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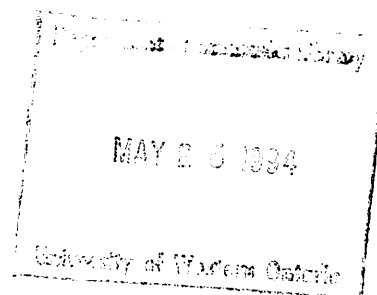
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**Credit Market Imperfections and
the Heterogeneous Response of
Firms to Monetary Shocks**

by

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May 1994



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Credit market imperfections and the heterogeneous response of firms to monetary shocks

Jonas D.M. Fisher*

May, 1994

Abstract

This paper seeks to account for three pieces of empirical evidence on the heterogeneous response of firms to monetary contractions: (i) borrowing by small firms falls relative to large firms, (ii) sales by small firms falls relative to large firms, and (iii) the spread between interest rates paid by bank-dependent versus non-bank-dependent firms rises. This evidence has been interpreted by several authors as supportive of theories of the monetary transmission mechanism that emphasize frictions in credit markets. I describe a quantitative dynamic general equilibrium monetary model in which two particular credit market frictions play key roles: limited participation and costly state verification. After its parameters are identified from U.S. data the model is found to be consistent with these empirical findings as well as other evidence on the response of the economy to monetary shocks.

JEL Classification: E32, E44, E51

Keywords: limited participation, costly state verification, monetary transmission mechanism

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1. Introduction

This paper seeks to account for three pieces of empirical evidence on the heterogeneous response of firms to monetary shocks. First, borrowing by small firms falls relative to large firms following a monetary contraction. Second, following a monetary contraction, sales of small firms fall relative to large firms. Third, the spread between the interest rate on loans paid by bank-dependent firms versus non-bank-dependent firms rises in a monetary contraction. Findings like these have been interpreted by several authors, including Bernanke (1993), Gertler and Gilchrist (1992, 1993) and Kayshap and Stein (1992), as being supportive of theories of the monetary transmission mechanism that emphasize a role for frictions in credit markets. In this paper I describe a quantitative dynamic general equilibrium monetary model in which two particular credit market frictions play key roles: limited participation and costly-state verification. After its parameters are identified from U.S. data, the model is found to be consistent with these empirical findings as well other evidence on the response of the economy to monetary shocks.

One way to interpret the model developed here is in terms of the credit channel theory of the monetary transmission mechanism. According to this theory, monetary policy influences aggregate activity by reducing the real supply of funds available to firms that must rely on banks for external finance. As described by Bernanke (1993), Bernanke and Blinder (1988, 1992), and Kayshap and Stein (1993), the credit channel theory can be decomposed into two distinct elements. First, there must be a mechanism by which the monetary authority is able to influence the real supply of funds available to banks for lending to firms. Second, frictions must exist that force some firms to rely on banks, rather than public debt markets, for external finance. The model I construct incorporates these two elements and as such can be seen as a general equilibrium articulation of the credit channel theory.

In the model, the friction underlying the monetary authority's ability to influence intermediary liquidity is the limited participation assumption associated with Lucas (1990)

and Fuerst (1992). This assumption states that the frequency at which firms and financial intermediaries adjust their financial positions is greater, on average, than for households. For this reason the impact of unanticipated monetary disturbances is likely to be felt first by firms and intermediaries.¹ This may imply that unanticipated monetary contractions reduce the real supply of funds available to banks for lending to firms.² Other frictions, such as sticky wages or prices, may deliver the same result.

Following Townsend (1979), Gale and Hellwig (1985), Gertler and Gilchrist (1992) and many others, the friction underlying bank reliance is costly state verification. Some firms have private information on their operations. Banks possess a monitoring technology that allows them to acquire this information at a cost. Since it is assumed that all firms must raise funds externally to finance their ongoing operations, this friction has implications for how firms operate. It alters the way loans are intermediated between privately informed firms and banks, relative to the way loans are intermediated for firms whose operations are common knowledge. It can have the implication that privately informed firms are more sensitive to changes in the real supply of loans than other firms.

I develop the model in the next two sections of the paper. A one-period model of costly state verification is embedded into an otherwise dynamic general equilibrium monetary model with limited participation. In section four attention is turned to assessing the empirical performance of the model. I accomplish this task in three steps. First, I must assign values to the model's parameters. Using a combination of aggregate time series evidence and micro cross-section evidence, I am able to identify the model's parameters.

One interesting parameter I must identify is the proportion of intermediate goods producing firms in the model that are credit constrained. If the model is required to be consistent with evidence on the nominal liabilities of failed firms to nominal GDP ratio, this fraction is estimated to be only 8.2 per cent. Based on the estimated parameters the model also has

¹Christiano, Eichenbaum and Evans (1993) report evidence from the Flow of Funds accounts that generally supports this assumption.

²The hypothesis that monetary disturbances impact agents asymmetrically has also been analysed by Grossman and Weiss (1982) and Rotemberg (1984).

some interesting steady state properties. For example, credit constrained firms are found to be “rationed,” in terms of the loans they would demand absent incentive compatibility constraints, by 29 per cent of the size of loan they are actually granted.

The second step is to ask whether the model accounts for evidence on how output, employment and nominal interest rates respond to a monetary disturbance. Consistent with results reported in Christiano (1991) and Christiano and Eichenbaum (1992a, 1992b), the model reproduces empirical findings that output and employment fall and interest rates rise following a monetary contraction.³

The third step involves addressing the question of whether the model can account for the evidence on the heterogeneous impact of monetary policy on firms. Gertler and Gilchrist (1992) and Oliner and Rudebusch (1992) have reported that borrowing by small firms falls relative to large firms following monetary contractions. Similar findings for noncorporate versus corporate firms are reported by Christiano, Eichenbaum and Evans (1993). Evidence that sales of small firms falls relative to large firms in a monetary contraction is reported by Gertler and Gilchrist (1992). Finally, Berger and Udell (1992) and Kayshap, Wilcox and Stein (1991) report evidence that suggests the spread between interest rates paid by bank-dependent and non-bank-dependent firms increases in a monetary contraction.

A version of the model without costly state verification predicts firms respond equally in terms of debt and sales to an unanticipated monetary contraction and pay identical rates of interest on loans. When costly state verification is re-introduced it is found that the model replicates the qualitative features of empirical heterogeneity findings. Borrowing and sales of firms that have private information that is costly to verify fall relative to firms whose operations are common knowledge following an unanticipated decline in monetary growth. It is also found that the spread between the interest rates these two types of firm pay increases.

In addition to these characteristics the model has a rich array of other empirical predictions, which are also described in section 4. For example, in response to an unanticipated monetary contraction in the model, bankruptcies rise sharply. Time series evidence that is

³See Christiano and Eichenbaum (1992d) for a discussion of the relevant empirical evidence.

consistent with this finding is reported in Fisher (1994). In addition, the model predicts asymmetric responses in downturns and upturns that are brought on by monetary policy disturbances. For example, sales and borrowing of privately informed firms fall by more in a contraction, on average, than they rebound in an expansion. The opposite is true for firms whose operations are common knowledge. In the fifth and final section of the paper I summarize and interpret the findings, and describe some shortcomings of the analysis.

Many papers, including Bernanke and Gertler (1989), Farmer (1985, 1988), Fuerst (1992b), Gertler (1992), Stiglitz and Weiss (1981) and Williamson (1987), have emphasized the theoretical potential for imperfections in credit markets to influence aggregate activity. With two exceptions, the purely real effects these imperfections may have are emphasized in these papers, rather than their role in propagating monetary disturbances *per se*. One exception is Fuerst (1992b). He develops a model in which costly state verification and limited participation interact in a model of money to generate outcomes similar to Roosa's (1966) availability doctrine. The second exception is Farmer (1988) who analyses a model of layoffs with private information and an *ad hoc* money market. A common theme among these papers is an "excess sensitivity" result which states that agents that are faced with imperfect credit markets are more sensitive to external disturbances than similar agents who do not face the imperfections. This kind of result plays a crucial role in the analysis presented here.

2. A partial equilibrium model of the loan market

In this and the following section I outline a model in which monetary shocks have a heterogeneous impact on firms. The discussion proceeds in two stages. In this section I present a static, partial equilibrium, real model of borrowing in the presence of asymmetric information. After reviewing some of the properties of the optimal loan contract in this environment, the second stage of the discussion proceeds in section 3. This is where I describe how the contracting environment is embedded in a dynamic general equilibrium monetary model.

To model borrowing with asymmetric information, I adopt a static costly state verification framework that is due to Townsend (1979) and Gale and Hellwig (1985). This framework

is used for four main reasons. First, it has received considerable attention in the literature on the macroeconomic implications of credit market imperfections and in the literature on financial contracting. As such it represents a prominent benchmark. Second, it has the desirable property that it delivers a standard debt contract as an equilibrium phenomenon. Third, from a quantitative perspective, the static loan environment is a natural place to start since it can be viewed as maximizing the distortion due to asymmetric information.⁴ Finally, it is a parsimonious modeling environment which makes it straightforward to incorporate into a quantitative general equilibrium setting.

The model consists of large numbers of banks and firms. Firms are profit maximizers. They have access to a stochastic revenue technology which specifies that an investment level of l yields $\theta F(l)$ revenues, where θ is a positive random variable with uniform distribution function $G(\theta)$ on the support $[\underline{\Theta}, \bar{\Theta}]$ and a mean of unity. In order to produce at all, a firm must pay an overhead cost $\xi \geq 0$. The deterministic portion of revenues are then Cobb-Douglas in the level of investment net of this cost. That is,

$$F(l) = \begin{cases} 0, & l \leq \xi \\ (l - \xi)^{1/\psi}, & l > \xi \end{cases}, \quad (2.1)$$

where $\psi > 1$. Notice that the average revenue functions of these firms are negatively related to investment after a certain investment level is attained. We can interpret this property as arising from at least two kinds of situations. First, firms may participate in a competitive product market and operate with a technology that exhibits diminishing returns beyond some capacity level determined by overhead costs. In a second interpretation, one which is adopted in the next section, individual firms may face downward sloping product demand curves and use a constant returns to scale production technology augmented by overhead costs. For now, either interpretation is valid.

Because an individual firm has no resources of its own, it must seek external finance.

⁴This view follows from the well known fact that dynamic interaction between borrowers and lenders may mitigate agency costs associated with asymmetric information (see Townsend, (1982)).

Rather than construct an environment where banks emerge endogenously, as in Diamond (1984), I assume that banks exist and are the source of external finance for firms. Each bank is assumed to hold a sufficiently large and diversified portfolio of investments to achieve perfect risk pooling. Banks are interest rate takers in the market for deposits. The gross interest rate in this market, denoted by R , is the opportunity cost of funds for banks.

A firm's investment decision is complicated by two factors other than its lack of internal finance: (i) the level of investment must be chosen before θ is observed, and (ii) only firms costlessly observe their own revenues. Banks possess a monitoring technology that involves forgoing $\mu l > 0$ goods to observe the revenues of a firm in which it has made an investment of l . I assume that banks can commit to a monitoring strategy and that stochastic monitoring is not feasible. Gale and Hellwig (1985) have shown, for an environment consistent with this one, that the optimal financial arrangement between a bank and a firm is a standard debt contract. Here, this means three things. First, the firm agrees to pay $R^b l$ if it is solvent, and defaults otherwise. The variable R^b is the gross interest rate on the loan. Second, banks monitor only in the event of default. Finally, when default occurs banks appropriate all the revenues from the firm.

To solve for equilibrium in the credit market we can make use of the form the optimal contract must take. The firm will be insolvent if and only if $\theta F(l) < R^b l$. Define the variable γ by the condition $\gamma F(l) = R^b l$. The expected payoff to a firm on an investment of l , given the interest rate R^b , is then given by $\int_{\gamma}^{\bar{\theta}} [\theta F(l) - R^b l] dG(\theta)$. This expression states that the firm's expected payoff is the expected value of its surplus of revenues over costs, conditional on it being solvent. The expected return to the bank on an (l, R^b) pair is given by $[1 - G(\gamma)] R^b l + \int_{\bar{\theta}}^{\gamma} \theta F(l) dG(\theta) - \mu G(\gamma) l$. Here, from the definition of γ , $G(\gamma)$ denotes the probability of bankruptcy. This expression says that the return to the bank is the expected payments from solvent firms plus the expected revenues of insolvent firms, less the expected costs of monitoring.

The optimal contract maximizes the expected payoff to the firm, subject to the condition that the bank earns at least its opportunity cost. For the purposes of studying the properties

of equilibrium it is convenient to write the bank and firm expected return expressions in terms of (l, γ) instead of (l, R^b) . We can do this by using the definition of γ stated above. The problem used to compute equilibrium in the credit market can then be written:

$$\max_{\{l > \xi, \underline{\theta} < \gamma < \bar{\theta}\}} F(l) \left\{ \int_{\gamma}^{\bar{\theta}} \theta dG(\theta) - \gamma[1 - G(\gamma)] \right\} \quad (2.2)$$

subject to

$$\frac{F(l)}{l} \Gamma(\gamma) - \mu G(\gamma) \geq R, \quad (2.3)$$

where $\Gamma(\gamma) \equiv \int_{\underline{\theta}}^{\gamma} \theta dG(\theta) + \gamma[1 - G(\gamma)]$.⁵ For the case when $\xi = 0$, Fuerst (1992b) shows that equilibrium loan size is decreasing in R while bankruptcy is increasing. For a broad range of parameter configurations, including configurations consistent with parameters used in the general equilibrium analysis of section 4, it can be shown numerically that these results hold for $\xi > 0$.

What is crucial for the analysis that follows is not that loan size falls when R is increased, but the extent of the reduction relative to the perfect information case. We now consider factors that determine the relative sensitivity of loans in the asymmetric information versus the perfect information case. Perfect information (PI) occurs when monitoring costs are zero. In this case, the firm acts as if it is an interest taker in the market for loans. Under PI, then, loan size is determined by the point at which the expected marginal revenue from a loan equals its marginal cost, R :

$$F'(l) = R. \quad (2.4)$$

Here $'$ denotes the derivative operator.

An analogous condition holds for the asymmetric information (AI) case. In equilibrium the bank's return constraint (2.3) will bind. We can write it as

$$\frac{F(l)}{l} \Gamma(\gamma) = R + \mu G(\gamma). \quad (2.5)$$

⁵See appendix A for a discussion of the existence and uniqueness of the solution to this problem.

This condition says the average revenue from loans, as observed by a bank, must equal the bank's opportunity cost of funds plus the average cost of monitoring.

Consider the effect on l of an increase in R . It is useful to compare the (absolute value of the) interest rate elasticity of l under PI and AI, e_R^{PI} and e_R^{AI} , respectively. Condition (2.4) implies e_R^{PI} is determined by the elasticity of $F'(l)$ with respect to l . In comparing e_R^{PI} to e_R^{AI} , I decompose the effect of the change in R on l under AI into two parts: the direct effect on l holding γ constant and the indirect effect via changes in γ induced by the change in R .

Consider the direct effect first. Suppose for the moment that γ is unaffected by the change in R and imagine computing the elasticity of equilibrium loan size in this case, \hat{e}_R^{AI} . When $\xi = 0$, $F(l)/l$ differs from $F'(l)$ by only a constant so that the loan size elasticities of these objects are identical (and equal to a constant). This implies

$$\hat{e}_R^{AI} = \frac{R}{R + \mu G(\gamma)} e_R^{PI} < e_R^{PI}.$$

Thus the direct effect of an interest rate change under AI implies a smaller per cent change in loan size for a given per cent change in the gross interest rate, relative to PI.

Taking into account the indirect effect of the change in γ on l may not alter this partial result. Changes in γ push $\Gamma(\gamma)$ and $\mu G(\gamma)$ in the same direction and their net effect on condition (2.5) is small.⁶ This intuition underlies my finding, for a broad range of parameter values (again, including parameters consistent with those used in section 4), that when there are no overhead costs, $e_l^{AI} < e_l^{PI}$. That is, when $\xi = 0$, AI leads to a *dampening* of the sensitivity of loans to changes in the interest rate relative to PI.

The role of overhead costs in this setting is to make $F(l)/l$ less elastic with respect to l than $F'(l)$. The less loan elastic $F(l)/l$ is, the more l must change to induce a given change in the bank's rate of return on loans. This increases the direct effect of the change in R on l . In addition, positive overhead costs tend to make the marginal loan revenue schedule more loan elastic relative to the no overhead costs case. This dampens the response of loans under

⁶Since $\Gamma'(\gamma) = [1 - G(\gamma)] > 0$ and $\mu G'(\gamma) = \mu[\bar{\Theta} - \gamma]/[\bar{\Theta} - \underline{\Theta}] > 0$, a rise in γ increases $\Gamma(\gamma)$ and $\mu G(\gamma)$ in equilibrium. Neither term changes enough relative to the other to have a determinant affect.

PI. It is easy to show that $\hat{e}_R^{AI} > e_R^{PI}$ if overhead costs are sufficiently large (see appendix A). I have found that $e_R^{AI} > e_R^{PI}$ for a broad range of parameter values as well.⁷ In these situations, otherwise identical firms are more sensitive to changes in interest rates (brought on, say, by a reduction in the real supply of funds available to intermediaries for lending to firms) if their revenues are costly to verify, relative to the case where their revenues are common knowledge.

3. A dynamic general equilibrium model of the credit channel

I embed the static model of asymmetric information in the credit market within an otherwise dynamic general equilibrium monetary model. Analogous to Fuerst (1992b), I suppose firms contract for funds on a period-by-period basis without any dynamic interaction with their lenders. Since I am interested in how firms that face a monitoring problem behave relative to firms that do not have to be monitored directly, I include both types of firm in the model. This raises the question of what frictions would underlie the coexistence of both types of firm. To resolve this question I assume that these firms act as monopolistic competitors in an intermediate goods producing industry. They each produce a distinct good which final good producers demand because in their production functions the distinct intermediate goods are imperfectly substitutable.⁸ With this in mind, I turn now to an overview of the model. This is followed by a more formal description of the model.

The model is populated by households, intermediate goods producing firms of type a and of type b , type f firms that produce a final good, financial intermediaries and a monetary authority. Each period a continuum of distinct intermediate goods, y_i , for $i \in [0, 1]$ are

⁷Gertler and Gilchrist (1991) argue that in their model loans are more sensitive to a change in the opportunity cost of bank funds, relative to the PI case. They adopt a technology for the representative firm in which $F(l)/l$ is inelastic relative to $F'(l)$. Farmer (1985), Williamson (1987), Bernanke and Gertler (1989) and Gertler (1992) also display “excess sensitivity” results in real settings.

⁸The monopolistic competition environment is similar to one analysed recently by Hornstein (1993). Dixit and Stiglitz (1977) and Spence (1976) are the original contributors.

produced. Using capital and labor as inputs, goods indexed by $i \in A \equiv [0, \lambda]$ are produced by firms of type a and goods indexed by $i \in B \equiv (\lambda, 1]$ are produced by firms of type b . Type f firms use these intermediate goods to produce a final good for sale in a competitive market. The final good is transformed into consumption and investment goods in the usual way. The household consumes and, because it owns the capital stock, invests in capital.⁹ It also sells labor services in the labor market and, by virtue of its ownership of capital, rents capital. To capture the notion that working capital is required for production, I assume that type a and b firms must hire factors of production on a pay-as-you go basis. Without other sources of finance, these firms must turn to intermediaries for funds. The intermediaries accept deposits from households and uses them to lend to the firms that require financing.

At the beginning of a period, the representative household holds the entire money stock, m .¹⁰ At this time the household allocates its cash between two uses. It lends n dollars to intermediaries at the gross nominal interest rate R , and sets aside $m - n$ dollars to purchase consumption goods with. By assumption the household must use cash to purchase consumption goods. This cash-in-advance constraint can be satisfied using current wage earnings as well as $m - n$. The household does not face a cash-in-advance constraint for investment – investment is a credit good.¹¹

In addition to cash raised from households, another source of funds for intermediaries is a lump sum injection of cash, x' , by the monetary authority. Since information on the operations of type a firms is common knowledge, they act as interest rate takers when borrowing funds from intermediaries. Type b firms, however, face the same problem as the firms in the previous section. I assume that *each period* type b firms face the contract problem outlined before. This is feasible because I also assume that some proportion of

⁹This ownership assumption is made for two related reasons. First it simplifies the contract problem of privately informed firms. Second it allows me to ignore the problem of how the capital of firms that default on their loans is reallocated.

¹⁰All nominal variables are measured relative to the beginning of period per household money stock. This is to accomodate growth in the aggregate stock of money. When nominal variables are normalized in this way the optimization problems of the agents in the model can be written as stationary problems.

¹¹Investment is a credit good and the household is allowed to spend wage earnings contemporaneously to reduce the impact of inflation on the capital to output ratio and average employment, respectively. See Christiano (1991) and Stockman (1981).

the intermediaries have access to the necessary monitoring technology. Type f firms do not face any cash-in-advance constraints and so do not have to borrow cash from intermediaries. They use “trade credit” to purchase goods from intermediate goods producers.

At the end of the period, accounts are settled and the entire money stock flows back to the household. Type a and b firms do not receive any cash revenues during the period. To repay principal and interest on their loans, type a firms transfer claims over their accounts receivables (the trade credit used by final good producers) to intermediaries. In the event of solvency, type b firms do this as well. In the event of default, all accounts receivables for these firms are transferred to intermediaries. Type f firms take in revenues in both cash and credit. That is, they receive cash for the consumption goods they sell to households, but not for the investment goods. However, since both the cash and the credit used to purchase goods from these firms represent claims over goods, these can be used to settle their trade credit accounts. These accounts must be settled with type a and b firms, and, since some claims over this trade credit are transferred to the bank, with the bank as well.

There is an indeterminacy in the model about the actual flow of cash from final good producers to these other agents. However, all of this cash, by the beginning of the next period, will end up with the household. This is because the firms and intermediaries are owned by the household and so pay dividends on their operations to it.¹² These dividends are paid in the form of cash and claims on goods remaining with the firms and the bank. The remaining cash flows back to the household in the form of payments for the capital it rents during the period.¹³

Monetary injections are the only source of aggregate uncertainty in the model. Lump

¹²Since households are assumed to hold perfectly diversified portfolios of type b firms, they will earn the average return from these firms (households do not face any idiosyncratic risk). If type b firms represent young, unestablished enterprises, then this may be a problematic assumption. This is because the decision to start up a business is often made by a household or a small number of households that cannot diversify the idiosyncratic risk involved. The diversification assumption is made for tractability, but it is recognized that by doing so I ignore the implications of a potentially important element of risk.

¹³The complexity of this decentralization scheme arises because the total cash revenues of firms in the model do not equal or exceed the interest and principal on the loans they must take out to finance production. In Christiano and Eichenbaum (1992a), for example, this condition is satisfied (on average). In their model firms have enough cash on hand at the end of the period (on average) to repay their loans with interest entirely with cash.

sum monetary injections normalized by the per household stock of money equal the growth rate of money. I assume the growth rate of money evolves according to:

$$x' = (1 - \rho)\bar{x} + \rho x + \epsilon. \quad (3.1)$$

In this expression \bar{x} is the unconditional mean of monetary growth, x equals last period's growth rate, $\rho \in (-1, 1)$, and ϵ is an independent mean zero random variable. This specification for money growth has been adopted in many monetary business cycle models.¹⁴

I now present a more formal description of the model. In this description I use conventional dynamic programming notation. In particular, time subscripts are not used and, with the exception of monetary growth, primes denote the next period value of a variable. The discussion focuses on the optimization problems of representative agents in an arbitrary period. Before describing these optimization problems it is convenient to define the following information sets: $\Omega_0 \equiv \{K, m, x\}$ and $\Omega_1 \equiv \Omega_0 \cup \{x'\}$. Here, K , denotes the aggregate stock of capital.

3.1. Final good producers

There are a continuum of final good producers indexed by $j \in [0, 1]$. Given amounts of the intermediate goods, y_{ij} , $i \in [0, 1]$, output of final good producer j is given by

$$Y_j = \left[\int_0^\lambda \phi_{ij} y_{ij}^{1/\psi} di + \int_\lambda^1 \phi_{ij} y_{ij}^{1/\psi} di \right]^\psi,$$

where $\psi > 1$. The ϕ_{ij} , $i \in [0, 1]$, terms are intermediate good-specific random productivity shocks idiosyncratic to final good producer j .¹⁵ They are realized before final good producers' input demand decisions are made. Since the shocks are idiosyncratic to any given final

¹⁴See for example, Cho and Cooley (1990), Cooley and Hansen (1989) and Christiano (1991) and King (1992).

¹⁵These and other idiosyncratic shocks to be described below must satisfy a certain property for the model to be consistent with perfect competition and zero profits in the final good producing sector. In appendix A I describe what this property is.

goods producer, aggregate demand for any particular intermediate good cannot be inferred without knowledge of the actual shocks for all final goods producers. I assume no agent in the economy has this information or can collect it costlessly. We will see below that these assumptions help ensure revenues for each type b firm are private information. For current purposes, I need only assume that for each $j \in [0, 1]$, realizations of ϕ_{ij} are independently and identically distributed for all $i \in [0, 1]$.

Final good producers choose inputs to maximize profits. They do so conditional on information contained in Ω_1 and after the input-specific productivity shocks are realized. The implied input demand functions for each final good producer j are as follows:

$$y_{ij} = \left[\frac{\phi_{ij} p}{p_i} \right]^{\frac{\psi}{\psi-1}}, i \in [0, 1]. \quad (3.2)$$

Here, p denotes the (money) price of the final good and p_i , $i \in [0, 1]$ denotes the price of the i 'th intermediate good. Acting competitively, final good producers take $\{p, (p_i : i \in [0, 1])\}$ as given.

3.2. Intermediate goods producers

Intermediate goods producers formulate factor demands and requirements for external finance before the resolution of two forms of uncertainty. First, they do not know the amount of aggregate demand that will be forthcoming once they have decided on a production plan. For intermediate goods producer $i \in [0, 1]$, aggregate demand for its product is given by

$$y_i = \int_0^1 y_{ij} dj = \left[\frac{\phi_i p}{p_i} \right]^{\frac{\psi}{\psi-1}} Y^f, \quad (3.3)$$

where $\phi_i \equiv \left[\int_0^1 \phi_{ij}^{\frac{\psi}{\psi-1}} dj \right]^{\frac{\psi-1}{\psi}}$ and $Y^f \equiv \int_0^1 Y_j dj$.¹⁶ For intermediate goods producer i , aggregate demand uncertainty is summarized by the random variable ϕ_i , $i \in [0, 1]$.

¹⁶In this derivation I use the fact that $\int_0^1 \phi_{ij}^{\frac{\psi}{\psi-1}} Y_j dj = \int_0^1 \phi_{ij}^{\frac{\psi}{\psi-1}} dj \cdot \int_0^1 Y_j dj$. This is reasonable given the infinitesimal contribution of any one intermediate good to the production function of a final good producer.

The second source of uncertainty is with respect to the productivity of factors of production once they are hired. This is summarized by the independently and identically distributed random variable ω_i , $i \in [0, 1]$. In particular, intermediate goods producer i is endowed with the technology

$$\omega_i[F(k_i, h_i) - \xi_i]. \quad (3.4)$$

Here k_i and h_i are the capital input and labor input for intermediate goods producer i and F is the production function that is common across all intermediate goods producers. The production function is given by $F(k_i, h_i) = k_i^\alpha h_i^{1-\alpha}$, $\alpha \in (0, 1)$. Finally, for $i \in A$ I assume that $\xi_i = \xi^a$ and for $i \in B$ I assume that $\xi_i = \xi^b$, where ξ^a and ξ^b are positive scalars representing overhead costs.¹⁷

The uncertainty faced by intermediate goods producer i can be summarized by the random variable $\theta_i \equiv \phi_i \omega_i^{1/\psi}$. This variable reflects the combined influence of demand and supply shocks for intermediate goods producer i . I assume θ_i for $i \in [0, 1]$ are independently and identically distributed according to the uniform distribution G which has positive support, a mean of unity and standard deviation equal to σ .

The situations of the two types of intermediate goods producers are similar in two other respects. First, both types must use external finance to pay factors of production. Second, both types, given a level of financing by an intermediary, l_i , hire factors of production according to

$$k_i = \alpha l_i / r; \quad (3.5)$$

$$h_i = (1 - \alpha) l_i / w, \quad (3.6)$$

for $i \in [0, 1]$, where r and w denote the rental rate on capital and the wage rate, respectively. These expressions follow from solving the input demand problems for a given intermediate goods producer conditional on a level of financing. I now consider the unique aspects of the situations encountered by the two types of intermediate goods producers.

¹⁷The technology given in (3.4) corresponds closely to technologies considered by Hornstein (1993) and Rotemberg and Woodford (1992) in models with imperfectly competitive intermediate goods producers.

3.2.1. Type a firms

Firms of type a are distinguished by the fact that individual realizations of ϕ_i , ω_i and θ_i are common knowledge. This implies revenues for these firms are common knowledge also. Thus type a firms act as interest rate takers in the market for loans. These firms seek financing for their production plans so as to maximize expected revenues less financing costs. As a monopolistic competitor, a type a firm makes use of its known expected demand schedule when deciding on its production plan. Taking into account the assumptions on the underlying distribution of each θ_i , for $i \in A$, the inverse demand functions for these firms implicit in (3.3), the production function for these firms given in (3.4), and the input demand functions conditional on intermediary financing given in (3.5) and (3.6), the optimization problem a given type a firm solves is

$$\max_{l_i} pY^f \frac{\psi-1}{\psi} [z(r, w)l_i - \xi^a]^{\frac{1}{\psi}} - Rl_i, \quad i \in A, \quad (3.7)$$

where $z(r, w) \equiv [\alpha/r]^\alpha [(1-\alpha)/w]^{1-\alpha}$. This problem is solved conditional on information contained in Ω_1 . As such, type a firms take $\{p, Y^f, r, w, R\}$ to be known functions of the elements of Ω_1 . Assuming symmetric behavior among these firms, loan demand for a type a firm, implied by (3.7), is given by

$$l^a = \left[\frac{z(r, w)^{1/\psi} p}{\psi R} \right]^{\frac{\psi}{\psi-1}} Y^f + \frac{\xi^a}{z(r, w)}. \quad (3.8)$$

3.2.2. Type b firms

The distinguishing feature of type b firms is that realizations of ϕ_i , ω_i and θ_i , $i \in B$ are private information for them. One implication of this is that the aggregate demand for a given type b intermediate good is known only by its producer. Along with private information regarding idiosyncratic productivity shocks ω_i , $i \in B$, it is impossible for intermediaries to infer the revenues of a type b firm by examining properties of the equilibrium. We will see below that if any one of these disturbances were common knowledge there would not be a

private information problem.

Type b firms seek financing to maximize expected revenues net of costs. To observe the revenues of a type b firm that it has financed, an intermediary must pay a monitoring cost proportional to the scale of operations it has financed. To ensure incentive compatibility, loan size and the bankruptcy cut-off are determined by a contract problem exactly analogous to the one described in the previous section. Assuming symmetric behavior on the part of type b firms, the loan size granted to a typical type b firm, l^b and the bankruptcy cutoff for these firms, γ , solve

$$\max_{\{l^b > \xi^b, \underline{\Theta} < \gamma < \bar{\Theta}\}} pY^f \frac{\psi-1}{\psi} [z(r, w)l^b - \xi^b]^{\frac{1}{\psi}} \left\{ \int_{\gamma}^{\bar{\Theta}} \theta dG(\theta) - \gamma[1 - G(\gamma)] \right\} \quad (3.9)$$

subject to

$$\frac{pY^f \frac{\psi-1}{\psi} [z(r, w)l^b - \xi^b]^{\frac{1}{\psi}}}{l^b} \Gamma(\gamma) - \mu G(\gamma) - R = 0, \quad (3.10)$$

here $\underline{\Theta}$ and $\bar{\Theta}$ denote the lower and upper bound, respectively, of the support of the distribution G . The upper bound is implicitly defined by $\sigma^2 = [\bar{\Theta}^3 - (2 - \bar{\Theta})^3]/[6(\bar{\Theta} - 1)] - 1$ and the lower bound is given by $\underline{\Theta} = 2 - \bar{\Theta}$. The statement of this problem depends on the fact that intermediaries know the distribution function underlying the idiosyncratic component of revenues for a type b , G . Intermediaries and type b firms agree to a loan size and a bankruptcy cut-off after the monetary injection is observed. This means the contract problem is solved conditional on information in Ω_1 . The intermediaries and type b firms take $\{p, Y^f, r, w, R\}$ to be known functions of Ω_1 and treat these objects parametrically.

3.3. Intermediaries

Intermediaries accept deposits from households and make loans to type a and type b firms to maximize profits. In addition they are the conduit for cash injections by the monetary authority. Intermediaries cannot make loans that exceed the sum of their deposits and the cash injection in a given period. This means that in the aggregate, the following condition

must hold

$$\lambda l^a + (1 - \lambda)l^b \leq n + x'. \quad (3.11)$$

The first term on the left hand side of the inequality is the total of loans granted to type a firms and the second term is the total of loans granted to type b firms. In an equilibrium, (3.11) will hold with equality.

3.4. Households

The representative household is infinitely lived with period utility given by

$$U(C, H) = \ln(C) + \eta \ln[T - H]. \quad (3.12)$$

Here C denotes consumption, T denotes the time endowment, and H denotes hours worked. Let $V(k, m; K, x)$ denote the household's value function when at the beginning of the period cash balances equal m , capital holdings equal k , the aggregate stock of capital is K , and monetary growth from the previous period is x . The problem solved by the representative household can be expressed as the following dynamic program

$$V(k, m; K, x) = \max_{\{k', n\}} E \left\{ \max_{\{C, H\}} \{U(C, T - H) + \beta V(k', m'; K', x')\} \mid \Omega_0 \right\} \quad (3.13)$$

subject to

$$pC \leq m - n + wH \quad (3.14)$$

and

$$m' = \frac{m - n + Rn + rk + wH + \pi - pC - p[k' - (1 - \delta)k]}{1 + x'}. \quad (3.15)$$

In (3.13) $\beta \in (0, 1)$ is the subjective discount factor. Constraint (3.14) is the household's cash-in-advance constraint for consumption purchases. Expression (3.15) is its budget constraint. In the budget constraint, π denotes total nominal profits accruing to the household

from its ownership of firms of type a , b and f , and from intermediaries.¹⁸ Also δ is the rate of depreciation on capital. The multiplicative term $1/(1 + x')$ appears in (3.15) to account for monetary growth between periods. In solving its dynamic program, the household behaves competitively by taking $\{K', r, w, p, \pi, R\}$ as known parametric functions of Ω_1 .

Implicit in how I have stated the household's problem is that C and H are chosen based on Ω_1 , and n and k' are chosen based on Ω_0 . The conventions regarding when agents make their decisions in the model are important for the results I obtain. They are intended to capture the idea that agents make decisions at different frequencies in time relative to open market operations. Based on this interpretation, household portfolio decisions, including real and nominal savings, are made less frequently than open market operations are conducted. Decisions related to consumption and ongoing production of firms are made at the same frequency as open market operations. Thus there is limited participation in financial markets with households not participating as frequently as firms and intermediaries.

3.5. Market Clearing and Equilibrium

I consider a stationary bi-symmetric rational expectations equilibrium. Intermediate goods producers of type a act symmetrically as do type b intermediate goods producers. For type a firms this means the following. These firms all receive the same size loan, hire the same quantities of their factors of production, and, when they make their production plans, expect to charge the same price. Because of idiosyncratic uncertainty, however, actual production and actual prices charged by these firms differ in equilibrium. These prices are set by each type a firm to ensure that none of its production goes unsold.¹⁹ In equilibrium these prices are given by

$$p_i = \frac{\theta_i}{\omega_i} p Y^f \frac{\psi-1}{\psi} [z(r, w)l^a - \xi^a]^{\frac{1-\psi}{\psi}}, \quad i \in A. \quad (3.16)$$

¹⁸In equilibrium intermediary profits will equal the return from lending out the monetary injection. Firms of type f , because of constant returns to scale, will earn zero profits in equilibrium. Intermediate goods producers will earn profits in equilibrium (which will depend on the assumed overhead costs, ξ^a and ξ^b).

¹⁹Given that they must repay their loans in full and that revenues are increasing in the quantity of product sold, this corresponds to optimizing behavior on the part of these firms.

The situation of type b firms is analogous. In their case, prices in equilibrium are given by

$$p_i = \frac{\theta_i}{\omega_i} p Y^f \frac{\psi-1}{\psi} \left[z(r, w) l^b - \xi^b \right]^{\frac{1-\psi}{\psi}}, \quad i \in B. \quad (3.17)$$

From these expressions we can see that if any one of the idiosyncratic disturbances were to be common knowledge, then an intermediary could use prices to uncover the entire structure of demand and supply among intermediate goods producers it had financed. They could then use this information to infer the revenues of all the firms that it financed – there would not be any private information in this economy.

In equilibrium all markets clear. The market clearing conditions are:

$$\lambda k^a + (1 - \lambda) k^b = K \quad (3.18)$$

$$\lambda h^a + (1 - \lambda) h^b = H \quad (3.19)$$

$$rK + wH = n + x' \quad (3.20)$$

$$C + K' - (1 - \delta)K = Y \quad (3.21)$$

$$m' = 1. \quad (3.22)$$

Conditions (3.18) and (3.19), which follow from symmetric behavior by type a firms and by type b firms, are that capital and labor markets must clear. In these expressions k^a and k^b are the equilibrium choices for capital inputs of type a and type b firms, respectively, and h^a and h^b are the equilibrium choices for labor inputs of type a and type b firms, respectively. These values are determined from (3.5) and (3.6) using the equilibrium values of l^a and l^b . Condition (3.20) is that the loan market must clear. Goods market clearing is given by (3.21). In this expression, Y denotes aggregate supply and $K' - (1 - \delta)K$ is aggregate gross investment. Aggregate supply equals the total production of the final good less monitoring costs over a period. That is, $Y = Y^f - \mu G(\gamma)(1 - \lambda)l_b/p$.²⁰ Finally, (3.22) states that per

²⁰In appendix A it is shown that aggregate output of final goods producers can be written as $Y^f =$

household aggregate demand and supply of money must be equated.

A rational expectations equilibrium consists of functions $\{K, n\}$ of Ω_0 and functions $\{C, H, l^a, l^b, p, r, w, R, \gamma\}$ of Ω_1 such that agents optimize and markets clear. Obtaining these functions exactly is not possible. Instead I make use of a log-linearization approximation technique. Details are provided in appendix B. There I also discuss the existence and uniqueness of the approximate equilibrium.

4. Empirical Results

In this section I analyze the quantitative properties of the model. First, I describe how model parameter values were assigned. Second, I discuss selected features of the model's steady state implied by these parameter values. Third, I examine how variables of interest in the model respond to unanticipated monetary disturbances. Finally, I discuss the robustness of the findings to alternative parameter value assignments.

4.1. Parameter Values

I use a combination of estimation and *a priori* assumptions to assign values to parameters. I begin this subsection by discussing the values I assign to the parameters I do not estimate. I then discuss my estimates for the remaining parameters. An innovation of this subsection is that I am able to identify the parameters λ , μ and σ using properties of aggregate time series and micro-level cross-section data.

The parameter values used and the conditions for their identification are summarized in table 1. To illustrate features of the model, I consider two sets of parameter values. These parameter sets are distinguished by assumptions regarding μ . I investigate a version of the model without asymmetric information (the PI case) and a version with asymmetric information (the AI case). The choice $\mu = 0$ identifies the PI case. For the AI case I estimate μ in a manner to be described below. In most studies of business cycle models with

$$[\lambda(F(k^a, h^a) - \xi^a)^{1/\psi} + (1 - \lambda)(F(k^b, h^b) - \xi^b)^{1/\psi}]^\psi.$$

monopolistic competition, overhead costs are used to drive the share of profits in steady state, π^* , to zero.²¹ In the model studied here the contract problem is only well defined if type b firms make positive profits on average. For this reason I fix π^* to be a positive number (0.05) in the two cases.²² Throughout, β was set to $(1.03)^{-.25}$ and T was fixed to 1369 hours, a value used by Christiano and Eichenbaum (1992c). A time period in the model is thus a quarter.

The remaining parameters are ψ , ξ^a , ξ^b , λ , μ , δ , α , η , σ , \bar{x} and ρ . In appendix A it is shown that ψ corresponds to the average markup over marginal cost of intermediate goods producers. Thus, ψ was selected by setting it equal to the average markup in the U.S. economy. As described by Rotemberg and Woodford (1992, 1993), there are a wide range of estimates of this average for the U.S. economy. For the cases described here I fix the markup at the value (1.2) used by Rotemberg and Woodford (1992). Below I discuss how different values of ψ affect the findings reported here. The overhead costs, ξ^a and ξ^b , were chosen to be consistent with the fixed value of π^* . In doing so I assumed that the share of profits attributable to each type of intermediate goods producer equals its proportion of the total number of intermediate goods producers. I used steady profits to approximate average profits in these calculations.

I estimated λ and μ by requiring that the model be consistent with two particular features of the data. First, I required that the model be consistent with an estimate of the ratio of nominal liabilities of failed firms to nominal GDP. The Dun and Bradstreet Corporation maintains a quarterly time series of nominal liabilities of failed firms which can be used to estimate this ratio. For the period 1984-1990 the average of this ratio is 0.234 per cent.²³

²¹See Rotemberg and Woodford (1993) and the references given there.

²²It should be noted that the main findings for the model I report below are enhanced if I select π^* to be a number closer to zero. The main findings reported below are also robust to choosing π^* as high as 0.09.

²³The Dun and Bradstreet failure liability series is based on their survey of businesses in each four digit SIC classification. Their definition of a business failure is as follows: Businesses are classified as having failed that "ceased operations following assignment or bankruptcy; ceased operations with losses to creditors after such actions as foreclosure or attachment; voluntarily withdrew leaving unpaid debts; were involved in court actions such as receivership, reorganization or arrangement; or voluntarily compromised with creditors" (*Business Failure Record*, 1990). The series extends back before 1984 but is based on a more limited variety of enterprises in this period.

Second, I required that the model be consistent with micro studies that estimate the ratio of direct bankruptcy costs to total assets of firms near the time of failure. One measure of this ratio is given by Guffey and Moore (1991). They examined the U.S. trucking industry (an intermediate good producing industry) and estimated the ratio to be 9.1 per cent. I measure the value of assets of failed firms in the AI model by the accounts receivables of failed firms that are forfeited to banks. The total assets of failed firms measured in this manner are given by

$$A^* = (1 - \lambda) \left[\int_{\underline{\theta}}^{\gamma} \theta dG(\theta) \right] pY^f \frac{\psi-1}{\psi} (F(k^b, h^b) - \xi^b)^{\frac{1}{\psi}}.$$

Thus, λ and μ were estimated by equating the model implied unconditional means of these two ratios to their empirical counterparts. I used the steady state liability and Guffey-Moore ratios to approximate these unconditional means. My point estimates for λ and μ using this procedure are 0.918 and 0.093, respectively.

The estimate for λ may seem quite high. It implies that less than 10 per cent of intermediate good producing firms in the model are credit constrained. This proportion can be regarded as a lower bound on the empirical proportion of firms that are credit constrained. Two facts justify this conclusion. First, only intermediate good producing firms can be constrained in this model. Clearly, there is nothing special about the contract problem examined here that limits its applicability to intermediate good producers. Second, liabilities of failed firms in the model are relatively large because I have required intermediate good producers to finance all their factors of production through intermediaries. In reality firms make use of internal funds to finance their operations. Incorporating this fact in the analysis might increase the estimate of the proportion firms that are credit constrained by reducing the liabilities of a typical failed firm.

The remaining parameters were estimated directly using aggregate U.S. time series data on Y , I , K , H , x , R and R^b . The data on Y , I and K are updated versions of series discussed in Christiano (1988). The reader is referred to that paper for a description of these series. The per capita hours worked series I make use of is Hansen's (1992) efficiency weighted household survey series. I use growth in the monetary base to measure x (Citibase

mnemonic FMFBA). The rate on 6 month commercial paper is used to measure R (FYCP). Finally, I use the commercial bank lending rate to measure R^b .²⁴ All the data are quarterly. The sample period for estimation is 1967:I–1988:I.²⁵

The parameters \bar{x} and ρ are estimated as the mean rate of monetary growth and the first order autocorrelation coefficient on money growth. The point estimates for these parameters are 0.014 and 0.32, for \bar{x} and ρ , respectively.²⁶ The parameter δ was set equal to the sample average rate of depreciation on capital, *i.e.* the sample average of $1 - (K' - I)/K$. This yields a point estimate for δ equal to 0.025. My point estimates of α and η were designed to equate the model's implications for the unconditional means of K/Y and H with the sample averages of my empirical measures of these variables. I approximate the model's mean values of K/Y and H by the steady state. For both cases the point estimate for α is 0.37 and the point estimate for η is 2.99.

The estimate of the mean spread between the commercial bank lending rate and 6 month commercial paper is 157 basis points, at an annual rate. Given every other parameter, the model implied spread $R^b - R$ is governed by the riskiness of the technology available to type b firms. Thus, I used σ to equate the unconditional mean spread in the model with the mean spread estimated from the data. By using the steady state spread to approximate this unconditional mean, the point estimate of σ using this procedure is 0.038.

4.2. Features of the steady state

In table 2 I report selected steady state variables from the AI and the PI versions of the model. Consider first the top two rows of this table. These indicate the steady state bankruptcy rate and the nominal income share of bankruptcy costs, respectively. The bankruptcy rate is 0.315 per cent. The average failure rate computed from the Dun and Bradstreet failure rate series is 0.974 per cent for the period 1984-1990. Since the Dun and Bradstreet series is

²⁴This can be found in various issues of the Federal Reserve's *Survey of Terms of Lending at Commercial Banks*.

²⁵This is the longest period that includes a complete series for each data set I make use of.

²⁶My qualitative findings are not changed if the value of ρ is larger than the one reported here. Larger values of ρ are found when the monetary base growth process is estimated over a longer sample.

based on a survey representative of all U.S. firms and not just intermediate good producers, the bankruptcy rate implied by the model appears to be low relative to the data. I had more difficulty finding a measure of bankruptcy costs as a per cent of GDP. However, the value of 0.02 per cent reported in the table does not seem unreasonably large. In the steady state of the PI version of the model these variables are of course both equal to zero.

The next four rows of this table indicate how introducing monitoring costs affects the relative size of loans granted to the two types of intermediate goods producers, total sales for these firms and the degree of “rationing” of type b firms.²⁷ In the PI version of the model both kinds of firm are identical in all respects. Hence, they receive equal sized loans in equilibrium. The share of total sales attributable to each type of firm corresponds to their proportion of the total number of intermediate goods producers. When monitoring costs are introduced, the size of loan going to a typical type b firm falls to 78.4 per cent of the loan size granted to a typical type a firm. Thus, type b firms are smaller than type a firms in this version of the model. This effect on loan size has the obvious implication for sales shares.

It is straightforward to compute the size of loan demanded by a type b firm at the steady state value of R^b , taking into account the probability of bankruptcy, but *not* the affect on bank rate of return (see appendix A). The difference between this amount and the actual loan granted, $l^* - l^b$, can be thought of as the amount of steady state (intensive margin) rationing. From the table we see that introducing monitoring costs implies the amount of rationing exceeds 29 per cent of the loan actually granted. Evidently, on the basis of the parameter estimates used here, type b firms are substantially rationed in terms of what they would prefer absent incentive compatibility constraints.

The next two rows in this table indicate overhead ratios for typical type a and type b firms. The overhead ratios are important because they help to determine the relative loan size elasticities that were described in section 3.2. In appendix A the analogous elasticities for the general equilibrium model are displayed and it is shown that, other things equal, the responsiveness to a change in R of loans to type b firms is increasing in the size of the

²⁷Formulas for total sales of type a and type b firms are given in appendix A.

overhead ratio and for type *a* firms it is decreasing. To get a sense of whether the overhead ratios reported in this table are reasonable one can compare them to overhead labor ratios for manufacturing firms reported by Davis and Haltiwanger (1991). They compute the number of non-production workers to be roughly 40 per cent of the total workforce. Taking into account wage differentials, this would seem to be a lower bound on the overhead labor ratio. If overhead capital is used proportionately by these firms, then the overhead ratios of 12.4 to 14.8 per cent reported in the table do not seem exceptionally large. This should be borne in mind when the excess sensitivity findings are reported below.

The final two rows indicate steady state nominal savings and welfare. The average amount of nominal savings determines the impact of limited participation in the model. By requiring the share of profits in total income, π^* , to be the same in the two versions of the model, nominal savings are equalized. This ensures that differences in the cyclical properties of the two versions of the model can be attributed entirely to monitoring costs. Welfare comparisons are somewhat problematic. However, since the share of profits in the two versions of the models are identical and, with the exception of the overhead ratios, preferences and technology are identical, it seems reasonable to compare the two steady state welfare measures. From the table we see that the monitoring costs reduce steady state welfare by only 0.02 per cent.

4.3. Impulse responses

In this subsection I discuss impulse responses for various variables in different versions of the model. Attention is focused on the impact effects of unanticipated monetary disturbances, rather than entire dynamic responses. This is done because the propagation mechanisms in this model are relatively weak so that differences between the versions of the model are greatest in the period of impact.²⁸ Presumably a model with more sophisticated propagation

²⁸The differences between models with limited participation and without are almost entirely confined to the period of the monetary shock. This is discussed in Christiano (1991). Loans in the current model are contracted on a period-by-period basis and limited participation only lasts one period. Thus the current model shares this property.

could extend beyond the period of impact, and even amplify, the impact effects emphasized here.²⁹

The main findings of the paper are reported in table 3. Here impact responses following an unanticipated 1 percentage point reduction in money growth are reported for the AI and PI cases. Consider first the top three rows of the table where the responses of Y , H and R are reported. We see that output and hours fall and the interest rates rise in both the AI and PI versions of the model. With households' nominal savings fixed before the monetary disturbance, firms and intermediaries must absorb the entire amount of the monetary contraction. Funds are at a premium because less cash is in the money market than anticipated. Since firms require funds to finance their ongoing operations, they are willing to pay the premium. The premium they are willing to pay is large enough that it dominates the Fisher anticipated inflation effect and R rises following the contraction. With higher operating costs (and a reduction in the equilibrium real supply of loanable funds), firms cut back on production and hours worked falls.³⁰

Notice that the aggregate output and interest rate responses in the AI case are more pronounced than in the PI case. Since the limited participation effect is constant across the two experiments, these differences are entirely due to monitoring costs. The interest rate difference can be described as follows. After the contraction, limited participation implies the real supply of credit must fall. This means that, in equilibrium, banks must reduce loans to type b firms. Reducing loans to type b firms increases the return on loans to banks

²⁹It is well known that the propagation mechanisms in the class of models of which the current one is a member are weak. A lot of ongoing research is focused on rectifying this problem. Two papers are directly related to the current analysis along this dimension. Christiano and Eichenbaum (1992b) discuss a way of propagating the impact of limited participation over many periods that could in principle be applied here. Dynamic contracting, as for example in Gertler (1992), might help in propagating the impact effects emphasised here as well.

³⁰If the limited participation assumption is dropped (so that nominal savings are chosen after the realization of the monetary disturbance), then output and hours rise and interest rates fall following the negative shock. These are standard results for cash-in-advance economies which arise because of the dominating influence of anticipated inflation effects. Because monetary growth is persistent, agents forecast a lower rate of inflation following the monetary contraction. The interest rate R is determined by Fisherian fundamentals in a standard cash-in-advance setting and very little happens to the real interest rate. This leads to a reduction in R . Firms must borrow to finance their operations and so the interest rate is a part of their operating costs. With lower operating costs, hours and output expand.

because of diminishing average revenues. In addition, the optimal contract implies R^b and the bankruptcy cut-off increase as well. These factors combine to increase banks' sure rate of return which puts upward pressure on the default-free interest rate in addition to the limited participation effect. The extra responsiveness of output in the AI case turns out to be mainly because of increased costs from bankruptcy. Hours in fact respond the same in the two cases.³¹

The impact of an unanticipated monetary contraction on the activities of type b firms relative to type a firms are indicated by the entries in the next five rows of table 3. Here impact responses of relative debt flows, sales for all type a and type b firms, and average real profits for firms of type a , π^a , and type b , π^b , are reported. In the PI case relative debt flows are unaffected by a monetary contraction, regardless of the extent of participation in financial markets. This is because the firms in this case are identical in all respects. With limited participation and monitoring costs, the relative amount of debt flowing to type b firms falls by close to 0.12 per cent.

The intuition underlying this finding follows directly from the elasticity discussion of section 2. For the parameter values of this experiment, the partial equilibrium interest rate elasticity of loan size is larger for type b firms than for type a firms. As previously described, limited participation leads to a reduction in the real supply of funds available to intermediaries for lending to firms. Intuition from static demand analysis suggests, then, that firms with the more elastic loan schedules will respond greatest in per cent terms to the reduction in the supply of loanable funds.

The response of sales and profits in the type a sector and the type b sector are similar to the relative debt flow findings and for the most part can be explained by them. In the PI case the sales responses are the same and equal to the aggregate output responses. Profits fall by an equal amount for both types of intermediate goods producer. With monitoring costs, sales in the type b sector are hit twice as hard as sales in the type a sector. In addition, compared to the PI case, the response of the type a firms is dampened and for the type b

³¹This can be proved analytically along the lines outlined in Christiano and Eichenbaum (1992c),

firms it is exacerbated. For type b firms profits fall by 1.4 per cent (0.436 per cent in the PI case) compared to 0.637 per cent for type a firms (also 0.436 per cent in the PI case). While both types of firm must struggle more in the AI model following a contraction, type b firms clearly are hit much harder.

To summarize, limited participation and monitoring costs deliver findings consistent with empirical work on how broad aggregates respond to a monetary contraction, and results consistent with empirical work that describes the heterogeneous response of firms to monetary contractions. The model with limited participation and monitoring costs is consistent with the notion that after unanticipated monetary contractions, firms that face credit constraints are hit harder than firms that do not face these constraints. The firms in the model that face the credit constraints are smaller than the firms that do not. Thus the results are broadly consistent with the Gertler and Gilchrist (1992) findings on the behavior of sales and outstanding debt for small versus large firms.

Several other results reported in table 3 are worth noting. First, there is a sharp rise in bankruptcy (13.6 percent) in the AI case following the contraction. This is qualitatively consistent with the estimates of increases in bankruptcy following unanticipated contractions reported in Fisher (1994). Second, the $R^b - R$ spread rises by more than 25 basis points in the AI case. This is consistent with findings reported by Berger and Udell (1992) and Kayshap, Stein and Wilcox (1993).³² Third, the degree of rationing actually falls in the AI case with limited participation. It may be bad times for type b firms, but their excess demand for loans is in fact lower than otherwise.

The AI case with limited participation exhibits some interesting asymmetries with respect

³²Berger and Udell (1992) examine a large and detailed sample of commercial bank loans and find that a measure of what I am calling the $R^b - R$ spread is positively related to variables used to proxy for a "credit crunch." They argue that their results contradict asymmetric information stories of credit market imperfections that emphasize credit rationing. The models of credit rationing they cite imply exogenous changes in the opportunity cost of funds of funds for banks are not translated directly into changes in the interest rate charged to credit rationed firms. If one presumes that a credit crunch increases the opportunity cost of funds to banks (as it does here), then these models imply the spread should fall in a contraction. Clearly this implication is not a feature of the environment studied here. Kayshap, Stein and Wilcox (1993) find that the spread between the prime rate at commercial banks (roughly R^b) and the commercial paper rate (R) rises following an unanticipated monetary contraction.

to the responses of certain variables in contractions versus expansions. These findings are summarized in table 4.³³ There we see that the default free interest rate, the outstanding debt of type *b* versus type *a* firms, the sales of type *b* firms, the default free interest rate spread, profits of type *a* and type *b* firms and bankruptcies all respond by more following unanticipated monetary contractions compared to expansions. Interestingly, sales of type *a* firms rise by more in an expansion than they fall in a contraction. Also, after an unanticipated monetary expansion relatively more of the injected funds go to type *a* firms rather than type *b* firms. This last finding can be understood by recognizing that other variables besides *R* influence the contract problem in the general equilibrium setting (see (3.9) and (3.10)). Evidently general equilibrium effects on *p*, *r* and *w* outweigh the partial equilibrium interest rate effect described in section 2 so that loans are *less* responsive for type *b* firms relative to type *a* firms in an expansion – the opposite of what occurs in a contraction.³⁴

4.4. Sensitivity to perturbations in parameter values

In Fisher (1994) the sensitivity of the findings reported in the last two subsections to perturbations in π^* , ψ , μ and λ are analyzed. Here I provide a brief account of some of the main findings from that analysis. In doing so I focus on the impact response findings. Aside from choosing alternative values for these parameters, the method for selecting parameters was unchanged from section 3.1.

Perturbations in these parameters influence the size of the limited participation effect. The size of this effect can be measured by the average amount of cash households deposit with intermediaries. Other things equal, the smaller the amount of cash the households deposit with intermediaries, the more impact an unanticipated contraction will have on firms.³⁵ Increasing the share of credit constrained firms (decreasing λ), for example, has the effect of

³³The log-linear approximation technique employed restricts the control variables ($\ln K'$, $\ln H$, $\ln n$) to respond symmetrically in contractions versus expansions. However, the variables listed in table 3.4 are all nonlinear functions of these variables and so they are not so restricted.

³⁴This finding is sensitive to the value assigned to λ . For example, with $\lambda = 0.60$ the sign of the response of l^b/l^a to an expansionary monetary shock is positive.

³⁵This follows because, when the pool of cash made available by households for firms to borrow is made smaller, the per cent change in this pool following an unanticipated monetary disturbance is larger.

increasing the limited participation effect. This arises because the credit constrained firms receive smaller loans than non-credit constrained firms. The larger is the number of credit constrained firms, then, the smaller is the total amount of real loans to the intermediate goods producing sector. To accommodate this, households end up depositing less with intermediaries, on average. Higher monitoring costs have a similar impact on steady state nominal household savings since they imply smaller loans to credit constrained firms.

According to Fisher (1994), the excess sensitivity findings for type b firms (debt flow responses, sales responses and profit responses) are increasing in the markup, decreasing in the profit share parameter and increasing in monitoring costs. With the exception of the monitoring cost result, these findings can be attributed almost entirely to the implied steady state effect on overhead ratios. Increasing the average markup requires higher overhead ratios to keep profits at the fixed share of income. Similarly, increasing the profit share parameter implies lower overhead ratios. These changes have the usual effect on loan size elasticities. Increasing the monitoring cost adds to the second order effects of changes in bankruptcy on the loan contract which amplify the excess sensitivity findings. Interestingly, the excess sensitivity findings are *decreasing* in the share of credit-constrained firms in the model. That is, the fewer credit constrained firms there are, the more sensitive this class of firm is to an unanticipated monetary disturbance.

The last finding described in Fisher (1994) that I will stress relates to the role that differential fixed costs can have in the absence of monitoring costs. From the loan size elasticity discussion in section 2 we know that the partial equilibrium responsiveness of loans to a change in the default free interest rate is decreasing in the size of fixed costs for non credit-constrained firms. This would suggest differential fixed costs of the sort reported in table 1 might lead to excess sensitivity with zero monitoring costs. It is shown in Fisher (1994) that this is in fact true, but the degree of excess sensitivity is quantitatively insignificant relative to that generated by positive monitoring costs. In particular, similarly parameterized AI and PI examples in which overhead costs are the same are compared. In this case excess sensitivity for type b firms is almost 50 times greater in the AI case compared

to the PI case.

5. Concluding remarks

A quantitative dynamic general equilibrium model resembling the credit channel of the monetary transmission mechanism has been constructed. Parameters of the model were identified by using macro and micro observations. On the basis of the parameter estimates obtained, the model is consistent with empirical evidence on the responses of aggregate output, employment and interest rates to unanticipated monetary shocks. It is also consistent with empirical evidence that has been put forward as supportive of the credit channel being operative in the U.S. economy. Namely, the heterogeneous response of firms to monetary shocks.

In terms of providing a general equilibrium accounting of the heterogeneity evidence, therefore, the analysis appears to have been fruitful. From the perspective of providing a basis for evaluating the quantitative importance of the credit channel theory, however, the model developed here has some shortcomings. In this concluding section I describe three of these shortcomings. I then discuss how they influence the conclusions one can draw from the analysis and what features of the model can be improved to overcome them.

One shortcoming is that the quantitative impact of monetary shocks in terms of the variability of output appears to be small relative to observed cyclical output volatility. The impulse responses discussed in the last section were computed for unanticipated monetary injections that are large relative to the aggregate stock of money –considerably larger than the size of a typical injection instigated by the Federal Reserve. Despite this fact, output responds by much less than one per cent to such a monetary shock. This would suggest that the mechanisms in the model are quite weak in terms of generating output variability.

A second and related shortcoming is that the introduction of credit-constrained firms adds to output variability mainly through its effects on bankruptcy costs. At the same time, incorporating this distinguishing feature of the credit channel theory is crucial for accounting for the heterogeneity evidence within the context of the model. Thus, the model

would seem to suggest that the credit channel theory is useful for understanding certain compositional aspects of the monetary transmission mechanism, but may not be important for understanding the behavior of broad aggregates.

The third shortcoming I wish to emphasize concerns the direction of the response of loans in the model. Evidence reported by Christiano, Eichenbaum and Evans (1993), Gertler and Gilchrist (1991), and Oliner and Rudebusch (1992) indicates that at the very least loans for larger firms rise following an unanticipated money contraction. In addition aggregate bank loans do not begin to fall until several periods following a contraction (Bernanke and Blinder (1992)). In the model presented here loans uniformly fall immediately following a contraction.

On initial reading, the first two of these shortcomings may not appear to be shortcomings at all. One may want to conclude that the model implies the credit channel theory is not important for accounting for output variability due to monetary shocks. There are two main problems with this conclusion. First, it ignores the fact that the propagation mechanisms in the model are extremely weak: monetary disturbances in the model have their main impact only in the period of the shock. An interpretation of the findings that is more favorable to the credit channel theory is that the mechanisms in the model are part of an initial impulse effect that other mechanisms amplify and propagate over many periods. Thus, an improvement to the model which would help provide a better evaluation of the credit channel theory would be to incorporate realistic propagation. One potentially important method for propagation, in the spirit of the credit channel theory, is the internal funds mechanism described by Bernanke and Gertler (1989).

Second, the limited participation assumption as a source of monetary nonneutrality limits the potential for costly state verification to amplify the effects of monetary shocks.³⁶ Other mechanisms for monetary nonneutrality, such as sticky wages or prices, may leave room for quantitatively significant amplification effects due to this friction. While this is certainly

³⁶This is because the response of hours to a monetary disturbance in the model is determined entirely by the limited participation effect, as in Christiano and Eichenbaum (1992c).

a topic worthy of further research, a caveat suggested by the current analysis should be considered. It was found here that the proportion of credit constrained firms in the economy is small. In addition, the degree of heterogeneity in the response of firms to monetary shocks in the model is inversely related to the proportion of firms that are credit constrained. Evidence of strong heterogeneity among firms responses may thus be indicative of credit constrained firms being a small fraction of the total number of firms in the economy. Any future analysis, then, must contend with the possibility that firms that face credit market frictions may be too small to be quantitatively significant in terms of accounting for the behavior of broad aggregates.

The failure of the model with respect to the response of loans is perhaps the most troubling. It suggests that in the periods immediately following a monetary contraction it is the demand for loans that changes as much as it is the supply. This may mean some other friction, in addition to or instead of, limited participation, works to initially propagate a monetary shock. However, the feature of the credit channel that gives rise to heterogeneous responses of firms which was emphasized here may still be operative. In addition, other frictions related to the credit market imperfections stressed here could work to propagate any initial response generated by other frictions. Not until these considerations have been addressed will it be possible to fully assess the quantitative importance of the credit channel and credit market imperfections for understanding the monetary transmission mechanism.

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A. Miscellaneous details of the model

In this appendix I collect results related to the loan contract model and the general equilibrium monetary model. First I describe in more detail the contract problem as it applies to the monetary model. Then I derive formulas used in the analysis of the monetary model and report properties that the idiosyncratic shocks must satisfy for the model to be consistent with perfect competition and zero profits in the final good producing sector.

A.1. Details of the contract environment

In this section I discuss three details of the contract environment. First I discuss existence and uniqueness of the optimal contract. Second I describe the loan size elasticities for the type a and type b firms of the monetary model. Finally, I describe the derivation of the demand for loans by type b firms in the case that the rate of return constraint is ignored.

To analyze the contract environment I focus on the version of the contract problem given in section 3. For convenience I reproduce that problem here. The loan size granted to a typical type b firm, l^b and the bankruptcy cutoff for these firms, γ , solve

$$\max_{\{l^b > \xi^b, \underline{\Theta} < \gamma < \bar{\Theta}\}} q [z l^b - \xi^b]^{\frac{1}{\psi}} \left\{ \int_{\gamma}^{\bar{\Theta}} \theta dG(\theta) - \gamma [1 - G(\gamma)] \right\} \quad (\text{A.1})$$

subject to

$$\frac{q [z l^b - \xi^b]^{\frac{1}{\psi}}}{l^b} \Gamma(\gamma) - \mu G(\gamma) - R = 0, \quad (\text{A.2})$$

here I have dropped the dependence of z on r and w , and $q \equiv p Y^f \frac{\psi-1}{\psi}$. I analyze this problem conditional on fixed values of z and q . In particular, having computed the steady state of the monetary model, I analyze the contract problem in partial equilibrium conditional on the implied steady state values of z and q .

The objective function for this problem is quasiconcave over its domain, but the rate of return constraint is not. Thus standard global quasiconcave programming arguments cannot be applied. However, it is easy to show that the local conditions for optimality are satisfied at a solution to the problem's first order conditions, for values of z and q that correspond to parameter sets considered in the main text and in the sensitivity analysis described in Fisher (1994). By numerical analysis it can also be shown that this local solution corresponds to a global solution and that if there is an interior optimum, it is unique. I have carried out this analysis for values of z and q implied by the parameter values used in the main text and the values used in Fisher (1994).

To establish existence and uniqueness numerically, I reduced the problem to a one variable optimization problem by using (A.2) to define l^b as an implicit function of γ . It is then straightforward to plot the objective as a function of γ only. This function is always smooth and hump shaped on the support of G . Thus the one dimensional problem always has a unique solution characterized by its first order conditions. This means the first order conditions of the original problem completely characterize its solution as long as the implied solution is interior.

The first order conditions of the problem defined by (A.1) and (A.2) are

$$\frac{qz}{\psi} [zl^b - \xi^b]^{\frac{1}{\psi}-1} [1 - \Gamma(\gamma)] + v\Gamma(\gamma) \frac{q}{l^b} \left[\frac{zl^b}{\psi} [zl^b - \xi^b]^{\frac{1}{\psi}-1} - [zl^b - \xi^b]^{\frac{1}{\psi}} \right] = 0 \quad (\text{A.3})$$

$$ql^b [zl^b - \xi^b]^{\frac{1}{\psi}} [1 - G(\gamma)] - v \left[q [zl^b - \xi^b]^{\frac{1}{\psi}} [1 - G(\gamma)] - \mu g(\gamma) l^b \right] = 0 \quad (\text{A.4})$$

and (A.2). Here v is the multiplier on the rate of return constraint and g is the probability density function of the composite idiosyncratic shock. Eliminating the multiplier from these equations one arrives at

$$\frac{q [zl^b - \xi^b]^{\frac{1}{\psi}}}{l^b} \Gamma(\gamma) - \mu G(\gamma) - R = 0, \quad (\text{A.5})$$

$$\frac{q}{l^b} [zl^b - \xi^b]^{\frac{1}{\psi}} \left[\Gamma(\gamma) \frac{zl^b - \xi^b}{zl^b} - \frac{1}{\psi} \right] [1 - G(\gamma)] + \frac{\mu g(\gamma)}{\psi} [1 - \Gamma(\gamma)] = 0. \quad (\text{A.6})$$

This system of equations implicitly defines the optimal contract. The optimal loan size for a type a firm is given by the solution to

$$\frac{qz}{\psi} [zl^a - \xi^a]^{\frac{1}{\psi}-1} - R = 0. \quad (\text{A.7})$$

I now show how the direct effect elasticity from section 2, \hat{e}_R^{AI} , and the full effect elasticity e_R^{PI} depend on overhead the overhead ratio. Denote the overhead ratio for type a firms as $\Upsilon^a \equiv \xi^a / (zl^a)$ and for type b firms as $\Upsilon^b \equiv \xi^b / (zl^b)$. From equation (A.7) it is straightforward to show

$$e_R^{PI} = \left| \frac{R \partial l^a}{l^a \partial R} \right| = \frac{\psi}{\psi - 1} [1 - \Upsilon^a].$$

Clearly, $\partial e_R^{PI} / \partial \Upsilon^a < 0$. That is, the sensitivity of type a firms to changes in the default-free interest rate is decreasing in the size of overhead costs. As in section 2 \hat{e}_R^{AI} is computed using equation (A.5) assuming that γ is fixed. It is straightforward to show

$$\hat{e}_R^{AI} = \left| \frac{R \partial l^a}{l^a \partial R} \right|_{\gamma \text{ fixed}} = \frac{R}{R + \mu G(\gamma)} \cdot \frac{\psi}{\psi - 1} \cdot \frac{1 - \Upsilon^b}{1 - \frac{\psi}{\psi-1} \Upsilon^b}.$$

Using this expression one can compute $\partial \hat{e}_R^{AI} / \partial \Upsilon^b > 0$. These facts underlie my statements in the main text that, for sufficiently large overhead costs, type b firms display (partial equilibrium) excess sensitivity to changes in the default free interest rate relative to type a firms.

To derive the amount of “credit rationing” at the steady state of the monetary model, I examine the following modified version of the contract problem.

$$\max_{\{l^* > \xi^b, \underline{\theta} < \gamma^* < \bar{\theta}\}} q [zl^* - \xi^b]^{\frac{1}{\psi}} \left\{ \int_{\gamma^*}^{\bar{\theta}} \theta dG(\theta) - \gamma^* [1 - G(\gamma^*)] \right\} \quad (\text{A.8})$$

subject to

$$\frac{\gamma q [z l^* - \xi^b]^{\frac{1}{\psi}}}{l^*} = R^b, \quad (\text{A.9})$$

where R^b is assumed to be parametric to the firm. This is a hypothetical problem in which the firm is assumed to be an interest rate taker at the steady state value of R^b , takes into account how its choice of loan size influences the probability of bankruptcy, but ignores the influence of its choices on intermediary returns. It is straightforward to confirm that the solution to this problem is completely characterized by its first order conditions. These can be simplified to the following system of equations that implicitly define the values $\{l^*, \gamma^*\}$:

$$\frac{z}{\psi} [1 - \Gamma(\gamma^*)] - \gamma^* [1 - G(\gamma^*)] \left[\frac{z l^* - \xi^b}{l^*} - \frac{z}{\psi} \right] = 0$$

and (A.9). These equations were used to compute the amounts of steady state rationing reported in section 4.

A.2. Results for the general equilibrium model

In this section I derive formulas for total sales of type a and type b firms and for aggregate output of the final good producing sector. For the model to be consistent with perfect competition and zero profits in the final good producing sector, the idiosyncratic shocks must satisfy a consistency property. This is reported here also. Finally, I show that ψ equals the average markup over marginal cost for the intermediate goods producing firms.

Total nominal sales for type a firms equal $\int_0^\lambda p_i y_i di$. Using the price formula for type a firms given in section 3, the nominal sales of a typical type a firm are given by

$$p_i y_i = \frac{\theta_i}{\omega_i} p Y^f \frac{\psi-1}{\psi} [z(r, w) l^a - \xi^a]^{\frac{1-\psi}{\psi}} \omega_i [F(k^a, h^a) - \xi^a].$$

Since $z(r, w) l^a = F(k^a, h^a)$, this reduces to

$$p_i y_i = \theta_i p Y^f \frac{\psi-1}{\psi} [F(k^a, h^a) - \xi^a]^{\frac{1}{\psi}}.$$

I assumed in section 3 that the expected value of θ_i equals unity. Thus

$$\int_0^\lambda p_i y_i di = \lambda p Y^f \frac{\psi-1}{\psi} [F(k^a, h^a) - \xi^a]^{\frac{1}{\psi}}.$$

An expression for total nominal sales of type b firms can be similarly derived. That is,

$$\int_\lambda^1 p_i y_i di = (1 - \lambda) p Y^f \frac{\psi-1}{\psi} [F(k^b, h^b) - \xi^b]^{\frac{1}{\psi}}.$$

To arrive at formulas for real sales, just divide these expressions by the money price of the final good, p .

Now, by constant returns to scale, final good producers make zero profits in equilibrium.

This means

$$pY^f - \int_0^1 p_i y_i di = 0.$$

Using the formulas derived above,

$$pY^f = \lambda pY^f \frac{\psi-1}{\psi} [F(k^a, h^a) - \xi^a]^{\frac{1}{\psi}} + (1-\lambda)pY^f \frac{\psi-1}{\psi} [F(k^b, h^b) - \xi^b]^{\frac{1}{\psi}}.$$

Dividing through by p and by $Y^f \frac{\psi-1}{\psi}$, and then applying the exponent ψ to both sides of the resulting expression, one arrives at the following formula for total final good production,

$$Y^f = \left[\lambda [F(k^a, h^a) - \xi^a]^{\frac{1}{\psi}} + (1-\lambda) [F(k^b, h^b) - \xi^b]^{\frac{1}{\psi}} \right]^\psi.$$

I now describe the consistency property that the idiosyncratic shocks must satisfy. By constant returns to scale and perfect competition, each final good producer must earn zero profits in equilibrium. For each final good producer $j \in [0, 1]$ this implies

$$p \left[\int_0^\lambda \phi_{ij} y_{ij}^{1/\psi} di + \int_\lambda^1 \phi_{ij} y_{ij}^{1/\psi} di \right]^\psi - \int_0^1 p_i y_{ij} di = 0.$$

If we substitute the input demand function for producer j given by (3.2), substitute the formula for the equilibrium price of input i , $i \in A$ and of input i , $i \in B$, given by (3.16) and (3.17), respectively, multiply the resulting expression by $p \frac{1}{\psi-1} Y^f$, and rearrange, one arrives at

$$\begin{aligned} & \left[\int_0^\lambda \phi_{ij}^{\frac{\psi}{\psi-1}} \Delta_i^{\frac{1}{1-\psi}} y^{a^{1/\psi}} di + \int_\lambda^1 \phi_{ij}^{\frac{\psi}{\psi-1}} \Delta_i^{\frac{1}{1-\psi}} y^{b^{1/\psi}} di \right]^\psi \\ & - Y^f \frac{\psi-1}{\psi} \left[\int_0^\lambda \phi_{ij}^{\frac{\psi}{\psi-1}} \Delta_i^{\frac{1}{1-\psi}} y^{a^{1/\psi}} di + \int_\lambda^1 \phi_{ij}^{\frac{\psi}{\psi-1}} \Delta_i^{\frac{1}{1-\psi}} y^{b^{1/\psi}} di \right] = 0, \end{aligned} \quad (\text{A.10})$$

where $\Delta_i \equiv \theta_i/\omega_i$, $\forall i \in [0, 1]$, $y^a \equiv F(k^a, h^a) - \xi^a$ and $y^b \equiv F(k^b, h^b) - \xi^b$.

To ensure that this condition holds in equilibrium I assume the following property is satisfied by the idiosyncratic disturbances in the model:

$$E_i \left[\phi_{ij}^{\frac{\psi}{\psi-1}} \right] = E_i \left[\Delta_i^{\frac{1}{1-\psi}} \right]^{-1}, \quad \forall j \in [0, 1]. \quad (\text{A.11})$$

To see that this property is sufficient notice that, for each $j \in [0, 1]$,

$$\frac{\int_0^\lambda \phi_{ij}^{\frac{\psi}{\psi-1}} \Delta_i^{\frac{1}{1-\psi}} di}{\lambda} = \frac{\int_0^\lambda \phi_{ij}^{\frac{\psi}{\psi-1}} di}{\lambda} \cdot \frac{\int_0^\lambda \Delta_i^{\frac{1}{1-\psi}} di}{\lambda} = 1.$$

The first equality follows from the independence of ϕ_{ij} and Δ_i for each fixed $j \in [0, 1]$. The

second equality follows directly from (A.11). It should now be clear that

$$\int_0^\lambda \phi_{ij}^{\frac{\psi}{\psi-1}} \Delta_i^{\frac{1}{1-\psi}} di = \lambda. \quad (\text{A.12})$$

A similar derivation leads to

$$\int_\lambda^1 \phi_{ij}^{\frac{\psi}{\psi-1}} \Delta_i^{\frac{1}{1-\psi}} di = 1 - \lambda. \quad (\text{A.13})$$

It is now easy to see that the zero profit condition is satisfied in equilibrium if (A.11) holds by examining (A.10), (A.12) and (A.13).

I now derive the result that the average markup for intermediate goods producers is given by ψ . First, consider the case of a typical type a firm. Combining the price formula for type a firms with (A.7) we have

$$p_i = [\theta_i \psi] \left[\frac{1}{\omega_i z(r, w)} R \right], \quad i \in A. \quad (\text{A.14})$$

The second term on the right hand side of the inequality is the marginal cost of funds, adjusted so that it is in real terms. The first term is the markup for a typical type a firm, κ_i , $i \in A$. Now, the average markup for type a firms, κ^a , is the expected value of the markup for a typical type a firm. But this is just $\kappa^a = \psi$.

An expression for type b firms analogous to (A.14) can be derived as follows. First, combining (A.5), (A.6) and (A.9) it is straightforward to show that

$$p Y^f \frac{\psi-1}{\psi} [z(r, w)l^b - \xi^b]^{\frac{1-\psi}{\psi}} = \frac{\psi}{z(r, w)} \cdot \frac{R + \mu G(\gamma)}{1 - \frac{\mu g(\gamma)}{1-G(\gamma)} \cdot \frac{\gamma}{R^b} \cdot [1 - \Gamma(\gamma)]}.$$

Substituting this expression into the price formula for a typical type b firm yields

$$p_i = [\theta_i \psi] \left[\frac{1}{\omega_i z(r, w)} \cdot \frac{R + \mu G(\gamma)}{1 - \frac{\mu g(\gamma)}{1-G(\gamma)} \cdot \frac{\gamma}{R^b} \cdot [1 - \Gamma(\gamma)]} \right], \quad i \in B.$$

The second term on the right hand side of the equality in this expression can be thought of as the marginal cost of funds for a type b firm. The adjustment so that marginal cost is written in real terms is more complicated than for type a firms because of the filter of the contract. For marginal cost appropriately measured, then, the markup for a typical type b firm is $\kappa_i = \theta_i \psi$, $i \in B$. The average markup for type b firms, κ^b , is thus the same as for type a firms: $\kappa^b = \psi$.

Computing the average markup for intermediate goods producers as a whole involves averaging over the markups for type a and type b firms. This is just $\lambda \kappa^a + (1 - \lambda) \kappa^b = \psi$, the desired result.

B. Solution method

In this appendix I describe how the different versions of the monetary model were solved. The method is closely related to procedures outlined in King, Plosser and Rebelo (1988) and Blanchard and Kahn (1980). However, the state-space methods these authors have applied are not directly applicable in the limited participation framework. Here I describe a vectorized version of Christiano (1991)'s undetermined coefficient method which is designed to accommodate the limited participation assumption. In addition to describing the details of the solution procedure I discuss the existence and uniqueness of the approximate solution.

B.1. The Euler equations

Each version of the model shares a common set of Euler equations. These are derived from the representative household's problem, given in (3.13), (3.14) and (3.15). They are

$$\frac{1}{C} \frac{w}{p} - \frac{\eta}{T - H} = 0, \quad (\text{B.1})$$

$$E \left\{ \frac{1}{C'} \frac{1}{p'} \frac{p}{1 + x'} - \beta \frac{1}{C''} \frac{1}{p''} \frac{r' + (1 - \delta)p'}{1 + x''} \mid \Omega_1 \right\} = 0. \quad (\text{B.2})$$

$$E \left\{ \frac{1}{C} \frac{1}{p} - \beta \frac{1}{C'} \frac{1}{p'} \frac{R}{1 + x'} \mid \Omega_0 \right\} = 0, \quad (\text{B.3})$$

Equation (B.1) is just the efficiency condition for household labor supply. The second Euler equation is the households efficiency condition for capital accumulation. Equation (B.3) displays the household's efficiency condition for nominal savings (the choice of n). The PI version of the model is solved by log-linearizing these Euler equations around the steady state. For the AI version of the model I add another equation to this system:

$$l^a - Y^f \left[\frac{pz(r, w)^{1/\psi}}{\psi R} \right]^{\frac{\psi}{\psi-1}} - \frac{\xi^a}{z(r, w)} = 0. \quad (\text{B.4})$$

This is the equation for loan demand of a typical type a firm that was originally given in (3.8). To apply the log-linearization procedure it is necessary to reduce the above equations into functions of a small number of state and control variables. I now describe how to go about doing this for the different versions of the model.

I consider the case of the perfect information model first. In this case I need to show that equations (B.1), (B.2) and (B.3) can be written as

$$e^1(H, n, x') = 0, \quad (\text{B.5})$$

$$E \left\{ e^2(K, K', K'', H, H', n, n', n'', x', x'', x''') \mid \Omega_1 \right\} = 0, \quad (\text{B.6})$$

$$E \left\{ e^3(K, K', H, n, n', x', x'') \mid \Omega_0 \right\} = 0. \quad (\text{B.7})$$

To accomplish this task I make use of the following equations, which must hold in equilibrium:

$$l^b - Y^f \left[\frac{pz(r, w)^{1/\psi}}{\psi R} \right]^{\frac{\psi}{\psi-1}} - \frac{\xi^b}{z(r, w)} = 0. \quad (\text{B.8})$$

$$\lambda l^a + (1 - \lambda)l^b = n + x \quad (\text{B.9})$$

$$Y = \left[\lambda(F(k^a, h^a) - \xi^a)^{1/\psi} + (1 - \lambda)(F(k^b, h^b) - \xi^b)^{1/\psi} \right]^\psi \quad (\text{B.10})$$

$$pC = 1 - n + wH \quad (\text{B.11})$$

along with (B.4), (3.5), (3.6), (3.18), (3.19), (3.21), (3.20) and (3.22). Equation (B.8) is the loan demand equation for a typical type b firm in the perfect information model. In equation (B.9) I have reproduced (3.11) as an equality. Equation (B.10) is the expression, derived in appendix A, for aggregate output of the final good producing sector. Since monitoring costs are zero in the perfect information model, this equals aggregate supply. Equation (B.11) just states that the household's cash-in-advance constraint must bind in an equilibrium (this will be true if $R > 1$, which is always the case in the parameterizations considered in section 4 of the main text).

Equation (B.5) can be derived as follows. First, combine (3.6), (B.9) and (3.19) to arrive at

$$w = \frac{(1 - \alpha)(n + x')}{H}. \quad (\text{B.12})$$

Now, by examining (B.12) and (B.11) it is easy to see how to formulate (B.5). The derivation of (B.6) is as straightforward. First, combine (3.5), (B.9) and (3.18) to arrive at

$$r = \frac{\alpha(n + x')}{K}. \quad (\text{B.13})$$

Second, by combining (B.4), (B.8) and (B.9) it follows that

$$l^a = n + x' + \frac{(1 - \lambda)(\xi^a - \xi^b)}{z(r, w)} \quad (\text{B.14})$$

and

$$l^b = n + x' + \frac{\lambda(\xi^b - \xi^a)}{z(r, w)}. \quad (\text{B.15})$$

It should now be clear that by combining (B.12), (B.13), (B.14), (B.15), (3.5), (3.6) and (B.10) it is possible to write aggregate supply as a function of K , H , n and x' . If we combine this function for aggregate supply with (3.21) we arrive at an expression for consumption as a function of K , K' , H , n and x' . Using (B.11), (B.12) and the consumption function we can derive p as a function of K , K' , H , n and x' . The function (B.6) can be formulated by using (B.11), (B.13) and the function for p . To derive (B.7) use (B.4) and (B.14) to compute R as a function of K , K' , H , n and x' . By combining this function with (B.11) one arrives at (B.7).

In the asymmetric information case I need to show that (B.1), (B.2), (B.3) and (B.4) can

be written as

$$\hat{e}^1(H, n, x') = 0, \quad (\text{B.16})$$

$$E \left\{ \hat{e}^2(K, K', K'', H, H', n, n', n'', l^a, l^{a'}, x', x'', x''') \mid \Omega_1 \right\} = 0, \quad (\text{B.17})$$

$$E \left\{ \hat{e}^3(K, K', H, n, n', l^a, x', x'') \mid \Omega_0 \right\} = 0, \quad (\text{B.18})$$

$$\hat{e}^4(K, K', H, n, l^a, x', x'') = 0. \quad (\text{B.19})$$

To accomplish this task I make use of

$$l^b = \frac{n + x' - \lambda l^a}{(1 - \lambda)}, \quad (\text{B.20})$$

$$Y^f = \left[\lambda(F(k^a, h^a) - \xi^a)^{1/\psi} + (1 - \lambda)(F(k^b, h^b) - \xi^b)^{1/\psi} \right]^\psi, \quad (\text{B.21})$$

$$Y = Y^f - \frac{\mu G(\gamma)(1 - \lambda)l^b}{p}, \quad (\text{B.22})$$

(3.5), (3.6), (3.18), (3.19), (3.21), (3.20), (3.22), (A.5), (A.6), (B.4), (B.9), (B.10), (B.11), (B.12) and (B.13). Equation (B.20) is just a rearrangement of (B.9), (B.21) is the expression for aggregate production in the final good sector, and (B.22) is the expression defining aggregate output.

The key step in deriving (B.16)-(B.19) is to formulate the following

$$Y = a_1 Y^f(K, H, n, l^a, x') + a_2 [k' - (1 - \delta)K], \quad (\text{B.23})$$

where

$$a_1 = \frac{1 - n + (1 - \alpha)(n + x')}{1 - n + (1 - \alpha)(n + x') + \mu G(\gamma)(1 - \lambda)l^b}$$

and

$$a_2 = \frac{\mu G(\gamma)(1 - \lambda)l^b}{1 - n + (1 - \alpha)(n + x') + \mu G(\gamma)(1 - \lambda)l^b}.$$

Here I have written total final good production as a function of K , H , n , l^a and x' . This is possible by making use of (3.6), (3.5), (B.12), (B.13) and (B.21). To arrive at (B.23) I combine (3.21), (B.11) and (B.22). With (B.23) in hand it is straightforward to compute p as a function of K , K' , H , n , l^a , x' and γ using (B.11) and (3.21). By substituting $Y^f(K, H, n, l^a, x')$, the expression for p and (B.20) into (A.6) we arrive at an expression that defines γ implicitly as a function of K , K' , H , n , l^a and x' .³⁷ Substituting this function along with $Y^f(K, H, n, l^a, x')$, the expression for p and (B.20) into (A.5) we can compute R as a function of K , K' , H , n , l^a and x' . We now have all the ingredients needed to formulate (B.16)-(B.19). This can be carried out by following steps similar to those used to formulate (B.5)-(B.7), with the obvious modifications.

³⁷To compute γ in this way I use the GAUSS non-linear equation solver NLSYS.

B.2. The log-linearization procedure

I describe how the log-linearization procedure is applied in the perfect information version of the model and then discuss how to modify this procedure for the case of asymmetric information. The first step is to rewrite (B.5)-(B.7) as

$$e^1(\exp \widehat{H}, \exp \widehat{n}, x') = 0, \quad (\text{B.24})$$

$$E \left\{ e^2(\exp \widehat{K}, \exp \widehat{K}', \exp \widehat{K}'', \exp \widehat{H}, \exp \widehat{H}', \exp \widehat{n}, \exp \widehat{n}', \exp \widehat{n}'', x', x'', x''') \mid \Omega_1 \right\} = 0, \quad (\text{B.25})$$

$$E \left\{ e^3(\exp \widehat{K}, \exp \widehat{K}', \exp \widehat{H}, \exp \widehat{n}, \exp \widehat{n}', x', x'') \mid \Omega_0 \right\} = 0, \quad (\text{B.26})$$

where $\widehat{y} \equiv \ln y$. We seek a system of decision rules of the form

$$z_t = Az_{t-1} + B\eta_t \quad (\text{B.27})$$

where

$$\eta_t = H\eta_{t-1} + \varepsilon_t. \quad (\text{B.28})$$

Here, $z_t = [\widehat{K}_{t+1} - \widehat{K}, \widehat{H}_t - \widehat{H}, \widehat{n}_t - \widehat{n}, x_t - \bar{x}]'$, where \widehat{K} , \widehat{H} and \widehat{n} correspond to the logs of steady state capital stock, hours and nominal savings, respectively. Also, $\eta_t = [x_t, x_{t-1}]'$ and

$$H = \begin{bmatrix} 0 & 1 \\ 0 & \rho \end{bmatrix}.$$

I will first describe the computation of the feedback part of the decision rule, A , and then I will turn to how the feedforward term B is computed.

By a certainty equivalence argument, to compute A we can consider a deterministic version of (B.24)-(B.26) where the expectations are ignored. Now, by computing the first-order Taylor series expansion around the (log of the) steady state, the functions e^1 , e^2 and e^3 in this deterministic formulation can be written,

$$ay_{t+1} + by_t = 0, \quad (\text{B.29})$$

where $y_t = [\widehat{K}_t - \widehat{K}, \widehat{K}_{t+1} - \widehat{K}, \widehat{H}_t - \widehat{H}, \widehat{n}_t - \widehat{n}, x_t - \bar{x}]'$,

$$a = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & e_3^2 & e_5^2 & 0 & e_8^2 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$b = \begin{bmatrix} 0 & 0 & e_2^1 & e_3^1 & 0 \\ e_1^2 & e_2^2 & e_4^2 & e_6^2 & e_7^2 \\ e_1^3 & e_2^3 & e_3^3 & e_4^3 & e_5^3 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}.$$

In these expressions e_i^j denotes the partial derivative of e^j , $j = 1, 2, 3$, with respect to its i 'th argument.

I used a procedure described by Jeff Campbell to remove the singularities from this system. The first step in this procedure is to compute the singular value decomposition of a ,

$$USV' = a, \quad (\text{B.30})$$

where here $'$ denotes the transposition operator. Substituting from (B.30) into (B.29) we have

$$USV'y_{t+1} + by_t = 0.$$

Multiplying through both sides of this equation by U' and making use of the fact that $U'U = I$ by definition of the singular value decomposition, we arrive at

$$SV'y_{t+1} + U'by_t = 0.$$

Define $k = \text{rank}(\text{null } a)$. Then if S is ordered so that zero singular values are placed last on its diagonal, then the bottom k rows of $U'by_t$ equal zero. For convenience denote $L = U'$. Then we have that $Lby_t = 0, \forall t$. If we define $\tilde{a} = a + L'Lb$, then this means we can rewrite (B.29) as

$$\tilde{a}y_{t+1} + by_t = 0, \quad (\text{B.31})$$

since

$$[a + L'Lb]y_{t+1} = ay_{t+1} + L'Lby_{t+1} = ay_{t+1}.$$

In the computations carried out for the results reported in section 4 of the main text, \tilde{a} is always invertible. Thus, from (B.31),

$$y_{t+1} = -\tilde{a}^{-1}by_t.$$

Let $\Pi = -\tilde{a}^{-1}b$ and write $\Pi = P\Lambda P^{-1}$, where P is the matrix of right eigenvectors of Π arranged in columns and Λ is a diagonal matrix with the eigenvalues of Π on its diagonal. Suppose the eigenvalues have been placed in order of decreasing absolute value. Using this decomposition we have

$$y_{t+1} = P\Lambda^t P^{-1}y_0.$$

Multiplying through by P^{-1} , and using the definition $\tilde{y}_t = P^{-1}y_t$ gives us

$$\tilde{y}_{t+1} = \Lambda^t \tilde{y}_0.$$

If m elements of y_{t+1} are nonpredetermined then we require $m - k$ eigenvalues of Π to be explosive (*i.e.*, to be greater than $\beta^{-1/2}$ in absolute value). The other k conditions come

from $Lby_t = 0, \forall t$. Let q denote the matrix composed of the rows of P^{-1} corresponding to the explosive eigenvalues. Then,

$$\begin{bmatrix} q \\ Lb \end{bmatrix} y_t = 0$$

is the condition used to solve for the feedback part of the decision rule. Partition $y_t = [y_{p,t}, y_{n,t}]'$, where $y_{n,t} = [\widehat{K}_{t+1} - \widehat{K}, \widehat{H}_t - \widehat{H}, \widehat{n}_t - \widehat{n}]'$ and $y_{p,t} = [\widehat{K}_t - \widehat{K}]$, the vector of non-determined and predetermined variables, respectively. Also, partition accordingly

$$[q' : b'L']' = [Q'_p : Q'_n]'$$

Then the non-predetermined variables can be solved for as follows

$$y_{n,t} = -Q_n^{-1}Q_p y_{p,t} = D y_{p,t}.$$

We can now form the feedback part of the decision rule as $A = [D : 0_{3 \times 2}]$. For all sets of parameter values considered in section 4 of the main text the number of explosive eigenvalues exactly equal the number of non-predetermined variables less the rank of the null space of the matrix a . Thus in these cases the approximate solutions exist and are all unique.

The first step in the computation of the feedforward part of the decision rule is to use the first-order Taylor series approximation to the functions e^1 , e^2 and e^3 to write these functions as

$$e = \alpha_0 z_{t+2} + \alpha_1 z_{t+1} + \alpha_2 z_t + \alpha_3 z_{t-1} + \beta_0 \eta_t + \beta_1 \eta_{t+1} + \beta_2 \eta_t, \quad (\text{B.32})$$

where $e = [e^1, e^2, e^3]'$ and

$$\alpha_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & e_8^2 \\ 0 & 0 & 0 \end{bmatrix}, \quad \alpha_1 = \begin{bmatrix} 0 & 0 & 0 \\ e_3^2 & e_5^2 & e_7^2 \\ 0 & 0 & e_5^3 \end{bmatrix}, \quad \alpha_2 = \begin{bmatrix} 0 & e_1^1 & e_2^1 \\ e_2^2 & e_4^2 & e_6^2 \\ e_2^3 & e_3^3 & e_4^3 \end{bmatrix},$$

$$\alpha_3 = \begin{bmatrix} 0 & 0 & 0 \\ e_1^2 & 0 & 0 \\ e_1^3 & 0 & 0 \end{bmatrix}, \quad \beta_0 = \begin{bmatrix} 0 & 0 \\ 0 & e_{11}^2 \\ 0 & 0 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} 0 & 0 \\ 0 & e_{10}^2 \\ 0 & e_7^3 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} 0 & e_3^1 \\ 0 & e_9^2 \\ 0 & e_6^3 \end{bmatrix}.$$

Making use of (B.27) and (B.28), (B.32) can be written

$$e = [\alpha_0 A^3 + \alpha_1 A^2 + \alpha_2 A + \alpha_3] z_{t-1} + [(\alpha_0 B + \beta_0) H^2 + (\alpha_0 A B + \alpha_1 B + \beta_1) H + (\alpha_0 A^2 B + \alpha_1 A B + \alpha_2 B + \beta_2)] \eta_t.$$

By the construction of A the first term to the right of the equality in this expression is zero. It is convenient to rewrite this expression as

$$e = \tilde{Q} \eta_t,$$

where,

$$\tilde{Q} = [Q_0 B + \beta_0] H^2 + [Q_1 B + \beta_1] H + [Q_2 B + \beta_2].$$

Here,

$$Q_0 = \alpha_0, \quad Q_1 = \alpha_0 A + \alpha_1 \quad \text{and} \quad Q_2 = \alpha_0 A^2 + \alpha_1 A + \alpha_2.$$

For the Euler equations to be satisfied for all possible realizations of x_t and x_{t-1} it must be the case that $\tilde{Q} = 0$. We will use this condition to find the undetermined elements of B . The first step in this procedure is to use the following fact

$$\text{vec } \tilde{Q} = [H^{2'} \otimes Q_0 + H' \otimes QL + I_2 \otimes Q_2] \text{vec } B + \text{vec } [\beta_0 H^2 + \beta_1 H + \beta_2].$$

Write this as

$$\text{vec } \tilde{Q} = \tilde{q} \text{vec } B + \tilde{d}, \tag{B.33}$$

using the obvious notation. Now, define

$$\tilde{Q} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \\ Q_{31} & Q_{32} \end{bmatrix}, \quad \text{and} \quad \hat{Q} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} + \rho Q_{22} & 0 \\ Q_{31} + \rho Q_{32} & 0 \end{bmatrix}.$$

Also, select τ so that $\tau \text{vec } \tilde{Q} = z(\text{vec } \hat{Q})$, where $z(x)$ operates to remove the zero elements from x . This requires choosing

$$\tau = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & \rho & 0 \\ 0 & 0 & 1 & 0 & 0 & \rho \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

For $\tilde{Q} = 0$ it must be the case that $\tau \text{vec } \tilde{Q} = 0$. Using (B.33), this results in

$$\text{vec } B = -[\tau \tilde{q}]^{-1} \tau \tilde{d}.$$

Rearranging the elements of $\text{vec } B$ computed from this expression gives us the desired feed-forward part of the decision rule, B .

In the asymmetric information version of the model I require log-linear decision rule of the form (B.27) and (B.28). In this case

$$z_t = [\widehat{K}_{t+1} - \widehat{K}, \widehat{H}_t - \widehat{H}, \widehat{l}_t^a - \widehat{l}^a, \widehat{n}_t - \widehat{n}, x_t - \bar{x}]',$$

where \widehat{l}^a is the steady state value of the log of the loan size for a typical type a firm. It is straightforward to apply the procedure described above to compute the matrices A and B in the required decision rules. All we need in order to do this is to specify the matrices a , b ,

$\alpha_i, i = 0, 1, 2, 3$, and $\beta_i, i = 0, 1, 2$. These are as follows

$$a = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{e}_3^2 & \tilde{e}_5^2 & 0 & \tilde{e}_8^2 & \tilde{e}_{10}^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 0 & 0 & \tilde{e}_1^1 & \tilde{e}_2^1 & 0 & 0 \\ \tilde{e}_1^2 & \tilde{e}_2^2 & \tilde{e}_4^2 & \tilde{e}_6^2 & \tilde{e}_7^2 & \tilde{e}_9^2 \\ \tilde{e}_1^3 & \tilde{e}_2^3 & \tilde{e}_3^3 & \tilde{e}_4^3 & \tilde{e}_5^3 & \tilde{e}_6^3 \\ \tilde{e}_1^4 & \tilde{e}_2^4 & \tilde{e}_3^4 & \tilde{e}_4^4 & 0 & \tilde{e}_5^4 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix},$$

$$\alpha_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{e}_8^2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \alpha_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \tilde{e}_3^2 & \tilde{e}_5^2 & \tilde{e}_7^2 & \tilde{e}_{10}^2 \\ 0 & 0 & \tilde{e}_5^3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\alpha_2 = \begin{bmatrix} 0 & \tilde{e}_1^1 & \tilde{e}_2^1 & 0 \\ \tilde{e}_2^2 & \tilde{e}_4^2 & \tilde{e}_6^2 & \tilde{e}_9^2 \\ \tilde{e}_2^3 & \tilde{e}_3^3 & \tilde{e}_4^3 & \tilde{e}_6^3 \\ \tilde{e}_2^4 & \tilde{e}_3^4 & \tilde{e}_4^4 & \tilde{e}_5^4 \end{bmatrix}, \quad \alpha_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \tilde{e}_1^2 & 0 & 0 & 0 \\ \tilde{e}_1^3 & 0 & 0 & 0 \\ \tilde{e}_1^4 & 0 & 0 & 0 \end{bmatrix},$$

$$\beta_0 = \begin{bmatrix} 0 & 0 \\ 0 & \tilde{e}_{13}^2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \beta_1 = \begin{bmatrix} 0 & 0 \\ 0 & \tilde{e}_{12}^2 \\ 0 & \tilde{e}_8^3 \\ 0 & 0 \end{bmatrix}, \quad \beta_2 = \begin{bmatrix} 0 & \tilde{e}_3^1 \\ 0 & \tilde{e}_{11}^2 \\ 0 & \tilde{e}_7^3 \\ 0 & \tilde{e}_6^4 \end{bmatrix}.$$

For each parameterization considered for the AI version of the model, the number of explosive eigenvalues equal the number of non-predetermined variables less the rank of the null space of the matrix a . Thus, there always exists a unique approximate solution for the AI version of the model.

Table 1

Baseline parameters and conditions for their identification
in the Asymmetric Information and Perfect Information models

Parameter	Identification condition	PI model	AI model
β	fixed <i>a priori</i>	$1.03^{-0.25}$	$1.03^{-0.25}$
T	fixed <i>a priori</i>	1369	1369
π^*	fixed <i>a priori</i>	0.05	0.05
ψ	$E \{ \psi - E_i \kappa_{i,t} \} = 0$	1.2	1.2
ξ^a	$E \left\{ \frac{\pi^a}{p} - \pi^* Y \right\} = 0$	127.6	130.4
ξ^b	$E \left\{ \frac{\pi^b}{p} - \pi^* Y \right\} = 0$	127.6	122.7
λ	$E \left\{ \frac{L_t}{P_t Y_t} - E \frac{(1-\lambda)G(\gamma)l^b}{PY} \right\} = 0$	0.918	0.918
μ	$E \left\{ E_i COST_{i,t} - E \frac{\mu(1-\lambda)G(\gamma)l^b}{A^*} \right\} = 0$	0	0.093
δ	$E \left\{ 1 - \delta - \frac{K_{t+1} - I_t}{K_t} \right\} = 0$	0.025	0.025
α	$E \left\{ \frac{K_t}{Y_t} - E \frac{K}{Y} \right\} = 0$	0.37	0.37
η	$E \{ H_t - EH \} = 0$	2.99	2.99
σ	$E \left\{ (R_t^B - R_t^{CP}) - E(R^b - R^a) \right\} = 0$	NA	0.038
\bar{x}	$E \{ x_t - \bar{x} \} = 0$	0.014	0.014
ρ_x	$E \{ (x_t - (1 - \rho_x)\bar{x} - \rho_x x_{t-1}) x_{t-1} \} = 0$	0.32	0.32

Notes: The term $E_i \kappa_{i,t}$ denotes the empirical average markup across firms at time t . The term $E_i COST_{i,t}$ denotes the empirical average of bankruptcy costs as a share of assets across failed firms. The variable L_t denotes total liabilities of failed firms. Expected values of variables without time subscripts indicate unconditional means implied by the model. These are approximated by steady-state values. Values with t subscripts correspond to empirical measures of the indicated variables. See the text for a description of the data used to estimate particular parameters. Finally, for the PI case, the parameter λ was fixed *a priori* to the value estimated for the AI case.

Table 2

Features of the steady state in the
Asymmetric Information and Perfect Information models

Variable	PI model	AI model
$(1 - \lambda)G(\gamma)$	0	0.315
$\mu(1 - \lambda)G(\gamma)l^b/(pY)$	0	0.02
l^b/l^a	100	78.4
$(1/p) \int_0^\lambda p_i y_i di$	836.1	845.9
$(1/p) \int_\lambda^1 p_i y_i di$	75.1	60.6
$(l^* - l^b)/l^b$	0	29.2
$\xi^a/(z(r, w)l^a)$	12.3	12.4
$\xi^b/(z(r, w)l^b)$	12.3	14.8
n	0.861	0.861
$\frac{\ln C + \eta \ln(T-H)}{(1-\beta)}$	3719.2	3718.5

Note: All entries are in per cent except the value added, welfare, and nominal savings entries. These are measured in levels.

Table 3

Impact responses following an unanticipated 1 per cent reduction in money growth in the Asymmetric Information and Perfect Information models

Variable	PI model	AI model
Y	-0.131	-0.134
H	-0.181	-0.181
R	129	135
l^b/l^a	0	-0.117
$(1/p) \int_0^\lambda p_i y_i di$	-0.142	-0.124
$(1/p) \int_\lambda^1 p_i y_i di$	-0.142	-0.241
π^a	-0.436	-0.627
π^b	-0.436	-1.40
$(1 - \lambda)G(\gamma)$	0	13.6
$R^b - R$	0	25.4
$(l^* - l^b)/l^b$	0	-0.228

Note: All entries are in per cent except entries involving interest rates. These are measured in basis points at an annual rate.

Table 4

Impact responses following unanticipated 1 per cent reductions versus unanticipated 1 per cent increases in monetary growth in the Asymmetric Information model

Variable	Contraction	Expansion
R	135	-121
l^b/l^a	-0.117	-0.082
$(1/p) \int_0^\lambda p_i y_i di$	-0.124	0.136
$(1/p) \int_\lambda^1 p_i y_i di$	-0.241	0.059
π^a/p	-0.627	0.230
π^b/p	-1.40	0.47
$(1 - \lambda)G(\gamma)$	13.6	-5.3
$R^b - R$	25.4	-9.6

Note: All entries are in per cent except entries involving interest rates. These are measured in basis points at an annual rate.