1993

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ISSN: 0318-725X
ISBN: 0-7714-1587-7

RESEARCH REPORT 9315

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September 1993

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DERIVING AN ESTIMATE OF THE OPTIMAL AUCTION: AN APPLICATION TO BRITISH COLUMBIAN TIMBER SALES*

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September 1993

Abstract

In this paper, I use a simple game-theoretic model of behaviour at English auctions within the independent private values paradigm to put structure upon data from a sample of timber sales held in the province of British Columbia where, to a first approximation, the independent private values paradigm appears appropriate. I then estimate several different empirical specifications and use the methods of Vuong to decide which model is closer to the truth than the others. Under different assumptions concerning the seller's valuation of the timber, estimates of the optimal auction are calculated.

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*This paper represents a revision of some research contained in University of British Columbia Discussion Papers 89-14 and 91-19. That work was initially funded by grants from the University of British Columbia and the Forest Economics and Policy Analysis Research Unit, and subsequently by the SSHRC of Canada. I should like to thank the staff of the British Columbia Ministry of Forests for their help in creating the data set used, especially Bill Howard, Mark Ismay, John Nichol, and Peter Spearman. I am especially grateful to Quang Vuong and two anonymous referees for comments.

1. Introduction

During the past fifteen years, one of the most exciting areas of theoretical research in economics has been systematic study of mechanism design and, in particular, its application in industrial organization to the study of regulation; see Laffont (1992) for an up-to-date summary of current research. This research concerning mechanism design is in turn related to that investigating auctions; see McAfee and McMillan (1987) for a recent survey. Despite the explosion of theoretical work in these two areas, only recently have economists attempted to implement any of this research empirically.[1] In this paper, I add to the recent empirical literature concerning auctions and mechanism design by using a structural econometric approach first proposed by Paarsch (1992) in conjunction with information derived from a sample of timber sales held in the province of British Columbia, Canada to estimate the optimal auction for timber.

The paper is in five more sections. In section 2, I describe the environment within which British Columbian timber sales are held and the data available concerning these sales, while in section 3 I specify a theoretical model of bidding at English auctions (one of the mechanisms used to sell timber in British Columbia) within the independent private values paradigm (IPVP). In section 4, I link the theoretical model of the English auction to data available concerning actual timber sales to construct the empirical specifications, while in section 5 I present my empirical results and use them to derive estimates of the optimal auction under different assumptions concerning the seller's valuation for timber. In section 6, I summarize and conclude the paper. The construction of the data set is described in an appendix.

2. Timber Sales in British Columbia

In British Columbia, the *Forest Act* of 1979 permits small businesses to acquire the right to harvest timber on Crown (government) land. The Minister of Forests sets aside a portion of each year's allowable cut to sell to eligible loggers and sawmillers through a series of public auctions held under the Small Business Forest Enterprise Program (SBFEP). Several criteria for eligibility exist. For example, to be eligible a

[1] Some notable exceptions are Paarsch (1989,1991,1992); Laffont, Ossard, and Vuong (1991); Elyakime, Laffont, Oisel, and Vuong (forthcoming); and Wolak (forthcoming).

person must be an independent logger (in Category 1) or a mill owner (in Category 2) over nineteen years of age with at least two years of experience. In addition, a registration fee, which amounted to $100 per annum in 1987, must be paid. Registrants can participate at auctions anywhere in the province, but over 90 percent of all sales in any particular Forest District involve only bidders from that district. During the period considered in this paper (1984 to 1987), the program also prohibited any registrant from holding more than two SBFEP sales at any one time. Below, I shall focus upon sales to loggers (Category 1 sales) because they comprise the bulk of timber sold under the SBFEP.

Although both English and first-price sealed-bid auctions have been used to sell the right to harvest timber on Crown land, for reasons which will be made clear below, I shall focus solely upon English auctions. The reader should note that the choice of mechanism by the Ministry of Forests appears unrelated to such economic variables as log prices. Using data which are discussed in detail below, I estimated a simple Probit model of mechanism choice where the dependent variable equalled zero if an English auction was used and one if a first-price sealed-bid auction was used. The covariates included such observables as "average" log prices, "average" minimum acceptable bids, volume of timber, distance to nearest timber processing facility, and dummy variables for the season of the sale. None of these covariates had a significant coefficient estimate and jointly they were insignificant at size 0.01. Together, they predicted the choice of mechanism correctly about 60 percent of the time. With a fair coin I could expect to predict this choice correctly 50 percent of the time, suggesting that these covariates add little to prediction and supporting the notion of random assignment.

The type of bidding admitted at the English auctions is quite simple. Essentially, the Ministry of Forests assigns a minimum price per cubic metre of timber harvested. This minimum price, often referred to as the "upset" price, will vary across species and depends upon past lumber prices. The upset price is known in advance to all potential bidders. Bidders may then verbally tender an additional amount per cubic metre of timber harvested, called the "bonus" bid. Bonus bids are uniform across species. Although the auction rules in British Columbia vary slightly across Forest Districts, bidders are typically required to tender increments of no less than $0.01 per cubic metre of timber harvested. The total amount bid is called the "stumpage rate", and it will vary across species as it is the sum of the species-specific upset price and the uniform bonus bid.

Each potential bidder has a considerable amount of information concerning the timber for sale. For example, from the timber cruise report and other supporting documents, he can obtain detailed information concerning the location of the timber, the surrounding terrain, and access to the timber by roads.[2] Typically, timber sold under the SBFEP is in areas that have quite well developed road networks, so road constructions costs are usually neglible. An unbiased estimate of the volume of standing timber by species and grade (known as the "cruised" volume) is also available from the Forest Service. Some error may exist in the cruised-volume estimate, but potential bidders can and do inspect sales themselves. In any case, there is no reason to believe that any one potential bidder has more information than the others concerning the volume or quality of timber for sale.

For 129 sales of timber at English auction, I observe the district, year, and month in which the sale was held; the volume of timber by species; the upset price by species; the distance to the nearest timber processing facility; the number of actual bidders; and the final recorded bonus bids for each of the bidders. From other sources, which are described in detail in an appendix to the paper, I have derived measures for the price of timber and the number of potential bidders. As will become obvious below, having a measure for the number of potential bidders makes this study special since such a covariate is central to undertaking any structural econometric analysis of auctions.

3. Bidding at English Auctions within the IPVP

In this section, I outline the equilibrium bidding strategy at English auctions within the IPVP and describe the structure of the optimal auction.[3] I begin by assuming that auctions can be modelled as non-coöperative games. To specify a game one must identify the players, characterize the information each player has, describe the strategies available to each player, describe how each player is rewarded, and characterize the equilibrium.

[2] The timber cruise report is a document prepared for the Ministry of Forests in which the timber for sale is described.

[3] For more details than are provided below, see the recent survey by McAfee and McMillan (1987).

I consider auctions at which a single seller wishes to dispose of one object to N potential bidders (players). The i^{th} player is assumed to know his valuation v_i, but not those of his opponents. Heterogeneity in valuations is modelled as a continuous random variable V having probability density function $\phi(v)$ and cumulative distribution function $\Phi(v)$ which have support upon $[\underline{v}, \overline{v}]$. The valuations of players are assumed to be independent draws from $\Phi(\cdot)$. Together, the above assumptions constitute the IPVP.

The strategies available to the players are their bids. The seller is assumed to have a reservation valuation $v_0 \in [\underline{v}, \overline{v}]$ which he imposes in the form of a known minimum price that must be bid. English auctions can be modelled in several different ways. Milgrom and Weber (1982) describe one approach which involves assuming that the sale price is measured on a thermometer that is set initially at the minimum price v_0. Players with valuations below v_0 do not bid. As the thermometer rises, remaining players drop out at their valuations. The winner is the player with the highest valuation, and he pays the second highest valuation.[4] Formally, for players who bid, the dominant bidding strategy $\sigma(v)$ given valuation v is

$$\sigma(v) = v \quad v_0 \leq v \leq \overline{v}.$$

Notice that risk aversion with respect to winning the auction does not affect the equilibrium bidding strategy.

The only behavioural hypotheses of the model are that potential bidders bid independently and that losers tell the truth. By focusing upon English auctions, one can avoid the stronger Bayesian-Nash assumption needed to solve for the equilibrium bid function at first-price sealed-bid auctions. Of course, this strength derives from the IPVP assumption. Within other paradigms (*e.g.*, the common value paradigm or affiliated private values paradigm), one would need to employ the Bayesian-Nash concept of equilibrium to solve the game.

Anecdotal evidence suggests that English auctions are typically used in environments where little information useful to all of the bidders concerning the value of the object is revealed in the course of the auction. Sales of oil and gas leases, for example, are not undertaken using English auctions because the proprietory information of any

[4] Note that if all save one player drop out at v_0, then the winner pays v_0, while if all drop out the object goes unsold.

particular bidder concerning the probability of discovering oil (and thus the value of the lease) would be revealed in the course bidding. That English auctions are used to sell timber lends support to the IPVP assumption.

In the case of British Columbian timber sales, several other factors suggest that the IPVP is a good approximation to the environment within which loggers bid for timber. First, log prices are generally fixed to loggers either by contract or by list prices at sawmills. Moreover, during the period considered in this analysis (1984 to 1987) considerable price stability existed, so any asymmetries in expectations concerning future prices are unlikely to have been important. Also, because each potential bidder knows a considerable amount about the timber to be sold any asymmetries of information concerning the volume and quality of timber are likely minimal. Therefore, a natural explanation for differences in bidding behaviour is differences in harvesting costs which are likely individual-specific effects, and independent across potential bidders.

Because the winner is the bidder with the highest valuation and because he pays what his nearest opponent would have been willing to pay, the equilibrium pay-off to the i^{th} player is

$$v_i - v_{(2:N)}$$

if he wins and zero otherwise. Here $v_{(i:N)}$ denotes the i^{th} highest order statistic for a sample of size N from the distribution of v.

Riley and Samuelson (1981) have shown that deriving the optimal auction within the IPVP involves choosing a reserve price r to maximize the expected revenues $\mathrm{E}[R]$ from the sale where

$$\mathrm{E}[R] = N \int_r^{\overline{v}} [v\phi(v) + \Phi(v) - 1]\Phi(v)^{N-1}\, dv.$$

Thus, the optimal reserve price r^* satisfies

$$r^* = v_0 + \frac{[1 - \Phi(r^*)]}{\phi(r^*)}. \tag{3.1}$$

Suppose, for example, that V is distributed uniformly upon the interval $[0, 1]$, then r^* solves

$$r^* = v_0 + \frac{1 - r^*}{1} \qquad \text{or} \qquad r^* = \frac{v_0 + 1}{2}.$$

When $v_0 = 0$, $r^* = 0.5$, and when $v_0 = \mathrm{E}[V] = 0.5$, $r^* = 0.75$. Clearly, to calculate r^* requires information concerning $\Phi(\cdot)$, the latent distribution of heterogeneity.

To uncover the distribution of valuations, I shall use information from the distributions of all bids and of just the winning bids. An attractive feature of using data from English auctions is that, for those who bid, their tenders map out the valuation distribution. Note, however, that the number of actual bidders (participants) n at an auction is endogenous, and typically less than N: Only those potential bidders with valuations exceeding v_0 participate. Thus, the observed bid distribution is a truncated one. In addition, the distribution of the winning bid (when it exceeds v_0) is the distribution of the second-highest order statistic for a sample of size N from the distribution of v, and thus depends upon the amount of potential competition N.

4. Empirical Models

In order to uncover $\Phi(v)$, the latent valuation distribution discussed in section 3, I must map the observed data into a stochastic specification for v, derive the implications of this structure for the data generating process, propose methods for estimating these data generating processes, and derive the exact specifications to be estimated. I break the description of this work into four subsections.

4.1. Mapping the Observed Data into a Stochastic Specification

To develop an empirically tractable model of bidding for timber within the environment described in section 2 using the theoretical model described in section 3, several assumptions must be made. First, I assume that only one stand of timber is to be auctioned, and that on that stand at most k different species of timber exist. Letting p_j denote the price of species j (measured in dollars per cubic metre) and q_j denote the volume of species j (measured in cubic metres of timber), I assume next that a logger's valuation of a sale v depends upon total revenues

$$\sum_{j=1}^{k} p_j q_j$$

and total harvesting costs. Total harvesting costs are assumed to depend upon the total volume of timber harvested, but not to vary with the species composition. Such costs are also assumed to depend upon the distance to the nearest timber processing facility (sawmill). Letting q denote the total volume of timber to be harvested

$$q = \sum_{j=1}^{k} q_j$$

and d denote the distance in kilometres to the nearest timber processing facility, total harvesting costs are denoted $C(q,d)$. I assume that timber prices are known perfectly and are the same to all potential bidders. Thus, for any particular logger, the value of a timber sale v is then

$$v = \sum_{j=1}^{k} p_j q_j - C(q,d).$$

Introducing the weights $\{\lambda_j\}_{j=1}^{k}$ where $\lambda_j = q_j/q$, one can write v as

$$v = \sum_{j=1}^{k} (p_j \lambda_j - a) q$$

where $a = C(q,d)/q$ denotes average harvesting costs for the sale. I assume that variations in average harvesting costs a across bidders can be modelled as a continuous random variable A having probability density function $f(a)$ and cumulative distribution function $F(a)$.[5]

Conditional upon d, $\{p_j\}_{j=1}^{k}$, and $\{q_j\}_{j=1}^{k}$, bidding will depend upon how expensive it is for loggers to harvest. In the case of British Columbian timber sales, loggers must bid a non-negative bonus b above the species-specific upset prices set by the Ministry of Forests $\{u_j\}_{j=1}^{k}$. Hence, the species-specific stumpage rates $\{s_j\}_{j=1}^{k}$ tendered to the Crown must satisfy

$$s_j = u_j + b \geq u_j \quad j = 1, \ldots, k.$$

What makes the bidding problem tractable in the case of British Columbian timber sales is the fact that the bonus bid b must be the same across all species, so only one decision exists, the choice of b.

[5] Note that $\phi(v)$ is related to $f(a)$ by

$$\phi(v) = \left|\frac{da}{dv}\right| f(a) = \frac{f(\sum_{j=1}^{k} p_j \lambda_j - \frac{v}{q})}{q}.$$

In general, the i^{th} participant at an English auction will bid up to the point where zero profit obtains

$$\sum_{j=1}^{k}(p_j - a_i - s_j)q_j = 0. \tag{4.1}$$

Dividing both sides of (4.1) by q implies that the bonus bid b_i is a function (β) of the average harvesting cost a_i for participant i

$$b_i = \beta(a_i) = \sum_{j=1}^{k}(p_j - u_j)\lambda_j - a_i = \hat{a} - a_i$$

where

$$\hat{a} = \sum_{j=1}^{k}(p_j - u_j)\lambda_j.$$

An English auction ends when the bidder with the lowest average harvesting costs (the highest valuation) bids just over the final offer of his opponent who has the second-lowest average harvesting costs (the second-highest valuation). Letting $\{a_{(i:N)}\}_{i=1}^{N}$ denote the N average harvesting costs indexed in ascending order, the winning bonus bid w is

$$w = \beta(a_{(2:N)}) = \hat{a} - a_{(2:N)}.$$

4.2. Data Generating Processes of All Bids and Winning Bids

The bonus bidding rule defined above is a monotonically decreasing function of a over relevant as: a bidder continues either until he wins or until zero profit obtains. The lower is a bidder's a (the higher is his v), the longer he remains at the auction. Because the bonus bidding rule is a function of a random variable, it too is a random variable and its distribution is related to $F(a)$ (and $\Phi(v)$).

4.2.1. All Bonus Bids

The i^{th} potential bidder will participate at an auction if

$$A_i \leq \hat{a}.$$

Introducing the indicator variable

$$P_i = \begin{cases} 1 & \text{if } A_i \leq \hat{a}, \\ 0 & \text{otherwise,} \end{cases}$$

one can calculate the probability of potential bidder i participating at the auction

$$\Pr[P_i = 1] = \Pr[A_i \leq \hat{a}] = F(\hat{a}).$$

The probability of potential bidder i not participating is then

$$\Pr[P_i = 0] = \Pr[A_i \geq \hat{a}] = [1 - F(\hat{a})].$$

When potential bidder i participates, the density of his bonus bid $g(b_i)$ is related to the density of average harvesting costs via

$$g(b_i) = f(\hat{a} - b_i) \quad 0 < b_i \leq \hat{a}.$$

Thus, by independence, the joint density of bidding and non-participation for the sequence $\{a_i\}_{i=1}^{N}$ is

$$\prod_{i=1}^{N}[1 - F(\hat{a})]^{(1-P_i)} g(b_i)^{P_i} = [1 - F(\hat{a})]^{(N-n)} \prod_{P_i=1} f(\hat{a} - b_i) \tag{4.2}$$

where the subscript $P_i = 1$ on the product operator $\prod$ denotes that the product is taken over all potential bidders who participated at the auction. Unfortunately, sales at which none of the potential bidders participates are unobserved. Thus, (4.2) must be scaled by one minus the probability of none of the potential bidders' participating; *viz.*,

$$1 - \Pr[(A_1 > \hat{a}) \cap \cdots \cap (A_N > \hat{a})] = 1 - \prod_{i=1}^{N} \Pr[A_i > \hat{a}] = 1 - [1 - F(\hat{a})]^N, \tag{4.3}$$

yielding the following density for auctions with some positive bids:

$$\frac{[1 - F(\hat{a})]^{(N-n)} \prod_{P_i=1} f(\hat{a} - b_i)}{1 - [1 - F(\hat{a})]^N}. \tag{4.4}$$

At some English auctions, however, only one potential buyer is willing to bid. As mentioned above, the dominant strategy for that person is to submit the minimum acceptable bid, in the case of timber a bonus bid of zero. To calculate the probability of this event, note that the number of participants at an auction

$$n = \sum_{i=1}^{N} P_i$$

is distributed binomially with parameters N and $\Pr[P_i = 1] = F(\hat{a})$. Thus, the probability of only one potential bidder participating is

$$N[1 - F(\hat{a})]^{N-1} F(\hat{a})$$

which, when scaled by (4.3), yields

$$\frac{N[1 - F(\hat{a})]^{N-1} F(\hat{a})}{1 - [1 - F(\hat{a})]^N}. \tag{4.5}$$

Collecting (4.4) and (4.5), the density of observed bids is

$$\frac{\left[N[1 - F(\hat{a})]^{N-1} F(\hat{a})\right]^D \left[[1 - F(\hat{a})]^{N-n} \prod_{P_i=1} f(\hat{a} - b_i)\right]^{(1-D)}}{1 - [1 - F(\hat{a})]^N} \tag{4.6}$$

where I have introduced the indicator variable

$$D = \begin{cases} 1 & \text{if } n = 1, \\ 0 & \text{otherwise.} \end{cases}$$

4.2.2. Winning Bonus Bids

Often, however, information concerning non-winning bids is unavailable or uninformative. For example, some of those in attendance at the auction may not cry out, so their valuations will not be observed. Thus, observed n may measure actual n with error. This phenomenon may not be that severe, but another can be important. In particular, for those who do cry out their last recorded bids may be far from their true valuations because in the course of the auction other bidders have cried out bids which exceed the maxima they were willing to pay. Thus, recorded bids may measure actual valuations with error. For these reasons, one may want to focus solely upon the winning bid where the measurement of n is irrelevant and where presumably the last two bidders continued to bid as long as it was in their interest to do so.

The winning bonus bid at an English auction is, of course, a simple function of the $\{a_i\}_{i=1}^N$. Thus, its distribution is also related to $F(a)$. Consider a positive winning bonus bid w at an English auction

$$w = \beta(a_{(2:N)}) = \hat{a} - a_{(2:N)} > 0.$$

The density of w, denoted $h(w; N, \hat{a})$, is related to the density of the second-lowest order statistic of average harvesting costs for a sample of size N. Whence

$$h(w; N, \hat{a}) = N(N-1)[1 - F(\hat{a} - w)]^{N-2} F(\hat{a} - w) f(\hat{a} - w) \quad 0 < w \leq \hat{a}. \tag{4.7}$$

As shown above, the probability of a winning bonus bid of zero is

$$h(0; N, \hat{a}) = N[1 - F(\hat{a})]^{N-1} F(\hat{a}). \tag{4.8}$$

In some cases for the data considered below, the observed winning bid is significantly larger than the second highest bid. For example, in the data set considered below between 15 and 20 percent of the sample had winning bids which were more than 5 percent higher than the next highest bid. Although the bulk of these observations occurred when the values of bonus bids were small implying that increments to the bonus bids by integer cents appeared large in percentage terms, the winning bid can potentially measure the second-order statistic of average harvesting costs with error. To admit the presence of this error, I make use of the fact that the average harvesting costs for the second highest bidder (whose last bid was $b_{(2:N)}$, and who did not respond to the final bid of the winner, who bid $b_{(1:N)} = w$) satisfy the following inequalities:

$$\hat{a} - b_{(1:N)} \leq a_{(2:N)} \leq \hat{a} - b_{(2:N)}.$$

Thus, the "density" for a winning bonus bid when more than one logger bids is defined by

$$H\big(b_{(1:N)}; N, \hat{a}\big) - H\big(b_{(2:N)}; N, \hat{a}\big) \tag{4.9}$$

where

$$H(w; N, \hat{a}) = \int_0^w N(N-1)[1 - F(\hat{a} - z)]^{N-2} F(\hat{a} - z) f(\hat{a} - z)\, dz \quad 0 \leq w \leq \hat{a}.$$

Collecting (4.8) and (4.9) in conjunction with (4.3) yields the following density of an observed winning bid:

$$h(w; N, \hat{a}) = \frac{\Big[N[1 - F(\hat{a})]^{N-1} F(\hat{a})\Big]^D \Big[H(w) - H\big(b_{(2:N)}\big)\Big]^{(1-D)}}{1 - [1 - F(\hat{a})]^N} \quad 0 \leq b_{(2:N)} < w. \tag{4.10}$$

4.3. Methods of Estimating the Empirical Models

To construct the optimal auction one must recover information concerning the latent unobserved variable a. A natural way to proceed in recovering an estimate of a's distribution would be to examine the empirical distribution of bonus bids and then to map back to the distribution of a. For example, in the absence of covariates and given a large enough sample, one could perform this exercise non-parametrically. Unfortunately, like most data sets concerning auctions the one considered below is relatively small, 129 auctions with 424 observed final bids. In addition, at least six types of observed heterogeneity across sales (to be discussed below) appear important, making estimating $f(a)$ using non-parametric methods difficult to do reliably.[6] Thus, I have chosen to estimate $f(a)$ using parametric methods.

To employ the parametric approach, I follow Paarsch (1992) by assuming that $f(a)$ comes from a particular family of flexible distributions which can be characterized up to some unknown parameter vector $\underline{\theta}$

$$f(a) = f(a; \underline{\theta}).$$

Thus, the parameter vector $\underline{\theta}$ will imbed itself in (4.6) and (4.10). Using the method of maximum likelihood, I can then back out estimates of $\underline{\theta}$.

4.4. Specifications to be Estimated

In order to write down an exact specification for the empirical model of an English auction considered above, I must specify the six types of observed heterogeneity mentioned above and how precisely they affect harvesting costs and bonus bidding. I must also choose a family for $F(a)$.

The six types of observed heterogeneity are as follows: First, upset rates vary across timber sales. Second, log prices can vary across species, and they have varied somewhat over the period considered, so bidding can vary systematically across sales with different species compositions and in different time periods. Third, the volume of timber varies across sales, and this can affect average harvesting costs. Fourth,

6 The fastest rate of convergence for non-parametric methods when $f(a)$ has one derivative is $T^{2/(2+K)}$ where T is the sample size and where K is the dimensionality of the heterogeneity.

the distance to the nearest timber processing facility varies from sale to sale. Fifth, because logging is a regional industry (less than ten percent of all timber sales in any particular Forest District involve bidders from outside of that district), the level of potential competition can vary from district to district, as well as over time, since no SBFEP registrant can hold more than two SBFEP sales at one time. In short, the number of potential bidders can vary across sales. Finally, because no formal market exists for timber harvested in the interior of the province (unlike on the coast), only an estimate for the price of timber (the construction of which is discussed in the appendix) is available. Thus, the possibility that timber prices for coastal and interior sales may differ systematically must be admitted.

The following specifications admit the six types of observed heterogeneity discussed above. To introduce upset and timber price variation into the above framework I allow upset and timber prices to vary across sales. For a sample of $t = 1, \ldots, T$ sales, I denote the upset and timber prices for species j at the t^{th} auction by u_{jt} and p_{jt} respectively, yielding

$$\hat{a}_t = \sum_{j=1}^{k} (p_{jt} - u_{jt}) \lambda_{jt}$$

where $\lambda_{jt} = q_{jt}/q_t$. I assume that the total harvesting function $C(d, q)$ are a quadratic form in d and q. Thus,

$$C(d, q) = \gamma_0 + \gamma_{d1} d + \gamma_{q1} q + \gamma_{dq} dq + \gamma_{d2} d^2 + \gamma_{q2} q^2.$$

Average harvesting costs for the t^{th} sale a_t will depend upon d_t and q_t according to

$$a_t = \gamma_{q1} + \gamma_{dq} d_t + \gamma_{q2} q_t + \gamma_0 q_t^{-1} + \gamma_{d1} d_t q_t^{-1} + + \gamma_{d2} d_t^2 q_t^{-1}. \tag{4.11}$$

A changing number of potential bidders across auctions is also admitted, denoted N_t, and discussed extensively in section A.6 of the appendix. A dummy variable I_t which equals one for interior sales and zero otherwise is introduced to capture any systematic differences in the level of timber prices between the coast and the interior.

A number of ways of introducing randomness into (4.11) exist. For example, in earlier work (*viz.*, Paarsch [1989, 1991]) I assumed that γ_{q1} followed a Weibull distribution, while the remaining γs were unknown constants to be estimated. When $\gamma_{q2} = \gamma_{d1} = \gamma_{d2} = 0$, γ_{q1} can be interpreted of as a random marginal harvesting cost. Of course, an alternative would be to assume that the fixed costs of harvesting γ_0 are random and that γ_{q1} is a parameter to be estimated. Ex ante, none of these alternatives appears preferable, although some of them are more computationally parsimonious than others. While some of the alternatives may nest others, most do not. Thus, I use the methods of Vuong (1989) to decide upon the empirical specification which is closer to the truth than the others.

In general, I assume that the generic form of parameter randomness for γ follows a Weibull distribution which has cumulative distribution function

$$F_\gamma(c;\theta_1,\theta_2) = 1 - \exp\left(-\theta_1 c^{\theta_2}\right) \quad \theta_1 > 0,\ \theta_2 > 0.$$

The Weibull family has a flexible shape; see Figure 1 for graphs of γ's probability density function conditional upon different $\underline{\theta} = (\theta_1, \theta_2)$ pairs. The Weibull is also attractive because the cumulative distribution function of the second-order statistic has a closed-form solution which makes implementing it on a computer straightforward.[7]

5. Empirical Results and Estimates of the Optimal Reserve Price

Using the data which have been described briefly in section 2 and which are described in complete detail in an appendix to the paper, I estimated several different empirical specifications. As space precludes me from presenting all of my empirical work, in this section I focus upon the two most promising specifications. In particular, I consider empirical models in which either γ_{q1} or γ_0 follows the Weibull law.[8] I then use the procedures of Vuong (1989) to decide between the two specifications.

My parameter estimates for the random γ_{q1} specification of all bids and just the winning bids are presented Tables 1 and 2 respectively. The first thing to note is

[7] This is particularly attractive since the quadrature required for other distributional assumptions often leads to solutions for the maximum likelihood estimator which are ill-behaved numerically.

[8] In some of my work concerning the all-bid specification, I also considered mixtures of Weibull distributions of the form

$$f_\gamma(c) = \omega f_{\gamma 1}(c) + (1-\omega) f_{\gamma 2}(c) \quad 0 \le \omega \le 1$$

where $f_{\gamma 1}(c)$ and $f_{\gamma 2}(c)$ are different Weibull probability density functions. However, when estimating such specifications, I found that they continually attempted to collapse to one-branch Weibulls.

that in both of these specifications I was unable to estimate the γ_{d2} coefficient. The Newton-Raphson routine continually strayed into areas of the parameter space where the Hessian of the likelihood function was numerically singular.

In column (1) of Table 1, I present the least restrictive parameter estimates. Note that the estimate of γ_0, which is the fixed costs of harvesting, is quite large at \$28,811 dollars. Note too that the estimates of γ_{dq} and γ_{d1} are negative, with the latter being estimated quite imprecisely. When I imposed the constraint that $\gamma_{d1} = 0$, the logarithm of the likelihood function fell a little, but the calculated $\chi^2(1)$ statistic of the constraint was 0.9426, which has a p-value of between 0.50 and 0.75. When γ_{d1} is constrained to zero, the estimate of γ_0 falls to \$6881.77, see column (2) of Table 1, which is a reasonable amount for the sales considered in this sample. The estimate of γ_{dq}, the marginal cost of transporting one cubic metre of timber one kilometre, remained negative but insignificant. I subsequently imposed the restriction that $\gamma_{dq} = 0$. The resulting decrease in the logarithm of the likelihood function resulted in a calculated $\chi^2(1)$ statistic of 0.6776, which has a p-value of between 0.50 and 0.75. Subsequently, I imposed the restriction that $\gamma_{q2} = 0$. The resulting decrease in the logarithm of the likelihood function yielded a calculated $\chi^2(1)$ statistic of 23.9824, the p-value for which is less than 0.005. Thus, the preferred estimates for this specification are given in column (3) of Table 1.

Note that in column (3) the estimate of δ suggests that the price series for interior timber is different from that for the coast, about \$10 more on average. Note too that the negative estimate for γ_{q2} implies increasing returns-to-scale in the harvest of SBFEP timber. Moreover, the cost advantage for large sales is substantial. In particular, according to these estimates the marginal cost of harvesting falls about \$16 per cubic metre between the average sale in the sample (having about 10,000 cubic metres) and the largest sale in the sample (having about 50,000 cubic metres).

Consider now the parameter estimates concerning the winning bid specification which are presented in Table 2. Note that for all four columns of this table the estimate of γ_0 is lower than its counterpart in Table 1. This is basically true for the estimates of δ, θ_1, and θ_2 too. The estimates of γ_{q2} are consistently similar to those in Table 1, and the insignificance of γ_{d1} and γ_{dq} carried over to this specification. I used a Wald test to test the hypothesis that the coefficient estimates from Table 1 column (3) are the same as those from column (3) of Table 2 assuming that the two sets of estimates were independent.[9] The calculated $\chi^2(5)$ statistic was 25.2889, the p-value for which is less than 0.005.

That observed n can mis-measure actual n and that the observed bids can mis-measure the actual valuations provide one explanation for the differences between the estimates presented in Tables 1 and 2. Because the estimates of the winning bid specification are less likely to be affected by these sorts of mis-measurement, I prefer the estimates of Table 2. Of course, the differences could be arising because of a mis-specification in the way in which the randomness is introduced. Thus, I considered many other ways in which to introduce the randomness, the most promising of which was through γ_0. In Tables 3 and 4, I present my parameter estimates for the random γ_0 specification of all bids and just the winning bids. As with the random γ_{q1} specification, I found it numerically impossible to obtain estimates of γ_{d2}. Hence, the absence of estimates for this parameter from the tables.

Tables 3 and 4 warrant several remarks. First, the estimates of γ_{q1} are consistently between \$16 and \$18 per cubic metre of timber harvested, which is reasonable. Second, the estimates of δ are consistently negative and between \$2 and \$3 per cubic metre, in contrast to the consistently positive estimates reported in Tables 1 and 2. Third, whereas the estimate of γ_{q2} is negative in Table 3 and of the same magnitude as those in Tables 1 and 2, the estimate estimate of γ_{q2} in Table 4 is positive, and significant; the p-value for the likelihood ratio statistic of its being zero is below 0.005. Finally, one could not reject the hypotheses that either γ_{d1} or γ_{dq} or both equalled zero. Because the estimates of the winning bid specification are less likely to be affected by the sorts of mis-measurement discussed above, I prefer the estimates of Table 4.

But of the two winning bid specifications, which one is preferred? I calculated Vuong's (1989) test statistic of the γ_{q1} specification for the winning bid versus the γ_0 specification for the winning bid. This statistic was 2.9327, which is distributed standard normal under the null hypothesis, has a p-value of less than 0.005. Such evidence suggests that the γ_{q1} specification is closer to the true specification than is the γ_0 specification.

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[9] The two sets of estimates are likely positively correlated, but deriving an estimate of their covariance has proven difficult to calculate. Nevertheless, this test statistic likely understates the magnitude of the true test statistic, so the test is likely conservative.

Using the parameter estimates from column (3) of Table 2 in conjunction with the sample means of the covariates, I calculated $\hat{\phi}(v)$, an estimate of the probability density function of valuations evaluated at the "average" covariates of sales in the sample. This is presented in Figure 2. Note that a large portion of the mass is negative, a fact consistent with anecdotal evidence concerning the type of sales encountered in the SBFEP. In particular, it is often claimed that the major Tree Farm Licence holders give up only marginal timber for disposition in SBFEP. Thus, SBFEP sales are typically of marginal profitability.

Assuming different values for v_0, the reservation valuation for the timber by the seller, I can also estimate the optimal reserve price r^* using $\hat{\phi}(v)$. For example, if $v_0 = 0$, then an estimate of the optimal reserve price $\hat{r}^*$ is \$158,998.54. In the current sample, the "average" total upset price is \$24,234.60, suggesting that the Forest Service is too lenient in the setting of the reserve price for timber. On the other hand, if v_0 is set equal to the "average" total upset price, then $\hat{r}^*$ is even higher at \$183,162.61. Suppose, however, that the "average" total upset price is, in fact, the optimal reserve price. What then would be the revealed reservation valuation of the seller? In this case, an estimate of the seller's reserve valuation $\hat{v}_0$ would be $-\$135,156.45$.

6. Summary and Conclusion

In this paper, I have used a simple game-theoretic model of behaviour at English auctions within the independent private values paradigm to put structure upon data from a sample of timber sales held in the province of British Columbia where, to a first approximation, the independent private values paradigm appears appropriate. Estimates of several different empirical specifications were presented and the methods of Vuong (1989) were used to select a preferred specification. Under different assumptions concerning the seller's valuation of the timber, estimates of the optimal auction were calculated. These estimates suggest that the current practice of imposing relatively low upset rates is sub-optimal. Using my preferred empirical specification, I estimate that the optimal upset rates should be somewhere between \$15.68 and \$18.06 per cubic metre instead of \$2.39. In addition, the cost structure of a representative firm suggests that the Forest Service could enhance the stumpage rates which it garners by increasing the volumes per site for sale from 10,000 cubic metres to 50,000 cubic metres.

A. Appendix

In this appendix, I document the development of the data set used, describing the sources from which the data were taken as well as the transformations used in obtaining the final data set. The important descriptive statistics are also presented in Table A.1.

A.1. Bonus Bids

For data concerning the bonus bids tendered at auctions, I travelled around the province of British Columbia, Canada to nine Forest Districts, and searched through the District records on file for a sample of timber sales. The data set covers auctions held between January 1984 and December 1987 inclusive. The districts considered are the Arrow, Campbell River, Kamloops, Kootenay Lake, Lillooet, Merritt, Port Alberni, Prince George East, and Prince George West. I chose these particular districts because the SBFEP is well-established in each.

The sampling scheme was not scientific. I searched through as many files as time permitted at each district. The binding constraints were funding related. I eliminated sales for which files were unavailable during the collection period. There are potentially several reasons why files might be unavailable, but the most common in my case was that employees of the Ministry of Forests were using them in their work and could not release them. In one case, however, a file was evidence in a criminal case. Sales were *not* selected by species composition or by the number of actual bidders at the auction. For each sale, a copy of the original bid record for the auction was made. In all, the data set contains information concerning 129 English auctions.

A.2. Upset Rates

For each species in a sale, the Ministry of Forests calculates a minimum acceptable price per cubic metre which is the upset rate. Data concerning these rates were retrieved from the Harvest Database maintained by the provincial Ministry of Forests in Victoria, British Columbia, Canada.

A.3. Cruised Timber Volumes

Data concerning the volume of standing timber on each sale by species were also retrieved from the Harvest Database. These data are derived from information contained in the timber cruise report.

A.4. Percentage Small/Large Log Composition and Lumber Recovery Factors

Unlike on the coast, no log market exists in the interior of British Columbia. Consequently, valuing the timber on any sale in the interior, using market prices for logs, is impossible. Moreover, log prices from the coastal log market are unlikely to form a useful proxy since interior timber is quite different from coastal timber. One way of circumventing this problem is to convert a market price for interior lumber into a price index for logs.[10] I discuss the creation of this index separately in section A.7. Here I simply describe the variables used.

The volume of lumber to be recovered from timber sales in the interior is calculated by using the percentage of small and large logs estimated on the sale, as well as by the lumber recovery factors (LRFs) for each type of log by species. These too are contained in the timber cruise report.

Small and large logs are defined by diameter, with small logs having diameters less than thirty centimetres and large logs having diameters greater than thirty centimetres. The volume of timber to be recovered differs by log size and species, and so too do the LRFs. These data were also retrieved from the Harvest Database.

A.5. Log and Lumber Prices

To value coastal sales, I used log prices from the Vancouver Log Market. These data were provided by the Council of Forest Industries. I considered seven different species: balsam, fir, hemlock, pine, red cedar, spruce, and yellow cedar. Some of the sales included timber for species such as alder, aspen, and poplar which I assumed were to be used as firewood. I ignored these species, since their volumes are negligible (less than two percent on average).

The lumber price series I have chosen to use in valuing interior timber is the real average monthly price per thousand board feet (1M) for one box car of Spruce-Pine-Fir (SPF), Western, Kiln Dried (KD), 2x4s, Standard and Better (Std&Btr), Random Lengths (R/L), and is taken from the trade publication *Madison's Canadian Lumber Reporter*, weekly issues, 1984 to 1987.

This price series is listed as "less 5 & 2 percent" discounts, and is FOB mill. Moreover, it is quoted in nominal U.S. dollars. To convert the series into Canadian dollars I used the Canadian/U.S. exchange rate series B3400 from the CANSIM database.

[10] I shall discuss one potential lumber price below.

I converted all nominal data (bonus bids and upset rates as well as log and lumber prices) into real terms by dividing by the Canadian Consumer Price Index (CPI) with January 1987 = 1.0.

A.6. Number of Potential Bidders

All participants in the SBFEP must be registered at a district office, and their names and addresses are kept in the Small Business Forest Enterprise Program Registry. Eligible registrants may bid at any auction in the province, but over ninety percent of all sales in a particular district involve only bidders from that district. In my sample, all bidders are from the district in which the sale was held, and in a few cases from an adjacent district in the same region. For example, because the Prince George East and West districts border one another, four registrants from the East district bid at West auctions.

At any auction, only a subset of the potential bidders participates. There are at least two reasons for this: First, conditional upon timber prices and upset rates as well as the cost structures of SBFEP registrants, it is unprofitable to bid. Second, no registrant in the SBFEP can hold more than two SBFEP sales at one time.

In creating the variable "number of potential bidders," I wanted to include those bidders who chose not to participate, but to exclude those who were ineligible. I have assumed that only those registrants from the home district holding fewer than two SBFEP sales at the time of a sale are potential bidders at that auction.

A.7. Data Transformations

Other than converting the nominal data into real Canadian dollars, few transformations of the data described above were required. The main exception is log prices for the interior. No formal log market exists for interior timber, like the market on the coast. Since some of the analysis rests upon the variable p, a proxy is required. I have chosen to convert the price for one thousand board feet (Mftb) of Spruce-Pine-Fir 2x4s (SPF) into such a proxy.

Suppose there are k different species indexed $j = 1, \ldots, k$. I assume that this SPF price applies across all species. Such an assumption is not as restrictive as it may sound since the bulk of interior wood is either fir, pine, or spruce. Moreover, a great deal of timber in the interior is small in diameter, so 2x4s would be a likely use for the logs.

The SPF prices must be converted into the appropriate units. The units of timber prices, upset rates, and bonus bids ($\{p_j\}_{j=1}^{k}$, $\{u_j\}_{j=1}^{k}$, and b) are dollars ($)

per cubic metre (m^3), while those for timber volumes ($\{q_j\}_{j=1}^k$) are m^3. The units of SPF, on the other hand, are \$/Mftb. When divided by 1000, they are \$/ftb. I require a conversion factor, the units of which are ftb/m^3. The lumber recovery factors discussed in subsection A.4 (which, for small and large logs respectively, I denote by $\{\text{LRF}_{sj}\}_{j=1}^k$ and $\{\text{LRF}_{lj}\}_{j=1}^k$) are measured in ftb/$m^3$. Define $q = \sum_{j=1}^k q_j$ and introduce the weights $\{\lambda_j\}_{j=1}^k$ where $\lambda_j = q_j/q$. Now the weights $\{\lambda_j\}_{j=1}^k$ are pure numbers as are the proportions of small and large logs (denoted ω_s and $(1-\omega_s)$). Thus the conversion factor κ,

$$\kappa = \sum_{j=1}^{k}[\omega_s \text{LRF}_{sj} + (1-\omega_s)\text{LRF}_{lj}]\lambda_j, \tag{A.1}$$

has units ftb/m^3. The log price index I use for interior sales is

$$\frac{\text{SPF}}{1000}\cdot\kappa = \frac{\$}{\text{ftb}}\cdot\frac{\text{ftb}}{\text{m}^3} = \frac{\$}{\text{m}^3}. \tag{A.2}$$

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Table 1
Maximum Likelihood Estimates
Weibull γ_{q1} All Bids Specification

Specification	(1)	(2)	(3)	(4)
γ_0	28811.4217	6881.7729	6829.1127	6681.2312
	(767.5713)	(725.2934)	(729.1425)	(502.3718)
γ_{dq}	−0.0362	−0.0332	0	0
	(0.0412)	(0.0398)	–	–
γ_{q2}	−0.0004	−0.0004	−0.0004	0
	(0.0001)	(0.0001)	(0.0001)	–
γ_{d1}	−365.4039	0	0	0
	(737.9325)	–	–	–
δ	10.6213	9.0298	10.6801	6.7497
	(3.2892)	(3.0033)	(2.2883)	(2.0071)
θ_1	0.5726	0.5480	0.5414	0.6815
	(0.1065)	(0.1031)	(0.1030)	(0.1113)
θ_2	4.9493	5.3493	5.3683	4.5043
	(0.3304)	(0.3221)	(0.3268)	(0.2008)
Log-Likelihood Function	−1034.9412	−1035.4125	−1035.7513	−1047.7425

Table 2
Maximum Likelihood Estimates
Weibull γ_{q1} Winning Bid Specification

Specification	(1)	(2)	(3)	(4)
γ_0	7902.5234	2741.2723	2558.9421	4172.0912
	(1897.0159)	(1719.9229)	(1707.5317)	(1576.1391)
γ_{dq}	−0.0585	−0.0765	0	0
	(0.0513)	(0.0461)	–	–
γ_{q2}	−0.0003	−0.0003	−0.0004	0
	(0.0001)	(0.0001)	(0.0001)	–
γ_{d1}	−91.4523	0	0	0
	(117.5302)	–	–	–
δ	6.3484	6.4853	9.1051	7.2131
	(2.9201)	(2.9203)	(2.4804)	(2.4350)
θ_1	0.2364	0.2283	0.2302	0.2221
	(0.0567)	(0.0544)	(0.0546)	(0.0514)
θ_2	3.6252	3.7367	3.6576	3.0985
	(0.3776)	(0.3448)	(0.3529)	(0.2850)
Log-Likelihood Function	−623.9791	−624.2920	−625.5583	−629.9851

Table 3
Maximum Likelihood Estimates
Weibull γ_0 All Bids Specification

Specification	(1)	(2)	(3)	(4)
γ_{q1}	18.3781	18.6081	16.8661	16.9241
	(0.8969)	(0.9047)	(0.3643)	(0.3207)
γ_{q2}	−0.0003	−0.0003	−0.0002	0
	(0.0007)	(0.0007)	(0.0004)	–
γ_{dq}	−0.0523	−0.0501	0	0
	(0.0821)	(0.0312)	–	–
γ_{d1}	−31.9681	0	0	0
	(78.9152)	–	–	–
δ	−3.5691	−3.4346	−2.1401	−2.2139
	(1.0258)	(1.0187)	(0.4957)	(0.4634)
θ_1	0.0755	0.0756	0.0796	0.0774
	(0.0129)	(0.0130)	(0.0110)	(0.0094)
θ_2	1.5825	1.5876	1.5511	1.5718
	(0.1184)	(0.1196)	(0.1149)	(0.1024)
Log-Likelihood Function	−4542.2702	−4542.5601	−4543.3245	−4545.5763

Table 4
Maximum Likelihood Estimates
Weibull γ_0 Winning Bid Specification

Specification	(1)	(2)	(3)	(4)
γ_{q1}	18.9215	17.4001	16.7361	17.1123
	(0.9163)	(0.6482)	(0.6146)	(0.2670)
γ_{q2}	0.0001	0.0001	0.0001	0
	(0.0000)	(0.0000)	(0.0000)	–
γ_{dq}	−0.0149	−0.0168	0	0
	(0.0213)	(0.0151)	–	–
γ_{d1}	−1.3692	0	0	0
	(7.0138)	–	–	–
δ	−2.6923	−2.5984	−2.1233	−2.3032
	(0.4912)	(0.4106)	(0.4919)	(0.3482)
θ_1	0.0504	0.0515	0.0517	0.0512
	(0.0079)	(0.0066)	(0.0064)	(0.0062)
θ_2	0.6537	0.6628	0.6709	0.6856
	(0.0667)	(0.0660)	(0.0600)	(0.0582)
Log-Likelihood Function	−660.5701	−660.8835	−661.2851	−663.1142

Table A.1
Sample Descriptive Statistics — English Auctions
Sample Size = 129, January 1987 CPI = 1.0

Variable	Mean	St.Dev.	Minimum	Maximum
Winning Bonus Bid	6.89	7.01	0.00	28.16
"Average" Upset	2.39	1.59	0.30	10.07
"Average" Stumpage	9.29	7.36	0.30	31.87
"Average" Price	46.89	6.69	34.14	67.48
Actual Bidders	3.29	2.00	1.00	9.00
Potential Bidders	92.39	31.88	27.00	185.00
Total Cruised Volume	10140.04	9720.55	130.00	53300.00
Conversion Factor*	126.75	91.95	0.00	217.10
Haul Distance	37.80	28.37	1.00	136.00

* Conversion factors apply only to interior sales.
Zeros apply to coastal sales, of which there are forty-four.

Figure 1: Weibull Probability Density Functions for Various Parameter Pairs

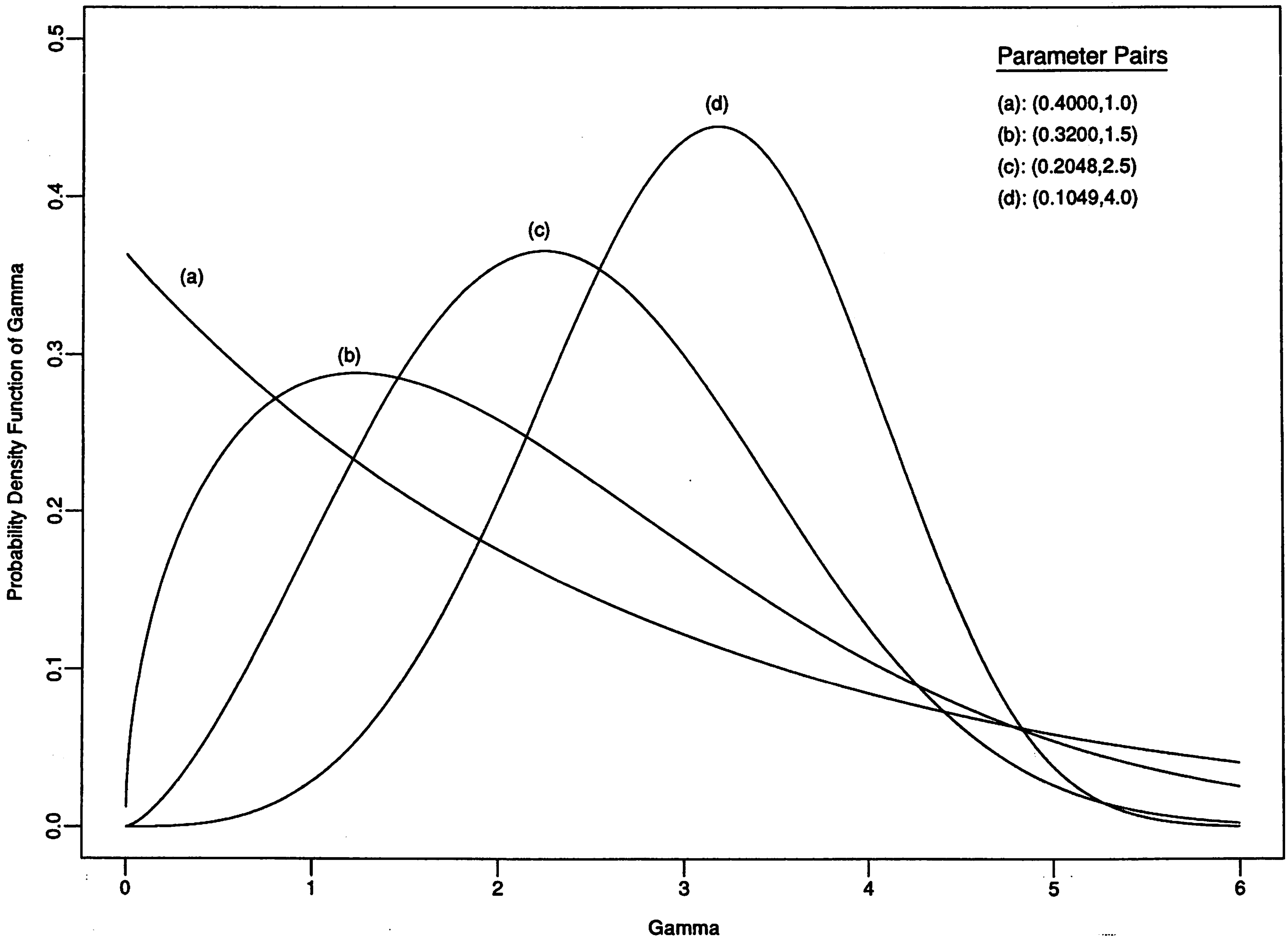

Figure 2: Probability Density Function of 'Average Sale' - Table 2, Column (3) Estimates