1994

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ISSN:0318-725X
ISBN:0-7714-1669-5

RESEARCH REPORT 9412

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May 1993

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Abstract

We study the core of an insurance economy based on the Rothschild and Stiglitz (1976) model. The core is characterized by the set of sustainable insurance contracts. Feasible insurance contracts satisfy resource and incentive feasibility constraints given the distribution of types in the economy. Alternative insurance markets must specify, for each distribution of types, a set of contracts that satisfy resource and incentive feasibility constraints. A set of contracts is sustainable if for some distribution of types there is no resource and incentive feasible set of contracts that is preferred by any agent. The standard set of separating contracts are shown to be sustainable, and pooling contracts are shown to be not sustainable.

1. Introduction

An economy with incomplete information is one in which individuals have private information at the time they engage in trade or other economic activities with other individuals. An individual's "type" is a description of his private information. A simple example of a market with incomplete information is the model of market for insurance studied in Rothschild and Stiglitz 1976 where there are two types of individuals, facing high and low risks of an accident, and insurers who are unable to observe individuals' types. The purpose of this paper is to illustrate, by means of the Rothschild and Stiglitz model, the incomplete information cores studied in Myerson 1988 and Green 1993.

Incomplete information cores extend to markets with incomplete information the definitions of the core that are well developed for markets with perfect information. The simplest type of core is that of an exchange economy. Such a core is defined using a mathematical model that characterizes for each coalition of individuals the payoffs that are resource feasible for the coalition. It is assumed that coalitions use only their own resources. A coalition of individuals can improve upon an allocation if its members have a resource feasible allocation that each of its members prefers. Thus, the core is a weak solution concept; if an allocation is in the core then we know that there is no allocation that is resource feasible for, and preferred by, any subset of individuals.

There are two common interpretations of the core, the first of which is that the core is the equilibrium of a coalitional bargaining process whereby proposals and counter proposals are put forward for resource feasible allocations for the individuals in the economy. It seems reasonable to assume that a coalition would not agree to an allocation that they could improve upon and, therefore, the only reasonable allocations are those in the core.

A second interpretation of the core is based on a literal reading of the mathematical model; the core is interpreted as the set of allocations relative to which no coalition has a resource feasible allocation that its members prefer. With this interpretation, the model does not describe a coalitional bargaining process or any other set of economic institutions that facilitate trade in the economy; the core is defined only with respect to the parameters of the economy that represent preferences and determine what trades and other economic activities are resource feasible. The economic institutions that facilitate trade may be a coalitional bargaining process or *any* other set of institutions. The role of the core is to evaluate the allocations that arise through the use of economic institutions described in other models. If a set of economic institutions yields an equilibrium outcome that is in the core then we know that, no matter how rich the set of institutions available, no subset of individuals can improve upon the equilibrium because no preferred allocation is resource feasible.

Defining the core for markets with incomplete information requires developing a mathematical model that describes for each coalition of individuals the payoffs that are both resource and incentive feasible. The difficulty is that the set of trades that are resource and incentive feasible depends on the distribution of types in the coalition, and individuals' types are unobservable. For example, in the Rothschild and Stiglitz model the set of resource feasible insurance contracts, that is the set of insurance contracts that allow insurers to earn nonnegative profits, depends on the relative proportions of high and low risk types in a coalition. The set of resource and incentive feasible contracts for the economy as a whole may be defined using the fact that the distribution of types in the economy is assumed to be common knowledge. However, a small subset of individuals seeking to improve upon the payoffs they would receive from contracts traded in the market may contain any distribution of types.

When we define the core for a market with perfect information, we usually say that a coalition can improve upon an allocation if it has a resource feasible allocation that its members prefer. That is, members of deviating coalitions compare the payoffs they obtain in the coalition to the payoffs in some status quo allocation for the coalition consisting of all individuals (See Myerson 1991). In their definitions of incomplete information cores based on the Rothschild and Stiglitz model, Boyd, Prescott and Smith (1988) and Marimon (1988) address the problem of identifying the distribution of types in a deviating coalition by having each type of individual compare the payoffs they obtain in the coalition to the payoffs they would obtain in the *complementary* coalition, the coalition made up of all individuals less those in the deviating coalition. They assume that when a coalition deviates from a market, the coalition is large enough that it may change the set of contracts that are resource and incentive feasible in the market. The distribution of types in a deviating coalition is restricted by assuming that participation in a deviating coalition is voluntary and open to all, and imposing sorting constraints that require that no type of individual strictly prefer to be in the complementary coalition if he is in the deviating coalition, and visa versa. Every individual of a type that would gain, and no individual of a type that would lose, in a deviating coalition is present in it. While this approach is appealing, it rules out small coalitions that could improve upon an allocation. The core defined in Marimon 1988 has another feature that is inconsistent with the usual definitions of perfect information cores; it does not require that deviating coalitions use only their own resources. A deviating coalition specifies an allocation for both its own members and the members of the complementary coalition and may use the resources of the complementary coalition as long as each member of the complementary coalition obtains an allocation that he weakly prefers to his endowment.

The core of the insurance economy studied in this paper is based on the idea of sustainability studied in Myerson 1988 and Green 1993. A sustainable set of contracts is interpreted as a core outcome. In this paper I consider pooling and separating contracts. It is assumed that the numbers of individuals in any deviating coalition is small relative to the number of individuals in the economy. This implies that the contracts available in the market is unchanged by the activities of a deviating coalition. Members of a deviating coalition are assumed to be able to return to the market to trade whatever contracts are traded there. Deviating coalitions are also assumed to use only their own resources. A deviating coalition with a given distribution of types can improve upon the contracts traded in the market if there is a set of resource and incentive feasible contracts that is weakly preferred by high and low risk types and insurers, and strongly preferred by at least one type or by insurers. A set of contracts traded in the market is sustainable if, for some distribution of types, it cannot be improved upon by a deviating coalition. This distribution may be interpreted as a belief. If individuals hold such a belief, then there is nothing to be gained by deviation.

In the next section of the paper the Rothschild and Stiglitz model is reviewed. In section 3. the meaning of sustainability in the insurance model is discussed. In section 4. pooling contracts are shown to be not sustainable and the standard set of separating contracts is shown to be sustainable.

2. The Rothschild and Stiglitz model

In the Rothschild–Stiglitz model, each individual has income W when lucky enough to avoid an accident. If an accident occurs, the individual's income is $W - d$. Potential buyers of insurance differ only in their probabilities of having an accident. These probabilities are private information. High risk types will have an accident with probability p^H, and low risk types will have an accident with probability p^L where, of course, $p^H > p^L$. λ is the commonly known proportion of high risk individuals in the population. Without insurance, for each type i in $\{H, L\}$, expected utility is given by

$$(2.1) \qquad \left(1 - p^i\right) U(W) + p^i U(W - d),$$

where $U(\cdot)$ is the utility of money income. Individuals prefer more income to less, and are risk averse: $U' > 0$ and $U'' < 0$. In this paper the utility functions are normalized so that the expected utility of being uninsured is zero. If an individual buys an insurance contract, he pays a premium α_1, and in the event of an accident is paid $\hat{\alpha}_2$. An insurance contract is described by a vector $\alpha = (\alpha_1, \alpha_2)$, where α_2 is the insured individual's net receipt, $\hat{\alpha}_2 - \alpha_1$, in the event of an accident. Insurers are assumed to be risk neutral and face zero costs. When an individual with accident probability p^i buys insurance contract α, expected profit is given by

$$\pi\left(p^i, \alpha\right) = \left(1 - p^i\right)\alpha_1 - p^i\alpha_2. \tag{2.2}$$

3. Sustainable insurance contracts

The contracts traded in the insurance market must be resource and incentive feasible. A set of insurance contracts is resource feasible if insurers earn nonnegative profits selling it. The set of contracts $\{\alpha^H, \alpha^L\}$ is resource feasible for the market if

$$\lambda\left[(1 - p^H)\alpha_1^H - p^H\alpha_2^H\right] + (1 - \lambda)\left[(1 - p^L)\alpha_1^L - p^L\alpha_2^L\right] \geq 0. \tag{3.1}$$

This set of contracts is incentive feasible if both high and low risk types prefer them to being uninsured and neither type prefers the contract intended for the other type. Define $V^i(\alpha^i)$ to be the expected utility that an individual of type i obtains with contract $\alpha^i = (\alpha_1^i, \alpha_2^i)$, for i in $\{H, L\}$. That is,

$$V^i(\alpha^i) = (1 - p^i)U(W - \alpha_1^i) + p^iU(W - d + \alpha_2^i). \tag{3.2}$$

Then, the set of contracts $\{\alpha^H, \alpha^L\}$ is incentive feasible if

$$V^i(\alpha^i) \geq 0, \quad i \in \{H, L\}, \text{ and,} \tag{3.3}$$

$$V^i(\alpha^i) \geq V^i(\alpha^j), \quad i, j \in \{H, L\}. \tag{3.4}$$

A deviating coalition can seek to improve upon the set of contracts $\{\alpha^H, \alpha^L\}$ traded in the market. In this paper it is assumed that, if a deviating coalition is formed, it always contains at least one insurer. The insurer in a deviating coalition has the option of offering either type a null contract, in which case that type just faces the contracts available in the market.

Let λ' be the proportion of high risk types in the deviating coalition. The set of contracts $\{\beta^H, \beta^L\}$ is resource feasible for $(\lambda', 1-\lambda')$ if

$$\lambda' \left[(1-p^H)\beta_1^H - p^H\beta_2^H\right] + (1-\lambda')\left[(1-p^L)\beta_1^L - p^L\beta_2^L\right] \geq 0. \tag{3.5}$$

If, for i in $\{H, L\}$, β^i is a null contract, then $\beta_1^i = \beta_2^i = 0$ in (3.5).

The set of contracts $\{\beta^H, \beta^L\}$ is incentive feasible for $(\lambda', 1-\lambda')$ if both high and low risk types weakly prefer them to the contracts traded in the market, and neither type prefers the contract intended for the other type. That is, the set of contracts $\{\beta^H, \beta^L\}$ is incentive feasible for $(\lambda', 1-\lambda')$ if

$$V^i(\beta^i) \geq V^i(\alpha^i), \quad i \in \{H, L\}, \text{ and,} \tag{3.6}$$

$$V^i(\beta^i) \geq V^i(\beta^j), \quad i, j \in \{H, L\}. \tag{3.7}$$

If, for i in $\{H, L\}$, β^i is a null contract, then $\beta^i = \alpha^i$ in (3.6) and (3.7).

Given the distribution $(\lambda', 1-\lambda')$ of types in a deviating coalition, the set of contracts $\{\alpha^H, \alpha^L\}$ can be **improved upon** by the deviating coalition if there is a set of contracts $\{\beta^H, \beta^L\}$ that is resource and incentive feasible for $(\lambda', 1-\lambda')$ such that

$$\exists i \in \{H, L\}, \text{ such that } V^i(\beta^i) > V^i(\alpha^i), \text{ or,} \tag{3.8}$$

$$\lambda' \left[(1-p^H)\beta_1^H - p^H\beta_2^H\right] + (1-\lambda')\left[(1-p^L)\beta_1^L - p^L\beta_2^L\right] > 0.$$

The insurer specifies, for each possible distribution of types $(\lambda', 1-\lambda')$ such that $0 < \lambda' < 1$, a set of resource and incentive feasible contracts that it will offer. Let $\Delta(\{H, L\})$ be the set of probability distributions on the set $\{H, L\}$. That is, $\Delta(\{H, L\}) = \{(\lambda', 1-\lambda') | 0 \leq \lambda' \leq 1\}$. The insurer is required to offer contracts such that for some upper-hemicontinuous mapping

$$F : \Delta(\{H, L\}) \twoheadrightarrow \Re^2,$$

$(\tilde{V}^H, \tilde{V}^L) \in F(\lambda', 1-\lambda')$ implies that the insurer offers a set of contracts $\{\beta^H, \beta^L\}$ that are resource and incentive feasible for $(\lambda', 1-\lambda')$ and for which

$$V^H(\beta^H) = \tilde{V}^H \quad \text{and} \quad V^L(\beta^L) = \tilde{V}^L.$$

This allows us to compute the payoffs to a type that enters in zero proportion by assuming the type enters in positive proportion and taking limits. The purpose of this is illustrated in the discussion of pooling contracts below.

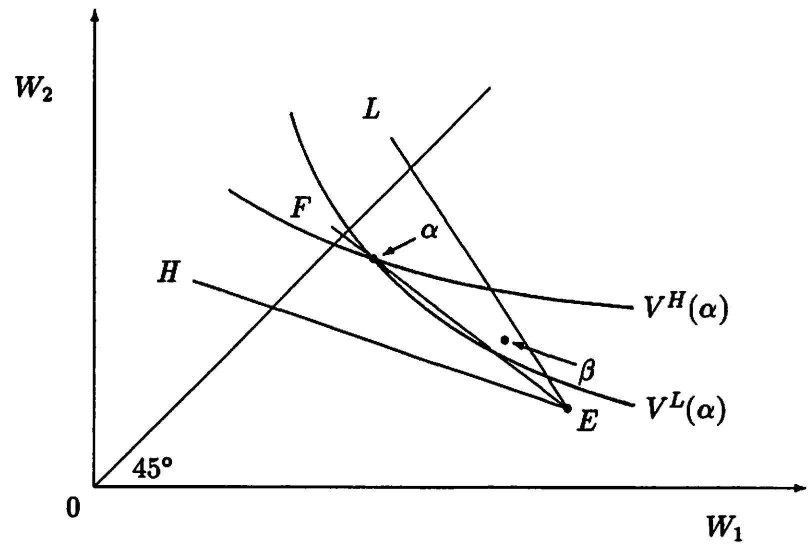
The set of contracts $\{\alpha^H, \alpha^L\}$ is **sustainable** if, for some distribution $(\lambda', 1-\lambda')$, it cannot be improved upon by a deviating coalition.

4. Sustainable insurance contracts

Pooling contracts In figure 4.1, W_1 denotes an individual's income in the event of no accident, and W_2 denotes his income when there is an accident. E denotes each individual's income when uninsured. EL and EH are the *fair odds* lines for low and high risk individuals. EL and EH are defined by the set of contracts that earn zero expected profit when purchased by low and high risk individuals respectively. Given that λ is the proportion of high risk individuals in the population of potential buyers of insurance contracts, the average probability of accident is $\bar{p} = \lambda p^H + (1-\lambda)p^L$. An insurer that sells the pooling contract $\alpha = (\alpha_1, \alpha_2)$ to a representative subset of individuals earns zero profit if

$$(4.1) \qquad (1-\bar{p})\alpha_1 - \bar{p}\alpha_2 = 0 \qquad \text{or, equivalently} \qquad \frac{\alpha_2}{\alpha_1} = \frac{1-\bar{p}}{\bar{p}}.$$

This ratio defines the fair odds line EF; any pooling contract on the fair odds line yields an expected profit of zero, and any pooling contract below it earns a positive profit. Contract α shown in figure 4.1 is incentive compatible because both high and low risk types prefer it to remaining uninsured. To see that contract α is not sustainable, consider an insurer in a deviating coalition that, for every $(\lambda', 1-\lambda')$ in $\Delta(\{H, L\})$ offers the contract β shown in figure 4.1 to low risk types and a null contract to high risk types. For every $(\lambda', 1-\lambda')$ in $\Delta(\{H, L\})$, this set of contracts



A pooling contract
Figure 4.1

is incentive feasible since low risk types prefer β to α, and high risk types prefer α to β, is resource feasible since $(1 - p^L)\beta_1 + p^L\beta_2 > 0$, and improves upon α since $V^L(\beta) > V^L(\alpha)$.

To see why the payoffs to types that are present in a deviating coalition in zero proportion are specified by assuming they are present in positive proportion and then taking limits, notice that for every λ' in $[0, 1)$, the payoff to low risk types in the deviating coalition is $V^L(\beta)$, and the profit to insurers is proportional to

$$(1 - \lambda')\left[(1 - p^L)\beta_1 + p^L\beta_2\right] > 0.$$

If the payoff to types that are present in zero proportion were zero, then we could trivially sustain the pooling contract α by assuming that no low risk types join the deviating coalition.

Separating contracts The standard set of separating contracts are shown in figure 4.2. High risk types obtain full insurance with contract α^H. The contract offered to low risk individuals cannot be more attractive to high risk individuals than the contract α^H. Therefore, α^L is the best contract that can be made available to low risk individuals. Insurers break even because high and low risk types are purchasing contracts on their respective fair odds lines.

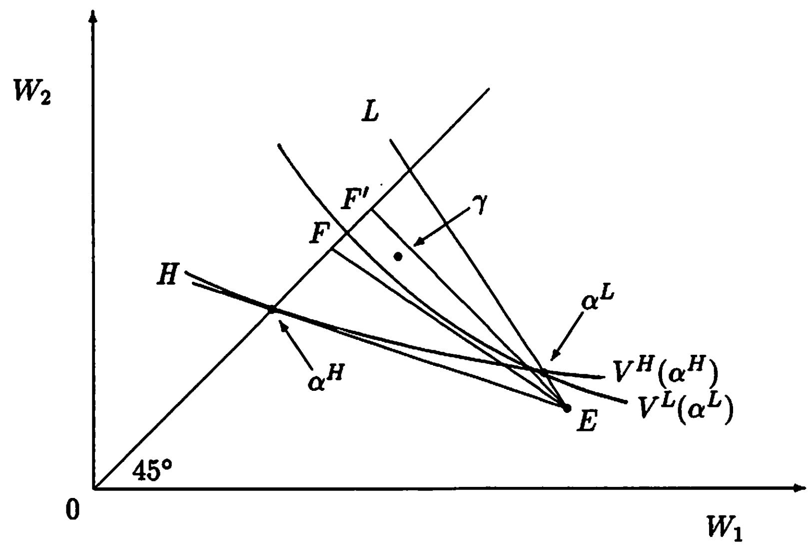
Suppose that a deviating coalition has a small enough proportion of high risk types that the pooled fair odds line in the deviating coalition is given by EF' in figure 4.2. Such a coalition could improve upon the separating contracts $\{\alpha^H, \alpha^L\}$. For example, the pooling contract shown, γ, is resource and incentive feasible, is profitable for insurers, and is preferred by both types.

Now suppose that the deviating coalition contains a large enough proportion of high risk types that the pooled fair odds line in the deviating coalition is given by EF in figure 4.2. Then no pooling contract is resource and incentive feasible. A pooling contract γ' on EF would give low risk types a payoff $V^L(\gamma') < V^L(\alpha^L)$ and therefore fail to be incentive feasible. If an insurer offered such a contract, it would be accepted by only high risk types and would therefore earn negative profits. Excluding null contracts, the only set of contracts that is resource and incentive feasible for a deviating coalition when the proportion of high risk types is too high is one identical to the set of separating contracts $\{\alpha^H, \alpha^L\}$.

The standard set of separating contracts $\{\alpha^H, \alpha^L\}$ is sustained by the belief that λ' is close enough to 1 that no set of contracts that is resource and incentive feasible for $(\lambda', 1-\lambda')$ earns a nonnegative profit and is preferred by either high or low risk types. One may ask why individuals would not hold more optimistic beliefs about the distribution of types that would participate in a deviating coalition if it formed. If beliefs were such that the pooled fair odds line were EF', then both high and low risk types would want to participate in the deviating coalition and any distribution of types at all could choose to participate. Therefore favorable beliefs are not necessarily consistent with the distribution of types that would choose to participate. The only beliefs that we can justify are those that are consistent with the optimal choices that they imply.

The set of separating contracts $\{\alpha^H, \alpha^L\}$ is sustained by beliefs that are consistent with no one choosing to participate in a deviating coalition. This argument is analogous to the argument whereby sequential equilibrium strategies are supported by beliefs off the equilibrium path.

Another way that sustainability may be viewed in this example is that any



Separating contracts
Figure 4.2

set of contracts that is not sustainable may be improved upon no matter what the distribution of types that chooses to participate in a deviating coalition. Sustainable contracts are just those for which this is not true. That is, sustainability only eliminates those contracts that can be improved upon no matter what the distribution of types that participate in an alternative. It may be the case that some sustainable allocations are unreasonable because, for whatever reason, the beliefs that sustain some allocations are unreasonable. From this perspective, sustainability should therefore be viewed as a weak concept. That is, it provides a minimum criterion for establishing what is contained in an incomplete information core.

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