1980

A Note on Pareto Optimal Insurance

Sam Bucovetsky

20113

ISSN: 0318-725X
ISBN: 0-7714-0165-5

RESEARCH REPORT 8007

A NOTE ON PARETO OPTIMAL INSURANCE

by

Sam Bucovetsky

February, 1980

A NOTE ON PARETO OPTIMAL INSURANCE

Sam Bucovetsky
Department of Economics
The University of Western Ontario
London, Ontario N6A 5C2 Canada

Abstract

This note examines the characteristics of Pareto optimal insurance contracts in the Rothschild and Stiglitz model of imperfect information. These contracts can be characterized by a subsidy from one risk group to another. The contracts consist of an interval between the minimal and maximal subsidies. At the maximal subsidy, Pareto optimality may involve full insurance by the low-risk, and excessive insurance by the high-risk. The differing equilibrium concepts of Rothschild and Stiglitz, Wilson and Miyazaki are then related to Pareto optimality.

*Comments by M. Hoy are gratefully acknowledged, without implicating him in any inadequacies herein.

A NOTE ON PARETO OPTIMAL INSURANCE

1

The purpose of this note is to relate various concepts of equilibrium in markets with imperfect information to the Pareto optimal outcomes. The framework used will be that of Rothschild and Stiglitz (1976). There are two classes of people, whose probability of some loss d differs. Both classes have identical concave von Neumann-Morgenstern utility functions for wealth in the two states of the world. Initial wealth W is the same for each class. An insurance contract (α_1,α_2) is a premium α_1 to be paid by the insured if there is no loss, and an indemnity (net of the premium) α_2 to be paid to the insured in the event of a loss. Thus net wealth, given the contract α, is $W - \alpha_1$ in the event of no loss, and $W - d + \alpha_2$ in the event of the loss. Let p_i be the probability of loss for class i $(p_1 > p_2)$. This is assumed exogenous. Finally, let λ be the proportion of high-risk (class 1) customers.

Firms are risk-neutral, so that expected profits are all that matter. Thus in the absence of information problems, the optimal zero-profit contract is one with $\alpha_1 = p_i d$, $\alpha_2 = (1-p_i)d$ for each i, providing full insurance. But if identifying low-risk individuals is impossible, it is not feasible to provide optimal insurance to each group, as the high-risk group will prefer the low-risk contract. A pair of contracts (α^1,α^2) (superscripts refer to groups) will be described as feasible if

(1) the high-risk prefer their contract to the low-risk contract

(2) the low-risk prefer their contract to the high-risk contract

(3) the contracts together make zero expected profit.

$$(1) \quad p_1U(W-d+\alpha_2^1) + (1-p_1)U(W-\alpha_1^1) \geqq p_1U(W-d+\alpha_2^2) + (1-p_1)U(W-\alpha_1^2)$$

$$(2) \quad p_2U(W-d+\alpha_2^2) + (1-p_2)U(W-\alpha_1^2) \geqq p_2U(W-d+\alpha_2^1) + (1-p_2)U(W-\alpha_1^1)$$

$$(3) \quad \lambda(p_1\alpha_1^1 - (1-p_1)\alpha_2^1) + (1-\lambda)(p_2\alpha_1^2 - (1-p_2)\alpha_2^2) = 0$$

As defined here, feasibility has nothing to do with competitive behavior by firms. It is merely a requirement for zero profits in the aggregate. Figure 1 illustrates this. On the axes are wealth in the two states. The indifference curves of group i have the slope $-\frac{1-p_i}{p_i}$ at the 45^o line. The point E is the no-insurance point. The lines BE and AE are the sets of contracts making zero profits for groups 1 and 2 respectively.[1] Lines GD and FC have the same slopes as BE and AE. Thus expected profits on GD are constant (and positive in Figure 1) if people of group 1 choose contracts there. Any pair of contracts on lines such as GD and FC will satisfy condition (3) provided that FC and GD are on opposite sides of the zero-profit lines, and the length of the line segments BG and AF are in the ratio $(1-\lambda)/\lambda$. Alternatively, BG and AP must intersect along the market-odds line PE.[2] The indifference curves in the figure also indicate that conditions (1) and (2) are satisfied by the contracts yielding the points C and D as wealth in the two states.

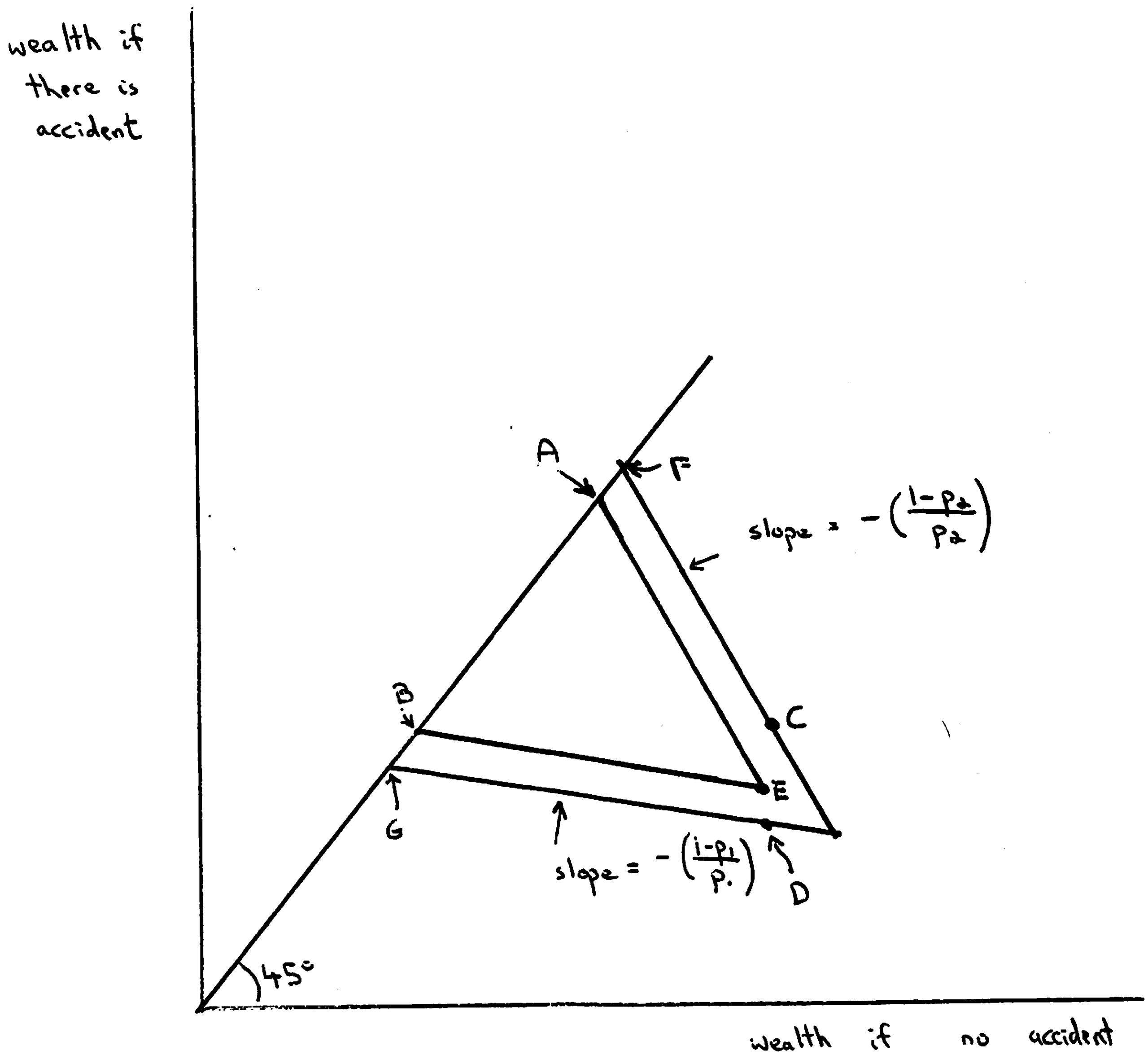
There is nothing in the definition of feasibility which requires a contract to be preferred to the initial endowment. Clearly, however, if insurance is not compulsory, no one would choose a contract which was not preferred. In such a case, two further conditions will be required.

$$(4) \quad p_1U(W-d+\alpha_2^1) + (1-p_1)(W-\alpha_1^1) \geqq p_1U(W-d) + (1-p_1)U(W)$$

$$(5) \quad p_2U(W-d+\alpha_2^2) + (1-p_2)(W-\alpha_1^2) \geqq p_2U(W-d) + (1-p_2)U(W)$$

A pair of contracts (α^1,α^2) satisfying (1)-(5) will be referred to as voluntarily feasible.

figure 1


wealth if there is accident
A
F
slope = $-\left(\frac{1-p_2}{p_2}\right)$
B
C
E
G
slope = $-\left(\frac{1-p_1}{p_1}\right)$
D
45°
wealth if no accident

I first wish to characterize Pareto optima. Any feasible set of contracts defines a subsidy s from low-risk to high-risk.[3] This is just the distance BG in Figure 1 (note that there the subsidy is negative). The zero-profit condition (3) can be written

$$(1-\lambda)(p_2\alpha_1^2 - (1-p_2)\alpha_2^2) = \lambda s \tag{6}$$

It is a standard result (to be proved below)[4] that an optimal set of separating contracts involves full insurance for the high-risk. In such a case their utility must be $U(W+s-p_1d)$, since there the expected loss is $p_1(d-s+p_1d) + (1-p_1)(1-p_1d) = s$. Any pareto optimal contact must then maximize the low-risk's utility subject to it not being chosen by the high-risk (condition (1)), and making appropriate profits (condition (6)). However, this may not be sufficient for optimality. The utility of the high-risk increases with s. For the solution described to be Pareto-optimal, then, the utility of the low-risk must decrease with s.

That this possibility does not always occur is proved by Rothschild and Stiglitz. They show it may be possible for a separating "equilibrium" pair of contracts, each making zero profits, to be upset by a firm offering a pair of cross-subsidizing contracts. They also show that this can occur if and only if the original separating "equilibrium" is not Pareto optimal. In this "perverse" situation, then, the utility of the low-risk people increases with the subsidy s they pay, at least in a neighborhood of s = 0.

2

In this section, properties of the Pareto optimal pairs of contract are described for "low" values of the subsidy. Suppose a subsidy of s will be given to the high-risk, represented by the distance BG in Figure 2.

If the full-insurance point G is chosen by the high-risk, then the FC which leaves the high-risk indifferent between the two contracts, and makes profits $(\frac{\lambda}{1-\lambda})$ s on low-risk people is C. Now suppose some other contract on the subsidy line GD is given to the high-risk. It clearly must give them lower utility.[5] Moreover the indifference curve (for the high-risk) through such a point (H in Figure 2) cuts the low - risk group's iso-profit line at a point like J, which is to the right of C. This gives low-risk people lower utility (concavity implies utility of group i decreases as one moves along any line of slope $-\frac{1-p_i}{p_i}$ away from the 45^o line). Thus the pair of contracts (G,C) dominate any other feasible pair yielding the same subsidy. For future reference, however, it might be noted that this analysis will not work if the subsidy is so large that G is to the right of F in Figure 2. Such a subsidy will be analyzed below.

Using the notation of Rothschild and Stiglitz, for a given subsidy s, let

$$X = W - \alpha$$

$$Y = W + s - p_1 d$$

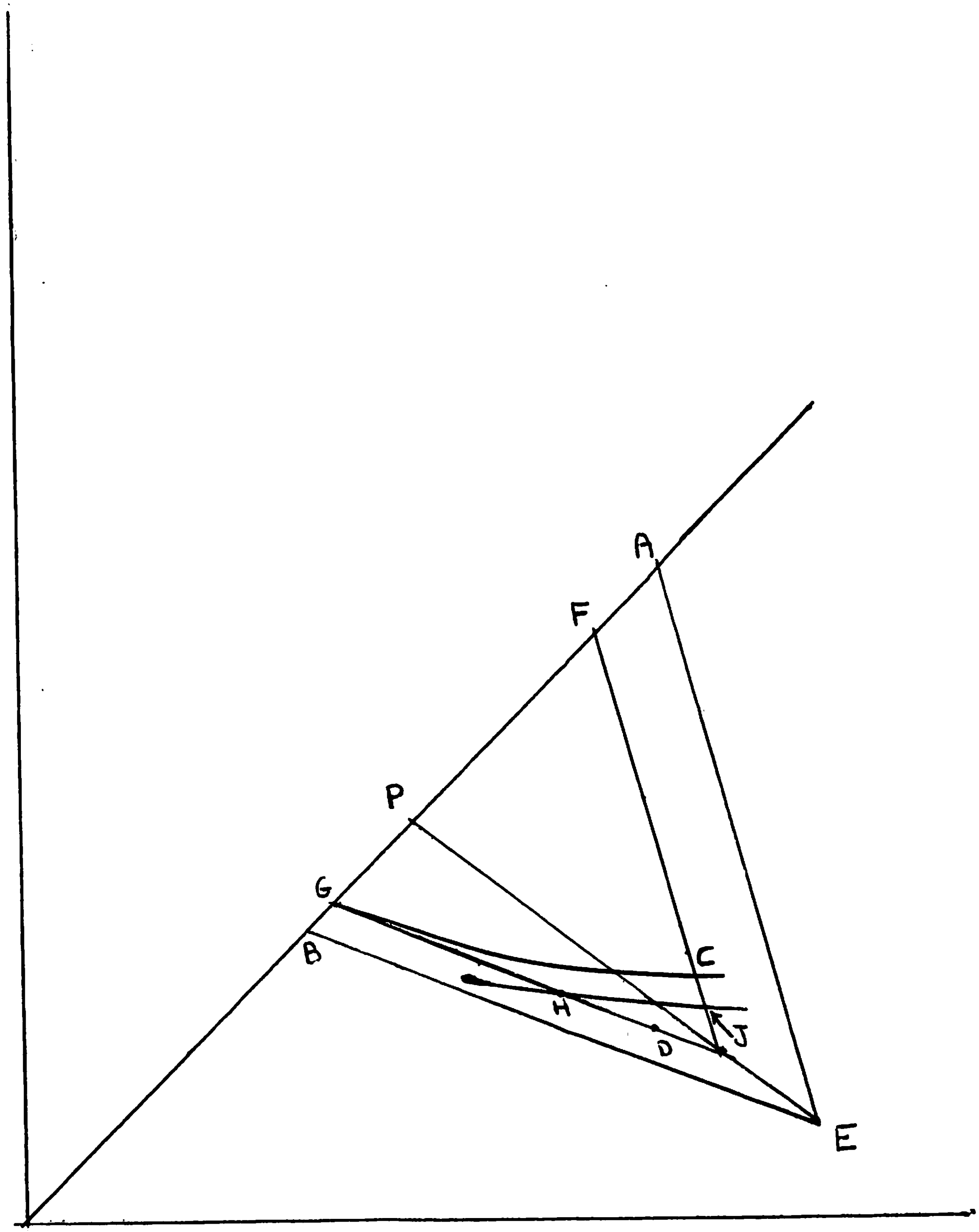
$$Z = W - d + \frac{1-p_2}{p_2}\alpha - \frac{\lambda}{1-\lambda} s$$

where α is the premium paid in the event of no loss by the low-risk group, defined by[6]

$$p_1 U(Z) + (1-p_1)U(X) = U(Y) \tag{7}$$

Therefore, the point G in Figure 2 is (Y,Y) and the point C is (X,Z). Equation (7) implicitly defines the premium α as a function of the subsidy s, and thus can be used to assess the effects of changing the subsidy on the utility of

figure 2



the low-risk group. Differentiating (7),

(8) $$\frac{d\alpha}{ds}\{\frac{p_1}{p_2}(1-p_2)U'(Z) - (1-p_1)U'(X)\} = U'(Y) + p_1 \frac{\lambda}{1-\lambda} U'(Z)$$

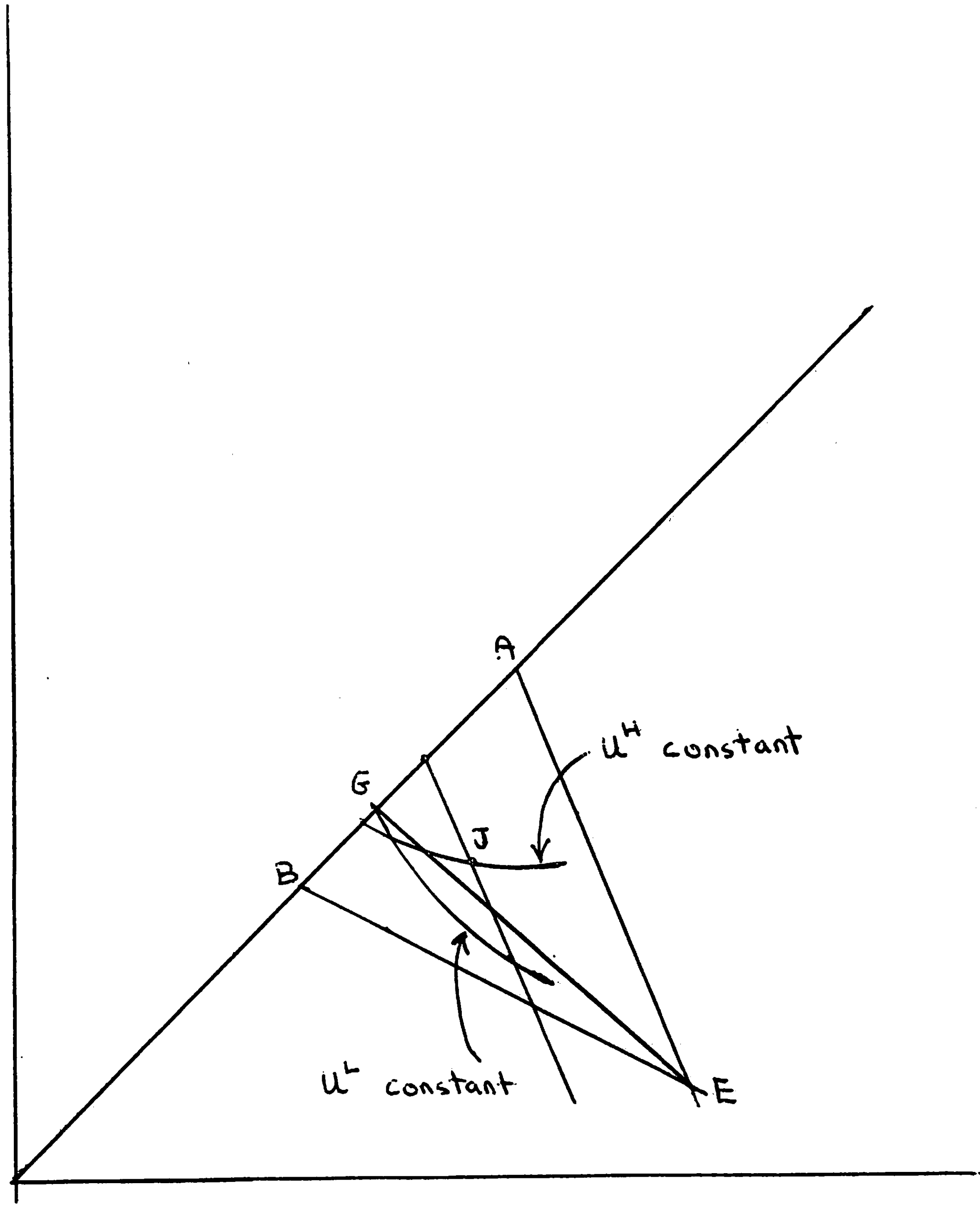
Since $Z < X$, and $p_1 > p_2$, the term in brace brackets is positive so that $\frac{d\alpha}{ds} > 0$. The change in low-risk utility U_2, as s changes, can be written

(9) $$\frac{dU_2}{ds} = U'(Z) \frac{\lambda}{1-\lambda} + (1-p_2) \frac{d\alpha}{ds}(U'(Z) - U'(X))$$

The first term indicates the loss in utility due to the higher price of insurance. The second represents the gain due to being able to buy more insurance. (For (7) to hold, Z must increase with s.) If the whole expression $\frac{dU_2}{ds}$ is positive, then the outcome (G,C) is not Pareto optimal, since a higher subsidy will increase both groups' utility. If it is negative, then the point is a local Pareto optimum. If there is a unique s for which $\frac{dU_2}{ds} = 0$, then a local Pareto optimum is also globally Pareto optimal. The reason for this is it will be shown below that for large values of s, $\frac{dU_2}{ds} < 0$.

Suppose now that the subsidy s is so large that the points G and F coincide, as in Figure 3. Denote this subsidy by s*. Then the point G is on the pooling-equilibrium line of contracts which break even if both groups purchase. The Pareto-optimum (given the subsidy s*) is to offer both groups G. This is just a limiting case for the solution when G is to the left of F. The indifference curve for low-risk people through G lies below the pooling-equilibrium line, and the high-risk indifference curve

figure 3



lies above. Thus a slightly smaller subsidy will give rise to a low-risk contract like J, which is preferred by the low-risk to G. Hence near the pooling solution the utility of the low-risk falls with the subsidy.

Thus Theorem 1, which proves U_2 to be concave as a function of s, indicates that there is some minimum optimal subsidy s_o, which is less than s*, and may be negative. For all values of s between s_o and s*, there is a Pareto optimal pair of contracts, with full insurance for the high-risk. The corresponding low-risk contract does not involve full insurance (for $s < s^*$), and leaves the high-risk group indifferent between the two contracts. The utility of the low-risk group declines with s. There will be a competitive equilibrium if and only if $s_o \leq 0$.

3

Theorem 1

If the subsidy s is such that $\frac{dU_2}{ds} = 0$ (and $s < s^*$) then

$$\frac{d^2U_2}{ds^2} < 0 .$$

Proof: Differentiate (8) with respect to s

$$(10) \quad \frac{d^2\alpha}{ds^2}\{p_1 \frac{(1-p_2)}{p_2} U'(Z) - (1-p)U'(X)\} + \frac{d\alpha}{ds}\{p_1 \frac{(1-p_2)}{p_2} U''(Z)[\frac{1-p_2}{p_2}\frac{d\alpha}{ds} - \frac{\lambda}{1-\lambda}] + (1-p_1)U''(X)\frac{d\alpha}{ds}\} = U''(Y) + p_1 \frac{\lambda}{1-\lambda} U''(Z)[\frac{1-p_2}{p_2}\frac{d\alpha}{ds} - \frac{\lambda}{1-\lambda}]$$

Differentiate (9) with respect to s

$$(11) \quad \frac{d^2U_2}{ds^2} = - p_2U''(Z)\frac{\lambda}{1-\lambda}[\frac{1-p_2}{p_2}\frac{d\alpha}{ds} - \frac{\lambda}{1-\lambda}] + (1-p_2)\frac{d^2\alpha}{ds^2}(U'(Z) - U'(X)) + (1-p_2)\frac{d\alpha}{ds}U''(Z)[\frac{1-p_2}{p_2}\frac{d\alpha}{ds} - \frac{\lambda}{1-\lambda}] + (1-p_2)(\frac{d\alpha}{ds})^2 U''(X)$$

This can be rearranged to

$$\text{(12)}\quad \frac{d^2U_2}{ds^2} = p_2U''(Z)\left[\frac{1-p_2}{p_2}\frac{d\alpha}{ds} - \frac{\lambda}{1-\lambda}\right]^2 + (1-p_2)\frac{d^2\alpha}{ds^2}(U'(Z)-U'(X)) + (1-p_2)\left(\frac{d\alpha}{ds}\right)^2 U''(X)$$

Since $Z < X$ and U is concave, if $\frac{d^2\alpha}{ds^2}$ is negative, then $\frac{d^2U_2}{ds^2} < 0$. So attention can be restricted to the case where $\frac{d^2\alpha}{ds^2} > 0$.

$$\text{(13)}\quad \frac{p_1}{p_2}\frac{d^2U_2}{ds^2} = p_1U''(Z)\left[\frac{1-p_2}{p_2}\frac{d\alpha}{ds} - \frac{\lambda}{1-\lambda}\right]^2 + \frac{d^2\alpha}{ds^2}p_1\frac{1-p_2}{p_2}U'(Z) - \frac{d^2\alpha}{ds^2}p_1\frac{1-p_2}{p_2}U'(X) + \frac{p_1}{p_2}(1-p_2)\left(\frac{d\alpha}{ds}\right)^2 U''(X)$$

Since $\frac{d^2\alpha}{ds^2} > 0$ and $\frac{p_1}{p_2}(1-p_2) > (1-p_1)$

$$\text{(14)}\quad \frac{p_1}{p_2}\frac{d^2U_2}{ds^2} < p_1U''(Z)\left[\frac{1-p_2}{p_2}\frac{d\alpha}{ds} - \frac{\lambda}{1-\lambda}\right]^2 + \frac{d^2\alpha}{ds^2}\left\{p_1\frac{1-p_2}{p_2}U'(Z) - (1-p_1)U'(X)\right\} + (1-p_1)\left(\frac{d\alpha}{ds}\right)^2 U''(X)$$

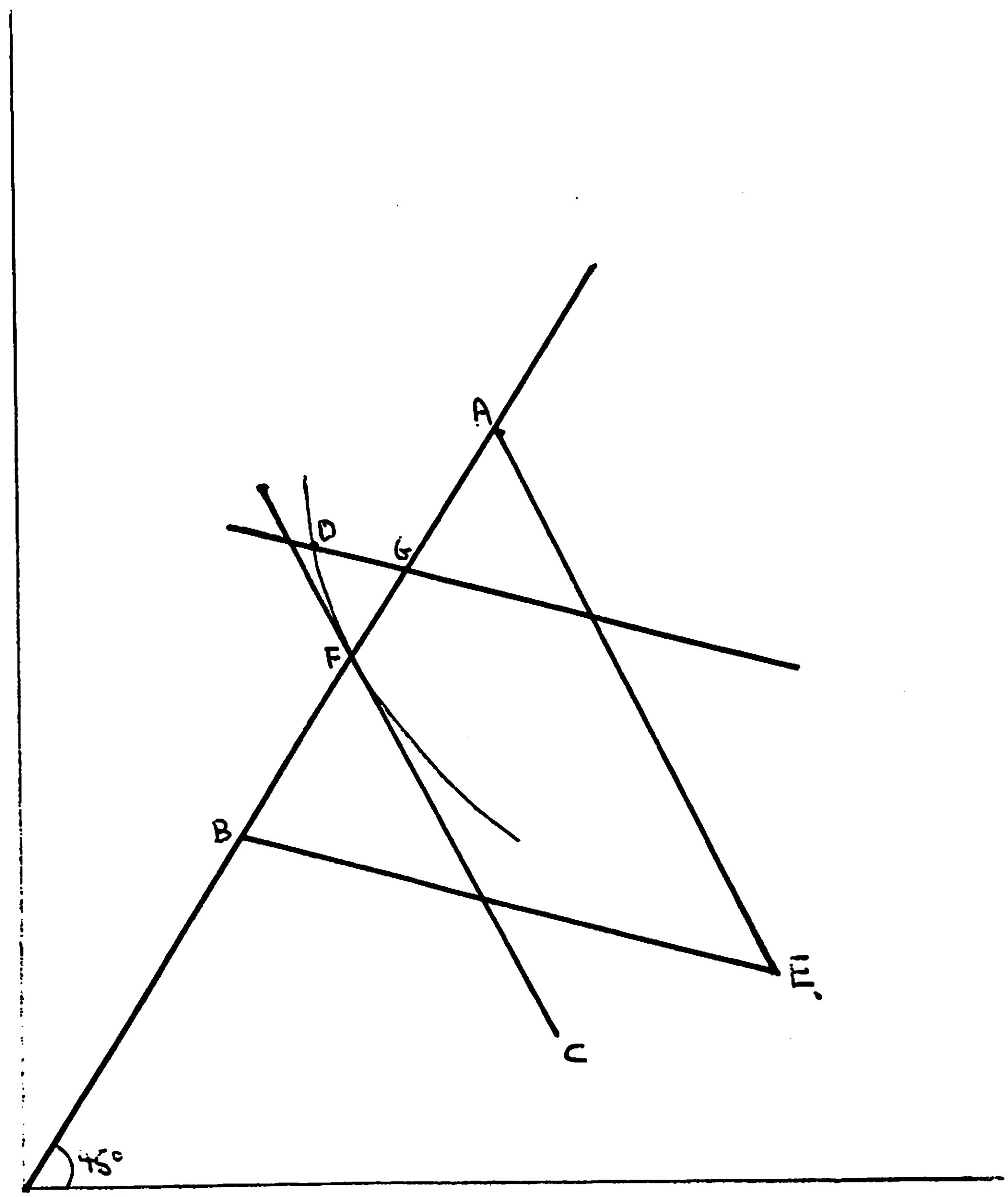
But from (10) the right-hand side of (14) equals $U''(Y)$ which is negative.

4

So far, nothing has been said about the possibility of larger subsidies than s*. And nothing has been said in the literature about this possibility. When the subsidy is this large, the problem is no longer one of preventing the high-risk from choosing a preferred policy intended for the low-risk, but the converse. In other words, constraint (2) becomes binding.

This situation is illustrated in Figure 4. It will now be argued that the pair of contracts illustrated F and D are Pareto optimal. This pair involves complete insurance for the low-risk, and excessive insurance for the high-risk.

figure 4



First, the contract F maximizes the utility of group 2, given the subsidy inherent in FC. Any other contract, such as J in Figure 5, will give them less utility. But the contract on GD which the low-risk find indifferent to J is to the left of D, and thus is less preferred by the high-risk.[7] Thus any other feasible pair (such as (J,H) in Figure 5) yielding subsidy s will be dominated by (F,D). Therefore if there is an optimum involving a subsidy of s, it will be the one illustrated in Figure 4.

As s increases, clearly the utility of group 2 declines, as F moves left along the 45° line. But only if the utility of group 1 increases with s will the contracts be Pareto optimal. This must be true for s near (and greater than) s*. To see this, notice that the intersection of the lines FC and GD (marked K in Figure 6) occurs on the pooling line. Also, the fact that the low-risk group's indifference curves are steeper than $-\frac{1-p_2}{p_2}$ to the left of the 45° line means the high-risk contract D will be to the right of this intersection K. D is thus preferred by the high-risk to K (since high-risk utility increases as G is approached along the line GD). But for K near the 45° line, it will be preferred by the high-risk to the point P where the pooling line intersects the full-insurance line. This is because the high-risk indifference curve is shallower than the pooling line at P. Hence we have D preferred to K which is preferred to P, by the high-risk. But for s near s*, K will be close to P. And P is the contract offered to the high-risk when s = s*. Therefore utility of the high-risk group increases with their subsidy s.

figure 5

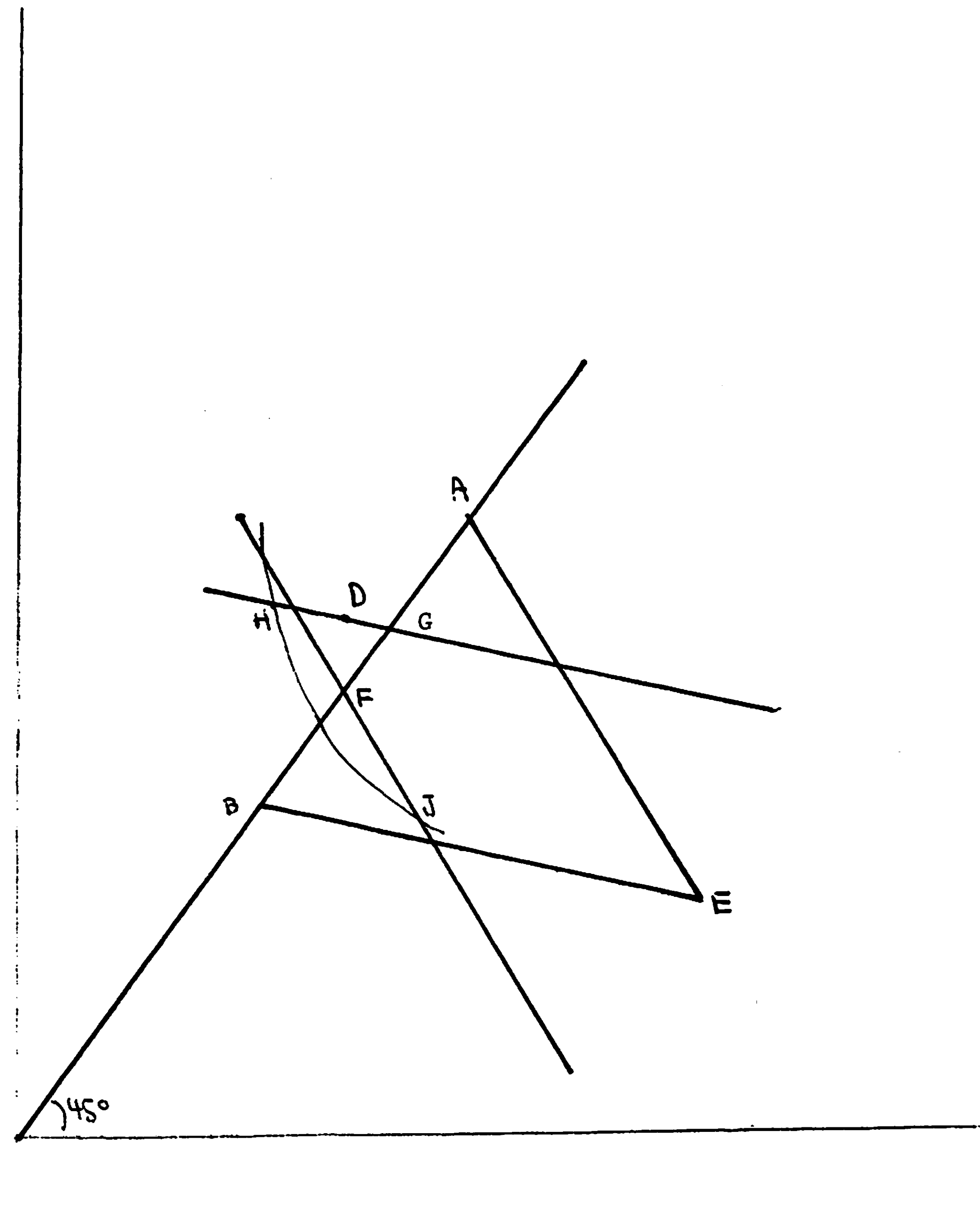
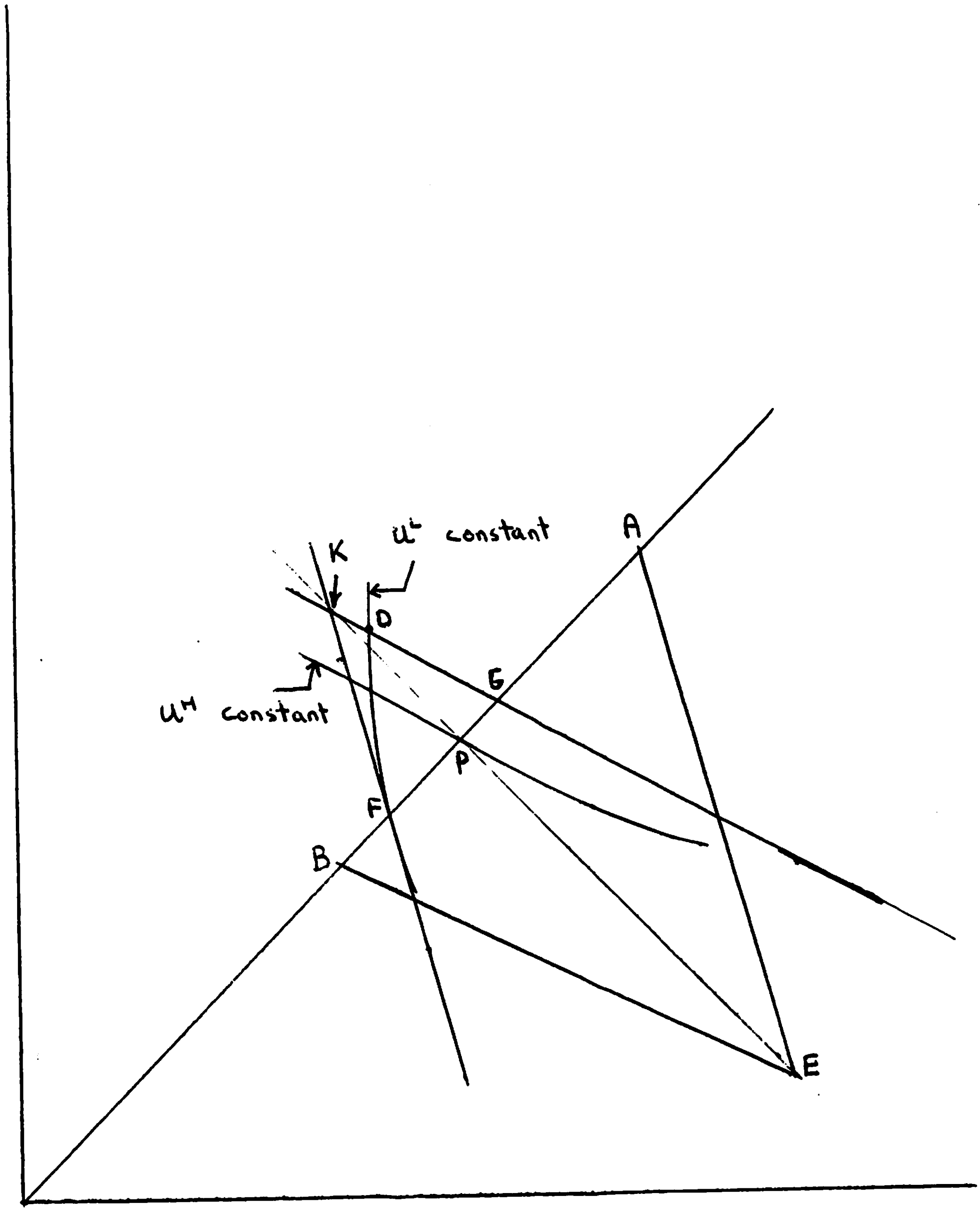


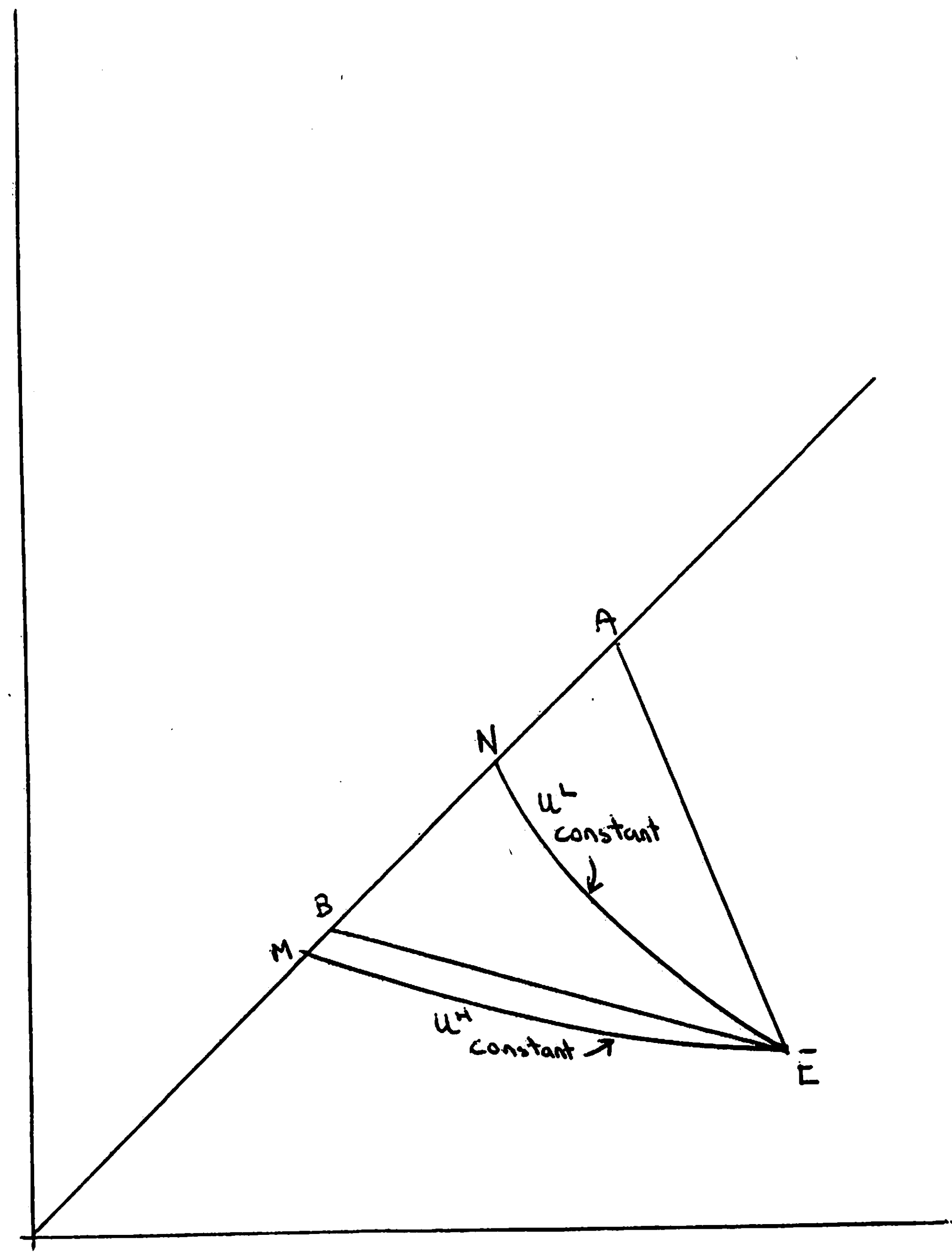
figure 6



This completes the description of Pareto-optimal pairs of contracts which break even. With voluntary feasibility, there is some minimal subsidy which leaves the high-risk indifferent to no insurance, and a maximal subsidy which leaves the low-risk indifferent. These are denoted M and N in Figure 7. M must be to the left of the point B (of zero subsidy), but N may be to the right or left of the pooling line. Each point defines a subsidy. There is some minimal subsidy (which may be negative), at which the utility of the low-risk does not vary with the subsidy. Similarly, there is a maximal subsidy, at which the righ-risk's utility does not vary with the subsidy. Any subsidy between these limits (which meets the voluntarism requirement above) yields a Pareto-optimal pair of contracts. There is some positive subsidy s* at which the pair of contracts is identical at P, the intersection of the 45° line with the pooling line. For all lower subsidies, the contracts are full insurance for the high-risk, and a partial contract for the low-risk which leaves the high-risk indifferent. For higher subsidies, the low-risk get full insurance, and the high-risk more than full insurance, in a contract which leaves the low-risk indifferent.

If N is to the left of P, then such contracts will not be voluntarily feasible. But there will always be some voluntarily feasible Pareto optimum. For if (B,C) is the zero-subsidy optimum, C is preferred by the low-risk to no insurance. Hence either it is Pareto optimal, or there is some larger subsidy which will yield an optimum.

figure 7



5

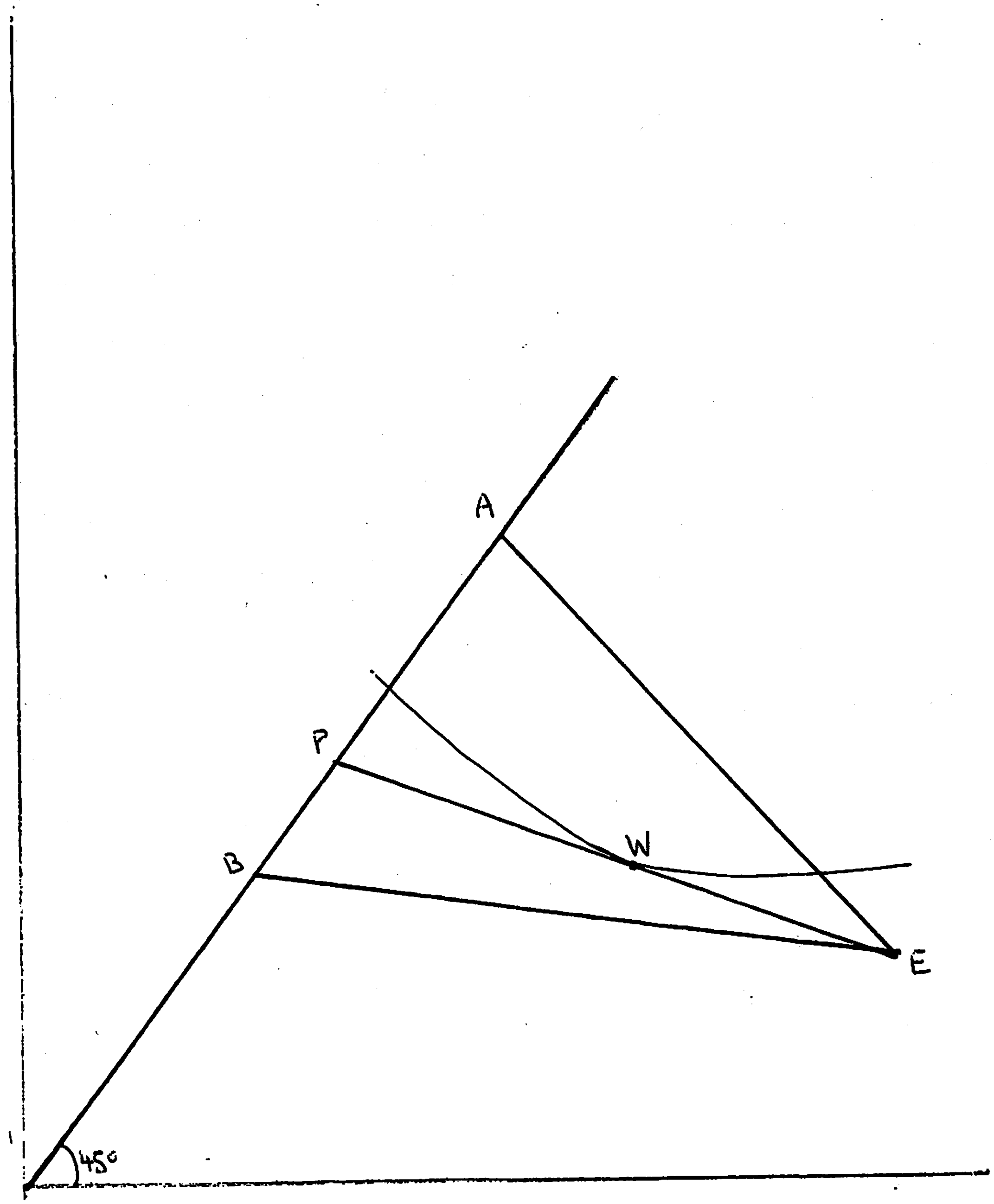
As shown in Rothschild and Stiglitz, this zero-subsidy optimum is the only possible equilibrium with free entry. And it will be an equilibrium precisely when it is an optimum; namely when the minimum feasible subsidy is non-positive, or, alternatively when it maximizes the welfare of the low-risk among all feasible contracts which do not lose money on the low-risk.

An alternative equilibrium concept is Wilson's. This was defined as a state where no single new contract could make profits when all existing contracts which develop losses due to its entry are withdrawn. Miyazaki modified this definition to a state where no set of new contracts could make profits when all existing contracts which develop losses due to their entry are withdrawn. If a competitive equilibrium exists it must be a Wilson equilibrium. If not, then the point W in Figure 8, where the low-risk indifference curve is tangent to the pooling line, must be the Wilson equilibrium. Any new contract which attracts the low-risk from W will cause W to lose money and be withdrawn, and thus must itself attract all customers. To break even it must be on or below the pooling line, which means there is no such contract which will attract the low-risk.

But what about a contract like B in Figure 8? If B is preferred by the high-risk, it will break even. And W will make profits with just the low-risk. Therefore for W to be a Wilson equilibrium, it must be preferred to B by the high-risk.

Suppose now B is preferred to W by the high-risk. Consider the original no-subsidy pair (B,C). It must be shown that this is a Wilson equilibrium. It must be true in this case that C is preferred by the low-risk to any pooling contract.[8] Consider introducing a new contract D. If it attracts the low-risk

figure 8



it must be preferred to C. But any contract preferred to C by the low-risk either loses money (is to the right of AE), or is also preferred by the high-risk. And no pooling contract will be preferred by the low-risk. Thus no contract more attractive to the low-risk than C can be introduced. And no contract more attractive to B to the high-risk can break even, since B maximizes high-risk utility subject to breaking even. Hence either (B,C) or W is always a Wilson equilibrium. Note the above proof depending crucially on cross-subsidizing pairs of contracts not being introduced. The pair (B,C) may be a Wilson equilibrium without being a competitive equilibrium.

Miyazaki's equilibrium concept does allow pairs of contracts to be introduced. A Miyazaki equilibrium must clearly be a Pareto optimum. In fact, it is the Pareto optimum which maximizes low-risk utility, subject to not losing money on the low-risk contract. To prove that this is an equilibrium, first suppose that this Pareto optimum is the zero-subsidy one. Then any other pair of contracts which breaks even cannot attract both groups. And no single new contract which attracts the low-risk can make money on them without also attracting the high-risk. In this case, withdrawal of losing contracts is not a worry to potential entrants, since both existing contracts break even.

If the zero subsidy set of contracts is not a Pareto-optimal one, then the potential Miyazaki equilibrium involves losses on the high-risk. Thus any pair of new contracts which attracts the low-risk will cause withdrawal of the existing high-risk contract. Thus the new pair must break even, and be preferred by the low-risk if it is to succeed, which contradicts the definition of the original pair of contracts. A Miyazaki equilibrium exists, therefore. It is unique if any other Pareto optimum can be upset by a new pair of contracts (Theorem 1 shows that there is a unique Pareto optimum which maximizes the utility of the low-risk subject to making non-negative profits on them). Any pair which makes profits on the high-risk can clearly be easily upset. And any pair which makes profits on the low-risk can be upset by the Miyazaki equilibrium just described. This establishes uniqueness.

We then have

(1) A Wilson equilibrium exists, and is Pareto optimal only if it separates. (This necessary condition is not sufficient.)

(2) A Miyazaki equilibrium always exists, always is Pareto optimal, always separates, and always makes non-negative profits on the low-risk.

(3) A Wilson equilibrium is Pareto optimal if and only if it is a Miyazaki equilibrium.

(4) A competitive equilibrium is Pareto optimal if it exists (and is then a Wilson- and Miyazaki-equilibrium).

The possibility of non-existence of competitive equilibrium in this model (which is enhanced as more risk-classes are added) suggests the possibility of government intervention of some sort. But the analysis here indicates that a government agency constrained only to break even on its insurance ventures may have a wide menu of policies to choose from. There is nothing to recommend any of the Pareto-optimal policies without explicit value judgments. Certainly lump-sum transfers between groups are not feasible (since the groups are not identifiable). If there really are no characteristics which are correlated with the probability of an accident, the principle of horizontal equity might suggest the zero-subsidy contract pair. But although this involves no cross subsidy, it gives higher utility to the low-risk. If it is felt that a high propensity to have accidents should not be punished (the probability of accidents is exogenous), then horizontal equity might suggest the same contract for both groups, namely the pooling Pareto optimum P. This yields the same expected utility to both groups, since it involves full insurance.

It is also the only Pareto optimum which is also Pareto optimal under full information. Such Pareto optima must involve full insurance for both groups. Thus one has that if one's welfare function is such as to require equal expected utility for all risk classes, limited information is not a problem. The social optimum is the same.[9] Of course it is only attainable in a decentralized competitive market if there is full information.

Footnotes

[1]This diagram is due to Rothschild and Stiglitz.

[2]This line has slope $-(1-p)/p$, where $P = \lambda p_1 + (1-\lambda)p_2$.

[3]The subsidy s is the expected loss on the high-risk contract $p_1\alpha_2^1 - (1-p_1)\alpha_1^1$.

[4]Except for very high subsidies, also noted below.

[5]The indifference curve UH is tangent to GD at G.

[6]i.e., the high-risk are indifferent between (Y,Y) and (X,Z).

[7]Utility for the high-risk decreases along GD, as one moves left from the tangency G.

[8]The high-risk prefer it (since they are indifferent between C and B), and the low-risk have steeper indifference curves through C.

[9]Dahlby has demonstrated that this solution also maximizes expected consumers' surplus, subject to the zero-aggregate-profit constraint.

Bibliography

Dahlby, B: "Adverse Selection and the Case for Compulsory Health Insurance," University of Alberta Research Paper # 79-22.

Miyazaki, H: "The Rat Race and Internal Labor Markets," Bell Journal of Economics, 8, 1977.

Rothschild, M. and J. Stiglitz: "Equilibrium in Competitive Insurance Markets: An Essay in the Economics of Imperfect Information," Quarterly Journal of Economics, 90, 1976.

Wilson, C: "A Model of Insurance Markets with Incomplete Information," Journal of Economic Theory, 16, 1977.