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by

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February 1998

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Discrimination and Skill Differences in an Equilibrium Search Model

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Abstract: In this paper we analyze an equilibrium search model with three sources for wage and unemployment differentials among workers with the same (observed) human capital but different appearance (race): unobserved productivity (skill), search intensities and discrimination (Becker 1957) due to an appearance-based employer disutility factor. Because these sources affect the earnings distributions differently, empirical identification of these potential sources for the explanation of wage and unemployment differentials is possible. We show that the structural parameters of the model, including the firm's disutility from certain workers, are identifiable using standard labor market survey data. We demonstrate identification using data from the National Longitudinal Survey of Youth. Estimation of these parameters by matching moments from a sample of black and white high school graduates implies: a) blacks have a 9% lower productivity level than whites; b) the disutility factor in employer's preferences is 28% of the white's productivity level; and c) 53% of firms have a disutility factor in their utility toward blacks.

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1. Introduction

Substantial evidence exists on large wage differentials across workers with the same observable productive characteristics (human capital, experience, etc.) but with different appearance (race, gender, etc.). In addition, wage differentials are often accompanied by unemployment rate and job duration differentials. For example, among young male high school graduates the average hourly wage for blacks is about 15% lower than the equivalent for whites. And the unemployment rate of blacks is twice that of whites (15% vs. 7.5%).[1] Two common explanations for these phenomena are unobserved productivity differences and discrimination. The main difficulty is to empirically distinguish between these explanations, since standard reduced form wage regressions cannot separately identify unobserved productivity and discrimination effects (Eckstein and Wolpin, 1996). Using readily available data there is a need for a structural model that distinguishes between the effects of unobserved productivity and discrimination by identifiable parameters.[2]

In this paper we analyze an equilibrium labor market search model that contains both discrimination and skill differences among workers of different appearance.[3] We follow Becker's (1957) theory of discrimination by assuming that there exists a positive fraction of firms/managers that have a disutility taste parameter toward workers with a certain appearance, called type B workers. These workers may also have a lower productivity (skills) than the other type of workers, type A. Workers search for jobs while unemployed and while they are working. Job offer rates are lower for less productive and disliked workers (type B). Employers maximize utility (a function of profits and any disutility) by choosing a wage for workers depending on their skill and appearance. In equilibrium the utility from each type of worker is equalized across like firms. The steady state earnings distributions and unemployment durations are solved endogenously for workers of different

[1] See table 1 below. The empirical literature on the economics of discrimination is vast. Cain's (1986) survey emphasized the earnings gaps. Most of the research on measuring the level of discrimination focused on new data and methods that control for heterogeneity between workers. Recently, Neal and Johnson (1996) found that a large portion of the black-white wage gap can be explained by differences in skill as measured by AFQT scores. Donohue and Heckman (1991) found that the relative position of blacks has improved over time with substantial advancement being made from 1960-1980 with the enforcement of anti-discrimination legislation in the South.

[2] Alternatively the availability of firm data on productivity matched with worker data on wages helps to differentiate between these two explanations (see e.g. Hellerstein and Neumark 1995).

[3] The model is based on the Mortensen (1990) equilibrium search model. Bowlus (1997) used this model to study male-female wage, turnover and duration differentials. Bowlus. Kiefer and Neumann (1997) used the same framework to study black-white wage and duration differences.

appearance and skill. Using the analytical solutions for these distributions, we show that the disutility taste parameter, the fraction of firms with this parameter and the skill differential can be identified using standard labor market survey data. This is possible because unobserved productivity differences and discrimination affect the earnings distributions in distinctly different ways. That is, the presence of discrimination implies the existence of kinks in the earnings distribution, while productivity differences imply a particular stochastic dominance between the earnings distributions of the worker types. Even without productivity differences, the model predicts wage and unemployment differentials that are consistent with the main facts presented for blacks and whites. That is, the mean wage offer and mean earnings of type B workers are lower than those for type A workers, and the unemployment rate and mean unemployment duration are higher for type B workers.

The empirical relevance of this model depends on the possibility of identifying the parameters related to discrimination and skill differences from standard available data as well as matching the main facts. The standard data consist of worker's unemployment durations, job durations, and wages. To show the empirical relevance we estimate the parameters using data on black and white male high school graduates from the National Longitudinal Survey of Youth (NLSY). The unemployment and wage differentials in the sample are consistent with evidence from many other sources. Matching first moments from this data with predicted moments from the model we show that there exists a set of parameters that fit the mean unemployment and wage differentials observed in the NLSY sample and are consistent with the model. Both discrimination and skill differences are important in explaining the black-white wage and unemployment differentials. In the data blacks earn 15% less than whites. We estimate that their productivity is about 9% lower. Furthermore, 53% of the firms have disutility from employing blacks, and their disutility factor is 28% of the white productivity level.

The combination of search and discrimination in our model is most closely related to Black (1995). In Black's equilibrium search model a fraction of firms refuse to hire some workers on the basis of appearance. This leads to a lower reservation wage for those workers and hence a lower mean wage. Our model deviates from Black's in two important ways, first the disutility from hiring type B workers is allowed to be low enough such that all employers hire type B workers albeit at a possibly lower rate, and second workers are allowed to search both on and off the job. Both aspects are important for our results.

In our model we assume that the fraction of firms that have disutility from certain workers is a fixed proportion of the potential employers. Hence, there is no infinite number of potential firms that do not have this disutility and, therefore, the Arrow (1972) critique of Becker (1957) does not hold. That is, there is no infinite supply of firms that can enter the market and in the long run make the proportion of firms with a disutility effectively zero. Moreover, the presence of search friction in the model allows firms to have monopsony power, and therefore, the discriminatory behavior survives in equilibrium.

In the next section we describe the model and section 3 discusses its properties and their relation to the data. In section 4 we discuss identification and show the estimation results. Since the estimates imply discrimination is a factor, we conduct an analysis of equal pay policies in section 5. In comparing worker types of equal productivity we find that equal pay policies do not necessarily eliminate the wage differential. If discriminatory hiring practices are in place, equal pay policies may reduce but cannot eliminate the wage differential between type A and B workers. However, if the equal pay policy is supported by equal offer rates and employment rates for the different worker types, wages and unemployment will be equalized. This result is in contrast to the result by Coate and Loury (1993) who use a version of the statistical discrimination theory studied initially by Arrow (1973). In their model affirmative-action policies may imply that equally productive workers are perceived by employers to be unequally productive.[4]

2. The Model

There are M workers divided into two types: $(1-\theta)M$ are type A and θM are type B. The worker types differ by appearance as well as productivity. Type A (B) workers have productivity level P_A (P_B), where $P_A \geq P_B$ since we assume that type A workers may have a higher skill level. Firms are managed by owners, and they maximize utility that depends on profits and the owner/manager preferences over the types (A and B) of workers. A fraction γ_d of the

[4] There is a large theoretical literature on discrimination. Coate and Loury (1993) provide a nice survey of this literature and the results on affirmative-action policies.

owners/managers have a linear disutility d when having a type B worker (labelled as disutility firms) and $1-\gamma_d$ firms do not have a disutility (labelled as non-disutility firms). The number of firms is normalized to 1. θ and γ_d are exogenously given.

The arrival rates of offers from the firm types vary across worker type and state of employment. We assume that both firm types respond to productivity differences by searching less intensively for the lower productivity type. We also assume that disutility firms search less intensively for type B workers than non-disutility firms because of their distaste for type B workers. In the model these rates are all exogenous.[5] The arrival rate of offers to unemployed (employed) type A workers from both firm types is λ_0 (λ_1), and it is assumed that workers search more intensively while unemployed than while employed, $\lambda_0 > \lambda_1$.

The difference in arrival rates between type A and B workers due to productivity differences is governed by a proportional factor μ, $0 \leq \mu \leq 1$. Firms put less effort into their search for less productive workers, hence, only if $P_A = P_B$ do we set $\mu = 1$. This last case implies a model of discrimination only. The difference in arrival rates between type A and B workers due to disutility is governed by a proportional factor k, $0 \leq k \leq 1$. Disutility firms put less effort into their search for type B workers, the workers they dislike. Then, only if $d=0$ (or $\gamma_d=0$) do we set $k=1$, resulting in a model with pure productivity differences. If $k=0$ disutility firms do not search for and, therefore, do not hire type B workers. If $k=1$ the search intensities for type B workers by disutility and non-disutility firms are the same. In general, the arrival rate of offers to unemployed (employed) type B workers by non-disutility firms is $\mu\lambda_0$ ($\mu\lambda_1$) and by disutility firms is $\mu k\lambda_0$ ($\mu k\lambda_1$). For simplicity the job destruction rate, δ, is assumed to be the same for all workers and firms.[6]

Firms

Managers maximize utility (U) by setting wages for type A and B workers taking the reservation wages and wage offer distributions as given. Firms are allowed to only post one wage

[5] Robin and Roux (1997) endogenize the arrival rates in the Mortensen (1990) model. This important extension complicates the model but, given their work, is not expected to change our main results given the way we set the arrival rates below.

[6] This assumption is relaxed when we estimate the model.

offer for each worker type. That is, wage offers can be conditional on worker type but not on the state or current wage of a worker. Utility is additive in worker type. For non-disutility managers U_n is equal to the firm's profit function.

$$U_n(w_A,w_B) = (P_A - w_A)\, l_n^A(w_A) + (P_B - w_B)\, l_n^B(w_B) \tag{1}$$

where w_i is the wage offered to type i workers by the firm and $l_n^i(w_i)$ is the steady-state labor stock of type i workers for a non-disutility firm offering wage w_i (i=A,B). For disutility managers U_d is equal to profits minus the disutility (d>0) they receive from employing type B workers:

$$U_d(w_A,w_B) = (P_A - w_A)\, l_d^A(w_A) + (P_B - d - w_B)\, l_d^B(w_B). \tag{2}$$

We assume d is small enough such that disutility firms receive positive net utility from employing type B workers.

Workers

Workers maximize utility over an infinite horizon in continuous time by adopting a reservation wage strategy that is state dependent. The reservation wage while employed is the current wage, w. That is, the optimal strategy of an employed worker is to accept any outside wage offer greater than the current wage. The reservation wage while unemployed is solved by equating the value of unemployment with the value of being employed at the reservation wage. The value of being unemployed depends on the value of non-market time and the value of future possible states. These include receiving and accepting an offer from a non-disutility or disutility firm. If no offer is received or if an offer is rejected one continues to receive the value of being unemployed. Thus, the value of unemployment for a type A worker, V_U^A, is given by

$$\begin{aligned}(1+\beta dt)V_U^A &= b\,dt + \lambda_0(1-\gamma_d)\,dt\,E_w^n \max(V_E^A(w), V_U^A) \\ &\quad + \lambda_0\gamma_d\,dt\,E_w^d \max(V_E^A(w), V_u^A) + (1-\lambda_0 dt)V_U^A\end{aligned} \tag{3}$$

where β is the rate of time preference and b is the common value of non-market time. It is the sum of the value of non-market time, the probability of getting a job offer from a non-disutility firm and the expected value of that offer, the probability and the expected value of getting an offer from a disutility firm, and the probability and value of remaining unemployed.

Likewise the value of being employed is a function of the current wage and possible transitions such as accepting an outside offer and moving to another firm or having a job destroyed and moving to unemployment. The value of being employed at wage w (independent of firm type) for a type A worker, $V_E^A(w)$, is given by

$$\begin{aligned}(1+\beta dt)\,V_E^A(w) &= w\,dt + \lambda_1(1-\gamma_d)\,dt\,E_{w'}^{n}\max(V_E^A(w'),V_E^A(w)) \\ &\quad + \lambda_1\gamma_d\,dt\,E_{w'}^{d}\max(V_E^A(w'),V_E^A(w)) + \delta\,dt\,V_U^A \\ &\quad + (1-(\lambda_1+\delta)dt)\,V_E^A(w).\end{aligned} \tag{4}$$

It is the sum of the current wage, the probabilities and expected values of job offers from non-disutility and disutility firms, the probability and value of losing a job and the probability and value of remaining employed at wage w.

The value functions for type B workers differ from equations (3) and (4) because of arrival rate differences and possible wage offer differences. They are given by

$$\begin{aligned}(1+\beta dt)\,V_U^B &= b\,dt + \mu\lambda_0(1-\gamma_d)\,dt\,E_{w}^{n}\max(V_E^B(w),V_U^B) + \\ &\mu k\lambda_0\gamma_d\,dt\,E_{w}^{d}\max(V_E^B(w),V_U^B) + (1-(\mu\lambda_0(1-\gamma_d)+\mu k\lambda_0\gamma_d)dt)\,V_U^B\end{aligned} \tag{5}$$

and

$$\begin{aligned}(1+\beta dt)\,V_E^B(w) &= w\,dt + \mu\lambda_1(1-\gamma_d)\,dt\,E_{w'}^{n}\max(V_E^B(w'),V_E^B(w)) \\ &\quad + \mu k\lambda_1\gamma_d\,dt\,E_{w'}^{d}\max(V_E^B(w'),V_E^B(w)) + \delta\,dt\,V_U^B \\ &\quad + (1-(\mu\lambda_1(1-\gamma_d)+\mu k\lambda_1\gamma_d+\delta)dt)\,V_E^B(w).\end{aligned} \tag{6}$$

Because expectations are taken over wage offers, the worker's value functions are a function of the wage offer distributions of the firm types. Let $F_n^i(w)$ ($F_d^i(w)$) be the endogenously determined wage offer cumulative distribution function (*cdf*) of non-disutility (disutility) firms for type i (i=A,B) workers. These need not be the same, and in general are not, across the worker types. The reservation wage while unemployed for a worker of type i is the value of r_i that solves the equation, $V_E^i(r_i)=V_U^i$. For the value functions given here, r_A and r_B are given by (see Mortensen and Neumann (1988))

$$r_A = b + \int_{r_A}^{\infty} \frac{(\lambda_0-\lambda_1)((1-\gamma_d)(1-F_n^A(w)) + \gamma_d(1-F_d^A(w)))}{\beta+\delta+\lambda_1((1-\gamma_d)(1-F_n^A(w)) + \gamma_d(1-F_d^A(w)))}\, dw \tag{7}$$

and

$$r_B = b + \int_{r_B}^{\infty} \frac{\mu(\lambda_0-\lambda_1)(1-\gamma_d)(1-F_n^B(w)) + \mu k(\lambda_0-\lambda_1)\gamma_d(1-F_d^B(w))}{\beta+\delta+\mu\lambda_1(1-\gamma_d)(1-F_n^B(w)) + \mu k\lambda_1\gamma_d(1-F_d^B(w))}\, dw. \tag{8}$$

For the sake of simplicity, without loss of generality, we assume that β is equal to zero.

Equilibrium

We use the following standard equilibrium conditions (e.g. Mortensen (1990)) to solve for the steady state equilibrium wage offer distribution and labor supply:

a) The reservation wages of the two worker types are utility maximizing given their respective wage offer distributions.

b) The flows of workers in and out of each state are equal.

c) U_n is equalized across non-disutility firms and, given the reservation wage strategies of both worker types and the wage offer strategies of the disutility firms, U_n is maximized.

d) U_d is equalized across disutility firms and, given the reservation wage strategies of both worker types and the wage offer strategies of the non-disutility firms, U_d is maximized.

Because the utility functions are additive in worker types and firms are setting type-specific wage offers, the equilibrium can be solved as if the workers were in separate markets. That is, one can solve for the steady state flows and equilibrium wage offer distributions for type A and B workers separately.

Flow Conditions

In steady state the flows of workers in and out of unemployment and employment at each firm type must be equal. The flow of type A (B) workers into a firm offering wage w_A (w_B) must be equal to the flow out. Let UE^i be the steady-state number of type i unemployed workers and $G^i(w_i)$ be the fraction of type i workers earning w_i or less in steady-state (i=A,B), that is, $G^i(w_i)$ is the earnings *cdf*. Because the arrival rates are the same, the type A labor supply stocks are equal ($l_n^A(w_A)=l_d^A(w_A)=l^A(w_A)$) and the flow condition for each firm type is the same:

$$\begin{gathered}\lambda_0 UE^A + \lambda_1 G^A(w_A)((1-\theta)M - UE^A) = \\ \delta l^A(w_A) + \lambda_1(1-\gamma_d)(1-F_n^A(w_A))l^A(w_A) + \lambda_1\gamma_d(1-F_d^A(w_A))l^A(w_A).\end{gathered} \tag{9}$$

The left hand side of equation (9) represents the inflow of type A workers to a firm paying w_A and the right hand side the outflow. Disutility and non-disutility firm's labor stocks of type B workers are not equal at a given wage because of arrival rate differences. The equilibrium flow conditions that determine the labor stocks of type B workers in non-disutility and disutility firms are, respectively, given by

$$\begin{gathered}\mu\lambda_0 UE^B + \mu\lambda_1 G^B(w_B)(\theta M - UE^B) = \\ \delta l_n^B(w_B) + \mu\lambda_1(1-\gamma_d)(1-F_n^B(w_B))l_n^B(w_B) + \mu k\lambda_1\gamma_d(1-F_d^B(w_B))l_n^B(w_B)\end{gathered} \tag{10}$$

and

$$\begin{gathered}\mu k\lambda_0 UE^B + \mu k\lambda_1 G^B(w_B)(\theta M - UE^B) = \\ \delta l_d^B(w_B) + \mu\lambda_1(1-\gamma_d)(1-F_n^B(w_B))l_d^B(w_B) + \mu k\lambda_1\gamma_d(1-F_d^B(w_B))l_d^B(w_B).\end{gathered} \tag{11}$$

UE^A, UE^B, $G^A(w_A)$ and $G^B(w_B)$ can also be solved from flow conditions. Equations (12) and (13) equate the flows out of and into unemployment for type A and B workers, respectively.

$$\lambda_0(1-\gamma_d)(1-F_n^A(r_A))\,UE^A + \lambda_0\gamma_d(1-F_d^A(r_A))\,UE^A \;=\; \delta((1-\theta)M - UE^A) \tag{12}$$

$$\mu\lambda_0(1-\gamma_d)(1-F_n^B(r_B))\,UE^B + \mu k\lambda_0\gamma_d(1-F_d^B(r_B))\,UE^B \;=\; \delta(\theta M - UE^B) \tag{13}$$

The flow conditions that relate the offer distributions and the earnings distributions are given in (14) and (15). The left hand side of each equation gives the steady-state number of workers who receive acceptable wage offers below w from unemployment, and the right hand side contains the number of workers with wages below w who exit to unemployment or to higher paying firms.

$$\begin{aligned}&[\lambda_0(1-\gamma_d)(F_n^A(w_A)-F_n^A(r_A))+\lambda_0\gamma_d(F_d^A(w_A)-F_d^A(r_A))]\,UE^A = \\ &\quad \delta G^A(w_A)((1-\theta)M-UE^A)+[\lambda_1(1-\gamma_d)(1-F_n^A(w_A))+ \\ &\quad \lambda_1\gamma_d(1-F_d^A(w_A))]\,G^A(w_A)((1-\theta)M-UE^A)\end{aligned} \tag{14}$$

$$\begin{aligned}&[\mu\lambda_0(1-\gamma_d)(F_n^B(w_B)-F_n^B(r_B))+\mu k\lambda_0\gamma_d(F_d^B(w_B)-F_d^B(r_B))]\,UE^B = \\ &\quad \delta G^B(w_B)(\theta M-UE^B)+[\mu\lambda_1(1-\gamma_d)(1-F_n^B(w_B))+ \\ &\quad \mu k\lambda_1\gamma_d(1-F_d^B(w_B))]\,G^B(w_B)(\theta M-UE^B)\end{aligned} \tag{15}$$

Together equations (9)-(15) imply expressions for the labor stocks, unemployment levels and earnings distributions in terms of the wage offer distributions of the firms. We turn our attention now to the solution of the equilibrium wage offer distributions for type A and B workers. The separability of the production function and the supply of labor enables us to analytically solve the equilibrium wage distribution for each type of worker independently.

Wage Distribution: Type A Workers

Let $F^A(w_A)$ be the fraction of all firms paying w_A or less to type A workers

$$F^A(w_A) = (1-\gamma_d)F_n^A(w_A) + \gamma_d F_d^A(w_A). \tag{16}$$

Because the utility from type A workers is equal across the firm types, the wage offer distributions for type A workers are the same, i.e. $F_n^A(w_A)=F_d^A(w_A)=F^A(w_A)$. In addition, no firm offers a wage below the reservation wage, since these offers would always be refused, i.e. $F^A(r_A)=0$. Substituting these conditions into equations (9), (12) and (14) and solving for the labor stock function yields

$$l_n^A(w_A) = l_d^A(w_A) = l^A(w_A) = \frac{(1-\theta)M\kappa_0(1+\kappa_1)}{(1+\kappa_0)(1+\kappa_1(1-F^A(w_A)))^2} \tag{17}$$

where $\kappa_0=\lambda_0/\delta$ and $\kappa_1=\lambda_1/\delta$. $F^A(w_A)$ is solved from the condition that utility is equalized across managers, that is

$$(P_A-r_A)l^A(r_A) = (P_A-w_A)l^A(w_A). \tag{18}$$

Substituting in the labor stock function and solving for $F^A(w_A)$ yields

$$F^A(w_A) = \frac{1+\kappa_1}{\kappa_1} - \left(\frac{1+\kappa_1}{\kappa_1}\right)\left(\frac{P_A-w_A}{P_A-r_A}\right)^{\frac{1}{2}} \qquad r_A \leq w_A \leq wh_A \tag{19}$$

where wh_A is the highest wage paid to type A workers.[7] The reservation wage, r_A, is solved for by substituting $F^A(w_A)$ into equation (7) and wh_A from $F^A(wh_A)=1$. The resulting expressions are

$$r_A = \frac{(1+\kappa_1)^2 b + (\kappa_0-\kappa_1)\kappa_1 P_A}{(1+\kappa_1)^2 + (\kappa_0-\kappa_1)\kappa_1} \tag{20}$$

and

$$wh_A = P_A - \left(\frac{1}{1+\kappa_1}\right)^2 (P_A-r_A). \tag{21}$$

The earnings distribution, $G^A(w_A)$, can be solved from equation (14) and is given by

$$G^A(w_A) = \frac{1}{\kappa_1}\left[\left(\frac{P_A-r_A}{P_A-w_A}\right)^{\left(\frac{1}{2}\right)}-1\right] \qquad r_A \leq w_A \leq wh_A. \tag{22}$$

Figure 1 presents the earnings distribution function of type A workers. The convexity of this function implies that the density is monotonically increasing.[8]

[7]The wage offer distribution equilibrium is the same as in Mortensen's (1990) model with homogeneous firms and workers.

[8]This result is inconsistent with data on wages. The model would therefore need to be modified to empirically implement its full estimation. One possibility is to introduce firm heterogeneity in the form of additional types (e.g., Bowlus, Keifer and Neuman (1997)). Extending the model to include discrete firm heterogeneity does not affect the main results of this paper, and therefore, for simplicity, we maintain the single productivity level assumption.

Wage Distribution: Type B Workers

To solve for the type B wage distribution we follow Mortensen (1990) and show the distribution must be segmented for all $0 \le k \le 1$ with disutility firms only offering low wages and non-disutility firms offering higher wages.[9] Formally we state it in proposition 1.

Proposition 1: If $0 \le k \le 1$, then there exists an equilibrium that satisfies:

$$l_d^B(w_B) = \frac{\mu k \kappa_0 (1+\kappa_1^B)\theta M}{(1+\kappa_0^B)(1+\mu k \kappa_1 \gamma_d (1-F_d^B(w_B)) + \mu \kappa_1 (1-\gamma_d))^2} \qquad r_B \le w_B \le wh_d \tag{23}$$

$$l_n^B(w_B) = \frac{\mu \kappa_0 (1+\kappa_1^B)\theta M}{(1+\kappa_0^B)(1+\mu \kappa_1 (1-\gamma_d)(1-F_n^B(w_B)))^2} \qquad wh_d \le w_B \le wh_B \tag{24}$$

$$F^B(w_B) = \begin{cases} \dfrac{1+\kappa_1^B}{\mu k \kappa_1} - \left(\dfrac{1+\kappa_1^B}{\mu k \kappa_1}\right)\left(\dfrac{P_B - d - w_B}{P_B - d - r_B}\right)^{\frac{1}{2}} & r_B \le w_B \le wh_d \\[2ex] \dfrac{1+\mu\kappa_1}{\mu\kappa_1} - \left(\dfrac{1+\mu\kappa_1(1-\gamma_d)}{\mu\kappa_1}\right)\left(\dfrac{P_B - w_B}{P_B - wh_d}\right)^{\frac{1}{2}} & wh_d \le w_B \le wh_B \end{cases} \tag{25}$$

and

$$G^B(w_B) = \begin{cases} \dfrac{\kappa_0}{\kappa_1 \kappa_0^B}\left[\left(\dfrac{P_B - d - r_B}{P_B - d - w_B}\right)^{\frac{1}{2}} - 1\right] & r_B \le w_B \le wh_d \\[2ex] \dfrac{\kappa_0}{\kappa_1 \kappa_0^B}\left[\dfrac{(1+\kappa_1^B)}{(1+\mu\kappa_1(1-\gamma_d))}\left(\dfrac{P_B - wh_d}{P_B - w_B}\right)^{\frac{1}{2}} - 1\right] & wh_d \le w_B \le wh_B \end{cases} \tag{26}$$

where wh_B is the highest wage offered to type B workers; wh_d is the highest (lowest) wage offered to type B workers by disutility (non-disutility) firms; $\kappa_i^B = \mu\kappa_i(1-\gamma_d) + \mu k \kappa_i \gamma_d$ (i=0,1); and $F^B(w_B)$ is

[9] The assumption of a proportional reduction in the offer arrival rates is more restrictive than needed for this proposition to hold. A sufficient and more general condition for segmentation is $k_1 \le k_0$ where k_1 (k_0) is the disutility firm's reduction in the type B worker's offer arrival rate while employed (unemployed).

the market wage offer distribution, the fraction of all firms paying w_B or less to type B workers ($F^B(w_B)=(1-\gamma_d)F_n^B(w_B)+\gamma_d F_d^B(w_B)$).

Proof: See Appendix A.

Figure 1 presents the segmented type B earnings distribution. The shapes and locations of the type A and B distributions are different, due to both discrimination (d and γ_d) and the productivity differential, but in different ways, which enables us to identify these parameters. In figure 1 we have depicted the type A distribution as stochastically dominating the type B distribution. In the next section we prove that this is a property of the equilibrium.[10]

3. Equilibrium Properties[11]

Wage differentials

Wage differentials between type A and B workers, as well as unemployment rate and duration differences, can be generated in this model through three main mechanisms: productivity differences, search intensity differences and discrimination. In this section we present several propositions describing the main features of the equilibrium in section 2. The first key result is that the type A earnings distribution stochastically dominates the type B distribution.

Proposition 2: If $P_B \le P_A$, $0 \le \mu \le 1$, and $0 \le k \le 1$, then $r_B \le r_A$; $G^A(w) \le G^B(w)$ for all w; and the mean wage offer and mean earnings for type B workers are lower than those for type A workers.

Proof: See Appendix A.

The relationship between the earnings distributions is depicted in figure 1. The lower wages for type B workers stem from their lower productivity level, lower arrival rates, and the influence of disutility firms on the shape of the wage offer distribution. The latter implies that type B workers have lower wages even if their arrival rates and productivity levels are the same as type A workers.

[10] The wage distribution is also segmented if there is discrete firm heterogeneity. The segments are ordered according to net productivity levels (productivity - disutility). In this case a disutility firm with high productivity level may offer a higher wage than a non-disutility firm with a low productivity level.

[11] The properties of this section hold in the presence of discrete firm heterogeneity.

Arrival rate differences do imply an added effect of hindering the movement of type B workers up their wage offer distribution relative to type A workers. This, in turn, drives the earnings distributions further apart.

Because discrimination is a common explanation for wage differentials, proposition 3 establishes the effects of the disutility parameter, d, and the fraction of disutility firms, γ_d, on the wage differential.

Proposition 3: The wage differential between type A and B workers is positively related to d and γ_d.

Proof: See Appendix A.

Thus, the wage differential is a function of the disutility parameter, unlike in Black's (1995) model where only the fraction of disutility firms matters and the competitive framework where no differential emerges only segregated firms. Here complete segregation, i.e. firms with only type A workers, occurs only if μ or k equals zero. As long as k and μ are greater than zero, the labor stocks of all firms are composed of both type A and B workers. If k is less than one, then the fraction of type B workers at disutility firms is less than θ, the population proportion, and the fraction at non-disutility firms is greater than θ.[12] Of course, the productivity differential also affects the wage differential. A widening of this differential widens the wage differential. In addition proposition 4 characterizes the ordering of the productivity and wage differentials.

Proposition 4: If the only difference between types A and B is the productivity level, then the mean wage differential between the types is lower than the productivity differential.

Proof: See Appendix A.

This is a general point that says that search selection for jobs implies that mean wage differentials are lower than the difference in productivity.

[12] In making these compositional comparisons we assume that the placements of the firm in the type A and B wage offer distributions are the same. That is, $F^A(w_A)=F^B(w_B)$ for all firms. Without this assumption it is not possible to make comparisons across firm types with respect to labor stock composition.

Unemployment differentials

The different job offer arrival rates to type A and B workers generate differences in unemployment rates and average unemployment and job durations. If the search intensities for the worker types differ, the effective arrival rates of job offers are different across the worker types. The arrival rate of offers while unemployed for type A workers is λ_0, while it is $\mu\lambda_0(1-\gamma_d)+\mu k\lambda_0\gamma_d$ for type B workers. Since all wage offers are accepted during unemployed search and the arrival processes are Poisson, unemployment durations are exponential with means $1/\lambda_0$ and $1/(\mu\lambda_0(1-\gamma_d)+\mu k\lambda_0\gamma_d)$ for type A and type B workers, respectively. If $0\le k<1$ or $0\le\mu<1$, the mean duration of unemployment is higher for type B workers. Unemployment rates, ue_A and ue_B, are found by solving equations (12) and (13) for UE^A and UE^B and dividing by $(1-\theta)M$ and θM, respectively. A comparison of the two yields the following relationship:

$$ue_B = \frac{\mu\lambda_0(1-\gamma_d)+\mu k\lambda_0\gamma_d}{\delta+\mu\lambda_0(1-\gamma_d)+\mu k\lambda_0\gamma_d} \geq \frac{\lambda_0}{\delta+\lambda_0} = ue_A. \tag{27}$$

Job spell durations are also governed by exit rates. The average exit rates for type A and type B workers are, respectively, given by

$$\int_{r_A}^{wh_A} (\delta+\lambda_1(1-F^A(w_A)))g^A(w_A)dw_A = \frac{\delta(1+\kappa_1)}{\kappa_1}\ln(1+\kappa_1) \tag{28}$$

and

$$\int_{r_B}^{wh_B} (\delta+\mu\lambda_1(1-\gamma_d)(1-F_n^B(w_B))+\mu k\lambda_1\gamma_d(1-F_d^B(w_B)))g^B(w_B)dw_B = \frac{\delta(1+\kappa_1^B)}{\kappa_1^B}\ln(1+\kappa_1^B) \tag{29}$$

where $g^i(w_i)$ is the probability density function of the earnings distribution, $G^i(w_i)$, for type i workers (i=A,B). The expression $(1+x)*\ln(1+x)/x$ is increasing in x. Therefore, since $\kappa_1 \geq \kappa_1^B$, the average exit rate for type A workers is greater than that for type B workers. A higher average exit rate implies shorter average job spell durations for type A workers. Thus, the model is able to generate not only wage differentials but also many of the duration and rate differentials often found in conjunction with wage differences. Note, however, that if there are no search intensity differences (k=1 and μ=1), the mean durations, average exit and unemployment rates are equal.

Profit differentails

Because of the presence of d, the total utility a disutility manager receives from hiring type B workers is lower than that received by non-disutility managers.[13] This is true even if the arrival rates are the same (k=1). Since the utility from hiring type A workers is the same across the firm types, this results in a lower level of utility overall for disutility managers.[14] Profits for non-disutility firms are the same as the utility of a non-disutility manager posting wage offers (w_A,w_B), and therefore, profits are equalized across non-disutility firms. Profits for disutility firms do not equal the utility of their managers because of the disutility. Across the firm types profits are the same from type A workers, but not from type B workers. Proposition 5 characterizes the properties of the profit functions.

Proposition 5: The profit function of disutility firms is non-decreasing in w_B for $r_B \leq w_B \leq wh_d$. Disutility firms earn lower profits than non-disutility firms.

Proof: See Appendix A.

This last result is a general finding in the discrimination literature - that is disutility firms make lower profits. Arrow (1972, 1973) criticized Becker's (1957) model for not being sustainable in the long run because with free entry non-disutility firms can buy out disutility firms. Note that the key assumption here is that there exists a positive fraction of the population of firms that have disutility from type B workers. If there are infinitely many non-disutility firms, then this assumption would not hold.

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[13] This relationship is easiest seen by comparing the utility of a disutility and a non-disutility manager offering wage wh_d to type B workers. The disutility manager has a lower per worker utility because of the presence of d and, if k<1, a lower labor stock of type B workers as well.

[14] If arrival rates were tied to total utility then this ordering may give some justification, besides prejudicial behavior, for disutility managers to offer lower arrival rates to type B workers.

4. Identification and Estimation

Identification

Key to understanding the role discrimination plays in determining wage differentials is the ability to differentiate between discrimination and unobserved productivity differences, and to differentiate between competing discrimination models. In this paper we are concerned with the former and show that for the model presented one can identify all of the underlying parameters using standard labor market data.[15]

The ability to differentiate between unobserved productivity differences and discrimination in our model is possible because each affects the earnings distributions of type A and B workers differently. To see this it is helpful to examine the pure productivity case (d=0) and the pure discrimination case (P_A=P_B) separately. The earnings distributions for type A and B workers are shown in figures 2 and 3 for the pure productivity and pure discrimination cases, respectively. Note that in the pure productivity case the locations of the distributions differ, but their shapes are similar. This is not true in the pure discrimination case. Here a kink emerges in the type B earnings distribution at $G^B(wh_d)$. Thus while the earnings distributions for type A workers are the same under the two cases, they substantially differ for type B workers.

In the pure productivity case (figure 2) the distance between the two distributions is governed by the difference in P_A and P_B. In addition, μ determines the curvature of the type B distribution relative to the type A distribution. The larger the reduction in search intensity, the less convex the type B distribution is. In the pure discrimination case (figure 3) the distance between the two distributions is influenced by the disutility parameter d as it affects the reservation wage of type B workers, the highest wage paid to type B workers, and the lower portion of the type B earnings distribution, i.e., all wages paid by disutility firms, including the location of the kink point. The kink is also directly related to the fraction of disutility firms in the market, γ_d. And finally the curvature of the lower portion of the type B earnings distribution relative to the type A distribution is influenced by k, the reduction in search intensity for type B workers by disutility firms.

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[15] In terms of the discrete firm heterogeneity case, identification is still possible as long as the unobserved productivity differences are the same for all firms and the disutility parameters are the same for all disutility firms.

The differing shapes of the type B earnings distributions lead to strong predictions regarding trends in the wage differential as one moves up the earnings distribution. That is, as one examines higher percentiles of the earnings distribution different predictions emerge as to the direction the differential is moving. These are outlined in proposition 6.

Proposition 6: In the pure productivity case with $\mu=1$ the wage differential between type A and B workers increases as the earnings percentile increases. In the pure discrimination case with $k=1$ the wage differential increases as the earnings percentile increases until $G^B(wh_d)$, after which it decreases.

Proof: See Appendix A.

When both unobserved productivity differences and discrimination are present (figure 1), it is not possible to sign the direction of the wage differential. However, it is clear that only if discrimination is present can the wage differential fall.

Because of these differential effects wages can be used to identify the productivity and discrimination parameters. They can also help with the identification of the arrival rate parameters. However, the main source of identification for the arrival rates comes from information on durations and transitions across states. Type A worker's durations and transitions are governed by, and therefore can identify, the main arrival rate parameters of λ_0, λ_1 and δ. The rates, durations and transitions for type B workers are functions of their effective arrival rates $\mu\lambda_0(1-\gamma_d)+\mu k\lambda_0\gamma_d$ and $\mu\lambda_1(1-\gamma_d)+\mu k\lambda_1\gamma_d$ and δ. Thus, differences in rates, durations and transitions across the types identify the parameter combination $\mu(1-\gamma_d(1-k))$. Since μ and γ_d can be identified from wage data, k is then recoverable from any duration or transition difference across type A and B workers.

We have shown that unobserved productivity differences and discrimination affect the earnings distribution differently in our model and can therefore be identified. It should be noted that this is only possible if the fraction of disutility firms is less than 1. If not, then one cannot distinguish between a market with unobserved productivity differences P_A and P_B and a market with discrimination parameters $d=P_A-P_B$ and $\gamma_d=1$. There are other scenarios that may seem similar to the one presented here. For example, type B workers could come in two productivity types - P_B and P_B-d, while type A workers are only of one type. This case is not equivalent to ours as the productivity differences now all reside with the workers and are not specific to any firm. It resembles the pure productivity case with P_B replaced by the expected productivity of type B workers and therefore does not result in a kinked distribution. To get back to our case one would have to assume that type B workers are only less productive at a certain fraction of firms, γ_d. Now the productivity differences are both worker and firm specific. If the latter is the case, then there needs to be a reason that the type B workers are less productive at some firms than others, when type A workers are equally productive at all firms. One could use a discrimination argument, say customer discrimination, and the analysis would follow through exactly as we have here. Alternatively one could assume separate markets for type A and B workers with some low productivity firms only offering wages to type B workers. However, this leads to segregated firms and is not consistent with a single market endogenously producing wage differentials.

Estimation

We use standard black and white male high school graduate worker data from the NLSY.[16] The estimation is based on matching first moments of unemployment and wages to the moments predicted by the model. This is a simple way to demonstrate the identification of the discrimination and productivity parameters and to check whether the model can match the observed wage and unemployment differentials.[17]

The means, shown in table 1, are from black and white male high school graduates in the NLSY who are participating in the private, full-time labor market. For this group of young black and white males several differentials emerge. Black males have an unemployment rate that is twice that for white males with mean unemployment durations almost 2 months longer. Their mean job duration is shorter and a higher fraction of their jobs end in unemployment. In addition, the mean weekly earnings for black males is 85% that of white males. Thus, within the framework of our model we define white males as type A workers and black males as type B workers. To be able to

[16] Appendix B contains the full description of the sample.

[17] At this stage the specification is not rich enough to be interesting to fit the entire densities of wages and durations by maximum likelihood, as in Bowlus, Kiefer and Neumann (1995,1997) and van den Berg and Ridder (1997). However, because we are matching means, moving to a more complicated setting with firm heterogeneity is unlikely to have much effect on our results. This is because in the presence of firm heterogeneity the means of the earnings and wage offer distributions can be written as functions of the homogeneous means evaluated at the average productivity level plus a term involving the productivity distribution itself. We thank Gerard van den Berg for this result.

match the black unemployment rate as well as the black mean unemployment duration, we allow the job destruction rate δ to vary across worker type in the estimation.

Using mean moments instead of densities, we set up a just identified system of equations.[18] To start we identify λ_0 from the mean unemployment duration for whites. As stated before, any difference in unemployment rates, durations or transitions can identify $\mu(1-\gamma_d(1-k))$. We use the difference in mean unemployment durations to estimate this parameter combination. Equation (27) shows that we can estimate δ_A and δ_B from the unemployment rates of whites and blacks, respectively. The fraction of completed white job spells that end in a transition to unemployment is used to estimate λ_1. We could have also used the mean job duration for whites. However, we found that it was difficult to find a parameter set that could match all of the remaining moments if we used job spell durations.

Turning to the wage moments we simplify the model by assuming that a minimum wage is effective. The main reason is that without measurement error in wages the lowest observed wages are the consistent estimators for the reservation wages. This assumption is not compatible with $r_A > r_B$ and, therefore, we adopt the minimum wage model. The only difference from the model above is that the minimum wage, $\underline{w}$, is now the lowest wage offered by firms and is known a priori. We set the weekly minimum wage at \$134.00. P_A is identified from the mean earnings of whites. Four remaining wage moments are used to identify P_B, d, γ_d and μ. They include the mean earnings of black workers, the mean wage offer of black workers, and the median and 75th percentile wage differentials between black and white workers of 0.831 and 0.862, respectively, from the earnings distributions. Note that the differential is decreasing as the percentile increases. From proposition 6 we know that this type of relationship is only consistent with the model if discrimination is present in the labor market. And, finally, given μ and γ_d we can recover k from our estimate of $\mu(1-\gamma_d(1-k))$. The following restrictions on the parameters are imposed: $P_B>d+\underline{w}$, $P_A>\underline{w}$, $d\geq 0$, $0\leq\gamma_d\leq 1$, $0\leq\mu\leq 1$, $0\leq k\leq 1$, $\lambda_0>0$, $\lambda_1>0$, $\delta_A>0$ and $\delta_B>0$.

[18] Since we do not intend to test the quality of the fit we do not report standard errors. The calculation of standard errors in this case is complicated by our use of percentile wage differentials in our set of moments and would require the use of bootstrapping to recover the covariance matrix for the moments.

Estimation of the parameters allows us to distinguish between the competing hypotheses for the observed wage differential. On the one hand, if d=0 and k=1 or if γ_d=0, then the differences observed in table 1 between black and white males can be attributed to productivity differences only. On the other hand, if P_A=P_B and μ=1, then discrimination is the only source for differences between blacks and whites in the sample.

Column 1 of table 2 shows the parameter estimates that result from the above process. The parameter estimates indicate that productivity, discrimination and search intensity differences play significant roles in the determination of the wage differential. The productivity level of blacks is 91% that of whites. In line with this all firms search slightly less intensively for blacks with μ at .95. We find that 53% of the firms are disutility firms with a disutility level (d) that is 28% of the white productivity level. Disutility firms are also found to substantially reduce their search efforts for black workers with k at .57. It is interesting to note that while disutility firms make up just over 50% of the market, they employ only 13% of black workers ($G^B(wh_d)$=.126). The arrival rates while employed are high enough that most blacks are able to move out of disutility firms and into higher paying non-disutility firms. However, they have a much higher job destruction rate than whites and therefore spend more time in unemployment.

In columns 2 and 3 of table 2 we present the estimates assuming models of pure discrimination and pure productivity differences, respectively. When the restrictions P_A=P_B and μ=1 are in place, the disutility parameter falls but the fraction of disutility firms rises substantially. In contrast, when d=0, γ_d=0 and k=1, the productivity differential increases only slightly but the reduction in search is now much larger with μ at .73. Given the particular specification of the model and the equilibrium distributions, this latter case is equivalent to the case with no productivity differential (P_A=P_B) and all firms are disutility firms (γ_d=1). That is, we can match perfectly well both cases to the same set of moments. However, testing which of the models is a better approximation of reality, requires more elaborate econometric methods.

5. Equal Pay Policies

As we have seen, even if worker types are equally productive, the presence of disutility firms in the labor market generates wage differentials if firms are free to set wage offers conditional on appearance. This differential is widened if, in addition, the disutility firms search less intensively for type B workers. In this section we examine the effects of imposing equal pay restrictions on the firms. That is, each firm must post and pay only one wage. To focus and simplify we do so within the pure discrimination model ($P_A=P_B=P$ and $\mu=1$). There are two cases. The first case is where the offer arrival rates are equal across firms ($k=1$). The existence of disutility firms does not justify this case as possible without policy intervention such as anti-discriminatory hiring legislation. The second case is where search intensities are different ($k<1$). In each case we restrict the job destruction rates to be equal ($\delta=\delta_A=\delta_B$).

Equal Pay When Offer Rates Are Equal

Because the arrival rates are equal, the labor supply behavior of the worker types is the same. The only difference between the firm types is their expected utility per worker: P for a non-disutility firm and $P(1-\theta)+(P-d)\theta=P-\theta d$ for a disutility firm. Like the wage offer distribution for type B workers in section 2, the equilibrium wage offer distribution (for both type A and type B workers), $F(w)$, is segmented with the disutility firms offering lower wages than the non-disutility firms.

$$F(w) = \begin{cases} \dfrac{1+\kappa_1}{\kappa_1} - \left(\dfrac{1+\kappa_1}{\kappa_1}\right)\left(\dfrac{P-\theta d-w}{P-\theta d-r}\right)^{\frac{1}{2}} & r \leq w \leq wh_d \\ \dfrac{1+\kappa_1}{\kappa_1} - \left(\dfrac{1+\kappa_1(1-\gamma_d)}{\kappa_1}\right)\left(\dfrac{P-w}{P-wh_d}\right)^{\frac{1}{2}} & wh_d \leq w \leq wh \end{cases} \tag{30}$$

where

$$wh_d = P-\theta d - \left(\frac{1+\kappa_1(1-\gamma_d)}{1+\kappa_1}\right)^2 (P-\theta d-r) \tag{31}$$

$$wh = P - \left(\frac{1}{1+\kappa_1(1-\gamma_d)}\right)^2 (P-wh_d) \tag{32}$$

and

$$r = \frac{(1+\kappa_1)^2 b + (\kappa_0-\kappa_1)\kappa_1(P-\theta d)}{(1+\kappa_1)^2+(\kappa_0-\kappa_1)\kappa_1} + \frac{(1+\kappa_1)^2(\kappa_0-\kappa_1)\kappa_1(1-\gamma_d)^2 d}{((1+\kappa_1)^2+(\kappa_0-\kappa_1)\kappa_1)(1+\kappa_1(1-\gamma_d))^2}. \tag{33}$$

In this case blacks and whites face the same wage offer distribution as well as the same arrival rates and therefore have the same reservation wage and mean wage offer and earnings. The wage differential is eliminated under equal pay.

To illustrate the effects of equal pay, we return to our empirical example in section 4. In table 2 we presented estimates of the pure productivity model allowing k to vary and the δ's to differ. Here we have assumed k=1 and $\delta_A=\delta_B$. To account for this aspect we impose these additional restrictions and estimate the pure productivity model without equal pay. The resulting parameter estimates are the same as those in the original pure productivity case (column 2, table 2) with the following exceptions: δ_B=.0041, d=54.44 and γ_d=.9243. If we now impose equal pay on the firms, the mean earnings of whites falls from \$270.17 to \$264.62 and the mean earnings of blacks increases from \$230.26 to \$264.62.[19] White workers are hurt by the equal pay legislation while black workers gain. Interestingly both firm types are indifferent to the policy change. The increase in wages they must now pay to black workers is exactly offset by the decrease in wages to white workers.

Equal Pay When Offer Rates Are Not Equal

When disutility firms practice discriminatory hiring (k<1), imposing an equal pay policy does not necessarily result in an elimination of the wage differential. Suppose we take an extreme example of disutility firms refusing to hire type B workers (k=0).[20] Then, in the absence of equal pay legislation the equilibrium is the same as in section 2 with k set equal to 0. With disutility firms not

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[19]These calculations were done assuming θ=.14, the fraction of blacks in the NLSY sample. Increasing θ lowers the mean earnings and offers.

[20]The solution to the case of equal pay when 0<k<1 is currently unsolved. However, the example of k=0 is sufficient to show equal pay policies do not always eliminate the wage differential within this framework.

offering wages to type B workers, the non-disutility firms now operate in the entire wage range for type B workers from r_B to wh_B. The resulting wage offer distribution for type B workers is

$$F^B(w_B) = \frac{1+\kappa_1(1-\gamma_d)}{\kappa_1(1-\gamma_d)} - \left(\frac{1+\kappa_1(1-\gamma_d)}{\kappa_1(1-\gamma_d)}\right)\left(\frac{P-w_B}{P-r_B}\right)^{\frac{1}{2}} \qquad r_B \leq w_B \leq wh_B \tag{34}$$

where

$$r_B = \frac{(1+\kappa_1(1-\gamma_d))^2 b + (\kappa_0-\kappa_1)\kappa_1(1-\gamma_d)^2 P}{(1+\kappa_1(1-\gamma_d))^2 + (\kappa_0-\kappa_1)\kappa_1(1-\gamma_d)^2} \tag{35}$$

and

$$wh_B = P - \left(\frac{1}{1+\kappa_1(1-\gamma_d)}\right)^2(P-r_B). \tag{36}$$

In this case the disutility parameter, d, does not enter into the wage offer distribution. All differences between type A and B workers are driven off the effective arrival rate differences.

Requiring the non-disutility firms to pay the same wage to both worker types, results in the following: (i) the reservation wage of type B workers is still lower than that of type A workers due to the lower effective arrival rates of offers; (ii) it is possible that some non-disutility firms specialize in hiring an all type B work force by offering wages in the $[r_B, r_A)$ range; (iii) disutility firms only offer wages above r_A since they wish to only attract type A workers; (iv) disutility and non-disutility firms allocate themselves along the type A wage range so as to equalize utility among hiring type A workers; and (v) non-disutility firms allocate themselves along the type B wage range (above r_A) so as to equalize the utility generated from hiring type B workers.

Condition (ii) is only true if it is possible to generate as much utility in this range as that generated by offering r_A and attracting both type A and B workers. Simulations show this only occurs when θ and γ_d are large. Since the wage differential remains even without this feature, we assume for simplicity that the conditions for wages in this range to be offered are not met. Because of (iv), type A workers face the same wage offer distribution as they do in the unequal pay case. Given the type A distribution and (v), the resulting wage offer distribution for type B workers is

$$F^B(w) = \frac{1+\kappa_1(1-\gamma_d)}{\kappa_1(1-\gamma_d)} - \frac{1+\kappa_1(1-\gamma_d)}{\kappa_1(1-\gamma_d)}(\frac{P-w}{P-r_A})^{\frac{1}{2}} \qquad r_A \leq w \leq wh_B \tag{37}$$

where r_A is given by equation (20) and wh_B equals

$$wh_B = P - (\frac{1}{1+\kappa_1(1-\gamma_d)})^2(P-r_A). \tag{38}$$

It is straightforward to show that wh_B is less than wh_A (equation (21)). So that while the mean wage offer of type B workers has increased, due to the infeasibility of offering wages below r_A, it is still lower than the mean wage offer of type A workers.

Hence, equal pay policies reduce but do not eliminate the wage differential. In this case, type A workers are indifferent regarding equal wage policies, while type B workers prefer equal pay. Disutility firms are also indifferent seeing the same utility from type A workers, but non-disutility firms reject the equal pay policy since they have to pay type B workers a higher wage and still attract the same labor stock.[21] It is possible in this case for equal pay to have no effect on the wage distribution. In the presence of a binding minimum wage, the lowest wage paid to both worker types under equal pay is still the minimum wage. Replacing r_A and r_B with $\underline{w}$ in equations (34) and (37) shows equal pay brings about no change in the type B wage offer distribution. Because we have a minimum wage in our empirical example in section 4, imposing equal pay would not change the mean earnings of whites and blacks if disutility firms refused to hire blacks (k=0).[22] Thus, the equal pay legislation would be completely ineffective.

Finally, so far the imposition of equal pay has not affected the unemployment rate gap between type A and B workers. If in the example with k=0 we had not assumed conditions were such that the lowest wage paid to type B workers was r_A, but rather r_B ($<r_A$), then the policy would have lowered the unemployment rate gap. The unemployment rate for type B workers would have remained unchanged, but the rate for type A workers would have increased to

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[21] Note, again, that the separability of the production function is a key assumption regarding the segmentation of the labor market.

[22] This minimum wage result is special to the k=0 case.

$$\frac{\delta}{\delta+\lambda_1(1-F(r_A))} \tag{39}$$

since $F(r_A)$ is now greater than zero. That is, there are some wage offers made by non-disutility firms to type A workers that are rejected.

6. Conclusions

In this paper we analytically solve for the equilibrium wage distribution of a search model where workers have different skills that are perfectly correlated with their appearance. Firms are heterogeneous with respect to their preferences regarding worker's appearance and, given the search friction, firms offer different wages to workers of potentially the same quality. As a result, the equilibrium earnings function includes the mean effects of skill differences, appearance differences (race or sex) and an "unobserved" variance that is due to search frictions.

We show that it is possible using standard labor market survey data to identify these three different effects. In particular, the disutility parameter, that affects a fraction of firms and is the reason for the existence of discrimination, is distinguishable from the unobserved productivity differential. This is due to the two sources having different effects on the earnings distributions of the discriminated workers. The presence of discriminatory behavior results in a kinked earnings distribution with the wage differential a function of the preference parameter. This parameter cannot be estimated without a complete specification of the relationship between the preferences of firms and their strategies regarding the wages of workers. We show in this paper how one can estimate the unobserved productivity and discrimination parameters and how changes in both can affect the wage differential. Moreover, we show the implication of the model on equal pay policies.

The model as presented here is consistent with many aspects of the data, but not all. In particular, the earnings distributions are convex in shape not concave as in the data. However, we feel the next step is not to go further with estimating this particular model but rather to nest other possible discrimination models within a single framework and determine which models are consistent with the data. This must come first before one pursues the best fit of the data with any one particular model.

Figure 1. Earnings Distributions of Type *A* and *B* Workers

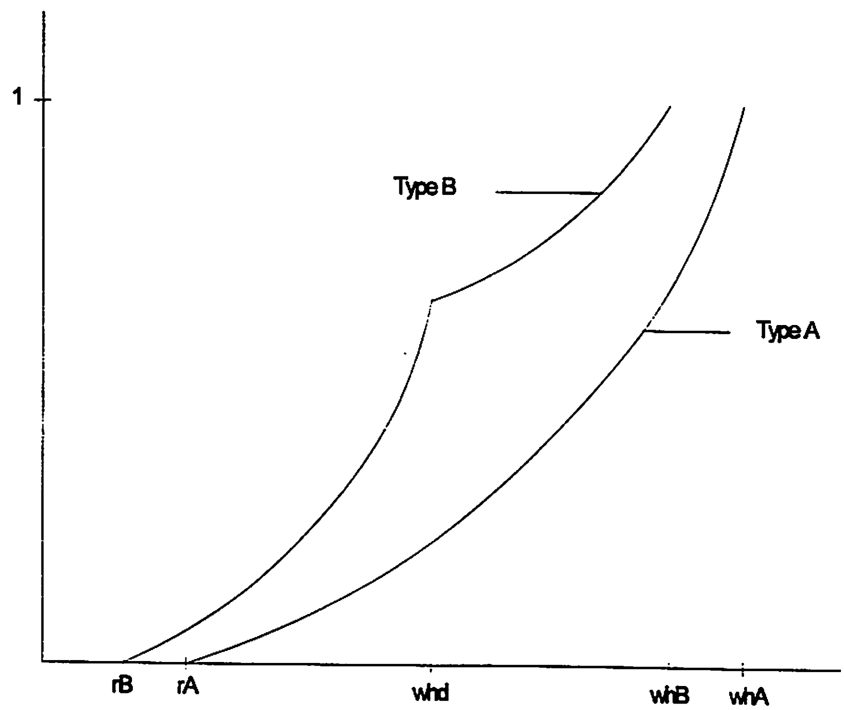

Figure 2. Pure Productivity Case

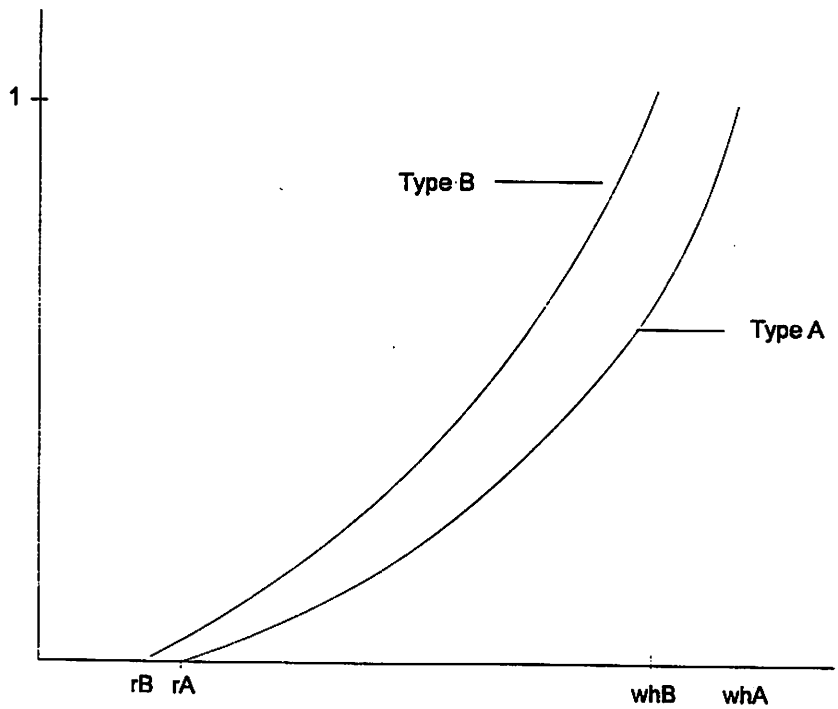

Figure 3. Pure Discrimination Case

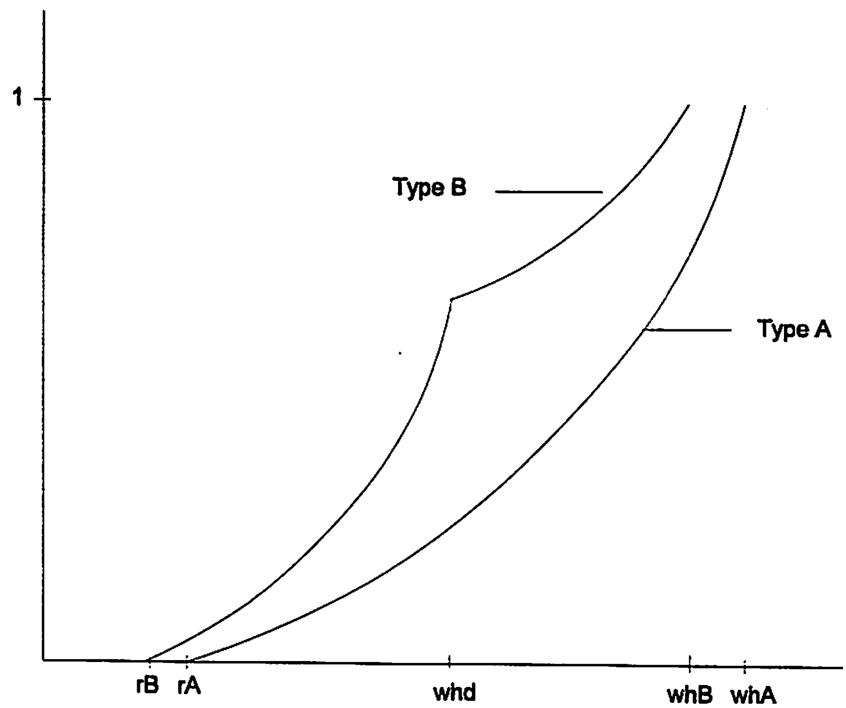

Table 1. NLSY Moments for Male High School Graduates - 1985-1988		
	Whites	Blacks
Unemployment rate - April 1985	0.078	0.157
Unemployment duration in weeks	20.46	27.88
Fraction completed job spells ending in unemployment	0.411	0.595
Mean weekly earnings - April 1985	270.17	230.26
Mean weekly offers	236.78	197.47
Job spell durations in weeks	219.53	168.44

Table 2. Parameter Estimates Using Moments from NLSY			
Parameter	Discrimination & Productivity (1)	Pure Discrimination (2)	Pure Productivity (3)
λ_0	0.0489	0.0489	0.0489
λ_1	0.0311	0.0311	0.0311
δ_A	0.0041	0.0041	0.0041
δ_B	0.0067	0.0067	0.0067
$\underline{w}$	134.00	134.00	134.00
P_A	288.16	288.16	288.16
P_B	263.14	288.16	258.35
μ	0.9514	1	0.7339
d	83.03	61.01	0
γ_d	0.5281	0.7345	0
k	0.5670	0.6376	1

Appendix A. Proofs

Proof of Proposition 1:
Let w_B^n (w_B^d) be a utility maximizing wage for non-disutility (disutility) firms. Then by equations (7), (8), and utility maximization,

$$\begin{aligned}(P_B-w_B^n)l_n^B(w_B^n) &\geq (P_B-w_B^d)l_n^B(w_B^d)\\ (P_B-d-w_B^d)l_d^B(w_B^d) &\geq (P_B-d-w_B^n)l_d^B(w_B^n)\end{aligned} \tag{A.1}$$

implying

$$\begin{aligned}&(P_B-w_B^n)l_n^B(w_B^n) - (P_B-d-w_B^n)l_d^B(w_B^n)\\ &\geq (P_B-w_B^d)l_n^B(w_B^d) - (P_B-d-w_B^d)l_d^B(w_B^d) \qquad \forall\ w_B^n,\ w_B^d.\end{aligned} \tag{A.2}$$

Under the assumption of proportional arrival rates equations (10) and (11) give us $l_d^B(w_B) = k{*}l_n^B(w_B)$. Thus equation (A.2) becomes

$$\begin{aligned}&(P_B-w_B^n)l_n^B(w_B^n) - (P_B-d-w_B^n)kl_n^B(w_B^n)\\ &\geq (P_B-w_B^d)l_n^B(w_B^d) - (P_B-d-w_B^d)kl_n^B(w_B^d).\end{aligned} \tag{A.3}$$

Define $X(w_B^n)$ equal to the left hand side of equation (A.3)

$$X(w_B^n) = (P_B-w_B^n)l_n^B(w_B^n) - (P_B-d-w_B^n)kl_n^B(w_B^n). \tag{A.4}$$

The derivative of $X(w_B^n)$

$$X'(w_B^n) = ((P_B-w_B^n)(1-k)+kd)l_n^{B'}(w_B^n) - (1-k)l_n^B(w_B^n) > 0 \tag{A.5}$$

is strictly positive because by utility maximization we have $(P_B - w_B^n)\ l_n^{B\prime}(w_B^n) = l_n^B(w_B^n)$. Suppose the wage offer distribution is not segmented, i.e. $\exists\ w_B^d \in (\underline{w}_B^n, \overline{w}_B^n]$ where $\underline{w}_B^n$ ($\overline{w}_B^n$) is the lower (upper) support of the non-disutility firm's set of utility maximizing wage offers. Then by equation (A.5)

$$\begin{aligned}&(P_B-\underline{w}_B^n)l_n^B(\underline{w}_B^n) - (P_B-d-\underline{w}_B^n)l_d^B(\underline{w}_B^n)\\ &< (P_B-w_B^d)l_n^B(w_B^d) - (P_B-d-w_B^d)l_d^B(w_B^d) \qquad w_B^d > \underline{w}_B^n\end{aligned} \tag{A.6}$$

which violates equation (A.2). Thus the distribution is segmented with disutility firms offering wages in the lower range:

$$\begin{array}{ll} F_d^B(w_B) = F_n^B(w_B) = 0 & w_B \le r_B \\ F_d^B(w_B) > 0;\ F_n^B(w_B) = 0 & r_B < w_B \le wh_d \\ F_d^B(w_B) = 1;\ F_n^B(w_B) > 0 & wh_d \le w_B \le wh_B \\ F_d^B(w_B) = F_n^B(w_B) = 1 & w_B \ge wh_B \end{array} \tag{A.7}$$

where wh_B is the highest wage offered to type B workers and wh_d is the highest (lowest) wage offered to type B workers by disutility (non-disutility) firms.
Substituting the conditions in equation (A.7) into equations (10), (11), (13) and (15) and solving for the labor stocks yields equations (23) and (24). The equalization of utility within manager type conditions are given by

$$(P_B-d-r_B)l_d^B(r_B) = (P_B-d-w_B)l_d^B(w_B) \tag{A.8}$$

and

$$(P_B-wh_d)l_n^B(wh_d) = (P_B-w_B)l_n^B(w_B). \tag{A.9}$$

Substituting in the labor stock functions from equations (23) and (24) yields the wage offer distributions for disutility firms

$$F_d^B(w_B) = \frac{1+\kappa_1^B}{\mu k\kappa_1\gamma_d} - \left(\frac{1+\kappa_1^B}{\mu k\kappa_1\gamma_d}\right)\left(\frac{P_B-d-w_B}{P_B-d-r_B}\right)^{\frac{1}{2}} \qquad r_B \le w_B \le wh_d \tag{A.10}$$

and non-disutility firms

$$F_n^B(w_B) = \frac{1+\mu\kappa_1(1-\gamma_d)}{\mu\kappa_1(1-\gamma_d)} - \left(\frac{1+\mu\kappa_1(1-\gamma_d)}{\mu\kappa_1(1-\gamma_d)}\right)\left(\frac{P_B-w_B}{P_B-wh_d}\right)^{\frac{1}{2}} \quad wh_d \le w_B \le wh_B \tag{A.11}$$

where wh_d is given by the solution to $F_d^B(wh_d)=1$

$$wh_d = P_B-d - \left(\frac{1+\mu\kappa_1(1-\gamma_d)}{1+\kappa_1^B}\right)^2(P_B-d-r_B) \tag{A.12}$$

and wh_B is given by the solution to $F_n^B(wh_B)=1$

$$wh_B = P_B - \left(\frac{1}{1+\mu\kappa_1(1-\gamma_d)}\right)^2(P_B-wh_d). \tag{A.13}$$

The reservation wage for type B workers is solved for by substituting the wage offer distribution expressions (A.10) and (A.11) into equation (8) and is given by:

$$r_B = \frac{xb + yP_B}{x+y} - \frac{zd}{(x+y)(1+\mu\kappa_1(1-\gamma_d))^2} \tag{A.14}$$

where $x = (1+\kappa_1^B)^2$, $y = \mu^2\kappa_1(\kappa_0 - \kappa_1)(1 - \gamma_d + k\gamma_d)^2$, and $z = \mu^2\kappa_1(\kappa_0 - \kappa_1)((1 - \gamma_d + k\gamma_d)^2(1+\mu\kappa_1(1-\gamma_d))^2 - (1-\gamma_d)^2(1+\kappa_1^B)^2)$. The market wage offer distribution, the fraction of all firms paying w_B or less to type B workers ($F^B(w_B)=(1-\gamma_d)F_n^B(w_B)+\gamma_d F_d^B(w_B)$), is then given by equation (25) and the type B earnings distribution from equation (15) is given by equation (26).

Proof of Proposition 2:
For the reservation wage ordering to be true the following must hold

$$r_B = \frac{xb + yP_B}{x+y} - \frac{zd}{(x+y)(1+\mu\kappa_1(1-\gamma_d))^2} < \frac{(1+\kappa_1)^2 b + (\kappa_0-\kappa_1)\kappa_1 P_A}{(1+\kappa_1)^2 + (\kappa_0-\kappa_1)\kappa_1} = r_A \tag{A.15}$$

where x, y and z are defined as in the proof of proposition 1. Rearranging terms yields:

$$\frac{y(1+\kappa_1)^2(P_B-b) - x(\kappa_0-\kappa_1)\kappa_1(P_A-b) + y(\kappa_0-\kappa_1)\kappa_1(P_B-P_A)}{(1+\kappa_1)^2 + (\kappa_0-\kappa_1)\kappa_1} < \frac{zd}{(1+\mu\kappa_1(1-\gamma_d))^2}. \tag{A.16}$$

Given $0 \le \mu \le 1$, $0 \le k \le 1$, and $\lambda_0 > \lambda_1$, z is positive; the denominator of the left hand side is positive; and the first term in the left hand side is negative because $P_B \le P_A$ and

$$x(\kappa_0-\kappa_1)\kappa_1 - (1+\kappa_1)^2 y = (\kappa_0-\kappa_1)\kappa_1(1-\mu(1-\gamma_d(1-k)))(\mu(1+2\kappa_1)(1-\gamma_d(1-k))+1) > 0. \tag{A.17}$$

Thus, the right hand side is positive and the left hand side is negative and the inequality holds.

For stochastic dominance define w^α such that $G(w^\alpha)=\alpha$. Then using the earnings distributions for type A and B workers given in equations (22) and (26), we have

$$w_A^\alpha = P_A - (P_A - r_A)\frac{1}{(1+\alpha\kappa_1)^2} \qquad 0 \le \alpha \le 1 \tag{A.18}$$

and

$$w_B^\alpha = \begin{cases} P_B-d-(P_B-d-r_B)\left(\dfrac{1}{1+\alpha\kappa_1^B}\right)^2 & 0\leq\alpha\leq G^B(wh_d) \\ P_B-(P_B-wh_d)\left(\dfrac{1+\kappa_1^B}{(1+\mu\kappa_1(1-\gamma_d))(1+\alpha\kappa_1^B)}\right)^2 & G^B(wh_d)\leq\alpha\leq 1. \end{cases} \tag{A.19}$$

It is straightforward to show

$$w_B^\alpha < w_A^\alpha \qquad \forall\ \alpha\in[0,1] \tag{A.20}$$

if $P_A=P_B$. Since the derivative of w^α with respect to P is positive and $P_B\leq P_A$,

$$w_B^\alpha < w_A^\alpha \quad \forall\ \alpha\in[0,1] \ \Rightarrow\ G^A(w) \leq G^B(w) \quad \forall\ w\in[r_B,wh_A]. \tag{A.21}$$

The mean earnings result follows directly from the stochastic dominance of the type A worker's earnings distribution. Mean wage offers are given by the following expressions for type A and type B workers, respectively,

$$\begin{aligned} E_o^A(w_A) &= \gamma_d\int_{r_A}^{wh_A} w_A f_d^A(w_A)dw_A + (1-\gamma_d)\int_{r_A}^{wh_A} w_A f_n^A(w_A)dw_A \\ &= \int_{r_A}^{wh_A} w_A f^A(w_A)dw_A \end{aligned} \tag{A.22}$$

and

$$E_o^B(w_B)=\frac{k\gamma_d}{k\gamma_d+1-\gamma_d}\int_{r_B}^{wh_d} w_B f_d^B(w_B)dw_B + \frac{(1-\gamma_d)}{k\gamma_d+1-\gamma_d}\int_{wh_d}^{wh_B} w_B f_n^B(w_B)dw_B. \tag{A.23}$$

Equations (A.22) and (A.23) contain the expected wage offer from each firm type multiplied by the probability of an offer from that type. Solving equations (A.22) and (A.23) yields the following

$$E_o^A(w_A) = \frac{\kappa_1(3+2\kappa_1)P_A+(3+3\kappa_1+\kappa_1^2)r_A}{3(1+\kappa_1)^2} \tag{A.24}$$

and

$$E_o^B(w_B) = \frac{\kappa_1^B(3+2\kappa_1^B)P_B+(3+3\kappa_1^B+\kappa_1^{B2})r_B}{3(1+\kappa_1^B)^2} + \frac{\mu k\kappa_1\gamma_d d(2(1+\mu\kappa_1(1-\gamma_d))+\mu k\kappa_1\gamma_d-2(1+\mu\kappa_1(1-\gamma_d))^2(1+\kappa_1^B)^2)}{3\kappa_1^B(1+\mu\kappa_1(1-\gamma_d))^2(1+\kappa_1^B)^2}. \tag{A.25}$$

Since $r_B < r_A$, $\kappa_1^B \le \kappa_1$, $P_B \le P_A$ and the expression in E_o^B containing d is negative, $E_o^A(w_A) > E_o^B(w_B)$. QED.

Proof of Proposition 3:
The mean of the type A earnings distribution is

$$E^A(w_A) = \int_{r_A}^{wh_A} w_A g^A(w_A)dw_a = \frac{P_A\kappa_1+r_A}{1+\kappa_1}. \tag{A.26}$$

Since this mean is not a function of d or γ_d, only the mean of the type B earnings distribution needs to be examined to determine what happens to the wage differential. The type B mean is given by the following expression:

$$\begin{aligned} E^B(w_B) = {} & (1-\gamma_d)\frac{1+\kappa_1^B}{1-\gamma_d+k\gamma_d}\left[\frac{\mu\kappa_1(1-\gamma_d)P_B}{(1+\mu\kappa_1(1-\gamma_d))^2} + \frac{r_B}{(1+\kappa_1^B)^2}\right] \\ & + \gamma_d\frac{k}{(1-\gamma_d+k\gamma_d)(1+\kappa_1^B)}\left[\frac{\mu k\kappa_1\gamma_d(P_B-d)}{1+\mu\kappa_1(1-\gamma_d)} + r_B\right] \\ & + \gamma_d(1-\gamma_d)\frac{\mu k\kappa_1(1+\kappa_1^B)(2+2\mu\kappa_1(1-\gamma_d)+\mu k\kappa_1\gamma_d)}{(1-\gamma_d+k\gamma_d)(1+\kappa_1^B)^2(1+\mu\kappa_1(1-\gamma_d))^2}(P_B-d). \end{aligned} \tag{A.27}$$

From equation (A.14) we have that r_B is decreasing in d. Thus $E^B(w_B)$ is decreasing in d, and the wage differential is increasing in d. The derivative of $E^B(w_B)$ with respect to γ_d is also negative. To see this note that if γ_d=0. then the expression in (A.27) reduces to that in (A.26). If γ_d=1, (A.27) becomes

$$E^B(w_B) = \frac{(P_B-d)\mu k\kappa_1+r_B}{1+\mu k\kappa_1}. \tag{A.28}$$

It is straightforward to show that (A.28) is smaller than (A.26). Equation (A.27), the expression for the mean earnings of type B workers, falls between (A.26) and (A.28) approaching (A.28) as γ_d increases. Thus the wage differential is increasing in γ_d. QED.

Proof of Proposition 4:
Under the assumptions k=1, μ=1 and d=0 mean earnings for type A and B workers are given by

$$E^{i}(w_i) = \frac{P_i\kappa_1 + r_i}{1+\kappa_1} \quad i=A,B \tag{A.29}$$

where r_i equals

$$r_i = \frac{(1+\kappa_1)^2 b + (\kappa_0-\kappa_1)\kappa_1 P_i}{(1+\kappa_1)^2 + (\kappa_0-\kappa_1)\kappa_1} \quad i=A,B. \tag{A.30}$$

Substituting r_i into equation (A.29) we have

$$\frac{E^{B}(w_B)}{E^{A}(w_A)} > \frac{P_B}{P_A} \tag{A.31}$$

if and only if $P_A > P_B$ which is true by assumption. QED.

Proof of Proposition 5:
Since profits from type A workers are equalized across disutility firms we only need to concern ourselves with the profits from type B workers. For k=0, disutility firms do not hire type B workers and thus profits do not vary with respect to w_B. For k>0, we need to show

$$(P_B-w_B)l_d^B(w_B) < (P_B-w_B')l_d^B(w_B') \quad \forall\ w_B, w_B' \in [r_B, wh_d],\ w_B<w_B'. \tag{A.32}$$

By utility equalization we have

$$(P_B-d-w_B)l_d^B(w_B) = (P_B-d-w_B')l_d^B(w_B') \quad \forall\ w_B, w_B' \in [r_B, wh_d]. \tag{A.33}$$

Solving for $(P_B-w_B')l_d^B(w_B')$ and substituting into the profit condition yields

$$d(l_d^B(w_B') - l_d^B(w_B)) > 0 \qquad \forall\ w_B, w_B' \in (r_B, wh_d],\ w_B<w_B' \tag{A.34}$$

which is positive because $l_d^B(w_B)$ is increasing in w_B.

To prove the second statement we need to only compare the profits of the two firm types at wh_d, since profits are equalized across non-disutility firms and increasing in the wage for disutility firms. For profits to be greater for non-disutility firms the following must hold

$$(P_B - wh_d)l_n^B(wh_d) > (P_B - wh_d)l_d^B(wh_d). \tag{A.35}$$

It does because the labor stock is greater at non-disutility firms due to their higher offer arrival rates. QED.

Proof of Proposition 6:
For the pure productivity case with $\mu=1$ the wage differential for the α-percentile is given by

$$\frac{w_B^\alpha}{w_A^\alpha} = \frac{P_B - (P_B - r_B)(1+\alpha\kappa_1)^{-2}}{P_A - (P_A - r_B)(1+\alpha\kappa_1)^{-2}}. \tag{A.36}$$

The derivative of the α-percentile wage differential with respect to α is

$$\frac{\partial \frac{w_B^\alpha}{w_A^\alpha}}{\partial\alpha} = \frac{2(1+\alpha\kappa_1)\kappa_1(P_B r_A - P_A r_B)}{((1+\alpha\kappa_1)^2 P_A - P_A + r_A)^2} \tag{A.37}$$

which is negative because $P_B r_A - P_A r_B = (1+\kappa_1)^{-2} b(P_B - P_A) < 0$ by assumption. Hence under the pure productivity case the wage differential between type A and B workers gets larger as α increases.

For the pure discrimination case with $k=1$ the wage differential for the α-percentile is given by

$$\frac{w_B^\alpha}{w_A^\alpha} = \begin{cases} \dfrac{P - d - (P - d - r_B)(1+\alpha\kappa_1)^{-2}}{P - (P - r_A)(1+\alpha\kappa_1)^{-2}} & 0 \le \alpha \le G^B(wh_d) \\[2ex] \dfrac{P - (P - wh_d)\left(\dfrac{1+\kappa_1}{(1+\kappa_1(1-\gamma_d))(1+\alpha\kappa_1)}\right)^2}{P - (P - r_A)(1+\alpha\kappa_1)^{-2}} & G^B(wh_d) \le \alpha \le 1. \end{cases} \tag{A.38}$$

The derivative of the wage differential with respect to α for $\alpha < G^B(wh_d)$ is

$$\frac{\partial \frac{w_B^\alpha}{w_A^\alpha}}{\partial\alpha} = \frac{2(1+\alpha\kappa_1)\kappa_1((P-d)r_A - Pr_B)}{((1+\alpha\kappa_1)^2 P - P + r_A)^2} \tag{A.39}$$

which is negative because $(P-d)r_A-P_B<0$. Thus for percentiles below $G^B(wh_d)$ the wage differential is increasing. The derivative of the wage differential with respect to α for $\alpha \geq G^B(wh_d)$ after substituting in for wh_d and simplifying is

$$\frac{\partial \frac{w_B^\alpha}{w_A^\alpha}}{\partial \alpha} = \frac{2(1+\alpha\kappa_1)\kappa_1 P(r_A-r_B+d((\frac{1+\kappa_1}{1+\kappa_1(1-\gamma_d)})^2-1))}{((1+\alpha\kappa_1)^2P-P+r_A)^2} \tag{A.40}$$

which is positive because $r_A \geq r_B$ and $\gamma_d \leq 1$. Thus for percentiles above $G^B(wh_d)$ the wage differential is decreasing. QED.

Appendix B. Data Description

The means in Table 1 are from black and white males in the National Longitudinal Survey of Youth's cross-section and supplemental samples. To be in our sample a respondent must have been interviewed in 1986, graduated from high school after 1977 and before 1985 (GED recipients are dropped), not gone on to any further education before 1989, and not served in the military between 1985-1988. The respondent must either be employed in a full-time job (≥ 35 hours/week) in the private sector in April 1985 or if unemployed have found a private sector full-time job before December 1988 and must not have spent more than half a year's time (>26 weeks) out of the labor force in each year between 1985-1988.

For each respondent who meets these criteria we collect the following. The state - unemployment or employment - they are in the first week of April 1985.[23] If employed we collect the wage on the current job and the duration of the job (we know the exact starting date so there is no left censoring). If the job ends prior to December 1988[24], the transition to unemployment or another job is noted and, if the transition is to unemployment, the duration to the next full-time job is recorded. If the respondent is unemployed in April 1985, we record the wage on the first full-time job after April 1985, the job duration, the transition and the unemployment duration if available. All durations are in weeks and all wages are converted to weekly wages (all wages are in 1985 dollars). Because of problems with measurement error we treat as missing all wage responses that do not fall within upper (95th percentile) and lower (5th percentile) bounds collected from the U.S. March Current Population Survey outgoing rotation groups. We also treat as missing any wage observations below the legislated minimum wage of $3.35*40 hours = $134.00.

The means in Table 1 are calculated as follows: the unemployment rate is the fraction of respondents in the sample who do not have a job in the first week of April 1985; the mean unemployment duration is the mean of the unemployment durations following the job spells; the fraction of completed spells ending in unemployment is the number of respondents employed in April 1985 that transition to unemployment before December 1988 divided by the total number of respondents employed in April with jobs that end prior to December 1988: the mean earnings is the mean of weekly wages from jobs that are ongoing in April 1985: the mean wage offer is the mean of the weekly wages of jobs that start after April 1985: and the mean job duration is the mean of job spell durations ongoing in April 1985.

[23] April 1985 was chosen to produce unemployment rates consistent with annual averages.

[24] If the job does not end prior to December 1988, it is treated as censored.

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