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1. INTRODUCTION

It is well known that a product-market monopolist selling a homogeneous good can extract more rents from consumers with private information than by charging a uniform price. The properties of optimal nonuniform pricing schemes have been analysed for the case of a continuum of consumers by Spence (1977), Mussa and Rosen (1978), Maskin and Riley (1984), and Goldman et al (1984), among others. One result of these models is that, to maximize profits, a constant-marginal-cost-monopolist will typically charge declining marginal and average prices, i.e. offer quantity discounts (see Spence, 1977, pp. 11-13). Another is that, since the marginal prices paid by all consumers but the "highest" are above marginal cost, the resulting monopoly policy is different from and less efficient than what would occur in a competitive market.

Perhaps surprisingly, the above results have apparently not been used to analyse the behavior of factor market monopolists such as trade unions. This oversight has occurred despite the fact that the theory of implicit employment contracts with asymmetric information (e.g. Hart 1983, Green and Kahn 1984, Rosen 1985) analyses an essentially isomorphic problem, where the continuum of consumers is replaced by a continuum of states the firm may be in.[1] This paper develops a model of trade union wages and employment that is based on nonuniform pricing theory. Its main conclusions are as follows.

First, under fairly general conditions implementation of a union's optimal nonuniform pricing policy for labor requires the use of a "seniority rule", which operates as follows: (1) Each worker is assigned a seniority index, the value of which is monotonically related to the amount of firm-specific human capital he possesses. (2) The union assigns a wage to each worker in such a way that the wages of more "senior" workers who have more specific skills are "marked up" more relative to their opportunity wages than are junior workers' wages. And (3) the firm is required to hire and lay off workers strictly according to the seniority index, with the more able, more expensive workers getting the highest employment probabilities. Thus, the present paper is apparently the first to offer a formal explanation of why unions impose seniority systems on firms.[2]

Second, the model has some interesting implications for union politics, an issue which is typically dealt with in one of two rather unsatisfactory ways in existing models. The most common approach is simply to assume a union has preferences over total wages and employment without specifying how these preferences arise from the utility functions of individual union members, or to make an assumption like random layoffs or complete work sharing which eliminates conflict of interest among union members, so that the union's preferences are those of "representative member". The second approach is explicitly to model the union's seniority system for layoffs while continuing to assume the union must charge the firm the same wage for all its workers (e.g. Grossman 1983, Blair and Crawford 1984). This assumption generates conflict between union members with different seniority ranks, which is then assumed to be "resolved" via some political process like majority voting. Such a process may of course demand considerable union resources to operate, and may have results which are difficult or costly to enforce.

The current model shows that, in certain situations, the introduction of different wages for different workers into a seniority model, rather than complicating matters, can greatly simplify the union's political problem because it gives rise to a kind of "independence" property: Subject to certain restrictions, the union's optimal wage for each worker will be independent of all other workers' wages and identical to that worker's own most preferred wage, given his seniority rank. Thus, within limits, the optimal union wage policy could be unanimously approved by all members, and a union can be operated with a rather minimal need for internal political decisionmaking.[3]

A final set of results consists of some potentially testable implications of the model. These include the prediction that unionism lowers the equilibrium employment probabilities of (almost) all workers in the firm, and makes the level of employment more cyclically volatile than in nonunion firms. Also, the union-nonunion average wage differential is predicted to move countercyclically.

The paper is organized as follows. First, in section 2, I present the structure of the basic model. This is a purely static model, with a continuum of union members whose hours of work are fixed if they are employed. For simplicity, each of these workers is thought of as having a kind of "property right" to the payments the firm makes for his services, so that the union cannot force the firm to pay one worker (or union official) for work done by another. In addition, all workers are all equally endowed with human capital, perfectly substitutable, and risk neutral. I also ignore bargaining considerations between firm and union by assuming no isoprofit constraint is binding on the union's optimal policy.

Section 3 characterizes the solution to the basic model and establishes the paper's main results. Perhaps its most interesting point is that, even in a one-period model with identical workers, a "seniority system" regulating the order of layoffs and rehires is typically in the union's interest.

Section 4 of the paper extends the basic model by relaxing some of its main assumptions in turn. These are: (1) the lack of ex ante union-firm bargaining, (2) identical workers, (3) fixed hours per employed worker, (4) "property rights", and (5) other considerations.

The paper concludes in Section 5 with a brief discussion of some issues concerning empirical testing of the model.

2. THE BASIC MODEL: STRUCTURE

Imagine a risk-neutral firm employing one variable input, labor. Its production function is given by $F(L)$, where $L \in R_+$ is the amount of labor hired, measured in efficiency units. Define $f(L) \equiv F'(L)$, and assume $f(L)$ is decreasing and continuous.[4] Throughout the paper, it will be useful to index units of labor, or human capital by ℓ, i.e. to use $L = \int_{\ell=0}^{L} d\ell$. Thus one can think of $f(\ell)$ as the marginal product of the ℓth unit of human capital hired, and analogously, of $w(\ell)$ as the marginal cost to the firm of the ℓth unit of human capital.

It is convenient to think of this firm as purely competitive in product markets, but as earning some rents to a fixed factor (say, entrepreneurial ability) which a union seeks to extract. The output price, θ, faced by the firm is unknown ex ante when the union designs its nonlinear pricing system, but is observed by the firm ex post in the single period when production occurs. Let θ be distributed on $[\underline{\theta}, \bar{\theta}]$, with $\bar{\theta} > \underline{\theta} > 0$. Denote the density of θ by $m(\theta)$, its right-hand cumulative by $M(\theta) = \int_{u=\theta}^{\bar{\theta}} m(u)du$, and assume $m(\cdot)$ is continuous. Throughout the paper, I assume the firm is able to take two kinds of actions to maximize profits, both of them <u>ex post</u> (after θ has been revealed and after the union has committed itself to its nonuniform pricing schedule). These are (1) choosing the level of total labor input L, and (2) if not restricted in its choice by union rules, choosing the <u>composition</u> of L, i.e. which workers are to be employed.[5]

The union facing this firm is a continuum of members, each identified by a value of $z \in (0,Z]$, where Z exceeds the highest employment level ever chosen by the firm.[6] Union members are risk neutral, and all have the same utility function:

$$V(\omega,e) = \omega + \bar{\omega}[1-e] \tag{1}$$

where ω is a worker's total compensation from employment, e is hours worked, or "effort", and $\bar{\omega}$ is a parameter measuring the value of leisure or of alternative employment, assumed to be the same for all workers. Worker z's level of firm-specific skill as well as his labor supply "technology" are summarized by the function h(e,z), which gives the total amount of human capital supplied by worker z when working e hours.

Since a worker's compensation may depend in general on both his identity and on the state of nature, his expected utility can be written:

$$U(z) = \int_{\theta=\underline{\theta}}^{\bar{\theta}} [\omega(z,\theta) - \bar{\omega}e(z,\theta)]m(\theta)d\theta \tag{2}$$

The union is assumed to behave as though it maximizes the following welfare function:

$$\mathcal{W} = \int_{z=0}^{Z} A[z, U(z)]dz \tag{3}$$

where $A_2 \geq 0$, and $A_{22} \leq 0$.[7] Of course this objective function admits a number of interesting special cases,[8] one of which will serve as a useful reference point below. This is the case where $A_{12} = A_{22} = 0$, an objective one would expect the union to adopt if it were indifferent to distribution, or, contrary to the "property-rights" assumption described below, if it had unlimited access to lump sum inter-worker transfers which would serve to "linearize" its utility-possibility frontier. This is because $A_{12} = A_{22} = 0$ implies the union maximizes expected total rents extracted from the firm.

In addition to the above underlying structure, in the basic model I make a number of additional simplifying assumption which are dropped in turn in Section 4. The first of these is that hours per employed worker are exogenously fixed, without loss of generality at e=1; the second that all workers are equally able, i.e. h(e,z) is independent of z and without loss of generality h(1,z) = 1.

Third, I assume in the basic model that each union member has a kind of "property right" to the payments made by the firm for his services. In other words, whenever worker z supplies a certain amount of labor to the firm, worker z receives whatever the firm pays for that labor.[9] In the context of the basic model, this implies simply that:

$$y(z,\theta) = \omega(z,\theta) = \begin{cases} 0 \text{ if worker } z \text{ is unemployed}^{10} \\ w(\ell(z)) \text{ if worker } z \text{ is employed} \end{cases} \quad (4)$$

In other words, because in the basic model each unit of human capital hired ℓ, corresponds to a specific worker, z, worker z's total income in state θ, $\omega(z,\theta)$, equals the amount paid by the firm to worker z in state θ, $w(z,\theta)$, which is also the entire amount paid to the union for the unit of human capital he supplies.

A final restriction used to simplify the basic model is that no *ex ante* expected profit constraint is imposed.

What tools are available to the union seeking to extract rents from this firm? In the basic model I consider two instruments, consisting of functions which are chosen before θ is realized. The first of these is a wage schedule, $w(z)$, specifying each worker's wage. The second is a seniority assignment

$n(z)$, and seniority rule which together operate as follows:

1) The union assigns a "seniority index" $n \in (0,Z]$ [11] to each of its members, by choosing a one-to-one mapping $n(z)$, or (equivalently) its inverse $z(n)$.

2) The union imposes on the firm a "seniority rule", which states that, whenever worker n' is hired, all workers with $n < n'$ must be hired also.

Given this system, it is natural to refer to workers with low values of the seniority index as "senior" workers; thus a worker's "seniority index", n, will be inversely related to his level of "seniority", in common parlance. Indeed, because workers are identical in the basic model, the indexes ℓ and n are interchangeable, and I shall refer throughout section 3 to "worker ℓ" as the worker who is identified, via the seniority rule, with the ℓth unit of human capital hired by the firm. Thus the index n will be dropped and replaced by ℓ until it is needed in Section 4, and $\ell(z)$ will denote the seniority-assignment rule until then.

The rationale for considering a seniority rule is most easily seen by considering the firm's response to an arbitrary $w(z)$ in the absence of such a rule. Because, in any state, the firm will then always hire the cheapest workers first, the firm's total outlays on labor, $W(L)$ will necessarily be increasing at a nondecreasing rate with L, i.e. the union can enforce quantity premia but not quantity discounts. On the other hand, by using the seniority rule, the union can impose an arbitrary total outlay schedule, $W(L)$

on the firm, making the union's problem here into a general optimal nonuniform pricing problem. In section 3 I shall solve this problem, noting that, if in its solution, the optimal W(L) involves any quantity discounts, the seniority rule just described will be strictly necessary for such a schedule to be enforced.

Given all the assumptions of the basic model, the union's maximization problem can now be written:[12]

$$\max_{w(\ell),\ \ell(z)} \mathcal{W} = \int_{z=0}^{Z} A[z,\ p(\ell(z))w(\ell(z)) + [1-p(\ell(z))]\bar{\omega}]dz \tag{5}$$

subject to:

$$w(\ell) \geq \bar{w},\ \forall \ell \tag{6}$$

where $p(\ell)$ is the employment probability of worker ℓ given the entire seniority-wage profile and given optimizing behaviour by the firm, and $\bar{w} = \bar{\omega}$ is the opportunity cost to the union of each unit of human capital sold to the firm. It may be worth noting here that (5) - (6) implicitly embody the assumption $W(0) = 0$, since $W(L) = \int_{\ell=0}^{L} w(\ell)d\ell$.

3. THE BASIC MODEL: SOLUTION AND RESULTS

A necessary condition for a maximum of (5) with respect to both $w(\ell)$ and $\ell(z)$ is that $w(\ell)$ maximizes (5) given an <u>optimal</u> assignment of seniority rights $\ell(z)$. I make use of this property in this section by solving (5)-(6) conditional on an arbitrary $\ell(z)$ and noting that the resulting conditions must hold given the optimal $\ell(z)$ as well. Thus (5) is replaced by:

$$\max_{w(\ell)} \mathcal{W} = \int_{\ell=0}^{Z} \alpha[\ell,\ p(\ell)w(\ell) + (1-p(\ell))\bar{w}]d\ell \tag{7}$$

with $\alpha_2 \geq 0$, $\alpha_{22} \leq 0$. In the case of the rent-maximizing union this procedure completely characterizes the optimal seniority wage profile, since in that problem, when workers are equally productive and have the same $\bar{\omega}$ as they do here, the union's optimal wage policy will be <u>independent</u> of $\ell(z)$. In general however first-order conditions for $w(\ell)$ must be interpreted with more care because they are conditional on $\ell(z)$. I now state:

<u>PROPOSITION 1</u>: For any $w(\ell)$, which may possess any finite number of discontinuities or isolated points, the employment probability of worker ℓ will be given by either: (i) the probability his own VMP exceeds his own wage, i.e.:

$$p(\ell) = M\left(\frac{w(\ell)}{f(\ell)}\right) \tag{8}$$

or (ii) the probability that the combined VMP's of the workers in some interval $[\underline{\ell}, \bar{\ell}]$, which contains ℓ, exceed their combined VMP's, i.e.:

$$p(\ell) = M\left(\frac{\int_{s=\underline{\ell}}^{\bar{\ell}} w(s)ds}{\int_{s=\underline{\ell}}^{\bar{\ell}} f(s)ds}\right) \equiv M(\theta^*), \text{ for } \underline{\ell} \leq \ell \leq \bar{\ell}. \tag{9}$$

<u>PROOF</u>

See Appendix.

If worker ℓ's employment probability is given by (i) in Proposition 1, I shall say he belongs to a U-interval, for "unrelated" workers. This is because, as is shown below, the optimal wage for worker ℓ is independent of wages charged for all other workers in the interval. The other distinguishing characteristic of U-intervals is that, throughout a U-interval, $\frac{d\theta^*}{d\ell} > 0$, where $\theta^*(\ell)$ is defined as the lowest state in which worker ℓ is employed. Thus employment adjusts continuously to the state, and each worker in the interval has a different employment probability.

If worker ℓ's employment probability is given by (ii), he is said to be in a B-interval, for "bundled" workers, since all workers in the interval share the same employment probability. Thus, over B-intervals, $\frac{d\theta^*}{d\ell} = 0$, and employment adjusts discontinuously to the state. B-intervals can occur when wages decline rapidly with ℓ, as is shown in Figure 1(a), in which area A=B, and θ^* gives the common employment probability of all workers in the B-interval. I shall henceforth call such intervals "interior" B-intervals. Indeed, downward discontinuities in $w(\ell)$ always create an interior B-interval, as shown by the second B-interval in Figure 1(b) with θ_2^*. B-intervals can also occur when θ is at an endpoint and, for example p=1 as is shown by the first bundling interval in Figure 1(b), with θ_1^*.[13]

<u>COROLLARY</u>

The union's optimal wage and employment policy within any B-interval or within any U-interval is independent of the choices it makes anywhere outside that interval.

<u>PROOF</u>

To see this, simply write out the expression for the union's maximand, 4, for the interval in question, as is done in (10) and (14) below.

Proposition 1 and its corollary mean it is legitimate to construct the union's optimal seniority-wage policy for a given $\ell(z)$ by considering optimal policies in U-intervals and B-intervals in isolation and then examining when and where each type of interval will occur, which I do in turn below.

3.1 OPTIMAL POLICY IN A U-INTERVAL

Within a U-interval, $\ell \in (\underline{\ell}, \bar{\ell})$ the union's objective function is just:[14]

$$= \int_{\ell=\underline{\ell}}^{\bar{\ell}} \alpha\{\ell, [w(\ell) - \bar{w}]M(\frac{w(\ell)}{f(\ell)})\}d\ell \qquad (10)$$

This is independent of wage policy outside the interval. Pointwise optimization of (10) with respect to $w(\ell)$, assuming the individual-rationality constraint does not bind, yields the first-order condition,

$$M(\frac{w(\ell)}{f(\ell)}) = (w(\ell) - \bar{w})m(\frac{w(\ell)}{f(\ell)}) \cdot (\frac{1}{f(\ell)}), \ \forall \ell \qquad (11)$$

The optimal policy summarized in (11) has two key properties. The first is that, if worker ℓ is in a U-interval, the optimal wage for worker ℓ is both independent of the form of the union's welfare function $\alpha[\ell, U(\ell)]$ and of the wages charged for all other workers. The second is apparent when we consider the following problem:

$$\underset{w}{\text{Max}} \ (w - \bar{w})M(\frac{w}{f(\ell(z))}) \qquad (12)$$

i.e. choose the single monopoly wage which maximizes worker z's expected utility conditional on his seniority assignment, which has the same first-order condition as (10) for each ℓ. Thus, the union's optimal wage for worker z is identical to the wage he would choose to set for himself, conditional on his seniority level.

The second-order condition for a maximum of (10) is given by:

$$2m^2 + Mm' > 0. \qquad (13)$$

I shall assume henceforth that the distribution of θ satisfies this condition globally, which rules out the possibility of upward discontinuities in $w(\ell)$ in the interior of U-intervals, such as that shown in Figure 1(b).

3.2 OPTIMAL WAGE POLICY IN A B-INTERVAL

Suppose the union chose to implement a "bundling" policy for some "interior" group of workers $[\underline{\ell}, \bar{\ell}]$.[15] What is the optimal wage and employment policy in such an interval? The union's maximization problem in this interval is now:

$$\max_{w(\ell),\theta^*} \mathcal{W} = \int_{\ell=\underline{\ell}}^{\bar{\ell}} \alpha\{\ell,\ [w(\ell) - \bar{w}]M(\theta^*)\}d\ell \tag{14}$$

subject to:
$$\int_{\ell=\underline{\ell}}^{\bar{\ell}} w(\ell) = \theta^* \int_{\ell=\underline{\ell}}^{\bar{\ell}} f(\ell) \tag{15}$$

$$\int_{s=\underline{\ell}}^{\ell} w(s) \geq \theta^* \int_{s=\underline{\ell}}^{\ell} f(s), \quad \forall \ell \in [\underline{\ell}, \bar{\ell}] \tag{16}$$

and
$$w(\ell) \geq \bar{w}, \quad \forall \ell \in [\underline{\ell}, \bar{\ell}]. \tag{17}$$

Condition (15) is an isoperimetric constraint giving the (common) employment probability, θ^*, of all the workers in the interval, as stated in Proposition 1. It guarantees that the firm is indifferent between $\underline{\ell}$ and $\bar{\ell}$ in state θ^*. Condition (16) incorporates the fact that, if $[\underline{\ell}, \bar{\ell}]$ is truly a single bundling interval, the firm must (at least weakly) prefer its endpoints to any interior point. It is easily shown to be nonbinding in the optimal policy if seniority assignments, $\ell(z)$, have been optimally chosen.[16] Individual rationality is incorporated in (17).

Thus (14)-(17) can be solved by maximizing:

$$\mathcal{W} = \int_{\ell=\underline{\ell}}^{\bar{\ell}} \alpha\{\ell,[w(\ell) - \bar{w}]M(\theta^*)\} + \lambda[\theta^* f(\ell) - w(\ell)] + \phi(\ell)[w(\ell) - \bar{w}]\}d\ell \tag{18}$$

First-order conditions are:

$$\alpha_2\{\ell,\ [w(\ell) - \bar{w}]M(\theta^*)\} - \lambda + \phi(\ell) = 0, \quad \forall \ell \in [\underline{\ell}, \bar{\ell}] \tag{19}$$

$$-\int_{\ell=\underline{\ell}}^{\bar{\ell}} \alpha_2\{\cdot\}\ (w(\ell) - \bar{w})m(\theta^*)d\ell + \lambda \int_{\ell=\underline{\ell}}^{\bar{\ell}} f(\ell)d\ell = 0 \tag{20}$$

$$\phi(\ell)[w(\ell) - \bar{w}] = 0 \tag{21}$$

Solving for $\alpha_2\{\ \cdot\ \}$, and substituting (19) and (21) into (20) yields:

$$M(\theta^*) = m(\theta^*) \frac{\int_{\ell=\underline{\ell}}^{\bar{\ell}} [w(\ell) - \bar{w}]d\ell}{\int_{\ell=\underline{\ell}}^{\bar{\ell}} f(\ell)d\ell} \tag{22}$$

which is the same condition as (11), except that it applies to the entire group of workers in the interval.

Thus, employment policy in a B-interval is determined purely by the efficiency condition (22), which is the condition for maximization of union rents subject to the constraint that all workers in the interval $[\underline{\ell}, \bar{\ell}]$ be assigned the same employment probability, or the same θ^*. This efficiency condition uniquely determines the wage bill $\int_{\ell=\underline{\ell}}^{\bar{\ell}} w(\ell)$ that can be charged for all workers in $[\underline{\ell}, \bar{\ell}]$. Wage policy on the interval, given this total wage bill, the individual rationality constraint (17), and constraint (16), is then dictated purely by distributional considerations.

3.3 OPTIMAL SENIORITY POLICY

In this section I present several main results concerning the union's optimal overall policy. It is useful to begin with some definitions.

Definition 1: (Feasibility of an optimal U-policy) Denote the solution to first-order condition (11) at a given ℓ by $w^*(\ell)$, and define $\hat{\theta}^*(\ell) = \frac{w^*(\ell)}{f(\ell)}$. Then an optimal U-policy is feasible at ℓ if and only if the unconstrained solution to (11) is feasible and consistent with a U policy, i.e. it exhibits $\frac{d\hat{\theta}^*}{d\ell} > 0$. This requires:

$$\underline{\theta} \leq \hat{\theta}^*(\ell) \leq \bar{\theta} \tag{23}$$

and $$\frac{\hat{\theta}^*(\ell)}{M(\hat{\theta}^*(\ell))} m(\hat{\theta}^*(\ell)) > 1 \tag{24}$$

Condition (23) ensures that $\hat{\theta}^*(\ell)$ is not beyond the boundary of the feasible set, while Condition (24) ensures that it is strictly increasing in ℓ. More intuitively perhaps, (24) simply guarantees that the first-order condition (11) for worker ℓ's individual monopoly pricing problem (12) can be satisfied by a finite $w^*(\ell)$, by requiring that the elasticity of demand for worker ℓ's services exceed 1.

Definition 2: (Wage Trends over B-intervals). Consider the wages of two workers, $\underline{\ell} - \varepsilon$ and $\bar{\ell} + \varepsilon$, on either side of a B-interval $(\underline{\ell}, \bar{\ell})$, where ε is arbitrarily small. Then wages will be said to increase on average with seniority over the B-interval if $w(\underline{\ell} - \varepsilon) > \tilde{w} > w(\bar{\ell} + \varepsilon)$, where $\tilde{w}$ is the average wage of all workers in $(\underline{\ell}, \bar{\ell})$, i.e. $\tilde{w} = \frac{1}{\bar{\ell}-\underline{\ell}} \int_{\ell=\underline{\ell}}^{\bar{\ell}} w(\ell)d\ell$.

I now present four main results.

RESULT 1 (Optimal Wage Policy)

Wages will increase monotonically with seniority in U-intervals and will increase "on average" with seniority over B-intervals if the following condition holds:

$$\theta m^2 + Mm + \theta Mm' > 0 \tag{25}$$

PROOF

In a U-interval, it is easy to show that:

$$\frac{dw}{df} = \frac{\theta m^2 + Mm + \theta Mm'}{2m^2 + Mm'} \tag{26}$$

where the denominator is positive by the second-order condition, (13). In a B-interval, rewrite (22) as:

$$M\left(\frac{\tilde{w}}{\tilde{f}}\right) = (\tilde{w} - \bar{w})\, m\left(\frac{\tilde{w}}{\tilde{f}}\right) \frac{1}{\tilde{f}} \tag{27}$$

with $\tilde{f}$ defined analogously to $\tilde{w}$. Now since $f(\underline{\ell} - \varepsilon) > \tilde{f} > f(\bar{\ell} + \varepsilon)$,

then $w(\underline{\ell} - \varepsilon) < \tilde{w} < w(\bar{\ell} + \varepsilon)$, provided (25) holds globally.

Q.E.D.

Is the condition for wages to rise with seniority, (25), likely to be satisfied? Sufficient conditions for (25) to hold are alternatively: $m' \geq \frac{m}{\theta}$, or $m^2 + Mm' \geq 0$. The first of these requires that $\theta \cdot m(\theta)$ is increasing with θ. The second can be expressed as $\frac{d}{d\theta}\left(\frac{m(\theta)}{M(\theta)}\right) \geq 0$, i.e. the hazard function is nondecreasing. This nondecreasing hazard condition is satisfied globally by a large number of distributions, including the normal.

Condition (25) is of course satisfied locally for any distribution whenever $m' > 0$, as well as at $\bar{\theta}$, where $M=0$. Thus wages must rise with seniority among the most junior workers in the union, as well as for a finite group of senior workers, for any unimodal $m(\theta)$.

Of course, one important implication of Result 1 concerns the value of the seniority rule to the union. Since a seniority rule is <u>always</u> needed to implement B-intervals, and since it has positive value in U-intervals when (25) holds and zero value otherwise, it is clear that a comprehensive seniority rule (covering all workers in the union) has value to the union under fairly general conditions. Indeed, the distribution of θ need only satisfy condition (25) for <u>some</u> values of θ.

Another implication of Result 1 is that the average union-nonunion wage differential should move countercyclically, i.e. fall with θ, regardless of whether B-intervals occur or not. To see this, note that the nonunion average wage is constant over the cycle at $\bar{w}$, that the unionized firm's total wage bill $W(L) = \int_{\ell=0}^{L} w(\ell)d\ell$ increases at a decreasing rate with employment in U-intervals and on average over B-intervals, that employment is always procyclical, (from lemma A3 in the appendix), and that <u>only the endpoints</u> of B-intervals are ever realized as optimal employment levels. Thus the quantity discounts implemented by the union will manifest themselves in higher average prices paid by the union firm for labor in bad states of nature.

<u>RESULT 2</u> (Efficiency of the Optimal U-policy)

Consider any group of workers (perhaps the entire union) for whom an optimal U-policy is feasible. Then a rent-maximizing union will always choose to implement the optimal U-policy for those workers.

PROOF: In Appendix.

Result 2 has two main implications. The first of course is that, under what appear to be reasonable conditions, rent-maximizing unions will not "bundle" workers.

The second is that, since the optimal U-policy is independent of the union's distributional preferences, changes in union preferences away from rent-maximization will not affect the union's optimal policy unless they induce a change of regime and cause bundling of workers. Since, as Kuhn (1986) shows for the two-worker case, such deviations from rent-maximization must typically be large if they are to induce a "regime-switch", it appears that the optimal U-policy is likely to be the union's best policy for a wide variety of possible union objective functions. Indeed, since (as was shown above) the optimal U-policy sets each worker's wage at the level he would choose himself, conditional on his seniority rank, the union's optimal policy, independent of its distributional preferences, is likely to have the following property: Suppose each worker z had the right to unilaterally choose the union's wage profile, $w(\ell)$, subject to the restriction that the profile chosen does not "bundle" him with other workers. Then, regardless of the assignment of seniority indexes, each worker will choose the same wage profile, namely the optimal U-policy. In this sense, the union's policy will be unanimously supported by its members.

The fact that the union's seniority-wage policy is likely to be independent of its distributional preferences (i.e. the form of $\alpha(\cdot, \cdot)$) underscores clearly the difference between the present model of union seniority and a well-known informal view of union seniority that views such systems as arising from the fact that unions "care" more about their older members (see, for example Freeman and Medoff, 1979). Seniority is used here primarily because it increases the efficiency of rent-extraction by a union in a world of asymmetric information, and even in the absence of lump sum transfers between workers the union's preferred policy may often be invariant to its distributional goals. This occurs essentially because under the nonuniform pricing technology there is a "kink" in the union's utility-possibility frontier at the efficient point (see Kuhn (1986) for an elaboration).

RESULT 3 (Underemployment)

Regardless of whether or not bundling intervals occur, the union's optimal employment policy has the following properties:

Relative to a competitive firm, a union lowers the equilibrium employment probabilities of all workers except (i) the least senior worker ever hired in the nonunion firm, whose employment probability is unchanged; (ii) (perhaps) the least senior worker in any bundling interval, whose employment probability may be unchanged by the union; and (iii) (possibly) some subset of the set of workers who are employed with certainty in the nonunion firm, if this set is nonempty. Some of these latter workers may remain employed with certainty under the union but if such a group exists, it is smaller in the union.

PROOF: In Appendix.

An interesting implication of Result 3 is that, regardless of how much a union "cares" about its senior members, it is never optimal to assign them greater job security than they would have in a nonunion setting. Another implication is that the level of employment in union firms is likely to be more sensitive to the "business cycle", θ than in nonunion firms, in the sense that union employment equals nonunion employment in the best state, but that the union extends the range of equilibrium employment levels downward and lowers every worker's employment probability.

<u>RESULT 4</u> (Assignment of Seniority Indexes)

If $\alpha_{22} = 0$, the union's optimal assignment of seniority indexes <u>within</u> any U- or B-interval is simply to assign higher seniority to workers with higher "welfare weights". This monotonicity property does not necessarily hold across intervals of different types.

<u>PROOF</u>

If $\alpha_{22} = 0$, it must be that the union's maximand, defined on z, can be written $\mathcal{W} = \int_{z=0}^{\infty} \gamma(z)U(z)$ where γ is a "welfare weight" and U the expected utility of worker z. Since, using the envelope theorem it is easy to show that $\frac{\partial U}{\partial \ell} > 0$ in a U-interval, and since (16) requires utility to increase "on average" within a B-interval, the union can do no better within a given interval than to have $\ell(\gamma(z))$ monotonically decreasing everywhere. Finally, to see why this monotonicity may not hold across intervals, consider the optimal wage policy in a B-interval which will involve giving all the <u>surplus</u> extracted on this interval to the most senior worker in it, $\underline{\ell}$, and setting $w = \bar{w}$ for everyone else. Clearly the seniority rank just below $\underline{\ell}$ is very undesirable and could easily be dominated by a lower rank outside the B-interval. Q.E.D.

Result 4 indicates that a union with strong distributional preferences may wish to satisfy these by assigning high seniority ranks, (but low wages) just below workers it cares about a great deal, to workers it cares about not at all. Since this seems quite implausible, it suggests that B-intervals are unlikely to exist in reality.

4. EXTENSIONS AND ALTERNATIVE FORMULATIONS

In this section I extend the model in several directions, considering only the case where the union maximizes rents and there are no interior bundling intervals in the optimal policy. This greatly simplifies the analysis and, given the likely consequences of bundling explored in Section 3 (particularly Result 4) seems a reasonable restriction.

4.1 UNION-FIRM BARGAINING AND THE ROLE OF ISOPROFIT CONSTRAINTS

In order to place the foregoing analysis into a context of union-firm bargaining, it is useful to sketch briefly the nature of the utility-possibility frontier between the firm and a rent-maximizing union that exists under the nonuniform pricing technology. The general shape of such a frontier is shown in Figure 2, which considers the case where $\lim_{\ell\to 0} \underline{\theta} f(\ell) > \bar{w}$, i.e. it is jointly efficient to employ some group of workers $\ell \in (0, \ell']$ in all states. This case is shown in Figure 1(a), which indicates that, in this case, the union can extract any amount of rents up to area C without compromising efficiency.[17] Thus, the utility-possibility frontier in Figure 2 has a 45^{o} section between points a and b. (If $\lim_{\ell\to\theta} \underline{\theta}\, f(\ell) \leq \bar{w}$, such a section will not exist). If the union extracts more than A from the firm, the previous analysis shows it will typically both (a) set wages above $\bar{w}$ for workers with $\ell > \ell'$, and (b) extend the range of realized employment levels downward (i.e. lower ℓ', perhaps to zero). This compromises overall efficiency, giving the utility-possibility frontier a steeper slope than 45^{o}, between points b and c. Maximum rents are extracted from the firm at point c, and since of course one can always design wage profiles that eliminate all employment, the utility-possibility frontier must return to the origin beyond c.

For simplicity, Sections 2 and 3 characterized the behavior of a union that was able to achieve point c. This section considers what happens when the firm has sufficient bargaining power to require the solution to be on bc, by imposing a binding ex ante, isoprofit constraint in the optimization. The results can thus be thought of as the nonuniform pricing policy a firm and union agree to during ex ante contract negotiations, and against which both parties know the firm will optimize ex post, during the life of the contract.

When expected profits are constrained to equal $\bar{\Pi}$ and there are no interior B-intervals, the union's rent-maximization problem can be written:

$$\max_{\ell',w(\ell)} \int_{\ell=0}^{\ell'} [f - \bar{w}]d\ell + \int_{\ell=\ell'}^{\hat{\ell}} (w-\bar{w})M(\frac{w}{f})d\ell \tag{28}$$

subject to:

$$\int_{\ell=\ell'}^{\hat{\ell}} \int_{\theta=\frac{w}{f}}^{\bar{\theta}} [\theta f - w]\, m(\theta)d\theta d\ell = \bar{\Pi} \tag{29}$$

where $\hat{\ell}$ is defined by $\bar{\theta}f(\hat{\ell}) = \bar{w}$, which identifies the least senior worker ever employed in the union. (See Result 3).

First-order conditions for the above problem with respect to $w(\ell)$ are:

$$(1 - \lambda)M(\frac{w}{f}) = (w - \bar{w})\frac{m}{f} \tag{30}$$

where λ, the multiplier on the isoprofit constraint (29), is easily shown to be strictly between zero and one for all solutions on segment bc.

The optimal wage profile described by (30) is essentially identical in structure to that derived in Section 3 with the exception that wages are lower for every worker. To see this, totally differentiate (30) and solve for:

$$\frac{dw}{df} = \frac{1}{m\Delta} (\theta m^2 + Mm + \theta Mm') \tag{31}$$

$$\frac{dw}{d\lambda} = \frac{-1}{\Delta(1-\lambda)} fM < 0 \tag{32}$$

where $\Delta > 0$ by the second order condition for a maximum. The condition for wages to rise with seniority (i.e. for a binding seniority rule to be optimal) is the same as before; however the union's optimal wage for each ℓ decreases with the firm's "bargaining power", λ. As a consequence, whenever $\lambda > 0$, the union's optimal wage for member ℓ is below what that member would unilaterally choose himself, given his seniority rank, and the convenient "unanimity" property of Section 3 no longer holds. Thus the main effect of a binding isoprofit constraint is to create an additional need for unions to "discipline" their members, keeping each individual's wage below his privately optimal level. Such discipline problems may indeed be important issues in actual union politics.

4.2 WHY ATTACH SENIORITY INDEXES TO YEARS OF SERVICE IN THE FIRM? SPECIFIC HUMAN CAPITAL IN THE MODEL

4.2.1 Amending the basic model

Suppose now that, instead of being equally endowed, union members are endowed with varying amounts of firm-specific human capital, h, given by the function $h(z)$, $h \in [0,\bar{h}]$. Workers all continue to have identical opportunity incomes given by $\bar{\omega}$, however. Given a seniority assignment rule $n(z)$, the total amount of human capital available when N workers are employed is now:

$$L(N) = \int_{n=0}^{N} h(n)dn \tag{33}$$

Equation (33) implies the following relationship between the indexes ℓ and n:

$$\frac{dn}{d\ell} = \frac{1}{h} \tag{34}$$

which now replaces the property n=ℓ used in Sections 2 and 3.

When $A_{12} = A_{22} = 0$, the union maximizes:

$$\mathcal{W} = \int_{n=0}^{Z} [\omega(n)p(n) + \bar{\omega}(1-p(n))]dn \tag{35}$$

where $\omega(n)$ is the total compensation of worker n when employed. Using (34) this can be written:

$$\mathcal{W} = \int_{\ell=0}^{\bar{L}} [\omega(\ell)p(\ell) + \bar{\omega}(1-p(\ell))] \frac{1}{h(\ell)} d\ell \tag{36}$$

or

$$\mathcal{W} = \int_{\ell=0}^{\bar{L}} [w(\ell)p(\ell) + \bar{w}(\ell)(1-p(\ell))]d\ell \tag{37}$$

where $w(\ell) = \frac{\omega(\ell)}{h(\ell)}$ is the marginal cost to the firm of the ℓth unit of human capital, and $\bar{w}(\ell) = \frac{\bar{\omega}(\ell)}{h(\ell)}$ is the marginal opportunity cost to the union of that unit of human capital. $\bar{L}$ is the total amount of labor at the union's disposal, which is no longer necessarily equal to Z. Thus, the union's problem in determining the optimal marginal outlay schedule $w(\ell)$, is the same as in the basic model, except that $\bar{w}$ now varies with ℓ and is endogenously determined by the seniority assignment rule through $h(\ell)$.

4.2.2 Finding the optimal seniority assignment

To find the optimal assignment of seniority indexes in the presence of specific human capital, note first that the union's total opportunity cost of labor schedule is $\bar{W}(L) = \omega N(L)$, where $N(L)$ is the inverse of $L(N)$ in (33). Also note that $\bar{W}(L)$ must satisfy the constraint $\bar{W}(L) \leq \bar{W}_{min}(L)$, where $\bar{W}_{min}(L)$ is the lowest feasible total opportunity cost of L units of human capital, which can be achieved only by assigning seniority indexes (inversely) according to in-firm productivity. Now suppose we allow the union to choose $\bar{W}(L)$ freely, subject only to his inequality constraint. Then if the union's optimal $W(L)$ is achievable via a one-to-one seniority mapping $n(z)$, it must be the optimal $\bar{W}(L)$ for all possible seniority mappings.

The rent-maximizing union thus solves:

$$\max_{W(\ell),\ \bar{W}(\ell)} \int_{\ell=0}^{\bar{L}} [W'(\ell) - \bar{W}'(\ell)]M\left(\frac{W'(\ell)}{f(\ell)}\right) d\ell \tag{38}$$

$$\text{subject to:} \quad \bar{W}(\ell) \geq \bar{W}_{min}(\ell) \tag{39}$$

Defining the control variables $\bar{w} = \bar{W}'$ and $w = W'$ this yields the Hamiltonian:

$$\mathcal{H} = [w - \bar{w}]M[\tfrac{w}{f}] + \lambda_1 w + \lambda_2 \bar{w} + \phi\,[\bar{W} - \bar{W}_{min}] \tag{40}$$

To see what the optimal $\bar{w}$ will be here, it is sufficient to note that, at the optimum, the inequality constraint (39) must always be binding (this can be seen by deriving first-order conditions, or by noting that the Hamiltonian is globally only weakly concave in the controls). Thus, the optimal

$\bar{W}(\ell) = \bar{W}_{min}(\ell)$, which is of course achievable only via the one-to-one seniority assignment rule that assigns higher seniority to workers with the most specific capital.

4.2.3 Changes in Results

The introduction of specific human capital has two main types of effects on the current model's conclusions. The first of these concerns the union's optimal policy at any one time given the distribution of specific human capital among its members. What the above analysis shows is that it is clearly in the collective interest of current union members to assign seniority indexes directly according to the amount of specific human capital each worker has. Since firm-specific skills are typically thought to accumulate with years of service in the firm, this may explain why seniority is also typically assigned according to the same criterion. It also has some direct implications for the shape of seniority-wage profiles when training occurs. To see this, consider what happens to the union's optimal price per unit of human capital, w; to the union "markup", $w - \bar{w}$; and to the wage of worker n, $\omega(n) = w(n)h(n)$, as seniority increases, when seniority indexes are assigned optimally.

The answer to the first question is given by

$$\frac{dw}{df} = \frac{\partial w}{\partial f} + \frac{\partial w}{\partial \bar{w}}\frac{d\bar{w}}{df} = B + \frac{m^2}{2m^2+Mm'} \cdot \frac{d\bar{w}}{df} \tag{41}$$

where $B \equiv \dfrac{\theta m^2 + Mm + \theta Mm'}{2m^2 + Mm'}$.

Since the first term in (41) is expected to be positive but the second is negative, it is possible that, in firms where a lot of training occurs, i.e. where specific skills increase rapidly with seniority, the union's optimal policy involves a lower price per unit of human capital for more senior workers. This means that union seniority rules may not always be binding on the firm in such situations, since the firm will prefer to hire workers in the order prescribed by the union, even when the nondecreasing hazard condition (25) holds.

The answer to the second question is given by:

$$\frac{d}{df}(w - \bar{w}) = B + \frac{d\bar{w}}{df}\left\{\frac{m^2}{2m^2 + Mm'} - 1\right\} \tag{42}$$

which is positive under the nondecreasing-hazard condition $m^2 + Mm' \geq 0$. Thus the union charges higher absolute markups for more senior workers.

Finally, one can derive,

$$\frac{d}{df}(wh) = hB + h\,\frac{m^2}{2m^2 + Mm'}\,\frac{d\bar{w}}{df} + w\,\frac{dh}{df} \tag{43}$$

Using $\bar{w} = \dfrac{\bar{\omega}}{h(f)}$, and hence $\dfrac{d\bar{w}}{df} = -\dfrac{\bar{w}}{h}\dfrac{dh}{df}$, (43) becomes:

$$\frac{d}{df}(wh) = hB + \frac{dh}{df}\left[w - \bar{w}\cdot\frac{m^2}{2m^2 + Mm'}\right] \tag{44}$$

which is again positive under the nondecreasing hazard condition. Thus, even though the price charged for human capital may decline with seniority when h(f) increases rapidly, the total compensation of workers will increase with seniority regardless of the shape of h(f), as long as m(θ) satisfies the nondecreasing hazard condition (25).

The second main effect of specific human capital on the current model's conclusions arises when time is introduced into the model, and one thinks of the union as designing an optimal seniority policy each period. Since workers will now progress through seniority ranks over their lifetime, it seems possible, given the right restrictions and low enough worker discount rates, that not just the union's wage policy conditional on seniority, but also the policy of assigning seniority ranks according to years of service, could be approved by all union members because of its efficiency advantages.[18]

4.3 Variable hours per Worker

4.3.1 Amending the basic model

In this section I consider a situation where the labor-supply technology, $h(e(z),z)$ is fully general--in other words hours of work $e(z)$ can vary continuously for each worker, and workers may differ in firm-specific skills by having $h_z \neq 0$, $h_{ez} \neq 0$. To render the hours/bodies issue nontrivial, I suppose, due to fixed "setup" costs per worker, that average h per worker, $\frac{h(e,z)}{e}$ has a unique (local and global) maximum at some $e > 0$ for each z. To focus on essentials, the analysis is conditional on a given assignment of seniority rights $n(z)$.

Throughout this section, the union is assumed to have access to two policy instruments (in addition to the seniority rule). These are, first, a wage schedule $w(n,e(n))$, $\forall n \in (0,Z]$ specifying the total amount the firm must pay worker n if it wishes to employ him for $e(n)$ hours, and second, an "hours rule" $e(n,L)$, $\forall n \in (0,Z]$, $\forall L \in (0,\bar{L}]$, giving each worker's total hours when L units of labor are employed. Since hours are zero for all unemployed workers, the hours rule can also be summarized by a function $N(L)$ giving total employment, and e(n,L) defined only for employed workers $n \in (0,N(L)]$.

Given an hours rule, the total human capital employed by the firm can be written:

$$L = \int_{n=0}^{Z} h(e(n,L),n)dn = \int_{n=0}^{N(L)} h(e(n,L),n)dn \tag{45}$$

Its total opportunity cost to the union is given by:

$$\bar{W}(L) = \int_{n=0}^{Z} \bar{\omega}\, e(n,L)dn = \int_{n=0}^{N(L)} \bar{\omega}\, e(n,L)dn \tag{46}$$

The sum of union members' expected utilities in this case is:

$$= \int_{n=0}^{Z} \int_{\theta=\underline{\theta}}^{\bar{\theta}} \{\omega[n,\theta] - \bar{\omega}e[n,\theta]\}m(\theta)d\theta dz. \tag{47}$$

Given an hours rule e(n,L), this is again equivalent[19] to total rents as written

in (37), where $\bar{w}(\ell)$, the marginal opportunity cost to the union of a unit of human capital, is now given by:

$$\bar{w}(L) = \frac{d}{dL} \int_{n=0}^{Z} \bar{\omega}e(n,L)dn = \bar{\omega} \int_{n=0}^{Z} e_L(n,L)dn. \tag{48}$$

Thus the union's maximization problem when hours are variable can once again be reduced to a simple nonuniform pricing problem in which both the marginal price of human capital charged the firm, $w(\ell)$ and the marginal cost of human capital to the union, $\bar{w}(\ell)$, are endogenous. Since choosing $\bar{w}(\ell)$ is equivalent to choosing an hours rule, the optimal hours rule can then be inferred from the union's optimal $\bar{w}(\ell)$.

4.3.2. Finding the optimal hours rule

Not surprisingly, it is easy to show that the rent-maximizing union's optimal hours rule, like its optimal seniority assignment, is the one that minimizes the total opportunity cost of providing L units of human capital (i.e. (46)) for every L, for the same reasons as before. The proof is omitted here to save space.

4.3.3 Changes in results

Aside from the fact that, given that the union can impose an hours rule, the nonuniform pricing model applies when hours are variable as well, two additional results emerge from consideration of the variable-hours case. The first is apparent when the union's cost minimization problem (minimize (46) subject to (45)) is solved for the case of identical workers ($h_n \equiv 0$). It is then easy to show that optimal hours per employed worker are constant,[20] and thus the union's optimal "hours rule" is simply to prohibit hours reductions and require the firm to make all adjustments to labor input on the extensive margin. Second, when workers are not identical, more complex hours rules are required, in which, if specific skills increase with seniority, hours of employed workers are likely to be procyclical. Thus one would expect unions to try to impose different kinds of "hours rules" in firms where considerable training occurs than elsewhere. Possible examples of such rules are discussed in Slichter, Healy, and Livernash (1960, pp. 150-54), although it is unclear whether the rather crude rules in actual use are sufficient to achieve an efficient hours-bodies mix in all states.[21] To the extent that they do not and that efficient hours rules are too costly to enforce, unions will face a tradeoff between rent-extraction and efficiency that is not analysed here, but constitutes an interesting topic for future research.

4.4 Alternative Allocations of "Property Rights"

4.4.1 Amending the basic model

In the basic model, the "property rights" assumption implied that, because he can be paid by the firm only for his own services, a worker's total compensation can take on only two values: $\omega=w$ if employed, and $\omega=0$ if not.

When the property rights assumption in the basic model is replaced by the general function $\omega = \omega(\ell, L)$, the union's maximand becomes:

$$= \int_{\ell=0}^{Z} [\int_{\theta=\underline{\theta}}^{\bar{\theta}} \omega(\ell, L(\theta))m(\theta)d\theta + [1-p(\ell)]\bar{w}]d\ell \qquad (49)$$

which can again be rewritten as (37). Thus the model's predictions for the optimal wage schedule $w(\ell)$ charged the firm for each worker ℓ are unchanged, as are the conditions under which a seniority rule is needed. However since the only restriction on wages received by worker ℓ, $\omega(\ell, L)$ is that:

$$\int_{\ell=0}^{L} \omega(\ell, L)d\ell = \int_{\ell=0}^{L} w(\ell)d\ell \qquad (50)$$

very little about its precise form can be said by the current model, without imposing further restrictions. One example of such restrictions is considered below.

4.4.2 An Example: Fringes and Layoff Pay

Suppose that the function $\omega(\ell, L)$ was constrained to depend on L in only a very simple way: As long as total employment, L, is above some threshold level, L_o, each worker ℓ receives total compensation of $s(\ell)$ when employed, and $t(\ell)$ when not. Whenever L falls below L_o, layoff pay for all workers falls to zero.[22] I focus on states in which $L > L_o$, and adopt the following conventions:

$$s(\ell) \equiv a(\ell) + b(\ell) \tag{51}$$

$$t(\ell) \equiv a(\ell) = c(\ell) \tag{52}$$

where $a(\ell)$ represents fringe benefits--which are received whether on layoff or not--, $b(\ell)$ represents wages, and $c(\ell)$ is privately financed, unfunded unemployment compensation. What does the current model predict about the shapes of these functions?

Implementation of the union's optimal pricing policy when the firm provides fringes and layoff pay (and when the nondecreasing hazard condition is satisfied) requires only that the marginal cost of human capital to the firm, $w(\ell) = s(\ell) - t(\ell) = b(\ell) - c(\ell)$, be increasing in seniority. Thus the current model places <u>no</u> restrictions on the pattern of fringe benefits across workers, but requires that the difference between wages and unemployment compensation be increasing in seniority. This can of course be accomplished via increasing wages combined with unemployment compensation that is independent of seniority, or constant wages combined with unemployment compensation that <u>declines</u> with seniority. It also implies that, if unemployment compensation rises with seniority, wages must increase <u>at an even greater rate</u>.

It is perhaps worth noting that all of the above-mentioned policies require the use of a seniority rule, even if they set juniors' wages above seniors'. This is because in all situations, the marginal cost of employing a worker is designed to increase with seniority. Also, note that, compared to the basic model, expanding the union's strategy set to include payments to laid-off workers does <u>not</u> mean the union can extract any more rents than before, at least if we maintain the assumption we have used throughout, that $W(0) = 0$, i.e. the union cannot charge the firm anything if the firm employs no one. Indeed since the firm's only concern is $W(L)$--total compensation paid when ℓ workers are employed, it is clear that the union's optimal marginal-cost-of human capital schedule is the same with these policies as without; they simply represent alternative ways to implement it.

4.5 Other Extensions

Three remaining extensions of the basic model deserve some comment. First, employee and employer risk aversion can be introduced into the model, but are unlikely to substantially alter its conclusions. This is because the incentive-compatibility constraints imposed by the fact that firms maximize profits ex post severely restrict the firm's ability to insure workers against income fluctuations. Indeed, when no isoprofit constraint is binding on the union in the basic model, the only effect of risk aversion will be to lower each individual's optimal wage demand in order to achieve more job security. In the presence of a binding isoprofit constraint, the model would take on more of the properties of implicit contracts models with asymmetric information, such as Hart (1983), or Green and Kahn (1983), for example.

Second, complementarity between workers may be introduced, for example by proposing that the firm hires two different kinds of labor, L_1 and L_2, and has a production function $F(L_1, L_2)$, $F_{12} > 0$. The two types of labor might be thought of as workers in different departments of the firm, whose outputs both contribute to the final product. It seems likely that depending on the nature of the unobserved shock(s) affecting the firm in the case a union might either prefer to impose separate seniority ladders for the two departments (which allows the firm to control the relative size of the two departments) or, alternatively, use "plantwide" seniority, which could be used by the union, with the right assignment of seniority indexes, to control relative employment levels in the two departments. Since both kinds of systems do exist, it may be interesting to derive conditions under which one is preferred to the other.

Third, recall that one inconvenient feature of the union's problem in Section 3 was that optimal U-policies (in which each worker's wage is set separately), which always yield higher rents than B-policies, are not necessarily always feasible under a seniority-wage system. Thus it seems reasonable to ask if, by expanding the set of policy instruments available to the union, one can make the U-regime feasible everywhere.

One way this can be done in a world of identical workers can be thought of as a system of assigning wages to "jobs", not workers. Specifically, think of the firm's marginal-product-of-labor schedule, $f(\ell)$, as consisting of a series of tasks, or jobs, each of which requires one worker. The function $f(\ell)$ orders jobs in terms of declining productivity. In this interpretation of $f(\ell)$, one can think of the model of Sections 2 and 3 as a model in which the union attaches wages to workers, but the firm, once it has hired the desired number of workers, is free to assign these workers to tasks. Naturally, it always fills the highest-productivity tasks first in any state, so the union is never able to achieve a higher employment probability for low-productivity tasks than for high-productivity tasks.

Suppose now that the union can assign wages directly to jobs rather than to workers. Then the above problem disappears; wages can be set to produce any employment probability for any task. The U-regime applies everywhere, and the union's rent-maximizing policy is to set a separate monopoly wage for each task. A seniority rule for tasks is not required; indeed it may be optimal in some states to let the firm leave "gaps" in the distribution of jobs filled, with low-productivity jobs filled but higher-productivity jobs empty. Thus an optimal nonuniform pricing scheme can be implemented via a wages-for-jobs system without the need for a seniority rule, and because rents can be extracted using a U-policy than a B-policy, the union is better off.

If workers are not identical, it may still be in the union's interest to have wages depend on the job to which the worker is assigned, but one would also expect the wages charged for a particular job to depend on the skill level of the worker assigned to it. Thus one would expect wages to depend both on the job and the worker's identity, or years of service, as they do in reality. In addition, the union once again has an interest in controlling the order in which workers are hired, in order to minimize $\bar{W}(L)$, and thus one would once again expect a seniority rule to be optimal. Indeed, since it can now be optimal for the union to leave "gaps" in the job distribution in some states, the seniority system is likely to contain a set of "bumping" provisions giving seniors whose jobs disappear in some states the right to the jobs of more junior workers, which have not disappeared. Such schemes, which economize on the union's opportunity cost of human capital while allowing an optimal nonuniform pricing system to operate, seem to be quite common in North American unions. Thus, a "wages-for-jobs" model of unionism seems a promising topic for further study.

5. TESTING THE MODEL--SOME CONSIDERATIONS

While the foregoing analysis does offer a number of empirical predictions, some of which are consistent with existing evidence and others which appear to be amenable to fairly straightforward tests, it is important to mention, in conclusion, two complicating factors that are likely to mitigate the usefulness of such simple tests. These are: First, the rate of specific human capital accumulation within the firm is both (a) not measurable directly and (b) likely to differ between union and nonunion firms. This implies that many of the predictions of <u>any</u> theory regarding the marginal cost of human capital in both types of firms are not testable directly. Concerning the current model, as long as specific skills do not <u>decumulate</u>

with age, the prediction that wages rise with tenure remains unaffected, but the basic model's more interesting predictions concerning, for example, the union-nonunion average wage differential, are not testable directly. This is because the average wage per worker (or per hour) is no longer related in the same way to the average wage per unit of human capital in union and nonunion firms.

Second, the underlying structure of production, i.e. the production function F and the distribution of the shock θ, may differ systematically between union and nonunion firms. Indeed if unions tend to select themselves into environments where their nonuniform-pricing technology is most useful, the current model <u>itself</u> has implications, not yet explored, for the expected incidence of unionism across firms of different types. This again tends to limit the usefulness of simple tests of the paper's hypotheses, which are based on the assumption of identical structures of production in union and nonunion firms. As a simple example, imagine for a moment that unions can extract more rents from firms with greater variability in the shock θ. Then, evidence of greater cyclical variation in union employment might simply reflect this selection process rather than the effect of union wage policy <u>per se</u>. If unions do relatively better in stable environments (which is possible since, for example, if θ takes on only one value, unions can extract <u>all</u> the firm's surplus), then the model's predictions regarding equilibrium union-nonunion differences in employment become ambiguous.

Thus, while the current model offers a new framework for analysing and understanding many union policies which have not been addressed formally to date, it is clear that a convincing test of the model's main predictions still needs to address some rather difficult and unresolved issues. The author hopes that some progress in this direction will occur soon.

APPENDIX

A1 PROOF OF PROPOSITION 1

Proposition 1 summarizes the implications of the firm's profit-maximizing employment decisions in the presence of an arbitrary $w(\ell)$, for the employment probability of a worker as a function of his seniority rank, $p(\ell)$. The proof below applies to $w(\ell)$ which may be discontinuous, nonmonotonic, and may possess isolated points. It is constructive: lemma 1 establishes the firm's optimal employment rule while the remaining lemmas draw out its implications for $p(\ell)$. Proposition 1 consists of the last two lemmas, numbers 5 and 6.

Definition A1

When faced by a given wage profile, $w(\ell)$, the firm's profit over the interval $[\underline{\ell}, \bar{\ell}]$ in state θ can be written as:

$$\pi(\underline{\ell}, \bar{\ell}, \theta) = \int_{\ell=\underline{\ell}}^{\bar{\ell}} [\theta f(\ell) - w(\ell)]d\ell \tag{A1}$$

It is easy to see that $\pi(\underline{\ell}, \bar{\ell}, \theta)$ is strictly increasing in θ, and convenient to define $f(0) = w(0) = 0$ for notational simplicity. (In reality there is no worker with $\ell=0$ exactly).

Lemma 1

The firm's optimal employment level in the state θ is given by $\ell^*(\theta)$ if and only if:

$$\pi(\ell, \ell^*(\theta), \theta) \geq 0, \qquad \forall \ell < \ell^* \tag{A2}$$

$$\pi(\ell, \ell^*(\theta), \theta) \leq 0, \qquad \forall \ell > \ell^* \tag{A3}$$

$$\theta f(\ell^*(\theta)) \geq w(\ell^*(\theta)) \tag{A4}$$

Proof

Conditions A2 and A3 follow directly from the definition of maximum profits, $\pi(0, \ell^*(\theta), \theta) \geq \pi(0, \ell, \theta)$, $\forall \ell$, by subtraction. Condition A4 follows from (A2) for $\ell = \ell^*(\theta) - \varepsilon$, where ε is arbitrarily small. It implies simply that, if $\theta f(\ell^*(\theta)) < w(\ell^*(\theta))$, the firm could do better by hiring all workers on $(0, \ell^*(\theta)]$ except worker $\ell^*(\theta)$.

Lemma A2

A worker, ℓ, is employed in state θ, if and only if $\ell^*(\theta) \geq \ell$.

Proof

This follows directly from the definition of the seniority rule.

Lemma A3

If a worker is employed in state θ, he is also employed in all states $\theta' > \theta$. In other words, $\ell^*(\theta)$ is nondecreasing.

Proof

Recall that $\ell^*(\theta)$ is optimal employment in state θ, and consider another state $\theta' > \theta$. Also consider an arbitrary worker $\ell \leq \ell^*(\theta)$ who was employed in θ, and the profits earned on the interval $[\ell, \ell^*]$ in the two states.

In state θ, because $\ell^*(\theta)$ is an optimum, $\pi(\ell, \ell^*, \theta) \geq 0$, by (A2). Also because π is increasing in θ, $\pi(\ell, \ell^*, \theta') > \pi(\ell, \ell^*, \theta)$ ≥ 0. Now, if ℓ were an optimal employment level in state θ', we would require $\pi(\ell, \ell^*, \theta') \leq 0$ (by A3), which contradicts the above. Therefore no $\ell < \ell^*(\theta)$ can be an optimal employment level in a state $\theta' > \theta$.

Definition A2

Define $\theta^*(\ell)$ as the lowest state in which worker ℓ is employed. Since, by lemma A3, he is employed in all higher states we may write his employment probability as, simply, $p(\ell) = M[\theta^*(\ell)]$.

Lemma A4

$\theta^*(\ell)$ is nondecreasing.

Proof

This follows directly from lemma A3.

Definition A3

A worker, ℓ, is defined as belonging to a "bundling interval," if and only if his "critical state", $\theta^*(\ell)$, and hence his employment probability, is the same as some other worker's.

Lemma A5

If a worker, ℓ, belongs to a bundling interval, $[\underline{\ell}, \bar{\ell}]$ his "critical state", $\theta^*(\ell)$, is given by:

$$\theta^*(\ell) = \frac{\int_{s=\underline{\ell}}^{\bar{\ell}} w(s)ds}{\int_{s=\underline{\ell}}^{\bar{\ell}} f(s)ds}, \text{ where } \underline{\ell} \leq \ell \leq \bar{\ell} .$$

The quantities $\underline{\ell}$ and $\bar{\ell}$ are the bounds of the "bundling interval", in which worker ℓ is contained. Note that all workers in this interval share the same $\theta^*(\ell)$.

Proof

If worker ℓ' shares the same value of θ^* as any other worker, ℓ'', then by lemma 4, all the workers between ℓ' and ℓ'' must share that value of θ^*. In other words, ℓ' and ℓ'' are on a flat section of $\theta^*(\ell)$. Consider now the upper and lower bounds of this interval, $\underline{\ell}$ and $\bar{\ell}$. By definition $\ell^*(\theta)$ jumps upwards, from $\underline{\ell}$ to $\bar{\ell}$, at the point $\theta = \theta^*(\ell)$, where

$\underline{\ell} \leq \ell \leq \bar{\ell}$. In state $\theta^{*}(\ell)$, the firm is therefore indifferent between $\underline{\ell}$ workers or $\bar{\ell}$ workers. This implies, from (A2)

$$\theta^{*} = \frac{\int_{s=\underline{\ell}}^{\bar{\ell}} w(s)ds}{\int_{s=\underline{\ell}}^{\bar{\ell}} f(s)ds}, \text{ as asserted.}$$

<u>Lemma A6</u>

If a worker, ℓ, does not belong to a bundling interval, his critical state, $\theta^{*}(\ell)$, is given by $\theta^{*} = \dfrac{w(\ell)}{f(\ell)}$.

<u>Proof</u>

By lemma A4, the fact that worker ℓ does not share critical state, θ^{*}, with any other worker, implies that $\theta^{*}(\ell)$ is strictly increasing, either continuously or discontinuously, at ℓ. This implies, in turn, that $\ell^{*}(\theta^{*}(\ell))$ does <u>not</u> jump upward at $\theta^{*}(\ell)$. Now since employment cannot jump at $\theta = \theta^{*}(\ell)$, and since by definition worker ℓ is employed in state $\theta^{*}(\ell)$ but not in state $\theta(\ell) - \varepsilon$, worker ℓ must be the "marginal" worker hired in state $\theta^{*}(\ell)$, i.e. $\ell^{*}(\theta^{*}(\ell)) = \ell$. Because $\ell = \ell^{*}(\theta^{*}(\ell))$ is the optimal employment level in state $\theta^{*}(\ell)$, (A4) implies that $\theta^{*}(\ell)f(\ell) \geq w(\ell)$.

Now suppose the preceding inequality were strict, i.e. $\theta^{*}(\ell)f(\ell) > w(\ell)$, and consider what happens to optimal employment as we lower the state infinitesimally below $\theta^{*}(\ell)$ to θ'. Since $\theta^{*}(\ell)$ is defined as the <u>lowest</u> state in which ℓ is employed, optimal employment must fall, and therefore $\ell = \ell^{*}(\theta(\ell))$ must violate one of the conditions for an optimum, (A2) - (A4), in state θ'. It will not violate (A3) since π increases in θ. It will not violate (A4) because the inequality was strict in state $\theta^{*}(\ell)$. It must therefore violate (A2), i.e. we must have $\pi(\ell', \ell^*(\theta^*(\ell)), \theta') < 0$ for some $\ell' < \ell^*(\theta^*(\ell))$, which in turn implies $\pi(\ell', \ell^*(\theta^*(\ell)), \theta^*(\ell)) = 0$, i.e. the firm is indifferent between ℓ' and $\ell^*(\theta^*(\ell))$ in state $\theta^*(\ell)$. This means that worker ℓ is part of bundling interval, which is a contradiction. Therefore the inequality cannot be strict, and we must have $\theta^*(\ell) = \dfrac{w(\ell)}{f(\ell)}$.

Q.E.D.

AII <u>PROOF OF RESULT 2</u>

Consider an arbitrary interval $[\underline{\ell}, \bar{\ell}]$ throughout which an optimal U-policy is feasible. Consider also an arbitrary wage profile $w^B(\ell)$ which induces bundling of all workers on $[\underline{\ell}, \bar{\ell}]$, i.e. induces $\dfrac{d\theta^*(\ell)}{d\ell} = 0$, $\forall \ell \in [\underline{\ell}, \bar{\ell}]$, and, note that it yields total rents of:

$$_B = \int_{\ell=\underline{\ell}}^{\bar{\ell}} (w^B(\ell) - \bar{w}) \cdot M\left[\frac{\int_{s=\underline{\ell}}^{\bar{\ell}} w^B(s)ds}{\int_{s=\underline{\ell}}^{\bar{\ell}} f(s)ds}\right]d\ell \tag{A5}$$

The profile w^B yields the same expected total rents as another profile, $w^{B'}$, which also bundles all workers on $[\underline{\ell}, \bar{\ell}]$, and is constructed as follows: $\int_{\underline{\ell}}^{\bar{\ell}} w^B(\ell)d\ell = \int_{\underline{\ell}}^{\bar{\ell}} w^{B'}(\ell)d\ell$, i.e. the total wage bill over the interval is the same, and $\dfrac{w^{B'}(\ell)}{f(\ell)} = \theta^*$, where θ^* is constant over the interval. Total expected rents extracted using $w^{B'}(\ell)$ can be written:

$$ {}_{B'} = \int_{\ell=\underline{\ell}}^{\bar{\ell}} [w^{B'}(\ell) - \bar{w}]\, M\, [\frac{w^{B'}(\ell)}{f(\ell)}]. \tag{A6} $$

Thus the level of rents achievable via <u>any</u> wage profile, w^{B}, that induces bundling of workers over the entire interval $[\underline{\ell}, \bar{\ell}]$ can be expressed in the form of (A6), with $w^{B'}$ defined as above.

Now consider the optimal U-policy on $[\underline{\ell}, \bar{\ell}]$ and note that by definition it solves:

$$ \underset{w(\ell)}{\text{Max}} \quad {}_{U} = \int_{\ell=\underline{\ell}}^{\bar{\ell}} [w(\ell) - \bar{w}]M(\frac{w(\ell)}{f(\ell)})d\ell \tag{A7} $$

Call the solution to (A7) $w^{*}(\ell)$. Clearly since $w^{*}(\ell)$ maximizes W_U, since (A6) and (A7) are isomorphic, and since $w^{B'}(\ell)$ and $w^{*}(\ell)$ differ [one induces $\frac{d\theta^{*}}{d\ell} > 0$, the other $\frac{d\theta^{*}}{d\ell} = 0$], ${}_{B} < {}_{U}$.

Finally, since the above argument applies to <u>any</u> interval and any profile that induces bundling, it must apply to optimal bundling profiles over any interval on which the union might choose to bundle workers. Q.E.D.

AIII <u>PROOF OF RESULT 3</u>

Focus initially on worker $\hat{\ell}$, defined by $\bar{\theta}f(\hat{\ell}) = \bar{w}$. This is the least senior worker ever hired in a nonunion firm; it is clear from the first-order condition (14) that if this worker belongs to a U-interval in the union firm, then $w(\hat{\ell}) = \bar{w}$, and thus he is also the least senior worker ever hired in a union firm. If on the other hand, $\hat{\ell}$ belongs to a B-interval $[\underline{\ell}, \bar{\ell}]$, note first that he must be the least senior worker in that interval, i.e. $\hat{\ell} = \bar{\ell}$, because inducing the firm to hire $\hat{\ell} > \bar{\ell}$ in state $\bar{\theta}$ would violate the individual rationality constraint. Finally, note that θ^* on this interval $[\underline{\ell}\ \hat{\ell}]$, must equal $\bar{\theta}$, the same as if the U-interval applied to worker $\hat{\ell}$, for the following reason: $\hat{\ell}$ is an optimal employment level for the firm in state θ^*, which requires (from A4) that $\theta^* f(\hat{\ell}) \geq \bar{w}$, or $\theta^* \geq \frac{\bar{w}}{f(\hat{\ell})} = \bar{\theta}$. Since $\bar{\theta}$ is the upper limit of θ, the equality must hold.

Focus now on an arbitrary worker $\ell < \hat{\ell}$. I show below (i) that if ℓ is in a U-interval, $p(\ell)$ is always strictly lower in the union, and (ii) if ℓ is in a B-interval, $[\underline{\ell}, \bar{\ell}]$ $p(\ell)$ is strictly lower in the union unless $\ell = \bar{\ell}$, when $p(\ell)$ may be the same in union and nonunion firms.

(i) In a nonunion firm, $\theta^*(\ell) = \frac{\bar{w}}{f(\ell)}$. In a union, if this worker belongs to a U-interval, he has $\theta^*(\ell) = \frac{w(\ell)}{f(\ell)}$. But the first-order condition requires $w(\ell) > \bar{w}$ except for the most junior worker ever employed (where M=0); thus $\theta^*(\ell)$ is strictly greater in the union firm.

(ii) If, in the union, worker ℓ belongs to a B-interval, we know that $\theta^*(\ell)$ is constant for $\underline{\ell} \leq \ell \leq \bar{\ell}$. Since $\theta^*(\ell)$ in a nonunion firm increases smoothly with ℓ, consider worker $\bar{\ell}$. If his θ^* is lower in the union, then θ^* of all workers in the interval $[\underline{\ell}, \bar{\ell}]$ must be so as well.

Now since $\bar{\ell}$ is an optimal employment level for the firm in state θ^*, we have from (A4), that $\theta^* f(\bar{\ell}) \geq w(\bar{\ell})$, or

$\theta^* \geq \dfrac{w(\bar{\ell})}{f(\bar{\ell})} \geq \dfrac{\bar{w}}{f(\bar{\ell})}$, where the second inequality is from the individual rationality constraint. Finally, consider any worker $\ell < \bar{\ell}$ still in the B-interval. His employment probability is still θ^* in the union, but strictly greater than $\dfrac{\bar{w}}{f(\bar{\ell})}$ in the nonunion as long as $\theta > \underline{\theta}$. Therefore the inequality must be strict for all workers in the interval but worker $\bar{\ell}$, if we are not on the lower bound of $\theta, \underline{\theta}$.

Q.E.D.

FOOTNOTES

[1]The main difference between the current, nonuniform pricing model and implicit contract theory is the following. Because implicit contract theory focuses on the relationship between a firm and a "representative" worker, it typically imposes no limits on the enforceability of lump-sum transfers from the firm to workers. Indeed when no such limits exist, contract theory (e.g. Hall and Lilien, 1979) has shown that optimal nonuniform pricing will yield a first-best outcome in spite of asymmetric information unless firms are risk averse or the worker's preferences exhibit "income effects".

Unfortunately, while unlimited lump sum transfers from the firm to an individual worker might be realistic, they are less so when we consider transfers to a union of workers which is interested in extracting the most rents it can from the firm. In particular, it seems reasonable in this context to impose the condition that the union cannot charge the firm anything if none of its members are employed--i.e. the firm can, at worst, always shut down to escape the union. When such a condition is imposed (as is done here), the results of nonuniform pricing theory--in which distortions exist even when both parties are risk neutral--once again apply. Interestingly, Hayes (1984) derives these kinds of distortions in the case of strikes, but does not relate them to the infeasibility of lump sum transfers.

[2]Grossman (1983) and Blair and Crawford (1984) both model the effects of seniority systems in which all workers are paid the same wage without attempting to explain why such systems are imposed. Carmichael (1983) provides a rationale for nonunion seniority systems. The current paper is not inconsistent with the use of seniority as a criterion for layoffs and rehires in nonunion firms, but does predict a role for seniority rules in union firms

that does not arise elsewhere . Evidence (e.g. Freeman and Medoff, 1984, pp. 123-126) that seniority criteria for layoffs and rehires are more important in union than nonunion firms seems to support this view.

[3]This result actually emerges quite immediately from the setup of the model itself, which is somewhat different from the well-known but cumbersome formulation usually employed in the nonuniform pricing literature. In contrast to earlier approaches, e.g. Spence (1977) and Mussa and Rosen (1978), and to the implicit-contracts literature I characterize the solution to the buyer's problem by a probability that each successive unit of the good is purchased. This yields a very intuitive interpretation of the first-order conditions for an optimum and of the nature of the "bundling" of successive units of the good that can occur, which does not appear to be noted elsewhere.

[4]Differentiability of f(L) is not required in the proofs of the paper's main results.

[5]Another aspect of the composition of L the firm may be able to choose is the hours-employment mix. This issue does not arise in the basic model because hours per employed worker are assumed fixed. It does not arise in the expanded model with variable hours (Section 4.3) because the union is given control over this decision there.

[6]It is analytically more convenient to treat the fact that some of these workers may not "belong" to the union, in the sense that the union does not care about them, by assigning tham a zero welfare weight in the union's objective function.

[7] Loosely speaking, one can think of this maximand as representing the outcome of a cooperative game among the set of union members: by maximizing (3) I simply guarantee that the outcome of this game is on the utility-possibility frontier for union members.

[8] One example is a "boss-dominated" union--in which case A_2=0 for all members but one (who may or may not ever be employed).

[9] With the exception of union dues, which are typically small and cover only the operating costs of the union, this appears to be a realistic assumption. Indeed these property rights are given some legal sanction by provisions such as those in 8(b) 5 of the National Labor Relations Act which attempt to limit intra-union transfers by prohibiting "discriminatory" variation in union dues across members, as well as prohibiting "excessive" union dues.

[10] Note that the property-rights assumption rules out unfunded layoff pay to workers, which is introduced in section 4.4. Funded layoff pay, in which the firm contributes a portion of the employed worker's wages in one period to a fund for unemployment compensation in another period, is not necessarily ruled out.

[11] The fact that there is no "zeroth" worker incorporates the notion that the union cannot impose a charge on the firm if it does not operate.

[12] Interestingly, if the union maximizes rents, it is easy to see that its optimal policy derived by solving the maximization problem (5) - (6) is also the best it can do via <u>any</u> mechanism, given the information constraints of the problem. This result is well known from implicit contract theory (see for example Hart 1983, or Rosen 1985) and can be shown using the Revelation Principle by deriving the optimal direct, incentive-compatible mechanism and showing that it can be achieved via the appropriate outlay schedule $w(\ell)$.

Unfortunately, this result does not appear to hold in general, suggesting that better (but likely extremely complicated) mechanisms than the one considered here exist.

[13]Figure 1(b) also shows how, when $w(\ell)$ jumps upwards, the firm's optimal employment level may be independent of θ for some θ, both on the interior of U-intervals as well as at the boundary between intervals of a different types.

[14]For convenience I treat bundling intervals as including their endpoints, and U-intervals as not.

[15]The analyses of "corner" B-intervals is identical except for a binding inequality constraint on θ and is omitted to save space.

[16]Suppose, with $\ell(z)$ as well as wage policy chosen optimally, there was a B-interval in which, for some ℓ, (16) was binding. Now consider re-assigning seniority indexes only among workers in this interval but keeping each individual's wage the same. This leaves (22) unchanged, affects nothing outside the interval, and can always be done in such a way that the unconstrained solution to (19)-(21) is feasible, and therefore constitutes a superior policy.

[17]The well-known result in implict contract theory (see Hall and Lilien 1979) that first-best contracts can be implemented even under asymmetric information as long as firms are risk neutral is, essentially, based on an implicit assumption that unlimited fixed charges (possibly greater than area C in Figure 1(a)) can be imposed on the firm without inducing it to shut down in some states. Such an assumption would of course make the utility-possibility frontier linear everywhere (not just between a and b), and make all possible divisions of rents between the firm and union consistent with efficiency in production.

[18]Using years of service to allocate seniority may also have a useful incentive effect: it ensures that all current members of the union eventually get a high-paying senior job, and since this "bonus" is received at the end of each worker's life, each worker's incentives to undermine the union while a member are accordingly diminished. Indeed Lazear (1983) has made this kind of argument concerning the allocation of union versus nonunion jobs within an industry.

[19]Equation (47) can be written, using a continuous-hours analogue of the property-rights assumption:

$$\mathcal{W} = \int_{n=0}^{Z} \int_{\theta=\underline{\theta}}^{\bar{\theta}} \{\int_{\ell=0}^{L(\theta)} w(\ell)h_e(e,n)e_\ell(n,\ell)d\ell - \bar{\omega}e(n,L(\theta))\}m(\theta)d\theta dz$$

Noting that $\int_{n=0}^{Z} h_e(e,n)e_\ell(n,\ell)d\ell = 1$, (this follows from differentiation of 45) this can be rewritten:

$$\mathcal{W} = \int_{\theta=\underline{\theta}}^{\bar{\theta}} \int_{\ell=0}^{L(\theta)} w(\ell)d\ell m(\theta)d\theta - \int_{\theta=\underline{\theta}}^{\bar{\theta}} \int_{n=0}^{Z} \bar{\omega}e(n,L(\theta))dnm(\theta)d\theta$$

$$= \int_{\theta=\underline{\theta}}^{\bar{\theta}} [W(L(\theta)) - \bar{W}(L(\theta))]m(\theta)d\theta$$

which is just another way of expressing total rents extracted from the firm, (37).

[20]First-order conditions for a minimum of (46) subject to (45) can be written:

$$\frac{1}{\bar{w}(L)} = h_e(e(n),n) = \frac{h(e(N),\ N)}{e(N)} \ \forall\ n \in [0,N]$$

The marginal h of <u>each</u> worker hired at any given L equals the average h of the last worker hired at that L. Since this is true for worker N as well, his effort level must be that which maximizes his average h, i.e. $\frac{h(e)}{e}$. If all workers are identical, this is independent of the identity of the last worker hired, and therefore hours per worker, e(n) are independent of the amount of human capital supplied, L.

[21]While rules relating e(n) to L are, strictly speaking, rarely (if ever) seen in unions, it is worth noting that, given a (sufficiently well-behaved) function N(L), they are equivalent to a rule of the form e(n,N), $\forall n \in (0,N)$, $\forall N$. In other words, the union can simply stipulate, for each level of employment N, how hours are to be divided among the employed workers. Actual rules of this kind include minimum weekly hours per employee, and rules governing the allocation of overtime among workers.

[22]If individual workers' layoff pay is not allowed to be a continuous function of the level of employment, (which seems a realistic restriction) then this kind of discontinuity is needed to satisfy the condition W(0)=0. Levels of employment below L_o need of course never be realized; indeed one can imagine a situation where L_o is the lowest realized employment level, workers $\ell \in (0,L_o]$ are all employed at their reservation wage with probability one, while the combined layoff pay of all workers $\ell \in (L_o,Z]$ just exhausts the rents produced by the other, senior, group of workers in state $\underline{\theta}$. In other words, layoff pay can be used as a way of getting the firm to pay <u>junior</u> workers the rents accruing to the employment of seniors.

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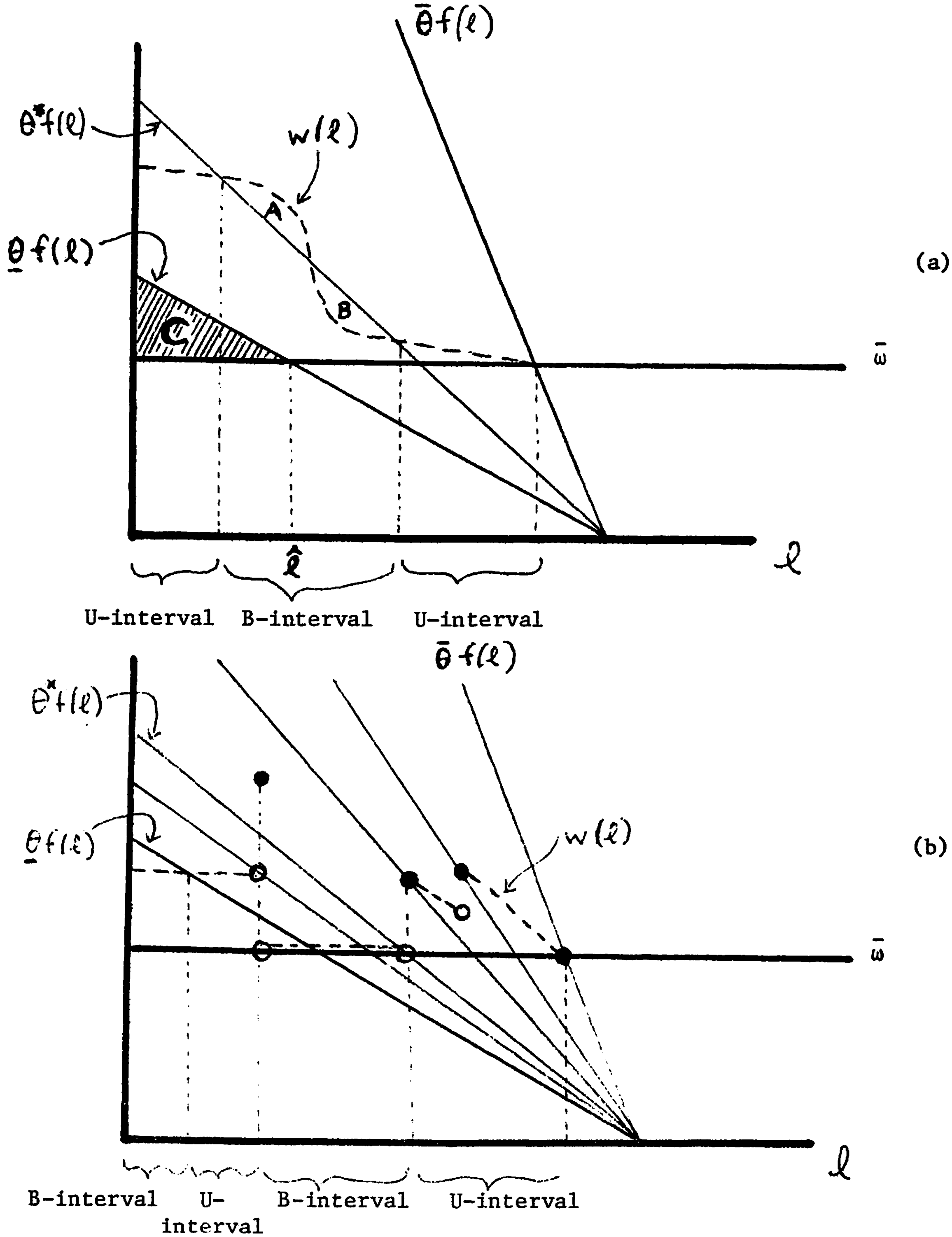

Figure 1: Examples of U- and B-intervals

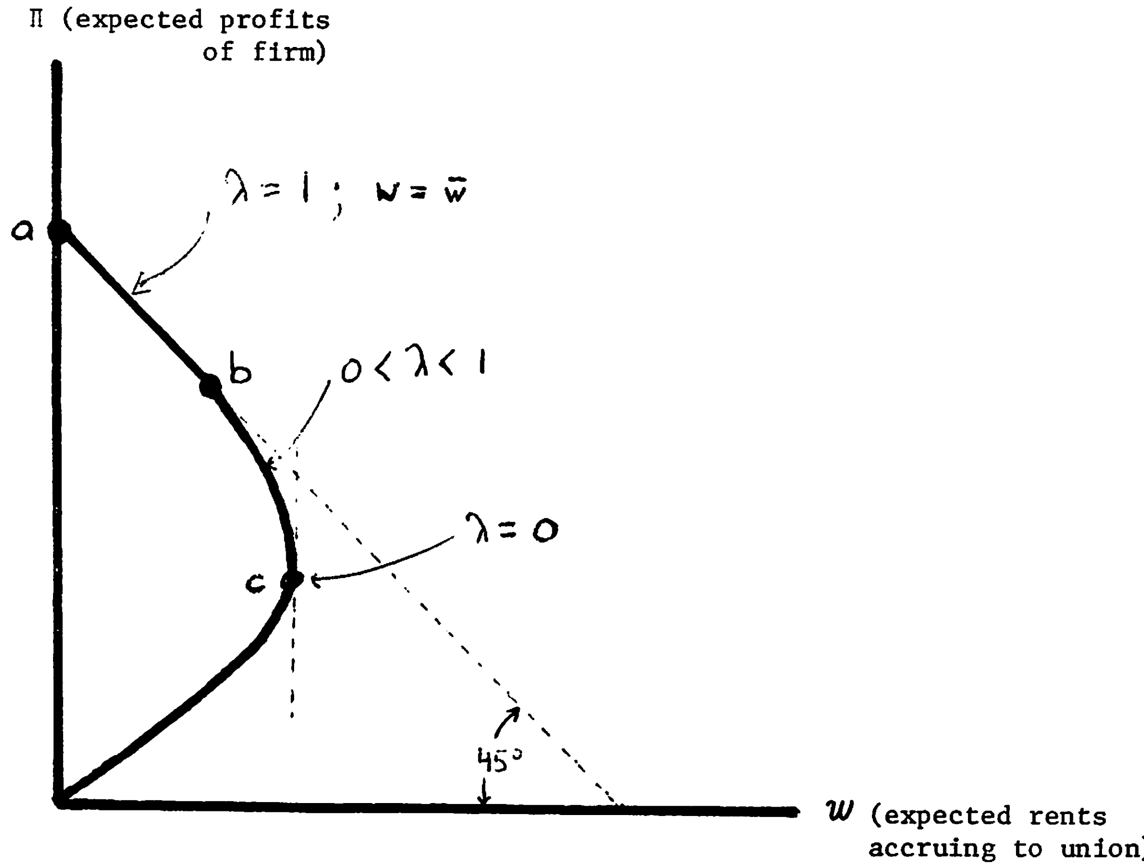

Figure 2: Utility-possibility frontier between Firm and Union