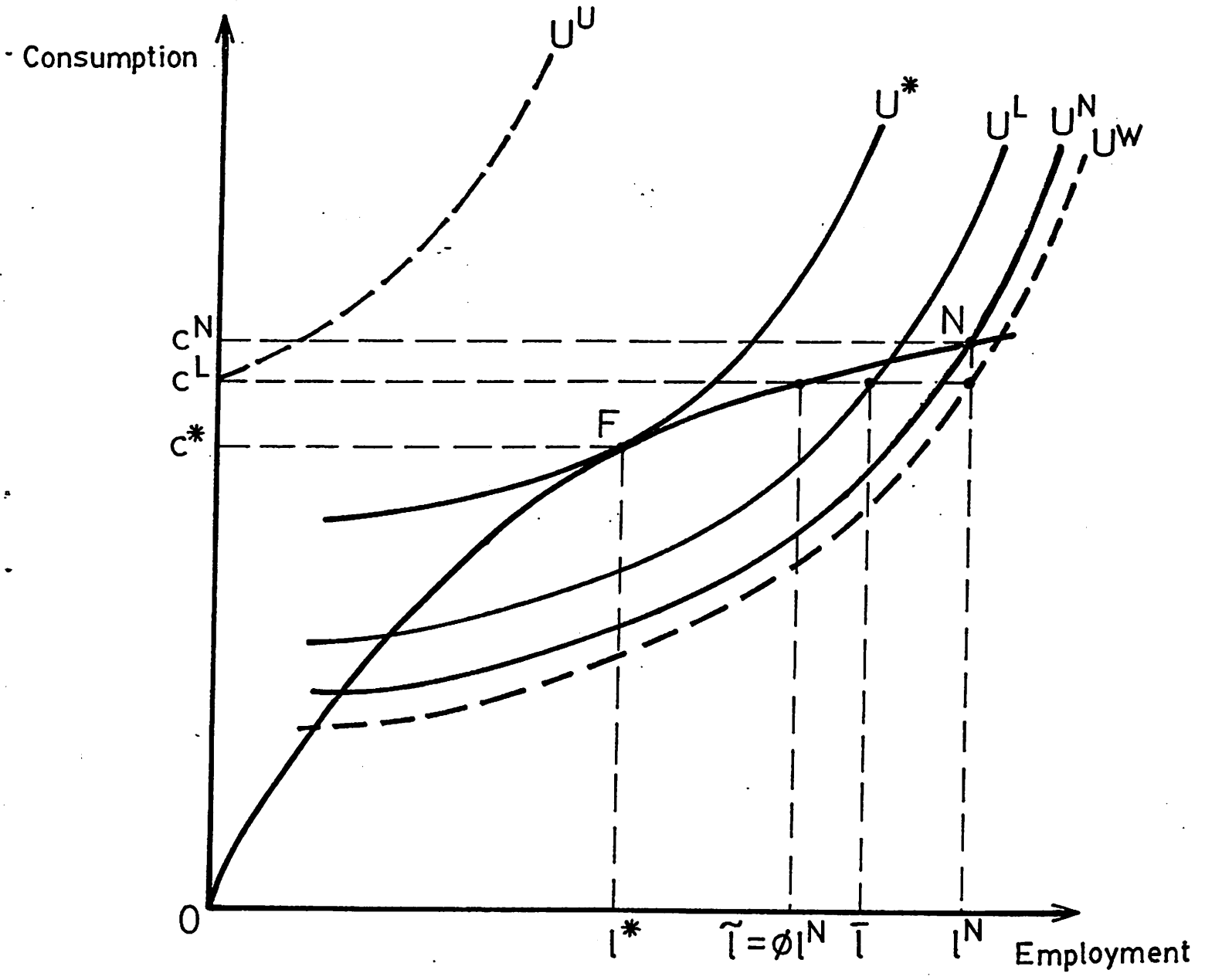


Figure 1

denoted by $F(\mu_t | \mu_{t-1})$. This distribution function will be taken to have the property that the higher the period- $t-1$ growth rate of money, the greater the probability that the period- t money supply growth rate will be big, so that formally $F_2(\cdot | \cdot) < 0$. This amounts to requiring that the money supply's growth rate exhibits positive autocorrelation. Last, the realizations for the μ_t 's are assumed to be drawn from the time-invariant set $Q = [\underline{\mu}, \bar{\mu}]$.

Before proceeding to an analysis of private sector decision-making, a brief statement about the timing of transactions will be given. The circular flow of money in the model closely parallels that found in the paradigms developed by Helpman (1981), and Lucas and Stokey (1984)--see also Aschauer and Greenwood (1983). An individual enters a period with a certain amount of currency left over from the previous period. At the beginning of this period the individual receives in cash an earnings-cum-dividend payment from the firm arising from its sales during the previous period. At this time the agent knows the value of all period- t exogenous variables including the current growth of the money supply and his state of employment for the period. He then enters the asset market, redeems the bonds he purchased during the previous period for money, and acquires new holdings of bonds (which may be negative) and money. This permits the agent to borrow money on the bond market. During the remainder of the period, the agent uses his holdings of currency to buy his consumption quantities of output, and either works or stays at home depending on his state of employment. Note that while the individual's consumption within the period is constrained by the amount of money he chose to hold at the beginning of the period, at that time he was free to pick with full current information the quantity of money required to finance his planned consumption. Within the period the individual is spending

currency by purchasing commodities while the firm is accumulating cash via its sales of goods. The firm holds its sales income until the beginning of period $t+1$, at which time these proceeds are distributed via wage-cum-dividend payments to individuals who have entered this period with any unspent cash balances leftover from the previous period. The whole process then starts over again.

Finally, a list of assumptions governing the specification of tastes, technology, and the stochastic structure of the environment in the model, whose content for the most part is of secondary importance for the economic interpretation of the results subsequently obtained but which is required for the formal technical analysis, is provided in Appendix A.

III. PRIVATE SECTOR DECISION-MAKING

The decision-making of consumer-workers and firms in competitive equilibrium can be summarized by the outcome of the following "representative" agent/firm's programming problem (P1) shown below with the choice variables being c_t^j, m_t^j, b_t^j for $j=w, u$, and $\phi_t^w = (1-\phi_t^u)$.

$$W(a_t, s_t) = \max \left\{ \sum_j \phi_t^j [U(c_t^j) + V(\ell_t^j)] + \beta \int_Q \int_Z W(a_{t+1}, s_{t+1}) dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) \right\} \quad (P1)$$

s.t.

$$\sum_j \phi_t^j [m_t^j + b_t^j] = \frac{P_{t-1}(s_{t-1})}{P_t(s_t)} f(\phi_{t-1}^w, \ell_{t-1}, K, \lambda_{t-1}) + \tau_t + \sum_j \phi_{t-1}^j \left[\frac{P_{t-1}(s_{t-1})}{P_t(s_t)} \right. \\ \left. \cdot (m_{t-1}^j - c_{t-1}^j) + (1+r_{t-1}(s_t, s_{t-1})) b_{t-1}^j \right] \equiv a_t \quad (1)$$

$$c_t^j \leq m_t^j \quad \forall j = w, u \quad (2)$$

where $s_t \equiv (\mu_t, \lambda_t)$ denotes the economy's state vector.⁴ The first constraint is the representative agent/firm's budget constraint. The left-hand side of this equation illustrates his uses of financial wealth at the beginning of period t , which take the form of acquisitions of money and bonds for the two states of nature, while the right-hand side portrays his sources of funds at this time, which in total value are defined to be a_t . The second equation represents the agent/firm's cash-in-advance constraints and states that period- t consumption in state j cannot exceed his period- t money holdings in this state.

The following interesting facts characterize the upshot of the above optimization problem.⁵ First, for any given period t , the marginal utility of consumption is equalized across the employed and unemployed states so that

$$U'(c_t^w) = U'(c_t^u) \quad \forall t \quad (3)$$

This implies that the lottery provides equal consumption for those working and not working, i.e., $c_t^w = c_t^u \equiv c_t$. Thus, the income-employment contract offered by the firm to consumer-workers effectively provides for unemployment insurance benefits that allow individuals to maintain their consumption in the unemployed state.⁶

Next, so long as the implicit nominal interest rate, i_t , in the economy is positive, the cash-in-advance constraints (2) will always hold with equality. From the above maximization problem, it is straightforward to deduce that the implicit nominal interest rate facing the representative agent/firm is defined by

$$1+i_t = \frac{U'(c_t)/P(s_t)}{\beta \int_Q \int_Z (U'(c_{t+1})/P(s_{t+1})) dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t)} \quad (4)$$

The numerator of this expression is merely the utility worth of a dollar today, while the denominator is the expected utility worth of a dollar tomorrow. The ratio, therefore, measures the relative price of a current dollar in terms of future dollars which is of course the gross nominal interest rate. So long as $i_t > 0$, which as in Lucas (1982) will be assumed henceforth, bonds dominate money as an abode of purchasing power and consequently individuals will not hold money across adjacent periods. Thus, the firm is the only economic actor which holds cash balances across periods. This "corner" could be incorporated into the model in a manner similar to that of Lucas and Stokey (1984). Since this problem is not germane to any of the issues being addressed here, the assumption that $i_t > 0$ for all t does not seem particularly severe.

Finally, the gist of the representative agent/firm's consumption/employment decision-making can be encapsulated into the following efficiency condition:

$$[V(0) - V(1)] = \beta \int_Q \int_Z U'(c_{t+1}) \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \frac{\partial f(\phi_t^W, K, \lambda_t)}{\partial \phi_t^W} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) \quad (5)$$

This is the central equation of the model. The left-hand side illustrates the expected utility loss agents realize when the probability of each agent working, ϕ_t^W , increases. This is simply the difference in utility between not working and working. The right-hand side shows the expected utility gain, through the rise in consumption, associated with a rise in this probability. Specifically, as ϕ_t^W becomes bigger a greater fraction of the population is working. As a consequence, output in period t rises by $\partial f(\phi_t^W, K, \lambda_t) / \partial \phi_t^W$ and the firm's period- t nominal sales rise by $P_t(s_t) \partial f(\phi_t^W, K, \lambda_t) / \partial \phi_t^W$. As has been mentioned, the income derived from these sales is not distributed by the firm

to agents until the beginning of period $t+1$ and at that time will be worth $[P_t(s_t)/P_{t+1}(s_{t+1})] \partial f(\phi_t^w, K, \lambda_t) / \partial \phi_t^w$. The expected discounted utility value of these extra earnings to individuals is given by the right side of equation (5).

This equation also displays another notable feature. The right-hand side displays the uncertainty associated with technology, λ_t , and money growth, μ_t , shocks. This is aggregate non-diversifiable uncertainty. The left-hand side reflects the uncertainty associated with the employment lottery. This is contrived uncertainty which agents themselves manufacture. Because of the nonconvexity in labor supply, it is optimal for agents to construct uncertainty which in the aggregate is diversifiable.

IV. THE MODEL'S GENERAL EQUILIBRIUM

In the model's general equilibrium the goods and money markets must clear each period. Thus, $\sum_j \phi_{ct}^j = f(\phi_t^w, K, \lambda_t)$ and $\sum_j \phi_{mt}^j = M_t^S / P_t(s_t)$ for all t .⁷ Noting that consumption is equalized across the employed and unemployed states and that equation (2) holds with equality, the following expression is obtained: $M_t^S / P_t(s_t) = f(\phi_t^w, K, \lambda_t)$. By utilizing these facts equation (5) can be rewritten as

$$\left[\frac{f(\phi_t^w, K, \lambda_t)}{\partial f(\phi_t^w, K, \lambda_t) / \partial \phi_t^w} \right]$$

$$= A \int_Q \int_Z \left[\frac{U'(f(\phi_{t+1}^w, K, \lambda_{t+1})) f(\phi_{t+1}^w, K, \lambda_{t+1})}{\mu_{t+1}} \right] dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) \quad (6)$$

where $\mu_{t+1} \equiv M_{t+1}^s / M_t^s$, and $A \equiv \beta/[V(0) - V(1)]$. Equation (6) implicitly defines a solution for the current equilibrium employment rate ϕ_t^w . It can be shown that there exists a stationary function $\phi(\cdot)$ which maps period- t money growth rates and technology parameters $(\mu_t, \lambda_t) \equiv s_t$ into equilibrium employment rates, ϕ_t^w , so that formally one may write $\phi_t^w = \phi(s_t)$.⁸ (Henceforth ϕ_t^w will be written more simply as ϕ_t .) The existence proposition will now be stated formally.

Proposition 1: There exists a unique bounded continuous function $\phi: Q \times Z \rightarrow [0,1]$ satisfying equation (6).

Proof: See Appendix A.

Corollary: The fixed point described in Proposition 1 does not have $\phi(\cdot) = 0$ or $\phi(\cdot) = 1$ as a solution.

Proof: Again, see Appendix A.

The corollary states that the equilibrium employment rate, and therefore the unemployment rate, always lies between zero and unity. This result is obtained by placing restrictions on the structure of the economic environment (see Appendix A for details) which effectively ensure that the model's necessary efficiency condition (5) will never be satisfied at either zero or full employment. Loosely speaking, these corner solutions are ruled out by making sure that the equilibrium return to working becomes profitable enough as the employment rate approaches zero to induce work effort, and likewise becomes sufficiently small as full employment is reached so as to dissuade further employment.

V. THE STOCHASTIC PROPERTIES OF UNEMPLOYMENT AND INFLATION

Of particular interest in dynamic models of this sort are the stochastic properties of aggregate endogenous variables. For this model in particular it is of interest to study the covariation between unemployment and inflation both conditioned and unconditioned upon the monetary and real shocks. Common Keynesian folklore dictates that unemployment and inflation should display negative correlation. Although this could indeed be true, but it would still not imply the existence of an exploitable trade-off between these variables. It will be demonstrated later in this section that indigenous forces in a competitive economy can result in a negative association between unemployment and inflation in the model. In spite of this, policy-engineered increases in the rate of monetary expansion, if they have an impact on the real side of the economy, cause the unemployment rate to rise. That is, the operational Phillips curve the policymaker faces is always non-negatively sloped. A more formal statement of this observation will now be made.

Proposition 2: The unique equilibrium employment rate function, $\phi(\mu, \lambda)$, is decreasing in the growth rate of the money supply, μ .

Proof: See Appendix A, once again.

An immediate consequence of the above proposition is that current equilibrium output, $y_t = f(\phi(\mu_t, \lambda_t)l, K, \lambda_t)$, and hence consumption, c_t , are decreasing functions in the contemporaneous growth rate of the money supply, μ_t , since $f(\cdot)$ is increasing in ϕ . Hence employment, output, and consumption all move in tandem in response to a money supply shock. It is now straightforward to see that an upward shift in today's growth rate of the money supply, μ_t , is associated with a rise in the current inflation rate, π_t , since the latter is determined by the equation⁹

$$(1+\pi_t) \equiv P_t/P_{t-1} = \mu_t f(\phi(\mu_{t-1}, \lambda_{t-1})^{\lambda}, K, \lambda_{t-1}) / f(\phi(\mu_t, \lambda_t)^{\lambda}, K, \lambda_t). \quad (7)$$

The intuition underlying the above proposition is straightforward. A shift upward in the current money supply growth rate signals an increased probability of higher future money supply growth rates. This portends greater inflation. Now recall that the firm holds agents' nominal earnings for one period before distributing them. Thus, the expected purchasing power of these earnings will be reduced by the higher expected inflation. The expected marginal return to working consequently is eroded, causing a drop in employment, output, and consumption. Market activity--here, the operation of a firm--requires the use of currency and is taxed by inflation while non-market activity--leisure--does not require the use of currency and hence escapes the inflation levy. As a result, when the rate of inflation rises individuals on average move out of market activity (production and consumption) into non-market activity (leisure).¹⁰ The role that greater current money growth plays in signalling the higher probability of increased future money growth should be emphasized. If current and future monetary growth rates were independently distributed, then it is easy to discern from equation (6) that a large realization of the currency supply growth rate would have no implications for this period's equilibrium employment rate--the only effect would be a once-and-for-all increase in the price level.¹¹

Inflation would have a similar deleterious effect on output in both the standard overlapping generation model with money and a transaction cost model where money is held so as to minimize the amount of output absorbed in the exchange process. Again, this occurs since the private return to work effort is eroded by inflation. Thus, the result obtained here would not appear to be an artifact of the assumed cash-in-advance structure.

Recent work in macroeconomics has stressed how business cycle fluctuations can arise from purely real phenomenon (for example, Kydland and Prescott (1982) and Long and Plosser (1983)). The technology parameter λ in the model can be thought of as the driving force behind the real side of the economy. It is of interest to consider the effects that such shocks can have upon inflation and unemployment in the model. King and Plosser (1984) found that there is a positive correlation between output and a measure of inside money. The importance of this finding for the association between inflation and unemployment cannot be overemphasized. It implies that some of the cyclical behavior of monetary aggregates is an endogenous component of the business cycle--as opposed to being wholly determined by outside forces such as policymakers. To incorporate this finding into the analysis suppose that the period- t money supply growth rate, μ_t , can be written as

$$\mu_t = \theta_t + \eta(\lambda_t)$$

The term $\eta(\lambda)$ reflects that part of the gross growth rate in the money supply which is determined in association with real factors in the economy, and is a positive and strictly increasing, continuously differentiable function of λ . The term θ represents the component of the growth rate of money which is determined by exogenous forces. It is assumed that the stochastic process governing θ_t is determined by the distribution function $F(\cdot|\cdot)$ described above. A needed consistency requirement is that $F(\cdot|\cdot)$ be redefined over the new domain $Q'XQ'$ where $Q' \equiv [\underline{\theta}, \bar{\theta}]$, and $\underline{\mu} - \eta(\underline{\lambda}) = \underline{\theta} < \bar{\theta} = \bar{\mu} - \eta(\bar{\lambda})$.

This slight extension of the model does not involve introducing any new technical considerations into the earlier analysis. In particular, the existence proof of an equilibrium for the economy proceeds along lines identical to those outlined in Proposition 1, with Proposition 2 implying that $\phi(\cdot)$ is decreasing in θ .

t

It is easy to show that the equilibrium employment rate can rise in response to a positive technology shock. All that is required is for a positive shock to increase the marginal product of labor by an extent sufficient to induce a rise in the equilibrium employment rate. Assuming this requirement--which is made more precise in Appendix A--allows the following proposition to be stated:

Proposition 3: The unique equilibrium employment rate function, $\phi(\theta, \lambda)$, is strictly increasing in the technological parameter, λ .

Proof: One more time, see Appendix A.

The above result also implies that equilibrium output, $y=f(\phi(\theta, \lambda)l, K, \lambda)$, and thus consumption, c , are increasing functions in the level of productivity, λ . Consumption, output, and employment consequently will all move in concert in response to a productivity shock.

The theory outlined so far will now be illustrated by way of a concrete example.

Example: Let $U(c) = \ln(c)$, $f(\phi l, K, \lambda) = [K^\rho + \lambda(\phi l)^\rho]^{1/\rho}$ for $\rho \in (0, 1)$, $[V(0) - V(l)] > 1$, and $\underline{\theta} > \beta$. It is straightforward to show that equation (6) can be written as

$$\begin{pmatrix} K \\ l \end{pmatrix}^\rho \begin{pmatrix} \phi_t \\ \lambda_t \end{pmatrix}^{1-\rho} + \phi_t = \frac{\beta \int_Q \int_Z \frac{1}{\mu_{t+1}} dG(\lambda_{t+1}) dF(\theta_{t+1} | \theta_t)}{[V(0) - V(l)]} < 1$$

$$[\text{with } \mu_{t+1} = \theta_{t+1} + \eta(\lambda_{t+1}) > \beta].$$

This is an implicit equation in ϕ . It is clear that ϕ_t is decreasing in θ_t [see footnote (15) for further details] and increasing in λ_t . Also, it is straightforward to deduce that $\phi_t \in (0, 1)$. Furthermore, since $\underline{\theta} > \beta$ it can easily be seen from (4) that the example is portraying an equilibrium where the cash-in-advance constraints are holding with equality.

The last unresolved question is the relationship between the technology shock and inflation in the model. To determine the impact of the period- t technology shock, λ_t , on the inflation rate, π_t , differentiate equation (7) to get

$$\frac{d\pi_t}{d\lambda_t} = \begin{bmatrix} y_{t-1} \\ y_t \end{bmatrix} \begin{bmatrix} \mu_t \\ \lambda_t \end{bmatrix} \left\{ \xi(\mu_t, \eta_t) \xi(\eta_t, \lambda_t) - [\xi(y_t, \lambda_t) + \xi(y_t, \phi_t) \xi(\phi_t, \lambda_t)] \right\} > 0 \quad (8)$$

where $y_t \equiv f(\phi_t, \lambda_t, K, \lambda_t)$, $\eta_t = \eta(\lambda_t)$, $\phi_t = \phi(\theta_t, \lambda_t)$, and, for example, $\xi(y_t, \lambda_t)$ represents the elasticity of y_t with respect to λ_t , i.e., $\xi(y_t, \lambda_t) \equiv [\lambda_t / f(\phi_t, \lambda_t, K, \lambda_t)] [\partial f(\phi_t, \lambda_t, K, \lambda_t) / \partial \lambda_t]$. [In the above condition it is assumed that the production function, $f(\cdot)$, is restricted in a manner such that the elasticity expressions in the brackets are always bounded in value.] As equation (8) illustrates, a stimulative productivity shock may be linked with a contemporaneous rise in inflation rate if it is associated with a sufficiently large increase in the quantity of inside money. Specifically, in the current setting all that is required is for the money supply to rise proportionately more than income when a high productivity shock is realized. More formally, the restriction shown below, which will now be assumed, is what is needed.

$$\xi(\mu_t, \eta_t) \xi(\eta_t, \lambda_t) > \xi(y_t, \lambda_t) + \xi(y_t, \phi_t) \xi(\phi_t, \lambda_t) \quad \text{for all } \lambda_t \in Z \text{ and } \theta_t \in Q'$$

The model's import will now be discussed. Consider a policymaker in a heretofore non-interventionist economy. By eyeing the relationship between money and employment he may be tempted to conclude that an expansionary monetary policy can stimulate (mitigate) employment (unemployment) since

$$\text{Cov}(\phi(\theta_t, \lambda_t), \eta(\lambda_t) | \theta_t) = \int_Z \phi(\theta_t, \lambda_t) [\eta(\lambda_t) - \bar{\eta}] dG(\lambda_t) > 0,$$

$$\text{for } \theta \in Q' \text{ and where } \bar{\eta} \equiv \int_Z \eta(\lambda) dG(\lambda),$$

with the sign of the above expression following from the fact that the covariance between the two strictly increasing functions of a variable must be positive.¹² Yet in reality there is no trade-off between inflation and unemployment facing the policymaker in the assumed environment. An activist monetary policy, represented by the distribution function $F(\cdot)$ in the model, in fact has a detrimental impact on employment in the economy as

$$\text{Cov}(\phi(\theta_t, \lambda_t), \theta_t | \lambda_t, \theta_{t-1}) = \int_{Q'} \phi(\theta_t, \lambda_t) [\theta_t - \bar{\theta}] dF(\theta_t | \theta_{t-1}) \leq 0,$$

$$\text{for } \lambda_t \in Z \text{ and where } \bar{\theta} \equiv \int_{Q'} \theta dF(\theta_t | \theta_{t-1}),$$

noting that the covariance between a variable and a decreasing function of the same variable is nonpositive. Finally, with an activist policymaker in the economic environment the observed relationship between employment and money growth--and consequently the Phillips curve relationship--may appear to be either weak or noisy since it will depend on the relative strengths of the endogenous and exogenous components of the money supply. Specifically,

$$\begin{aligned} \text{Cov}(\phi(\theta_t, \lambda_t), \mu_t | \theta_{t-1}) &= \int_Z \text{Cov}(\phi(\theta_t, \lambda_t), \theta_t | \lambda_t, \theta_{t-1}) dG(\lambda_t) \\ &+ \int_{Q'} \text{Cov}(\phi(\theta_t, \lambda_t), \eta(\lambda_t) | \theta_t) dF(\theta_t | \theta_{t-1}) > 0 \\ &\quad [\text{recall } \mu_t = \theta_t + \eta(\lambda_t)] \end{aligned}$$

where the integrals of the conditional covariances have the same signs as the conditional covariances themselves.¹³

VI. CONCLUSIONS

A stochastic dynamic equilibrium model was presented to examine the relationship between unemployment and inflation. The introduction of a nonconvexity into agents' labor supply decisions resulted in a certain fraction of the population being unemployed at any particular time. This permitted the study of the determinants of unemployment within the context of a representative agent model. It was argued that indigenous forces in a competitive economy could result in the traditional negative relationship between inflation and unemployment. In particular, shocks which result in increased productivity could stimulate the return to working, thereby reducing unemployment and boosting output, and given sufficient endogeneity in the money supply, result in increased inflation. This conditional negative covariation between unemployment and inflation did not imply an exploitable trade-off between the two variables. In fact, any engineered inflation by authorities in the model had an adverse impact on unemployment by reducing the return to market activity. The policymaker, while perhaps observing a negatively sloped Phillips curve, in actuality faced Friedman's positively sloped one.

APPENDIX A

The details of the formal line of analysis used to justify the results outlined in the text are provided in this appendix. So as to facilitate cross-referencing with text, the presentation is done on a section-by-section basis.

Section II: The Physical Environment

To begin with, some assumptions delimiting the specification of tastes, technology, and the stochastic nature of the environment will be stated.

Assumption (A1) $U(c)$ is twice continuously differentiable and strictly concave.

Also, let $\lim_{c \rightarrow 0} [cU'(c)] > 0$, and

$$0 < \left[1 + \frac{cU''(c)}{U'(c)} \right] \leq \alpha, \text{ where } \alpha \text{ is chosen below.}^{14}$$

(This implies $[cU'(c)]$ is increasing in c). Last, assume

$$-\infty < V(l) < V(0) < \infty.$$

Assumption (A2) Let λ_t be determined by a stochastic process whose

distribution function is $G(\lambda_t)$. Let $Z = [\underline{\lambda}, \bar{\lambda}]$ so then

$G(\cdot): Z \rightarrow [0,1]$. For all continuous functions $w(\lambda)$ the integral $\int_Z w(\lambda) dG(\lambda)$ is assumed to exist.

Assumption (A3) $f(\cdot, \cdot, \cdot)$ is increasing, twice continuously differentiable, strictly concave in all its arguments.

Assume $\lim_{\phi \rightarrow 0} \left[\frac{\partial f(\phi, K, \lambda)}{\partial \phi} \right] = \infty$, and $f(0, K, \lambda) = 0, \forall \lambda \in Z$.

Assumption (A4) $F(\mu_t | \mu_{t-1})$ is assumed to be the transition distribution

function for μ_t . Let $Q = [\underline{\mu}, \bar{\mu}]$ and hence for all $\mu_t \in Q$,

$F(\cdot | \mu_t): Q \rightarrow [0,1]$. It is assumed that for all $\mu_t \in Q, \lambda \in Z$

$$0 < \left[\frac{\beta(\bar{y}U'(\bar{y}))}{[V(0)-V(\bar{\lambda})]} \int_Q \frac{1}{\mu_{t+1}} dF(\mu_{t+1} | \mu_t) \right] \leq \gamma < \left[\frac{f(\phi\bar{\lambda}, K, \lambda)}{\frac{\partial f(\phi\bar{\lambda}, K, \lambda)}{\partial \phi}} \right] \Big|_{\phi=1} \equiv \kappa < \infty.$$

where $\bar{y} \equiv f(1, \bar{\lambda}, K, \bar{\lambda})$, and γ will be specified below.

Also, for all continuous functions $w(\mu)$, the function

$\int_Q w(\mu) dF(\mu | \mu')$ is assumed to be continuous.

Assumption (A5) Suppose the distribution function $F(\cdot | \cdot)$ satisfies

$$0 < -F_2(\cdot | \cdot) < F_1(\cdot | \cdot) < 1.$$

This amounts to requiring that the growth rate of the currency supply follows a stable stochastic process with positive autocorrelation.

Assumption (A6) For $\phi \in (\phi^*, 1)$ and $\lambda \in Z$ let

$$\frac{\partial^2 f(\phi\bar{\lambda}, K, \lambda)}{\partial \phi \partial \lambda} > \left[\frac{\partial f(\phi\bar{\lambda}, K, \lambda)}{\partial \phi} \frac{\partial f(\phi\bar{\lambda}, K, \lambda)}{\partial \lambda} \right] / f(\phi\bar{\lambda}, K, \lambda).$$

where ϕ^* is given a precise value below.

Finally, all integrals should be interpreted as being Lebesgue integrals with the qualification "almost everywhere" being omitted.

Section III: Private Sector Decision-Making

Next the first-order conditions associated with the dynamic programming problem (P1) are shown below where φ_t and α_t^j are the Lagrange multipliers associated with the constraints (1) and (2)

$$\phi_t^j U'(c_t^j) = \phi_t^j \beta \int_Q \int_Z W(a_{t+1}, s_{t+1}) \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) + \alpha_t^j \quad j=w,u \quad (1A)$$

$$\alpha_t^j + \phi_t^j \beta \int_Q \int_Z W(a_{t+1}, s_{t+1}) \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) = \varphi_t \phi_t^j \quad j=w,u \quad (2A)$$

$$\begin{aligned} \phi_t^j \beta \int_Q \int_Z (1+r_t(s_{t+1}, s_t)) W(a_{t+1}, s_{t+1}) \frac{P_t(s_t)}{P_{t+1}(s_{t+1})} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) \\ = \varphi_t \phi_t^j \quad j=w,u \quad (3A) \end{aligned}$$

$$\begin{aligned} U(c_t^w) - U(c_t^u) + V(l) - V(0) - \beta \int_Q \int_Z W(a_{t+1}, s_{t+1}) \left[\frac{P_t(s_t)}{P_{t+1}(s_{t+1})} \right] \left\{ \frac{\partial f(\phi_t, l, K, \lambda)}{\partial \phi_t} \right. \\ \left. + (m_t^w - c_t^w) - (m_t^u - c_t^u) + [1+r_t(s_{t+1}, s_t)](b_t^u - b_t^w) \right\} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) \\ = \varphi_t [b_t^w - b_t^u - m_t^w + m_t^u] \quad (4A) \end{aligned}$$

First, combining equations (1A) and (2A) yields

$$U'(c_t^j) = \varphi_t \quad j = w,u \quad (5A)$$

from which equation (3) in the text is obtained. Second, note that cash-in-advance constraints (2) will always hold with equality provided that $\alpha_t^j > 0$. Now from (2A), (5A) and the additional fact that $W_a(\cdot, t+1) = \varphi_{t+1}$, it is easy

to determine that $\alpha_t^j > 0$ if and only if $1+i_t > 1$, where the gross nominal interest rate, $1+i_t$, is defined by equation (4) in the text. Finally, through the use of equations (3A) and (5A) and the fact that equation (2) holds with equality, equation (4A) can be simplified to get (5) in the text.

Section IV: The Model's General Equilibrium

Proposition 1, which establishes the existence of a unique stationary equilibrium employment rate function, can now be proved. In the course of the proof a restriction on the values of α and γ , mentioned in assumptions (A1) and (A4) will be developed.

Proposition 1: Under assumptions (A1)-(A4) there exist parameters α , γ such that there is a unique bounded continuous function $\phi: Q \times Z \rightarrow [0,1]$ satisfying equation (6).

Proof: Define the function

$$H(\phi_t, \lambda_t) \equiv \frac{f(\phi_t, l_t, K_t, \lambda_t)}{\partial f(\phi_t, l_t, K_t, \lambda_t) / \partial \phi_t}$$

with $H(0, \lambda_t) = 0$ for all $\lambda_t \in Z$. Note $H: [0,1] \times Z \rightarrow [0, \kappa]$ and $H(\cdot, \cdot)$ is strictly increasing in its first argument. Since $f(\cdot, \cdot, \cdot)$ is twice differentiable, by the Inverse Function Theorem there exists a function $H_\lambda^{-1}(\cdot)$ such that $H_\lambda^{-1}(H(\phi, \lambda)) = \phi$. Also $H_\lambda^{-1}(\cdot)$ is both differentiable and increasing.

Furthermore, since $\frac{\partial H(\phi, \lambda)}{\partial \phi} \geq 1$, the derivative of $H_\lambda^{-1}(\cdot)$ is in the

interval $[0,1]$. Equation (6) can now be rewritten as

$$\phi(s_t) = H_{\lambda_t}^{-1} \left(A \int_Q \int_Z \frac{U'(f(\phi(s_{t+1}), K, \lambda_{t+1})) f(\phi(s_{t+1}), K, \lambda_{t+1}))}{\mu_{t+1}} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t) \right) \quad (6A)$$

Let \mathcal{F} be the space of bounded continuous functions $h: Q \times Z \rightarrow [\phi^*, 1]$ with norm $\|h\| = \sup_{\substack{\mu \in Q \\ \lambda \in Z}} |h(\mu, \lambda)|$, where the constant ϕ^* is defined as

$$\phi^* \equiv \min_{\substack{\mu \in Q \\ \lambda \in Z}} \left\{ H_{\lambda}^{-1} \left(A \lim_{c \rightarrow 0} U'(c) c \right) \int \frac{1}{\mu'} dF(\mu' | \mu) \right\} > 0.$$

Equation (6A) describes a mapping $\hat{\phi} = T(\phi)$ from \mathcal{F} into \mathcal{F} . It will now be shown that T is a contraction operator.

To prove this let

$$G(\phi; \mu) \equiv \int_Q \int_Z \frac{A U'(f(\phi, K, \lambda)) f(\phi, K, \lambda)}{\mu'} dG(\lambda) dF(\mu' | \mu).$$

Now

$$\begin{aligned} \frac{\partial G(\phi; \mu)}{\partial \phi} &\leq \max_{\substack{\lambda \in Z \\ \phi \in [\phi^*, 1]}} \left\{ \left[\frac{1}{c} \frac{\partial c}{\partial \phi} \right] \left[1 + \frac{c U''(c)}{U'(c)} \right] \left[A \bar{y} U'(\bar{y}) \int \left(\frac{1}{\mu'} \right) dF(\mu' | \mu) \right] \right\} \Bigg|_{c=f(\phi, K, \lambda)} \\ &\leq (\alpha \gamma) \max_{\lambda \in Z} \left[\frac{\frac{\partial f(\phi, K, \lambda)}{\partial \phi}}{f(\phi, K, \lambda)} \right] \Bigg|_{\phi=\phi^*} \end{aligned}$$

by assumptions (A1) and (A4).

Finally, by the Mean Value Theorem, equation (6A) implies

$$\|T(\phi^1) - T(\phi^2)\| \leq \max \left| \frac{\frac{\partial H_{\lambda}^{-1}(G(\phi; \mu))}{\partial G}}{\partial G} \cdot \frac{\partial G(\phi; \mu)}{\partial \phi} \right| \|\phi^1 - \phi^2\| \equiv \Lambda \|\phi^1 - \phi^2\|$$

where the maximum is taken over all ϕ in the convex hull spanned by ϕ^1, ϕ^2 .

Since $H_{\lambda}^{-1}(\cdot)$ has a derivative between 0 and 1, by choosing $(\alpha\gamma)$ sufficiently small we have $\Lambda < 1$. Hence the operator T has a unique fixed point on \mathfrak{F} (by the Banach Fixed Point Theorem).

Corollary: The fixed point described in Proposition 1 does not have $\phi(\cdot) = 0$ or $\phi(\cdot) = 1$ as a solution for any realizations of the state space (except, of course, on a set of measure zero).

Proof: That $\phi(\cdot) = 0$ is not a solution was shown in the proposition and is a consequence of assumptions (A1) and (A3). That $\phi(\cdot) = 1$ cannot be a solution can be seen from equation (6). Assumption (A4) implies that the left side of equation (6) evaluated at $\phi = 1$ must be strictly greater than the right side. Hence $\phi(\cdot) = 1$ cannot be a solution.

Remark: The corollary shows there is no need formally to add a restriction to the representative agent/firm's dynamic programming problem (P1) that $\phi \in [0,1]$.

Section V: The Stochastic Properties of Unemployment and Inflation

Finally, the next two propositions establish that the equilibrium employment rate function is decreasing in the rate of monetary expansion and strictly increasing in the state of technology.

Proposition 2: Under the hypothesis of assumptions (A1)-(A5), the unique equilibrium employment rate function, $\phi(\mu, \lambda)$, is decreasing in the growth rate of the money supply, μ .

Proof: Let $\tilde{\phi}(\mu_{t+1}, \lambda_{t+1})$ be decreasing in μ_{t+1} . Then

$$\left[\frac{U'(f(\tilde{\phi}(s_{t+1}), \lambda_{t+1}))f(\tilde{\phi}(s_{t+1}), \lambda_{t+1}))}{\mu_{t+1}} \right]$$

is decreasing in μ_{t+1} since $cU'(c)$ is increasing in c . By an argument

similar to that employed in Lucas (1978), Lemma 1, this implies that

$$\int_Q \int_Z \frac{U'(f(\tilde{\phi}(s_{t+1}), \lambda_{t+1}))f(\tilde{\phi}(s_{t+1}), \lambda_{t+1}))}{\mu_{t+1}} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t)$$

is also decreasing in μ_t .¹⁵ Finally, since $H_\lambda^{-1}(\cdot)$ is increasing, the

mapping T , described in the previous proposition, given by equation (6A) maps decreasing functions into decreasing functions. Thus if

$\tilde{\phi} = \lim_{n \rightarrow \infty} T^n \tilde{\phi}$ there exists an equilibrium in which $\tilde{\phi}: Q \times Z \rightarrow [0,1]$

and where $\tilde{\phi}$ is decreasing in its first argument. Further, $\tilde{\phi}$ is a solution to equation (6A) and hence is an equilibrium employment rate function.

Before proceeding to the statement and proof of the next proposition, recall that at this stage of the analysis in the text the gross growth rate of the money supply has been redefined to be the sum of two components so that $\mu_{t+1} = \theta_{t+1} + \eta(\lambda_{t+1})$, where it is now θ_{t+1} which is governed by the distribution function $F(\theta_{t+1} | \theta_t)$ defined over the new domain $Q' \times Q'$ with

$Q' = [\underline{\theta} = \underline{\mu} - \eta(\underline{\lambda}), \bar{\theta} = \bar{\mu} - \eta(\bar{\lambda})]$. Thus for all $\theta_t \in Q'$, $F(\cdot | \theta_t): Q' \rightarrow [0,1]$. As

is readily apparent, θ now plays the economic role that μ did previously

with the previous technical analysis remaining intact, except for the innocuous replacement of μ by θ in $F(\cdot|\cdot)$ and Q by Q' , etc.

Proposition 3: Given assumptions (A1)-(A6), the unique equilibrium employment rate function, $\phi(\theta, \lambda)$, is strictly increasing in the technology parameter, λ .

Proof: Let $D: Q' \rightarrow R_{++}$ be continuous, and $\hat{\phi}(\theta, \lambda)$ be the implicit solution to

$$\frac{f(\hat{\phi}\lambda, K, \lambda)}{\partial f(\hat{\phi}\lambda, K, \lambda)/\partial \hat{\phi}} = D(\theta). \quad (7A)$$

Since $f(\cdot)$ is twice continuously differentiable, an application of the Implicit Function Theorem guarantees that $\hat{\phi}(\cdot)$ is continuously differentiable in λ and together with assumption (A6) implies $\hat{\phi}(\cdot)$ is strictly increasing in this variable. Finally, by letting $D(\theta)$ represent the right-hand side of equation (6), with μ replaced by θ , it is clear that the equilibrium employment rate function $\phi(\theta, \lambda)$ falls within the class of functions satisfying equation (7A). [Note that assumption (A6) ensures that a positive technological shock increases the marginal product of labor by an extent sufficient to induce a rise in the equilibrium employment rate.]

The above line of analysis could readily be extended to investigate other government policy issues. For instance, it would appear to be relatively straightforward to address the impact of various government expenditure (both productive and unproductive) and labor-income taxation programs. Also information-type variables, which are relevant to agents

only in that they help to forecast the future state of the economy, could be included. In such an extension, a single functional equation resembling (6) would again characterize the model's general equilibrium. Other extensions such as making the capital stock endogenous would seem to complicate the analysis substantially. Now there would be more than one functional equation describing the model's general equilibrium and this would likely involve a discrete jump in the degree of analytical complexity.

APPENDIX B

This appendix will be devoted to an examination of the effect that a nonconvexity in agent's labor-leisure choice set has on the economy's general equilibrium. Again there is a continuum of agents on the unit interval. Each agent has preferences

$$U = \frac{1}{\alpha} c^{\alpha} - \frac{1}{\rho} l^{\rho} \quad (1B)$$

where c the consumption of an agent, and l his labor effort. There is a firm in this economy which has access to a constant-return-to-scale production function

$$f(K, \tilde{l}) = \lambda K^{\beta} \tilde{l}^{1-\beta}$$

where K is aggregate quantity of capital employed and \tilde{l} equals the aggregate quantity of labor employed. Hence the firm maximizes

$$\lambda K^{\beta} \tilde{l}^{1-\beta} - w\tilde{l} - rK \quad (2B)$$

where w is the wage rate of labor and r represents the rental rate on capital.

Each agent is assumed to supply K units of capital inelastically. In a competitive equilibrium the agent faces the budget constraint

$$c = w\tilde{l} + rK \quad (3B)$$

The economy is now parameterized as follows:

$$\alpha = .75, \quad \rho = 1.25, \quad \beta = .25, \quad \lambda = 15.0, \quad K = 100.$$

Economy 1: Each agent maximizes (1B) subject to (3B) and $c, \tilde{l} \geq 0$, with the firm maximizing (2B). The resulting equilibrium consumption, labor effect, and utility levels are respectively (all numbers are rounded off to two decimal places),

$$c^* = 814.97, \quad \tilde{l}^* = 44.33, \quad U^* = 111.86.$$

The prices associated with these choices are

$$w^* = 13.79 \quad r^* = 2.04.$$

This equilibrium is illustrated by point F in Figure 1 in the text.

Economy 2: Each agent maximizes (1B), subject to (3B) and the added constraint, $l \in \{0, l^N\}$. The firm again maximizes (2B). That is, each agent has a dichotomous choice problem with respect to his labor effort. For $l^N = 73.14$, the equilibrium quantities and prices are

$$\begin{aligned} c^N &= 1186.47, & \tilde{l}^N &= 73.14 & U^N &= 98.40 \\ w^N &= 12.17, & r^N &= 2.97. \end{aligned}$$

This equilibrium is illustrated by point N in Figure 1. To have all agents employed the utility return for all agents must exceed or equal that of being unemployed.

Economy 3: Now consider introducing a lottery into the environment of Economy 2. Each agent receives a constant level of consumption c^L and each agent faces probability ϕ of being called to work l^N units. Aggregate employment is then $\tilde{l} = \phi l^N$ and in equilibrium $c^L = \lambda(k)^\beta (\phi l^N)^{1-\beta}$. The equilibrium values for this economy are

$$\begin{aligned} \phi &= .76, & c^L &= 963.98, & \tilde{l} &= 55.46 \\ U^U &= 230.67 & U^W &= 59.52, & U^L &= 100.9 \\ w^L &= 13.04 & r^L &= 2.41 & l^N &= 73.14 \end{aligned}$$

There are two types of agents ex post: those that work and those that don't. Those agents who work receive an ex post utility level of U^W while those agents who do not work receive an ex post utility level of U^U . This is shown in Figure 1. Each agent then attains an ex ante expected utility level of $U^L = \phi U^W + (1-\phi)U^U$. Under the lottery scheme ϕ is chosen so as to maximize U^L .

Note that the utility level with the lottery, U^L , exceeds the utility level without the lottery, U^N . Also, note that due to risk aversion an agent would be indifferent between working the amount $\bar{l} > \phi l^N$ and taking his chances with the lottery.

FOOTNOTES

¹Each agent's consumption and labor effort could also be indexed by his (random) period- t position, say σ , on the interval $[0,1]$, so for instance $c_t(\sigma)$ and $l_t(\sigma)$ would represent agent σ 's period- t quantities of these variables. It turns out that there is really no need to do this--c.f. footnote 4 below.

²A precise physical environment giving rise to a cash-in-advance economy has not been specified. A modification of the above economy along the lines of Townsend (1980, p. 284) could produce a cash-in-advance economy as an outcome of the assumed structure of the environment. Such a modification, while making the model more cumbersome, would not seem to change any of the results obtained here.

³This particular formulation is used because of its tractability. Agents could be allowed to choose their labor supply over some more general discrete (nonconvex) set. Also, it could be assumed that agents who don't work the maximal number of hours in the market place, instead engage in some amount of home production. These extensions would not affect the main conclusions obtained.

⁴The fact that the decentralized decision making by a continuum of agents in competitive equilibriums with nonconvexities and lotteries can be summarized by a simple representative agent's programming problem is discussed formally in Rogerson (1985).

⁵A full derivation of these facts is provided in Appendix A.

⁶Note that since the employed and unemployed have the same level of consumption, the former are worse off ex post since they are working. Also, the employed are being paid less than the value of their marginal product. In a weak sense then, one could label them as being "involuntarily employed."

With a non-separable momentary utility function, it is possible to have the unemployed consuming less than the employed. In particular, if consumption and work effort are contemporaneous net complements in the Edgeworth-Pareto sense in the momentary utility function, then the unemployed will consume less than the employed. The next question to ask is whether unemployed agents' consumption can be reduced to such an extent so as to make them worse off than the employed agents. The problem which arises is that the stronger the degree of Edgeworth-Pareto complementarity between consumption and labor in the momentary utility function, the more likely it becomes that leisure is an inferior good, an undesirable implication (see Greenwood and Huffman (1986)). An intuitively appealing characterization of the determination of the relative welfare levels of the employed versus unemployed agents has not yet been obtained. For the purpose here of examining the responsiveness of the "natural" rate of unemployment to real and monetary shocks, progress on this issue probably isn't too crucial.

⁷Utilizing the notation of footnote 1 these conditions can be perhaps more transparently written as $\int_0^1 c_t(\sigma) d\sigma = f(\phi_t^W, l, K, \lambda_t)$ and $\int_0^1 m_t(\sigma) d\sigma = M_t^S/P_t(s)$, where $(c_t(\sigma), m_t(\sigma)) = (c_t^W, m_t^W)$ for $\sigma \in [0, \phi_t^W]$, and with $(c_t(\sigma), m_t(\sigma)) = (c_t^U, m_t^U)$ for $\sigma \in (\phi_t^W, 1]$.

⁸It can now be easily discerned that to have a positive nominal interest rate, as defined by (4), the distribution functions $F(\cdot|\cdot)$ and $G(\cdot)$ should be restricted to ensure that

$$\frac{U'(f(\phi(\mu_t, \lambda_t), l, K, \lambda_t))f(\phi(\mu_t, \lambda_t), l, K, \lambda_t))}{\beta \int_Q Z U'(f(\phi(\mu_{t+1}, \lambda_{t+1}), l, K, \lambda_{t+1}))f(\phi(\mu_{t+1}, \lambda_{t+1}), l, K, \lambda_{t+1})) / \mu_{t+1} dG(\lambda_{t+1}) dF(\mu_{t+1} | \mu_t)} > 1.$$

⁹By conditioning equation (7) on the value of μ_{t-1} , it can be seen that the expected rate of inflation is also an increasing function of the present money supply growth rate.

¹⁰Note that a low realization of μ will signal a more deflationary trend for the economy. This will increase the expected return to working and consequently cause the unemployment rate to fall. Similarly, distribution functions for μ having negative serial correlation properties could occasionally result in a high money shock being associated with a drop in unemployment, again because of the increased probability of a more deflationary price path for the economy. Among the class of deflationary monetary policies the best is to follow the optimum quantity of money rule, which results in the gross (net) interest rate being set equal to unity (zero) --see Aschauer and Greenwood (1983). This class of deflationary monetary policies, while potentially lowering unemployment, is certainly not what most people have in mind when thinking about expansionary monetary policies.

¹¹One might be tempted to conclude that if the money supply growth rates exhibited negative serial correlation, then the employment rate must necessarily be increasing in the current money growth rate. This is false. Such correlations, in general, turn out to be of indeterminate sign.

¹²Trivially, the conditional covariance between period- t the unemployment rate, $(1 - \phi(s_t))$, and the endogenous money supply growth rate shock, $\eta(\lambda_t)$, can be written as $\text{Cov}((1 - \phi(\theta_t, \lambda_t)), \eta(\lambda_t) | \theta_t) = -\text{Cov}(\phi(\theta_t, \lambda_t), \eta(\lambda_t) | \theta_t) < 0$.

¹³As has been noted earlier, current equilibrium output, y_t , is a decreasing function in θ_t and an increasing function in λ_t so that one

could write $y_t = y(\theta_t, \lambda_t)$. Hence employment, output, and consumption

(-)(+)

always move positively with one another in the model. Thus, one could replace employment, ϕ_t , with output, y_t (or consumption, c_t), in the above covariance formulae and preserve the story being told.

¹⁴ The assumption that $\lim_{c \rightarrow 0} [cU'(c)] > 0$ is innocuous. In fact, the propositions proved in the paper can be shown to hold for utility functions of the isoelastic type, $U(c) = \frac{c^\alpha}{\alpha}$ where $\alpha \in (0,1)$, can

¹⁵Specifically, make the following definition: $h(s_{t+1}) \equiv$

$U'(f(\tilde{\phi}(s_{t+1}), L, K, \lambda_{t+1})) / f(\tilde{\phi}(s_{t+1}), L, K, \lambda_{t+1}) / \mu_{t+1}$. Now let $\pi = F(\mu_{t+1} | \mu_t)$ and invert this function to get $\mu_{t+1} = J(\pi, \mu_t)$. Note $J_2(\cdot) = -F_2(\cdot) / F_1(\cdot)$.

Hence by the standard change in variable transform $D(\mu) \equiv \int_Q \int_Z h(\mu_{t+1}, \lambda_{t+1})$

$\cdot dG(\lambda_t) dF(\mu_{t+1} | \mu_t) = \int_0^1 \int_Z h(J(\pi, \mu_t), \lambda_{t+1}) dG(\lambda_{t+1}) d\pi$. Consequently, it

follows that $D'(\mu_t) = \int_0^1 \int_Z h_1(J(\pi, \mu_t), \lambda_{t+1}) J_2(\pi, \mu_t) dG(\lambda_{t+1}) d\pi \leq 0$,

since it was assumed that $0 < -F_2(\cdot) / F_1(\cdot) < 1$, and $h_1(\cdot) \leq 0$ for $\mu \in Q$ and $\lambda \in Z$.

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