1978

An Analysis of the Demand and Supply of Shiftworkers

C. M. G. F. Robinson

20171

RESEARCH REPORT 7817

AN ANALYSIS OF THE DEMAND AND SUPPLY OF SHIFTWORKERS

by

C. M. G. F. Robinson

April 1978

AN ANALYSIS OF THE DEMAND AND SUPPLY OF SHIFTWORKERS*

C. M. G. F. Robinson
Department of Economics
University of Western Ontario

April 1978

*This research is part of my Ph.D. dissertation submitted to the University of Chicago, December 1977. I should like to thank, without implicating, my advisors, Professors G. S. Becker, G. R. Ghez and A. Zellner, and to acknowledge generous financial support from the Hillman Fund, the Lilly Honor Fund and the Sloan Foundation.

I

In all modern industrialized nations for which figures are readily available the incidence of shiftworking is increasing. A growing proportion of a nation's labour hours is being supplied (and demanded) outside the "normal working day."[1] Moreover this growth in recent times has been substantial--in some cases quite striking. For example, in Great Britain the number of people engaged in shiftworking almost doubled in the decade 1954 to 1964 and continues to rise. The secular increase in the relative number of night time labour hours (shiftwork by labour) has been accompanied by an increase in capital utilization--the relative number of night time machine hours actually used (shiftwork by capital). In the United States, for example, Foss (1963) estimates that whereas in 1929 the average machine was being worked only 15.9% of the time, by 1954 this figure had risen to 20.9%. This is a reflection of a connection between capital utilization and shiftworking explored in this study. The planned rate of capital utilization is viewed, following Marris (1964), Winston (1974a,b) and others[2] as an economic variable. The higher the profit maximizing rate of capital utilization the higher, _ceteris paribus_, will be the demand for night time labour hours. This will be translated into higher actual night time labour hours provided the supply is not completely inelastic. Conversely, a shift in the supply curve of night time

---

[1]The terms "normal working day" and "night time working" are defined formally in Parts II and III below where their relation to shiftworking is considered. It may be noted here, however, that an increase in "shiftworking" as reported in official statistics will imply an increase in hours worked outside the "normal working day" or in "night time working".

[2]Betancourt and Clague (1975).

labour hours will affect the profit maximizing level of capital utilization. The rate of capital utilization and (labour) shiftworking are thus jointly determined.

The effect of nightwork on the physical and mental health of the industrial worker is the subject of a substantial literature.[1] More recently increasing attention has been devoted to the "social" aspects of nightwork.[2] The large and growing literature on shiftwork includes studies on the attitudes of shiftworkers and their wives to shiftwork, the "ergonomics of shiftwork", the technical aspects of shift rotation and so forth. Two striking features of the literature, however, are the paucity of contributions by economists and the absence of any systematic explanation of the published data. Indeed, prior to 1970 it would appear that with the important exception of Marris no economist had devoted serious study to shiftwork. Recent analyses by Winston supplemented by Betancourt and Clague have demonstrated that the dependence of the planned rate of capital utilization on relative factor prices has far-reaching implications for a wide variety of theoretical models and policy issues.[3] The "rhythmic cost model" underlying this analysis has not, however, been subjected to empirical test.

---

[1] The bibliographies of Marc (1975), Sergean (1971) and the National Board for Prices and Incomes (1970), contain a large number of studies on this subject.

[2] In the United States one of the major works in this field is Mott, et al. (1965).

[3] For example, estimates of the elasticity of substitution between labour and capital from studies using measures of capital in place of proxy capital services become suspect. In fact incorrect results will be obtained whenever a capital stock measure is used to proxy a service flow and the relation between stock and flow is changing. A case in point is the study of Solow (1957) on technical progress in which at least part of the increased output reported due to technical change may be attributed to the increase in the capital utilization rate over the study period.

The plan of the present paper is as follows. In Part II the long-run planned demand for night time labour hours is derived using a "rhythmic input cost" model. This analysis builds on and extends the work of Winston, Marris and Betancourt and Clague. These authors do not deal directly with the demand for night time labour hours but their work contains an implicit demand theory. The supply of night time labour hours is the topic of Part III. There appears to be little or no explicit theoretical economic literature dealing with this problem but Part III demonstrates that it may readily be handled by an application of the theory of the allocation of time--in this instance the allocation of time <u>across the day</u>. Finally, Part IV is devoted to a test of the implications of the rhythmic input cost model using industrial cross-section data from the United Kingdom and the United States.

II

THE DEMAND FOR NIGHT TIME LABOUR HOURS

The demand for night time labour hours is determined simultaneously with the demand for night time hours of machine operation--i.e., the daily rate of capital utilization. In most of the literature on capital utilization the emphasis is on the short run, dealing with unexpected events which force firms to operate at a utilization rate different from that planned when the plant was built.[1] This paper, however, is concerned mainly with the <u>planned</u> demand for night time hours of labour and hence with the <u>planned</u> rate of capital utilization at the time of the investment decision. In order to focus on this

---

[1] Winston (1974a) contains a good review of previous work and notes the tendency until recently to consider less than full capital utilization a temporary, unplanned phenomenon.

aspect the analysis of Section A below is conducted on the assumption that demand for the product is known and unchanging and that all expectations are realized. The consequences of relaxing these assumptions are discussed in Section B.

A. The Long-Run Demand for Night Time Labour Hours

Most discussions of shiftwork distinguish between continuous- and non-continuous-process industries. While a formal definition is seldom given, typically a technological distinction is implied and the occurrence of a continuous process in a particular industry is taken as exogenous. In this study a continuous process is defined as one which once halted requires in excess of twelve hours operation to produce a positive output.[1] Operation outside the "normal" working day is then a necessary condition for a positive output. The "normal" working day may be thought of as some 8-10 hour period, say 8:00 a.m. to 5:00 p.m. between which most people work. It is defined more precisely below with reference to the periodicity of the rhythm in the price of labour hours across the day. For convenience any hours supplied outside the "normal" working day will be termed "night time hours of labour." For some products it may be technically impossible to obtain any output other than by a continuous process: in such cases the occurrence of a continuous process is taken as an exogenous factor exerting an independent effect on the demand for night time labour hours. For other products however there may be a choice of processes, some continuous, some not. The occurrence of a continuous process in these cases ceases to be given data and becomes a variable to be determined, simultaneously with the demand for night time labour hours,

[1] There are extreme cases where a process once halted takes several months to restart; however, about 12 hours is sufficient for nightwork to be unavoidable.

by relative costs. The derivation of the long-run demand for night time labour hours pursued in this section is directed towards the latter group.

The Criterion for Operating Multiple Shifts

A major consequence of allowing hours of operation to vary during the day is that it introduces a complication in the relation between a firm's capital stock and the service flows obtained from that stock. In short, a firm's capital stock becomes an unreliable proxy for its capital service flow. Similarly the true capital services input price to the firm and the definition of its elasticity of substitution between "capital" and "labour" are complicated. Accordingly the analysis of this section begins with explicit consideration of the relation between capital stock and service flow and the price of capital services.

The purchase price of a "unit of capital" (standard machine) is denoted $\bar{P}_m$;[1] the cost of holding the capital for a day is therefore $P_k = \bar{P}_m(i+d)$ where i is the daily rate of interest and d is the daily rate of obsolescence. Physical depreciation is assumed zero. Machines may either be purchased or rented, though if rented must be held for the whole day. A stock of standard machines, $\bar{K}$, yields a flow of capital services, K, per day which obeys:

$$\text{Max } K = v\bar{K}$$

where v is determined by the period of analysis and maintenance requirements in that period. For simplicity maintenance requirements are assumed to be zero so that Max $K = \bar{K}$ and the utilization rate is then defined as:

(1) $$u = K/\text{Max } K \qquad 0 < u \leq 1$$

---

[1]The notation of Winston (1974b) is adhered to wherever convenient.

where K is the actual machine hours used in the period. The period of analysis considered here will be the 24 hour day. The relation between the capital stock and service flow is therefore:

(2) $$K = u\bar{K}$$

Since daily expenditure on capital services is given by $P_k\bar{K}$ the per unit input price of capital services is given by:

(3) $$P_k\bar{K}/K = P_k/u$$

All firms are assumed to be price takers in factor markets so that P_k is given to the firm. However, u is a choice variable under the control of the firm hence (3) indicates that firms in effect determine their input price of capital services.

The functions governing production are assumed to have as arguments service flows of the inputs labour and capital and to exhibit constant returns to scale in these flows. It is also assumed that for any given product the function is the same for each hour of production irrespective of the time of day. Two forms of production are distinguished: those which are continuous *ex post* as well as *ex ante* and those which may be continuous only *ex ante*.[1] The criterion is derived first for the latter group. In this case total costs incurred by a firm operating a single shift (the "day" shift) only are given by:

(4) $$TC^s = wL_D^s + P_k\bar{K}^s = wL_D^s + (P_k/u_D^*)K_D^s$$

where the superscript s indicates that the operation is single shift and the

---

[1]Only the latter are considered by Winston and Betancourt and Clague. Winston (1974a) discusses the relation between zero *ex post* substitution and the "clay" phase of "putty-clay" models and shows that they are not the same phenomenon.

subscript D indicates that it is the "day time" shift, so that L_D^s is the amount of labour services hired during "day time" hours by a firm operating a single shift, $\overline{K}^s$ is the firm's capital stock and K_D^s is the flow of capital services used during the shift. The fraction of the 24-hour day regarded as "day time" is denoted u_D^* and the per unit price for labour services used during "day time" hours is denoted w. u_D^* corresponds to the "normal working day". Its most important dimension from the point of view of this paper is its length. However it also has the dimension of "location"during the 24 hours. While "location" may vary somewhat from firm to firm across industries (e.g., from 8:00 a.m.-4:00 p.m. to 9:00 a.m.-5:00 p.m.) without affecting the demand for night time labour hours, any variation in magnitude--i.e., in the number of hours for which the wage w applies--will have some demand implications.

Corresponding costs for a firm operating throughout the 24-hour period, through the use of multiple shifts, are given by:

$$(5) \qquad TC^m = wL_D^m + (1+b)wL_N^m + P_k\overline{K}^m = wL_D^m + (1+b)wL_N^m + P_kK^m/(u_D^*+u_N^*)$$

where the superscript m indicates that the operation is multiple shift and the subscript N refers to "night time" so that, for example, L_N^m is the labour input used during "night time" hours by a firm operating multiple shifts. The parameter b is the percent premium payable for labour services used outside of "day time" hours and u_N^* is the fraction of the 24-hour day for which this premium applies. Since $u_N^* + u_D^* = 1$ and zero <u>ex post</u> substitutability implies:

$$L_N^m = (u_N^*/u_D^*)L_D^m \quad \text{and} \quad K_N^m = (u_N^*/u_D^*)K_D^m$$

expression (5) may be rewritten:

$$(6) \qquad TC^m = (1+bu_N^*)wL_D^m/u_D^* + P_kK_D^m/u_D^*$$

Profit maximizing firms will either work throughout the 24 hours or just during the "day time" period; they will never work part of a period since within a period the only input price which varies is that of capital services and it falls continuously throughout the period. Thus the two options whose total costs are given by (4) and (6) are the only ones that have to be considered. This is a consequence of the assumed "rhythm" across the day in the price of labour services taking the form of a step function with only two-step heights, w and w(1+b).[1] In fact there may be several step heights--possibly so many that the function may be nearly continuous.[2] For the purposes of exposition however it is convenient to consider only two-step heights. This may then be generalized to n step heights since the decision to prefer n+1 to n periods of operation during the day, for n>1, will involve the same analysis as in the present case of n=1.

Multiple shift operation will be preferred if it results in lower average costs than single shift operation, i.e., if:

$$(7) \qquad TC^s/Y_D^s > TC^m/Y^m$$

where Y_D^s is the output produced during "day time" hours by the single shift operation and $Y^m = Y_D^m + Y_N^m$ is the total 24-hour output produced under multiple shift operation. Since by assumption average costs equal marginal costs, the actual levels of output are immaterial and are left indeterminate.[3] Substituting from (4) and (6) yields as the criterion:[4]

---

[1] See Figure on page 11 below.

[2] This case is considered in Winston and McCoy (1974).

[3] The relation between shiftworking and scale is analyzed in Robinson (1978) where a positive relation is derived and tested on several data sets.

[4] See Robinson (1977), Appendix A for derivation of this and the remaining results presented in this section.

$$(8) \qquad [w + r^s K_D^s / L_D^s] AP_L^m / AP_L^s > \bar{w}^m + r^m K_D^m / L_D^m$$

where $r^s (= P_k / u_D^*)$ is the per unit price for capital services under single shift operation, $r^m (= P_k)$ is the corresponding price under multiple shift operation, $\bar{w}^m (= w(1 + bu_N^*)$ is the per unit price of labour services under multiple shift operation, and AP_L^s and AP_L^m are the average products of labour under single and multiple shift operation respectively.

These criteria may be specialized for particular ex ante production functions. The most general form of production function considered in this section is the constant elasticity of substitution function (CES). By imposing cost minimizing conditions in this case (8) specializes to:

$$(9) \qquad (1+bu_N^*)^{\sigma-1} \{(\delta/1-\delta)^{\sigma} (w/P_k)^{\sigma-1} (u_D^{*\sigma-1} -1) + 1\} \begin{array}{l} > 1 \text{ if } \sigma < 1 \\ < 1 \text{ if } \sigma > 1 \end{array}$$

where σ is the ex ante elasticity of substitution and δ is the distribution or factor intensity parameter. If the ex ante production function is of the Cobb-Douglas form the criterion simplifies to:

$$(10) \qquad (1+bu_N^*) > u_D^{*-(1-\alpha)/\alpha}$$

where α is the elasticity of output with respect to the labour input. Finally, if the elasticity of substitution is zero both ex ante and ex post, then (8) reduces to:

$$(11) \qquad P_k a_1 / u_D^* a_k > wb$$

where a_1 and a_k are the fixed input coefficients for labour and capital respectively.

A pervasive pressure on a productive enterprise to operate for 24 hours per day arises because of the positive interest rate on capital and/or a positive rate of obsolescence.[1] In the absence of "off-setting" factors all profit maximizing firms would operate continuously. The off-setting factor considered explicitly in this paper is the premium paid for labour hours supplied outside the "normal" working day. If this premium, b, is set equal to zero the shiftwork criteria developed above will always be satisfied and multiple shifts will always be worked.

In all cases of zero <u>ex post</u> substitution, therefore, a firm will operate continuously and demand night time and day time labour hours in the ratio u_N^*/u_D^* when b=0. The same is also true for production functions which are continuous both <u>ex ante</u> and <u>ex post</u>. In this case multiple shifts will clearly be preferred to single shift operation since under either scheme the production conditions are identical while under multiple shift operation one of the input prices--that of capital services--is lower. For the same total cost a higher output is therefore always obtainable under multiple shifts which must therefore be preferred by profit maximizing firms. Given that multiple shifts will always be chosen the capital services price in both "day time" and "night time" hours is P_k: if b=0 the labour services price is also the same in both periods hence the optimal factor proportions will be constant across the 24-hour day. The relative demand for night time labour hours in this case, like the others, is thus $L_N/L_D = u_N^*/u_D^*$.

---

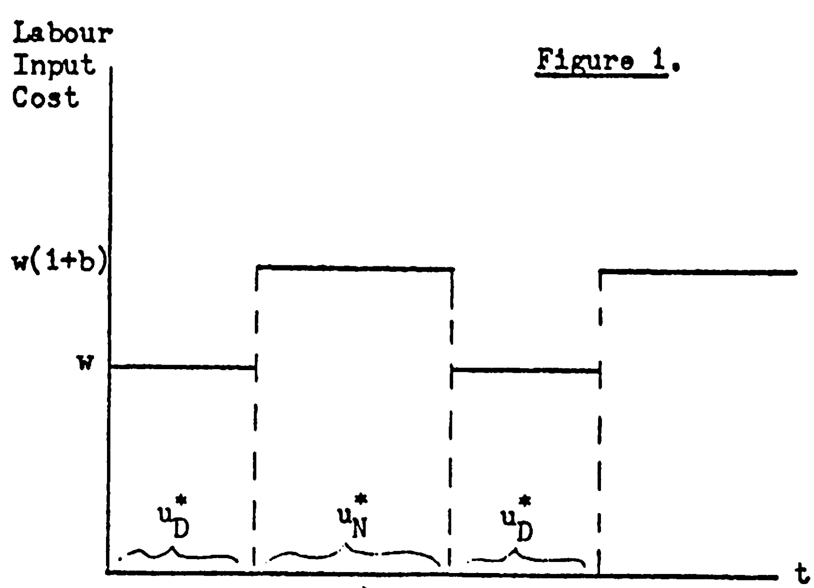
[1] At a more basic level it operates because of the nature of the contractural arrangements under which capital services are obtained. If the capital services were typically hired on the same basis as labour services the incentive to operate for 24 hours would be removed. Why the contractural arrangements are different for capital, and in particular why capital must normally be held for at least a day, is presumably due to differential costs in delivering and setting up most capital equipment as compared to labour "equipment".

A Simple Rhythmic Input Cost Model

It was shown above that in the absence of off-setting factors the positive rate of interest and/or obsolescence induces firms to operate continuously. Since this is not observed in practice some off-setting factor is presumed to be at work. Marris (1964) has pointed to the night time wage premium as the obvious candidate for the most pervasive off-setting factor and hence an important reason why firms' planned utilization rate would be less than 100%. The night time premium is an example of a factor price which rather than being constant across the day exhibits a regular rhythm. If a constant percentage premium, b, is payable on all hours supplied during "night time" then the price of an hour of labour services will have the daily rhythm illustrated in Figure 1 below. Models advanced to explain the rate of capital utilization by reference to such premia have been termed "rhythmic cost models" by Winston (1974a). In this section the simple rhythmic cost model implied by setting $b > 0$ in the criteria developed above is considered, beginning with the zero ex post substitutability production functions.

The criterion for shiftworking to be profitable in the most general ex post substitutability case--that of CES--was given by (9). This formulation

Figure 1.



indicates that the profitability of shiftwork depends positively on the instantaneous capital intensity parameter $(\delta/1-\delta)$ and negatively on the night time premium b. The effect of the factor price ratio, P_k/w, is ambiguous, being positive when $\sigma < 1$ and negative when $\sigma > 1$. In view of the evidence currently available on the magnitude of σ in manufacturing industry it will be assumed throughout that $\sigma \leq 1$.[1] The positive effect of the instantaneous factor intensity parameter indicates that firms manufacturing products which require large amounts of capital services input relative to labour services input in a given hour of operation have a greater incentive to run their production processes continuously by using a given number of machines for more hours in the day. It does not indicate that firms with such processes will use more machines per man over a 24-hour period, nor therefore that their ratio of capital expenditure to labour expenditure will be higher. This will be indeterminate.[2]

The dependence of the effect of the factor price ratio on σ is due to the dual role played by factor prices in the shiftwork criterion. Consider an increase in P_k: this will raise the profitability of shiftwork by increasing the capital savings accruing from the change to multiple shifts for a given (single shift) capital/labour ratio; at the same time, however, it will reduce the profitability of shiftwork by lowering the (single shift)

---

[1] In the original article introducing the CES function the authors concluded: "We have produced some evidence that the elasticity of substitution between capital and labour in manufacturing may be typically less than unity," Arrow, Chenery, Minhas and Solow (1961), p. 246. More recently Moroney (1971), provides a considerable body of evidence supporting the hypothesis that for the majority of U.S. manufacturing σ lies between zero and unity.

[2] The ratio of machines to men resulting from a move to multiple shift operation may, [illegible] is small, be smaller than the single shift ratio because of the addition of an extra shift of men.

capital/labour ratio and hence the gains from shiftwork. The strength of the latter effect is determined by σ. Supply side influences on the shift-work decision may operate via their effects on the night time premium, b, and the length of the working day, u_D^*.

The Cobb-Douglas case (see (10) on page 9 above) illustrates the behaviour of the criterion at the upper bound of the range of σ considered. The factor price ratio and instantaneous capital intensity effects are now both channeled through the single parameter α. For given factor-prices, the greater is the capital intensity the smaller is α and, by (10), the more profitable is multiple shift operation. The lower bound on the range of σ is zero which implies fixed proportions. The criterion in this case was (11) which, since the flow capital/labour ratio is the same in single and multiple shift operation, may be written:

$$(12) \qquad P_k K/u_D^* L > wb \quad \text{or} \quad P_k K/wL > bu_D^*$$

Factor intensity, the conventional factor price ratio and b are all represented and operate in the same direction as in the general CES case for $\sigma < 1$. The fixed proportions case however emphasizes the importance of the absolute premium wb. Given factor prices a change in b changes the absolute night premium. The first part of (12) indicates that a change in the absolute premium holding the relative premium constant will always affect the value of the inequality while a change in the relative premium holding the absolute premium constant leaves the value of the inequality unchanged.

By virtue of the zero <u>ex post</u> substitution assumption the relative demand for night time labour hours by the firms for which the criterion is satisfied will be $L_N/L_D = u_N^*/u_D^*$ and for the remainder it will be zero. It remains to consider the continuous <u>ex post</u> production functions. If <u>ex post</u>

substitution is permitted so that capital services may be used in any ratio with labour services and all marginal products are positive then no machine would be ever left idle in any hour and all productive processes would be run continuously. Contrary to the previous cases however the instantaneous capital-labour ratio will not be the same throughout the day and the relative demand for night time labour hours will no longer be given by the ratio u_N^*/u_D^* but rather will depend on b and on the elasticity of substitution.

Rhythmic Prices for Other Factors of Production

The simple rhythmic cost model illustrates how a night time labour premium tends to inhibit continuous production. However, labour is not the only input which may be more costly during night time and a night premium for any other input would be an inhibitory force in exactly the same way. For example, let a given set of (mandatory or efficient) working conditions relating to heat, light, etc., be regarded as a factor of production then if more fuel (supplied at a constant price) is required to achieve a given set of working conditions at night time than during the day time this factor will be more expensive during night time. (Of course if the fuel, say electricity, is supplied at a cheaper rate during night time hours then this inhibiting factor may be partially offset--or even reversed.) A comparison of the construction industry versus bakeries or steel making may be used to indicate the circumstances under which this factor would be important. In the construction industry the work is done mainly outdoors requiring essentially no light other than daylight during the day; for night time work however floodlights would be required making the provision of light considerably more costly at night. In bakeries or steel making the work is done largely indoors so that indoor lighting will in general be required during the day with only a marginal increase being necessary for night time operation. Considering the provision of heat the bakeries and steel making concerns may well be using resources to reduce the heat generated in their production processes so that for both day and night operation there is essentially no difference in producing the desired level of heat--indeed if reducing heat is the problem the desired level will be more cheaply achieved during the colder night time hours. On construction sites on the other hand making up for the loss of daytime temperature during the night would be very costly: the solution would perhaps involve heavier clothing reducing the worker's efficiency, more frequent rest breaks, etc.

The provision of material inputs may be more or less costly for night time operation--i.e., the price of material inputs may also exhibit a regular rhythm across the day. As an example of the former a supplier may deliver only during the day or may require a premium for night time deliveries; to carry production over the night shifts it would therefore be necessary either to hold larger inventories which is not in general costless or to order the more expensive night time deliveries. A prime example of the latter is the supply of electricity which may be cheaper during night time hours.

Skilled and Unskilled Labour

The simple rhythmic input cost model introduced above may be readily extended to cover skill differences in the labour input. Under zero *ex post* substitution the total costs under single and multiple shift operation become:

(18) $$TC^s = wL_D^s + (1+h)wH_D^s + P_k\overline{K}^s = wL_D^s + (1+h)wH_D^s + r^sK_D^s$$

and

(19) $$TC^m = wL_D^m + (1+b)wL_N^m + (1+h)wH_D^m + (1+c)(1+h)wH_N^m + P_k\overline{K}^m$$

$$= (1+bu_N^*)wL_D^m/u_D^* + (1+cu_N^*)(1+h)wH_D^m/u_D^* + r^mK_D^m/u_D^*$$

where L is unskilled labour, H is skilled labour, h is the skill premium and b and c are the night premia for unskilled and skilled labour respectively. In this case average costs are lower for multiple shift operation if the following criterion is satisfied:

$$(20) \qquad \{w + (1+h)w \frac{H_D^s}{L_D^s} + r^s \frac{K_D^s}{L_D^s}\} \frac{AP_L^m}{AP_L^s} > w^m + \bar{w}_h^m \frac{H_D^m}{L_D^m} + r^m \frac{K_D^m}{L_D^m}$$

where $\bar{w}^m = (1+bu_N^*)w$ is the per unit price of the unskilled labour input under multiple shift operation and $\bar{w}_h^m = (1+cu_N^*)(1+h)w$ is the corresponding price for skilled labour. As in the homogeneous labour input case this may be specialized for particular <u>ex ante</u> production functions. For example, under fixed proportions, and the assumed lower bound for the elasticities of substitution, (20) reduces to:

$$(21) \qquad P_kK/u_D^*w(L+H) > [(1+h)c-b](H/H+L) + b$$

Comparing this with the expression obtained for the homogeneous labour case (12) the ratio of capital to (homogeneous) labour services is replaced by the ratio of capital to total skilled and unskilled labour services and a new term capturing the effect of the skill proportion, $[(1+h)c-b](H/H+L)$, is added to the right-hand side. The effect of relative factor prices, capital intensity, b and u_D^* is unchanged from the homogeneous labour case. The effect of the skill proportion, H/H+L, depends upon the sign of the square bracket. If absolute night time premia paid to skilled workers exceed those paid to the unskilled the sign will be positive. Assuming this to be the case[1] the higher the proportion of skilled to unskilled workers required by the production process the less likely it is that the process will be run continuously.

[1] The evidence for the U.K. suggests that this is true. See Robinson (1977), Chapter IV.

In the Cobb-Douglas case (20) reduces to:

$$(22) \qquad (1+bu_N^*)^{-\alpha_u} > (1+cu_N^*)^{\alpha_s} u_D^{*(1-\alpha_u-\alpha_s)}$$

where α_u is the exponent on L and α_s is the exponent on H in the three factor Cobb-Douglas production function. This is essentially the same as the criterion in the homogeneous labour case (10) except for the new term $(1+cu_N^*)^{\alpha_s}$ which captures the effect of the share of skilled labour in total costs.

B. Some Short-Run Aspects

When the assumptions of Section A are relaxed a great variety of new factors enter the picture, many of which have been discussed at length in the literature on capital utilization which has tended to emphasize the short run. In this section two aspects of the new situation are briefly examined: first the effect of unexpected shocks on the demand for night time hours, and second the effect of expected future variations in output on the type of plant that is built.

Consider a firm whose plant was built for continuous operation on the basis of an expected stable demand which faces an unexpected decline in demand. Assume also that this decline is expected to be temporary so that the capital stock and perhaps some skilled labour is to be retained. Under these conditions the shift that will be dropped is the night shift(s) since in the short run the labour costs are more expensive on that shift. The relative demand for night time labour hours will thus tend to fall. Conversely, a firm whose plant was built for single shift operation facing an unexpected increase in demand will, if the price rise is sufficient, introduce a night shift(s) thereby increasing the relative demand for night time labour hours. The ratio of night to day employment and the average real hourly wage will therefore tend to exhibit a pro-cyclical pattern.[1]

Finally, suppose that future variations in demand are expected at the time a plant is built. This introduces the possibility that the firm will be induced to build a flexible plant capable of operating at "reasonably" low cost for the majority of output levels it expects to produce.[2] This in turn introduces the possibility that the rhythm in the input cost of labour services faced by profit maximizing firms will be complicated. Under the assumptions of Section A overtime working by the day time workers at a premium rate to the employer in excess of the premium rate for "nightworkers" will always be inferior to introducing a second shift. Therefore the rhythm in the labour input cost faced by profit maximizers was not affected by overtime rates in excess of night time labour rates since they would have no relevance for profit maximizing firms. Under the assumptions of this section however it is possible that overtime rates in excess of night time rates will affect the labour input cost rhythm faced by profit maximizing firms because of the inducement to build a flexible plant. In these circumstances the optimal way to meet a fluctuating demand may be to build a flexible plant and regulate the supply of labour to it by means of overtime since this may be cheaper than raising and depressing the basic wage several times a year. This plant and its implied labour input cost rhythm would have to be compared to other alternatives by the profit maximizing firm and a key feature of the comparison would involve the more complicated input cost rhythms.

---

[1] Lucas (1970) uses this argument to explain the observed pro-cyclical pattern in the real wage which he points out is inconsistent with existing theories.

[2] See Stigler (1939).

In practice, a considerable amount of "overtime" is observed both in the U.K. and the U.S. often at rates to the employee in excess of the rates payable to shift workers outside the day shift. Moreover, while it varies cyclically, the overtime appears to have a substantial permanent component. It is argued here that this permanent overtime may be analyzed and explained within the framework of Section A and need not be attributed to the flexible plant argument of the previous paragraph. First there is the possibility that day workers will supply some overtime at rates below the premia required by non-day shiftworkers. The rhythm in the input price of labour would then have three steps instead of two and there will be some combination of the factors entering the shiftwork criteria for which operation during the day time only will be optimal, some for which day time plus overtime will be optimal, and some for which continuous operation will be optimal. Second, even if no overtime would be supplied at rates to the employee less than those paid to non-day shiftworkers, it remains possible that overtime may be supplied at rates to the employer less than those paid to non-day shiftworkers because of the fixed costs of employing a worker. The importance of this argument depends on the size of the shiftwork premia. Officially recorded premia tend to be small; however it may be shown that official premia tend to grossly understate the true premia for hours supplied outside the normal working day, which are typically 25 percent or more.[1]

---

[1] See Robinson (1977), Chapter IV for estimates of the 'true' shiftwork premia. The substantial size of the true premia has two important consequences: (i) shiftwork may be an important omitted variable in wage generating equations, (ii) the positive correlation between unionization and shiftwork may have resulted in part of the shiftwork premia being attributed to unionization effects on wages.

III

THE SUPPLY OF NIGHT TIME LABOUR HOURS

The observed incidence of night time working across industries, occupations and regions and the secular path of the shiftwork ratio depend in principle on an interaction of demand and supply factors. In this section attention is devoted to the supply factors. For a married person the decision to supply labour outside "normal" working hours has a major impact on the spouse (and children) and on the type of household production undertaken by the family--much more so in general than the decision to work for firm A rather than firm B or at occupation X rather than Y. Some shift arrangements will result in the husband and wife sleeping together much less frequently, having different meal times, etc. Others will result in household production that specializes, using either all wife's time or all husband's time. A full analysis, therefore, would analyze this decision explicitly within a family context. The implications of such an analysis, however, regarding age, probability of being married, marital stability, number of children, number of friends, major leisure activities and so forth relate to individual based data whereas the empirical work in Part IV concerns primarily industrial cross-section data. Consequently most of the following discussion is based, for simplicity, on a single person household model.

The recent capital utilization literature provided some implicit analysis of the demand for night time labour hours. The supply side, however, has not been investigated within an economic framework. The bulk of the literature on nightwork as it relates to the supplier is the work of sociologists, occupational psychologists, etc. who are interested in the physical and psychological health aspects, the "social consequences" and husbands' and wives' "attitudes" to shiftwork as revealed in their answers to questionnaires.

The Individual Choice of Work Pattern

The decision to supply night time labour hours may conveniently be analyzed in a variety of frameworks depending on the variables of particular interest. In this paper the framework adopted is that of utility maximizing individuals facing over an appropriately chosen "period" a continuous mix of night-day work alternatives. It will then be assumed that in the region of equilibrium the individual chooses non-zero amounts of both "normal" and "non-normal" working and consumption time. Thus, in more concrete terms, let the period be a month: the individual may choose to work, say, two weeks on day- and two on non-day shifts, or one week on day- and three on non-day shifts, etc. If his desire for non-normal working hours is very small this may be accommodated by having him work say ten hours of "overtime" in non-normal working hours per month. A case can be made for the individual actually facing a very considerable array of alternatives in this regard.

Following Ghez and Becker (1975) individuals are assumed to derive utility from commodities produced in the home using inputs of time and market goods and services. In order to focus on night-day differences only two commodities are considered:[1]

$$Z_N = Z_N(T_N^c, X_N)$$

(1)

$$Z_D = Z_D(T_D^c, X_D)$$

where Z_D is a composite commodity produced with the inputs of daytime consumption time, T_D^c, and daytime market goods and services, X_D, and Z_N is the night

[1] As before "night" refers to hours outside the normal working day.

time composite commodity which is produced with night time consumption time, T_N^c, and night time market goods and services, X_N. These commodities are the arguments in the typical individual's utility function:

(2) $$U = U(Z_D, Z_N)$$

which is to be maximized with respect to Z_D and Z_N subject to the production functions (1) and the constraints on the inputs.

The constraints on the inputs are the familiar time and goods constraints of the household-producer literature, specialized in this case by the night-day distinction and may be combined in the single budget constraint:

(3) $$p_D X_D + p_N X_N + w_D T_D^c + w_N T_N^c = w_D T_D + w_N T_N + V = S$$

where p_D and p_N are the per unit prices of day and night goods (and services) respectively, w_D and w_N are the respective hourly wage rates for day and night time, V is non-labour income per period, T_D and T_N are the respective amounts of total day and night time, and S is "full" money income. The first order necessary conditions for a maximum of (2) subject to (3) imply;

(4) $$\frac{p_D}{\partial Z_D/\partial X_D} = \frac{w_D}{\partial Z_D/\partial T_D^c} = m_D; \quad \text{and} \quad \frac{p_N}{\partial Z_N/\partial X_N} = \frac{w_N}{\partial Z_N/\partial T_N^c} = m_N$$

where m_D and m_N are the marginal costs of producing Z_D and Z_N respectively. If the production functions are both assumed homogeneous of degree one then the m's will be constant, independent of the levels of Z_N and Z_D. In this case the model may be put in the "standard" demand theory form with the utility function (2) maximized subject to the full money income constraint:

(5) $$m_D Z_D + m_N Z_N = S$$

The demand functions for the Z's then obey the standard properties of being homogeneous of degree zero in proportionate changes in all prices and money income and having negative own (compensated) price elasticities. These demand functions imply derived demand functions for the inputs T_D^c and T_N^c and hence supply functions of day and night time labour hours, T_D^w and T_N^w (where $T_D^w = T_D - T_D^c$ and $T_N^w = T_N - T_N^c$) which are the focus of interest in this paper.

Properties of the Supply Functions of Night and Day Time Labour Hours

Consider first the effect of a one percent rise in w_N holding w_D constant. This will have three effects: (i) it will riase real full income, thereby increasing the derived demand for both T_N^c and T_D^c; (ii) it will increase the shadow price of Z_N thereby inducing substitution (in consumption) away from Z_N and hence lowering the derived demand for T_N^c relative to T_D^c; (iii) finally, it will raise the input price of T_N^c thus inducing substitution in production away from T_N^c and towards X_N. The overall percentage change in the ratio T_N^c/T_D^c is given by:

(6) $$(w_N T_N^w/S)(B_N - B_D) - s_{TN}\sigma_{ND} - \sigma^N(1-s_{TN})$$

where B_N and B_D are the income elasticities of demand for Z_N and Z_D respectively, s_{TN} is the share of time inputs in the production of Z_N, σ^N is the elasticity of substitution in production of Z_N between time and goods, σ_{ND} is the elasticity of substitution in consumption between Z_D and Z_N. Both elasticities are defined positive for convenience. If the income and substitution elasticities are unity the implied change in the ratio of night to day time labour supplied is:

(7) $$(T_D^c/T_D^w)(w_N T_N/S) + (T_N^c/T_N^w)\{1 - w_N T_N/S\} > 0$$

so that an increase in w_N will increase the relative supply of night time labour hours. Analogously the effect on T_N^w/T_D^w of an increase in w_D holding w_N constant is given by:

(8) $$- (T_N^c/T_N^w)(w_D T_D/S) - (T_D^c/T_D^w)\{1 - w_D T_D/S\} < 0$$

The effect of a one percent increase in both w_D and w_N--i.e., a rise in the absolute night time premium, holding the relative premium constant is obtained (in the unit elasticity case) by adding (7) and (8) which yields:

(9) $V/S\{(T_N/T_N^w) - (T_D/T_D^w)\}$

Since $T_N > T_D$ (i.e., the period for which w_N prevails is outside the "normal working day" which is less than half the 24-hour period) and for the majority of the population $T_D^w > T_N^w$, (9) will be positive or zero depending on whether V is positive or zero. Thus in general it may be expected that the relative supply of night time labour hours will increase in response to increases in the absolute premium even if the percentage premium is unchanged.

Thus far only differences in the prices of the inputs T_D^c and T_N^c have been considered. It is also possible that there may be potentially important differences in the productivities of T_D^c and T_N^c. For example, T_D^c may be more productive in some cases than T_N^c if congestion is harmful to production. (Thus in some recreational activities overcrowding of the facilities may detract from the enjoyment of the activity.) Conversely, it may be less productive for commodities where congestion is beneficial. (A visit to a half-empty theatre or restaurant may be less enjoyable than to a full one.) These possibilities may be handled in the above framework by introducing inputs as efficiency units.

Changes in the prices p_D and p_N have three effects: (i) they alter the relative prices of commodities, m_D/m_N, and hence cause substitution between Z_D and Z_N according to their price elasticities; (ii) they alter the relative input prices of time and market goods and services and thus induce substitution in production; (iii) finally, they affect the level of real full income by changing the price index (a weighted average of the m's). The overall effect of a change in p_N/p_D is therefore, in general, ambiguous. However, the magnitude of the effect of changes in p_N and p_D may be considered small enough to be ignored since only a portion of market goods and services vary in price across the day and purchases of these goods constitute only a small fraction of total expenditure.

The Market Supply of Night Time Labour Hours

The relative supply of night time labour hours will be determined by the distribution of preferences in the population, the wage rates w_D and w_N, prices p_D and p_N, and non-labour income V, together with productivity differences in the inputs T_D^c, T_N^c, X_D, and X_N. The elasticity with respect to changes in the absolute night time premium will depend on the relative number of night time hours being supplied. As this number increases the supply response will decline since the value of the second term in (16) will fall. To the extent that the unitary elasticities assumption is true the relative supply curve, plotted against the absolute premium, will be fairly stable since secular increases in V or the levels of w_D and w_N will have no effect, and as argued above changes in the relative price of market inputs will be unimportant; this leaves only changes in the distribution of preferences and input productivities to shift the curve. A secular increase in the relative supply of night time labour hours should therefore be accompanied by a secular increase in the absolute premium.

The preference map for a typical member of the population may be summarized in the usual way by the implied income and substitution elasticities which have already been discussed. An additional summary characteristic in this case is the degree of day time preference defined as the extent to which the slope of the indifference curves at the 45° line differs from (minus) unity. It will be assumed throughout that this rate of day time preference is stable over time.

The Supply of Night Time Hours to a Single Industry

Given an unsegmented labour market, if no industry employs a large fraction of the total night time workers in any given occupation then each will face a supply curve of night time labour hours of each occupational type which is infinitely elastic at the market premium for that occupation. If however the labour market is distinctly segmented by, say, region and industries were unevenly distributed across regions then individual industries could face different relative supply curves of night time labour. It may be argued, for example, that other things equal young, unmarried people will wish to "consume" when most people of their age are "consuming" to maximize their chances of obtaining a good spouse. In the language of the model developed above, for them the productivity of T_N^c will be greater than that of T_D^c. Industries located primarily in areas in which this group are over-represented could then face a higher absolute night time premium.

IV

EMPIRICAL ANALYSIS: THE INDUSTRIAL CROSS SECTIONS

The rate of capital utilization or, looking at the other side of approximately the same coin, the amount of shiftworking, varies across firms within the same industry (as defined in official statistics) and across industries. The rhythmic input cost model maintains that an important part of this variation is due to differences in planned rates of capital utilization or multiple shift operation at the time of the investment decision; idle capital is not, therefore solely due to *ex post* reactions to unexpected events, nor are night shifts solely due to unexpected increases in demand or their absence to unexpected decreases. In this section industrial cross-section data are used to test the implication of the rhythmic input cost models.

A Simple Model

The firms making up an "industry" as defined in official Standard Industrial Classifications are assumed to have similar, but not identical production functions.[1] For example, in the fixed proportions case they will have similar but not identical capital/labour service flow ratios. In the simple case of homogeneous labour and capital inputs and fixed proportions production conditions the relative demand for night time labour hours by firm i was:[2]

$$(1) \qquad (L_N/L_D)_i^d = u_N^*/u_D^* \quad \text{if} \quad wb < A_i$$
$$= 0 \qquad \text{if} \quad wb > A_i$$

where $A_i = P_k a_1/u_D^* a_k$. This results in a step function at the level of the individual firm. The industry demand curve depends on the distribution of A_i in the firms considered to fall within the industry. Assuming an approximately continuous unimodal distribution the industry demand curve will be a smooth negatively inclined curve. The larger is the mean of the distribution, $\bar{A}$, *ceteris paribus*, the further to the right will be the industry demand curve, which may therefore be specified as:

$$(2) \qquad (L_N/L_D)^d = (u_N^*/u_D^*) \int_{wb}^{\infty} p(A)dA = f(wb, \bar{A}, \underline{\theta}), \quad f_1 < 0, \ f_2 > 0$$

---

[1] Official "industry" statistics present a variety of problems from the point of view of testing economic theories because the definition of an industry tends to be centred on the material from which the product is made rather than on the purpose the product serves.

[2] See criterion (11) on page 9 above.

where $\underline{\theta}$ is a vector of parameters (other than the mean) characterizing p(A). Note that since $a_1/a_k = K_D/L_D$, $A_i = (P_k K_D/u_D^* L_D)_i = (P_k \bar{K}/L_D)_i$, i.e., A_i is the ratio of total expenditure on capital services to day time employment. Moreover, since A_i is determined (for given u_D^* and P_k) by the instantaneous capital intensity of the process, the positive relation between (L_N/L_D) and $\bar{A}$ derived for the fixed proportions case will hold for all production conditions considered in Part II.

It was argued in Part III that unless the labour market was markedly segmented and industries were unevenly divided across geographical areas the relative supply of night time labour hours to a single industry would be completely elastic at the market differential, denoted (wb)*. Supply factors will therefore determine wb while the industry demand curve determines the relative quantity of night time labour hours. Thus the simple model may be represented by the following three equation system:

(3) $$(wb)_j = (wb)^*$$

(4) $$(L_N/L_D)^d_j = g(wb_j, \bar{A}, \underline{\theta})\, e^{u'_j} \qquad \text{DEMAND EQUATION}$$

(5) $$(L_N/L_D)^d_j = (L_N/L_D)^s_j = (L_N/L_D)_j \qquad \text{EQUILIBRIUM CONDITION}$$

where u'_j is a disturbance term.

Substituting from (3) and (5) into (4) the model collapses to the single equation:

(6) $$(L_N/L_D)_j = g[(wb)^*, \bar{A}_j, \underline{\theta}]e^{u'_j}$$

The preferred function form for (6) is:

(7) $$\log(L_N/L_D)_j = c_0 + c_1 \log \bar{A}_j + u_j$$

which imposes the constraint that (L_N/L_D) tends to zero as $\bar{A}$ tends to zero.[1] Estimation of this model permits a test of the rhythmic input cost hypothesis which implies $c_1 > 0$.

The appropriate estimation method depends to a large extent on the properties of the disturbance in the true relation, u_j, and the degree of measurement error. Assume initially that all the variables are measured without error. The disturbance in the equation to be estimated is the disturbance from the demand equation. The primary theoretical reason for the inclusion of u_j in the demand equation is the fact that variation in the ex ante planned rate of capital utilization or shiftworking due to input price rhythms is not expected to account for all of the variation in shiftworking at a point in time. Some of the variation will be due to differences in planned rates due to factors other than input price rhythms, e.g., regular demand rhythms, but more importantly for the cross-section data some of the variation will be due to reactions to unexpected increases or decreases in product demand at the time the data was collected. This introduces the possibility that industries whose demands are closely linked may have disturbances with non-zero covariances--industries producing complements will tend to have positively correlated disturbances and those producing substitutes, negatively correlated ones. However, investigation of this possibility has produced no evidence of non-zero covariances.[2] There also appears to be no evidence of

---

[1]It also results in residuals which appear to be free of heteroscedasticity and which exhibit a random pattern when plotted against the fitted value. However, it does not impose an upper bound on (L_N/L_D) as $\bar{A} \to \infty$, and hence must be regarded as a local approximation to the relation between (L_N/L_D) and $\bar{A}$ over the range of (L_N/L_D) exhibited in the data. Estimates were computed with a functional form which also imposed an appropriate upper bound. While the direction of effect of the independent variables considered was unaffected, the goodness of fit was reduced--see Robinson (1977).

[2]For example, when the industries were grouped into their two-digit groups the disturbances showed no tendency to have the same sign within groups.

significant heteroscedasticity. In the absence of measurement error it may therefore be argued that u_j obeys the assumptions of the standard multiple regression model, i.e.,

$$(8) \qquad Eu_j = 0 \text{ and } Eu_j^2 = \sigma^2, \text{ all } j; \quad Eu_j u_i = 0 \text{ all } j \neq i$$

and hence that ordinary least squares may be applied to (7) for a consistent estimate of the demand parameter c_i.

Secondly, the possibility of measurement error must be considered. Since $\bar{A}$ is the ratio of total expenditure on capital services to day time employment, L_D appears on both sides of (7). If all the variables are correctly measured this presents no difficulty. However, if L_D is measured with error the ordinary least squares estimate of c_1 will be biased towards unity.[1] Accordingly, both ordinary least squares and instrumental variable estimation methods were used in estimating (7) and the results are presented in Table 1 below.[2]

For the U.K. the data were obtained from the Ministry of Labour Shift-work Surveys of 1954 and 1964 and from the Censuses of Production for corresponding years.[3] The U.S. data set was constructed from material available in

---

[1]See the Appendix for an evaluation of the bias. Simultaneous equation bias may also result if the labour and capital inputs are not exogenous. They will be exogenous if firms are assumed to maximize expected profits (see Zellner, Kmenta and Dreze, "Specification and Estimation of Cobb-Douglas Production Function Models," Econometrica, 34, (1966)).

[2]See the Appendix for discussion of the instruments, and evaluation of biases.

[3]The 1963 Census of Production had to be used with the 1964 Shiftwork Survey but since the relative ranking of industries by shiftwork changes slowly any errors due to this imperfect sychronization are assumed to be small.

Table 1

Industrial Cross Section Regression Results for the Simple Model

Data Set	Estimation Method[a]	Estimated Equation[b]			R^2	N
	OLS	$\log(L_N/L_D) =$	-2.86 (4.02)	+ 1.24 $\log \bar{A}$ (6.55)	.34	85
U.K. 1954	IV(1)		-1.20 (1.49)	+ 0.79 $\log \bar{A}$ (3.67)	n.a.	85
	IV(2)		-2.16 (2.88)	+ 1.05 $\log \bar{A}$ (5.25)	n.a.	85
	OLS	$\log(L_N/L_D) =$	-1.92 (3.20)	+ 0.95 $\log \bar{A}$ (6.62)	.32	108
U.K. 1964	IV(1)		-0.63 (0.70)	+ 0.62 $\log \bar{A}$ (3.35)	n.a.	108
	IV(2)		-2.13 (3.27)	+ 1.00 $\log \bar{A}$ (6.41)	n.a.	108
	OLS	$\log(L_N/L_D) =$	-3.46 (3.25)	+ 1.10 $\log \bar{A}$ (6.10)	.47	44
U.S. 1963	IV(1)		-1.15 (0.87)	+ 0.71 $\log \bar{A}$ (3.11)	n.a.	44
	IV(2)		-2.66 (2.39)	+ 0.97 $\log \bar{A}$ (5.10)	n.a.	44

Notes: a. The instrument used for IV(1) was the logarithm of the share of capital in total costs and for IV(2) it was the logarithm of output per man.

b. The absolute value of the t-statistic for the null hypothesis that the coefficient is zero is reported in parentheses under the coefficient.

the Industry Wage Survey series and the 1963 Census of Manufactures and is described in Robinson (1977). The results presented in Table 1 indicate that as far as it goes the simple model is in broad agreement with the data since the estimate of c_1 is always positive and the one-tailed test suggested by the theory leads to a rejection of the null hypothesis that $c_1 = 0$ at the one percent level.

An Expanded Model

In Part II the proportion of skilled workers required in a production process was shown to have a negative effect on the likelihood of multiple shiftworking in the fixed proportions case provided that the absolute night premium paid to skilled workers exceeds that paid to unskilled workers. The same is true, though the effect is weaker, if the elasticity of substitution is between zero and unity. If it is unity the effect will only be negative if the relative night premium is higher for skilled workers. Evidence presented in Robinson (1977) indicates that (for the U.K.) a higher absolute but equal or lower relative premium is paid to skilled workers. The proportion of skilled workers in the industry, $\bar{S}$, should thus be included in the industry demand equation and provided $\sigma < 1$ for most industries the expected sign of its coefficient will be negative. A similar case, at least for the U.K., can be made for the inclusion of the proportion of women in the industry, $\bar{F}$, since those firms whose processes are suitable for female labour will face a large absolute night time premium for their overall labour input since by law the female labour would have to be replaced by the (presumably) more expensive male labour for the night shifts.[1] Incorporating these points into the

[1] The Factories Act 1961 and related legislation place restrictions on the employment of women outside the daytime hours in factories and elsewhere. Section 117 of the Act enables the Secretary of State for Employment to grant, under certain conditions, exemptions from these restrictions by granting special exemption orders in respect of employment in particular factories. Some 60,000

empirical analysis (7) should be replaced by:

$$(9) \qquad \log(L_N/L_D)_j = c_0 + c_1 \log \bar{A}_j + c_2 \log \bar{F}_j + c_3 \bar{S} + u_j$$

The discussion of the disturbances in the simple model applies equally to the expanded version represented by (9) and should be borne in mind when interpreting the estimates, from the U.K. data sets, of the parameters of (9) reported in Table 2 below.[1] The predictions of the rhythmic cost model were that $c_1 > 0$, $c_2 < 0$ and $c_3 < 0$. The results reported in Table 2 indicate that these predictions are in good accord with the data since all coefficients have the predicted sign and are significant at the 5% level or better. However, some qualifications should be made. As an industry enters a downturn L_N/L_D will fall as the night shift will tend to be laid off first; at the same time both the capital/labour ratio and the skill ratio will rise (the latter due to specific human capital investments in skilled workers). $\bar{S}$ and (L_N/L_D) will thus tend to be negatively associated because of this short-run effect and the estimate of c_3 may therefore over-estimate the long-run effect of $\bar{S}$ predicted by the rhythmic input cost model. Similarly, the estimate of c_1 may be an under-estimate of the long-run effect since the short-run

---

women were permitted to work double-day shifts and night shifts under such orders in force at the end of 1970. While less widespread in the U.S. restrictions on the employment of women at night are not non-existent. Thus a recent study by the Bureau of Labor Statistics noted that: "When these surveys were conducted, a few States prohibited or regulated the employment of women on evening or night shifts and a number of collective bargaining agreements in women's apparel manufacturing banned late shifts." (O'Connor (1970).)

[1]Lack of data prevented the estimation of (9) for the U.S. and c_3 for U.K. 1954.

Table 2

Industrial Cross Section Regression Results for the Expanded Model

Data Set	Estimation Method[a]	Dependent Variable	Estimated Coefficients[b] c_0	c_1	c_2	c_3	R^2	N
	OLS	$\log(L_N/L_D)$	-1.31 (1.37)	1.13 (5.97)	-0.35 (2.33)	-	.38	85
	IV(1)	$\log(L_N/L_D)$	-0.06 (0.06)	0.84 (4.21)	-0.40 (2.65)	-	n.a.	85
	IV(2)	$\log(L_N/L_D)$	-0.09 (0.09)	0.85 (4.23)	-0.40 (2.65)	-	n.a.	85
	OLS	$\log(L_N/L_D)$	0.45 (0.47)	0.82 (5.14)	-0.46 (3.36)	-0.002 (2.64)	.42	108
	IV(1)	$\log(L_N/L_D)$	2.08 (1.96)	0.50 (2.80)	-0.52 (3.74)	-0.002 (3.30)	n.a.	108
	IV(2)	$\log(L_N/L_D)$	1.19 (1.20)	0.68 (4.06)	-0.49 (3.55)	-0.002 (2.96)	n.a.	108

Notes: a. The matrix of instrumental variables was the same as the matrix of independent variables with log $\bar{A}$ replaced by the logarithm of the share of capital in total costs, IV(1), or the logarithm of output per man, IV(2).

b. The absolute value of the t-statistic for the null hypothesis that the coefficient is zero is reported in parentheses.

considerations imply a tendency for (L_N/L_D) to be negatively related. Several attempts were made to adjust for these short-run effects but the parameter estimates remained unchanged (see Robinson (1977)). Finally, note that at least part of the negative association between $\overline{S}$ and (L_N/L_D) may be due to the higher fixed costs attached to employing skilled workers which makes an extension of the working period by means of "permanent overtime" relatively more attractive.

Since the new variables, $\overline{S}$, and $\overline{F}$, enter the regression significantly the results of Table 2 are preferred to those of Table 1. Regarding the coefficient on the instantaneous capital labour ratio, c_1, however, comparison of Tables 1 and 2 indicates little change in its value with the addition of the new variables.

Appendix

Potential Biases in the Coefficients and the Use of Instrumental Variables

Since L_D is used in the calculation of $\bar{A}$ it appears on both sides of the estimating equation (9) in Part IV. If it is measured without error this causes no problem; however, if measurement error is present the estimate of c_1 in equation (9) is biased towards unity. This may be shown as follows. Let the measurement error be given by:

(A1) $$\log L_D = \log L_D^* + \varepsilon$$

where L_D^* is the true value, L_D is the measured value and ε is the measurement error whose expected value is zero. Since L_D appears in the denominator of $\bar{A}$, express measured $\bar{A}$ as:

(A2) $$\log \bar{A} = \log (X|L_D)$$

The true model is then:

(A3) $$\log (L_N/L_D^*) = c_0 + c_1 \log (X/L_D^*) + u$$

But the OLS estimate of c_1 using measured variables is:

$$\hat{c}_1 = \frac{\Sigma[\log L_N/L_D - \log L_N/L_D]\ [\log X/L_D - \log X/L_D]}{\Sigma[\log X/L_D - \log X/L_D]^2}$$

which has expected value:

(A4) $$E\hat{c}_1 = \frac{E[\log L_N/L_D - E \log L_N/L_D]\ [\log X/L_D - E \log X/L_D]}{E[\log X/L_D - E \log X/L_D]^2}$$

Substituting from (A2), and assuming u and ϵ are uncorrelated we have:

$$\text{(A5)} \qquad E\hat{c}_1 = \frac{E[\log L_N/L_D^* - E \log L_N/L_D^*]\ [\log X/L_D^* - E \log X/L_D^*] + \sigma_\epsilon^2}{E[\log X/L_D^* - E \log X/L_D^*]^2 + \sigma_\epsilon^2}$$

$$= (\sigma^2/(\sigma^2 + \sigma_\epsilon^2))c_1 + (\sigma^2/(\sigma^2 + \sigma_\epsilon^2))$$

where $\sigma^2 = E[\log X/L_D^* - E \log X/L_D^*]$. This indicates that the expected value of the estimate $\hat{c}_1$ is a weighted average of the true parameter value, c_1, and unity, with the weights depending on the variance of the measurement error relative to the variance of the true value of the independent variable.

The possibility of biases in the estimation of c_1 led to the computation of instrumental variable estimates. The choice of instruments required that they be independent of the disturbance in the estimating equation. If measurement error is present the instrument should be independent of this, as well as the disturbance in the true model. The instruments used were measures of output per man and share of capital in total costs, assumed to be independent of any error in the measurement of total day time workers. These were both obtained from the Census of Manufactures data corresponding most closely in time to the date of the observation on shiftwork ratios. Since the former were averages over a year while the latter were obtained at a single point in time,there appeared minimal danger of bias problems arising from the contemporary nature of the instruments. And indeed this was confirmed when instruments calculated from previous Censuses of Manufactures--predetermined variables--were used, and yielded the same results.

REFERENCES

Arrow, K. J., Chenery, H. B., Minhas, B. S., Solow, R. S., "Capital-Labour Substitution and Economic Efficiency," Review of Economics and Statistics 43: 3 (August, 1961).

Becker, G. S. and G. R. Ghez, The Allocation of Time and Goods Over the Life Cycle, NBER: 1975.

Betancourt, R. and C. Clague, "An Economic Analysis of Capital Utilization," Southern Economic Journal 42: 1 (July, 1975): 69-78.

Foss, M., "The Utilization of Capital Equipment: Post-war Compared with Pre-war," Survey of Current Business 43; 6 (June, 1963): 8-16.

Great Britain, Ministry of Labour, "Shift Working," Gazette 42:10 (October, 1954): 337-342.

Lucas, R. E., "Capacity, Overtime, and Empirical Production Functions," American Economic Review 60: 2 (May, 1970): 23-27.

Marc, M., Shiftwork, Geneva: International Labour Office, 1975.

Marris, R., The Economics of Capital Utilization, University of Cambridge Department of Applied Economics Monograph No. 10, Cambridge: Cambridge University Press, 1964.

Moroney, J. R., The Structure of Production in American Manufacturing, Chapel Hill: University of North Carolina Press, 1971.

Mott, P. E., et al., Shiftwork--The Social, Psychological and Physical Consequences, Ann Arbor: University of Michigan Press, 1965.

O'Connor, C. M., "Late Shift Employment in Manufacturing Industries," Monthly Labour Review 93: 10 (November, 1970): 37-42.

Robinson, C. M. G. F., "The Allocation of Time Across the Day: An Analysis of the Demand and Supply of Shiftworkers, unpublished Ph.D. dissertation, University of Chicago, December, 1977.

Robinson, C. M. G. F., "Capital Utilization and the Long-Run Average Cost Curve," 1978.

Sergean, R., Managing Shiftwork, London: Gower Press, 1971.

Solow, R., "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, 39: 3 (August, 1957).

Stigler, G. J., "Production and Distribution in the Short Run," Journal of Political Economy, 47: 3 (June, 1939).

Winston, G. C., "The Theory of Capital Utilization and Idleness," Journal of Economic Literature, 12: 4 (December, 1974): 1301-1320.

Winston, G. C., "Capital Utilization and Optimal Shiftwork," Bangladesh Economic Review, 2: 2 (April, 1974): 515-557.

Winston, G. C. and T. O. McCoy, "Investment and the Optimal Idleness of Capital," Review of Economic Studies 41(3): 127 (July 1974): 419-428.

Zellner, A., Kmenta, J., Dreze, J., "Specification and Estimation of Cobb-Douglas Production Function Models," Econometrica 34 (1966).