1990

# Endogenous Policy in a Computational General Equilibrium Framework

Thomas Rutherford

Stanley L. Winer

23971

ISSN:0318-725X
ISBN:0-7714-1208-8

RESEARCH REPORT 9007

# ENDOGENOUS POLICY IN A COMPUTATIONAL GENERAL EQUILIBRIUM FRAMEWORK

by

Thomas Rutherford
and
Stanley L. Winer

June 1990

Department of Economics

University of Western Ontario

London, Ontario, Canada

N6A 5C2

# Endogenous Policy in a Computational General Equilibrium Framework

by

Thomas Rutherford* and Stanley L. Winer**

February 1990

Revised, May 1990

* University of Western Ontario, London, Ontario, N6A 5C2

** Carleton University, Ottawa, Ontario K1S 5B6 and University of Western Ontario

Winer's contribution stems in part from joint work with Walter Hettich on related topics. We are indebted for helpful comments to Larry Goulder, Glenn Harrison, Walter Hettich, Randall Holcombe, Lars Mathiesen, Michael Parkin, Herbert Scarf, and participants in seminars at the Univerities of Bern, Western Ontario and Perugia. Rutherford's research was supported by the Canadian Natural Science and Engineering Research Council under operating grant T306A1.

**ABSTRACT**

The analysis of policy in a computational general equilibrium (CGE) context usually proceeds on the assumption that the process determining policy instruments is exogenous to the equilibrium structure of the economy. This paper contributes to the art of CGE policy analysis by explicitly bringing the determination of policy outcomes into the general equilibrium structure. We specify and benchmark a CGE model in which fiscal structure is determined in a competitive political equilibrium. We also consider the meaning of policy analysis in the broader CGE model, and we illustrate a different type of analysis which is appropriate when policy outcomes are endogenously determined by political as well as by economic processes.

## 1 INTRODUCTION

The body of research known as computational or applied general equilibrium modelling, as surveyed by Shoven and Whalley (1984), has substantially expanded our ability to explore the economic consequences of alternative policy blueprints. The analysis of policy in a computational general equilibrium (CGE) context has usually proceeded on the assumption that the process determining policy instruments is exogenous to the equilibrium structure of the economy.[1] The pioneers of CGE modelling have of course been aware of the large body of public choice theory (see Mueller (1989) for an up-to-date text) that has attempted to endogenize policy outcomes, but they have generally chosen to expand the scope of CGE models in other important and challenging directions. The purpose of this paper is to contribute to the methodology of CGE policy analysis by explicitly bringing the determination of policy instruments into the general equilibrium structure.

We begin in section two with a brief survey of existing traditions in the analysis of collective choice processes. In section three we rely on the tradition we think most appropriate in order to develop a CGE model in which fiscal policy is determined in a competitive political equilibrium. To narrow the focus of the paper, collective choice is confined to the determination of two ad valorem tax rates and the aggregate size of a pure public good. We consider the interesting issues that arise in specifying and benchmarking such a model of policy outcomes in an applied general equilibrium setting.

Policy analysis when policy is endogenously determined by political as well as by economic processes will not be complete if collective choice is simply ignored. In section four the appropriate nature of policy analysis in the broader CGE model is discussed and some tentative steps in the direction of a somewhat different style of policy analysis are implemented. The suggested analysis deals explicitly with the fact that policy is part of an equilibrum and may not be easily altered

---

[1] An exception is the CGE model of Zodrow (1988) which incorporates a median voter. See also Harrison and Rutstrom (1990) and Markusen and Wigle (1989) who incorporate Nash equilibrium strategies in the study of international trade conflicts.

independently of political and economic forces. This analysis also acknowledges that not all political processes are regarded as being equally legitimate or desireable. A final section summarizes the results of the paper and indicates directions for further research which we think may lead to interesting, useful and practical developments in the art of CGE policy analysis.

Appendix A expands on the manner in which public goods are integrated into the general equilibrium framework. The GAMS code (Brooke, Kendrick, and Meeraus (1988)) and the MPS/GE code (Rutherford (1987, 1988)) for our expository model are given in Appendix B which is available upon request.

## 2 CHOOSING AN APPROPRIATE MODEL OF GOVERNMENT

There are three distinct and well known equilibrium approaches to the political economy of public policy, each of which might provide a basis for endogenizing tax instruments in a CGE framework; the median voter model, Leviathan, and expected vote maximization. Applications of the median voter model to taxation are surveyed in Ingberman and Inman (1988), Leviathan's choice of tax structure is analyzed by Brennan and Buchanan (1980), and the determination of the tax structure in the expected vote maximization tradition is reviewed by Hettich and Winer (1990).[2]

A central question that must be addressed by any model of political equilibrium is how the collective choice process effectively aggregates the heterogeneous preferences of citizens in the determination of public policy. In the median voter model, equilibrium outcomes are as if the government maximizes the utility of the median voter, the voter whose most preferred policy or ideal point is the median of all ideal points in the economy. Information on the characteristics of different voters is therefore not required in order to predict policy outcomes; only the characteristics of the decisive voter need be known.

A critical assumption behind this median voter result is that the policy space on which

---

[2] An explicit comparison of all three models is found in the Hettich and Winer paper.

voting takes place is uni-dimensional. When voters assess two or more tax instruments or parameters simultaneously, pure majority rule does not lead to a policy that defeats all others in a pairwise sequence of votes, and it is well known that there is no stable equilibrium. Since almost all interesting investigations of tax systems involve consideration of at least two and usually more tax parameters, the median voter model is not a viable candidate for the collective choice process in the CGE context.

The Leviathan model generates well defined predictions about policy even when the issue space is multi-dimensional. Leviathan's objective is to maximize revenues rather than votes. The tax structure chosen in pursuit of this objective is analogous to the pricing policies of a perfectly discriminating multi-plant monopolist. Tax rates are adjusted to take into account each individual's unique Laffer curve or tax rate-tax revenue relationship, in the same manner as a discriminating monopolist charges different prices to different consumers depending on their elasticities of demand.

While heterogeneity of economic exit in response to taxation is acknowledged in the Leviathan model, use of this model for positive analysis implies that electoral and institutional restraints are weak enough for the ruling party to be able to ignore the political voice of individual citizens when selecting tax policies. Hence the question of how the collective choice process in a *democratic* society aggregates preferences is by-passed, and as a consequence, while the model may still be of much interest for normative theorizing, its predictions concerning tax structure lack realism.

Expected vote maximization has a stable equilibrium when the issue space is multi-dimensional and, in addition, the constraints placed on policy choices by democratic institutions are explicitly acknowledged. It is for these two reasons that we use a variant of this model to endogenize policy outcomes. Use of the expected vote maximization model is based on the view that political outcomes can be described as an equilibrium in the sense of a balancing of heterogeneous interests. The key to a formal proof of the existence of such an equilibrium is the assumption that voting is probabilistic. This ensures that each political party's expected vote is a continuous function of its policy platform rather than a discontinuous function as with deterministic voting, thereby eliminating voting cycles.[3] Under appropriate conditions concerning the nature of uncertainty about how individual voters will cast their ballots, competition for office induces an equilibrium in the space of policies that can be described by maximizing a weighted sum of individual utilities subject to the equilibrium structure of the economy. (See for example Coughlin, Mueller and Murrell, (1990 a,b) and Wittman (1984)).

In order to briefly derive the particular synthetic optimization problem that can be used to describe the political equilibrium, we follow Coughlin, Mueller and Murrell in assuming that the electorate can be partitioned into H interest groups with $n_h$ identical voters in group h.[4] The utility function of the representative voter in interest group h, $U_h$, is assumed to be composed of two parts:

$$U_h = u_h(G, x_h) + \xi_h \, .$$

The first part $u_h$ depends directly on the level of public services G and private net consumption of a vector of goods and factors $x_h$, and indirectly via economic structure on a vector of tax rates t. The second part $\xi_h$ represents the evaluation of each political party by voter h on non-policy matters such as ideology or personality. The difference in the non-policy evaluations of the incumbent (i) and the opposition (o),

$$B_h = \xi_{hi} - \xi_{ho}$$

represents an expected utility bias in favour of the current government that is independent of G, X

---

[3] With deterministic voting it is always possible for a candidate to propose a new policy that will attract a majority of voters, regardless of the policy proposed by the opposition.

[4] The formal model of expected vote maximization has its origins in the work of Fair (1978), Coughlin and Nitzan (1981), Barooah and Van der Ploeg (1983), Ledyard (1984), Enelow and Hinich (1984) and Wittman (1984), as well as in Coughlin et al. (1990). See Mueller (1989, 215-6) for a survey. For formal application to situations involving interest groups see also Austen-Smith (1987). On a less formal basis similar models have been used Downs (1957) and many others.

and t.

The probability that voter h supports the incumbent, $P_{hi}$, is assumed to be a function of the utility differentials:

$$P_{hi} = \begin{cases} 1 & \text{if } u_h(G_o, x_{ho}) - u_h(G_i, x_{hi}) < B_h \\ 0 & \text{otherwise} \end{cases}$$

The bias $B_h$ is a random variable having a uniform distribution over the real interval $(\ell_h, r_h)$.[5] Furthermore, it is assumed that

$$\ell_h < u_{hi} - u_{ho} < r_h .$$

Thus every voter always has a positive probability of voting for each party, and consequently neither party can afford to completely write off any interest group in its choice of politically optimal policies.

Each party chooses policies so as to continuously maximize its expected plurality or equivalently its expected vote which, for the incumbent, is given by:

$$EV^i(G,t) = \sum_h n_h - \sum_h \theta_h \, (u_{ho} - u_{hi} - \ell_h)$$

where $\theta_h = n_h \, \alpha_h$ and $\alpha_h = 1 \,/\, (r_h - \ell_h)$.[6] Maximization of this objective subject to a government budget restraint and the structure of the economy requires that policy instruments be adjusted until their net vote-productivity is equalized. If we write the budget restraint and GE structure of the economy as $F(u,t,G,x) = 0$, first order conditions for the optimal choice of tax rates by either party are of the form:

---

[5] The admissible types of probability functions in the vote maximization model is a subject of current discussion. See Enelow and Hinich (1989), Feldman and Lee (1988) and Lindbeck and Weibull (1987).

[6] $EV^o$ is equal to $\sum_h n_h - EV^i$. Expected vote maximization is equivalent to expected plurality maximization since a plurality for i (and analogously for o) is just $EV^i - (\sum_h n_h - EV^i)$.

$$\sum_h \theta_h \frac{\partial u_h}{\partial t_j} - \lambda \frac{\partial F}{\partial t_j} = 0 \; ; \qquad j = 1,2,... \tag{1}$$

Similar conditions define the politically optimal level of G. Nash equilibrium in such strategies is assured because of the zero-sum nature of the competition for votes, and because utility functions and the density functions for $B_h$ are continuous (implying the continuity of expected vote functions).

Equilibrium policy choices of the two parties will be unique and identical if the expected vote functions are strictly concave in policy instruments. Given the uniform density function for $B_h$, this concavity depends exclusively on the concavity of the $u_h$, an aspect of the utility functions to which we shall return below.

The description of equilibrium policy choices by (1) above provides the basis for the CGE model of policy outcomes constructed in this paper. Given a set of tax rates and a public good, equilibrium in a competitive political system can be represented by the solution to (Coughlin *et al.* (1990a), Couglin and Nitzan (1981)):

$$\max_{\{t,G\}} \sum_h \theta_h u_h(G, x_h) \tag{2}$$

$$\text{subject to} \qquad F(u, t, G, X) = 0 \tag{3}$$

where $F(\cdot) = 0$ represents the general equilibrium structure of the economy, including the government budget restraint.[7] This synthetic problem can be conveniently used to compute equilibrium policy outcomes because the first order conditions for the problem (2) - (3) are clearly identical to those in (1). In other words, unless policies are adjusted until the weighted sum of voter utilities in (2) is maximized, it would be possible for a party to increase its expected vote by proposing policies that are likely to improve the welfare of some voters without reducing the welfare of others. Competition in the pursuit of political power insures that in equilibrium no such policies exist.

---

[7] It is implicitly assumed in (3) that $x_h > 0$. If $x_h \geq 0$, (3) must be written $F(x) \geq 0 \perp x \geq 0$, where $\perp$ denotes complementary slackness.

It should be noted that since a weighted sum of utilities is maximized in equilibrium, the choice of policies in this political equilibrium are Pareto Efficient for the restricted policy set {t,G}.[8] This characteristic of the model will be useful in calibrating the weights $\theta_h$. In a more general setting (eg., Hettich and Winer (1988)) the nature of policy instruments would be unrestricted and the *set of instruments as well as instrument values* would be determined in equilibrium. We shall leave exploration of this substantially more complex situation for future research.

## 3 A CGE MODEL WITH AN ENDOGENOUS PUBLIC SECTOR

### 3.1 *Model Structure*

In this section we develop an explicit formulation of the equilibrium structure represented by (2) and (3). We begin with a general specification in which any number of public goods, private goods, production sectors, individuals, interest groups (of identical individuals) and tax instruments is accommodated. Special attention is given to the method by which public goods are incorporated into the model, an issue often neglected in CGE policy evaluation but which cannot be avoided here if we wish to fully endogenize the public sector.[9] The generality of the model provides a clear framework for the analysis and numerical solution of a wide class of models of which the one implemented in this paper is a special case. This particular model is outlined immediately following the general structure.

Our general formulation accommodates complementary slackness associated with negative unit profits (idle activities) and excess commodity supplies (zero prices). It imposes restrictions on preferences (which must be homothetic or quasi-homothetic) and technology (which exhibits either

[8] The support function (2) is consistent with a franchise that is universal. It may not be so, and in that case the efficiency of the equilibrium must be understood to be defined relative to the extent of the franchise as well as relative to the set of policy instruments.

[9] Piggott and Whalley (1987) have noted the potential importance of the endogeneity of the level of public services in the context of standard CGE tax policy analysis.

constant or decreasing returns to scale). We begin with some notation.

*Dependent variables* determined in equilibrium include:

| | |
|---|---|
| $r_h$ | Real income index for a representative individual in interest group h. |
| $u_h$ | Utility index for individual in group h, $= u_h(r_h)$. |
| $y_j$ | Activity level of private sector j. |
| $\pi_i$ | Market price of private good i. |
| $v_{kh}$ | Marginal utility of public good k for the representative individual of group h. |
| $Y_h$ | Nominal income for group h inclusive of the value of public goods. |
| $e_h$ | Real income deflator for group h (i.e. the minimum expenditure for one unit of $r_h$; $e_h = Y_h / r_h$). |
| $g_1$ | Level of provision, public good 1. |

*Policy instruments* for which values are determined by political equilibrium:

| | |
|---|---|
| $t_{ij}$ | Ad valorem tax rate on private good i in sector j. |
| $g_k$ | Level of provision, public good $k > 1$. |

*Secondary variables and parameters* of the model include:

| | |
|---|---|
| $A = (a_{ij})$ | Price-responsive net output coefficient for good i in sector j. |
| $B = (b_{ik})$ | Input coefficient for private good i per unit output of public good k. |
| $C = (c_{ih})$ | Price-responsive demand coefficient, private good i for group h. |
| $\omega_h$ | Endowment vector, group h. |
| $\Omega = (\sum_h \omega_h)$ | Aggregate endowment vector. |
| $G = (\gamma_{kh})$ | Public goods demand coefficient, good k, group h (i.e. the quantity of $g_k$ required per unit of real income). |
| $T = (\tau_{ij})$ | Matrix of tax payments, good i in sector j. If the ad valorem tax rate on good i inputs to sector j is $t_{ij}$, then $\tau_{ij} = -a_{ij}\, t_{ij}$. |

The incumbent party or government chooses tax rates $t_{ij}$ and levels of public goods provision $g_k$ which maximize "political support", S(u), subject to the structure of the economy. The model is given below, where associated with each equilibrium condition is the variable (on the left) which exhibits complementary slackness with that equation:[10] [11]

$$\underset{\{t_{ij},\, g_k\}}{\text{Maximize}} \qquad S(u) = \textstyle\sum_h \theta_h\, u_h(x_h,g) \tag{4}$$

subject to

- Market clearance for private goods:

$$\pi_i: \qquad A\,y \;-\; B\,g \;-\; C\,r \;+\; E \;\geq 0 \tag{5a}$$

- Exhaustion of product in private production:

$$y_j: \qquad -\,A_j^T\pi \;+\; T_j^T\,\pi \;\geq 0 \tag{5b}$$

- Budget constraint for government which reduces the number of "free" instruments by one:

$$g_1: \qquad \pi^T\,T\,y \;-\; \pi^T\,B\,g \;=0 \tag{5c}$$

- Income definitions (including private and public "endowments"):

$$Y_h: \qquad e_h^T\,\pi \;+\; g^T\,v_h \;-\; Y_h \;=\; 0 \tag{5d}$$

- Definition of the expenditure function:

$$e_h: \qquad G_h^T\,v_h \;+\; C_h^T\pi \;-\,e_h \;= 0 \tag{5e}$$

- Walras' law for each household:

$$r_h: \qquad Y_h/e_h - r_h \;=0 \tag{5f}$$

- Determination of private valuations of public goods:

$$v_{kh}: \qquad g_k \;-\; G_{kh}\,r_h \;\;\geq\;\; 0 \tag{5g}$$

---

[10] It should be noted that we obtain complementary slackness through imposition of budget constraints on both private consumers and the government. Heady and Mitra (1988), in an optimal tax model complementary to the present one, omit the government budget constraint and must then impose complementary slackness explicitly.

[11] As in most CGE models, an equilibrium determines only relative prices. In our calculations, we fix the price of one commodity to unity and omit the associated market clearance condition. The choice of numeraire does not alter the equilibrium.

The solution to the maximization of the political support function **(4)** subject to equations **(5a)** - **(5g)** yields the equilibrium values of all private sector dependent variables as well as the equilibrium values of policy instruments.

In addition to acknowledging the importance of collective choice in determining policy outcomes, the model incorporates public goods in a manner not usually found in the literature. The usual model of private good demands in the presence of public goods relies on individual expenditure functions $e_h(\pi,g)$, where g is exogenous, from which are derived private good demands $x_{hi}(\pi,g) = \partial e_h / \partial \pi_i$. In that case, private valuations of public goods, $v_h$, are implicitly defined by $v_{hk}(\pi,g) = \partial e_h / \partial g_k$, but do not appear explicitly in the model structure. We wish to explicitly consider the role played by private valuations of public goods (as distinct from the taxes levied to pay for such goods) since in an economy with an endogenous public sector these valuations are, and will be shown to be, crucial to calibration and to the computation of equilibria.

To incorporate private valuations of public goods we treat public goods quantities and private evaluations of these goods symmetrically with private goods quantities and prices, while continuing to acknowledge that public goods quantities are determined by a collective choice process. We first define nominal income $Y_h$ in equation **(5d)** to be inclusive of the private total valuations of public goods, $g^T v_h$, assuming for the moment that private valuations of public goods, $v_h$, are known. We also define an expenditure function that depends on $v_h$, $e_h(\pi,v)$ in **(5e)**, in contrast to $e_h(\pi,g)$. The function $e_h(\pi,v)$ defines the hypothetical expenditure required to provide one unit of utility if it were possible to buy public goods at prices $v_h$. This expenditure function together with nominal income determines a real income index, $r_h$ in **(5f)**. Given homotheticity of preferences, the index $r_h$ can then be used to determine the aggregate demand for private goods, C r in **(5a)**, and a synthetic private demand for public goods, $G_h$ $r_h$ in **(5g)**. Finally, given the supply of public goods $g_k$ which emerges from the collective choice process, condition **(5g)** determines the private valuations of public goods $v_h$ that are consistent with private and public goods quantities and private good prices. Further details on the treatment of public goods are provided in Appendix A.

### 3.2 *An Expository Model: Benchmarking*

The specific model we have implemented contains one pure public good, two tax rates, and three interest groups, the Rich, the Middle Class and the Poor. The interest groups are differentiated by relative political influence, by capital ownership and by preference for public as opposed to private goods. Since there is only one public good, maximization of the political support function is explicitly carried out with respect to the tax rates, and the level of the public good is determined endogenously by the government budget restraint.

In the benchmark equilibrium each factor input to production is subject to an ad valorem tax at a different rate. As the benchmark Social Accounting Matrix (SAM) in Figure 1 indicates, the tax rate on capital services ($t_K$) is 20% and that on labour services ($t_L$) is 10%. This difference in tax rates drives a wedge between relative factor prices and the ratio of marginal products.

All production sectors are based on constant-returns-to-scale technologies. This is not exceedingly restrictive; decreasing returns functions can be represented through explicit specification of omitted factors. We do not make provisions for increasing returns although this is a straightforward extension (see Rutherford (1988)). Production of the private good X is described by a single level C.E.S. function (see Figure 2):

$$X = f(K,L) = (\alpha_K K^{1/2} + \alpha_L L^{1/2})^2 \tag{6}$$

where K and L are capital and labour inputs respectively. The parameters $\alpha_K$ and $\alpha_L$ are determined by the benchmark SAM when the model is calibrated. (The elasticity of substitution, $\sigma_x$, is a free parameter which has been set at 1.5.)

Production of the public good G requires input of the private good according to:

$$G = X^{1/2} \tag{7}$$

This function is represented in the model by a CRS activity in which X and a fictitious fixed capital

**Figure 1**

Benchmark Equilibrium Structure Social Accounting Matrix

| | Private Sector | Public Sector | Poor | Middle | Rich | Govt |
|---|---|---|---|---|---|---|
| X | 29.6 | -3.6 | -10 | -8 | -8 | |
| G | 0 | 7.2[2] | (-3.5)[1] | (-2.0)[1] | (-1.5)[1] | -7.2 |
| K | -10 | | | 4 | 6 | |
| L | -16 | | 12-2[3] | 4.5-.5[3] | 2.5-.5[3] | |
| T-K | -2 | | | | | 2.0 |
| T-L | -1.6 | | | | | 1.6 |
| K-G | | -3.6 | | | | 3.6 |

**Key:**

X = consumption good;

G = pure public good;

K = capital;

L = labour;

T-K = tax on K (at rate $t_k$=20%)

T-L = tax on labour (at rate $t_L$=10%);

K-G = artificial capital good used in producing G.

**Notes:**

1. These entries reflect the exogenously specified marginal valuations by the different interest groups of the given supply of public goods.

2. The provision of public goods from an input of X units of private goods is $X^{1/2}$. We represent this function through the constant returns to scale Cobb-Douglas production function $Y(X,K_G) = A\ X^{1/2}\ K_G^{1/2}$. Earnings on the artificial capital good correspond to profits which would be associated with the DRS function, hence the formulations are identical.

3. Labour-leisure demand is represented by an agent "purchasing" her own labour at the net of tax market wage rate.

input $K_G$ are combined in a Cobb-Douglas function[12]

$$G = A\ X^{1/2}\ K_G^{1/2} \tag{8}$$

As noted above, differences in endowments and preferences in addition to inequalities in political influence distinguish interest groups. The SAM records that the Rich are relatively better endowed with capital than the Middle Class, while the Poor have no capital, and earn relatively low incomes solely from the sale of labour services. Preferences are represented by nested CES functions. For the representative of interest group h,

$$\begin{aligned} u_h &= u_h(G, X_h, L_h) \\ &= \{\beta_c\ (\gamma_x\ X_h^{\sigma-1} + \gamma_G\ G^{\sigma-1})^{(s-1)/(\sigma-1)} + \beta_L\ (E_{Lh} - L_h)^{(s-1)}\}^{1/(s-1)} \end{aligned} \tag{9}$$

where $(E_{Lh} - L_h)$ is the difference between the labour endowment and labour supply. This specification of preferences allows for excess burdens to arise as a result of the influence of taxation on labour-leisure choices. The parameters $\beta_c$, $\beta_L$, $\gamma_x$ and $\gamma_G$ are determined by the benchmark SAM and by the assumed values of the free elasticity parameters $\sigma_g$ and $s$ given in Figure 2. The elasticity of substitution between public and private goods, $\sigma_g$, is assumed to decline continuously with income.

Neither Figure 1 nor Figure 2 describes the weights $\theta_h$ in the political support function **(4)**. These weights may either be imposed on the model (in which case we might need to adjust the public goods valuations to replicate the benchmark), or they may be determined as part of the calibration procedure. Since little empirical research is available that might give us some guidance in choosing the $\theta$'s, we calibrate these parameters in the manner discussed in the next section.[13]

---

[12] The functional form in (7) is satisfactory for the implementation of the model using GAMS. The second formulation is appropriate for MPS/GE in which production functions are restricted to constant returns to scale. Note that minimization of the total cost $(X + K_G)$ of one unit of G requires $X/(X + K_G) = 3.6/7.2 = 1/2$ in the SAM, where $P_X = P_K = 1$.

[13] See Van Velthoven (1989), Van Winden and Renaud (1987), Barooah and Van der Ploeg (1983) and Van Winden (1983) for preliminary attempts at estimation of the $\theta$'s and for a discussion of the difficulties in doing so.

**Figure 2**

The Structure of Technology and Preferences

<u>Technology</u>

*Private Good*

```
   X
  / \   σx = 1.5
 K   L
```

$$X = (\alpha_K K^{1/2} + \alpha_L L^{1/2})^2$$

*Public Good*

$$G = A X^{1/2}$$

*The public goods production function is represented by a CRS activity in which X and a "fictitious" capital input, $K_G$, are combined in a Cobb-Douglas function.*

<u>Preferences</u>

```
        U
       / \   s = 1.1
  σg  /\  L
     X  G
```

| | $\sigma_g$ |
|---|---|
| *Poor* | 0.3 |
| *Middle Class* | 0.7 |
| *Rich* | 1.1 |

$$U = \{\beta_c (\gamma_X X_h^{\sigma-1} + \gamma_G G^{\sigma-1})^{(s-1)/(\sigma-1)} + \beta_L (E_{Lh} - L_h)^{(s-1)}\}^{1/(s-1)}$$

### 3.3 *Calibration of the Political Weights*

The $\theta_h$'s in the political support function represent the manner in which changes in individual welfare as a result of government policy are translated into effective political influence. These weights must be calibrated so that the benchmark equilibrium tax structure $\bar{t} = (\bar{t}_L, \bar{t}_K)$ and level of public goods $\bar{G}$ given in the SAM solves the problem of maximizing support subject to the equilibrium structure of the economy when the $\theta_h$'s are set at their calibrated values. That is, we seek $\bar{\theta}$ such that $\bar{t}$ solves

$$\max_{\{t_L, t_K\}} \quad \bar{\theta}^T u \quad \text{subject to (5a) - (5g)} \tag{10}$$

where $\bar{\theta}^T = (\bar{\theta}_P, \bar{\theta}_M, \bar{\theta}_R)$ is the vector of calibrated political weights and $u^T = (u_P, u_M, u_R)$ is a vector of representative voter utilities. $\bar{G}$ does not appear in (10) because it is determined by the government budget restraint **(5g)**.

Let M(t) be the 2 x 3 matrix of equilibrium utility gradients with respect to the tax rates:

$$M_{ih}(t) = [ \frac{\partial u_h}{\partial t_i} \quad ; \; i \in \{L, K\}, \;\; h \in \{P, M, R\} ]$$

Formally, $M = - (\nabla_u F)^{-1} \nabla_t F$, but these gradients may be approximated by numerical differencing:

$$M_{ih} \approx \frac{u_h(\bar{t} + \Delta_i) - u_h(\bar{t})}{|\Delta_i|}$$

where $\Delta_i$ represents a vector of taxes in which all components except the $i^{th}$ are zero.[14]

The first order condition for the problem in (10) is:

$$M(\bar{t}) \cdot \bar{\theta} = 0 \tag{11}$$

where M is evaluated at the benchmark equilibrium. For (11) to be solved by some $\bar{\theta} > 0$, it is clear that for each i some $M_{ih}$ must be negative and some must be positive. Understandably, political equilibrium requires disagreement among voters concerning the best direction in which to alter each tax rate.

Finally, since we wish to impose the normalization rule

$$\sum_h \theta_h = 1 \tag{12}$$

it is apparent from condition (11) that a unique solution for the $\bar{\theta}_h$'s requires that the total number of policy instruments equal the number of interest groups, as in the present model.

Combining (11) and the normalization rule yields the calibration rule for $\bar{\theta}$:

---

[14] In our calculations we use the difference approximation as well as a second method based on calculating the synthetic optimization problem: max $u_h$ s.t. $F(u,x,t) = 0$ and $t = \bar{t}$, from which $M_{ih}$ can be identified as the Lagrange multiplier associated with the constraint, $t_i = \bar{t}_i$.

$$\bar{\theta}(\bar{t}) = \begin{bmatrix} M(\bar{t}) \\ e^T \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{13}$$

where $e^T$ is a $1 \times 3$ row vector of 1's. At the benchmark values, $(\bar{t}_L, \bar{t}_K) = (.10, .20)$, we have:

$$M(\bar{t}) = \begin{array}{c} \begin{array}{ccc} P & M & R \end{array} \\ \begin{bmatrix} -.147 & -.047 & .246 \\ .249 & -.150 & -.231 \end{bmatrix} \end{array} \begin{array}{c} \\ (t_L) \\ (t_K) \end{array}$$

and (13) then yields $(\bar{\theta}_P, \bar{\theta}_M, \bar{\theta}_R) = (0.44, 0.25, 0.31)$ as the vector of calibrated political weights for the present model.[15] [16]

It is important to observe that for an arbitrary benchmark dataset, the solution to (13) may yield $\bar{\theta}_h < 0$ for some h. This indicates that the benchmark in question cannot be represented as the equilibrium of a competitive political system. Some examples of datasets for which political

---

[15] Here and below, utility is cardinalized by assuming $u_h = \log r_h$, i.e. as the natural logarithm of "real income", unless otherwise indicated.

[16] An alternative approach to the calibration of the $\theta$'s uses the solution to a synthetic problem:

$$\text{Max } \textstyle\sum_h u_h \quad \text{subject to} \quad F(u, t, G, X) \geq 0 \quad \text{and} \quad u_h \geq \bar{u}_h,$$

where $F(\cdot)$ represents the equilibrium structure (5a) - (5g). In the solution to this problem, $\theta$ can be calculated from the Lagrange multipliers for the lower bounds on utility levels. To see this, write the Lagrangian as:

$$L = \textstyle\sum_h u_h - \lambda^T F(\cdot) + \sum_h \alpha_h (u_h - \bar{u}_h) .$$

The first order conditions associated with $u_h$ are:

$$1 - \lambda^T \nabla_u F(\cdot) + \alpha_h = 0,$$

where $\nabla_u F$ is the gradient of F with respect to utilities. Now consider the original problem of maximizing support (4) subject to the GE structure, $F(\cdot)$. If $\bar{\theta}$ is the vector of benchmark political weights, the first order conditions for $u_h$ are:

$$\bar{\theta}_h - \lambda^T \nabla_u F(\cdot) = 0.$$

Comparison of the two sets of first-order conditions indicates that the benchmark weights are proportional to $1 + \alpha_h$, or normalized by setting $\bar{\theta}_h = (1 + \alpha_h) / (H + \sum_j \alpha_j)$. The correspondence of the political weights and Lagrange multipliers in an appropriately specified maximization problem has also been discussed by Van Velthoven (1989, Ch. 3).

weights cannot be successfully calibrated are provided below. The general lesson is that it may not be possible to successfully calibrate political influence weights to an arbitrary benchmark social accounting matrix.

Before turning to the meaning of policy analysis in the present model where policy outcomes are endogenous, we briefly explore two additional issues central to the construction of such models: the nature of private valuations of public goods, and the concavity of the political support function.

### 3.3 *Private Valuations of Public Goods and Benchmark Calibration*

A complete model of fiscal structure cannot be constructed without explicitly acknowledging that taxes are levied for a purpose, and that purpose is in part to provide goods and services which are valued by private individuals. If independent estimates of the $\theta$'s were available, it might be possible to calibrate the marginal valuations of public goods from these. Given an estimate $\bar{\theta}$, (11) may be written as a nonlinear system of equations, $M(\bar{t},v)\,\bar{\theta} = 0$, which could (in some cases) be solved for $v = \bar{v}$. However, since the $\theta$'s must be calibrated and empirical estimates of private valuations of public goods are not available, it is necessary to make reasonable guesses about them.

To acquire some insight into the role played by the private valuation of public goods specified in the SAM, we have conducted a sensitivity analysis centered on the extent to which the Samuelson rule for public goods provision is satisified by the benchmark dataset. The Samuelson rule in the present context is that the sum of marginal valuations equals the marginal cost of producing G; i.e. $\sum_h v_h = MC_G$. This condition is not satisfied in the benchmark SAM, where $\sum_h v_h = 7$ and $MC_G = 7.2$. It is well known (see, for example, Atkinson and Stern (1974)) that in the presence of distorting taxes, the Samuelson rule is neither necessary nor sufficient for constrained optimization of functions like (4). In the present context, heterogeneity of voters with respect to political influence in addition to the excess burden of taxation also plays a role in explaining why the unweighted sum, $\sum_h v_h$, will not form part of the description of equilibrium policy.[17]

But while the Samuelson condition does not have to be satisfied, we have developed expermiental evidence that substantial deviations from the Samuelson rule in our expository model may, in fact, preclude calibration of the $\theta$'s. Moreover, it turns out that even when the Samuelson rule is satisfied, certain distributions of private valuations of the public good across the groups will prevent successful calibration. Figure 3 illustrates the potential non-existence of a mapping from given private marginal evaluations to $\bar{\theta} > 0$ via the calibration rule. For each set of valuations in the simplices illustrated in the Figure, we have computed $\bar{\theta}(v)$ using (13). The region in which calibration yields at least one negative $\theta_h$ clearly grows as $\bar{v} = \sum_h v_h$ diverges from $MC_G$.

The diagrams in the last column in figure 3 indicate the range of $\bar{\theta}$'s that result from successful calibration when the valuations range over all possible distributions for a given $\bar{v}$. It is apparent from this figure that, in the present model, when $\bar{v} \neq MC_G$ the admissible $\bar{\theta}$'s are quite

---

[17] To see this, consider a modified version of the model based on Atkinson and Stern (1974). In this model, producer prices for the goods and factors are assumed constant and equal to consumer prices less the specific tax i.e. $p_k = \pi_k - t_k$. Competition is perfect; production exhibits constant returns, and all individuals are on their budget restraints. Consider the case in which there is one public good G and good 1 is chosen as the numeraire with $p_1 = 1$ and $t_1 = 0$. Since $p_k$ is assumed to be constant, market clearing and zero profit conditions are automatically satisfied. Moreover, since markets clear, Walras' law implies that the government's budget constraint must also be satisfied. Under these conditions, equilibrium policy choices can be represented by

$$\max_{\{t,\, G\}} \; \sum_h \theta_h V^h(\pi,G) \quad \text{subject to} \quad f(G,X) = 0 \quad (\perp \lambda \geq 0)$$

where $V^h$ is the indirect utility function of a representative voter and $f(G,X) = 0$ is a single nonlinear equation which represents the production possibilities frontier. Assuming the marginal utility of income $\alpha$ is identical across groups, the first order condition for G can be written as:

$$\frac{\partial f/\partial G}{\partial f/\partial X_1} = \frac{\alpha}{\lambda} \sum_h \theta_h \left( \frac{\partial V^h/\partial G}{\alpha} \right) + \frac{\partial}{\partial G} \{ \textstyle\sum_k t_k X_k \}. \tag{20}$$

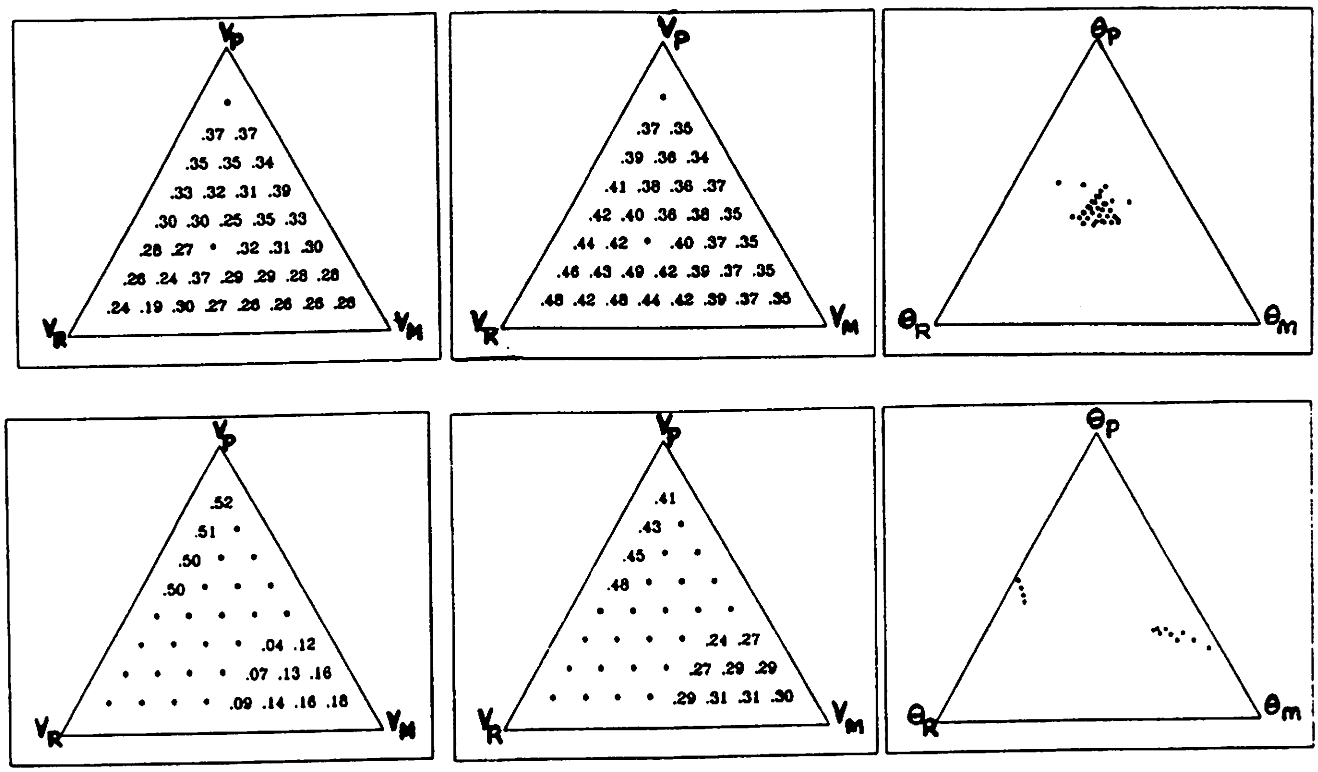
The left hand side of this equation is the marginal rate of transformation (MRT) of G for $X_1$. On the right hand side, the term $(\partial V/\partial G\} / \alpha$ is the marginal rate of substitution (MRS) of G for $X_1$ and it is weighted by political influence $\theta_h$. The second term represents the tax revenue effect of altering the level of the public good. The simple Samuelson rule is of course MRT $= \sum_h MRS_h$, where only the unweighted sum of $MRS_h$ is used.

## Figure 3

The Role of Marginal Valuations of the Public Good in Calibrating Political Influence Weights

$\bar{v} = MC_G$

$\bar{v} = \frac{3}{4} MC_G$




$\theta_{Rich}$ $\theta_{Poor}$ Resulting $\theta$ Values

*Notes:*

The first two columns present values of $\theta_h$ which arise from particular distributions of $v_{Gh}$ on the $\bar{v}$ simplex ($\bar{v} = \sum_h v_{Gh}$). In these diagrams, the *relative value* of $v_{Gh}$ for any point ($v_{Gh} / \bar{v}$) is inversely proportional to the distance to vertex h. An asterisk ("•") indicates that the corresponding distribution of marginal valuations is not consistent with successful calibration of the $\theta_h$'s (i.e. some $\theta_h < 0$). The final column plots values of $\theta_h$ for successful calibrations onto the unit simplex ($\sum_h \theta_h = 1$).

sensitive to changes in the distribution of marginal evaluations. Only when $\bar{v} = MC_G$ are the range of $\bar{\theta}$'s that result from successful calibration relatively insensitive to the distribution of marginal valuations. In general, it appears that assumptions about private valuations of public goods will influence the outcome of the calibration procedure substantially.

### 3.4 *Concavity and Policy Response*

Uniqueness of the support maximizing policy choices depends on the concavity of the expected vote function or, equivalently, of the support function (4), an issue discussed by Enelow and Hinich (1989), Feldman and Lee (1988) and Lindbeck and Weibull (1987). The assumption that the non-policy bias terms $B_h$ are distributed uniformly leads to a support function which is linear in the $\theta$'s. Hence, as noted earlier, concavity of the objective function depends crucially on the nature of the utility functions, and in particular on how the CES utility functions (9) are cardinalized when the model is implemented.

If indifference curves are numbered so that the marginal utility of real income is constant (i.e. $u_h = r_h$) the support function is quite flat in the neighbourhood of the benchmark values of policy instruments $(\bar{t}_L, \bar{t}_k, \bar{G}) = (10\%, 20\%, 3.6)$.

It is of interest to compare the response of equilibrium policies to exogenous shocks when the support function is quite flat around the benchmark equilibrium with the response when the support function has a greater degree of concavity, such as when $u_h = \log r_h$. We should expect larger changes in equilibrium policy outcomes in the former than in the later case. Since the most novel feature of the present model is its political context, we investigate the following exogenous shocks: (i) an increase in the effective power of the lowest income individuals such as would follow an extension of the franchise, and (ii) a rise in the size of the middle class. Both shocks represent long-term developments in Western society that may have had offsetting general equilibrium effects on the trend rate of tax on capital. It should be recalled that the Poor own no capital, so they will tend to favour higher taxes on capital to finance public services. The Middle Class own both labour and capital and are therefore less inclined to expropriate the Rich.

An extension of the franchise can be represented by an increase in $\theta_P$ above its benchmark value $\overline{\theta}_P$. We increase the size of the Middle Class through proportional growth of its labour, capital and political influence ($\theta_M$). Figures 4 and 5 illustrate the effect of each shock separately. In these experiments, we see that model behavior crucially depends on cardinalization of utility. It can be seen that equilibrium tax rates respond quickly to small changes in $\theta_P$ and $\theta_M$ when $u_h = r_h$. (The tax on capital falls to zero when the weight on the Middle Class is increased by only 10 percent.) Tax rates are less volatile when $u_h = \log r_h$. In either case, taxes generally move in the expected directions. These experiments suggest a simple general equilibrium story to explain why extension of the franchise did not, apparently, lead to expropriation of capital. It may be of interest to explore this old but still unresolved issue further using the sort of model presented here.

## 4 THE ANALYSIS OF ENDOGENOUS POLICY

The usual approach to tax policy analysis in a CGE framework involves exogenously changing one or more policy instruments according to normative criteria that are not generally concerned with the quality of the collective choice process, and then investigating the ramifications of this change for economic welfare. When policy instruments are determined endogenously as part of a competitive political equilibrium, however, it is not clear whether such a CGE policy analysis is entirely adequate. In this section, following Winer and Hettich (1988), we operationalize two kinds of analysis which may be somewhat better suited to a world in which policy outcomes are part of a general equilibrium.

The first kind of analysis we consider is intended to help overcome the potential inconsistency of a normatively desirable tax blueprint with the set of tax structures that can be supported as equilibria of the economy. The second type of analysis represents a broadening of normative policy evaluation to explicitly incorporate views about the political process. Both of these

**Figure 4**

Rise of the Middle Class

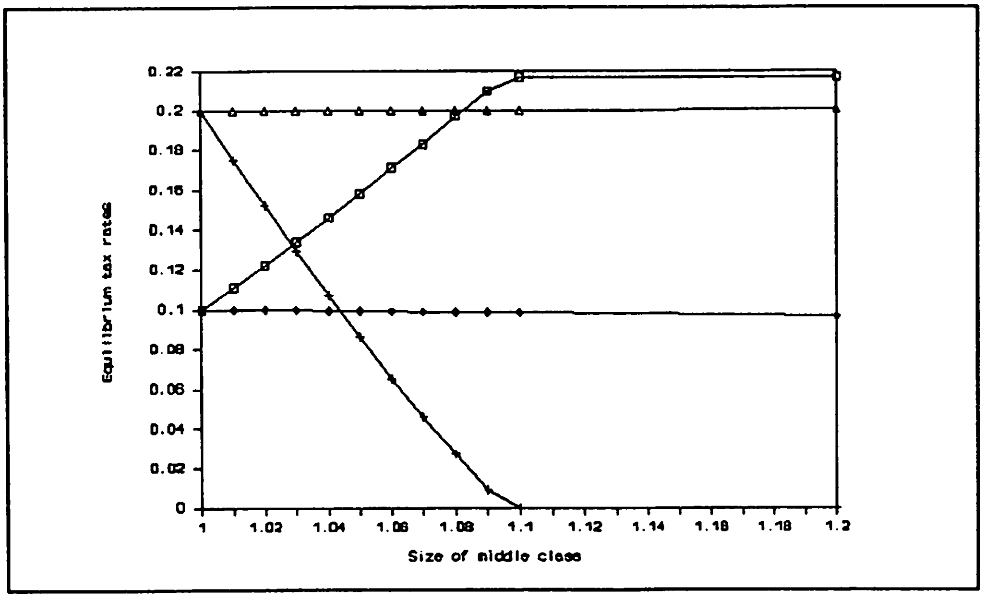
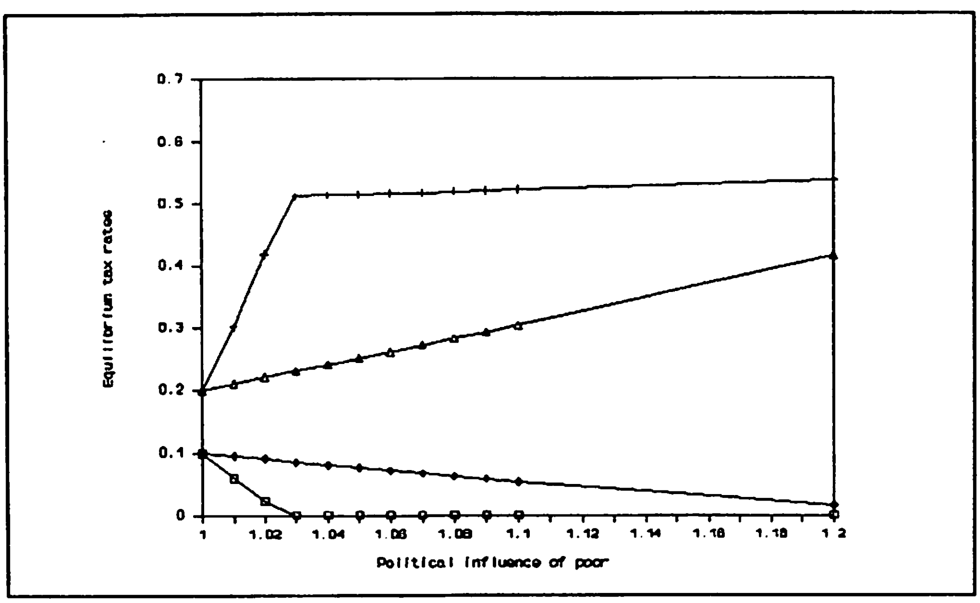



**Figure 5**

Extension of the Franchise




*Linear Utility:* $t_K = +$ ; $t_L = \square$ *Log Utility:* $t_K = \Delta$ ; $t_L = \diamond$

methods are intended as complements to, rather than as replacements for, traditional approaches such as those based on social welfare functions or on tax principles such as ability to pay. Neither approach is original to us; the second has an especially long history going back at least to Wicksell, and the first method has often been practised by astute policy advisors. Their explicit implementation in a CGE framework is, we think, novel.

### 4.1 *Evaluating the Political Feasibility of a Tax Blueprint*

Tax policy in the benchmark equilibrium is efficient for the restricted set of policy instruments $\{t_L, t_K, G\}$ and for given weights $\bar{\theta}$. However, a normative tax theory such as one based on maximizing a social welfare function may suggest another set of tax instruments or different values for existing ones, and in such cases it will be of interest to know what changes in political factors are necessary or sufficient to support the alternative tax blueprint as an equilibrium. This sort of investigation can help to sort out normatively desirable reforms which are unstable or unlikely to be adopted from those which are politically feasible because they do not require drastic changes in political institutions or in the relative influence of different groups. The interesting question of what metric should be used to judge political feasibility arises here.

In order to demonstrate how one might implement this approach, we investigate the nature of the political weights $\theta$ required to support a broadly based expenditure tax ($t_L = t_k$) in place of the benchmark tax system which involves distortionary taxation of factor inputs ($t_L \neq t_k$). We begin our investigation by considering the global properties of the model. In Figure 6 the shaded region consists of all tax rate pairs $(t_L, t_k)$ that can be reached as a solution to the problem of maximizing support subject to the GE structure of the economy. That is, this region represents all policy outcomes for which the calibration rule (13) yields positive $\theta$'s.[18] [19] It should be noted that this set

[18] At any point in the political equilibrium set, there is no direction of movement that is unanimously preferred. The same technique used to compute this set could also be used to compute Weymark's (1980) unanimity set or Wicksellian set defined for each point outside of the political equilibrium set.

is invariant to alternative cardinalizations of utility.

Figure 6 will be recognized as a standard diagram used in the analysis of electoral outcomes. Here it is numerically constructed, to our knowledge for the first time in the CGE context. The outer edge of the political equilibrium set is represented by the relation: $(\min_i\{\theta_i(t)\}=0)$. This curve is continuous and apparently smooth. (The field values defined by $\min_i\{\theta_i(t)\}$ were employed for contouring.) The set of political equilibria in Figure 6 excludes very high tax rates on both labour and capital, as well as equilibria in which tax rates on both factors are very low. In these cases, it is possible to find alternative tax rates which improve everyone's welfare and which thereby increase political support. For example, with very high tax rates, say $t_L = t_k = 75\%$, all groups can be made better off by lowering tax rates and the level of public goods. If marginal valuations of the public good G were higher some groups might be prepared to endure the losses in full income required to support the larger public sector made possible by such high tax rates.

The benchmark equilibrium is shown by point $\bar{t}$ in Figure 6, where $t_L = 10\%$ and $t_k = 20\%$. The curves $\bar{u}_R$, $\bar{u}_M$ and $\bar{u}_P$ through $\bar{t}$ are indifference curves for the Rich, Middle Class and Poor respectively, while the arrows originating at $\bar{t}$ indicate the direction in which each group would like to see tax policy evolve. The fact that $\bar{u}_M$ and $\bar{u}_P$ intersect at $\bar{t}$ indicates that this point would not be a stable equilibrium if voting was strictly deterministic. The points $P^*$, $M^*$ and $R^*$ are ideal points for the representative agent of the respective interest group. $M^*$ is the median in each dimension

---

[19] The set of possible political equilibria was constructed in the following manner. We formed a square grid with 0.1% spacing on the region bounded by

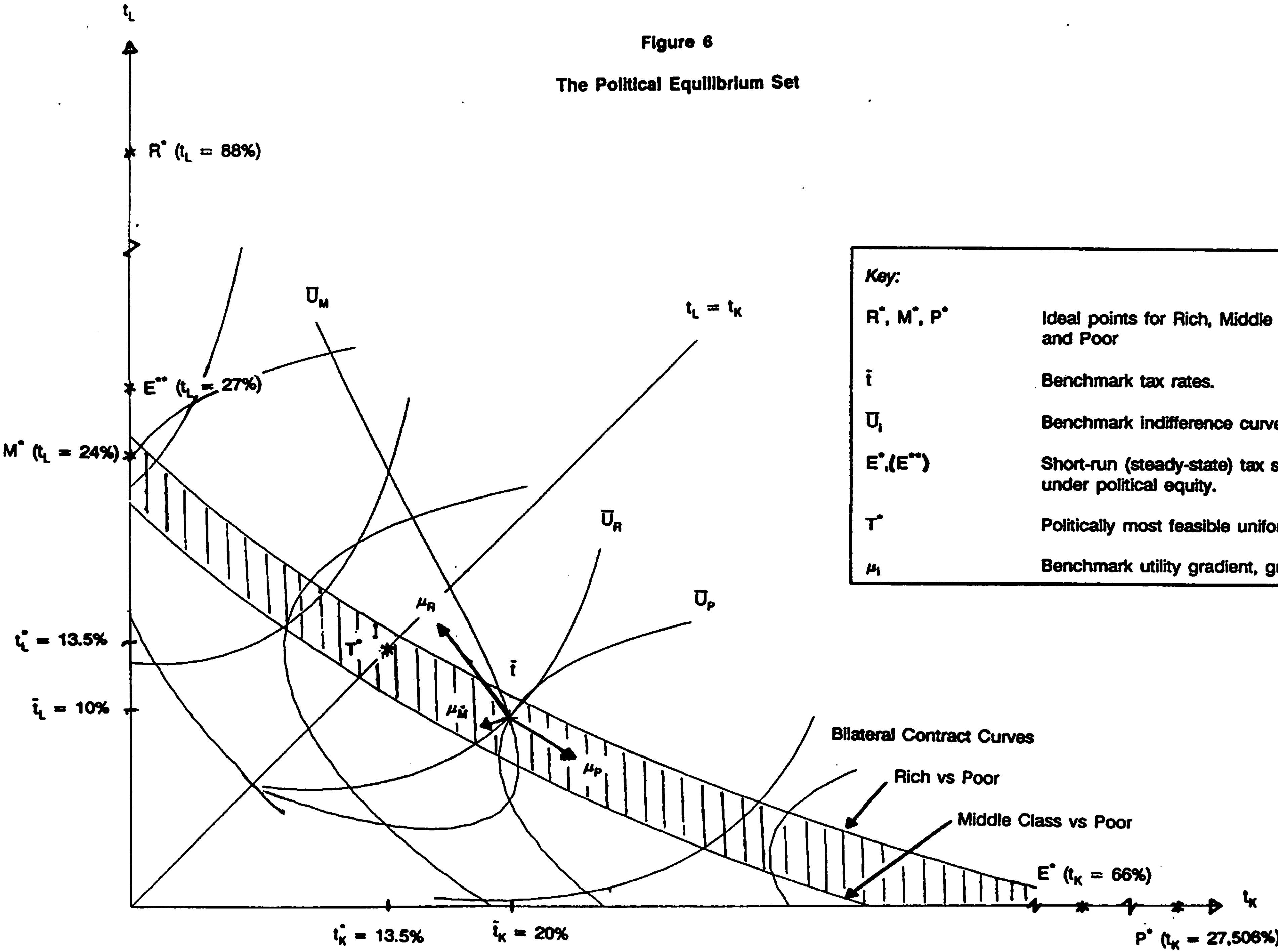
$$0\% \le t_K \le 60\% \qquad 0\% \le t_L \le 40\%$$

$$10\% - 0.666\,\bar{t}_K \le \bar{t}_L \le 40\% - 0.8\,\bar{t}_K$$

This produced 950 points in the $(t_L, t_K)$ plane. At each of these points the equilibrium prices and quantities were computed, as well as at neighbouring points $(t_K + \varepsilon, t_L)$ and $(t_K, t_L + \varepsilon)$ where $\varepsilon = 10^{-5}$. Computation of equilibria using MPS/GE was completed to an accuracy of $10^{-9}$ in about 45 minutes on a 386 running at 25 MHz. Nominal utility levels at grid points and adjacent perturbations were employed to approximate the matrix of utility gradients with respect to tax instruments M(t). Associated $\theta$'s were then computed using the calibration rule (13).

**Figure 6**

**The Political Equilibrium Set**




*Key:*

| | |
|---|---|
| R*, M*, P* | Ideal points for Rich, Middle Class and Poor |
| $\bar{t}$ | Benchmark tax rates. |
| $\bar{U}_i$ | Benchmark indifference curve, group i. |
| E*,(E**) | Short-run (steady-state) tax structure under political equity. |
| T* | Politically most feasible uniform tax. |
| $\mu_i$ | Benchmark utility gradient, group i. |

and could be a 'structure-induced' equilibrium (Shepsle (1979)) with strictly deterministic voting *provided that voting was effectively restricted to one tax rate at a time.* Under probabilistic voting, $\bar{b}$ is the stable equilibrium in spite of the multi-dimensional issue space.

When we consider uniform taxation we must look at the set of equilibria which lie along the 45 degree line labelled $t_L = t_K$ in Figure 6. There are at least two ways in which to evaluate the political feasibility of elements of this set. We may either assess a particular uniform tax equilibrium in terms of the losses in political support required to implement the change (Figure 7), or we can evaluate the requisite changes in the distribution of political influence which would produce an equilibrium in this set using a metric of political feasibility (Figures 8 and 9). Both methods suggest that the 'most politically feasible' uniform tax rate is roughly 13½%, as shown by point $T^*$ in Figure 6. This minimizes loss in support (see Figure 7) and involves increasing the tax on labour from its benchmark level ($\bar{t}_L = 10\%$) and decreasing the tax on capital ($\bar{t}_K = 20\%$), reflecting the fact that the Poor suffer as a result of the adoption of uniform taxation in the expository model. Support declines at $T^*$ relative to the benchmark status quo in Figure 7 because political optimization has been carried out subject to an additional constraint on tax structure.

Figure 9, which is derived from the information in Figure 8, illustrates how definitions of political feasibility based on the distribution of the $\theta$'s can be used to measure the 'distance' of a particular uniform tax equilibrium from the status quo, $\bar{\theta}$. The two metrics shown are the $L_2$ norm ( $\sum_h (\theta_h - \bar{\theta}_h)^2$ ) and the $L_\infty$ norm ( $\text{argmax}_h\ |\theta_h - \bar{\theta}_h|$ ). In the expository model, minimization of either of these metrics yields the same conclusion as that based on minimizing loss in support.

Figure 10 shows the size of government associated with different uniform tax rates. At the 'politically most feasible' rate of 13½%, the level of provision of G is 0.99 times the benchmark level. Apparently, the uniform tax blueprint would not require much adjustment in the size of government.

**Figure 7**

Normalized Political Support
under Uniform Taxation

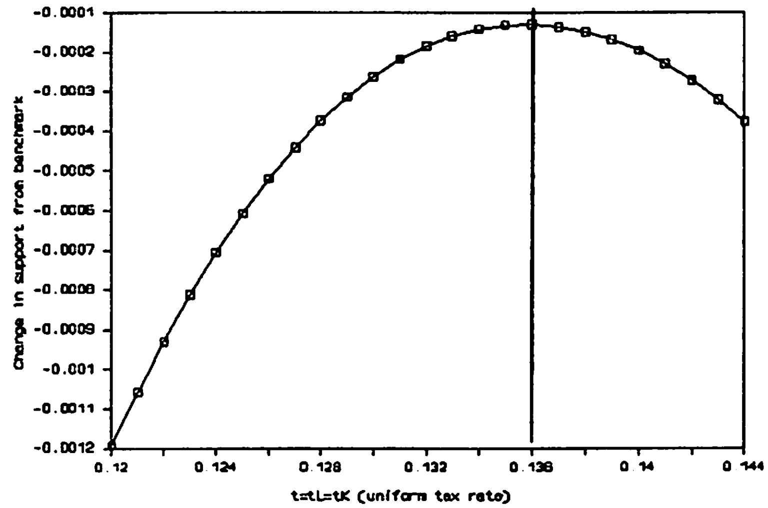
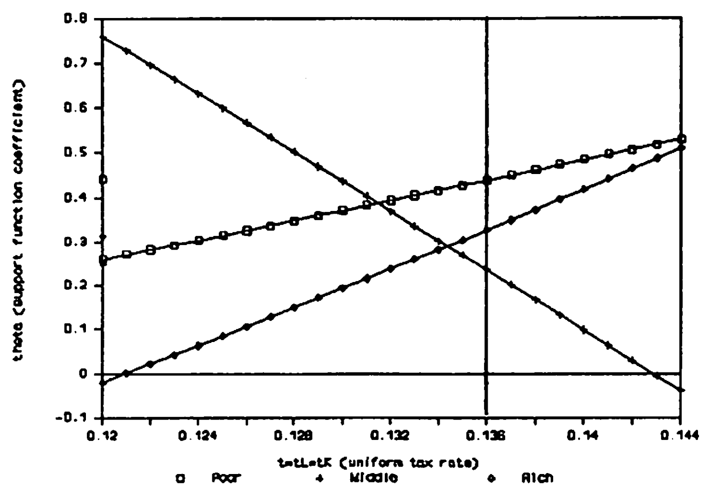


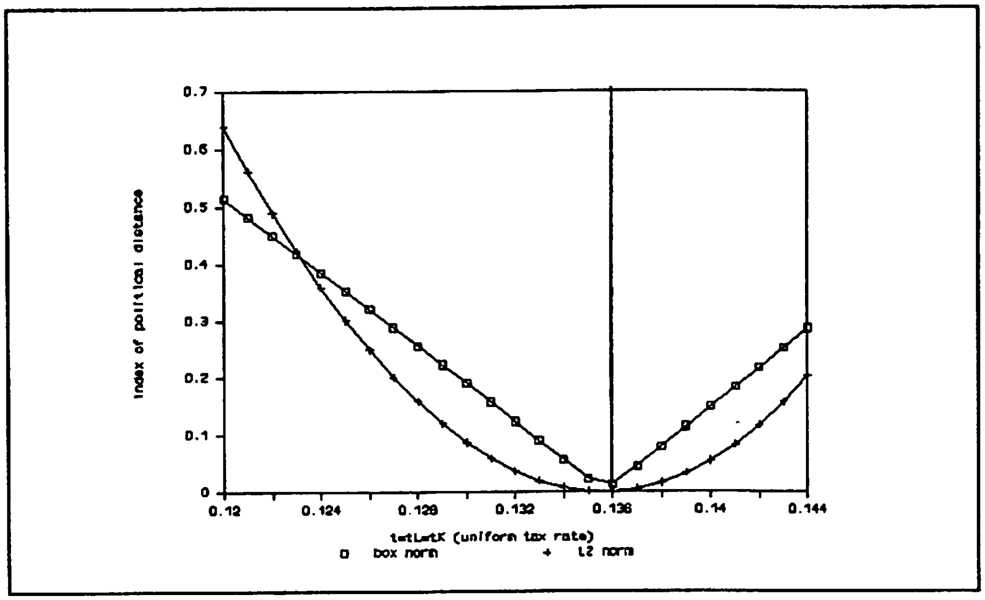

**Figure 8**

Equilibrium Political Weights
under Uniform Taxation



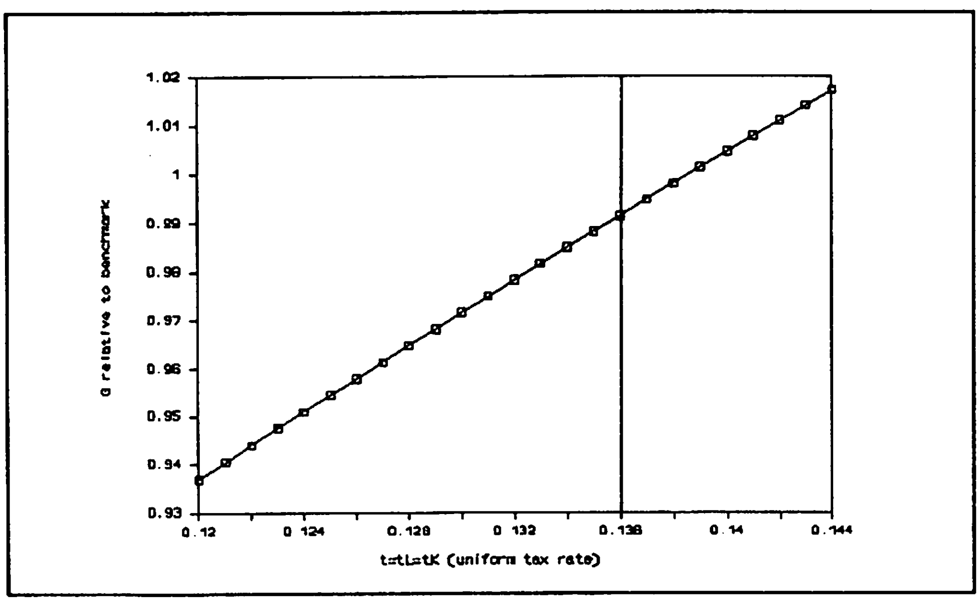
## Figure 9

Political Feasibility
of Uniform Taxation




## Figure 10

The Size of Government
under Uniform Taxation



4.2 *The Role of Democratic Political Theory in CGE Policy Anaysis*

The analysis outlined above is concerned with the political feasibility of alternative equilibrium tax structures. It is important to recognize, however, that whatever the "political distance" of these alternatives from those in the benchmark, a tax blueprint which is desirable from the perspective of a traditional normative theory of taxation may not be consistent in equilibrium with changes in political institutions that are desirable on broader philosophical grounds. If this is in fact so, it would be an interesting conclusion, helping tax theorists to realign their normative assumptions with more general criteria of human welfare, and especially with those criteria more directly concerned with the quality of the democratic process.

It is possible in the present framework to inquire about the nature of taxation that is consistent with democratic political theory (such as in Dahl (1988)), and to compare these tax structures with those based on traditional tax principles.[20] One might, for example, compare the uniform tax alternative constructed above with the tax structure that would be consistent with equality of effective political influence for some specific definition of the franchise. A tax system consistent with equality of effective individual influence could be defined as the vector t in the political equilibrium set of Figure 6 that is implied by political weights $\theta_h = n_h/N$. For our purposes, the number of individuals in group h may be proxied by labor endowments in the social accounting matrix. In our model, the equilibrium tax structure which is compatible with equality in this sense is $(t_L,t_K)$ = ( 0%, 63%), represented by point $E^*$ in Figure 6. Evidently, the benchmark tax structure, the most feasible uniform tax and the tax structure consistent with political equality differ substantially from each other in this simple model.

In the static model (4) - (5), the physical capital stock, unlike the supply of labour services, is fixed regardless of the tax rate $t_K$. This is an unrealistic characterization of the long-run properties of an economy in which an increase in $t_K$ would be expected to lead to a decrease in

[20] On this point see also Kolm (1987).

the net return to capital and thereby to a reduction in investment. Such a long-run response will create a trade-off between political equality and long-run average income, and ought to be part of any comprehensive assessment of a proposal to increase effective political equality. In order to consider the implications of the dynamic linkage between equality and the size of the capital stock, we modified the model formulation to reinterpret the benchmark equilibrium data as though the economy were in a stationary equilibrium with an annual depreciation rate of 10% and a discount rate of 2%. The discount rate may be thought of as a weighted average of discount rates of heterogeneous voters. Here it is simply imposed on the model, though in a more complete dynamic formulation the government's effective choice of discount rate would also be part of the equilibrium.

Adopting the invariant capital stock formulation of Hansen and Koopmans (1972) as extended to the general equilibrium setting by Rutherford (1988, section 3.5), we find that in the dynamic model with the given discount rate, there is a significant change in the nature of political equilibrium. The equilibrium tax structure consistent with complete political equality moves from $(t_L,t_K)$ = ( 0%, 63%) at $E^*$ in Figure 6 to ( 27%, 0%) at $E^{**}$. In the long-run model, the size of the capital stock is quite sensitive to the rate of return and political competition insures that labor supply, being much less price responsive, bears the entire tax burden.

## 5 SUMMARY AND CONCLUDING REMARKS

This paper is motivated by the view that public policy is determined endogenously as part of the competitive struggle for political office. We have implemented one model of a democratic political economy in which equilibrium tax rates and the level of public services take on values which maximize a political support function subject to the general equilibrium structure of the private economy. Political support in equilibrium is represented here by a weighted sum of the utilities of representative interest group members, where weights reflect the size of interest groups and the nature of density functions underlying voting behaviour. Use of such a model is appropriate to the extent that equilibrium policy choices in a democratic society reflect a balancing of politically relevant interests. This is characteristic of a probabilistic voting model in which political parties continuously attempt to maximize their expected plurality. Implementation of the model in a CGE framework raises a number of interesting issues that carry with them implications for empirical research in public economics and for theoretical work on voting models, as well as for the art of CGE modelling and policy analysis.

Calibration of the weights reflecting political influence to a benchmark social accounting matrix is feasible. A necessary but not sufficient condition for calibration is that the number of instruments is equal to the number of interest groups.[21] It would be most interesting to compare political influence calibrated in this fashion with independent estimates. At this time it is not possible to do so; the empirical research aimed at measuring such weights is just beginning to emerge.

Not every social accounting matrix can be calibrated, even when the numbers of interest groups and instruments are equal. The distribution of marginal valuations of the public good as well as the departure of their sum from the marginal cost of public goods supply appear to be very important in determining whether calibration is successful. Reliable estimates of private valuations of public services would be very useful in choosing between alternative admissible calibrations.

The calibration exercise suggests that resolution of the debate concerning the concavity of the political support function is of substantial practical importance, and for obvious reasons. Proof of existence of the political equilibrium (i.e. as in Coughlin et al (1990a)) only requires continuity of utility functions and assumptions about the nature of density functions underlying voting behaviour. Implementation in a CGE model requires substantial concavity to enable computation of the

[21] Additional information about the relative influence of groups would permit reductions in the number of instruments.

equilibrium and to 'slow down' the equilibrium response of policy to exogenous shocks.

We have argued that policy analysis when policy is endogenous, as it surely is, may not be appropriately conducted by simply altering (equilibrium) values of policy instruments. It is reasonable in the political economy setting to ask about the conditions under which a tax reform considered desirable on the basis of some normative principle is consistent with equilibrium, and we have indicated how such an analysis could be conducted in a CGE framework. We have shown how the "political feasibility" of a desired reform may be calculated in terms of requisite changes in the relative influence of different interests required to induce the reform as an equilibrium. The choice of a metric for the political distance of a reform from the benchmark status quo is, of course, a difficult issue. But it seems reasonable to suppose that proposals for reform made in the absence of any consideration of political feasibility are not likely to survive in a democratic society. We have also suggested that it would be interesting to compare GE outcomes when policy is chosen according to a social welfare function or other traditional tax principles with policy outcomes consistent with normative priciples that are more directly grounded in democratic political theory. Such comparisons can be operationalized in the CGE framework presented here and may provide fresh insight to those who wish to blend social welfare concerns with concerns about the quality of the democratic process.

There are several directions in which our framework may be developed further. Clearly it would be desirable to endogenize policy in a more realistic model such as GEMTAP (Ballard et al (1985)) using a more sophisticated specification of the structure of interest groups and calibrating political weights for successive benchmark datasets. We have employed a particular representation of political equilibrium and it would be of interest to investigate other representations in the multi-dimensional issue space, heterogeneous voter context. Situations in which idle or unused instruments in the benchmark become employed in a counterfactual would represent a first step toward endogenizing the instrument set. The framework presented here might also be used to empirically investigate the economic consequences of alternative constitutions, an aspect of constitutional design which is often neglected. Finally, our initial investigations of intertemporal issues suggests that extension to a dynamic, overlapping generations setting could be quite intriguing. These extensions may require development of new computational methods, as the present model is highly nonlinear, non-differentiable and non-convex. It remains to be seen if GAMS/MINOS is capable of processing larger models or if other methods must be developed. Despite computational challenges however, we see no serious barriers to further equilibrium policy analysis using probabilitic voting models within a CGE framework.

## REFERENCES


Atkinson, A.B. and Nicholas H. Stern (1974), "Pigou, Taxation and Public Goods", *Review of Economic Studies*, 41 (125), January, 119-128.

Austen-Smith, David (1987), "Interest Groups, Campaign Contributions and Probabilistic Voting", *Public Choice*, 54 (2), 123-139.

Ballard, Charles L., Don Fullerton, John Shoven and John Whalley (1985), *A General Equilibrium Model for Tax Policy Evaluation*, University of Chicago Press.

Barooah, Vani and Frans Van der Ploeg (1983), *Political Aspects of the Economy*, Cambridge University Press.

Brennen, Geoffrey and James M. Buchanan (1980), *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, Cambridge University Press.

Brooke, Anthony, David Kendrick and Alexander Meeraus (1988), *GAMS: A User's Guide*, Scientific Press.

Coughlin, Peter, Dennis Mueller and Peter Murrell (1990a), "A Model of Electoral Competition with Interest Groups", *Economics Letters*, forthcoming.

____________________ (1990b), "Electoral Politics, Interest Groups, and the Size of Government", *Economic Inquiry*, forthcoming.

Dahl, Robert A. (1989), *Democracy and Its Critics*, Yale University Press.

Downs, Anthony (1957), *An Economic Theory of Democracy*, Harper & Row.

Enelow, James and Melvin Hinich (1984), *The Spatial Theory of Voting*, Cambridge University Press.

____________________ (1989), "A General Probabilistic Spatial Theory of Elections", *Public Choice*, 61 (2), May, 101-114.

Fair, Ray (1978), "The Effect of Economic Events on Votes for President", *Review of Economics and Statistics*, 60, May, 159-173.

Feldman, Allan and K. Lee (1988), "Existence of Electoral Equilibria with Probabilistic Voting", *Journal of Public Economics*, 35, 205-227.

Hansen, Terje and Tjalling Koopmans, (1972), "On the Definition and Computation of a Capital Stock Invariant under Optimization", *Journal of Economic Theory*, 5, 487-523.

Harrison, Glenn and Lisa Rutstom (1990), "Trade Wars and Trade Negotiations: A Computational Approach", *The Economic Journal*, forthcoming.

Heady, Christopher and Pradeep Mitra (1986), "The Quantitative Analysis of Optimal Taxation and Public Production", in D.W. Pearce and N.J. Rau (eds.), *Economic Perspectives*, 45-90.

Hettich, Walter and Stanley L. Winer (1990), "Basic Issues in the Positive Political Economy of

Income Taxation" in R. Bird and S. Cnossen (eds), *The Personal Income Tax: Phoenix from the Ashes?*, North-Holland, forthcoming.

______________________ (1988), "Economic and Political Foundations of Tax Structure", *American Economic Review*, 78, September, 701-712.

Ingberman, Daniel and Robert Inman (1988), "The Political Economy of Fiscal Policy" in Paul G. Hare (ed.) *Surveys in Public Sector Economics*, Basil Blackwell, 105-160.

Kolm, Serge-Christophe (1987), "Public Economics", in *The New Palgrave*, 1047-1053.

Ledyard, J.O. (1984), "The Pure Theory of Large Two-Candidate Elections," *Public Choice*, 44(1), 7-41.

Lindbeck, Assar and Jorgen W. Weibull (1987), "Balanced Budget Redistribution as the Outcome of Political Competition", *Public Choice*, 52 (3), 273-297.

Markusen, James and Randy Wigle (1989), "Nash Equilibrium Tariffs for the United States and Canada: The Roles of Country Size, Scale Economies and Capital Mobility", *Journal of Political Economy*, 97 (2), 368-386.

Mueller, Dennis (1989), *Public Choice II*, Cambridge University Press.

Piggot, John and John Whalley (1987), "Interpreting Net Fiscal Incidence Calculations", *Review of Economics and Statistics*, 59 (4).

Renaud, Paul and Frans Van Winden (1987), "Tax Rate and Government Expenditure", *Kyklos*, 40, 349-367.

Rutherford, Thomas (1987), *Applied General Equilibrium Modelling*, PhD Thesis, Stanford.

______________________ (1988), *General Equilibrium Modelling with MPS/GE*, Department of Economics, University of Western Ontario.

Shepsle, K.A. (1979), "Institutional Arrangements and Equilibrium in Multidimensional Voting Models", *American Journal of Political Science*, February, 27-59.

Shoven, John B. and John Whalley (1984), "Applied General-Equilibrium Models of Taxation and International Trade: An Introduction and Survey", *Journal of Economic Literature*, 22, September, 1007-1051.

Van Velthoven, Ben C.J. (1989), *The Endogenization of Government Behaviour in Macroeconomic Models*, Springer-Verlag.

Van Winden, Frans (1983), *On the Interaction Between State and Private Sector*, North Holland.

Weymark, John (1981), "Undominated Directions of Tax Reform", *Journal of Public Economics*, 16(3), 343-370.

Winer, Stanley L. and Walter Hettich (1988), "The Structure of the Sieve: Political Economy in the Explanation of Tax Systems and Tax Reform", *Osgoode Hall Law Journal*, 26 (2), summer, 409-422.

Wittman, Donald (1984), "Multi-Candidate Equilibria", *Public Choice*, 43 (3), 287-291.

Zodrow, George R., "Eliminating State and Local Tax Deductibility: A General Equilibrium Model of Revenue Effects", in Harvey Rosen (ed.), *Fiscal Federalism: Quantitative Studies*, University of Chicago Press, 177-214.

## Appendix A

### Public Goods in General Equilibrium: A Complementarity Formulation

The preferences of consumer *h* are represented by a utility function $u_h(\cdot)$. We assume that this function is a monotone increasing function of private and public good consumption (vectors $x_h$ and g respectively). As a simplification, suppose that $u_h$ is homothetic or quasi-homothetic. We model agents as though they choose levels of private goods consumption taking private goods prices and the quantity of public goods as given:

$$\max_{x} \; u(x, \bar{g}) \qquad \text{(A.1)}$$
$$\text{s.t.} \qquad \pi^T x \le \pi^T \omega$$

where $\bar{g}$ is the level of public provision, $\pi$ is the vector of private goods prices, e is the agent's endowment vector, and the *h* subscript is omitted. For theoretical analysis, **(A.1)** may be a perfectly adequate characterization of the demand function $x(\pi,\bar{g})$. For computation using complementarity methods (i.e., in the presence of inequalities such as **(5a)**, **(5b)** or **(5g)**), it is helpful to instead work with a demand correspondence $x(\pi,v)$ which explicitly incorporates private valuations of the public good v. Such a formulation also brings private valuations of public goods explicitly into the model structure so that their role in determining equilibrium outcomes may be more easily studied. This appendix outlines how we define and utilize demand functions defined as $x(\pi,v)$ instead of $x(\pi,\bar{g})$.

Monotonicity of u implies that **(A.1)** is equivalent to:

$$\max_{x,g} \; u(x, g) \qquad \text{(A.2)}$$
$$\text{s.t.} \qquad \pi^T x \le \pi^T \omega$$
$$g \le \bar{g}$$

The solution to **(A.2)** will include one Lagrange multiplier for the private income constraint, $\lambda > 0$, and a vector of Lagrange multipliers, $\nu = (\nu_k)$, for the public goods constraints. We may interpret $\nu_k$ as the agent's marginal utility of the kth public good. Alternatively, we may interpret $v_k = \nu_k/\lambda$ as the amount of money which the agent would be willing to pay for an additional unit of the kth public good.

If the vector of public goods valuations, v, were given exogenously, **(A.2)** could be reformulated as:

$$\max_{x,g} \; u(x, g) \qquad \text{(A.3)}$$
$$\text{s.t.} \qquad \pi^T x + v^T g \le \pi^T \omega + v^T \bar{g} = Y.$$

This transformation recovers the "standard" consumer's problem for an Arrow-Debreu general equilibrium model, based on a vector of commodity prices $(\pi,v)$ and a vector of commodity endowments $(\omega,\bar{g})$. The agent's "effective gross income", Y, is determined by private goods prices, public goods valuations, and the level of provision of the public good.

Equations (5d)-(5g) exploit the homotheticity of $u(\cdot)$ in order to construct an expenditure function which depends on the prices of private goods and the marginal valuations of public goods. This index is defined by:

$$e(\pi,v) = \min \ \pi^T C + v^T G \qquad \text{(A.4)}$$

$$\text{s.t.} \quad u(C, G) = 1$$

where C and G are the levels of private good and public good consumption required for a unit of utility. An agent's "real income" may then be defined by $r = Y / e(\pi,v)$. Demands for private goods are given by $C(\pi,v)\, r$ and effective demands for public goods are given by $G(\pi,v)\, r$, as in (5a) and (5g) respectively. Private valuations of public goods are adjusted in (5g) in order to equilibrate effective demand and public good supply. There is one such constraint for each agent and public good.

The text discusses the importance of the private valuations, v, which through the above formulation are explicitly introduced into the model structure. Computationally, this formulation seems to improve convergence of the Newton solution algorithm. We have no formal demonstration of this effect, but our computational experience leads to this conclusion. We conjecture that because $u(\cdot)$ is a convex function of C and G, the Jacobian of the demand equations is "more" positive definite when the v's appear explicitly in the model.

APPENDIX B

## A. GAMS CODE

```
$TITLE: BENCHMARK CODE FOR GE MODEL WITH ENDOGENOUS GOVERNMENT POLICY
*
*       This file benchmarks a general equilibrium tax model
*       to uncover the range of political weights consistent
*       with the benchmark tax rates and provisions of public goods.
*
*       The model is standard applied general equilibrium with
*       taxes, implementated in Robinson-Devarajan style, with
*       equilibrium conditions represented by a square system
*       of nonlinear equations.  Solution by MINOS using projected
*       Lagrangian method is NOT guaranteed.  A sparse representation
*       of the nonlinear constraints, in practice, seems to improve
*       robustness.
*
*       -------------------------------------------------------------
*       Define sets and subsets:
*
*       C includes all private goods, public goods and factors.
*
*       G(C) is the set of private goods, PG(G) is the set of public
*       goods and F(C) is the set of primary factors.  The elements
*       of these subsets must be ordered consistently with C.
*       (i.e. the elements must appear in the same sequence as they
*       appear in C).
*
*       FM(C,F), GM(C,G) and PM(C,PG) are "logical" indicator sets.
*       FM() sets up a correspondence between elements of C and F(C),
*       GM() connects C and G(C), and PM() connects C and PG(C).  For
*       example, FM(C,F) = YES if commodity C and factor F reference
*       the same item.
*
SETS    C       COMMODITIES    /X, L, K, G/,
        G(C)    PRIVATE GOODS  /X/,
        PG(C)   PUBLIC GOODS   /G/,
        F(C)    FACTORS        /L, K/,
        N       NESTS          /N1, N2/,
        I       INTEREST GROUPS /RICH, MIDDLE, POOR/,
        FM(C,F)  FACTOR MARKET INDICATOR
        GM(C,G)  GOODS MARKET INDICATOR
        PM(C,PG) PUBLIC GOODS MARKETS;
*
*       ALIAS sets up alternative indices for these sets.
*
ALIAS (I,II), (C,CC), (F,FF);
*
*       The following, somewhat cryptic GAMS code, sets up the
*       indicator sets.
*
        FM(C,F)$F(C)  =YES$(ORD(F) EQ SUM(CC$(ORD(CC) LE ORD(C)),1$F(CC)));
        GM(C,G)$G(C)  =YES$(ORD(G) EQ SUM(CC$(ORD(CC) LE ORD(C)),1$G(CC)));
        PM(C,PG)$PG(C)=YES$(ORD(PG) EQ SUM(CC$(ORD(CC) LE ORD(C)), 1$PG(CC)));
*
*       Specify some tolerances used to improve robustness of solution.
*
SCALAR  ZTOLO  LOWER BOUND TO PREVENT EXPONENTS OF ZERO /1.E-12/
        PMIN   LOWER BOUND ON PRICES /1.E-2/
        PMAX   UPPER BOUND ON PRICES /2.E+2/
        QMAX   UPPER BOUND ON ACTIVITY LEVELS (RELATIVE) /5.0/
        ONE    UNITY /1.E0/;
```

```
*       ------------------------------------------------------------
*       Benchmark data begins here:
*
PARAMETERS

        TBAR(F)    BENCHMARK TAX RATE       /L 0.1, K 0.2/,
        ESUBP(G)   PRODUCTION ELASTICITIES /X 1.5/,
        ESUBT(I)   TOP LEVEL ELASTICITIES  /POOR 1.1, MIDDLE 1.1, RICH 1.1/;
*
*       NEST() is an indicator set which specifies how
*       demands are aggregated in preferences.  Every good
*       which is demanded must enter an aggregate.  (Note: To have
*       one or more goods enter in the top level, place them
*       in a nest with the same elasticity of substitution as at
*       the top level.)
*
SET     NEST(C,I,N)   NESTING OF COMMODITIES IN UTILITY FUNCTIONS
       /  X.POOR.N1,     G.POOR.N1,    L.POOR.N2,
          X.MIDDLE.N1,   G.MIDDLE.N1,  L.MIDDLE.N2,
          X.RICH.N1,     G.RICH.N1,    L.RICH.N2/;

TABLE  ESUB(I,N)  DEMAND ELASTICITIES (THOSE DIFFERENT FROM ZERO).
                N1     N2
        POOR    0.3    1.1
        MIDDLE  0.7    1.1
        RICH    1.1    1.1

*
*       GD() specifies commodity inputs to public goods.
*       We assume Leontief (fixed-coefficient) structure.
*
TABLE GD(G,PG)  PUBLIC GOOD FORMATION ACTIVITIES
                G
        X       3.6

TABLE ABAR(F,G)  BENCHMARK FACTOR INPUTS TO PRIVATE PRODUCTION.
                X
        L       16.
        K       10.

*
*       Factor supplies to production represents gross endowments
*       less final demand.  (In this data set, net labor supply
*       equals 10, 4 and 2 for POOR, MIDDLE and RICH interest groups).
*
TABLE E(F,I)  BENCHMARK FACTOR ENDOWMENTS
                POOR     MIDDLE   RICH
        L       12.      4.5      2.5
        K       0        4        6

*
*       Demands for public goods in dbar represent the agent's
*       marginal valuation of the public provision.  The
*       Samuelson condition is satisfied exactly if the sum of
*       these marginals equals the total cost of provision
*       (cf. column sums of table GD(G,PG))
*
TABLE DBAR(C,I)  BENCHMARK FINAL DEMANDS
                POOR     MIDDLE   RICH
        X       10.0     8.0      8.0
        L       2.0      0.5      0.5
        G       3.5      1.5      2.0
*
*       End of benchmark data.
```

```
*       ------------------------------------------------------------
*       Check benchmark data:
*
*       At present, we only verify Walras' law for market demand.
*       We will in a subsequent version incorporate a number of
*       other checks, such as Walras' law by household,
*       goods supply = goods demand, factor supply = factor demand,
*       and government expenditure = tax revenue.)
*
SCALAR BMERR   BENCHMARK INCONSISTENCY CHECK;
BMERR = SUM(I, (SUM(C$(NOT PG(C)), DBAR(C,I)) - SUM(F, E(F,I)))**2);
DISPLAY BMERR;
ABORT$(BMERR GT ZTOLO) " WALRAS LAW VIOLATED.";

*       ------------------------------------------------------------
*       Set up two more logical sets which are useful in specification
*       of equilibrium conditions:
*
SET     DC(C,I)         INDICATOR OF DEMANDED COMMODITIES
        EXIST(N,I)      INDICATOR OF EXISTING NESTS;

        DC(C,I)    = YES$(DBAR(C,I) GT ZTOLO);
        EXIST(N,I) = YES$(SUM(C, NEST(C,I,N)));

*       ------------------------------------------------------------
*       Declare and specify function parameters derived from the
*       benchmark data:
*
PARAMETERS

        XBAR(G)         BENCHMARK OUTPUT
        ALPHA(F,G)      CES COEFFICIENTS IN PRODUCTION.
        SBAR(PG)        BENCHMARK PUBLIC GOODS PROVISION
        UBAR(I)         TOTAL VALUE OF CONSUMPTION.
        UNBAR(N,I)      UTILITY AGGREGATES BY NEST.
        VBAR(PG,I)      VALUATION OF PUBLIC GOOD PG BY INTEREST GROUP I.
        CBAR(C,I)       REFERENCE SECOND LEVEL UNIT INPUTS
        GAMMA(N,I)      CES COEFFICIENTS IN PREFERENCES (TOP LEVEL)
        BETA(C,I)       CES COEFFICIENTS IN PREFERENCES (SECOND LEVEL)
        THETA(I)        WEIGHT OF HOUSEHOLD H IN GOVERNMENT OBJECTIVE;
*
*       Calibrate private production parameters.
*
        XBAR(G) = SUM(F, ABAR(F,G) * (ONE + TBAR(F)));
        ABAR(F,G)  = ABAR(F,G) / XBAR(G);
        ALPHA(F,G) = (ONE + TBAR(F)) * ABAR(F,G)**(ONE/ESUBP(G));
*
*       Calibrate public production parameters.
*
        SBAR(PG) = SUM(G, GD(G,PG));
        GD(G,PG) = GD(G,PG) / SBAR(PG);
*
*       Calibrate utility parameters.
*
        UBAR(I) = SUM(C, DBAR(C,I));
        UNBAR(N,I) = SUM(C$NEST(C,I,N), DBAR(C,I));
*
*       Redefine quantities of public goods as true quantities
*       and install private valuation as vbar(pg,i).
*
        VBAR(PG,I) = SUM(C$PM(C,PG), DBAR(C,I)/SBAR(PG));
        DBAR(C,I)$PG(C) = SUM(PG$PM(C,PG), SBAR(PG));

        GAMMA(N,I) = UNBAR(N,I)**(ONE/ESUBT(I));
```

```
        CBAR(C,I) = SUM(N$NEST(C,I,N), DBAR(C,I)/UNBAR(N,I));
        BETA(C,I)$PG(C) = SUM(PG$PM(C,PG), SUM(N$NEST(C,I,N),
                VBAR(PG,I) * CBAR(C,I)**(ONE/ESUB(I,N)) ));
        BETA(C,I)$(NOT PG(C)) =
                SUM(N$NEST(C,I,N), CBAR(C,I)**(ONE/ESUB(I,N)) );
*
*       For debugging purposes, a print-out may be needed.
*
*       DISPLAY DBAR, UBAR, UNBAR, VBAR, CBAR, ESUBT, ESUB, GAMMA, BETA;
*
*       At this point, all parameters are calibrated.
*
*       -------------------------------------------------------------
*       Specify model fomulation:
*
*       We define the model as a square nonlinear system of
*       equations, including both primal and dual variables.
*
*       Variables and their corresponding equations are connected
*       by numbers.  If the equilibrium conditions were to
*       accomodate "boundaries"  (i.e. weak inequalities and
*       complementary  slackness), this would be the correspondence
*       between variables and constraints.
*
POSITIVE VARIABLES
        X(G)     OUTPUT LEVELS, PRIVATE GOODS (1.G).
        A(F,G)   UNIT FACTOR INPUTS (2.F.G).
        PU(I)    PRICE OF UTILITY LEVEL (3.I)
        PN(N,I)  PRICE OF AGGREGATE (4.N.I)
        UN(N,I)  UNIT INPUTS OF AGGREGATES (5.N.I)
        SC(C,I)  SECOND LEVEL INPUT COEFFICIENTS (6.C.I)
        D(C,I)   CONSUMPTION DEMAND (7.C.I)
        P(G)     PRICE OF PRIVATE GOODS (8.G)
        W(F)     FACTOR PRICE (9.F)
        V(PG,I)  PRIVATE VALUATIONS OF PUBLIC GOODS (10.PG.I)
        WT(F)    TAX-INCLUSIVE FACTOR PRICE (11.F)
        U(I)     UTILITY INDEX (12.I)
        S(PG)    PUBLIC GOODS SUPPLY (13)
        T(F)     TAX RATES BY FACTOR;

VARIABLES
        SUPPORT          POLITICAL SUPPORT FUNCTION;

EQUATIONS
        COSTX(G)         COBB-DOUGLAS PRICE EQUATION FOR X (1.G)
        UNITX(F,G)       FACTOR DEMAND EQUATIONS (2.F.G)
        UCOST(I)         UTILITY "COST" FUNCTION (3.I)
        NCOST(N,I)       AGGREGATE COST FUNCTIONS (4.N.I)
        UNITN(N,I)       DEMAND FUNCTIONS FOR AGGREGATES (5.N.I)
        UNITC(C,I)       DEMAND FUNCTIONS FOR UNIT INPUTS (6.C.I)
        UNITD(C,I)       COMPOSITE DEMAND (7.C.I)
        GMKT(G)          GOODS MARKET CLEARANCE (8.G)
        FMKT(F)          FACTOR MARKET CLEARANCE (9.F)
        PMKT(PG,I)       PRIVATE VALUATIONS OF PUBLIC GOODS (10.PG.I)
        WTDEF(F)         TAX INCLUSIVE FACTOR PRICE DEFINITION (11.F)
        INCOME(I)        INCOME EQUATION (12.I)
        INCOMEG          INCOME EQUATION FOR GOVERNMENT (13)
        OBJDEF           OBJECTIVE FUNCTION;
```

```
*       Cost equations for production.
*
WTDEF(F)..
        WT(F) =E= W(F) * (ONE + T(F));

COSTX(G)..
        P(G)**(ONE-ESUBP(G)) =E=
        SUM(F, ALPHA(F,G)**ESUBP(G) * WT(F)**(ONE-ESUBP(G)));
*
*       Factor demands.
*
UNITX(F,G)..
        WT(F) * A(F,G)**(ONE/ESUBP(G)) =E= ALPHA(F,G) * P(G);

*
*       Final demand functions (nested CES)
*
*       PU(I) is the expenditure function for a unit of utility.
*       (N.B. These nested ces functions are homothetic)
*
UCOST(I)..
        UBAR(I) * PU(I)**(ONE-ESUBT(I)) =E=
        SUM(N$EXIST(N,I), GAMMA(N,I)**ESUBT(I) * PN(N,I)**(ONE-ESUBT(I)));

*
*       PN(N,I) is the expenditure function for a unit of aggregate N.
*
NCOST(N,I)$EXIST(N,I)..

        PN(N,I)**(ONE-ESUB(I,N)) =E=
        SUM(C$NEST(C,I,N), BETA(C,I)**ESUB(I,N) *
              ( SUM(G$GM(C,G),   P(G)**(ONE-ESUB(I,N))) +
                SUM(F$FM(C,F),   W(F)**(ONE-ESUB(I,N))) +
                SUM(PG$PM(C,PG), V(PG,I)**(ONE-ESUB(I,N))) ));
*
*       Demand functions for aggregates.
*
UNITN(N,I)$EXIST(N,I)..

        PN(N,I) * UN(N,I)**(ONE/ESUBT(I)) =E= GAMMA(N,I) * PU(I);

*
*       Demand functions for goods per unit of associated aggregate.
*
UNITC(C,I)$DC(C,I)..

        BETA(C,I) * SUM(N$NEST(C,I,N), PN(N,I)) =E=
                    SUM(N$NEST(C,I,N), SC(C,I)**(ONE/ESUB(I,N))) *
                    ( SUM(G$GM(C,G),   P(G)) +
                      SUM(F$FM(C,F),   W(F)) +
                      SUM(PG$PM(C,PG), V(PG,I)) );

*
*       Gross demands determined by combining utility level,
*       demands by aggregate and demand for goods per unit aggregate.
*
UNITD(C,I)$DC(C,I)..

        D(C,I) =E= U(I) * SC(C,I) * SUM(N$NEST(C,I,N), UN(N,I));
```

```
*       Market clearance conditions.
*
*       Private markets for the public good.
*
PMKT(PG,I)..

        SUM(C$PM(C,PG), D(C,I)) =E= S(PG);
*
*       Private goods (omit market clearance for the numeraire
*       commodity, as it is automatically satisfied through Walras law).
*
GMKT(G)$(ORD(G) NE 1)..

        X(G) =E= SUM(C$GM(C,G), SUM(I, D(C,I))) +
                 SUM(PG, S(PG)**2 * GD(G,PG) / SBAR(PG));
*
*       Factor markets.
*
FMKT(F)..

        SUM(G, X(G) * A(F,G)) =E= SUM(I, E(F,I) - SUM(C$FM(C,F), D(C,I)));
*
*       Income equation for government. (To reduce nonlinearity,
*       we specify tax revenue as the difference between the
*       value of private goods output and the cost of factor
*       inputs.)
*
INCOMEG..

        SUM(G, X(G) * P(G)) -

        SUM((I,F), W(F) * (E(F,I) - SUM(C$FM(C,F), D(C,I))))

        =E= SUM((PG,G), P(G) * GD(G,PG) * S(PG)**2 / SBAR(PG));
*
*       Income equation for interest group I.
*
INCOME(I)..

        SUM(F, E(F,I) * W(F)) + SUM(PG, V(PG,I) * S(PG))
        =E= PU(I) * U(I) * UBAR(I);

*
*       Taxes and public goods provision are chosen to
*       maximize expected votes (political support).
*
OBJDEF..

        SUPPORT =E= SUM(I, THETA(I) * U(I));

MODEL TAXEQL/ALL/;
*
*       -----------------------------------------------------------
*       Initialize using benchmark values.
*
        X.L(G)    = XBAR(G);
        A.L(F,G)  = ABAR(F,G);
        PU.L(I)   = ONE;
        PN.L(N,I) = ONE;
        UN.L(N,I) = UNBAR(N,I);
        SC.L(C,I) = CBAR(C,I);
        D.L(C,I)  = DBAR(C,I);
        P.L(G)    = ONE;
        W.L(F)    = ONE;
```

```
        V.L(PG,I) = VBAR(PG,I);
        WT.L(F)   = ONE + TBAR(F);
        U.L(I)    = ONE;
        S.L(PG)   = SBAR(PG);
        T.L(F)    = TBAR(F);
*
*       -----------------------------------------------------------
*       Set some lower bounds to prevent divide by zero etc.
*
        P.LO(G)  = PMIN;
        P.UP(G)  = PMAX;
        W.LO(F)  = PMIN;
        W.UP(F)  = PMAX;
        WT.LO(F)  = PMIN;
        WT.UP(F)  = PMAX;
        A.LO(F,G)   = ZTOLO;
        PU.LO(I) = PMIN;
        PU.UP(I) = PMAX;
        PN.LO(N,I) = PMIN;
        PN.UP(N,I) = PMAX;
        V.LO(PG,I) = PMIN;
        V.UP(PG,I) = PMAX;
        UN.LO(N,I) = ZTOLO;
        SC.LO(C,I) = ZTOLO;

        S.UP(PG)   = QMAX * SBAR(PG);
        U.UP(I)    = QMAX;
        UN.UP(N,I) = QMAX * UNBAR(N,I);
        SC.UP(C,I) = QMAX * CBAR(C,I);
        D.UP(C,I)  = QMAX * DBAR(C,I);
*
*       -----------------------------------------------------------
*       Fix the price of the numeraire commodity.
*
        P.FX(G)$(ORD(G) EQ 1) = 1.0;
*
*       -----------------------------------------------------------
*       Fix values for aggregates and demands which are not used.
*
        UN.FX(N,I)$(NOT EXIST(N,I)) = 0;
        D.FX(C,I)$(NOT DC(C,I)) = 0;
        SC.FX(C,I)$(NOT DC(C,I)) = 0;
*
*       -----------------------------------------------------------
*       Perform evaluations to determine gradients of U
*       with respect to tax rates and public provisions.
*
        T.FX(F) = TBAR(F);
        S.FX(PG)$(ORD(PG) GT 1) = SBAR(PG);
        THETA(I) = 0;

PARAMETER MU(I,*)       PARTIALS OF UTILITY OF GOVT. INSTRUMENTS;

LOOP(II,
        THETA(II) = 1.0;
        SOLVE TAXEQL USING NLP MAXIMIZING SUPPORT;
        THETA(II) = 0.0;
        MU(II,PG)$(ORD(PG) GT 1) = S.M(PG);
        MU(II,F) = T.M(F) );
```

```
*       --------------------------------------------------------
*       Solve the Pareto problem to infer the political weights.
*
        THETA(I) = 1;
        U.LO(I)  = 1;
        T.LO(F)  = 0;
        T.UP(F)  = INF;
        T.L(F) = TBAR(F);
        SOLVE TAXEQL USING NLP MAXIMIZING SUPPORT;

        THETA(I) = (1.0 - U.M(I))/SUM(II, 1.0-U.M(II));

        OPTION DECIMALS=8;
        DISPLAY MU;
        DISPLAY THETA;
*
*       Terminate with warning if Pareto criterion is violated.
*
        ABORT$(ABS(SUPPORT.L-CARD(I)) GT ZTOLO)
                " WARNING: BENCHMARK NOT PARETO EFFICIENT";
```

## B. MPS/GE BENCHMARK DATASET

```
*       Benchmark Code for a Simple GE Model with Endogenous Government Policy
*
*       In this model, a government levies taxes on factor inputs
*       to production and uses the revenue to produce a single
*       pure public good.  The good enters into both agent's utility.
*
*       In the benchmark we assume that factor taxes are the only
*       available means of raising government revenue.  In equilibrium,
*       there will be a trade-off between inefficiencies associated with
*       tax distortions and the welfare benefits of public goods supply.
*
*       Primary factors and output good are the usual, K, L and
*       X, respectively.  U-P, U-M, and U-R are commodities with prices
*       corresponding to the marginal utilities of the POOR, MIDDLE
*       and RICH consumers.
*
*       GG      Government expenditure.
*       G-P     Public good, household 1.
*       G-M     Public good, household 2.
*       G-R     Public good, household 3.
*
$COMMODITIES:
  X  K  L KG U-P  U-M  U-R  GG  G-P 3.5  G-M 2.0  G-R 1.5
*
*       S-X is the private production sector.
*       W-i is the welfare index for interest group i.
*
*       S-G is the sector through which public goods are produced.
*
$SECTORS:
  S-X  S-G  W-P  W-M  W-R
*
*       IRS is the tax agent who collects revenue and whose income
*  .            determines the level of public good provision.
*
$CONSUMERS:
  POOR  MIDDLE  RICH  IRS
*       G is an auxiliary variable through which we ration supplies
*       of public goods.
*
$AUXILIARY:
  G  1.0

*       Good X is produced through CES technology
*       with distortionary taxes on factor inputs, with tax revenues
*       paid to the government tax collector.
*
$PROD:S-X  s:1.5
  O:X  X:29.6
  I:K  X:10.0  P:1.2  A:IRS  T:0.2
  I:L  X:16.0  P:1.1  A:IRS  T:0.1
*
*       Public goods are produced through a Leontief activity.  The
*       level of supply is determined by the income of consumer IRS,
*       which is simply the tax revenue.
*
$PROD:S-G   s:1.0
  O:GG  X:7.2
  I:X   X:3.6
  I:KG  X:3.6
```

```
*       Welfare functions: homothetic, nested CES.
*
$PROD:W-P  s:1.1  a:0.3
  O:U-P  X:15.5
  I:X    X:10.0  a:
  I:G-P  X: 1.0  P:3.5  a:
  I:L    X: 2.0

$PROD:W-M  s:1.1  a:0.7
  O:U-M  X:10.5
  I:X    X:8.0  a:
  I:G-M  X:1.0  P:2.0  a:
  I:L    X:0.5

$PROD:W-R  s:1.1
  O:U-R  X:10.0
  I:X    X:8.0
  I:L    X:0.5
  I:G-R  X:1.0 P:1.5
*
*       Consumers are endowed with primary factors and (rationed)
*       private supplies of the public good.  These commodities are
*       introduced in order to incorporate the supply of G into each
*       of the consumer's subproblems.  The shadow price on these
*       private supplies reflects the agent's marginal valuation of
*       the public good.  This valuation is unobserved in the benchmark
*       equilibrium and must be calibrated in accordance with the
*       political weights and assumed optimizing behaviour of the
*       government.
*
$DEMAND:POOR
  E:G-P  R:G
  E:L    X:12.0
  D:U-P

$DEMAND:MIDDLE
  E:G-M R:G
  E:K   X:4.0
  E:L   X:4.5
  D:U-M
$DEMAND:RICH
  E:G-R R:G
  E:L   X:2.5
  E:K   X:6.0
  D:U-R
*       IRS IS AN ARTIFICIAL AGENT INTRODUCED TO ENFORCE
*       A BALANCED BUDGET FOR THE GOVERNMENT.
*
$DEMAND:IRS
  E:KG   X:3.6
  D:GG
*
*       FIX THE AUXILIARY VARIABLE G EQUAL TO THE ACTIVITY LEVEL S-G
*
$CONSTRAINT:G
  Z:G
  K:-1.0  Z:S-G
*
*       REPLICATE THE BENCHMARK.
*
$CASEID:   0

$SOLVE:
```