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by

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This paper analyzes the optimal mix and overall level of taxes and government expenditure where one of the taxes is that on real money balances, the inflation tax (Friedman (1953)). The criterion for judging optimality will not be the maximization of revenue raised or the minimization of the ratio of welfare loss to revenue raised which has been commonly employed in many studies of this question (see Bailey (1956), Marty (1967), Friedman (1971), Barro (1972), Marty (1973), Auernheimer (1974)). Rather it will be the Pareto efficiency criterion which has been employed in the related optimum quantity of money (the dual of the optimal inflation rate) literature. However, unlike in the optimum quantity of money studies (see especially Friedman (1969), Feige and Parkin (1971) and also Samuelson (1968), Clower (1969), Johnson (1969)) lump sum taxes will not be permitted as a means whereby the government may balance its budget. All the means available to the government for raising revenue produce welfare losses and the problem is to trade off one against the other(s) in an optimal manner. This places the analysis presented here in the tradition of work begun in the more general public finance setting by Ramsey (1927), developed by Dixit (1970), Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971), Sandmo (1974) and applied to deal explicitly with the optimal inflation tax by Phelps (1973) and Marty (1976). However, a greater degree of generality is achieved here than in these two earlier studies. First, as in the Diamond and Mirrlees case, utility is obtained from public consumption. This enables conditions to be derived for the optimal level as well as mix of taxes. Secondly, the analysis deals explicitly with production in which returns are non-constant so that equilibrium relative supply prices are

not independent of the tax-public spending regime. Thus a more genuine general equilibrium analysis of the problem of the optimal inflation tax is developed.

The analysis has several limitations which need to be noted. First, there are only two goods, a private consumption good and a public consumption good. Thus, capital accumulation and growth are ignored. Secondly, there is only one financial asset, money. This prevents an analysis of the choice between tax, money creation and bond financing of public expenditure. Such an analysis would be crucial if the central concern of the paper was the positive question concerning the effects on aggregate demand of a bond financed compared with a money financed tax change. However, for the normative question which is of concern here, the exclusion of bonds does not seem to be a matter of such central importance, although no doubt some details would change and the analysis would become more cumbersome if they were included. Thirdly, the analysis deals only with steady states and not with optimal transition paths following the disturbance of a steady state (see Auernheimer (1974)). Fourthly, only a constant rate income tax will be considered as an alternative to an inflation tax. Fifthly, there is no discussion of the detailed reasons why money yields utility. Rather, real balances are simply assumed to yield utility and enter, along with private consumption, public consumption and leisure in the household's utility function.

The economy is composed of three sectors - households, firms and government. Households supply labour and demand a private consumption good and real balances. Firms demand labour and supply goods (private and public). Households pay taxes to the government which in turn buys the

public consumption good from firms. The government is the sole supplier of money. Given the pattern of taxes and public consumption, households and firms maximize their utility and real profit respectively. Changes in taxes and government spending produce utility and profit maximizing responses from households and firms. Because of these responses to changes in government policy it is necessary, in analyzing optimal policy to establish the optimizing behaviour of private agents conditional on government policy. This is dealt with in section I. Section II analyzes the social optimization problem and compares the results derived with those presented in Phelps (1973) and Marty (1976). Section III summarizes the conclusions.

The main conclusion reached within the framework of the limitations outlined above is that the optimal mix of taxes, which ultimately depends on preferences and technology, does not, in general, involve setting the inflation tax at its first-best level of zero. This general result is in agreement with Phelps but not with Marty. However, the properties of the optimal inflation tax differ in some important respects from those derived in a more limited framework by Phelps. In particular, Phelps' conclusion that the optimal inflation tax will be zero if the elasticity of the compensated labour supply with respect to the income tax is perfectly inelastic is shown to be not in general correct. For that first-best result to be optimal in the second-best world considered here the supply of labour must fall off with increased inflation and the ratio of the slopes of equilibrium level of labour with respect to the income and inflation tax $\frac{L}{L} \frac{T}{\pi}$ equals the ratio of equilibrium gross wages to equilibrium real balances $\frac{wL}{m}$.

In general, the optimal inflation tax and income tax in the second-best world depend on the slopes (or elasticities) of the equilibrium quantities of consumption, real balances and labour employed each with respect to the inflation tax and income tax. Without detailed specification of both demand and supply functions and thus of these equilibrium quantity functions the general equilibrium analysis does not give clear cut optimal values for the government's policy instruments. However the rule for the optimal mix of taxes is that the ratios of the elasticities of the equilibrium quantity of the public good with respect to each tax to the fraction of real revenue raised via that tax minus the marginal cost of the tax to households arising via the impact of the tax on the relative wage, are equal. The optimal level of these taxes is such that this ratio equals the ratio of the marginal utility derived from private consumption to that derived from the public good.

In addition it is possible to interpret how the optimal income tax and inflation tax vary with changes in the slopes of these equilibrium quantities. In a simplified case in which supply prices are constant, the equilibrium quantity of labour is fairly inelastic with respect to the inflation tax and the equilibrium quantity of real balances is fairly inelastic with respect to the income tax, the following relationships between the optimal taxes and the elasticities are found to hold: the optimal inflation tax is lower and the optimal income tax is higher the more inelastic is the supply of labour with respect to the income tax and the more elastic are the demand for real balances with respect to each of the taxes and the more elastic is the supply of labour with respect to the inflation tax.

I. The Private Optimization Problem

(a) The Firm

The representative firm chooses the supply of the consumption good C^S and the public good G^S as well as the demand for labour L^d and the allocation of that labour between the production of the two goods (as shown by L_g , the labour employed in producing the public good) so as to maximize real profits, R . It takes as given the real wage rate w and the relative price of the public good q and is constrained by two production functions. As labour is assumed to be the only factor of production real profits are simply the difference between real output $C^S + qG^S$ and the real wage bill wL^d . That is, the firm maximizes

$$R = C^S + qG^S - wL^d \quad (1)$$

subject to the production functions

$$C^S = C(L^d - L_g) \quad \text{with } C_L > 0, C_{LL} < 0 \quad (2)$$

$$G^S = G(L_g) \quad \text{with } G_L > 0, G_{LL} < 0 \quad (3)$$

given w and q .

Substituting (2) and (3) into (1) real profits may be written as $R = C(L^d - L_g) + qG(L_g) - wL^d$, and are maximized when

$$\left. \begin{array}{l} \text{(i)} \quad C_L - w = 0 \\ \text{(ii)} \quad qG_L - C_L = 0 \\ \text{(iii)} \quad C^S = C(L^d - L_g) \\ \text{(iv)} \quad G^S = G(L_g) \end{array} \right\} \quad (4)$$

Equation 4(i) is the familiar condition that the marginal product of labour equal the real wage rate. Equation 4(ii) which may be written as

$$q = \frac{C_L}{G_L}, \quad (5)$$

is simply the condition that the marginal rate of transformation between the two goods $(\frac{C^L}{G^L})$ be equal to their relative price. The above first-order conditions may be solved for the output supply and labour demand functions and will have the following form:

$$\left. \begin{aligned} C^S &= C^S(w, q) \\ G^S &= G^S(w, q) \\ L^d &= L^d(w, q) \end{aligned} \right\} (6)$$

The signs of the partial derivatives of (6) implied by the technology structure are as follows:

$$C_w^S, G_w^S, L_w^d < 0; \quad G_q^S > 0, L_q^d > 0.$$

(b) The Household

The representative household chooses its demand for the consumption good C^d , supply of labour L^S (or equivalently, demand for leisure) and demand for nominal money balances M^d given the real wage rate w , the money price of the private consumption good P (treated as the price level), real profits, R , government provision of the public good G and the rate of income tax τ so as to maximize utility. The utility function is assumed to be intertemporally additive with a constant rate of time preference δ and with consumption C^d , public good G , labour L^S and real balances M^d/P as the arguments. That is, the household's utility function is given by

$$V = \int_0^{\infty} e^{-\delta t} U(C^d, G, L^S, M^d/P) dt \quad (7)$$

where $U_C > 0$, $U_G > 0$, $U_m > 0$ and $U_L < 0$, and $m = M/P$. The household's choice of C^d , L^S and M^d is constrained by the requirement that after-tax

labour income $(1 - \tau)wL^s$ and real profits R be allocated either to private consumption C^d or additions to real balances \dot{M}^d/P . That is, the household's budget constraint is

$$(1 - \tau)wL^s + R - C^d - \dot{M}^d/P = 0 \quad (8)$$

In formal terms, the household's optimization problem is to choose C^d , L^s and \dot{M}^d , given w , P , G and τ to maximize

$$V = \int_0^{\infty} e^{-\delta t} U(C^d, G, L^s, \dot{M}^d/P) dt \quad (9)$$

subject to

$$(1 - \tau)wL^s + R - C^d - \dot{M}^d/P = 0 \quad (10)$$

Using (8) to eliminate C^d from (9), the problem reduces to maximizing with respect to L^s and \dot{M}^d ,

$$H = e^{-\delta t} [U((1 - \tau)wL^s + R - \dot{M}^d/P, G, L^s, \dot{M}^d/P)]$$

given w , P , R , G and τ . The necessary conditions for optimality are

$$\left. \begin{aligned} \text{(i)} \quad & U_C(1 - \tau)w + U_L = 0 \\ \text{(ii)} \quad & U_m - U_C(\delta + \pi) = 0 \\ \text{(iii)} \quad & (1 - \tau)wL^s + R - C^d - \dot{M}^d/P = 0 \end{aligned} \right\} \quad (11)$$

where π is the rate of inflation \dot{P}/P .

Equations 11(i) and (ii) may be written more familiarly, but equivalently,

as

$$U_C = - \frac{U_L}{(1 - \tau)w} = \frac{U_m}{\delta + \pi} \quad (12)$$

That is, the marginal rate of substitution between consumption and leisure, $\frac{-U_L}{U_C}$, is equal to the net of tax real wage rate, and the marginal rate of substitution between consumption and real balances $\frac{U_m}{U_C}$ is equal to what, in this model, is the equivalent of the nominal rate of interest $\delta + \pi$.

The conditions 11(i), (ii) and (iii) may be solved for the demand functions C^d , M^d and the supply function L^s . In the steady state M^d/P is constant so that $\dot{M}^d = \pi M^d$. The individual demand and supply functions are then of the form

$$\left. \begin{aligned} C^d &= C^d((1 - \tau)w, R, \pi, G) \\ M^d/P &= M^d((1 - \tau)w, R, \pi, G)/P \\ L^s &= L^s((1 - \tau)w, R, \pi, G) \end{aligned} \right\} (13)$$

The sign restrictions on the partial derivatives of (13) will depend on the strength of assumptions made about the properties of $U(C^d, G, L^s, M^d/P)$.

Private agents maximize utility and real profits given the government's policy. This private optimization behaviour is summarized in the following demand and supply functions:

$$\left. \begin{aligned} C^s &= C^s(w, q) \\ G^s &= G^s(w, q) \\ L^d &= L^d(w, q) \\ C^d &= C^d((1 - \tau)w, R, \pi, G) \\ M^d/P &= M^d((1 - \tau)w, R, \pi, G)/P \\ L^s &= L^s((1 - \tau)w, R, \pi, G) \end{aligned} \right\} \begin{array}{l} (6) \\ (13) \end{array}$$

II. The Social Optimization Problem

The government chooses its provision of the public good G and sets the money supply M^s and the rate of income tax τ so as to maximize its social welfare function. This function will be the utility function of the representative household. A large number of constraints apply to the social optimization problem. First, all markets must be cleared. This requires that

$$L^s = L^d (=L) \quad (14)$$

$$C^s = C^d (=C) \quad (15)$$

$$G^s = G \quad (16)$$

$$M^s/P = M^d/P (=m) \quad (17)$$

Secondly, the government's budget determines the provision of the public good G . Its budget constraint is given by

$$qG = \tau wL^s + \frac{\dot{M}^s}{P} \quad (18)$$

but $\frac{\dot{M}^s}{P} \equiv \frac{\dot{M}^s}{M^s} \frac{M^s}{P} \equiv \mu m^s$, where μ is the rate of growth of the nominal money supply. Restricting the analysis to that of stationarity in all real variables, the real money supply is constant only if the rate of monetary growth μ equals the inflation rate π . Then the government's provision of the public good is given by

$$G = \frac{1}{q} [\tau wL^s + \pi m^s] \quad (19)$$

The money market always clears through adjustment of the price level. Then (19) can be rewritten as

$$G = \frac{1}{q} [\tau wL^s((1-\tau)w, R, \pi, G) + \pi m^d((1-\tau)w, R, \pi, G)] \quad (20)$$

or simply as

$$G = G((1 - \tau)w, R, \pi, q) \quad (21)$$

Thirdly, the government is constrained by the optimizing behaviour of the private sector. These constraints are the solutions to the private optimizing problems which may be stated in either of the forms derived in the preceding section; that is, either as the set of first-order conditions for private optimality or as the set of market supply and demand functions for which those first-order conditions may be solved. Use of the demand and supply functions makes the analysis simpler to interpret and is especially useful for comparing the results of the present analysis with those of Phelps and Marty. These private optimizing constraints are

$$\left. \begin{aligned} C^d &= C^d((1 - \tau)w, R, \pi, G) \\ M^d/P &= M^d((1 - \tau)w, R, \pi, G)/P \\ L^s &= L^s((1 - \tau)w, R, \pi, G) \end{aligned} \right\} (13)$$

$$\left. \begin{aligned} C^s &= C^s(w, q) \\ G^s &= G^s(w, q) \\ L^d &= L^d(w, q) \end{aligned} \right\} (6)$$

and profit is defined by (1)

$$\begin{aligned} R &= C^s(w, q) + qG^s(w, q) - wL^d(w, q) \\ &= R(w, q) \end{aligned} \quad (22)$$

Substituting for G from (21) and R from (22) in the set of constraints (13), the market clearing conditions are:

$$\begin{aligned}
 C^d(w, q, \tau, \pi) &= C^s(w, q) \\
 G(w, q, \tau, \pi) &= G^s(w, q) \\
 L^d(w, q) &= L^s(w, q, \tau, \pi) \\
 m^d(w, q, \tau, \pi) &= m^s
 \end{aligned}
 \tag{23}$$

As stated above, the price level adjusts so as to clear the money market.

As all economic agents (the household, firm and the government) satisfy their budget constraints and the money market always clears, the remaining three market-clearing conditions are not linearly independent. "Walras' Law implies that if all economic agents satisfy their budget constraint and all markets but one are in equilibrium, then the last market is also in equilibrium. It also implies that when all markets clear and all economic agents but one are on their budget constraints, then the last economic agent is on his budget constraint." Diamond and Mirrlees (1971, p. 4). Thus any two of the first three constraints (23) can be solved for the two variables w and q for any given setting of the government's policy instruments τ and π . Then the market clearing or equilibrium quantities C , G , L and m are functions only of τ and π .

The social optimization problem facing the government is to choose the taxes τ and π , so as to maximize social welfare subject to the three sets of constraints outlined above. This is identical to maximizing social welfare with respect to the taxes τ and π where the arguments of the social welfare functions are evaluated at their market clearing or general equilibrium values. That is, the government chooses τ and π in order to maximize

$$V = U(C(\tau, \pi), G(\tau, \pi), L(\tau, \pi), m(\tau, \pi))
 \tag{24}$$

This optimization is not subject to any additional constraints. As shown above these equilibrium or market clearing quantities are such that all the above constraints are satisfied. That is, they are such that (i) all markets clear; (ii) the government's budget constraint is satisfied; and (iii) private agents' optimization behaviour is taken into account. These equilibrium functions are combinations of demand and supply functions and not merely the demand function of goods and the supply function of labour as in the case of Phelps and Marty. In their analyses the demand for labour and the supply of goods are perfectly elastic and thus place no restriction on the equilibrium quantities. Thus all equilibrium quantities in their analyses are determined by only one side of the market. The first-order necessary conditions for optimality are:

$$\left. \begin{aligned} V_{\tau} &= U_C C_{\tau} + U_G G_{\tau} + U_m m_{\tau} + U_L L_{\tau} = 0 \\ V_{\pi} &= U_C C_{\pi} + U_G G_{\pi} + U_m m_{\pi} + U_L L_{\pi} = 0 \end{aligned} \right\} (25)$$

That is, the optimal settings of τ and π are such that the net marginal social welfare generated by the imposition of each tax, when all markets clear and all economic agents satisfy their budget constraints, is zero. The optimal provision of the public good is then determined from these optimal settings of the taxes.

The household's optimizing behaviour is also satisfied by these equilibrium quantities. Use can be made of these conditions (26) to simplify the necessary conditions (25). The household's optimizing behaviour is given by

$$\begin{aligned}
 \text{(i)} \quad & U_C(1 - \tau)w(\tau, \pi) + U_L = 0 \\
 \text{(ii)} \quad & U_m - U_C(\delta + \pi) = 0
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} \text{(i)} \\ \text{(ii)} \end{aligned}} \right\} (26)$$

Using (26) to eliminate U_m and U_L from (25) the first-order conditions for social optimality reduce to

$$\begin{aligned}
 \text{(i)} \quad & C_\tau - (1 - \tau)w(\tau, \pi)L_\tau + (\pi + \delta)m_\tau = -\frac{U_G}{U_C} G_\tau \\
 \text{(ii)} \quad & C_\pi - (1 - \tau)w(\tau, \pi)L_\pi + (\pi + \delta)m_\pi = -\frac{U_G}{U_C} G_\pi
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} \text{(i)} \\ \text{(ii)} \end{aligned}} \right\} (27)$$

In order to solve equations (27) for the optimal mix of taxes τ and π it is first necessary to determine the eight slopes C_τ , L_τ , m_τ , G_τ , C_π , L_π , m_π , and G_π . These are derived in the appendix. In order to derive these slopes two simplifying assumptions have been made. First, the relative price of the public consumption good is constant and equal to 1. Secondly, the utility function is separable such that there exists no cross elasticities between the private choice variables C , L , m and the government's choice variable G . Thirdly, the discount rate is assumed to be zero. These slopes are

$$\begin{aligned}
 C_\tau &= C_1^d [(1 - \tau)w_\tau - w(\tau, \pi)] + C_2^d R_w w_\tau \\
 L_\tau &= L_w w_\tau \\
 m_\tau &= M_1^d [(1 - \tau)w_\tau - w(\tau, \pi)] + M_3^d R_w w_\tau \\
 C_\pi &= C_1^d [(1 - \tau)w_\pi] + C_2^d R_w w_\pi + C_3^d \\
 L_\pi &= L_w w_\pi \\
 m_\pi &= M_1^d [1 - \tau]w_\pi + M_2^d + M_3^d R_w w_\pi \\
 G_\tau &= w(\tau, \pi)L^d(w) + \tau w_\tau L^d(w) + \tau w(\tau, \pi)L_w w_\tau \\
 &\quad + \pi M_1^d [(1 - \tau)w_\tau - w(\tau, \pi)] + \pi M_3^d R_w w_\tau \\
 G_\pi &= M^d(w[1 - \tau], \pi) + \tau w_\pi L^d(w) + \tau w L_w w_\pi + \pi M_1^d [1 - \tau]w_\pi \\
 &\quad + \pi M_2^d + \pi M_3^d R_w w_\pi
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} C_\tau \\ L_\tau \\ m_\tau \\ C_\pi \\ L_\pi \\ m_\pi \\ G_\tau \\ G_\pi \end{aligned}} \right\} (28)$$

and

$$w_{\tau} = \frac{-L_1^S w}{L_W^d - (1 - \tau)L_1^S - L_3^S R_W} \quad (29)$$

$$w_{\pi} = \frac{L_2^S}{L_W^d - (1 - \tau)L_1^S - L_3^S R_W} \quad (30)$$

Substituting these quantities into (27) gives a set of equations in τ and π . These equations are too general to be solved for the optimal taxes. Despite this some properties of the solution can be determined.

For any setting of the government's policy instruments τ and π , the household must satisfy its budget constraint. That is,

$$(1 - \tau)wL + R = C + m\pi \quad (31)$$

Furthermore, the budget must be satisfied for any change in the setting of τ and π . That is,

$$(i) \quad (1 - \tau)w_{\tau}L + (1 - \tau)wL_{\tau} - wL + R_{w_{\tau}} = C_{\tau} + m_{\tau}\pi \quad (32)$$

$$(ii) \quad (1 - \tau)w_{\pi}L + (1 - \tau)wL_{\pi} + R_{w_{\pi}} = C_{\pi} + m_{\pi}\pi + m$$

Using (32) to eliminate C_{τ} , C_{π} , L_{τ} and L_{π} the set of necessary conditions (27) reduces to

$$(i) \quad (1 - \tau)w_{\tau}L - wL + R_{w_{\tau}} = -\frac{U}{U_C} G_{\tau} \quad (33)$$

$$(ii) \quad (1 - \tau)w_{\pi}L - m + R_{w_{\pi}} = -\frac{U}{U_C} G_{\pi}$$

Rearranging (33) gives the general rule which the optimal taxes satisfy. That is,

$$\frac{G_{\tau}}{wL - (1 - \tau)w_{\tau}L - R_{w_{\tau}}} = \frac{G_{\pi}}{m - (1 - \tau)w_{\pi}L - R_{w_{\pi}}} = \frac{U}{U_C} \quad (34)$$

Or, in terms of elasticities of these equilibrium quantities, the optimal level of taxes is such that

$$\frac{\frac{\eta_{G,T}}{G} - \left[\frac{(1-\tau)wL + R\eta_{R,W}}{G} \right] \eta_{W,T}}{\frac{\tau wL}{G}} = \frac{\frac{\eta_{G,\pi}}{G} - \left[\frac{(1-\tau)wL + R\eta_{R,W}}{G} \right] \eta_{W,\pi}}{\frac{m\pi}{G}} = \frac{U_C}{U_G}. \quad (35)$$

That is, the optimal mix of taxes is such that the ratios of the elasticities of the equilibrium quantity of the public good with respect to each tax to the fraction of real revenue raised by that tax minus the marginal cost of the tax to households arising through the impact of the tax on the relative wage rate, are equal. The optimal level of these taxes is such that the above ratios equal $\frac{U_C}{U_G}$.

In the Phelps and Marty partial equilibrium analyses w is assumed constant and G is chosen arbitrarily. The partial equilibrium rule for the optimal mix of taxes under their conditions can be deduced from (35) and is given by

$$\frac{\frac{\eta_{G,T}}{G}}{\frac{\tau wL^s}{G}} = \frac{\frac{\eta_{G,\pi}}{G}}{\frac{m\pi}{G}} \quad (36)$$

That is, the optimal mix of taxes is such that the ratios of the elasticities of the arbitrarily chosen level of public consumption with respect to each tax to the fraction of revenue raised via that tax are equal. This result agrees with that presented by Phelps. But attention is drawn to the important differences between the rules derived for optimal taxes under the general and partial equilibrium framework. First, the general equilibrium model determines not only the optimal level of each tax but also the optimal level of public

consumption. The partial equilibrium model determines the optimal levels of the taxes in order to finance some arbitrarily selected level of public consumption. Secondly, all quantities determined by the general equilibrium model are equilibrium quantities which depend on both demand and supply functions. In the partial equilibrium model all quantities are determined by only one side of the market, such as the labour supply, the demand for real balances.

A second property of the general equilibrium solution is the condition under which the optimal inflation tax is zero. To derive this condition substitute for G_{τ} and G_{π} from (28) and $-L$ for R_w in equations (33). Then this gives the necessary conditions for an optimal inflation tax of zero to be

$$(i) \quad -\tau w_{\tau} L - wL = \frac{-\frac{U_G}{U_C}}{1 - \frac{U_G}{U_C}} \tau w L_{\tau} \quad (37)$$

$$(ii) \quad -\tau w_{\pi} L - m = \frac{-\frac{U_G}{U_C}}{1 - \frac{U_G}{U_C}} \tau w L_{\pi}$$

Dividing (i) by (ii) to eliminate $\frac{U_G}{U_C}$, the necessary condition for the optimal inflation tax to be zero is

$$\frac{\tau w_{\tau} L + wL}{\tau w_{\pi} L + m} = \frac{L_{\tau}}{L_{\pi}} \quad (38)$$

But

$$\frac{L_{\tau}}{L_{\pi}} = \frac{L w_{\tau}}{L w_{\pi}} = \frac{w_{\tau}}{w_{\pi}} \quad (39)$$

Substituting for $\frac{L}{L_{\pi}}$ and simplifying, equation (38) reduces to

$$\frac{L}{L_{\pi}} = \frac{w_{\tau}}{w_{\pi}} = \frac{wL}{m} \quad (40)$$

or, in terms of elasticities

$$\frac{\eta_{L,\tau}}{\eta_{L,\pi}} = \frac{\eta_{w,\tau}}{\eta_{w,\pi}} = \frac{\tau wL}{m\pi} = \infty \quad (41)$$

Then the optimal inflation tax is zero if either the equilibrium real wage is perfectly elastic with respect to income tax or perfectly inelastic with respect to the inflation tax. Or that the equilibrium level of employment is perfectly elastic with respect to income tax or perfectly inelastic with respect to the inflation tax. This requires also that the equilibrium level of employment is neither perfectly elastic nor inelastic with respect to the real wage rate.

As these equilibrium quantities depend on supply and demand functions these results can be expressed more clearly in terms of demand and supply elasticities. Substituting for w_{τ} and w_{π} from (29) and (30) condition (40) reduces to

$$\frac{-L_1^s w}{L_2^s} = \frac{wL}{m} \quad (42)$$

which can be expressed in terms of elasticities as

$$\frac{\eta_{L^s, (1-\tau)w}}{\eta_{L^s, \pi}} = \frac{(1-\tau)wL}{m\pi} = \infty \quad (43)$$

That is, the optimal inflation tax is zero if the supply of labour is either perfectly elastic with respect to the after-tax real wage rate or perfectly

inelastic with respect to the inflation tax at zero inflation tax. This requires from (29) and (30)

$$L_w^d - (1-\tau)L_1^s - L_3^s R_w$$

or

$$\eta_{L,w}^d - \eta_{L,(1-\tau)w}^s - \eta_{L,R}^s \eta_{R,w}$$

be finite and non-zero. This condition is fulfilled if (i) none of these elasticities including the supply of labour with respect to the after-tax real wage rate, are infinite and (ii) the elasticities $\eta_{L,w}^d$, $\eta_{L,(1-\tau)w}^s$, $\eta_{L^s,R}$ are not all zero. Then provided these conditions (i) and (ii) hold, the optimal inflation tax is zero only if the labour supply is perfectly inelastic with respect to the inflation tax, when the optimal inflation tax is zero. But from equation (42)

$$\frac{-L_1^s w}{L_2^s} = \frac{wL}{m}$$

$\frac{-L_1^s w}{L_2^s}$ must be positive for the optimal inflation tax to be zero. Since L_1^s is positive this condition requires L_2^s to be negative. That is, the first-best solution of a zero inflation tax is optimal in this second-best world only if the supply of labour falls off with increased inflation. If this condition holds then the elasticity of labour supply with respect to the inflation tax is perfectly inelastic at zero inflation tax. That is, if as the inflation tax is raised above zero the supply of labour declines, the

equilibrium real wage rises and the demand for labour declines such that the equilibrium level of employment falls. Aggregate output must decline. That is for any given level of income tax social welfare must decline as a result of imposing an inflation tax. Then the optimal inflation tax is zero.

The question now raised is: given this condition for a zero inflation tax in the general equilibrium analysis, does it also hold for the partial equilibrium analyses of Phelps and Marty. With the real wage rate constant demand for labour is perfectly elastic so that the equilibrium level of employment is determined by the supply function of labour. The necessary conditions for the optimal inflation tax of zero is derived from (37) and are, in terms of elasticities,

$$\begin{aligned}
 \text{(i)} \quad -1 &= \frac{-\frac{U_G}{U_C}}{1 - \frac{U_G}{U_C}} \eta_{L^S, \tau} \\
 \text{(ii)} \quad -\frac{\pi}{\tau w L} &= \frac{-\frac{U_G}{U_C}}{1 - \frac{U_G}{U_C}} \eta_{L^S, \pi}
 \end{aligned}
 \tag{44}$$

Then provided $\frac{U_G}{U_C}$ is not zero or infinite the above result is seen to hold in 44(ii).

This however is not the result obtained by either Phelps or Marty. The Phelps' result is that the first-best result is optimal when the compensated

supply of labour is perfectly inelastic¹ with respect to the income tax.

Marty's result is that the optimal inflation tax is always zero. From

equation 44(i)

$$\frac{\frac{U_G}{U_C}}{1 - \frac{U_G}{U_C}} = \frac{1}{\eta_{L^S, \tau}} \quad (45)$$

If $\eta_{L^S, \tau}$ is zero then $\frac{\frac{U_G}{U_C}}{1 - \frac{U_G}{U_C}}$ must be infinite. The only way this can be true

is for $1 - \frac{U_G}{U_C}$ to be zero. This implies that the government provides, via income

tax alone, the same quantity of the public good as the household would choose to buy privately. Thus the public good is not distinguishable from the private good. But consider 44(ii). On rearranging it becomes

$$\left(1 - \frac{U_G}{U_C}\right)\pi = \tau w L \frac{U_G}{U_C} \eta_{L^S, \pi} \quad (46)$$

with the left-hand side zero because both $\left(1 - \frac{U_G}{U_C}\right)$ and π are zero. The

right-hand side can only be zero if labour supply is also perfectly inelastic with respect to the inflation tax. That is, if the public good is a good

which is distinguishable from all private goods in the sense that the government's provision of it is different to that which the household would buy privately

then the optimal inflation tax is zero only if the supply of labour declines as the inflation tax rises. Thus these results are not the same. Phelps'

result turns on the elasticity of labour supply with respect to the income tax whilst the result obtained here hinges on the elasticity of labour supply with respect to the inflation rate.

Marty imposes the restriction of a proportional tax structure. It is this added restriction that leads to the conclusion that the optimal inflation tax is zero. Given the arbitrarily chosen positive level of government revenue or public consumption, the proportional tax $\frac{t_i}{t_i + p_i}$, where t_i is the tax levied, p_i is the marginal cost of producing good i , must be a non-zero constant k . Solving the social welfare problem is to solve initially for the optimal size of k . But irrespective of the level of k found, the optimal tax on real balances or of the inflation tax is such that $\frac{\pi}{\pi + p_i} = k$. Or, on rearranging $(\frac{1-k}{k})\pi = p_i$. The marginal cost of producing real balances is zero so that $(\frac{1-k}{k})\pi$ equals zero which leads to Marty's conclusion that the inflation tax π is always zero. Thus Marty's conclusion that the optimal inflation tax is always zero comes directly from the added restrictive assumption that the tax structure is proportional.

Another property of the optimal mix of taxes can be derived by asking: What is the effect on the optimal income and inflation taxes of a small change in the slopes of the equilibrium level of real balances and the equilibrium level of labour with respect to both taxes. To simplify this question in order to obtain non-ambiguous results, it is assumed that real wage is constant. This assumption implies that, along with constant real prices of both goods, the equilibrium level of labour is determined only by the supply side of the labour market and, in addition, real profits are zero.

Substituting for G_τ and G_π from $G = \tau w L^S + \pi m^d$ and $w_\tau = w_\pi = 0$ the set of necessary conditions (33) becomes

$$(i) \quad w L^S(w(1-\tau), \pi) \left[1 - \frac{U_G}{U_C}\right] = \frac{U_G}{U_C} [\tau w L_\tau^S + \pi m_\tau^d] \quad (47)$$

$$(ii) \quad m^d(w(1-\tau), \pi) \left[1 - \frac{U_G}{U_C}\right] = \frac{U_G}{U_C} [\tau w L_\pi^S + \pi m_\pi^d]$$

Dividing (i) by (ii) and totally differentiating gives

$$\begin{aligned} & (w L_\tau^S [\tau w L_\pi^S + \pi m_\pi^d] + w L_\pi^S w L_\tau^S - m_\tau^d [\tau w L_\tau^S + \pi m_\tau^d] - \pi w L_\tau^S) d\tau + (w L_\pi^S [\tau w L_\pi^S + \pi m_\pi^d] \\ & + w L_\pi^S m_\pi^d - m_\pi^d [\tau w L_\tau^S + \pi m_\tau^d] - \pi m_\tau^d) d\pi = \pi \tau w d(L_\tau^S) + \pi \tau d(m_\tau^d) \\ & - w L_\tau^S \tau w d(L_\pi^S) - w L_\pi^S \pi d(m_\pi^d) \end{aligned} \quad (48)$$

As the coefficients of both $d\tau$ and $d\pi$ are ambiguous it is, even with the above simplifications, still impossible to obtain a clear indication of small change in any one of these slopes on either of the optimal taxes. In addition the signs of the taxes are still unknown. Making the additional assumption that the supply of labour and the demand for real balances are fairly inelastic with respect to the inflation and income taxes respectively, it is easily deduced from (47) that the optimal level of both taxes are positive and the effect of a small change in the slopes of the supply of labour and demand for real balances with respect to the two taxes on the optimal level of taxes are as presented in the following table.

Table I

		∂_{τ}	∂_{π}
∂L_{τ}^s	(< 0)	(+)	(-)
∂L_{π}^s	(< 0)	(-)	(+)
∂m_{τ}^d	(> 0)	(+)	(-)
∂m_{π}^d	(< 0)	(-)	(+)

That is, when the relative price of public good and real wage are constant, the supply of labour and demand for real balances are fairly inelastic with respect to the inflation tax and the income tax respectively,

(i) The optimal income tax is higher the more inelastic is the supply of labour with respect to the income tax and the more elastic are the demand for real balances with respect to each of the taxes and the more elastic is the supply of labour with respect to the inflation tax.

(ii) The optimal inflation tax is higher the more elastic is the supply of labour with respect to the income tax and the more inelastic are the demand for real balances with respect to both taxes and the supply of labour with respect to the inflation tax.

III. Conclusions

The analysis shows that the optimal mix of income and inflation taxes is determined by preferences and technology and that "in general" the first-best result that the optimal inflation tax is zero does not carry over to this second-best world. The condition that must be satisfied for the first-

best result to be optimal in the second-best world is that the supply of labour falls off with increased inflation and that the ratio of the slopes of equilibrium level of labour with respect to the income and inflation tax $\frac{L}{L_T}$ equals the ratio of equilibrium gross wages to equilibrium real balances $\frac{wL}{m}$.

This result does not agree completely with those of the earlier studies by Phelps (1973) and Marty (1976). The main reason for the discrepancy lies in the fact that each of these papers is formulated as a general equilibrium model but each fails to achieve complete generality because all supply prices are held constant. That is, any tax is not permitted, by assumption, to distort production efficiency through the misallocation of resources. This, as has been shown by Dixit (1970), effectively reduces the analysis to that of partial equilibrium. As the comparison of the results has shown Phelps' and Marty's conclusions are specific cases of the above more general result. Phelps concludes that both the optimal income tax and inflation rate are always positive unless the compensated labour supply becomes completely inelastic with respect to the income tax, in which case the optimal inflation rate is zero. Marty on the other hand concludes that the inflation tax is always zero if the tax structure is proportional. Incorporating all of their partial equilibrium assumptions does give the first of Phelps' result that both optimal taxes are set at a positive level. However, Phelps' condition for the optimal inflation tax to be zero is not derived. Phelps' condition of a perfectly inelastic compensated labour supply with respect to the income tax does not give the result that the optimal inflation tax is zero but requires that the supply of labour increases with an increase in the income tax. If however, the labour supply is perfectly inelastic with respect

to the income tax, the government's provision of the public good is the same as the amount of this good that the household would buy privately. Thus the household receives the amount of the good from the government, paid for by the income tax, that it would buy out of its own untaxed budget. There is nothing specific to this second good which characterizes it as a public good. Marty's conclusion of a zero inflation tax arises from his added restriction of proportional tax structure.

The signs of the optimal taxes remain indeterminate without more detailed specification of all demand and supply functions. However movements in these optimal taxes as the elasticities of the demand and supply functions vary have been determined for the less general case in which the real wage is constant and the elasticities of supply of labour and the demand for real balances with respect to the inflation and income tax respectively are small. These movements are:

(i) The optimal income tax is higher the more inelastic is the supply of labour with respect to the income tax and the more elastic are the demand for real balances with respect to each of the taxes and the more elastic is the supply of labour with respect to the inflation tax.

(ii) The optimal inflation tax is higher the more elastic is the supply of labour with respect to the income tax and the more inelastic are the demand for real balances with respect to both taxes and the supply of labour with respect to the inflation tax.

FOOTNOTE

¹A compensated labour supply which is perfectly inelastic with respect to the income tax requires that at the given wage rate, the elasticity of the labour supply with respect to the income tax is positive. That means the operational part of the labour supply curve is the backward bending part or alternatively that the labour supply is independent of the compensation.

APPENDIX

Derivation of the slopes of the equilibrium quantities

It is assumed that $q = 1$ and that the utility function is separable.

$$(1) \quad C = C^s(q, w) = C^d((1 - \tau)w, \pi, R(w))$$

that is $C_\tau = C_1^d[(1 - \tau)w_\tau - w] + C_3^d R w_\tau$

$$C_\pi = C_1^d[(1 - \tau)w_\pi] + C_2^d + C_3^d[R w_\pi]$$

$$(2) \quad L = L^s((1 - \tau)w, \pi, R) = L^d(w)$$

$$L_\tau = L_w^d w_\tau$$

$$L_\pi = L_w^d w_\pi$$

$$(3) \quad m = m^d((1 - \tau)w, \pi, R(w)) = m^s(\tau, \pi)$$

$$m_\tau = m_1^d[(1 - \tau)w_\tau - w] + m_3^d R w_\tau$$

$$m_\pi = m_1^d[(1 - \tau)w_\pi] + m_2^d + m_3^d R w_\pi$$

$$(4) \quad \text{To calculate } w_\tau \text{ and } w_\pi,$$

$$L^d(w) = L^s[(1 - \tau)w, \pi, R(w)]$$

Differentiating with respect to τ

$$L_w^d w_\tau = L_1^s[(1 - \tau)w_\tau - w] + L_3^s R w_\tau$$

$$\text{that is } w_\tau = \frac{-L_1^s w}{L_w^d - (1 - \tau)L_1^s - L_3^s R w}$$

Differentiating the labour market clearing equation with respect to π

$$L_{w\pi}^d = L_1^s [(1-\tau)w_\pi] + L_2^s + L_3^s R w_\pi$$

that is
$$w_\pi = \frac{L_2^s}{L_w^d - (1-\tau)L_1^s - L_3^s R w}$$

$$(5) \quad G^s(w) = G = \tau w L + \pi m \\ = \tau w L^d + \pi m^d$$

Differentiating

$$G_\tau = w L^d + \tau w_\tau L^d + \tau w L_w^d w_\tau \\ + \pi M_1^d [(1-\tau)w_\tau - w] + \pi M_3^d R w_\tau$$

$$G_\pi = M^d + \tau w_\pi L^d + \tau w L_w^d w_\pi \\ + \pi M_1^d [1-\tau]w_\pi + \pi M_3^d R w_\pi$$

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